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Modeling the Conditional Covariance Between Stock and Bond Returns: A Multivariate GARCH Approach

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Abstract

To analyze the intertemporal interaction between the stock and bond market returns, we assume that the conditional covariance matrix follows a multivariate GARCH process. We allow for asymmetric effects in conditional variances and covariances. Using daily data, we find strong evidence of conditional heteroskedasticity in the covariance between stock and bond market returns. The results indicate that not only variances, but also covariances respond asymmetrically to return shocks. Bad news in the stock and bond market is typically followed by a higher conditional covariance than good news. Cross asymmetries, i.e. asymmetries followed from shocks of opposite signs, appear to be important as well. Covariances between stock and bond returns tend to be relatively low after bad news in the stock market and good news in the bond market. A financial application of our model shows that optimal portfolio shares can be substantially affected by asymmetries in covariances. Moreover, our results show sizable gains due to asymmetric volatility timing.

KEYWORDS: Multivariate GARCH, Volatility Transmission, Asymmetric Effects

JEL CLASSIFICATION CODES: G12, C22.

The development of multivariate GARCH models represented a major step forward in the modeling of volatility. These models allow for time-varying conditional variances as well as covariances. Conditional variances and covariances of asset returns are of considerable importance for the pricing of financial securities, and (co)variances are key inputs to asset allocation and risk management in financial institutions. Consequently, accurate models and forecasts of conditional variances and covariances are crucial. However, while there is a vast amount of literature on modeling returns and volatility, these are often restricted as they either examine the stock market or the bond market separately.[1] Little attention has been paid to the interaction between the two markets. Only since the last decade financial economists have begun to model these temporal dependencies. For example, Breen, Glosten and Jagannathan (1989) show that there is a negative relation between short term interest rates and future stock index returns, and Schwert (1989) documents that U.S. stock and bond returns and volatilities move together. A recent study by Fleming, Kirby and Ostdiek (1998) examines volatility interaction of stock, bond and money markets using a stochastic volatility model. Although they find a strong link in volatility between the three markets, they do not consider the conditional covariance between the stock and bond market returns. Studies that explicitly consider time-varying conditional covariances, using multivariate GARCH models, include Bollerslev, Engle and Wooldridge (1988), Ng (1991), Karolyi (1995) and Kroner and Ng (1998). However, these studies do not explicitly examine the interactions between the stock and bond market. Only Bollerslev, Engle and Wooldridge (1988) consider the stock and bond market. However, they concentrate on testing the CAPM and their model does not allow for leverage effects.

The purpose of our study is to analyze the intertemporal interactions of stock and bond returns. To this end we allow the conditional covariance matrix of stock and bond market returns to vary over time, according to a multivariate GARCH model. We extend the model by allowing for asymmetric effects of return shocks on the conditional covariance between stock and bond returns. Because these effects on covariances between stock and bond returns in a multivariate GARCH model appear to be neglected in the literature, this paper is a first step towards filling this gap. To model the asymmetric effects on conditional covariances we develop a new approach by extending the Glosten, Jagannathan and Runkle (1993) specification to a multivariate setting. The resulting model is able to capture asymmetries within and between stock and bond markets which allows us to find novel results that cannot be obtained from standard symmetric covariance models. We use daily data from 1982 to 2001 to examine the intertemporal interaction between the returns on the Standard and Poor's 500 index, the NASDAQ Composite index, and the returns on a short and long term bond. Finally, we apply our model to tactical asset allocation showing the importance of our model in financial applications. We show how the asymmetry introduced in the covariances affect optimal portfolio shares.

Although it is often recognized that variances and covariances of returns change over time (see, e.g., French, Schwert and Stambaugh, 1987, and Schwert, 1989) their determinants are not yet well identified. Among the econometric volatility models, the family of GARCH models, as introduced by Engle (1982) and generalized by Bollerslev (1986), seems to be the most fruitful. For an extensive literature overview we refer to Bollerslev, Chou and Kroner (1992) and Bollerslev, Engle and Nelson (1994). GARCH models are able to capture the phenomenon that volatilities of asset returns are clustered over time. Univariate GARCH models have appeared to be quite successful in predicting volatility. A drawback of standard GARCH models is that the arrival of "good" and "bad" news in the market (unexpected positive and negative returns, respectively) are assumed to have a symmetric impact on volatility, while typically unexpected decreases in prices tend to rise the predictable volatility more than unexpected increases of similar magnitude. This asymmetric effect of shocks in the second moment of stock returns is a well-known phenomenon in financial modeling. This effect is more pronounced during stock market crashes. For example the 20% drop on October 19, 1987 led to a huge increase in volatility. On the other hand, good news does not cause a sharp decrease in volatility. Recent studies have shown that more accurate volatility predictions can be obtained when asymmetric responses of volatility to news are taken into account. While many different extensions of the model have been suggested (for an excellent overview see Engle and Ng, 1993, or Bollerslev, Engle and Nelson, 1994), particularly nice extensions are the exponential GARCH, introduced by Nelson (1991), and the Glosten, Jagannathan and Runkle (1993) model. Empirical studies show that these models, which allow for the possibility that positive and negative shocks in returns affect volatility differently, work very well in practice.

While there is a large body of literature on asymmetric volatility in univariate ARCH models, there exists only few studies on the asymmetric effects in multivariate models. For an excellent overview of recent multivariate GARCH models and their properties, we recommend Bauwens, Laurent and Rombouts (2003). They present a comprehensive state-of-the-art survey. Surprisingly little attention has been paid to the asymmetric effects in the covariance between stock and bond market returns. As a portfolio manager's optimal portfolio depends on the predicted covariance between assets, relaxing the symmetric specification may lead to superior investment choices. Other examples of applications in finance can be found in the field of risk management and derivative pricing. One of the few examples that imposes asymmetric effects in multivariate models is Kroner and Ng (1998). They use data on large and small firms to compare four popular multivariate GARCH models. Their model is very general, but it does not allow for covariance asymmetries due to shocks of opposite signs. Another example is Braun, Nelson and Sunier (1995), who estimate a bivariate exponential GARCH model with asymmetries in stock return betas for different sectors. Their study does not explicitly consider asymmetries in covariances. Moreover, in order to examine asymmetries between different asset classes, their method is not very suitable.

The remainder of this paper is organized as follows. In Section 1 we describe the multivariate model which enables us to analyze time-varying covariances. Section 2 describes the data used in our analysis and presents empirical results based on estimating the time-varying covariance models. To show the importance of our extended model, we also present a financial application in the field of tactical asset allocation. This section concludes with comparing the results of this study with previous studies. Conclusions are offered in the final section.

1 Modeling Time-Varying Asymmetric Covariances

In this section we present the conditional volatility equation. To obtain a measure of risk in the *multivariate* case, we need to model the conditional covariances. We do this by modeling the volatility by a *multivariate* GARCH process. This way we can easily examine the conditional covariance structure and interactions between the stock and bond market.

Following, e.g., Karolyi (1995) and Kroner and Ng (1998), we assume that the mean equation follows a $VAR(p)$ process (for $i = 1, ..., N$):

$$r^e_{i,t+1} = \mu_i + \sum_{l=0}^{p} \sum_{j=1}^{N} \psi_j r^e_{j,t-l} + \varepsilon_{i,t+1}, \tag{1}$$

where $r^e_{i,t+1}$ denotes the return on asset i in excess of the riskfree return. The excess return on asset i depends on a constant, μ_i, and p lags of return on asset i as well as p lags of the other assets (r_j; $j = 1, ..., N$). Finally, $\varepsilon_{i,t+1}$ represents the unexpected excess return on asset i, i.e. $r^e_{i,t+1} - E_t\{r^e_{i,t+1}\}$. Thus $\varepsilon_{i,t+1}$ represents the "news" corresponding to asset i that is arrived in the corresponding market. Model (1) enables us to test the importance of the influence of past returns on current levels of returns. Next, we describe how the conditional covariances evolve over time.

We model the time-varying covariances by a multivariate GARCH process. While the GARCH specification does not follow from any economic theory, it provides a good approximation to the heteroskedasticity typically found in financial time-series data. The univariate GARCH(1,1) can be generalized to a multivariate setting[2] (see, e.g., Bollerslev, Engle and Wooldridge, 1988). The matrix Σ_{t+1}, containing the conditional covariances, is assumed to follow a simple multivariate GARCH(1,1) model, which can be compactly written in vector form as:

$$\text{vech}(\Sigma_{t+1}) = \mathbf{c} + B^* \text{vech}(\Sigma_t) + A^* \text{vech}(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'), \tag{2}$$

where vech denotes the operator which stacks columns of the lower triangle (those elements on and below the main diagonal) of a $N \times N$ symmetric matrix as an $N(N+1)/2 \times 1$ vector.[3] Further, $\boldsymbol{\varepsilon}_t$ denotes the vector of error terms at time t. The vector $\mathbf{c}$ has dimension $N(N+1)/2 \times 1$, and matrices A^* and B^* have dimension $N(N+1)/2 \times N(N+1)/2$. While this model is a natural extension of the univariate GARCH model and is easy to understand, there are two major problems in estimating this model. The first problem concerns the number of parameters to be estimated and the second problem concerns the positive-definiteness constraints to be imposed on the conditional covariance matrix.

Obviously a disadvantage of the multivariate approach is that the number of parameters to be estimated in the GARCH equation increases rapidly (for example, with $N = 4$ there are 210 parameters to be estimated), which limits the number of assets that can be included. In order to reduce the number of parameters to be estimated, it is advisable to impose some restrictions on A^* and B^*, without lowering the explanatory power of the model significantly. Following Bollerslev, Engle and Wooldridge (1988), we assume that matrices A^* and B^* are diagonal. Thus, (2) can be written, after conveniently rearranging the parameter indices, as:

$$\sigma_{ij,t+1} = \gamma_{ij} + \beta_{ij}\sigma_{ij,t} + \alpha_{ij}\varepsilon_{i,t}\varepsilon_{j,t}, \qquad i,j = 1,...,N, \tag{3}$$

with $\sigma_{ij,t+1} = Cov_t\{r_{j,t+1}, r_{i,t+1}\}$. For $N = 4$ this reduces the number of parameters to 30. Despite the fact that this number is reduced substantially, this specification is only useful for a limited number of asset classes as typically used by pension funds. Recently, Engle (2002) proposed a new class of multivariate GARCH models in which the number of parameters grows linearly with the number of assets. Therefore his model is relatively parsimonious and, in contrast to our specification, suitable for a large set of assets.

Model (3) is called the diagonal VECH model. By diagonalizing the model we constrain the dynamic dependence and may introduce biases in the estimates of the other parameters. For instance, only shocks in asset i can influence the conditional variance of asset i. This assumption is quite restrictive and is obviously a disadvantage of the diagonal VECH model. However we expect the potential biases to be small as models allowing for such spillover effects, such as the BEKK model (see Engle and Kroner, 1995) show that these effects are typically small. Moreover, a recent study by Ferreira and Lopez (2004) shows that among the most popular multivariate models the diagonal VECH seems to provide the best out-of-sample (co)variance forecasts for interest rates. Moreover, Bollerslev, Engle and Wooldridge (1988) use the diagonal VECH model to estimate the trade-off in variance among three assets: a stock index, a bond and a Treasury bill. To guarantee that the conditional covariance matrix is positive definite we estimate the model using constrained maximum likelihood.[4]

Since the conditional variance is a function of the magnitudes of the lagged error terms and not their signs, GARCH models are not capable to capture the so-called *leverage effect*. This asymmetric volatility phenomenon, first noted by Black (1976), refers to the tendency that good and bad news in returns have a different impact on conditional volatility in stock markets. More specifically, bad news is followed by larger volatility than good news. The rationale of this phenomenon, according to Black (1976), is that a lower stock price increases the debt-equity ratio of a company (i.e. the financial leverage of the firm increases) and this again increases the risk of holding stocks of this company. Because firms have many fixed costs, a price decrease has a larger impact on volatility than a price increase of the same magnitude. It is however not likely that the large response of stock volatility can be explained by leverage alone (see Black, 1976).

Several recent papers put forward alternative explanations. Campbell and Hentschel (1992) and Bekaert and Wu (2000), e.g., use a volatility feedback approach. This implies that changes in volatility affect the level of required stock returns. Campbell and Hentschel show that volatility feedback explanation is able to explain the asymmetries in volatilities. An alternative interpretation is provided by a psychological explanation: the *following-the-herd effect*. That is, during a stock market crash, investors might pay less attention to the fundamentals, and sell their stocks when (they think that) other investors are selling stocks. This leads to a relatively high volatility when bad news arrives in the market. This idea is very similar to Veronesi (1999), who shows, using a rational equilibrium asset pricing model where the drift of fundamentals shifts between two unobservable states, that stock prices overreact to bad news in good times and underreact to good news in bad times. Veronesi (1999) shows that this model is able to explain the asymmetric effect in stock returns.

