Tilburg University

Stability, governance and effectiveness

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Publication date:
2006

Link to publication in Tilburg University Research Portal

Citation for published version (APA):
Lazarova, E. A. (2006). *Stability, governance and effectiveness: Essays on the service economy*. CentER, Center for Economic Research.

STABILITY, GOVERNANCE AND EFFECTIVENESS:

ESSAYS ON THE SERVICE ECONOMY

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P

ter verkrijging van de graad van doctor aan de Universiteit van Tilburg, op gezag van de rector magnificus, prof. dr. F.A. van der Duyn Schouten, in het openbaar te verdedigen ten overstaan van een door het college voor promoties aangewezen commissie in de aula van de Universiteit op woensdag 20 december 2006 om 10.15 uur door

E A L

geboren op 23 februari 1978 te Minsk, Wit-Rusland.

P : prof.dr. P. H. M. Ruys

prof.dr. P. E. M. Borm

prof.dr. R. P. Gilles

A

> I am not young enough to know everything.
>
> James Matthew Barrie (1860–1937)

Working on my PhD thesis was an experience that I could only compare to the process of growing up. In this process my supervisors had a fundamental role. I would like to thank Pieter Ruys, Peter Borm, and Rob Gilles for their guidance, time, and effort during our meetings.

During our first meetings with Pieter he often warned me that the theme I had taken up was a risky investment. I must admit I did not quite understand why and I was bravely claiming: ”Well, I am here to try.” Indeed, there have been difficult times. I would like to thank Pieter for continuously insisting that I look for some economic intuition in my work during those times. I would also like to thank Pieter and his wife Ireen for their kindness, understanding, and support during my stay in Tilburg and in particular during my prolonged illness in my second year.

Entering Peter’s office with the intention to seek help was, I must admit, not an easy decision. I remembered his presentation during the introduction of the research groups, where he introduced the Game Theory group as having one unifying property– all the researchers have a solid mathematical background. Needless to say, I could not meet the high requirements. I would like to thank Peter for agreeing to work with me in the first place and for his enormous patience during our work. I would like to thank him for the numerous productive meetings, numerous suggestions, numerous corrections, and not the least for the re-occurring question: ”Do we have some good news today?” when I was entering his office in total despair. I would like to thank him for the many lessons that I learnt and the directions he has given me.

The workshop on network formation that Rob gave in Tilburg was the beginning of my interest in this literature. The idea which provided some safe haven to the risky assets we were holding in our research basket with Pieter was indeed drafted at the last minutes of this workshop. I would like to thank Rob for introducing me to the literature on network formation and for working with me. I would like to thank him for his critique and meticulous scrutiny of my ideas. I would also like to thank Rob for the conversations over lunch during his visits to Tilburg where apart from the research agenda we also discussed politics, economics, and social peculiarities.

Besides being my supervisors, Pieter, Peter, and Rob were also my co-authors and I learnt a lot by working with them. I am also thankful to my other co-authors whom I had the chance to meet in Tilburg. I would like to thank Bas van Velzen for keeping my work parsimonious and precise. I am grateful to Maria Montero for sharing her experience, her enthusiastic collaboration, and for her encouragement during my job search. Hans Reijnierse's expertise in game theory as well as his extraordinary humanity were a great lesson to me. I would like to thank Hans for agreeing to work with me. Together with Ilaria Mosca we shared the process of learning. Ilaria gave me the courage to embark on the econometrics projects by agreeing to share the risks and the joys.

Together with Pieter, Peter, Rob, and Maria, René van den Brink, Ruud Hendrickx and Arthur van Soest agreed to join the committee. I would like to thank them for this, and for reading and discussing my work.

I would also like to thank Corrado Di Maria, Willemien Kets, Arthur van Soest, and Gema Zamarro for their constructive comments and suggestions at various stages of my work during the years.

I am grateful to Annemiek Dankers, Heidi Ket, and Jolande Peeters from the Department of Econometrics and Operations Research, and Ank de Vries-Habraken and Jolanda Schellekens-Bakhuis from CentER for their administrative support.

Apart from working on my thesis, I learnt a lot from the people with whom I spent the last four years in Tilburg. I am thankful to my office-mates Arantza Estevez and Chendi Zhang who were both very kind and encouraging people. It was easy for me to go through the rainy days in the Netherlands with such sunny people in the office. With Marina Velikova we shared not only the Bulgarian language, but a great deal of laughter, talking, and dancing. I cherished her integrity, strength, and smile. Andrei Vasnev had a special place as my yoga teacher and as a very good friend. Speaking to Mohammed Ibrahim was always bringing peace to my mind and I am especially thankful to him for his support during my illness. Amar Sahoo, Andrea Krajina, Cristina Majo, Edwin van der Werf, Gema Zamarro, Jetske Bouma, Jutta Hartmann, Martin and Marta Kahanec, Norbert and Krisztina Hari, Vera Hegedus, and Willemien Kets were all very special to me and each contributed a verse in the midst of the daily routine.

Here, I would also like to thank Boryana Inkova and Koen Giesen who welcomed me in their home during my first weeks in the Netherland and during my stay in Nijmegen for treatment. To my friends from outside of Tilburg: Dmytro Babik, Elisa Galeotti, Nadya Dimitrova, Katrin Surolejska, Violina Georgieva, and Vladislav Bonev, I am grateful for their care and encouragement.

Deep is my gratitude to my family: скъпа мамичко, ти изстрада всяка минутка заедно с мен; татенце, ти ми даваше кураж; мила моя Аничка, ти винаги знаеше кога да се обадиш и как да попиташ. Вие бяхте до мен по пътя, безкрайно съм ви благодарна, че сте до мен и в деня на заштитата. Моето семейство е моята сила.

Together with my family, I would like to thank Corrado who became more than a friend to me: tu mi sei stato vicino in ogni momento di questo cammino e senza di te, tu lo sai, questo giorno non sarebbe arrivato. Ma e' per il modo in cui risponderesti alla domanda: "Ogni cosa va bene per scrivere libri, ma sei capace di dizzare le tue orecchie?" (James Matthew Barrie 1860–1937) che io ti ringrazio: grazie per aver portato gioia nei miei giorni.

CONTENTS

I

> […] in the universe of not-understood phenomena, service activities form one continent, only slightly explored from different angles.
>
> Sven Illeris (1996) pp.8

The etymology of the word *service* is from the Latin *servitium* meaning "condition of a slave, body of slaves". As early as the XIV century, the verb "serve" was recorded in the expression "to attend a (customer)".[1] Through the centuries the noun "service" became used to signify an ever widening range of notions. The Merriam-Webster online dictionary[2] presents 11 entries for the noun "service". Among these, the entries more closely related to services as economic activities are:

(i) the occupation or function of serving;

(ii) contribution to the welfare of others;

(iii) the act of serving as a useful labor that does not produce a tangible commodity;

(iv) a facility supplying some public demand;

(v) a facility providing maintenance and repair.

Notably, these notions refer to services as immaterial products that generate value as well as to physical objects used to facilitate the generation of some immaterial object of mainly *subjective* value, *e.g.*, public telephone boots, parks, hospitals, watch repairs. It might be due to the wide range of activities that fall under the category of "services", that economists initially defined economic services as the "tertiary" or "residual" sector in the economy, see Fuchs (1968). As Illeris (1996) discusses in his

[1] See the Online Etymology Dictionary at `http://www.etymonline.com`.
[2] See `www.m-w.com`.

book, several authors criticize this definition because it brings together very heterogeneous activities. Attempts have also been made to provide an alternative definition of economic services based on their unifying characteristics. In this respect the discussion by Hill (1977) takes a prominent place in the literature. According to him,

> A service is a change in the condition of an economic unit, which results from the activity of another economic unit.

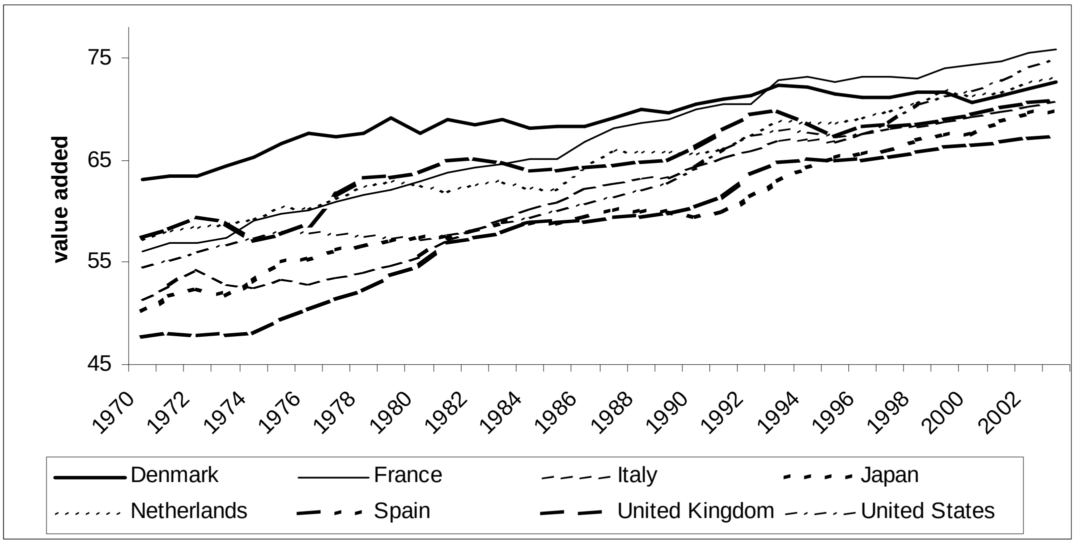
Economic services are hence regarded as *relational activities* of one economic unit with another such that the former one 'serves' the latter. Often the 'production' and the 'consumption' processes are one and the same. This property of economic services is called by Illeris (1996) "*uno actu principle*". This is most evident in education, where the transfer of knowledge is contingent on the characteristics of both teacher and student. Furthermore, as Hill (1977) underlies the fact that in (physically) changing its condition, the "good (person) does not lose its identity". This property is what distinguishes production of goods from services that affect goods such as repairs. It also implies that usually the anonymity assumption between the "producer" and "consumer" valid for commodities does not hold for services. Furthermore, it follows that the "output" of a service is difficult to quantify and standardize as it is unique to the pair of economic units involved. As a result, the decision to be involved in a service activity is almost always a decision under uncertainty. Last, as Illeris (1996) point out, the effect of a service performed may be irreversible, or it may have *long-term effects* such as the performance of surgery or receiving education. Hence, unlike in the analysis of commodities, in the analysis of economic services one cannot assume free-disposal.

The above properties call for a methodologically distinct framework of economic services relative to consumption goods. However, mainstream economic theory seems to be uninterested in developing such framework, and, if anything, treats economic services as "immaterial goods".[3] As Fuchs (1968) put it, economic services were treated as the "stepchild of economic research".

One reason for this attitude might be that the labor involved in the production of services has been long ago characterized as "unproductive labor" by classical economists starting from Adam Smith.[4] Though, it has also been acknowledged, *e.g.*, Hill (1977), that some services may lead to an increase in labor productivity in the long run. For example, health care services allow individuals to work longer, while education improves their skills.

[3] See Hill (1977) for a discussion.
[4] See Smith (1937).

Figure 1: Value Added in Services as a Percentage of Total Value Added



Source: OECD

Furthermore, since the service sector is very labor intensive, it does not exhibit economies of scale, and it is less likely to benefit from increase in productivity due to innovation. This led the famous neoclassical economist William Baumol to claim that service activities cause a 'cost-disease' that would lead to economic stagnation, *e.g.*, Baumol and Bowen (1966), Baumol (1967). The reason for this is that the costs in this sector will grow faster than the real output due to the differences in productivity growth between the service and non-service sectors.

To this, a number of authors[5] respond that the productivity growth in the manufacturing sector is in fact possible due to existence of relational activities in the service sectors. Hence, the higher costs in the service sector should also be taken to reflect the additional benefits in terms of productivity growth in the non-service sectors.

Indeed, one would expect an unproductive activity to be driven to a halt, while the growth of the service sector is noticeable. As it is shown on Figure 1, the value added in the service sector[6] has reached about $\frac{2}{3}$ of the total value added in several industrialized countries representative of the OECD members. These statistics alone justify our interest in further investigating issues peculiar to services.

[5]For an extensive discussion and reference list see Illeris (1996).

[6]The data combines the figures for value added in transport, trade, hotels and restaurants, for value added in banks, insurance, real estate and other business services, and value added in Government, health, education and other personal services as a percentage of total value added for each country.

The issues studied in the essays that make up this thesis are multifaceted. The main themes of stability, governance, and effectiveness are recurrent in different contexts: relational activities, group cooperation, country-level management and a very specific case of health care provision. The methodological tools employed are varied, too. In the theoretical essays contributions are made to the literature on network and endogenous coalition formation. The empirical essays employ linear and non-linear regression models and use aggregate as well as micro-level data.

Different definitions pertain to the notion of *stability* in the essays. The underlying concept, however, is the same: it is an expression of equilibrium that allows us to make theoretical predictions with respect to emerging outcomes. Another unifying feature is that we are studying stable outcomes against one-player deviations (as opposed to groups deviations).

Governance is a complex notion in itself. Ruys (2006) defines the term governance of an organization as

> [...] the distribution and exercise of authority of bodies and institutions within and outside an organization, aiming at realizing the mission of the organization and its related values and services. This includes the transactions between the competent parties and the way the exercise of power is balanced and monitored by these parties.

Governance thus signifies the various rules governing the value generation processes discussed in the essays. In Chapter 1 labor specialization, that is the dichotomy in roles, and in Chapter 2 authority emerge as necessary components of governance for the existence of stable productive activities. In Chapter 3, governance takes the form of contractual arrangements and in Chapter 4 this is a bargaining process. In Chapter 5 we use the definition of governance of the World Bank which more specifically refers to the quality of institutions at a national level, to investigate the governance effect on life expectancy.

Effectiveness, on the other hand is a concept that is relevant only for the last two empirical essays in the thesis. In the measure of governance used in Chapter 5, the notion of effectiveness in the functioning of institutions is crucial. In Chapter 6 the specific case of treatment of patients with acid-suppressing drugs is studied. There, effectiveness refers to the ability of the health care system to bring about high healing rates at minimal costs.

The thesis is organized in three parts. The first part, entitled “Stability and Social Recognition” was inspired by the desire to explore the possibilities of a methodological framework in which “relational activities” can be studied taking into account that each activity is peculiar to the economic agents participating in it. Conceptually, our stepping stone is the research program of Professor Xiaokai Yang, which was seminally developed in Yang (1988) and subsequently brought to fruition in numerous research papers.[7] The core of this research program is the application of an inframarginal analysis to the decision model underlying a consumer-producer, within a system of perfectly competitive market. In turn, this approach is used to model the Smith-Young approach to the relationship of specialization, the social division of labor, and increasing returns to scale, Smith (1776), Young (1928), and Stigler (1951), and collective production, Yang (2003). We study relational activities in general — and consumer-producer entities in particular — in a *pre-market* setting. In doing so, we are able to show that the emergence of labor specialization and socially recognized authority is not contingent on the functioning of competitive markets and an existing price mechanism. These phenomena, instead, are linked to the viability of productive systems. It should be noted that the general framework is designed to investigate economic services as relational activities. The applications we develop are based on commodity exchange and production. Since these activities are investigated in a pre-market setting, however, we are able to capture their relational aspects. Furthermore, in Chapter 1, we are able to outline the transition from subjective exchange to objective trade that paves the path to the emergence of markets.

In Chapter 1, we study production processes carried out by matchings, *i.e.*, a relational activity between two individuals. Within this non-market environment, we discuss the emergence of economic specialization and ultimately of economic trade and a social division of labor. We base our approach on three stages in organizational development: the presence of a stable relational structure; the presence of relational trust and subjective specialization; and, finally, the emergence of objective specialization through the social recognition of subjectively defined economic roles.

In Chapter 2, we extend our notion of production processes to include such carried out by teams, *i.e.*, relational activities between several individuals, organized in a primitive firm. We show that the presence of a socially recognized authority ensures the formation of productive teams.

In terms of the methodological tools, these chapters build upon the fast growing network formation literature.[8] The approach used here is one of non-cooperative link formation. In Chapter 1 we use the pairwise stability concept developed by Jackson

[7] We refer to Yang (2001, 2003) and Cheng and Yang (2004) for a comprehensive review of the work that has been accomplished in this research program.

[8] See Dutta and Jackson (2003) for a collection of essays in wide ranging topics applying cooperative and non-cooperative approaches.

and Wolinsky (1996) to conduct the analysis. In Chapter 2, we modify their stability concept to allow for greater deviational possibilities of star central players. The common underlying question that is investigated is under what conditions on the potential network structure, there emerges a stable network of particular form: in Chapter 1, this is a network of pairs and in Chapter 2, this is a network of stars. Technically, the question is reminiscent to the one studied by Pápai (2004). Pápai (2004) identifies necessary and sufficient conditions on the permissible coalition structure in a hedonic coalition formation model that ensure existence of stable coalition structures for any preference profile. Our work differs from hers in two respects. First, our analysis is based on networks. Second, we are investigating stability of particular network patterns, as discussed above. This allows us to relax some of the conditions identified by Pápai (2004) that ensure stability of the network of pairs in Chapter 1. In Chapter 2, the fact that we employ a network approach, allows us to study patterns that do not have a direct analogue in coalition formation models, since coalition formation models cannot discriminate between a coalition of, say, three players in which all three players are connected and such in which one player acts an intermediary for the other two who are not connected.

The second part, "Stability and Endogenous Coalition Formation" consists of two game theoretical essays, in which we develop new stability concepts relevant to environments of cooperation. Indeed, cooperative game theory is another fruitful framework to study services, as it allows the analysis of the service output as a value produced by the cooperation among the participating economic units, *i.e.*, to put into practice the "uno actu principle" discussed earlier. In these two essays, there is a common underlying question: when a set of individuals are faced with a cooperative situation, which groups will form, and how will members of the group split the proceeds. Aumann and Dreze (1974) brought up this question, recognizing that answering it would be equivalent to performing a *general equilibrium* type of analysis in a game theoretical framework. Almost 20 years later, Maschler (1992) claimed that in his opinion this question had not been answered satisfactory in the literature. Instead, researchers have performed either one of two types of partial equilibrium analysis: given a fixed preference profile, they study the formation of coalitions, *i.e.*, the recent hedonic coalition formation literature, *e.g.* Banerjee, Konishi and Sonmez (2001), Bogomolnaia and Jackson (2004), Pápai (2004); or given a fixed organization structure, they study the way total coalition value is allocated among the members, *i.e.*, cooperative game theory literature. In the two essays, new solution concepts are introduced that are capable of answering the simultaneous question.

In Chapter 3, we revisit the work of Dreze and Greenberg (1980) in which endogenous coalition formation problems are studied based on three stability notions that reflect three different contractual arrangements with respect to one-person deviations. In particular we modify these definitions in such a way that individual rationality is implied by individual stability. Contrary to Dreze and Greenberg's (1980) claim, we show that contractually stable outcomes exist in any coalitional game. We, furthermore, show that any coalition structure of maximum social worth is both contractually and compensation stable. Applying the general framework to an example of mutual insurance in agricultural production, we find that, in each type of contractual setting, there are stable individually rational pooling outcomes while, on the contrary, individually rational separating outcomes are not stable.

In Chapter 4, we discuss stability notions based on bargaining. This analysis is applicable to situations in which no binding contracts are possible such as when either the effort of an individual, or the outcome of cooperation is not observable and verifiable. Here, we offer a new solution concept and we discuss its relation with existing bargaining sets such as the Maschler bargaining set, developed by Aumann and Maschler (1964) and the Zhou bargaining set, developed by Zhou (1994). The novelty in our solution concepts is that it explicitly takes into account differences of deviation possibilities within an already formed group and outside the group. Such distinction is necessitated by existing legal restrictions, physical restrictions, asymmetric information availability, and other phenomena that give rise to different transaction costs within and outside an organization entity. We illustrate these concepts by applying them to weighted majority games, used in modeling voting situations, and to a new class of coalitional games called cooperation games applicable to discussions of organization of services producing units such as medical centers, research teams, etc.

The third part, "Governance and Effectiveness", consists of two empirical works, which aim at gathering evidence on the functioning of services and its real life implications.

In Chapter 5, we use a cross-country comparison to investigate the impact of national-level governance on socio-economic development. As a measure of socio-economic development we take life expectancy, which as argued by Amartya Sen is a variable that better reflects social welfare, compared to monetary variables such as Gross Domestic Product.[9] The point of departure of the econometric analysis is the seminal work of Rodgers (1979) on the absolute and relative income hypotheses. We find that substituting the governance index for the Gini index of income inequality

[9] See Sen (1981), Sen (1987).

is statistically the preferred regression model. Our findings lend support to the argument that governance matters. Further investigation provides evidence for two types of threshold effects: in terms of both absolute income and governance. For those countries below a threshold, absolute income is the most significant determinant of life expectancy, while for those above it, governance matters the most. The regression analyses are conducted on a sample of 112 states, which is representative of a wide range of absolute income and governance levels. It employs Ordinary Least Squares methods.

In Chapter 6, the focus is on a specific case in health care provision, that is the effective treatment of patients with acid-suppressing drugs. Stomach related diseases, such as Gastroesophageal Reflux Disease, H. Pylori, and Non-Steroidal Anti-Inflammatory Drugs-induced gastropathy are usually treated with acid-suppressing drugs. These diseases can be treated while being in acute form, or, if not detected on time, in their chronic form. Chronic illnesses, however, have higher burden on the health care system and lead to reduced quality in life for the patients. There are two types of prescription acid-suppressing drugs: H2 blockers (H2B) and Proton Pump Inhibitors (PPI) and we investigate their usage in the health care system. Clinical trials suggest that PPI are more effective in both healing and reducing the symptomatic levels.[10] However, in practice H2B' are also widely used due to its lower costs. For our analysis, we use administrative data provided by a Dutch health insurance group. The Dutch case is interesting to study because General Practitioners (GP) are encouraged to prescribe H2B to patients with first-time complaints. However, this may not be a cost-effective treatment in practice because the GP have imperfect knowledge of the symptomatic history of the patient, hence, might treat patient with the cheaper but less effective drug while the more effective drug could have prevented the transition to a chronic disease.

Methodologically, we employ a binary choice model for the probability that a patient a hospital and a duration model with time varying regressors to analyse the time before a patient enters the hospital. The estimates show that there are patients who had they been treated with PPI drug they would have had a lower probability of hospitalization. The interpretation of these estimates heavily rely on the validity of the assumptions of the regression models.

[10] As reported by Sridhar, Huang, O'Brien and Hunt (1996), Briggs, Goeree, Blackhouse and O'Brien (2002), Vanderhoff and Tahbour (2002), van Pinxteren, Numans, Bonis and Lau (2004).

Part I

Stability and Social Recognition

CHAPTER

ONE

S , S S R :
E P -M S [*]

1.1 On Specialization, Institutions and Social Organization

Smith (1776) argued in his seminal work *Wealth of Nations* that the social division of labor is limited by the extent of the market so that the benefits of specialization to an individual are determined largely by the existing social division of labor in the economy. (This is also known as the *Smithian Theorem*.) Young (1928) extended this into a synergetic argument that the extent of the market also depends upon the level of social division of labor. Thus, the presence of increasing returns to scale leads to specialization and further social division of labor. In turn, a high level of social division of labor leads to increasing economies of specialization that form further incentives to specialize and deepen the social division of labor.

In the present chapter we intend to sketch an argument that extends the Smithian theorem beyond the setting of a competitive market economy based on a system of perfectly competitive markets. Our argument is that the Smith-Young mechanism also applies to social organizations and institutional settings other than that of a system of perfectly competitive markets.

Indeed, we argue that the process of specialization occurs at different levels of embeddedness of the individual consumer-producer and that only at its most advanced state—namely that of objective specialization—this process results into a social di-

[*] This chapter is based to a great extent on Gilles, Lazarova and Ruys (2006).

vision of labor. Thus, a social division of labor can indeed exist and generate economic development and growth in the context of more primitive economic institutions and systems of imperfectly competitive markets. This development mechanism is *not* based on the endogenous selection of a specialization by an individual based on the prevailing market prices; instead, each individual selects from a given set of complementary social economic roles, each corresponding to some specialism. Each of these social economic roles is collectively recognized as such and, regarding each of these social roles, there is a common knowledge.

Yang and Borland (1991) already showed that the Smith-Young mechanism functions as a determining factor in economic growth. Indeed, the mechanism of ever-deepening economic specialization and the accompanying development of the social division of labor leads to significant growth. In economic history and the new institutional economics this has been accepted as the main engine behind the rise of the western economies. (North and Thomas 1973, North 1990, Greif 1994, North 2005)

Recently, Ogilvie (2004), Acemoglu, Johnson and Robinson (2005) and Greif (2006) have extended this argument and pointed to economic organizations other than the perfectly competitive market in which the Smith-Young mechanism causes economic development and growth. Acemoglu et al. (2005) mainly point to the development of property rights and the underlying political institutions as causes of economic growth. Empirical evidence of past performance of western economies back up these arguments.

Our focus is on a rather primitive economy: economic agents directly interact with each other without reference to a central organization such as a system of competitive markets. Instead, individual economic agents engage in binary, value-generating relationships—to which we refer as *matchings*. Matchings have to be understood as binary productive engagements, which are not necessarily trade relationships. It is assumed in this very primitive economy that every individual activates exactly one value-generating matching.

Our theory is developed along two different lines of thought. The first line is that of a formal theory in which we develop precise mathematical definitions and show two main theorems. Our notion of equilibrium uses pair-wise stability developed by Jackson and Wolinsky (1996). The first theorem gives conditions under which equilibrium in a specific matching economy can be sustained; the second theorem gives a generic existence result that supports the emergence of a social division of labor. The result in the second theorem is closely related to the result obtained by Pápai (2004), however, the conditions that we provide are less restrictive since the focus of attention is coalitions of two players[1] and not coalitions of arbitrary number of players.

The second line of thought develops an application of our theory to a specific case to illustrate the notions of subjective and objective specialization. Our main argument is that there are two different types of stability possible within a matching economy.

Subjective stability: Individuals engage in binary value-generating relationships, and stability is attained if individuals are not willing to become autarkic or switch partners for higher benefits. The presence of stability is thus "subjective" in the sense that it is completely based on the properties of the productive abilities and utility functions of the individuals in the economy.

If a state of subjective stability is attained in the economy, the individuals might develop mutually beneficial trade within the relationship that they are engaged in. Moreover, individuals might specialize their productive activities within the (subjective) setting of the matching that they are engaged in. This is called *subjective* specialization since it is founded on the specific properties of the matching in which they generate their utilities.

We emphasize that subjective specialization does *not* induce a social division of labor since individuals are not engaged at a higher social plane; their economic interaction is explicitly limited to be within their matchings only. In that regard the organization of the economy remains scattered and there are no widespread gains from trade.

Generic stability: Only if generic stability is possible, economic agents can truly specialize in an objective fashion and there emerges a social division of labor. A matching economy attains generic stability if for *every profile* of utility functions and production sets, there exists a stable matching pattern. Our main theorem states that such generic stability is attained if there is a social organization of the economy based on at least two socially recognized roles. Hence, there exist at least two complementary socio-economic roles such that value-generating relationships solely exist between individuals with different social roles. Hence, only after complementary social roles are established, a true endogenous social division of labor can emerge in which individuals specialize in these roles.

Our main existence theorem on generic stability thus identifies that a binary social division of labor is a pre-requisite for stability. This amends the Smithian theorem in the sense that there has to exist a finite set of socio-economic roles

[1]A matching can be treated as coalition structure that contains coalitions of two members.

into which individuals can specialize, to establish stability in the social organization of the economy. The emergence of a set of socially recognized roles is, thus, a necessary condition for stability in the economy.

Although our model of a matching economy describes a very primitive society, we believe that it makes possible some deep conclusions. Our approach also resolves the indeterminacy problem identified by Gilles and Diamantaras (2005). They argued that the theory of the Smith-Young development mechanism is founded on a circular argument: prices of traded goods determine individuals' specialization and, thus, prices determine the social division of labor. This, in turn, determines which goods are produced and traded, thus determining the extent of the market. This brings up the question who or what ultimately determines which goods are traded and how economic development is accomplished.

In our current model we put this determinacy problem at the center of our analysis. Indeed, our main result states that generic stability requires the existence of a certain set of established social roles from which individuals can choose when they specialize. Each social role stands for a certain social-economic specialization and in equilibrium the number of agents of each role is balanced. Only then an effective social division of labor emerges and the society can engage into an effective process of economic development and growth. Ultimately this development is founded on the enhancement and extension of the commonly known set of economic roles.

Ultimately we conclude that economic development and growth is caused by organizational and institutional change (Acemoglu et al. 2005), rather than technical change only (Romer 1986, Romer 1990). We believe that technical change is a consequence and expression of the effectiveness of the social organization of the economy.

In Section 2 we provide some technical definitions. In Section 3 we present our framework of a matching economy based on binary value-generating activities among economic agents, we define stability as our main equilibrium notion and develop the application to a primitive hunter-gatherer economy. Section 4 discusses the existence of stable matching patterns and the emergence of subjective specialization. In Section 5 we introduce generic stability and the possibility of objective specialization. This in turn implies the emergence of a social division of labor in such a matching economy. We summarize our main line of thought in Section 6.

1.2 Technical Preliminaries

Let $N = \{1, ..., n\}$ be a finite set of *individuals*. At this stage we do not make any assumptions about these individuals regarding their individual abilities. Hence, in this general model we do not explicitly assume that these individuals are consumer-producers or that they are even able to specialize in any form.

Instead we endow these individuals with the abilities to engage into relational economic activities that generate economic values or wealth.[2] Therefore, these individuals are assumed to have *relational* abilities. (These relational abilities have to be understood as special forms of more generalized social-economic abilities.) These relational abilities in turn might be based on individualistic abilities; this approach is explored in some examples throughout this chapter. Note that we do not assume or impose that these relational activities take place in the context of a market. Instead we assume that these relational abilities describe the economy itself.

Formally, we let the set $\Gamma \subset \{ij \mid i, j \in N\}$ be a set of potential relational activities between the individuals in N. Here, for two distinct individuals $i \in N$ and $j \in N$ with $i \neq j$ we define by $ij \in \Gamma$ that these individuals i and j are able to engage in a "value-generating relational activity". We indicate this potential relational engagement $ij \in \Gamma$ as a *potential matching* of i and j. This is formalized as follows.

Definition 1.2.1 *A* ***set of potential matchings*** *on the set of individuals N is given as $\Gamma \subset \{ij \mid i, j \in N\}$ such that*

(i) for every individual $i \in N$: $ii \in \Gamma$ and

(ii) for every individual $i \in N$ there exists some $j \in N$ with $j \neq i$ and $ij \in \Gamma$.

Every relationship ij in the set Γ on N is denoted as a ***set of potential matchings****.*

We emphasize that any potential matching is symmetric in the sense that a matching between individuals i and j is exactly the same matching as the one between individuals j and i. On the other hand, individuals i and j need not have the same utility from this potential matching, as it will become evident later.

It is also possible that an individual $i \in N$ does not engage in an economic activity with any of the other economic individuals. In this regard i attains a *relationally*

[2] The most primitive form of a matching is that of cooperation in some production activities. More advanced forms include the simple exchange or *trade* of two commodities. The gains from trade then form the values that are generated between the two traders.

autarkic position.[3] Mathematically this is represented by the pairing of i with himself, *i.e.*, by the matching ii. The definition of the set of potential matchings Γ assumes that each player $i \in N$ has the possibility to exclude himself from the relational activities in this economy and assume a relationally autarkic position, indicated by $ii \in \Gamma$. We define

$$\Gamma_0 = \{ii \mid i \in N\} \subset \Gamma$$

as the collection of relationally autarkic positions.

Another interpretation is that the set of potential matchings Γ represents the social capital that is present within the population N. It describes what is the potential set of matching partners for each individual, *i.e.*, the complete description of her potential social interactions. Some of these potential interactions may generate positive utilities and others negative. Most importantly, it is assumed that no two individuals i and j with $ij \notin \Gamma$ can even engage in an economic value-generating relation. This indeed corresponds to the notion of social capital as used in the social sciences. (Portes 1998, Putnam 2000, Dasgupta 2005)

The relative position of an individual in Γ defines his matching possibility set as it will become clear in the analysis. The *set of connected players* in a set of potential matchings Γ is given by $N(\Gamma) = \{i \in N \mid \text{ there exists } j \in N \text{ with } j \neq i \text{ such that } ij \in \Gamma\}$. By definition since Γ is a set of potential matchings $N(\Gamma) = N$. For every individual $i \in N$, we introduce i's *neighborhood* in Γ as the set of individuals who can be partners of player i in potential matchings, *i.e.*,

$$N_i(\Gamma) = \{j \in N \mid ij \in \Gamma \text{ with } \; i \neq j\}.$$

The set of potential matchings that individual i can engage in, can now be formulated as

$$L_i(\Gamma) = \{ij \in \Gamma \mid j \in N_i(\Gamma)\}.$$

Let $m \in \mathbb{N}$. A *path* between individuals i and j in the set of potential matchings Γ is a sequence of distinct individuals $P(ij) = (i_1, i_2, \ldots, i_m)$ such that $i_1 = i$, $i_m = j$, $i_k \in N$ and $i_k i_{k+1} \in \Gamma$ for all $k \in \{1, \ldots, m-1\}$. The *length* of the path $P(ij)$ is said to be the number of links $m - 1$ that make up this path.

A *cycle* in the structure Γ is a sequence of distinct players $C = \{i_1, i_2, \ldots, i_m\}$ with $m \geqslant 4$ such that $i_1 = i_m$, $i_k \in N$ and $i_k i_{k+1} \in \Gamma$ for all $k \in \{1, \ldots, m-1\}$. Now the *length* of the cycle C is given as $m - 1$. Thus, a cycle is a path from an individual to

[3] Throughout the chapter we distinguish two types of autarky: relational autarky and exchange autarky. *Relational autarky* refers to the state of isolation of a player within the set of potential matchings Γ, while *exchange autarky* refers to a state of nonparticipation in any of the exchange processes in the economy. Obviously, relational autarky implies exchange autarky.

herself, which consists of at least three distinct players. We emphasize that each cycle has length of at least three, *i.e.*, a cycle consists of at least three distinct relations.

Definition 1.2.2 *We say that a sub-structure $\Omega \subset \Gamma$ of the set of potential matchings Γ on N is* ***odd acyclic*** *if Ω does not contain any cycle C of length $\ell \geqslant 3$ such that ℓ is an odd integer.*

Odd acyclicity turns out to be a crucial property in the further development of our theory.

1.3 A Matching Economy

The focus of our work is on relation activities in which each individual activates *exactly one* of her potential matchings. This fundamental hypothesis is founded on the fact that we model a very primitive economy without the presence of advanced economic or social institutions. In such a primitive economy it is natural to assume that individuals only interact with a single other individual at a time and that more complex interactions require more advanced social institutions than assumed within our context.

Such relational activities give rise to patterns of activated links in which each player activates only one link out of all her potential links. Such value generating activities, we call *matching patterns*.

Definition 1.3.1 *A* ***matching pattern*** *is a subset of the set of potential matchings $\pi \subset \Gamma$ such that every individual is either paired with exactly one other individual or remains relationally autarkic, i.e., $\pi \subset \Gamma$ is such that $|L_i(\pi)| = |N_i(\pi)| = 1$[4], for all $i \in N(\pi)$.*
We denote by $\Pi(\Gamma) = \Pi$ the class of all potential matching patterns within Γ.

To complete our model we assume that every individual $i \in N$ is endowed with complete and transitive preferences over the possible matching patterns $\pi \in \Pi(\Gamma)$ in which she can engage. Thus, by finiteness of Γ, these preferences can be represented by a *hedonic utility function* given by $u_i \colon L_i(\Gamma) \cup \{i, i\} \to \mathbb{R}$. Let $u = (u_1, \ldots, u_n)$ denote a profile of utility functions for every player $i \in N$ and let $\mathcal{U}$ be the set of all permissible profiles of hedonic utility functions representing complete and transitive preferences.

[4] *The convention in the network formation literature is to assume that the cardinality of the neighborhood of a player who is autarkic equals 1.*

Definition 1.3.2 *A **matching economy** is defined to be a triple $\mathbb{E} = (N, \Gamma, u)$ in which N is a finite set of individuals, Γ is a set of potential matchings on N, and $u \in \mathcal{U}$ is a profile of hedonic utility functions on Γ.*
*The pair (N, Γ) is also called the **matching structure** of the matching economy $\mathbb{E} = (N, \Gamma, u)$.*

A matching economy essentially is based on potential binary activities that generate economic values. For example, a trade economy can be represented as a matching economy between buyers and sellers who can trade physical goods to generate gains from trade. We emphasize here that a trade economy with two commodities—one *desirable* and *money*—imposes that the potential matching structure Γ is bipartite and that there are in fact two social types of individuals, namely buyers of the desirable and sellers of the desirable. This in turn implies that Γ is odd-acyclic. This imposes very strong properties on the matching economy as we explore in subsequent sections of this chapter.

In a matching pattern one and only one matching is selected and executed by each individual. For ease of notation we denote the indirect utility an individual i has when participating in a matching pattern π with $ij \in \pi$ for some $j \in N$ as $u_i(\pi)$, *i.e.*, $u_i(\pi) \equiv u_i(ij)$, for all $i \in N$. For a given matching pattern, the indirect utility level for players are given in a *utility profile* $u(\pi) = (u_1(\pi), \ldots, u_n(\pi)$. [5]

With the tools developed so far we are able to introduce two relational stability concepts. Again we let the matching economy $\mathbb{E} = (N, \Gamma, u)$ be given throughout. For matching pattern $\pi \in \Pi$, a potential matching $ij \in \Gamma \setminus \pi$ is a *blocking matching* if $u_i(ij) > u_i(\pi)$ as well as $u_j(ij) > u_j(\pi)$.

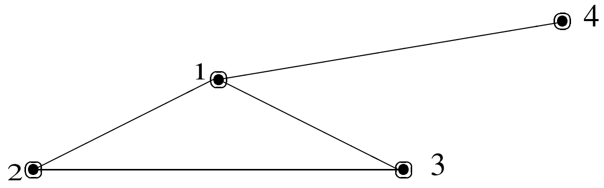
Having defined a blocking matching as a *strict* binary Pareto improvement, we follow the concepts used in the literature on matching (Roth and Sotomayor 1990). We point out that our notion of stability is closely related to that of stability in network formation (Jackson and Wolinsky 1996). With this concept we can define our stability property of a matching pattern.

Definition 1.3.3 *Let (N, Γ, u) be a matching economy. A matching pattern $\pi \in \Pi$ is **stable** if all matchings in π satisfy the **individual rationality** (IR) and **no blocking** (NB) conditions:*

IR $u_i(\pi) \geqslant u_i(ii)$ *for all $i \in N$,* *and*

[5] We emphasize that the hedonic utility profile considered here allows an individual to consider only one matching at a time, since we do not allow an individual to engage in multiple matchings at the same time.

Figure 1.1: The set of potential matchings G in Example 1.3.4.



NB *there is no blocking matching with regard to* π*, i.e., for all* $i, j \in N$*,* $i \neq j$*, with* $ij \in \Gamma \setminus \pi$*:*

$$u_i(ij) > u_i(\pi) \text{ implies that } u_j(ij) \leqslant u_j(\pi). \tag{1.1}$$

Stable matching patterns in $\mathbb{E}$ *are denoted by* $\pi \in \Pi^\star(N, \Gamma, u)$.

Condition (IR) is an individual rationality requirement, that states that an individual cannot be matched with another individual without her consent, *i.e.*, if an individual is better-off under relational autarky, she will pursue that.

In (NB) stands for a non-blocking condition requiring that a blocking matching does not exist with respect to matching pattern $\pi \in \Pi$. Under (NB) if an individual prefers to be matched with an alternative individual than the one with whom she is matched under matching pattern π, then that alternative individual does not agree to engage with her. This condition is closely related to the condition of link addition proofness in network formation. Link addition proofness is at the foundation of the notion of pairwise stability in network formation, seminally introduced by Jackson and Wolinsky (1996).

To illustrate our definition of stability, we discuss an abstract example.

Example 1.3.4 Consider an economy $\mathbb{E}_1 = (N, \Gamma, u)$ with $N = \{1, 2, 3, 4\}$, set of potential matchings $\Gamma = \{12, 23, 13, 14, 11, 22, 33, 44\}$, and the profile of utility functions u given in the table below.[6]

$j =$	1	2	3	4
$u_1(1j)$	0	1	2	3
$u_2(2j)$	1	0	2	–
$u_3(3j)$	2	-1	0	–
$u_4(4j)$	0	–	–	1

Given Γ we now derive the collection of all possible matching patterns, which is given by

$$\Pi = \{\{11, 22, 33, 44\}; \{11, 23, 44\}; \{12, 33, 44\}; \{13, 22, 44\}; \{23, 14\}\}.$$

[6]In this table a dash in a cell indicates that no potential matching between individuals i and j exists.

We now identify the stable matching patterns in this example. Let us start the discussion with individual 1. She prefers to be matched to individual 4 since her utility in this matching is the highest. However, individual 4 prefers to be by herself rather than to be matched with 1 ($u_4(14) < u_4(44)$). Hence a matching between individuals 1 and 4 violates the individual rationality condition for individual 4.
Excluding link 14, individual 1 prefers to be matched with individual 3. Since individual 1 is also individual 3's most preferred partner, a matching between them cannot be blocked by individual 2. Finally, individuals 2 and 4 do not have a potential matching, hence in the matching pattern they should be in a state of relational autarky.
Therefore, the unique stable matching pattern is given by $\pi^* = \{13, 22, 44\}$. ♦

Our main application of the general relational framework developed is that of a relational economy of consumer-producers. We follow the new classical framework developed in Yang (2001) and Yang (2003). The new classical approach is firmly founded on the premise that consumer-producers specialize within a social context of a structure of (market) interactions and, thus, attain higher welfare levels.

Here we start at an even more primitive level of reasoning. Before there is actual specialization, there are consumer-producers with simple *skills* on which these specializations can be based. We recognize that skills, unlike commodities, are intrinsic to a consumer-producer and cannot be exchanged. They can, however, be shared. Sharing one's skills with another individual is a process that does not make the giver any poorer in the skill.[7] As established by Yang and Borland (1991) and Yang (2003), learning-by-doing is an important mechanism in the process of growth. However, in Yang's framework this process is individual-specific, *i.e.*, economic individuals are not allowed to learn from each other. In our framework, we go beyond this restriction by allowing limited learning between individuals. When two individuals engage in a relational activity, they do not actually exchange consumption goods, as in the case of Yang; instead their learning externalities increase their productivity through the (limited) sharing of the skills accumulated by their partners.

These ideas are illustrated in Example 1.3.5 below. There is a finite set of consumer-producers. Each individual is endowed with one unit of productive time. There are two types of skills, hunting (H) and gathering (G), complementing the production of two types of consumption goods, meat and vegetables. When individuals are engaged in a matching they acquire also some of the skills acquired by their partner. Thus, there are relational externalities in the acquisition of skills.

[7] A commodity, in comparison, if shared makes the giver poorer in the possession of that commodity. This is to say that while commodities are pure private goods, skills are non-rival in nature.

Individuals principally engage in the individual accumulation of hunting and gathering skills. We also implement that they can decide to match with another individual and enjoy the relational externalities in the acquisition of skills with this other individual; skills are actually shared. This sharing is based on some learning process between the matched individuals. Within such a "sharing" matching, each individual produces meat and vegetables by hunting and gathering, respectively. Before making a decision to match, each individual can calculate the potential production output and the level of utility attainable in each potential matching.

At this point in the development of a society, it is *not* assumed that matched individuals actually engage in the exchange of the produced goods if this is beneficial for both parties. Instead they remain exchange autarkic[8] and only share their skills in the way described above.

Example 1.3.5 (A relational economy with consumer-producers)
Let $N = \{1, 2, 3\}$ be the set of three individuals. Each individual is endowed with one unit of time which she can use to acquire some amount of gathering skills G_i and some amount of hunting skills H_i. Skill acquisition is linear in time, *i.e.*, $G_i = l_i$ and $H_i = 1 - l_i$ where $l_i \in [0, 1]$ is the labor time used by individual i in acquiring gathering skills G_i. Each individual i is therefore endowed with a technology to produce two types of consumption goods: vegetables (the amount of vegetables is denoted by v) and meat (the amount of meat is denoted by m) by using some amounts of gathering skills G_i and hunting skills H_i, respectively.
Furthermore, the interaction between these individuals is introduced as a complementarity in skill acquisition; individuals can acquire some of the skills of their matching partner. This is described by two *learning parameters* $\alpha^i_{ij}, \beta^i_{ij} \in [0, 1]$, which are individual and pair specific. The parameters α (respectively β) describe the transfer of gathering (respectively hunting) skills from an individual's partner to that individual. The corresponding production functions in a matching between two individuals $i, j \in N$ are now introduced as

$$g_i(ij) = (G_i(1 + \alpha^i_{ij} G_j))^2 \quad \text{and}$$
$$h_i(ij) = (H_i(1 + \beta^i_{ij} H_j))^2, \quad \text{for all } i, j = 1, 2, 3.$$

In this example we assume that the learning parameters are given in the following table:

[8]As introduced before, we use the term "exchange autarkic" to express that an individual is self-sufficient without engaging in trade to obtain certain commodities.

ij	α^i_{ij}	α^j_{ij}	β^i_{ij}	β^j_{ij}
11	0	0	—	—
12	0.3	0.6	0.3	0.6
13	0.5	0.3	0.8	0.3
22	0	0	—	—
23	0.3	0.8	0.3	0.5
33	0	0	—	—

Individuals are endowed with homothetic preferences over the consumption of meat and vegetables given by

$$\phi_i(v_i\, m_i) = \sqrt{v_i\, m_i} \tag{1.2}$$

where v_i denotes the consumption of vegetables by individual i and m_i denotes the consumption of meat by individual i.

