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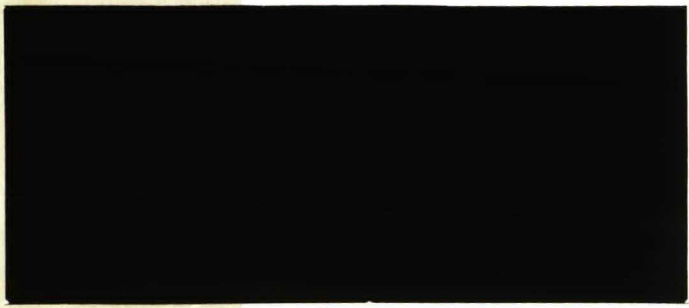
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EQUILIBRIUM SELECTION IN STAG HUNT GAMES

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by Hans Carlsson
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Equilibrium Selection in Stag Hunt Games[*]

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Abstract

A Stag Hunt Game is an n-person symmetric binary choice game. Each player can play either a safe strategy that yields a certain payoff irrespective of what the opponents do, or a risky strategy that yields a payoff that increases monotonically with the number of players that follow this strategy. There are two strict Nash equilibria, viz. the two symmetric pure strategy profiles. For such a game, we compute and compare the solutions according to the equilibrium selection theories of Harsanyi and Selten (1988), Güth and Kalkofen (1989) and Güth (1990). A further comparison is obtained by applying the global payoff uncertainty approach of Carlsson and Van Damme (1990). If there are two players all solutions coincide, but if the number of players exceeds two, then, in general, all solutions differ.

1 Introduction

A weakness of the Nash equilibrium concept for noncooperative games is that it frequently does not generate a unique outcome. In the literature, therefore, a great many concepts have been introduced that refine the Nash concept by imposing additional rationality restrictions or by requiring additional robustness properties. However, frequently even the strongest refinements that have been proposed — such as Kohlberg and Mertens' (1986) stability concept — do not succeed in determining unique solutions. This is true, in particular, for any game that has several strict Nash equilibria since all such equilibria survive any of the established refinement tests.

Many economic situations give rise to games with multiple strict Nash equilibria. As an example consider the game displayed in Figure 1. (Various economic scenarios that can be associated with this game are described in Section 2.)

Figure 1: Game $g(x)$ $(0 < x < 1)$

In the game $g(x)$, each player has to choose among two strategies: α is a safe strategy that yields x irrespective of what the opponent does. The strategy β might yield the higher payoff of 1 — if the opponent chooses β as well — but it is risky since it yields only 0 if the opponent chooses the safe strategy. It is easily seen that $g(x)$ has two strict Nash equilibria, viz. $\bar{\alpha}$ (both players choose α) and $\bar{\beta}$ (both choose β). Hence, when playing this game, players face a coordination problem. The dilemma is whether one should go for the equilibrium $\bar{\beta}$ with the highest possible payoff or whether, in view of the strategic uncertainty, one should rather play according to the safe equilibrium $\bar{\alpha}$.

Many game theorists would argue that the Pareto-dominant equilibrium $\bar{\beta}$ is the natural focal point in the game $g(x)$. In support of this view, they might invoke Schelling's principle of tacit bargaining: Since the players know that they would talk each other into $\bar{\beta}$ if they could communicate, they will also be able to reach this conclusion without actually communicating. From the viewpoint of a strict methodological individualism, however, these arguments in favor of $\bar{\beta}$, which almost amount to simply postulating collective rationality, are not wholly convincing. A satisfactory solution to the equilibrium selection problem should have a better foundation in individual decision making.

One can also question whether $\bar{\beta}$ is always the intuitively most appealing solution: For instance, would you accept a bet where you win $10000 in case another person B, with whom you cannot communicate, accepts a similar bet, but lose $90000 if B declines the bet? We believe many people would hesitate to accept *even if* prior communication with the opponent was allowed. Finally, one can note that subjects participating in experiments frequently do not succeed in coordinating on the Pareto-dominant equilibrium (see Cooper et al. (1990) and Van Huyck et al. (1990)).