Among financial economists there is no consensus yet about the explanation of the asymmetric volatility phenomenon, and the rationale of the asymmetry is a hot topic nowadays in financial economics. While the leverage argument can only partly explain the asymmetric nature of the volatility response to return shocks, in this paper we use the leverage effect as a synonymous for the asymmetric effect in (co)variances. We do not concentrate on the rationale behind this phenomenon. Instead we focus on estimating the importance of asymmetric effects in conditional covariances.

Numerous studies have shown that introducing a certain asymmetry in GARCH models to capture the leverage effects in conditional volatility, can substantially improve univariate models. These models are often referred to as leverage or asymmetric volatility models. One of the most successful asymmetric specification in univariate models is Nelson's (1991) EGARCH (which stands for Exponential GARCH), in which a logarithmic transformation is applied. This guarantees that variances are non-negative.

A generalization of EGARCH is however inconvenient in a multivariate setting, because this would imply that all covariances between returns are positive. Nevertheless, Braun, Nelson and Sunier (1992) use a bivariate EGARCH model estimate the variances of the market portfolio and a second asset. To estimate the conditional beta between the market portfolio and a second asset they use a different specification without logarithms. This specification, which is not a very natural extension, seems less appropriate to model asymmetric covariances. Instead we generalize the Glosten Jagannathan and Runkle (1993) (GJR henceforth) specification. We show that asymmetries in covariances are likely to exist if there is asymmetry in variances (see Appendix A). The generalized GJR model becomes[5]:

$$\begin{aligned}\sigma_{ij,t+1} &= \gamma_{ij} + \beta_{ij}\sigma_{ij,t} + \alpha_{1ij}\varepsilon_{i,t}\varepsilon_{j,t} + \alpha_{2ij}I_{\varepsilon_{i,t}}\varepsilon_{i,t}I_{\varepsilon_{j,t}}\varepsilon_{j,t} \\ &\quad +\alpha_{3ij}I_{\varepsilon_{i,t}}\varepsilon_{i,t}(1-I_{\varepsilon_{j,t}})\varepsilon_{j,t} + \alpha_{4ij}(1-I_{\varepsilon_{i,t}})\varepsilon_{i,t}I_{\varepsilon_{j,t}}\varepsilon_{j,t} \qquad (4)\end{aligned}$$

$i, j = 1, ..., N$. The indicator variable $I_{\varepsilon_{k,t}}$ is equal to 1 if $\varepsilon_{k,t} < 0$ (and zero otherwise), $k = i, j$, such that the space can be partitioned into four quadrants[6] in the $\{\varepsilon_i,\varepsilon_j\}$ plain. Let us partition this plane into: $Q(+,+)$, $Q(+,-)$, $Q(-,+)$, and $Q(-,-)$, denoting the quadrant, corresponding to the signs of $(\varepsilon_i,\varepsilon_j)$: a "+" for a positive and a "−" for a negative shock. In (4), $I_{\varepsilon_{i,t}}\varepsilon_{i,t}I_{\varepsilon_{j,t}}\varepsilon_{j,t}$ is nonzero for pairs of $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$ in $Q(-,-)$. This term assigns an asymmetric covariance effect on shocks in the same direction ($Q(+,+)$ vs. $Q(-,-)$). On the other hand, $I_{\varepsilon_{i,t}}\varepsilon_{i,t}(1-I_{\varepsilon_{j,t}})\varepsilon_{j,t}$ is nonzero for pairs in $Q(-,+)$, while $(1-I_{\varepsilon_{i,t}})\varepsilon_{i,t}I_{\varepsilon_{j,t}}\varepsilon_{j,t}$ is nonzero for pairs in $Q(+,-)$. These terms assigns an asymmetric covariance effect on shocks in the opposite direction ($Q(+,-)$ vs. $Q(-,+)$). We will refer to these latter effects as *cross-asymmetry effects* or simply *cross effects*. Kroner and Ng (1998) present an asymmetric covariance model without these effects. However when modeling the covariance between stock and bond returns these cross effects should not be neglected, as shocks of opposite signs are more common (see data section). Our model provides a generalization of the asymmetric GJR model by allowing explicitly for asymmetric conditional covariance terms. We will refer this model as the *asymmetric diagonal VECH* model.

In order to discuss the properties of the model we rewrite the model in matrix notation. Ding and Engle (2001), e.g., show that the diagonal VECH model of Bollerslev, Engle and Wooldridge (1988) can be written in matrix notation as

$$\Sigma_{t+1} = C + B \odot \Sigma_t + A \odot \varepsilon_t \varepsilon_t^{'}, \tag{5}$$

where Σ_t is the conditional covariance matrix at time t, C, A and B are all $(N \times N)$ parameter matrices and $\odot$ denotes the Hadamard product (element by element matrix multiplication). Since Σ_{t+1} must be symmetric, so must be the parameter matrices and only the lower portions of these matrices need to be parameterized and estimated. Silberberg and Pafka (2001), for example, prove that a sufficient condition to assure the positive definiteness of the covariance matrix Σ_{t+1} in (5) is that the constant term C, is positive definite and all the other coefficient matrices, A and B, are positive semidefinite.

Now consider the asymmetric diagonal VECH model. In matrix notation, the model can be written as

$$\begin{aligned} \Sigma_{t+1} &= C + B \odot \Sigma_t + A_1 \odot \varepsilon_t \varepsilon_t^{'} + A_2 \odot (\varepsilon_t^{-} \varepsilon_t^{-'}) \\ &\quad + A_3 \odot \mathcal{T}(\varepsilon_t^{-} \varepsilon_t^{+'}) + A_4 \odot \mathcal{T}(\varepsilon_t^{+} \varepsilon_t^{-'}), \end{aligned} \tag{6}$$

where C, B, A_1, A_2, A_3 and A_4 are $(N \times N)$ parameter matrices, $\mathcal{T}$ is the operator that permutes rows of a square matrix, in such a way that the lower triangular part of the matrix is substituted by the upper triangular part of the matrix (see He and Teräsvirta, 2002) and $\varepsilon_t^{-} = [I_{\varepsilon_{1,t}} \varepsilon_{1,t}, ..., I_{\varepsilon_{N,t}} \varepsilon_{N,t}]'$ and $\varepsilon_t^{+} = [(1 - I_{\varepsilon_{1,t}}) \varepsilon_{1,t}, ..., (1 - I_{\varepsilon_{N,t}}) \varepsilon_{N,t}]'$.

In order to derive sufficient conditions to assure positive definiteness of the covariance matrix Σ_{t+1} in (6), we have to show that the individual matrices in (6) are positive semidefinite as symmetry and positive semi definiteness are preserved by matrix addition (see, e.g., Silberberg and Pafka, 2001). Ding and Engle (2001) show that $A_2 \odot (\varepsilon_t^{-} \varepsilon_t^{-'})$

is positive semidefinite if and only if A_2 is positive semidefinite. Moreover, they show that if two matrices are positive semi-definite matrices, then their Hadamard product is positive semidefinite as well. Therefore, if we can show that $\mathcal{T}(\varepsilon_t^- \varepsilon_t^{+'})$ and $\mathcal{T}(\varepsilon_t^+ \varepsilon_t^{-'})$ are both positive semidefinite, then to guarantee positive semidefiniteness of the Hadamard product, both A_3 and A_4 have to positive semidefinite. It is easy to show that the matrices $\mathcal{T}(\varepsilon_t^- \varepsilon_t^{+'})$ and $\mathcal{T}(\varepsilon_t^+ \varepsilon_t^{-'})$ are indefinite. It appears impossible to derive sufficient conditions to guarantee that the asymmetric diagonal VECH model provides positive definite conditional covariance matrices. Consequently, we impose no a-priori restrictions on the parameters, such that we do not employ restrictions that might violate the data. However, during estimation we impose that the coefficients behave in such a way that the one-step ahead forecast of the conditional covariance matrix is positive definite. A drawback of this approach is that it does not guarantee that multiple-step ahead forecasts for the conditional covariance matrices are positive definite. We argue however, that in practice, the asymmetric diagonal VECH model generates positive definite covariance matrices. Simulations, forecasting up to eight months in the future, showed that the resulting covariance matrices in our application are positive definite.

The stationarity condition for Σ_t can be directly obtained from (6). However we need the additional assumption that errors are distributed equally around zero, i.e. there are the same number of observations left and right from zero. Then it is straightforward to show that for the univariate GJR model the conditional variance is stationair if $\alpha_1 + \frac{1}{2}\alpha_2 + \beta$ is less than 1. Likewise, assuming that errors are equally distributed around zero for the quadrants, the asymmetric diagonal VECH model in (6) is weakly stationary if the eigenvalues of $A_1 + \frac{1}{2}A_2 + \frac{1}{4}A_3 + \frac{1}{4}A_4 + B$ are less than 1 in modulus. This would imply that for all assets the unconditional covariance matrix exists. Note that the condition for the univariate GJR model is nested. In the next section we will check these conditions and examine whether the model is able to explain the variance and covariance between a short term bond the long term bond return and the return on the S&P 500 and NASDAQ index.

2 Empirical Results

2.1 Data

In order to examine the asymmetric volatility in the stock and bond market, our data include the daily excess returns on two stock market indices and two bonds. More specifically, the return on a short term bond implied by the 1 year Treasury bond (denoted by $r_{1,t}$), the return on a long term (10 year) Treasury bond (denoted by $r_{2,t}$), the return on the Standard and Poor's 500 index (denoted by $r_{3,t}$) and the return on the NASDAQ index (denoted by $r_{4,t}$). For reasons of convenience, we will refer to these asset returns as the short bond returns, the long bond returns, and the S&P 500 and NASDAQ returns. All returns were converted to excess returns (denoted by $r^e_{1,t}$, $r^e_{2,t}$, $r^e_{3,t}$ and $r^e_{4,t}$ respectively) using the riskfree rate approximated by the 3 month Treasury bill rate. We adjust for weekends and holidays in the daily returns calculations (Appendix B provides details on the calculations). The bond market data were obtained from the federal reserve bank in Chicago, while the data on the S&P 500 and the NASDAQ indices were provided by Datastream and the National Association of Securities Dealers Inc. respectively. The data cover the period January 4, 1982 - August 31, 2001 (4908 observations), such that we can examine some volatile periods (1987-1988, 1990 and 1998) and less volatile periods (1991-1995). Table 1 provides a summary of the descriptive statistics at the daily frequency.[7] A stylized fact of asset returns is excess kurtosis, which indicates that its empirical distribution has fatter tails than a normal distribution. Moreover, financial asset returns exhibit volatility clustering. In Figure 1 we see that large returns tend to be followed by large returns (of either sign). The attractiveness and empirical success of GARCH models is that they are able to explain to a large extent the volatility clustering behavior and the excess kurtosis of the empirical distribution of returns.

[Table 1 about here]

[Figure 1 about here]

To obtain some idea of the number of observations in the quadrants $Q(+,+)$, $Q(+,-)$, $Q(-,+)$, and $Q(-,-)$, Figure 2 presents return shocks for all asset combinations, obtained from estimating the mean equations. We see from this figure that combined shocks between stock and bonds have many observations in the cross-asymmetric quadrants $Q(+,-)$ and $Q(-,+)$. Thus there is a considerable amount of return shocks with opposite signs in our sample.

[Figure 2 about here]

Table 2 presents first-order autocorrelation and cross-autocorrelation for the four assets. The table shows that the correlation between lagged bond returns and current stock returns is always much higher than the correlation between lagged stock returns and current bond returns. For instance, the correlation between lagged return on the 1 year Treasury bond, $r^e_{1,t-1}$, and the return on the S&P 500, r^e_{3t}, is 0.821 while the correlation between lagged return on the S&P 500, $r^e_{3,t-1}$, and the 1 year Treasury bond, r^e_{1t}, is only -0.004, and not statistically significant at the 5% level. Higher order autocorrelation and cross-autocorrelations (not reported here) also showed statistically significant correlations, although generally smaller. Whereas Lo and MacKinlay (1990) document an asymmetry in the weekly cross-autocorrelation between big firms and small firms, we find a similar effect for daily stock and bond returns. Lagged returns on bonds are correlated with current returns on stocks, but not vice versa. Thus, based on these statistics a VAR model to describe the first moments seems appropriate.

[Table 2 about here]

Because shocks of the mean equation are the main actors in the multivariate model, it is important that the mean equation is not misspecified. We have estimated VAR models up to six lags and tested the individual and joint significance of the coefficients. Appropriate model selection criteria are the Akaike Information Criterion (AIC) and the Schwarz Information Criterion (SIC); see Table 3. We choose the value of p that minimizes the AIC and the SIC. The AIC selects $p = 5$, whereas the SIC selects $p = 1$. It is well known that the SIC penalizes additional parameters more heavily than the AIC, as the SIC prefers more parsimonious models. Based on the selection criteria and the results of the statistical tests, we choose the VAR(5) specification. This specification was also employed in Karolyi (1995). On the basis of the AIC, Karolyi (1995) finds the VAR(5) as preferred mean specification using daily returns on the S&P 500 and TSE 300. Finally, note that Kroner and Ng (1998) include ten lags in the VAR specification without testing for the optimal number of lags.

