Index options

Boes, M.J.

Publication date:
2006

Document Version
Publisher's PDF, also known as Version of record

Index Options: Pricing, Implied Densities, and Returns

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Proefschrift

ter verkrijging van de graad van doctor aan de Universiteit van Tilburg, op gezag van de rector magnificus, prof. dr. F.A. van der Duyn Schouten, in het openbaar te verdedigen ten overstaan van een door het college voor promoties aangewezen commissie in de aula van de Universiteit op vrijdag 13 januari 2006 om 14.15 uur door

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geboren op 10 september 1978 te Sint-Jansklooster.

Promotor: prof. dr. Bas J.M. Werker
Copromotor: dr. Feike C. Drost

Voor alle geïnteresseerden

Voorwoord (Acknowledgements)

Het is uiteindelijk dan toch zover gekomen. Na bijna vier jaar hard werken, zit mijn AiO-schap er nagenoeg op. Het waren vier jaar met ups en downs waarbij vooral het laatste deel om uiteenlopende redenen lastig is geweest.

De doorgemaakte intellectuele groei heb ik als meest positieve aspect van het AiO-schap ervaren. De mensen die daarvoor het meest verantwoordelijk zijn, te weten Bas Werker en Feico Drost, wil ik daarvoor hartelijk bedanken. Ook ben ik aan hen veel dank verschuldigd vanwege het feit dat ze zoveel tijd en energie in mijn begeleiding gestoken hebben. Ik ben blij dat ze gedurende de hele periode het geloof in mij hebben behouden. Door hen kon ik mij geen betere werkomstandigheden wensen.

De samenwerking met ABN-Amro Structured Asset Management bleek een vruchtbare te zijn. Het bracht mij een welkome afwisseling van mensen, kantoor en werkbenadering. Het voortdurend heen en weer pendelen tussen de theorie en de dagelijkse praktijk van vermogensbeheer heb ik als bijzonder prettig ervaren. Ik wil dan ook Cees Dert en Bart Oldenkamp bedanken dat ze zich vier jaar geleden hebben ingezet om deze constructie mogelijk te maken. Verder kijk ik er naar uit om vanaf 1 oktober, na $5\frac{1}{2}$ jaar, eindelijk binnen een vast dienstverband mee te draaien. Mijn toekomstige collega's wil ik vanaf deze plaats hartelijk danken voor het goede contact en de subtiele (voetbal)discussies de afgelopen jaren. In het bijzonder wil ik Mark Petit en Anne de Kreuk danken voor het lezen van bepaalde delen van mijn proefschrift.

Het eerste contact met de Universiteit van Tilburg, in die tijd nog Katholieke Universiteit Brabant, is totstandgekomen via Theo Nijman. Hem wil ik bedanken voor het ontspannen eerste gesprek waardoor de keuze voor Tilburg eigenlijk direct gemaakt was.

Daarnaast wil ik ook de andere commissieleden Andre Lucas, Frans de Roon en Robert Tompkins danken dat ze in de promotiecommissie zitting hebben willen nemen.

Ik zou nog een uitgebreide opsomming van mensen kunnen geven die het leven in Tilburg en ver daarbuiten veraangenaamd hebben. Aangezien ik zeker een aantal zou vergeten, houd ik het bij een bijzonder gezelschap. Ik wil mijn ouders, de rest van mijn familie en mijn vrienden bedanken voor alle steun, geduld, warmte, gezelligheid en gastvrijheid in de afgelopen vier jaar.

Daarbij heb ik de afgelopen jaren mogen genieten van een geweldige kamergenote. Bedankt, Marta, voor al die persoonlijke en minder persoonlijke praatsessies van ons. Verder wil ik ook Anna, Esther, Evgenia, Jeroen, Laurens en Rob bedanken voor het prettige contact dat is ontstaan de afgelopen jaren. Ook is het prettig thuiskomen in de wetenschap dat iemand als Petra naast je woont. Petra, oneindig veel dank voor je emotionele steun tijdens de laatste paar maanden. Tenslotte wil ik Shaastie bedanken voor al haar geloof in en begrip voor mij tijdens de afgelopen jaren.

Amsterdam, augustus 2005

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CHAPTER 1

Introduction

1.1 Motivation

The usage of derivatives in risk management and portfolio management has expanded tremendously over the last couple of decades. The increased liquidity in standard derivative contracts (options and futures) and the development of more complex exotic derivative products are mainly caused by the general developments of financial institutions, investors' needs, and regulations.

From a risk management perspective, derivatives are used to control the uncertainty in the value of investment portfolios. For instance, credit derivatives provide financial institutions a tool to manage the credit risk of their investments by insuring against adverse movements in the credit quality of the borrower. Other possible sources of uncertainty that are hedgeable by derivatives include changing interest rates, exchange rates, and stock prices.

Derivatives are not only used for hedging purposes but have also become a direct source of revenue in portfolio management. In equity markets, for example, (institutional) investors are interested in products that have a high expected return and a limited downside risk. Portfolios of standard options can be constructed in such a way that these features are present in the portfolio payoff profile. Combinations of (barrier) options, default-free bonds, and cash constitute the basis of the so-called guaranteed products and click funds. These type of investment products were very popular in the late nineties and have attracted a significant share of new capital to equity markets in at least The Netherlands. Financial products that provide leveraged equity combined with limited downside risk are still popular after the world wide downfall of equity markets between 2000 and 2003 and after the increased stringent guidelines that institutional investors, like pension funds and insurance companies, are faced with. More recently, derivative products related to the volatility of equity markets have gained more popularity. The obvious consequence of using derivatives in portfolio management is that these financial instruments become objects of risk management themselves.

Evidence for the increased liquidity in derivative products can, for example, be found in the Bank for International Settlements publications (BIS) and on Bloomberg. The quarterly BIS publications on International Banking and Financial Developments report an outstanding exchange traded futures amount of \$2.3 trillion at the end of 1991 up to \$6.0 trillion in 1995 and \$17.7 trillion at the end of 2003. The interest rate futures market is by far the largest and most liquid among futures markets. Similar growth patterns are recognized in option markets. According to the same source, the outstanding amount of exchange traded options (interest rate, currency and equity) was \$1.3 trillion by the end of 1991 and subsequently increased to \$3.1 trillion in 1995 and reached \$31.3 trillion at the end of 2003. Again, the interest rate options are the most actively traded. The notional amount of outstanding OTC contracts grew from \$72.1 trillion in 1998 to \$197 trillion in 2003. The numbers on turnover show that besides the size of the markets, trading activity has also increased. Bloomberg reports a similar growth in European and Asian markets. The numbers reported above confirm the spectacular growth in liquidity and trading activity in derivative markets during the last two decades.

The increased importance of derivatives in financial management is the main motivation for this thesis. A strong emphasis is thereby placed on the information revealed by exchange traded plain vanilla index option prices. Although numerous papers have appeared in the financial and econometrics literature utilizing the information of option prices and returns on the option's underlying asset, there are still a number of questions which remain unanswered. This thesis fills some of these gaps that still exist in the current financial literature.

Financial modeling deals with two issues concerning risky assets: pricing and hedging. In order to calculate the model price of a derivative security three theoretical concepts are of crucial importance. These are (1) the real-world dynamics of the derivative's underlying security, (2) the pricing kernel, and (3) the risk-neutral dynamics of the derivative's underlying security. The pricing kernel consists of the risk premia of the systematic risks that are implied by the underlying economy and the attitude towards risk of the representative investor in that economy. Financial theory shows that in equilibrium two of three aforementioned concepts imply the remaining third concept. For instance, in the widely celebrated Black-Scholes model the assumption that both the real-world dynamics and the risk-neutral dynamics of the derivative's underlying security are geometric Brownian Motion, can be used to infer that the representative investor in the underlying economy has constant relative risk aversion.

In the option pricing literature numerous papers are motivated by the failure of the Black-Scholes model in describing all features in observed stock (index) returns and option prices. Within the current financial literature three different streams can be identified that deal with the failures of the Black-Scholes model. First, alternative models are proposed that relax the geometric Brownian Motion assumption of the option's underlying value.[1] These models mostly introduce additional systematic risk factors. In addition, assumptions are imposed on the risk premia that are required on these factors. The dynamics of the option's underlying value and the imposed risk premia together determine the risk-neutral dynamics and, therefore, theoretical option prices. To see whether the proposed model corresponds to the empirical regularities in the data, model-based option prices are compared to the option prices observed in practice. The second stream in the option pricing literature utilizes the direct relation between empirical option prices and the risk-neutral dynamics of the option's underlying asset. From observed plain vanilla option contracts so-called Arrow-Debreu securities are constructed. These securities define the risk-neutral probabilities of future values of the underlying. This stream of literature estimates the risk-neutral distribution of the underlying value nonparametrically while in the first stream a parametric specification of the risk-neutral distribution is imposed. The last stream of research uses the Black-Scholes model as the benchmark. This model is appealing to derivative practitioners because of its simplicity. However, instead of a constant volatility practitioners often use an option implied volatility as a model input. This implied volatility is assumed to be a (deterministic) function of the option's strike price and maturity. Research in this area aims to find empirical regularities in the dynamics of the option implied volatilities. This thesis contributes to the first and second stream of literature while the third stream falls outside the scope of this thesis.

Early finance theory is founded on the work of Markowitz (1952) who was the first

[1] Bates (2003) divides the first stream further into univariate models, stochastic volatility models, and jump models.

to analyze the concept of the risk-return trade off in a portfolio of stocks instead of picking the best stock from a set of available stocks. The idea was further developed and eventually resulted in the Capital Asset Pricing Model (CAPM) in which the single stock expected return is determined by the stock's correlation with the market. In this relatively simple model the only source of systematic risk, for which compensation is required, is market risk. Though in other respects conceptually different, the CAPM feature of one single systematic risk factor is shared with the Black-Scholes option pricing model. This feature allows for a testing framework that identifies market completeness and/or the redundancy of options. Current literature only provides a limited amount of papers that study the nature of option returns and the correlation between option returns and the option's underlying. More insight in these issues is of significant importance in the practical implementation of portfolio management (and hence risk management).

Motivated by the increasing liquidity in option markets, this thesis aims to use the information contained in these option prices to solve some remaining issues in the option pricing literature and to study the nature of option returns in more detail.

1.2 Overview and Contribution of Thesis

Chapter 2 gives an extensive overview of the literature that is related to empirical option prices. Option pricing literature has developed from the early Black and Scholes (1973) paper to the general class of affine jump-diffusions (Duffie, Pan, and Singleton (2000)) and the literature on option pricing under Lévy processes (see, e.g., Cont and Tankov (2004)). Attention is also paid to econometric issues, like parameter estimation with latent state variables, that come into play when more sophisticated models are applied. Difficulties that arise due to the incompleteness of markets are treated as well. Extraction of the risk-neutral distribution from observed option prices is based on the theoretical results in Breeden and Litzenberger (1978). Early empirical applications of the theoretical concept can be found in Shimko (1993) and Aït-Sahalia and Lo (1998). The comparison between model distributions and empirical distributions, the implications for empirical risk aversion, and the profitability of trading strategies based on the differences between these distributions can be found, among others, in Aït-Sahalia and Lo (2000) and Aït-Sahalia, Wang, and Yared (2001). Coval and Shumway (2001) is one of the few papers that studies the nature of option returns instead of looking for the best description of observed option prices. An extension of the empirical work in Coval and Shumway (2001) can be found in Driessen and Maenhout (2004).

In Chapter 3 the influence of overnight trading halts on option prices is considered. The chapter is based on Boes, Drost, and Werker (2004). The motivation for this chapter lies in the fact that traditional asset pricing models ignore trading halts in overnight periods while literature shows that distributional properties of asset returns in nontrading periods differ considerably from the asset returns during trading periods (see, for instance, French and Roll (1986)). Chapter 3 proposes an option pricing model that takes the nontrading overnight periods explicitly into account. More specifically, the change in the index between the closing one day and the opening the other day is modeled by means of a single jump. During the trading day, changes in the index price are described by a stochastic volatility model that includes random jumps. After a change of measure, theoretical option pricing formulas are derived. These prices are used to estimate the risk-neutral parameters by using S&P-500 index option prices. The main conclusion of Chapter 3 is that overnight jumps have a non-trivial impact on S&P-500 index option prices: the overnight jump component accounts for approximately one quarter of total jump variation. Moreover, an option pricing model including overnight jumps next to stochastic volatility and random jumps provides the best fit for SPX options.

Chapter 4 proposes a new methodology for the estimation of the joint risk-neutral density of excess index returns and future spot volatility by using plain vanilla options that are written on that index. The main advantage of this approach is that besides the risk-neutral distribution of returns, the risk-neutral density of the future instantaneous volatility can also be estimated. The risk-neutral density of future volatility can be used to price derivative products that have a payoff dependent on future spot volatility. The chapter originates from Boes, Drost, and Werker (2005). The current literature, initiated by Shimko (1993) and Aït-Sahalia and Lo (1998), is based on the insight provided in Banz and Miller (1978) and Breeden and Litzenberger (1978) that the risk-neutral density of returns is the second derivative of a call option pricing formula with respect to the strike price. Such an approach is not feasible in estimating the risk-neutral volatility distribution since there are no derivatives that have a payoff perfectly correlated with future spot volatility. Theoretically, the methodology in Chapter 4 is an application of the First Fundamental Theorem of Asset Pricing. The method is verified in a Heston (1993) world. The results show that the method is able to extract the joint density of excess returns and future spot volatility out of the Heston (1993) model option prices. A similar conclusion is drawn even if the true spot volatilities are replaced by the estimated EGARCH volatilities. Furthermore, the results confirm a right-shift in the future volatility distribution for higher initial volatility levels, but additionally reveal positive risk-neutral volatility skewness. Moreover, volatility skewness is more pronounced in low volatility periods. This is consistent with a large aversion towards unexpected positive volatility shocks. With respect to the risk-neutral return distribution, estimation results confirm overall negative skewness and show that conditional on decreasing volatility levels, the negative return skewness disappears. Concerning the risk-neutral dependence between return and volatility, the results show that this dependence is negative. Compared to parametric models, the outcomes imply that risk-neutral volatility of volatility is much smaller than predicted by the Heston (1993) model. This indicates the necessity of a jump component in the risk-neutral return process. Finally, the results indicate that the risk-neutral volatility of volatility cannot be described by a single diffusion risk-neutral volatility process.

Chapter 5 is a small note on parameter estimation in stochastic volatility models. Parameter estimation in these models is cumbersome since the instantaneous volatility appears in moment conditions while this variable is unobserved. Solutions that are proposed in the literature include for example, computer intensive simulation methods like Simulated Method of Moments or Efficient Method of Moments. Other methods construct a noisy estimator of the instantaneous volatility utilizing high frequency data and subsequently apply standard GMM techniques. Chapter 5 shows in a simulation study that taking unconditional moments instead of conditional moments results in a bad empirical identification of the parameters in the stochastic volatility process. Furthermore, results of a simulation study show that instruments based on GARCH parameter estimates lead to a significant reduction of the standard errors of the parameter estimates of the stochastic volatility model in comparison to the standard errors resulting from using traditional instruments like lagged squared returns. However, standard errors are still too large for the estimates to be of practical relevance.

Chapter 6 treats the mean-variance characteristics of option returns. As was pointed out in Coval and Shumway (2001), there is an enormous literature on the pricing of options under all kinds of advanced models, but there is a limited amount of papers available that treats option *returns* both theoretically and empirically. Chapter 6 provides a methodology, based on characteristic functions, that allows for the calculation of the conditional expectation and the conditional variance of returns on options that are not necessarily held to maturity. Using the same methodology, the covariance between the stock and the option and the covariance between options that have different strike prices can be calculated. The theoretical derivations are applied in the area of mean-variance investment analysis. The first application, based on Leland (1999), treats the issue of performance measurement of option based strategies under mean-variance preferences. Leland (1999) argues that under the assumption of perfect markets and independently and identically distributed returns on the market portfolio, CAPM β is an invalid measure of risk and CAPM α an inappropriate performance measure for option based strategies. The results of Chapter 6 show that these conclusions still hold after the assumption of independently and identically distributed returns is relaxed. However, if only market risk is priced, CAPM α can be used as a performance measure for returns on delta-hedged straddles. The second application compares optimal asset allocation for mean-variance investors and power utility investors in a setting where investors have access to the derivatives markets. Mean-variance investors optimally hold short straddle positions when the volatility risk premium is negative. In this way, mean-variance investors earn the risk premium on stochastic volatility. In case of a crash risk premium mean-variance investors optimally take short positions in out-of-the-money puts if the compensation for crash risk is sufficiently high.

Chapter 7 summarizes the thesis and provides some directions for future research.

CHAPTER 2

Literature Overview

2.1 Option Pricing

The introduction illustrated that derivative markets have expanded spectacularly in the past couple of decades. This growth is not only recognized in derivatives markets but also in the academic derivatives literature. This chapter gives a detailed overview of the progress that has been made in modeling observed asset returns, option prices, and option returns.

2.1.1 Price processes and theoretical option pricing

This section reviews the literature on modeling stock prices and option prices using continuous time stochastic processes. Bachelier (1900) is one of the first studies that applied stochastic process theory to financial markets. The paper proposes to model stock prices as a Brownian Motion with drift. A fundamental property of these type of processes is that the processes become negative with probability one. This drawback was corrected in Samuelson (1965) by modeling stock prices as geometric Brownian Motion. Black and Scholes (1973) derived theoretical option prices under the geometric Brownian Motion assumption. In the seventies and early eighties the model seemed to provide a reasonable description of both daily stock index returns and observed option prices. Monday October 19, 1987, also called Black Monday, had a huge impact on

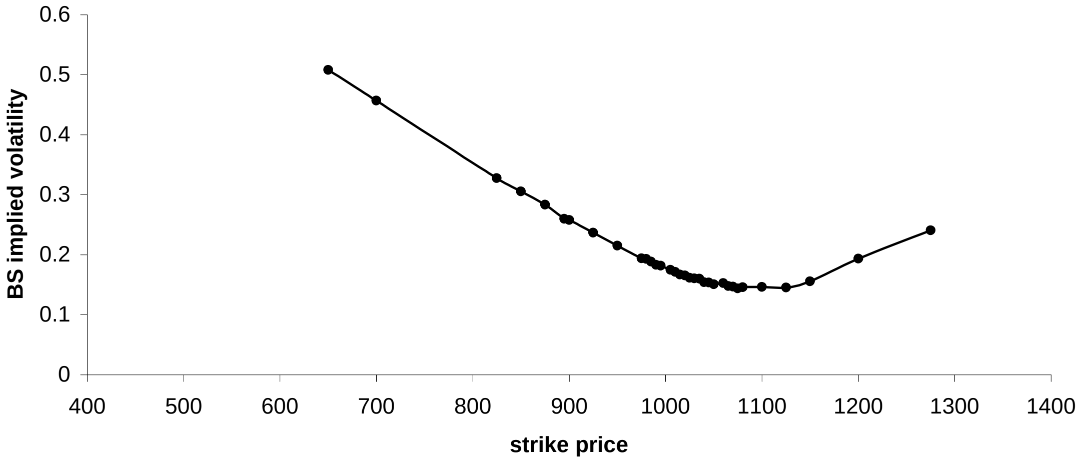


Figure 2.1: Black-Scholes implied volatility skew using S&P-500 index options with one month to maturity on October 22, 2003. The closing index value on this day was 1032.

financial markets. On this day, the Dow Jones Industrial Average lost 22.6% and the S&P-500 index dropped 20.5%. The effect of this event became clearly visible in option markets. Pictures in Bates (2000) show that before the stock market crash the Black-Scholes implied volatility was approximately constant across strike price. However, after the crash a pronounced implied volatility skew appeared in option markets. An example of a volatility skew on the S&P-500 is shown in Figure 2.1. This figure is based on put option data from October 22, 2003. The non-constancy of implied volatility and the changing shape of the implied volatility smile across maturities and the dynamics of the smile through time is in contrast to the assumptions of the Black-Scholes model. The average difference between the at-the-money implied volatility of an option and the realized volatility of the option's underlying asset, e.g., a stock index (see Figure 2.2) provides another argument against the Black-Scholes assumptions. In addition, asset return data reveal that historical volatility is non-constant: volatilities cluster and short horizon stock (index) returns exhibit heavy tails. These empirical observations contradict the Black-Scholes assumptions of constant volatility and normally distributed asset returns.

Academic literature responds and continues to respond by proposing alternatives to the Black-Scholes model of stock prices ranging from the Heston (1993) stochastic volatility model to the more complicated Lévy jump models as for example in Carr, Geman, Madan, and Yor (2003). Self contained references in the latter area are Schoutens (2003) and Cont and Tankov (2004). In the Black-Scholes world the defined market is complete

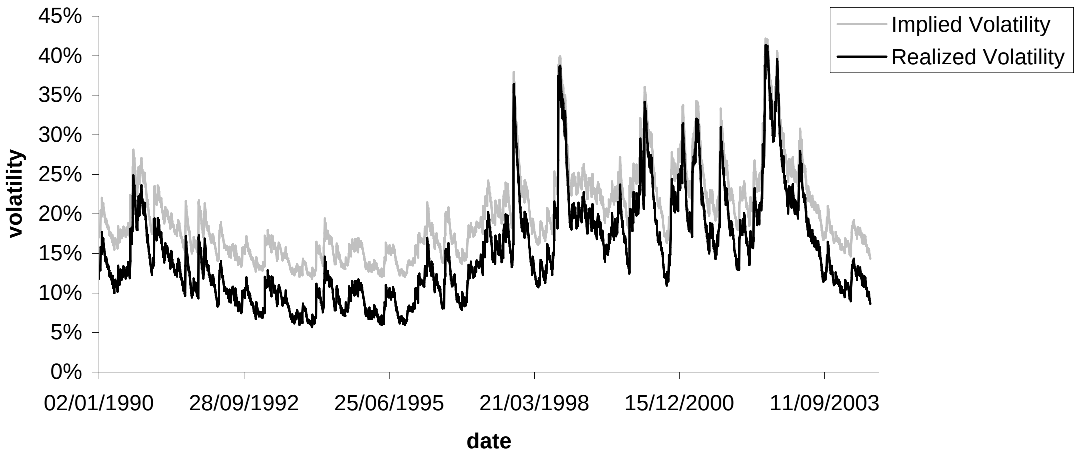


Figure 2.2: Black-Scholes at-the-money implied volatilities and one-month realized volatilities using S&P-500 data over the period January 1990 to July 2004.

with respect to the risk free asset and the stock. This feature disappears in the stochastic volatility and jump models. In stochastic volatility models an extra asset, e.g. a call option needs to be introduced in order to restore the completeness of the market. In most jump models there are infinite sources of uncertainty and hence an infinite number of assets is necessary for completeness. This and other issues show that complicated models induce an increasing theoretical and numerical complexity.

2.1.2 Stochastic volatility models

Comparing the high standard deviation of asset returns in 2002 and 2003 to the extreme low volatility of asset returns in 2004 leads to the conclusion that variability of asset returns changes stochastically over time. One of the well known classes of continuous time models allowing for changing volatilities is the class of bivariate diffusion models. The stochastic differential equations of the stock price and volatility in this class of models are usually of the type

$$\begin{aligned} \frac{\mathrm{d}S_t}{S_t} &= \mu\left(t, S_t, \sigma_t\right)\mathrm{d}t + \sigma_t \mathrm{d}W_t^S, \\ \mathrm{d}\sigma_t &= a(t, S_t, \sigma_t)\mathrm{d}t + b\left(t, S_t, \sigma_t\right)\mathrm{d}W_t^V, \end{aligned} \tag{2.1}$$

where W^S and W^V are Brownian Motions. The Brownian Motions are allowed to have a constant correlation coefficient ρ. Note that the introduction of the stochastic volatility process implies that the price process is not a Markov process, i.e. the probability distribution of the future stock price depends not only on the current stock price but also on the current level of volatility. The models of Hull and White (1987), Stein and Stein (1991), and Heston (1993) belong to this general class of stochastic volatility models.[1] These models differ in the specification of the volatility process and in the assumption about the correlation between the Brownian Motions. In the Heston (1993) model the assumed variance process is based on the Cox, Ingersoll, and Ross (1985) interest rate process

$$\mathrm{d}\sigma_t^2 = -\kappa\left(\sigma_t^2 - \sigma^2\right)\mathrm{d}t + \sigma_\sigma \sigma_t \mathrm{d}W_t^V,$$

with κ as the reversion speed of the process to the long run mean variance σ^2. Application of Ito's Lemma yields the corresponding volatility process

$$\mathrm{d}\sigma_t = -\frac{1}{2}\kappa\left(\sigma_t - \left(\frac{\sigma^2}{\sigma_t} - \frac{\sigma_\sigma^2}{4\kappa\sigma_t}\right)\right)\mathrm{d}t + \frac{1}{2}\sigma_\sigma \mathrm{d}W_t^V,$$

which clearly fits in specification (2.1). The difference between the Hull and White (1987) and Heston (1993) model lies in the specification of the variance process. In the Hull and White (1987) model, for instance, the volatility of volatility is a linear function of the instantaneous variance whilst in the Heston (1993) model this function is linear in the instantaneous *volatility*. The Stein and Stein (1991) model differs from the Heston (1993) model in the sense that the Stein and Stein (1991) model imposes a zero correlation between the Brownian Motions while Heston (1993) allows for a non-zero correlation coefficient.

For the purpose of option pricing, bivariate diffusions are a convenient class of processes since partial differential equation (PDE) methods can be utilized to calculate option prices. Analogous to Merton (1973), Heston (1993) reports the no-arbitrage PDE for the value of any asset that depends both on the stochastic underlying value and the stochastic variance. Under the assumption that the variance risk premium is linear to the instantaneous variance, Heston (1993) determines closed form solutions for option prices which can be obtained by Fourier inversion.[2]

[1] Other examples include Johnson and Shanno (1987), Scott (1987), and Wiggins (1987).

[2] Hull and White (1987), Johnson and Shanno (1987), Scott (1987), and Wiggins (1987) use other methods to calculate theoretical option prices. For instance, in Scott (1987) a Monte Carlo technique is employed and Wiggins (1987) uses a higher-dimensional finite-difference approach. In these papers option prices are determined under the assumption that volatility risk is not priced. The Fourier inversion technique is, among others, also applied in Stein and Stein (1991), Bakshi, Cao, and Chen (1997), Scott (1997), Bakshi and Madan (2000), Bates (2000), Duffie, Pan, and Singleton (2000), and Dai and Singleton (2000).

An alternative approach for calculating option prices is the risk-neutral valuation method. This method is based on the First Fundamental Theorem of Asset Pricing which states that the absence of arbitrage opportunities is equivalent to the existence of an equivalent martingale measure.[3] More formally: a market model is arbitrage free if and only if there exists a probability measure $\mathbb{Q}$ equivalent to the real world probability measure $\mathbb{P}$ such that all discounted asset prices are martingales. The mathematical tool that is used to change the measure is Girsanov's theorem. The mechanism is easily demonstrated in the Heston (1993) model

$$\frac{\mathrm{d}S_t}{S_t} = \mu \mathrm{d}t + \sigma_t \mathrm{d}W_t^S, \tag{2.2}$$

$$\mathrm{d}\sigma_t^2 = -\kappa\left(\sigma_t^2 - \sigma^2\right)\mathrm{d}t + \sigma_\sigma \sigma_t \left(\rho \mathrm{d}W_t^S + \sqrt{1-\rho^2}\mathrm{d}W_t^V\right), \tag{2.3}$$

where W^S and W^V are independent Brownian Motions under the probability measure $\mathbb{P}$. In comparison to the original Heston (1993) model, (2.3) is slightly reformulated with two independent Brownian Motions. The model can be rewritten as

$$\begin{aligned} \frac{\mathrm{d}S_t}{S_t} &= r\mathrm{d}t + \sigma_t\left\{\mathrm{d}W_t^S + \left(\frac{\mu - r}{\sigma_t}\right)\mathrm{d}t\right\}, \\ \mathrm{d}\sigma_t^2 &= -\left(\kappa + \eta^V\right)\left(\sigma_t^2 - \frac{\kappa\sigma^2}{\kappa+\eta^V}\right)\mathrm{d}t + \sigma_\sigma\sigma_t\left(\rho\left\{\mathrm{d}W_t^S + \left(\frac{\mu-r}{\sigma_t}\right)\mathrm{d}t\right\}\right. \\ &\quad \left. + \sqrt{1-\rho^2}\left\{\mathrm{d}W_t^V + \frac{1}{\sqrt{1-\rho^2}}\left(\frac{\eta^V\sigma_t}{\sigma_\sigma} - \left(\frac{\mu-r}{\sigma_t}\right)\rho\right)\mathrm{d}t\right\}\right). \end{aligned}$$

Applying Girsanov's theorem to this set of equations yields

$$\begin{aligned} \frac{\mathrm{d}S_t}{S_t} &= r\mathrm{d}t + \sigma_t \mathrm{d}\tilde{W}_t^S, \\ \mathrm{d}\sigma_t^2 &= -\left(\kappa+\eta^V\right)\left(\sigma_t^2 - \frac{\kappa\sigma^2}{\kappa+\eta^V}\right)\mathrm{d}t + \sigma_\sigma\sigma_t\left(\rho\mathrm{d}\tilde{W}_t^S + \sqrt{1-\rho^2}\mathrm{d}\tilde{W}_t^V\right), \end{aligned}$$

where $\tilde{W}^S$ and $\tilde{W}^V$ are independent Brownian Motions under a probability measure $\mathbb{Q}$ equivalent to the probability measure $\mathbb{P}$. This example clearly demonstrates that the market is not complete with respect to the stock and the bond. The Second Fundamental Theorem of Asset Pricing states that a market is complete if and only if there is a unique

[3] Note that this is true in discrete time models with finitely many states. In continuous time, the existence of an equivalent martingale measure still implies absence of arbitrage but the converse does not hold in general. No-arbitrage is not sufficiently strong to imply the existence of an equivalent martingale measure and should be replaced by the stronger concept of no free lunch with vanishing risk, see Delbaen and Schachermayer (1994) and Delbaen and Schachermayer (1998).

equivalent martingale measure. In model specification (2.3), there exists another risk-neutral process

$$\begin{aligned}\frac{\mathrm{d}S_t}{S_t} &= r\mathrm{d}t + \sigma_t \mathrm{d}\overline{W}_t^S, \\ \mathrm{d}\sigma_t^2 &= -\left(\kappa+\eta^V\right)\left(\sigma_t^2 - \frac{\left(\kappa\sigma^2 - \sigma_\sigma\left(\mu - r\right)\rho\right)}{\kappa+\eta^V}\right)\mathrm{d}t + \sigma_\sigma\sigma_t\left(\rho \mathrm{d}\overline{W}_t^S + \sqrt{1-\rho^2}\mathrm{d}\overline{W}_t^V\right),\end{aligned}$$

where $\overline{W}^S$ and $\overline{W}^V$ are independent Brownian Motions under a probability measure $\mathbb{Q}^*$ equivalent to the probability measure $\mathbb{P}$. This model is different from the Heston (1993) model and therefore implies different model option prices. However, the prices in this model are still arbitrage free. The notion of incompleteness becomes more important in jump models where markets are usually incomplete with respect to any finite number of traded assets.

A final pricing method is based on the pricing kernel process. The pricing kernel equivalent of the First Fundamental Theorem of Asset Pricing is that absence of arbitrage is equivalent to the existence of a nonnegative pricing kernel. For a given nonnegative pricing kernel π, time t no-arbitrage value X_t of a derivative with payoff X_T at time T is

$$X_t = \mathrm{E}_t^{\mathrm{P}}\left(X_T \frac{\pi_T}{\pi_t}\right).$$

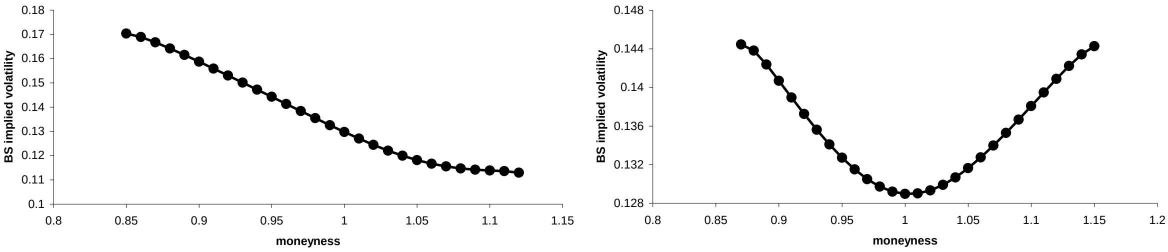
In bivariate diffusion models with independent Brownian Motions, the process π is described as (assuming a constant risk free interest rate r)

$$\mathrm{d}\pi_t = -r\pi_t \mathrm{d}t - \zeta_t^S \pi_t \mathrm{d}W_t^S - \zeta_t^V \pi_t \mathrm{d}W_t^V,$$

with ζ_t^S and ζ_t^V as the market prices of market risk and variance risk, respectively.

The reason to treat the Heston (1993) model extensively is that the model is empirically reasonable and analytically tractable. The model, for instance, allows for a non-zero correlation between the Brownian Motions which is important in explaining observed implied volatility patterns. Furthermore, Duffie and Kan (1996) shows that the model belongs to the general class of affine jump-diffusions. Figure 2.3 shows that a negative correlation between the Brownian Motions leads to a downward sloping implied volatility skew, while $\rho = 0$ implies a symmetric smile. Since in option markets both volatility skews and smiles are observed, flexibility in the correlation parameter is called for.

Another important issue in bivariate diffusion models is the volatility risk premium parameter η^V. A negative value of this parameter leads to a higher long run mean of the



(a) Negative correlation (b) Zero correlation

Figure 2.3: Black-Scholes implied volatilities resulting from option prices calculated from the Heston model. Model prices for a negative and zero correlation between the driving Brownian Motions are determined for options that have one remaining month to maturity.

risk-neutral variance process than the long run mean in case the risk in variance is not priced ($\eta^V = 0$). Hence, model option prices increase when the volatility risk premium parameter decreases. This is intuitively clear since options provide a desired protection against high volatility states.

There are several ways to extract information on the volatility risk premium from empirical data. First, a structural model implies that the theoretical option prices are a function of the risk-neutral parameters that contain the risk prices. Option prices can be used to choose model parameters in such a way that some criterion on the option pricing errors is minimized. Bakshi, Cao, and Chen (1997) chooses the risk-neutral parameters by using option data only. Since the objective parameters are not separately identified, this approach does not give any information about the sign of the risk premium. In Chernov and Ghysels (2000) and Pan (2002) both option prices and stock (index) returns are used to estimate the parameters and therefore reported estimates include both the objective parameters and risk-neutral parameters. These papers report estimates that imply a negative volatility risk premium in the standard Heston (1993) model, i.e. option prices used in these studies are best described by a higher long run mean in the variance process. However, the outcomes strongly depend on the model specification. Hence, the results should be treated with care.

Secondly, option positions can be constructed such that these positions are (instantaneous) delta-neutral. The returns generated from these strategies are related to the variance of the underlying value. Bakshi and Kapadia (2003) considers a dynamic strategy in which equity call options are delta hedged. The paper derives a theoretical relationship between the variance risk premium and the gains on option portfolios. Empirical results reveal that the variance risk premium is negative. In Coval and Shumway (2001) a similar approach is taken but instead of considering single call options the paper uses delta-neutral at-the-money straddles. These turn out to have a payoff directly related to the variance of the underlying asset. That paper also suggests that the most plausible explanation for the results is a negative volatility risk premium. Finally, Bondarenko (2004) chooses an approach that is completely model free and also comes to the conclusion that the variance risk premium is negative and large in magnitude.

Chernov (2002) gives yet another argument for a negative volatility risk premium. The paper derives, in a stochastic volatility framework, an approximate relation between the expected integrated volatility, the Black-Scholes implied volatility, and the covariance between the stochastic discount factor and the integrated volatility

$$\mathrm{E}_t^{\mathrm{P}}\left(\frac{1}{h}\int_t^{t+h}\sigma_u^2\mathrm{d}u\right)\approx\sigma_{t,t+h}^{BS}-e^{-rh}\mathrm{Cov}_t\left(\frac{\pi_{t+h}}{\pi_t},\frac{1}{h}\int_t^{t+h}\sigma_u^2\mathrm{d}u\right),$$

with $\sigma_{t,t+h}^{BS}$, the time t Black-Scholes implied volatility from an at-the-money option that matures at time $t+h$. The empirical observed positive difference between the at-the-money implied volatility and the realized volatility (see Figure 2.2) is in the stochastic volatility setting explained by a positive covariance between future variance and the pricing kernel. A positive covariance corresponds to a negative volatility risk premium in the Heston (1993) stochastic volatility model.

2.1.3 Jump processes

The addition of jumps to a continuous time asset price process was first motivated in Merton (1976). The paper introduces a model in which the continuous part (modeled by a constant variance geometric Brownian Motion) represents the normal vibrations of the stock prices due to, for instance, changes in the economic outlook. The jump part of the model describes the abnormal vibrations of the stock, i.e. the arrival of important new information about the stock that causes a significant change to the stock price value. Since then the empirical significance of jumps and the implications for option pricing have been studied extensively in the finance and econometrics literature. Nowadays, two main strands of literature can be identified. First, models that are based on the class of affine jump-diffusion models as described in Duffie, Pan, and Singleton (2000). One of the characteristics of these models is that there are finitely many jumps in every time interval. Furthermore, the distribution of jump sizes is assumed to be known which simplifies the understanding of the dynamic model structure. The second stream of literature uses more general Lévy processes as building blocks of the stock price process. The main distinction with the former class of models is that there are possibly an infinite number of jumps in every time interval. These models are often called infinite activity models.

The expression affine jump-diffusion models stems from the affine dependence of the drift vector, the instantaneous covariance matrix, and the jump intensities on the state vector. Earlier papers like Heston (1993) for derivative pricing and Cox, Ingersoll, and Ross (1985) for interest rates already present models that fit into the class of affine jump-diffusion models. In Duffie, Pan, and Singleton (2000) the affine jump-diffusion state-process model is presented as follows. Assume that X is a stochastic process in some state space $D \subset \mathbb{R}^n$ following the dynamics

$$\mathrm{d}X_t = \mu\left(X_t\right)\mathrm{d}t + \sigma(X_t)\mathrm{d}W_t + \mathrm{d}Z_t,$$

with W a standard Brownian Motion in $\mathbb{R}^n$. The function $\mu\left(\cdot\right) : D \rightarrow \mathbb{R}^n$ represents the time trend of the process and the function $\sigma\left(\cdot\right) : D \rightarrow \mathbb{R}^{n\times n}$ is the diffusion function. The process Z is a pure jump process and is assumed to follow a Poisson process with time varying intensity $\lambda\left(\cdot\right)$. The jump sizes are independent of all other random variables at the time the jump occurs. The functions $\mu, \sigma\sigma^T, \lambda$ and the discount rate function $R : D \rightarrow \mathbb{R}$ are assumed to be affine in the state variables in X

$$\begin{aligned}
\mu(x) &= K_0 + K_1 x, \text{ for } K = (K_0, K_1) \in \mathbb{R}^n \times \mathbb{R}^{n\times n}, \\
\left(\sigma\left(x\right)\sigma\left(x\right)^T\right)_{ij} &= \left(H_0\right)_{ij} + \left(H_1\right)_{ij} x, \text{ for } H = (H_0, H_1) \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times n\times n}, \\
\lambda(x) &= l_0 + l_1 x, \text{ for } l = (l_0, l_1) \in \mathbb{R} \times \mathbb{R}^n, \\
R(x) &= \rho_0 + \rho_1 x, \text{ for } \rho = (\rho_0, \rho_1) \in \mathbb{R} \times \mathbb{R}^n.
\end{aligned}$$

Together with the jump size distribution, the parameters (K, H, l, θ) determine the distribution of X. Consider now a function $\psi\left(\cdot\right) : \mathbb{C}^n \times D \times \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{C}$ defined by

$$\psi\left(u, X_t, t, T\right) = \mathrm{E}\left(\exp\left(-\int_t^T R(X_s)\mathrm{d}s\right) e^{uX_T} \middle| \mathcal{F}_t\right), \tag{2.4}$$

for $t \leq T$. In this formula $\mathcal{F}_t$ denotes all information available at time t. The discounting factor makes $\psi\left(\cdot\right)$ different from the standard conditional characteristic function. Duffie, Pan, and Singleton (2000) shows that under technical regularity conditions (which are omitted here, see for details Duffie, Pan, and Singleton (2000))

$$\psi\left(u, X_t, t, T\right) = e^{\alpha(t)+\beta(t)x},$$

where β and α satisfy the complex-valued ordinary differential equations

$$\begin{aligned}\dot{\beta}(t) &= \rho_1 - K_1^T\beta(t) - \frac{1}{2}\beta(t)^T H(t)\beta(t) - l_1\left(\theta\left(\beta(t)\right) - 1\right),\\ \dot{\alpha}(t) &= \rho_0 - K_0^T\beta(t) - \frac{1}{2}\beta(t)^T H(t)\beta(t) - l_0\left(\theta\left(\beta(t)\right) - 1\right),\end{aligned}$$

with boundary conditions $\beta(T) = u$ and $\alpha(T) = 0$. Function $\psi(\cdot)$ proves to be useful for option pricing. Duffie, Pan, and Singleton (2000) first derives the expected present value of a call option's payoff $C(d, c, T, \chi)$ with maturity T, i.e. for each given $(d, c, T) \in \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}_+$

$$\begin{aligned}C(d,c,T,\chi) &= \mathrm{E}\left(\exp\left(-\int_t^T R(X_s)\mathrm{d}s\right)\left(e^{d\cdot X_T} - c\right)^+ \middle| \mathcal{F}_t\right),\\ &= \mathrm{E}\left(\exp\left(-\int_t^T R(X_s)\mathrm{d}s\right)\left(e^{d\cdot X_T} - c\right)^+ 1_{d\cdot X_T \geq \log c} \middle| \mathcal{F}_t\right),\\ &= G_{d,-d}\left(-\log c; X_0, T, \chi\right) - cG_{0,-d}\left(-\log c; X_0, T, \chi\right), \qquad (2.5)\end{aligned}$$

where χ contains all model parameters and, under some condition, $G_{a,b} = (\cdot; x, T, \chi) : \mathbb{R} \to \mathbb{R}_+$

$$\begin{aligned}G_{a,b}\left(y; X_0, T, \chi\right) &= \frac{\psi\left(a, X_0, 0, T\right)}{2}\\ &\quad -\frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\left[\psi\left(a + ivb, X_0, 0, T\right)e^{-ivy}\right]}{v}dv, \qquad (2.6)\end{aligned}$$

with $\mathrm{Im}(c)$ the imaginary part of the complex number c. If there is a jump component in the class of affine jump-diffusion models the market model is incomplete with respect to any finite number of traded assets due to the infinite number of uncertainties following from the jump part. Consequently, there exists an infinite number of equivalent martingale measures that give no-arbitrage prices. If the equivalent martingale measure is chosen such that the structure of the model is preserved (i.e., the state-process model still fits in the class of affine jump-diffusion under this chosen equivalent martingale measure) then (2.5) and (2.6) can be applied, using a given $\chi^{\mathbb{Q}}$ (vector containing risk-neutral model parameters) instead of χ, to determine the time 0 price of a call option with strike price c and maturity T.

For example, suppose that in a Heston (1993) world (see previous section) the value of a call option on S with strike price K and maturity T needs to be calculated. The Heston (1993) model follows from taking $n = 2$, $X = (\log S, \sigma^2)$, $d = (1, 0)$, and $c = K$ in the more general affine jump-diffusion model. Heston (1993) proves that the theoretical option price (using constant interest rates and notation as in previous section)

$$C = S_0 P_1 - Ke^{-rT} P_2,$$

with

$$
\begin{aligned}
P_1 &= \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \mathrm{Re} \left[\frac{\varphi_0 \left(\phi_S - i \right) e^{-i\phi_S \log K}}{i\phi_S \varphi_0 \left(-i \right)} \right] \mathrm{d}\phi_S, \\
P_2 &= \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \mathrm{Re} \left[\frac{\varphi_0 \left(\phi_S \right) e^{-i\phi_S \log K}}{i\phi_S} \right] \mathrm{d}\phi_S,
\end{aligned}
$$

and

$$
\begin{aligned}
\varphi_0 \left(\phi_S \right) &= \mathrm{E}_0 \left\{ \exp \left(i\phi_S \log S_T \right) \right\}, \\
&= \exp \left(C \left(T; \phi_S \right) + D \left(T; \phi_S \right) \sigma_0^2 + i\phi_S \log S_0 \right),
\end{aligned}
$$

where

$$
\begin{aligned}
C \left(T; \phi_S \right) &= ri\phi_S T + \frac{\kappa \sigma^2}{\sigma_\sigma^2} \left\{ \left(\kappa - \rho \sigma_\sigma i\phi_S + d \right) T - 2 \log \left(\frac{1 - g e^{dT}}{1 - g} \right) \right\}, \\
D \left(T; \phi_S \right) &= \frac{\kappa - \rho \sigma_\sigma i\phi_S + d}{\sigma_\sigma^2} \frac{1 - e^{dT}}{1 - g e^{dT}},
\end{aligned}
$$

and

$$
\begin{aligned}
g &= \frac{\kappa - \rho \sigma_\sigma i\phi_S + d}{\kappa - \rho \sigma_\sigma i\phi_S - d}, \\
d &= \sqrt{\left(\rho \sigma_\sigma i\phi_S - \kappa \right)^2 + \sigma_\sigma^2 \left(i\phi_S + \phi_S^2 \right)}.
\end{aligned}
$$

Application of affine jump-diffusion models to asset return data (mostly S&P-500 index returns) shows that there is a consensus about the added value of jumps under the objective probability measure. Andersen, Benzoni, and Lund (2002), Pan (2002), Eraker (2004), and Eraker, Johannes, and Polson (2003) report benefits of adding jumps in returns to the Heston (1993) stochastic volatility model. Although these studies use different data periods and estimation techniques (see next section), conclusions about the impact of jumps in the return process are similar. This is considered as strong evidence for the presence of jumps in the S&P-500 index price process.

Adding a jump part to the asset return process is not sufficient to capture all empirical regularities in the data. For instance, Jones (2003) finds a higher volatility of volatility during more volatile periods in the stock market. Pan (2002) also reasons that the volatility of volatility might be stochastic. One way to add more flexibility to the variance process is by means of a jump process. However, empirical evidence for the presence of jumps in the volatility process is mixed. Eraker, Johannes, and Polson (2003) and Eraker (2004) find strong evidence for jumps in the volatility process while the evidence in Chernov, Gallant, Ghysels, and Tauchen (2003) and Pan (2002) is less clear or completely absent. In Broadie, Chernov, and Johannes (2004) the positive skewness and excess kurtosis in model implied variance increment point towards jumps in volatility under the objective probability measure.