The optimal acquisition of skills

The optimal investment in hunting and gathering skill of each individual depends on the specialization decisions made by other individuals. First, we consider the case in which individuals maximize their utility in the relationally autarkic case.[9] The relationally autarkic utility maximization problem for all $i = 1, 2, 3$ is given by

$$\max_{0 \leqslant l_i \leqslant 1} \phi_i(v_i(l_i)\, m_i(l_i)) = \sqrt{v_i\, m_i}$$

subject to

$$v_i = g_i(ii) = (G_i)^2 = l_i^2$$
$$m_i = h_i(ii) = (H_i)^2 = (1 - l_i)^2.$$

The solution yields $l_i = \frac{1}{2}$ for all individuals $i = 1, 2, 3$. Hence, they invest equally in acquiring gathering and hunting skills, *i.e.*, $G_i = H_i = \frac{1}{2}$.

Second, given the externality parameters α and β, we can calculate the optimal investment of an individual in acquiring hunting and gathering skills *given the skill levels of her partner*. To take a generic case, let the partner j of individual i have acquired skill levels H_j and G_j respectively. Then the utility maximization problem of individual i is given by

$$\max_{0 \leqslant l_i \leqslant 1} \phi_i(v_i(l_i)\, m_i(l_i)) = \left[\, l_i(1 + \alpha^i_{ij} G_j)\,\right] \cdot \left[\,(1 - l_i)(1 + \beta^i_{ij} H_j)\,\right] \tag{1.3}$$

[9] This captures the extremely pessimistic case in which individuals believe that they cannot match to any other individual. This can also be considered to be the outcome of the maximization problem of extremely risk-averse individuals, or individuals who have very low degree of trust in the abilities of the other individuals.

Irrespective of the parameter values α^i_{ij} and β^i_{ij} and of the levels H_j and G_j, this reduces to the same optimization problem as under relational autarky. Thus, individuals remain exchange autarkic irrespective of the complementarities in the relationships with their partners. So, again the optimal investment in acquisition of skills is given by $l_i = \frac{1}{2}$ implying that $H_i = G_i = \frac{1}{2}$.

The resulting matching economy

Given the optimal acquisition of skills, we first compute the optimal production outputs for vegetables and meat for all potential relationships. Subsequently, we determine the resulting potential utility values.

In fact, given $H_i = G_i = \frac{1}{2}$ for all individuals $i \in N$, the potential production levels of meat and vegetables by each individual in each potential matching are now given by

ij	$g_i(ij)$	$h_i(ij)$	$g_j(ij)$	$h_j(ij)$
11	0.25	0.25	—	—
12	0.3306	0.3306	0.4225	0.4225
13	0.3906	0.49	0.3306	0.3306
22	0.25	0.25	—	—
23	0.3306	0.3306	0.49	0.3906
33	0.25	0.25	—	—

We emphasize again that, since all individuals remain exchange autarkic, no trade will ensue. Moreover, note that there is no mutually beneficial trade between any two individuals because in any pair one of the individuals has bigger quantities of both goods. In fact, we assume that all individuals believe that they will not engage in trade after creating a relationship with another individual.[10] Hence, we can derive the hedonic utility function based on the utility of consumption in a straightforward way, *e.g.*, $u_1(13) = \phi_1(13) = \sqrt{g_1(13) \cdot h_1(13)} = \sqrt{0.3906 \times 0.49} = 0.4375$. Similarly, the remainder of all utility levels are computed and presented in the table below.

j	1	2	3
$u_1(1j)$	0.25	0.3306	0.4375
$u_2(2j)$	0.4225	0.25	0.3306
$u_3(3j)$	0.3306	0.4375	0.25

.

The absence of stability

We claim that in the resulting matching economy, there does *not* exist a stable matching pattern. Hence, in this economy based on the acquisition of complementing skills,

[10] As argued in the introduction, trade can only emerge within stable relations. Thus, only within a stable matching pattern such trade can evolve. We also refer to Examples 1.4.7 and 1.4.8 for further details.

there does not exist a stable matching.
As the utility levels show, individual 1 prefers to be matched with individual 3 rather than with individual 2. Individual 2 prefers to be matched with individual 1 rather than with individual 3. Individual 3 prefers to be matched with individual 2 rather than with individual 1. Finally, all individuals prefer to be matched with a partner rather than to stay relationally autarkic. Hence, we conclude that there is no stable matching pattern. ♦

1.4 Existence of Stability and Subjective Specialization

In our previous discussion, we have shown that in a primitive economy with limited specialization, there might be no equilibrium emerging in the form of a stable matching pattern. Here we investigate sufficient conditions for the existence of stable matching patterns. We also discuss the implications of our findings with regard to specialization in a relational economy.

Below we define a specific subclass of matching patterns. A similar class of matchings has been defined by Sotomayor (1996) in her proof of existence of stable matching patterns in a bipartite matching economy. (Sotomayor refers to these patterns as "simple"; we do not adopt this terminology.)

Definition 1.4.1 *A matching pattern $\pi \in \Pi$ is* ***weakly stable*** *in $\mathbb{E} = (N, \Gamma, u)$ if for all individuals the Individually Rationality (IR) condition holds and whenever a blocking matching $ij \in \Gamma \setminus \pi$ exists, at least one of the partners in ij is relationally autarkic under π, i.e.,*

$$u_i(ij) > u_i(\pi) \text{ and } u_j(ij) > u_j(\pi) \text{ imply that } \{ii, jj\} \cap \pi \neq \varnothing. \tag{1.4}$$

We denote this as $\pi \in \Pi_w(N, \Gamma, u) = \Pi_w \subset \Pi$.

In a weakly stable matching pattern at least one of the partners in a blocking matching is autarkic, hence if we are to delete all the relationally autarkic individuals from such a pattern the remaining matchings will be stable. Further, note that the set of weakly stable matching patterns Π_w is non-empty as it contains at least the autarkic matching pattern $\Gamma_0 = \{ii \mid i \in N\} \subset \Gamma$. We use these properties of Π_w to show the existence of stable matching patterns.

We first establish the following trivial insight, which follows immediately from Definitions 1.3.3 and 1.4.1.

Lemma 1.4.2 *Every stable matching pattern is also weakly stable.*

Our analysis requires the introduction of some auxiliary notions. We define for any sub-collection of matching patterns $\Theta \subset \Pi$ its *cover* by

$$\overline{\Theta} = \left(\bigcup_{\pi \in \Theta} \pi \right) \setminus \Gamma_0. \tag{1.5}$$

where $\Gamma_0 = \{ii \mid i \in N\} \subset \Gamma$ denotes the set of relationally autarkic positions. Now (1.5) defines the cover of $\Pi_w(N, \Gamma, u)$ to be

$$\overline{\Pi}_w = \left[\left(\bigcup_{\pi \in \Pi_w} \pi \right) \setminus \Gamma_0 \right] \subset \Gamma. \tag{1.6}$$

Similar to the odd acyclicity property of the structure of potential matchings, we define odd acyclicity property of a matching economy:

Definition 1.4.3 *A matching economy $\mathbb{E} = (N, \Gamma, u)$ is* ***odd acyclic*** *if for the class of weakly stable matching patterns $\Pi_w(N, \Gamma, u)$ it holds that its cover $\overline{\Pi}_w \subset \Gamma$ defined in equation (1.6) is odd acyclic.*

We first show that it is possible that the class of all possible matching patterns Π is not odd acyclic—and, thus, $\overline{\Pi} \equiv \cup_{\pi \in \Pi} (\pi \setminus \Gamma_0) = \Gamma \setminus \Gamma_0$ contains an odd length cycle—while the economy $\mathbb{E}$ itself is odd acyclic.

Example 1.4.4 Consider the matching economy $\mathbb{E}_1$ given in Example 1.3.4. Now, the cover $\overline{\Pi}$ of the collection of possible matching patterns contains an odd length cycle between individuals 1, 2, and 3. Indeed, $\{12, 23, 31\} \subset \overline{\Pi} = \{12, 13, 14, 23\}$.
On the other hand, given the utility profile u, the set of weakly stable matching patterns Π_w is given by

$$\Pi_w = \{\{11, 22, 33, 44\}; \{12, 33, 44\}; \{13, 22, 44\}\}.$$

Now $\overline{\Pi}_w = \{12, 13\}$ and therefore it does not contain a cycle. Thus, the matching economy $\mathbb{E}_1$ is odd acyclic. ♦

Our main existence theorem states that stable matching patterns exist if the collection of weakly stable matching patterns satisfy the odd acyclicity condition. We refer to Chung (2000, Theorem 1) for a similar result for the case of a pure matching problem.[11]

[11] In his stability result Chung (2000) imposes the odd-acyclicity condition on the preference profile of the agents.

Theorem 1.4.5 *If a matching economy $\mathbb{E} = (N, \Gamma, u)$ is odd acyclic, then it holds that*

$$\Pi^{\star}(N, \Gamma, u) \neq \emptyset.$$

Proof. First, we consider the case that the cover of the collection of weakly stable matching patterns $\overline{\Pi}_w$ does not contain *any* cycle. Subsequently, we investigate the case that $\overline{\Pi}_w$ only contains cycles that have an even number of links.

A: $\overline{\Pi}_w$.

Assume that $\overline{\Pi}_w$ does not contain any cycle, and suppose that no stable matching pattern exists. Then for any weakly stable matching pattern $\pi \in \Pi_w$ there is a blocking matching. By the definition of a weakly stable matching pattern, in such a blocking matching at least one of the individuals is relationally autarkic under π. Hence, without loss of generality, we can take a weakly stable pattern $\pi \in \Pi_w$ for which ij is a blocking matching, $ii, jk \in \pi$, and there is a match of i with j leaving k alone and keeping all other matchings the same. Matching pattern π', obtained in this way, must be weakly stable, *i.e.*, $\pi' \in \Pi_w$ since there can be only one new blocking matching and it contains individual k, who is relationally autarkic under π'.

Since π' is not stable, individual k can form a blocking matching with another individual, say l, such that $lk \notin \pi'$. By forming the pair kl, a new matching pattern is formed $\pi'' = \pi' \cup \{kl\} \in \Pi_w$. Note that $l \neq i$ since Π_w does not contain a cycle. Now the matching pattern π'' can in turn be blocked by a matching ps where $ps \notin \pi''$. Thus, a new matching pattern $\pi''' = \pi'' \cup \{ps\} \in \Pi_w$ is generated where $p \neq j$, since $\overline{\Pi}_w$ does not contain a cycle.

Iterating a sequence of matching patterns $\pi^{(k)}$ with $k \in \mathbb{N}$ according to the construction outlined above, we reach a contradiction to the acyclicity due to the finiteness of the number of individuals.

B: $\overline{\Pi}_w$.

Next we assume that $\overline{\Pi}_w$ is odd acyclic. Let $\overline{\Pi}_w$ consist of a single cycle, *i.e.*, $\overline{\Pi}_w = (i_1 i_2, i_2 i_3, ..., i_{k-1} i_k)$ such that $i_k = i_1$, $k \geqslant 3$, and $k - 1$ is an even integer.

We consider two cases, distinguished by the preference profile of the individuals represented by the utility function. In the first case the proof of the existence of a stable matching pattern is reduced to the analysis of an acyclic cover of the collection of weakly stable matching patterns. In the second case we propose an algorithm and prove that it leads to identifying a stable matching pattern.

Case I: $\overline{\Pi}_w = \{i_1 i_2, i_2 i_3, ..., i_{k-1} i_k\}$, $i_k = i_1$, $k \geqslant 3$, and $k - 1$ is an even integer and let there be at least one pair $ij \in \overline{\Pi}_w$, such that individual i is in the set of most preferred partners of individual j and individual j is in the set of most preferred partners of in-

dividual i, *i.e.*, $j \in B_i(\Gamma) \equiv \{k \in N_i(\Gamma) \cup \{i\} \mid u_i(ik) \geqslant u_i(ih)$ for all $h \in N_i(\Gamma) \cup \{i\}\}$ and $i \in B_j(\Gamma)$. Note that individual i is not necessarily different from individual j. However, if $i = j$, then the set of individual i's most preferred partner must contain also his two neighbors along a cycle.
Then it follows that ij is an element of any stable matching pattern, otherwise it will form a blocking matching. Next consider the set of weakly stable matching patterns which does not contain the pair ij. Thus truncated, the cover of the class of weakly stable matching patterns, $\overline{\Pi}_w \setminus ij$, is acyclic and the existence of a stable matching pattern, $\pi^\star$, follows from the discussion of the first part of the proof.

Case II: Assume that $\overline{\Pi}_w = \{i_1 i_2, i_2 i_3, \ldots, i_{k-1} i_k\}$, $i_k = i_1$, such that there is no matching ij for which $j \in B_i(\Gamma)$ and $i \in B_j(\Gamma)$. Note that this precludes any of the individuals from having relational autarky as the most preferred state.
Without loss of generality, consider a a profile of utility functions $u = (u_{i_1}, \ldots, u_{i_{k-1}})$ such that $u_{i_s}(i_s i_{s+1}) > u_{i_s}(i_{s-1} i_s)$, for all $s = 1, \ldots, k-1$ where $i_0 = i_{k-1}$. Consider the following algorithm for selecting a matching pattern:

> Take any individual $i_s \in \{1, \ldots, k-1\}$ and match her with her most preferred partner, $B_{i_s} = i_{s+1}$, hence $i_s i_{s+1} \in \pi$;
> Then consider the most preferred partner of individual i_{s+1} and match her with her most preferred partner, *i.e.*, $B_{i_{s+1}}$ is i_{s+2}, and $B_{i_{s+2}} = i_{s+3}$, therefore $i_{s+2} i_{s+3} \in \pi$;
> Continue until all individuals are matched in π. Note that all individuals in π are in a matching with another individual, thus, $\pi \in \Pi_w$ if and only if π is stable.

Now, suppose that π is not a stable matching pattern. Then there exists a blocking matching $i_s i_{s+1}$ for $s = 1, \ldots, k-1$ such that $u_{i_s}(i_s i_{s+1}) > u_{i_s}(\pi)$ and $u_{i_{s+1}}(i_s i_{s+1}) > u_{i_{s+1}}(\pi)$, which contradicts the construction of π in which one of every two consecutive individuals is matched with her most preferred partner in π. Thus, $\pi \in \Pi_w$ is a stable matching pattern. ∎

In fact in the last case of the proof of Theorem 1.4.5 there are two distinct stable matching patterns. One is selected if the starting individual in the algorithm has an odd index on the cyclical path. The other stable matching pattern is selected if the starting individual in the algorithm has an even index on the cyclical path

The converse of Theorem 1.4.5 is not necessarily true, *i.e.*, if a stable matching pattern exists with respect to some $\Gamma \subset \Gamma_N$ then it might be that $\overline{\Pi}_w$ contains a cycle of odd length. This is illustrated in the following example.

Example 1.4.6 Consider again the matching economy $\mathbb{E}_1$ as discussed in Example 1.3.4 with the potential matching structure depicted in Figure 1.1. Now we modify the profile of utility functions over potential matchings as follows:

j	1	2	3	4
$u_1(1j)$	0	1	2	3
$u_2(2j)$	1	0	2	–
$u_3(3j)$	2	1	0	–
$u_4(4j)$	0	–	–	1

In this modified matching economy $\mathbb{E}_2$ there exists a unique stable matching pattern $\pi^\star = \{13, 22, 44\}$.[12] However, the cover of the set of simple matching patterns, $\overline{\Pi}_w$, generates a cycle. Indeed,

$$\Pi_w = \{\{11, 22, 33, 44\}; \{13, 22, 44\}; \{11, 23, 44\}; \{12, 33, 44\}\}; \tag{1.7}$$

and therefore $\overline{\Pi}_w = \{12, 13, 23\}$ gives rise to an odd cycle itself. ♦

1.4.1 Basis for Exchange

In Example 1.3.5 we showed that there might not exist stable matching patterns in relational settings with complementarities in skill acquisition. In such a matching economy, all individuals could establish mutually beneficial relationships with another individual based on relational complementarities in the acquisition of skills. However, in that example, the lack of mutual consent of most preferred partners precludes them from establishing these relationships. The absence of a stable matching pattern implies that there is essentially a state of chaos in such a society.

The next example extends the discussion in Example 1.3.5 and shows that in many cases there might emerge stable matching patterns within such situations. It develops a case of an economy in which the learning parameters allow the formation of a stable matching pattern consisting of mutually beneficial relationships.

Example 1.4.7 (Existence of stable matching patterns)
Consider the matching economy that has been developed in Example 1.3.5. We modify this example to allow the existence of a stable matching pattern. For this we modify the learning parameters as given in the table below:

[12] We refer the reader to the discussion of Example 1.3.4 to see why this is a stable matching pattern.

ij	α^i_{ij}	α^j_{ij}	β^i_{ij}	β^j_{ij}
11	0	0	—	—
12	0.3	0.6	0.3	0.6
13	0.5	0.8	0.8	0.5
22	0	0	—	—
23	0.3	0.3	0.3	0.3
33	0	0	—	—

As in Example 1.3.5 individuals remain exchange autarkic under the given circumstances and have an optimal investment in the acquisition of skills given by $l_i = \frac{1}{2}$. Hence, all individuals $i \in \{1, 2, 3\}$ attain skill levels $G_i = H_i = \frac{1}{2}$. This results into the following production levels:

ij	$g_i(ij)$	$h_i(ij)$	$g_j(ij)$	$h_j(ij)$
11	0.25	0.25	—	—
12	0.3306	0.3306	0.4225	0.4225
13	0.3906	0.49	0.49	0.3906
22	0.25	0.25	—	—
23	0.3306	0.3306	0.3306	0.3306
33	0.25	0.25	—	—

These production levels now result into the following potential consumption utility levels:

j	1	2	3
$\phi_1(1j)$	0.25	0.3306	0.4375
$\phi_2(2j)$	0.4225	0.25	0.3306
$\phi_3(3j)$	0.4375	0.3306	0.25

.

It is clear that given the set of potential matchings and a profile of hedonic utility functions such that $u_i(ij) = \phi_i(ij)$ for a player $i \in N$ and a potential matching $ij \in \Gamma$, there exists a stable matching pattern. Indeed, the pattern $\pi^* = \{13, 22\}$ is stable. This stable matching pattern results into utility levels given by $u_1^* = u_3^* = 0.4375$ and $u_2^* = 0.25$. ♦

Only after stable matchings have been formed, individuals can engage in mutually beneficial exchange within such relationships. Without the support of a stable relationship, there would neither exist nor emerge any trust among the individuals and therefore there would be no institutional basis for exchange.

However, within a stable matching, trade is founded on a moderate level of trust and both individuals can be assumed to engage in exchange. This is the subject of the next extension of Example 1.4.7:

Example 1.4.8 (Justification of limited exchange)
Consider the matching economy discussed in Example 1.4.7. This matching economy admits a stable matching pattern $\pi^* = \{13, 22\}$. The only relevant stable matching that emerges within this pattern is 13. Both individuals 1 and 3 can indeed engage in mutually beneficial exchange within this relationship.
Note that within 13, $g_1 = 0.3906$, $h_1 = 0.49$, $g_3 = 0.49$, and $h_3 = 0.3906$. It is clear that the exchange resulting within the relationship 13 ultimately leads to final consumption levels given by

$$v_1 = v_3 = \tfrac{1}{2}\left(g_1(13) + g_3(13) \right) = 0.4403, \text{ and}$$
$$m_1 = m_3 = \tfrac{1}{2}\left(h_1(13) + h_3(13) \right) = 0.4403.$$

This in turn leads to after-exchange hedonic utility levels given by $\hat{u}_1 = \hat{u}_3 = 0.4403 > 0.4375 = u_1^* = u_3^*$. Hence, there are mutual gains from exchange within the stable relationship between individuals 1 and 3. ♦

1.4.2 The Emergence of Subjective Specialization

Example 1.4.8 indicates that within a stable matching, there naturally emerges a moderate level of trust and, consequently, the possibility of mutually beneficial exchange. If such a stable matching is sustained, individuals will identify that *specialization* of their skills leads to further deepening of the gains from exchange. Indeed, after both parties engage in exchange, individual 1 will identify that increasing his skill level in hunting will increase his meat production further. Similarly, individual 3 will identify the complementary effect of increasing her gathering skills to increase her vegetable production.

This implies that further deepening of the stable trade relationship between individuals 1 and 3 results into mutual specialization. We emphasize that this specialization is induced at the most primitive level by the nature of the complementarities between these individuals. Indeed, that individual 1 specializes in hunting is a consequence of $\alpha^1_{13} < \beta^1_{13}$. Hence, there are social foundations to this specialization; specialization is still founded on the specific interaction within the relationship between 1 and 3. In this regard this type of specialization is completely *subjective*; this specialization only occurs within the context of the matching 13 and has no consequences beyond that relationship.

Another motivation for the foundation of such subjective specialization is to say that there are Ricardian comparative advantages for individual 1 to specialize in hunting *only within* the context of the relationship between 1 and 3.[13]

Example 1.4.9 (Subjective specialization)
Consider the matching economy developed in Examples 1.4.7 and 1.4.8. Within the matching 13 both individuals now develop a deepening of their economic relationship. As described in our previous discussion this ultimately leads to a moderate level of trust and the development of subjective specialization; both individuals specialize their production based on the environment of the matching 13 only.

Endogenous specialization under limited exchange
Individual 1 considers the exchange opportunities with individual 3 and consequently optimizes her investment in the acquisition of gathering and hunting skills. Hence, given the investment of individual 3 in the acquisition of gathering skills l_3, individual 1 solves the following problem:

$$\max_{0 \leqslant l_1 \leqslant 1} \phi_1(v_1, m_1) = \sqrt{v_1 \cdot m_1} \tag{1.8}$$

subject to

$$v_1 = \tfrac{1}{2}\left[\, l_1(1+\alpha^1_{13} l_3)\,\right]^2 + \tfrac{1}{2}\left[\, l_3(1+\alpha^3_{13} l_1)\,\right]^2$$
$$m_1 = \tfrac{1}{2}\left[\,(1-l_1)(1+\beta^1_{13}(1-l_3))\,\right]^2 + \tfrac{1}{2}\left[\,(1-l_3)(1+\beta^3_{13}(1-l_1))\,\right]^2$$

This optimization problem is based on the exchange opportunities emerging *within*[14] the matching 13. It is assumed that both individuals equally divide the gains from exchange.
In a fully equivalent fashion we can determine the optimization problem of individual 3:

$$\max_{0 \leqslant l_3 \leqslant 1} \phi_3(v_3, m_3) = \sqrt{v_3 \cdot m_3} \tag{1.9}$$

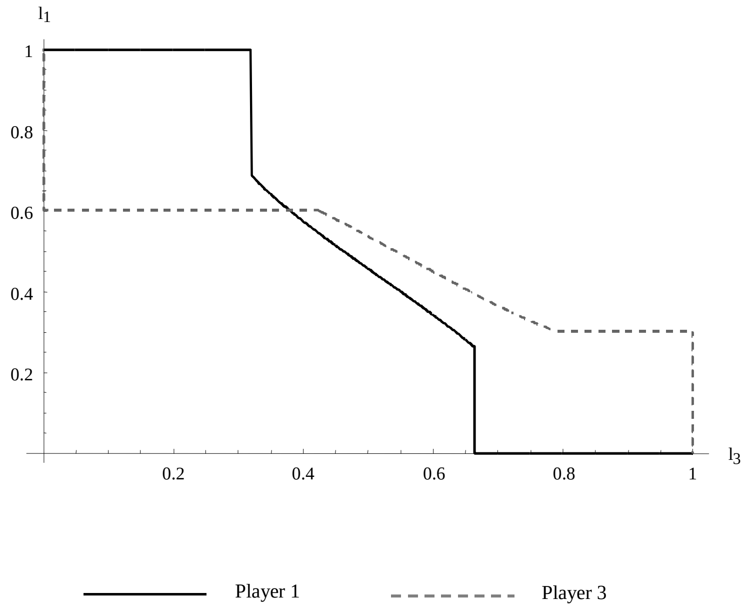
subject to

$$v_3 = \tfrac{1}{2}\left[\, l_1(1+\alpha^1_{13} l_3)\,\right]^2 + \tfrac{1}{2}\left[\, l_3(1+\alpha^3_{13} l_1)\,\right]^2$$
$$m_3 = \tfrac{1}{2}\left[\,(1-l_1)(1+\beta^1_{13}(1-l_3))\,\right]^2 + \tfrac{1}{2}\left[\,(1-l_3)(1+\beta^3_{13}(1-l_1))\,\right]^2$$

[13] For a comprehensive discussion we also refer, *e.g.*, to Yang (2003, Chapter 3.2).

[14] Since exchange may occur only within the matching, there is no reason to assume other than equal bargaining power between the individuals. This will not be the case in Example 1.5.5 in which *objective* specialization leads to potential exchange *outside* a given matching, the is *trade*. In that case, the solution to the maximization problem will be given by the Walrasian allocation.

Figure 1.2: The reaction curves in Example 1.4.9.



The reaction functions of players 1 and 3 for the values of the learning parameters given in Example 1.4.7 are presented in Figure 1.2. The continuous line represents the optimal investment in gathering skills by player 1 given the investment in gathering skills by player 3 and similarly the dashed line represents the optimal investment in gathering skills for player 3 given the investment of player 1.

This mutual optimization problem has three solutions, namely the two cases of full specialization: one in which player 1 specializes in gathering and player 3 in hunting and the other in which player 1 specializes in hunting and player 3 in gathering; and an equilibrium of relative specialization in which player 1 specializes relatively more in gathering and player 3 specializes relatively more in hunting.[15] Any of these three solutions indicates a certain level of subjective specialization.

In the two extreme solutions given by $(l_1, l_3) = (1, 0)$ and $(l_1, l_3) = (0, 1)$, in which both individuals fully specialize either gathering or hunting the attained utility levels are $u_1 = u_3 = 0.5$. In the solution of relative specialization given by $(l_1, l_3) =$

[15]We can reformulate this in game theoretic terms. Indeed, individuals 1 and 3 engage in a two-player normal form game with strategies l_1 and l_3 respectively. The two optimization problems formulate a Nash equilibrium in this game. Thus, we identify three Nash equilibria in pure strategies for this interaction game.

$(0.6, 0.4)$ the attained utility levels are $u_1 = u_3 = 0.4344$. Clearly full specialization leads to a higher attainable utility level.

Optimal subjective specialization
We can also compute the Pareto optimal outcome which is given as the solution to the following problem:

$$\max_{0 \leqslant l_1 \leqslant 1,\, 0 \leqslant l_3 \leqslant 1} \sqrt{v_1 \cdot m_1} + \sqrt{v_3 \cdot m_3}$$

subject to

$$v_1 = v_3 = \tfrac{1}{2}\left[\, l_1(1 + \alpha^1_{13} l_3)\,\right]^2 + \tfrac{1}{2}\left[\, l_3(1 + \alpha^3_{13} l_1)\,\right]^2$$
$$m_1 = m_3 = \tfrac{1}{2}\left[\, (1 - l_1)(1 + \beta^1_{13}(1 - l_3))\,\right]^2 + \tfrac{1}{2}\left[\, (1 - l_3)(1 + \beta^3_{13}(1 - l_1))\,\right]^2 .$$

By substituting the given parameter values for α and β, there are two solutions namely the solutions that correspond to full specialization identified by $(l_1, l_3) = (1, 0)$ and $(l_1, l_3) = (0, 1)$. ♦

1.5 Objective Specialization

The previous discussion clarifies the emergence of stable matching patterns and of subjective specialization. This emergence is essentially based on features *within* the pattern of stable matchings. For an economy to have persistent access to gains from specialization, the social structure of the economy has to *generically* admit stable matchings. Hence, whatever capabilities and desires of the individuals—represented by their utility functions and (possibly) other individualistic features—a stable matching pattern has to exist in the matching economy.

Technically, this brings up the question under which conditions on (N, Γ) there exists a stable matching pattern for *every* possible matching economy (N, Γ, u), where u is an arbitrary utility profile. This line of research follows the research agenda set in the matching literature. Here we are able to invoke the main result of Pápai (2004) obtained in hedonic coalition formation framework. However, first we outline her main framework, to facilitate comparison.

Pápai (2004) investigates the existence of a unique stable coalition structure in a coalition formation model where a *coalition structure* is a partition of the player set into pairwise disjoint sets such that each set is an element of a given set of permissible coalitions, and a *coalition formation model* is defined as a collection of permissible coalitions that a given finite set of players may form. Players have strict, complete, and transitive preferences over the set of coalitions in the collection of permissible coalitions in which they are members. A pair of coalition formation model and a preference profile define a *coalition formation problem*. Pápai (2004) takes a coalition formation model to be *stable* if there is a stable coalition structure for all all coalition formation problems. A coalition structure is stable for a given coalition formation problem if given a preference profile, there is no coalition such that all players in this coalition strictly prefer to be its members than to be part of the coalition whose members they are in the given coalition structure. The necessary and sufficient condition that she identifies is called *single-lapping property*. Its implication to the network framework is that the set of potential matchings does not contain a cycle.

Our notion of generic stability is clearly related to the notion of stable coalition formation model discussed by Pápai (2004). As defined below, generic stability is a property of the matching structure. Since our analysis is focused on matching patterns,that in hedonic coalition terms represent a set of permissible coalitions that contains coalitions of no more than two players, we are able to provide less stringent necessary and sufficient conditions for stability relative to Pápai (2004).

Formally we define:

Definition 1.5.1 *A matching structure (N, Γ) is* ***generically stable*** *if for every utility profile $u \in \mathcal{U}$ it holds that $\Pi^{\star}(N, \Gamma, u) \neq \emptyset$.*

Our main existence theorem can now be stated as follows:

Theorem 1.5.2 *The matching structure (N, Γ) is generically stable if and only if the set of potential matchings Γ is odd acyclic.*

Proof.
If: From Definition 1.4.1 it follows that if Γ is odd acyclic, then $\overline{\Pi}_w$ is odd acyclic too. The sufficiency of odd acyclicity condition on the set of simple matching patterns for the existence of a stable matching pattern follows from Theorem 1.4.5 directly applied to Definition 1.5.1. This implies that odd acyclicity of Γ is a sufficient condition for the existence of a stable matching pattern for any utility profile $u \in \mathcal{U}$.

Only if: Suppose that there exists a stable matching pattern for all utility profiles $u \in \mathcal{U}$. Next suppose to the contrary that the potential matching structure Γ is not odd acyclic. Without loss of generality, we may assume that Γ contains a single odd cycle $C = \{i_1, i_2, \ldots, i_k\}$ such that $i_1 = i_k$ and $k - 1$ is an odd integer.
Take the utility profile $u \in \mathcal{U}$ such that $u_{i_s}(i_s i_{s+1}) > u_{i_s}(i_{s-1} i_s) > u_{i_s}(i_s i_s)$, for all $s = 1, \ldots, k-1$ where $i_0 = i_{k-1}$. In every weakly stable matching pattern $\pi \in \Pi_w(u)$ with respect to the utility profile u there is at least one individual, i_s for $s = 1, \ldots, k-1$, on the cycle Γ who is relationally autarkic since there are odd number of individuals in the cycle. Thus individual i_s can form a blocking matching with individual i_{s-1} since

$u_{i_s}(i_{s-1}i_s) > u_{i_s}(\pi)$, *i.e.*, every individual prefers to be matched with another individual rather than be relationally autarkic, and $u_{i_{s-1}}(i_{s-1}i_s) > u_{I_{s-1}}(\pi)$ since individual i_s is the most preferred partner of individual i_{s-1} given preference profile u. Thus, no stable matching pattern exists in Γ with respect to the given preference profile u.
We now conclude that (N, Γ) cannot be generically stable, which establishes a contradiction. Hence, we have shown the assertion. ■

Theorem 1.5.2 provides a complete characterization of generically stable matching structures. This is a very strong result with some deep consequences. Before discussing the consequences of this insight to the discussion of specialization, we turn to the interpretation of the odd acyclicity property.

Theorem 1.5.3 *Let Γ be a set of potential matchings on N. A sub-structure $\Theta \subset \Gamma$ is odd acyclic if and only if (N, Θ) is bipartite in the sense that there exists a partitioning $\{N_1, N_2\}$ of N such that*

$$\Theta \setminus \Gamma_0 \subset N_1 \otimes N_2 = \{\, ij \mid i \in N_1 \text{ and } j \in N_2 \,\}. \tag{1.10}$$

Proof. It is obvious that every bipartite structure Θ on N is odd acyclic, since all cycles have to be of even length. So, we only have to show the converse.
Let Θ be odd acyclic on N. Without loss of generality we may assume that $\Theta \neq \varnothing$, $\Theta \cap \Gamma_0 = \varnothing$, and that Θ is completely connected in the sense that for all $i, j \in N$ with $i \neq j$ there is a path $P(ij) \subset \Theta$ between i and j.
Select some $i_0 \in N$. Assume that $j \in N$ is such that there exist two distinct paths $P_a = P_a(i_0 j)$ and $P_b = P_b(i_0 j)$ between i_0 and j. We now claim that the length of both P_a as well as P_b are either odd or even. Indeed, if the length of P_a is odd and the length of P_b is even, then $P_a \cup P_b \subset \Theta$ defines a cycle from i_0 to i_0 that has an odd length. This violates odd acyclicity of Θ.
Now define $N_1 \subset N$ as follows: For every $j \in N$ we let $j \in N_1$ if and only if the unique length of a path $P(i_0 j)$ is odd. Subsequently we define $N_2 = N \setminus N_1$, consisting of all individuals that have paths of even length with i_0.
Finally, we claim that for any $ij \in \Theta$ it holds that either $i \in N_1$ and $j \in N_2$ or $j \in N_1$ and $i \in N_2$. This follows immediately from the observation that for all $i, j \in N_1$ a path $P(ij)$ between them has to have even length. (Otherwise, there would be an even- as well as an odd-length path between i_0 and i.) Similarly, for all $i, j \in N_2$ a path $P(ij)$ between them has to have even length. ■

Theorem 1.5.3 states that odd acyclicity of a sub-structure of the set of potential matchings Γ is equivalent to this sub-structure being bipartite. The latter refers to familiar structures in matching theory (Roth and Sotomayor 1990) and imposes that relations are only possible between individuals of a different, distinct "type". We develop an interpretation of this requirement in the next sections of this chapter.

Our main insight provided in Theorem 1.5.2 can now be re-stated using the characterization in Theorem 1.5.3:

Corollary 1.5.4 *The matching structure* (N, Γ) *is generically stable if and only if* (N, Γ) *is bipartite in the sense that there exists a partitioning* $\{N_1, N_2\}$ *of* N *such that*

$$\Gamma \setminus \Gamma_0 \subset N_1 \otimes N_2 = \{\, ij \mid i \in N_1 \text{ and } j \in N_2 \,\}. \tag{1.11}$$

We now turn to the discussion of the application of this insight to the economies with skill complementarities developed in Examples 1.3.5, 1.4.7, 1.4.8 and 1.4.9.

As stated before, certain sets of skill complementarities might result into the emergence of stable matching patterns. These stable matching patterns in turn give rise to subjective specialization and mutually beneficial exchange. This does not mean that there result widespread gains from exchange. For such enhanced economic development it is necessary that there emerges an objective or socially recognized division of labor.

In particular, we argue that the deepening of the stable matching patterns through subjective specialization in turn leads to the emergence of odd acyclic structures of potential matchings. This emergence is based on the social recognition of the roles that are based on the subjective specialization of individuals in such stable matching patterns. This is discussed next. The economic activities are thus recognized as *trade*.

Example 1.5.5 (Objective specialization)
Consider the stable matching pattern $\pi^* = \{13, 22\}$ discussed extensively in Examples 1.4.7, 1.4.8 and 1.4.9 as the unique stable matching pattern. Within this stable matching pattern, the matching 13 is the only binary, value-generating relationship. In Example 1.4.8 it was sketched that within this relationship there would result mutually beneficial exchange opportunities if sufficient trust among the individuals 1 and 3 was established. Also, within this matching, individual 1 generated a higher output of meat ($h_1(13) = 0.49$) than of vegetables ($g_1(13) = 0.39$) and individual 3 generated a higher output of vegetables ($g_3(13) = 0.49$) than of meat ($h_3(13) = 0.39$) due to the actual values of the complementarity parameters α and β.
Subsequently, in Example 1.4.9 we discussed the emergence of subjective specialization within the matching 13. We identified three different subjective specialization configurations. Such subjective specialization is based on sufficiently high levels of

trust and the presence of a trade relation between individuals 1 and 3.

At present we argue that further deepening of the efficiency in this economy is only possible through the establishment of a true social division of labor. Given the initial output levels, the subjective specialization will develop into the direction as indicated through these output levels. Hence, individual 1 probably specializes subjectively on hunting only, while individual 3 specializes subjectively on gathering only. If these subjective specializations are recognized socially, individual 1 becomes a "hunter" and individual 3 becomes a "gatherer". Being a hunter now becomes a socially recognized economic role, as does being a gatherer. Only after the establishment of such complementary social roles there emerges a *social division of labor*.[16]

In the application, players 1 and 3 can achieve social recognition as a gatherer and a hunter and re-evaluate their potential utility level from a matching with another player. Now, let player 1 assume the role of a gatherer and player 3 the role of a hunter. Hence, there emerge three social roles within this simple economy: **H** stands for a hunter, **G** stands for a gatherer, and **A** stands for an individual in a position of autarky. The assumed skill acquisition of each role is respectively $G_{\mathbf{G}} = H_{\mathbf{H}} = 1$, $H_{\mathbf{G}} = G_{\mathbf{H}} = 0$, and $G_{\mathbf{A}} = H_{\mathbf{A}} = \frac{1}{2}$. The production level of each potential matching is then given by:

ij	$g_i(ij)$	$h_i(ij)$	$g_j(ij)$	$h_j(ij)$
GG	1	0	—	—
GA	1.3225	0	0.64	0.25
GH	1	0	0	1
AA	0.25	0.25	—	—
AH	0.25	0.64	0	1.3225
HH	0	1	—	—

In objective specialization each individual now expects to be trading when she engages in a matching. Also, under objective specialization, unlike under subjective specialization, the level of trust expands to the whole set of players, *i.e.*, to the whole economy. This is why an individual believes fully that she can be matched with another player with whom trade is beneficial in a stable matching. In fact, there is *common knowledge* that gatherers and hunters can be matched in highly productive social (trade) relationships. Individuals who assume social roles, have socially justified beliefs that a stable matching pattern exists.

In our example, after objective specialization and the establishment of a social division

[16]We emphasize here that the establishment of a social role requires the the social recognition of each role and the separation of the related specialism from each individual. Thus, the social recognition of an economic role induces a dichotomy of this role and other aspects of her life for every individual that assumes this role.

of labor, there emerge three types of individuals: hunters, gatherers and individuals in autarky. A hunter (or gatherer) believe that they will trade half a unit of meat (vegetables) for half a unit of vegetables (meat) in a potential matching with a gatherer (or hunter). The trade between a hunter (or gatherer) and an individual in autarky will take the terms of 0.66125 units of vegetables (meat) for 0.084235 units of meat (vegetables). These are calculated to be the optimal trade patterns in the matchings **AG** and **AH**, respectively.

It should be noted here that the emergence of trade between individuals with different social roles is fundamentally different from commodity exchange between subjectively specialized individuals. Social recognition indeed alleviates the informational burden and implements certain expectations.

These production levels now result into the following potential utility levels from consumption after trade:

j	**H**	**G**	**A**
$\phi(\mathbf{H}j)$	0	0.5	0.2360
$\phi(\mathbf{G}j)$	0.5	0	0.2360
$\phi(\mathbf{A}j)$	0.4644	0.4644	0.25

.

Clearly, gatherers and hunters prefer to be engaged in matching with each other rather than to be in relation with an individual in autarky. Hence, the unique stable matching pattern can be identified as $\{\mathbf{GH}, \mathbf{AA}\}$, which corresponds to $\{13, 22\}$ in the original setting.

Within the developed example, there now can emerge a market. If sufficiently large number of individuals assume the social roles of hunter and gatherer and other economic institutions such as the protection of property rights, monetary instruments, and the creation of actual market places are established, then there might emerge a market at which hunters and gatherers can trade vegetables and meat for a well established and unique market price. ♦

Objective specialization excludes relationships between individuals with the same social role as being potentially beneficial economic matchings. This implicitly reduces the set of potential matchings to an odd acyclic or bipartite structure in which only matchings between individuals with two different roles are recognized.

Finally, Theorem 1.5.2 does not guarantee the uniqueness of the stable matching pattern. For uniqueness we need to impose two additional restrictions on the set of potential matchings, namely that $u \in \mathcal{U}_s$ with $\mathcal{U}_s \subset \mathcal{U}$ being the set of all utility representations of strict preferences only and that the set of potential matchings Γ is (fully) acyclic, *i.e.*, also cycles with even number of links in the path are not allowed. This result is a direct application of Pápai (2004) uniqueness theorem and, hence, the proof is omitted here.

Proposition 1.5.6 *Let $\mathcal{U}_s \subset \mathcal{U}$ be the class of utility functions of strict preferences. The matching structure (N, Γ) is generically stable with $|\Pi^\star(N, \Gamma, u)| = 1$ for every utility profile $u \in \mathcal{U}_s$ if and only if Γ is acyclic.*

1.6 Concluding Remarks

In this chapter we introduced a four stage approach to the emergence of a social division of labor based on the objective specialization of individuals. As a fifth stage we can add the emergence of market institutions themselves. This approach clarifies that the presence of a social division of labor is in fact a prerequisite for the creation or emergence of a functioning price mechanism. Summarizing these four stages are:

Stage I: Non-equilibrium. In a primitive relational economy without objective specialization, there are conditions that do not support an equilibrium. This leads to a situation of indeterminancy to which we can refer as a permanent relational chaos or indeterminancy. Individuals would like to benefit from the learning externalities and thus prefer to be matched with another individual than to be fully self-reliant for the provision of necessities for survival and stay in exchange autarky. However, there is a lack of double coincidence of a desired matching. (Example 1.3.5)

Stage II: Primitive equilibrium. Within a primitive relational economy there might exist conditions that allow the emergence of a stable social interaction pattern. Such a stable pattern is only founded on subjective and personal features, not on any objective or social conditions.
Within this stage we distinguish two sub-stages.
(II-A) At first there only emerges a stable pattern in which interpersonal spillovers are exploited. This first level of stable social interaction facilitates the emergence of a moderate level of subjective trust among the matched individuals. (Example 1.4.7)
(II-B) Next, the emergence of sufficient subjective trust among the individuals that are engaged with each other, supports the introduction of exchange among those individuals; the exploitation of interpersonal spillovers is extended into

the exchange of economic commodities leading to even higher levels of utilities. The emergence of exchange is an important step into the development of an economy. (Example 1.4.8)

Stage III: Subjective specialization. After exchange has been established there is the possibility for a further deepening of interpersonal trust within the stable relationships in the economy. This facilitates the emergence of subjective specialization in which individuals based on the demands of their interpersonal relationships specialize their economic activities. Hence, within the context of a stable exchange relationship with other individuals, an individual chooses a production plan to optimize his utility level.

This process of subjective specialization is similar to the specialization process based on inframarginal analysis developed by Yang, *e.g.*, Yang (2001) and Yang (2003),—as a formalization of the Smith-Young development mechanism—within the context of a perfectly competitive price mechanism. However, subjective specialization does *not* take place within the context of a functioning price mechanism, but rather within the interpersonal relational setting of each individual separately. (Example 1.4.9)

Stage IV: Objective specialization. The emergence of subjectively specialized individuals can lead to the recognition of social economic roles in the society at large. Individuals who specialize on hunting skills in the context of their individual relationships, become socially recognized as “hunters”. Thus, hunters are identified and appointed in the society as producers of meat. Subsequently, there emerge social rules related to the social role of a hunter as a producer of meat. The engagement of a “hunter” with a “gatherer” in an economically beneficial (exchange) relationship may thus become the foundation for economic development. Individuals subsequently specialize in an objective fashion: they now select from a limited set of social roles and engage in an objective fashion with other individuals in their respective social roles to generate mutual economic benefits. It is only within this context of objective specialization that there emerges a social division of labor which further development acts as an engine for economic growth—described in the context of a market by the Smith-Young mechanism. (Example 1.5.5)

Stage V: Market emergence. We argue that only after the establishment of a social division of labor based on the social recognition of certain economic roles, there can emerge a functioning market or price mechanism. Besides the social division of labor there have to be established other economic institutions. Only after these other conditions are met, there might emerge a price mechanism through which further economic growth and development is made possible in the form of the Smith-Young mechanism based on the extent of the market.

In this chapter we only have developed the most basic principles of this descriptive theory. The main conclusion is that economic development and growth is closely related to the development of the social roles in an economy. These social roles have a public nature and as such are subject to a purely public economic theoretical analysis or an evolutionary treatment. This is closely related to the conclusion in Gilles and Diamantaras (2005). Further development of the abstract theory of matching economies is required before we can expect a full and working understanding of the five-stage process of market development summarized above. This is left to future research.

CHAPTER

TWO

S S R A :
C P P -M S [*]

2.1 On Social Complexity and Productive Complexity

Complex economic organizations, such as multi-hierarchy firms are taken to be a common economic entity. Coase (1937) was the first to question the origin of the firm and by raising this question he started a substantial volume of literature that discusses the organizational form of the firm as an alternative to the market organizations. However, complex productive organizations have emerged even before the establishment of markets as institutions. While such pre-market organizations have been ignored in the above works, they will be in the center of attention here. The current work develops along two dimensions. On one hand, we develop a general framework that allows us to study the emergence of complex productive entities, and on the other hand, we apply this framework to a specific collective production process, via which, according to anthropologists, complexity emerged in primitive hunter-gatherer societies.