The above discussion indicates the hazards of taking short cuts in matters of equilibrium selection and the need for a more fundamental approach where solutions to the selection problem are derived from individualistic assumptions. In view of the importance of games with multiple strict equilibria, there have been remarkably few contributions to this approach. Recently, however, the theory of equilibrium selection has received a strong impetus from Harsanyi and Selten's (1988) book. Apart from the Harsanyi/Selten theory, we will here discuss modifications to that theory by Güth and his co-authors as well as the rather different approach initiated in Carlsson and Van Damme (1990).

A common feature of these approaches is that the selection of a particular equilibrium results from the individual players' strategic uncertainty. There are, however, considerable differences: Theorists within the Harsanyi/Selten approach content themselves to postulate a specific form of uncertainty, typically a uniform prior in some fixed scheme

of expectations formation, but in Carlsson and Van Damme, the uncertainty is derived from more fundamental assumptions about how the players' information is generated. Hence, while in the former approach the equilibrium selection rule has a somewhat ad hoc character, we derive the selection rule within a strictly noncooperative framework by perturbing the game to be solved and embedding it in a game of incomplete information.

A drawback of our earlier paper is that it only covers 2×2 games and hence, does not describe a general equilibrium selection theory. Here, however, we demonstrate that some properties we derived in the earlier paper can be generalized to a certain class of n-person symmetric binary choice problems. In particular, we show that introducing slight payoff uncertainty allows the equilibrium selection problem to be resolved by a process of iterative elimination of strictly dominated strategies (Proposition 4.1). In our 1990 paper we showed that for 2-player 2×2 games equilibrium selection on the basis of Harsanyi and Selten's risk dominance criterion[1] can be justified by considerations of slight payoff uncertainty. Our aim in this paper is to show that these approaches do no longer yield the same solutions if the number of players exceeds two. Along the way we also show that the modifications of the Harsanyi/Selten theory that have been proposed by Güth yield still different outcomes.

The remainder of the paper is organized as follows. Section 2 describes the class of n-person binary choice games that we will study and mentions some contexts in which these games arise. The approaches to equilibrium selection that have been proposed in Harsanyi and Selten (1988), Güth and Kalkofen (1989) and Güth (1990) are outlined in Section 3, while Section 4 is devoted to extending the approach to equilibrium selection that has been put forward in Carlsson and Van Damme (1990). This section contains the paper's main result (Proposition 4.1). Section 5 concludes the paper with a general discussion on equilibrium selection and an illustration of the fact that different forms of

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[1] It is important to note that the theory of Harsanyi and Selten does not rely on risk dominance alone but also invokes the principle of payoff dominance. Although the authors are aware of its questionable justification, they decide to rank payoff dominance above risk dominance; see Harsanyi and Selten (1988, Sects. 10.11 and 10.12).

payoff uncertainty may give rise to very different results.

2   Stag Hunt Games

We consider games where a number, n, of identical players, are to choose simultaneously between two actions, α and β. The players' identical preferences are determined by a parameter x $(0 < x < 1)$ and a nondecreasing function $p : [0,1] \rightarrow [0,1]$ with $p(0) = 0$ and $p(1) = 1$. If a player chooses action α his payoff is x no matter what his opponents' choices are. If he chooses β then his payoff is $p(k/n)$ where k is the number of players that choose β. The normal form game described by the above data is denoted by $g(n, x, p)$ and will be called a Stag Hunt Game. In the remainder of the paper we will restrict ourselves to the case where

$$x \neq p(k/n) \quad \text{for all} \ \ k \in \{0, 1, \ldots, n\} \tag{2.1}$$

in order to avoid uninteresting case distinctions. In this case we have

Proposition 2.1 . *The stag hunt game $g(n, x, p)$ has two Nash equilibria in pure strategies, viz. $\bar{\alpha}$ and $\bar{\beta}$. Both equilibria are strict.*