[Table 3 about here]

2.2 Results

In this section the estimation results of the temporal interaction between U.S. stock and bond markets are presented. Moreover, we examine the economic significance of asymmetric responses of conditional covariances to return shocks. The covariance equations are estimated by maximum likelihood.[9]. In order to use maximum likelihood we need to make distributional assumptions about the error terms. If we assume that $\boldsymbol{\varepsilon}_{t+1}|\mathcal{I}_t \sim N(\mathbf{0}, \Sigma_{t+1})$, the loglikelihood function (for the sample $1, ..., T$) is given by

$$\ell(\boldsymbol{\theta}) = -\frac{1}{2}TN \log 2\pi - \frac{1}{2}\sum_{t=1}^{T} \log \det \Sigma_t(\boldsymbol{\theta}) - \frac{1}{2}\sum_{t=1}^{T} \boldsymbol{\varepsilon}_t^{'}(\boldsymbol{\theta})\Sigma_t^{-1}(\boldsymbol{\theta})\boldsymbol{\varepsilon}_t(\boldsymbol{\theta}), \tag{7}$$

where $\boldsymbol{\theta}$ denotes the vector of unknown parameters, the $N \times 1$ vector $\boldsymbol{\varepsilon}_t(\boldsymbol{\theta})$ contains the error elements $\varepsilon_{i,t}(\boldsymbol{\theta}) = r_{i,t}^e - \mu_i - \sum_{j=1}^{N} \psi_j r_{j,t-1}^e$, $i = 1, ..., N$, and $\Sigma_t(\boldsymbol{\theta})$ contains the covariance terms $\sigma_{ij,t}(\boldsymbol{\theta})$, as defined in (4). The conditions under which the maximum likelihood is consistent and asymptotically normal are derived by Bollerslev and Wooldridge (1988).

The estimates are obtained by numerical methods using the Berndt, Hall, Hall and Haussman (1974) (BHHH) optimization algorithm, which approximates the Hessian with the first derivatives. Without any restrictions, the multivariate VECH model is likely to produce nonpositive definite matrices, so that the maximum likelihood method fails to compute an optimum. To guarantee positive definiteness of the conditional covariance matrix, we use the constrained maximum likelihood optimization procedure of GAUSS and impose that the smallest eigenvalue of each covariance matrix has to be positive during estimation. The existing literature generally puts additional structure on the parameters to ensure that matrices are positive definite (see, e.g., Engle and Kroner, 1995 and Bollerslev, 1990). While it can be useful to impose sensible restrictions for forecasting purposes, there is also the danger of employing a priori restrictions that violate the data. We therefore prefer our less restrictive approach, at the price of a higher computational cost. Note that estimation of multivariate GARCH with a lot of parameters is typically demanding in computer time. In order to improve convergence, a sensible choice of starting values is important. We use starting values based on unconditional sample statistics and preliminary estimates of univariate GARCH models. A range of starting values was used to ensure that the estimation procedure converged to a global maximum. We repeated the estimations with random re-starts of the starting values, conditioned to the range of two times the standard error of the univariate estimates. None of the estimation results indicated any local maximum. The results also seem robust to alternating convergence criterions and optimizing methods. Consequently, we are confident that we have found a global maximum.

In order to help building some intuition on the multivariate model parameters, we also present the estimates of the univariate asymmetric model specification as introduced by Glosten, Jagannathan and Runkle (1993). The estimation results of the volatility models are given in Table 4[9] . The first column in this table presents the results from estimating the Glosten, Jagannathan and Runkle (1993) model. This corresponds to model (4) when $i = j$. The second column refers to the diagonal VECH specification, i.e. model (4) without asymmetric terms in the (co)variance equations. The third column of Table 4 presents the results of the asymmetric Diagonal VECH model. The stationary conditions for the asymmetric diagonal VECH model are met as the eigenvalues of $A_1 + \frac{1}{2}A_2 + \frac{1}{4}A_3 + \frac{1}{4}A_4 + B$ range between 0.954 and 0.992. This implies that for all assets the unconditional covariance matrix exists. The resulting conditional covariance was positive definite and simulations, forecasting up to eight months in the future, shows that the resulting covariance matrices are positive definite matrices.

[Table 4 about here]

As the diagonal VECH model is nested in the asymmetric diagonal VECH model, we can easily test one against the other using the likelihood ratio test. The results clearly suggest that asymmetric effects are important when modeling the conditional covariances between stock and bond market returns. The likelihood ratio test statistic is 140.66, and with the degrees of freedom being equal to 22, the null hypothesis is soundly rejected at conventional significance levels. This means that the model specification with asymmetric effects in covariances is superior to the diagonal VECH model. Consequently, economic interpretations are mainly concentrated upon the asymmetric specification. We also estimated a version of the asymmetric diagonal VECH model in which the cross-asymmetry terms are set to zero.[10] The likelihood ratio test statistic corresponding to the hypothesis that all the parameters of the cross-asymmetry terms are equal to zero is 28.68.[11] Consequently the null hypothesis is easily rejected at conventional significance levels. Thus cross asymmetries in stock and bond market returns are important.

There are a number of compelling observations to be made concerning the estimation results, and subsequently we schedule our comments in the following order: first, the dynamics in the covariance structure (Subsection 2.2.1), second, the asymmetric effects in the variances (Subsection 2.2.2), and finally, the asymmetric effects in the covariances (Subsection 2.2.3).

2.2.1 Dynamics in Volatility

In this section we consider the estimation results of the parameters that govern the dynamics in the variances and covariances. It appears that covariances change substantially over time, as most of the corresponding estimated parameters are statistically significant at the five percent level. Hence, the constant covariance hypothesis can be rejected. This result is consistent with the findings of Bollerslev, Engle and Wooldridge (1988), Harvey (1989) and Bodurtha and Mark (1991), who also document strong evidence in favor of heteroskedastic covariances.

The estimates for the coefficients on the product of the return shocks (i.e. the $\varepsilon_i\varepsilon_j$'s) in asymmetric diagonal VECH specification range from 0.044 to 0.068 for the variances, and from 0.012 to 0.050 for the covariances. The estimates for the variance are close to -and not significantly different from- the univariate GJR estimates. A positive estimate for the ARCH term in the covariance equation means that two shocks of the same sign affect the conditional covariance between the corresponding assets positively, while two shocks of opposite signs have a negative effect on the forecasted covariance. Apparently, two negative (or positive) shocks lead to a significant increase in next period's covariance. However, this interpretation only holds if we neglect the asymmetries in covariance. We will see below that the introduction of these asymmetric effects lead to more complex relationships. Finally, the estimates for the coefficients on lagged volatility (i.e. the $\sigma_{ij,t}$'s) are statistically significant and range from 0.893 to 0.934 for the variances and from 0.910 to 0.960 for the lagged conditional covariances. Obviously, not only variances, but also covariances tend to cluster over time. Note that the estimates for the coefficients on lagged variance are very similar to the univariate estimates. The results suggest that when comparing stock and bond volatility, past shocks seem to explanatory power is somewhat stronger for stock returns. Similarly, past volatility seems to have a greater explanatory power for bond return volatility.

Figure 3 and 4 present the plots of the conditional variance and covariance forecasts over time, based on the estimation results of the asymmetric diagonal VECH model. The figures show that the conditional variances and covariances are not constant over time and are especially volatile during the periods 1987-1988 (the October 1987 crash), 1990-1991 (recession and Gulf war), and 1998-2000 (the Millennium crash). Like Schwert (1989) we find that U.S. stock and bond return volatilities tend to move together.[12] Furthermore, the figures suggest that in general covariances between assets are higher (lower) in times of high (low) volatility. Looking at Figure 4, we see that the conditional covariance between bond returns, between stock returns and between bond and stock returns are highly clustered over time.

[Figures 3 and 4 about here]

To examine whether the time-variability in covariances is solely due to the variation in variances, we consider the conditional correlation coefficients. Let $\rho_{ij,t+1}$ denote the conditional correlation coefficient between return i and j at time $t+1$:

$$\rho_{ij,t+1} = \frac{Cov_t\{r_{i,t+1}, r_{j,t+1}\}}{\sqrt{Var_t\{r_{i,t+1}\}}\sqrt{Var_t\{r_{j,t+1}\}}}. \tag{8}$$

If $\rho_{ij,t+1}$ is constant over time, the variability in covariances is solely due to variation in variances. In that case, modeling of time-varying covariances is not very interesting, as all the dynamics are captured in variances. Figure 5 presents the estimated correlation coefficients, and shows that correlation coefficients vary considerably over time. This is in line with Tse (2000), who rejects for various countries that conditional correlations are constant over time. Tests of constancy of our correlation coefficients (not reported), by performing regression of the correlation coefficients on a constant and lagged correlation coefficients, clearly show that the correlation coefficients are not constant over time. Consequently, the variability in covariances is not solely due to time-varying variances, and modeling time-varying covariances is important.

[Figure 5 about here]

2.2.2 Asymmetric Effects in Variances

In this subsection, we address the degree of importance of the asymmetric effects in the variances (i.e. $(I_{\varepsilon_{i,t}}\varepsilon_{i,t})^2$, $i = 1, 2, 3$). The results in Table 4 indicate that these effects are especially pronounced in the variance of the stock indices. For example, the estimated coefficient of the variable that captures the negative shocks in the S&P 500 return is equal to 0.066, which means that negative return shocks in the S&P 500 are followed by a relatively high conditional variance. Both the univariate and multivariate results show that the asymmetric effects are only statistically significant in the stock market. Given existing results in the literature (see, e.g., Glosten, Jagannathan and Runkle, 1993, and Engle and Ng, 1993), it is not surprisingly that we find this asymmetric effect in the variance of the stock index. A drop in stock prices lead to an increase in leverage, making stocks riskier. The news impact curves for the four assets using the estimates from Table 4 are given in Figure 6. The solid lines represent the symmetric impacts on volatility of shocks in the asset returns, calculated using the diagonal VECH specification. The dashed lines represent the asymmetric impact on volatility, which are calculated using the estimates of the asymmetric diagonal VECH model. The figure illustrates that the model predicts that a negative return shock is followed by a higher subsequent volatility than a positive return shock of the same magnitude. While this effect is small and insignificant for the bonds, it is substantial for the S&P 500 and NASDAQ returns.

[Figure 6 about here]

2.2.3 Asymmetric Effects in Covariances

Next, we focus on the asymmetries in *covariances*. The results in Table 4 show that not only variances, but also covariances exhibit significant leverage effects. The asymmetric effects for shocks with the same sign (i.e. $I_{\varepsilon_{i,t}}\varepsilon_{i,t}I_{\varepsilon_{j,t}}\varepsilon_{j,t}$, $i \neq j$) seem to be important, as the corresponding estimated coefficients are statistically significant for four out of six cases. While the asymmetric effects in the covariances involving the short bond return are statistically negligible, the leverage effect in the covariance between the other assets are statistically significant. A positive sign of the coefficients indicates that next day's conditional covariance between returns is higher when there are two negative shocks rather than two positive shocks. Below, interpretations will be given using estimated news impact curves and surfaces. The cross effects in the asymmetry, i.e. when shocks in the two assets are of opposite signs (i.e. $I_{\varepsilon_{i,t}}\varepsilon_{i,t}(1 - I_{\varepsilon_{j,t}})\varepsilon_{j,t}$ and $(1 - I_{\varepsilon_{i,t}})\varepsilon_{i,t}I_{\varepsilon_{j,t}}\varepsilon_{j,t}$ $i \neq j$), also appear to be important. An estimated negative sign of the parameters of $I_{\varepsilon_{i,t}}\varepsilon_{i,t}(1 - I_{\varepsilon_{j,t}})\varepsilon_{j,t}$ for example indicates that the conditional covariance between returns is higher when there is a negative shock in i and a positive shock in j rather than a positive shock in i and a negative shock in j of the same magnitude.

[Figure 7 about here]

The estimated news impact surfaces imposing symmetry, based on the diagonal VECH estimation results, are shown in Figure 7, while Figure 8 presents estimated news impact surfaces which allow for asymmetries, obtained from the asymmetric diagonal VECH model. The interpretation of these surfaces is more difficult than the news impact curves, as there are two shocks instead of one. The symmetric news impact surface for short and long bonds in Figure 7 shows that the conditional covariance is high after shocks of the same sign, while shocks in opposite direction lower the conditional covariances. This is because bond returns are positively correlated (see Figure 5). As these assets move together, shocks in the same direction involves a higher risk than shocks in opposite direction. This makes sense, as it is riskier to invest in two assets that are highly positively correlated than to invest in two assets that are less correlated.