Conclusions about the importance of jumps in asset returns and volatility for the fit of option prices is mixed as well. Bakshi, Cao, and Chen (1997) and Broadie, Chernov, and Johannes (2004) find significant improvement in the pricing of options by adding jumps in the return process of a stochastic volatility model. On the other hand, Bates (2000), Pan (2002), and Eraker (2004) find only minor benefits. Furthermore, Eraker (2004) reports that the addition of jumps in volatility does not lead to an improvement in fit while Broadie, Chernov, and Johannes (2004) finds a relative improvement of almost 20% due to jumps in volatility. Finally, there are also some contrasting results reported on the several risk premia. As mentioned in the previous section, Coval and Shumway (2001), Chernov and Ghysels (2000), and Bakshi and Kapadia (2003) provide strong empirical indications of a significant negative volatility risk premium. However, in Broadie, Chernov, and Johannes (2004) the diffusive volatility risk premium is insignificant. This is explained by the additional jump component in the volatility process. In general, empirical results indicate that the expected value of future instantaneous variance is higher under the risk-neutral measure than under the objective measure. In models that include jumps in the asset return process, Pan (2002) finds a significant jump risk premium and an insignificant volatility risk premium. This in contrast to Eraker (2004) that reports empirical evidence on a significant volatility risk premium and an insignificant jump risk premium. The lack of consensus is mainly due to the different option data that are used in the various studies. Most papers use data over a small sample period or only use a small part of the information contained in the data. The next section shows that an estimation algorithm that fully exploits the information in return data and option data is still unavailable.

The second category of jump models is called infinite activity models. As mentioned before, these models assume an infinite number of jumps in each time interval. The commonly assumed market models consist of one risk free bond and a risky asset S which is modeled by

$$S_t = S_0 e^{X_t}, \tag{2.7}$$

where $X = \{X_t, t \geq 0\}$ is a Lévy process. The motivation for using a Lévy process is that in contrast to Brownian Motion the Lévy process is allowed to have discontinuities. Brownian Motion is the only continuous Lévy process. In order to ensure the independent and stationary increments assumption of a stochastic process, the time t

distribution of X has to be infinitely divisible. These type of processes are, in general, called Lévy processes, after Paul Lévy. An excellent self-contained treatment of the application of Lévy processes in financial modeling can be found in Cont and Tankov (2004). Schoutens (2003) offers a more applied overview of Lévy processes in finance. Technical details are omitted since the application of Lévy processes is beyond the scope of this thesis. The most well-known choices for the process X are the symmetric variance gamma process (Madan and Seneta (1990)), the general variance gamma process (Madan, Carr, and Chang (1998)), the normal inverse Gaussian process (Bandorff-Nielsen (1997) and Bandorff-Nielsen (1998)), the CGMY process (Carr, Geman, Madan, and Yor (2003)), and the generalized hyperbolic Lévy processes (Bandorff-Nielsen (1978), Eberlein (2001), and Raible (1998)). The main drawback of these Lévy models is that stochastically changing volatility is not allowed for. In the Lévy literature two methods are proposed to correct for this. First, like in the Heston (1993) model a stochastic volatility process is added to the asset return process. The basic selected process is of the Ornstein-Uhlenbeck type where the process is driven by a positive Lévy process. References that illustrate this method include, among others, Bandorff-Nielsen and Shephard (2001) and Bandorff-Nielsen and Shephard (2003). The second method is to apply a stochastic time change to the Lévy process X. The stochastic time clock is usually modeled by an integrated CIR process or an Ornstein-Uhlenbeck. The main reference for this second method is Carr, Geman, Madan, and Yor (2003).

In option pricing the same problems arise as for the affine jump-diffusion models. Unless the process X is Brownian Motion, the Lévy market model is incomplete. This, again, means that the equivalent martingale measure is not unique, i.e. a wide range of no-arbitrage option prices can be calculated. One way to construct an equivalent martingale measure in the exponential Lévy model (2.7) is to use the so-called Esscher transform. In short, the method works as follows. Suppose that $f_t(x)$ is the conditional objective density of random variable X_t. Then a new density $f_t^\theta(x)$ can be defined as

$$f_t^\theta(x) = \frac{\exp(\theta x)\, f_t(x)}{\int_{-\infty}^{\infty} \exp(\theta y) f_t(y) \mathrm{d}y},$$

for some real number $\theta \in \left\{\theta \in \mathbb{R} | \int_{-\infty}^{\infty} \exp(\theta y) f_t(y) \mathrm{d}y < \infty\right\}$. The parameter θ is chosen in such a way that the discounted asset price is a martingale. Gerber and Shiu (1996) provides an economic argument for choosing the Esscher transform martingale measure. Another possible solution is to add an extra drift parameter to the Lévy process. After estimating the model parameters, this drift parameter can be adjusted in such a way that the discounted stock price process becomes a martingale. In case the Esscher transform equivalent measure is used, the time t call price $C_t(T,K)$ with strike price K and maturity T is given by

$$C_t(T,K) = \int_{\log \frac{K}{S_t}}^{\infty} f_T^{(\theta^*+1)}(x)\mathrm{d}x - e^{-r(T-t)}K \int_{\log \frac{K}{S_t}}^{\infty} f_T^{\theta^*}(x)\mathrm{d}x,$$

where θ^* is the choice of θ which makes the discounted asset price a martingale.

2.1.4 Econometric issues

The issue of parameter estimation in continuous time models is intensively studied in the financial econometrics literature. Stock (index) return data can only be utilized to identify the parameters of the objective probability distribution. In the Black-Scholes model parameter estimation is relatively simple. Standard maximum likelihood is applied to the data to get consistent and efficient estimates of the drift parameter and the variance parameter. The extension of the Black-Scholes to a stochastic volatility model creates difficulties for parameter estimation. Namely, (conditional) probability distributions and moment conditions depend on the unobservable volatility factor. The consequence is that maximum likelihood estimation becomes computationally infeasible. As a result, several methods have been proposed in the literature that deal with the problem of latent factors. The methods that only employ stock (index) return data are roughly divided in simulation based methods, characteristic function based methods, and Bayesian Markov Chain Monte Carlo methods.

Simulation methods are feasible if simulation of the model processes is relatively easy. The basic idea of the simulated method of moments procedure (Duffie and Singleton (1993)) is that sample moments are matched with simulated moments. These simulated moments are determined using a simulated time series of the assumed underlying stochastic processes. The efficient method of moments (Gallant and Tauchen (1996)) is an extension of the simulated method of moments by generating moment conditions from an auxiliary model that approximates the distribution of the observed data. Under certain conditions the parameter estimates obtained by applying efficient method of moments are as efficient as maximum likelihood parameter estimates. The resulting estimator is closely related to the indirect inference estimator proposed by Gourieroux, Monfort, and Renault (1993). The efficient method of moments is, among others, applied in Andersen, Benzoni, and Lund (2002) for affine jump-diffusion models and in Chernov and Ghysels (2000) for the Heston stochastic volatility model. Another simulation based method is simulated maximum likelihood as described in Brandt and Santa-Clara (2002).

In this approach the likelihood function is evaluated in a consistent approximation of the transition density of the diffusion. Applications of the method to models that contain both jumps and stochastic volatility can be found in Piazzesi (2000), Durham (2000), and Brandt and Santa-Clara (2002).

In the two classes of models discussed in the previous section the (joint) characteristic function of the random state variables is known in closed form. In Das (1996) and Bates (1996a) the characteristic function is used for parameter estimation in continuous time models. These papers employ inversion techniques to obtain the density function from the characteristic function. Because of the computational complexity of inversion, new estimation techniques were developed that utilized the characteristic function directly. Examples can be found in Singleton (2001), Jiang and Knight (2002), and Chacko and Viceira (2003). The difference between the methods in these papers lies in the treatment of latent variables. The methods integrate out the latent variable from the characteristic function in some sense and therefore become conditional only on the current value of the stock price.

The final class of methods discussed here are the Bayesian Markov Chain Monte Carlo (MCMC) methods. MCMC is based on the Hammersley-Clifford theorem which states that a joint distribution can be characterized by the complete set of conditional distributions. Relying on this result, MCMC generates samples from a given target distribution. In financial applications this means that the distribution of the state variables and the parameters are characterized by, first, the distribution of the state variables conditioned on the data and the parameters, and secondly on the distribution of the parameters given the state variables and the data. The method is successful because the conditional distributions are relatively easy to compute compared to the joint density. From a financial point of view, the main advantage of the MCMC methods is that both the model parameters and state variables are estimated. For instance, no additional filtering rule is necessary to obtain an estimate of instantaneous volatilities. Results in Jacquier, Polson, and Rossi (1994) and Andersen, Chung, and Sorensen (1999) show that MCMC outperforms (in terms of mean squared error) GMM, QMLE, and EMM. A practical application to S&P-500 returns using a model that allows for jumps in returns and volatility is found in Eraker, Johannes, and Polson (2003).

Efficiency of the parameter estimates could be improved by using the cross-section of option data in addition to return data. Chernov and Ghysels (2000) applies EMM to asset return data and at-the-money Black-Scholes implied volatilities. In Pan (2002) the implied state GMM methodology is introduced. This method uses at most two options to identify the risk premia in the model. Furthermore, given model parameters, option prices provide an estimate of the instantaneous volatility process. The implied volatility is then assumed to be known and is subsequently used as an input to several moment conditions. Eraker (2004) shows how to estimate parameters by the MCMC method using both option prices (approximately three on a day) and returns. A common feature between the procedures is that the information of only a few options is employed. The main reason for this is that computing time increases heavily with the inclusion of more options.

Finally, there are studies that only use the entire cross-section of option prices for parameter estimation. The consequence is that only risk-neutral parameters are estimated and therefore risk premia and objective parameters are often not separately identified. In Bakshi, Cao, and Chen (1997) parameters are estimated in a model with stochastic interest rates, stochastic volatility, and jumps in the return process utilizing the information in the entire cross-section of option prices between 1988 and 1991. In Bates (2000) futures option prices between 1988 and 1993 are used. Two issues concerning this methodology need to be addressed. First, the choice of the criterion function and the options that are used for optimization. The choice of the criterion function depends on the application at hand. If the main interest is the estimation of the tails of the distribution, in-the-money options (most illiquid, see Bondarenko (2003b)) are left out and relative pricing errors are minimized. On the other hand, if interest lies on the center of the distribution, absolute pricing errors are used instead of relative pricing errors. Secondly, as was pointed out by Bates (2000), an appropriate statistical theory of option pricing errors is lacking. This implies that the calculation of standard errors or confidence bands of parameters is a non-trivial task. Broadie, Chernov, and Johannes (2004) solves this issue by using a nonparametric bootstrapping procedure.

2.1.5 Implied price processes

As was already mentioned in the introduction of this thesis, financial theory is centered around the concepts of (1) the representative agent's preferences in combination with an equilibrium model, (2) the asset price dynamics, and (3) the risk-neutral dynamics. Theoretical literature states that in equilibrium two of the three aforementioned concepts imply the third.

The previous section treated the issues concerning parametric specifications of asset price dynamics intensively. In this section attention is shifted towards the risk-neutral dynamics that are implied by derivative prices. The first stream of literature that utilized the information of derivatives aims to construct the risk-neutral price process of the option's underlying asset. The basis of the method goes back to Cox, Ross, and Rubinstein (1979) that gives the discretization of the continuous time Black-Scholes model by means of a binomial tree. Rubinstein (1994) builds on this method by constructing binomial trees using observed option prices. By assuming that all paths reaching the same terminal node have the same probability, a unique implied binomial tree is derived. The no-arbitrage tree is constructed using backward recursion. Derman and Kani (1994) proposes another method for the construction of the binomial tree. This method employs a forward construction procedure that utilizes the information of options with different maturities. A number of numerical difficulties arise when the procedure is implemented using observed option prices. To solve these problems, Derman, Kani, and Chriss (1996) proposes to use trinomial trees instead of binomial trees. The underlying assumption of implied (binomial) trees is that these are discretizations of a one-dimensional diffusion in which the volatility is a deterministic function of the asset price and time.[4] This is a rather restrictive and empirically implausible assumption (see Dumas, Fleming, and Whaley (1998)). There are a number of studies that extend to stochastic volatility in tree methods. The most appealing among these is the method proposed in Britten-Jones and Neuberger (2000). The paper describes all continuous price process that are compatible with observed option prices without making the restrictive assumption that volatility is a function of asset price and time. Unfortunately, a formal empirical test of the concepts in Britten-Jones and Neuberger (2000) is not yet provided in literature. Although tree methods induce numerous numerical difficulties, the positive properties should not be forgotten. Once the risk-neutral price process can be obtained from option prices the task of pricing all kinds of exotic options is fairly simple. Jackwerth (1999) provides a more detailed overview of tree methods.

Another stream of literature concentrates on the information contained in option prices on the future stock price distribution. The (conditional) density of the underlying model factors under the risk-neutral dynamics is called the risk-neutral density or state price density. In a model where asset prices can take every possible positive value, the state price density is the continuous state analogue of the prices of Arrow-Debreu securities. These are contingent claims that have a unit payoff in a given state and

[4]Univariate diffusion models relax the geometric Brownian Motion assumption in the Black-Scholes model. Other examples of univariate diffusion models include the constant elasticity of variance model in Cox and Ross (1976) and Cox and Rubinstein (1985) and the leverage effect model in Geske (1979) and Rubinstein (1983).

nothing in each other possible state. The conditional risk-neutral density proves to be useful in valuing derivative securities at a particular point in time. Breeden and Litzenberger (1978) shows that there is an obvious link between the state price density and option prices. The paper shows that the risk-neutral density of the underlying value of the option at maturity of the option is the second derivative of a call option pricing formula with respect to the strike price of the option. This can easily be seen by considering (in a discrete setting) an option portfolio that gives the butterfly spread as a payoff, i.e. $1/c$ of call options with strike $K-c$ and with strike $K+c$, and additionally -2/c call options with strike K. If the distance between two successive states is equal to c then the payoff of this portfolio equals 1 in case the underlying value takes value K at maturity and value 0 otherwise. Assuming a constant risk free interest rate r the First Fundamental Theorem of Asset Pricing implies

$$\frac{C\left(S_t, K-c, T-t\right)-2C\left(S_t, K, T-t\right)+C\left(S_t, K+c, T-t\right)}{c}=e^{-r(T-t)}\mathbb{Q}_t\left(S_T=K\right), \tag{2.8}$$

where $C\left(S_t, K, T-t\right)$, is the time t value of a call option with strike K and maturity $T-t$ given that the time t value of the underlying is S_t. For the continuous state setting, the risk-neutral probability is transformed to a density value in a standard way. Letting c go to zero then gives

$$e^{r(T-t)}\frac{\partial^2 C\left(S_t, K, T-t\right)}{\partial K^2}=q_{S_T}(K), \tag{2.9}$$

where $q_{S_T}(\cdot)$ denotes the conditional risk-neutral density of S_T in a continuous state setting.

The methods that make use of (2.9) are nonparametric methods. These methods can be divided in a number of classes of which the kernel methods and curve-fitting methods are most important. An example of a methodology that fits into the first class is found in Aït-Sahalia and Lo (1998). This paper proposes a semiparametric approach for finding an estimate of the option-pricing function. In the second step (2.9) is utilized to obtain an estimate of the risk-neutral density $q(\cdot)$. In the most general formulation the method is not unconditional. However, under the assumption that the option-pricing function is homogeneous of degree one, the resulting implied risk-neutral density is independent of the initial stock price. Jackwerth and Rubinstein (1996) provides a methodology that belongs to the class of curve-fitting methods. The risk-neutral probability distribution is calculated by minimizing the distance between the risk-neutral probabilities (which are the decision variables in the optimization) and a prior distribution under the restriction that options and the option's underlying value are priced correctly. In contrast to Aït-Sahalia and Lo (1998), Jackwerth and Rubinstein (1996) only use option data at a particular point in time to estimate the conditional risk-neutral density of the underlying asset, i.e. using data on another day results in a different implied distribution. Another popular method (see for instance Shimko (1993)) is to fit the implied volatility smile/skew by, for example, a polynomial. Subsequently, the implied volatilities are translated into option prices after which (2.9) is applied to obtain the risk-neutral distribution of the future stock price. Besides the nonparametric methods, parametric methods are also developed. These methods will not be treated in this thesis but Jackwerth (1999) gives a detailed overview. More recent contributions are found in Bondarenko (2000) that proposes a new nonparametric method for the calculation of risk-neutral densities and in Panigirtzoglou and Skiadopoulos (2004) which is the first paper that treats the dynamics of risk-neutral densities. Empirical application of several methods in Coutant, Jondeau, and Rockinger (1998) and Anagnou, Bedendo, Hodges, and Tompkins (2002) show that if there are a sufficient number of options available, the different methodologies produce similar results.

The estimated risk-neutral densities after the 1987 crash appear to be strongly negatively skewed, see for instance the results in Aït-Sahalia and Lo (1998) and Jackwerth and Rubinstein (1996). This typical post-crash shape of the implied risk-neutral distribution using S&P-500 options is also found in Weinberg (2001) and Anagnou, Bedendo, Hodges, and Tompkins (2002). The shape of the risk-neutral density is an immediate consequence of the volatility smile or skew that is present in options markets since the stock market crash in 1987. These patterns are not only observed in the United States but also in Japanese, German, and British markets (see Tompkins (2001a)). One of the possible explanations for the changing shape of the implied volatility curve around the crash is that investors' attitude toward risk has changed after the crash. This explanation was a motivation for several studies (Aït-Sahalia and Lo (2000), Jackwerth (2000), Bliss and Panigirtzoglou (2004), and Anagnou, Bedendo, Hodges, and Tompkins (2002)) that extract risk aversion coefficients from estimators of both the risk-neutral and the objective density.

The empirical work on implied risk aversion is based on the fact that in the economy that is described by Jackwerth (2000), the coefficient of absolute risk aversion RA can be expressed in terms of the risk-neutral and the statistical density

$$RA = \frac{p'(S_T)}{p(S_T)} - \frac{q'(S_T)}{q(S_T)},$$

where $p(\cdot)$ is the objective density and $q(\cdot)$ defined as before. Of course, a lot of critical points can be made about the choice of the underlying economy but intuitively the difference between the risk-neutral density and the statistical density provides information on general risk aversion towards the uncertainty in stock markets. Jackwerth (2000) uses a kernel estimator to find an estimate of $p(\cdot)$. Using this estimator and an estimate of the risk-neutral density, the paper finds that before the 1987 stock market crash, the risk aversion function is reasonably consistent with economic theory. However, after 1987, the risk aversion function become negative and increasing in certain states. Similar conclusions are drawn in Aït-Sahalia and Lo (2000). Brown and Jackwerth (2001) calculate the empirical pricing kernel using estimates of the objective and the risk-neutral distribution. The shape of the empirical pricing kernel is consistent with the findings in Jackwerth (2000) and Aït-Sahalia and Lo (2000). Although the literature entitles these observations as puzzles, a closer look to the estimation methodologies provides more insight. There is just one restriction in estimating the risk-neutral distribution: to avoid arbitrage opportunities the expected instantaneous return on the asset should be the risk free rate. Usually more assumptions (about the underlying economy) are implicitly imposed when objective parameters are estimated. This gives reason for the different shapes of the implied density and the objective density. Trading strategies based on the differences between the objective and risk-neutral distribution appear to be extremely profitable, see (Aït-Sahalia, Wang, and Yared (2001)). The profitability of these strategies is mainly explained by the relatively high price that is received for shorting an out-of-the money put option. Coval and Shumway (2001) empirically shows that simple short option strategies give extraordinary returns. These trading strategies are no pure arbitrage strategies since the return need not to be positive in all states of the world.

Derivatives prices do not only provide information on the risk-neutral density but can also be used to hedge realized variance of the underlying asset. Regarding variance, the literature concentrates mostly on how option prices can be used to determine the risk-neutral expectation of realized variance or quadratic variation of the option's underlying asset. This is theoretically illustrated in, for instance, Britten-Jones and Neuberger (2000). The paper shows that in a diffusion setting with zero interest rates the risk-neutral expectation of realized variance between times t and T equals

$$\mathrm{E}_t^{\mathbb{Q}}\left[\int_t^T \sigma_u^2 du\right] = 2\int_0^\infty \frac{C(S_t, K, T-t) - C(S_t, K, 0)}{K^2}\mathrm{d}K. \tag{2.10}$$

A similar kind of expression is obtained by calculating the reference level of a variance swap. A variance swap is a contract that pays off the difference between the realized variance RV of the underlying asset between time t and time T and the reference level L. When the contract is initiated the contract (like a forward contract) has value 0 and therefore the theoretical value of L is

$$L = \mathrm{E}_t^{\mathbb{Q}}(RV_{t,T}) = \frac{1}{T-t}\mathrm{E}_t^{\mathbb{Q}}\left[\int_t^T \sigma_u^2 \mathrm{d}u\right], \tag{2.11}$$

where the second equality is only true in a diffusion setting without any jumps. This setting also allows to rewrite (2.11) as

$$\begin{aligned} L &= \frac{2}{T-t}\mathrm{E}^{\mathbb{Q}}\left[\int_t^T \frac{\mathrm{d}S_t}{S_t} - \log\frac{S_T}{S_t}\right], \\ &= \frac{2}{T-t}\left\{r(T-t) - \left(\frac{S_t}{F_t}e^{rT} - 1\right) - \log\frac{F_t}{S_t} + e^{r(T-t)}\int_0^{F_t}\frac{1}{K^2}P(S_t, K, T-t)\,\mathrm{d}K \right. \\ &\quad \left. + e^{r(T-t)}\int_{F_t}^{\infty}\frac{1}{K^2}C(S_t, K, T-t)\,\mathrm{d}K\right\}. \end{aligned} \tag{2.12}$$

In this formula F_t represents the time t reference level of a standard forward contract that expires at time T. Setting interest rates at zero and applying put call parity to (2.12) leads to (2.10). The results in Coval and Shumway (2001) and Carr and Wu (2004) show that strategies whose payoffs are correlated with the quadratic variation of the underlying assets give on average high returns. This conclusion provides some evidence that investors are not only concerned about the uncertainty in the return but are also influenced by the uncertainty about the return variance. Carr and Wu (2004) finds by using the structure of variance swaps that uncertainty in the return variance of the S&P-500 and Dow Jones index is priced. The previous section has shown that these results are confirmed by studies that use parametric option pricing models.

2.1.6 Implied volatility modeling

Most practitioners do not think in terms of complicated stochastic processes when they are managing (the risk of) their option portfolios. Often the observed implied volatility surface in combination with the so-called 'sticky-strike' rule or 'sticky-delta' rule is used to get an estimate of the future value of the options under management. Scientific research that aims to fit observed option prices perfectly (in both the strike price and the maturity dimension) goes back to the lattice methods in Derman and Kani (1994) and Rubinstein (1994). The previous section mentioned that the underlying assumption in these tree methods is that volatility is a deterministic function of the asset price and time. Empirical evidence of the time-instability of parameters in deterministic volatility models is given in Dumas, Fleming, and Whaley (1998). Furthermore, that paper shows that time-varying parameters lead to substantial changes in hedge parameters which is undesirable from a risk management perspective. As a result, models are constructed that not only aim to fit the observed Black-Scholes implied volatilities but also model the dynamic evolution of the implied volatility surface. This type of research gives a number of reasons for using Black-Scholes implied volatilities instead of local or instantaneous volatilities. First, Black-Scholes implied volatilities are easily retrieved from market data. No model is presumed because of the one-to-one correspondence between market prices and Black-Scholes implied volatilities. Secondly, implied volatilities provide information on the state of the option market that is familiar to market practitioners. Finally, there is a high correlation between shifts in the levels of implied volatilities across maturities and strike prices. This indicates that the joint dynamics of implied volatilities across strikes and maturities can be described in a parsimonious way.

Empirical research in this area is mostly focused on the term structure of at-the-money implied volatilities or on the dynamics of the volatility skew/smile across strike where maturity is held fixed. Principal component analysis is usually applied to implied volatility surfaces that are retrieved from empirical data. The term structure of implied volatilities is among others studied in Heynen, Kemna, and Vorst (1994), Hardle and Schmidt (2000), and Avellaneda and Zhu (1997). Avellaneda and Zhu (1997), for instance, model the at-the-money implied volatility with a GARCH process. Subsequently, principal component analysis is applied to the term structure of the implied volatility. Das and Sundaram (1999) consider higher moments like skewness and kurtosis that are implied by option prices. That paper shows that the empirical properties of the data are not matched by the predictions of simple models.

The dynamics of the implied volatility smile/skew are treated, among others, in Skiadopoulos, Hodges, and Clewlow (1999) and Alexander (2001). Skiadopoulos, Hodges, and Clewlow (1999) identifies two significant principal components by performing principal component analysis of volatility smiles on S&P-500 options. The analysis in Alexander (2001) is more or less the same as in Skiadopoulos, Hodges, and Clewlow (1999) but the deviation of implied volatilities from the at-the-money volatility is used instead.

Finally, Cont and Fonseca (2002) looks simultaneously at all available maturity and moneyness combinations in order to retrieve the joint dynamics of all implied volatilities. The method in this paper is based on a Karhunen-Loeve decomposition of the daily variations of implied volatilities obtained from market data.

2.2 Expected Option Returns and Factor Models

The previous section stipulated that the risk-neutral probability distribution of an asset implied by option prices written on that asset often differs considerably from reasonable estimators of the objective distribution. The most important observation is that the left tail of the option's implied risk-neutral is extremely fat in comparison to the left tail of the objective distribution. In a discrete state world this implies that the low states of the asset earn a negative return because of the high price that is paid for those states. Option strategies that take a short position in the expensive states and a long position in the cheap states lead to impressive average returns. Since the work of Markowitz (1952) the view is advocated that returns on a strategy should be related to the risk of the investment. However, after a correction for risk, the previously mentioned option strategies still show a remarkable performance. For instance, Bondarenko (2004) reports a Jensen's α for shorting at-the-money put options of 23% (on a monthly basis) using S&P-500 futures options between 1987 and 2000. Using a similar data set, Driessen and Maenhout (2004) finds that shorting a single out-of-the money put option or combinations of options (i.e. straddles) give Sharpe ratios of approximately 0.30 (the Sharpe ratio of the index in that period was 0.18). The empirical performance of these type of option strategies motivated a number of papers in the financial literature on option strategies and the relation to factor models, introducing new expressions like "overpriced puts puzzle", "empirical pricing kernel puzzle", and "option pricing anomalies". This section gives a short overview of the papers that are available in this area.

The previous section obviously shows that literature was mainly interested in option pricing while a thorough study of the dynamics of an option price, i.e. the return on the option, was lacking for a long time. This is surprising since option returns should provide additional information on the risks that are priced in an economy. One of the few formal treatments of option returns related to systematic risks is provided in Coval and Shumway (2001). The theoretical part of the paper shows, that under the general condition that the pricing kernel is negatively correlated with the price of a given security, any call option written on that security has a positive expected net return that is increasing in the strike price of the option. The underlying financial intuition is that a negative correlation between the pricing kernel and the security implies that low values of the securities are considered as the bad states of the world. Call options deliver payoff in the good states of the world and therefore should earn a higher return than the risk free rate. The theoretical result for the put option is the other way around. Any put option written on a security of which the price is negatively correlated with the stochastic discount factor will have an expected return below the risk free rate. Furthermore, the return on the put option is increasing in strike price. The intuition is that put options provide protection against the bad states of the world. Using S&P-500 index options between January 1990 to October 1995 the paper claims, without a formal testing procedure, that average option returns are too low to be consistent with the Black-Scholes model. The results on individual option returns indicate that besides market risk, different risk factors are priced. In order to investigate the claims, option positions are constructed in such a way that at initiation the position is not sensitive to changes in the underlying asset. Although the delta of the position is not zero instantaneously, the return in a Black-Scholes world should not deviate too much from the risk free rate. Coval and Shumway (2001) reports significant negative returns on these (initial) delta-neutral straddles indicating (and nothing more since the position is not delta-neutral instantaneously) that there is a negative volatility risk premium. A negative volatility risk premium means that high volatility is disliked by investors. Since straddles have a higher expected payoff as volatility increases, straddles provide protection against volatility risk and therefore should earn a return below the risk free rate if this risk is priced. The empirical procedure in Coval and Shumway (2001) does not isolate the volatility risk factor completely but straddle returns are a very strong indication of a negative volatility risk premium. Conclusions do not change if a crash put is added to a straddle. The reason for considering the influence of a crash put on straddle returns is that the straddle position is not instantaneously insensitive for large jumps in the options' underlying asset. A crash risk premium and a volatility risk premium have the same effect on straddle returns and therefore conclusions about the volatility risk premium can only be made if these effects are separated. A deep out-of-the-money put protects against market crashes and thus is a tool to extract the crash risk premium from the straddle return. Since both crash puts and crash neutral straddles earn a negative return on average, there is strong evidence for the existence of a crash risk premium and a volatility risk premium.

Using a data set that contains two crashes (S&P-500 index options between January 1987 and June 2001) Driessen and Maenhout (2004) finds similar average returns for protective puts, straddles, and crash neutral straddles. All the results in Coval and Shumway (2001) and Driessen and Maenhout (2004) are driven by high put and straddle prices. Driessen and Maenhout (2004) tries to answer the question what type of investors optimally take long positions in puts and straddles given that the returns on short positions are so high. The paper shows that standard expected utility preferences and non-expected utility specifications do not lead to demand for out-of-the-money put options. Only an application of cumulative prospect theory results in positive demand for puts and straddles. Hence, in standard utility frameworks there are no reasonable risk aversion parameters that lead to a demand for long positions in out-of-the-money puts and straddles. Otherwise stated, with respect to these equilibrium models options but specifically put options are mispriced. The studies in Jones (2004), Bondarenko (2003a), and Bondarenko (2003b) come to a similar conclusion: no model from a broad class of models is able to explain the high prices of some particular options. Bondarenko (2003b) uses a class of models in which the pricing kernel only depends on the market returns while Jones (2004) allows for additional sources of priced risk. In comparison to the study in Bondarenko (2003b) this leads to a reduction of pricing errors but the factor is not able to explain the returns on short term deep out-of-the-money puts and longer term out-of-the-money puts simultaneously.

Despite the results in Coval and Shumway (2001), Driessen and Maenhout (2004), Bondarenko (2003b), and Jones (2004), the conclusion that option prices are set irrationally seems strong. Another possible explanation is that there is no rational model currently available that describes the extreme aversion of investors to low states of the asset. Broadie, Chernov, and Johannes (2004) finds, for instance, that pricing performance is improved if a jump volatility risk premium is allowed for. Furthermore, the paper finds empirical evidence for jumps in volatility and time-varying risk premia. Pricing errors depend on the level of volatility which indicates that risk premia in some way depend on volatility. Hence, more research is needed on more flexible and general equilibrium models before concluding that option prices are set irrational. Market microstructural effects may also play an important role as pointed out in Bollen and Whaley (2004). This paper documents that buying pressure has a significant effect on the shape of the Black-Scholes implied volatility curve. In order to make the right conclusions about the 'fairness' of option prices, these kind of effects should be taken into account.

CHAPTER 3

The Impact of Overnight Periods on Option Pricing

3.1 Introduction

As a result of the shortcomings in the classical Black-Scholes model for option pricing, two streams of literature can be identified. The first stream extends the Black-Scholes framework to time varying volatility and the occurrence of random jumps in the underlying stock price process. Hull and White (1987) derives option prices in a stochastic volatility model under the assumption that volatility risk is idiosyncratic. Heston (1993) gives closed form option pricing formulas using a mean-reverting volatility process and an explicit volatility risk premium. Parallel to this, Merton (1976) motivates that the occurrence of abnormal events can be modeled by a jump component in the underlying stock price process. That paper discusses the implications for option pricing in case jumps are modeled as a compound Poisson process and under the assumption that jump risk is not priced in the market.[1] The models derived in Heston (1993) and Merton (1976) can be merged in the affine jump-diffusion framework of Duffie, Pan, and Singleton (2000), where asset returns and variances are driven by a finite number of state variables. The second stream of literature uses more general Lévy processes instead of Brownian Motion and the compound Poisson process as driving factors for asset returns. If the parsimonious variance gamma process is assumed to be the stochastic

[1] Cox and Ross (1976) is another early paper that treats the option valuation problem for jump processes.

process for underlying stock returns, Madan, Carr, and Chang (1998) derives closed form expressions for the density of asset returns and option prices. Stochastic volatility models driven by Lévy processes are studied in Carr, Geman, Madan, and Yor (2003), among others.

From the empirical results concerning the aforementioned models, it is evident that jumps are important in explaining characteristics of asset returns and option prices, see, for example, Bakshi, Cao, and Chen (1997), Pan (2002), Andersen, Benzoni, and Lund (2002), and Madan, Carr, and Chang (1998). Using a parametrically specified pricing kernel, Pan (2002) provides evidence that jump risk is priced in the SPX options market. The results in Coval and Shumway (2001) are indicative of a negative volatility risk premium. This conclusion is based on returns of option positions that are (at initiation) only sensitive for volatility risk and jump risk. The Lévy literature also provides support for priced volatility and jump risk since the parameter estimates under the objective and the risk-neutral measure are generally significantly different. For instance, Madan, Carr, and Chang (1998) finds significant negative skewness under the risk-neutral probability measure while this is not present in their objective parameter estimates. The differences between the objective and the risk-neutral distributions are indicative of the presence of a price for crash risk in options markets. However, it is not always obvious how market prices of risk can be inferred from the estimation results, because a parametric pricing kernel that defines risk prices, is usually not specified in this literature. On the whole, it is clear from both streams of literature that jumps, next to stochastic volatility, are important in explaining observed patterns in asset returns and option prices.

The present chapter considers the jump process in more detail by focusing on jumps in asset prices that are inherent to overnight market closure.[2] Most of the empirical research cited above, uses daily returns. These returns are calculated using the last tick price on the exchange of each trading day. However, the exchange is closed a large part of the day and information that arrives during the closing time cannot be immediately incorporated in stock prices. For instance, European investors use information revealed in US stock markets, by submitting orders to their exchange before the opening. This means that the opening price of the exchange reflects overnight information. The effect of market closure on stock (index) returns has been considered extensively in

[2]The idea of modeling the overnight nontrading period by a jump component is not new. Oldfield and Rogalski (1980) already proposes a general model for stock returns that includes jumps for market closures.

the literature. Important findings are that (1) open-to-open returns are more volatile than close-to-close returns (see, for instance, Amihud and Mendelson (1987), Stoll and Whaley (1990), Amihud and Mendelson (1991), and Cao, Choe, and Hatheway (1995)), (2) weekend returns are lower than weekday returns (see, for example, French (1980), Gibbons and Hess (1981), and Keim and Stambaugh (1984)), and (3) returns over trading periods are more volatile than returns over nontrading periods (see, among others, Fama (1965), French and Roll (1986), Oldfield and Rogalski (1980), and Amihud and Mendelson (1991)).[3] However, the influence of market closure on option pricing is not treated yet.

In this chapter the difference in information is stressed by using different processes driving intraday and overnight returns, respectively.[4] In particular, in the spirit of Andersen, Benzoni, and Lund (2002) the model consists of a continuous part with stochastic volatility (reflecting the normal vibrations in the stock price) and a jump part (modeling the arrival of important new information) during the day. Furthermore, the "normal" overnight change in the stock price is modeled by means of a single jump. Additional random jumps due to important news releases are not excluded in the overnight period. The theoretical and empirical implications of this added factor on option prices are investigated.

The results show that, for the SPX market over two separate periods, both random jumps and overnight jumps are important for option pricing. In particular, the overnight jump component accounts for approximately one quarter of total jump variation. Moreover, the inclusion of overnight jumps leads to different parameter estimates for the stochastic volatility and random jump part of the stock price process. This will have important consequences for hedging these risks.

The organization of the chapter is as follows. Section 2 provides the theoretical formulation of the model under the risk-neutral measure. A closed-form option pricing formula in the spirit of Heston (1993) is also provided. Section 3 describes the data and discusses the estimation procedure. In Section 4 the empirical results are presented. Section 5 concludes. Mathematical details are gathered in the appendix.

[3] As a result of these observed patterns, theoretical models are developed to explain them. See, for instance, Slezak (1994) and Hong and Wang (2000).

[4] The usage of different processes for trading and nontrading periods is already motivated in Oldfield and Rogalski (1980).

3.2 The Overnight Jump Model

3.2.1 Stock price process

Financial markets all over the world do not allow for continuously trading stocks, interest rates products, and derivatives. Trading usually starts in the morning hours local time and ends in the late afternoon or in the evening. Of course, it is possible for individual and institutional investors to do 24 hours trading all over the world: by the time London closes, Wall Street is already open and when the US markets stop trading, Asian exchanges have already opened their doors. Due to increasing globalization and financial market integration, economies and firms from various countries are interrelated. As a consequence, changes in the value of financial instruments on different exchanges are not independent. This does not only hold if exchanges are open simultaneously, but also if one market is closed. In case an exchange is closed, relevant news cannot be immediately incorporated in prices. For instance, a high closing of stocks traded on the Dow Jones usually has a positive effect on stock price openings in Europe.[5] All news that is important for the value of a particular stock should ideally be processed in the opening price of the stock. The difference between the closing price and the opening price the next day can be seen as a measure of the revealed information all over the world during the overnight period.[6]

Up to now, the overnight period in financial markets has not been considered in the derivative pricing literature. This chapter tries to fill this gap by explicitly modeling this period through an additional jump process. The jump in the stock price process exactly

[5] Connolly and Wang (2000) concludes that intraday returns on foreign markets have a significant impact on domestic intraday returns and domestic overnight returns. The impact on the domestic overnight returns seems to be the highest. Furthermore, the US market has a greater influence on the UK and Japanese market than the other way around.

[6] There are important differences in market opening procedures between exchanges. Specifically, on the NYSE a stabilized auction market opens trading while on the NASDAQ a quote-driven, dealer market mechanism is used for all transactions during the trading day. However, even though there is no formal call market opening on the NASDAQ, the open of trade is preceded by a pre-opening session that facilitates price discovery. Greene and Watts (1996) and Masulis and Shivakumar (1997) examine the differences in close-to-open price reaction to overnight news announcements across these markets. Greene and Watts (1996) finds that the opening procedure on the NASDAQ leads to prices that incorporate more of the overnight information. In addition, Masulis and Shivakumar (1997) reports that the NASDAQ reacts faster to overnight seasoned equity offering announcements. Cao, Ghysels, and Hatheway (2000) concludes that the more rapid price adjustment on the NASDAQ is a consequence of the pre-opening session.

models the observed overnight return. Of course, closed markets also imply that an overnight jump has to be added to the money market process. However, as the interest rate sensitivity of stock derivatives is usually found to be rather low, the implications of this will be rather limited.

The money market process is given by, assuming a possibly different (annualized) risk-free interest rate r during the trading day and r^o during the overnight period

$$\frac{\mathrm{d}B_t}{B_{t_-}} = r\mathrm{d}t + \mathrm{d}\sum_{i=1}^{\lfloor 252t \rfloor} \left\{ \exp\left(\frac{r^o}{252}\right) - 1 \right\}, \tag{3.1}$$

i.e. $B_t = \exp\{rt + r^o \lfloor 252t \rfloor / 252\}$, where $\lfloor \cdot \rfloor$ denotes the floor function.

In this chapter the equivalent martingale method is used for pricing options. In comparison to the standard Black and Scholes (1973) framework, there are additional risk factors that make the market incomplete with respect to the traded financial securities. A consequence is the non-uniqueness of the equivalent martingale measure $\mathbb{Q}$. Motivated by, for example, the Breeden (1979) consumption based model, the value process of the underlying stock in transaction time under the risk-neutral probability measure $\mathbb{Q}$ is defined by

$$\frac{\mathrm{d}S_t}{S_{t_-}} = r\mathrm{d}t + \sigma_t \mathrm{d}W_t^S + \mathrm{d}\sum_{i=1}^{N_t} (Y_i - 1) - \mathrm{d}A_t + \mathrm{d}\sum_{i=1}^{\lfloor 252t \rfloor} (V_i - 1), \tag{3.2}$$

$$\log Y_i \sim N\left(\log(1+\mu_{RJ}) - \frac{1}{2}\sigma_{RJ}^2, \sigma_{RJ}^2\right),$$

$$\log V_i \sim N\left(\frac{r^o}{252} - \frac{1}{2}\frac{\sigma_{OJ}^2}{252}, \frac{\sigma_{OJ}^2}{252}\right),$$

where $\{W_t^S\}$ is a standard Brownian Motion independent of the Poisson process $\{N_t\}$ with

$$N_t \sim \text{Poiss}\left((1-c)\lambda t + c\lambda \frac{\lfloor 252t \rfloor}{252}\right).$$

Both $\{W_t^S\}$and $\{N_t\}$ are also assumed to be independent of sequences of jumps $\{Y_i\}$ and $\{V_i\}$. Note that the volatility model with jumps of Bakshi, Cao, and Chen (1997) and Andersen, Benzoni, and Lund (2002) is obtained by setting the parameter c equal to zero and by deleting the last sum covering the overnight jump part in (3.2). The time-varying volatility process $\{\sigma_t^2\}$ will be defined below.

Note that the random jump distribution of the Y's is parameterized such that a single jump multiplies, in expectation, the price by $1 + \mu_{RJ}$. On a yearly basis, due to the random number of jumps, this implies an expected instantaneous drift term $\{A_t\}$, see the appendix, that needs to be compensated in (3.2) to keep the martingale property of the discounted price process.

The contribution of this chapter consists of an extra jump term that is added to the stock price process. For simplicity weekends are counted as a single night and there are 252 days a year. At each time which is a multiple of 1/252, an overnight period is inserted. Each overnight period results in a stock return that is reflected by the jump V_i. Note that the random jump process (interpreted before as, for example, news releases) will also be active during the overnight periods but possibly at a different rate. The parameter c allows the random jumps to have a different intensity during the trading day compared to the overnight period. The expected number of *random* Y-jumps during one calendar year (in addition to the 252 V-jumps) is equal to λ. Finally, note that, as required, the $\mathbb{Q}$-expected yearly return on the stock price in our model is given by

$$\mathrm{E}_t S_{t+1}/S_t = \exp\left\{r + r^o\right\}.$$

The specification of the stochastic variance process in (3.2) is taken from Heston (1993)

$$\begin{aligned} \mathrm{d}\sigma_t^2 &= -\kappa\left(\sigma_t^2 - \sigma^2\right)\mathrm{d}t + \sigma_\sigma \sigma_t \mathrm{d}W_t^V, \quad (3.3)\\ \mathrm{Cov}_t\left(\mathrm{d}W_t^V, \mathrm{d}W_t^S\right) &= \rho \mathrm{d}t, \end{aligned}$$

where $\left\{W_t^V\right\}$ is a standard Brownian Motion independent of the Poisson process $\{N_t\}$, κ is the speed of mean reversion, σ^2 is the long run mean of the variance, and σ_σ the volatility of volatility. This specification allows a negative premium for volatility risk, see, for example, Bakshi and Kapadia (2003) for theoretical and empirical evidence. It has been often observed that a large decline in the stock price is accompanied by a positive shock in volatility levels. This is captured by means of the parameter ρ.

3.2.2 Option pricing

Given the risk-neutral processes in (3.2) and (3.3), a standard plain vanilla call option can be priced using

$$C_t(K,T) = B_t E_t\left(\frac{\max\left(S_T - K, 0\right)}{B_T}\right),$$

where T is the maturity and K is the strike price of the option. Following Heston (1993), Appendix 3.A shows that the pricing formulas for the value of a call option C and a put option P at time t can be simplified as

$$\begin{aligned} C_t(K,T) &= S_t P_1 - K e^{-r(T-t)-nr^o/252} P_2, \quad (3.4)\\ P_t(K,T) &= K e^{-r(T-t)-nr^o/252}\left(1 - P_2\right) - S_t\left(1 - P_1\right), \quad (3.5) \end{aligned}$$

where the probabilities P_1 and P_2 are given by (3.7) and (3.8), and $n = \lfloor 252T \rfloor - \lfloor 252t \rfloor$ denotes the remaining number of overnight periods till maturity.[7] The proof uses the independence of the overnight process and the intraday process and the fact that the trading day part of the model is an affine jump-diffusion in the spirit of Duffie, Pan, and Singleton (2000).

3.3 Data and Estimation Issues

In the previous section was motivated that different processes describe the intraday and overnight returns. In the empirical application the focus is on the S&P-500 index in two periods: a low volatility period from January 1, 1992 until August 27, 1997 and a high volatility period from July 9, 1999 until November 27, 2003.

To assess the effects of market closure in an intuitive informal way, Table 3.1 shows the sample statistics of the close-to-close, open-to-close, and close-to-open returns series for the respective sample periods.[8] Similar to Compton and Kunkel (2003), the numbers in Table 3.1 show that for both sample periods the close-to-open average return is higher than the average open-to-close return and that this higher average return is accompanied with a lower standard deviation. However, this is only a qualitative statement because the hypothesis of equal medians in close-to-open and open-to-close return series cannot be rejected at reasonable significance levels for both sample periods.[9,10] On the other hand, the hypothesis of equal variances of the open-to-close and close-to-open return series is rejected at reasonable significance levels.[11] Furthermore, outcomes of standard

[7] The resulting option pricing formulas in Appendix 3.A show that, except for parameter c, all model parameters have a different effect on option prices, i.e. all model parameters, except c, are separately identified in the overnight option pricing model. In the estimation procedure parameter c is fixed at the proportion of the day that markets are closed.

[8] To avoid potential stale-price problems associated with openings of US indices (see Stoll and Whaley (1990)), the opening price is taken as the value of the S&P-500 index taped together with the first option quote.

[9] The Wilcoxon signed rank test is used to test the hypothesis that the paired difference between the trading return and the nontrading returns has median zero. This test does not require the assumption that the population is normally distributed.

[10] Compton and Kunkel (2003) finds for several European countries that the location of close-to-open returns differs significantly from the location of the open-to-close returns. There are several possibilities to explain why the overnight return has a higher average and lower standard deviation than the intraday return.

[11] The hypothesis is tested by using the Levene test because this test is less sensitive to the normality assumption than the Bartlett test. The p-values are available upon request.

	January 1992–August 1997			July 1999–November 2003		
	close-close	open-close	close-open	close-close	open-close	close-open
average	13.2%	5.5%	7.7%	-4.3%	-3.5%	-0.8%
std.dev	10.5%	9.9%	2.7%	20.6%	18.9%	7.9%
skewness	-0.28	-0.26	-2.54	0.13	0.21	0.25
kurtosis	4.8	4.7	40.8	4.6	5.9	10.4

Table 3.1: Summary statistics S&P-500 returns during the low volatility period January 1, 1992–August 27, 1997, and the high volatility period July 9, 1999–November 27, 2003.

tests show that close-to-open and open-to-close returns have significant skewness and significant excess kurtosis in both sample periods. As a result, application of the Jarque-Bera test leads to rejection of the normality hypothesis for close-to-open and open-to-close returns in both sample periods.