We develop a theoretical framework in which economic value is generated in the interaction between people. An individual has a restricted domain of activities in which she can engage. The restriction may be coming from exogenously given social or production-specific rules. Given their potential domain, individuals are seeking to activate those relations, that allow them to participate in a productive process. The productive processes studied here may involve more than a pair of individuals, that is why we refer to them as complex processes. What is essential is that these individuals who engage in a productive process are organized in a specific way that follows from

[*]This chapter is based to a great extent on Lazarova, Gilles and Ruys (2006).

our definition of complexity. Complexity in an organization is translated in graph-theoretic terms as a directed network of star structures that we call a *star pattern*. The direction of a productive link signifies that one of the individuals has authority over the other. The utility of an individual engaged in production depends on her productive ability and on the abilities of the other players with whom she is connected as well as her relative position in the star, *i.e.*, whether she is located in the center or on one of the leaves. In the general framework, however, we do not explicitly model utility as a function of productivity abilities but use a *hedonic team utility profile* as an indirect utility measure. This makes our main theoretical results applicable to a wide range of frameworks. The potential activities and the preferences of players over the production teams in which they can engage define what we call a *team economy*.

Like in the preceding chapter, we study two types of stability: stability within a specific team economy, and *generic stability* that renders stability to the structure of potential activities. As in the previous chapter, generic stability implicitly requires that a structure of potential activities is such that no matter how the productive abilities are distributed among its individuals, there is a stable star (production) pattern. The lack of a stable star pattern is interpreted as chaotic behavior and unpredictability of the production outcome given a structure of potential activities and a distribution of the productive abilities. In terms of the individual possibilities for deviation, stability refers to the lack of pairs of players who prefer to deviate from the established pattern. This notion of stability is largely based on the notion of pairwise stability first introduced by Jackson and Wolinsky (1996) and also used in the preceding chapter. Here, however, the notion of stability is adapted to allow for greater possibilities for deviation of these players who are located in the center of a star component. Such players may add links, without severing existing links, unlike the rest of the players who are located at the leaves of a star, because in doing do so they still preserve the star structure of the activity pattern.

Our main theoretical results refer to two types of stability: economy-specific and generic. The concept of generic stability used here is closely related to the generic stability used in Chapter 1: it is a property of the potential structure that ensures the existence of a stable pattern for any preference profile. In Chapter 1, this is a matching pattern and here this is a star pattern. We provide a sufficient condition on the set of potential activities that ensures the existence of a stable star pattern in a given economy. Furthermore, we provide necessary and sufficient conditions on the set of potential activities that ensure the existence of stable star patterns for any team utility profile, *i.e.*, that ensure generic stability. These necessary and sufficient conditions require that the set of potential links does not have a cycle with a number of players different from a multiple of three, where the multiple is higher than one.[1] The implications of these conditions for the structure of potential activities is, that it may be a tree or it may contain special cycles which connect some players whose number is a multiple of three. Though related to the main theorem on existence of stable coalition structures derived by Pápai (2004)[2], our theoretical results are fundamentally different and can only be derived in a graph-theoretic framework. The graph-theoretic framework allows us to distinguish between coalitions of, for example, three players in which there are links between all three of them or in which there are links between only two of them and the player in the middle acts as a star central player.

The relation between the establishment of a hierarchical social structure and productive complexity via labor specialization has been documented in some anthropological works. This motivated us to develop an application of the general framework closely related to the discussion of the emergence of complexity in primitive societies carried out in some anthropological literature.

In some of anthropological works hierarchical organization is seen as an element of complexity, *e.g.*, in Arnold (1993) we read

> I define chiefly complexity to include three recognizable organizational characteristics: hereditary inequality, hierarchical organization (including some political authority on a multicommunity scale), and the elite ability to exercise partial control over domestic labor.

The hereditary property of the hierarchical organization implies that the hierarchical organization is ex-ante fixed on a set potential of activities. That is, when individuals decide on the type of activities to do, they are limited by their relative position in the socially recognized hierarchy. Note that here the hierarchical organization refers to a social hierarchy, not necessarily induced by the productive capabilities of individuals. The third element in Arnold's definition of complexity is what we define as a complex value generating relation, *i.e.*, the relation in which an individual has decision power on the production units intrinsic to another individual.

The process via which complex relations have emerged has been attributed by some anthropologists to the socially recognized specialization of labor. For an extensive discussion of these theories we again refer to Arnold (1993):

[1] The condition on the number of players connected in a cycle is a technical restriction that is founded on the limited types of complex patterns, which we investigate. We will come back to this point in the discussion in Section 2.7.

[2] See Chapter 1, Section 1.5 for an outline of the framework used by Pápai (2004).

> I suggest that the mechanism by which change occurs from egalitarian to non-egalitarian relations involves the changing organization of human labor, specifically the institutionalized separation of some labor and products from sole family or kin-group control into the hands of higher authority.

In a comparative case study of two hunter-gatherer societies in Papua New Guinea, Bedamuni and Kubo, distinguished by the levels of social and productive complexity (Bedamuni exhibit higher level of complexity and Kubo exhibit lower level of complexity), Minnegal and Dwyer (1998) summarize their expectations as follows:

> An increase in the investment of labor in particular tasks usually necessitates a change in the allocation of time to those tasks and thus in the organization of labor. We expected Bedamuni to be investing more in subsistence tasks than Kubo but, more importantly, to be working in different ways. We expected to see a clearer division of labor among Bedamuni than among Kubo, a sharper differentiation of roles between sexes and age classes and work more often performed by specialised task groups than by households. In short, we expected a rationalization of subsistence tasks among Bedamuni, increased differentiation of roles within production units.

When studying the process of the emergence of complex activities, one also needs to answer the question what triggered this process. Again we base the discussion on anthropological evidence. According to Arnold (1993), most anthropologists agree that the emergence of more complex production activities was "a product of necessity rather than choice". Among the ultimate triggers according to Arnold are the high population density and resource imbalances. This implies that even though individuals had the technology to engage in complex production, they did not do so, until it was necessary for them to do it in order to survive. An explanation that we put forward is that the engagement in complex collective production involves reliance on the actions of other individuals and, hence, trust in the social responsibility that the other engaged individuals will have to fulfill their duties in a situation in which there are no established enforcement institutions.

Based on these anthropological insights, we analyse value generation processes which take the form of collective production in a subsistence economy of consumers-producers. We will introduce in more detail collective production in the following section. Here we would like to point out that the implications of our theoretical results on the application conform to the insights present in the anthropological literature. A necessary and sufficient condition for the presence of stable collective production process is that the set of potential activities has to be void of components of *directed cycles* in which the number of individuals connected is different from a multiple of three with a multiple higher than one. Since there is a specific relation between the direction of the network and the team utility function, the condition is based on the definition of a cycle that also takes into account the direction of the network, *i.e.*, there cannot be two individuals who have (indirect) authority over each other. This implies that the set of potential activities must be of a hierarchical structure which indeed implies the presence of a socially recognized authority.

The remainder of the chapter is organized as follows. Section 2.2 discusses the notion of collective production and the presence of authority. Section 2.3 provides the necessary graph-theoretic toolbox. Section 2.4 presents the general framework and applies this framework to the application of complex production. Sections 2.5and 2.6 offer our main theoretical results and illustrative examples. Section 2.7 concludes with a discussion on the benefits and limitations of this framework.

2.2 Collective Production and Authority

Collective production is a process in which producer-consumers individuals may engage to generate value. Collective production is, thus, a form of an organization that precedes the emergence of firms. A prominent reference for a discussion of firms formed by consumers-producers is the works of Professor Xiaokai Yang. Yang (2003) defines the *firm* to be "a structure of transactions based on the division of labor". Yang also recognizes that within a firm there is a clear distinction between two roles: the role of an employer and the role of an employee such that the employer has a decision power over the employee's labor. We refer to the ability of one individual to make a decision of the participation in the productive process by another individual as *authority*. Collective production is modeled as a stable configuration of agents generating value based on the division of labor with a well-defined flow of authority. The main difference between the organization of collective production and a firm as defined by Yang (2003) is that in our setting there are *no markets*. Hence there is no external mechanism or, for that matter, an invisible hand that determines a rate of remuneration in return for one's labor. Furthermore, the output produced by collective effort is consumed by the participants in the process rather than sold on an external market.

Moreover, in Yang's analysis of a firm based on producer-consumers there is a uniform and pre-determined relation between the labor specialization and the authority roles of an "employer" or an "employee" that an individual can assume. The allocation of authority roles in our setting is idiosyncratic to each pair of individuals. So, it is possible that one player has authority over another player, while a third player has authority over the first.

The relation between our framework of collective production and Yang's analysis of a firm, will become clearer in the discussion below. The collective production is based on that of an economy of producer-consumers discussed by Yang (2003). In Chapter 6 of his book Yang offers a stylized framework in which there is a finite set of individuals who consume a consumption good. To produce the consumption good they need both labor and an intermediate good. To produce the intermediate good they need only labor as an input. There is increasing returns to scale in the production of both goods. Yang's framework concerns the emergence of firms in a market setting and as such it models players who maximize utility and profits taking prices as given. The objective here is to understand the emergence of complexity at a more primitive level prior to the establishment of the market institutions. Players in our framework maximize their free-time subject to sustaining some minimal consumption level called *subsistence level* $\bar{y}$.

Like Yang, in our application we assume that individuals are homogeneous in their preferences and production possibility set.[3] We denote the amount of the consumption good that they produce by $y \in \mathbb{R}_+$. There is a minimum level of the consumption good $\bar{y}$ that individuals must consume, otherwise they perish. To produce the consumption good individuals use as inputs their labor measured as time and some amount $x \in \mathbb{R}_+$ of an intermediate good . The production of the intermediate good involves only labor, *i.e.*, time. Formally the production functions for the intermediate good and the consumption good are given as $x = (l_x)^\alpha$ and $y = x\, l_y$, where l_x is the amount of time spent in producing x and l_y is the amount of time spent in producing y and $\alpha > 1$ captures the increasing returns to scale in production.[4] All individuals have preferences for leisure $f \in [0, 1]$ represented by a linear utility function $\phi(f) = f$. They have one unit of time available which they can allocate among having leisure, producing the intermediate good, or producing the consumption good, *i.e.*, $f + l_x + l_y = 1$. Note that the limited resource of time makes the utility function bounded from

[3]Since players are homogeneous in their preferences and in their production possibility set we will avoid using individual specific index when no confusion arises.

[4]This production possibility set is analogous to the one discussed by Yang (2003) in the chapter on the emergence of the firm. Yang's formulation is more general, *i.e.*, the production function of consumption good is given by $y = x(l_y)^\beta$. Here we take $\beta = 1$ for ease of exposition.

below and from above, *i.e.*, $\phi(0) \leq \phi(f) \leq \phi(1)$ for $f \in [0,1]$. We can interpret $u(1)$ as the bliss point of an individual and $\phi(0)$ the point of exhaustion.[5]

In situations in which individuals, acting alone, cannot achieve the subsistence consumption level, by necessity, they need to engage in collective production. The most primitive form of collective production is the one in which only pairs of individuals engage.[6] We conjecture that more complex patterns, such as star production patterns, emerge only after the establishment of stable matching patterns. As we have already seen in Chapter 1, a bi-partite set of potential links is a sufficient and necessary condition for the existence of stable matching patterns. Hence, here we take for granted that the economic agents will choose to specialize in one of two roles: a producer of the intermediate good or a producer of the consumption good.

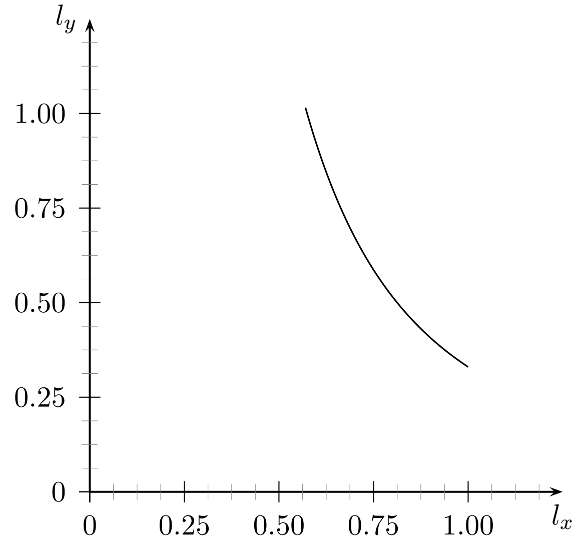
When two individuals, one specialized in the production of the intermediate good and the other specialized in the production of the consumption good decide to engage in collective production to produce two times the subsistence level, they need to resolve the following question: who works how many hours. On Figure 2.1 is given the reaction functions of two players who need to collectively produce two times the subsistence level, given their production possibility level as described above in a situation in which none of them can produce the subsistence level acting alone. As we see, the two reaction functions completely coincide. Hence, there is a continuum of points along which the two players can achieve the desired level of consumption.

The two end points represent two cases of particular interest: the one in the lower right corner is the case when the producer of the intermediate good devotes all her endowment of time in the production, and the one in the upper left corner is the complementary case when the producer of the consumption good devotes all her time endowment in the production. These cases correspond to the two types of firms analyzed by Yang (2003): the one in which the individual specialized in the production of the consumption good acts as an “employer” and the individual specialized in the production of the intermediate good acts as an “employee”; and the second one in which the individual specialized in the production of the intermediate good acts as an “employer” and the individual specialized in the production of the consumption good acts as an “employee”. In a firm, the individual acting as an “employer” has decision power over the amount of the production input of the “employee”. Hence,

[5]We have assumed that the utility function of free time is linear for analytical tractability. Alternatively, we could have taken any function defined on the interval [0,1]. For instance, we could have taken a cumulative distribution function, which probability density function has thick tails. Such function is fits well with the prospect theory described by Kahneman and Thversky (1979).

[6]The activity patterns of pair has been the focus of the previous chapter. Recall that there our main application was based on a framework of learning externalities.

Figure 2.1: Reaction Function of a Pair of Individuals in Collective Production ($\alpha = 2$)



an “employer” requires that an “employee” invests all her time endowment into the production. The two possibilities that Yang (2003) discusses are that either all players specialized in the production of the consumption good act as “employers” or players specialized in the production of the intermediate good act as “employers”.

In our analysis we will concentrate on these two cases as well, however, unlike Yang (2003) we will not assume a uniform relation between authority and labor specialization. That is, in our framework whether a person has authority over another person would depend on some exogenous, person-specific characteristics and not on her choice of labor specialization. In other words, it is not necessarily the case that all individuals specialized in the production of, say, the consumption good, have authority over the individuals specialized in the production of the intermediate good. In our framework, it is assumed that one of the individuals has authority over the other in each pair of individuals engaged in collective production, and it is not important what kind of labor specialization the individual has. So, anyone in a pair of players can have authority, what is important is that one of them does.

Clearly, collective production is founded on trust between the participants. One of the individuals has to submit to the authority of the other. Moreover, she has to trust that she will be provided with the subsistence amount of consumption at the end of the production process. The trust required in establishing collective production explains why collective production is driven by necessity, as pointed out above. If a risk averse individual can achieve the subsistence level acting alone, she will not trust another individual with providing the necessary amount of the consumption good. Under severe circumstances, such as the need to survive[7], trust is a necessity, and thus, collective production can be established.

Collective production may involve the cooperation between more than two individuals. In our framework, this more complex production process is modeled as a collection of pairs of individuals who have a unique common member. The common member is located in the center of the productive unit and has an additional role of *coordinating* its activities. In this respect, again, our framework is more general than the one developed by Yang (2003) as we do not assume that authority emanates from the individual who has a coordinating role. Instead, we allow for an individual who connects with many other individuals[8] to be simultaneously under the authority of some and to have authority over others.

2.3 Technical Preliminaries

Let $N = \{1, ..., n\}$ be a finite set of economic agents or *players*. Players engage in authoritarian relational activities represented by *directed links* between them, *e.g.*, between two distinct players $i, j \in N$ there may be a directed link (i, j) which indicates that player i has authority over player j and player j does not have authority over player i, *i.e.*, $(i, j) \neq (j, i)$. A directed link between two players refers to a value generating activity between them such that one of the players has authority over the other. In terms of our application, a directed link between two players refers to collective production in which these two players are involved, and the direction runs from the player who has decision power over the labor hours to the other player in the pair. The set of all possible directed links among players in the set N is given as $L_N = \{(i, j) \mid i, j \in N \text{ and } i \neq j\}$. The relational activity represented by the link to oneself we denote by $\{i, i\}$ and we call *autarkic*. An autarkic link is not directed by definition. The set of all autarkic positions is denoted by $D_0 = \{\{i, i\} \mid i \in N\}$.[9]

Definition 2.3.1 *A* ***set of potential links*** *on the set of individuals N is given as $D \subseteq$*

[7]The development of trust in this application is fundamentally different than the development of trust discussed in the previous chapter, where prolonged cooperation between pairs of individuals allowed them to develop trust and hence subjective specialization, which then developed into objective specialization.

[8]As in the previous chapter, here again only individuals with a different labor specialization can be linked. This is why the only individual in a group who has multiple links has a different specialization from the one of the players with whom she is linked.

[9]Despite the fact that we denote the set of autarkic pairs with the same letter as a set of directed oriented links, we re-emphasize that the set of autarkic pairs is *undirected*.

$L_N \cup D_0$ *such that*

(i) $D_0 \subsetneq D$*;*

(ii) for all $i \in N$ *there is a distinct player* $j \in N$ *such that either* $(i, j) \in D$ *or* $(j, i) \in D$*;*

(iii) $(i, j) \in D$ *implies* $(j, i) \notin D$*.*

We define a set of potential links to be an exogenously given subset of the set of all possible relations between the players in N such that no two players have direct authority over each other and that, in addition, contains the autarkic positions for all players. The set of potential links defines the potential activities that can be carried out by players. Since it is a subset of the set of all possible links, it is designed to capture physical, institutional, or any other restrictions that may prohibit the occurrence of activities between certain players. A pair (N, D) is called a *directed activity structure*. In terms of our application one can think of the pair (N, D) as a social structure.

Next we introduce some technical definitions from network theory, which will be used later in defining an economy and for deriving the main theoretical results.

Let (N, D) be a directed activity structure. For convenience, when the direction of the link between two distinct players $i, j \in N$ present in D is not important we will use the underlying undirected network $\varDelta$ corresponding to D, *i.e.*, an undirected link between two players $i, j \in N$ is given by $\{i, j\} \in \varDelta$ if $(i, j) \in D$ or $(j, i) \in D$ or $i = j$. Recall that since D is a set of potential links, it cannot be the case that $\{(i, j), (j, i)\} \subseteq D$. As in the previous chapter we will use the shorthand notation ij for the undirected link $\{i, j\} \in \varDelta$.

The location of a player within a network is an important characteristic. Let N be a finite set of players and let $D \subseteq D_N$ be a set of potential links and let $\varDelta$ be the corresponding undirected network of D. Let $i \in N$ be a player such that there is a distinct player $j \in N$ with $ij \in \varDelta$. The *set of connected players* in D is given by $N(D) = \{i \in N \mid \text{ there exists } j \in N \text{ with } j \neq i \text{ such that } ij \in \varDelta\}$. By definition, since D is a set of potential links $N(D) = N$. We define player i's *neighborhood* in D as

$$N_i(D) = \{j \in N \mid j \neq i \text{ and } ij \in \varDelta\}.$$

We can also express the neighborhood of a player in terms of its link based analogue, *i.e.*,

$$L_i(D) = \{(i, j) \in D \text{ and } (k, i) \in D \mid j \neq i,\ j \in N_i(D) \text{ and } k \neq i,\ k \in N_i(D)\}.$$

We define a *path* between any two distinct players $i \in N$ and $j \in N$ in G as a sequence of distinct players $P_{ij}(D) = (i_1, i_2, \ldots, i_m)$ with $i_1 = i$, $i_m = j$, $i_k \in N$ and $i_k i_{k+1} \in \Delta$ for all $k \in \{1, \ldots, m-1\}$. We call a *cycle* a path of a player from herself to herself which contains at least two other distinct players, *i.e.*, $C(D) = (i_1, i_2, \ldots, i_m)$ with $i_1 = i$, $i_m = i$, $m \geq 4$, $i_k \in N$, and $i_k, i_{k+1} \in \Delta$ for all $k \in \{1, \ldots, m-1\}$. We also define paths which follow the direction of the network, we call such paths *directed paths*, *i.e.*, a directed path between two distinct players $i \in N$ and $j \in N$ in D is a sequence of distinct players $P^d_{ij}(D) = (i_1, i_2, \ldots, i_m)$ with $i_1 = i$, $i_m = j$, $i_k \in N$ and $(i_k, i_{k+1}) \in D$ for all $k \in \{1, \ldots, m-1\}$. Similarly, we define a *directed cycle* $C^d(D) = (i_1, i_2, \ldots, i_m)$ with $i_1 = i$, $i_m = i$, $m \geq 4$, $i_k \in N$ and $(i_k, i_{k+1}) \in D$ for all $k \in \{1, \ldots, m-1\}$. We emphasize that each (directed) cycle has length of at least three, *i.e.*, a cycle consists of at least three distinct links. Note that any directed cycle is a cycle, however, not every cycle is a directed cycle.

Furthermore, there may be players in N between whom there is no path in a set of potential links D; such players are located in different components of D. A network $d \subseteq D$ is a *component* of D if for all $i \in N(d)$ and $j \in N(d)$ there is a path $P_{ij}(D)$ connecting players i and j and for all $i \in N(d)$ and $j \in N(D)$, $ij \in \Delta$ implies that $ij \in \delta$ with δ being the undirected analogue of the component d. The *set of all components* in a directed network D is denoted by $\mathsf{c}(D) = \{d \mid d \subseteq D\}$. Note that $D = \cup_{d \in \mathsf{c}(D)} d$.

Last, we describe the preferences of players over her potential activities. Players have complete and transitive preferences over the players in their neighborhood given by the set of potential links D and over the autarkic relation. These preferences can be represented by a hedonic utility function. The hedonic utility function is an *indirect* utility function that captures the utility of the value generating activities. For instance, in the application of collective production the hedonic utility function is an indirect utility function that measures the utility of free time a player has when participating in collective production with another player. Let (N, D) be a directed activity structure. For every $i \in N$, there is a hedonic utility function $u_i \colon L_i(D) \cup \{ii\} \to \mathbb{R}$.[10] Let u be a profile of n hedonic utility functions. Let $\mathcal{U}$ be the set of permissible profiles of hedonic utility functions.

Below we present an example of collective production as outlined in Section 2.2. First we derive the hedonic utility profile in the state of autarky and then we derive individuals hedonic utility functions on the set of potential links. In the first case,

[10]Note that we do not require that a player has a higher utility from a link in which she has the authority than from a link in which another player has authority over her. Our goal is to keep the basic framework as general as possible.

autarky is a positive value generation state, hence, labor specialization and collective production do not occur between two or more distinct individuals because there are no conditions to develop social trust.

Example 2.3.2 The set of players is given by $N = \{1, 2, 3, 4\}$ individuals who engage in subsistence activities. Consider the collective production process described in Section 2.2.

Given the preference profile and the production possibility set each individual solves the following maximization problem.

$$\max_{\substack{0 \leqslant l_x \leqslant 1; \\ 0 \leqslant l_y \leqslant 1;}} \quad \phi(f) = f = 1 - l_x - l_y$$

subject to

$$\begin{aligned} y &\geq \bar{y}, \\ y &= x\, l_y, \\ x &= l_x^{\alpha} \\ l_x + l_y &\leq 1. \end{aligned}$$

The individual problem can be rewritten in terms of minimizing time spent in production:

$$\min_{\substack{0 \leqslant l_x \leqslant 1; \\ 0 \leqslant l_y \leqslant 1;}} \quad l_x + l_y$$

subject to

$$\begin{aligned} l_x^{\alpha} l_y &= \bar{y}, \\ l_x + l_y &\leq 1. \end{aligned}$$

Assuming the existence of a solution, the solution to the problem under autarky is given by $l_x(ii) = (\alpha\, \bar{y})^{\frac{1}{\alpha+1}}$ and $l_y(ii) = (\alpha^{-\alpha}\, \bar{y})^{\frac{1}{\alpha+1}}$, which yields $\phi(ii) = 1 - \bar{y}^{\frac{1}{\alpha+1}} \left(\alpha^{\frac{1}{\alpha+1}} + \alpha^{\frac{-\alpha}{\alpha+1}}\right)$ for all players $i \in N$.

Finally we set the hedonic utility in the state of autarky equal to the utility of free time when a player is self-supplying both the intermediate and the consumption good. So, $u_i(ii) = \phi(ii)$ for all players $i \in N$.

We do not consider any other potential activities in this example, since as discussed above any collective activity between two distinct players requires trust, which only develops under extreme circumstances of necessity to survive. ♦

Example 2.3.2 describes the behavior of players who under autarky can sustain the minimal subsistence level $\bar{y}$. That is, it is assumed that the individual maximization problem has an interior solution and thus every individual in a state of autarky is self-sufficient. This assumption is violated when the minimal subsistence level cannot be met by an individual producing alone, *i.e.*, when $\bar{y} > \frac{\alpha^{\alpha}}{(\alpha+1)^{\alpha+1}}$. Next we consider this case when players need to engage in collective production, hence, specialize in order to achieve the subsistence level.

Example 2.3.3 Consider the economy described in Example 2.3.2. Furthermore assume $\bar{y} > \frac{\alpha^{\alpha}}{(\alpha+1)^{\alpha+1}}$. Hence, players cannot achieve the subsistence level of consumption $\bar{y}$ in a state of autarky; however, two players producing together can obtain two times the subsistence level $\bar{y} \leq \frac{1}{2}$.[11] In order to achieve the minimum required level of consumption they need to exploit the increasing returns to scale in the production of the intermediate goods by having some players specialize in the production of the intermediate good while others in the production of consumption good. For convenience, we will refer to player specialized in the production of the intermediate good as "he" and those specialized in the production of the final good as "she". We assume that players 1 and 3 specialize in the production of the intermediate good while players 2 and 4 specialize in the production of the consumption good.

Consider the set of potential links $D = \{(1,4),(4,3),(2,1),(3,2), ii_{i\in N}\}$. Note that the set of potential links does not contain links between players who have the same specialization in production, *e.g.*, between two producer of the intermediate good, because such links are not value generating. The direction of a link in the set of potential links reflects "authority". A player who has authority over another player has decision power over that player's productive time. In return for the labor hours, the player who has authority over the other player provides her or him with the subsistence level of consumption.

We can distinguish between two cases. The first case is when a producer of the final consumption good has authority over a producer of an intermediate good. The optimization problem for the producer of the consumption good in which she chooses the amount of labor that the producer of the intermediate good should invests as well as her own labor investment is given as:

$$\max_{\substack{0\leqslant l_x\leqslant 1;\\ 0\leqslant l_y\leqslant 1;}} \quad \phi(f) = f = 1 - l_y$$

[11]Note that the maximum of the function $l_x^{\alpha}(1 - l_x)$ for $l_x \in [0, 1]$ is obtained at the point $l_x = \frac{\alpha}{1+\alpha}$.

subject to

$$y \geq 2\overline{y},$$
$$y = x\, l_y,$$
$$x = l_x^{\alpha}.$$

Since the player producing the consumption good has preferences only over her own free time and complete decision rights over the labor hours worked by the intermediate good producer, she will require the producer of the intermediate good to allocate all his endowment of time in the production of the intermediate good $l_x = 1$ and will herself contribute just as much as it is sufficient to produce the subsistence level for both players, namely, $l_y = 2\overline{y}$. This is the solution to the collective production by potential links $(2, 1)$ and $(4, 3)$. Hence, $l_{x,1}(2, 1) = l_{x,3}(4, 3) = 1$ $l_{y,2}(2, 1) = l_{y,4}(4, 3) = 2\overline{y}$ where $l_{z,i}$ denotes the amount of time player $i \in N$ invests in the production of the good $z \in \{x, y\}$. The utility levels from these links of each player are thus $\phi_1((2, 1)) = \phi_3((4, 3)) = 0$ and $\phi_2((2, 1)) = \phi_4((4, 3)) = 1 - 2\overline{y}$.

The second case is when a producer of the intermediate good has a decision power over the hours worked by the player specialized in the production of the consumption good. In return for the labor hours worked by the producer of the consumption good, the producer of the intermediate good provides her with an amount of the consumption good equal to the subsistence level $\overline{y}$. The optimization problem of the producer of the intermediate good in which he chooses the amount of labor that the producer of the consumption good should invest as well as his own labor investment is given as:

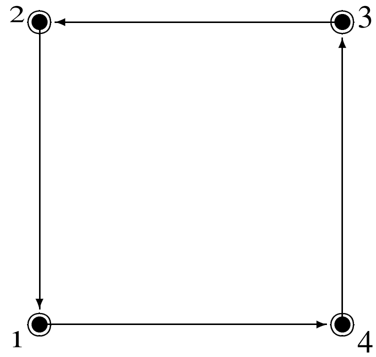
$$\max_{\substack{0 \leqslant l_x \leqslant 1; \\ 0 \leqslant l_y \leqslant 1;}} \quad \phi(f) = f = 1 - l_x$$

subject to

$$y \geq 2\overline{y},$$
$$y = x\, l_y,$$
$$x = l_x^{\alpha}.$$

Since the player producing the intermediate good has preferences only over his own free time and complete decision rights over the labor hours worked by the consumption good producer, he will require from the producer of the consumption good to allocate all her endowment of time into production, and he will himself contribute just as much as it is sufficient to produce the subsistence level for both players, namely, $l_x = (2\bar{y})^{\frac{1}{\alpha}}$. This is the solution to the collective production by potential links $(1,4)$ and $(3,2)$. Hence, $l_{x,1}(1,4) = l_{x,2}(3,2) = (2\bar{y})^{\frac{1}{\alpha}}$, $l_{y,2}(3,2) = l_{y,3}(1,4) = 1$. The utility levels of each player from these links are thus $\phi_1((1,4)) = \phi_3((3,2)) = 1-(2\bar{y})^{\frac{1}{\alpha}}$ and $\phi_2((3,2)) = \phi_4((1,4)) = 0$.

Due to the increasing returns to scale in the production of the intermediate good, it is easy to see that the first case is Pareto dominating the second case for $\bar{y} < \frac{1}{2}$ and the two cases yield the same total utility when $\bar{y} = \frac{1}{2}$.



Given the assumed specialization and the direction of authority, one can derive the hedonic utility function such that $u_i((i,j)) = \phi_i((i,j))$ and $u_j((i,j)) = \phi_j((i,j))$ for any link $(i,j) \in D$ and any two distinct players $i, j \in N$. Since in the state of autarky a player cannot reach a subsistence level we will denote this state as $-\infty$.

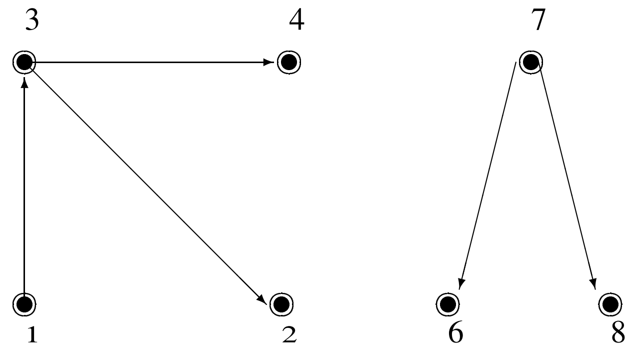
(i,j)	$ii_{i\in N}$	(1,4)	(2,1)	(3,2)	(4,3)
$u_1((i,j))$	$-\infty$	$1-(2\bar{y})^{\frac{1}{\alpha}}$	0	–	–
$u_2((i,j))$	$-\infty$	–	$1-2\bar{y}$	0	–
$u_3((i,j))$	$-\infty$	–	–	$1-(2\bar{y})^{\frac{1}{\alpha}}$	0
$u_4((i,j))$	$-\infty$	0	–	–	$1-2\bar{y}$

Note that the way we have constructed the example, player 1 prefers to engage in production with player 4, player 2 prefers to engage in production with player 1, player 3 prefers to engage in production with player 2, and player 4 prefers to engage in production with player 3. ♦

In Example 2.3.3 we have derived the hedonic utility functions of players.[12] Our goal is to develop a framework in which collective production is carried out in teams of two or more individuals. This framework is developed in the following section.

[12]Example 2.3.3 may be used as another application of the framework presented in the previous chapter.

Figure 2.2: A Set of Potential Links with Star Components



2.4 A Team Economy

The set of potential links and hedonic utility functions described in the previous section are used as a basis for developing a framework where more complex activity patterns can be studied. In particular, the complex activity patterns that we study are represented by networks consisting of star components.

Definition 2.4.1 *Let (N, D) be an activity structure. We say that the subset $S^\star \subseteq D$ has a **star structure** or that it is a **star** if there is at most one player $i \in N(S^\star)$ such that $|N_i(S^\star)| > 1$ and that for all $j \in N(S^\star) \setminus \{i\}$ it holds that $N_j(S^\star) = \{i\}$.*

Let (N, D) be an activity structure. We denote by $\mathcal{S}(D)$ the set of all subsets of D that have a star structure, *i.e.*, $\mathcal{S}(D) = \{S^\star \subseteq D \mid S^\star \text{ is a star}\}$. The definition of a star includes subsets in which there are only two connected players, hence, $\{(i, j)\}_{(i,j)\in D} \subseteq \mathcal{S}(D)$.

A graphical representation of a set of potential links which contains two components with star structure is given in Figure 2.2. As shown, in the general framework, it may be that the player in the star component who has multiple links has authority in some links while other players have authority over her in other links. For example, in the pattern $\{(1, 3), (3, 2), (3, 4)\}$ player 3 has multiple links and she has authority over players 2 and 4, however, player 1 has authority over her.

Complex activities are represented by a subset of the set of potential links whose components have a star structure. To such activities we refer as star patterns.

Definition 2.4.2 *Let (N, D) be a directed activity structure. A **star pattern** $H^\star \subseteq D$ is a subset of the set of potential links such that each player $i \in N$ is either connected in a component of a star strucutre of $H^\star$ or stays autarkic, i.e., $\mathsf{c}(H^\star) \subseteq \mathcal{S}(D)$.*

The class of all possible star patterns on D is denoted by $\mathcal{H}^\star(D) = \{H^\star \mid \mathsf{c}(H^\star) \subseteq \mathcal{S}(D)\}$.

Let (N, D) be an activity structure and let $H^\star$ be a star pattern. We define a *star central player* to be a player $i \in N(d^\star)$ for some $d^\star \in \mathsf{c}(H^\star)$ such that $|N_i(d^\star)| > 1$ if $|N(d^\star)| > 2$ and all players $i \in N(d^\star)$ if $|N(d^\star)| = 2$. The *set of star central players* in a star pattern $H^\star$ is denoted by $N^\star(H^\star)$.

Last, we discuss the preferences of players defined on the possible star patterns. To do so, we use an indirect value function, which is based on a player's hedonic preferences over potential links represented by her indirect utility function u_i for some player $i \in N$. In a star pattern a player can be linked to another player either directly, by activating their potential link, or indirectly via the star central player in the component. This is reflected in the *hedonic team utility function* to which we will refer as *team utility function* for brevity. Let $\mathcal{S}_i(D)$ be the set of all possible subsets of the set of potential links D in which player i is connected, then the team utility function of any player $i \in N$ is given by $v_i \colon \mathcal{S}_i(D) \cup \{ii\} \to \mathbb{R}$ such that for some subset $S_i \in \mathcal{S}_i(D) \cup \{ii\}$:

$$v_i(S_i) \begin{cases} = u_i(ii) & \text{if } S_i = ii; \\ = u_i((i,j)) & \text{if } (i,j) \in S_i \text{ and } |N_i(S_i)| = 1; \\ = u_i((j,i)) & \text{if } (j,i) \in S_i \text{ and } |N_i(S_i)| = 1; \\ \geq \sum_{j\colon (i,j)\in S_i} u_i((i,j)) + \sum_{j\colon (j,i)\in S_i} u_i((j,i)) & \text{if } |N_i(S_i)| > 1. \end{cases}$$

The team utility function captures the value generating abilities of a player being autarkic or being connected in a star component. The underlying value generation process may take various forms. The collective production process analyzed in the application is one example of such process. The team utility function as assumed requires that a player has value only from links with players with whom she is linked *directly*.[13] Furthermore, it is allowed that a star central player receives some extra value above the sum of her utility from the relations with players in her neighborhood. Hence the value function satisfies a *superadditivity property*, i.e., $v_i(S_i \cup T_i) \geq v_i(S_i) +$

[13] Alternatively, it may be that a player has utility from being linked to another player *indirectly*, *e.g.*, $v_i(H^\star) = \sum_{j\in N\colon j\neq i} \delta^{t(ij)-1} u_i((i,j)) + \sum_{j\in N\colon j\neq i} \delta^{t(ji)-1} u_i((j,i))$ where $\delta \in (0,1)$ is the distance discounting parameter and $t(ij)$ is the geodesic distance between players i and j such that it equals the number of links in the shortest path between the players. Given our definition of a star pattern the distance between any two players can be maximum two links. If these players are not connected in the set of potential links D the geodesic distance is set to infinity. Such type of utility function has been used by Jackson and Wolinsky (1996) in a undirected network setting when discussing the *Connections Model*.

$v_i(T_i)$ for all $S_i, T_i \in \mathcal{S}_i(D)$ with $S_i \cap T_i = \emptyset$. The superadditivity property reflects synergies which are assumed to be allocated to the star central player who acts as a coordinator in the value generation process.

The profile of team utility functions for all players is denoted by v. The set of all permissible profiles of team utility functions defined on the activity structure (N, D) is denoted by $\mathcal{V}(D)$. In a star pattern each player is engaged in a relational activity. For ease of notation we define the *indirect* team utility level that a players obtains when participating in a pattern $H^\star$ as $v_i(H^\star)$. For a given star pattern the indirect team utility levels of all players are summarized in a ***team utility profile*** $v(H^\star) = (v_1(H^\star), \ldots, v_n(H^\star))$.

Next, we define an economy in which activity patterns of a star structure are analyzed in terms of their stability properties.

Definition 2.4.3 *A **team economy** is defined to be a quadruple $\mathbb{E}^T = (N, D, u, v)$ in which (N, D) is a directed structure, $u \in \mathcal{U}(D)$ is a profile of utility functions on D and $v \in \mathcal{V}(D)$ is a profile of team utility functions on D.*

A team economy is defined to be the set of potential actions and the potential value that a player can obtain in activating one of her possible activity patterns. A star pattern defined on this economy represents a *de facto* activated activity pattern by all individuals. We analyse a team economy in terms of the stability properties of the possible activity patterns.

Definition 2.4.4 *Let $\mathbb{E}^T = (N, D, u, v)$ be a team economy. Let $H^\star \in \mathcal{H}^\star(D)$. We say that the star pattern $H^\star$ is **stable** for the economy (N, D, u, v) if it satisfies the individual rationality [IR] and two no blocking [NB] and [NB*] conditions as specified below:*

IR *for all $i \in N$ it holds that $v_i(H^\star) \geqslant u_i(ii)$;*

NB *for all $i, j \in N$ with $(i, j) \in D \setminus H^\star$ and $i, j \notin N^\star(H^\star)$:*

$$v_i(H^\star) < u_i((i, j)) \quad \textit{implies} \quad u_j((i, j)) \leq v_j(H^\star);$$

NB* *for all distinct players $i, j \in N$ with $(i, j) \in D \setminus H^\star$*

(i) and $j \in N^\star(H^\star)$:

$$v_i(H^\star) < u_i((i, j)) \quad \textit{implies} \quad \max\{v_j(H^\star \cup \{(i, j)\}), u_j((i, j))\} \leq v_j(H^\star);$$

(ii) or $i \in N^\star(H^\star)$*:*

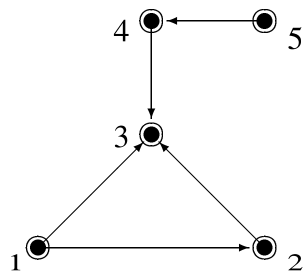
$$v_j(H^\star) < u_j((i,j)) \quad \textit{implies} \quad \max\{v_i(H^\star \cup \{(i,j)\}), u_i((i,j))\} \le v_i(H^\star).$$

The condition [IR] is a standard condition for stability that allows players to opt out of an economic activity if she is better off being autarkic. The condition [NB] rules out blocking out possibilities between two distinct players, none of whom is a star central player in the pattern $H^\star$. It requires that there are no pairs of players who prefer to be linked to each other rather than to the players with whom their are linked in the pattern $H^\star$. The condition [NB*] rules out blocking possibilities between two distinct players at least one of whom is a star central player in the pattern $H^\star$. Note the greater deviational possibilities for star central players: such players can add a links with another player with or without severing her current links. This condition requires that there are no two distinct players at least one of whom is a star central player who want to be linked to each other irrespective of whether (one of)[14] the star central player(s) keeps her existing links or not.

To illustrate the concepts here we provide an abstract example.

Example 2.4.5 Let $N = \{1,2,3,4,5\}$ and $D = \{(1,2),(1,3),(2,3),(4,3),(5,4)\} \cup \{ii \mid i = 1,\dots,5\}$ as shown on the figure below. Every player has utility of zero when she is in a state of autarky. The hedonic utility functions over the set of potential links between distinct players are given in the table below.

(i,j)	(1,2)	(1,3)	(2,3)	(4,3)	(5,4)
$u_1((i,j))$	2	1	–	–	–
$u_2((i,j))$	1	–	2	–	–
$u_3((i,j))$	–	1	2	1	–
$u_4((i,j))$	–	–	–	2	1
$u_5((i,j))$	–	–	–	–	2



The set of all subsets of D that have star structure is given below:

$$\begin{aligned}\mathcal{S}(D) = \; & \{\{(1,2)\},\{(1,3)\},\{(2,3)\},\{(4,3)\},\{(5,4)\},\{(1,2),(1,3)\},\{(1,2),(2,3)\},\\ & \{(1,3),(2,3)\},\{(1,3),(4,3)\},\{(2,3),(4,3)\},\{(4,3),(5,4)\},\{(1,3),(2,3),\\ & (4,3)\}\}.\end{aligned}$$

[14]Recall that in a star pattern at most one player has more than one partner in each component. Hence, it is not allowed in a star pattern to have two star central players deviating by keeping their existing links and at the same time establishing a link between each other.

Thus, the class of star patterns defined on the set D is:

$$\begin{aligned}\mathcal{H}^\star = \; & \{\{(1,2),\{3,3\},\{4,4\},\{5,5\}\};\ \{(1,2),(1,3),\{4,4\},\{5,5\}\};\ \{(1,2),\{3,3\},(5,4)\};\\ & \{\{1,1\},(2,3),\{4,4\},\{5,5\}\};\ \{(1,3),(2,3),\{4,4\},\{5,5\}\};\ \{(1,2),(1,3),(5,4)\};\\ & \{(1,3),\{2,2\},\{4,4\},\{5,5\}\};\ \{(1,3),\{2,2\},(4,3),\{5,5\}\};\ \{(1,3),(2,3),(5,4)\};\\ & \{(1,2),(2,3),\{4,4\},\{5,5\}\};\ \{\{1,1\},\{2,2\},(4,3),\{5,5\}\};\ \{(1,2),(4,3),\{5,5\}\};\\ & \{\{1,1\},(2,3),(4,3),\{5,5\}\};\ \{(1,3),(2,3),(4,3),\{5,5\}\};\ \{\{1,1\},(2,3),(5,4)\};\\ & \{(1,2),(2,3),(5,4)\};\ \{(1,3),\{2,2\},(5,4)\};\ \{(1,2),(4,3),\{5,5\}\};\text{and } \{ii\}_{i\in N}\ \}.\end{aligned}$$

For any team utility function satisfying the superadditivity property $v \in \mathcal{V}(D)$ based on the hedonic utility function u as given above. There are two stable patterns: $\{(1,2),(2,3),(5,4)\}$ and $\{(1,3),(2,3),(4,3),\{5,5\}\}$. With respect to the pattern $\{(1,2),(2,3),(5,4)\}$ player 4 would prefer to severe her link with player 5 and activate the link $(4,3)$, however, player 3 prefers her current activity in the component $\{(1,2),(2,3)\}$ to the link $(4,3)$. Similarly in the pattern $\{(1,3),(2,3),(4,3),\{5,5\}\}$ player 5 would prefer to be linked with 4 in the component $(5,4)$, however, player 4 prefers her current link with player 3 to the one with player 5.
There are no other stable patterns. For instance, consider the star patterns in which players 2 and 3 are not linked. Such patterns cannot be stable because player 2 prefers to be linked to player 3 and player 3 prefers to be linked to player 2 more than to any other player. Moreover, in all other patterns in which players 2 and 3 are linked but patterns $\{(1,2),(2,3),(5,4)\}$ and $\{(1,3),(2,3),(4,3),\{5,5\}\}$, the non-blocking conditions are not satisfied. Consider, for example, the pattern $H^\star = \{(1,3),(2,3),(5,4)\}$. It is not stable because player 4 prefers to be linked to player 5 than to player 5, $v_4(H^\star) = u_4((5,4)) < u_4((4,3))$, and player 3 prefers to be a star central player in the component $\{(1,3),(2,3),(4,3)\}$ than in the star component $\{(1,3),(2,3)\}$, $v_3(H^\star) < v_3(H^\star) + u_3((4,3)) \leq v_3(\{(1,3),(2,3),(4,3)\})$. ♦

Below, we proceed with a discussion of the team economy based on collective production.

Example 2.4.6 Consider the economy described in Example 2.3.3. Next, we present the production process when players are connected in a star pattern. First, we will derive the team utility profiles with respect to all possible star patterns. Then, we show that in this team economy, there are no stable star patterns.