[Figure 8 about here]

Figure 7 shows very clearly that, when one uses a symmetric model, shocks of the same magnitude (in absolute value) in both assets, e.g. 3% or −3%, imply an identical impact on the conditional covariance. Figure 8 presents the news impact surfaces, allowing for asymmetries. Most surfaces show that the covariance is higher for shocks in $Q(-,-)$ than for shocks in $Q(+,+)$ of the same magnitude. The cross effects in asymmetries appear to be important as well. The first plot, for example, contains the asymmetric news impact surface for the short and long bond returns. The slope of the covariances in $Q(-,+)$ is not downward anymore in most cases. A negative (positive) shock to the short bond return, combined with a positive (negative) shock in the long bond return, results in a relatively high conditional covariance. Apparently, a negative shock in the short bond is followed by a relatively high degree of risk in the bond market. This finding is to be expected because of the convexity in the relation between the price of a bond and the yield. A decrease in interest rates has a bigger impact on the price than an increase of the same size. Our model captures this bond market property, while this result cannot be found using standard symmetric covariance models. Figure 8 further uncovers that there are (cross) asymmetries in the conditional covariance between bond and stock returns. We see that the covariances between stocks and bonds tend to be relatively low after bad news in the stock market and good news in the bond market. Thus, we find evidence that the cross-asymmetry is important when modeling covariances between stock and bond returns.

2.3 Specification Tests

When modeling the conditional covariance, it is important whether the specification is a statistically adequate representation of the data. In particularly, it must be the case that the standardized residuals, $\breve{\varepsilon} = \hat{\Sigma}_t^{-1/2}\hat{\varepsilon}_t \sim$ i.i.d.$(0, I)$. In Table 5 we present the test statistics for the (normalized) covariance for the four assets combinations. The tests to evaluate the adequacy of the model are based on the standardized residuals and the standardized products of residuals from the asymmetric covariance model. We consider the mean, standard deviation, skewness and kurtosis. In addition we present the Ljung-Box tests for serial correlation in the normalized cross-product of residuals.

[Table 5 about here]

The t-statistics in Table 5 indicate that the mean standardized residuals are not significantly different from zero. In addition, the mean squared standardized residuals and the mean product of standardized residuals are not significantly different from one. As these results satisfy the Bollerslev and Wooldridge (1992) moment conditions we can be confident that the QML estimates are consistent. Note that the skewness and excess kurtosis of the standardized residuals are lower than the ones for the excess returns series (see Table 1). This implies that much of the excess kurtosis in daily returns is attributable to conditional heteroskedasticity. The remaining excess kurtosis is not due to the October 1987 crash, as excluding the extreme returns around this period, resulted in only slightly smaller excess kurtoses. Since we use QML estimation, non-normality is not crucial, since standard errors are adjusted to take into account possible non-normality.

Next, we test for serial correlation in the standardized residuals, their squares and standardized products of residuals. The Ljung-Box (1978) test is a popular diagnostic for models with time-varying conditional second moments because it addresses whether the model has adequately captured the serial correlation in the second moments. These statistics for 6, 12, 18 and 24 lags are reported. These reveal that there remains almost no autocorrelation in the model. However, for some lags there is still some significant autocorrelation. It would be unreasonable to expect an empirical model to completely account for the higher moments, since we use daily returns that are highly leptokurtic. Moreover, using daily return data one sometimes finds autocorrelation at rather long lags. We find almost no serial correlation in the standardized squared residuals and only for the standardized cross-product of the short bond return and the S&P 500 returns, we find evidence of serial correlation. Overall, the results suggest that residuals from the estimated model are well-behaved and that the model provides adequate descriptions of the daily stock and bond returns.

2.4 Tactical Asset Allocation

Multivariate GARCH models can be applied to, e.g., futures hedging, asset pricing modeling, Value-at-Risk, volatility transmitting and asset allocation. In this section we will concentrate on the latter. Note that asset allocation is only relevant if conditional correlations vary over time. In Section 2.2.1 we found that conditional correlations are not constant over time. In addition, Figure 9 below shows that the conditional correlation between bond and stock returns exhibit asymmetries. Thus it seems a relevant question what impact these asymmetries have on asset allocation.

[Figure 9 about here]

The results from the previous sections do not necessarily imply economically useful implications for forecasting volatility. Studies that explicitly examines the economic significance of volatility timing are, for example, Fleming, Kirby and Ostdiek (2001), Marquering and Verbeek (2004) and Patton (2004). None of these studies examine the economic significance of asymmetric volatility timing. The article by Patton (2004) studies the statistical and economic importance of two other symmetries: the skewness in stock returns and the asymmetry that stock returns are more dependent during market downturns than during market upturns. Patton (2004) finds that investors with knowledge of such asymmetries might end up with greater economically gains. In a recent study by Detemple, Garcia and Rindisbacher (2003), dynamic optimal portfolio shares are computed, using the same four asset classes as in this study. Detemple, Garcia and Rindisbacher (2003) show that their calculation of optimal portfolios offers great flexibility and can be adapted to many asset allocation problems. Without resorting to such an elaborate dynamic portfolio, we evaluate in this section the performance of the model by determining the economic value of a trading rule exploiting the model forecasts of the conditional covariance matrices. To examine the economic gains of constructing a portfolio using the asymmetric model we compare it with one using the restricted symmetric model.

We partly follow the approach by Fleming, Kirby and Ostdiek (2001) by evaluating the impact of volatility timing on the economic performance of a dynamic asset allocation strategy[13]. This approach has the advantage that is relatively simple, tailor made for volatility timing and the optimal portfolios do not involve extreme weights (in contrast to e.g. standard mean-variance portfolios). Fleming, Kirby and Ostdiek (2001) use a utility-based measure to determine the economic value of a dynamic strategy based on volatility timing (of daily returns) relative to a passive strategy. Their approach can also be applied to compare two dynamic strategies (see, e.g., Marquering and Verbeek, 2004). We consider an investor who minimizes his portfolio variance subject to a particular target expected rate of return (μ_p). This optimization problem can be written as:

$$\min_{w_{t+1}} \; w'_{t+1}\Sigma^{-1}_{t+1}w_{t+1}, \tag{9}$$

$$\text{s.t. } w'_{t+1}\mu + (1 - w'_{t+1}\iota)r_{f,t+1} = \mu_p,$$

where $\mu = E\{r_{t+1}\}$ and w_{t+1} is the vector of portfolio weights on the risky assets. The proportion invested in the riskfree asset is $w_{0,t+1} = 1 - w'_{t+1}\iota$. Solving (9) for w_{t+1} gives us the optimal weights:

$$w^*_{t+1} = \frac{(\mu_p - r_{f,t+1})\Sigma^{-1}_{t+1}(\mu - r_{f,t+1}\iota)}{(\mu - r_{f,t+1}\iota)'\Sigma^{-1}_{t+1}(\mu - r_{f,t+1}\iota)}. \tag{10}$$

To calculate the portfolio weights of the optimal portfolio, we need the conditional forecasts of the covariance matrix. We employ a *symmetric* time-varying, and an *asymmetric* time-varying covariance matrix. The investor determines the optimal mix of five assets: the riskfree asset, the S&P 500 index, the NASDAQ index, a 1 year Treasury bond, and a 10 year Treasury bond. Ideally, out-of-sample forecasts, generated by the model, are used to evaluate the performance. However, this means that for each observation the model has to be re-estimated, which is computationally very demanding. Therefore we re-estimated the model on a sample of the first 19 years of data and using these estimates we generate out-of-sample one-day ahead forecasts for the conditional covariance matrix for the last part of the sample (January, 2001 - August 2001).

We compare the dynamic strategy which entails the asymmetric effects with the dynamic strategy that only considers the symmetric covariances. If the asymmetric extension has no economic value, the ex-post performance of the two strategies should be the same. Making this comparison requires a performance measure that captures the trade-off between risk and return.

Assume that the investor's realized utility in period $t+1$ can be written as:

$$U(W_{t+1}) = W_t r_{p,t+1} - \frac{a}{2}(W_t r_{p,t+1})^2, \tag{11}$$

where W_{t+1} is the investor's wealth at period $t+1$, a is his absolute risk aversion, and

$$r_{p,t+1} = w_{t+1}^{*\prime} r_{t+1} + (1 - w_{t+1}^{*\prime}\iota) r_{f,t+1}$$

is the period $t+1$ return on his portfolio p. We hold aW_t constant, which is equivalent to setting the investor's relative risk aversion, $\gamma_t = aW_t/(1-aW_t)$ equal to some fixed value γ. With relative risk aversion held constant, we can use the average realized utility to consistently estimate the expected utility generated by a given level of initial wealth (normalized to 1). In particular we have

$$\hat{U}_p(\gamma) = \frac{1}{T}\sum_{t=0}^{T-1}\left[r_{p,t+1} - \frac{\gamma}{2(1+\gamma)} r_{p,t+1}^2\right]. \tag{12}$$

The above approach enables us to compare alternative investment strategies by calculating the associated average utility levels.

First we look at the time series of portfolio weights resulting from the portfolio decisions made using the asymmetric model. To consider the impact of the introduced asymmetries we also present a figure with the impact of the asymmetries in the optimal portfolio shares. Figure 10 shows the time series of portfolio weights for the four asset classes for investors with a relative risk aversion coefficient of 10. On average, the investor takes a short position in the 10 year bond and S&P 500, and a long position in the 1 year bond and NASDAQ. Further note that most of the time, the optimal weight for the 1 year bond is relatively high. Finally, due to an increasing volatility in NASDAQ returns, the weights in this asset class decrease over the second half of the 1990's and the beginning of the new millennium. Figure 11 illustrates how the asymmetry induced in the covariances affect the optimal portfolio shares. The mean change in portfolio weights is positive for 10 year bond and S&P 500 and negative for the 1 year bond and NASDAQ. This shows that there is substantial volatility transmission in stock and bond markets. Differences in portfolio weights are only economically interesting if they lead to differences in portfolio performance. We proceed to examine this below.

[Figures 10 and 11 about here]

For different values of the relative risk aversion coefficient and the target return, Table 6 presents the average realized utility values per month over an in-sample period (January, 1982 – December, 2000) and an out-of-sample period (January, 2001 - August, 2001). For example, an investor with $\gamma = 1$ and target return of 10, the utility increases by 2.2% in-sample (2.1% out-of-sample) if he switches from using a symmetric to an asymmetric volatility model. Likewise, for an investor with $\gamma = 10$ and target return of 15%, the utility increases by 5.7% in-sample (5.9% out-of-sample) if he switches to an asymmetric model. These numbers indicate sizeable gains due to asymmetric volatility timing. While these gains are lower for low target-return investors, all of the asymmetric dynamic strategies clearly outperforms the symmetric ones. This finding corresponds to Patton (2004), who finds statistically and economically gains for investors using "asymmetric dependence"; that is, returns are mode dependent during market downturns than during market upturns.

[Table 6 about here]

2.5 Comparison with Previous Studies

After having analyzed the asymmetric volatility for the U.S. stock and bond market, we shall now compare our results with other related studies. As mentioned in the introduction, little attention has been paid to the interaction between the stock market or the bond market. Moreover, most multivariate GARCH models do not allow for asymmetries. Consequently, this study contributes to several aspects of financial economics. Table 7 summarizes the main contributions of this study. As shown by the example in the previous section, these findings have important implications for, e.g., portfolio managers applying tactical asset allocation and risk management.

[Table 7 about here]

3 Conclusions

In this paper we analyzed the bond and stock market interactions by modeling the time-varying covariances between stock and bond market returns. The main contribution of this paper is that it extends the multivariate model by allowing for asymmetric effects in covariances between stock and bond returns. We showed that asymmetric effects are present in the covariances between stock returns and returns on a second asset. To model the asymmetric effects on conditional covariances we have developed a novel approach by generalizing the Glosten, Jagannathan and Runkle (1993) specification towards a multivariate setting. The model is estimated using daily returns on the S&P 500 index, NASDAQ Composite index, and a short and long Treasury bond.

The main empirical findings can be summarized as follows. As the conditional covariances change substantially over time, the constant covariance hypothesis should be rejected. With respect to asymmetric effects in variances, we find that daily returns on the S&P 500 index and the NASDAQ index exhibit significant leverage effects. Not only variances, but also covariances between stock and bond returns exhibit significant asymmetries. Overall, our findings imply that a symmetric specification is too restrictive to model the conditional covariances. Especially bad news in the stock market is followed by a much higher conditional covariance than good news in the stock market. This holds irrespectively the sign of the bond market shock. The cross effects in asymmetries appear to be important as well. Covariances between stock and bond returns tend to be relatively low after bad news in the stock market and good news in the bond market. Thus, we find evidence that the cross-asymmetry terms are important when modeling covariances between asset returns. Overall, the results indicate that the performance of the asymmetric diagonal VECH model of conditional second moments is quite well.