The standard deviations in Table 3.1 indicate that the overnight return is an important part of the total daily return in both the first and the second period. As the sample standard deviation of the overnight returns is lower than the standard deviation of the intraday returns, one may conclude that information important for S&P stocks generally arrives during trading hours. Information of significant importance during the night often leads to a high, either positive or negative, return on the S&P-500 explaining the high kurtosis values of overnight returns in Table 3.1.

Finally, daily closing option quotes of SPX options for both sample periods are available. These data are extracted from the proprietary ABN-Amro Asset Management option database. Following Bakshi, Cao, and Chen (1997), for each day in the sample, only the midprice based on the last reported bid-ask quote (prior to 3:00 PM Central Standard Time) of each option contract is used for estimation. Of course, the aforementioned S&P-500 index levels are measured at the same time. Following Jackwerth and Rubinstein (1996), the dividend amount and timing expected by the market is assumed to be identical to the dividends actually paid on the index. Interpolated LIBOR rates are used as a proxy of the intraday risk-free rate. In addition, information on overnight interest rates in the US market is extracted from Bloomberg.

Table 3.2 and Table 3.3 provides descriptive statistics on call and put option prices (stated in terms of Black-Scholes implied volatilities) that

1. have time-to-expiration of greater than or equal to six calendar days,

		January 1992–August 1997				July 1999–November 2003			
CALLS		days to expiration			subtotal	days to expiration			subtotal
	$Ke^{-r(T-t)}/S$	<60	60-180	>180		<60	60-180	>180	
ITM	< 0.97	0.209	0.171	0.167		0.316	0.277	0.252	
		14736	14828	6830	36394	12550	10908	2872	26330
ATM	0.97-1.03	0.137	0.138	0.147		0.222	0.225	0.238	
		14611	13693	5571	33875	7929	6802	2874	17605
OTM	> 1.03	0.124	0.118	0.124		0.241	0.212	0.207	
		4768	9380	5836	19984	8318	10021	2130	20469
	subtotal	34115	37901	18237	90253	28797	27731	7876	64404
PUTS		days to expiration			subtotal	days to expiration			subtotal
	$Ke^{-r(T-t)}/S$	<60	60-180	>180		<60	60-180	>180	
OTM	< 0.97	0.190	0.173	0.172		0.307	0.279	0.249	
		12895	14723	7074	34692	12121	11667	2955	26743
ATM	0.97-1.03	0.137	0.139	0.151		0.220	0.222	0.233	
		14690	13771	5710	34171	7926	6773	2874	17573
ITM	> 1.03	0.162	0.125	0.130		0.244	0.220	0.205	
		8500	11259	6122	25881	10081	10623	2155	22859
	subtotal	36085	39753	18906	94744	30128	29063	7984	67175

Table 3.2: Summary statistics on SPX call and put option implied volatilities. The reported numbers are implied volatilities of options on the S&P-500 index corresponding to the average last tick before 3:00 PM and the total number of observations for each maturity category. The sample periods are January 1, 1992, to August 27, 1997, and July 9, 1999, to November 27, 2003, respectively.

		January 1992–August 1997				July 1999–November 2003			
CALLS		days to expiration			subtotal	days to expiration			subtotal
	$N(d_2)$	<60	60-180	>180		<60	60-180	>180	
ITM	≥ 0.60	0.191	0.162	0.160		0.303	0.274	0.251	
		19319	17737	8101	45157	14434	11206	2969	28609
ATM	0.40-0.60	0.154	0.150	0.149		0.232	0.231	0.235	
		5647	8862	4753	19262	4259	6532	3589	14380
OTM	< 0.40	0.122	0.118	0.126		0.234	0.210	0.196	
		9149	11302	5383	25834	10104	9993	1318	21415
	subtotal	34115	37901	18237	90253	28797	27731	7876	64404
PUTS		days to expiration			subtotal	days to expiration			subtotal
	$N(-d_2)$	<60	60-180	>180		<60	60-180	>180	
OTM	< 0.40	0.174	0.164	0.165		0.295	0.276	0.248	
		17495	17710	8422	43627	14005	11963	3052	29020
ATM	0.40-0.60	0.154	0.151	0.153		0.230	0.228	0.230	
		5688	8890	4820	19398	4262	6531	3589	14382
ITM	> 0.60	0.148	0.124	0.132		0.238	0.219	0.198	
		12902	13153	5664	31719	11861	10569	1343	23773
	subtotal	36085	39753	18906	94744	30128	29063	7984	67175

Table 3.3: Summary statistics on SPX call and put option implied volatilities. The reported numbers are implied volatilities of options on the S&P-500 index corresponding to the average last tick before 3:00 PM and the total number of observations for each maturity category. The sample periods are January 1, 1992, to August 27, 1997, and July 9, 1999, to November 27, 2003, respectively.

2. have a bid price of greater than or equal to 3/8\$,

3. have a bid-ask spread of less than or equal to 1\$,

4. have a Black-Scholes implied volatility greater than zero and less than or equal to 0.70, and satisfy the arbitrage restriction,

$$C_t(K,T) \geq \max\left(0, S_t e^{-\delta(T-t)} - K e^{-r(T-t)}\right),$$

for call options and a similar restriction for put options. In this formula K is the option exercise price, δ the dividend rate, and r the continuously compounded intraday risk-free rate.

From the numbers in Table 3.2 and Table 3.3, well known patterns in implied volatilities across strikes and maturities are recognized.[12] The volatility skew or smile is clearly present for most option categories. The exceptional categories are less frequently traded. From the return data in Table 3.1, it is clear that the 1992–1997 sample period can be characterized as a low volatility period and the 1999–2003 sample as a high volatility period. This characterization of both periods also becomes clear from the implied volatilities in Table 3.2, since they are consistently on a higher level across strike prices and maturities in the 1999–2003 sample period. Christensen and Prabhala (1998), among others, provide evidence for a high correlation between realized volatility and Black-Scholes implied volatility.

In this chapter information about $\mathbb{Q}$-parameters is extracted from the option prices since our focus is on the influence of overnight jumps on these options. The practical implementation of the estimation procedure is straightforward and follows Bakshi, Cao, and Chen (1997). For a particular day t, a set of N options is chosen for which the

[12]In Table 3.2 and Table 3.3 two measures of moneyness are employed. Table 3.2 uses the discounted ratio of the strike price to the underlying (see, for instance, Fung and Hsieh (1991) and Bakshi, Cao, and Chen (1997)). However, this measure does not take the time to maturity of the option into account (see Natenberg (1994) and Tompkins (2001a)). Therefore, a second measure of moneyness is reported in Table 3.3. This is the Black-Scholes (risk-neutral) probability of ending in the money, i.e. $N(d_2)$ for calls and $N(-d_2)$ for puts, where d_2 is given by

$$d_2 = \frac{\log(S/K) + \left(r - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

with σ as the Black-Scholes at-the-money implied volatility. This volatility is extracted from an option series with shortest maturity longer than one week. The tables show that there are only small differences in the implied volatility patterns for the two different measures of moneyness.

closing price is observed. Henceforth, the i-th option price in this set will be denoted by O_{it}^{obs}. For all options, related values as strike price, remaining time to maturity, risk-free interest rates, and (dividend discounted) value of the underlying are observed as well. Subsequently, a model price of option i at time t, say O_{it}^{model}, that is a function of the structural $\mathbb{Q}$ parameter vector $\theta = (\mu_J, \sigma_{RJ}, \lambda, \sigma_{OJ}, \kappa, \sigma, \sigma_\sigma, \rho)$ and the unobservable instantaneous variance σ_t^2, is calculated. For a particular time t the estimated parameter vector is determined from

$$\left[\hat{\theta}_t, \hat{\sigma}_t^2\right] = \arg\min_{\theta, \sigma_t^2} \sum_{i=1}^{N} \left(\frac{O_{it}^{\text{model}} - O_{it}^{\text{obs}}}{O_{it}^{\text{obs}}} \right)^2 . \tag{3.6}$$

This objective function implies that the focus is on fitting the steepness of the observed (Black-Scholes) implied volatility skews or otherwise stated the tails of the market implied risk-neutral distribution, see Britten-Jones and Neuberger (2000).[13] The procedure is repeated for each day in both samples resulting in two time series of estimators. Similar procedures are applied to option pricing models in Bakshi, Cao, and Chen (1997) and Madan, Carr, and Chang (1998). In the implementation of the procedure above only out-of-the money options (for low strikes put options and for high strikes call options) are used, since these options are generally more liquid than in-the-money options (see Bondarenko (2003b)).

3.4 Empirical Results

This section provides the estimation results obtained by applying the data and estimation techniques as described in Section 3.3 to the model formulated in Section 3.2. First, as a benchmark, results are presented for the standard stochastic volatility model (SV) and the stochastic volatility model with random jumps (SVRJ). These results are followed by a discussion of the results in the extended model including overnight jumps. The

[13] Surprisingly, the topic of specification of the loss function is not heavily debated in the option valuation literature. This in contrast to other topics like model specification and parameter estimation in continuous time models. However, specification of the loss function is not an unimportant issue since the loss function amounts to the specification of a statistical model (Engle (1993)). Christoffersen and Jacobs (2004) is one of the few studies that treats the loss function in more detail. The choice of minimizing relative price errors instead of absolute pricing errors (Bakshi, Cao, and Chen (1997), Heston and Nandi (2000), and Figlewski (2002)) is made because an absolute pricing error based loss function assigns much weight to long maturity contracts. A similar disadvantage appears for relative pricing errors (emphasis on short maturity and out-of-the-money contracts) but the disadvantage is circumvented (partly) by excluding options with very short maturities and very low prices.

	January 1992–August 1997				July 1999–November 2003			
	SV	SVRJ	SVOJ	SVRJOJ	SV	SVRJ	SVOJ	SVRJOJ
μ_{RJ}		-6.3%		-7.2%		-11.5%		-8.5%
		(3.9%)		(3.6%)		(7.6%)		(5.8%)
σ_{RJ}		8.8%		6.7%		13.8%		11.4%
		(4.2%)		(2.7%)		(10.0%)		(6.8%)
λ		0.60		0.54		0.63		1.18
		(0.05)		(0.42)		(0.11)		(1.05)
σ_{OJ}			7.5%	5.1%			6.9%	7.7%
			(2.8%)	(2.6%)			(5.2%)	(4.6%)
κ	1.67	3.55	1.62	3.32	1.60	3.90	1.72	3.27
	(0.96)	(1.00)	(0.13)	(1.08)	(0.51)	(0.11)	(0.22)	(3.55)
σ	16.0%	11.6%	16.0%	11.2%	15.9%	11.3%	16.0%	11.9%
	(4.3%)	(3.5%)	(1.0%)	(4.8%)	(1.5%)	(3.1%)	(1.7%)	(5.2%)
σ_σ	61.1%	40.0%	92.4%	50.6%	86.8%	39.3%	79.2%	56.7%
	(18.7%)	(30.0%)	(32.2%	(28.9%)	(18.7%)	(11.2%)	(38.9%)	(40.3%)
ρ	-0.69	-0.59	-0.88	-0.71	-0.64	-0.53	-0.89	-0.70
	(0.15)	(0.20)	(0.13)	(0.23)	(0.18)	(0.07)	(0.15)	(0.22)
σ_t	14.4%	11.7%	11.9%	9.5%	24.8%	20.2%	22.3%	17.4%
	(3.0%)	(3.2%)	(3.4%)	(3.4%)	(5.3%)	(5.0%)	(7.0%)	(7.4%)
SSE	0.70	0.16	0.40	0.12	0.70	0.22	0.49	0.10
	(0.58)	(0.15)	(0.35)	(0.10)	(0.66)	(0.31)	(0.53)	(0.13)

Table 3.4: Implied average parameter estimates in SV, SVRJ, SVOJ, SVRJOJ models using option data on the S&P–500 from the low volatility period January 1, 1992, until August 27, 1997, and the high volatility period July 9, 1999, until November 27, 2003. Standard deviations of the daily parameter estimates are given in brackets.

results are presented both in a setting with only stochastic volatility during the day (SVOJ) as well as in a setting where random jumps are possible (SVRJOJ).

3.4.1 Standard option pricing models

This subsection presents the results for the SV-model and the SVRJ-model in order to make these comparable to those of Bakshi, Cao, and Chen (1997). Their model specification and their estimation techniques are similar to the ones that are employed in this chapter. For both sample periods described in Section 3.3, Table 3.4 gives an overview of the estimation results of the risk-neutral parameters.

For the SV-model, Table 3.4 confirms that the average instantaneous volatility in the 1992–1997 sample is low in comparison to, for example, the estimated values in Bakshi, Cao, and Chen (1997) over the period June 1988 to May 1991. In the 1999–2003 sample the average instantaneous volatility is higher. In comparison to Bakshi, Cao, and Chen (1997), parameters σ_σ, κ, and σ are also estimated differently.[14] One obvious explanation for these differences is the different sample periods used. Furthermore, Bakshi, Cao, and Chen (1997) focuses on absolute pricing errors while in this chapter relative pricing errors are considered, see (3.6). By using relative pricing errors the misspecification of the SV-model becomes more apparent since a high value of σ_σ is necessary to fit empirically observed implied volatility curves.

To address this issue in more detail, consider the usual situation where the option implied volatility curve for short term options is downward sloping in the strike price for low levels of the strike price.[15] The steepness of the implied volatility curve provides information about the risk-neutral distribution of the underlying index at the maturity date. The steeper the implied volatility curve for a certain strike price region, the more probability mass in that particular region of the implied risk-neutral distribution.

There is an enormous literature on methodologies that extract information about the risk-neutral distribution from option prices, see for example Britten-Jones and Neuberger (2000).[16] Because squared relative errors are minimized, the fit of cheaper options

[14]Bakshi, Cao, and Chen (1997) estimates σ_σ equal to 0.39 while this parameter in Broadie, Chernov, and Johannes (2004) is estimated at a level of 2.82 in a stochastic volatility model.

[15]For shorter maturities the option implied volatility curve has usually a smile shape (see Table 3.2 and Tompkins (2001a)) and hence the option implied volatility curve is not downward sloping over the whole range of strike prices.

[16]For an overview of methods see Coutant, Jondeau, and Rockinger (1998), Jackwerth (1999), Anagnou, Bedendo, Hodges, and Tompkins (2002), Bliss and Panigirtzoglou (2002), and Panigirtzoglou and Skiadopoulos (2004).

(short term OTM puts and calls) is relatively more important compared to the more expensive options in the sample (long term ATM and ITM puts and calls). Stated differently, the focus is more on the tails of the market implied risk-neutral distribution. The negative slope of the implied volatility curve for short term options forces the optimization algorithm to choose parameter values that are able to generate negative skewness in the risk-neutral distribution. The desired skewness can be obtained both from ρ and σ_σ. In more detail, the SV-estimates would imply a volatility of volatility $\sigma_\sigma \sigma_t$ of 9% in the low volatility period and a volatility of volatility of 21% in the high volatility period while using empirical data volatility of volatility is estimated around 5% in low volatility markets and 12% in high volatility markets.[17,18]

The estimation results show that part of the misspecification in the SV-model is solved by adding random jumps to the option's underlying value. Compared to the SV-estimates, the parameter estimates of σ_σ and ρ are much smaller in the SVRJ-model which is due to the appearance of (on average) negative jumps that capture (part of) the negative skewness in the implied risk-neutral distribution.[19] The three parameter random jump size process combined with stochastic volatility is superior to the SV-model in describing the tails of the market implied risk-neutral distribution and fitting the option data.

Comparing the results for both sample periods, the parameter estimates show that the instantaneous volatility in the SVRJ-models is lower on average than in the SV-model. This is intuitively correct since the total variation in the underlying value is now divided in the variation of a jump component and the variation that stems from the stochastic volatility part of the model. The variance in the log-return due to the jumps is given by

$$\mathrm{Var}\left(\sum_{i=1}^{N_{t+1}-N_t} \log Y_i\right) = \lambda \sigma_{RJ}^2 + \lambda \left(\log\left(1+\mu_{RJ}\right) - \frac{1}{2}\sigma_{RJ}^2\right)^2.$$

[17]These estimates are based on the standard deviation of the at-the-money Black-Scholes implied volatilities of the data described in Section 3.3.

[18]Although the estimator of σ_σ differs from the estimate in Bakshi, Cao, and Chen (1997), Bates (2000), and Broadie, Chernov, and Johannes (2004), the conclusion is the same: the volatility of volatility parameter σ_σ is estimated at a too high level to be consistent with time series estimates in, for instance, Andersen, Benzoni, and Lund (2002), Chernov, Gallant, Ghysels, and Tauchen (2003), and Eraker, Johannes, and Polson (2003). The latter study reports the highest estimate of $\sigma_\sigma = 0.14$ in a stochastic volatility model.

[19]A similar conclusion can be found in Bakshi, Cao, and Chen (1997) and Bates (2000). These studies also find that adding jumps to the risk-neutral return process leads to lower estimates of ρ and σ_σ.

	1992-1997		1999-2003	
	SVRJ	**SVRJOJ**	**SVRJ**	**SVRJOJ**
Continuous part	0.014	0.009	0.041	0.030
Random Jump part	0.007	0.006	0.023	0.026
Overnight Jump part		0.003		0.006
Total	0.021	0.017	0.063	0.061
Volatility	11.7%	9.5%	20.2%	17.4%
Objective function	0.163	0.123	0.216	0.103

Table 3.5: Variance decomposition of the SVRJ-model and the SVRJ-model for the 1992–1997 and 1999–2003 sample periods. The numbers are based on the implied parameter estimates of Table 3.4.

The full variance decomposition for the SVRJ-model is presented in Table 3.5. This table shows that the variance due to the random jump part is given by 0.007 and 0.023 in the respective sample periods. Taking σ_t^2 as a proxy of the variance of the continuous part of the underlying value process, approximately one third of the total variance is due to random jumps. Moreover, if the variance of the random jump part is added to the estimate of σ_t^2, then, for both samples, the total variance in the SV-model is almost identical to the total variance in the SVRJ-model.

Summarizing, the results of this subsection show that the parameter estimates in the SVRJ-model are more in line with the findings of Bakshi, Cao, and Chen (1997) than in the SV-model case. The addition of the random jump component stabilizes the stochastic volatility parameters to more reasonable levels and, hence, reduces the misspecification of the model.[20]

3.4.2 Option pricing models with overnight jumps

As the goal of the present chapter is to assess the importance of overnight trading halts for derivative pricing, the estimation results for SVOJ- and the SVRJOJ-models are compared with the results in the previous subsection.

As a first remark, note that the yearly log-return on a risk-free investment of one

[20]Note that compared to time-series estimates the volatility of volatility parameter is still estimated at a too large value. This indicates misspecification of the risk-neutral volatility process that possibly could be solved by adding jumps to the volatility process. Eraker, Johannes, and Polson (2003) finds strong evidence for jumps in volatility by using index returns.

dollar in the model with overnight jumps is equal to $r + r^o$. Since trading takes place approximately 6.5 hours a day, the annualized risk-free rate during trading periods and the annualized overnight risk-free rate is divided in the proportions $\frac{1}{4}$ and $\frac{3}{4}$, respectively. Secondly, observe that the parameter c is not present in the option pricing formulas (3.4)–(3.5), i.e. the different risk-neutral jump intensities during trading periods and overnight periods cannot be identified from option data, only λ, the total expected number of random jumps during a calendar year, is determined by option prices.

Table 3.4 shows that the parameter estimates in the SVOJ-model are quite similar to the ones resulting from the SV-model. Again, just as discussed for the SV-model, the parameters σ_σ and ρ are too extreme in the SVOJ-model. This leads to the conclusion that the inclusion of overnight jumps only, fails to produce the desired skew in the risk-neutral distribution. Moreover, as already observed in the SVRJ-model, the attributed proportion of the total variance due to jumps is approximately one third. Especially in the second sample period, the SVOJ-model fails to reproduce this result. Taking σ_t as a proxy of the standard deviation of the continuous part, the total variance is given by $\sigma_t^2 + \sigma_{OJ}^2$. Using this it follows that the jump proportion of the variance is slightly less than one third (28%) in the first period but that it is far too low (9%) in the second period. Since jumps play a more dominant role in high volatility periods, this once more indicates that the SVOJ-model is misspecified. A final objection against the SVOJ-model is the fit to the option data. Of course, the SVOJ-model beats the classical SV-model but the increased fit due to overnight jumps, although not negligible, is low in comparison to the inclusion of random jumps as in the SVRJ-model. All this leads to the conclusion that replacement of the random jumps in the SVRJ-model by a single overnight jump is not sufficient. However, the question whether overnight jumps influence option prices, remains open. This issue will be tackled in the next paragraph.

The estimation results for the SVRJOJ-model clearly outperform the models discussed before. In comparison to the SV-, SVRJ-, and the SVOJ-models, the SVRJOJ-model improves the fit of option prices in both sample periods considerably. The addition of random jumps to the SVOJ-model has the same effect on parameters σ_σ and ρ as the addition of random jumps to the SV-model. The reasoning is also the same: the random jump part captures (part of) the negative skewness in the risk-neutral distribution required to fit option prices that otherwise could only partly be captured by large changes in the parameters σ_σ and ρ. Comparing the remaining parameters in the SVRJOJ-model with the SVRJ-model leads to some first obvious conclusions. Since overnight jumps are included, the parameter estimates of the random jump distribution are less dominant and since the total variance has to be divided over three terms, the estimated variance of the continuous part diminishes. One striking difference is the change in the estimated intensity λ. In the first sample period, the estimated value decreases as expected since additional jumps are added. However, in the high volatility period, the intensity is almost doubled compared to the SVRJ-model. This effect is greatly offset by the much lower value of σ_{RJ}. Probably, in high volatility periods, the model fits much more smaller jumps and due to the effect of the overnight jump, the SVRJOJ-model is better able to identify the smaller jump intensity.[21]

In the same spirit as in the previous subsection, the total variance of the log-return can be split into three parts: a first component arrives from the stochastic volatility term σ_t, and the two remaining components stem from both the random jumps and the overnight jumps. The trading period's variance consists of the variance of the continuous component (stochastic volatility) and (part of) the random jump component. The nontrading overnight period variance is due to the remaining part of the random jump component and the overnight jumps. Similar to the continuous trading model without overnight periods, the variance in the in the log-return due to the jumps in the extended models SVOJ and SVRJOJ is given by

$$\mathrm{Var}_t\left(\sum_{i=1}^{N_{t+1}-N_t}\log Y_i\right) = \lambda\sigma_{RJ}^2 + \lambda\left(\log\left(1+\mu_{RJ}\right) - \frac{1}{2}\sigma_{RJ}^2\right)^2 + \sigma_{OJ}^2.$$

Given the estimates of the SVRJOJ-model in Table 3.4, the variance decomposition is provided in Table 3.5. The estimated variances due to the jumps are 0.009 and 0.032, in the respective periods. These values can be split into a variance of 0.006 (0.026) due to the random jumps and 0.003 (0.006) due to the overnight jumps in the first (second) sample period. The proportion of the total variance due to jumps has increased to around 50% in both sample periods. On average 25% of this part has to be attributed to the overnight jumps, once more indicating that the inclusion of overnight jumps is nonnegligible.

This section showed that the most appealing model is clearly the SVRJOJ-model, allowing for difference in intraday asset return variance and overnight asset return variance. The SVRJOJ-model fits empirical option prices best in two different sample periods.[22] Since this model contains the overnight jump part, which covers approxi-

[21]Note that the addition of overnight jumps comes at the cost of a worse empirical identifiability of λ. This is reflected by the higher standard deviation of the estimate of λ in the SVRJOJ-model compared to the SVRJ-model.

[22]As the focus of this chapter is on identifying overnight jump influences, an out-of-sample analysis

mately one quarter of total jump variance, the estimation results show that overnight periods are important and have a considerable impact on option prices. The economic content of this result is that the risk of overnight closures is identifiable from option prices. Investors that have positions in these options are faced with an additional and undiversifiable source of risk which was previously attributed to random jump risk.[23]

3.5 Summary

This chapter presented an option pricing model that explicitly models the influence of nontrading overnight periods on option prices. One of the main conclusions is that both random jumps during trading periods and the overnight jump are important in explaining observed option prices. The results show that in two sample periods, of which the first can be characterized as a period of low volatility and the second as a period of high volatility, the added jump component covers a significant amount of the variation in the underlying value (risk-neutral) process. In more detail, the results show that the overnight jump part covers approximately one quarter of total jump variation. Moreover, fifty percent of the daily variance is explained by jumps, either random or overnight. Furthermore, the empirical results reveal that the model including the overnight jump component gives a better fit of empirical option prices than the traditional asset pricing models. Finally, the results show that a model containing only overnight jumps in combination with stochastic volatility has the same problem as a pure stochastic volatility model: the estimated volatility of volatility is too large in comparison to the volatility of volatility extracted from volatility series.

3.A Option Pricing Formulas

The theoretical formula for a plain vanilla call option is derived given the risk-neutral processes in (3.2) and (3.3). The put price follows similarly. Using Ito's Lemma, the stochastic differential of $\log S_t$ is

$$\mathrm{d}\log S_t = \left(r - \frac{1}{2}\sigma_t^2\right)\mathrm{d}t + \sigma_t \mathrm{d}W_t^S + \mathrm{d}\left(\sum_{i=1}^{N_t} \log Y_i\right) - \mathrm{d}A_t + \mathrm{d}\left(\sum_{i=1}^{\lfloor 252t \rfloor} \log V_i\right).$$

of pricing errors or hedging errors based on the parameter estimates in Table 3.2 is omitted.

[23]The pricing of this additional risk is postponed to future research.

Following Scott (1997), the call option value formula is given by

$$\begin{aligned} C_t(K,T) &= B_t \mathrm{E}_t\left(\frac{\max\left(S_T - K, 0\right)}{B_T}\right) \\ &= S_t P_1 - e^{-r(T-t)-nr^o/252} K P_2, \end{aligned}$$

where

$$\begin{aligned} P_1 &= \int_X^\infty \frac{S_T}{\mathrm{E}_t(S_T)} p_t(S_T) dS_T, \\ P_2 &= \mathbb{P}_t(S_T > K). \end{aligned}$$

Since the probability density function is unknown under our assumptions regarding the evolution of stock and money market, Fourier inversion techniques are used to derive expressions for P_1 and P_2 (see Bakshi and Madan (2000)). For P_2 this gives

$$P_2 = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \mathrm{Re}\left(\frac{\exp\left(-i\alpha \log K\right) \varphi\left(\alpha\right)}{i\alpha}\right) \mathrm{d}\alpha, \tag{3.7}$$

where $\varphi\left(\alpha\right)$ denotes the characteristic function of the random variable $\log S_T$, i.e. $\varphi\left(\alpha\right) = \mathrm{E}_t \exp\left(i\alpha \log S_T\right)$. The probability P_1 will be obtained later from P_2. Given the process of $\log S_t$ above, $\varphi\left(\alpha\right)$ can be written as, with $\tau = T - t$

$$\begin{aligned} \varphi(\alpha) &= \mathrm{E}_t\left\{\exp\left(i\alpha \log S_T\right)\right\}, \\ &= \mathrm{E}_t\left\{\exp\left(i\alpha\left[\log S_t + r\tau - \frac{1}{2}\int_t^T \sigma_u^2 \mathrm{d}u + \int_t^T \sigma_u \mathrm{d}W_u^S + \right.\right.\right. \\ &\quad \left.\left.\left. + \sum_{i=N_t+1}^{N_T} \log Y_i - (A_T - A_t) + \sum_{i=\lfloor 252t\rfloor+1}^{\lfloor 252T\rfloor} \log V_i\right]\right)\right\} \\ &= \mathrm{E}_t\left\{\exp\left(i\alpha\left[\log S_t + r\tau - \frac{1}{2}\int_t^T \sigma_u^2 \mathrm{d}u + \int_t^T \sigma_u \mathrm{d}W_u^S\right]\right)\right\} \times \\ &\quad \mathrm{E}_t\left\{\exp\left(i\alpha\left[\sum_{i=N_t+1}^{N_T} \log Y_i - (A_T - A_t)\right]\right)\right\} \mathrm{E}_t\left\{\exp\left(i\alpha \sum_{i=\lfloor 252t\rfloor+1}^{\lfloor 252T\rfloor} \log V_i\right)\right\}. \end{aligned}$$

The characteristic functions of the various parts will be derived separately. The first part is equal to formula (17) in Heston (1993), i.e.

$$\begin{aligned} &\mathrm{E}_t\left\{\exp\left(i\alpha\left(\log S_t + r\tau - \frac{1}{2}\int_t^T \sigma_u^2 \mathrm{d}u + \int_t^T \sigma_u \mathrm{d}W_u^S\right)\right)\right\} \\ &= \exp\left(C\left(\tau;\alpha\right) + D\left(\tau;\alpha\right)\sigma_t^2 + i\alpha \log S_t\right), \end{aligned}$$

where

$$\begin{aligned} C(\tau;\alpha) &= ri\alpha\tau + \frac{\kappa\sigma^2}{\sigma_\sigma^2}\left\{(\kappa - \rho\sigma_\sigma i\alpha + d)\,\tau - 2\log\left(\frac{1 - ge^{d\tau}}{1-g}\right)\right\}, \\ D(\tau;\alpha) &= \frac{\kappa - \rho\sigma_\sigma i\alpha + d}{\sigma_\sigma^2}\frac{1 - e^{d\tau}}{1 - ge^{d\tau}}, \end{aligned}$$

and

$$\begin{aligned} g &= \frac{\kappa - \rho\sigma_\sigma i\alpha + d}{\kappa - \rho\sigma_\sigma i\alpha - d}, \\ d &= \sqrt{(\rho\sigma_\sigma i\alpha - \kappa)^2 + \sigma_\sigma^2\,(i\alpha + \alpha^2)}. \end{aligned}$$

The random jump part of the model is described by means of a compensated compound Poisson process. The lognormal distribution of the jump sizes Y_i determines the characteristic function as, still with $\tau = T - t$

$$\begin{aligned} &\mathrm{E}_t\left\{\exp\left(i\alpha\left[\sum_{i=N_t+1}^{N_T}\log Y_i - (A_T - A_t)\right]\right)\right\} = \\ &= \exp\left\{(A_T - A_t)/\mu_{RJ}\left[(1+\mu_{RJ})^{i\alpha}\exp\left(\left(\frac{i\alpha}{2}\right)(i\alpha - 1)\,\sigma_{RJ}^2\right) - 1\right] - i\alpha\,(A_T - A_t)\right\}, \end{aligned}$$

where the compensator is given by

$$A_t = \lambda\mu_{RJ}\left[(1-c)\,t + c\lfloor 252t\rfloor/252\right].$$

Note that for integer values of 252τ, this expression does not depend on c. The expression for the characteristic function of the fixed jump part is more tractable since (relative to the random jump part) one source of randomness disappears. The characteristic function then can be calculated, using the lognormal jump sizes V_i, as

$$\mathrm{E}_t\left\{\exp\left(i\alpha\sum_{i=\lfloor 252t\rfloor+1}^{\lfloor 252T\rfloor}\log V_i\right)\right\} = \exp\left(i\alpha n r^o/252 - \frac{1}{2}\alpha\,(\alpha + i)\,n\sigma_{OJ}^2/252\right).$$

where $n = \lfloor 252T\rfloor - \lfloor 252t\rfloor$. The characteristic function of the terminal stock price is determined and can be used to obtain P_2 in the option pricing formula.

In order to obtain P_1 observe the following lemma with $Y = \log S_T$.

Lemma 3.1. *Let Y be a random variable whose distribution has density p and characteristic function φ and for which $\mathrm{E}\{\exp(\mathrm{Y})\} < \infty$. Define the distribution F by its survival function*

$$1 - F(z) = \int_z^\infty \frac{\exp(y)}{\mathrm{E}\{\exp(\mathrm{Y})\}}p(y)\,dy.$$

Then, F has characteristic function $\tilde{\varphi}$ with

$$\tilde{\varphi}(\alpha) = \frac{\varphi(\alpha - i)}{\mathrm{E}\{\exp(\mathrm{Y})\}}.$$

Proof. Let Z have distribution function F and density

$$f(z) = \frac{\exp(z)\, p(z)}{\mathrm{E}\{\exp(Y)\}}.$$

Now

$$\begin{aligned}
\tilde{\varphi}(\alpha) &= \mathrm{E}\exp(i\alpha Z) = \int_{-\infty}^{\infty} \exp(i\alpha z)\, \frac{\exp(z)\, p(z)}{\mathrm{E}\{\exp(Y)\}} \mathrm{d}z \\
&= \int_{-\infty}^{\infty} \frac{\exp(i(\alpha - i) z)}{\mathrm{E}\{\exp(Y)\}} p(z)\, \mathrm{d}z = \frac{\mathrm{E}\exp\{i(\alpha - i) Y\}}{\mathrm{E}\{\exp(Y)\}} \\
&= \frac{\varphi(\alpha - i)}{\mathrm{E}\{\exp(Y)\}},
\end{aligned}$$

which concludes the proof of the Lemma. □

Comparable to (3.7), this leads to

$$P_1 = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \mathrm{Re}\left(\frac{\exp(-i\alpha \log K)\, \varphi(\alpha - i)}{i\alpha\varphi(-i)} \right) \mathrm{d}\alpha. \tag{3.8}$$

CHAPTER 4

Nonparametric Risk-Neutral Return and Volatility Distributions

4.1 Introduction

During the past decades, a considerable amount of financial research has been devoted to the informational content of derivative prices. These prices depend on one or more underlying financial quantities and, therefore, price changes give information about the stochastic evolution of these quantities. This information is not only used in academic research but also in the everyday practice of risk management, investment strategies, and monetary policies. This chapter focuses on the information revealed by option prices about risk-neutral return and volatility distributions. An extensive literature exists on inference concerning the risk-neutral density of future stock prices/returns. However, this chapter studies the information contained in option prices concerning the joint density of returns and volatility in a nonparametric way.

Initially, stock index options were used to discover the relation between the Black-Scholes implied volatility and the realized volatility. The Black-Scholes at-the-money implied volatility is often regarded as the option market's forecast of future realized volatility over the time to maturity of the option. Jorion (1995) concludes, using foreign currency options, that implied volatility is an efficient but biased estimator of future realized volatility. However, Day and Lewis (1992) and Lamoureux and Lastrapes (1993) find that in addition to implied volatility historical volatility contains information on future volatility. Thus these papers conclude that implied volatility is an inefficient predictor of future realized volatility. A stronger conclusion comes from the study by Canina and Figlewski (1993) which claims that there is no correlation between future realized volatility and implied volatility. Finally, by using a longer time series of volatilities and a lower frequency Christensen and Prabhala (1998) finds that implied volatility is a less biased estimator for future realized volatility than is reported in previous studies. Summarizing, various tests show that implied volatility is a biased predictor of future realized volatility.[1] A possible explanation is the existence of a negative volatility risk premium in stochastic volatility models (see Chernov (2002)). Hull and White (1987) shows that in a stochastic volatility setting the price of a call option equals the expected value of the Black-Scholes formula evaluated in the average integrated volatility.[2] This expectation should be taken under the risk-neutral measure which can be separated in an expectation under the objective measure and a risk premium term. Hence, if volatility risk is priced, the future realized volatility will deviate from the Black-Scholes implied volatility.

Furthermore, the information contained in index option prices was utilized to infer the risk-neutral density of the future value of the option's underlying index. The risk-neutral density is also known as the state price density or the implied density. The name "state price density" derives from the insight that a set of option prices with all possible strikes determines the continuous state equivalent of Arrow-Debreu securities.[3] A popular device to extract risk-neutral return distributions from option prices is based on Breeden and Litzenberger (1978). That paper shows that under the condition that a continuum of European options with the same maturity date and strike prices from zero to infinity exists, the risk-neutral return density can be obtained as the second derivative of the call option pricing function with respect to the strike price. However, in practice, only a finite number of options is available. Since the work of Shimko (1993), Rubinstein (1994), and Jackwerth and Rubinstein (1996) numerous papers have appeared that provide numerical techniques to solve this problem. For an overview of the various methods, see Coutant, Jondeau, and Rockinger (1998), Jackwerth (1999), Bliss and Panigirtzoglou (2002), and Panigirtzoglou and Skiadopoulos (2004).[4] These methods

[1]See also Poteshman (2000), Bandi and Perron (2001), and Blair, Poon, and Taylor (2001).

[2]This result is derived under the assumption of no correlation between the stock (index) return and the instantaneous volatility.

[3]The work of Ross (1976) provides the insight that there should be a way to extract state-contingent prices from option prices. The relationship is made explicit in Banz and Miller (1978) and Breeden and Litzenberger (1978).

[4]The focus of this chapter is on nonparametric techniques. However, there are also studies that utilize

aim to extract the implied state price density at a single point in time. Panigirtzoglou and Skiadopoulos (2004) investigate the dynamics of the implied distributions and provide algorithms that make their results applicable to areas like option pricing and risk management.

The evolution of estimated risk-neutral return distributions has been studied extensively. In particular, Jackwerth and Rubinstein (1996) shows that, before the 1987 stock market crash, both the risk-neutral return distribution and the objective distribution are close to log-normal (over a 1-month horizon). However, after the 1987 crash the objective distribution still appears log-normal but the shape of the implied distribution has changed considerably.[5] Weinberg (2001), Anagnou, Bedendo, Hodges, and Tompkins (2002), and Bliss and Panigirtzoglou (2002) and find a typical post-crash shape of the implied risk-neutral distribution using S&P-500 index options.[6] Bates (2000) provides three possible explanations for a shift in the implied distribution after the crash: a change in the investors' assessment of the stochastic process that the S&P-500 index follows, a change in aggregate risk aversion, and mispricing. The first and second explanation led to a number of papers that estimate the risk aversion of a representative investor as implied by the risk-neutral return distribution and an estimated objective distribution. The resulting implied risk aversion function seems to be inconsistent with theory. For example, the risk aversion functions estimated in Jackwerth (2000) imply that investors are more risk averse at high and low levels of wealth. A similar conclusion can be drawn from the results in Hodges, Tompkins, and Ziemba (2003). Aït-Sahalia and Lo (2000) finds decreasing but non-monotonic implied risk aversion functions as wealth increases.[7] In a non-published paper, Brown and Jackwerth (2001) calls the phenomenon that implied risk aversion functions do not fit with the requirements of

parametric density functions to model the implied density. For instance, Ritchey (1990), Bahra (1997), Melick and Thomas (1997), Soderlind and Svensson (1997), and Gemmill and Saflekos (2000) propose a mixture of lognormals.

[5] Rubinstein (1994) states that the change in the shape of the implied volatility smile after the crash, which is directly related to the change in the shape of the implied distribution, seems to indicate an increasing crash-o-phobia.

[6] In several studies the forecasting power of different implied distributions is tested. Most of these studies reject the hypothesis that the option implied distribution is an accurate forecast of the distribution of the future value of the underlying asset. See Anagnou, Bedendo, Hodges, and Tompkins (2002) for a detailed overview of these studies.

[7] Other studies that examine implied risk aversion functions include Aït-Sahalia, Wang, and Yared (2001), Coutant (2001), Weinberg (2001), Perignon and Villa (2002), Rosenberg and Engle (2002), Bliss and Panigirtzoglou (2004).

economic theory even "the pricing kernel puzzle". The final explanation is suggested by market practitioners who claim that large demand causes the high out-of-the-money put option prices. Bollen and Whaley (2004) indeed shows that buyer-initiated trading in out-of-the-money puts dominates the market. This may lead to the conclusion that the changing patterns in implied volatility provide only limited information on the distribution of the underlying index.

The existing methods all consider risk-neutral return distributions only. At the same time, the methods mostly recognize that risk-neutral volatility distributions are important as well to test financial theory and in the application of risk management.[8] So far, the papers that study risk-neutral volatility distributions use parametric methods, as for example in Pan (2002). In contrast, this chapter extracts information on the joint risk-neutral density of returns and future spot volatility from plain vanilla option prices without making parametric assumptions. Under the assumption that option prices depend on moneyness, spot volatility, and time-to-maturity, this chapter provides a nonparametric (and thus model free) methodology that gives the empirical risk-neutral distribution of both asset returns and instantaneous volatility. While the risk-neutral volatility distributions are interesting per se for financial theory, they can also be used to test parametric stochastic volatility models and to obtain nonparametric estimates of prices of derivatives written on volatility, like variance swaps.

In particular, a number of interesting new facts about risk-neutral distributions are documented. First of all, while the risk-neutral return distributions exhibit significant negative skewness (a premium for crash risk), the results show that this skewness disappears in situations of decreasing volatility levels.[9] Apparently, decreasing volatility reduces the risk premium for crashes. At the same time, the empirical results reveal that increasing volatility, also in risk-neutral terms, goes together with decreasing asset prices. Concerning the risk-neutral volatility distribution, a clear positive skewness is documented that is not present in, for instance, the parametric Heston (1993) model. This indicates that the market is more averse towards high volatility states than is implied by the Heston (1993) model. At the same time, the Heston (1993) model is sufficiently flexible to describe the risk-neutral return distribution accurately but it does so by overestimating the volatility of volatility. Furthermore, the results show that the

[8] See the introduction of Aït-Sahalia and Lo (1998) for a detailed motivation for the importance of implied distributions.

[9] Given the statistical definition of skewness this result seems strange. However, the "volatility" that appears in the denominator of skewness differs from the volatility that is mentioned in the text. The distinction will become clear later in this chapter.

skewness of the risk-neutral volatility distribution depends on the overall volatility level which possibly indicates the presence of a jump component in the risk-neutral volatility process that has a volatility dependent jump intensity. Finally, the estimated volatility density indicates that the volatility risk premium depends in a non-linear way on the current level of volatility.

The rest of the chapter is organized as follows. Section 4.2 presents the proposed methodology for obtaining risk-neutral distributions of returns and volatilities jointly. Sections 4.3.1 and 4.3.2 show how the approach relates to existing methods that yield nonparametric estimates of risk-neutral return distributions. Moreover, Section 4.3.3 illustrates the scope of the method in three hypothesized stochastic volatility worlds. In Section 4.4 the method is applied to recent S&P-500 data which leads to several new insights in risk-neutral volatility distributions. Section 4.5 summarizes the main conclusions of this chapter.

4.2 Estimation Methodology

This section proposes a new methodology to extract risk-neutral return and spot volatility distributions from plain vanilla options. As explained in the introduction, nonparametric estimates of risk-neutral return distributions, as they are available in the literature, use butterfly spreads or, more specifically, the second derivative of option prices with respect to the strike price of the option, see Breeden and Litzenberger (1978). Such approaches, by construction, only lead to risk-neutral return distributions. They could be used to infer risk-neutral volatility distributions if derivatives were traded whose payoff depends on volatility. However, such derivatives are not (liquidly) traded. The approach in this chapter is based on the straightforward observation that standard plain vanilla options *before* maturity have a value that does depend on both the asset's price and the instantaneous volatility of the asset.

To be more precise, consider a financial market for an asset whose price at time t is denoted by S_t. Assume that the spot volatility in this market is stochastic and denote it by σ_t. Fix a horizon Δ. The return of the asset over the interval $(t, t+\Delta]$ is written $R_{t:t+\Delta} := \log S_{t+\Delta}/S_t$. Also assume that interest rates are constant at a level r and denote excess returns by $\tilde{R}_{t:t+\Delta} := R_{t:t+\Delta} - r\Delta$. The risk-neutral distribution (joint in prices and volatilities) is denoted by $\mathbb{Q}$. The main interest of this chapter is the conditional distribution of $(S_{t+\Delta}, \sigma_{t+\Delta})$, given all information available at time t. This information set is denoted by $\mathcal{F}_t$. The risk-neutral process of prices and volatilities are assumed to be Markovian and homogeneous with respect to the initial price level throughout this chapter.

Assumption 1. The risk-neutral distribution of $\left(\tilde{R}_{t:t+\Delta}, \sigma_{t+\Delta}\right)$ conditional on all the information available at time t, is equal to this distribution given σ_t. Formally, for all $\Delta > 0$

$$\mathcal{L}_{\mathbb{Q}}\left(\begin{bmatrix} \tilde{R}_{t:t+\Delta} \\ \sigma_{t+\Delta} \end{bmatrix} |\mathcal{F}_t\right) = \mathcal{L}_{\mathbb{Q}}\left(\begin{bmatrix} \tilde{R}_{t:t+\Delta} \\ \sigma_{t+\Delta} \end{bmatrix} |\sigma_t\right).$$

Under Assumption 1, time t call option prices, with maturity T and exercise price K can be written as

$$\begin{aligned} C_t(K,T) &= \mathrm{E}^{\mathbb{Q}}\left\{\exp(-r(T-t))\max\left\{S_T - K, 0\right\}|\mathcal{F}_t\right\} \\ &= S_t \mathrm{E}^{\mathbb{Q}}\left\{\max\left\{\exp(\tilde{R}_{t:T}) - \frac{\exp(-r(T-t))K}{S_t}, 0\right\}|\mathcal{F}_t\right\} \\ &= S_t c(m_t, T-t; \sigma_t), \end{aligned} \tag{4.1}$$

for some deterministic function c, time t moneyness $m_t = \exp(-r(T-t))K/S_t$, and time-to-maturity $T-t$. Assumption 1 excludes the possibility that current price *levels* influence the excess return volatility. This assumption underlies most of the empirical asset pricing models, both in continuous time and in discrete time.[10]

Relation (4.1) cannot be used to estimate risk-neutral volatility distributions directly due to the occurrence of the current volatility level σ_t only, without simultaneous reference to future volatility levels. However, the First Fundamental Theorem of Asset Pricing does provide such a relation as it implies, for $T \geq t+\Delta \geq t$

$$C_t(K,T) = \mathrm{E}^{\mathbb{Q}}\left\{\exp(-r\Delta)C_{t+\Delta}(K,T)|\mathcal{F}_t\right\}.$$

Substituting (4.1) for $C_t(K,T)$ and $C_{t+\Delta}(K,T)$ yields

$$S_t c(m_t, T-t; \sigma_t) = \mathrm{E}^{\mathbb{Q}}\left\{\exp(-r\Delta)S_{t+\Delta}c(m_{t+\Delta}, T-(t+\Delta); \sigma_{t+\Delta})|\mathcal{F}_t\right\},$$

or, using $m_{t+\Delta} = \exp(-r(T-(t+\Delta)))K/S_{t+\Delta} = m_t \exp(-\tilde{R}_{t:t+\Delta})$

$$c(m_t, T-t; \sigma_t) = \mathrm{E}^{\mathbb{Q}}\left\{\exp(\tilde{R}_{t:t+\Delta})c(m_t\exp(-\tilde{R}_{t:t+\Delta}), T-(t+\Delta); \sigma_{t+\Delta})|\mathcal{F}_t\right\}. \tag{4.2}$$

[10] An example of a model that does not satisfy Assumption 1 is a stochastic volatility model where the drift or diffusion function of the stochastic volatility process depends on the current price level S_t. These type of models are usually not examined (empirically) in the literature and therefore Assumption 1 is not too restrictive.