First, consider the star component $\{(2,1),(3,2)\}$. Recall that the collective production in the link $(2,1)$ is governed by player 2 who is a producer of the consumption good and that the link $(3,2)$ is governed by player 3 who is a producer of the intermediate good. Furthermore, in the star component $\{(2,1),(3,2)\}$ three units of the subsistence level $\bar{y}$ must be produced. The team utility function of each player is set to equal her/his utility from free time, *i.e.*, $v_i(H^\star) = \phi_i(d_i)$ for $i \in N$ where $d_i \subseteq H^\star$ is the component of the activity pattern $H^\star$ in which player i is linked $i \in N(d_i)$. To keep the utility levels of players 1 and 3 the same[15] as in the links $(2,1)$ and $(3,2)$, it is assumed that they contribute to the collective production process in $\{(2,1),(3,2)\}$ the same time as in the production processes of $(2,1)$ and $(3,2)$, respectively, *i.e.*, player 1 contributes $l_{x,1}(2,1) = 1$ and player 3 contributes $l_{x,3}(3,2) = (2\bar{y})^{\frac{1}{\alpha}}$. Player 2 as a star central player has to contribute the sufficient amount of labor necessary to produce three times the subsistence level $\bar{y}$ given the amount of intermediate goods produced by players 1 and 3, which is $l_{y,2}(\{(2,1),(3,2)\}) = 3\bar{y}\left(1 + (2\bar{y})^{\frac{1}{\alpha}}\right)^{-\alpha}$. The value levels of each player in the star component $\{(2,1),(3,2)\}$ can be calculated in straightforward manner: $v_1(\{(2,1),(3,2)\}) = u_1((2,1)) = 0$, $v_2(\{(2,1),(3,2)\}) = \phi_2(\{(2,1),(3,2)\}) = 1 - 3\bar{y}\left(1 + (2\bar{y})^{\frac{1}{\alpha}}\right)^{-\alpha}$ and $v_3(\{(2,1),(3,2)\}) = u_3((3,2)) = 1 - (2\overline{(y)})^{\frac{1}{\alpha}}$. Simulations show that $v_2(\{12,23\}) > u_2(12) + u_2(23) = u_2(12)$ for $\alpha \in [1,10]$ and $\bar{y} \in (\frac{\alpha^\alpha}{(\alpha+1)^{\alpha+1}}, \frac{1}{2}]$.

Next, consider the star component $\{(2,1),(1,4)\}$ in which player 1 is a star central player. Recall that players 2 and 4 are producers of the consumption good and player 1 is a producer of the intermediate good. Furthermore, players 1 and 4 contribute the same amount of time in the collective production process of the pattern $\{(2,1),(1,4)\}$ as they did in the collective productions $(2,1)$ and $(1,4)$, namely, $l_{y,2}(\{(2,1),(1,4)\}) = 2\bar{y}$ and $l_{y,4}(\{(2,1),(1,4)\}) = 1$. The star central player, player 1, contributes the minimum amount of labor in the production of the intermediate good such that given the amount of labor contributed by players 2 and 4, they can produce three times the subsistence level $\bar{y}$. Player 1 hence allocates $l_{x,1}(\{(2,1),(1,4)\}) = \left(\frac{3\bar{y}}{1+2\bar{y}}\right)^{\frac{1}{\alpha}}$ hours in the production of the intermediate good. The team utility levels of players in the pattern $\{(2,1),(1,4)\}$ are $v_1(\{(2,1),(1,4)\}) = 1 - \left(\frac{3\bar{y}}{1+2\bar{y}}\right)^{\frac{1}{\alpha}}$, $v_2(\{(2,1),(1,4)\}) = u_2((2,1)) = 2\bar{y}$ and $v_4(\{(2,1),(1,4)\}) = u_4((1,4)) = 0$. Note that for $\bar{y} > \frac{1}{4}$, $u_1(\{(2,1),(1,4)\}) > u_1((2,1)) + u_1((1,4)) = u_1((1,4))$.

Given the symmetry of the set of potential links, the allocation of labor in the component $\{(3,2),(4,3)\}$ will be analogous to the one in the component $\{(2,1),(1,4)\}$, and the allocation of labor in the component $\{(1,4),(4,3)\}$ will be analogous to the one in the component $\{(2,1),(3,2)\}$.

The team utility profiles of the star patterns in which players are linked in pairs

[15] Recall that by the definition of the team utility function a player does not have utility from an indirect link with another player.

Table 2.1: Team Utility Profiles in Example 2.4.6

$H^\star$	$v_1(H^\star)$	$v_2(H^\star)$	$v_3(H^\star)$	$v_4(H^\star)$
$ii_{i\in N}$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
$\{(2,1),\{3,3\},\{4,4\}\}$	0	$1-2\overline{y}$	$-\infty$	$-\infty$
$\{\{1,1\},\{2,2\},(4,3)\}$	$-\infty$	$-\infty$	0	$1-2\overline{y}$
$\{(2,1),(4,3)\}$	0	$1-2\overline{y}$	0	$1-2\overline{y}$
$\{(1,4),\{2,2\},\{3,3\}\}$	$1-(2\overline{y})^{\frac{1}{\alpha}}$	$-\infty$	$-\infty$	0
$\{\{1,1\},(3,2),\{4,4\}\}$	$-\infty$	0	$1-(2\overline{y})^{\frac{1}{\alpha}}$	$-\infty$
$\{(1,4),(3,2)\}$	$1-(2\overline{y})^{\frac{1}{\alpha}}$	0	$1-(2\overline{y})^{\frac{1}{\alpha}}$	0
$\{(2,1),(1,4),\{3,3\}\}$	$1-\left(\frac{3\overline{y}}{1+2\overline{y}}\right)^{\frac{1}{\alpha}}$	$1-2\overline{y}$	$-\infty$	0
$\{(2,1),(3,2),\{4,4\}\}$	0	$1-\frac{3\overline{y}}{\left(1+(2\overline{y})^{\frac{1}{\alpha}}\right)^{\alpha}}$	$1-(2\overline{y})^{\frac{1}{\alpha}}$	$-\infty$
$\{\{1,1\},(3,2),(4,3)\}$	$-\infty$	0	$1-\left(\frac{3\overline{y}}{1+2\overline{y}}\right)^{\frac{1}{\alpha}}$	$1-2\overline{y}$
$\{(1,4),\{2,2\},(4,3)\}$	$1-(2\overline{y})^{\frac{1}{\alpha}}$	$-\infty$	0	$1-\frac{3\overline{y}}{\left(1+(2\overline{y})^{\frac{1}{\alpha}}\right)^{\alpha}}$

have been calculated in Example 2.3.3 as these in fact are based on the hedonic utility functions. Note that in a star pattern $H^\star$ for a player i who is connected to exactly one distinct player in $H^\star$, *e.g.*, $N_i(H^\star) = \{j\}$ with $j \in N$, it holds that $v_i(H^\star) = u_i(ij)$. The team utility profiles in all possible star patterns are summarized in Table 2.1.

We conjecture that in this example, there is no stable star pattern for a relevant range of the parameter values. For $\overline{y} \in (\frac{1}{4}, \frac{1}{2})$ and $\alpha \in [1, 10]$, we can show that the NB conditions are not satisfied. To see why this is the case, first note that for the specified range of the parameters a player's most preferred pattern is one in which she or he is the only star central player. Furthermore, every player prefers to be linked with another player over being autarkic. Next, consider the pattern $\{(2,1),(3,4)\}$. It is not stable because for players 1 and 4 the no-blocking condition [NB*] is not satisfied, *i.e.*, $v_1\{(1,4),(4,3)\}) > v_1((2,1))$ and $v_4(\{(1,4),(4,3)\}) > v_4((4,3))$. Furthermore, consider the star pattern $\{(1,4),\{2,2\},(4,3)\}$. It is not stable because players 2 and 3 do not satisfy the no-blocking condition [NB], *i.e.*, $v_2((2,3)) > v_2(\{2,2\})$ and $v_3((3,2)) > v_3(\{(1,4),(4,3)\})$. Similarly, one can show that no other pattern is stable. ♦

2.5 Existence of Stable Patterns

In Example 2.4.6 we have illustrated a situation in which there is no stable star pattern. This raises the question whether there is a sufficient restriction on the set of potential links that ensures the existence of stable pattern in a given team economy. The answer for the general case is given in the following theorem and for the application concerning collective production it is given in Proposition 2.5.5.

Theorem 2.5.1 *Let (N, D, u, v) be a a team economy. There exists a stable star pattern in (N, D, u, v) if the set of potential activities D does not contain a cycle or if it contains a cycle with a number of connected players $m - 1 = 3s$ with $s \in \{2, 3, \ldots\}$.*

Note that the sufficient condition of existence does not take into account the direction of the cycle, *i.e.*, it requires the absence of *any* cycles; not only *directed* ones. This is because in our general framework we have not assumed any relation between the direction of a link and the hedonic utility that a player has from this link. As we will see in Proposition 2.5.5 when we discuss the sufficient conditions for the existence of a stable pattern in our application, it is the presence of *directed cycles* that matters. This is because in collective production there is a direct relation between the direction of a link and the utility of the players connected by this link.

Before proceeding to the presentation of a constructive proof of Theorem 2.5.1, we need a supplementary result. Let (N, D) be an activity structure and let $u \in \mathcal{U}$ be a profile of hedonic utility functions, we denote by $B_i(N, D, u) = \{j \in N \text{ with } ij \in \varDelta \mid u_i(ij) \geq u_i(ik) \text{ for all } k \in N \text{ such that } ik \in \varDelta\}$ the *set of most preferred partners* of player i for all $i \in N$ where $\varDelta$ is the undirected network underlying the set of potential links D.

Lemma 2.5.2 *Let (N, D, u, v) be a team economy and let the set of potential links D does not contain a cycle. Then there is a pair of players $i, j \in N$ such that $j \in B_i(N, D, u)$ and $i \in B_j(N, D, u)$.*

Proof. Suppose not. Hence, for all players $i, j \in N$ such that $i \in B_j(N, D, u)$ it holds that $j \notin B_i(N, D, u)$. Consider player $i \in N$ and without loss of generality assume $B_i(N, D, u) = \{j\}$, it must be that $j \neq i$. Next, consider the set of most preferred partners of player j. Without loss of generality assume $B_j(N, D, u) = \{k\}$. It must be that $k \notin \{i, j\}$. Next, consider the set of most preferred partners of player k. Without loss of generality assume $B_k(N, D, u) = \{l\}$. It must be that $l \notin \{j, k\}$, moreover $l \neq i$ otherwise D contains a cycle. Hence, $l \notin \{i, j, k\}$. By continuing in a similar fashion, given that the player set N is finite, we establish a contradiction. ■

Proof of Theorem 2.5.1. Let $\mathbb{E}^T = (N, D, u, v)$ be a team economy. We will consider two cases. First we study sets of potential links that do not contain a cycle. Then we study sets of potential links that contain cycles with a number of connected players equal to $3s$ with $s \in \{2, 3, \ldots\}$. We will slightly abuse notation and we will use the nondirect equivalent for a link in constructing the stable star pattern.

Case I Let the set of potential links D not contain a cycle. We will construct a star pattern that satisfies the stability conditions.

Step I: Consider a set of mutually disjoint links $\{i, j\} \in D$ such that $i \in B_j(N, D, u)$ and $j \in B_j(N, D, u)$, and denote this set by $H' \subseteq D$. Note that by Lemma 2.5.2 $H' \neq \emptyset$. Moreover, no players linked in H' would want to delete their link to form a link with another player. Let $M' = \{i \in N \mid |N_i(H')| = 1\}$ be the set of all players who are linked to another distinct player in H'. The player set M' contains those players who are linked in H' and who may have in the future more than one link in a stable star pattern.

Step II: Next, consider a player i who is not linked in H' such that $B_i(N, D, u) \cap M' \neq \emptyset$. If there exists a player $j \in B_i(N, D, u) \cap M'$ such that $u_j(ij) > 0$, construct $H'' = H' \cup \{ij\}$ and $M'' = M' \setminus N_j(H')$. Note that since v satisfies the superadditivity property for each player i, $u_j(ij) > 0$ is sufficient to ensure that $v_j(H' \cup \{ij\}) > v_j(H')$. Players linked in H'' will not want to delete their links and players in M'' may have more than one link in a star pattern. If there is no player $j \in B_i(N, D, u) \cap M'$ such that $u_j(ij) > 0$, choose another player k who is not linked in H such that $B_k(N, D, u) \cap M' \neq \emptyset$.

Continue in the same fashion until there is no player i who is not matched in H^ν and $B_i(N, D, u) \cap M^\nu \neq \emptyset$ and there exists a player $j \in B_i(N, D, u) \cap M^\nu$ such that $u_j(ij) > 0$ where ν is the index of the preceding iterations. That is after the last iteration which is a finite number since N is finite, the only players not linked in H^ν are (1) these players whose best partners are not linked in H^ν and (2) these players some of whose best partners are linked in H^ν but the partners linked in H^ν do not want to add a link with them.

Step III: If there is no player i who is not linked by the last iteration in H^ν and $B_i(N, D, u) \cap M^\nu \neq \emptyset$ and there is a player $j \in B_i(N, D, u) \cap M^\nu$ such that $u_j(ij) > 0$, consider all disjoint links $\{i, j\} \in N \setminus (N(H^\nu) \cup \{\{ii\} \mid ii \in H^\nu\})$ who are not linked in H^ν such that player $i \in B_j(N \setminus (N(H^\nu) \cup \{\{ii\} \mid ii \in H^\nu\}), D, u)$ and player $j \in B_i(N \setminus (N(H^\nu) \cup \{\{ii\} \mid ii \in H^\nu\}), D, u)$ and denote this set by $G \subseteq D \setminus H^\nu$. Note that by Lemma 2.5.2, $G \neq \emptyset$. Moreover, no players linked in $\tilde{H} = H^\nu \cup G$ would want to delete a link or can form a blocking pair with another player. Let $T = \{i \in N \mid |N_i(G)| = 1\}$ be the set of all players who are linked to another distinct player in G. Let $\tilde{M} = M^\nu \cup T$.

The player set $\tilde{M}$ contains those players who are linked in $\tilde{H}$ who may have more than one link in a stable star pattern.
Step IV: Set $H' = \tilde{H}$ and $M' = \tilde{M}$. Go back to step II. Continue in the same fashion until there are no more players in step III. The process is finite since the player set N is finite.

Thus constructed the star pattern H is stable: players are linked to their most preferred partners out of the set of players who are also willing to be linked with them.

Case II Let the set of potential links D contain a cycle $C = (i_1, \dots, i_m)$ with $i_{k-1}i_k \in \Delta$ where Δ is the nondirected equivalent of the set D and $m \geq 7$ with $m - 1 = 3s$. Depending on the profile of team utility functions, we will distinguish two sub-cases.

Case II.1 First, consider a profile of team utility functions $v \in \mathcal{V}(D)$, such that (i) either there are two consecutive players along the cycle's path $i_{k-1}, i_k \in C$ for some $k = 1, \dots, m-1$ with $i_0 = i_{m-1}$ such that $i_{k-1} \in B_{i_k}(N, D, u)$ and $i_k \in B_{i_{k-1}}(N, D, u)$, (ii) or there is a pair of players one of whom is on cycle path and the other other not, *i.e.*, $i_k \in C$[16] for some $k = 1, \dots, m-1$ and $j \notin C$ such that $j \in B_{i_k}(N, D, u)$ and $i_k \in B_j(N, D, u)$. Then, we can use the algorithm described in Case I for constructing a stable star pattern since the profile of team utility functions ensures that the sets H' and G constructed in Step I and Step III, respectively, are not empty given the profile of team utility functions.

Case II.2 Last, consider a profile of team utility functions $v \in \mathcal{V}(D)$ such that there are no consecutive players along he cycle path $i_{k-1}, i_k \in C$ for some $k = 1, \dots, m-1$ with $i_0 = i_{m-1}$ such that $i_{k-1} \in B_{i_k}(N, D, u)$ and $i_k \in B_{i_{k-1}}(N, D, u)$, nor is there a pair of players one of whom is on cycle path and the other other not, *i.e.*, $i_k \in C$ for some $k = 1, \dots, m-1$ and $j \notin C$ such that $i_j \in B_{i_k}(N, D, u)$ and $i_k \in B_j(N, D, u)$. Then, without loss of generality[17], we can assume that $u_{i_k}(i_k i_k) \leq u_{i_k}(i_{k-1} i_k) < u_{i_k}(i_k, i_{k+1})$ for all $k = 1, \dots, m-1$ with $i_0 = i_{m-1}$.

A star pattern $H^\star$ that contains s number of components of the following type $\{\{i_1 i_2, i_2 i_3\},\ \{i_4 i_5, i_5 i_6\}, \dots \{i_{m-3} i_{m-2}, i_{m-2}, i_{m-1}\}\} \subseteq H^\star$ and all other players are linked following the algorithm presented in Case I is stable. All players who are not linked to their most preferred partner have their most preferred partner linked to her own most preferred partner, hence, they will not sever their links, moreover, these players are not star central players, hence, they cannot add a link without severing an existing link. ■

[16] By slightly abusing notation here C denotes the set of players in the sequence of players connected in the cycle.

[17] Alternatively, the profile of team utility functions $v \in \mathcal{V}(D)$ must be such that $u_{i_k}(\{i_k, i_k\}) \leq u_{i_k}(\{i_k, i_{k+1}\}) < u_{i_k}(\{i_{k-1}, i_k\})$ for all $k = 1, \dots, m-1$ with $i_0 = i_{m-1}$.

Example 2.5.3 Consider the economy discussed in Examples 2.3.3 and 2.4.6. We modify the economy described in these examples in such a way that a stable star pattern exists. For this purpose we modify the direction of authority in the set of potential links as $D = \{(1,4),(2,1),(2,3),(3,4)\}$.

In this setting the collective production processes in the matchings $(2,1)$ and $(2,3)$ are symmetric where player 2 has a decision power over the hours worked by players 1 and 3, respectively, and so are the collective production processes in the links $(1,4)$ and $(3,4)$, where, respectively, players 1 and 3 have decision power over the hours worked by player 4. These problems have already been analyzed in Example 2.3.3, hence, we know that $l_{x,1}(2,1) = l_{x,3}(2,3) = l_{y,4}(1,4) = l_{y,4}(3,4) = 1$, $l_{x,1}(1,4) = l_{x,3}(3,4) = (2\overline{y})^{\frac{1}{\alpha}}$, and $l_{y,2}(2,1) = l_{y,2}(2,3) = 2\overline{y}$.

Next, we discuss the collective production processes in the possible star components in which more than two players are linked. First consider the star component $\{(2,1),(2,3)\}$. The team utility of players 1 and 3 should be the same as in the links $(2,1)$ and $(2,3)$, hence, $l_{x,1}\{(2,1),(2,3)\} = l_{x,1}(2,1) = 1$ and $l_{x,3}\{(2,1),(2,3)\} = l_{x,3}(2,3) = 1$. Since collectively these players have to produce three units of the subsistence level $\overline{y}$, player 2 spends the minimum hours in the production of the consumption good such that given the amount of the intermediate good produced by players 1 and 3, they produce three times the subsistence level of the of the consumption good. So, $l_{y,2}(\{(2,1),(2,3)\}) = 2^{-\alpha}(3\overline{y})$. The utility levels from free time can be computed in a straightforward way $\phi_1(\{(2,1),(2,3)\}) = \phi_3(\{(2,1),(2,3)\}) = 0$ and $\phi_2(\{(2,1),(2,3)\}) = 1 - 2^{-\alpha}(3\overline{y})$.

Next, consider the star component $\{(1,4),(3,4)\}$, in which both producers of the intermediate good have authority over the producer of the consumption good and a star central player, player 4. We know from the analysis of collective production in $(1,4)$ and $(3,4)$ that $l_{x,1}(\{(1,4),(3,4)\}) = l_{x,1}(1,4) = (2\overline{y})^{\frac{1}{\alpha}}$ and $l_{x,3}(\{(1,4),(3,4)\}) = l_{x,3}(3,4) = (2\overline{y})^{\frac{1}{\alpha}}$. Player 4 can utilize her position as a star central player in which she acts as a coordinator of the collective production process, and she only works as much as to produce three times the subsistence level given the amount of time contributed by players 1 and 3, *i.e.*, $l_{y,4}(\{(1,4),(3,4)\}) = \frac{3}{2^{\alpha+1}}$. The utility levels from free time can be computed in a straightforward way $\phi_1(\{(1,4),(3,4)\}) = \phi_3(\{(1,4),(3,4)\}) = 1 - (2\overline{y})^{\frac{1}{\alpha}}$ and $\phi_4(\{(1,4),(3,4)\}) = 1 - \frac{3}{2^{\alpha+1}}$, which is higher than zero for all $\alpha \geq 1$.

Last, consider the star component $\{(1,4),(2,1)\}$. It has already been discussed in Example 2.4.6. Recall that players 2 and 4, respectively, contribute the following amounts of time $l_{2_y}(\{(1,4),(2,1)\}) = l_{2_y}(2,1) = 2\overline{y}$ and $l_{4_y}(\{(1,4),(2,1)\}) =$

Table 2.2: Team Utility Profiles in Example 2.5.3

$H^\star$	$v_1(H^\star)$	$v_2(H^\star)$	$v_3(H^\star)$	$v_4(H^\star)$
$ii_{i\in N}$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
$\{(2,1),\{3,3\},\{4,4\}\}$	0	$1-2\bar{y}$	$-\infty$	$-\infty$
$\{\{1,1\},\{2,2\},(3,4)\}$	$-\infty$	$-\infty$	$1-(2\bar{y})^{\frac{1}{\alpha}}$	0
$\{(2,1),(3,4)\}$	0	$1-2\bar{y}$	$1-(2\bar{y})^{\frac{1}{\alpha}}$	0
$\{(1,4),\{2,2\},\{3,3\}\}$	$1-(2\bar{y})^{\frac{1}{\alpha}}$	$-\infty$	$-\infty$	0
$\{\{1,1\},(2,3),\{4,4\}\}$	$-\infty$	$1-2\bar{y}$	0	$-\infty$
$\{(1,4),(2,3)\}$	$1-(2\bar{y})^{\frac{1}{\alpha}}$	$1-2\bar{y}$	0	0
$\{(1,4),(2,1),\{3,3\}\}$	$1-\left(\frac{3\bar{y}}{1+2\bar{y}}\right)^{\frac{1}{\alpha}}$	$1-2\bar{y}$	$-\infty$	0
$\{(2,1),(2,3),\{4,4\}\}$	0	$1-2^{-\alpha}(3\bar{y})$	0	$-\infty$
$\{\{1,1\},(2,3),(3,4)\}$	$-\infty$	$1-2\bar{y}$	$\left(\frac{3\bar{y}}{1+2\bar{y}}\right)^{\frac{1}{\alpha}}$	0
$\{(1,4),\{2,2\},(3,4)\}$	$1-(2\bar{y})^{\frac{1}{\alpha}}$	$-\infty$	$1-(2\bar{y})^{\frac{1}{\alpha}}$	$1-\frac{3}{2^{\alpha+1}}$

$l_{4_y}(1,4) = 1$. Player 1 contributes the minimum amount of time, so that given the hours worked by players 2 and 4, he can produce three times the subsistence level, *i.e.*, $l_{1_x}(\{(1,4),(2,1)\}) = \left(\frac{3\bar{y}}{1+2\bar{y}}\right)^{\frac{1}{\alpha}}$. The collective production in the star component $\{(2,3),(3,4)\}$ yields an analogous allocation of labor.

Having derived the utility from free time in collective production, we can compute the team utility profile in each possible star pattern. These values are presented in Table 2.2.

If $\bar{y} > \frac{1}{4}$, then $1-(2\bar{y})^{\frac{1}{\alpha}} < 1-\left(\frac{3\bar{y}}{1+2\bar{y}}\right)^{\frac{1}{\alpha}}$, players 1 and 3 will prefer to be star central players than to be linked only to player 4. For any $\alpha \geq 1$, $1-2\bar{y} < 1-2^{-\alpha}(3\bar{y})$, hence player 2 prefers to be a star central player than to be linked either to player 1 or to player 3 alone. Since $0 < 1-\frac{3}{2^{\alpha+1}}$ player 4 prefers to be a star central player than to be linked either to player 1 or player 3 alone. Hence for $\bar{y} > \frac{1}{4}$, there are three stable star patterns: $\{(1,4),(2,1),33\}$, $\{\{1,1\},(2,3),(3,4)\}$, and $\{(1,4),22,(3,4)\}$. Note that in all of these patterns the player who is in a state of autarky prefers to be linked to another player, however, no player with whom she has a potential link can increase her own utility by being linked with that autarkic player compared to her utility in the component in which she is linked. ♦

In Examples 2.4.6 and 2.5.3, we illustrated that depending on the direction of the set of potential links there can be no stable star pattern or there can be multiple stable star patterns despite that the set of potential links contains a cycle. In fact in this special setting we can sharpen the sufficiency requirements for the existence of a stable star pattern in comparison to Theorem 2.5.1. The most important characteristics of this setting are that players are homogeneous in terms of their preference for free time; they are homogenous within their production specialization, *i.e.*, a player can specialize to be either an intermediate good producer or a consumption good producers; any player has a higher utility from a link in which she has authority, than from a link in which another player has authority over her or him. A team economy that satisfies these characteristics, we name a *team economy based on collective production*.

Definition 2.5.4 *Let $\mathbb{E}^T = (N, D, u, v)$ be a team economy. Let $\{A, B\}$ be a set of roles, let $r\colon N \to \{A, B\}$ be a role-assignment function such that there are a non-empty set N_A of players assigned to the role A and a non-empty set N_B of players assigned to the role B with $N_A \cup N_B = N$ and $N_A \cap N_B = \emptyset$. A* ***team economy based on collective production*** *$\mathbb{E}^{TC} = (N, D^c, u^c, v^c)$ is a team economy such that the set of potential links contains only links between players of different role assignments and the autarkic states $D^c \subseteq \{N_A \bigotimes N_B\} \cup D_0$. All profiles of utility functions in the set of permissible hedonic profiles of utility functions $u^c \in \mathcal{U}^c(D^c)$ satisfy the following properties: there is a set of real numbers $\{\underline{u}_A, \underline{u}_B, , \overline{u}_A, \overline{u}_B\}$ such that for all players $i \in N_A$ and $j \in N_B$ with $(i, j) \in D$, it holds that $u_i((i, j)) = \overline{u}_A$ and $u_j((i, j)) = \underline{u}_B$; and for all players $i \in N_A$ and $j \in N_B$ with $(j, i) \in D$, $u_j((j, i)) = \overline{u}_B$ and $u_i((j, i)) = \underline{u}_A$ with $\underline{u}_A, \underline{u}_B, \overline{u}_A, \overline{u}_B \in \mathbb{R}$.*

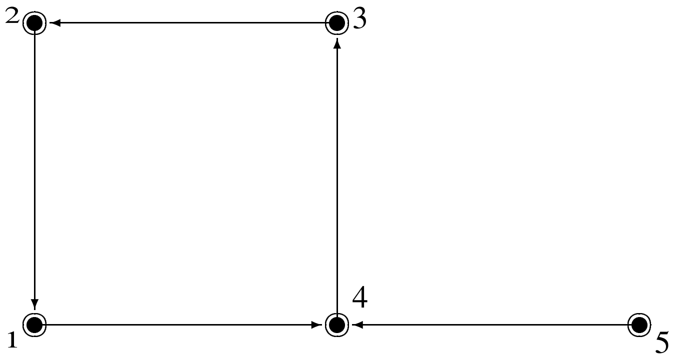
Below we present the result on existence of stable patterns in a team economy based on collective production.

Proposition 2.5.5 *There exists a stable star pattern in a team economy based on collective production, if the set of potential links does not contain a directed cycle.*

The proof is similar to the proof of Theorem 2.5.1 since the absence of directed cycles in this setting guarantees the analogous result to the result in Lemma 2.5.2.

The converse of Proposition 2.5.5, however, does not hold. That is, there are team economies of collective production in which the sets of potential links contains a directed cycle and there are stable activity patterns. Such a case is illustrated in Example 2.5.6.

Example 2.5.6 Consider the economy that has been developed in Examples 2.3.3 and 2.4.6. We modify these examples by introducing one more player who is specialized in the production of the intermediate good and who has a potential link only with player 4. The set of potential links is given by $D = \{(1, 4), (2, 1), (3, 2), (4, 3), (5, 4)\}$ and it is presented graphically below.



Due to the symmetry between the links $(1,4)$ and $(5,4)$, the team utility profiles of the star patterns that contain the components $d_1 = \{(5,4)\}$ and $d_2 = \{(4,3),(5,4)\}$ are analogous to the team utility profiles of the star patterns containing the components $d'_1 = \{(1,4)\}$ and $d'_2 = \{(1,4),(4,3)\}$ with $v_5(d_1) = v_1(d'_1)$. A new star component that we need to consider is the component $\{(1,4),(5,4)\}$. The same type of component in terms of the specialization of the players connected and in the direction of authority has already been analyzed in Example 2.5.3, *i.e.*, the component $\{(1,4),(3,4)\}$. Hence, we know that $v_5(\{(1,4),(5,4)\}) = v_3(\{(1,4),(3,4)\}) = 1-(2\overline{y})^{\frac{1}{\alpha}}$ and $v_4(\{(1,4),(5,4)\}) = 1-\frac{3}{2^{\alpha+1}}$. The only new type of pattern that has not been considered so far is $\{(1,4),(4,3),(5,4)\}$. Here, by assumption of the team utility profile $v_1(\{(1,4),(4,3),(5,4)\}) = v_5(\{(1,4),(4,3),(5,4)\}) = 1-(2\overline{y})^{\frac{1}{\alpha}}$, and $v_3 = (\{(1,4),(4,3),(5,4)\}) = v_3(4,3) = 0$. Player 4 has to contribute the minimum amount of hours in the production of the consumption good such that given the amounts of the intermediate good produced by players 1, 3 and 5, four times the subsistence level is being produced, *i.e.*, $l_{y,4}(\{(1,4),(4,3),(5,4)\}) = 4\overline{y}(1+2(2\overline{y})^{\frac{1}{\alpha}})^{-\alpha}$ and hence $v_4(\{(1,4),(4,3),(5,4)\}) = 1-4\overline{y}(1+2(2\overline{y})^{\frac{1}{\alpha}})^{-\alpha}$. In addition recall that the utility level in a state of autarky for each player equals $-\infty$.

Consider the star pattern $H^\star = \{(1,4),(3,2),(5,4)\}$. This star pattern is stable: the only players who might want to deviate are players 2 and 4, however, the non-blocking conditions are satisfied: $u_2(2,1) = 1-2\overline{y} > 0 = v_2(H^\star)$, however, $v_1(2,1) = 0 < 1-(2\overline{y})^{\frac{1}{\alpha}} = v_1(H^\star)$; and $v_4(H^\star \cup \{(4,3)\} = 1-4\overline{y}(1+2(2\overline{y})^{\frac{1}{\alpha}})^{-\alpha} > 1-\frac{3}{2^{\alpha+1}} = v_4(\{(1,4),(5,4)\})$ for instance for $\alpha = 2$, however, $v_3(H^\star) = 1-(2\overline{y})^{\frac{1}{\alpha}} > 0 = v_3(\{(1,4),(4,3),(5,4)\})$. ♦

2.6 Team Generic Stability

As we discussed in Section 2.1, the set of potential links defines the social constraints and the social flow of authority between people, while the star pattern which emerges defines the productive patterns within a society. It is important to identify conditions which ensure the existence of stable productive patterns in any social arrangements. Such conditions will ensure what we call *team generic stability* of the activity structure (N, D) with respect to the emergence of team patterns.

Definition 2.6.1 *Let (N, D) be an activity structure and let $\mathcal{V}(D)$ be a set of permissible profiles of team utility functions defined on D. The activity structure (N, D) is **team generically stable** if for every $v \in \mathcal{V}(D)$ there is a stable star pattern in the team economy $\mathbb{E}^T = (N, D, u, v)$.*

The main existence theorem follows. Similarly to Theorem 2.5.1, the condition is based on presence of cycles and the direction of the cycle is not important.

Theorem 2.6.2 *The activity structure (N, D) is team generically stable if and only if the set of potential links D does not contain a cycle or if it contains a cycle, it is a cycle with $m - 1 = 3s$ and $s \in \{2, 3, \ldots\}$.*

Proof. If: Let (N, D) be an activity structure and let $\mathcal{V}(D)$ be the set of permissible profiles of team utility functions defined on the set D.

The sufficiency of the conditions directly follows from Theorem 2.5.1 applied to every team economy (N, D, u, v) for every profile of team utility functions $v \in \mathcal{V}(D)$.

Only if: Let (N, D) be an activity structure and let $\mathcal{V}(D)$ be the set of permissible profiles of team utility function in D, and let $v \in \mathcal{V}(D)$ be a particular profile of team utility functions. We will show the necessity of the condition that D contains no cycles or that if it contains a cycle, it is a cycle with $m - 1 = 3s$ and $s = \{2, 3, \ldots\}$, by contradiction.

So, let there be a stable star pattern in any team economy (N, D, u, v) for all $v \in \mathcal{V}(D)$ and let the set of potential links contain a cycle $C = (i_1, i_2, \ldots, i_m\}$ with $i_k, i_{k+1} \in \Delta$ for all $k = 1, \ldots, m - 1$ and $m \geq 4$ and suppose $m - 1 \neq 3s$ with $s = \{2, 3, \ldots\}$. Consider a profile of team utility functions $v \in \mathcal{V}(D)$ such that the hedonic utility profile u is given by: $u_{i_k}(i_k, j) < u_{i_k}(i_k, i_k) < u_{i_k}(i_{k-1}, i_k) < u_{i_k}(i_k, i_{k+1})$ for all $k = 1, \ldots, m - 1$ with $i_0 = i_{m-1}$ and all $j \in N_{i_k}(D) \setminus \{i_{k-1}, i_{k+1}\}$. Let $H^\star$ be a stable star pattern in this economy. Note that in the stable star pattern $H^\star$ the largest number of players connected in the cycle C that form a component in a star pattern, is three. We will consider three cases.

First, suppose that $i_k i_k \in H^\star$ for some $k = 1, \ldots, m - 1$. Since $H^\star$ is a stable star pattern, the individual rationality condition is satisfied for all players in N. Hence, player i_{k-1} is in a state of autarky or connected to player i_{k-2} either in the component $g' = \{i_{k-1}i_{k-2}\}$ where she is a star central player, or in the component $g'' = \{i_{k-1}i_{k-2}, i_{k-2}i_{k-3}\}$ with $i_0 = i_{m-1}$, $i_{-1} = i_{m-2}$, and $i_{-2} = i_{m-3}$ where she is not a star central player. In all three cases one of the non-blocking conditions is violated: if player $i_{k-1} \notin N^\star(H^\star)$, then the condition [NB] is violated since by the construction of the profile of hedonic utility functions $u_{i_k}(i_{k-1}, i_k) > u_{i_k}(H^\star)$ and $u_{i_{k-1}}(i_{k-1}, i_k) > u_{i_{k-1}}(H^\star)$; in case $i_{k-1} \in N^\star(H^\star)$, then the condition [NB*] is violated since $u_{i_k}(i_{k-1}, i_k) > u_{i_k}(H^\star)$ and $v_{i_{k-1}}(H^\star \cup \{i_{k-1}, i_k\}) \geq u_{i_{k-1}}(i_{k-2}, i_{k-1}) + u_{i_{k-1}}(i_{k-1}, i_k) > u_{i_{k-1}}(H^\star)$. Since $H^\star$ is stable, then it cannot be that $\{i_k, i_k\} \in H^\star$ for some $i_k \in C$[18].

Next, suppose that there is no player along the cycle's path such that $i_k i_k \in H^\star$ and in addition $m - 1 \geq 4$. Since $H^\star$ is a stable star pattern, the individual rationality condition is satisfied for all players in N. Since $m - 1 \neq 3s$ with $s = \{2, 3 \ldots\}$, $m - 1 \geq 4$ and there is no player i_k along the cycle's path such that $i_k i_k \in H^\star$, there must be at least two distinct players along the cycle's path i_{k-1} and i_k for some $k = 1, \ldots, m-1$ and $k_0 = m - 1$ such that the component $\{i_{k-1}, i_k\} \in H^\star$. However, in that case, the non-blocking condition [NB*] is violated: $u_{i_{k-2}}(i_{k-2}, i_{k-1}) > u_{i_{k-2}}(H^\star)$ and $v_{i_{k-1}}(\{i_{k-2}i_{k-1}, i_{k-1}i_k\}) \geq u_{i_{k-1}}(i_{k-2}i_{k-1}) + u_{i_{k-1}}(i_{k-1}i_k) > u_{i_{k-1}}(H^\star)$ with $k_{-1} = m-2$ and where the first inequality follows from the superadditivity of the team utility function.

Last, consider the case when there is no player i_k along the cycle's path such that $i_k i_k \in H^\star$ and $m - 1 = 3$. Since $i_k i_k \notin H^\star$, the players in along the cycles path must be connected in a component $\{i_{k-1}i_k, i_k i_{k+1}\} \in H^\star$ for some $k = 1, 2, 3$ with $i_0 = i_3$ and $i_4 = i_1$. Again the non-blocking condition [NB*] is violated: $u_{i_{k+1}}(i_{k-1}i_{k+1}) > u_{i_{k+1}}(H^\star)$ and $v_{i_{k-1}}(\{i_{k-1}i_{k+1}, \ i_{k-1}, i_k\}) \geq u_{i_{k-1}}(i_{k-1}i_{k+1}) + u_{i_{k-1}}(i_{k-1}i_k) > u_{i_{k-1}}(H^\star)$ where the first inequality follows from the superadditivity of the team utility function. This exhausts the possible cases and thus establishes a contradiction to the existence of a stable pattern $H^\star$. ■

In team economies based on collective production, where we consider only a subclass of utility functions $\mathcal{U}^c$ such that the hedonic utility function is contingent on the direction of the directed links, the sufficient condition for the existence of a stable pattern requires the absence of *directed cycles*. Furthermore, as Example 2.5.6 showed, the absence of directed cycles is not a *necessary condition*. The necessary and sufficient condition presented in Proposition 2.6.4 requires that the set of potential links does not contain *components* that consist of directed cycles.

[18]Here again we slightly abuse the notation and use C to denote the set of connected players in the cycle.

First, we define the activity structure pertaining to a team economy based on collective production.

Definition 2.6.3 *Let (N, D) be an activity structure. Let $\{A, B\}$ be a set of roles, let $r\colon N \to \{A, B\}$ be a role-assignment function such that there are a non-empty set N_A of players assigned to the role A and a non-empty set N_B of players assigned to the role B with $N_A \cup N_B = N$ and $N_A \cap N_B = \emptyset$. The pair (N, D^c) is an* ***activity structure pertaining to an economy based on collective production*** *if $D^c \subseteq \{N_A \bigotimes N_B\} \cup D_0$.*

The result follows.

Proposition 2.6.4 *Let (N, D^c) be an activity structure pertaining to an economy based on collective production. Then (N, D^c) is team generically stable if and only if the set of potential links does not contain a component consisting of a directed cycle C^d with $m - 1 \neq 3s$ with $s \in \mathbb{N}$.*

The proof of Proposition 2.6.4 is very similar to the one of Theorem 2.6.2 and uses the results of Proposition 2.5.5. What should be kept in mind is that in team economies based on collective production only cycles of even number of players are possible. This follows from the restriction on the set of potential links that it can only include links between players of different role assignments.

2.7 Final Remarks

The results that we derive in our application to pre-market collective production, are in line with the anthropological insights that the recognition of authority is a necessary condition for the emergence of complex production. To ensure team generic stability in a team economy based on collective production a social structure must not contain components of directed cycles, which implies that there must be a well-defined flow of authority in all components of the social structure. The presence of directed cycles in a set of potential links has a very intuitive interpretation in term of the structure of an organization in which the potential links stand for value generating activities. Within such an organization a directed cycle would mean that players have authority over players who have indirect authority over them.

Note that in our application of collective production we have not assumed an ex-ante relation between a production role and authority, *i.e.*, it has not been assumed that all players specialized in the production in one of the types of goods, intermediate or consumption, have authority over the players specialized in the other type of group.

Such a one-to-one relation between labor specialization and authority, however, would imply that the set of potential links does not contain a directed cycle. Hence, what some anthropologists identify as the path for the emergence of complex relations, labor specialization, in our framework is an ex-ante realization of the necessary and sufficient condition for guaranteed ex-post stability.

A one-to-one relation between production roles and authority, however, is just one possible way of satisfying the necessary and sufficient condition for stability. There are other organizations of the sets of potential links that would satisfy this condition. For instance, one such structure is a set of potential links in which producers of intermediate goods belong to either one of two types: either they have authority over all the producers of the consumption good with whom they are linked, or all the producers of the consumption good with whom they are linked have authority over them.

Moreover, we should reiterate that, according to our analysis, it is not necessary that authority emanates from a star central player. This may be the case, if the set of potential links is such that there is a one-to-one relation between production role and authority as discussed above.

We should also point out the fact that in the theoretical results applicable to the general framework, Theorems 2.5.1 and 2.6.2, the direction of a cycle does not play a role. This is because in the general setting we have not assumed a relation between a player's possession of authority and the utility she derives from a link with another player. This makes this framework applicable to a broad class of situations, also such which can be modeled by undirected graphs.

Last, we should mention some limitations of our general framework. In particular, in our work we focus on very special class of activity patterns, which consists of star structures. For complex production processes, such as hierarchies of several levels, predominant in today's economic world, these tools are inadequate. A clear goal for future work is the development of a framework where more complex patterns can be analyzed.

Part II

Stability and Endogenous Coalition Formation

CHAPTER

THREE

C C F A P [*]

This chapter views coalition formation as a problem of cooperative nature in which two intertwined processes are simultaneously studied: the organization of economic agents into coalitions and the allocation of coalition values among the coalition members. Aumann and Dreze (1974) state that answering the simultaneous questions would be providing a general equilibrium type of analysis. Recently, several authors offer solution suitable to answer this question, *e.g.* Zhou (1994) and Morelli and Montero (2003) based on a notion of bargaining set. Instead, we investigate the stability of endogenous coalition formation under various contractual arrangements. Our approach is applicable to environments in which contracts are enforceable. We regard the coalition[1] and an individual member as two sides in a contract. The arrangement gives particular rights, such as the right to end the contract and the right to be compensated to one or both of the parties. Depending on the allocation of rights, we distinguish between three types of contractual arrangements leading to the notions of *individual, contractual*, and *compensation* stability. Individual and contractual stability give the individual member the right to end the contract but do not allow for compensation rights. Contractual stability setting in addition gives a veto right to the coalition. To end the contract a member needs the agreement of the rest of the coalition members. If they are worse-off after the change, they disagree. The third concept, compensation stability, gives both parties the right to end the contract and the right to be compensated in case they are worse-off after the change. Compensation stability has two complementary sides to it: pull-in and push-out stability. Pull-in stability re-

[*]This chapter is based to a great extent on Lazarova, Borm and van Velzen (2005).

[1]Here by coalition we mean the rest of the coalition members.

flects external stability by allowing a coalition to attract new members. On the other hand, push-out stability reflects internal stability by allowing coalitions to push out one of their members.

The focus on contractual arrangements is justified, on one hand, by the fact that contracts are common arrangements in economic organization in which economic value is generated by the cooperation of group members, *e.g.*, business alliances and consumer clubs. Moreover, it is motivated by the observation that agents, being part of large groups, are anonymous to each other. Thus, no group deviations are allowed in our setting. This makes other well established equilibrium notions, such as the core, an inadequate equilibrium concept for stability analysis.

The use of these type of stability concepts to study endogenous coalition formation has been extensively motivated by Dreze and Greenberg (1980). Dreze and Greenberg (1980) are also the first to define the notions of individual and contractual stability in a setting in which transfers between groups coalition structure elements are allowed, thus, an individual (a coalition) may compensate her former coalition members (the individual who leaves the coalition) in case that the change makes them (her) worse-off.

However, to the best of our knowledge the subsequent literature that was spurred by their seminal paper, *e.g.*, Banerjee et al. (2001), Bogomolnaia and Jackson (2004), and Pápai (2004), abstracted away from the problem of value allocation. In these works, it is assumed that each player has exogenously determined preferences over membership in all coalitions of which she can be a member.

In our setting, as well as in the seminal paper by Dreze and Greenberg (1980) a player's preferences are endogenous since they depend on her share from the coalition's value and her outside options.

To make this point more clear, we elaborate on the behavioral assumptions we make. We assume that each player makes a decision to join/leave a coalition based on her own perceived payoff without taking into account the effect of her actions on other players' payoffs. A player's payoff from a coalition membership is *endogenously* determined. It depends on her "power"[2] to obtain a share of the coalition value. This power-based measure, though related, is not entirely determined by the player's marginal contribution to the coalition value but it also depends on her outside options, *i.e.*, what other coalitions are being formed and how much she can possibly get as a

[2]Here by "power" we refer to the bargaining strength of a player in the value allocation process. However, since there is no explicit bargaining process that we analyse, we use this term loosely to indicate that some players obtain larger share of the value than other and that's what makes them more powerful.

share of their payoff, *i.e.*, the *endogenous coalition structure*. In this way a member of a coalition with high value might prefer to switch to a coalition with a smaller value because her payoff in the latter is higher than the share she gets from the former. By switching coalitions, though, a player changes the outside options for the rest of the players, which she does not take into account at the time of the move. Accordingly, by switching coalitions, the player also affects another player's relative power in bargaining for a share from the coalition value, not only in the two coalitions in which the membership has changed, but also in the rest of the coalitions in the coalition structure. Not taking into accounts the market-wide effects of her actions makes each player myopic.

We offer alternative formulations of the notions of individual, contractual, and compensation stability than the ones introduced by Dreze and Greenberg (1980). Our main critique to their definitions is that it only considers deviations that would lead to the same number of groups in the group structure. For example, in a coalition formation problem, the grand coalition, and any feasible payoff allocation will be individually stable since a deviation of a single player to the empty set is not contemplated. Clearly, this implication of their definition of individual stability is not recognized by Dreze and Greenberg (1980) since they claim that in Example 3.1 of their paper—a three-player "game of pairs" such that coalitions of two members have a value of 1, and the singletons and the grand coalition have values of 0—there is no individually stable equilibrium. This, as explained above, is not true.

The alternative definition of individual stability furthermore, implies individual rationality. Individual rationality is a natural requirement in an equilibrium concept as players should have the ability to opt out of a coalition and be alone if not restricted by binding contracts.

Moreover, in their work Dreze and Greenberg (1980) claim that they provide an example in which, supposedly, there is no contractually stable equilibrium in a setting in which no transfers between coalitions are allowed. We disagree with this claim. As we show in Theorem 3.2.2, even when allowing for possibilities of deviation to the empty set, there is a contractually stable outcome in any coalition formation problem.