Asymmetries in covariances have important implications for portfolio managers. From modern portfolio theory we know that investors should diversify between different asset classes. We have shown that investors can benefit from tactical asset allocation when asymmetric leverage effects in covariances are taken into account. Optimal portfolio shares can be substantially affected by asymmetries in covariances. Finally, our results show that there are sizable gains due to asymmetric volatility timing. For example, an investor with a relative risk aversion coefficient of ten percent and a target return of fifteen percent, the utility increases by six percent if he switches from using a symmetric to an asymmetric volatility model.

Appendix A

This appendix shows that if both stock and bond returns exhibit leverage effects, the conditional covariance between these assets responses asymmetrically to shocks, in such a way that the covariance will be relatively higher after two negative shocks. If there are asymmetric effects in conditional covariances, we must have that

$$Cov\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; \varepsilon^*_{i,t}, \varepsilon^*_{j,t}\} \neq Cov\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; -\varepsilon^*_{i,t}, -\varepsilon^*_{j,t}\}$$

and/or

$$Cov\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; -\varepsilon^*_{i,t}, \varepsilon^*_{j,t}\} \neq Cov\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; \varepsilon^*_{i,t}, -\varepsilon^*_{j,t}\}.$$

where $\varepsilon^*_{i,t}$ denotes a given positive shock in asset i at time t, i.e. in the interval $(0, \infty)$. We can show mathematically that if leverage effects in volatility exist, they also affect covariances. If leverage effects exists in the variance of asset i we have that

$$Var\{r_{i,t+1}|\mathcal{I}_t; -\varepsilon^*_{i,t}\} - Var\{r_{i,t+1}|\mathcal{I}_t; \varepsilon^*_{i,t}\} = \delta_{i,t} > 0. \tag{13}$$

Using the definition of the squared correlation coefficient:

$$\rho^2_{ij,t+1} = \frac{Cov^2_t\{r_{i,t+1}, r_{j,t+1}\}}{Var_t\{r_{i,t+1}\}Var_t\{r_{j,t+1}\}}, \tag{14}$$

we can write[14]

$$Cov^2\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; -\varepsilon^*_{i,t}, -\varepsilon^*_{j,t}\} - Cov^2\{r_{i,t+1}, r_{j,t+1}|\mathcal{I}_t; \varepsilon^*_{i,t}, \varepsilon^*_{j,t}\} =$$

$$\rho^2_{ij,t+1}Var\{r_{i,t+1}|\mathcal{I}_t; \varepsilon^*_{i,t}\}\delta_{j,t} - \rho^2_{ij,t+1}Var\{r_{j,t+1}|\mathcal{I}_t; \varepsilon^*_{j,t}\}\delta_{i,t} + \rho^2_{ij,t+1}\delta_{i,t}\delta_{j,t}, \tag{15}$$

where $\varepsilon^*_{i,t}, \varepsilon^*_{j,t} > 0$, and $\rho^2_{ij,t+1} > 0$. If index i denotes the stock index and index j a bond index, then $\delta_{j,t}$ corresponds to the leverage effect in bond returns. As this effect has not been documented before, we expect $\delta_{j,t}$ to be (close to) zero. It follows from (13) that, in general, the right hand side of (15) will not be equal to zero. If $\delta_{j,t}$ equals zero, the right hand side of (15) reduces to $\rho^2_{ij,t+1}Var\{r_{j,t+1}|\mathcal{I}_t; \varepsilon^*_{j,t}\}\delta_{i,t}$, which will be a positive number. As the conditional covariances between stock and bond returns are mostly positive numbers (see Figure 4), it follows from (15) that the conditional covariance between two assets given two negative shocks will be larger than given two positive shocks. More generally, if both stock and bond returns exhibit leverage effects, (15) implies that the conditional covariance between these assets responses asymmetrically to shocks, in such a way that the covariance will be relatively higher after two negative shocks.

Appendix B

In this appendix the calculations of the bond returns are given. We obtained the "daily constant maturity interest rate series" from the federal reserve bank in Chicago. To calculate the bond returns we have followed the method in Jones, Lamont and Lumsdaine (1998)[15]. The U.S. Treasury bonds have semi-annual coupon payments, and the coupon on the hypothetical bonds is half the stated coupon yield. Hence, the price of the bond at the beginning of the holding period is equal to its face value. We have calculated an end-of-period price on this bond using the next day's yield augmented with the accrued interest rate:

$$P_{n-\#hd,t+1} = \sum_{i=1}^{2n-1} \frac{\frac{1}{2}y_{nt}}{(1+\frac{1}{2}y_{n,t+1})^i} + \frac{1+\frac{1}{2}y_{nt}}{(1+\frac{1}{2}y_{n,t+1})^{2n}} + \frac{\#\text{ holding days}}{365} y_{nt}, \tag{16}$$

where $P_{n-\#hd,t+1}$ is the end-of-period price of the bond, n is the number of years the bond is referring to, t is the time and y_{nt} is the yield of an n-period bond at time t. The $\#hd-$return, is calculated as

$$r_{t+1} = P_{n-\#hd,t+1} - 1. \tag{17}$$

Finally, the excess returns are calculated using the 3-month interest rate as the risk free rate that accrues over the holding period, which varies from one to five days due to weekends and holidays.

$$r^e_{t+1} = r_{t+1} - \frac{\text{\# holding days}}{365} y_{3mo,t}.$$

The S&P 500 index data are obtained from Datastream, while the NASDAQ index data are obtained from the National Association of Security Dealers. The returns on the S&P 500 index and the NASDAQ index are calculated as

$$r_{index,t+1} = \frac{P_{index,t+1} - P_{index,t}}{P_{index,t}}. \tag{18}$$

Excess returns are calculated by substracting the risk free rate that accrues over the holding period

$$r^e_{index,t+1} = r_{index,t+1} - \frac{\text{\# holding days}}{365} y_{3mo,t}. \tag{19}$$

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Notes

1. Some well-known examples of stock market studies include Breen, Glosten and Jagannathan (1989), Campbell and Hentschel (1992), Engle and Ng (1993), Glosten, Jagannathan and Runkle (1993) and Kroner and Ng (1998). Literature on the modeling of bond returns include Engle, Lilien and Robins (1987), Engle, Ng and Rothschild (1990), Fama and French (1995) and Duffie and Singleton (1997).

2. While the majority of the GARCH literature focuses on the univariate properties, there now appears a vast amount of literature that considers multivariate extensions. Some examples include Harvey (1989), Bollerslev (1990), Bodurtha and Mark (1991), Ng (1991), Ng, Engle and Rothschild (1992), Braun, Nelson and Sunier (1995), Engle and Kroner (1995), Nijman and Sentana (1996) and Kroner and Ng (1998).

3. The multivariate model in (2) is called the VECH model.

4. We impose that the smallest eigenvalue of each covariance matrix has to be positive during estimation.

5. Note that the univariate GJR model is obtained when $i = j$.

6. Strictly, we should not talk about quadrants in this setting, but octants.

7. The data is available from the authors upon request.

8. See Bouwens, Laurent and Rombouts (2003) and Brooks, Burke and Persand (2003) for a detailed discussion on issues in estimating multivariate GARCH models.

9. Experiments with a covariance term in the mean equation showed that the relation between the expected market risk premium and the conditional market covariance is not statistically significant at the usual significance levels. Thus, our results suggest that, at the daily frequency, the expected returns are independent of the time-varying reward to risk.

10. Results can be obtained from the authors upon request.

11. The test statistic in this case is $2 \times (3,663.29 - 3,648.95) = 28.68$, and there are twelve degrees of freedom.

12. Moreover, Engle, Ng and Rothschild (1990) uncover that changes in U.S. bond volatility are closely linked across maturities.

13. Note that Fleming, Kirby and Ostdiek (2001) use a simple univariate 'rolling window' to estimate conditional volatility.

14. For simplicity, we assume symmetric time-varying correlations.

15. We thank Charles Jones, Owen Lamont and Charlotte Christiansen for their help with the program to construct the data.

Tables

Table 1: **Descriptive Statistics for Stock and Bond Excess Returns**

	1 yr bond	10 yr bond	S&P 500	NASDAQ
Mean	0.0038	0.0196	0.0377	0.0286
Std. Dev.	0.0751	0.4763	1.0400	1.3061
Minimum	−0.9306	−2.7149	−20.460	−11.405
Maximum	0.7905	4.8037	9.0979	14.158
Skewness	0.5626	0.1724	−1.6416	−0.1049
Kurtosis	20.854	7.5832	37.623	15.097
Jarque-Bera	65,448	4,320	26,171	29,935

Notes: This table gives descriptive statistics for the excess return on the S&P 500 index, the NASDAQ index, the 1 year Treasury bond and the 10 year Treasury bond for the period January 4, 1982 - August 31, 2001. All returns are daily returns in percentages.

Table 2: **Autocorrelations and Cross Autocorrelations**

	r^e_{1t}	r^e_{2t}	r^e_{3t}	r^e_{4t}
$r^e_{1,t-1}$	0.0881	0.5890	0.8209	0.7702
	(3.1435)	(4.4786)	(2.5090)	(2.3631)
$r^e_{2,t-1}$	0.0089	0.0740	0.1228	0.1636
	(3.0045)	(4.5411)	(2.5133)	(3.1353)
$r^e_{3,t-1}$	−0.0040	0.0014	0.0224	0.1825
	(−1.3353)	(0.0698)	(0.6084)	(4.9173)
$r^e_{4,t-1}$	−0.0026	0.0015	−0.0017	0.0798
	(−1.9458)	(0.1654)	(−0.0722)	(2.3102)

Notes: Autocorrelations on diagonals and cross-autocorrelations on off-diagonals. White heteroskedasticity-consistent t-statistics between brackets.

Table 3: **Akaike and Schwarz Information Criteria**

p	AIC	SIC
1	3.5576	3.5841*
2	3.5546	3.6023
3	3.5509	3.6199
4	3.5489	3.6392
5	3.5394*	3.6509
6	3.5413	3.6741

Notes: p denotes the lag in $VAR(p)$,while a '*' denotes the minimum value of the information criteria.

Table 4: **Estimation Results**

Explanatory	**Univ. GARCH**		**Diagonal VECH**		**Asym. Diag. VECH**	
Variables	Estimate	Std. Error	Estimate	Std. Error	Estimate	Std. Error
$Const_{11}$	0.0032*	(0.0009)	0.0041*	(0.0005)	0.0037*	(0.0004)
$Const_{21}$	.	.	0.0306*	(0.0031)	0.0265*	(0.0028)
$Const_{22}$	0.4642*	(0.0120)	0.4289*	(0.0037)	0.3839*	(0.0390)
$Const_{31}$	.	.	0.0153*	(0.0019)	0.0116*	(0.0035)
$Const_{32}$	.	.	0.2273*	(0.0285)	0.1848*	(0.0334)
$Const_{33}$	1.8532*	(0.4421)	1.0488*	(0.1488)	1.0718*	(0.1103)
$Const_{41}$	.	.	0.0162*	(0.0019)	0.0156*	(0.0043)
$Const_{42}$	.	.	0.2471*	(0.0275)	0.2185*	(0.0400)
$Const_{43}$	.	.	0.8678*	(0.1061)	0.7820*	(0.0856)
$Const_{44}$	2.1810*	(0.4716)	1.0297*	(0.1202)	0.9420*	(0.1052)
$\sigma^2_{1,t}$	0.9402*	(0.0094)	0.9351*	(0.0044)	0.9355*	(0.0026)
$\sigma_{12,t}$	.	.	0.9414*	(0.0032)	0.9417*	(0.0028)
$\sigma^2_{2,t}$	0.9330*	(0.0106)	0.9317*	(0.0034)	0.9330*	(0.0037)
$\sigma_{13,t}$	.	.	0.9625*	(0.0026)	0.9599*	(0.0046)
$\sigma_{23,t}$	.	.	0.9495*	(0.0033)	0.9499*	(0.0038)
$\sigma^2_{3,t}$	0.9077*	(0.0162)	0.9164*	(0.0079)	0.9146*	(0.0031)
$\sigma_{14,t}$	.	.	0.9519*	(0.0030)	0.9511*	0.0055)
$\sigma_{24,t}$	.	.	0.9366*	(0.0037)	0.9367*	(0.0053)
$\sigma_{34,t}$	.	.	0.9128*	(0.0069)	0.9095*	(0.0035)
$\sigma^2_{4,t}$	0.8474*	(0.0143)	0.8950*	(0.0076)	0.8927*	(0.0045)

Notes: This table reports the maximum likelihood estimation results of model (4) using data from January 4, 1982 to August 31, 2001 ($T = 4,908$). Index $i = 1$ refers to the short term bond, $i = 2$ to the long term bond, $i = 3$ to the S&P 500 index, and $i = 4$ to the NASDAQ index. Robust Bollerslev Wooldridge standard errors are reported in parentheses, while a '*' denotes statistical significance at the 5% level.