Denote the joint risk-neutral conditional density of $(\tilde{R}_{t:t+\Delta}, \sigma_{t+\Delta})$ at time t by $q(r, v|\sigma_t)$. Invoking the Markov property in Assumption 1 once more, expression (4.2) can be rewritten, for all $H \geq \Delta$, $m > 0$, and $\sigma > 0$, as

$$c(m, H; \sigma) = \int_z \int_v \exp(z) c(m \exp(-z), H - \Delta; v) q(z, v|\sigma) \mathrm{d}z \mathrm{d}v. \tag{4.3}$$

Relation (4.2), or the integral equivalent (4.3), does simultaneously involve current volatility levels σ_t and future volatility levels $\sigma_{t+\Delta}$. This observation can be used to infer risk-neutral volatility distributions over a given horizon Δ, jointly with the risk-neutral return distribution. In order to infer the risk-neutral return/volatility density q, the c function is estimated nonparametrically and subsequently, the integral equation (4.3) is solved for q.

Given empirically observed plain vanilla option prices, Aït-Sahalia and Lo (1998) is followed to come up with a nonparametric estimate of the function c. More precisely, given option prices $C_t(K, T)$, the c function can be expressed as

$$\frac{C_t(K, T)}{S_t} = c(m_t, T - t, \sigma_t) = BS\left(m_t, T - t, IV\left(m_t, T - t, \sigma_t\right)\right), \tag{4.4}$$

with

$$BS\left(m, T - t, \sigma^2\right) = \Phi\left(\frac{\log(m) + \sigma^2(T - t)/2}{\sigma\sqrt{T - t}}\right) - m\Phi\left(\frac{\log(m) - \sigma^2(T - t)/2}{\sigma\sqrt{T - t}}\right). \tag{4.5}$$

Here BS stands for the Black-Scholes formula (normalized by the current stock price level) and IV denotes the option's Black-Scholes implied volatility. The Black-Scholes implied volatility is assumed to depend on moneyness m_t, time-to-maturity $T - t$, and spot volatility σ_t.[11] A detailed comparison of this chapter's approach with that of Aït-Sahalia and Lo (1998) is provided in Section 4.3.2, but note already that in this chapter's method implied volatilities depend on spot volatilities and not on the stock price level (other than through moneyness). Since spot volatilities are unobserved, these are filtered using an estimated EGARCH model

$$\begin{aligned} R_{t:t+\Delta} &= \mu + \sqrt{h_t}\ \varepsilon_{t:t+\Delta}, \\ \varepsilon_{t:t+\Delta}|\mathcal{F}_t &\sim N(0, h_t), \\ \log h_t &= \omega + \beta \log h_{t-\Delta} + \alpha \left| \frac{\varepsilon_{t-\Delta:t}}{\sqrt{h_{t-\Delta}}} \right| + \gamma \frac{\varepsilon_{t-\Delta:t}}{\sqrt{h_{t-\Delta}}}. \end{aligned} \tag{4.6}$$

[11] Note here the difference between the spot volatility σ_t in (4.4) and the Black-Scholes implied volatility σ in (4.5). The spot volatility σ_t in (4.4) is used (together with moneyness and time to maturity) to obtain an estimate of the Black-Scholes implied volatility. This estimate is translated to an estimate of the c function by means of (4.5).

Nelson and Foster (1994) shows that the EGARCH volatilities filtered from (4.6) provide consistent (as the data frequency increases) estimates of the underlying spot volatility for general stochastic volatility diffusion models. The effect of the filtering step is assessed in Section 4.3.3.[12] Following Aït-Sahalia and Lo (1998) once more, kernel-based nonparametric regression is used to estimate the implied volatility of observed options as a function of moneyness, time-to-maturity, and spot-volatility. For each of these variables the kernel function is chosen as the fourth order kernel function that is given in the appendix of Aït-Sahalia and Lo (1998). The bandwidths used in these kernel functions are determined according to the procedures described in the aforementioned appendix.

Observe that the current volatility level σ_t or the filtered equivalent $\sqrt{h_t}$ is used as explanatory variable in the nonparametric regression for the implied volatility. At the same time, Assumption 1 is relied on to ignore possible dependence on current price levels. If deemed appropriately, price levels could be added in the regression as long as the curse of dimensionality does not affect the results, that is, as long as sufficient data points are available. Moreover, observe that the risk-neutral return distribution $\mathcal{L}_{\mathbb{Q}}\left(\tilde{R}_{t:t+\Delta}|\sigma_t\right)$ can be obtained directly from the nonparametric implied volatility estimate using the Breeden and Litzenberger (1978) result. This, however, does not lead to risk-neutral volatility distributions $\mathcal{L}_{\mathbb{Q}}\left(\sigma_{t+\Delta}|\sigma_t\right)$, nor to the risk-neutral dependence between returns and volatilities.

The estimate of the risk-neutral joint return/volatility distribution is obtained by solving the integral equation (4.3) for q using the estimated option price function $\hat{c}(m_t, T-t, \sigma_t)$. More precisely, a grid $z_0 < z_1 < z_2 < \ldots < z_M$ is chosen for excess asset returns and a grid $0 < v_0 < v_1 < \ldots < v_N$ for volatility levels.[13] Expression (4.3) is discretized as

$$\hat{c}(m, H; \sigma) = \sum_{i=1}^{M} \sum_{j=1}^{N} \exp(z_i) \hat{c}(m \exp(-z_i), H - \Delta, v_j) q(z_i, v_j|\sigma)(z_i - z_{i-1})(v_j - v_{j-1}). \tag{4.7}$$

Equation (4.7) provides, for each moneyness m and each time-to-maturity $H > \Delta$ a linear equation in $q(z_i, v_i|\sigma)$. A suitable grid of possible values is selected for both moneyness and time-to-maturity to obtain a system of linear equations. The grid exploits

[12] There are many other ways to obtain an estimate of spot volatility using high-frequency data (see, for instance, Jiang and Oomen (2004)) or option data (see Bakshi, Cao, and Chen (1997) and Pan (2002)). The simulation study later in this chapter shows that for several stochastic volatility models the EGARCH procedure as proposed above is an adequate solution.

[13] The endpoints z_M and v_N are based on the distribution of moneyness and estimated spot volatility in the available data set. By doing this, the estimated density becomes a truncated density.

the fact that it is coarser for low density areas. Details are available upon request. The resulting system of linear equations is solved numerically by means of a standard least-squares algorithm. In the algorithm is imposed that the probabilities $q(z_i, v_j)$ are positive, the unit-integral condition

$$\sum_{i=1}^{M}\sum_{j=1}^{N} q(z_i, v_j|\sigma)(z_i - z_{i-1})(v_j - v_{j-1}) = 1,$$

and the constraint that excess returns have zero risk-neutral expectations, i.e.

$$\sum_{i=1}^{M}\sum_{j=1}^{N} z_i q(z_i, v_j|\sigma)(z_i - z_{i-1})(v_j - v_{j-1}) = 0.$$

Since $\hat{c}$ is a nonparametric estimate, the numerical integral approximation (4.7) could lead to non-smooth densities. Therefore, a dimension-reduction and a smoothness penalty are used in the simulation and empirical sections. The dimension reduction is easily obtained using the observation above that the Breeden and Litzenberger (1978) result leads to $\mathcal{L}_{\mathbb{Q}}\left(\tilde{R}_{t:t+\Delta}|\sigma_t\right)$, i.e. the marginal distribution of $q(\tilde{R}_{t:t+\Delta}, \sigma_{t+\Delta}|\sigma_t)$ with respect to returns. This reduction is used throughout the chapter. Secondly, following Jackwerth and Rubinstein (1996), a smoothness condition on the solution q as a function of returns and volatilities is added. This smoothness condition is a penalty on the second derivative of q with respect to both returns and volatilities. Formally, the least-squares criterion function is extended with the terms

$$\sum_{i=1}^{M-1}\left[(z_{j+1} - z_j)\, q\left(z_{i-1}, v_j|\sigma\right) - (z_{j+1} - z_{j-1})\, q\left(z_i, v_j|\sigma\right) + (z_j - z_{j-1})\, q\left(z_{i+1}, v_j|\sigma\right)\right]^2,$$

$$\sum_{j=1}^{N-1}\left[(v_{j+1} - v_j)\, q\left(z_i, v_{j-1}|\sigma\right) - (v_{j+1} - v_{j-1})\, q\left(z_i, v_j|\sigma\right) + (v_j - v_{j-1})\, q\left(z_i, v_{j+1}|\sigma\right)\right]^2,$$

each with an appropriate penalty factor.

Summarizing, the proposed method consists of the following four steps.

1. Filter spot volatilities using an EGARCH model for observed returns as in (4.6);

2. Calculate a nonparametric estimate of Black-Scholes implied volatilities as in Aït-Sahalia and Lo (1998) using moneyness, time-to-maturity, and the EGARCH-filtered spot volatility as explanatory variables;

3. Apply the Breeden and Litzenberger (1978) result to obtain the marginal risk-neutral return distribution, conditionally on current values of spot volatility;

4. Solve the linear equations (4.7) to obtain the joint risk-neutral density q with respect to both returns and volatilities.

4.3 Relation with Existing Methods

The literature presents mainly two approaches to obtain nonparametric estimates of risk-neutral return distributions: the method discussed in Jackwerth and Rubinstein (1996) and the nonparametric Breeden and Litzenberger (1978) based approach of Aït-Sahalia and Lo (1998). The next section describes that the method proposed in this chapter is, with respect to the conditional information used to determine the risk-neutral return distribution, in between these two approaches. More importantly, however, the proposed method offers the additional advantage of estimating the risk-neutral volatility distribution and the risk-neutral return-volatility dependence structure. In that respect, the method applies to, e.g., the popular Heston (1993) stochastic volatility model, but, without any parametric assumptions. The present section discusses the relation of the proposed approach with both alternatives mentioned above and the performance of the proposed approach in a theoretical Heston (1993) world (Section 4.3.3).

4.3.1 Fully nonparametric methods

Shimko (1993) and Jackwerth and Rubinstein (1996) use option prices observed at a given date to infer risk-neutral probabilities of returns for a given future date. Essentially the (discrete) risk-neutral probability distribution of returns is determined such that all observed option prices today are within the bid-ask bounds. In the notation of Section 4.2, these methods provide an estimate of $\mathcal{L}_{\mathbb{Q}}\left(R_{t:t+\Delta}|\mathcal{F}_t\right)$ without any further restrictions on the conditioning information set $\mathcal{F}_t$. In particular, as noted in Aït-Sahalia and Lo (1998), no time-consistency is imposed in this method. As a result, the estimates of risk-neutral return probabilities will vary over time. The method is thus fully nonparametric, but only few observations (i.e., only options traded on a given day with a particular maturity) can be used in the estimation. The method proposed in this chapter builds on Assumption 1 which identifies the current spot volatility as the only relevant state variable for predicting risk-neutral return distributions. As mentioned before, such an assumption is common in most parametric stochastic volatility models.

4.3.2 Breeden and Litzenberger (1978) based methods

The nonparametric risk-neutral return distributions in Aït-Sahalia and Lo (1998) are based on the Breeden and Litzenberger (1978) result that the risk-neutral return distribution is proportional to the second derivative of plain vanilla call prices with respect to the exercise price. The functional relation between the option prices and relevant explanatory variables is estimated using nonparametric kernel regression of Black-Scholes implied volatilities on the futures price associated with the underlying asset, the exercise price, and time-to-maturity.[14] As mentioned above, other state variables can be added to the nonparametric regression. In particular, the current spot volatility level σ_t is added to set of state variables.

Compared to the method proposed in this chapter, Aït-Sahalia and Lo (1998) does allow for time-varying volatility of the GARCH type, i.e., where current *levels* of the stock price induce a certain volatility. However, in cases of stochastic volatility as a separate state variable, the method does not lead to risk-neutral return distributions conditional on a certain volatility level, but to unconditional distributions. Since spot volatility is considered as a separate state variable in this chapter, which is in line with the Heston (1993) model, option's implied volatilities is assumed to depend on futures prices and exercise prices through moneyness alone (see, also, Renault (1997)). Observe that in case of a stochastic volatility model, the Aït-Sahalia and Lo (1998) method will pick up of some of the stochastic volatility effects as stock prices and volatilities are (negatively) correlated.[15] This will be discussed in more detail in the next section where this chapter's method and the Aït-Sahalia and Lo (1998) method are considered in a theoretical Heston (1993) world.

4.3.3 Risk-neutral return/volatility distributions in the Heston model

Heston (1993) presents a parametric stochastic volatility model which is especially useful for calculating derivative prices due to the fact that the characteristic function of the

[14] Aït-Sahalia and Lo (1998) consider other (vectors of) explanatory variables as well, but futures price, exercise price, and time-to-maturity come out as preferred choice.

[15] Black (1976) and Christie (1982) find empirical evidence of this negative leverage effect for individual stocks. Tompkins (2001b) reports a similar conclusion for several futures markets in the nineties. Theoretical explanations for the negative correlation between equity (index) returns and instantaneous (conditional) volatility can, among others, be found in Black (1976), Poterba and Summers (1986), and Campbell and Hentschel (1992).

risk-neutral return distribution is known in analytical form. This latter property is a demonstration of the fact that the Heston (1993) model belongs to the class of affine jump-diffusions (see, Duffie and Kan (1996)). The Heston (1993) model is given by the dynamics, under the risk-neutral probability measure

$$\begin{aligned} \mathrm{d}S_t &= rS_t\mathrm{d}t + \sigma_t S_t \mathrm{d}W_t^S, \\ \mathrm{d}\sigma_t^2 &= \left(\kappa + \eta^V\right)\left(\frac{\kappa}{\kappa+\eta^V}\sigma^2 - \sigma_t^2\right)\mathrm{d}t + \sigma_\sigma \sigma_t \mathrm{d}W_t^\sigma, \\ \mathrm{Cov}\left\{\mathrm{d}W_t^S, \mathrm{d}W_t^\sigma\right\} &= \rho \mathrm{d}t. \end{aligned} \tag{4.8}$$

Under the objective probability measure, the dynamics can be obtained by setting $\eta^V = 0$ and $r = \mu$, the expected instantaneous return. For given parameters, the risk-neutral return distribution is known in closed form as the inverse of its Fourier transform, see Heston (1993). The risk-neutral distribution of spot volatility $\sigma_{t+\Delta}$ given σ_t is also known in analytical form, see Cox, Ingersoll, and Ross (1985). Moreover, the Heston (1993) model satisfies the Markovianity condition in Assumption 1.

In order to study the performance of the proposed method, five years (1260 trading days) of daily S&P-500 data are simulated using the Pan (2002) parameters, i.e. in the notation of (4.8), $\kappa = 6.4$, $\sigma = 0.124$, $\sigma_\sigma = 0.30$, $\rho = -0.53$, and $\eta^V = -3.1$. The interest rate is fixed at a constant annual level of 4%, the initial volatility level σ_0 is set equal to the unconditional mean $\sigma = 0.124$, and the expected instantaneous return is fixed at 10%.[16] The diffusion (4.8) is simulated using an Euler discretization with time steps of 1/200 of a day. The main interest is the risk-neutral return/volatility distribution over a period of one month. Given the fact that the parameters above are annualized, this implies that $\Delta = 1/12$. Using the simulated prices and volatilities, analytical option prices are calculated with the Heston (1993) formula. For each day options with on average four different maturities and, for each maturity, forty different strike prices are considered. The actual number of options available each day varies as a stylized option introduction scheme is implemented, in line with the methodology used by most exchanges. Finally, the method as described in Section 4.2 is applied.

Figure 4.3 shows the estimated risk-neutral return distributions for various initial volatility levels and the standard Aït-Sahalia and Lo (1998) estimate. This latter estimate is unconditional with respect to the initial spot volatility. The Aït-Sahalia and Lo (1998) estimate is provided to assess the effect of stochastic volatility on this estimate.

[16] The analytic joint density and copula that are implied by these parameter assumptions are given in Figure 4.1 and Figure 4.2. The shape of the copula is explained by the negative sign of the correlation parameter ρ.

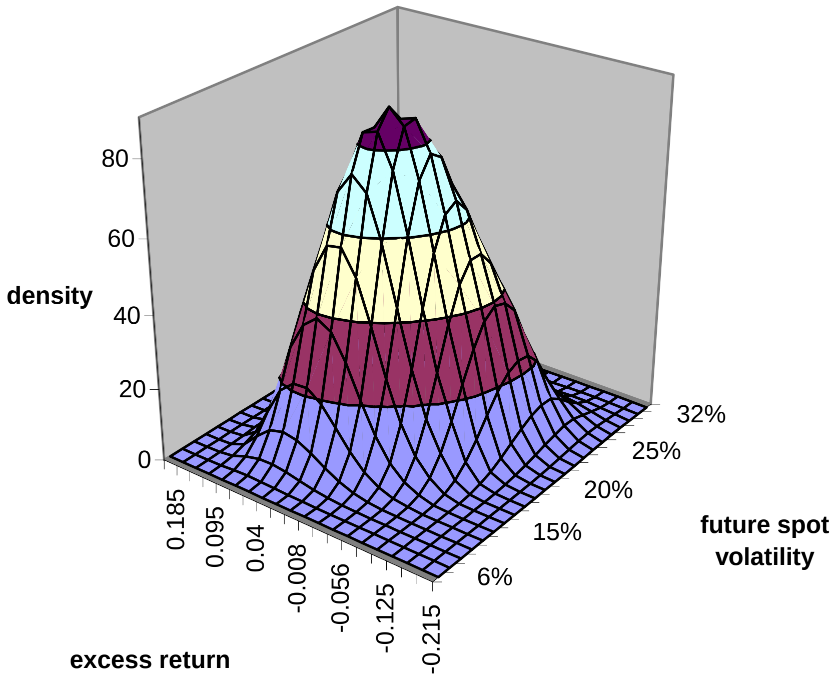


Figure 4.1: Theoretical joint risk-neutral return-volatility distribution over a horizon of one month in the Heston (1993) model with the Pan (2001) parameters.

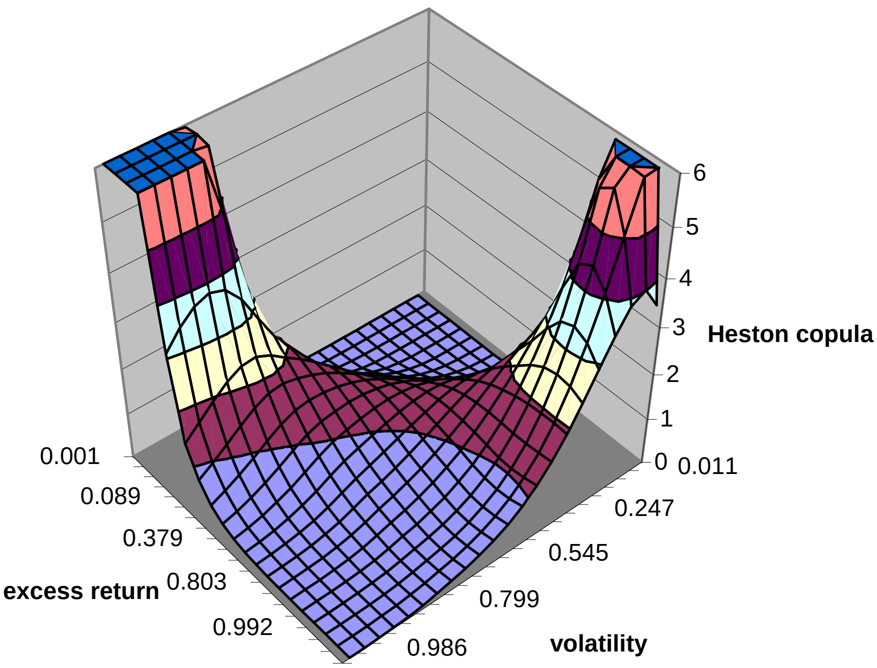


Figure 4.2: Theoretical copula over a horizon of one month in the Heston (1993) model with the Pan (2001) parameters.

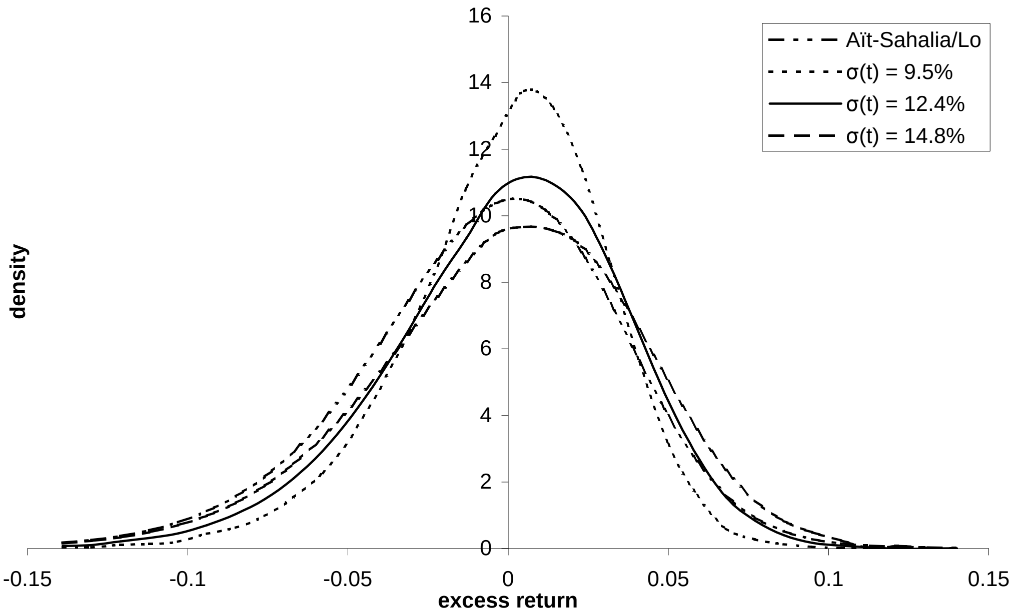


Figure 4.3: The risk-neutral return distribution estimates over a horizon of one month based on five years of simulated data in the Heston (1993) model with the Pan (2002) parameters. The solid line denotes the estimate given an initial volatility level of $\sigma_t = 12.4\%$. The dotted line corresponds to $\sigma_t = 9.5\%$ and the dashed line corresponds to $\sigma_t = 14.8\%$. The dotted-dashed line shows the (unconditional) Aït-Sahalia and Lo (1998) estimate.

The methodology proposed in this chapter is conditional on initial spot volatility levels and the low level of $\sigma_t = 9.5\%$ corresponds to the first quartile of the objective volatility distribution (as measured by the filtered EGARCH volatilities), while the high level of $\sigma_t = 14.8\%$ corresponds to the third quartile. Observe that the Pan (2002) estimates refer to a period of low overall volatilities. The figure shows some clear variation in the risk-neutral return distribution for various initial volatility levels. Moreover, in all cases the distribution is left-skewed as induced by the negative return/volatility correlation parameter ρ. As could be expected, the unconditional Aït-Sahalia and Lo (1998) estimate is closest to the estimate given an average initial volatility.

A possible issue in this chapter's methodology is the use of filtered EGARCH volatilities instead of (unobserved) actual spot volatilities. This simulation exercise allows to study the effect of using the EGARCH volatilities. Figure 4.4 shows (1) the true marginal risk-neutral return distributions, (2) the estimated marginal risk-neutral return distributions in case actual Heston spot volatilities are used and (3) the estimate using the

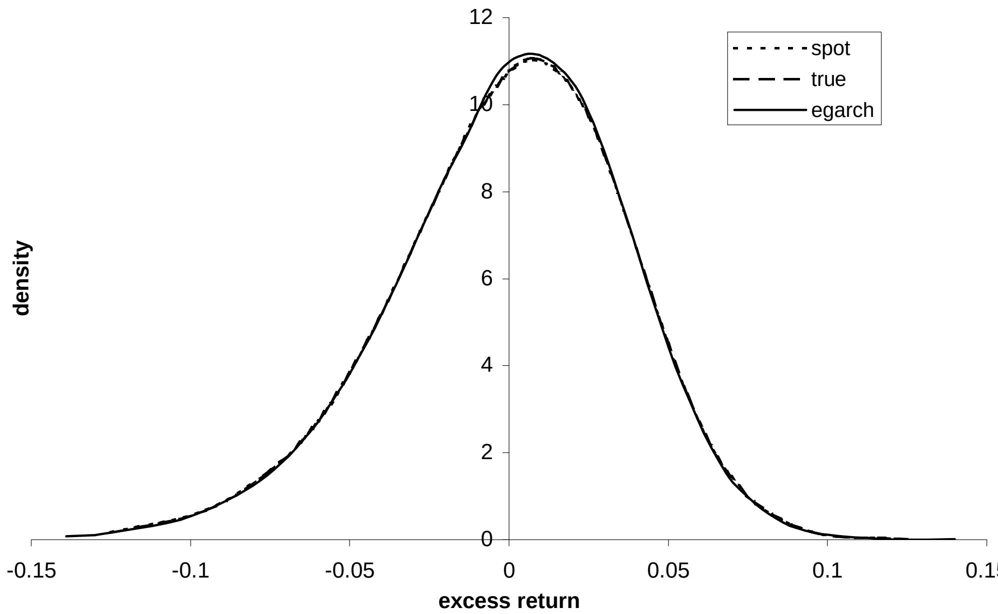


Figure 4.4: Risk-neutral return distribution estimates over a horizon of one month based on five years of simulated data in the Heston (1993) model with the Pan (2002) parameters. The solid line denotes the estimate based on EGARCH filtered volatilities. The dotted line shows the estimate based on the actual simulated spot volatilities. The dashed line shows the actual risk-neutral distribution. All distributions are conditional on a current level of spot volatility $\sigma_t = 12.4\%$.

EGARCH filtered volatilities. The graphs clearly show that the effect of using EGARCH filtered volatilities instead of the true underlying spot volatilities is negligible. Moreover, Figure 4.4 shows that the estimated densities closely follow the true risk-neutral volatility distribution. The graphs are conditional on an initial average volatility level $\sigma_t = 12.4\%$. For other initial volatility levels, the results are comparable.

Similarly, Figure 4.5 provides the risk-neutral volatility density for the same cases. Once more, the method proposed in this chapter succeeds in recovering the risk-neutral volatility distribution with high precision. This precision is due to the large number of observations that are available in this kind of analysis, due to the fact that, over a period of time, all options with more than one month to maturity are used. The effect of filtering spot volatilities is visible in these graphs. This shows that there are some statistically significant biases in the filtered volatilities. From a financial point of view, however, the differences are small, especially in the tails of the distributions. In particular, observe that in the right tail the EGARCH based estimator seems to perform

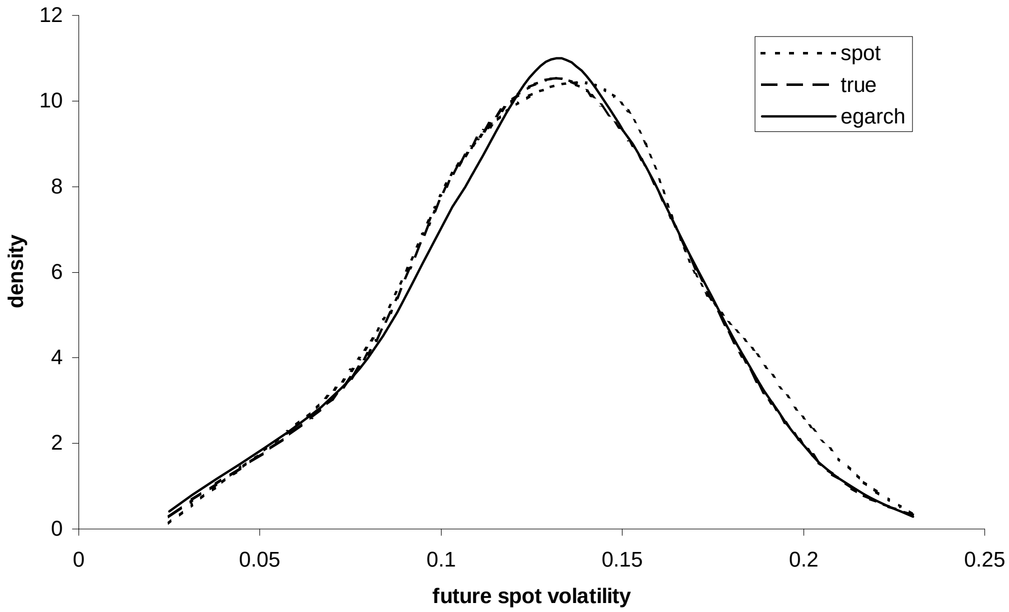


Figure 4.5: Risk-neutral volatility distribution estimates over a horizon of one month based on five years of simulated data in the Heston (1993) model with the Pan (2002) parameters. The solid line denotes the estimate based on EGARCH filtered volatilities. The dotted line shows the estimate based on the actual simulated spot volatilities. The dashed line shows the actual risk-neutral distribution. All distributions are conditional on a current level of spot volatility $\sigma_t = 12.4\%$.

better than an estimator based on the true instantaneous volatilities.

4.3.4 Risk-neutral volatility distributions in stochastic volatility models

This section analyzes the results of applying the previously discussed methodology to two stochastic volatility models that differ from the Heston (1993) model. First, the Hull and White (1987) model is considered. The dynamics of this model, under the risk-neutral measure, are given by

$$\begin{aligned} \mathrm{d}S_t &= rS_t\mathrm{d}t + \sigma_t S_t \mathrm{d}W_t^S, \\ \mathrm{d}\sigma_t^2 &= \kappa\sigma_t^2\mathrm{d}t + \sigma_\sigma\sigma_t^2\mathrm{d}W_t^\sigma, \\ \mathrm{Cov}\left\{\mathrm{d}W_t^S, \mathrm{d}W_t^\sigma\right\} &= \rho\mathrm{d}t. \end{aligned} \tag{4.9}$$

The paper provides a series solution for the value of an option that is written on S. However, this solution is derived under the assumption that $\rho = 0$. Furthermore, fast

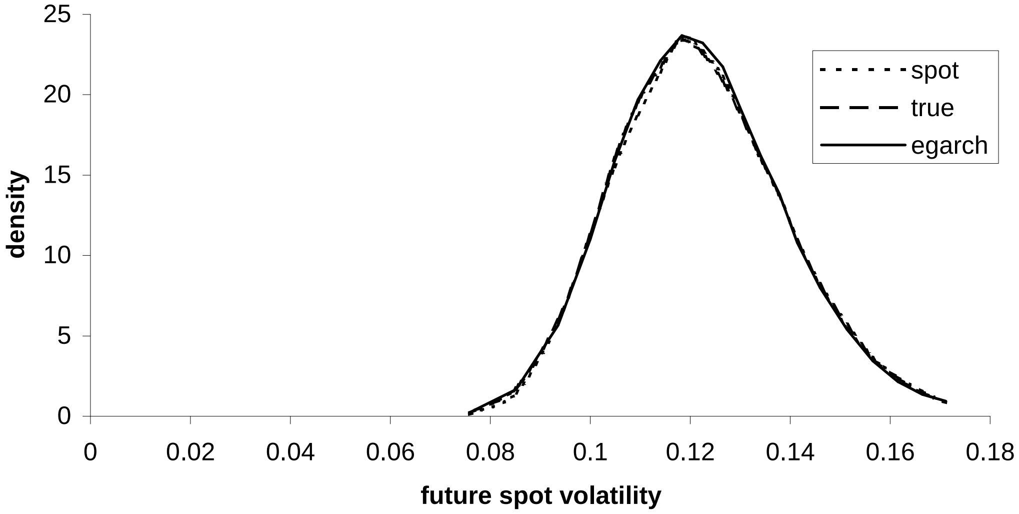


Figure 4.6: Risk-neutral volatility distribution estimates over a horizon of one month based on five years of simulated data in the Hull and White (1987) model. The solid line denotes the estimate based on EGARCH filtered volatilities. The dotted line shows the estimate based on the actual simulated spot volatilities. The dashed line shows the actual risk-neutral distribution. All distributions are conditional on a current level of spot volatility $\sigma_t = 12.4\%$.

convergence is only achieved for small values of $\sigma_\sigma^2(T-t)$, where $T-t$ is the time to maturity of the option.

The simulation experiment is designed in the same manner as in the previous section. The parameters used are based on Pan (2002) although adjustments are made to meet the assumptions in Hull and White (1987), i.e. $\kappa = 6.4$, $\sigma = 0.124$, $\sigma_\sigma = 1.00$, $\rho = 0.00$, and $\eta^V = 0.00$. Interest rates are again fixed at a level of 4%. Figure 4.6 presents the true marginal risk-neutral volatility distribution, the estimated marginal risk-neutral volatility volatility distribution if simulated spot volatilities are used, and the estimated volatility distribution using EGARCH volatilities.[17] The graphs are conditional on an initial volatility level of 12.4%. The conclusions drawn from Figure 4.6 are the same as in the Heston (1993) case of the previous section. The proposed methods succeeds in recovering the risk-neutral volatility distribution and the differences between the estimated densities using spot volatilities or EGARCH volatilities are small.

The second model considered in this section is the Stein and Stein (1991) stochastic

[17]Estimated risk-neutral return densities are not reported. The outcomes are similar to the results shown in Figure 4.4.

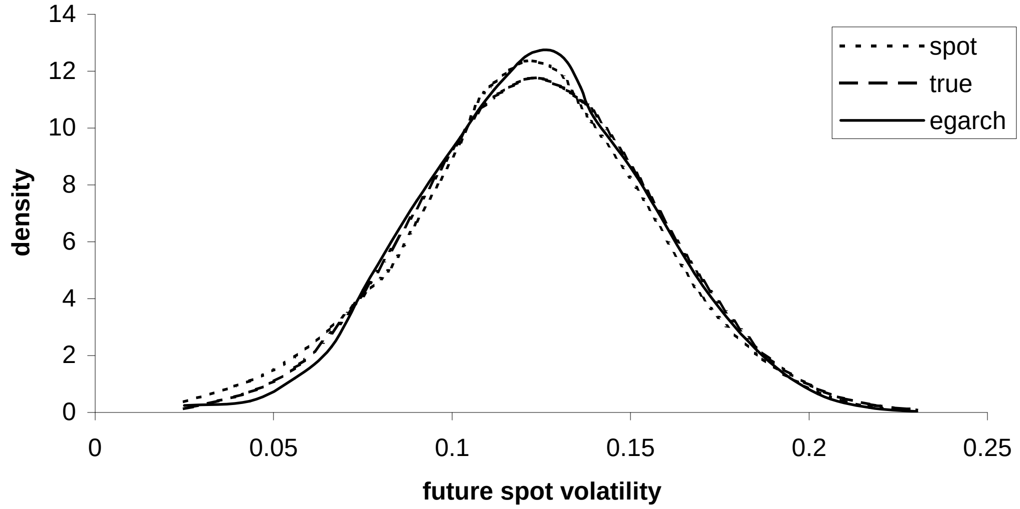


Figure 4.7: Risk-neutral volatility distribution estimates over a horizon of one month based on five years of simulated data in the Stein and Stein (1991) model. The solid line denotes the estimate based on EGARCH filtered volatilities. The dotted line shows the estimate based on the actual simulated spot volatilities. The dashed line shows the actual risk-neutral distribution. All distributions are conditional on a current level of spot volatility $\sigma_t = 12.4\%$.

volatility model. In Stein and Stein (1991) the stochastic evolution of the stock price process and the volatility process under the risk-neutral measure is

$$\begin{aligned} \mathrm{d}S_t &= rS_t\mathrm{d}t + \sigma_t S_t \mathrm{d}W_t^S, \\ \mathrm{d}\sigma_t &= \kappa\left(\sigma - \sigma_t\right)\mathrm{d}t + \sigma_\sigma \mathrm{d}W_t^\sigma, \end{aligned} \tag{4.10}$$

where the processes W^S and W^σ are assumed to be two independent Brownian Motions. Stein and Stein (1991) provides an analytical solution for the future stock price density. This solution can be used to derive theoretical prices in the model. In the simulation experiment which is conducted in the same fashion as in the previous section, parameters are based on Pan (2002) with again some necessary adjustments, i.e. $\kappa = 6.4$, $\sigma = 0.124$, $\sigma_\sigma = 0.15$, $\rho = 0.00$, and $\eta^V = 0.00$. Figure 4.7 shows marginal volatility densities for the three previously mentioned cases using an initial spot volatility of 12.4%. The obvious conclusion from this figure is that the proposed methodology is able to extract the true risk-neutral volatility distribution from simulated data independent of the chosen volatility measure.

4.4 Empirical Risk-Neutral Return/Volatility Distributions

This section provides risk-neutral return/volatility distribution estimates based on S&P-500 data. Section 4.4.1 discusses the data that are used in more detail and gives some summary statistics. In Section 4.4.2 is confirmed that the risk-neutral return distribution is negatively skewed. Moreover, the results show that the Heston (1993) model calibrated to this return distribution significantly overestimates the risk-neutral volatility of volatility. Section 4.4.3 shows the bivariate risk-neutral return and volatility distribution. The risk-neutral dependence between returns and volatilities is more apparent from the conditional distribution of returns given future volatility levels. Section 4.4.4 presents these results and shows that in situations of decreasing volatility, the return distribution is in fact slightly positively skewed.

4.4.1 Data description

The empirical results in the present chapter are based on European options traded on the Chicago Board Options Exchange over the period from July, 1999, to December, 2003. The option data are extracted from the ABN-Amro Asset Management database and contain daily closing quotes of SPX options for all trading days in the sample period. In addition, the closing S&P-500 index levels are provided. Following Jackwerth and Rubinstein (1996), dividend rates are calculated from the actual dividends paid out by the SPX stocks. The methodology presented in Section 4.2 does not treat dividends explicitly but in the empirical analysis index prices are replaced by index prices discounted by the dividend rate. Finally, interpolated LIBOR rates are employed as a proxy for the risk free rate.

Following Bakshi, Cao, and Chen (1997), only options are used that satisfy a number of criteria. More precisely, attention is restricted to calls and puts that

1. have time-to-expiration greater than or equal to six calendar days,

2. have a bid price greater than or equal to 0.03$,

3. have a bid-ask spread less than or equal to 1$,

4. have a Black-Scholes implied volatility greater than zero and less than or equal to 80% (annualized),

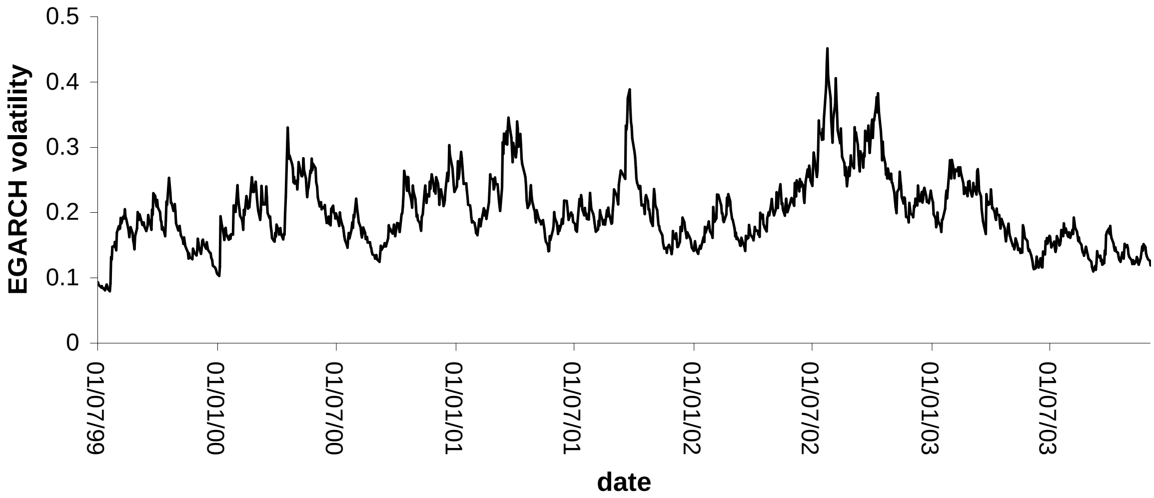


Figure 4.8: Estimated EGARCH volatilities over the sample period July, 1999, until December, 2003.

5. satisfy the arbitrage restriction,

$$C_t(K,T) \geq \max\left(0, S_t e^{-\delta(T-t)} - K e^{-r(T-t)}\right),$$

for call options and a similar restriction for put options. In this formula δ denotes the dividend rate.

Table 4.1 provides descriptive statistics on the resulting set of options.[18]

From Table 4.1 well-known patterns in implied volatilities across strikes and maturities are recognized. The volatility skew or smile is clearly present for all option categories. Unreported statistics on return data show that in the sample period the annualized standard deviation of returns equals 20.6%. Figure 4.8 shows the estimated EGARCH volatilities over the complete sample period.

[18]In Table 4.1 two measures of moneyness are employed. First, the discounted ratio of the strike price to the underlying (see, for instance, Fung and Hsieh (1991) and Bakshi, Cao, and Chen (1997)). However, this does not take the the time to maturity of the option into account (see Natenberg (1994) and Tompkins (2001a)). Therefore, a second measure of moneyness is reported in Table 4.1. This is the Black-Scholes (risk-neutral) probability of ending in the money, i.e. $N(d_2)$ for calls and $N(-d_2)$ for puts, where d_2 is given by

$$d_2 = \frac{\log(S/K) + \left(r - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

with σ as the Black-Scholes at-the-money implied volatility. This volatility is extracted from an option series with shortest maturity longer than one week. The table shows that there is hardly any difference in the implied volatility patterns for the two different measures of moneyness.

CALLS		days to expiration				days to expiration		
	$Ke^{-r(T-t)}/S$	<60	60–180	>180	$N(d_2)$	<60	60–180	>180
ITM	< 0.97	0.315	0.274	0.250	≥ 0.60	0.303	0.274	0.251
		12734	11877	4251		14434	11206	2969
ATM	0.97–1.03	0.221	0.231	0.217	0.40–0.60	0.232	0.231	0.235
		7949	6624	2350		4259	6532	3589
OTM	≥ 1.03	0.262	0.235	0.199	< 0.40	0.252	0.233	0.198
		12902	12879	1837		14892	13642	1380
	subtotal	33585	31380	8438		33585	31380	8438
PUTS		days to expiration				days to expiration		
	$Ke^{-r(T-t)}/S$	<60	60–180	>180	$N(-d_2)$	<60	60–180	>180
OTM	< 0.97	0.323	0.280	0.246	< 0.40	0.311	0.280	0.248
		14304	13086	4334		16005	12413	3052
ATM	0.97–1.03	0.233	0.220	0.214	0.40–0.60	0.230	0.228	0.230
		7947	6603	2350		4262	6531	3589
ITM	≥ 1.03	0.245	0.221	0.198	≥ 0.60	0.238	0.219	0.198
		9877	9824	1800		11861	10569	1343
	subtotal	32128	29513	8484		32128	29513	8484

Table 4.1: Summary statistics on SPX call and put option implied volatilities. Implied volatilities of options on the S&P-500 index corresponding to the last tick before 3:00 PM and the total number of observations for each maturity category are reported. The sample period is July 9, 1999, to November 27, 2003.

The (annualized) volatility during the sample period varies between 8.0% and 45.2% with an average of 19.4%. Figure 4.8 shows that, after the turbulence in 2001 (September 11) and 2002, volatility has decreased to low levels in 2003.

4.4.2 Risk-neutral return and volatility densities

This section presents risk-neutral distributions for both returns and volatilities individually.[19] Note that these distributions are conditional on an initial spot volatility level, as

[19]The proposed methodology of first calculating a nonparametric estimate of the Black-Scholes implied volatility function, subsequently determining the joint density of excess return and future volatility, while the data are clearly generated under the objective measure, makes it impossible to derive reliable

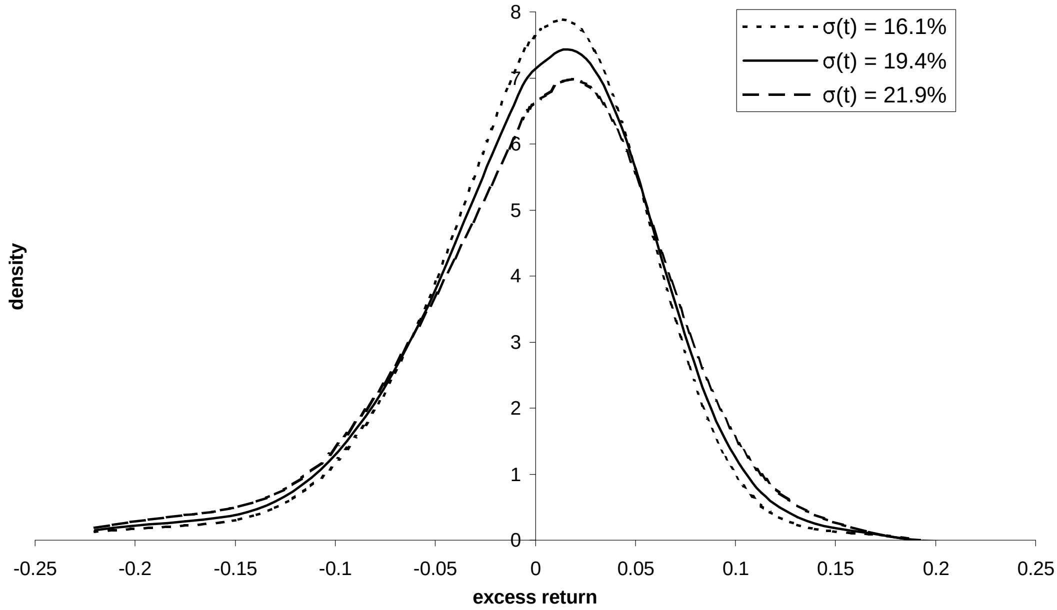


Figure 4.9: Estimated risk-neutral marginal excess return density over one month horizon based on S&P-500 data as described in the main text for three initial spot volatility levels.

implied by Assumption 1. In the present section the dependence between future returns and volatilities is not considered. This dependence will be discussed in Sections 4.4.3 and 4.4.4. Figure 4.9 presents the risk-neutral return distribution as implied by the empirical data.

The initial volatility level of $\sigma_t = 19.4\%$ corresponds to the average volatility level as follows from the filtered volatilities. The high and low volatility levels correspond to the 75% and 25% quantile respectively. Note that these levels are objective estimates which, due to negative volatility risk premiums lie below their risk-neutral counterparts. Observe that the volatility levels are much higher than those in Pan (2002) as the (much) more volatile 1999–2003 period is considered, while Pan (2002) covers the January, 1989, until December, 1996, period. The figure clearly confirms negative skewness in the risk-neutral return distribution for all initial volatility levels.

More interesting are the nonparametric risk-neutral volatility densities provided by this chapter's method. These are presented in Figure 4.10. The expected risk-neutral future volatility is in two of three initial volatility scenarios (much) larger than the

confidence bounds for the density estimate using standard techniques. Therefore, confidence bounds for the nonparametric density estimators are not provided.