With the alternative definition of individual stability, we find that there are coalition formation problems in which there are no individually stable outcomes. This is the case if the value of the grand coalition is less than the sum of the values of the singleton coalitions. Moreover, coalition structures of maximum worth may not be individually stable either. We use individual stability as a stepping stone for the construction of the other two stability concepts. Contractual stability and compensation stability have positive existence results. The contractual stability setting, however, does not allow for one-player value increasing deviations to take place, while compensation stability does.

As an application of our theoretical framework we consider group formation problem of agricultural cooperation driven by the benefit from mutual insurance. In this setting wheat producers experience the risk of a damage of their seeds. The value of a cooperating group is generated by the ability of group members to smoothen their wheat production by pooling the risk of a seed loss and sharing the harvest. The objective of a mutual insurance group is to maximize the welfare of the group members. In this setting the members of an insurance group are stakeholders. As such they divide the value of the group generated by their cooperation amongst themselves.[3]

To illustrate the risk of damage, we adopt the model studied by Rothschild and Stiglitz (1976). To the best of our knowledge all other works offering cooperative approach to insurance group formation, *e.g.*, Demange and Guesnerie (2001)[4], Kahn and Mookherjee (1995), Boyd et al. (1988), and Boyd and Prescott (1986) allow for group deviations and do not consider pooling production risk. These works focus on studying informational asymmetry and the properties of the core-stable insurance group structure under various channels of signalling. Instead we abstract from the asymmetric information problem to develop contract-based stability concepts.

We refer to the setting as "*mutual* insurance" to emphasize the cooperative nature of the problem and distinguish it from the third-party market insurance setting studied by Rothschild and Stiglitz (1976). We furthermore, consider production rather than a utility-maximization setting. We focus on the stability of two types of outcomes widely studied in the literature: pooling and separating outcomes. We find that given the assumption of risk averse players, all pooling outcomes are contractually and compensation stable, while no separating outcomes are individually or compensation stable. The individually rational pooling outcomes are individually stable and, moreover, this type of outcomes exists in every mutual insurance formation problem. Finally, no individually rational separating outcome is contractually stable either. What drives these results is the possibility for side-payments within groups and, in the case of the compensation setting, between groups.

Last we suggest an extension of the notions of stability based on contractual arrangements to two-sided problems that take into account coalitional formation on one side of the market and matching between the formed coalitions and players on

[3]The advantages of employing a cooperative approach to studying insurance organizations has been discussed by Boyd, Prescott and Smith (1988).

[4]Demange and Guesnerie (2001) explicitly model anonymous agents, however, this assumption only concerns the information channels rather than the group-wide deviation possibilities.

the other side of the market. The matching problem that we discuss here differs from the matching studied in Chapter 1 because here we consider only matchings between members of two dichotomous groups. Moreover, the attention focuses the inter-relation between the coalition formation and matching problems. We think that studying these problems further may bring better understanding of large range of service activities in which on one side of the market professional organizations are being formed which generate economic value by interacting with organization on the other side of the market *e.g.* insurance groups and health care providers. The chapter is structured in the following way. In Section 3.1 we define the coalition formation problem and present the three stability concepts. We present existence results as well as a technical discussion on the relation between the concepts in Section 3.2. In Section 3.3 we apply the problem to mutual insurance in agricultural production to which we adapt the model of Rothschild and Stiglitz (1976), and discuss the stable outcomes in terms of the risk composition of the groups. In Section 3.4, we extend the framework to coalitional matching problem and introduce possible notions of stability.

3.1 Contractual Settings

There is a finite set of players $N = \{1, 2, \ldots, n\}$. Players in the set N form coalitions. The collection of subsets of N is denoted by 2^N. A player cannot be a member of more than one coalition. A partition of N into non-empty coalitions forms a *coalition structure* and it is denoted by $\mathtt{P}$. The set of all possible coalition structures is denoted by $\mathcal{P}$. Each coalition generates value by the cooperation of its members. This value differs depending on the identity of the group members. For the purposes of the general analysis, it suffices to regard this relation as given by a value function $v : 2^N \to \mathbb{R}$. The pair (N, v) such that $v(\emptyset) = 0$ defines a coalitional game. Without loss of generality, we study zero-normalized problems, *i.e.*, $v(\{i\}) = 0$ for all $i \in N$.

An outcome of a coalitional game is represented by a *payoff configuration*. A payoff configuration is a pair $(\mathtt{P}, x)$ where $\mathtt{P} \in \mathcal{P}$ is a coalition structure of N and $x \in \mathbb{R}^N$ is an efficient payoff vector for $\mathtt{P}$, *i.e.*, $x(S) = v(S)$ for all $S \in \mathtt{P}$, where $x(S) := \sum_{i \in S} x_i$.

We refer to a coalition structure $\mathtt{P}$ which has a maximum total coalition structure value as a coalition structure of *maximum social worth*, *i.e.*, $\sum_{S \in \mathtt{P}} v(S) \geq \sum_{S \in \mathtt{P}'} v(S)$ for all coalition structures $\mathtt{P}' \in \mathcal{P}$. Similarly the payoff configuration $(\mathtt{P}, x)$ such that $\mathtt{P}$ is a coalition structure of maximum social worth and x is efficient is called a payoff configuration of *maximum social worth*.

Given the coalition value, a player's payoff depends on an endogenous allocation of the group value. A player $i \in N$ prefers to be a member of a coalition which yields a higher payoff to her. A fair player's payoff in a coalition will also depend on the exact coalition structure of the set of players, since the composition of the coalition structure defines the outside options. This point will become more clear with the discussion of the stability concepts below.

3.1.1 Individual Stability

In the contractual arrangement of individual stability the right to end the contract is given only to the individual players and no compensatory obligations are imposed. Individual stability thus entails that a player cannot obtain a higher payoff by joining another coalition structure element or by forming a singleton coalition.[5]

Definition 3.1.1 *Let (N, v) be a coalitional game. A payoff configuration (P, x) is **individually stable** if there are no $i \in N$ and $S \in \mathrm{P} \cup \{\emptyset\}$ with $i \notin S$ such that*

$$x_i < v(S \cup \{i\}) - v(S).$$

The following example is used to illustrate the concept of individual stability.

Example 3.1.2 Let $N = \{1, 2, 3\}$, $v(\{1, 2\}) = 2$, $v(\{1, 3\}) = 3$, $v(\{2, 3\}) = 4$, and $v(S) = 0$, otherwise.

In the given coalitional game there is only one individually stable payoff configuration, namely, $(\{N\}, 0)$. Clearly, this is an individually stable outcome: the players can deviate only by joining the empty set and obtain a payoff of zero.

No other payoff configuration is individually stable because there is always at least one player who wants to deviate. As an example consider the payoff configurations of maximum social worth $\big(\{\{1\}, \{2, 3\}\}, (0, \alpha, 4 - \alpha)\big)$ with $\alpha \in \mathbb{R}$. The best outside option for player 2 is to join 1 in the coalition $\{1, 2\}$ where player 2's marginal contribution is two. Thus for player 2 not to have incentives to deviate it must be that $x_2 = \alpha \geq 2$. Similarly, for player 3 not to deviate, it must hold that $x_3 = 4 - \alpha \geq 3$. The two conditions cannot hold simultaneously in any efficient payoff vector and hence this type of payoff configurations cannot be individually stable. ♦

[5]Implicit in the definition of individual stability is that a player can join a coalition only if her membership is unanimously approved by the current coalition members. This is to say, a player can join a coalition if the current members have at least as high payoff after she joins as they had before. This is why when a player decides on joining a coalition she bases her decision on her marginal contribution to the coalition value.

As seen above there are coalitional games in which no payoff configuration of maximum social worth is individually stable. The next example shows that there are coalitional games in which there are no individually stable payoff configurations.

Example 3.1.3 Let $N = \{1, 2, 3\}$, $v(\{1, 2\}) = 2$, $v(\{1, 3\}) = 3$, $v(\{2, 3\}) = 4$, $v(N) = -1$, and $v(S) = 0$, otherwise.

Note that the only difference with Example 3.1.2 is that here the grand coalition has a negative value. The payoff configuration that consists of the grand coalition and an efficient payoff vector cannot be individually stable: by the efficiency of the payoff vector follows that there is at least one player who has a negative payoff, thus, such player will deviate by forming a singleton coalition. ♦

3.1.2 Contractual Stability

Contractual stability is based on individual incentives under the additional condition that a deviating player needs to acquire permission from the coalition, whose member she is, in case she wants to end the contract, *i.e.* each individual member has the right to end the contract and each coalition has the right to veto the end of the contract. we assume that a coalition grants the permission, *i.e.*, does not use its veto power, only if the rest of the coalition members are as well-off without that particular player as when she is part of the coalition. Therefore, in a contractually stable payoff configuration no player can obtain a higher payoff by joining another coalition structure element without making the members of her current coalition worse-off.

Definition 3.1.4 *Let (N, v) be a coalitional game. A payoff configuration $(\mathtt{P}, x)$ is* ***contractually stable*** *if there are no $i \in N$ and $S, T \in \mathtt{P} \cup \{\emptyset\}$ with $i \in T$ and $S \neq T$ such that*

$$x_i < v(S \cup \{i\}) - v(S) \quad \textit{and} \quad x(T \setminus \{i\}) \leq v(T \setminus \{i\}).$$

The next example illustrates how contractual stability limits the deviating possibilities of a player in contrast to individual stability.

Example 3.1.5 Let (N, v) be the coalitional game of Example 3.1.2.

There are infinitely many contractually stable payoff configurations in this coalitional game. Any coalition structure with the exclusion of the coalition structure that consists of all singletons can be part of a contractually stable payoff configuration. Consider the following type of coalition structures: $\big(\{\{1, 3\}, \{2\}\}, (\alpha, 0, 3 - \alpha)\big)$ with $\alpha \in (0, 3]$. These are contractually stable payoff configurations: player 2's outside option is to join $\{1,3\}$ in the grand coalition but there her marginal contribution is negative. Neither player 1 nor 3 will give a permission to the other who has incentives to join player 2 since by being alone that player will have a strictly lower payoff.

A payoff configuration that is not contractually stable is $\left(\{\{1,3\},\{2\}\},(0,0,3)\right)$. Player 3's outside option to join player 2 in coalition $\{2,3\}$ gives her a higher payoff than what she has, *i.e.* $v(\{2,3\}) - v(\{2\}) = 4 > 3 = x_3$. Player 1 grants permission to player 3 to leave as she is indifferent between having a payoff of zero in the coalition $\{1,3\}$ or as a member of the singleton coalition $\{1\}$. ♦

The above example shows that in the contractual stability setting, there are situations in which a one-person deviation can lead to an increase in the total value of the coalition structure, yet, it is not performed because one of the parties of the contract is strictly worse-off. To overcome this restriction on profitable deviations, in the next contractual specification we allow for side payments between players after the contract between them has ended .

3.1.3 Compensation Stability

First, the two complementary sides of compensation stability, pull-in stability and push-out stability, are introduced.

In the contractual setting of pull-in stability, the individual player is the only party who can end the contract. In case the remaining group members are worse-off, the new coalition of the deviating player is obliged to compensate them for this loss. Thus, in a pull-in stable outcome there is no coalition structure element that by attracting a new member may increase its value enough to give higher payoffs to its members after possible compensation of the incoming player's previous coalition.

Definition 3.1.6 *Let (N,v) be a coalitional game. A payoff configuration (P,x) is* ***pull-in stable*** *if there are no $i \in N$ and $S,T \in \mathrm{P}$ with $i \in T$ and $S \neq T$ such that*[6]

$$v(S) \quad < \quad v(S \cup \{i\}) - x_i - \max\left\{0, x(T \setminus \{i\}) - v(T \setminus \{i\})\right\}.$$

In push-out stability the right to end the contract is given only to the coalition. A coalition wants to end the contract with one of its members if by doing so, it can increase the payoffs of the remaining members. In the push-out setting a compensation is required in case the member whose contract has been ended has a lower best outside option than her current payoff as a coalition member.

[6]*Since the empty set is not regarded as a coalition structure element, it is not included in the possible set of coalitions that can pull a player in.*

Definition 3.1.7 *Let (N, v) be a coalitional game. A payoff configuration (P, x) is* ***push-out stable*** *if there are no $i \in N$ and $S, T \in \mathrm{P} \cup \{\emptyset\}$ with $i \in T$ and $S \neq T$ such that*

$$x(T \setminus \{i\}) \quad < \quad v(T \setminus \{i\}) - \max\left\{0, x_i - \left(v(S \cup \{i\} - v(S)\right)\right\}.$$

Combining pull-in and push-out stability, we have compensation stability.

Definition 3.1.8 *An outcome of a coalitional game is* ***compensation stable*** *if it is pull-in and push-out stable.*

To illustrate that compensation setting may overcome the restriction on profitable deviations of the contractual stability setting, we consider the following example.

Example 3.1.9 Let (N, v) be as in Example 3.1.2.

There is one type of compensation stable payoff configurations, *i.e.*, the payoff configurations of maximum social worth $\left(\left\{\{1\}, \{2, 3\}\right\}, (0, \alpha, 4 - \alpha)\right)$ with $\alpha \in \mathbb{R}$.

These payoff configurations are pull-in stable. Coalition $\{2, 3\}$ does not want to attract player 1 as a member since the grand coalition has lower value than their current value. Player 1 can increase the value of its coalition by attracting either player 2 or 3. Yet, the increase is not enough to give her a higher payoff after she compensates the remaining member of coalition $\{2, 3\}$ for the change. These payoff configurations are also push-out stable. Neither player 2 nor 3 can increase her payoff by pushing the other player out to join player 1 in a coalition and compensate her for the change, if needed. Player 1 cannot be pushed out of the singleton coalition either. ♦

3.2 Existence and Relations

The discussion of existence and relations between the stability contracts is focused on the notions of compensation and contractual stability. Example 3.1.3 shows that there are coalitional games with no individually stable payoff configurations.

From the definitions of the stability concepts the following results can be obtained in a straightforward fashion.

Proposition 3.2.1 *Let (N, v) be a coalitional game. Then the following results hold:*

(i) Any individually stable payoff configuration is contractually stable;

(ii) Any individually stable payoff configuration is pull-in stable;

(iii) Any payoff configuration $(\{N\}, x)$ is pull-in stable;

(iv) Any payoff configuration $(\{i\}_{i\in N}, 0)$ *is push-out stable.*

We establish positive existence results with respect to compensation and contractual stability. In particular, all payoff configurations of maximum social worth are compensation and contractually stable.

Theorem 3.2.2 *Any payoff configuration of maximum social worth is both compensation and contractually stable.*

Proof. Let (N, v) be a coalitional game. Let $(\mathtt{P}^\star, x)$ be a payoff configuration of maximum social worth of (N, v).

Compensation stability: Suppose $(\mathtt{P}^\star, x)$ is not a compensation stable payoff configuration. Then either $(\mathtt{P}^\star, x)$ is not pull-in stable or $(\mathtt{P}^\star, x)$ is not push-out stable.

First, suppose $(\mathtt{P}^\star, x)$ is not a pull-in stable payoff configuration. Then there are $i \in N$ and $S, T \in \mathtt{P}^\star$ with $i \in T$ and $S \neq T$ such that

$$v(S) < v(S \cup \{i\}) - x_i - \max\Big\{0, x(T \setminus \{i\}) - v(T \setminus \{i\})\Big\}.$$

Using the efficiency of the payoff vector, the above inequality implies

$$v(S) + v(T) < v(S \cup \{i\}) + v(T \setminus \{i\}).$$

So the coalition structure $\mathtt{P} = \Big[\mathtt{P}^\star \setminus \{S, T\}\Big] \cup \{S \cup \{i\}, T \setminus \{i\}\}$ has a higher total value contradicting that $\mathtt{P}^\star$ is a coalition structure of maximum social worth.
Now suppose $(\mathtt{P}^\star, x)$ is not a push-out stable outcome. Then there are $i \in N$ and $S, T \in \mathtt{P}^\star \cup \{\emptyset\}$ with $i \in T$ and $S \neq T$ such that

$$x(T \setminus \{i\}) < v(T \setminus \{i\}) - \max\Big\{0, x_i + \Big(v(S \cup \{i\}) - v(S)\Big)\Big\}.$$

Using the efficiency of the payoff vector, the above inequality implies

$$v(T) + v(S) < v(T \setminus \{i\}) + v(S \cup \{i\}),$$

establishing a contradiction.

Contractual stability: Suppose $(\mathtt{P}^\star, x)$ is not a contractually stable outcome. Then there are $i \in N$ and $S, T \in \mathtt{P}^\star \cup \{\emptyset\}$ with $i \in T$ and $S \neq T$ such that

$$\begin{aligned} x_i &< v(S \cup \{i\}) - v(S) \\ x(T \setminus \{i\}) &\leq v(T \setminus \{i\}). \end{aligned}$$

Adding up the two inequalities and using the efficiency of the payoff vector, we find

$$v(T) + v(S) < v(T \setminus \{i\}) + v(S \cup \{i\}),$$

establishing a contradiction. ■

For establishing a relation between compensation stability and contractual stability we need to introduce one additional property. A payoff configuration $(\mathtt{P}, x)$ is *individually rational* if $x_i \geq v(\{i\}) - v(\emptyset)$ for all $i \in N$.

Proposition 3.2.3 *Any compensation stable payoff configuration which is individually rational is also contractually stable.*

Proof. Let (N, v) be a coalitional game. Let $(\mathtt{P}, x)$ be a compensation stable payoff configuration which is individually rational. Then for all $i \in N$ and $S, T \in \mathtt{P}$ with $i \in T$ and $S \neq T$:

$$v(S) \geq v(S \cup \{i\}) - x_i - \max\Big\{0, x(T \setminus \{i\}) - v(T \setminus \{i\})\Big\}.$$

This implies that for all $i \in N$ and $S, T \in \mathtt{P}$ with $i \in T$ and $S \neq T$

$$x_i \geq v(S \cup \{i\}) - v(S) \quad \text{or} \quad x(T \setminus \{i\}) > v(T \setminus \{i\}).$$

Since $(\mathtt{P}, x)$ is individually rational $x_i \geq 0 = v(\{i\}) - v(\{\emptyset\})$. We conclude that for all $i \in N$ and $S, T \in \mathtt{P} \cup \emptyset$ with $i \in T$ and $S \neq T$, it holds that

$$x_i \geq v(S \cup \{i\}) - v(S) \quad \text{or} \quad x(T \setminus \{i\}) > v(T \setminus \{i\}).$$

So, $(\mathtt{P}, x)$ is contractually stable. ■

Note that the proof of Proposition 3.2.3 in fact implies that any pull-in stable payoff configuration which is individually rational is contractually stable.

We have seen that individually stable outcomes may not always exist. However, if they do, they are necessarily individually rational, as stated in Proposition 3.2.4.

Proposition 3.2.4 *Any individually stable payoff configuration is individually stable.*

A trivial restriction on the characteristic value function that ensures the existence of individual stable outcomes is a restriction that ensures the existence of individually rational payoff configurations of which the grand coalition is an element.

Proposition 3.2.5 *Let (N, v) be a coalitional game such that $v(N) \geq \sum_{i \in N} v(\{i\})$. Then the payoff configuration $(\{N\}, x)$ with $x_i \geq v(\{i\})$ for every player $i \in N$ is an individually stable outcome of (N, v).*

On the other hand, contractual and compensation stable outcomes, can be found in any coalitional game. However, not all of these outcomes may be individually rational.

3.3 Mutual Insurance

We apply our stability concepts to the formation of mutual insurance groups in wheat production when the wheat producers experience the risk of a damage. This is an important question that is of concern to the Food and Agriculture Organization of the United Nations (FAO) since the risk of seed damage due to natural causes is seen as a major disrupting factor to the sustainability of food production in developing countries.[7] To illustrate this setting, we adopt the model of an insurance market studied by Rothschild and Stiglitz (1976). Here we will refer to a coalition as an *insurance group*. We use the three types of stability concepts to analyze payoff configurations which differ in terms of the risk composition of the insurance group. In particular, we discuss the pooling and separating outcomes which have received much attention in the literature. In Rothschild and Stiglitz (1976) framework, the separating type of outcomes is the only type that may be stable.

First, we describe the demand for insurance against seed damage. There is a finite set of players N. All players are expected output maximizers each with the same increasing and strictly concave production function Y defined over the amount of seeds a, *i.e.*, $Y : \mathbb{R} \to \mathbb{R}$ with $Y' > 0$ and $Y'' < 0$. Each player is endorsed with an initial amount of seeds w. With some probability π a player incurs a damage of his seed endowment which has an equivalent amount D with $D < w$. There are two groups of players, L and H, forming a partition of N. Players in L have a low probability π_L of incurring a damage. Those in H have a high probability π_H. So $\pi_L < \pi_H$.

We assume that each insurance group offers the same amount of insurance to all members equalling the total damage, *i.e.*, in case her seeds are damaged, a wheat producer will receive the same amount of good quality seeds as the amount of her damaged seeds. However, the group may require a different contribution "fee" per unit of insurance measured in terms of seeds. The contribution fee $q(S)$ of group S is

[7] See Sperling, Osborn and Cooper (2004) for a discussion on seed security and policies concerning local communities.

determined by a break-even condition given by:

$$|S|q(S)D - \pi_L|S \cap L|D - \pi_H|S \cap H|D = 0.$$

Hence the contribution fee charged by a group S is

$$q(S) = \frac{|S \cap L|}{|S|}\pi_L + \frac{|S \cap H|}{|S|}\pi_H. \tag{3.1}$$

So, the contribution fee depends only on the relative size of the risk-pool of the insurance group. In particular, $q(S) \in [\pi_L, \pi_H]$. It is lowest when an insurance group consists of low-risk players only, and it is highest when it consists of high-risk players only.

The value of an insurance group S is defined to be the total production of its members. Formally,

$$v_\pi(S) = |S|\ Y\Big(w - q(S)D\Big) \quad \text{for all } S \in 2^N \setminus \{\emptyset\}. \tag{3.2}$$

A coalitional game (N, v_π) derived from the above described tuple $(L, H, Y, w, D, \pi_L, \pi_H, q)$ with $N = L \cup H$ and v_π defined by (3.2) is called *a mutual insurance coalitional game* to which we refer as an *insurance game* for brevity. Note that the value function is not zero-normalized. However this affects neither the definitions of stability nor the results in Section 3.2.

The next example is used to illustrate the mutual insurance setting.

Example 3.3.1 Consider $L = \{1, 2\}$ and $H = \{3\}$, so $N = \{1, 2, 3\}$. The probabilities of incurring a damage are given by $\pi_L = 0.1$ and $\pi_H = 0.7$, respectively. Every player is endowed with the same amount of seed endowment $w = 10$ while $D = 9$. Every player has production abilities represented by the same increasing and strictly concave production function Y defined by $Y(a) = \sqrt{a}$ for $a \geq 0$.

The break-even contribution fee of each insurance group is calculated using Equation (3.1). The numbers are given below:

$$q(S) = \begin{cases} 0.1 & : \quad S = \{1\}, \{2\}, \{1, 2\}; \\ 0.3 & : \quad S = \{1, 2, 3\}; \\ 0.4 & : \quad S = \{1, 3\}, \{2, 3\}; \\ 0.7 & : \quad S = \{3\}. \end{cases}$$

Using the definition of group value given by Equation (3.2) and the break-even contribution fees, we obtain the following value function: $v_\pi(\{1\}) = v_\pi(\{2\}) = 3$, $v_\pi(\{3\}) = 1.9$, $v_\pi(\{1, 2\}) = 6$, $v_\pi(\{1, 3\}) = v_\pi(\{2, 3\}) = 5$, and $v_\pi(N) = 8.1$.

In this insurance game all individually stable payoff configurations $(\mathtt{P}, x)$ are of the form $\mathtt{P} = \{N\}, x(N) = 8.1$ and $x_1 \geq 3,\ x_2 \geq 3,\ x_3 \geq 1.9$.

The contractual and compensation stable payoff configurations $(\mathtt{P}, x)$ coincide and are given by $\mathtt{P} = \{N\}$ and $x(N) = 8.1$. ♦

In the rest of the section the analysis is focused on pooling[8] and separating types of payoff configurations. Below we give the formal definitions of these outcomes.

Definition 3.3.2 *Let (N, v_π) be an insurance game. A payoff configuration is called* ***pooling*** *if $\mathtt{P} = \{N\}$.*

Definition 3.3.3 *Let (N, v_π) with $N = L \cup H$ be an insurance game. A payoff configuration $(\mathtt{P}, x)$ is called* ***separating*** *if $\mathtt{P} = \{L, H\}$.*

We first show that the value function of an insurance game satisfies superadditivity.

Lemma 3.3.4 *Let (N, v_π) be an insurance game. Then for all $S, T \in 2^N$ with $S \cap T = \emptyset$*

$$v_\pi(S) + v_\pi(T) \quad \leq \quad v_\pi(S \cup T).$$

Proof. Let (N, v_π) be an insurance game derived from $(L, H, Y, w, D, \pi_L, \pi_H, q)$ with $N = L \cup H$ and v_π defined by Equation (3.2). Without loss of generality, consider $S, T \in 2^N \setminus \{\emptyset\}$ with $S \cap T = \emptyset$. Then

$$\begin{aligned}
v_\pi(S) + v_\pi(T) = & \\
& |S|Y\left(w - \frac{|S \cap L|\pi_L + |S \cap H|\pi_H}{|S|}D\right) + |T|Y\left(w - \frac{|T \cap L|\pi_L + |T \cap H|\pi_H}{|T|}D\right) \\
= \ & |S \cup T|\left\{\frac{|S|}{|S \cup T|}Y\left(w - \frac{|S \cap L|\pi_L + |S \cap H|\pi_H}{|S|}D\right)\right. \\
& \left. + \frac{|T|}{|S \cup T|}Y\left(w - \frac{|T \cap L|\pi_L + |T \cap H|\pi_H}{|T|}D\right)\right\} \\
\leq \ & |S \cup T|\left\{Y\left(w - \frac{|S \cap L|\pi_L + |S \cap H|\pi_H + |T \cap L|\pi_L + |T \cap H|\pi_H}{|S \cup T|}D\right)\right\} \\
= \ & v_\pi(S \cup T).
\end{aligned}$$

The above inequality follows from the strict concavity of Y. The inequality holds as equality in three cases: $S, T \subset L$; $S, T \subset H$; or $|S| = |T|$ with $q(S) = q(T)$. ■

[8]Unlike Rothschild and Stiglitz (1976) we refer to *pooling* payoff configuration only as those outcomes which contain the grand coalition.

Theorem 3.3.5 *In an insurance game we have*

(i) Any pooling payoff configuration is compensation stable;

(ii) Any pooling payoff configuration is contractually stable;

(iii) Any individually rational pooling payoff configuration is individually stable;

(iv) An individually rational pooling payoff configuration exists.

Proof. Lemma 3.3.4 implies that any pooling payoff configuration is a payoff configuration of maximum social worth. Therefore, *(i)* and *(ii)* follow from Theorem 3.2.2. *(iii)* is immediate from the definitions of individual rationality and individual stability: the only coalition to which a player may deviate from the grand coalition is the empty set, such deviation cannot lead to a higher payoff in an individually rational pooling outcome, while *(iv)* follows from the fact that $v_\pi(N) \geq \sum_{i \in N}(v_\pi(\{i\}) - v(\emptyset))$, which is a consequence of Lemma 3.3.4. ∎

Next, we discuss the stability of the separating payoff configurations.

Theorem 3.3.6 *In an insurance game we have*

(i) No separating payoff configuration is individually stable;

(ii) No separating payoff configuration is compensation stable;

(iii) No individually rational separating payoff configuration is contractually stable.

Proof. Let (N, v_π) be an insurance game derived from $(L, H, Y, w, D, \pi_L, \pi_H, q)$ with $N = L \cup H$ and v_π defined by Equation (3.2).

(i) Consider a separating payoff configuration $(\{L, H\}, x)$. We will show that the value of the high-risk insurance group is insufficient to give each member at least her outside option of joining the low-risk insurance group. For all $i \in H$, the outside option of joining the low-risk insurance group yields $v_\pi(L \cup \{i\}) - v_\pi(L)$. So for all $i \in H$

$$\begin{aligned}
& v_\pi(H) - |H|\Big(v_\pi(L \cup \{i\}) - v_\pi(L)\Big) = \\
& \quad |H|Y(w - \pi_H D) - |H|\left((|L|+1)Y\Big(w - \frac{|L|\pi_L + \pi_H}{|L|+1}D\Big) - |L|Y(w - \pi_L D)\right) \\
= & \quad |H|(|L|+1)\left\{\frac{1}{|L|+1}Y(w - \pi_H D) + \frac{|L|}{|L|+1}Y(w - \pi_L D) - Y\Big(w - \frac{|L|\pi_L + \pi_H}{|L|+1}D\Big)\right\} \\
< & \quad 0.
\end{aligned}$$

Here, the inequality follows from the strict concavity of Y.

(ii) Consider a separating payoff configuration $(\{L, H\}, x)$. We will first show that total coalition structure value increases when a high-risk player joins the low-risk group.

For any $i \in H$

$$\begin{aligned}
&v_\pi(L \cup \{i\}) + v_\pi(H \setminus \{i\}) - \Big(v_\pi(L) + v_\pi(H)\Big) \\
&= \; (|L|+1)Y\Big(w - \frac{|L|\pi_L + \pi_H}{|L|+1}D\Big) - Y(w - \pi_H D) - |L|Y(w - \pi_L D) \\
&= \; (|L|+1)\left\{Y\Big(w - \frac{|L|\pi_L + \pi_H}{|L|+1}D\Big) - \frac{1}{|L|+1}Y(w - \pi_H D) - \frac{|L|}{|L|+1}Y(w - \pi_L D)\right\} \\
&> \; 0.
\end{aligned}$$

The above inequality follows from the strict concavity of Y.

Using the efficiency of the payoffs we have

$$v_\pi(L \cup \{i\}) + v_\pi(H \setminus \{i\}) - \Big(v_\pi(L) + x_i + x(H \setminus \{i\})\Big) > 0.$$

The above inequality implies that at least one of the inequalities below holds for any $i \in H$

$$\Big(v_\pi(L \cup \{i\}) - v_\pi(L) - x_i\Big) > 0 \quad \text{or} \quad v_\pi(H \setminus \{i\}) - x(H \setminus \{i\}) > 0.$$

So for any $i \in H$,

$$\begin{aligned}
v_\pi(L) &< v_\pi(L \cup \{i\}) - x_i - \max\Big\{0, x(H \setminus \{i\}) - v_\pi(H \setminus \{i\})\Big\} \\
&\text{or} \\
x(H \setminus \{i\}) &< v_\pi(H \setminus \{i\}) - \max\Big\{0, x_i - \Big(v_\pi(L \cup \{i\}) - v_\pi(L)\Big)\Big\}.
\end{aligned}$$

Therefore, at least one of the pull-in and push-out conditions is violated.

(iii) Consider an individually rational separating payoff configuration $(\{L, H\}, x)$. By individual rationality, for all $i \in H$

$$x_i \geq v_\pi(\{i\}) = Y(w - \pi_H D).$$

By the efficiency of the payoff vector, for all $i \in H$

$$x(H) = v_\pi(H) = |H|Y(w - \pi_H D).$$

Combining both conditions, we obtain that $x_i = Y(w - \pi_H D)$ for all $i \in H$.

Hence, if any player wants to leave the high risk-group, the rest of the members will grant permission: for all $i \in H$

$$x(H \setminus \{i\}) = |H - 1|Y(w - \pi_H D) = v_\pi(H \setminus \{i\}).$$

To show that this outcome is not contractually stable, we need to show that the value of the high-risk group is insufficient to give all of its members their best outside option, *i.e.*, what they can get by joining the low-risk insurance group. This is already shown in the proof of *(i)* above. ■

3.4 Coalitional Matching Problem

The framework discussed in Section 3.1 can be extended to analyse the organization of a health care sector that in addition to insurance groups formation takes into account the matching between patients and doctors. To do so, we develop and analyze a *coalitional matching problem*. We also propose several possible adjustments of our notions of stability to capture the possibility for deviation on both sides of the matching problem. In the exposition, we focus only on the notion of individual stability. The notions of contractual stability and compensation stability can be re-formulated in a similar fashion.

There are two finite sets of players $N = \{1, \ldots, i, \ldots, n\}$ and $M = \{1, \ldots, k, \ldots, m\}$. Players in the set N form coalitions S. The collection of subsets of N is denoted by 2^N. A partition of N into non-empty coalitions is called coalition structure and is denoted by $\mathtt{P}$. The collection of all coalition structures on N is denoted as $\mathcal{P}$. Players in M cannot form coalitions.

In addition we define a bi-matrix A of size $M \times 2^N$, which has players $k \in M$ as row labels and coalitions $S \in 2^N$ as column labels. Each cell in the bi-matrix, a_{kS}, $k \in M$ and $S \in 2^N$, contains a pair of numbers $(u^S(k), v^k(S))$ that characterizes the matching between a player $k \in M$ and a coalition $S \in 2^N$: $u^S(k)$ is the utility player k when matched to coalition S and $v^k(S)$ is the value of S when matched to player k. It is assumed that $u^\emptyset(k) = 0$ and $v^k(\emptyset) = 0$ for all $k \in M$. The triple (N, M, A) denotes a *coalitional matching problem*.

An outcome of a coalitional matching problem is represented by a *payoff matching configuration*. A payoff matching configuration is a triple $(\mathtt{P}, \mu, r)$ with $\mathtt{P} \in \mathcal{P}$, a function $\mu : \mathtt{P} \to M$, and a payoff vector $r \in \mathbb{R}^{N \cup M}$ such that $r(S) = v^{\mu(S)}(S)$, for all

$S \in \mathtt{P}$ and $r_k = u^{\mu^{-1}(k)}(k)$ where $u^{\mu^{-1}(k)}(k) := \sum_{S \in \mathtt{P} | \mu(S)=k} u^S(k)$ for all $k \in M$. We call the pair $(\mathtt{P}, \mu)$ a *coalitional matching*.

Note that in a coalitional matching $(\mathtt{P}, \mu)$, it is possible for a player $k \in M$ to be matched to several coalition structure elements of P or not at all while all players in N are matched to a player in M.

Next, we propose two possible formulations of the notion of individual stability in the context of coalitional matching problems that takes into account deviational possibilities by players in M as well. The first definition is closely related to the one used in the matching literature. In particular we allow coalitions in P and players in M who are not matched to each other and prefer each other to their current matching partners to deviate.

Definition 3.4.1 *Let (N, M, A), be a coalitional matching problem. A payoff matching configuration $(\mathtt{P}, \mu, r)$ is* ***two-sided individually stable*** *if the following conditions are satisfied:*

(i) there are no $i \in N$ and $S \in \mathtt{P} \cup \{\emptyset\}$ with $i \notin S$ such that

$$r_i < v^{\mu(S)}(S \cup \{i\}) - v^{\mu(S)}(S);$$

(ii) there is no $k \in M$ such that $r_k < 0$;

(iii) there are no $S \in \mathtt{P}$ and $k \in M$ with $\mu(S) \neq k$ such that

$$v^{\mu(S)}(S) < v^k(S) \quad \text{and} \quad 0 < u^S(k).$$

Condition (i) is the same as the one in Definition 3.1.1. Condition (ii) guarantees individual rationality for the players in M. Condition (iii) is similar to the no-blocking condition in Sotomayor (1996). The following example illustrates the concepts.

Example 3.4.2 Let $N = \{1, 2, 3\}$ and $M = \{k, l\}$ and the bi-matrix A be as given below.

$M/2^N$	$\emptyset$	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
k	(0,0)	(1,0)	(1,0)	(1,0)	(2,5)	(2,5)	(2,5)	(1,0)
l	(0,0)	(2,1)	(2,1)	(2,1)	(1,4)	(1,4)	(1,4)	(0,0)

.

Consider cell $a_{k,\{1,2\}}$. It characterizes a matching between player k with the coalition $\{1, 2\}$. the pair of numbers $(2, 5)$ indicate that the utility of player k in the matching with the coalition $\{1, 2\}$ is 2, and the value of the coalition $\{1, 2\}$ in the matching with player k is 5. The rest of the cells can be read in a similar fashion.

In this coalitional matching problem there are no two-sided individually stable payoff matching configurations. First consider the coalition structure $\{\{1\},\{2\},\{3\}\}$. It cannot be an element of a two-sided individually stable payoff configuration because players 2 and 3 always prefer to form a coalition than stay singletons, $v^k(\{2,3\}) > v^k(\{2\})+v^k(\{3\})$ and $v^l(\{2,3\}) > v^l(\{2\})+v^l(\{3\})$. Next, consider the coalition structure $\{N\}$. It cannot be an element of a two-sided individually stable payoff configuration because at least one player prefers to deviate to a singleton coalition matched to player l because $v^k(N) = v^l(N) < v^l(\{1\}) + v^l(\{2\}) + v^l(\{3\})$. Finally, consider the coalition structure $\{\{1,2\},\{3\}\}$ and matching μ such that $\mu(\{1,2\}) = k$ $\mu(\{3\}) = l$. Note that μ is the only matching that satisfies condition (iii). These coalition structure and matching cannot be elements of a two-sided individually stable matching payoff configuration because condition (i) cannot be satisfied for players 1 and 2 simultaneously by any payoff vector, *i.e.*, $v^k(\{1,2\}) < v^l(\{1,3\}) - v^l(\{3\}) + v^l(\{2,3\}) - v^l(\{3\})$. Similarly, one can show that the coalition structures $\{\{1,3\},\{3\}\}$ and $\{\{1\},\{2,3\}\}$ cannot be elements of a two-sided individually stable matching payoff configuration either. ♦

There are other possible formulations of the notion of individual stability in the context of a coalitional matching problem.

Definition 3.4.3 *Let (N, M, A) be a coalitional matching problem. A payoff matching configuration (P, μ, r) is* ***two-sided individually stable with a possibility of blocking*** *if the following conditions are satisfied:*

(i) there are no $i \in N$, S, $T \in \mathrm{P} \cup \{\emptyset\}$, with $S \neq T$ and $i \in T$ such that

$$\begin{aligned} r_i &< v^{\mu(S)}(S \cup \{i\}) - v^{\mu(S)}(S) && \textit{and} \\ u^S(\mu(S)) &< u^{S\cup\{i\}}(\mu(S)) && \textit{if } \mu(S) \neq \mu(T) \\ u^S(\mu(S)) + u^T(\mu(T)) &< u^{S\cup\{i\}}(\mu(S)) + u^{T\setminus\{i\}}(\mu(T)) && \textit{if } \mu(S) = \mu(T); \end{aligned}$$

(ii) there is no $k \in M$ such that $r_k < 0$;

(iii) there are no $S \in \mathrm{P}$ and $k \in M$ with $\mu(S) \neq k$ such that

$$v^{\mu(S)}(S) < v^k(S) \quad \textit{and} \quad 0 < u^S(k).$$

The difference between Definition 3.4.1 and Definition 3.4.3 is in condition (i). The underlying assumption in Definition 3.4.1 is that players in the set M do not have a blocking power in the coalition formation process among the players in the set N. This assumption is relaxed in Definition 3.4.3.

The following example illustrates the difference between the two notions of stability.

Example 3.4.4 Let (N, M, A) be as in Example 3.4.2. Consider the coalition structure $\mathtt{P} = \{\{1,2\},\{3\}\}$, the matching function μ such that $\mu(\{1,2\}) = k$ and $\mu(\{3\}) = l$, and the payoff vector r such that $r_1 = 0$, $r_2 = 5$, $r_3 = 1$, $r_k = 2$, and $r_l = 1$. It is easy to see that the payoff matching configuration $(\mathtt{P}, \mu, r)$ is two-sided individually stable with a possibility of blocking: player 2 would like to join player 3 in a coalition matched to player *l*, however, *l* will be worse off if the deviation were executed, so she blocks it; for all other players conditions (i), (ii), and (iii) are satisfied. ♦

Our motivation for developing the new class of problems called coalitional matching problems is to study in a comprehensive manner the organization of a sector such as the health care sector. In reality health care is a sector which is heavily regulated and countries differ in the extent and type of regulation. This is why the choice of stability concept used in predicting an organizational outcome should be tailored to the particular case under investigation. Here we propose two such definitions. What is left for future work is to investigate the conditions for existence of a stable outcome in each case.

CHAPTER

FOUR

A B S B E I S E C F *

4.1 Introduction

Economic entities, such as medical practices, insurance groups, research teams and coalitional governments, involve agents who generate value by cooperating in groups. In some situations the groups that actually form will partition the whole population into smaller groups, while in others the population as a whole will form one cooperating group. Studying this endogenous formation of groups and predicting which groups will break up or will be stable is a captivating area of research. In politics it can predict which governments can be stable. In organizational science it can predict which researchers can be grouped together or alternatively should work alone. The value generated by a coalition in most cases cannot be traced back to the individual efforts. This brings about an additional question of how the group value should be translated into individual payoffs.

These two questions, of coalition formation and of value allocation, are interdependent and require a simultaneous answer as argued by Maschler (1992). They were addressed simultaneously in the seminal work of Aumann and Maschler (1964) where an outcome of a cooperative game consists of a coalition structure, *i.e.*, a partition of the player set into coalitions, and a payoff vector which divides the value of each coalition in the partition among its members. To analyze the stability properties of an outcome Aumann and Maschler (1964) introduce the *Maschler bargaining set*[1]. The

*This chapter is based to a great extent on Lazarova, Borm, Montero and Reijnierse (2006).

[1] Aumann and Maschler (1964) introduce several definitions of bargaining sets and study in depth only one of them, which is not the *Maschler bargaining set*. The Maschler bargaining set gains pop-

Maschler bargaining set is the set of outcomes which survive a specific bargaining process among all players. In this bargaining process over a given outcome, players put forward "objections" and "counterobjections" against other members of the same coalition in the coalition structure of the outcome. An objection consists of a new coalition, of which the objecting player is a member and the player against whom the objection has been raised is not, such that all members of the new coalition can obtain higher payoffs than what is allocated to them in the proposed outcome. The player against whom the objection has been raised can launch a counterobjection. A counterobjection consists of a coalition and a payoff vector such that the coalition members can obtain at least as high a payoff as in the original outcome and those of them who also participate in the coalition used in the objection can get at least as much as they would have obtained if the objection had been executed. The player who launches the counterobjection must be a member of the coalition used in the counterobjection, while the player who has raised the objection must be excluded from it. The bargaining set contains those outcomes for which each objection can be countered.

An early work by Peleg (1967) shows that any coalition structure is stable for a coalitional game with a non-empty imputation set in terms of the Maschler bargaining set, *i.e.*, there is a payoff vector which allocates the value of each coalition in the coalition structure among its members such that the coalition structure and this payoff vector constitute an outcome in the Maschler bargaining set. This finding precludes the use of the Maschler bargaining set in analyzing endogenous coalition formation. Zhou (1994) offers a new bargaining set which has the desirable property that in this setting it does not support all possible coalition structures. A more recent work by Morelli and Montero (2003) introduces another solution concept which selects "more desirable" outcomes out of those selected in the Zhou's bargaining set.

A common aspect of these bargaining sets is that they treat the deviation possibilities within a coalition structure element and between coalition structure elements in a symmetric way. This, in our opinion, is a serious limitation since in many economic situations transaction costs and institutional arrangements will require to make a distinction between the two. When considering a deviation within a group, all subsets of this group should be taken as a possible threat point against the group.[2] However, when considering a deviation involving more than one coalition structure element, our new bargaining set only allows a player to join an already formed group. As a motivation one can think of prohibitively high transaction costs in terms of licensing requirements, which make it impossible that new groups are formed based on sub-

ularity and is studied in more detail in later works, *e.g.*, Davis and Maschler (1967).

[2]This is also the case in all other existing bargaining sets.

groups of distinct coalition structure elements. The different treatment of internal and external stability distinguishes our bargaining set from the previously studied bargaining sets. Below we offer two examples that illustrate the difference between internal and external objections in an endogenous coalition formation setting.

Consider a parliament of representatives of four parties and a seat distribution such that no party can form a government on its own. Suppose that there is one big party and three small ones such that the big party with any of the three small parties can form a government, and so can the three small parties together. Consider an outcome in which a government is formed by the big party and one of the small parties. Our bargaining set predicts that the allocation of government value is different if the opposition parties act together or separately. In the first case any of the government parties may threaten to split off the government by joining the opposition to form a new government. In the second case, such a threat is only available to the big party.

Investigating the outcomes in our bargaining set in the general setting of weighted majority games is the first application that we offer. We show that in any weighted majority game, the minimal winning coalition formed by the players with the highest weights and all other players acting alone leads to a coalition structure that is supported by an outcome in our bargaining set.[3] This implies that a coalitional government based on the biggest parties in the parliament when the opposition parties do not cooperate is stable. However, in practice one also observes situations of united opposition. This raises the question whether in any weighted majority game, there is a stable partition comprised of a minimal winning coalition and its complement. We answer this question negatively for the general case but for the case of homogeneous weighted majority games the answer is positive.

Now consider a different setting of a group of researchers who have the same research capabilities and only differ in a cooperation parameter. Some researchers experience positive spillovers when working in teams and have team building abilities, while others tend to free-ride when they are in a team and thus carry negative cooperation effect. Consider a coalition structure consisting of teams of researchers. For a coalition structure to be internally stable, there should not be an internal objection of a researcher against another researcher member of the same team, which the latter researcher cannot counter. An internal objection in a coalition structure element is analogous to an objection in the Maschler bargaining set of the coalition-restricted cooperative game. A valid counterobjection in our setting has an additional requirement over the counterobjection defined in the Maschler bargaining set: the subset

[3]Burani and Zwicker (2003) show that the same type of coalitional structure is both core and Nash stable in the setting of hedonic coalition formation games with descending separable preferences.

of the team used in the objection and the one used in the counterobjection have at least one member in common. This type of modification was originally introduced by Zhou (1994) and it tailors the bargaining set to select coalition structures with higher total partition value. In addition, a researcher may raise an external objection against another researcher of her team by threatening to join another team in the coalition structure. Such an external objection can be countered if the researcher against whom it has been raised is at least as desirable to the outside team as the researcher who launches the objection. A coalition structure and a payoff vector such that for any internal objection there is an internal counterobjection and for every external objection there is an external counterobjection will constitute an outcome in our bargaining set.