Table 4: **Estimation Results** *(Continued)*

Explanatory Variables	Univ. GARCH Estimate	Univ. GARCH Std. Error	Diagonal VECH Estimate	Diagonal VECH Std. Error	Asym. Diag. VECH Estimate	Asym. Diag. VECH Std. Error
$\varepsilon_{1,t}^2$	0.0492*	(0.0114)	0.0551*	(0.0041)	0.0543*	(0.0031)
$\varepsilon_{1,t}\varepsilon_{2,t}$	.	.	0.0440*	(0.0027)	0.0413*	(0.0030)
$\varepsilon_{2,t}^2$	0.0412*	(0.0097)	0.0486*	(0.0028)	0.0449*	(0.0035)
$\varepsilon_{1,t}\varepsilon_{3,t}$	.	.	0.0193*	(0.0017)	0.0153*	(0.0038)
$\varepsilon_{2,t}\varepsilon_{3,t}$	.	.	0.0308*	(0.0021)	0.0199*	(0.0034)
$\varepsilon_{3,t}^2$	0.0261*	(0.0097)	0.0757*	(0.0084)	0.0436*	(0.0044)
$\varepsilon_{1,t}\varepsilon_{4,t}$	.	.	0.0215*	(0.0019)	0.0122*	(0.0044)
$\varepsilon_{2,t}\varepsilon_{4,t}$	.	.	0.0325*	(0.0022)	0.0182*	(0.0045)
$\varepsilon_{3,t}\varepsilon_{4,t}$	.	.	0.0764*	(0.0069)	0.0495*	(0.0044)
$\varepsilon_{4,t}^2$	0.0904*	(0.0167)	0.0971*	(0.0077)	0.0679*	(0.0054)
$(I_{\varepsilon_{1,t}}\varepsilon_{1,t})^2$	0.0074	(0.0125)	.	.	0.0025	(0.0039)
$I_{\varepsilon_{1,t}}\varepsilon_{1,t}I_{\varepsilon_{2,t}}\varepsilon_{2,t}$	.	.	.	.	0.0053	(0.0035)
$(I_{\varepsilon_{2,t}}\varepsilon_{2,t})^2$	0.0097	(0.0161)	.	.	0.0069	(0.0042)
$I_{\varepsilon_{1,t}}\varepsilon_{1,t}I_{\varepsilon_{3,t}}\varepsilon_{3,t}$	.	.	.	.	0.0103*	(0.0052)
$I_{\varepsilon_{2,t}}\varepsilon_{2,t}I_{\varepsilon_{3,t}}\varepsilon_{3,t}$	.	.	.	.	0.0216*	(0.0050)
$(I_{\varepsilon_{3,t}}\varepsilon_{3,t})^2$	0.1010*	(0.0303)	.	.	0.0661*	(0.0053)
$I_{\varepsilon_{1,t}}\varepsilon_{1,t}I_{\varepsilon_{4,t}}\varepsilon_{4,t}$	.	.	.	.	0.0100	(0.0066)
$I_{\varepsilon_{2,t}}\varepsilon_{2,t}I_{\varepsilon_{4,t}}\varepsilon_{4,t}$	.	.	.	.	0.0244*	(0.0065)
$I_{\varepsilon_{3,t}}\varepsilon_{3,t}I_{\varepsilon_{4,t}}\varepsilon_{4,t}$	.	.	.	.	0.0620*	(0.0055)
$(I_{\varepsilon_{4,t}}\varepsilon_{4,t})^2$	0.0931*	(0.0301)	.	.	0.0712*	(0.0067)
$I_{\varepsilon_{2,t}}\varepsilon_{2,t}(1-I_{\varepsilon_{1,t}})\varepsilon_{1,t}$	.	.	.	.	−0.0499*	(0.0145)
$I_{\varepsilon_{3,t}}\varepsilon_{3,t}(1-I_{\varepsilon_{1,t}})\varepsilon_{1,t}$	.	.	.	.	−0.0134*	(0.0059)
$I_{\varepsilon_{3,t}}\varepsilon_{3,t}(1-I_{\varepsilon_{2,t}})\varepsilon_{2,t}$	.	.	.	.	−0.0112	(0.0066)
$I_{\varepsilon_{4,t}}\varepsilon_{4,t}(1-I_{\varepsilon_{1,t}})\varepsilon_{1,t}$	.	.	.	.	−0.0076	(0.0060)
$I_{\varepsilon_{4,t}}\varepsilon_{4,t}(1-I_{\varepsilon_{2,t}})\varepsilon_{2,t}$	.	.	.	.	−0.0047	(0.0069)
$I_{\varepsilon_{4,t}}\varepsilon_{4,t}(1-I_{\varepsilon_{3,t}})\varepsilon_{3,t}$	.	.	.	.	−0.0399*	(0.0130)
$(1-I_{\varepsilon_{2,t}})\varepsilon_{2,t}I_{\varepsilon_{1,t}}\varepsilon_{1,t}$	.	.	.	.	−0.0288*	(0.0137)
$(1-I_{\varepsilon_{3,t}})\varepsilon_{3,t}I_{\varepsilon_{1,t}}\varepsilon_{1,t}$	.	.	.	.	0.0078	(0.0047)
$(1-I_{\varepsilon_{3,t}})\varepsilon_{3,t}I_{\varepsilon_{2,t}}\varepsilon_{2,t}$	.	.	.	.	0.0131*	(0.0054)
$(1-I_{\varepsilon_{4,t}})\varepsilon_{4,t}I_{\varepsilon_{1,t}}\varepsilon_{1,t}$	.	.	.	.	0.0105	(0.0059)
$(1-I_{\varepsilon_{4,t}})\varepsilon_{4,t}I_{\varepsilon_{2,t}}\varepsilon_{2,t}$	.	.	.	.	0.0152*	(0.0061)
$(1-I_{\varepsilon_{4,t}})\varepsilon_{4,t}I_{\varepsilon_{3,t}}\varepsilon_{3,t}$	.	.	.	.	−0.0605*	(0.0202)
Log Likelihood			−3,719.28		−3,648.95	

Notes: See the first part of the table.

Table 5: **Diagnostic Tests of the Generalized Residuals using the Asymmetric Diagonal VECH Model**

	$\breve{\varepsilon}_1$	$\breve{\varepsilon}_2$	$\breve{\varepsilon}_3$	$\breve{\varepsilon}_4$	$\breve{\varepsilon}_1^2$	$\breve{\varepsilon}_2^2$	$\breve{\varepsilon}_3^2$
Mean	0.019	0.015	0.006	0.010	1.014	1.008	1.011
Std. Dev.	1.007	1.004	1.006	1.005	2.481	2.105	2.504
Skewness	0.077	−0.017	−0.470	−0.521	7.927	8.663	17.16
Excess Kurtosis	3.982	2.364	4.148	2.502	93.64	162.6	4981.8
t-stat. for $H_0 : \overline{\breve{\varepsilon}_{i,t}} = 0$	1.3265	1.032	0.402	0.665	.	.	.
t-stat. for $H_0 : \overline{\breve{\varepsilon}_{i,t}\breve{\varepsilon}_{i,t}} = 1$	.	.	.	.	0.397	0.266	0.303
Ljung-Box Statistics							
$Q(6)$	3.725	2.813	3.254	0.465	3.432	11.18	2.078
$Q(12)$	14.74	13.06	10.99	8.230	6.416	15.68	6.540
$Q(18)$	35.09*	25.98	21.68	14.71	17.65	19.95	9.219
$Q(24)$	47.99*	29.55	29.62	37.76*	36.93*	31.84	12.78

	$\breve{\varepsilon}_4^2$	$\breve{\varepsilon}_2\breve{\varepsilon}_1$	$\breve{\varepsilon}_3\breve{\varepsilon}_1$	$\breve{\varepsilon}_3\breve{\varepsilon}_2$	$\breve{\varepsilon}_4\breve{\varepsilon}_1$	$\breve{\varepsilon}_4\breve{\varepsilon}_2$	$\breve{\varepsilon}_4\breve{\varepsilon}_3$
Mean	1.011	1.028	0.618	2.219	-3.240	0.278	1.031
Std. Dev.	2.140	2.655	66.42	111.2	251.4	65.31	2.752
Skewness	9.211	9.795	−34.03	52.28	−59.87	−33.30	13.32
Excess Kurtosis	144.0	187.1	2048	3366	3886	1774	299.2
t-stat. for $H_0 : \overline{\breve{\varepsilon}_{i,t}\breve{\varepsilon}_{i,t}} = 1$	0.352	0.737	−0.403	0.768	−1.181	−0.774	0.801
Ljung-Box Statistics							
$Q(6)$	7.037	6.893	18.10*	3.349	0.303	8.981	1.379
$Q(12)$	12.57	8.708	32.27*	3.396	2.762	12.04	5.084
$Q(18)$	20.06	16.57	40.51*	8.801	2.778	12.36	9.325
$Q(24)$	27.32	28.57	47.03*	9.095	2.894	16.74	14.27

Notes: This table reports summary statistics and Ljung-Box statistics for standardized residuals and standardized products of residuals. $Q(r)$ denotes the Ljung-Box test statistic for rth order serial correlation in the standardized cross-product of residuals. The 95% critical values for $Q(6)$, $Q(12)$, $Q(18)$ and $Q(24)$ are 12.6, 21.0, 28.9 and 36.4, respectively. '*' indicates statistical significance at the 5% level.

Table 6: **Economic Evaluation Results**

Target Return	Gamma	Without asymmetry	With Asymmetry	Incremental utility
		In-sample		
10	1	0.03113	0.03181	2.2%
15	1	0.03338	0.03502	4.9%
10	10	0.02960	0.03027	2.2%
15	10	0.02721	0.02877	5.7%
		Out-of-sample		
10	1	0.00910	0.00929	2.1%
15	1	−0.01661	−0.01525	8.2%
10	10	0.00733	0.00752	2.7%
15	10	−0.02722	−0.02561	5.9%

Notes: The average realized utilities, obtained using formula (12). Target returns are 10 and 15 percent, and relative risk aversions are 1 and 10. The table presents in-sample results (January, 1982 – December, 2000) and out-of-sample results (January, 2001 - August, 2001).

Table 7: **Overview of Contribution**

Existing results	**This study**
Many studies consider the conditional volatility in stock markets and bond markets separately (e.g. Breen, Glosten and Jagannathan, 1989 and Engle, Lilien and Roberts, 1987).	Only few studies consider the conditional covariance between stock and bond returns.
Leverage effect in variance of stock returns (Black, 1976).	Leverage effect in covariances between stock and bond returns.
A constant correlation model is able to describe the conditional covariances (Bollerslev, 1990).	Modeling covariances and correlations as a time-varying structure provides some interesting results that are not obtained from constant-correlation models.
Diagonal VECH model to describes conditional covariances between stock and bond returns (Bollerslev, Engle and Wooldridge, 1988).	An asymmetric diagonal VECH model outperforms the diagonal counterpart statistically.
Kroner and Ng (1998) find asymmetries in the covariance between portfolios of small cap and large cap firms.	This study allows for another source of asymmetry: asymmetry due to shocks of opposite signs.
The use of volatility timing, using a simple (symmetric) measure, can be economically useful (Fleming, Kirby and Ostdiek, 2001).	Asymmetric volatility timing economically outperforms symmetric volatility timing.

