Operations research, games and graphs

Tijs, S.H.; Borm, P.E.M.

Published in:
Zeitschrift for Operations Research: methods and Models for Operations Research

Publication date:
1993

ZOR – Methods and Models of Operations Research (1993) 38:109–111

Operations Research, Games and Graphs

1 Introduction

Typically, operations research studies situations in which one decision-maker, guided by an objective function, faces an optimization problem. The theory focuses on the question how to act in an optimal way and, in particular, on the issues of computational complexity and the design of efficient algorithms to implement these actions. Game theory considers situations involving at least two decision-makers (players), with possible diverging interests. The link with operations research lies in the fundamental assumption that a player's behaviour is determined by rational individual (utility maximizing) decision-making. Roughly speaking, game theory deals with mathematical models of competition and cooperation. Competitive or non-cooperative game theory studies situations in which the players can not make binding agreements and strategic behaviour and incentives play an important role. In cooperative game theory binding agreements can be made and often also side payments are allowed. Here, the main issue is allocation, either of costs or of revenues.

Since the early developments of operations research and game theory there has been a strong interplay between the two disciplines. Well-known is the interrelation between operations research and non-cooperative game theory. We only mention:

i) Duality results in mathematical programming theory and minimax results in zero sum game theory.
ii) Linear complementarity problems and the theory of bimatrix games.
iii) Markov decision theory and the theory of stochastic games.
iv) Optimal control theory and the theory of differential games.

Interrelation between operations research and cooperative game theory is of a more recent date and part of this issue of *Zeitschrift für Operations Research* highlights these relations.

Interestingly, one can say that an important part of the interplay between cooperative games and operations research stems from the basic discrete structure of a graph or a network that underlies various types of combinatorial

0340–9422/93/38:2/109–111 $2.50

optimization problems. If one assumes that at least two decision-makers are located at or control parts (e.g. points or arcs) of the underlying network, then a cooperative game can be constructed to this problem. In working together, the players can possible create extra gains or save costs and the problem arises how to share these revenues or how to allocate these costs. One way to answer these questions is to look at the properties (e.g. balancedness or convexity) of the games arising from that type of combinatorial optimization problems, and to apply an existing game theoretic solution concept (e.g. core, Shapley value or nucleolus), or to create a new allocation rule especially suited for this class of games.

Since the beginning of the seventies many situations were considered. Examples are:

i) Sequencing games, permutation games and assignment games.
ii) Traveling salesman games.
iii) Linear production games and linear programming games.
iv) Minimum spanning tree games and spanning network games.

In this issue Curiel, Potters, Rajendra Prasad, Tijs and Veltman elaborate on sequencing games arising from one machine scheduling problems where the jobs are assumed to be attached to players. Traveling salesman problems and traveling salesman games are considered both in the paper of Kuipers and in the paper of Faigle and Kern. The last two authors also apply their taxation model in relation with ε-balancedness to matching games and bin packing games. The paper of Feltkamp, Koster, van den Nouweland, Borm and Tijs shows balancedness for games arising from linear production situations in which players control resources and directed graphs describe the possibilities to transport products, resources and technology. Aarts and Driessen focus on spanning networks and minimum spanning tree games. It is shown how, by a transformation of the network, core elements can be determined.

The other four contributions have a somewhat different setting. The paper of Otten illustrates the fact that, in various cases, cost allocation rules especially developed for specific practical problems turn out to have a strong game theoretic appeal. In this particular case this boils down to an axiomatic characterization via reduced games. The contributions of Bergantinos, Carreras and Garcia Jurado and Jurg, Jansen and Tijs clearly display the interrelation between graphs and game theory. Bergantinos et al. explore cooperative games where incompatabilities between the players are modelled by an undirected graph. Jurg et al. provide a characterization of a class of finite noncooperative two-person games with pure Nash equilibria based on the (non-)existence of a cycle in a corresponding directed graph. Finally, the paper of Peters, van der Stel and Storcken discusses an alternative characterization of median voter schemes with applications in social choice theory and location theory.

In all, we hope that this special issue of Zeitschrift für Operations Research will invite the reader to take part in the research on the interplay between operations research, graphs and game theory.

Stef Tijs
Peter Borm