This example illustrates the second application that is studied. In this application researchers will differ not only in the direction of the cooperation effect, but also in the degree of positive or negative cooperation abilities that they exhibit. In the symmetric case in which each researcher has either a fixed "negative" (less than 1) or "positive" cooperation parameter (higher than 1), the free-riding ability weakly dominates the team-building ability, and in which there are at least two players who have a cooperation parameter higher than 1, we find that the unique coalition structure supported by the bargaining set is the one in which the "cooperative" researchers form a coalition, while the "non-cooperative" researchers are singletons. [4]

In addition to the distinction between internal and external deviations, we introduce two types of coalitional rationality conditions, splitting-proofness, which is a weak form of the coalitional rationality condition present in the bargaining set studied in depth by Aumann and Maschler (1964), and merging-proofness. These conditions require that total payoffs do not increase if a coalition structure element is split in two or if two coalition structure elements merge.

The remainder of the chapter is organized in the following way. In Section 4.2 we formally introduce our bargaining set. Section 4.3 and Section 4.4 consider applications to weighted majority games and games with cooperation , respectively.

4.2 The Bargaining Set

We first give some basic notions. Let $N = \{1, 2, \ldots, n\}$ be a finite set of players. Players can form coalitions $S \subseteq N$. The set of all possible coalitions is denoted by 2^N. A value function $v : 2^N \to \mathbb{R}$, $v(\emptyset) = 0$, provides the value each coalition generates by cooperation of its members. In addition and without loss of generality we assume

[4]This result is also reminiscent of the one described in Burani and Zwicker (2003).

$v(\{i\}) = 0$ for all $i \in N$. The pair (N, v) is a *coalitional game*. A partition of N into non-empty coalitions is called a *coalition structure*. The set of all possible coalition structures of N is denoted by $\mathcal{P}$.

An outcome of a coalitional game is represented by a *payoff configuration*. A payoff configuration is a pair (P, x) where $\mathrm{P} \in \mathcal{P}$ is a coalition structure of N and $x \in \mathbb{R}^N$ is an efficient payoff vector for P, *i.e.*, $x(S) = v(S)$ for all $S \in \mathrm{P}$, where $x(S) := \sum_{i \in S} x_i$. A payoff configuration (P, x) of a coalitional game (N, v) is *individually rational* if $x_i \geq 0$ for all $i \in N$.

For the sake of completeness and comparison we first present the Maschler and Zhou bargaining sets.

For the definition of the Maschler bargaining set we follow Maschler (1992). Let (P, x) be an individually rational payoff configuration of a coalitional game (N, v). Let players k and l be two distinct members of some coalition $S \in \mathrm{P}$. The pair (T, y) with $T \in 2^N$ and $y \in \mathbb{R}^T$ is called an *objection* of k against l in (P, x) if $k \in T$, $l \notin T$, $y(T) = v(T)$, $y_k > x_k$, and $y_i \geq x_i$ for all $i \in T$. The pair (Q, z) with $z \in \mathbb{R}^Q$ is called a *counterobjection* to the above objection (T, y) in (P, x) if $l \in Q$, $k \notin Q$, $z(Q) = v(Q)$, $z_i \geq x_i$ for all $i \in Q$, and $z_i \geq y_i$ for all $i \in Q \cap T$.

The Maschler bargaining set $\mathcal{M}(v)$ consists of those individually rational payoff configurations for which each objection can be countered.

The bargaining set introduced in Zhou (1994) differs from the Maschler bargaining set in the definitions of both an objection and a counterobjection. Let (P, x) be a payoff configuration of a coalitional game (N, v). The pair (T, y) with $y \in \mathbb{R}^T$ is called a $\mathcal{Z}$-*objection* of coalition $T \in 2^N$ in payoff configuration (P, x) if $y(T) = v(T)$ and $y_i > x_i$ for all $i \in T$. The pair (Q, z) with $z \in \mathbb{R}^Q$ is called a $\mathcal{Z}$ *-counterobjection* to the above objection (T, y) in (P, x) if $z(Q) = v(Q)$, $Q \setminus T \neq \emptyset$, $T \setminus Q \neq \emptyset$, $T \cap Q \neq \emptyset$, $z_i \geq x_i$ for all $i \in Q$, and $z_i \geq y_i$ for all $i \in Q \cap T$.

The Zhou bargaining set $\mathcal{Z}(v)$ consists of those payoff configurations for which each $\mathcal{Z}$-objection can be countered.

Note that the above condition will imply that any element of $\mathcal{Z}(v)$ is individually rational.

Our new bargaining set will combine separate notions of internal stability and external stability. First we present the internal bargaining set. The following two conditions reflect the notions of internal coalitional rationality and stability against internal deviations, respectively.

Definition 4.2.1 *Let (N, v) be a coalitional game. A coalition structure* P *is **splitting-proof** if for all $S \in \mathrm{P}$, and all disjoint coalitions $T, Q \subseteq S$, such that $T \cup Q = S$,*

$v(S) \geq v(T) + v(Q)$.

Let $(\mathtt{P}, x)$ be a payoff configuration of a coalitional game (N, v) and let k and l be two distinct members of the same coalition $S \in \mathtt{P}$. The pair (T, y) with $T \subseteq S \setminus \{l\}$, $k \in T$ and $y \in \mathbb{R}^T$ is called an *internal objection* of k against l in $(\mathtt{P}, x)$ if $y_i > x_i$ for all $i \in T$ and $y(T) = v(T)$. The pair (Q, z) with $z \in \mathbb{R}^Q$ is called an *internal counterobjection* to the above internal objection (T, y) in $(\mathtt{P}, x)$ if $z(Q) = v(Q)$, $l \in Q$, $Q \subseteq S \setminus \{k\}$, $Q \cap T \neq \emptyset$, $z_i \geq x_i$ for all $i \in Q$, and $z_i \geq y_i$ for all $i \in Q \cap T$. We say that an internal objection (T, y) of player k against l in $(\mathtt{P}, x)$ is *justified* if there is no internal counterobjection.

Definition 4.2.2 *Let (N, v) be a coalitional game. A payoff configuration $(\mathtt{P}, x)$ is in the* ***internal bargaining set*** *$\mathcal{B}^I(v)$ if*

(i) $\mathtt{P}$ is splitting-proof, and

(ii) there is no justified internal objection.

A payoff configuration $(\{N\}, x)$ of the internal bargaining set is in the Maschler bargaining set as well: the definition of objection is the same and that of counterobjection imposes the additional requirement that the coalition used in the objection and the coalition used in the counterobjection are not disjoint. Moreover, $\{N\}$ is required to be splitting-proof. For payoff configurations comprising other coalition structures than $\{N\}$, there is no general relation since we require the coalitions used both in the objection and the counterobjection to be subsets of one specific coalition structure element.

Compared to the Zhou bargaining set we have the following differences. First, any coalition is allowed to object in the Zhou bargaining set; in our bargaining set the coalition T used in the objection is a subset of one coalition structure element and, moreover, must exclude at least one player. On the other hand, countering an objection is easier in the definition of Zhou. We require the counterobjection to be launched by the player against whom the objection has been raised, whereas in the Zhou approach a counterobjection can be launched by any player. Furthermore, we require that the coalition used in the counterobjection must be a subset of the same coalition structure element of which T is a subset, whereas any coalition which has a non-empty intersection with T can be used to counterobject in Zhou's framework. Given the differences it is easy to see that a payoff configuration $(\{N\}, x)$ of the internal bargaining set is in the Zhou bargaining set as well: any $\mathcal{Z}$-objection can be translated into an internal objection, and the corresponding internal counterobjection can also be used as a $\mathcal{Z}$-counterobjection. However, the reverse does not necessarily hold. The next example clearly illustrates the differences.

Example 4.2.3 Let $N = \{1,2,3\}$, $v(\{i\}) = 0$ for all $i \in N$, $v(\{2,3\}) = 0$, $v(\{1,2\}) = v(\{1,3\}) = 20$, and $v(N) = 21$.

Consider the payoff configuration $(\{1,2,3\},(7,7,7))$. It is in the Zhou bargaining set: any $\mathcal{Z}$-objection $(\{1,2\},y)$ can be countered using $\mathcal{Z}$-counterobjection $(\{1,3\},z)$ with $z_1 = y_1$. Similarly, $\mathcal{Z}$-objections using coalition $\{1,3\}$ can be countered using coalition $\{1,2\}$. However, it is not in the internal bargaining set: player 1 has a justified internal objection $(\{1,2\},(10,10))$ against player 3.

Consider the payoff configuration $(\{\{1,2\},\{3\}\},(10,10,0))$. It is in the internal bargaining set because there are no internal objections. However, it is not in the Zhou bargaining set: the pair $(\{1,2,3\},(10\frac{1}{3},10\frac{1}{3},\frac{1}{3}))$ constitutes a $\mathcal{Z}$-objection that cannot be countered. ♦

The following result is immediate.

Proposition 4.2.4 *Let (N,v) be a coalitional game. Then*

(i) $(\langle N\rangle^5, 0) \in \mathcal{B}^I(v)$*;*

(ii) $(P,x) \in \mathcal{B}^I(v)$ *implies that* (P,x) *is individually rational.*

Next we present the external bargaining set. Similar to the internal bargaining set, the external bargaining set is based on two notions which reflect external coalitional rationality and stability against external deviations, respectively.

Definition 4.2.5 *Let (N,v) be a coalitional game. A coalition structure P is **merging-proof** if for all $S,T \in \mathrm{P}$ we have $v(S) + v(T) \geq v(S \cup T)$.*

Let (P,x) be a payoff configuration of a coalitional game (N,v) and let k and l be two distinct members of the same coalition $S \in \mathrm{P}$. The pair $(T \cup \{k\},y)$ with $y \in \mathbb{R}^{T\cup\{k\}}$ is called an *external objection* of k against l in (P,x) if $T \in \mathrm{P}$, $T \neq S$, $y(T \cup \{k\}) = v(T \cup \{k\})$ and $y_i > x_i$ for all $i \in T \cup \{k\}$. The pair $(T \cup \{l\},z)$ with $z \in \mathbb{R}^{T\cup\{l\}}$ is called an *external counterobjection* to the above objection $(T \cup \{k\},y)$ in (P,x) if $z(T \cup \{l\}) = v(T \cup \{l\})$, $z_k \geq x_k$, and $z_i \geq y_i$ for all $i \in T$. We say that an external objection (T,y) of player k against l in (P,x) is *justified* if there is no external counterobjection.

[5] $\langle S\rangle := \{\{i\}_{i\in S}\}$ *for all* $S \in 2^N$.

Definition 4.2.6 *Let (N, v) be a coalitional game. A payoff configuration $(\mathtt{P}, x)$ is in the* ***external bargaining set*** $\mathcal{B}^E(v)$ *if*

(i) $\mathtt{P}$ *is merging-proof,* *and*

(ii) there is no justified external objection.

Since no external objections can be launched against the grand coalition, we find the following result.

Proposition 4.2.7 *For a coalitional game (N, v) any payoff configuration $(\{N\}, x)$ lies within $\mathcal{B}^E(v)$.*

Note that the payoff configurations in the external bargaining set are not necessarily individually rational.

The new bargaining set $\mathcal{B}(v)$ consists of all payoff configurations that are both in the internal bargaining set and in the external bargaining set.

Definition 4.2.8 *For a coalitional game (N, v) the* ***bargaining set*** $\mathcal{B}(v)$ *is given by*

$$\mathcal{B}(v) = \mathcal{B}^I(v) \cap \mathcal{B}^E(v).$$

We say that the coalition structure $\mathtt{P}$ is *stable* if there is a payoff vector $x \in \mathbb{R}^N$ such that the payoff configuration $(\mathtt{P}, x)$ is an element of $\mathcal{B}(v)$.

Unfortunately, the bargaining set can be empty. Below we provide two examples of coalitional games in which the bargaining set is empty. Example 4.2.9 illustrates tension between the internal and external stability. In this example for any given coalition structure $\mathtt{P}$ for which there exists a payoff vector x such that $(\mathtt{P}, x) \in \mathcal{B}^I(v)$, it holds that $(\mathtt{P}, x) \notin \mathcal{B}^E(v)$. Similarly, for any given coalition structure $\mathtt{P}$ for which there exists a payoff vector z such that $(\mathtt{P}, z) \in \mathcal{B}^E(v)$, it holds that $(\mathtt{P}, z) \notin \mathcal{B}^I(v)$.

Example 4.2.9 Consider the coalitional game (N, v) given by $N = \{1, 2, 3, 4, 5, 6, 7\}$ with $v(\{1, 2\}) = v(\{1, 3\}) = v(\{2, 3\}) = v(\{5, 6\}) = v(\{5, 7\}) = v(\{6, 7\}) = 20$, $v(\{1, 2, 3\}) = v(\{5, 6, 7\}) = 21$, $v(\{1, 2, 3, 4\}) = v(\{4, 5, 6, 7\}) = 30$, $v(S) = 0$ for $|S| = 1$, and $v(S) < 0$, otherwise. For this game $\mathcal{B}(v) = \emptyset$.

First note that no coalition with a negative value can be a coalition structure element of a payoff configuration which belongs to our bargaining set since such a payoff configuration will not be individually rational.

Suppose $(\mathsf{P}, x) \in \mathcal{B}(v)$ with $\mathsf{P} = \big\{\{1,2,3,4\},\{5,6,7\}\big\}$. First suppose $x_4 = 0$. Then player 4 has a justified external objection $(\{4,5,6,7\}, y)$ against any other member of the coalition $\{1,2,3,4\}$. Next suppose $x_4 > 0$. Then it must be the case that at least one of the other players, 1, 2, or 3, receives a payoff of strictly less than 10, *e.g.*, player 1; and that the sum of the payoffs of two of these players must be strictly less than 20, *e.g.*, $x_1 + x_2 < 20$. Player 1 can then launch a justified internal objection against player 4 using coalition $\{1,2\}$ and a payoff vector that gives to both players more than what is allocated to them. Hence $(\mathsf{P}, x) \notin \mathcal{B}(v)$.

Next consider the coalition structure $\big\{\{1,2,3\},\{4\},\{5,6,7\}\big\}$. It is not stable since it is not merging-proof: $v(\{1,2,3,4\}) > v(\{1,2,3\}) + v(\{4\})$.

In a similar fashion it can be shown that no payoff configuration belongs to the bargaining set. ♦

Example 4.2.10 illustrates that the internal bargaining set is empty for the grand coalition of a strictly superadditive game.

Example 4.2.10 Consider $N = \{1,2,3,4,5\}$, $v(\{i\}) = 0$ for all $i \in N$, $v(\{1,3\}) = v(\{2,4\}) = 30$, $v(\{1,2,5\}) = v(\{3,4,5\}) = 60$ and $v(\{N\}) = 66$, $v(S)$ is minimal with respect to strict superadditivity, otherwise.

Since the coalitional game is strictly superadditive, the only coalition structure which is merging proof is $\{N\}$.

Consider the payoff configuration $(\{N\}, x)$. First, suppose $x_5 > 30$. So, $x_1 + x_3 < 30$ or $x_2 + x_4 < 30$. Without loss of generality let $x_1 + x_3 \leq x_2 + x_4$. Either player 1 or player 3 can launch a justified objection using coalition $\{1,3\}$ against player 5: either $30 - x_1 + x_4 + x_5 > 60$ or $30 - x_3 + x_2 + x_5 > 60$. Next suppose $x_5 = 30$. Without loss of generality let $x_1 + x_3 \leq x_2 + x_4$. Either player 1 or player 3 can launch a justified objection using coalition $\{1,3\}$ against player 2: either $30 - x_1 + x_2 + x_4 > 30$ or $30 - x_3 + x_2 + x_4 > 30$ Finally suppose $x_5 < 30$. Without loss of generality, let $x_1 + x_2 \leq x_3 + x_4$. Player 5 can launch a justified objection using coalition $\{1,2,5\}$ against either player 3 or player 4: either $60 - x_2 - x_5 + x_3 > 30$ or $60 - x_1 - x_5 + x_4 > 30$. ♦

However, the bargaining set is not empty in a coalitional game with three players.

Proposition 4.2.11 *Let (N, v) be a coalitional game with $N = \{1,2,3\}$. Let P be such that $\sum_{S \in \mathsf{P}} v(S)$ is maximized. Then there exists a payoff vector $x \in \mathbb{R}^N$ such that $(\mathsf{P}, x) \in \mathcal{B}(v)$.*

Proof. First note that $\mathtt{P}$ is splitting-proof and merging-proof. Recall that $\mathcal{M}(v)$ is nonempty for all coalition structures. So there is a payoff vector $x \in \mathbb{R}^3$ such that $(P, x) \in \mathcal{M}(v)$. We will show that $(P, x) \in \mathcal{B}(v)$.

Let $\mathtt{P} = \{N\}$. By individual rationality of $\mathcal{M}(v)$, if a player i has an objection against another player j at x it must be using coalition $\{i, k\}$. Because $\{N\}$ is the coalition structure with maximum total value, an objection of i against j can only exist if $x_j > 0$. A counterobjection in the Maschler sense must then use coalition $\{j, k\}$, which is also a valid counterobjection in the sense of $\mathcal{B}(v)$.

Similar arguments can be used in case $\mathtt{P} = \{\{i, j\}, \{k\}\}$. The case $\mathtt{P} = \langle N\rangle$ is straightforward since objections are not possible. ■

The result of Proposition 4.2.11 cannot be extended to coalitional game with four players as the next example illustrates.

Example 4.2.12 Consider the coalitional game (N, v) given by $N = \{1, 2, 3, 4\}$ with characteristic value function $v(\{1, 2\}) = v(\{1, 3\}) = v(\{2, 3\} = 20$, $v(\{N\}) = 21$, and $v(S) = 0$, otherwise.

Clearly $\mathtt{P} = \{N\}$ is the coalition structure with the maximal $\sum_{S \in \mathtt{P}} v(S)$. However, it is not stable. At least two of the three players 1, 2, and 3 have a justified objection against player 4 in any individually rational payoff configuration because the three inequalities $x_1 + x_2 \geq 20$, $x_1 + x_3 \geq 20$, and $x_2 + x_3 \geq 20$ cannot be satisfied simultaneously by any individually rational payoff vector and no coalition containing player 4 but $\{N\}$ has a strictly positive value. ♦

Yet, we establish a positive result for a superadditive coalitional game with four players.

Proposition 4.2.13 *Let (N, v) be a superadditive coalitional game with $N = \{1, 2, 3, 4\}$. Then there exists a payoff vector $x \in \mathbb{R}^N$ such that $(\{N\}, x) \in \mathcal{B}(v)$.*

Proof. Note that $\{N\}$ is splitting-proof since (N, v) is superadditive. Furthermore $\{N\}$ is merging-proof. Consider a payoff vector $x \in \mathbb{R}^N$ such that $(\{N\}, x) \in \mathcal{M}(v)$. We will show that $(\{N\}, x) \in \mathcal{B}(v)$.

There are no external objections. Suppose there is an objection in the Maschler sense of player k against l using (T, y). Then (T, y) is also an internal objection of player k against l. Since any payoff configuration in $\mathcal{M}(v)$ is individually rational $|T| \geq 2$. First suppose $x_l \geq 0$. Since $(\{N\}, x) \in \mathcal{M}(v)$ there is a counterobjection of player l in the Maschler sense using a pair (Q, z). Then it must be that $|Q| \geq 2$ since $v\{i\} = 0$ and $Q \cap T \neq \emptyset$, otherwise $v(Q) + v(T) \geq x(Q) + y(T) > x(Q) + x(T) = v(\{N\})$, which violates the superadditivity of (N, v). So, player l can use (Q, z) as an internal counterobjection. Next suppose $x_l = 0$. Then there must be another player $j \in N \setminus T$, $j \neq l$ such that $x_j > 0$, otherwise $v(T) = y(T) > x(T) = v(\{N\}$. So, $|T| = 2$. Then player k can launch an objection in the Maschler sense against player j using (T, y). Since $\{N, x\} \in \mathcal{M}(v)$, player j must have a counterobjection $(\hat{Q}, \hat{z})$ in the Maschler sense and as it was shown above it must be that $|\hat{Q}| \geq 2$ and $\hat{Q} \cap T \neq \emptyset$. If $l \in \hat{Q}$, then player l can use $(\hat{Q}, \hat{z})$ to counterobject. If $l \notin \hat{Q}$, then player l can use the pair $(\tilde{Q}, \tilde{z})$ with $\tilde{Q} = \hat{Q} \cup \{l\}$, $\tilde{z}_l = 0$, $\tilde{z}_i = \hat{z}_i$ for all $i \in \hat{Q}$ to counter object since since by superadditivity $v(\tilde{Q}) \geq v(\hat{Q})$. ■

4.3 Monotonic Proper Simple Games

First we provide some basic definitions. A coalitional game (N, v) is called *simple* if $v(\emptyset) = 0$, $v(N) = 1$, and $v(S) \in \{0, 1\}$, otherwise. A simple game is monotonic if $v(T) = 1$ whenever $v(S) = 1$ for some $S \subseteq T$. We denote the set of *winning coalitions* in a simple game by $\mathcal{W} := \{S \in 2^N \mid v(S) = 1\}$ and by $\mathcal{W}^m := \{S \in 2^N \mid v(S) = 1 \text{ and } v(T) = 0 \text{ for all } T \subsetneq S\}$ the set of *minimal winning coalitions*. A simple game is *proper* if for all $S, T \in \mathcal{W}$ it holds that $S \cap T \neq \emptyset$.

We can establish the following result with respect to monotonic proper simple games.

Theorem 4.3.1 *Let (N, v) be a monotonic proper simple game. Then*

$$(\{N\}, x) \in \mathcal{B}(v) \iff (\{N\}, x) \in \mathcal{M}(v).$$

Proof. We have already seen that for general coalitional games $(\{N\}, x) \in \mathcal{B}(v)$ implies $(\{N\}, x) \in \mathcal{M}(v)$. To show the converse, consider a payoff configuration $(\{N\}, x) \in \mathcal{M}(v)$. Proposition 4.2.7 gives that $(\{N\}, x) \in \mathcal{B}^E(v)$. Next we show that $(\{N\}, x) \in \mathcal{B}^I(v)$. In all proper simple games $\{N\}$ is splitting-proof. Furthermore, every internal objection can be countered. Let (T, y) be an internal objection of player k against player l. Since $(\{N\}, x) \in \mathcal{M}(v)$, there exists a counterobjection (Q, z) in the Maschler sense. We have to find a counterobjection $(\tilde{Q}, \tilde{z})$ in the new sense. If $Q \cap T \neq \emptyset$, we can take $(\tilde{Q}, \tilde{z}) = (Q, z)$. So assume $Q \cap T = \emptyset$. Since $(\{N\}, x)$ is individually rational, it follows that $T \in \mathcal{W}$. Then by properness $v(Q) = 0$ and $z_i = x_i$ for all $i \in Q$. Hence, there must be a player $j \in N \setminus T$ such that $x_j > 0$. Player k can launch an objection against player j by (T, y) as well. Let (Q', z') be the counterobjection of player j in the Maschler sense. Then, $Q' \in \mathcal{W}$ and by properness $Q' \cap T \neq \emptyset$. Monotonicity implies that $Q' \cup \{l\} \in \mathcal{W}$. So we can take $\tilde{Q} = Q' \cup \{l\}$ and $\tilde{z}$ such that $\tilde{z}_i = z'_i$ for all $i \in Q'$ and $\tilde{z}_l = x_l = 0$. ∎

Note that Theorem 4.3.1 implies that the bargaining set is non-empty for all monotonic proper simple games.

Next we provide a characterization of the payoff configurations in the bargaining set of a weighted majority game that contain a minimal winning coalition as a coalition structure element . A *weighted majority game* (N, v) is a simple game for which there exists a vector of weights $w \in \mathbb{R}^N_+$ and a threshold $q \in \mathbb{R}_+$ with $0 \leq q \leq w(N)$ such that $S \in \mathcal{W}$ if and only if $w(S) \geq q$. The pair $(q; w)$ is called a *representation* of weighted majority game (N, v). Let $S \in \mathcal{W}^m$ and let $\mathtt{P} \in \mathcal{P}$ with $S \in \mathtt{P}$. We denote the set of members of S who can make another coalition in $\mathtt{P}$ winning by joining it $E(S, \mathtt{P})$, *i.e.*,

$$E(S, \mathtt{P}) = \{i \in S \mid v(T \cup \{i\}) = 1 \text{ for some } T \in \mathtt{P} \setminus \{S\}\}.$$

Further we denote the indicator vector of a coalition $S \in 2^N$ by e_S, *i.e.*, $e_S(i) = 1$ if $i \in S$ and $e_S(i) = 0$ if $i \in N \setminus S$.

Theorem 4.3.2 *Let (N, v) be a proper weighted majority game. Let $S \in \mathcal{W}^m$ and let $\mathtt{P} \in \mathcal{P}$ be a coalition structure with $S \in \mathtt{P}$. Then $\left\{(\mathtt{P}, x) \mid x \in \mathbb{R}^N\right\} \cap \mathcal{B}(v)$ equals:*

(i) $\{(\mathtt{P}, x) \mid x \in \mathbb{R}^N_+, x(S) = 1, x(N \setminus S) = 0\}$ *if* $E(S, \mathtt{P}) = \emptyset$,

(ii) $\{(\mathtt{P}, e_i\}$ *if* $E(S, \mathtt{P}) = \{i\}$,

(iii) $\emptyset$ *if* $E(S, \mathtt{P}) \subsetneq S$ *and* $|E(S, \mathtt{P})| \geq 2$,

(iv) $\{(\mathtt{P}, \frac{1}{|S|} e_S)\}$ *if* $E(S, \mathtt{P}) = S$ *and* $|E(S, \mathtt{P})| \geq 2$.

Proof. Observe that $\mathtt{P}$ is robust against merging and splitting. Let $(\mathtt{P}, x)$ be a payoff configuration in the bargaining set. It is immediate that $x \in \mathbb{R}^N_+$, $x(S) = 1$, and $x(N \setminus S) = 0$. Clearly, there are no internal objections. Moreover, external objections can be made only by members of $E(S, \mathtt{P})$ who are allocated less than one.

If $E(S, \mathtt{P}) = \emptyset$, there are no further requirements for $(\mathtt{P}, x)$ being an element of the bargaining set. Hence we are in case (i).

If $E(S, \mathtt{P}) = \{i\}$ and furthermore $E(S, \mathtt{P}) \subsetneq S$, then player i has a justified external objection against any player in $S \setminus E(S, \mathtt{P})$, unless x_i equals 1. If $E(S, \mathtt{P}) = \{i\}$ and $E(S, \mathtt{P}) = S$, there are no external or internal objections possible. We are in case (ii).

If $E(S, \mathtt{P}) \subsetneq S$ and $|E(S, \mathtt{P})| \geq 2$, then there are at least two distinct players i and j in $E(S, \mathtt{P})$ who have justified external objections against any player in $S \setminus E(S, \mathtt{P})$, unless both i and j have payoffs equal to 1. Since by definition $v(S) = 1$, $x_i = x_j = 1$ is not feasible. This gives case (iii).

If $E(S, \mathrm{P}) = S$ and $|E(S, \mathrm{P})| \geq 2$, let $i \in S$ and let $(T \cup \{i\}, y)$ be an objection against another player j in S. Clearly, $y(T)$ equals $1 - x_i - \varepsilon$ for some $\varepsilon > 0$. A counterobjection exists if $1 - x_i - \varepsilon + x_j \leq 1$, *i.e.*, $x_j \leq x_i + \varepsilon$. From this argument, we may conclude that $x_i = x_j$ for all $i, j \in S$. Hence we are in case (iv). ■

The following example illustrates the above result.

Example 4.3.3 Let $(6; 3, 2, 2, 2, 1)$ be a representation of a proper weighted majority game.

There are three types of minimal winning coalitions: a coalition formed by all players with weight 2; coalitions formed by the player with weight 3 and two players with weight 2; and coalitions formed by the player with weight 3, a player with weight 2 and the player with weight 1. Depending on the type of the minimal winning coalition and the coalitions formed by the players outside the minimal winning coalition, we distinguish between six types of coalition structures that contain a minimal winning coalition. We will discuss an example of each of the six types.

First, consider the minimal winning coalition $S_1 = \{2, 3, 4\}$ and the coalition structure $\mathrm{P}_1 = \big\{\{2, 3, 4\}, \{1\}, \{5\}\big\}$. Clearly, $E(S_1, \mathrm{P}_1) = \emptyset$ and we are in case (i) of Theorem 4.3.2. The payoff configurations in the set $\{(\mathrm{P}_1, x) \mid x \in \mathbb{R}^N_+, x(S_1) = 1, x(N \setminus S_1) = 0\}$ are elements of the bargaining set.

Another coalition structure that contains the same minimal winning coalition is the coalition structure $\mathrm{P}'_1 = \big(\{\{2, 3, 4\}, \{1, 5\}\big\})$. Since $E(S_1, \mathrm{P}'_1) = S_1$, we are in case (iv) of Theorem 4.3.2. The payoff configuration $(\mathrm{P}'_1, (0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0)$ is in the bargaining set: internal objections are not possible and any external objection can be countered.

Next consider the minimal winning coalition $S_2 = \{\{1, 2, 3\}$ and the coalition structure $\mathrm{P}_2 = \big\{\{1, 2, 3\}, \{4\}, \{5\}\big\}$. Since $E(S_2, \mathrm{P}_2) = \emptyset$, we are in case (i) of Theorem 4.3.2

Furthermore, consider the coalition structure $\mathrm{P}'_2 = \big\{\{1, 2, 3\}, \{4, 5\}\big\}$. Here the set $E(S_2, \mathrm{P}'_2) = \{1\}$. Clearly, we are in case (ii) of Theorem 4.3.2. The payoff configuration $(\mathrm{P}'_2, (1, 0, 0, 0, 0, 0)$ is in the bargaining set since internal and external objections are not possible.

Last consider the minimal winning coalition $S_3 = \{1, 2, 5\}$ and the coalition structure $\mathrm{P}_3 = \big\{\{1, 2, 5\}, \{3\}, \{4\}\big\}$. Since $E(S_3, \mathrm{P}_3) = \emptyset$, we are in case (i) of Theorem 4.3.2.

Consider the coalition structure $\mathrm{P}'_3 = \big\{\{1, 2, 5\}, \{3, 4\}\big\}$. In this coalition structure $E(S_3, \mathrm{P}'_3) = \{1, 2\}$. Since $|E(S_3, \mathrm{P}'_3)| = 2$ and $E(S_3, \mathrm{P}'_3) \subsetneq S_3$, we are in case (iii) of

Theorem 4.3.2. There is no payoff configuration in the bargaining set pertaining to P_3 since either player 1 or player 2 can have a justified external objection against player 5. ♦

The next result concerns types of stable coalition structures of any proper weighted majority game.

Theorem 4.3.4 *Let $(q; w)$ with $w_1 \geq \ldots \geq w_n$ be a representation of a proper weighted majority game (N, v). Then the coalition structure $\{\{1, 2, \ldots, k\}, \{k+1\}, \{k+2\}, \ldots, \{n\}\}$ with k such that $\{1, 2, \ldots, k\} \in \mathcal{W}^m$ is stable.*

Proof. Consider the coalition structure $P = \{\{1, 2, \ldots, k\}, \{k+1\}, \{k+2\}, \ldots, \{n\}\}$ with k such that $\{1, 2, \ldots, k\} \in \mathcal{W}^m$. First, assume $w_1 + w_{k+1} < q$. Then the payoff configuration $(P, (1, 0, \ldots, 0))$ is an element of the bargaining set: no player can raise an objection. Next, assume $w_1 + w_{k+1} \geq q$. Hence, $w_1 + w_2 \geq q$ and therefore $k \leq 2$. If $k = 1$ or if both $k = 2$ and $w_2 + w_{k+1} < q$, $(P, (1, 0, \ldots, 0)) \in \mathcal{B}(v)$. If $k = 2$ and $w_2 + w_{k+1} \geq q$, then $(P, (\frac{1}{2}, \frac{1}{2}, 0, \ldots, 0))$ is an element of the bargaining set. Hence P is stable. ■

Theorem 4.3.4 implies that the coalition structure containing the minimal winning coalition formed by the players with the highest weights and the rest of the players as singletons is stable in any proper weighed majority game.

In the remainder of this section we address the question whether for any proper weighted majority game, there is a stable coalition structure that consists of a minimal winning coalition and its complement. The following example illustrates that this is not true for the general class of proper weighted majority games.

Example 4.3.5 Let the following $(11; 4, 4, 2, 2, 2, 1, 1, 1, 1, 1, 1)$ be a representation of a proper weighted majority game.

We provide the main arguments why coalition structures containing a minimal winning coalition and its complement are not stable. We distinguish between minimal winning coalitions with weight 11 and minimal winning coalitions with weight 12.

A minimal winning coalition with weight 11 always contains a player with weight 1 and at least two players with weights higher than 1. At least one of these two players with weight higher than 1 has a justified external objection against the player with weight 1, *e.g.* in the payoff configuration $(\{\{1, 3, 4, 5, 6\}, \{2, 7, 8, 9, 10\}\}(1, 0, \ldots, 0))$ player 3 has a justified objection against player 6.

A minimal winning coalition with weight 12 consists of all players with weight 4 and two players with weight 2. At least one the players with weight 4 has a justified external objection against a player with weight 2. For instance, consider the payoff configuration $(\{\{1,2,3,4\}, \quad \{5,6,7,8,9,10,11\}\},(1,0,\ldots,0))$ where player 2 has a justified external objection against player 3. ♦

There is however a stable coalition structure containing a minimal winning coalition and its complement for all proper and homogenous weighted majority games. A proper weighted majority game (N,v) is *homogeneous* if there exists a representation $(q;w)$ such that $w(S)=q$ for all $S\in\mathcal{W}^m$.

Theorem 4.3.6 *Let (N,v) be a proper homogeneous weighted majority game. Then there exists a minimal winning coalition S such that the coalition structure $\{S,N\setminus S\}$ is stable.*

Proof. Let $(q;w)$ be a representation of (N,v). For ease of exposition all players $i\in N$ for whom $w_i+w(N)-q\geq q$ are called *strong* and all others *weak*. Homogeneity implies that strong players with a minimal winning coalition S are exactly those players in the set $E(S,\{S,N\setminus S\})$.

If there are no strong players, it is easily derived that any coalition structure containing a minimal winning coalition is stable.

So we may assume there are strong players. If the set of strong players is winning, following the same line of argument as in the proof of case (iv) in Theorem 4.3.2, there exists a minimal winning coalition S consisting of strong players only, and, $(\{S,N\setminus S\},\frac{1}{S}e_S)\in\mathcal{B}(v)$. If the set of strong players is losing, take a minimal winning coalition $T\in\mathcal{W}^m$ which contains all strong players. Take any strong player $i\in T$. Then the coalition $(N\setminus T)\cup\{i\}$ is winning. This coalition contains a minimal winning coalition S with $i\in S$. Again using the same argument as in the proof of case (ii) of Theorem 4.3.2, one can check that $(\{S,N\setminus S\},e_i)\in\mathcal{B}(v)$. ■

Finally, it is easy to see that there is a stable coalition structure consisting of a minimal winning coalition and its complement for all strong weighted majority games. A weighted majority game (N,v) is *strong* if $S\notin W$ implies $N\setminus S\in W$. Since $E(S,\mathrm{P})=S$ for all $S\in W^m$ the coalition structure $\{S,N\setminus S\}$ is stable for any minimal winning coalition S: either $|S|=1$ and we are in case (ii) or $|S|\geq 2$ and we are in case (iv) of Theorem 4.3.2.

4.4 Team-builders and Free-riders in Coalitional Games

We now construct a special type of coalitional games that model settings in which the players only differ in a *cooperation parameter*. The cooperation parameter captures either negative marginal contribution or positive marginal contribution that a player carries in cooperation with others. If a player's marginal contribution to a group value is positive, we say that she has team-building abilities with respect to that group. Conversely, if a player's contribution is negative, we say that she has a free-riding tendency with respect to that group. If a player's marginal contribution to a group value is zero, then we say that the player exhibits neutral cooperation skills with respect to that group. A player may be a team-builder in one group and a free-rider in another group depending on the composition of the group. Players may not only vary in the direction but also in the degree of her cooperation contribution. Apart from their heterogeneity in the cooperation abilities, which becomes evident only when they are members of a group, players are homogeneous. These ideas are captured in the characteristic function of the coalitional game given below.

Let N be the set of players. The vector $\varepsilon \in \mathbb{R}^N_+$ with $0 \leq \varepsilon_i \leq 2$ for all $i \in N$ is the vector of cooperation parameters of the players. The *cooperation game* (N, v_ε) is defined by

$$v_\varepsilon(S) = |S|^{\bar{\varepsilon}(S)} - |S| \quad \text{with } \bar{\varepsilon}(S) := \tfrac{1}{|S|} \textstyle\sum_{i \in S} \varepsilon_i \quad \text{for all } S \in 2^N. \tag{4.1}$$

The first term captures the cooperation productivity. Note that all singletons have a zero value, hence, players are homogeneous in their individual productivity. Consider a coalition of more than one player. A coalition formed by players with $\varepsilon > 1$ has value higher than zero, while a coalition formed by players with $\varepsilon < 1$ has value lower than zero. A mixed coalition formed by players of both types may have either higher, equal, or lower value than zero, depending on whether the players with $\varepsilon > 1$ outweigh, balance, or have lower impact than the one of the players with $\varepsilon < 1$.

Example 4.4.1 Let $N = \{1, 2, 3, 4\}$ and $\varepsilon = (0.5, 1, 1, 1.5)$. Then the corresponding cooperation game (N, v_ε) is given by $v_\varepsilon(\{1, 2\}) = v_\varepsilon(\{1, 3\}) = -0.318$, $v_\varepsilon(\{2, 4\}) = v_\varepsilon(\{3, 4\}) = 0.378$, $v_\varepsilon(\{1, 2, 3\}) = -0.502$, $v_\varepsilon(\{2, 3, 4\}) = 0.603$, and $v_\varepsilon(S) = 0$ otherwise.

The payoff configuration $(\{\{1\}, \{2, 3, 4\}\}, (0, 0.201, 0.201, 0.201))$ is an element of the bargaining set $\mathcal{B}(v_\varepsilon)$ since it does not allow for any (internal or external) objections. ♦

To facilitate the exposition below, we introduce the following additional notation. The set of players with a cooperation parameter at least one is denoted by $H := \{i \in N \mid \varepsilon_i \geq 1\}$; the set of players with a cooperation parameter lower than one is denoted by $L := \{i \in N \mid \varepsilon_i < 1\}$. The vector ε is called *symmetric* if there are real numbers ε_H

and ε_L such that $\varepsilon_i = \varepsilon_H$ for all $i \in H$ and $\varepsilon_i = \varepsilon_L$ for all $i \in L$. A cooperation game is called *symmetric* if the underlying vector ε is symmetric.

Theorem 4.4.2 *Let (N, v_ε) be a symmetric cooperation game with cooperation parameters $\varepsilon_H \in [1,2]$ for players in H and $\varepsilon_L \in [0,1)$ for players in L. If $\varepsilon_H + \varepsilon_L \leq 2$, then the coalition structure $\{H, \langle L \rangle\}$ is stable. It is the unique stable coalition structure if additionally $\varepsilon_H > 1$ and $|H| \neq 1$.*

The proof requires three auxiliary results.

Lemma 4.4.3 *Let $s, t \in \mathbb{N}$ such that $s + t \geq 3$. Then*

$$(s+t)^{s-t} < s^s. \tag{4.2}$$

Proof. The following assertion is equivalent to (4.2)

$$\left(\frac{s+t}{s}\right)^s < (s+t)^t.$$

Newton's binomial formula gives that

$$\left(1 + \frac{t}{s}\right)^s = \sum_{k=0}^{s} \binom{s}{k} \left(\frac{t}{s}\right)^k < \sum_{k=0}^{s} \frac{t^k}{k!} < e^t.$$

Hence, for $s + t \geq 3$, (4.2) is valid. ∎

Lemma 4.4.4 *Let $s, t \in [1, \infty)$ and $\alpha \in [0, \infty)$. Then*

$$(s+t)^\alpha - s^\alpha - t^\alpha \begin{cases} > 0 & \text{for } \alpha \in (1, \infty) \\ = 0 & \text{for } \alpha = 1 \\ < 0 & \text{for } \alpha \in [0, 1) \end{cases} \tag{4.3}$$

Proof. Let $g : [1, \infty) \times [1, \infty) \longrightarrow \mathbb{R}$ be defined by

$$g(s,t) = (s+t)^\alpha - s^\alpha - t^\alpha.$$

The first order derivative with respect to s, yields

$$g'_s(s,t) = \alpha(s+t)^{\alpha-1} - \alpha s^{\alpha-1},$$

which is positive for $\alpha > 1$, zero for $\alpha = 1$ and negative for $\alpha < 1$. A similar analysis holds for the first order derivative with respect to t. Furthermore,

$$g(1,1) = 2^\alpha - 2,$$

which is positive for $\alpha > 1$, zero for $\alpha = 1$ and negative for $\alpha < 1$. This completes the proof. ∎

Lemma 4.4.5 *Let $i \in L$ and $S \subseteq N \setminus \{i\}$ such that $v_\varepsilon(S) \geq 0$. Then*

$$v_\varepsilon(S \cup \{i\}) \leq v_\varepsilon(S) + v_\varepsilon(\{i\}).$$

Proof. Denote $\overline{\varepsilon}(S)$ by ε. Because $v_\varepsilon(S) \geq 0$, we have that $\varepsilon \geq 1$. Define $g : [1,2] \longrightarrow \mathbb{R}$ by

$$g(\varepsilon) = (s+1)^{\frac{s\varepsilon+2-\varepsilon}{s+1}} - s^\varepsilon - 1.$$

Because $\varepsilon_L \leq 2 - \varepsilon$, we have $v_\varepsilon(S \cup \{i\}) - v_\varepsilon(S) - v_\varepsilon(\{i\}) \leq g(\varepsilon)$ for $\varepsilon \in [1,2]$. Hence it suffices to show that $g(\varepsilon) \leq 0$. If $s = 1$, then $g(\varepsilon) = 0$. If $s \geq 2$, denote $(s+1)^{\frac{s-1}{s+1}}$ by a. So $g(\varepsilon) = (s+1)^{\frac{2}{s+1}} \cdot a^\varepsilon - s^\varepsilon - 1$. Because of Lemma 4.4.3, we have that $a < s$. Furthermore,

$$g'(\varepsilon) = \ln(a) \cdot (s+1)^{\frac{2}{s+1}} \cdot a^\varepsilon - \ln(s) \cdot s^\varepsilon.$$

If $\varepsilon = 1$, this boils down to

$$\begin{aligned} g'(1) &= \ln(a) \cdot (s+1)^{\frac{2}{s+1}} \cdot a - \ln(s) \cdot s \\ &= \ln(s+1) \cdot \frac{s-1}{s+1} \cdot (s+1)^{\frac{2}{s+1}} \cdot (s+1)^{\frac{s-1}{s+1}} - \ln(s) \cdot s \\ &= \ln(s+1) \cdot (s-1) - \ln(s) \cdot s \\ &= \ln(s+1)^{(s-1)} - \ln s^s, \end{aligned}$$

which is negative because of Lemma 4.4.3. Suppose ε is such that $g'(\varepsilon) < 0$. Then

$$\begin{aligned} g''(\varepsilon) &= \ln^2(a) \cdot (s+1)^{\frac{2}{s+1}} \cdot a^\varepsilon - \ln^2(s) \cdot s^\varepsilon \\ &< \ln(s) \cdot \ln(a) \cdot (s+1)^{\frac{2}{s+1}} \cdot a^\varepsilon - \ln^2(s) \cdot s^\varepsilon \\ &= \ln(s) \cdot g'(\varepsilon) \\ &< 0. \end{aligned}$$

Summarizing we find $g'(1) < 0$ and $g''(\varepsilon) < 0$ for all $\varepsilon \in [1,2]$ with $g'(\varepsilon) < 0$. This implies that $g'(\varepsilon) < 0$ for all $\varepsilon \in [1,2]$. Since $g(1) = 0$, this completes the proof. ∎

Proof of Theorem 4.4.2. Let $\varepsilon_H + \varepsilon_L \leq 2$. Consider the payoff configuration $(\{H, \langle L \rangle\}, x)$ with $x \in \mathbb{R}^N$ given by $x := \frac{v(H)}{|H|} e_H$. We show that $(\{H, \langle L \rangle\}, x) \in \mathcal{B}(v_\varepsilon)$.[6] First, we show that there are no internal objections. Obviously, there are no internal objections in $\{i\}$ for all $i \in L$. Suppose there is an internal objection of player $k \in H$ against player $l \in H$ using coalition $T \subseteq H \setminus \{l\}$ with $k \in T$. Then $|T| \geq 2$, since

[6]There are other payoff vectors pertaining to the coalition structure $\{\{H\}, \langle L \rangle\}$ that lead to a payoff configuration in our bargaining set, *e.g.*, each element of the Core of the H-restricted game and a zero payoff to all players in L.

$v_\varepsilon(\{k\}) = 0$. Player l can use coalition $Q = (T \setminus \{k\}) \cup \{l\}$ to counterobject since $v_\varepsilon(T)$ and $v_\varepsilon(Q)$ coincide.

Next, we show that H is splitting-proof and $\langle L\rangle$ is merging-proof. For the remainder of the proof we denote the cardinalities of the coalitions called S and T by s and t, respectively. First, consider $S, T \subseteq H$ such that $S \cap T = \emptyset$ and $S \cup T = H$. Then

$$v_\varepsilon(H) - v_\varepsilon(S) - v_\varepsilon(T) = (s+t)^{\varepsilon_H} - s - t - (s^{\varepsilon_H} + t^{\varepsilon_H} - s - t) \geq 0,$$

where the last inequality follows from $\varepsilon_H \geq 1$ and Lemma 4.4.4.