Figures

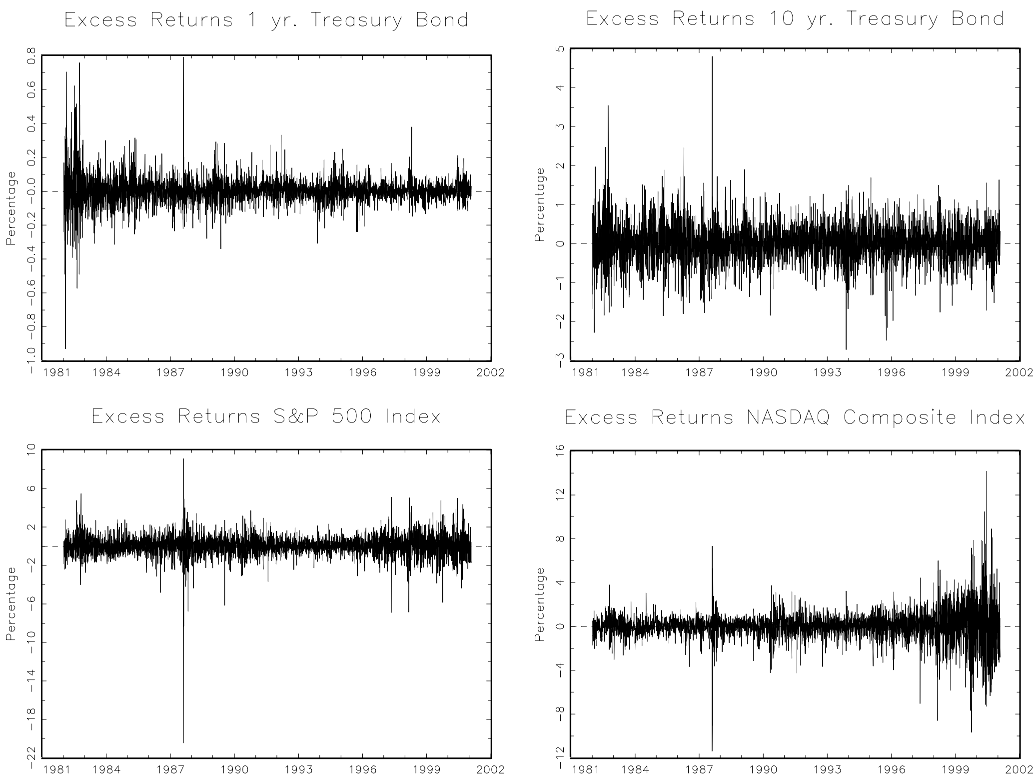

Figure 1: Excess Returns on the 1 and 10 year Treasury Bonds and on the S&P 500 and NASDAQ Index