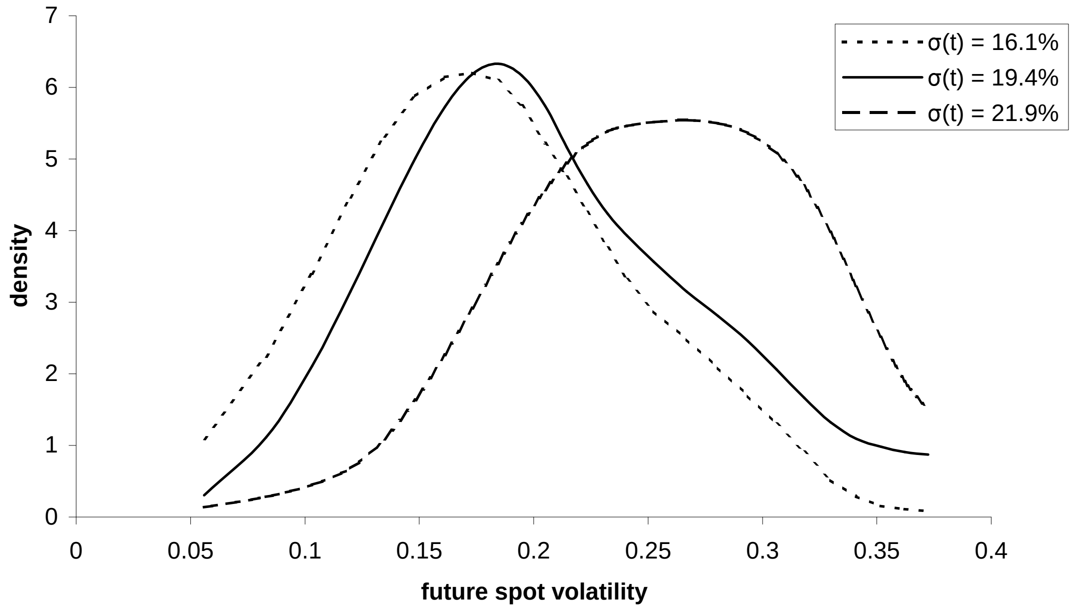


Figure 4.10: Estimated risk-neutral marginal volatility density over one month horizon based on S&P-500 data as described in the main text for three initial spot volatility levels.

objective average spot volatility of 19.4%. Therefore, the results in Figure 4.10 are a strong indication of a negative volatility risk premium. A negative volatility risk premium results in option prices higher than they would be in case of idiosyncratic volatility risk.[20] The higher price is a compensation for unhedged volatility risk that option traders typically face because they only delta hedge their short options positions, see also Bakshi and Kapadia (2003). The results confirm that higher initial volatility leads to a right-shift in the future volatility distribution. In addition to these long-established facts, Figure 4.10 depicts clear evidence of positive skewness in the risk-neutral volatility distribution. However, in high volatility states (as for $\sigma_t = 21.9\%$) the skewness seems to disappear. Note that this effect is not due to a mean-reversion in risk-neutral volatility distributions as the high volatility state leads to a larger (i.e., more negative) volatility risk premium, given the high expected future volatility levels in this case. These conclusions are consistent with a market aversion towards high volatility levels and an even larger aversion towards unexpected positive volatility shocks. From a parametric model perspective this result is indicative of a jump component in the

[20]In (4.8) a negative volatility risk premium, i.e. $\eta^V < 0$, leads (on average) to higher volatility levels than in case $\eta^V = 0$ (idiosyncratic volatility risk). The result follows then from the positive dependence of option prices on volatility.

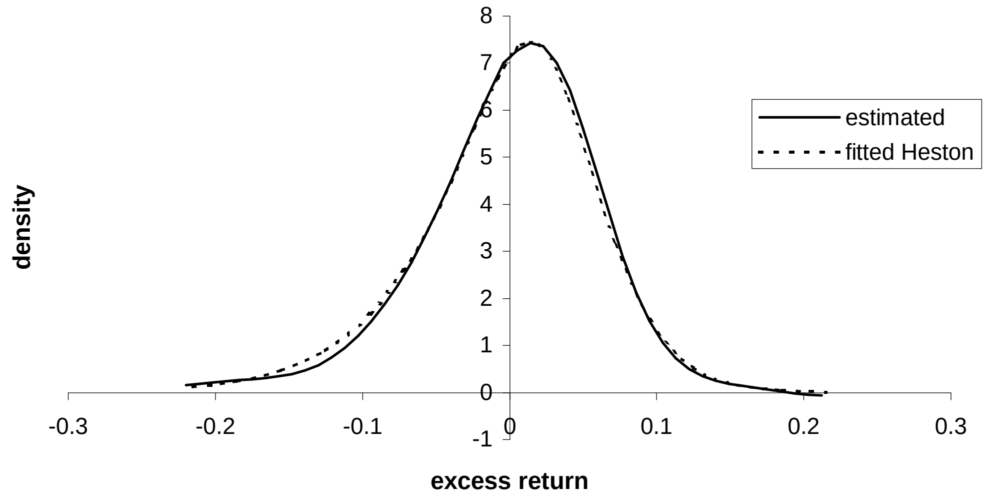


Figure 4.11: Risk-neutral return density from Heston (1993) models using parameters that fit the nonparametric estimate best (in quadratic mean sense). See main text for details.

risk-neutral volatility process. The changing skewness for the various volatilities can possibly be modelled by a jump intensity that depends on the volatility level.

The nonparametric technique can be used to infer the accuracy of parametric risk-neutral stochastic volatility models. To that end, the Heston (1993) model is used with parameter values that are chosen in such a way as to provide an optimal fit (in least squares sense) of the estimated risk-neutral return distribution as in Figure 4.9. This leads to the parameter choices $\kappa + \eta^V = 1.76$, $\kappa\sigma^2 = 0.25$, $\sigma_\sigma = 0.84$, and $\rho = -0.39$. Note that κ and σ^2 cannot be identified separately from the risk-neutral return distribution alone. The resulting risk-neutral return distribution from the Heston (1993) model with these parameter values is depicted in Figure 4.11. It is clear that the Heston (1993) model is capable of providing a very accurate description of risk-neutral return distribution for the sample. However, it fails in describing simultaneously the risk-neutral volatility distribution as shown in Figure 4.12, which shows the induced risk-neutral volatility distribution using the same parameter values. From this figure it is clear that an accurate fit of the return distribution, leads to a severe overestimation of the risk-neutral volatility of volatility.[21] Moreover, the Heston (1993) volatility distribution

[21] In the same way as for the risk-neutral density, an optimal fit of the volatility distribution was determined. The estimated σ_σ was indeed much lower (0.52) but still much larger than values reported in the time series literature, see for instance Eraker, Johannes, and Polson (2003). This suggests that more components in the volatility process are necessary to describe asset returns and the (volatility of the)

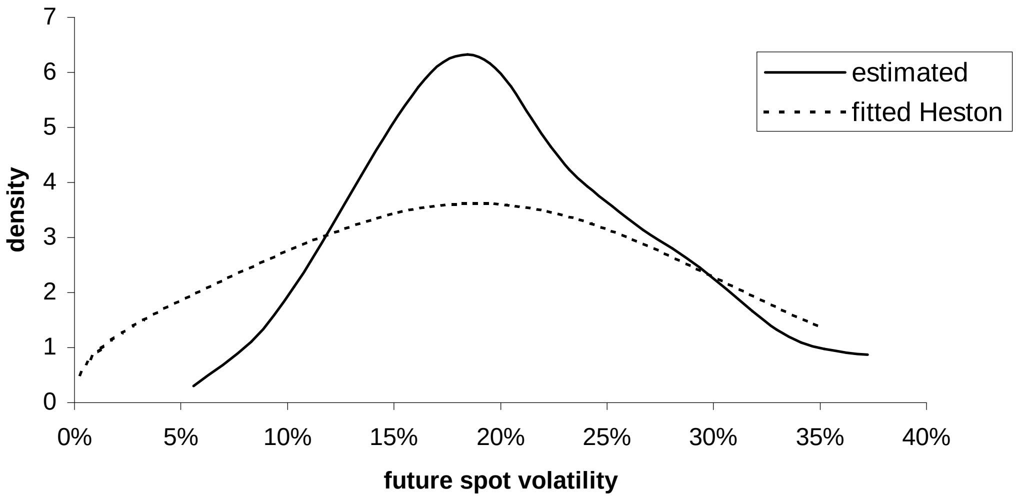


Figure 4.12: Risk-neutral volatility density from Heston (1993) models using the same parameters as in Figure 4.11.

does not provide the correct positive skewness that is apparent from the nonparametric estimate.

To assess the financial significance of the presented densities, consider a volatility swap which pays the difference between the actual spot volatility and a reference volatility level over a maturity of one month. The reference level is chosen in such a way that the current value of the contract is zero. Using the risk-neutral volatility density from the Heston (1993) model, the reference level would be 19.8%, while the nonparametric estimate yields a significantly higher reference level of 20.5%, mainly due to positive volatility skewness.

4.4.3 Risk-neutral bivariate return/volatility distribution

The method proposed in this chapter also leads to joint risk-neutral return and volatility distributions which can be used to study the risk-neutral dependence. Figure 4.13 graphs this joint estimate.[22] The graph clearly shows that standard Gaussian and other elliptical

empirical volatility distribution simultaneously. In parametric models this could be accomplished by, for instance, an additional Brownian component (see Chernov, Gallant, Ghysels, and Tauchen (2003)) or a jump component (see Broadie, Chernov, and Johannes (2004)) in the Heston (1993) volatility process. Once more, these are possible parametric adjustments of the Heston (1993) model that correspond to the nonparametric density estimates of this chapter.

[22]As for the simulated Heston (1993) model the corresponding copula is presented (Figure 4.14). This figure indicates that the dependence between returns and volatility is negative. Section 4.4.4 gives more

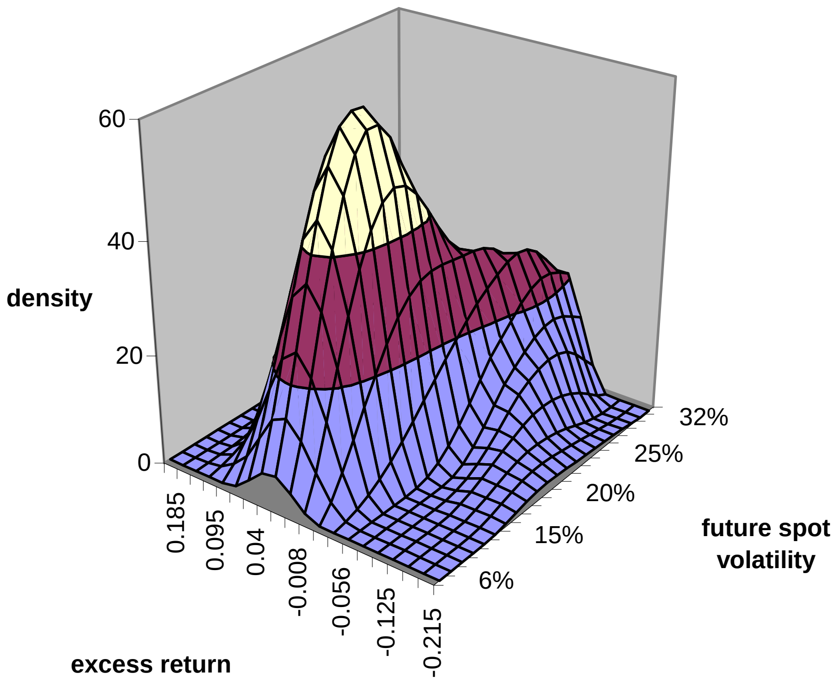


Figure 4.13: Nonparametric estimate of bivariate risk-neutral excess return/volatility density.

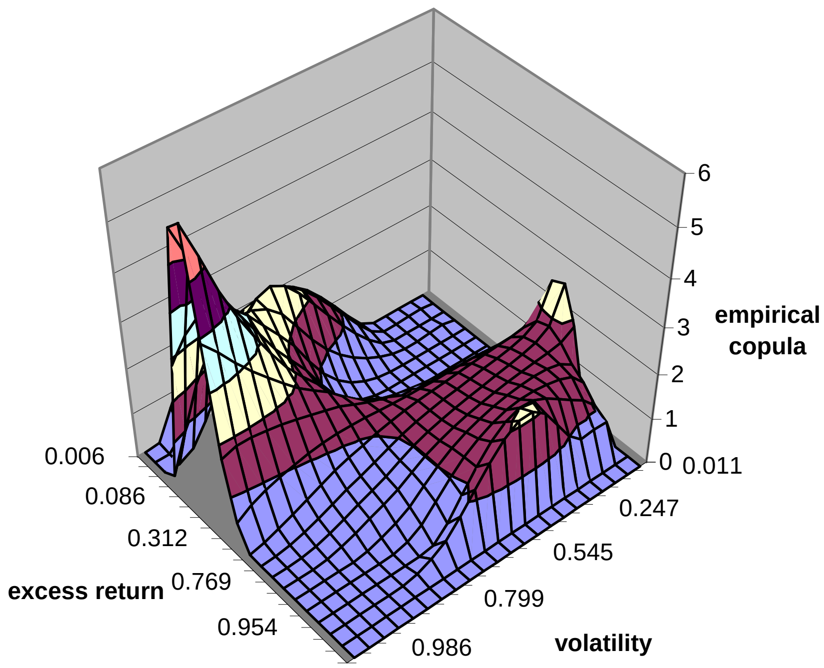


Figure 4.14: Empirical copula using estimated bivariate risk-neutral excess return/volatility density.

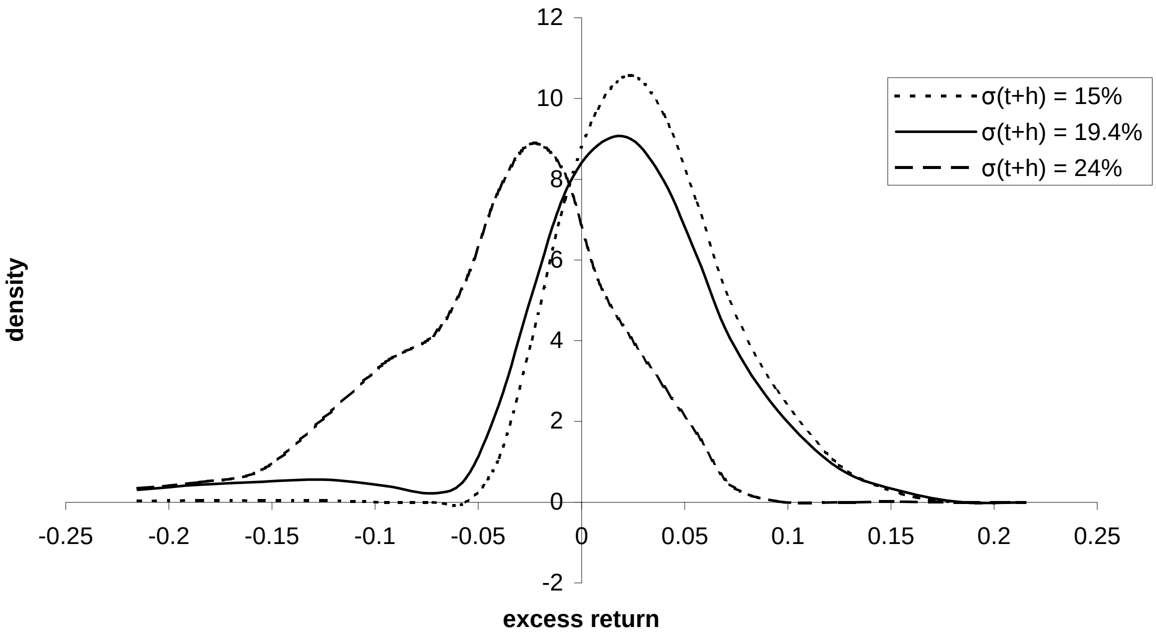


Figure 4.15: Risk-neutral conditional excess return distributions for an initial spot volatility level op 19.4% and several future spot volatility levels. Returns and volatilities are over a period of $h = 1$ month.

distributions will not provide a good fit to the bivariate distribution. However, it is difficult to asses the financial significance of these deviations. These are more apparent from conditional return distributions (conditional on *future* volatility levels) as provided in the next section.

4.4.4 Conditional risk-neutral return distributions

In order to assess the risk-neutral dependence structure of returns and volatilities, Figure 4.15 presents the return distribution conditional on the *future* level of the spot volatility. In line with previous results, see, for instance, Bakshi, Cao, and Chen (1997), Figure 4.15 provides clear evidence of negative risk-neutral correlation between (excess) returns and volatility. This follows from the fact that higher future volatility levels in Figure 4.15 lead to negative shifts in the return distribution. This is also confirmed when the excess return and spot volatility correlation is calculated.

At the same time, the method proposed in this chapter provides evidence that return distribution skewness depends on volatility changes. For a future volatility level of

insight on this dependence.

25%, which is close to the risk-neutral average spot volatility, the return distribution is clearly skewed to the left. Left skewness of the risk-neutral return distribution is usually associated with a significant crash risk premium. However, for low future volatility levels the return distribution not only shifts to the right, but it loses its negative skewness and even shows slight positive skewness. The intuition behind this result is that the crash risk premium is higher when there is more uncertainty in the market. This latter result has not been reported before (empirically).

4.5 Summary

A nonparametric technique to infer risk-neutral return and volatility distributions from plain vanilla option prices is presented. Using this technique and recent S&P-500 data, the results confirm negative skewness in the risk-neutral return distribution, negative volatility risk premiums, and negative risk-neutral return/volatility correlation. It is important to note that these results are obtained without using a parametrically specified model. At the same time, as the full joint risk-neutral return and volatility distribution is estimated, the results show positive skewness in the risk-neutral volatility distribution, which seems to decrease with volatility levels. Moreover, conditional on low future volatility levels, the return distribution is no longer negatively skewed but shows some slight positive skewness. These effects are consistent with volatility dependent risk premiums. Finally, the results are indicative of a volatility risk premium that depends on initial volatility in a non-linear way.

A future extension of the proposed technique could look at the possibility of jumps in both the underlying price process and in the underlying volatility process. Several parametric models have been proposed to include both. An issue in the implementation of the proposed method for these kinds of underlying methods is that the applied filtering procedure is only valid for continuous volatility processes and thus would have to be adapted. Recently, however, some techniques to distinguish jumps from continuous behavior have emerged, see, for instance, the work Bandorff-Nielsen and Shephard (2004).

CHAPTER 5

A Note on the Use of GARCH Instruments for Parameter Estimation in Stochastic Volatility Models

5.1 Introduction

The quest for an empirically sound description of equity-index returns (and option prices) in continuous time started right after the publication of the Black and Scholes (1973) model. A good description of the asset price process is not only important for the purpose of option pricing but is also of great relevance for risk management. The Black and Scholes (1973) model predicts normally distributed log-returns over a fixed horizon under the objective probability measure. This contradicts the empirical features of (short horizon) log-returns on individual stocks, stock indices, or exchange rates. Financial time series generally exhibit features as fat tails, negative skewness, and volatility clustering. These features are economically relevant and therefore numerous models have been proposed that allow for the observed regularities. One way to go is to introduce stochastic volatility in such a way that persistence in volatility is captured. Examples of stochastic volatility models can be found in Hull and White (1987) and Scott (1987). These papers derive theoretical option prices under the assumption that volatility risk is idiosyncratic. Wiggins (1987) derives an option pricing partial differential equation under particular assumptions for agents' risk preferences while Heston (1993) finds a closed form solution for option prices in the presence of stochastic volatility. The benchmark model (taken from Nelson (1990)) that is used throughout this chapter is

$$\mathrm{d}\log S_t = \mu \mathrm{d}t + \sigma_t \mathrm{d}W_t^S, \tag{5.1}$$

$$\mathrm{d}\sigma_t^2 = -\kappa\left(\sigma_t^2 - \sigma^2\right)\mathrm{d}t + \sqrt{2\kappa\theta}\sigma_t^2 \mathrm{d}W_t^V, \tag{5.2}$$

where $W = \left(W^S, W^V\right)$ is a two-dimensional standard Brownian motion. The long-run variance parameter σ^2 is positive as is the mean-reversion parameter κ. The parameter θ is a nuisance parameter that governs the volatility of volatility. The stochastic variance process (σ_t^2) is a mean-reverting process in the same spirit as Hull and White (1987), leading to stylized features as volatility clustering and fat tails. Although the focus of this chapter is on processes that are continuous, the methodology also applies to models that include jumps in asset prices and volatility.[1] Furthermore, the model allows for leverage effects as W^S and W^V may be correlated.

Statistical inference in continuous time models is challenging, mainly due to the fact that exact likelihoods (that need to be based on transition densities) for (jump-)diffusions are generally unknown in analytical form. Moreover, the presence of latent variables induces extra complications. As a result, several non-likelihood-based or simulation-based inference techniques have been proposed. For instance, the model (5.1)–(5.2) is derived in Nelson (1990) as the limit of a sequence of discrete-time GARCH(1,1) processes. This inspired empirical researchers to use the GARCH(1,1) model to estimate this diffusion. However, the moment conditions of the discrete time GARCH(1,1) model are not exactly satisfied in the continuous time model, see Drost and Werker (1996). The convergence result is used in Section 5.2 for the construction of informative instruments. Other approaches include the Efficient Method of Moments (EMM) which is applied by Chernov and Ghysels (2000) in the Heston stochastic volatility model and Andersen, Benzoni, and Lund (2002) in a jump-diffusion model; the Simulated Method of Moments (SMM) approach in Duffie and Singleton (1993); Markov Chain Monte Carlo (MCMC) methods applied by, e.g. Eraker, Johannes, and Polson (2003) in a jump-diffusion setting; and the spectral GMM estimator utilizing the characteristic function in Chacko and Viceira (2003) that applies the method to stochastic volatility and jump-diffusion models. Most methods are computationally demanding and cumbersome to implement in practice. This in contrast to the method described in Section 5.2 which can be best characterized as "simple". Following Meddahi and Renault (2004), moment conditions are constructed for the stochastic volatility model above that are independent of la-

[1] Examples can be found in Duffie, Pan, and Singleton (2000), Broadie, Chernov, and Johannes (2004), and Santa-Clara and Yan (2004).

tent variables by taking linear combinations of first and second conditional moments for different lags. The traditional problem with this method is that insufficiently informative instruments appeared to be available. Instead, instruments based on a Gaussian QMLE analysis of approximating GARCH(1,1) models are used in this chapter. Simulation results show that these are much more informative than classical instruments like lagged returns and their squares. However, the practical applicability of the method is questionable since the improvement does not lead to sufficiently precise parameter estimates.

The estimation proposal is detailed in Section 5.2 while the simulation study is presented in Section 5.3.

5.2 GMM Estimation

The main interest of this chapter is the estimation of model (5.1)–(5.2) based on regularly spaced observations in discrete time. First, the linear mean-reversion in the volatility equation (5.2) implies that exact conditional moment conditions for both instantaneous variances and log-returns over deterministic intervals are easily obtained. More precisely, for all fixed $h > 0$,

$$\begin{aligned}
\alpha_t(h) &:= \mathrm{E}_t\left(\log S_{t+h} - \log S_t\right) \\
&= \mathrm{E}_t \int_0^h \mu \mathrm{d}u + \sigma_{t+u} \mathrm{d}W_{t+u} \\
&= \mu h, && (5.3) \\
\beta_t(h) &:= \mathrm{E}_t\left(\log S_{t+h} - \log S_t - \mu h\right)^2 \\
&= \mathrm{E}_t \int_0^h \sigma_{t+u}^2 \mathrm{d}u \\
&= \int_0^h \sigma^2 + \exp(-\kappa u)\left(\sigma_t^2 - \sigma^2\right) \mathrm{d}u \\
&= \sigma^2 h + \frac{1-\exp(-\kappa h)}{\kappa h}\left(\sigma_t^2 - \sigma^2\right) h. && (5.4)
\end{aligned}$$

The conditional variance $\beta_t(h)$ depends on the latent instantaneous variance σ_t^2. In order to derive moment conditions in terms of observables alone, the dependency on σ_t^2 has to be removed. To eliminate σ_t^2 a well-known trick (see, e.g. Meddahi and Renault (2004)) is applied to the conditional moments $\beta_t(h)$ and $\beta_t(2h)$. More precisely, the moment condition (5.4) immediately implies

$$\frac{\beta_t(2h)}{1-\exp(-2\kappa h)} - \frac{\beta_t(h)}{1-\exp(-\kappa h)} = \sigma^2 h \left(\frac{2}{1-\exp(-2\kappa h)} - \frac{1}{1-\exp(-\kappa h)}\right). \quad (5.5)$$

Adding moment conditions beyond $\beta_t(2h)$ is possible as well. For horizons of length $h, 2h, \ldots, Kh$, we get

$$\mathrm{E}_t \begin{pmatrix} [\alpha_t(kh)]_{k=1,\ldots,K} \\ [\beta_t(kh)]_{k=1,\ldots,K} \end{pmatrix} = \begin{pmatrix} a_K & 0_K \\ b_K & c_K \end{pmatrix} \begin{pmatrix} 1 \\ \sigma_t^2 \end{pmatrix}, \tag{5.6}$$

for some K-vectors a_K, b_K, and c_K. The vector 0_K denotes the null vector and, for instance $a_K = \mu h(1, \ldots, K)^T$. In general, the vectors a_K, b_K, and c_K depend on h and the unknown parameters. To remove the latent volatility, again linear combinations of the conditional expectations are taken such that the resulting moment conditions are independent of σ_t^2. Note that this method could be easily generalized in case more latent variables are present. In that case, the above vectors generally become matrices, but the idea of taking linear combinations of moment conditions for different lags which do not depend on the unobservable variables remains unchanged. From a theoretical point of view, the inclusion of higher-order (i.e., $k = 1, \ldots, K$) moments would lead to more efficient estimates if the model (5.1)–(5.2) is correctly specified. However, the use of higher order moments will severely deteriorate the behavior of the estimates in case of small deviations of this model, while the first two conditional moments of the process describe the stylized features of log-returns.

The approach described here leads to exact moment conditions in terms of observables alone so that standard GMM can be applied by using some instruments. In estimating financial models these instruments are often chosen in an ad hoc manner although there is a literature on the optimal choice of instruments in a GMM framework (see Hansen (1982)). The reason behind this approach is that optimal instruments are often difficult to calculate in financial models. Taking ad hoc instruments leads to a consistent and asymptotically normal inference procedure, but the efficiency is quite low. In this chapter so-called 'GARCH' instruments are used. The motivation for this choice is based on the second estimation method that is sometimes used and which will be discussed now.

This second estimation method is based on Nelson (1990) that shows that the continuous time processes (5.1) and (5.2) can be obtained as the limit of a sequence of discrete-time GARCH processes. This has led people to estimate the approximating GARCH model, e.g. by a Gaussian likelihood method and to infer the continuous time parameters from these estimates. The disadvantage of such an approach is that the approximating nature of the GARCH processes induces a discretization type bias in the estimates. In fact Wang (2002) has shown that, while the discrete time GARCH processes do converge to the continuous time processes (5.1) and (5.2) in a probabilistic sense, there is no convergence in a statistical sense. This means that the associated estimation problems in the discrete time GARCH model and the continuous time model are by no means similar. For this reason, estimates of (5.1) and (5.2) based on GARCH QMLE will not be considered in the sequel of this chapter. However, the QMLE procedure is used to obtain more efficient instruments that will be employed in the GMM procedure described above. To be more precise, the QMLE technique estimates the GARCH parameters $\tilde{\theta} = (\omega, \alpha, \beta)$ in the "approximating" model

$$\log S_{t+h} - \log S_t = \mu h + v_t(\tilde{\theta})\varepsilon_{t+h}, \tag{5.7}$$

$$v_t^2(\tilde{\theta}) = \omega + \alpha \left(\log S_t - \log S_{t-h}\right)^2 + \beta v_{t-h}^2(\tilde{\theta}), \tag{5.8}$$

where the innovations ε_t are assumed to be i.i.d. with mean zero. The Gaussian QMLE for this GARCH model can be seen as a moment estimator based on

$$\mathrm{E}_t \left(\log S_{t+h} - \log S_t - \mu h\right)^2 = v_t^2(\tilde{\theta}) \tag{5.9}$$

with 'GARCH' instruments

$$\frac{\partial}{\partial \tilde{\theta}} \log v_t^2(\tilde{\theta}). \tag{5.10}$$

Note that these instruments are observable as a consequence of the recursion (5.7) and can easily be calculated using the same recursion (5.7). Nelson (1990) indeed stresses that a GARCH model with non-latent volatility can have a continuous time limit with latent volatility. The methodology proposed in this chapter utilizes the exact moment conditions (5.3) and (5.5) combined with instruments (5.10) that are based on Gaussian likelihood based inference in the approximating GARCH process. Observe that these moment conditions only identify μ, σ^2, and κ, but not the diffusion specification of the instantaneous variance σ_t^2. For the parameters that are identified, the simulation study in the next section shows that the proposed method dominates the GMM method with ad hoc instruments for relevant parameter configurations. In particular, the diffusion term of the instantaneous variance need not be specified. Simulation based methods like EMM/Indirect Inference do need a parametrization of this term. Methods based on conditional characteristic functions are also of a parametric nature.

5.3 Simulation Results

This section investigates the empirical applicability of the method that is described in the previous section by means of a simulation study. For simplicity reasons the drift parameter μ is restricted to be equal to 0 in the simulations. While most empirical papers show comparable estimates of the mean of the variance process for S&P-500 index data, there is wide variation in the estimates for the rate of mean-reversion and the volatility-of-volatility. For example, Andersen, Benzoni, and Lund (2002) find a mean reversion parameter equal to 3.251 and a volatility of volatility parameter of 0.185, while Chernov and Ghysels (2000) estimate $\kappa = 0.926$ and a volatility of volatility equal to 0.063. Chacko and Viceira (2003) confirm this poor identification and argue that, in order to capture the regularities in the variance process, a low speed of mean-reversion must be offset by a low value of the volatility of volatility and a high speed of mean reversion needs to be compensated by a high volatility of volatility. This argumentation is confirmed in the literature by the estimation results of stochastic volatility models using S&P-500 return data. Given these results two benchmark values for the parameter vector are chosen in the simulation study below, namely a high persistence/low volatility of volatility case and a low persistence/high volatility of volatility case. The parameter estimates of the aforementioned studies are used to simulate the daily return patterns. Furthermore, the mean of the variance process σ^2 is estimated as 0.014 in Andersen, Benzoni, and Lund (2002) and 0.0164 in Chernov and Ghysels (2000).

In the simulation study conducted in this chapter, for each replication, a return series is generated for the two above mentioned parameter vectors. Subsequently, GARCH(1,1) parameter estimates are determined for the simulated return series. Finally, the continuous time parameters are estimated using the exact moment conditions with GARCH instruments, as described in the previous section. This procedure is compared to using the ad hoc instruments $(1, r_{t-h}^2, r_{t-2h}^2)$ with $r_{t-h} = \log S_t/S_{t-h}$. The simulation study is based on 5,000 replications. Table 5.1 summarizes the results.

Table 5.1 shows that the use of exact moment conditions with GARCH instruments, leads to a significant reduction in variability of the estimates. With the notable exception of the median absolute deviation (MAD) for the high persistence case, all results imply that the use of GARCH instruments leads to point estimates which are on average closer to the true value and lower variability around this true value. This effect is especially strong for the estimates of the mean-reversion coefficient κ, where the use of ad hoc instruments often leads to extreme point estimates. These outliers in the estimators distribution do not occur when using GARCH instruments.

The second conclusion that can be drawn from Table 5.1 is that the constructed unconditional second moment does not provide sufficient empirical identification of the volatility process parameter κ. As was mentioned before, κ measures the speed of mean reversion in the *variance* process. Intuitively, changes in κ do not have a considerable influence on the second moment of *returns*, leading to a questionable empirical identification. The logical next step is to construct higher order moment conditions (of returns or variance) independent of the latent state variables that are more sensitive for changes in the mean reversion parameter κ than the set of moment conditions used in this chapter. The practical implementation of this extension is left for future work.

5.4 Summary

The main conclusions of this chapter can be summarized in only a few sentences. This chapter proposes a simple methodology for dealing with conditional moments that contain latent variables. The idea is to construct moment conditions independent of the latent variables by taking linear combinations of the conditional moments. The simulation experiment shows that for the purpose of parameter estimation in stochastic volatility models, the use of these moment conditions in combination with ad-hoc instruments is useless. Using so-called GARCH-instruments instead leads to a considerable decrease in standard deviations of the estimated parameters. However, the empirical identifiability of the mean-reversion parameter remains poor.

	Five years of daily data							
	ABL ad hoc		ABL GARCH		CG ad hoc		CG GARCH	
	κ	σ	κ	σ	κ	σ	κ	σ
True value	3.25	0.014	3.25	0.014	0.92	0.016	0.92	0.016
Average	47.41	0.207	4.52	0.039	89.64	0.198	2.95	0.026
Median	0.18	0.017	3.45	0.013	0.27	0.116	0.50	0.016
St.dev.	91.96	0.252	6.42	0.088	580.31	0.209	12.07	0.038
MAD	20.00	0.009	5.02	0.005	0.72	0.151	3.01	0.005
2.5 perc	-49.72	0.010	-6.03	0.006	-256.74	0.000	-14.29	0.000
97.5 perc	289.54	0.765	20.07	0.276	1455.34	0.687	34.49	0.133
	Ten years of daily data							
	ABL ad hoc		ABL GARCH		CG ad hoc		CG GARCH	
	κ	σ	κ	σ	κ	σ	κ	σ
True value	3.25	0.014	3.25	0.014	0.92	0.016	0.92	0.016
Average	32.01	0.155	3.78	0.025	57.02	0.214	1.69	0.021
Median	2.79	0.015	3.39	0.014	0.18	0.140	0.78	0.016
St.dev.	62.36	0.230	3.57	0.038	158.08	0.218	3.58	0.023
MAD	33.44	0.004	3.39	0.003	0.47	0.185	1.89	0.004
2.5 perc	-58.32	0.011	-2.50	0.009	-3.81	0.012	-4.88	0.000
97.5 perc	187.16	0.696	11.89	0.148	436.98	0.687	10.78	0.099

Table 5.1: Simulation results for exact GMM in the model (5.1)-(5.2), with ad hoc and GARCH instruments. The true underlying parameters are chosen as the estimates presented in Andersen, Benzoni, and Lund (2002), ABL, or Chernov and Ghysels (2000), CG. The upper panel refers to the use of five years of daily return data, while the lower panel refers to ten years of daily data. MAD denotes the Median Absolute Deviation, rescaled so that for the normal distribution it equals the standard deviation. The rows “2.5 perc” and “97.5 perc” refer to the respective empirical quantiles of the estimated parameters over the replications. The results are based on 5,000 replications.

CHAPTER 6

Mean-Variance Properties of Option Returns

6.1 Introduction

The mean-variance analysis of Markowitz (1952) is the first study that analyzes the risk-return trade-off for a portfolio of stocks. When compared to dynamic expected utility models, mean-variance analysis provides an intuitive and simple approach to the concept of diversification. The idea of Markowitz (1952) is further developed in Sharpe (1964), Lintner (1965), and Mossin (1966), leading to the classical Capital Asset Pricing Model (CAPM). The CAPM can be characterized as a single-period model in which the only source of systematic risk is the risk in the market portfolio. One of the assumptions underlying the CAPM is that each investor optimally holds a mean-variance efficient portfolio. Sufficient conditions for this assumption to hold are strong: (i) all asset returns are elliptically distributed or (ii) investors have a quadratic utility function.

The CAPM has been subject to criticism both theoretically and empirically. First, the elliptical distribution assumption of asset returns or portfolio returns is often not satisfied.[1] Furthermore, Dybvig and Ingersoll (1982) shows that, in complete markets, mean-variance preferences lead to arbitrage opportunities.[2] Jarrow and Madan (1997)

[1] For instance, if returns of underlying assets are normally distributed then returns of portfolios containing options written on these assets or returns of dynamic strategies will not be normally distributed. Empirical evidence of non-normality of option portfolios can be found in Merton, Scholes, and Gladstein (1978), Merton, Scholes, and Gladstein (1982), and Coval and Shumway (2001)

[2] This is an immediate consequence of the shape of the utility function. Quadratic utility implies

argues that completeness is not a necessary condition for these arbitrage opportunities to occur. That paper shows that the presence of arbitrage opportunities in the CAPM is due to the mean-variance preferences.

The discussion of the single-period nature of the model leads to the continuous time asset pricing models in Merton (1971) and Merton (1973). Fama and French (1992) and Fama and French (1993) provide empirical evidence that the single market factor cannot explain the difference in return between portfolios constructed on the basis of the ratio of book value of equity to market value of equity.

Despite the theoretical and empirical objections against the CAPM, performance measures based on the CAPM (like CAPM α and the Sharpe ratio) are still widely used by practitioners. Academic studies have shown that these performance measures should be treated with caution when the shape of the return distribution is far from normal.[3] Highly non-normal return distributions can be created by taking positions in options. Leland (1999) shows that under the assumption of perfect markets and independently and identically distributed (i.i.d.) returns of the *market portfolio*, the performance of derivative portfolio managers will be mismeasured by CAPM α.[4] That paper shows how to adjust CAPM β in the Black-Scholes world such that CAPM β can be interpreted as a risk measure.

The problems that occur when performance of option strategies is evaluated under the CAPM assumptions are clear now. However, the implications of mean-variance assumptions on optimal asset allocation, in a setting where options are treated as a separate asset class, have not been considered yet. Although there are studies that treat the issue of optimal positioning in options, only a few papers concentrate on the demand for options.[5] Options are often excluded from the analysis because of computational com-

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negative state prices for high return states because of negative marginal utility in these states.

[3]With respect to manipulation of the Sharpe ratio, see Henriksson and Merton (1981), Dybvig and Ingersoll (1982), Bernardo and Ledoit (2000), Spurgin (2001), and Goetzmann, Ingersoll, Spiegel, and Welch (2002). Goetzmann, Ingersoll, Spiegel, and Welch (2002) derives rules that define derivative strategies which maximize the Sharpe ratio. That paper finds that the optimal strategy is to sell out-of-the-money calls and out-of-the-money puts.

[4]The CAPM assumptions are not only violated because of the derivatives based portfolios but also because of the assumptions of i.i.d. returns of the market portfolio and perfect markets. This assumption implies (see Rubinstein (1976), Brennan (1979), and He and Leland (1993)) that the representative must have a power utility function. An implication of power utility is that investors treat upside and downside risks differently.

[5]Derivatives in an asset allocation context are mostly used to measure the economic value of market-timing performance, see Merton (1981), Henriksson and Merton (1981), and Evnine and Henriksson (1987).

plexities. If options are redundant securities, i.e. in complete markets, this exclusion is justified. However, the empirical results in, for instance, Bakshi and Kapadia (2003) indicate that options are non-redundant. The optimal asset allocation decision should, therefore, be based on the specific risk-return characteristics of options. Dert and Oldenkamp (1996) and Dert and Oldenkamp (2000) use empirically observed option prices to determine optimal investment portfolios from a universe of assets that consists of a risk free asset, a stock index, and options that are written on this index. Optimal portfolios are obtained by maximizing the expected return at the investment horizon under the restriction that the realized portfolio return at the horizon is not smaller than a specified guaranteed return. Carr, Jin, and Madan (2001) provides closed form solutions for the optimal derivative contracts when the utility function is in the HARA class and the (risk-neutral) process for the derivative's underlying asset is assumed to be in the variance gamma class. In a single period economy, Carr and Madan (2001) shows how investors can determine their optimal positions in three asset classes (risky asset, riskless asset, and options). Based on the findings of the empirical literature with respect to volatility risk and jump risk[6], Liu and Pan (2003) derives optimal dynamic derivative strategies in a model that incorporates three separate risk sources.[7] In addition, that paper determines the portfolio improvement from adding options to the investment opportunity set by comparing the certainty-equivalent wealth of investors with and without the opportunity to invest in options. Finally, Driessen and Maenhout (2004) uses an empirical approach to calculate optimal portfolio weights. As a consequence, no assumptions on price dynamics or risk prices need to be imposed. The empirical results of the related study in Driessen and Maenhout (2004) show that constant relative risk aversion investors, loss-averse investors, and disappointment-averse investors optimally take short positions in out-of-the-money puts and at-the-money straddles, indicating

[6]There is mixed empirical evidence on the presence and magnitude of volatility and jump risk premia. Chernov and Ghysels (2000) and Bakshi and Kapadia (2003) find a negative volatility risk premium but these studies do not include jumps in the model. Pan (2002) reports a significant negative volatility risk premium when jumps are excluded but the volatility risk premium becomes insignificant after jumps are included in the model. In a recent study Broadie, Chernov, and Johannes (2004) provides empirical evidence for a jump risk premium in the option's underlying asset and for a jump risk premium in the variance process. The results in Demertefi, Derman, Kamal, and Zou (1999), Coval and Shumway (2001), Bondarenko (2004), Carr and Wu (2004), and Dert, Pergamentsev, Petit, and Tolenaar (2004) also indicate that either volatility risk and/or jump risk is priced.

[7]One of the differences between Carr and Madan (2001) and Liu and Pan (2003) is that the former derives optimal portfolio weights in a general equilibrium setting while the analysis in Liu and Pan (2003) is of a partial equilibrium nature.

that the risk premia in option pricing models are large.[8]

This chapter examines the mean-variance properties of option returns in a world that allows for systematic volatility risk and jump risk. In order to investigate these properties the conditional expectation of option returns, the (conditional) variance of option returns, the conditional covariance between an option return and the return on the option's underlying asset, and the conditional covariance between two different options need to be determined. This chapter provides a methodology, based on the characteristic function, that can be employed to calculate the previously mentioned quantities for all models that fit in the class of affine-jump diffusions. The emphasis of this chapter will be on the two applications of mean-variance theory as described above: performance measurement and portfolio selection. The performance measurement application is an extension of the work in Leland (1999). Analytical expressions for CAPM α of single option returns are derived under more realistic assumptions for the return on the market. The portfolio allocation application is best compared to the study in Liu and Pan (2003). However, the model considered in this chapter's application is more general than the model in Liu and Pan (2003).

Adding single options to the investment opportunity set seems to be inconsistent with the theory underlying the CAPM. Single option returns are obviously not elliptically distributed and, secondly, the model structure implies that under reasonable parameter choices the utility function of the investor is strictly increasing. However, the mean-variance investor should also profit from the addition of options to the investment opportunity set in settings where volatility risk and jump risk are priced since options can be used for taking exposure in every risk factor. Optimal portfolio weights derived in the expected utility framework of Liu and Pan (2003) are compared to the optimal positions of a mean-variance opportunity. To deal with the theoretical inconsistencies that arise from considering single option returns, optimal portfolio weights of delta-hedged straddles are determined. Returns on delta-hedged straddles are more symmetric than single option returns. Again the optimal portfolio weights are compared to the results in Liu and Pan (2003).

The theoretical results show that as a performance measure of option returns, CAPM α should be treated carefully. The reason for this is that the CAPM is linear while option returns are highly non-linear. Despite the shortcomings of the CAPM in evaluation option based investment strategies, it is important to examine the properties of CAPM

[8] The usage of derivatives in asset allocation context is, for instance, also considered in Leland (1980), Brennan and Solanki (1981), and Haugh and Lo (2001).

α and CAPM β because the CAPM is widely applied in practice. Properties of CAPM α and CAPM β are not only considered for single option returns but also for returns on delta-hedged straddles. The outcomes reveal that CAPM α and CAPM β of delta-hedged straddle returns get their usual interpretation when the hedging frequency increases in a world with nonsystematic volatility risk and nonsystematic jump risk. Furthermore, the usage of the CAPM regression equation for validating option pricing models can be tricky because the underlying OLS estimation assumptions are often violated in stochastic volatility and jump models.

Like power utility investors, mean-variance investors optimally take short positions in volatility under the assumption of a negative volatility risk premium. The investment vehicle considered in this analysis is an at-the-money delta-neutral straddle since the return on a straddle is highly correlated with realized variance. Varying several parameters in the stochastic volatility model leads qualitatively to the same optimal straddle weights as in Liu and Pan (2003). In the presence of jumps, out-of-the-money options are used to create jump exposure because these options disentangle jump risk and diffusive risk most effectively. Differences are observed between optimal portfolio choice of power utility investors and mean-variance investors. Mean-variance investors tend to take the risky side of the out-of-the-money put at lower jump risk compensation than power utility investors. For some settings, power utility investors use the put options only as a hedge for long stock positions. Finally, large efficiency gains of adding options to the investment opportunity set are observed in a world with a negative volatility risk premium and a jump risk premium. This gain is realized if the mean-variance investor takes a short position in the delta-neutral straddle.

The chapter is organized as follows. Section 2 gives a brief summary of the main theoretical findings on option returns followed by a treatment of the methodology that is used to determine expected option returns and the variance of option returns in Section 3. Section 4 provides intuition for the influence of volatility risk premia on the expectation and variance of option returns in a jump-diffusion model that fits in the class of affine jump-diffusion models. The link between a single-period CAPM performance measure and continuous time option pricing models is discussed in Section 5. Mean-variance portfolio asset allocation is performed in Section 6. The main findings of the chapter are summarized in Section 7. All proofs are given in the appendix.

6.2 Review

Although there are quite some studies[9] that examine the empirical properties of option returns or returns of option based strategies, only a few theoretical results on the (conditional) probability distribution of option returns are available. One of these results can be found in Coval and Shumway (2001). This section discusses the findings of Coval and Shumway (2001) in more detail and demonstrates these for the Black-Scholes model.