Next, consider $i, j \in L$. Then

$$v_\varepsilon(\{i,j\}) - v_\varepsilon(\{i\}) - v_\varepsilon(\{j\}) = (2)^{\varepsilon_L} - 2 < 0,$$

where the last inequality follows from $\varepsilon_L < 1$.

In order for $\{H, \langle L\rangle\}$ to be merging-proof, $v_\varepsilon(H \cup \{i\}) \leq v_\varepsilon(H) + v(\{i\})$ for all $i \in L$. This is the case because of lemma 4.4.5.

Finally, we show that there are no external objections by players in H. Since $v_\varepsilon(\{i,j\}) \leq 0$ for all $i \in H$ and $j \in L$, no player with ε_H can launch an external objection.

In order to show the part of the proposition concerning uniqueness, we need to prove that no coalition structure but $\{H, \langle L\rangle\}$ is stable under the additional conditions $\varepsilon_H > 1$ and $|H| \geq 2$. Let (P', x) be an individually rational payoff configuration of (N, v).

Firstly, if there are coalitions $S, T \in \mathrm{P}'$ with $S, T \subset H$ and $S \neq T$, then P' is not merging-proof with respect to these coalitions: Lemma 4.4.4 gives

$$v_\varepsilon(S+T) - v_\varepsilon(S) - v_\varepsilon(T) = (s+t)^{\varepsilon_H} - s^{\varepsilon_H} - t^{\varepsilon_H} > 0.$$

Next, suppose there is a player $i \in L$ and a non-empty coalition $S \subseteq N \setminus \{i\}$ such that $S \cup \{i\} \in \mathrm{P}'$. Because x is individually rational, $v_\varepsilon(S \cup \{i\}) \geq 0$ and hence, $v_\varepsilon(S) \geq 0$ as well. Moreover, lemma 4.4.5 gives $v_\varepsilon(S \cup \{i\}) \leq v_\varepsilon(S) + v_\varepsilon(\{i\})$. For $S \cup \{i\}$ to be splitting proof, we need an equality. This is only the case when $s = 1$ and $\varepsilon_H = 2 - \varepsilon_L$. So let us focus on this case. Let $S = \{j\}$, and $\varepsilon_j = \varepsilon_H = 2 - \varepsilon_L$. We have $v_\varepsilon(S \cup \{i\}) = 0$, so $x_i = x_j = 0$ as well. Since $|H| \geq 2$, there must be another coalition $T \in \mathrm{P}'$ with $T \cap H \neq \emptyset$. Player j has a justified external objection using coalition $T \cup \{j\}$ against player i, since

$$v_\varepsilon(T \cup \{j\}) - v_\varepsilon(T) = (t+1)^{\frac{t\bar{\varepsilon}(T)+\varepsilon_H}{t+1}} - t^{\bar{\varepsilon}(T)} - 1 > 0.$$

The inequality follows from $1 \leq \varepsilon(T) \leq \varepsilon_H$ and Lemma 4.4.4. So, P' is not stable. ■

Lemma 4.4.5 and the proof of Theorem 4.4.2 show that in a symmetric cooperation game (N, v_ε) with $\varepsilon_H + \varepsilon_L \leq 2$ and $\varepsilon_H > 1$, all players in H have positive marginal contribution with respect to any coalition with a nonnegative value, except for singleton coalitions consisting of a member of L. All players in L have negative marginal contribution with respect to any coalition.

Moreover, the payoff configuration (P, x) with $P = \{H, \langle L \rangle\}$ and $x \in \mathbb{R}^N$ given by $x := \frac{v(H)}{|H|} e_H$ of the coalitional game (N, v) is also an element of the Maschler and Zhou bargaining sets. To see that $(P, x) \in \mathcal{M}(v_\varepsilon)$ recall that all objections in terms of our bargaining set are also possible in terms of the Maschler bargaining set and that it is more difficult to counter an objection in our bargaining set than in the Maschler setting. There are other types of objections that are possible in terms of the Maschler bargaining set, which are not possible in terms of our bargaining set: a team-building player k can launch an objection against another team-building player l with respect to (P, x) using a coalition T with $k \in T$ which contains more than one free-riding player and more than one team-building player. Such an objection can be countered by player l using coalition $Q = T \setminus \{k\} \cup \{l\}$ since $v(T)$ and $v(Q)$ coincide.

To see that $(P, x) \in \mathcal{Z}(v_\varepsilon)$ note that coalitions composed by the same number of free-riding players and team-building players have equal values. Hence any $\mathcal{Z}$-objection (T, y) can be countered unless $T = \{i\}$ for any $i \in N$, $T = \{N\}$, $T = \{L\}$, or $T = \{H\}$. Since the payoff configuration (P, x) is individually rational T cannot be a singleton. Since $v(\{N\}) \leq v(\{H\})$ as it can be deduced from the Proof of Proposition 4.4.2, T cannot be the grand coalition. Similarly, $\mathcal{Z}$-objection by means of the coalition $\{L\}$ is not possible. Since $\{H\} \in P$, $\mathcal{Z}$-objection by means of the coalition $\{H\}$ is not possible.

Next we will discuss the necessity of the various conditions provided in Theorem 4.4.2 for the stability of the coalition structure $\{H, \langle L \rangle\}$. The first example shows that when the cooperation game is not symmetric, $\{H, \langle L \rangle\}$ may not be stable.

Example 4.4.6 Let $N = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $\varepsilon = (1.1, 2, 2, 2, 2, 2, 2, 2, 2, 2)$ and $\tilde{H} := \{i \in N \mid \varepsilon_i = 2\}$. The value function can be calculated using Equation (4.1). Here we give just some of the values. $v_\varepsilon(\{i\}) = 0$ for all $i \in N$; $v_\varepsilon(N) = 71.283$, $v_\varepsilon(\tilde{H}) = 72$. Note that $H = N$. The coalition structure $\{N\}$ is not stable since it is not splitting-proof. ♦

The next example illustrates the necessity of the condition that $\varepsilon_H + \varepsilon_L \leq 2$.

Example 4.4.7 Let $N = \{1, 2, 3\}$, and $\varepsilon = (0.9, 0.9, 1.9)$. The value function is calculated using Equation (4.1) which yields $v_\varepsilon(\{1, 2\}) = -0.134$, $v_\varepsilon(\{1, 3\}) = v_\varepsilon(\{2, 3\}) = 0.639$, $v_\varepsilon(\{1, 2, 3\}) = 0.876$ and $v_\varepsilon(S) = 0$, otherwise. The coalition structure $\{\{1\}, \{2\}, \{3\}\}$ is not stable since it is not merging proof. ♦

The next example shows that the condition $\varepsilon_H > 1$ is necessary to obtain uniqueness.

Example 4.4.8 Let $N = \{1, 2\}$ and $\varepsilon = (1, 1)$. So, $v(S) = 0$ for all $S \in 2^N$. Both coalition structures $\{N\}$ and $\{\{1\}, \{2\}\}$ are stable since $(\{N\}, (0, 0)) \in \mathcal{B}(v_\varepsilon)$ and $(\{\{1\}, \{2\}\}, (0, 0)) \in \mathcal{B}(v_\varepsilon)$. ♦

In addition, to guarantee uniqueness in Theorem 4.4.2, it is required that $|H| \neq 1$. The next example illustrates the necessity of this condition.

Example 4.4.9 Let $N = \{1, 2\}$ and $\varepsilon = (0.9, 1.1)$. Using equation (4.1), we obtain $v_\varepsilon(S) = 0$ for all $S \in 2^N$. Both coalition structures $\{N\}$ and $\{\{1\}, \{2\}\}$ are stable since $(\{N\}, (0, 0)) \in \mathcal{B}(v_\varepsilon)$ and $(\{\{1\}, \{2\}\}, (0, 0)) \in \mathcal{B}(v_\varepsilon)$. ♦

Part III

Governance and Effectiveness

CHAPTER

FIVE

G S D : E W [*]

5.1 Introduction

By the beginning of the nineties "governance" took a permanent place in the reports of policy makers. At the European Union level "effective implementation" of EU rules was declared to be one of the most important criteria for accession by the General Affairs Council in December 2000. A country has 'effective implementation capacity', according to Nicolaides (2001), if it has institutions which have the necessary resources and legal discretion and which act and adjust under the proper incentives.[1] At the World Bank (WB) the decision of aid allocation to developing countries is also based on the governance quality of the applicants.

One may ask what justifies the central place of "governance" in the political lexicon. To answer this question in the beginning of the 1990's the WB launched a project with the goal to make a rigorous and testable theory about "how governance affects the socioeconomic and economic development." A part of the project is the compilation of a governance indices database for a large sample of countries, which now allow the researchers to embark on empirical investigations of the effects of governance on the actual economy. One such investigation by (Kaufmann and Kray 2003) shows that governance has a positive causal effect on aggregate income growth. In the current work, the impact of governance on socioeconomic development measured by life expectancy is studied.

[*] This chapter is based to a great extent on Lazarova and Mosca (2006).

[1] See Nicolaides (2001) for broader discussion of the link between effective implementation capacity and governance.

Before proceeding to a discussion of the data and the empirical results we review the conceptual understanding of governance and suggest paths via which it affects socioeconomic development. Governance is a concept that emerged in the development literature around the late 1980s. The first classical political science essays discussed the topic of "governability," which made the rule of law the core of development. With the end of the Cold War, "governability" was then substituted by the concept of governance, defined as redesigning or reinventing public administration to meet the new challenges, and be able to deal with them, in the era of globalization.

The concept of governance has gained momentum and a wider meaning over the years. Various international organizations have given different definitions of the term. In particular, the WB, whose data of governance is used in this work, understands 'by governance '[...] the manner in which power is exercised in the management of a country's economic and social resources for development" (WB (1992)). The WB definition clearly differentiates a political from an economic dimension.

The United Nations Development Programme's (UNDP) definition of governance goes beyond the idea of the state and takes into consideration the private sector that creates jobs and income, which make the economy work, and the civil society that facilitates political and social interactions, mobilizing groups to participate in economic, political and social arenas. Similar to the WB's view the state has again the role to create a conducive political and legal environment. Good governance is achieved when the three parts reach a constructive interaction (UNDP (1997)).

The World Health Organisation (WHO) takes the concept of governance a step further by adding a new focus on enabled participation, and equitable and sustainable outcomes in different sectors (WHO (1998)).

Governance is thus a complex notion of a country's effective organization and one would expect that the style of governance has far reaching consequences in many areas of human life. For instance Das Gupta (1999) argues that governance that fosters local accountability and social mobility may also influence fertility rates while Kaufmann and Kray (2003) have shown that it influences income growth.

The legacy of using variables such as life expectancy as an indicator of socioeconomic development is established in the works of Amartya Sen.[2] Indeed the groups of the elderly and of the youngest members of a society are some of the most vulnerable of the social groups and as such they are dependent on facilitated access to resources. That is why their health capital, measured as life expectancy and infant mortality rate, respectively, are indicative to both the economic and the socio-ethical development of

[2]Among others see Sen (1981), Sen (1987).

a society.

One would expect to see broad differentials of life expectancy of rich and poor countries mainly due to the scarcity of resources in the latter. Even taking into account these scarcities though there remain differentials between the two groups as well as within these groups which remain unexplained.

There are a number of branches in the literature aiming at explaining these differentials.[3] A large volume of empirical literature started with the seminal work of Rodgers (1979), investigating the effect of absolute and relative income on aggregate health.[4] Some recent work based on cross-country regressions that find support for the relation between absolute income and health is van Doorslaer, Wagstaff et al. (1997); while Kennelly, O'Shea and Earvey (2003), Kennedy, Kawachi and Prothrow-Smith (1996), and Chiang (1999) find support for the causal effect of income inequality on health. A much studied theory in this branch of the literature is the epidemiological transition theory established by Wilkinson (1996). The theory predicts the existence of an absolute income threshold below which only absolute income and above which only relative income matters for a nation's aggregate health capital[5]. However, this theory faces some criticism with respect to its empirical verification. In a recent work Gravelle, Wildman and Sutton (2002) argue that if the relation between individual health and income is non-linear one should not use aggregate data to test for a significant effect of the income distribution on the aggregate health capital, *e.g.* life expectancy. The essence of the aggregation problem is that if at the individual level the relation between personal income and health is not linear but concave, then one might find a significant negative effect of aggregate relative income on aggregate average level of health, while such an effect does not exist. Gravelle et al. (2002) support their conclusion with empirical evidence.

In our work we use Rodgers (1979) regression model as a point of departure. However, we suggest an alternative specification in which relative income is substituted for a governance index. In such a way we also take into account the critique of Gravelle et al. (2002). Our main hypothesis is that the way a state formulates regulations has a direct impact on a country's socioeconomic development. It is a matter of good governance if public funds reach the needy and essential drugs, effective clinical and medical services are provided. Bad governance, on the other hand, kills via corruption, bad management of people and institutions, and neglect.

[3] A considerable part of the literature contributes these differentials to some differences in eating habits and to the propensity of people to have an active social life. Here we only refer to those branches of the literature to which this work is more closely related.

[4] In this literature aggregate health is measured as life expectancy and infant mortality rate.

[5] Life expectancy is treated as a measure of a country's aggregate health in this theory.

As such our work is closely related to the branch of the literature that associates the differentials in socioeconomic development with differences in the economic and political freedom of a country. Economic freedom is considered to be the freedom of personal choice, the protection of private rights, and the freedom of exchange while political freedom is considered to be a democratic political system.

In an overview of the social choice theories relating political systems to economic development[6], Sirowy and Inkeles (1990), identify three major empirical predictions on the effect of political freedom on economic development: positive, if political freedom is believed to facilitate growth by allowing private parties to make the best choice in their own interest; negative, if political freedom is believed to facilitate the formation of special interest groups allowing those to divert funds to suboptimal ends; neutral, which is based on the argument that the notion of a political freedom is too broad to be related to economic growth in a systematic way. The empirical evidence as summarized in Esposto and Zaleski (1999) seems to support the third view.

Economic freedom, according to Esposto and Zaleski (1999), has a clear-cut positive predicted effect on socioeconomic development since it guarantees the channelling of funds to their most productive ends. Since political liberties, though important, do not have a clear-cut prediction, they should be separated from the channel running through the economic institutions, argue these author. Indeed, their empirical findings are of a positive effect running from an economic freedom index to quality of life measured in terms of life expectancy.

Socioeconomic development is arguably affected not only by economic mechanisms but also by social mechanisms. The governance index employed in the current work is based on the effectiveness of the social and economic mechanisms rather than on their mere presence. As such, this index overcomes the problems of other social and political indices as described by Esposto and Zaleski (1999), and, we believe, it has a clear cur positive predicted effect on socioeconomic development. We also test the validity of the epidemiological threshold theory in our sample, and, in addition we offer an alternative conjecture based on the governance differentials.

The rest of the chapter is organized in the following way. In Section 5.2 the data and the regression model are presented. In Section 5.3 the regression results are discussed.

[6]Here the term economic development is meant to capture both monetary growth as well as socioeconomic development. In this literature often the authors study quality of life rather than socioeconomic development. In practice, they use life expectancy among the indices of quality of life.

5.2 Data and Empirical Model

Data on four data series has been collected. Life expectancy (LE) measures the average life expectancy at birth in a country. Absolute income per capita of a country is measured by the Gross Domestic Product per capita measured at purchasing power parity (PPP), expressed in constant international dollars (GDP). Relative income is measure by the Gini index of income inequality (GINI). The data on governance (Gov) is composed by four separate governance indices discussed below. The database consists of 116 countries of which 112 have data on governance.[7] The years covered are 1996, 1998 and 2000. The choice of this time period is due to the governance data availability. Data were obtained by several sources, as reported in Table 5.1.

Table 5.1: Data Sources

Series	Source
LE	US Census International database
	`http://www.census.gov/ipc/www/idbsprd.html`
GDP	World Bank Development Indicators 2005
GINI	World Bank Development Indicators 2003 and some country statistics
Gov	World Bank Institute
	`http://www.worldbank.org/wbi/governance/govdata/index.html`

The governance indicator that we use is a multidimensional concept. The WB defines six exclusive components of it, however, we use only four of them in our analysis as political stability and voice and accountability have misleading interpretation. A broad discussion of the governance indices can be found Kaufmann, Kray and

[7]The countries in the sample are Algeria, Armenia, Austria, Azerbaijan, Bangladesh, Belarus, Belgium, Bolivia, Brazil, Bulgaria, Burkina Faso, Burundi, Cambodia, Cameroon, Canada, Chile, China, Colombia, Costa Rica, Croatia, Czech Republic, Denmark, Djibouti, Dominican Republic, Ecuador, Egypt, El Salvador, Estonia, Ethiopia, Finland, France, Gambia, Georgia, Germany, Ghana, Greece, Guatemala, Guinea, Guyana, Honduras, Hong Kong China, Hungary, India, Indonesia, Iran Islamic Rep., Israel, Italy, Jamaica, Japan, Jordan, Kazakhstan, Kenya, Korea Rep., Kyrgyz Republic, Lao PDR, Latvia, Lesotho, Lithuania, Luxembourg, Macedonia FYR, Madagascar, Malawi, Malaysia, Mauritania, Mexico, Moldova, Mongolia, Morocco, Mozambique, Nepal, Netherlands, New Zealand, Nicaragua, Niger, Nigeria, Norway, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Portugal, Romania, Russian Federation, Senegal, Singapore, Slovak Republic, Slovenia, South Africa, Spain, Sri Lanka, St. Lucia, Sweden, Switzerland, Tajikistan, Thailand, Tunisia, Turkey, Turkmenistan, Uganda, Ukraine, United Kingdom, United States, Uruguay, Uzbekistan, Venezuela RB, Vietnam, Yemen Rep., Zambia, and Zimbabwe. There are four countries for which there is not data on governance, these are Djibouti, Lesotho, Mauritania, and St. Lucia.

Zoido-Lobatón (2003). We give a short summary for the indices that are used here in Table 5.2.

Table 5.2: The WB Governance Indices

Index	**Definition**
Government Effectiveness (GE)	looks at the ability of the government to implement socially sound policies, i.e. the level and quality of public service provision and the smooth functioning of the bureaucracy;
Regulatory Quality (RQ)	evaluates the policies themselves: whether they are facilitating or burdening social interactions and/or transactions;
Rule of Law (RL)	measuring the efficiency of the judiciary system and the enforcement of contracts;
Control of Corruption (CC)	self-explanatory; encompasses corruption initiated from both sides, e.g. the giver and the taker.

Since the four governance indices exhibit high correlation (see Table 5.3), they cannot be used simultaneously in the regression analysis. Instead we construct a composite governance index (GOV) equal to the average of the four separate governance indicators. The numbers in the correlation matrix show that life expectancy is correlated the most with the rule of law and out of the variables that will be used in the regression analysis with the composite governance index. Life expectancy is correlated the least with the relative income. Governance and the absolute income exhibit high correlation. Since in the regression model the reciprocal transformation of absolute income will be used, this is not an indication of a multicollinearity problem.

The composite governance index may vary in the range from -2.5 to 2.5 where a higher number stands for better quality. In our sample it varies between -1.56 and 2.25 as shown in Table 5.4. The country with the lowest governance index in the sample is Turkmenistan and the one with the highest is Singapore. Japan has the highest life expectancy while Lesotho the lowest. Malawi has the lowest life expectancy, 40.05, in the sample of countries which have data on governance. Luxemburg has the highest absolute income and Guatemala the lowest. Belarus is the country with the most equal distribution of income while Brazil has the highest Gini index.

Table 5.3: Correlation Matrix

Variable	LE	GDP	GINI	GOV	GE	RQ	RL	CC
LE	1.000							
GDP	0.657	1.000						
GINI	-0.318	-0.377	1.000					
GOV	0.659	0.863	-0.234	1.000				
GE	0.643	0.865	-0.262	0.973	1.000			
RQ	0.555	0.642	-0.034	0.877	0.780	1.000		
RL	0.684	0.867	-0.290	0.977	0.948	0.799	1.000	
CC	0.610	0.882	-0.290	0.955	0.932	0.728	0.946	1.000

Table 5.4: Summary Statistics

Variable	Observations	Mean	Std. Deviation	Minimum	Maximum
LE	137	65.966	10.512	39.51	80.21
GDP	137	7,827.478	8,652.766	386.39	44527
GINI	137	39.023	9.155	21.7	60.66
GOV	133	0.059	0.886	-1.56	2.25

For our estimations we use the ordinary least squares (OLS) model with robust standard errors. Since for most of the countries in the sample, GINI data is available only for one year, we cannot conduct panel analysis.

Our main hypothesis is that governance has a significant positive effect on aggregate health capital. Moreover, governance is a better predictor of life expectancy of a country than relative income. The basic Rodgers' (1979) model is given in Equation (5.1) and the alternative model is given in Equation (5.2).

$$LE_{it} = \beta_0 + \beta_1 \frac{1}{GDP_{it}} + \beta_2 \frac{1}{GDP_{it}^2} + \beta_3 GINI_{it} + \epsilon_{it} \quad (5.1)$$

$$LE_{it} = \beta_0 + \beta_1 \frac{1}{GDP_{it}} + \beta_2 \frac{1}{GDP_{it}^2} + \beta_3 GOV_{it} + \epsilon_{it} \quad (5.2)$$

where i stands for the country and t for the year, GDP, GINI, and GOV are defined above, and ϵ is the error term. To test which model fits the data better we use the J-test for a linear non-nested hypothesis first suggested by Davidson and MacKinnon (1981). We evaluate the statistical performance of the regression using the score (Lagrange multiplier) test against multiplicative heteroskedasticity, which is also interpreted as a test of omitted variable bias or non-linear functional form. (Breusch and Pagan 1979), (Cook and Weisberg 1983)

5.3 Estimation Results and Discussion

The regression results are given in Table 5.5.

Table 5.5: Regression Results: GINI vs Governance

Expl. Variable	[A] OLS(robust)	[B] OLS(robust)	[C] OLS(robust)
$\frac{1}{GDP}$	-38,641.41 $(-14.39)^\star$	-31,596 $(-6.47)^\star$	-31,847.55 $(-6.57)^\star$
$\frac{1}{GDP^2}$	1.30×10^7 $(10.43)^\star$	9.89×10^6 $(4.18)^\star$	9.80×10^6 $(4.24)^\star$
$GINI$	-0.113 $(-1.98)^\star$	-0.074 (-1.48)	
GOV		1.782 $(1.93)^{\star\star\star}$	1.874 $(2.12)^{\star\star}$
Constant	81.309 $(37.38)^\star$	78.355 $(31.17)^\star$	75.591 $(55.91)^\star$
N	137	133	133
Adj R^2	0.708	0.749	0.747
Het. Test	$7.95^\star$	$18.81^\star$	$22.65^\star$

t-statistics in parentheses;

* means significance at 99 percent confidence level;

** means significance at 95 percent confidence level;

*** means significance at 90 percent confidence level.

Column [A] shows the regression results of the standard Rodgers' (1979) model. The coefficients have the predicted sign and they are all significant at the 95 percent confidence level. In column [B] GOV is included in the standard model. The overall explanatory power of the model increases with 4.1 percent compared to column [A]. It is notable that the statistical significance of GINI in the model is zero, while GOV is statistically significant at the 90 percent confidence level, which provides some evidence in favour of our hypothesis. In column [C] we present the regression results of the alternative model given in Equation (5.2). There GOV is significant at the 95 percent confidence level.

Next we perform J test for non-nested hypothesis. First we consider

$$H_0 : \text{Equation}(5.1) \text{ is the valid model} \quad H_1 : \text{Equation}(5.2) \text{ is the valid model.}$$

The test statistics has the form $\frac{\hat{\gamma}}{se(\hat{\gamma})}$ where $\hat{\gamma}$ is the OLS estimate of the parameter γ in the following regression model:

$$LE_{it} = \beta_0 + \beta_1 \frac{1}{GDP_{it}} + \beta_2 \frac{1}{GDP_{it}^2} + \beta_3 GINI_{it} + \gamma \hat{y}_{it} + \epsilon_{it},$$

where $\hat{y}_{it}$ are the predicted values of LE_{it} from estimating Equation (5.2). The test statistics has a standard normal distribution. Using our sample, the value of the test statistics is 1.93 so we can reject the null hypothesis at the 90 percent confidence level.

Similarly, we test the hypotheses

$$H_0 : \text{Equation}(5.2) \text{ is the valid model} \quad H_1 : \text{Equation}(5.1) \text{ is the valid model.}$$

In this case we cannot reject the null hypothesis that the model (5.2) is the one better fitting the data (the *J*-statistics is 1.48).

Table 5.6: Regression Results: Threshold Effects

	[Threshold: GDP][a]		**[Threshold: GOV]**[b]	
Expl. Variable	**[LIC]**	**[HIC]**	**[LGC]**	**[HGC]**
$\frac{1}{GDP}$	-40,226.57 (-10.30)*	-67,664.62 (-1.08)	-32,771.64 (-7.53)*	-18,231.6 (-1.07)
$\frac{1}{GDP^2}$	1.30×10^7 (8.14)*	4.14×10^8 (2.21)**	1.02×^{7} (4.87)*	4.21×10^6 (0.19)
GOV	-0.216 (-0.12)	5.726 (1.84)***	0.243 (0.11)	3.240 (3.09)*
Constant	78.645 (45.40)*	70.750 (9.79)*	74.920 (39.34)*	73.068 (30.46)*
N	70	63	77	56
Adj. R^2	0.642	0.561	0.556	0.623
Het. Test	8.67*	43.87*	0.14	3.44***

[a] LIC and HIC are estimated using OLS(robust).

[b] LGC is estimated using OLS and HGC using OLS(robust).

t-statistics in parentheses;

* means significance at 99 percent confidence level;

** means significance at 95 percent confidence level;

*** means significance at 90 percent confidence level.

We also investigate the existence of a threshold effect in terms of both absolute income and quality of governance. These results are presented in Table 5.6. In the first two columns the estimate of the model with a threshold of absolute income of 5,000 PPP international dollars per capita are presented.[8] According to the epidemiological

[8]The same level of absolute income (5,000 PPP dollars per capita) is used by Gravelle et al. (2002) to investigate an absolute income threshold effect.

transition theory established, we test the hypothesis that for countries with income below this threshold only absolute income will matter, while for countries above the threshold only governance will matter for life expectancy.[9] We find some evidence in support of this hypothesis. In the regression results for the low-income countries (LIC) group, absolute income is significant at the 99 percent confidence level, while GOV is not statistically significant. Conversely, in the high-income countries (HIC) group GOV is significant at the 90 percent confidence level and only the second regressor based on absolute income is significant at the 95 percent confidence level.

In the last two columns the estimates of the model including a threshold based on the governance index equal to zero are presented. There are clear differences in the regression results between the two groups. In the low-governance-countries group (LGC), only absolute income is statistically significant at the 99 percent confidence level, while in the high-governance-countries group, (HGC) only governance is significant at the 99 percent confidence level.

Our empirical findings suggest that the favored specification of the econometric model investigating the determinants of life expectancy includes the governance index rather than the Gini index. Moreover, governance matters for a country's socioeconomic development when a certain level of wealth has been reached; prior to this threshold, absolute income is the main driving factor of improving life expectancy. The evidence for a threshold effect is statistically stronger when the sample is split on the basis of the level of governance. The findings reinforce the need to study better the paths via which governance impacts socioeconomic activities as well as the way improvements in the governance quality can be achieved.

[9] We also expect that the intercept is significant in all models. This is confirmed by the regression results: the constant term is significant at the 99 percent confidence level.

CHAPTER

SIX

E T H2 B P P I : A E S A D *

6.1 Introduction

There is a notable increase in the burden of chronic type of illness relative to acute disease throughout the world.[1] Davis, Wagner and Groves (1999) claim that people in most countries are suffering of non-communicable diseases, which are chronic. Some statistics show that an estimated 691 million of the world population suffers from high blood pressure, 29 million people suffers from dementia, 165 million are affected by rheumatoid arthritis, and the number of diabetics will reach 300 million people by 2025. Some of these diseases have an initial acute form, which, if untreated properly, transforms into a chronic problem. An important question, thus, is whether the medical system functions in a way that is able to prevent the worsening of the condition of a patient from acute to chronic.

Among these chronic diseases we should also count stomach-related troubles such as peptic acid diseases, which might develop into gastric (or, stomach) cancer – the second most common cancer in the world[2]– Gastroesophageal Reflux Disease (GERD), heartburn that represents the core symptom of GERD, and gastropathy.[3] Figure 6.1

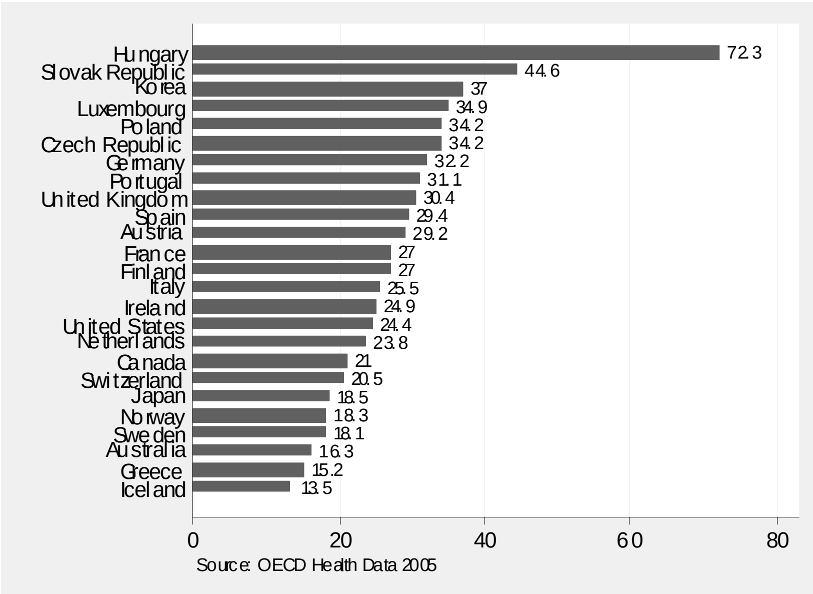
*This chapter is based to a great extent on a joint work with Ilaria Mosca and fruitful discussions with Arthur van Soest and Gema Zamarro.

[1]It is important to stress the difference between acute and chronic disease. With acute disease, the treatment aims at return to normal. With chronic disease, the patient's life is irreversibly changed, because neither the disease nor its consequences are static.

[2]Note that the first major cause of death is the circulatory system illnesses, and the second is cancer.

[3]GERD is also known as Acid Reflux Disease and describes the condition of backflow of stomach

Figure 6.1: Diseases Digestive System - Deaths /100,000 Population, 2001



represents the rate of deaths per 100,000 inhabitants due to digestive system illnesses in a number of OECD countries. Heartburn affects 25 percent of the adult population on a monthly basis while 5 percent suffer from it on a daily basis, according to the statistics provided by van Pinxteren et al. (2004). Some studies indicate that up to 44 percent of apparently healthy subjects experience heartburn at least once per month, *e.g.*, see Sridhar et al. (1996).

Other types of peptic acid diseases are peptic ulcers that develop due to an H.Pylori infection[4], and Non-Steroidal Anti-Inflammatory Drugs (NSAID)-induced gastropathy. Inflammation in the stomach (gastritis) as well as ulceration of the stomach or duodenum (peptic ulcer disease) is the result of an infection of the stomach caused by the bacterium H.Pylori, which is present in about 50 percent of all humans. In countries with high socio-economic standards of living the H.Pylori infection is considerably less common than in developing countries where virtually everyone may be infected. An antibiotic cure eradicates the bacteria in 90 percent of the cases; however, an indiscriminate use may also lead to severe problems with bacterial resistance against these important drugs. NSAID include drugs with analgesic, antipyretic, and anti-

acid into the esophagus, which frequently happens when the lower esophageal sphincter relaxes more often than it should and/or at inappropriate times.

[4]The discovery of the bacteria H.Pylori dates back to 1982 and is due to Robin Warren and Barry Marshall who won the Nobel Prize in Medicine in 2005.

Table 6.1: Clinical Characteristics of PPI and H2B

Proton Pump Inhibitors (PPI)	H2-Blockers (H2B)
• Control both basal and food-stimulated acid secretion. • Tolerance to PPI has not been observed. • More complete and longer lasting acid suppression than H2B. Moreover, they prevent relapses to GERD better. • Faster healing rates, shorter healing times, and faster symptom relief than H2B. • Some PPI may inhibit the absorption of other drugs.	• Relative short duration of action. • As a consequence multiple daily dozes are required. • H2B produce incomplete inhibition of postprandial acid secretion. • Tolerance to standard H2B generally develops within two weeks of repeated administration, resulting in a decline of acid suppression. • There are some drug interactions (*e.g.*, with oral asthma drugs, blood thinning drugs, and seizure drugs).

inflammatory effects to reduce pain, fever, and inflammation.[5]

All these diseases are treated with acid-suppressing medicaments, such as proton pump inhibitors (PPI), and H2-blockers (H2B) with a different success. Depending on the particular case, GERD can be completely cured, though relapses may occur. An early stage of H. pylori infection can also be cured. In the case of suspicion of the development of NSAID-induced gastropathy, an acid-suppressing drug may be prescribed preventively. If neglected, these diseases may lead to stomach ulcer and stomach cancer.

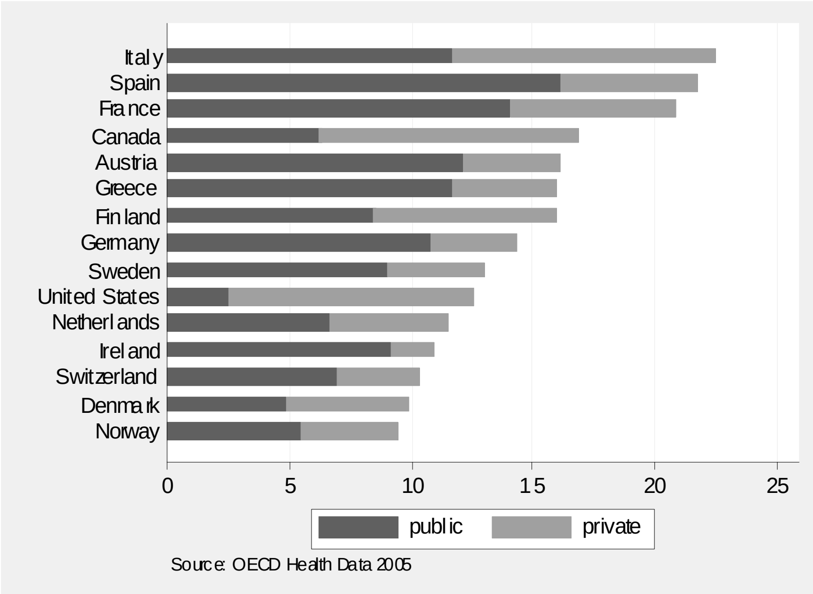
There is a growing and emerging literature that has summarized, quantified, and compared the efficacy of these medicaments, *e.g.*, van Pinxteren et al. (2004), Briggs et al. (2002), Vanderhoff and Tahbour (2002), Jones and Bytzer (2001), and Sridhar et al. (1996). The main results of all meta-analysis studies based on clinical research point into the direction of the superiority of PPI to H2B in the treatments of GERD, peptic ulcers, and NSAID-induced gastropathy.[6] Table 6.1 briefly summarizes the main clinical characteristics of PPI and H2B emerging from these studies.

In practice, there are two common therapeutic protocols: step-up therapy and step-

[5]The most known example of NSAID is the aspirin.

[6]Studies measuring the superiority of PPI to H2B show that a 20 mg/day dosage of PPI gives a healing rate from GERD of 56 and 75 percent for grade I disease at 4 and 8 weeks respectively. Healing for grade IV disease is 51 percent at 4 weeks and 71 percent at 8 weeks. With higher doses (40 mg/day) the healing rate increases for the lower grades of the disease to 74 percent at 4 weeks and 81 percent at 8 weeks according to Sridhar et al. (1996). The same authors claim that a 300 mg treatment with H2B has a healing rate between 8 and 19 percent at 6 weeks for grade III and IV disease and between 53 and 69 percent at 12 weeks for grade I disease.

Figure 6.2: Expenditure on Pharmaceuticals as a Percentage of Total Health Expenditure, 2002



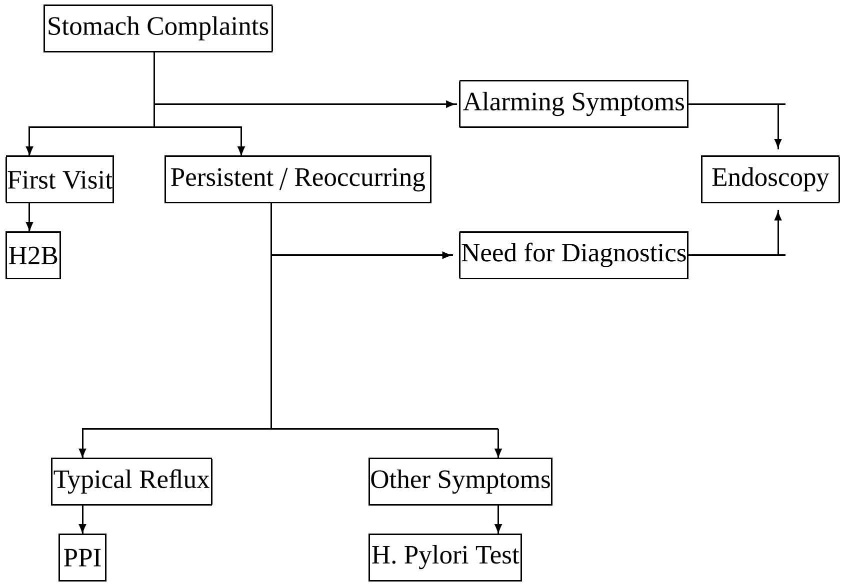
down therapy. The step-up therapy starts with standard dosages of H2B for symptom control, and if the symptoms worsen, PPI is prescribed. The step-down therapy starts with a full dose PPI therapy, and subsequently the dosage of PPI is decreased or H2B is prescribed for symptom control.

Evidently, despite the clinical superiority of PPI to H2B type of drugs, H2B type of drugs are being prescribed together with PPI by General Practitioners (GP). One reason for this is that the per-unit price of H2B's is much lower than the per unit price of PPI's. Indeed, given the fact that most developed countries have a relatively high quota of GDP spent on health care, as shown in Figure 6.2, controlling the spending on pharmaceutical drugs has been seen as a priority.

This is also the case in the Netherlands, where GP's are advised to follow the step-up therapy. In particular, the Dutch College of General Practitioners (NHG), (*Nederlands Huisartsen Genootschap*), provides the following guidelines for treatment of patients who have stomach related complaints (see Figure 6.3). According to them, a patient who comes for the first time with stomach related complaints should be prescribed H2B's, provided that the initial symptoms are not alarming, and, if the complaints persist, the physician should consider prescribing PPI type of medicaments.

The question that we ask is, is this practice cost-effective? Indeed, there might be

Figure 6.3: Guidelines for Treatment of Patients with Stomach Complaints



Source: NHG 2004

patients, who delay their visits to the GP until their illness has considerably advanced.[7] Such patients should be treated with the more effective PPI type of medication to reduce the chance of transition from a state of acute to chronic disease, while the GP might prescribe H2B's on the basis of first-time occurrence. Hence, in analyzing this question it should be taken into account not only the immediate costs of the drugs, but also the downstream cuts savings from using the more effective drug. What is also important to attest are the indirect costs borne by patients, which result from lost production caused by morbidity or mortality, and intangible costs– such as the decrease in the quality of life – that are very problematic to measure, and, therefore, to include in an economic evaluation study.

There are several studies which aim at analyzing the cost-effectiveness of alternative therapies based on H2B and PPI. For instance, Vanderhoff and Tahbour (2002) and Sridhar et al. (1996) argue that PPI's are more cost-effective when compared with H2B's. In their recommendations, they claim that although H2B's are less expensive than PPI's, the latter provide superior acid suppression, healing rates, and symptom relief. Therefore, PPI's could be more cost-effective especially for those patients

[7] The demand for health care might be distorted due to the imperfect knowledge of the patient of her own disease status.

suffering from severe acid-peptic disorders, because of their lower and less frequent dosages and the comparatively shorter duration of the required therapy.

The conclusions of the above studies are based on results of clinical experiments in which there is perfect knowledge about the condition and the diagnosis of the patient. In practice, however, the precise diagnosis might be unknown to the GP when prescribing the drugs. In our study, we use administrative data in order to address the cost-effectiveness issue of using alternative therapies as used by the medical practitioners. Our point of analysis is the medical practice rather than the clinical effectiveness of the two types of medicaments. The data are provided by the Dutch insurance company, the *CZ Insurance Group* (CZ). The Netherlands is an interesting case to study, because it is a country where the GPs prescribe both types of acid-suppressing drugs, and receive different incentives by the insurance companies to control spending on pharmaceuticals.[8,9] The advantages of using administrative data, rather than survey data is that we have a relatively good measure of the amount of medication that a patient has bought, and the time intervals in which medication treatment has taken place. However, we lack important information on the diagnosis for which the patient is being treated. Thus, the main econometric challenge is to isolate the effect of the effectiveness of the drug therapy from the effect of the severity of the case, given that PPI's are in practice prescribed to "more severe cases". Since we have two types of medicinal treatment information: what kind of drugs the patient has been prescribed and the amount of each type of drugs she has been prescribed over the course of tie, we expect that one of the two variables will pick up the severity effect, while the other the effectiveness effect. The econometric models that we employ are a binary choice model and a duration analysis model. In the binary choice model we analyse the probability that a patient enters hospital conditioning on her medicinal treatment history and some socioeconomic variables. The estimates of this model indicate that for individuals with low consumption of medicaments, the H2B's are indeed the more cost effective drugs, while for individuals with high consumption the PPI's are more costs effective. The binary choice model is restrictive because it does not take into account the time dimension of the treatment. In our sample we observe patients who take medication for years and others who just start taking medication at the end of the observation period. To take this into account we also use duration model in which we are able to model the heterogeneous treatment time among the individuals in the

[8]For example, to contain costs, the insurance company, that provided us with the data gives bonuses to GPs who incur lower spending on pharmaceuticals.

[9]In the Dutch Health System the GP has the role of a gate-keeper. Hence, a patient cannot visit a specialist without being referred by a GP.

sample. The point of analysis in the duration model in contrast to the binary choice model is the time period between the date when an individual starts taking medication and the date when she enters hospital. The estimates of the duration model indicate that taking relatively more PPI type of medication, indeed, reduces the hazard rate for entering the hospital, all else equal. Thus, confirming that therapeutic treatment based relatively more on PPI is more effective.

The remainder of this chapter is structured as follows. In Section 6.2 we describe our data set. In Section 6.3 we present the Logit and the Cox Proportional Hazard models and discuss the econometric results. In section 6.4 we summarize the main finding and discuss the limitations of the dataset.

6.2 Data

Our data set is compiled using the administrative database of the Dutch insurance group CZ for the period 2000–2005. The sample contains information about people who have been insured by the CZ in this period. The data set is based on four types of databases: insurance invoices, hospital invoices, a policy holder register, and a register containing the social codes of some policy holders. Using the pharmaceutical invoices we identify the sample of people insured by the CZ who purchased an acid-suppressing drug using their insurance policy in the period January 2000-December 2004.[10] These are 310,337 individuals, about whom we have information on the type of acid-suppressing drug (PPI versus H2B), the amount of pills, the total amount paid by the CZ, and the date of the purchase. However, due to recording mistakes, for some of these individuals, the quantity of either number of pills or amount reimbursed is equal to 0, we drop these observations from the data set (39,937 patients). Using a unique policy holder identifier, we link the pharmaceutical invoices to the hospital invoices and we identify the sub-group of people who were hospitalized for an illness related to the digestive system.[11] We keep in our database only patients about whom we have some information about the pharmaceutical treatment before hospitalization. This reduces the number of individuals in our data set to 265,862 out of whom 626 have been hospitalized. From the hospital invoices we derive information on the total cost of hospitalization, and the date of entering the hospital. Person-specific characteristics are collected using the CZ policy holders register. These characteristics include

[10] We do not have information on the purchases of acid-suppressing drugs such as over-the-counter H2 blocker type of drugs that have not been reimbursed by the CZ .

[11] We identified in the hospital invoices database, the in-patient treatment of individuals in the gastroentherological unit, and in the surgical units for a stomach-related treatment.

the birth date, the gender, the data of purchase of a CZ policy, whether the person has a family policy, whether a person is the main contract holder of the insurance policy, and whether the insurance policy is private or compulsory[12]. We keep in our data set only individuals who are matched to an entry in the register. This further excludes 5,265 individuals. We have also excluded individuals whose starting date of contract with CZ is after the first observed reimbursement for medication: 7,857 individuals. Finally, we have information on some additional socioeconomic indicators which values are available only for a subset[13] of the sample. As a baseline case, we take patients who have the social code “other”[14] These characteristics are classified in Table 6.2. To reduce the noise in the data, we have only kept in our database patients who have started taking pills in 2001 or after one year of the beginning of their contract if this date is after January 2000. In reducing the sample, the possibility for biasedness due to left-censoring is lower. In doing so, we have assumed that if a patient was not taking pills for one year, she had not been taking pills before, or, she has been healed in her previous treatment. Some summary statistics of the sample that will be used in the Logit estimations are shown in Table 6.3.

In the sample 0.17 percent of the patients who have only taken PPI have been hospitalized (195 patients out of 111,716); 0.02 percent of the patients who have only taken H2B have been hospitalized (6 patients out of 25,254); and, out of those who have taken both types of medicaments, 0.27 percent have been hospitalized (43 patients out of 16,086). So, simple between-group comparison indicates different frequency of the incidence of hospitalization among patients with different medicinal treatment history. As expected, the group of patients who have only taken H2B has the lowest proportion of patients who have been hospitalized because these are the patients who are to be treated for having light symptoms, according to the treatment guidelines.