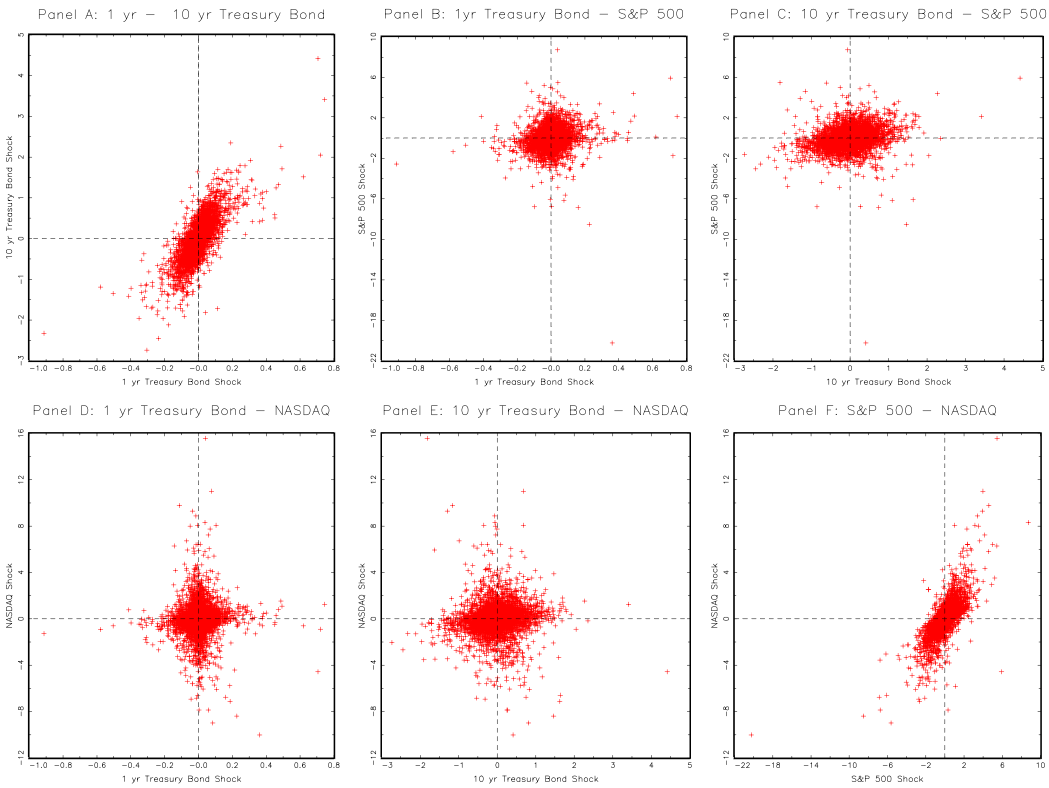

Figure 2: Return Shocks Within and Between Stock and Bond Markets

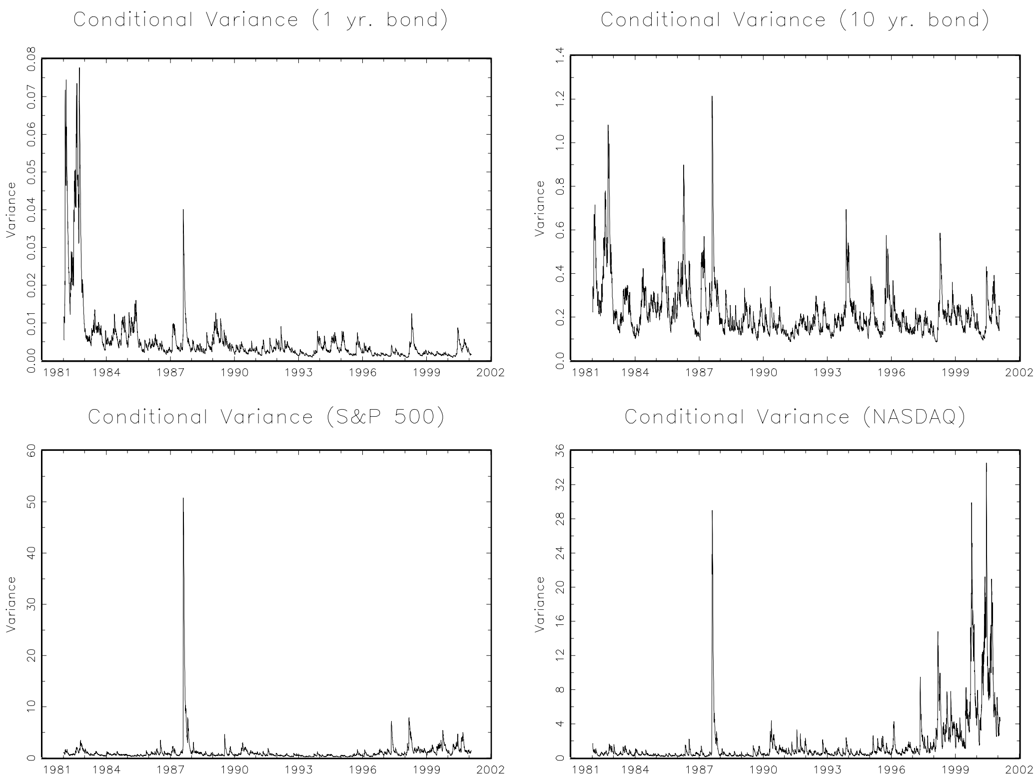

Figure 3: Conditional Variances

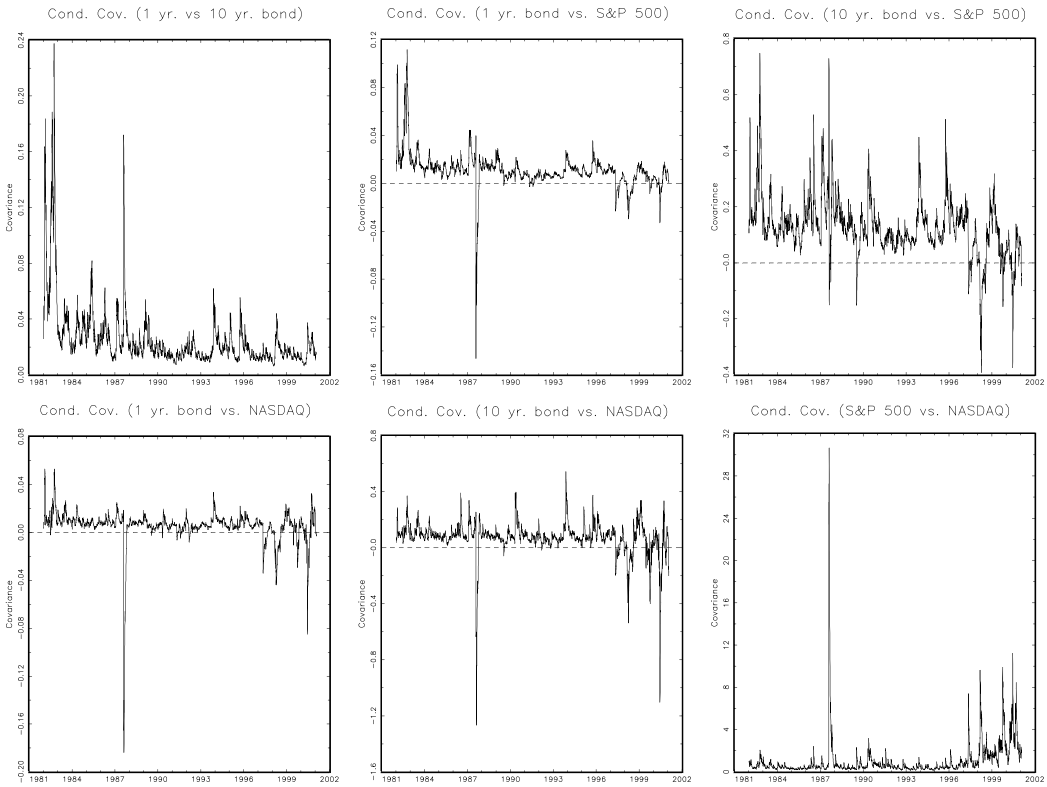

Figure 4: Conditional Covariances

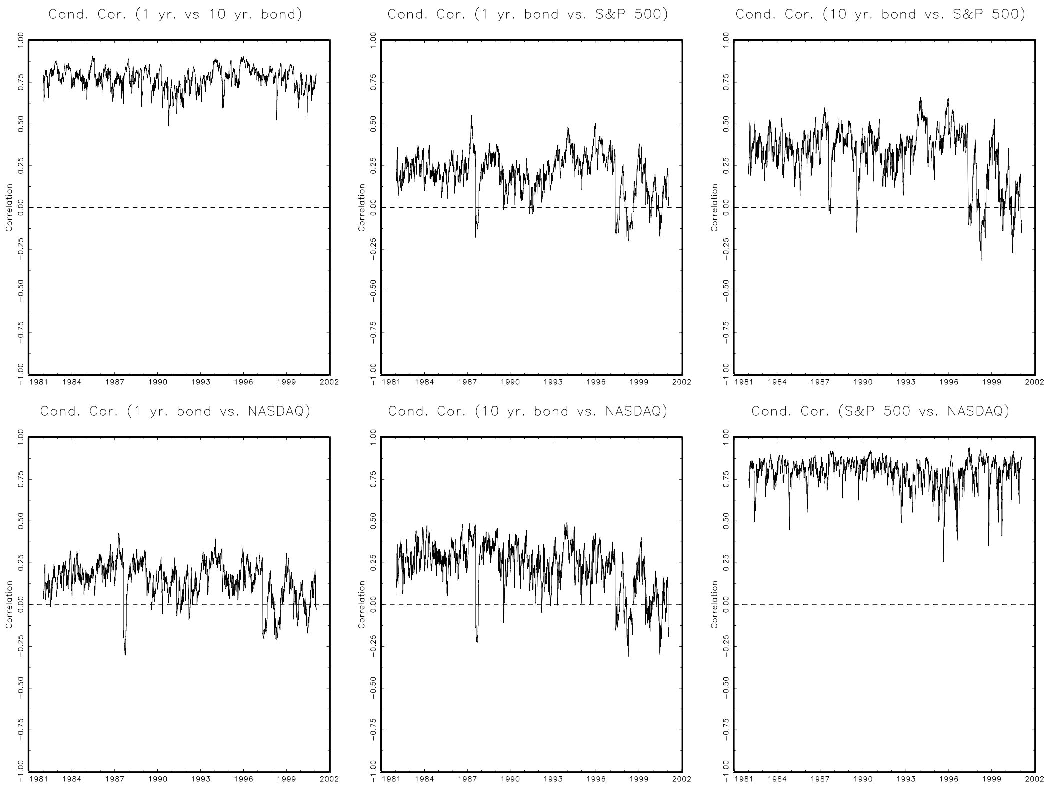

Figure 5: Conditional Correlations

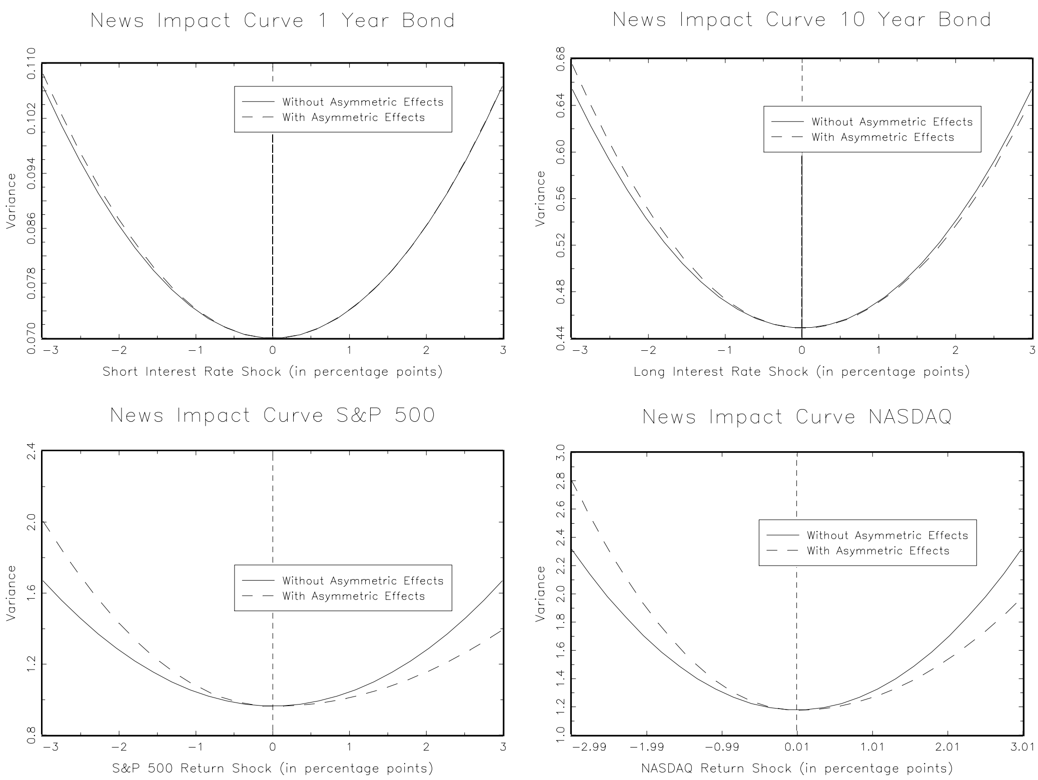

Figure 6: **Estimated News Impact Curves With and Without Imposing Symmetry**

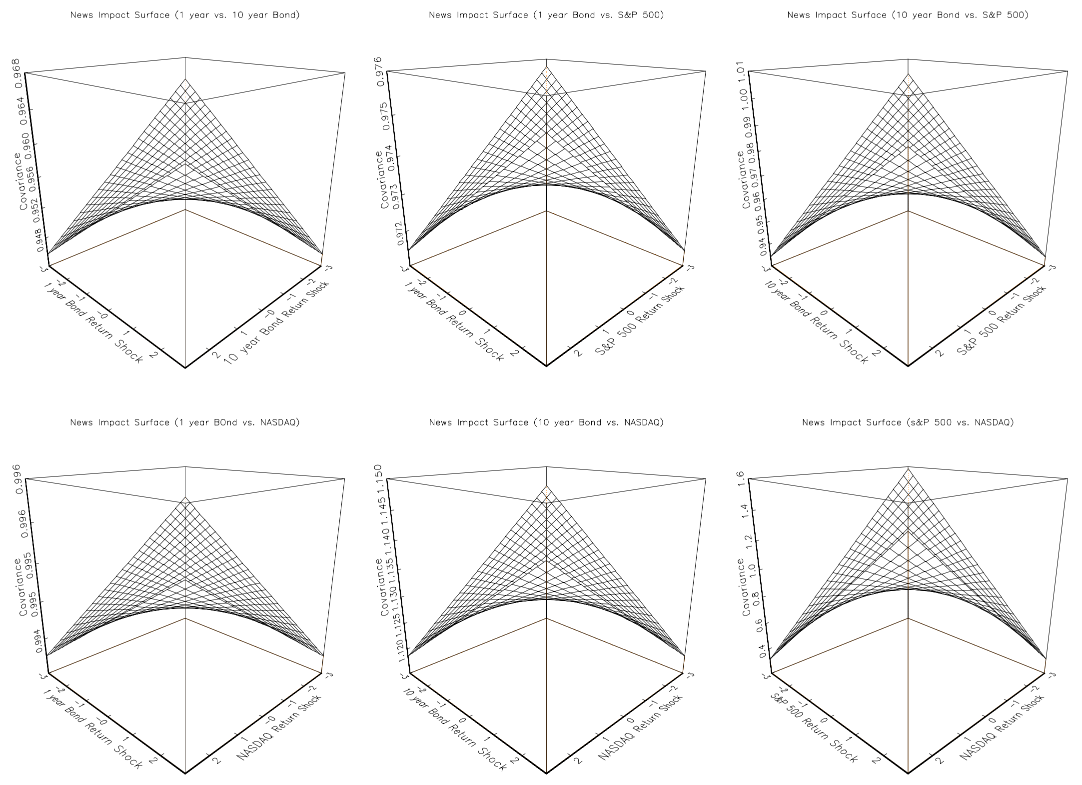

Figure 7: Estimated News Impact Surfaces from Diagonal VECH Model

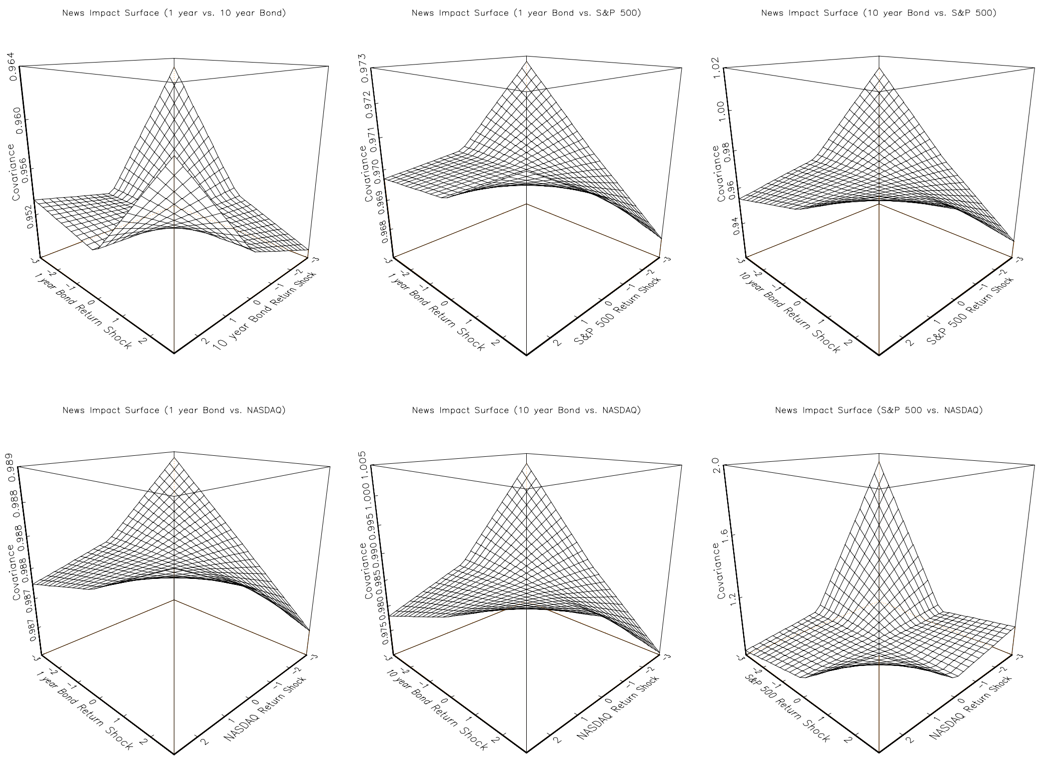

Figure 8: Estimated News Impact Surfaces from the Asymmetric Diagonal VECH Model

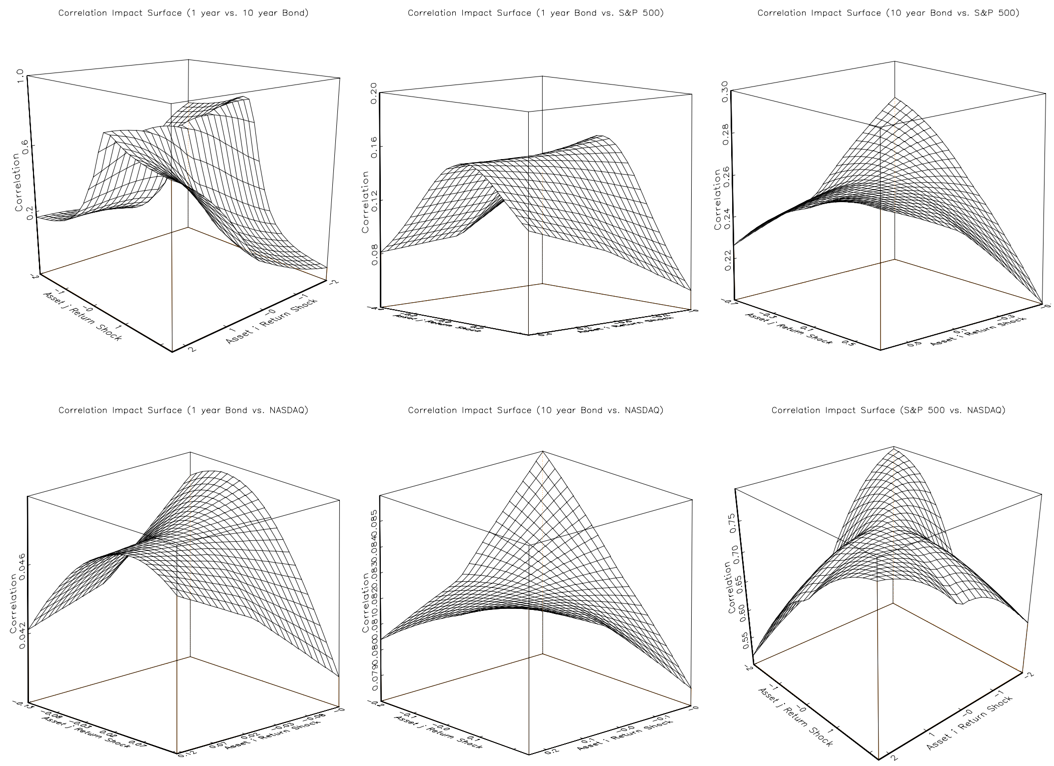

Figure 9: Estimated Correlation Impact Surfaces from the Asymmetric Diagonal VECH Model

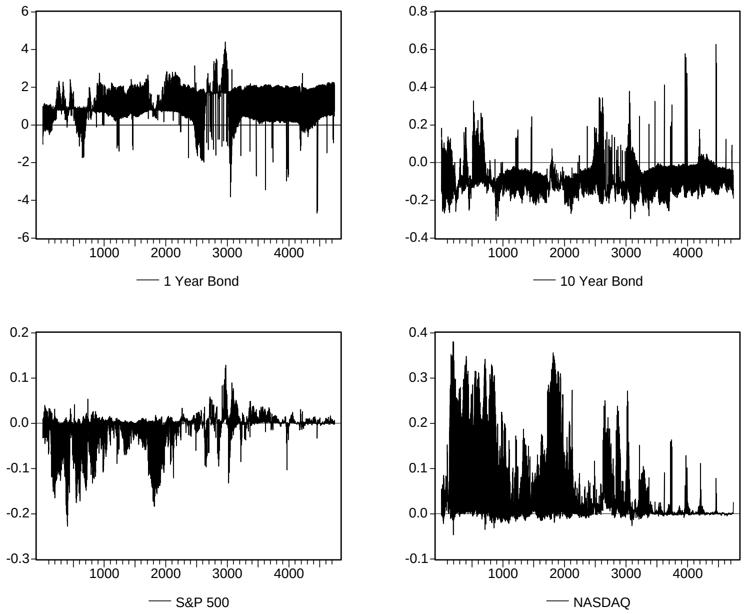

Figure 10: Optimal Portfolio Weights for Investors with Relative Risk Aversion of 10, using the Asymmetric Diagonal VECH Model

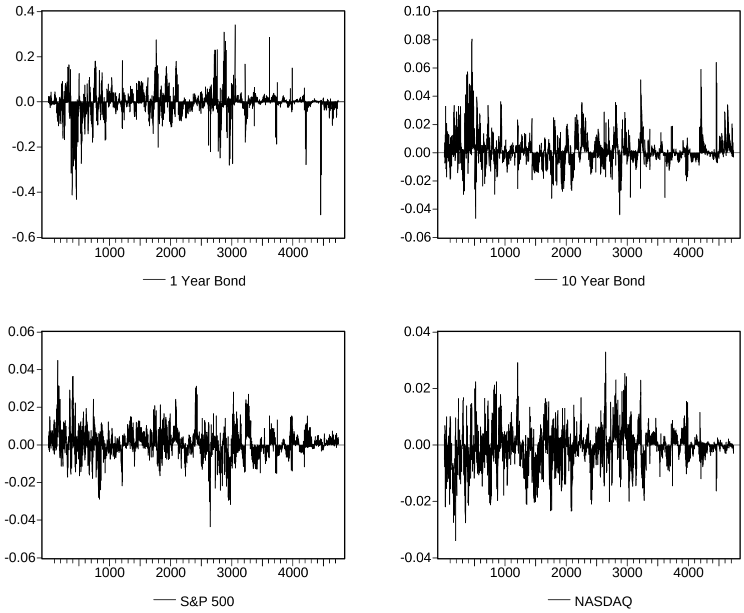

Figure 11: Changes in Optimal Portfolio Weights for Investors with Relative Risk Aversion of 10, using the Asymmetric instead of the Symmetric Diagonal VECH Model