Standard asset pricing theory shows that, under no-arbitrage, a positive stochastic discount factor π exists such that the time t price, X_t, of a financial claim X_T at time T is given by

$$X_t \pi_t = \mathrm{E}_t^{\mathrm{P}}(X_T \pi_T), \tag{6.1}$$

where the expectation is taken under the real-world probability measure $\mathbb{P}$. The stochastic discount factor takes high values in bad states of the world and low values in good states of the world. For instance, low values of stock indices are often considered to be bad states of the world. In such an economy there will be a negative correlation between the stock index and the stochastic discount factor. Coval and Shumway (2001) shows that if the underlying value of a call option is negatively correlated with the stochastic discount factor, expected call option returns will be higher than the expected return on the option's underlying asset. The same assumption also implies that the expected call option return is increasing in strike price. In order to illustrate this, note (assuming that $\pi_t = 1$) that the expected gross return $R_{t:T}^c(K)$ of a call option with strike price K and maturity T is given by

$$\mathrm{E}_t\left(R_{t:T}^c(K)\right) = \frac{\mathrm{E}_t(C_T)}{C_t} = \frac{\int_K^\infty (S_T - K)\, f_t(S_T)\mathrm{d}S_T}{\int_0^\infty \int_K^\infty \pi_T (S_T - K)\, f_t(S_T, \pi_T)\mathrm{d}S_T \mathrm{d}\pi_T},$$

where $f_t(\cdot)$ denotes the conditional probability density function of (S_T, π_T) at time t. The derivative of the expected net return with respect to the strike price K is

$$\frac{\partial \mathrm{E}_t\left(r_{t:T}^c(K)\right)}{\partial K} = \frac{-\mathrm{Cov}\left(\mathrm{E}\left(\pi_T | S_T\right), S_T - K | S_T > K\right)}{\left(\int_K^\infty (S_T - K)\,\mathrm{E}\left(\pi_T|S_T\right) \frac{f_t(S_T)}{1-F_t(K)}\mathrm{d}S_T\right)^2}, \tag{6.2}$$

where F_t is the cumulative distribution function corresponding to f_t.[10] Formula (6.2) shows that if the correlation between the stochastic discount factor at time T and the

[9] Merton, Scholes, and Gladstein (1978), Merton, Scholes, and Gladstein (1982), Sheikh and Ronn (1994), Coval and Shumway (2001), Hodges, Tompkins, and Ziemba (2003), Driessen and Maenhout (2004), and Jones (2004).

[10] For a more detailed derivation, see Coval and Shumway (2001).

underlying value at time T is negative, the numerator of the above expression is positive. A call option price with strike price zero is equivalent to a stock and hence expected net returns on a call option exceed that of the underlying security.

The general pricing formula (6.1) provides additional intuition for this result. The payoff of a call option is small when the underlying stock has a low value at maturity of the option. Under the assumption that the correlation between the stochastic discount factor and the option's underlying value is negative, the call option pays off zero in states where the stochastic discount factor is high, i.e. where payoff is most rewarded. The payoff of the underlying stock is also negatively correlated with the stochastic discount factor but the difference with the call option is that the stock keeps value. Therefore, call options are 'riskier' than their underlying value and should consequently earn a higher return than the stock they are written on.[11]

For a put option an analogous reasoning applies. The correlation between the payoff of a put option and the stochastic discount factor is positive since put options provide payoff when the underlying stock (index) takes low values. Therefore, put options are considered to be hedge instruments against the undesirable states. Consequently, put options earn a lower return than the risk free rate.

The main condition for the theoretical result in Coval and Shumway (2001) is that the stochastic discount factor is negatively correlated with the underlying value of the call option. This condition is equivalent to a positive equity risk premium in most asset pricing models.[12] This means that low levels of asset prices are disliked by investors and that investment opportunities which give protection against these states, like put options, generate a low expected return.

To illustrate the main results of Coval and Shumway (2001), the standard Black-Scholes model, in which the stock price follows a geometric Brownian Motion, is considered. In this Black-Scholes world the expected gross return of a strike K call option with maturity T is given by

$$\mathrm{E}_t\left(R_{t:T}^{c,BS}(K)\right) = \frac{S_t e^{\mu(T-t)}\Phi(d_{1,p}) - K\Phi(d_{2,p})}{S_t\Phi(d_{1,q}) - Ke^{-r(T-t)}\Phi(d_{2,q})},$$

[11]From a CAPM perspective: the covariance between the call option's payoff and the terminal stock price decreases with strike price but the covariance between call option's *return* and the return on the underlying asset is increasing in strike price.

[12]Parameters in jump models can be chosen such that the condition is not satisfied. However, these parameter sets are empirically irrelevant.

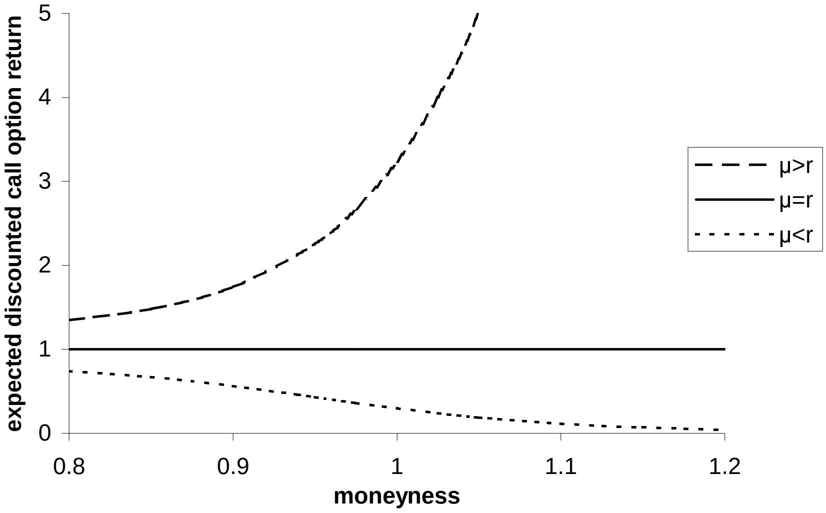


Figure 6.1: Expected discounted gross return (annualized) on a call option with one month to maturity for a positive equity risk premium, no equity risk premium, and a negative equity risk premium. Returns are calculated under the assumption of a Black-Scholes world.

where

$$\begin{aligned} d_{1,p} &= \frac{\log\left(\frac{S_t}{K}\right) + \left(\mu + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}}, \\ d_{1,q} &= \frac{\log\left(\frac{S_t}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{(T-t)}}, \\ d_{2,p} &= d_{1,p} - \sigma\sqrt{(T-t)}, \\ d_{2,q} &= d_{1,q} - \sigma\sqrt{(T-t)}, \end{aligned}$$

with μ and σ the standard Black-Scholes model parameters and $\Phi(\cdot)$ the standard normal cumulative distribution function. The risk free rate is denoted by r. If the strike price K goes to zero then

$$\lim_{K\to 0} \mathrm{E}_t\left(R_{t:T}^{c,BS}(K)\right) = e^{\mu(T-t)},$$

which is exactly the expected gross return on the stock. Note that in case market risk would not be priced, the gross return on each call option with maturity T equals $e^{r(T-t)}$ which is intuitively obvious since a zero market risk premium implies a risk-neutral world in this particular model.

The stochastic discount factor is important in the Coval and Shumway (2001) derivations. In the Black-Scholes world the stochastic discount factor process π can be derived

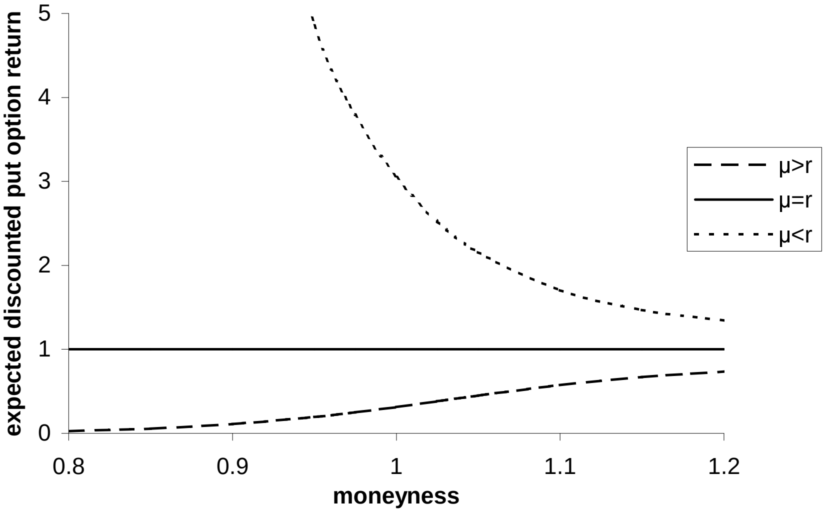


Figure 6.2: Expected discounted gross return (annualized) on a put option with one month to maturity for a positive equity risk premium, no equity risk premium, and a negative equity risk premium. Returns are calculated under the assumption of a Black-Scholes world.

as

$$\mathrm{d}\pi_t = -r\pi_t \mathrm{d}t - \left(\frac{\mu - r}{\sigma}\right)\pi_t \mathrm{d}W_t,$$

where W is a Brownian Motion under the real-world probability measure $\mathbb{P}$. The correlation between the stock price and the stochastic discount factor (for every t) is negative if $\mu > r$. Hence, the results in Coval and Shumway (2001) imply that under this condition call options will earn higher returns than the underlying value of the option. Figure 6.1 and Figure 6.2 illustrate theoretical expected discounted option returns for respectively calls and puts as a function of moneyness (where moneyness is defined as $\exp(-r(T-t))K/S_t$) in the Black-Scholes world.[13] The expected option returns are calculated for a one month maturity option that is held until maturity.

Figure 6.1 and Figure 6.2 confirm the intuition. If the expected rate of return on the underlying security is equal to the risk free rate (which equals 4% in Figure 6.1 and

[13]The reason for choosing this measure of moneyness will become clear later in this chapter when the conditional expectation of option returns is derived in a particular model. The disadvantage of this measure is that the term structure of implied volatilities is omitted. An alternative measure of moneyness which corrects for this is given in Natenberg (1994). However, this measure is based on total volatility over the life of the option. In empirical studies the Black-Scholes at-the-money implied volatility is often used to approximate total volatility. Such an approximation should be avoided in a theoretical study like this.

Figure 6.2) there is no premium required on the risk that is in the stock and therefore all assets should earn the risk free rate, i.e in this special case the real-world measure equals the risk-neutral measure. In case of a positive risk premium the expected return on the call option is increasing in strike price. This corresponds to the derivations in Coval and Shumway (2001). Furthermore, a call option with strike zero equals the stock and therefore the expected discounted gross return on a call option should converge to $e^{(\mu-r)(T-t)}$ as moneyness approaches zero. Finally, a put option with strike price infinity is a risk free asset and consequently the expected discounted gross return on a put converges to one as moneyness goes to infinity.

6.3 Methodology

In the theoretical analysis of Coval and Shumway (2001) expressions for expected option returns are derived in a setting where options are held to maturity. However, in that paper's empirical application, average option returns are calculated for option positions that are closed out before maturity. This section demonstrates how expected returns on options that are possibly not held to maturity dates, can be calculated explicitly. Moreover, the methodology can also be used to calculate (conditional) variances of option returns. The method is based on the joint characteristic function of the model's state variables. Because characteristic functions are model dependent this chapter's approach is less general than the approach in Coval and Shumway (2001). On the other hand, the resulting expressions can be used to determine the influence of risk premia on optimal portfolios. Furthermore, the moment conditions could also be useful for parameter estimation in the underlying continuous time model. The methodology will be demonstrated for the general class of affine jump-diffusion models as proposed in Duffie, Pan, and Singleton (2000). The practical applications will be based on a particular model that fits into this class.

6.3.1 Affine jump-diffusions

Duffie, Pan, and Singleton (2000) derives the time t price of a call option C_t with strike price K and maturity T in the class of affine jump models as

$$\begin{aligned} C_t &= \frac{\psi(1, X_t, t, T)}{2} - \frac{1}{\pi}\int_0^\infty \mathrm{Im}\left[\frac{\psi(1-i\phi_S, X_t, t, T)\, e^{i\phi_S \log K}}{\phi_S}\right] \mathrm{d}\phi_S \\ &\quad -K\left[\frac{\psi(0, X_t, t, T)}{2} - \frac{1}{\pi}\int_0^\infty \mathrm{Im}\left[\frac{\psi(-i\phi_S, X_t, t, T)\, e^{i\phi_S \log K}}{\phi_S}\right] \mathrm{d}\phi_S\right], \quad (6.3) \end{aligned}$$

where the vector X contains the state variables, ϕ_S takes real values, and $\mathrm{Im}(\cdot)$ denotes the imaginary part of a complex number. In this option pricing formula the function $\psi(\cdot)$ is defined by

$$\psi(u, X_t, t, T) = \mathrm{E}_t^{\mathbb{Q}}\left(\exp\left(-\int_t^T R(X_s)\mathrm{d}s\right) e^{u \log S_T}\right), \tag{6.4}$$

where S denotes the option's underlying asset, $R(\cdot)$ is a discounting function, and u takes complex values. The next lemma shows that (6.3) can be rewritten such that the resulting call option pricing formula looks more familiar.

Lemma 6.1. *The option price in (6.3) is rewritable in a Black-Scholes representation*

$$C_t = S_t P_{1,t} - K e^{\left(-\int_t^T R(X_s)ds\right)} P_{2,t}, \tag{6.5}$$

with

$$\begin{aligned} P_{1,t} &= \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{\varphi_t(\phi_S - i)\, e^{-i\phi_S \log K}}{i\phi_S \varphi_t(-i)}\right] \mathrm{d}\phi_S, \\ P_{2,t} &= \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{\varphi_t(\phi_S)\, e^{-i\phi_S \log K}}{i\phi_S}\right] \mathrm{d}\phi_S, \end{aligned}$$

where $P_{1,t}$ and $P_{2,t}$ are probabilities and $\varphi_t(\cdot)$ is the time t conditional characteristic function of $\log S_T$

$$\varphi_t(\phi_S) = \mathrm{E}_t^{\mathbb{Q}}\left(e^{i\phi_S \log S_T}\right).$$

Proof. Appendix 6.C. □

The derivations in the remainder of this chapter are based on option pricing formula (6.5) using a constant interest rate. In most empirical option pricing applications the statistical model is chosen such that the option pricing formula is homogeneous of degree one with respect to the pair (S_t, K).[14] The obvious consequence is that the option pricing formula can be rewritten as the product of the initial stock price and a function that depends on S_t only through the ratio K/S_t.

6.3.2 Expected option returns

Examination of the mean-variance properties of option returns requires knowledge of the expectation of option returns. This sections shows how this expectation can be obtained

[14] A sufficient condition for this assumption to hold is that the risk-neutral process of X is assumed to be Markovian and homogeneous with respect to the initial stock price level. See also, Garcia and Renault (1995) and Garcia and Renault (1998).

for every model that fits into the class of affine jump-diffusion models. The resulting expectations are conditional on the time t values of the state variables in X. Using (6.5) at time $t+h$, the expected gross return between time t and time $t+h$ on a call with strike price K and maturity $T \geq t+h$ is

$$\mathrm{E}_t^{\mathrm{P}}\left(\frac{C_{t+h}}{C_t}\right) = \mathrm{E}_t^{\mathrm{P}}\left(\frac{\mathrm{E}_{t+h}^{\mathbb{Q}}\left(\max(S_T - K, 0)\right)}{C_t}\right) = \mathrm{E}_t^{\mathrm{P}}\left(\frac{S_{t+h}P_{1,t+h} - Ke^{-r(T-(t+h))}P_{2,t+h}}{C_t}\right),$$

where

$$\begin{aligned} P_{1,t+h} &= \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{\exp\left(-i\phi_s \log K\right)\varphi_{t+h}\left(\phi_s - i\right)}{i\phi_s\varphi_{t+h}\left(-i\right)}\right] \mathrm{d}\phi_s, \\ P_{2,t+h} &= \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{\exp\left(-i\phi_s \log K\right)\varphi_{t+h}\left(\phi_s\right)}{i\phi_s}\right] \mathrm{d}\phi_s. \end{aligned}$$

The affine model structure implies

$$\begin{aligned} P_{1,t+h} &= \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{\exp\left(-i\phi_s \log K\right)\exp\left(\alpha^{1,q} + \beta^{1,q}\tilde{X}_{t+h} + i\phi_s \log S_{t+h}\right)}{i\phi_s}\right] \mathrm{d}\phi_s, \\ P_{2,t+h} &= \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{\exp\left(-i\phi_s \log K\right)\exp\left(\alpha^{2,q} + \beta^{2,q}\tilde{X}_{t+h} + i\phi_s \log S_{t+h}\right)}{i\phi_s}\right] \mathrm{d}\phi_s, \end{aligned}$$

where $\tilde{X}$ contains all state variables except S and $\alpha^{1,q}$ and $\beta^{1,q}$ are complex valued functions that depend on the model parameters and $T-(t+h)$. The necessary expectations for calculating the expected call option return can be expressed as

$$\begin{aligned} \mathrm{E}_t^{\mathrm{P}}\left(S_{t+h}P_{1,t+h}\right) &= a + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{be^{\alpha^{1,q}}\mathrm{E}_t^{\mathrm{P}}\left(\exp\left\{i\frac{\beta^{1,q}}{i}\tilde{X}_{t+h} + i\left(\phi_s - i\right)\log S_{t+h}\right\}\right)}{i\phi_s}\right] \mathrm{d}\phi_s, \\ \mathrm{E}_t^{\mathrm{P}}\left(P_{2,t+h}\right) &= \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{be^{\alpha^{2,q}}\mathrm{E}_t^{\mathrm{P}}\left(\exp\left\{i\frac{\beta^{2,q}}{i}\tilde{X}_{t+h} + i\phi_s \log S_{t+h}\right\}\right)}{i\phi_s}\right] \mathrm{d}\phi_s, \quad (6.6) \end{aligned}$$

where

$$\begin{aligned} a &= \frac{1}{2}\mathrm{E}_t^{\mathrm{P}}\left(S_{t+h}\right), \\ b &= \exp\left(-i\phi_s \log K\right). \end{aligned}$$

Subsequently, the model structure is utilized once more to calculate $\mathrm{E}_t^{\mathrm{P}}\left(S_{t+h}P_{1,t+h}\right)$ and $\mathrm{E}_t^{\mathrm{P}}\left(P_{2,t+h}\right)$ explicitly (details omitted). The particular chosen asset pricing model defines the parameters of the process and the closed form expressions of expected option returns.

6.3.3 Variance and covariance of option returns

Not only expected option returns are necessary for mean-variance analysis but the variance of option returns, the covariance between option returns and returns on the risky asset, and the covariance between option returns of different strikes need to be determined as well. This section shows that these quantities can be calculated in the same way as in the previous section.

For the (conditional) variance of call option returns only the expectation of the squared future call price is unknown. Given the option price representation in (6.5), the second moment of the future call price, for $t = 0$, is given by

$$\begin{aligned} \mathrm{E}_0^{\mathrm{P}}\left(C_h^2\right) &= \mathrm{E}_0^{\mathrm{P}}\left(S_h P_{1,h} - Ke^{-r(T-h)} P_{2,h}\right)^2, \\ &= \mathrm{E}_0^{\mathrm{P}}\left(S_h^2 P_{1,h}^2\right) - 2Ke^{-r(T-h)}\mathrm{E}_0^{\mathrm{P}}\left(S_h P_{1,h} P_{2,h}\right) + K^2 e^{-2r(T-h)}\mathrm{E}_0^{\mathrm{P}} P_{2,h}^2. \end{aligned} \quad (6.7)$$

Appendix 6.B shows that calculating the expectation of the squared probabilities is more cumbersome than the expectation of the probabilities in the previous section. However, the specific structure of the affine jump-diffusion models can be used once more to obtain analytical expressions for all conditional expectations in (6.7).

Finally, the methodology can also be used to calculate the covariance between the future value of the risky asset and the future option price, and the covariance between two future option prices that have different strikes or maturities. Consider, for instance, the covariance between the future value of the risky asset and the future call option price (option with strike price K and maturity T), for $t = 0$

$$\begin{aligned} \mathrm{Cov}_0\left(C_h, S_h\right) &= \mathrm{Cov}_0\left(S_h P_{1,h}, S_h\right) - Ke^{-r(T-h)}\mathrm{Cov}_0(P_2, S_h), \\ &= \mathrm{E}_0\left(S_h P_{1,h} S_h\right) - \mathrm{E}_0\left(S_h P_{1,h}\right)\mathrm{E}_0\left(S_h\right) \\ &- Ke^{-r(T-h)}\left(E_0\left(S_h P_{2,h}\right) - \mathrm{E}_0\left(S_h\right)\mathrm{E}_0\left(P_{2,h}\right)\right). \end{aligned} \quad (6.8)$$

Obviously, all expectations in (6.8) can be calculated by means of the methodology presented in the previous section.

6.4 Model

The previous section demonstrated how the expectation and variance of option returns can be determined for affine jump-diffusion models. In this section a particular model that belongs to the class of affine jump-diffusion models is chosen. The model will be used in the remainder of this chapter for the inspection of the mean-variance properties of option returns. The second part of this section shows how the risk premia in the chosen benchmark model affect expected option returns.

The following stock price model is used to assess the link between several risk premia and option returns

$$\begin{aligned}\frac{\mathrm{d}S_t}{S_{t_-}} &= \left(r + \eta^S + \mu_J\left(\lambda - \tilde{\lambda}\right)\right)\mathrm{d}t + \sigma_t \mathrm{d}W_t^S - \lambda\mu_J \mathrm{d}t + \mathrm{d}\sum_{i=1}^{N_t}\left(Y_i - 1\right),\\ \mathrm{d}\sigma_t^2 &= -\kappa\left(\sigma_t^2 - \sigma^2\right)\mathrm{d}t + \sigma_\sigma\sigma_t\left(\rho \mathrm{d}W_t^S + \sqrt{1-\rho^2}\mathrm{d}W_t^V\right),\\ \log Y_i &\sim N\left(\log(1+\mu_J) - \frac{1}{2}\sigma_J^2, \sigma_J^2\right),\end{aligned} \tag{6.9}$$

where W^S and W^V are uncorrelated Brownian Motions, and N is a pure-jump Poisson process under the objective probability measure $\mathbb{P}$. In addition to the risky stock there is a risk free asset that has a constant rate of return r. The equity premium is determined by the compensation for risk in the price diffusion W^S and by the compensation for jump risk in N. The compensation for jump risk is rather intuitive. Consider, for instance, the empirically relevant case $\mu_J < 0$. If $\tilde{\lambda} > \lambda$ the effect on the equity premium is positive indicating that investors require a premium for being unprotected against market crashes.[15]

The model in (6.9) is incomplete with respect to any finite number of traded assets. Assume that a risk-neutral measure is chosen such that the Poisson description of the jump part remains unchanged. Furthermore, suppose that the variance of jump sizes does not change after the change of measure, i.e. the return jump volatility risk premium is zero. Applying a change of measure under these assumptions leads to

$$\begin{aligned}\frac{\mathrm{d}S_t}{S_{t_-}} &= r\mathrm{d}t + \sigma_t \mathrm{d}\tilde{W}_t^S - \tilde{\lambda}\tilde{\mu}_J \mathrm{d}t + \mathrm{d}\sum_{i=1}^{\tilde{N}_t}\left(\tilde{Y}_i - 1\right),\\ \mathrm{d}\sigma_t^2 &= -\left(\kappa + \eta^V\right)\left(\sigma_t^2 - \frac{\kappa\sigma^2}{\kappa+\eta^V}\right)\mathrm{d}t + \sigma_\sigma\sigma_t\left(\rho \mathrm{d}\tilde{W}_t^S + \sqrt{1-\rho^2}\mathrm{d}\tilde{W}_t^V\right),\\ \log \tilde{Y}_i &\sim N\left(\log(1+\tilde{\mu}_J) - \frac{1}{2}\sigma_J^2, \sigma_J^2\right),\end{aligned} \tag{6.10}$$

where $\tilde{W}^S$ and $\tilde{W}^V$ are uncorrelated Brownian Motions, and $\tilde{N}$ is a pure-jump Poisson process under a risk-neutral probability measure $\mathbb{Q}$ equivalent to the objective probability measure $\mathbb{P}$. The proposed model fits in the class of affine jump diffusions. Note that the

[15] In the next sections asset allocation results will be compared to the outcomes in Liu and Pan (2003). The model in (6.9) is more general than the model in Liu and Pan (2003) since the former allows for stochastically varying jumps.

influence of volatility risk premia and jump risk premia on option returns is measurable by means of the parameters η^V, $\tilde{\lambda}$, and $\tilde{\mu}_J$.

The link between the data-generating dynamics in (6.9) and the risk-neutral process in (6.10) is the pricing kernel process π. In differential form, the pricing kernel process implied by the change of measure in (6.9) and (6.10) is given by

$$\mathrm{d}\pi_t = -r\pi_t \mathrm{d}t - \zeta_t^S \pi_t \mathrm{d}W_t^S - \zeta_t^V \pi_t \mathrm{d}W_t^V + \pi_t \mathrm{d}\left(H_t - \tilde{\lambda}t\right) - \pi_t \mathrm{d}\left(N_t - \lambda t\right), \tag{6.11}$$

with

$$\begin{aligned}
\zeta_t^S &= \frac{\eta^S}{\sigma_t}, \\
\zeta_t^V &= \frac{1}{\sqrt{1-\rho^2}}\left(\frac{\eta^V \sigma_t}{\sigma_\sigma} - \zeta_t^S \rho\right), \\
H_t &= \sum_{i=1}^{N_t} \frac{\tilde{\lambda} f(Y_i + (\tilde{\mu}_J - \mu_J))}{\lambda f(Y_i)},
\end{aligned}$$

where f is the lognormal jump size density. Pan (2002) questions the joint empirical identifiability of the jump timing risk premium and the jump size risk premium. Therefore, that paper assumes that all jump risk premia are absorbed by the jump size risk premium. In this chapter a similar view is adopted, but with the distinction that the jump risk premium is contained in the jump timing risk premium. Under this assumption, the pricing kernel process (6.11) simplifies to

$$\mathrm{d}\pi_t = -r\pi_t \mathrm{d}t - \zeta_t^S \pi_t \mathrm{d}W_t^S - \zeta_t^V \pi_t \mathrm{d}W_t^V + \frac{\tilde{\lambda} - \lambda}{\lambda} \pi_t \mathrm{d}\left(N_t - \lambda t\right), \tag{6.12}$$

where W^V is a Brownian Motion under the objective probability measure $\mathbb{P}$ and N is a Poisson process with intensity λ.[16] Appendix 6.A provides a closed form solution of the expected call option return in this particular asset pricing model. The appendix additionally shows how to calculate the variance of the call option return.

The assumed stochastic processes under the risk-neutral measure imply that the resulting option valuation formulas are homogeneous of degree one with respect to (S_t, K). Hence, the time t price of a call option with strike price K and maturity T can be rewritten as

$$C_t\left(K, T, \sigma_t^2\right) = S_t c_t(m_t, T, \sigma_t^2),$$

[16] The change of measure and corresponding pricing kernel process imply that this chapter's risk-neutral stochastic volatility process is slightly different from the risk-neutral stochastic volatility process in Liu and Pan (2003).

where $m_t = \exp(-r(T-t))K/S_t$. Intuitively, a similar property is expected for option returns in these type of models.

Lemma 6.2. *In the benchmark model, the discounted expected gross return of an option is a function of the model parameters, instantaneous variance, option's moneyness, option's maturity, and investment horizon, i.e.*

$$E_t\left(e^{-rh}\frac{C_{t+h}}{C_t}\right) = g\left(\Theta, \sigma_t^2, m_t, T-t, h\right),$$

where Θ *contains all model parameters and* g *is defined in (6.33).*

Proof. Appendix 6.C. □

The consequence of Lemma 6.2 is that in order to make proper qualitative or quantitative statements about average option returns the observations should be categorized not only in terms of moneyness (as is done in Coval and Shumway (2001)) but also in variance classes.

The next lemma describes that a similar result as Lemma 6.2 also applies for the conditional variance of option returns and the conditional covariance between option returns.

Lemma 6.3. *The conditional variance of the discounted call option return, put option return, and the covariance between the call option return and the put option return (where the strike price and maturity of the call and put are equal) depends only on the current stock price through moneyness, i.e.*

$$\begin{aligned}
\mathrm{Var}_t\left(e^{-rh}\left(\frac{C_{t+h}}{C_t}\right)\right) &= f_1\left(\Theta, \sigma_t^2, m_t, T-t, h\right), \\
\mathrm{Var}_t\left(e^{-rh}\left(\frac{P_{t+h}}{P_t}\right)\right) &= f_2\left(\Theta, \sigma_t^2, m_t, T-t, h\right), \\
\mathrm{Cov}_t\left(e^{-rh}\left(\frac{C_{t+h}}{C_t}\right), e^{-rh}\left(\frac{P_{t+h}}{P_t}\right)\right) &= f_3\left(\Theta, \sigma_t^2, m_t, T-t, h\right).
\end{aligned}$$

6.4.1 Expected option returns and the volatility risk premium

In order to get a deeper understanding of the benchmark model in (6.9) and (6.10), this section discusses the influence of the volatility risk premium. The influence of the jump risk premium is examined in the next section. The price diffusion risk premium is not treated here since the results are analogous to the Black-Scholes results in Section 6.2.

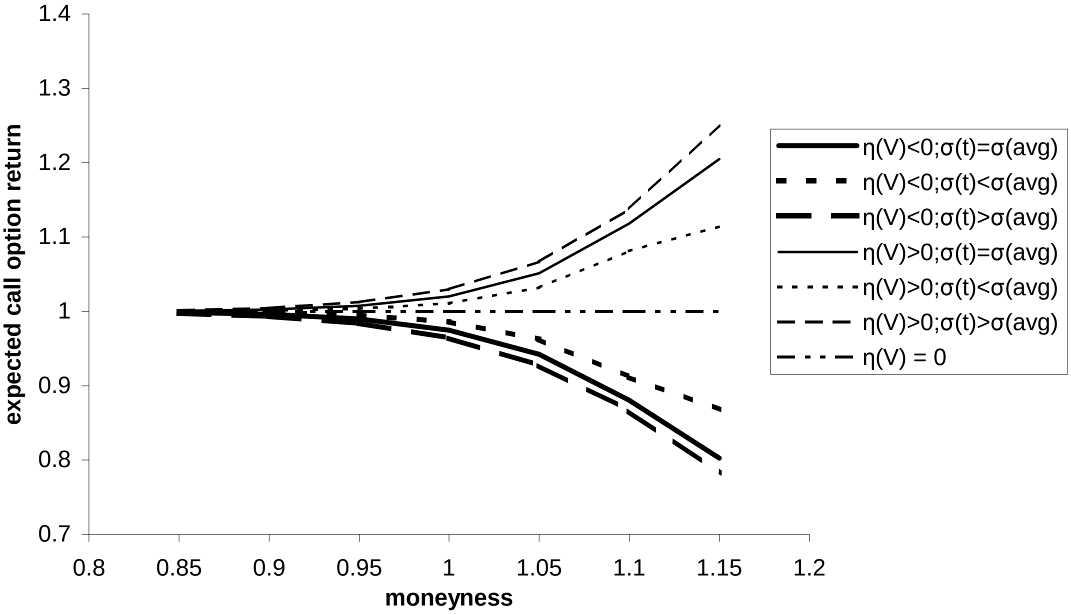


Figure 6.3: The influence of the volatility risk premium on expected discounted returns of a plain vanilla call option as a function of moneyness for several values of instantaneous variance. The price diffusion risk premium and the jump risk premium are set to zero.

The influence of the volatility risk premium on expected option returns is investigated in the benchmark model where only volatility risk is priced, i.e. the price diffusion risk premium and the jump risk premium are set to zero. Figure 6.3 and Figure 6.4 show expected option returns for respectively call options and put options as a function of moneyness for various levels of the instantaneous variance. The maturity of the options is two months and the options are held in portfolio for one month.

The intuition behind the results is best understood in a situation where interest rates are set to zero and options are not sold before maturity. Assume that the volatility risk premium is negative. In the benchmark model this implies a positive correlation between the pricing kernel and future variance for all t. The payoff of the option at maturity does not depend on the variance level at maturity but the probability distribution of the option's underlying depends on future volatility. A negative volatility risk premium leads to a risk-neutral distribution of the option's underlying value that has a higher variance than the variance of the objective distribution of the asset price at the option's maturity. This means that the stochastic discount factor is high when the option's underlying asset takes high or low values at the option's maturity. Both puts and calls provide payoff when the stochastic discount factor is high and therefore serve as protection tools against high

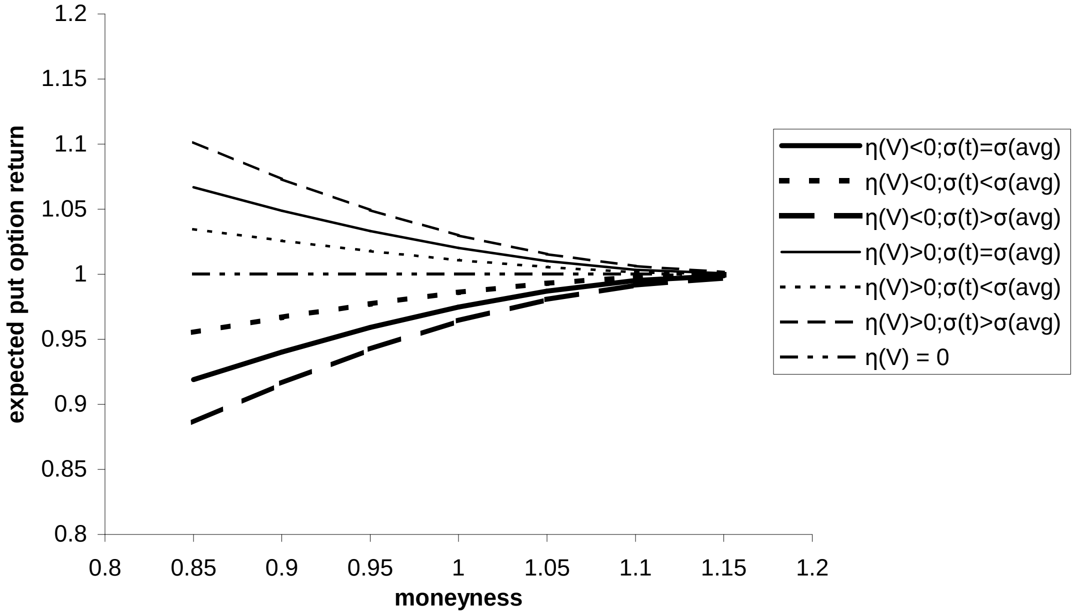


Figure 6.4: The influence of the volatility risk premium on expected discounted returns of a plain vanilla put option as a function of moneyness for several values of instantaneous variance. The price diffusion risk premium and the jump risk premium are set to zero.

levels of future volatility. Consequently, the expected return on calls and puts should be below the risk free rate. Furthermore, for call options with higher strike levels the option's payoff is concentrated in states for which the state price is highest compared to the state's expected return. As a result, the call option return is expected to decrease in moneyness. A similar reasoning applies for put options with lower strikes.

For a positive volatility risk premium a similar reasoning applies. The payoff of both puts and calls are negatively correlated with the stochastic discount factor. Hence, expected returns on these instruments should be above the risk free rate. In a similar fashion, the expected call option return is expected to be increasing in moneyness and the expected put returns should be decreasing in moneyness. Figure 6.3 and Figure 6.4 show that this also applies to option positions that are closed out before maturity.

In the benchmark model the variance risk premium is proportional to the instantaneous variance. Hence, a lower initial variance value reduces the difference between the $\mathbb{P}$-distribution and the $\mathbb{Q}$-distribution of the option's underlying value at maturity. Hence, a lower starting level of variance should lead to option returns closer to the risk free rate for all possible strikes. Figure 6.3 and Figure 6.4 clearly confirm this reasoning.

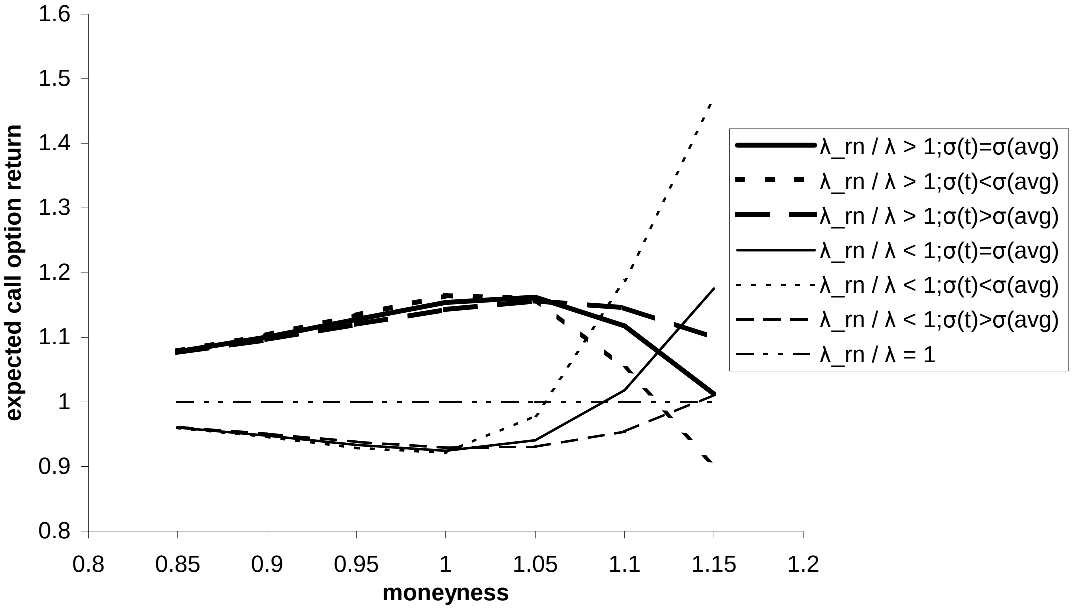


Figure 6.5: The influence of the jump timing risk premium on expected discounted returns of a plain vanilla call option as a function of moneyness for several values of instantaneous variance. The price diffusion risk premium and the volatility risk premium are set to zero.

6.4.2 The influence of the jump risk premium

This section shows that a crash risk premium combines the effects of the price diffusion risk premium and the volatility risk premium on expected option returns, since the jump risks influences the expectation of the option's underlying asset at the option's maturity under the objective probability measure as well as the variance of the underlying value at maturity under the risk-neutral measure.

The results in Pan (2002) show that jump size risk is priced in option markets. As was mentioned in the previous section, this chapter adopts the assumption that all jump risk is concentrated in jump timing risk. The ratio $\tilde{\lambda}/\lambda$ is a possible measure for the magnitude of the jump risk in the benchmark model under this assumption. Using several values for this ratio and all other risk premia equal to zero, Figure 6.5 and Figure 6.6 show expected option returns for respectively calls and puts as a function of moneyness for three different levels of instantaneous variance.

Crucial for the shapes of the curves in Figure 6.5 and Figure 6.6 is the assumption that $\mu_J < 0$. The intuition for the hump-shaped expected call option return curve is as follows. If $\tilde{\lambda}/\lambda > 1$ then the expectation of the option's underlying asset at maturity is

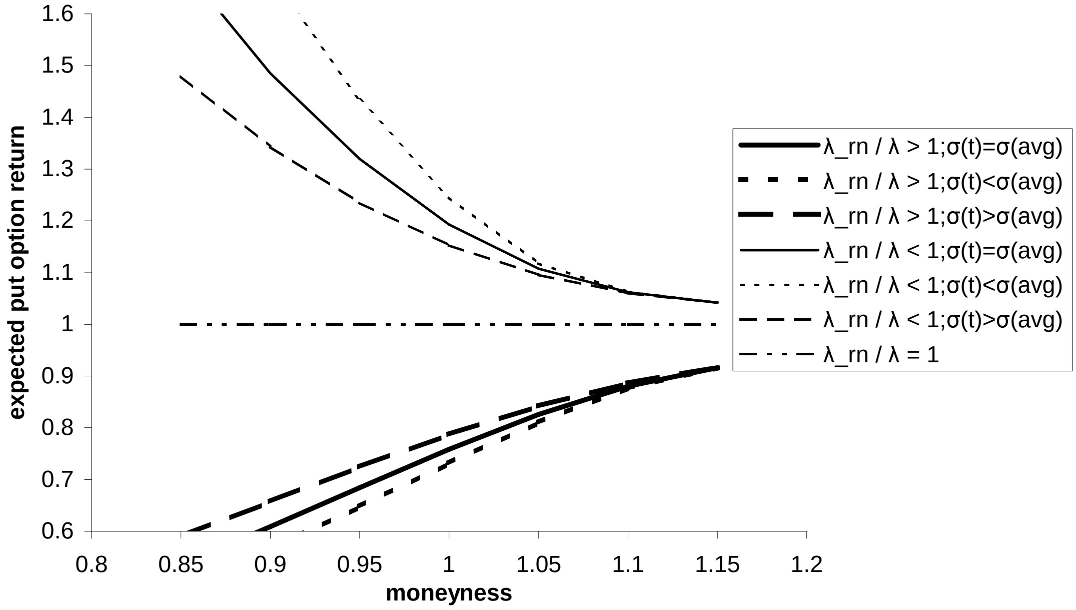


Figure 6.6: The influence of the jump timing risk premium on expected discounted returns of a plain vanilla put option as a function of moneyness for several values of instantaneous variance. The price diffusion risk premium and the volatility risk premium are set to zero.

higher under the objective measure than under the risk-neutral measure. However, there is a second effect that is of importance for the case $\tilde{\lambda}/\lambda > 1$. Under this assumption, the variance of the option's underlying asset at maturity will be higher under the risk-neutral measure than under the objective measure. As a result, the stochastic discount factor is increasing for high values of the option's underlying asset. Hence, far out-of-the-money calls pay off in states where the stochastic discount factor is increasing. The obvious consequence is that expected call option returns start to decrease at high strike prices. Note that in case μ_J is chosen to be positive, the expected call option return curve will be monotonic and the expected put option return curve hump-shaped.

The effect of a change in the starting instantaneous variance is less clear as in the previous cases since the jump risk premium does not depend on the current instantaneous variance. However, both the risk-neutral and the objective distribution of the option's underlying value at maturity obviously depend on the current variance level. Consider the case where $\mu_J < 0$ and $\tilde{\lambda}/\lambda > 1$, i.e. the variance of the option's underlying asset at maturity is higher under the risk-neutral measure than under the objective measure. For a smaller value of the instantaneous variance, the relative impact of the jump premium on the variance of the option's underlying asset is higher. Therefore, the stochastic discount factor is steeper in higher values of the option's underlying asset than in the average instantaneous variance case. Thus, the hedging characteristic for far out-of-the-money calls becomes more important and, therefore, these calls should have a lower expected return than in case the initial variance equals the average variance.

6.5 Mean-Variance Performance Measurement

The previous section provided the necessary tools and intuition for the examination of mean-variance properties of option returns. This section concentrates on the intercept of the CAPM regression equation (CAPM α) which is often used as a performance measure. Leland (1999) shows that in a world of i.i.d. returns on the market portfolio, CAPM α should not be used as a performance measure for buy-and-hold derivative strategies. This section shows that the same result holds if the assumption of i.i.d. market returns is relaxed.

The concept of risk-return trade off for a portfolio of stocks was first studied in Markowitz (1952). Earlier research aimed to find the 'best' stock among a set of available stocks. In the framework of Markowitz (1952) investors optimally hold a mean-variance efficient portfolio. A portfolio is called mean-variance efficient if the portfolio has the highest return for a given variance level. In order to come to a model that has equilibrium content, additional assumptions need to be imposed such that each investor holds a minimum-variance portfolio. Sharpe (1964) and Lintner (1965) show that if (i) all investors optimally hold mean-variance efficient portfolios, (ii) all investors have a common time horizon and homogenous beliefs, (iii) each asset is infinitely divisible, and (iv) a risk free asset that can be bought or sold in unlimited amounts exists, then the portfolio of all invested wealth (also called the market portfolio) is a mean-variance efficient portfolio. Extending this notion results in the classical Sharpe-Lintner Capital Asset Pricing Model. One of the assumptions underlying the CAPM is that each investor optimally holds a mean-variance efficient portfolio. A quadratic utility function and an elliptical distribution for asset returns are sufficient conditions under which this behavior maximizes expected utility.[17]

[17] The inclusion of options in the investment consideration set leads to problems if these options need to be priced by means of the one-period CAPM. Jarrow and Madan (1997) shows that the CAPM implies the existence of arbitrage opportunities for economies in which options are traded with an unbounded strike range, i.e. the linearity of the pricing kernel in the market return leads to negative

The Sharpe-Lintner CAPM regression equation for the expected gross return on a call option is given by (using continuously compounded interest rates)

$$\frac{C_{t+h}}{C_t} - e^{rh} = \alpha_t + \beta_t \left(\frac{S_{t+h}}{S_t} - e^{rh} \right) + \varepsilon_{t,t+h}, \tag{6.13}$$

where $\varepsilon_{t,t+h}$ is an error term with expectation zero and uncorrelated with the increments in the option's underlying value S. Under these assumptions, the CAPM β boils down to

$$\beta_t = \frac{\mathrm{Cov}_t \left(\frac{C_{t+h}}{C_t}, \frac{S_{t+h}}{S_t} \right)}{\mathrm{Var}_t \left(\frac{S_{t+h}}{S_t} \right)}. \tag{6.14}$$

Given β_t in (6.14), calculation of the intercept α_t is trivial

$$\alpha_t = \mathrm{E}_t \left(\frac{C_{t+h}}{C_t} - e^{rh} \right) - \beta_t \mathrm{E}_t \left(\frac{S_{t+h}}{S_t} - e^{rh} \right). \tag{6.15}$$

In the benchmark model (6.9) and (6.10), α_t and β_t depend only on S_t through moneyness. Therefore, the time t dependence of α_t and β_t stems from the moneyness of the option and the instantaneous variance level at time t. Furthermore, α_t and β_t are functions of model parameters, the investment horizon h, and the option's time to maturity T.

6.5.1 CAPM and the Black-Scholes model

This section studies the effect of measuring the performance of buy-and-hold option strategies by CAPM α in a world where options are priced under the Black-Scholes assumptions. The focus of this chapter is similar to Leland (1999) since the Black-Scholes assumptions imply i.i.d. returns on the market portfolio. The main difference with Leland (1999) is that in this section option positions are not necessarily held to maturity.[18] This makes the calculations more complicated, but the conclusions should remain the same.

values for far out-of-the-money calls. These problems do not arise if the instantaneous planning horizon version of the CAPM is considered (Merton (1973)).

[18]This is an important distinction from a practical perspective. On the one hand portfolio managers do not want to trade too often since trading costs can have a significant impact on investment performance (see Davis and Norman (1990), Aiyagari and Gertler (1991), He and Modest (1995), and Heaton and Lucas (1996)) which justifies the use of buy-and-hold strategies. On the other hand, the investment horizon may be shorter than the maturity of the traded options. Instead of excluding these options from the investment consideration set, this chapter determines the risk-return characteristics of these assets.