To perform robustness analysis of the Logit estimation, we also analyse the probability that a patient enters hospital within one year from the first purchase of acid-suppressing drugs. This prevents us from using data for patients who have first purchased the medication in 2004 and reduces the sample to 113,431 patients of which

[12]The Netherlands was characterized until 2005 by a two-tier health insurance system. People with an annual income above a yearly specified threshold set by the government had to insure themselves privately (*particulier*). The rest of the population had the so-called compulsory insurance (*ziekenfonds*). The basic differences were in the premium paid and the greater variety of co-payment packages available to the privately insured.

[13]By definition, these codes should be available only for people who have a compulsory insurance. However, in the database we see people who do not have a compulsory insurance but do have a social code for being disabled or below a social minimum.

[14]In the baseline group we include two patients who have the social code “foreign”.

Table 6.2: Definitions of Some of the Variables

Series	**Definition**
PPI	a dummy that equals 1 if PPI
H2B	a dummy that equals 1 if H2B
PPIall	a dummy that equals 1 for the patients who have only taken PPI
H2Ball	a dummy that equals 1 for the patients have only taken H2B
HOSP	a dummy that equals 1 for the patients who have been hospitalized
DAYSBED	the number of days a patient stayed in a hospital
PILLSppi	the total number of PPI pills purchased by a patient
PILLSh2b	the total number of H2B pills purchased by a patient
COSTSppi	the total cost of the PPI medication purchase
COSTSh2b	the total cost of the H2B medication purchase
COSTSh	the total cost of the hospitalization
ZKFOND	a dummy that equals 1 for the compulsory insured patients
FEM	a dummy that equals 1 for the female patients
SINGLE	a dummy that equals 1 for the patients who are single
HEAD	a dummy that equals 1 for the patients who are the head of a family insurance package
AGE	a patient's age on the day of first medication purchase measured in years
DISABLE*	a dummy that equals 1 for the patients with disabilities
D65*	a dummy that equals 1 for patients above 65 years of age
SEMP*	a dummy that equals 1 for patients who are self-employed
EMP*	a dummy that equals 1 for patients who are employees
NSC*	a dummy that equals 1 for patients without a social code
BPL*	a dummy that equals 1 for patients below a poverty line
OOB*	a dummy that equals 1 for patients who are out of the budget of the CZ

* These variables are only available for patients with a social code

138 have been hospitalized within 1 year.

For the purposes of the duration analysis, we use detailed data on the medicinal treatment history of each patient who has bought an acid-suppressing type of drug for the first time at least one year after she enters the sample. We use information on the date each patient purchased the drugs and the amount of pills she purchased at each instant of time. Hence, for each of the 153,056 patients there are different numbers of observations depending on the number of times she has purchased acid-suppressing drugs. We perform a continuous time analysis with time-varying regressors. One of the time-varying regressors for patients who have bought both types of drugs during their medicinal treatment is the dummy variable PPI which equals 1 if the drug that

Table 6.3: Summary Statistics

Variable	Observations	Mean	Std. Deviation	Minimum	Maximum
HOSP	153,056	0.002	0.041	0	1
DAYSBED[a]	244	19.01639	17.29244	0	120
COSTSh[a]	244	12,833.47	10,888.28	654	79,084
PPI	153,056	0.730	0.444	0	1
H2B	153,056	0.165	0.37	0	1
PILLSppi[b]	127,802	149.882	270.161	1	5,230
PILLSh2ra[c]	41,340	104.302	248.679	1	6,904
COSTSppi[b]	127,802	218.9616	414.4518	0.03	8,303.87
COSTSh2ra[c]	41,340	53.93564	112.0024	0.3	2228.88
AGE	153,056	51.566	18.340	1.001	102.933
ZKFOND	153,056	0.853	0.354	0	1
FEM	153,056	0.554	0.497	0	1
HEAD	153,056	0.711	0.453	0	1
SINGLE	153,056	0.423	0.494	0	1
NSC	153,056	0.140	0.347	0	1
DISABLE	153,056	0.134	0.341	0	1
D65	153,056	0.197	0.398	0	1
BPL	153,056	0.0483	0.214	0	1
SEMP	153,056	.044	0.205	0	1
EMP	153,056	0.393	0.488	0	1

[a] measured for those patients who stayed in hospital;

[b] measured for those patients who have taken a PPI drug;

[c] measured for those patients who have taken a H2B drug;

has been bought is a PPI type of drug and 0 if it is a H2B type of drug. We furthermore include two new time-varying regressors. These are the rate at which a patient takes the purchased pills per day and the proportion of instances in which a PPI type of drug is purchased relative to the total number of times a patient has purchased an acid-suppressing type of drug. In measuring the rate at which pills are taken per day, it is assumed that the patient takes the total amount of pills purchased at a constant rate, *e.g.*, if a patient buys 30 pills on January 1, 2001 and then the same patient buys pills again on January 30, 2001 we assume she has taken 1 pill per day in the period January 1–30, 2001. This assumption may not be a good approximation for a number of cases. For instance, a patient may purchase the total dose for a treatment on several installments, *e.g.*, a patient bought 300 pills in one day and then in the following day she bought 300 pills again. This might be due to the unavailability of

Table 6.4: Descriptive Statistics of the Duration Sample

Category	Total	per subject Mean	Minimum	Median	Maximum
No. of subjects	152,935				
No. of records	579,997	3.792	1	2	204
(first) entry time		0	0	0	0
(final) exit time		727.070	1	726	1,461
time at risk	1.112×10^8	727.070	1	726	1,461
failures	242	0.0016	0	0	1

the medication in the pharmacy. To correct for this, we manipulate the data in the following way: if a patient has a rate of more than 15 pills per day, and the type of pills purchased in the beginning and at the end of the observed period is the same[15], then we record both purchases at the beginning of the first period and re-estimate the pills rate. This manipulation occurred for 971 patients out of 2,905 patients who have a rate of taking pills higher than 15 per day. The remainder of the cases could not be manipulated because either they took two different types of medicaments in a relatively short period, or because the high rate of taking pills occurred close to the censoring time, *i.e.*, December 2004. To avoid bias due to outliers, patients who had a rate of taking pills more than 100 per day have been dropped from the sample. The resulting sample consists of 152,935 patients. The minimum number of observations per patient is 1 and the maximum is 204 as presented in Table 6.4. In Table 6.5 we

Table 6.5: Summary Statistics for Time-varying Regressors

Variable	Observations	Mean	Std. Deviation	Minimum	Maximum
PPI	579,997	0.836	0.371	0	1
PIILSRATE	579,997	1.123	2.738	0.0007	91
PPIHISTRL	579,997	0.813	0.359	0	1

present the summary statistics of the time-varying regressors described above. The variable $PILLSRATE$ measures the number of pills purchased divided by the number of days between the date the pills were purchased and the next date of purchase. The

[15]There are a number of cases in which a patient buys a PPI type of drug and in a few days buys an H2B type of drug or vice versa.

variable *PPIHISTRL* measures the number of instances when a PPI type of drug was purchased relative to the total number of times an acid-suppressing type of drug has been purchased as measured at the current date of purchase.

To conduct a robustness check of the obtained results, we furthermore extend the duration analysis to incorporate the possibility of a patient to exit the risk pool of patients entering hospital prior to January 2005. That is, we allow patients to stop being at risk either because they have been cured or because they die prior to entering hospital and before the end of the observation period. We assume that a patient exits the risk pool, if she does not purchase acid-suppressing drugs for a year after she has taken the amount of medication last purchased.[16] This seems to be a reasonable assumption since according to Jones and Bytzer (2001) the cessation of medicinal therapy of patients with chronic condition of GERD results in symptomatic relapse within 6 months in approximately 90 percent of the patients with oesophagitis, and 75 percent of the patients without.

In our sample, there are 91,823 instances at which a patient ceases medicinal treatment for more than one year. Out of these, in 18,385 of the cases the patient resumes the medicinal treatment after at least one year. In our analysis, patients whom we observe not taking the drugs for at least one year and who do not resume treatment, will be assumed to leave the risk pool after consuming the amount of drugs last purchased. For patients who interrupt treatment for at least a year and then resume treatment, it is assumed that we observe two different incidents of illnesses, *i.e.*, for the time between the first episode and the relapse we assume the patient is *not* at risk of entering hospital.

6.3 Regression Models and Estimation Results

We assume that each *patient* $i = 1, \dots, n$ is characterized by her disease status denoted by Y_i^* which is a function of some patient specific attributes, *i.e.*,

$$Y_i^* = \beta X_i + \gamma M_i + u_i, \tag{6.1}$$

where X_i is the vector of the socioeconomic characteristics of patient i (*e.g.*, age, marital status, type of insurance, social code, etc.), and M_i is the vector of variables that reflect the medicinal treatment history, (*e.g.*, type of medication, quantity of medicaments, etc.), β and γ are vectors of parameter values for the respective attributes, and

[16]The date when a patient is assumed to have consumed her purchased pills is calculated on the basis of a one-pill-per-day rate of taking medication.

u_i is a random term. The patient's disease status is unobservable. What we observe is whether a patient undergoes a hospital treatment or not (which is the point of analysis in the Logit model) and the duration of her medicinal treatment before entering hospital (which is the point of analysis in the Cox Proportional Hazard model). Below we will present the assumptions and the estimation results of the Logit and the Cox Proportional Hazard models.

6.3.1 Logit Estimates on the Probability to Enter Hospital

Since Y^* is unobservable, we use its dichotomous realization where a patient either enters hospital for treatment or does not. Let Y_i be a binary variable such that $Y_i = 1$ if $Y_i^* > \bar{Y}^*$ and $Y_i = 0$ if $Y_i^* \leq \bar{Y}^*$ for the threshold of disease status $\bar{Y}^* = 0$[17] above which a patient is hospitalized, and below which she is not. We will further assume for the purposes of this section that u_i in Equation 6.1 is a logistically distributed random variable for all patients $i = 1, \ldots, n$ with a cumulative distribution function $\Lambda(u_i) = \frac{e^{u_i}}{1+e^{u_i}}$. Hence,

$$\begin{aligned} P(Y_i = 1 \mid X_i, M_i) &= P(Y_i^* > 0 \mid X_i, M_i) = P(\beta X_i + \gamma M_i + u_i > 0 \mid X_i, M_i) \\ &= P(u_i > -(\beta X_i + \gamma M_i \mid X_i, M_i) = P(u_i < \beta X_i + \gamma M_i \mid X_i, M_i) \\ &= \Lambda(\beta X_i + \gamma M_i \mid X_i, M_i), \end{aligned}$$

which provides the underlying probabilistic structure of the regression model. It is also assumed that the logistically distributed error terms u_i are independently distributed of the patient-specific attributes and the treatment-specific variables. The results of the estimated Logit model are presented in Table 6.6. With respect to the regressors capturing the medicinal treatment history, the estimation results indicate that the patients who have taken both H2B and PPI types of medication have the highest probability to enter hospital. The second highest is the probability to enter hospital of the patients who have taken only PPI type of medication, and the lowest[18] is the probability of the patients who have taken only H2B. The marginal effects show that on average taking only PPI leads to a 0.0006 decrease in the probability compared to taking both types of medication all else equal, while taking only H2B leads on average to a 0.0014 decrease in the probability all else equal. These numbers should be considered relative to the sample percentage of people who have been hospitalized which is 0.16 percent.

[17] As it is shown in Green (2003) page 669, assuming that the value of the threshold is zero goes without loss of generality.

[18] The 95 percent confidence interval for the coefficient of PPI lies above the 95 percent confidence interval of the coefficient of H2B.

Table 6.6: Logit Estimates of the Probability to Enter Hospital

P(Y=1)	Coefficient Estimates		Marginal Effects Estimates		
Variable	**b**	**z-val**	$\Delta y \backslash \Delta x$	**z-val**	$\overline{x}$
PPIall[a]	-0.522	(-2.89)**	-0.0006	(-2.51)*	0.730
H2Ball[a]	-2.414	(-5.51)*	-0.0014	(-9.93)*	0.165
PILLSppi	-0.0007	(-2.34)**	-7.06×10^{-7}	(-2.33)**	125.151
PILLSh2b	0.0004	(0.92)	3.71×10^{-7}	(0.92)	28.172
AGE	0.057	(2.20)**	0.0001	(2.25)**	51.566
AGE^2	-0.0003	(-1.43)	-3.41×10^{-7}	(-1.45)	2,995.44
HEAD[a]	0.235	(1.05)	0.0002	(1.10)	0.712
FEM[a]	-0.249	(-1.40)	-0.0003	(-1.37)	0.554
SINGLE[a]	-0.405	(-2.32)**	-0.0004	(-2.34)**	0.423
ZKFOND[a]	0.231	(0.69)	0.0002	(0.75)	0.853
NSC[a]	1.003	(1.88)***	0.0015	(1.17)	0.140
DISABLE[a]	0.611	(1.15)	0.0008	(0.92)	0.134
D65[a]	1.084	(2.04)**	0.0016	(1.44)	0.197
BPL[a]	1.164	(2.07)**	0.0021	(1.28)	0.0483
SEMP[a]	0.658	(1.09)	0.0009	(0.82)	0.044
EMP[a]	0.276	(0.52)	0.0003	(0.51)	0.393
OOB[a]	-0.164	(-0.15)	-0.0002	(-0.16)	0.0104
constant	-8.623	(-8.64)*	–	–	–
N	153,056				
LR Test	161.98	p-value	(0.00)*		

[a] marginal changes calculated for discrete changes; * means significance at 99 percent confidence level;
** means significance at 95 percent confidence level; *** means significance at 90 percent confidence level.

Recall, that H2B's have been advised for prescription to individuals with lower symptomatic levels, hence, what the variable H2Ball might be capturing is the low illness severity of the patient rather than the effectiveness of the drug. Since we do not have an explicit measure of case severity, this might be a sign of the violation the basic assumptions that the regressors and the disturbances are independent. The violation of this assumption would render the estimates inconsistent. The result that the baseline group, the patients who have taken both types of drugs have the highest probability to enter hospital might be attributed to an initial "misjudgement" by the GP, *i.e.*, these are patients with higher relative severity of illness who have been given H2B, while PPI should have been prescribed. In the data we do not consistently observe that patients are first given H2B and then PPI. An alternative way of looking at these results is to treat the dummy variables H2Ball and PPIall as controlling for the severity of the case and the variables PILLSh2b and PILLSppi controlling for the effectiveness of the type of drug. The estimates show that the volume of H2B type of medication taken on average does not influence the probability to enter hospital, while the volume of the PPI medication has a negative effect: taking 1,000 more PPI pills (per treatment) on average leads to a 0.000706 decrease in the probability to enter hospital all else equal. The problem with this analysis is that the baseline group, the patients who have taken both types of medications, has to be treated as a group of patients with intermediate severity, however, it exhibits the highest estimated probability to enter hospital.

Among the socioeconomic variables, age has the expected positive effect on the probability to enter hospital. In addition, there is an added increase in the probability to enter hospital for patients who are above 65 years of age. Having no social code, which is a crude indicator of patients with high income, as well as being below the poverty line have statistically significant positive effects on the probability to enter hospital at the 5 percent significance level. On the other hand, being single has a negative effect on the probability to enter hospital: on average it leads to 0.0004 decrease. The Likelihood Ratio test for the validity of the model presented in Table 6.6 tests the null hypothesis that all the slope coefficients are jointly equal to zero, against the alternative hypothesis that at least one of them is not equal zero. The test statistics is calculated according to $2(\mathcal{L}_U - \mathcal{L}_R)$ where $\mathcal{L}_U$ is the log-likelihood of the model presented in Table 6.6 and $\mathcal{L}_U$ is the log-likelihood of the model that explains the probability of hospitalization by a constant, is distributed as a χ^2_{17}. The estimates indicate that the statistical performance of the regression model is good: given the p-value of the test, we can reject the null hypothesis at 1 percent significance level.

Furthermore, we perform the Hosmer-Lemeshow goodness-of-fit test in which the fitted values are clustered in 10 groups on the basis of similar predicted probabilities of hospitalization.[19] The null hypothesis is that the data are generated by the regression model that is specified. The test statistics is based on comparison between the predicted frequencies of hospitalization and the actual frequencies observed in the sample within the 10 groups of observations. The test statistics is distributed as χ^2_8. In our case the p-value is 0.616 which means that we fail to reject the null hypothesis.

Using a Logit model to estimate the effect of medication history on the hospitalization outcome, may be restrictive since we do not take into account the differences in the length of period during which patients are at risk of entering hospital, *i.e.*, some patients have been treated for years while others have been taking medication for several months before entering hospital. Furthermore, we observe some patients purchasing medication since the beginning of the sample period while others purchase medication for the first time just moths before the end of the observation period. To

[19]This test was developed by Hosmer and Lemeshow (1980).

Table 6.7: Logit Estimates of the Probability to Enter Hospital within 1 Year

$P(Y_{1yr}) = 1$	Coefficient Estimates		Marginal Effects Estimates		
Variable	**b**	**z-val**	$\Delta y \backslash \Delta x$	**z-val**	$\bar{x}$
PPIall[a]	-0.349	(-1.20)	-0.0002	(-1.13)	0.702
H2Ball[a]	-2.027	(-3.66)⋆	-0.0008	(-5.60)⋆	0.171
PILLSppi	-0.002	(-4.07)⋆	-1.45×10^{-6}	(-4.04)⋆	149.769
PILLSh2b	-0.003	(-1.26)	-1.94×10^{-6}	(-1.30)	34.325
AGE	0.103	(2.62)⋆	0.0001	(2.80)⋆	51.624
AGE^2	-0.0006	(-1.88)⋆⋆⋆	-3.89×10^{-7}	(-1.97)⋆⋆	2,998.57
HEAD[a]	0.209	(0.67)	0.0001	(0.70)	0.718
FEM[a]	-0.359	(-1.53)	-0.0002	(-1.47)	0.552
SINGLE[a]	-0.131	(-0.58)	-0.0001	(-0.58)	0.563
ZKFOND[a]	0.465	(0.95)	0.0002	(1.11)	0.859
NSC[a]	1.167	(1.56)	0.001	(1.01)	0.128
DISABLE[a]	0.267	(0.42)	0.0002	(0.38)	0.139
D65	0.809	(1.31)	0.0007	(1.01)	0.206
BPL[a]	0.588	(0.83)	0.0005	(0.64)	0.050
SEMP[a]	0.036	(0.04)	2.31×10^{-5}	(0.04)	0.044
EMP[a]	0.049	(0.08)	3.1×10^{-5}	(0.08)	0.399
c	-10.473	(-6.99)⋆	–	–	–
N	113,431				
LR Test	137.35	p-value	(0.00)⋆		

[a] marginal changes calculated for discrete changes; ⋆ means significance at 99 percent confidence level;
⋆⋆ means significance at 95 percent confidence level; ⋆⋆⋆ means significance at 90 percent confidence level.

compare the effectiveness between types of treatments relative to the same treatment period, we analyze the probability that a patient enters hospital within one year after we first observe her purchasing acid-suppressing medication. The estimation results are presented in Table 6.7. The estimates of the effect of the volume of medication are robust: taking higher volumes of PPI on average reduces the probability to enter hospital. However, the results are not robust in terms of the effect of the type of medication, *i.e.*, taking only PPI type of drugs does not lead on average to a significantly different probability than taking both types of medications. In terms of the socioeconomic variables, only the effect of age is robust. The Likelihood ratio statistics for the validity of the model test as presented in Table 6.7 leads to the rejection of the null hypothesis that the slope coefficients are jointly equal to zero. The Hosmer-Lemeshow goodness-of-fit statistics also lends support for the validity of the model by concluding that we cannot reject the null hypothesis that the data is generated by the specified regression model based on p-value 0.283. In this model we could not estimate the OOB effect because no patients with this social code have been hospitalized within 1 year of treatment.

Using the estimations shown in Table 6.6, we perform a cost-effectiveness analysis. The costs of the medicinal therapy is the cost of drugs each patient purchased using her insurance, denoted by COST_i for each person *i*. It is measured as:

$$\text{COST}_i = \text{COSTppi}_i + \text{COSTh2b}_i,$$

where COSTSppi_i and COSTSh2b_i as defined above are the total amounts spent on PPI and H2B drugs by the patient *i*.

The "benefit" is the expected reduction in direct and indirect hospitalization costs. The expected avoided costs for a patient *i* denoted by EB_i is measured as:

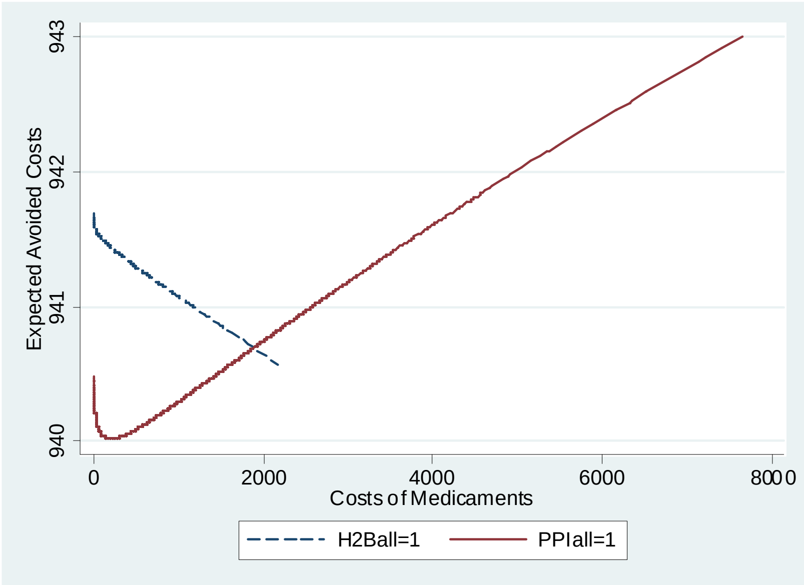
$$\text{EB}_i = \left(1 - \hat{P}(Y_{1yr} = 1)\right) \times \left(\overline{\text{COSTSh}} + \overline{\text{DAYSBED}} \times \bar{I}/365\right),$$

where $\hat{P}(Y_{1yr} = 1)$ is the predicted probability of a patient to enter hospital, $\overline{\text{COSTSh}}$ is the mean costs of hospitalization for the patients who entered hospital in the sample (12833.47 euros), $\overline{\text{DAYSBED}}$ is the mean number of days spent in hospital by these patients (19.01639 days), and $\bar{I}$ is the average annual income in the Netherlands in 2003 (17,700)[20]. The way we have constructed the expected benefit function, we have assumed that the number of days and the hospitalization costs do not depend on the medication history of the patient given that a patient has been hospitalized.

At this stage, we can distinguish and compare two different types of medicinal therapies: based only on H2B or only on PPI. On Figure 6.4, we present a fitted polynomial relation between the drug costs and expected cost avoidance for two groups of patient: those who in their medicinal therapy have taken only H2B type of drugs and those who have taken only PPI type of drugs. The results show that for lower levels of cost of drugs, H2B-only therapy yields higher future savings than PPI-only therapy while for higher values the PPI-only therapy is more effective. These results should be interpreted with caution. The gap between the cost savings from the different therapies for every given level of dug costs should not be attributed fully to the effectiveness of the drug. They are also due to differences in the severity of illness. As was discussed in the introduction, H2B type of acid-suppressing drugs have been advised for prescription to patients with relatively low symptomatic levels, while PPI to patients with relatively high symptomatic levels. Furthermore, recall the results in Tables 6.6 and

[20]The source of this number is the Dutch Statistical Office (*Centraal Bureau voor de Statistiek*).

Figure 6.4: Cost of Drugs and Expected Cost Saving: H2B vs PPI



6.7 according to which taking only H2B type of drugs reduces the probability of a patient to enter hospital, while taking a greater quantity of PPI type of drugs reduces this probability. This also implies that the differences along the lower cost range may be due to the differences in the severity of the case all else equal.

6.3.2 Cox Regression Estimates on the Hazard Rate of Entering Hospital

The focus of analysis is on the time interval of transition from the state before hospitalization, *i.e.*, the instant an individual becomes a patient, to hospitalization of each patient. Such intervals are called *spells*. The length of a spell is assumed to be a realization of a random variable T with a cumulative distribution function $F(t)$, *i.e.*, $P(T \leq t) = F(t)$, and a probability density function $f(t)$. The analysis entails estimating the *hazard rate* which summarizes the concentration of spell lengths at each instant of time conditional on not ending the spell up to that instant of time. In terms of probabilities, the hazard rate $\lambda(t)$ is given by

$$\lambda(t) = \frac{f(t)}{1 - F(t)}.$$

The estimation technique that we employ is the Cox Proportional Hazard Model with time-varying regressors. The seminal reference for introducing the Cox Proportional Hazard Model is Cox (1972). Our discussion of the assumptions of the Cox Proportional Hazard Models is based on Cameron and Trivedi (2005). The time-varying regressors in our case are the type of medication that a patient takes for those of the patients who take both H2B's and PPI's, and the rate of pills consumption in different periods. Our choice of using the Cox Proportional Hazard Model is based on the fact that it has become a standard estimation technique in the literature due to its less restrictive assumptions on the distributional parameters, and its relatively straightforward estimation.

For the Cox Proportional Hazard Model with time-varying regressors, the hazard rate is given by

$$\lambda(t \mid Z(t)) = \lambda_0(t)\phi(Z(t), \delta, u),$$

where $\lambda(t \mid Z(t))$ is the hazard rate conditional only on the *current* values of the regressors with $Z(t) = (X, M(t))$ and $\delta = (\beta, \gamma)$, and u is a disturbance term, $\lambda_0(t)$ is the baseline hazard which is only a function of time and cannot be estimated in the Cox Proportional Hazard Model, and $\phi(Z(t), \delta)$ is a function only of the *current* values of the regressors. To present the partial likelihood function used in the estimation of the parameters of the model, we need to present the notions of the set of subjects at risk, $R(t)$, and the set of subjects whose spells end at instant t, $D(t)$, respectively. Let $t_1 < t_2 < \ldots < t_k$[21] be the observed discrete time intervals in the state before hospitalization, then

$$\begin{aligned} R(t_j) = & \quad \{l \mid t_l \geq t_j\}: && \text{set of spells at risk at } t_j. \\ D(t_j) = & \quad \{l \mid t_l = t_j\}: && \text{set of spells completed at } t_j, \\ d(t_j) = & \quad \textstyle\sum_l \mathbf{I}(t_l = t_j): && \text{number of spells completed at } t_j, \end{aligned}$$

where **I** is an index function. The partial likelihood is given as

$$\ln L_p = \sum_{j\in 1}^{k} \left[\sum_{m\in D(t_j)} \ln \phi(Z_m(t_j), \delta) - d(t_j) \ln \left(\sum_{l\in R(t_j)} \phi(Z_l(t_j), \delta) \right) \right].$$

For our estimation, we take $\phi(Z(t), \beta) = \exp(Z(t)'\delta) = \exp(X'\beta + M(t)^{\gamma})$ An essential assumption of the Cox Proportional Hazard Model is the proportionality assumption of the hazard rate. The tests are developed by Grambsch and Therneau (1994) and described in the Stata Manual (1999). We test for the validity of this assumption

[21] Note that $k \leq n$ since there may be patients who have the same length of spells.

using the scaled Schoenfeld for the separate test for each regressor and the unscaled Schoenfeld residuals for the global test. The Schoenfeld residuals are proposed by Schoenfeld (1982). There is a separate residual for each individual patient and for each regressor and they are not defined for censored individuals. The Schoenfeld residual is the value z_{ik} of the regressor for person i at the time when person i enters hospital t_i minus the weighted average of this regressor for the patients who are at risk at this instant of time weighted by each individual's likelihood to enter hospital at this instant of time P_j. Formally,

$$s_{z_i k} = z_{ik} - \sum_{j \in R(t_i)} Z_{jk} P_j,$$

The scaled adjustment of the Schoenfeld residuals is formulated as

$$r_{z_k S_i^*} = \hat{\delta_{z_k}} + \triangle(\mathbf{S}\hat{\mathbf{V}}^{-1})_{z_{ik}},$$

where $\triangle$ is the total number of patients who enter hospital, $\mathbf{S}$ is the matrix of Schoenfeld residuals and $\hat{\mathbf{V}}$ the estimate of the variance matrix. These residuals should have slope zero when plotted against time if the proportional hazard assumption is satisfied. The H_0 hypothesis is that the proportional hazard assumptions is satisfied and the H_1 is that it is not. The test statistics is distributed as χ_1^2 for the individual tests and χ_q^2 for the global test where q is the number of regressors.

Like in the binary choice models used in the previous section, the independence between the error term and the regressors is also a crucial assumption that ensures the consistency of the estimates. Given the unobserved severity of the illness, which might be correlated with the type of medication used, there are some concerns that this assumption is not satisfied in our model.

In Table 6.8, Column A are presented the results of the basic model in terms of hazard ratios, *i.e.*, for each regressor z_j, we show $\exp \delta_j$, which is the coefficient with which the hazard ratio changes as a result of a change in z_j. Whether the hazard rate increases is determined by a one-sided test on $\exp \delta_j > 1$. Alternatively, the hazard rate decreases if $\exp \delta_j < 1$.

As expected, a higher rate of taking pills is indicative of higher severity of the illness and it is associated with a higher hazard rate to enter hospital. This effect is not estimated to be different for PPI and H2B types of drugs because the hazard ratio of the interaction term between PILLSRATE and the dummy PPI is not significantly different from 0. Another regressor, that captures the severity of the case of the patient is the dummy PPI: taking PPI type of drug rather than H2B increases the hazard rate to enter hospital. The effectiveness of the drug might be captured by the history of medicinal treatment. Taking a relatively higher proportion of PPI type of drugs than H2B type of drugs decreases the hazard rate.

Table 6.8: Cox Proportional Hazard Estimates

	Basic Model [A]		**Cause-Specific [B]**	
Variable	**Hazard Ratio**	**z-val**	**Hazard Ratio**	**z-val**
PPI^a	24.168	(6.25)⋆	10.938	(3.58)⋆
PILLSRATE	1.120	(5.16)⋆	1.108	(10.42)⋆
PPI*PILLSRATE	1.001	(0.05)	0.999	(−0.10)
PPIHISTRL	0.161	(-4.08)⋆	0.302	(-2.12)⋆⋆
AGE	1.000	(3.28)⋆	1.039	(1.70)⋆⋆⋆
AGE^2	1	(-3.04)⋆	1.000	(-1.06)
$HEAD^a$	1.327	(1.25)	1.300	(1.13)
FEM^a	0.845	(-0.94)	0.827	(-1.01)
$SINGLE^a$	0.675	(-2.23)⋆⋆	0.643	(-2.32)⋆⋆
$ZKFOND^a$	0.720	(-1.73)⋆⋆⋆	0.815	(-1.11)
D65	2.755	(4.40)⋆	1.738	(2.44)⋆⋆
N	599,997		599,997	
LR Test	247.55		653.04	
p-value	(0.00)⋆		(0.00)⋆	

⋆ means significance at 99 percent confidence level;

⋆⋆ means significance at 95 percent confidence level;

⋆⋆⋆ means significance at 90 percent confidence level.

The estimated effects of the regressors based on socioeconomic characteristics[22] show similar results to those presented in Table 6.6. Age has a significant positive effect on the hazard rate. Being single or having a compulsory insurance, on the other hand, on average decreases the hazard rate to enter hospital everything else equal. The effect of the type of insurance might be indicative of different incentives in the medical practice to hospitalize patients with private versus patients with compulsory insurance.

We perform a formal test on the validity of the proportionality assumptions. The test shows that we cannot reject the proportional hazard assumption at the 95 percent confidence level for all regressors but AGE, AGE^2, FEM, and SINGLE.[23] The global

[22]We have not included among the regressors the remainder of the dummies based on social codes since they were insignificant in the Logit analysis and in order to save computational time.

[23]The p-values of the respective χ^2 statistics of these variables are 0.004, 0.038, 0.001, and 0.030.

test on proportionality rejects the null hypothesis of proportional hazard rates at the 99 confidence level (χ^2-statistics equals 99.78). Hence, estimates may not be consistent.

The results of the test on the proportionality assumptions indicate that the regressors might have time-varying effects on the hazard rate. For instance, it might be that there are differences at the initial severity of the disease for female and male patients, however, these differences disappear if the female and male patient undergo medicinal treatment for a long period of time. It will be the case if, for example, female patients are more likely to visit doctors at early stages of their illness while male patients are more likely to wait longer before they visit a GP. The differences in hazard rate after 3 years of medicinal treatment conditional on being treated after 3 years, however, might fade away.

The limitation of the above analysis is that in constructing the model whose estimates are presented in Table 6.8, we do not allow for patients to exit the risk pool to enter hospital before the end of the observation period. However, a patient might have taken an acid-suppressing drug preventively against NSAID-induced ulcer, or to eradicate early stage of an H. Pylori infection, or early stage of GERD, or a patient might have died.

To conduct a robustness analysis to the result above, we perform a competing risk duration analysis. The seminal reference for the methodological tools in the analysis of duration models in the presence of competing risks is Prentice, Kalbfleisch et al. (1978). The theoretical discussion presented here is based on Cleves (1999) and Gichangi and Vach (2005). The basic assumption of this model is that an individual can exit the risk pool due to a number of reasons, called *causes*, such that if he exits due to one cause, he does not continue to be at risk of any other cause. We regard patients to be at risk of two possible causes: hospitalization and all others, *e.g.*, healing or death. We will index failure due to the cause hospitalization with a subscript h, and failure due to alternative causes, with a subscript a. The cause-specific hazard rate is given by

$$\lambda_h(t \mid Z_h(t)) = \lambda_o(t)\phi(Z_h(t), \delta),$$

where $\lambda_h(t \mid Z_h(t))$ is the hazard rate of entering hospital conditional only on the *current* values of the regressors relevant for the process that causes hospital treatment with $Z_h(t) = (X, M(t))$ and $\delta = (\beta, \gamma)$, $\lambda_o(t)$, as discussed above, is the baseline hazard which is only a function of time and cannot be estimated in the Cox Proportional Hazard Model,[24] and $\phi(Z_h(t), \delta)$ is a function only of the *current* values of regressors.

[24]Note that in estimating the general hazard rate of failure, one can allow for cause-specific baseline hazard rates.

The main difference between the cause-specific partial likelihood function in a competing risk model and the partial likelihood function in the standard Cox Proportional Hazard Model is in the definition of the subjects who are at risk at every given instant of time. We denote the set of subjects at risk of failure due to hospitalization at time t as $R_h(t)$, and the set of subjects whose spells end due to hospitalization at instant t, $D_h(t)$. Let $t_{h,1} < t_{h,2} < \ldots < t_{h,k}$ be the observed discrete time intervals in the state before hospitalization, and let $t_{a,1} < t_{a,2} < \ldots < t_{a,s}$ be the observed discrete time intervals in the state before alternative causes of failure, then

$$R_h(t_{h,j}) = \{l \mid t_{h,l} \geq t_{h,j} \text{ and } t_{a,j} > t_{h,j}\} \text{ :set of spells at risk of hospitalization at } t_{h,j}.$$
$$D_h(t_{h,j}) = \{l \mid t_{h,l} = t_{h,j} \text{ and } t_{a,j} > t_{h,j}\} \text{ :set of spells completed at } t_{h,j} \text{ in hospital,}$$
$$d_h(t_{h,j}) = \sum_l \mathbf{I}(t_{h,l} = t_{h,j} \text{ and } t_{a,j} > t_{h,j}) \text{ :number of spells completed at } t_{h,j} \text{ in hospital,}$$

where **I** is an index function.

The partial likelihood is given as

$$\ln L_{h,p} = \sum_{j\in 1}^{k} \left[\sum_{m\in D_h(t_{h,j})} \ln \phi(Z_{h,m}(t_{h,j}), \delta) - d_{h,j} \ln \left(\sum_{l\in R_h(t_{h,j})} \phi(Z_{h,l}(t_{h,j}), \delta) \right) \right].$$

Using maximum likelihood method, one may obtain unbiased estimates of the δ coefficients assuming that the k observations are independent. However, given the likelihood function, there are patients who may be observed under two different events of being at risk of hospitalization, *i.e.*, these are patients who are healed the first time they are at risk, and experience a relapse after more than one year. Hence, the observation on k subjects may be divided into n independent groups where $n < k$ is the number of patients. To estimate the model we use the robust covariance matrix given by

$$V = I^{-1}\, G'\, G\, I^{-1},$$

where $I^{-1} = \partial^2 \ln L(\delta)/\partial\delta\partial\delta$ and G is an $n \times p$ matrix of the group-efficient score residuals with p being the number of regressors.

The estimates of the parameters are presented in Table 6.8, Column B. The estimations confirm the findings with respect to the medicinal treatment variables: PPI and pills rate have a significant positive effect on the hazard rate, while having taken relatively more PPI type of drugs decreases the hazard rate, all else equal. With respect to the socioeconomic characteristics, the results of the dummy variable SINGLE are confirmed: being single leads to a lower hazard rate. Furthermore, patients above 65 have on average higher hazard rate to enter hospital, all else equal.

Based on the global rank test of the validity of the proportionality assumption, we can reject the hypothesis that they are valid (p-value=0.013). The regressor specific rank tests, show that the null hypothesis is rejected only for FEM (p-value=0.00004) at the 99 percent confidence level and HEAD (p-value=0.050) at the 95 percent confidence level.

6.4 Conclusion

The estimation results provide some evidence that, as prescribed by the guidelines, PPI's have been prescribed to patients with higher disease severity. Indeed, the estimates of the baseline Logit model and those of the duration model show that taking a PPI type of drug is an indirect measure of the severity of the patient illness. We see that taking a PPI type of drug increases the likelihood of a bad outcome. The estimation results also lend support to the findings in the clinical trial literature that PPI type of medication is more effective than H2B's. In the Logit analysis, we see that a higher quantity of PPI's drugs decreases the probability of entering hospital, while the quantity of H2B's does not matter. In the durational analysis, we see that taking relatively higher proportion of PPI drugs during a medicinal treatment, all else equal, decreases the hazard rate to enter hospital. With some degree of caution, one may interpret this result as an evidence of the ineffectiveness of the system. Had the practical evidence been that the relative amount of PPI type of drugs consumption does not matter, we could have concluded that there is no evidence that there are no patients who everything else equal would have had lower probability of hospitalization had they been given a PPI type of drug rather than H2B.

The cost-benefit analysis indicates that for patients who have taken relatively less medication, medicinal therapy based on H2B's is estimated to be more cost-effective. On the other hand, patients who require more prolonged medicinal therapy, PPI therapy is estimated to be more cost-effective. In our sample we identify an intermediate range of costs of medication, in which some patients have been prescribed H2B types of medication, while PPI therapy would have been more cost effective. It should be pointed out that in this analysis we do not control for the severity of each case. In drawing conclusions about the cost-effectiveness of the different therapies, we have been careful to make it clear that the effectiveness of the drugs is estimated for a given cost-level. Assuming that patients who have similar spending on medication, have similar disease severity, and thus using our estimates to derive an estimate of the "inefficiency" of the therapeutic practice, is not warranted given the limitations of the data. Moreover, for this analysis we use the predicted probability of hospitalization of each patient, which might not be a consistent measure of the true probability as discussed below.

The estimation results, should be interpreted with caution. An important limitation of the data is that it does not include an objective measure of the disease severity of each patient. This unobservable heterogeneity among patients is thus captured by the error term. Given that there is a relationship between the type of acid-suppressing drug and the unobservable health status, as prescribed by the guidelines, in our regression models there may be a correlation between a regressor and the error term, which may render the coefficient estimates inconsistent.

Finally, there are some interesting results with respect to the socioeconomic variables that may become a focus of future research. In particular, the estimated lower hazard rate to enter hospital of patients who are single and the consistent failure of the proportionality assumption of the dummy variable for gender, may be signals for consistently different behavior of these groups in the demand for health care. The result that patient with different type of insurance have different hazard rate of hospitalization is also interesting as this might be indicative of the presence of underlying incentives to have privately insured patients more often treated in hospitals than patients who have a compulsory insurance everything else being equal.

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N S

Stabiliteit, Governance en Effectiviteit: Essays op het gebied van de Diensteneconomie

De vragen die in de essays in dit proefschrift bestudeerd worden, kennen vele facetten. De belangrijkste thema's, stabiliteit, governance, en effectiviteit komen in verschillende contexten terug: relationele activiteiten, samenwerking binnen groepen, nationaal bestuur en een bepaalde klasse voorzieningen in de gezondheidszorg. Ook de methoden in dit proefschrift zijn gevarieerd. De theoretische artikelen dragen bij aan de literatuur op het gebied van netwerkvorming en endogene coalitievorming. De empirische essays gebruiken lineaire en niet-lineaire regressiemodellen en maken gebruik van zowel geaggregeerde data als data op microniveau.

Het eerste deel van dit proefschrift, "Stabiliteit en Sociale Erkenning" is geïnspireerd door het streven om de mogelijkheden van een methodologisch raamwerk te verkennen waarin "relationele activiteiten" kunnen worden bestudeerd, waarbij de identiteit van individuen die deelhebben aan de activiteit van invloed is op de uitkomst ervan.

We bestuderen relationele activiteiten in het algemeen – en consument-producent-eenheden in het bijzonder – in een *vóór-marktomgeving*. In Hoofdstuk 1 bestuderen we productieprocessen die in bilaterale verbanden worden uitgevoerd. In deze omgeving die gekenmerkt wordt door de afwezigheid van markten analyseren we het ontstaan van economische specialisatie en uiteindelijk handel en sociale deling van arbeid. We baseren onze analyse op drie stadia van organisatie-ontwikkeling: de aanwezigheid van stabiele relationele structuren, de aanwezigheid van relationeel vertrouwen en subjectieve specialisatie, en, uiteindelijk, het ontstaan van objectieve specialisatie door de sociale erkenning van subjectief gedefinieerde economische functies.

In Hoofdstuk 2 breiden we onze analyse uit tot de analyse van productieprocessen die in teamverband worden uitgevoerd, dat is, relationele activiteiten tussen meerdere personen die georganiseerd zijn in een primitief bedrijf. We laten zien dat de aanwezigheid van een autoriteit die sociaal erkend wordt ervoor zorgt dat productieve teamverbanden gevormd worden.

Het tweede deel, “Stabiliteit en Endogene Coalitievorming” bestaat uit twee speltheoretische essays, waarin we nieuwe stabiliteitsconcepten ontwikkelen voor coöperatieve omgevingen. De essays kennen een gezamenlijke onderliggende vraag: wanneer een verzameling individuen geconfronteerd wordt met een coöperatieve omgeving, welke groepen zullen dan gevormd worden, en hoe zullen de groepsleden de opbrengsten van de coalitie verdelen?

In Hoofdstuk 3 zijn de nieuw ontwikkelde oplossingsconcepten gebaseerd op contractuele overeenkomsten die de groep en ieder groepslid behandelen als twee contractpartijen. De contracten verschillen in termen van welke partij het recht heeft de overeenkomst te beëindigen, en of een partij die slechter af is door de beëindiging van het contract recht heeft op compensatie. Wanneer we het algemene raamwerk toepassen op het voorbeeld van onderlinge verzekering in landbouwproductie vinden we dat er voor elke klasse contractuele overeenkomsten stabiele pooling uitkomsten bestaan die individueel rationeel zijn, terwijl individueel rationele separating uitkomsten daarentegen niet stabiel zijn.

In Hoofdstuk 4 bespreken we stabiliteit die gebaseerd is op onderhandelingen. Deze analyse is van toepassing op situaties waarin bindende contracten niet mogelijk zijn, zoals wanneer individuele investeringen in de samenwerking of de opbrengst ervan niet kunnen worden waargenomen en geverifieerd. Ons oplossingsconcept houdt expliciet rekening met het verschil in deviatiemogelijkheden binnen een reeds gevormde groep en buiten de groep. Ter illustratie passen wij de concepten toe op gewogen meerderheidsspelen en op een nieuwe klasse spelen, de klasse van spelen met samenwerkingseffecten.

Het derde deel, “Governance en Effectiviteit” bestaat uit twee empirische artikelen die onderzoeken hoe diensten functioneren en de implicaties hiervan in het dagelijkse leven.

In Hoofdstuk 5 onderzoeken we de invloed van governance op nationaal niveau op de sociaal-economische ontwikkeling van landen door middel van een econometrische vergelijking tussen landen. Als maat voor de sociaal-economische ontwikkeling van een land nemen we de levensverwachting in het land. Het model met de governance index als verklarende variabele is op statistische gronden te verkiezen boven een model met de Gini-index van inkomensongelijkheid. Ook dragen we bewijs aan voor twee soorten drempeleffecten: zowel in termen van inkomen in absolute zin als in termen van governance. Voor de landen die onder de drempelwaarde zitten is inkomen in absolute termen de meest significante determinant van levensverwachting, terwijl voor de landen boven de drempelwaarde governance het belangrijkste is.

In Hoofdstuk 6 richten we ons op een specifieke casus in de gezondheidszorg, namelijk de effectieve behandeling van Nederlandse patiënten die aan ziekten lijden als Gastro-Oesofagale Reflux, H. Pylori, en gastropathie geïnduceerd door niet - steroïdale anti-inflammatoire geneesmiddelen met antacidum. We onderzoeken de relatieve effectiviteit van twee typen antacida die op recept verkrijgbaar zijn: H2-receptorantagonist, en proton pomp-remmers. Klinische testen suggereren dat protonpomp-remmers effectiever zijn in zowel het genezen van de aandoeningen als het verminderen van de symptomen. Deze Nederlandse casus is interessant omdat huisartsen wordt aanbevolen om H2-remmers voor te schrijven aan patiënten die voor de eerste keer met klachten bij hen komen. Echter, in de praktijk is dit wellicht geen kosteneffectieve behandeling: sommige patiënten wachten misschien lang voordat zij naar de huisarts gaan en hebben daardoor ernstiger klachten. Omdat huisartsen geen volledige kennis hebben van de symptoomgeschiedenis, zouden ze de patiënt kunnen behandelen met het minder effectieve medicijn.

Met een binair keuzemodel voor de kans dat een patiënt in het ziekenhuis wordt opgenomen en een duurmodel met regressoren die in de tijd variëren om de periode voor de opname te analyseren, laten we zien dat voor personen met een lage medicijnconsumptie de H2-remmer inderdaad het kosteneffectievere medicijn is, terwijl voor patiënten met een hoge medicijnconsumptie de protonpomp-remmer meer kosteneffectief is.