In the Black-Scholes world the stochastic evolution of a call option price is described by

$$\mathrm{d}C_t = \Delta_t^{BS}\mathrm{d}S_t + r\left(C_t - \Delta_t^{BS}S_t\right)\mathrm{d}t,$$

where r denotes the risk free rate and Δ_t^{BS} the sensitivity of the call option price for changes in the option's underlying asset at time t. The equation implies that the call can be replicated by continuously taking positions in the stock and the risk free asset, i.e. options are redundant assets. Consider the discrete time version of the model.[19] The call option price at time $t+1$ is approximated as

$$C_{t+1} \approx C_t + \Delta_t^{BS}\left(S_{t+1} - S_t\right) + r\left(C_t - \Delta_t^{BS}S_t\right). \tag{6.16}$$

The excess call option return is easily derived as

$$\frac{C_{t+1}}{C_t} - (1+r) \approx \frac{\Delta_t^{BS}S_t}{C_t}\left(\frac{S_{t+1}}{S_t} - (1+r)\right). \tag{6.17}$$

For obvious reasons, the expression $\Delta_t^{BS}S_t/C_t$ is often referred to as the CAPM β in the Black-Scholes model. However, relation (6.16) is only a discrete time approximation of the call option return in the Black-Scholes world. The exact representation is, for $(h \leq T)$, given by (6.13). For $h > 0$, α_t is unequal to zero and $\beta_t \neq \Delta_t^{BS}S_t/C_t$. When the holding period h approaches to zero, α_t converges to zero and β_t to $\Delta_t^{BS}S_t/C_t$.[20] This result implies that in the Black-Scholes world, for $0 < h \leq T$, a call option return is not fully explained by the return on the risky and the riskless asset. Hence, simple buy-and-hold option strategies generate positive CAPM α.[21] This seems counterintuitive since options are fairly priced and therefore CAPM α should be equal to zero. Leland (1999) provides the explanation for this phenomenon.[22] Returns on options are obviously not elliptically distributed in the Black-Scholes world. Moreover, the Black-Scholes assumptions imply that the representative investor must have power utility. Hence, none of the sufficient conditions which guarantees mean-variance preferences is satisfied. As a

[19]If market returns are lognormally distributed and the representative investor has power utility, the Black-Scholes option pricing formula gives fair prices, in discrete time, for options written on the market, see Rubinstein (1976).

[20]Black and Scholes (1973) already reports that the CAPM β of an instantaneous option return equals $\Delta_t^{BS}S_t/C_t$.

[21]For instance, in a Black-Scholes world with a price diffusion risk premium of 6%, an at-the-money call option with two months to maturity that is held for one month leads to $\alpha_t = -0.31\%$ (-3.65% annualized).

[22]Leland (1999) does not consider single option returns but treats covered call and portfolio protection strategies.

consequence is that CAPM β is an inappropriate risk measure and, therefore, CAPM α an invalid performance measure. The fact that the representative investor must have power utility in the Black-Scholes world can be used to explain the sign of α_t for option based strategies. Strategies that have a non-linear convex payoff as function of the market payoff give a negative α_t.[23] This is best understood when a put option is considered. The put option protects against low states of the market portfolio. This is appreciated more by power utility investors than by mean-variance investors, i.e. a power utility investor accepts a lower return on the put option than a mean-variance investor. Hence, the CAPM β is higher than the "true" β. Because the expected future option price is calculated under the Black-Scholes assumptions, CAPM α will be negative for a put option. Similar reasonings can be constructed for a single call and portfolio protection strategies. Strategies with a non-linear concave payoff as function of the market payoff lead to positive values of α_t.[24]

In empirical studies of option returns, CAPM regression equation (6.13) is sometimes used to validate option pricing models.[25] The results in this section indicate that statistical tests should be performed with care. For instance, for buy-and-hold call option strategies where h is not too small, the estimate of β should not be tested against $\Delta_t^{BS} S_t / C_t$ when the Black-Scholes model is validated.

6.5.2 CAPM and the Heston model

The analysis in Leland (1999) is performed under the assumptions of perfect markets and i.i.d. market returns. In this section the latter assumption is relaxed by investigating CAPM α and CAPM β in the Heston model. The Heston model allows for stochastically varying volatilities. The movements of the stock and the instantaneous variance are not driven by the same process and therefore the model is incomplete (with respect to risky asset and the riskless asset) whether the risk in volatility is priced or not. The dynamics of the call option price in the Heston model are given by (see Bakshi and Kapadia (2003))

$$\mathrm{d}C_t = \Delta_t^H \mathrm{d}S_t + r\left(C_t - \Delta_t^H S_t\right)\mathrm{d}t + \frac{\partial C_t}{\partial \sigma_t^2}\left(\eta^V \sigma_t^2 \mathrm{d}t + \sigma_\sigma \sigma_t \rho \mathrm{d}W_t^S + \sigma_\sigma \sigma_t \sqrt{1-\rho^2}\mathrm{d}W_t^V\right), \tag{6.18}$$

where notation of the benchmark model (6.9) and (6.10) is used. If volatility risk is idiosyncratic, the discrete time version of the model approximates the future call option

[23] Not only a single call and put option but also a put option combined with the market generate a negative α_t.

[24] See, for instance, the numbers of the covered call strategy in Table I of Leland (1999).

[25] See Coval and Shumway (2001) and Driessen and Maenhout (2004).

	ATM call			**OTM put**		
	$\mathrm{E}_t\left(\frac{C_{t+h}}{C_t}\right)$	α_t	β_t	$\mathrm{E}_t\left(\frac{P_{t+h}}{P_t}\right)$	α_t	β_t
Black-Scholes	4.28%	-0.10%	26.22	-8.64%	-0.61%	-48.18
Heston, $\rho = 0$	4.36%	-0.10%	26.77	-6.94%	1.17%	-48.61
Heston, $\rho < 0$	4.70%	0.72%	23.85	-5.31%	3.58%	-53.29
Heston, $\rho > 0$	4.03%	-0.92%	29.70	-11.16%	-3.99%	-42.98

Table 6.1: CAPM parameters and expected option returns for an at-the-money call option and an out-of-the-money put option under the Black-Scholes assumptions and the Heston assumptions. The numbers are calculated using an (annualized) rate of return on the risky asset of 2% and a 0% risk free rate.

price as

$$
\begin{aligned}
C_{t+1} \quad \approx \quad & C_t + \Delta_t^H \left(S_{t+1} - S_t\right) + r\left(C_t - \Delta_t^H S_t\right) + \sigma_\sigma \sigma_t \rho \frac{\partial C_t}{\partial \sigma_t^2}\left(W_{t+1}^S - W_t^S\right) \\
& + \sigma_\sigma \sigma_t \sqrt{1-\rho^2} \frac{\partial C_t}{\partial \sigma_t^2}\left(W_{t+1}^V - W_t^V\right). \qquad (6.19)
\end{aligned}
$$

Constructing the excess gross return gives

$$
\frac{C_{t+1}}{C_t} - (1+r) \approx \frac{\Delta_t^H S_t}{C_t}\left(\frac{S_{t+1}}{S_t} - (1+r)\right) + \varepsilon_t, \qquad (6.20)
$$

with ε_t representing the idiosyncratic risk in the variance process. This random variable has expectation zero. However, relation (6.19) is only a discrete time approximation of the call option return in the Black-Scholes world. The exact representation of the call option return is again given by (6.13). The methodology of Section 6.3 can be applied to determine closed form expressions for functions α_t and β_t. The difference with the Black-Scholes model in the previous section is that the time t dependence of α_t and β_t also originates from the instantaneous variance. The resulting expressions for α_t and β_t show that, for $h > 0$, α_t differs from zero and β_t is unequal to $\Delta_t^H S_t / C_t$. Under the assumption of non systematic volatility risk, α_t converges to zero when the investment horizon h goes to zero. Hence, the expected return on a call option is fully explained by the expected return of the option's underlying asset when $h \to 0$.[26]

[26]Note that the future value of the call option is not perfectly replicable by the stock and the risk free asset and, therefore, the option is non-redundant even when the investment horizon h goes to zero.

Table 6.1 gives an overview of the results under different parameter assumptions.[27] The results show that for $\rho = 0$ and the at-the-money call option, only small differences between the Heston outcomes and the Black-Scholes outcomes occur. Larger differences appear for the out-of-the-money put option. For this put option, α_t turns out to be positive. This is explained by the value of β_t which implies that the expected return on the put option should be lower than -6.94%. To compensate for this α_t is assigned a positive value. Hence, the CAPM β_t is not an adequate risk measure for single option returns in the Heston world and, therefore, α_t is not suitable as a performance measure for buy-and-hold option based strategies in the Heston world.[28] Again, the explanation is that the utility function implied by the Heston model is not quadratic.[29] This holds for all cases considered in Table 6.1. Each choice of ρ implies different probability distributions of the risky asset under both the objective measure and the risk-neutral measure. Therefore, marginal utility as a function of the risky asset also changes with ρ. For instance, the numbers for $\rho = 0$ and $\rho < 0$ indicate that the marginal utility for low levels of the risky asset is higher when volatility and the returns on the risky asset are uncorrelated.

For the validation of option pricing models an additional complexity arises when $\rho \neq 0$. Under this condition, the underlying assumption in (6.13) of no correlation between the return on the option's underlying asset and the error term is violated if $\rho \neq 0$ in the Heston model. Therefore, the more general representation of β_t should be used

$$\tilde{\beta}_t = \frac{\mathrm{Cov}_t\left(\frac{C_{t+h}}{C_t}, \frac{S_{t+h}}{S_t}\right)}{\mathrm{Var}_t\left(\frac{S_{t+h}}{S_t}\right)} - \frac{\mathrm{Cov}_t\left(\varepsilon_{t,t+h}, \frac{S_{t+h}}{S_t}\right)}{\mathrm{Var}_t\left(\frac{S_{t+h}}{S_t}\right)}. \tag{6.21}$$

If h goes to zero then $\tilde{\beta}_t$ in (6.21) converges to $\Delta_t^H S_t / C_t$. Ignoring the correlation between the error and the option's underlying would lead to a converging value of β_t

[27]The stochastic volatility parameters are based on the parameters in Pan (2002), i.e. $\kappa = 6.4$, $\sigma_\sigma = 0.30$, and $\sigma^2 = 0.015$. The starting level of variance is chosen equal to the long term mean level of the variance process. This is also the variance used for the Black-Scholes results. Furthermore, the chosen options have a maturity of two months and are held for one month in the portfolio. Finally, the rate of return on the risky stock is assumed to be 2% and the rate of return on the risk free asset is set at 0%.

[28]Note that the volatility risk premium is equal to zero which implies that α_t should be zero if risk is measured correctly.

[29]See also Leland (1999). That paper proposes to adjust CAPM β in a world of i.i.d. returns on the market portfolio such that it can be interpreted again as a risk measure. The topic of how to adjust CAPM β in the Heston world is left for future research.

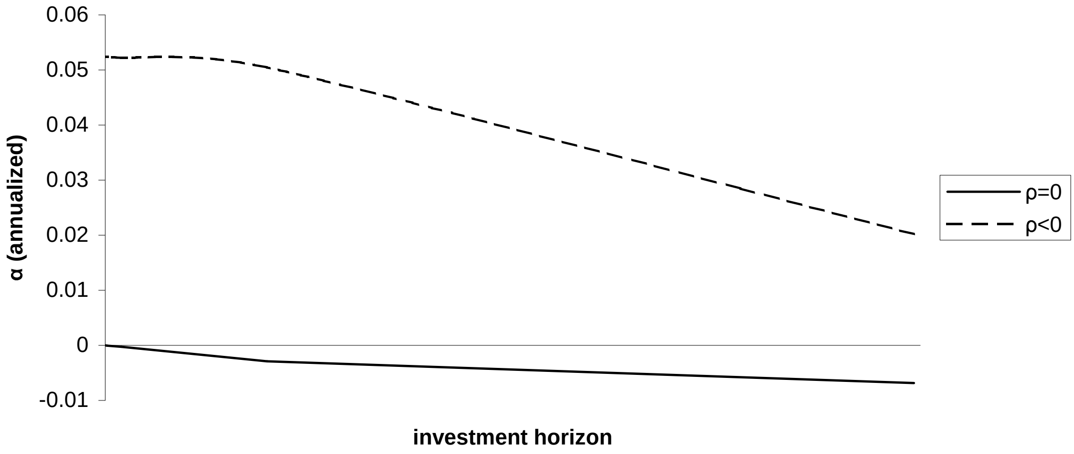


Figure 6.7: Convergence of α_t in regression equation (6.13) for $\rho = 0$ and $\rho < 0$ in the Heston model. The at-the-money call option has a maturity of two months. Volatility risk and jump timing risk are not priced. The premium on price diffusion risk is set to 6%.

smaller than $\Delta_t^H S_t / C_t$ for the case $\rho < 0$. The issue is illustrated in Figure 6.7 which presents the converging value of α_t when $h \to 0$ for $\rho = 0$ and for $\rho < 0$. Figure 6.7 shows that ignoring the correlation between the error term and the regressor in (6.13) may lead to a serious bias in α_t.

In the case of priced volatility risk, the values of α_t for the single option strategies of Table 6.1 change considerably. The volatility risk premium will have an effect on fair expected option returns in the Heston model. Furthermore, the value β_t will change after allowing for a volatility risk premium. Although the previous analyses concluded that CAPM α fails as a performance measure, a non-zero volatility risk premium creates even more complexities since α_t contains the fair compensation for taking volatility risk. Hence, even in the case of continuous trading, the expected call option return is not completely explained by the expected return on the option's underlying stock (index). The inclusion of an asset in (6.13) that has a payoff dependent on the instantaneous volatility would lead to $\alpha_t \to 0$ if $h \to 0$. This is obvious since adding a single volatility-dependent asset would complete the Heston market. The influence of volatility risk premia on optimal asset allocation will be treated in Section 6.6.

	ATM call			OTM put		
	$\mathrm{E}_t\left(\frac{C_{t+h}}{C_t}\right)$	α_t	β_t	$\mathrm{E}_t\left(\frac{P_{t+h}}{P_t}\right)$	α_t	β_t
$\lambda = 0.19, \tilde{\lambda} = 4.52, \mu_J = -0.8\%$	3.84%	-0.58%	26.50	-8.99%	-1.11%	-47.23
$\lambda = 0.19, \tilde{\lambda} = 4.52, \mu_J = -3.0\%$	-1.55%	-5.52%	23.77	-31.01%	-25.75%	-31.53
$\lambda = 0.19, \tilde{\lambda} = 4.52, \mu_J = 1.0\%$	3.56%	-0.84%	26.34	-9.48%	-1.65%	-46.93
$\lambda = 0.19, \tilde{\lambda} = 0.19, \mu_J = -0.8\%$	4.36%	-0.10%	26.76	-6.94%	1.16%	-48.58

Table 6.2: CAPM parameters and expected option returns for an at-the-money call option and an out-of-the-money put option in a model that allows for stochastic volatility and jumps of a fixed size. The numbers are calculated using an (annualized) rate of return on the risky asset of 2% and a 0% risk free rate.

6.5.3 CAPM and the Poisson-jump model

This section briefly investigates the value that CAPM α takes in models that allow for jumps. Throughout this section, the volatility risk premium parameter η^V and the correlation parameter ρ are set to zero. Consider the case where jump sizes are restricted to be constant. Given that volatility is assumed to be stochastic, this case implies that two options are necessary to complete the market. The option price dynamics in this situation are given by (see Liu and Pan (2003))

$$\begin{aligned} \mathrm{d}C_t &= \Delta_t^J \mathrm{d}S_t^c + r\left(C_t - \Delta_t^J S_t\right)\mathrm{d}t + \frac{\partial C_t}{\partial \sigma_t^2}\left(\eta^V \sigma_t^2 \mathrm{d}t + \sigma_\sigma \sigma_t \rho \mathrm{d}W_t^S + \sigma_\sigma \sigma_t \sqrt{1-\rho^2}\mathrm{d}W_t^V\right) \\ &\quad + \left[C(S_{t_-}(1+\mu_J), \sigma_t^2) - C(S_{t_-}, \sigma_t^2)\right](\mathrm{d}N_t - \lambda \mathrm{d}t) + S_t \lambda \mu_J \Delta_t^J \mathrm{d}t, \end{aligned} \tag{6.22}$$

where S_t^c denotes the continuous part of S_t. The methodology of Section 6.3 can be applied to determine closed form expressions for functions α_t and β_t in (6.13). Table 6.2 presents the expectation of the option return, α_t and β_t for several choices of λ, $\tilde{\lambda}$, and μ_J. The stochastic volatility parameters are based on Pan (2002).[30]

The results confirm the conclusions of the previous sections: CAPM β cannot be used as a risk measure and, therefore, CAPM α is an inadequate performance measure for nonlinear option based strategies. Again, the explanation is that the underlying assumptions of (6.13) do not match the underlying preference structure of the jump model. Valuation of jump models by means of (6.13) leads to the same problems as in the previous section. Results are not reported but (6.22) obviously shows that the

[30] The first row of results in Table 6.2 are based on the jump parameters as estimated in Pan (2002).

assumption of no correlation between the stock (index) return and the error term is violated in (6.13). Hence, when α_t and β_t are calculated in the traditional way by means of (6.15) and (6.14), the resulting values do not correspond to the true values of α_t and β_t in the Poisson-jump model with fixed jump sizes.

6.5.4 CAPM and delta-hedged straddles

Straddles are popular instruments nowadays[31] because the straddle value increases with the volatility of the underlying asset. Summary statistics in Driessen and Maenhout (2004) show that the skewness of the straddle return distribution is substantially lower than the skewness of the single option return distribution. As a result of these observations, this section examines the usefulness of CAPM α for discretely hedged straddles. The value of a discretely hedged straddle V_h at investment horizon h using options that expire at maturity date $T \geq h$ is given by

$$\begin{aligned} V_h &= C_h(K,T) + P_h(K,T) - \left(\frac{\partial\,(C+P)}{\partial S}\right)_{h-\Delta} S_h \\ &\quad + \left(X_0 - C_0(K,T) - P_0(K,T) + \left(\frac{\partial\,(C+P)}{\partial S}\right)_0 S_0\right) e^{rh} \\ &\quad + \sum_{j=1}^{N-1} S_{j\Delta} \left\{ \left(\frac{\partial\,(C+P)}{\partial S}\right)_{j\Delta} - \left(\frac{\partial\,(C+P)}{\partial S}\right)_{(j-1)\Delta} \right\} e^{r(h-j\Delta)}, \end{aligned}$$

where Δ denotes the time between subsequent portfolio adjustments and $N = h/\Delta$. The methodology of Section 6.3 can be employed to calculate all necessary quantities for analytical expressions of α_t and β_t. The results for several different models are presented in Table 6.3.

The results of the Black-Scholes case clearly indicate that the reduced skewness in the straddle return distribution has a big impact on α_t and β_t. When the hedging frequency increases, the position becomes less sensitive for changes in the option's underlying asset and, therefore, β_t is expected to decrease to zero. Furthermore, in the Black-Scholes world only market risk is priced and, thus, α_t should be equal to zero in (6.13). Hence, the results in Table 6.3 show that if the world would be Black-Scholes then α_t can be used as a performance measure. The same conclusion applies to a stochastic volatility model in which volatility risk is idiosyncratic and to a jump model with nonsystematic volatility risk en jump risk. In case volatility risk or jump risk is priced α_t does not

[31] From the statistics in Bondarenko (2003b) can be concluded that straddles are very liquidly traded.

hedge frequency	**21 days**		**7 days**		**1 day**	
	α_t	β_t	α_t	β_t	α_t	β_t
Black-Scholes	0.00%	0.036	0.00%	0.012	0.00%	0.00
SV, $\eta^V = 0$	0.00%	0.038	0.00%	0.014	0.00%	0.00
SV, $\eta^V < 0$	-0.29%	0.036	-0.29%	0.014	-0.29%	0.00
SVJ, $\lambda = \tilde{\lambda}, \mu_J < 0$	0.00%	0.038	0.00%	0.014	0.00%	0.00
SVJ, $\lambda > \tilde{\lambda}, \mu_J < 0$	-0.02%	0.037	-0.02%	0.014	-0.02%	0.00

Table 6.3: CAPM parameters of returns on at-the-money straddles for several option pricing models (SV for stochastic volatility models and SVJ for stochastic volatility models including jumps). The holding period is one month and the maturity of the options is two months. The hedging frequency is given in the first row. A hedging frequency of 21 days means that the option position is only hedged at initiation.

converge to zero if the hedging frequency goes to infinity because the compensation for these risks are contained in the intercept of the one-factor model (6.13).

6.6 Asset Allocation

The previous section examined the properties of a mean-variance based performance measure for several option strategies under several model assumptions. The main conclusion was that these model assumptions do not correspond to the mean-variance assumptions and, therefore, mean-variance based performance measures should not be used for the considered option strategies except for straddle strategies under some unrealistic model assumptions. This section considers the optimal portfolio choice for a mean-variance investor in settings where the mean-variance investor has access to the option market. The optimal portfolios are qualitatively compared to the optimal portfolio choice of a power utility investor. Optimal positions of power utility investors are reported in Liu and Pan (2003).

6.6.1 Asset allocation in stochastic volatility models

The focus in this section will be on mean-variance asset allocation in stochastic volatility models. As a consequence, jump parameters λ, $\tilde{\lambda}$, μ_J, and σ_J^2 are set to zero. In this section, the mean-variance investor can choose between the risky stock, the risk free

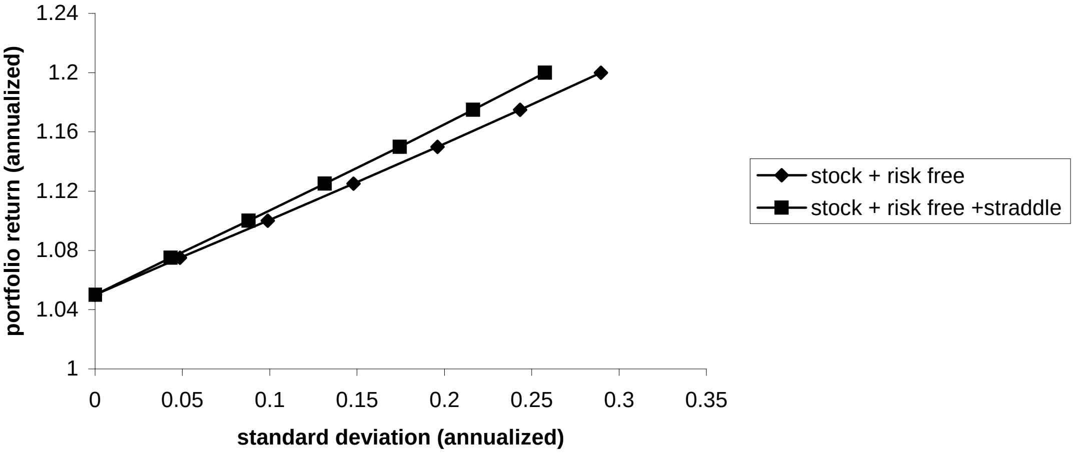


Figure 6.8: Efficient frontiers for several investment opportunities in a Heston world with a price diffusion risk premium of 6.76% and systematic volatility risk. The delta-neutral straddle has a maturity of two months and the investment horizon is chosen as one month.

asset, and straddles that are written on the risky stock. Straddles are made delta-neutral at initiation by choosing the strike price such that the delta of a call option is 0.5. Stochastic volatility parameters are taken from Liu and Pan (2003), i.e. $\kappa = 5$, $\sigma^2 = (0.13)^2$, $\sigma_\sigma = 0.25$, and $\rho = -0.40$. The price diffusion risk premium is chosen equal to 6.76% and the risk free rate is assumed to be 5%. For the cases where volatility risk is assumed to be systematic, η^V is set at -1.38.

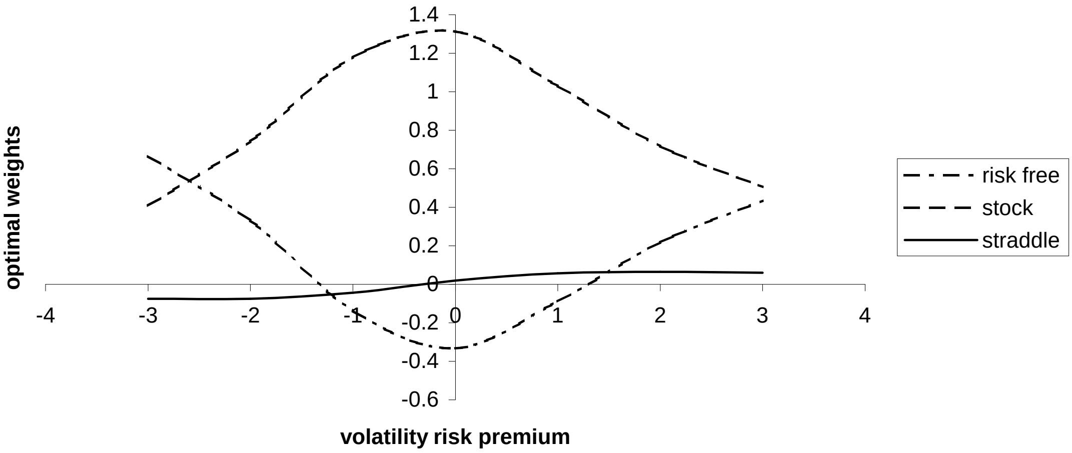
Figure 6.8 presents the mean-variance efficient frontiers resulting from investment opportunities sets excluding and including a delta-neutral straddle. The results are calculated under the assumption that volatility risk is priced. One conclusion that can be drawn from Figure 6.8 is that adding a delta-neutral straddle economically improves the efficiency of mean-variance efficient portfolios in case of systematic volatility risk.[32] For a portfolio return of 15%, the annualized standard deviation drops from 19.6% to 17.4% which is a substantial decrease. Optimally, the mean-variance investor takes a short position in the delta-neutral straddle for all levels of required return. The less risk averse the mean-variance investor, the smaller the weight in the straddles. Given that a negative

[32]Unreported results show that efficiency is not improved when volatility risk is idiosyncratic. Moreover, in case of systematic volatility risk efficiency cannot be further improved by adding more straddles to the investment opportunity set. This result appears since one volatility dependent asset completes the market. Finally, single option returns lead to efficiency improvements of the same magnitude as straddles.

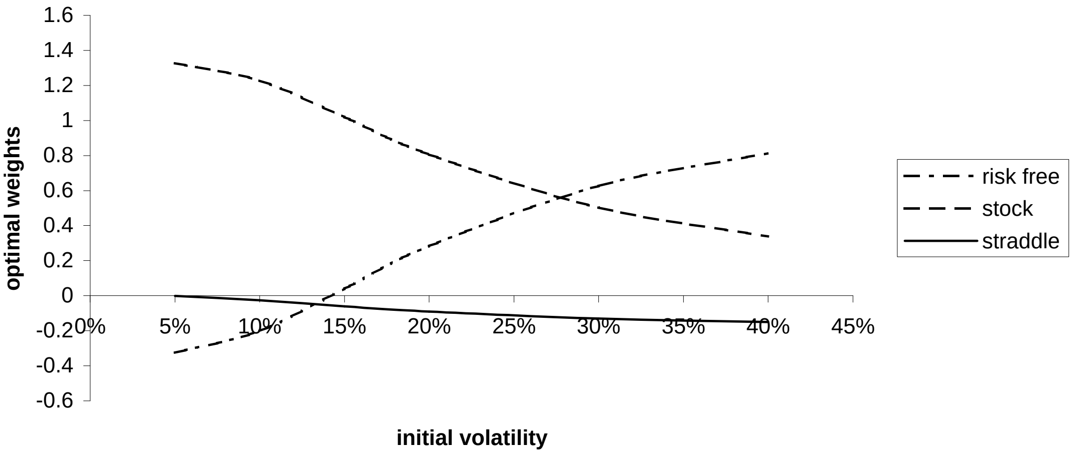
volatility risk premium implies a positive correlation between the stochastic discount factor and volatility, negative call option positions provide payoff in good volatility states. Therefore, the investor optimally takes the risky side of the position with respect to volatility. This explains the positive relation between risk aversion and the optimal weight in the straddle under the assumption of a negative volatility risk premium.

Figure 6.9 shows the sensitivity of optimal portfolio weights for changes in several underlying model parameters. The benchmark case parameters are set as described above. Furthermore, the required portfolio return and the initial spot volatility are assumed to be 15% (annualized). The results for the volatility risk premium η^V are qualitatively similar to the optimal weights in Liu and Pan (2003). Both the mean-variance investor and the power utility take a long position in the straddle when the volatility risk premium is positive and a short position when the volatility risk premium decreases to a negative value. The explanation is that investors take advantage of the specific risk-return characteristics of volatility by selling straddles ($\eta^V < 0$) or buying straddles ($\eta^V > 0$). The only qualitative difference can be identified for high and low values of the volatility risk premium. The reason is that the variance of a portfolio is an inappropriate risk measure for power utility investors. For low values of the volatility risk premium the power utility investor builds expected portfolio return by taking larger straddle weights (in absolute value) while the mean-variance investor stabilizes the portion of straddles in portfolios because of the large influence that option returns have on portfolio variance. For high values of the volatility risk premium a similar reasoning applies.

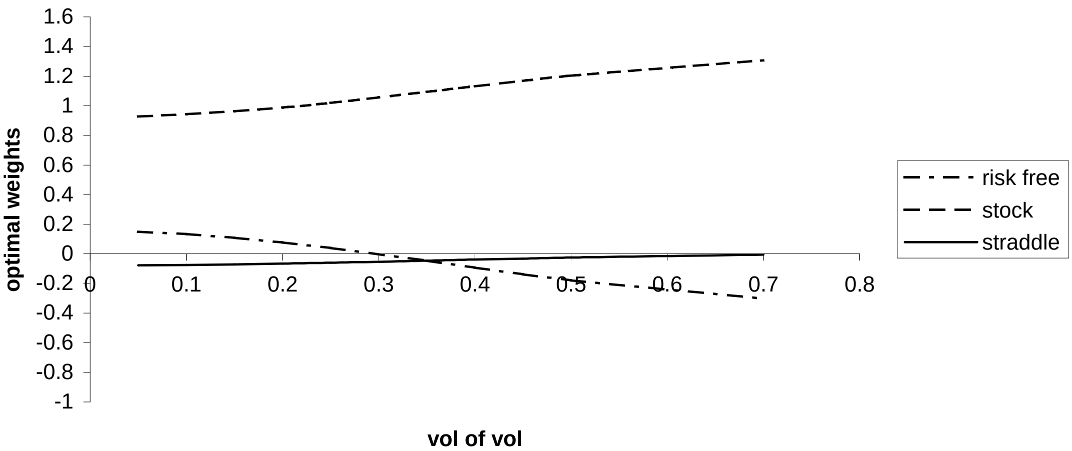
Although a higher value of the initial variance implies a lower expected straddle return, this higher value influences the variance of the stock the most. Thus, the delta-neutral straddle is more attractive relative to the risky stock for high volatility levels. This is reflected in the larger optimal straddle weights (in absolute value). Liu and Pan (2003) provides a different explanation but qualitatively the results are similar as reported here. An increase of parameter σ_σ has hardly any influence on the expected return of the straddle but the variance of the straddle return increases considerably. Hence, the risk-return characteristics become less attractive for the mean-variance investor and, therefore, lower straddle weights (in absolute value) are observed when the initial volatility increases. Similar patterns are reported in Liu and Pan (2003). The overall conclusion of this section is that the properties of the return distribution of straddles imply that asset allocation using the mean-variance criterion leads to sensible outcomes. Only for some specific (and unrealistic) cases the fundamental differences between the mean-variance framework and power utility approach appear.



(a) The influence of the volatility risk premium on optimal portfolio weights.



(b) The influence of the initial volatility on optimal portfolio weights.



(c) The influence of the volatility of volatility on optimal portfolio weights.

Figure 6.9: Optimal portfolio weights for a mean-variance investor. The benchmark case parameters are $\kappa = 5$, $\sigma^2 = (0.13)^2$, $\sigma_\sigma = 0.25$, $\rho = -0.40$, and the initial volatility is taken equal to 15%.

		case 1		case 2		case 3	
required return	$\tilde{\lambda}/\lambda$	stock	put	stock	put	stock	put
10%	1	0.80	0.56%	0.80	0.57%	0.79	0.51%
	2	0.63	-0.13%	0.72	0.17%	0.76	0.34%
	5	-0.02	-2.33%	0.19	-1.24%	0.54	-0.39%
15%	1	1.60	1.11%	1.60	1.13%	1.58	1.02%
	2	1.25	-0.26%	1.42	0.33%	1.51	0.68%
	5	-0.03	-4.62%	0.37	-2.46%	1.06	-0.77%
20%	1	2.36	1.64%	2.36	1.67%	2.33	1.50%
	2	1.85	-0.39%	1.02	0.49%	2.24	1.01%
	5	-0.07	-5.16%	0.54	-3.63%	1.57	-1.13%

Table 6.4: Optimal portfolio weights for three different jump scenarios, three different ways of jump risk compensation, and for three different required portfolio returns. The jump scenarios include (i) $\mu = -10\%$ and $\lambda = 1/10$, (ii) $\mu = -25\%$ and $\lambda = 1/50$, and (iii) $\mu = -50\%$ and $\lambda = 1/200$. The variance process and the equity risk premium are in each case adjusted such that the mean and the variance of the stock return remain the same.

6.6.2 Asset allocation in jump models

In this section optimal portfolio choice is investigated in a world where jumps in the risky stock can occur. There are no assets available that offer separate exposure to diffusive and jump risk. In the spirit of Liu and Pan (2003), the out-of-the-money put option is chosen as the asset that disentangles jump risk from diffusive risk most effectively. The value of this asset has low sensitivity to small movements in the underlying asset and a high sensitivity to big downward movements of the risky stock. The choice of the jump parameters is based on Liu and Pan (2003). This means that three different jump cases are considered: (i) $\mu = -10\%$ and $\lambda = 1/10$, (ii) $\mu = -25\%$ and $\lambda = 1/50$, and (iii) $\mu = -50\%$ and $\lambda = 1/200$. Although not very realistic, these parameters will be used to compare outcomes to Liu and Pan (2003). The variance of the return jumps is assumed to be zero, i.e. $\sigma_J^2 = 0$. The long term average of instantaneous volatility is adjusted such that total return volatility for each jump case equals 15%. The choice of the price diffusion risk premium also depends on the jump case. For each choice of the jump risk premium the total equity risk premium is fixed at 6.76% a year.

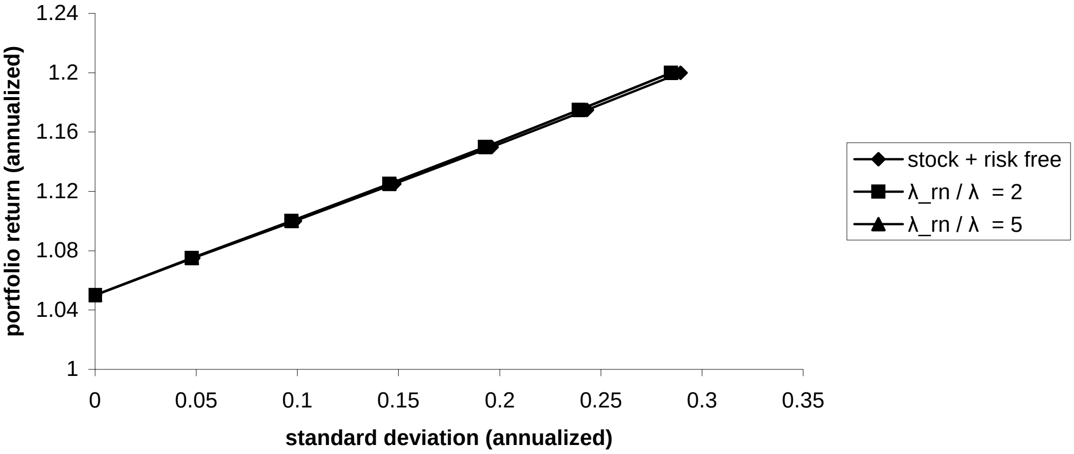


Figure 6.10: Efficient frontiers for several investment opportunities (risk free asset, stock, and 0.95 out-of-the-money put option) in a jump-diffusion world with an equity risk premium of 6.76% and idiosyncratic volatility risk.

The optimal portfolio weights of the risky stock and the 0.95 out-of-the-money put option are presented in Table 6.4. The put option has a maturity of two months and the investment horizon is chosen as one month. The results show some important qualitative differences with the outcomes of Liu and Pan (2003). In the first case, for instance, mean-variance investors take short positions in the stock as a hedge against the short positions in out-of-the-money put options. In contrast, power utility investors are not willing to take such a large position (in absolute value) in the put option because of the downside risk involved. As a consequence, Liu and Pan (2003) reports positive weights in the risky stock for all risk aversion levels. The case of very infrequent but very large negative jumps in the risky stock (case 3) shows also different positioning for power utility investors and mean-variance investors. Independent of the compensation and the level of risk aversion, power utility investors do not take negative jump exposure. The optimal policy for them is to hedge the long position in the stock by buying put options. The reported numbers in Table 6.4 show that mean-variance investors take, in case of sufficient compensation, short positions in put options. From the results in Table 6.4 can be concluded that at least for some cases the mean-variance criterion is not able to take the right side of the option position.

Another result of Liu and Pan (2003) is that the largest portfolio improvements occur for the final jump case combined with the lowest level of risk aversion. Figure 6.10 shows that, under these model assumptions, the efficiency improvement is small in the mean-variance setting. This occurs for several cases in Table 6.4. Hence, for some

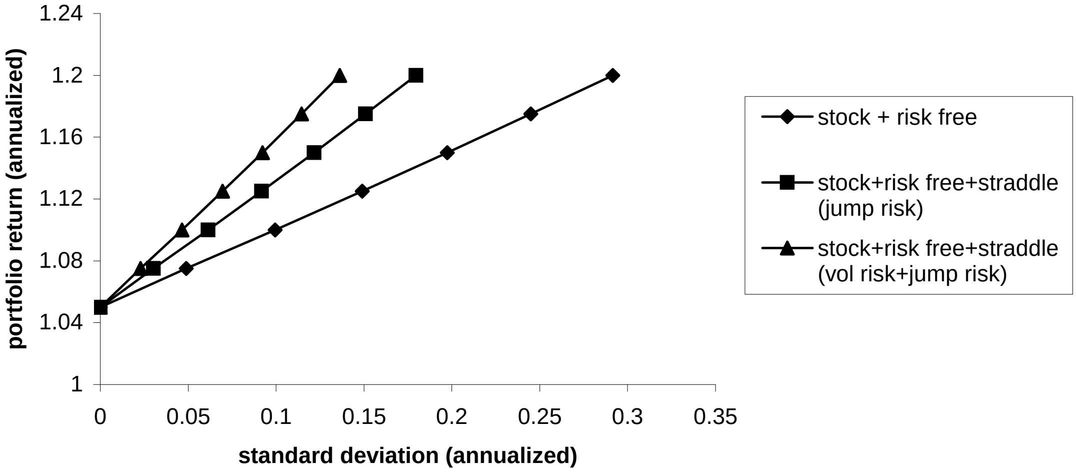


Figure 6.11: Efficient frontiers for several investment opportunities (risk free asset, stock, and 0.95 out-of-the-money put option) in a jump-diffusion world with an equity risk premium of 6.76% and idiosyncratic volatility risk.

cases mean-variance analysis cannot identify the economic value of taking jump risk exposure. In mean-variance sense the largest improvements are observed in the first case. The explanation of the differences between the power utility approach and the mean-variance approach lies again in the model assumptions with respect to investors' preferences.

Finally, the influence of jumps on optimal straddle positions is examined. Straddles are not the type of instruments used for taking jump exposure because these instruments provide exposure to both volatility risk and jump risk. Figure 6.11 presents the efficient frontiers for several assumptions regarding volatility premia and jump premia. In the first case only jump risk is priced and in the second case the uncertainty in volatility is priced as well. The jump parameters are based on the estimations in Pan (2002), i.e. $\lambda = 0.19$, $\tilde{\lambda} = 4.52$, $\mu_J = -0.8\%$, and $\sigma_J^2 = 0.0015$. From Figure 6.11 can be concluded that the priced jump component leads to a considerable improvement of portfolio efficiency. Unreported results show that for both cases the mean-variance investor optimally takes a short position in the delta-neutral straddle. In this way, the mean-variance investor profits from the compensation for volatility risk and jump risk. Note here that the power utility investor would most probably take smaller positions (in absolute value) than the mean-variance investor because of the aversion against crash states of the risky stock.

6.7 Summary

This chapter examines the mean-variance characteristics of option based investment strategies. The focus is first on performance measurement in the mean-variance model and subsequently on optimal portfolio choice in a setting where mean-variance investors have access to the options market. These analyses can be performed because this chapter provides a general methodology for the calculation of the conditional expectation, the conditional variance, and the conditional covariance of option returns for all option pricing models that can be classified in the class of affine jump-diffusion models. The resulting moment conditions depend on the spot volatility, model parameters, the holding period, the option's maturity, and moneyness. The findings show that in all models in which only market risk is priced, CAPM α cannot be used as a performance measure for nonsymmetric option return strategies. This conclusion changes when delta-hedged straddles are considered although CAPM α is still not useful when besides market risk, volatility risk or jump risk is priced. From the optimal portfolio allocation outcomes can be concluded that there are no qualitative differences in optimal portfolio weights between mean-variance investors and power utility investors when straddles are considered as separate investment opportunities. This conclusion holds in a stochastic volatility world. In a setting with stochastic volatility and jumps, power utility investors and mean-variance investors make qualitatively different investment decisions in some settings. Finally, large efficiency gains are observed for mean-variance investors that take short straddle positions in a world where both volatility risk and crash risk are priced.

6.A Benchmark Model Derivations

To calculate expected option returns knowledge on the joint characteristic function of $\log S_{t+h}$ and σ^2_{t+h} is required. For the benchmark model (6.9) the characteristic function can be derived from the general result in Duffie, Pan, and Singleton (2000) and is given by

$$\mathrm{E}_t^{\mathrm{P}}\left(\exp\left\{i\phi_V\sigma^2_{t+h}+i\phi_S\log S_{t+h}\right\}\right)=\exp\left\{A\left(\phi_V,\phi_S,h\right)+B\left(\phi_V,\phi_S,h\right)\sigma_t^2+i\phi_S\log S_t\right\}, \tag{6.23}$$

with

$$\begin{aligned}A(\phi_V,\phi_S,h) &= \mu i\phi_S h-\frac{\kappa\sigma^2}{\sigma_\sigma^2}\left[(\gamma_P+b_P)h+2\log\left\{1-\frac{(\gamma_P+b_P)+\sigma_\sigma^2 i\phi_V}{2\gamma_P}\left(1-e^{-\gamma_P h}\right)\right\}\right]\\&\quad+\lambda h\left[(1+\mu_J)^{i\phi_S}e^{\frac{i\phi_S}{2}(1+i\phi_S)\sigma_J^2}-1-i\phi_S\mu_J\right],\\B(\phi_V,\phi_S,h) &= -\frac{a\left(1-e^{-\gamma_P h}\right)-i\phi_V\left[2\gamma_P-(\gamma_P-b_P)\left(1-e^{-\gamma_P h}\right)\right]}{2\gamma_P-(\gamma_P+b_P)\left(1-e^{-\gamma_P h}\right)-\sigma_\sigma^2 i\phi_V\left(1-e^{-\gamma_P h}\right)},\end{aligned}$$

where

$$\begin{aligned}a &= i\phi_S+\phi_S^2,\\b_P &= \sigma_\sigma\rho i\phi_S-\kappa,\\\gamma_P &= \sqrt{b_P^2+a\sigma_\sigma^2},\\\mu &= r+\eta^S+\mu_J\left(\lambda-\tilde{\lambda}\right).\end{aligned}$$

The expectation of $P_{2,t+h}$ can be derived as

$$\mathrm{E}_t^{\mathrm{P}}\left(P_{2,t+h}\right)=\frac{1}{2}+\frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-i\phi_S\log K+C}\mathrm{E}_t^{\mathrm{P}}\left(\exp\left(i\frac{D}{i}\sigma_{t+h}^2+i\phi_s\log S_{t+h}\right)\right)}{i\phi_s}\right]\mathrm{d}\phi_s,$$

with

$$\begin{aligned}C(\alpha,\tau) &= ri\alpha\tau-\frac{\kappa\sigma^2}{\sigma_\sigma^2}\left[(\gamma_Q+b_Q)\tau+2\log\left\{1-\frac{(\gamma_Q+b_Q)}{2\gamma_Q}\left(1-e^{-\gamma_Q\tau}\right)\right\}\right]\\&\quad+\tilde{\lambda}\tau\left[(1+\mu_J)^{i\alpha}e^{\frac{i\alpha}{2}(1-i\alpha)\sigma_J^2}-1\right]-\tilde{\lambda}\tau i\alpha\mu_J, &(6.24)\\D(\alpha,\tau) &= -\frac{a\left(1-e^{-\gamma_Q\tau}\right)}{2\gamma_Q-(\gamma_Q+b_Q)\left(1-e^{-\gamma_Q\tau}\right)}, &(6.25)\end{aligned}$$

where

$$\begin{aligned}b_Q &= \sigma_\sigma\rho i\alpha-\left(\kappa+\eta^V\right),\\\gamma_Q &= \sqrt{b_Q^2+a\sigma_\sigma^2}.\end{aligned}$$

The expectation is obtained by evaluating (6.23) in $\left(\frac{D}{i},\phi_S,h\right)$. Finally, this leads to an expression which is again of the form

$$\mathrm{E}_t^{\mathrm{P}}\left(P_{2,t+h}\right)=\frac{1}{2}+\frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-i\phi_S\log K}\exp\left\{F+G\sigma_t^2+i\phi_S\log S_t\right\}}{i\phi_s}\right]\mathrm{d}\phi_s,$$

with

$$\begin{aligned}
F &= ri\phi_S\tau - \frac{\kappa\sigma^2}{\sigma_\sigma^2}\left[(\gamma_Q + b_Q)\tau + 2\log\left\{1 - \frac{(\gamma_Q + b_Q)}{2\gamma_Q}\left(1 - e^{-\gamma_Q\tau}\right)\right\}\right] \\
&\quad + \tilde{\lambda}\tau\left[(1+\mu_J)^{i\phi_S\left(1-e^{-\gamma_Q h}\right)} e^{\frac{i\phi_S}{2}(1-i\phi_S)\sigma_J^2} - 1\right] \\
&\quad - \tilde{\lambda}\tau i\phi_S\mu_J + \mu i\phi_S h - \frac{\kappa\sigma^2}{\sigma_\sigma^2}\left[(\gamma_P + b_P)h + 2\log\left\{1 - \frac{(\gamma_P + b_P) + \sigma_\sigma^2 D}{2\gamma_P}\left(1 - e^{-\gamma_P h}\right)\right\}\right] \\
&\quad + \lambda h\left[(1+\mu_J)^{i\phi_S} e^{\frac{i\phi_S}{2}(1-i\phi_S)\sigma_J^2} - 1\right] - \lambda h i\phi_S\mu_J, \qquad (6.26) \\
G &= -\frac{a\left(1 - e^{-\gamma_P h}\right) - D\left[2\gamma_P - (\gamma_P - b_P)\left(1 - e^{-\gamma_P h}\right)\right]}{2\gamma_P - (\gamma_P + b_P)\left(1 - e^{-\gamma_P h}\right) - \sigma_\sigma^2 D\left(1 - e^{-\gamma_P h}\right)}, \qquad (6.27)
\end{aligned}$$

where $\tau = T - h$. The expectation of $S_{t+h}P_{1,t+h}$ can be derived in a similar way

$$\mathrm{E}_t^{\mathrm{P}}\left(S_{t+h}P_{1,t+h}\right) = \frac{1}{2}S_t e^{\mu h} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-i\phi_S\log K + \tilde{C}}\mathrm{E}_t^{\mathrm{P}}\left(\exp\left(i\frac{\tilde{D}}{i}\sigma_{t+h}^2 + i(\phi_s - i)\log S_{t+h}\right)\right)}{i\phi_s}\right]\mathrm{d}\phi_s,$$

with

$$\begin{aligned}
\tilde{C}(\alpha,\tau) &= ri\alpha\tau - \frac{\kappa\sigma^2}{\sigma_\sigma^2}\left[\left(\tilde{\gamma}_Q + \tilde{b}_Q\right)\tau + 2\log\left\{1 - \frac{\left(\tilde{\gamma}_Q + \tilde{b}_Q\right)}{2\tilde{\gamma}_Q}\left(1 - e^{-\tilde{\gamma}_Q\tau}\right)\right\}\right] \\
&\quad + \tilde{\lambda}\tau(1+\mu_J)\left[(1+\mu_J)^{i\alpha} e^{\frac{i\alpha}{2}(1+i\alpha)\sigma_J^2} - 1\right] - \tilde{\lambda}\tau i\alpha\mu_J, \qquad (6.28) \\
\tilde{D}(\alpha,\tau) &= -\frac{\tilde{a}\left(1 - e^{-\tilde{\gamma}_Q\tau}\right)}{2\tilde{\gamma}_Q - \left(\tilde{\gamma}_Q + \tilde{b}_Q\right)\left(1 - e^{-\tilde{\gamma}_Q\tau}\right)}, \qquad (6.29)
\end{aligned}$$

where

$$\begin{aligned}
\tilde{a} &= -i\phi_S + \phi_S^2, \\
\tilde{b}_Q &= \sigma_\sigma\rho i\alpha - \left(\kappa + \eta^V\right) + \sigma_\sigma\rho, \\
\tilde{\gamma}_Q &= \sqrt{\tilde{b}_Q^2 + \tilde{a}\sigma_\sigma^2}.
\end{aligned}$$

Now, the joint characteristic function (6.23) needs to be evaluated in $\left(\frac{\tilde{D}}{i}, \phi_S - i, h\right)$. The final expression has got again the same convenient form

$$\begin{aligned}
\mathrm{E}_t^{\mathrm{P}}\left(S_{t+h}P_{1,t+h}\right) &= \frac{1}{2}S_t e^{\mu h} \\
&\quad + \frac{S_t e^{\mu h}}{\pi}\int_0^\infty \frac{e^{-i\phi_S\log K}\exp\left\{\tilde{F}\left(\phi_V,\phi_S,h\right) + \tilde{G}\left(\phi_V,\phi_S,h\right)\sigma_t^2 + i\phi_S\log S_t\right\}}{i\phi_S}\mathrm{d}\phi_s,
\end{aligned}$$

with

$$
\begin{aligned}
\tilde{F} &= ri\phi^S\tau - \frac{\kappa\sigma^2}{\sigma_\sigma^2}\left[\left(\tilde{\gamma}_Q + \tilde{b}_Q\right)\tau + 2\log\left\{1 - \frac{\left(\tilde{\gamma}_Q + \tilde{b}_Q\right)}{2\tilde{\gamma}_Q}\left(1 - e^{-\tilde{\gamma}_Q\tau}\right)\right\}\right] \\
&\quad + \tilde{\lambda}\tau(1+\tilde{\mu}_J)\left[(1+\tilde{\mu}_J)^{i\phi^S} e^{\frac{i\phi^S}{2}\left(1+i\phi^S\right)\sigma_J^2} - 1\right] - \tilde{\lambda}\tau i\phi^S\tilde{\mu}_J + \mu i\phi_S h \\
&\quad - \frac{\kappa\sigma^2}{\sigma_\sigma^2}\left[\left(\tilde{\gamma}_P + \tilde{b}_P\right)h + 2\log\left\{1 - \frac{\left(\tilde{\gamma}_P + \tilde{b}_P\right) + \sigma_\sigma^2\tilde{D}}{2\tilde{\gamma}_P}\left(1 - e^{-\tilde{\gamma}_P h}\right)\right\}\right] \qquad (6.30) \\
&\quad + \lambda h(1+\mu_J)\left[(1+\mu_J)^{i\phi_S} e^{\frac{i\phi_S}{2}(1+i\phi_S)\sigma_J^2} - 1\right] - \lambda h i\phi_S\mu_J, \\
\tilde{G} &= -\frac{\tilde{a}\left(1 - e^{-\tilde{\gamma}_P h}\right) - \tilde{D}\left[2\tilde{\gamma}_P - \left(\tilde{\gamma}_P - \tilde{b}_P\right)\left(1 - e^{-\tilde{\gamma}_P h}\right)\right]}{2\tilde{\gamma}_P - \left(\tilde{\gamma}_P + \tilde{b}_P\right)\left(1 - e^{-\tilde{\gamma}_P h}\right) - \sigma_\sigma^2\tilde{D}\left(1 - e^{-\tilde{\gamma}_P h}\right)}, \qquad (6.31)
\end{aligned}
$$

where, finally

$$
\begin{aligned}
\tilde{b}_P &= \sigma_\sigma\rho i\alpha - \kappa + \sigma_\sigma\rho, \\
\tilde{\gamma}_P &= \sqrt{\tilde{b}_P^2 + \tilde{a}\sigma_\sigma^2}.
\end{aligned}
$$

6.B Second Moment of Option Returns

Consider the second moment of a call option price at time $t+h$,

$$
\begin{aligned}
\mathrm{E}_t^{\mathrm{P}}\left(C_{t+h}^2\right) &= \mathrm{E}_t^{\mathrm{P}}\left(S_{t+h}P_{1,t+h} - KP_{2,t+h}\right)^2 \qquad (6.32) \\
&= \mathrm{E}_t^{\mathrm{P}}\left(S_{t+h}^2 P_{1,t+h}^2\right) - 2K\mathrm{E}_t^{\mathrm{P}}\left(S_{t+h}P_{1,t+h}P_{2,t+h}\right) + K^2\mathrm{E}_t^{\mathrm{P}}\left(P_{2,t+h}\right)^2.
\end{aligned}
$$

These expectations are treated separately,

$$
\begin{aligned}
\mathrm{E}_t^{\mathrm{P}}\left(P_{2,t+h}^2\right) &= \mathrm{E}_t^{\mathrm{P}}\left(\frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{\varphi_t(\phi_S)\, e^{-i\phi_S\log K}}{i\phi_S}\right]\mathrm{d}\phi_S\right)^2 \\
&= \frac{1}{4} + \mathrm{E}_t^{\mathrm{P}}\left(\frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{\varphi_t(\phi_S)\, e^{-i\phi_S\log K}}{i\phi_S}\right]\mathrm{d}\phi_S\right) \\
&\quad + \mathrm{E}_t^{\mathrm{P}}\left(\frac{1}{\pi^2}\int_0^\infty\int_0^\infty \mathrm{Re}\left[\frac{\varphi_t(\phi_S)\, e^{-i\phi_S\log K}}{i\phi_S}\right]\mathrm{Re}\left[\frac{\varphi_t(\alpha_S)\, e^{-i\alpha_S\log K}}{i\alpha_S}\right]\mathrm{d}\phi_s\mathrm{d}\alpha_s\right).
\end{aligned}
$$

Only the final term is not known yet. In order to calculate this expectation $\varphi_t(\phi_S)$ is replaced by e^{a+bi} and $\varphi_t(\alpha_S)$ by $e^{\tilde{a}+\tilde{b}i}$. Then

$$\begin{aligned}
\operatorname{Re}\left[\frac{e^{-i\phi_S \log K} e^{a+bi}}{i\phi_S}\right] &= e^a \operatorname{Re}\left[\frac{(\cos(\phi_S \log K) - i\sin(\phi_S \log K))(\cos(b) + i\sin(b))}{i\phi_S}\right] \\
&= e^a \left\{\frac{\cos(\phi_S \log K)\sin(b) - \sin(\phi_S \log K)\cos(b)}{\phi_S}\right\} \\
&= e^a \left\{\frac{\sin(b - \phi_S \log K)}{\phi_S}\right\}.
\end{aligned}$$

Using this result in the double integral above yields

$$\begin{aligned}
& \mathrm{E}_t^{\mathrm{P}}\left(\frac{1}{\pi^2}\int_0^\infty\int_0^\infty \operatorname{Re}\left[\frac{\varphi_t(\phi_S)e^{-i\phi_S\log K}}{i\phi_S}\right]\operatorname{Re}\left[\frac{\varphi_t(\alpha_S)e^{-i\alpha_S\log K}}{i\alpha_S}\right]\mathrm{d}\phi_s\mathrm{d}\alpha_s\right) \\
=\; & \mathrm{E}_t^{\mathrm{P}}\left(\frac{1}{\pi^2}\int_0^\infty\int_0^\infty e^{(a+\tilde{a})}\frac{\sin(b-\phi_S\log K)\sin\left(\tilde{b}-\alpha_S\log K\right)}{\phi_S\alpha_S}\mathrm{d}\phi_s\mathrm{d}\alpha_s\right) \\
=\; & \mathrm{E}_t^{\mathrm{P}}\left(\frac{1}{\pi^2}\int_0^\infty\int_0^\infty e^{(a+\tilde{a})}\left[\frac{\cos\left(b-\phi_S\log K-\tilde{b}+\alpha_S\log K\right)}{2\phi_S\alpha_S}\right.\right. \\
& \left.\left. -\frac{\cos\left(b-\phi_S\log K+\tilde{b}-\alpha_S\log K\right)}{2\phi_S\alpha_S}\right]\mathrm{d}\phi_s\mathrm{d}\alpha_s\right) \\
=\; & \mathrm{E}_t^{\mathrm{P}}\left(\frac{1}{\pi^2}\int_0^\infty\int_0^\infty e^{(a+\tilde{a})}\left[\frac{\operatorname{Re}\left[e^{-i\left\{b-\tilde{b}-(\phi_S-\alpha_S)\log K\right\}}\right]}{2\phi_S\alpha_S}-\frac{\operatorname{Re}\left[e^{i\left\{b+\tilde{b}-(\phi_S+\alpha_S)\log K\right\}}\right]}{2\phi_S\alpha_S}\right]\mathrm{d}\phi_s\mathrm{d}\alpha_s\right) \\
=\; & \mathrm{E}_t^{\mathrm{P}}\left(\frac{1}{\pi^2}\int_0^\infty\int_0^\infty e^{(a+\tilde{a})}\left[\frac{\operatorname{Re}\left[e^{i\left\{\tilde{b}-b-(\alpha_S-\phi_S)\log K\right\}}\right]}{2\phi_S\alpha_S}-\frac{\operatorname{Re}\left[e^{i\left\{b+\tilde{b}-(\alpha_S+\phi_S)\log K\right\}}\right]}{2\phi_S\alpha_S}\right]\mathrm{d}\phi_s\mathrm{d}\alpha_s\right) \\
=\; & \mathrm{E}_t^{\mathrm{P}}\left(\frac{1}{\pi^2}\int_0^\infty\int_0^\infty\left[\frac{\operatorname{Re}\left[\varphi(-\phi_S)\varphi(\alpha_S)e^{-i(\alpha_S-\phi_S)\log K}\right]}{2\phi_S\alpha_S}\right.\right. \\
& \left.\left. -\frac{\operatorname{Re}\left[\varphi(\phi_S)\varphi(\alpha_S)e^{-i(\alpha_S+\phi_S)\log K}\right]}{2\phi_S\alpha_S}\right]\mathrm{d}\phi_s\mathrm{d}\alpha_s\right) \\
=\; & \mathrm{E}_t^{\mathrm{P}}\left(\frac{1}{\pi^2}\int_0^\infty\int_0^\infty\left[\frac{\operatorname{Re}\left[\varphi(-\phi_S)\varphi(\alpha_S)e^{-i(\alpha_S-\phi_S)\log K}\right]}{2\phi_S\alpha_S}\right.\right. \\
& \left.\left. -\frac{\operatorname{Re}\left[\varphi(\phi_S)\varphi(\alpha_S)e^{-i(\alpha_S+\phi_S)\log K}\right]}{2\phi_S\alpha_S}\right]\mathrm{d}\phi_s\mathrm{d}\alpha_s\right).
\end{aligned}$$

This can be solved by using the joint characteristic function of $\log S_{t+h}$ and σ^2_{t+h}. Now, attention is turned to the first expectation at the right hand side of (6.32). The derivations are more or less the same given that the numerator of the integrand only consists of exponentials

$$\begin{aligned}
\mathrm{E}_t^{\mathrm{P}}\left(S_{t+h}^2 P_{1,t+h}^2\right) &= \mathrm{E}_t^{\mathrm{P}}\left(S_{t+h}^2\left(\frac{1}{2}+\frac{1}{\pi}\int_0^\infty \mathrm{Re}\left\{\frac{e^{\bar{a}+\bar{b}i}e^{-i\phi_S \log K}}{i\phi_S}\right\}\right)^2 \mathrm{d}\phi_S\right) \\
&= \frac{1}{4}\mathrm{E}_t^{\mathrm{P}}\left(S_{t+h}^2\right)+\mathrm{E}_t^{\mathrm{P}}\left(\frac{S_{t+h}^2}{\pi}\int_0^\infty \mathrm{Re}\left\{\frac{e^{\bar{a}+\bar{b}i}e^{-i\phi_S \log K}}{i\phi_S}\right\}\mathrm{d}\phi_S\right) \\
&\quad +\mathrm{E}_t^{\mathrm{P}}\left(\frac{S_{t+h}^2}{\pi^2}\int_0^\infty\int_0^\infty \mathrm{Re}\left\{\frac{e^{\bar{a}+\bar{b}i}e^{-i\phi_S \log K}}{i\phi_S}\right\}\mathrm{Re}\left\{\frac{e^{\tilde{a}+\tilde{b}i}e^{-i\alpha_S \log K}}{i\alpha_S}\right\}\mathrm{d}\phi_s \mathrm{d}\alpha_s\right).
\end{aligned}$$

Focusing again on the final expectation

$$\begin{aligned}
&\mathrm{E}_t^{\mathrm{P}}\left(\frac{S_{t+h}^2}{\pi^2}\int_0^\infty\int_0^\infty \mathrm{Re}\left[\frac{e^{\bar{a}+\bar{b}i}e^{-i\phi_S \log K}}{i\phi_S}\right]\mathrm{Re}\left[\frac{e^{\tilde{a}+\tilde{b}i}e^{-i\alpha_S \log K}}{i\alpha_S}\right]\mathrm{d}\phi_s \mathrm{d}\alpha_s\right) \\
= \; &\mathrm{E}_t^{\mathrm{P}}\left(\frac{1}{\pi^2}\int_0^\infty\int_0^\infty \frac{\mathrm{Re}\left[e^{\tilde{C}(-\phi_s)+\tilde{C}(\alpha_S)+\left(\tilde{D}(-\phi_s)+\tilde{D}(\alpha_S)\right)\sigma^2_{t+h}+i(\alpha_S-\phi_S-2i)\log S_{t+h}}e^{-i(\alpha_S-\phi_S)\log K}\right]}{2\phi_S\alpha_S}\right. \\
&\left. -\frac{\mathrm{Re}\left[e^{\tilde{C}(\phi_s)+\tilde{C}(\alpha_S)+\left(\tilde{D}(\phi_s)+\tilde{D}(\alpha_S)\right)\sigma^2_{t+h}+i(\alpha_S+\phi_S-2i)\log S_{t+h}}e^{-i(\alpha_S+\phi_S)\log K}\right]}{2\phi_S\alpha_S}\mathrm{d}\phi_s \mathrm{d}\alpha_s\right).
\end{aligned}$$

Finally, the cross-term in (6.32) is considered. The end result to which the joint characteristic function can be applied is

$$\begin{aligned}
&\mathrm{E}_t^{\mathrm{P}}\left(\frac{KS_{t+h}}{\pi^2}\int_0^\infty\int_0^\infty \mathrm{Re}\left[\frac{e^{\bar{a}+\bar{b}i}e^{-i\phi_S \log K}}{i\phi_S}\right]\mathrm{Re}\left[\frac{e^{\tilde{a}+\tilde{b}i}e^{-i\alpha_S \log K}}{i\alpha_S}\right]\mathrm{d}\phi_s \mathrm{d}\alpha_s\right) \\
= \; &\mathrm{E}_t^{\mathrm{P}}\left(\frac{K}{\pi^2}\int_0^\infty\int_0^\infty \frac{\mathrm{Re}\left[e^{\tilde{C}(-\phi_s)+C(\alpha_S)+\left(\tilde{D}(-\phi_s)+D(\alpha_S)\right)\sigma^2_{t+h}+i(\alpha_S-\phi_S-i)\log S_{t+h}}e^{-i(\alpha_S-\phi_S)\log K}\right]}{2\phi_S\alpha_S}\right. \\
&\left. -\frac{\mathrm{Re}\left[e^{\tilde{C}(\phi_s)+C(\alpha_S)+\left(\tilde{D}(\phi_s)+D(\alpha_S)\right)\sigma^2_{t+h}+i(\alpha_S+\phi_S-i)\log S_{t+h}}e^{-i(\alpha_S+\phi_S)\log K}\right]}{2\phi_S\alpha_S}\mathrm{d}\phi_s \mathrm{d}\alpha_s\right).
\end{aligned}$$

In the same spirit, the second moment of the future put price and the covariance between options can be calculated.

6.C Proofs

Proof of Lemma 6.1

For notational convenience the discount rate is taken to be equal to a constant rate r. First consider $P_{2,t}$. A proof is needed for the following

$$\begin{aligned} & \frac{\psi\left(0, X_t, t, T\right)}{2}-\frac{1}{\pi} \int_0^{\infty} \frac{\operatorname{Im}\left(\psi\left(-i \phi_S, X_t, t, T\right) e^{i \phi_S \log K}\right)}{\phi_S} \mathrm{d} \phi_S= \\ = & e^{-r(T-t)}\left(\frac{1}{2}+\frac{1}{\pi} \int_0^{\infty} \operatorname{Re}\left[\frac{\varphi_t\left(\phi_S\right) e^{-i \phi_S \log K}}{i \phi_S}\right] \mathrm{d} \phi_S\right) . \end{aligned}$$

Proving this part by part and starting with the first part

$$\frac{\psi\left(0, X_t, t, T\right)}{2}=\mathrm{E}^{\mathbb{Q}}\left(\exp \left(-\int_t^T r d s\right) \middle| \mathcal{F}_t\right)=\frac{e^{-r(T-t)}}{2}$$

where the first equality sign is by definition of $\psi(\cdot)$. For the second part observe that

$$\begin{aligned} & \frac{1}{\pi} \int_0^{\infty} \frac{\operatorname{Im}\left(\psi\left(-i \phi_S, X_t, t, T\right)\right) e^{i \phi_S \log K}}{\phi_S} \mathrm{d} \phi_S \\ = & \frac{e^{-r(T-t)}}{\pi} \int_0^{\infty} \frac{\operatorname{Im}\left(\varphi_t\left(-\phi_S\right)\right) e^{i \phi_S \log K}}{\phi_S} \mathrm{d} \phi_S \\ = & \frac{e^{-r(T-t)}}{\pi} \int_0^{\infty} \frac{\operatorname{Re}\left(\varphi_t\left(-\phi_S\right)\right) \sin \left(\phi_S \log K\right)+\operatorname{Im}\left(\varphi_t\left(-\phi_S\right)\right) \cos \left(\phi_S \log K\right)}{\phi_S} \mathrm{d} \phi_S \\ = & \frac{e^{-r(T-t)}}{\pi} \int_0^{\infty} \frac{\operatorname{Re}\left(\varphi_t\left(\phi_S\right)\right) \sin \left(\phi_S \log K\right)-\operatorname{Im}\left(\varphi_t\left(\phi_S\right)\right) \cos \left(\phi_S \log K\right)}{\phi_S} \mathrm{d} \phi_S \\ = & \frac{e^{-r(T-t)}}{\pi} \int_0^{\infty} \operatorname{Re}\left[\frac{\left(i \sin \left(\phi_S \log K\right)-\cos \left(\phi_S \log K\right)\right) \varphi_t\left(\phi_S\right)}{i \phi_S}\right] \mathrm{d} \phi_S \\ = & -\frac{e^{-r(T-t)}}{\pi} \int_0^{\infty} \operatorname{Re}\left[\frac{e^{-i \phi_S \log K} \varphi_t\left(\phi_S\right)}{i \phi_S}\right] \mathrm{d} \phi_S \end{aligned}$$

Consider now $S_t P_{1,T}$ in (6.3). For this part we need to prove that

$$\begin{aligned} & \frac{\psi\left(1, X_t, t, T\right)}{2}-\frac{1}{\pi} \int_0^{\infty} \frac{\operatorname{Im}\left(\psi\left(1-i \phi_S, X_t, t, T\right)\right) e^{i \phi_S \log K}}{\phi_S} \mathrm{d} \phi_S= \\ & S_t\left(\frac{1}{2}+\frac{1}{\pi} \int_0^{\infty} \operatorname{Re}\left[\frac{\varphi_t\left(\phi_S-i\right) e^{-i \phi_S \log K}}{\varphi_t(-i) i \phi_S}\right] \mathrm{d} \phi_S\right) \end{aligned}$$

Again proving this part by part and starting with the first part

$$\frac{\psi\left(1, X_t, t, T\right)}{2}=\frac{\mathrm{E}^{\mathbb{Q}}\left(\exp \left(-\int_t^T r d s\right) S_t \middle| \mathcal{F}_t\right)}{2}=\frac{e^{-r(T-t)} S_t e^{r(T-t)}}{2}=\frac{S_t}{2}$$

And finally,

$$
\begin{aligned}
& \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\left(\psi\left(1-i\phi_S,X_t,t,T\right)\right)e^{i\phi_S\log K}}{\phi_S}\mathrm{d}\phi_S \\
= & \frac{S_te^{-r(T-t)}}{\pi}\int_0^\infty \frac{\mathrm{Im}\left(\varphi_t\left(-\left(\phi_S+i\right)\right)e^{i\phi_S\log K}\right)}{S_t\phi_S}\mathrm{d}\phi_S \\
= & \frac{S_t}{\pi}\int_0^\infty \frac{\mathrm{Re}\left(\varphi_t\left(-\left(\phi_S+i\right)\right)\right)\sin\left(\phi_S\log K\right)+\mathrm{Im}\left(\varphi_t\left(-\left(\phi_S+i\right)\right)\right)\cos\left(\phi_S\log K\right)}{S_te^{r(T-t)}\phi_S}\mathrm{d}\phi_S \\
= & \frac{S_t}{\pi}\int_0^\infty \frac{\mathrm{Re}\left(\varphi_t\left(\phi_S-i\right)\right)\sin\left(\phi_S\log K\right)-\mathrm{Im}\left(\varphi_t\left(\phi_S-i\right)\right)\cos\left(\phi_S\log K\right)}{S_te^{r(T-t)}\phi_S}\mathrm{d}\phi_S \\
= & \frac{S_t}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{\left(i\sin\left(\phi_S\log K\right)-\cos\left(\phi_S\log K\right)\right)\varphi_t\left(\phi_S-i\right)}{S_te^{r(T-t)}i\phi_S}\right]\mathrm{d}\phi_S \\
= & -\frac{S_t}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{e^{-i\phi_S\log K}\varphi_t\left(\phi_S-i\right)}{\varphi_t\left(-i\right)i\phi_S}\right]\mathrm{d}\phi_S.
\end{aligned}
$$

Proof of Lemma 6.2

$$
\mathrm{E}_0\left(C_{ih}\right)=S_0e^{\mu h}V_1-K_ie^{-r(T_i-h)}V_2,
$$

where

$$
\begin{aligned}
V_1 &= \frac{1}{2}+\frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{\exp(-i\alpha\log K)\exp\left(i\alpha\log S_0+i\alpha\mu h+i\alpha r\left(T_i-h\right)+\tilde{F}+\tilde{G}\sigma_0^2\right)}{i\alpha}\right]\mathrm{d}\alpha \\
V_2 &= \frac{1}{2}+\frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{\exp(-i\alpha\log K)\exp\left(i\alpha\log S_0+i\alpha\mu h+i\alpha r\left(T_i-h\right)+F+G\sigma_0^2\right)}{i\alpha}\right]\mathrm{d}\alpha \\
\mu &= r+\eta^S+\mu_J\left(\lambda-\tilde{\lambda}\right),
\end{aligned}
$$

with F,$\tilde{F}$,G, and $\tilde{G}$ functions that only depend on T_i, h, and the model parameters. Expressions V_1 and V_2 are rewritable as function of moneyness

$$
\begin{aligned}
V_1 &= \frac{1}{2}+\frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{m_{i0}^{-i\alpha}\exp\left(i\alpha\left(\mu-r\right)h+\tilde{F}+\tilde{G}\sigma_0^2\right)}{i\alpha}\right]\mathrm{d}\alpha, \\
V_2 &= \frac{1}{2}+\frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{m_{i0}^{-i\alpha}\exp\left(i\alpha\left(\mu-r\right)h+F+G\sigma_0^2\right)}{i\alpha}\right]\mathrm{d}\alpha.
\end{aligned}
$$

Using this, the expected gross return on a call option is

$$
\mathrm{E}_0\left(e^{-rh}\frac{C_{ih}}{C_{i0}}\right)=\frac{S_0\left(e^{(\mu-r)h}V_1-m_{i0}V_2\right)}{S_0c_{i0}(m_{i0},T_i,\sigma_0^2)}=\frac{e^{(\mu-r)h}V_1(m_{i0},T_i,\sigma_0^2)-m_{i0}V_2(m_{i0},T_i,\sigma_0^2)}{c_{i0}(m_{i0},T_i,\sigma_0^2)}. \tag{6.33}
$$

CHAPTER 7

Conclusions and Future Research

The usage of option contracts in portfolio management has tremendously expanded in the last two decades. The payoff profile and the risk-return characteristics of options can be used by asset managers to construct financial products that tailor the needs of their clients. This thesis focuses on the informational content of standard European plain vanilla option contracts that are written on a stock index. The main findings, the practical relevance, and some directions for future research are summarized in this final chapter.

7.1 Summary and conclusions

In Chapter 2 we give an extensive overview of the continuous time option pricing literature. Starting from an earlier paper by Bachelier (1900) the continuous time literature on the modeling of index returns and option prices has expanded in various ways. We identify three streams of literature that utilize plain vanilla options in a methodologically different manner. First, option prices are used to calibrate parameters in parametric option pricing models. Usually, the information in both stock (index) prices and option prices is used for estimating the model parameters. Option prices are necessary to identify parameters that are not solely identifiable by stock (index) returns. Well-known examples can be found in Chernov and Ghysels (2000) and Pan (2002). Secondly, plain vanilla option prices contain information on the risk-neutral distribution of the stock (index) price at a future point in time. Numerous nonparametric methodologies have appeared in the literature that provide estimates of the future risk-neutral distribution of stock (index) prices. A leading reference in this area is Jackwerth and Rubinstein (1996). Finally, there is a stream of literature that studies the dynamics of the Black-Scholes implied volatility smile/skew. This literature is aimed to find estimates of future option prices given the history of implied volatility smiles and implied volatility term structures. Recently, there is a growing interest in the properties of empirical option *returns*. The most simple option strategies have impressive Sharpe ratios. However, theoretical explanations for the performance of option based investment strategies are still lacking. Coval and Shumway (2001) provides an intuitive treatment of empirical option returns on S&P-500 and S&P-100 index options.

In Chapter 3 we propose an option pricing model that allows for different behavior of stock prices during the periods that exchanges are closed. This is an option pricing model that explicitly takes this microstructural effect into account. The overnight nontrading periods are modeled by means of a single jump from the closing time on the one day to the opening time on the next day. We find that this additional jump component is of significant importance in explaining S&P-500 index option prices. To be more precise, the overnight jump component captures approximately one third of total jump variation in low volatility periods and about a quarter of total jump variation in high volatility periods. Moreover, we find that an option pricing including random jumps and overnight jumps outperforms standard stochastic volatility and jump models in terms of empirical fit of S&P-500 index option prices.

Chapter 4 presents a nonparametric technique for the estimation of the joint risk-neutral density of stock (index) return and future instantaneous volatility. This methodology uses the information in a set of option prices to estimate the future risk-neutral volatility *density* nonparametrically. We add a new dimension to the implied distribution literature. Concerning the marginal risk-neutral return we confirm negative skewness. This indicates that jumps with average negative jump size are necessary in a parametric study of option pricing. The results on the marginal risk-neutral density of future volatility strongly indicate the presence of a negative volatility risk premium. The volatility risk premium seems to depend on initial volatility in a non-linear way. Furthermore, the estimated risk-neutral probability of high volatility are high even when current volatility is low. This means that investors pay high prices for products that give protection against states of high volatility. Finally, we find that the Heston model is not able to describe the marginal risk-neutral return density and the marginal risk-neutral volatility density simultaneously. The results point to the direction of a parametric model including jumps in the return process and in the volatility process.

The issue of parameter estimation in models that contain latent variables is discussed in Chapter 5. These latent variables appear in the conditional moments of the stock return which makes GMM estimation rather complicated. We show that this issue can be solved theoretically by applying a simple trick. This results in moment conditions that are independent of the latent variables. From a simulation experiment in a stochastic volatility world we deduce two main findings. First, we find that parameters are estimated more precisely if 'GARCH'-instruments are used instead of the classical instruments. These 'GARCH'-instruments are based on the GARCH estimators of stochastic volatility time series. Secondly, the moment conditions in combination with the chosen 'GARCH'-instruments do not provide a proper empirical identification of the mean-reversion parameter in the stochastic volatility model. Apparently, this parameter can only be empirically identified by using conditional estimation techniques.

Chapter 6 studies mean-variance based performance measurement of option based strategies and mean-variance asset allocation in models that imply different preferences than mean-variance preferences. To perform these analyses, we provide a methodology that allows for the calculation of the conditional expectation and the conditional variance of returns on options that are not necessarily held to maturity. The methodology applies to all models that fit into the class of affine jump-diffusions. Additionally, the covariance between the stock and the option and between options that have different strikes can be calculated by means of the same method. We find that CAPM α cannot be used as a performance measure for nonsymmetric return strategies. For instance, shorting an out-of-the-money and fairly priced put option in the Black-Scholes world would generate positive α. Also for stochastic volatility and jump-diffusion models, simple α-generating strategies are easily found. As a second application, we calculate optimal asset allocation rules for an investor who has access to derivative markets. We find that both mean-variance investors and power utility investors take short positions in straddles if the volatility diffusion risk premium is negative in a stochastic volatility world. Overall, we find no qualitative differences between mean-variance investors and power utility investors when straddles are considered as separate investment opportunities. In jump models, mean-variance investors and power utility investors sometimes take different investment decisions. In a setting of very infrequent but very large downward movements in the risky stock, power utility investors always take a long position in the out-of-the-money put option in order to hedge the crash risk in the long stock position while, in case of sufficient compensation, the mean-variance investor is willing to take the short side of the put position. We observe large efficiency gains in the mean-variance sense when delta-neutral straddles are included in the investment opportunity set in a stochastic volatility world with stochastically varying jump sizes.

7.2 Directions for future research

Parameter estimation in models containing latent variables remains a hot issue. Although the method in Chapter 5 circumvents the problem by constructing moment conditions independent of the latent variables, the method has the disadvantage of poor empirical identification of the time scale parameters in stochastic volatility models. This issue can only be solved by considering conditional estimation techniques. These techniques require an estimate of the instantaneous variance time series. Pan (2002) uses option prices to extract an estimate of the variance series. In a second step, that paper uses the variance estimates as an input to conditional moments of the stock return distribution and the instantaneous variance distribution. In future work, the conditional moments could be replaced by conditional probabilities which would lead to a maximum likelihood estimation technique. Given that the characteristic functions of returns and future volatility is known in most models, this is conceptually not complicated. However, for the practical implementation of such a procedure more computational power is necessary. In addition, unconditional moments could be used as explicit restrictions in order to stabilize the optimization procedure. The conditional moments that are derived in Chapter 6 could be utilized in estimation procedures to get a better empirical identification of model parameters.

For the practical implementation of affine jump-diffusions in portfolio management, an easy-to-use and stable estimation procedure for affine jump-diffusion is of crucial importance. As a consequence of the results on implied distributions of future stock index values, which are confirmed by the findings in Chapter 4, investment strategies based on the difference between the empirical risk-neutral return distribution and a parametric estimate of the objective return distribution are extensively tested. These strategies typically take long positions in states that have a positive expected return (states for which the estimated objective probability is higher than the estimated risk-neutral probability) and short positions in states that have a negative expected return. Obviously, a good estimate of the parametric objective distribution is very important. Over longer investment horizons, the normal distribution provides a reasonable fit of empirical stock index returns. However, these long investment horizons are not always desirable. For shorter investment horizons the normality assumption of stock index returns is no longer appropriate and therefore models more dynamic than the Black-Scholes model are necessary to implement buy-and-hold investment strategies over shorter horizons.

The analysis in Chapter 4 can be extended in numerous different ways. One of the possibilities is to estimate the joint risk-neutral distribution of the return on a stock and the realized variance of a stock index. The increased liquidity in variance swaps indicates that the market is interested in products whose payoff is related to the realized variance of a stock index. Options on realized variance would probably be more popular than variance swaps. However, the theoretical valuation of options on realized variance is much more complicated than the valuation of variance swaps. This issue could be solved by the aforementioned extension of the methodology of Chapter 4.

The results in Chapter 4 also gives new insights to the model specification part of parametric option pricing. The results on the marginal volatility density imply that the volatility risk premium parameter should depend on the current level of instantaneous volatility. Furthermore, the same density clearly supports the inclusion of a jump component in the volatility process.

An obvious extension of the research in Chapter 6 is to use the derivations in an empirical study of option returns. The literature on option pricing returns reports surprising results on the performance of several option strategies. Is there a parameter set that gives a description of empirical option returns across moneyness categories for a fixed holding period? How do the optimal fitting parameters vary with changes in the investment horizon? Does there exist a parameter set that fits both option returns and stock (index) returns simultaneously?

Finally, in the world of active portfolio management CAPM α plays an important role. The higher α the more satisfaction among clients. Chapter 6 has shown that CAPM α is often a bad performance measure for option based strategies. Further research should be directed into an adjustment of CAPM β such that CAPM α can be interpreted as a performance measure again.

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Nederlandse Samenvatting (Dutch Summary)

In de laatste twee decennia is het gebruik van financiële afgeleide instrumenten op het gebied van financieel management enorm toegenomen. Derivaten worden bijvoorbeeld tegenwoordig veelvuldig gebruikt in de dagelijkse toepassing van risicobeheer en vermogensbeheer. De opmerkelijke ontwikkeling in de liquiditeit van derivaten is vooral te verklaren door de algemene ontwikkeling van financiële markten, behoeften van investeerders en regelgeving.

De voornaamste motivatie voor het schrijven van dit proefschrift schuilt in het toegenomen gebruik van derivaten. In dit proefschrift zal de nadruk liggen op de informatie die bevat is in de prijzen van één specifiek financieel product, namelijk de standaard Europese index optie. Een groot aantal artikelen is reeds verschenen dat de informatielading van Europese index opties bestudeert. Echter, er zijn ook nog steeds een aantal interessante onderzoeksvragen onbeantwoord gebleven. In dit proefschrift worden vragen beantwoord worden die bijvoorbeeld betrekking hebben op het prijzen van index opties, risico-neutrale kansverdelingen die worden geïmpliceerd door index opties en rendementen die behaald kunnen worden als index opties worden aangehouden.

Twee voorname vraagstukken binnen de literatuur van financiële producten zijn de prijsvorming van deze producten en het afdekken van de risico's die het aanhouden van deze producten met zich meebrengt. Om de prijs van een afgeleide product te kunnen uitrekenen zijn drie theoretische concepten van eminent belang. Dit zijn (1) het process dat de onderliggende waarde van de optie volgt in de werkelijke wereld, (2) de compensatie voor alle systematische risico's die in de gemodelleerde werkelijke wereld aanwezig zijn en (3) de stochastische ontwikkeling van de onderliggende waarde van de optie in de risico-neutrale wereld. Deze drie concepten definiëren een financieel model waarbinnen opties en allerlei andere derivaten gewaardeerd kunnen worden. Een belangrijk voorbeeld van zo een prijsvormingsmodel is het beroemde Black-Scholes model. De voorwaarden waaronder dit prijsvormingsmodel is afgeleid zijn dusdanig sterk dat het model praktisch niet (meer) toepasbaar is. Er zijn veel artikelen verschenen die de tekortkomingen van het Black-Scholes model behandelen en mogelijke alternatieven voordragen. Hoofdstuk 2 geeft een uitgebreid overzicht van deze artikelen. Daarbij wordt de literatuur die zich bezighoudt met de tekortkomingen van het Black-Scholes model opgedeeld in drie verschillende stromingen. Eerst worden artikelen behandeld die alternatieven voordragen zoals het stochastisch volatiliteitsmodel in Heston (1993) en de klasse van sprong-modellen in Duffie, Pan en Singleton (2000). Vervolgens is ruime aandacht geschonken aan een serie van artikelen die de risico-neutrale verdeling van aandelenrendementen probeert te onttrekken aan geobserveerde optieprijzen die geschreven zijn op dit aandeel. De verkregen impliciete verdeling is strijdig met de Black-Scholes aannamen. Tenslotte is kort de literatuur beschreven die de dynamiek van Black-Scholes impliciete volatiliteiten bestudeert, beschreven. Naast een uitgebreide opsomming van de prijsvormingsliteratuur zijn in hoofdstuk 2 ook artikelen behandeld die de rendementen op opties als uitgangspunt nemen.

In hoofdstuk 3 is het effect van gesloten aandelenmarkten op optieprijzen onderzocht. De motivatie voor dit onderzoek is gelegen in het feit dat traditionele waarderingsmodellen perioden waarin niet gehandeld wordt, buiten beschouwing laten terwijl de empirische literatuur heeft aangetoond dat de verdelingseigenschappen van handelsperioden substantieel verschillen van perioden waarin niet gehandeld wordt. In rendementen van opening naar opening valt bijvoorbeeld meer variatie waar te nemen dan in de rendementen van de slotkoersen. Een ander voorbeeld is dat de rendementen tussen de opening en de slotkoers beweeglijker zijn dan de rendementen tussen de slotkoers en de opening. Tot nu toe is het effect van gesloten financiële markten alleen onderzocht voor aandelenrendementen. Hoofdstuk 3 bestudeert vooral de invloed op optieprijzen. Daartoe is een optiewaarderingsmodel gepresenteerd waarin de niet-handelsperioden expliciet zijn meegenomen. Dit is gedaan door het verschil tussen de slotkoers van de ene handelsdag en de openingskoers van de volgende handelsdag te modelleren met één sprong in de aandelenindex. Gedurende de handelsdag is aangenomen dat de ontwikkeling van de aandelenindex wordt beschreven door een proces met stochastische volatiliteit waarin op ieder willekeurig tijdstip een sprong kan plaatsvinden. Het continue deel van het proces geeft de normale bewegingen in de aandelenindex weer terwijl het spronggedeelte het arriveren van belangrijke nieuwe informatie representeert. Gegeven het veronderstelde statistische proces voor een aandelenindex is het mogelijk (na een transformatie van de kansmaat) om theoretische optieprijzen uit te rekenen. De resulterende formules zijn vervolgens gebruikt om met behulp van S&P-500 index opties de risico-neutrale model-parameters te schatten. Dit wordt gedaan in twee verschillende dataperioden namelijk een periode van lage volatiliteit (1992-1997) en een periode waarin de volatiliteit "normale" waarden aanneemt (1999-2003). De belangrijkste conclusie van hoofdstuk 3 is dat de toegevoegde sprongcomponent voor niet-handelsperioden een belangrijke invloed heeft op S&P-500 index optieprijzen. Deze extra component beschrijft ongeveer een kwart van de totale variatie in de sprongen. Een andere belangrijke conclusie is dat een optiewaarderingsmodel dat stochastische volatiliteit, een willekeurige sprongcomponent en een vaste sprongcomponent bevat, de beste beschrijving geeft voor SPX opties.

In hoofdstuk 4 is een nieuwe methode geïntroduceerd waarmee de gezamenlijke risico-neutrale verdeling van indexrendementen en de toekomstige volatiliteit geschat kan worden. Hiertoe is alleen gebruik gemaakt van standaard opties die geschreven zijn op de desbetreffende index. De toegevoegde waarde van dit deel van het proefschrift is dat niet alleen de risico-neutrale verdeling van rendementen bepaald kunnen worden, maar ook de risico-neutrale verdeling van de toekomstige volatiliteit. Een methode waarmee de risico-neutrale verdeling van de toekomstige volatiliteit bepaald kan worden, is nog niet eerder gepresenteerd in de literatuur. De huidige literatuur baseert zich vooral op het resultaat dat de risico-neutrale verdeling van aandelen(index)-rendementen verkregen kan worden door de tweede afgeleide te nemen van de optiewaarderingsformule voor calls met betrekking tot de uitoefenprijs. Deze benadering is vruchteloos bij het bepalen van de risico-neutrale verdeling van volatiliteit omdat er geen derivaten voorhanden zijn waarvan de uitbetaling perfect is gecorreleerd met de toekomstige volatiliteit. Theoretisch gezien, is de methode die wordt geïntroduceerd in hoofdstuk 4, gebaseerd op de "First Fundamental Theorem of Asset Pricing". Deze methode is geverifieerd voor verschillende stochastische volatiliteitsmodellen waaronder het Heston (1993) model. De resultaten tonen aan dat de methode in staat is om de analytische gezamenlijke verdeling te onttrekken aan analytische prijzen die volgen uit het gekozen stochastische volatiliteitsmodel. Deze conclusie verandert niet wanneer in de schattingsprocedure niet de geobserveerde volatiliteiten worden gebruikt maar de EGARCH-schatters daarvan. Het toepassen van de methode op empirische data geeft een aantal nieuwe inzichten met betrekking tot de geschatte risico-neutrale verdeling van toekomstige volatiliteit. De resultaten laten namelijk zien dat de volatiliteitsverdeling naar rechts verschuift als de initiële volatiliteit een hogere waarde heeft. Verder heeft de geschatte volatiliteitsverdeling positieve scheefheid welke het meest aanwezig is in tijden van lage volatiliteit. Dit komt overeen met de theorie dat investeerders een grote aversie hebben jegens onverwachte positieve schokken in de volatiliteit. De eigenschappen van de geschatte risico-neutrale indexverdeling komen overeen met die reeds gerapporteerd zijn in de literatuur. Voor deze verdeling is bijvoorbeeld een negatieve scheefheid gevonden. Wanneer de niet-parametrische verdelingen geconfronteerd worden met parametrische optiewaarderingsmodellen dan blijkt dat de risico-neutrale volatiliteit van volatiliteit veel kleiner is dan wordt voorspeld door het Heston (1993) model. Dit is een sterke aanwijzing dat een sprongcomponent in het rendementsprocess moet worden opgenomen om de risico-neutrale rendementsverdeling te kunnen beschrijven. Tenslotte geven de schattingsresultaten aan dat de risico-neutrale volatiliteit van volatiliteit niet beschreven kan worden middels één enkel diffusieproces.

Hoofdstuk 5 behandelt het probleem van het schatten van parameters in stochastische volatiliteitsmodellen. Het schatten van parameters in deze modellen is ingewikkeld omdat de huidige volatiliteit verschijnt in momentencondities terwijl deze variabele in de werkelijkheid latent is. In hoofdstuk 5 is aangetoond dat het gebruik van onconditionele momenten in plaats van conditionele momenten leidt tot een slechte empirische identificatie van de modelparameters. De resultaten van een simulatiestudie laten zien dat instrumenten die samengesteld worden op basis van GARCH parameterschatters leiden tot een grotere efficiëntie van de parameterschatter dan wanneer traditionele instrumenten worden gebruikt. Echter, de standaardfouten zijn dusdanig hoog dat de schattingsprocedure geen praktische relevantie heeft.

In hoofdstuk 6 is de aandacht verschoven naar rendementen die behaald kunnen worden op het aanhouden van opties in een beleggingsportefeuille. In het bijzonder zijn de mean-variance eigenschappen van optie rendementen behandeld. Er zijn enorm veel artikelen die het optiewaarderingsvraagstuk behandelen, maar er zijn slecht enkele artikelen beschikbaar die de theoretische en empirische eigenschappen van optierendementen nader beschouwen. De mean-variance eigenschappen konden onderzocht worden, omdat in hoofdstuk 6 een methode is geïntroduceerd waarmee de (conditionele) verwachting, de (conditionele) variantie en de (conditionele) covariantie van optierendementen uitgerekend kan worden voor alle modellen die behoren tot de klasse van affine sprongmodellen. Hierbij is gebruik gemaakt van het feit dat de karakteristieke functie van de toekomstige waarde van de toestandsvariabelen bekend is voor deze klasse van modellen. De resulterende expressies worden gebruikt om de eigenschappen van de CAPM α en de

CAPM β te analyseren voor portefeuilles die opties bevatten. De uitkomsten tonen aan dat de CAPM α de natuurlijke interpretatie als prestatiemaat verliest wanneer optiestrategieën beschouwd worden. Dit kan worden verklaard doordat aan de ene kant het CAPM een lineair model is terwijl aan de andere kant optierendementen extreem niet-lineair zijn. Verder is ook aangetoond dat bij gebruik van de CAPM regressievergelijking bij het valideren van optiewaarderingsmodellen goed bekeken moet worden of aan de onderliggende OLS veronderstellingen is voldaan.

Wanneer de formules worden gebruikt om optimale portefeuilles te bepalen op basis van het mean-variance criterium, is aangetoond dat mean-variance investeerders kwalitatief gezien dezelfde portefeuillegewichten nemen in delta-hedged straddles als power-utility investeerders. Deze conclusie is afgeleid in een wereld waarin stochastische volatiliteit geprijsd is en er geen sprongen op kunnen treden in het rendementsprocess van de onderliggende waarde. Na het toevoegen van geprijsd sprongrisico veranderen de conclusies. Het blijkt dat in zo een setting en onder bepaalde voorwaarden mean-variance investeerders en power utility investeerders kwalitatief verschillende investeringsbeslissingen nemen. De mean-variance investeerder is eerder geneigd om het risico van een short out-of-the-money put optie in de portefeuille op te nemen. Tenslotte zijn grote efficiëntievoordelen waargenomen voor mean-variance investeerders die short straddle posities nemen waarin zowel de onzekerheid in volatiliteit als de onzekerheid in de sprongcomponent geprijsd zijn. Daarbij is ook van belang dat de spronggroottes stochastisch worden verondersteld.