Proof. Let s be a pure strategy combination in $g(n, x, p)$ and let k be the number of players choosing β. If $x > p(k/n)$, then a player choosing β (if there is any) has an incentive to switch to α. If $x < p(k/n)$, then a player choosing α (if there is any) can gain by unilaterally deviating to β. Hence, a necessary condition for a pure equilibrium is '$k = 0$ or $k = n$'. Straightforward verification shows that both possibilities do indeed yield strict equilibria.   □

The name Stag Hunt is motivated by the fact that the game $g(n, x, p)$ can be viewed as a formalization of a dilemma described in Rousseau's *Discours sur l'origine et les*

fondemens de l'inégalité parmi les hommes (see Rousseau (1971), also see Lewis (1969), Aumann (1990) or Crawford (1991)). Suppose each member of a group of hunters has to decide independently whether to cooperate with the others in hunting a stag (action β) or instead, to go off on his own and hunt hares (action α). While the latter does not require cooperation of the others, the probability that the stag hunt is successful depends, on the number of hunters that does not chase hares and is increasing in this number.

The Stag Hunt Game $g(n, x, p)$ can be viewed as a stylized model of many other interesting economic and political situations. For example, α may be interpreted as consuming own production and β may be interpreted as going to a market place to trade own production for more desired products. The attractiveness of the latter strategy depends on the chance of finding a trading partner, i.e. on how many people adhere to the same strategy. Alternatively, α may be interpreted as shirking while β stands for spending effort in a situation of team production: one's own effort is wasted unless a sufficient number of other people spend effort as well. The game $g(n, x, p)$ also arises in models of public goods. Assume that people have to decide whether to contribute to a public good (action β) or not (action α). If non-contributors can be excluded from consuming the public good, if contributions are not refunded and if the public good is provided only if enough people contribute, then, for interesting parameter values, the situation is represented by a game that is strategically equivalent to a Stag Hunt Game (see Harrison and Hirschleifer (1989)). Finally, games with the Stag Hunt structure have been used as models to explain the occurrence of Keynesian coordination failures (Cooper and John (1988), Van Huyck et al. (1990)), while in the international relations literature a model of this type is known as the Security Dilemma (Jervis (1987)).

Proposition 2.1 shows that in $g(n, x, p)$, Pareto efficiency is compatible with equilibrium play. (Note that, since $x < 1$, the unique Pareto efficient outcome is $\bar{\beta}$.) Hence, it is not necessary to make binding agreements in order to reach an efficient outcome. However, it is not clear whether players will be able to reach this outcome in a nonco-

operative context where no direct communication is possible. Choosing the cooperative action β is risky in the sense that it only pays if there are enough people who follow this action. If you expect only few other people to choose β, you are better off choosing α even though such expectations — if they are generally held — will lead the players to coordinate on the Pareto-inferior equilibrium $\bar{\alpha}$. Intuitively, one would think that the larger the value of x, the greater the chance that players will indeed end up in this (inefficient) equilibrium. The experimental results reported in Van Huyck et al. (1990) concerning similar games indeed point in this direction, but formal game theory provides no support: Both $\bar{\alpha}$ and $\bar{\beta}$ are strict equilibria — i.e. each player is strictly worse off is she deviates unilaterally — so each of these equilibria survives the most stringent refinements that have been proposed to date. To resolve the player's dilemma in $g(n, x, p)$ we, therefore, turn to theories of *equilibrium selection* in the following sections.

3 Equilibrium Selection

In this section we discuss some principles of equilibrium selection that are based on comparisons of riskiness of equilibria. All are variations of Harsanyi and Selten's (1988) concept of risk dominance and all yield the same result if the number of players is equal to two.[2] Specifically, if $n = 2$, then the game $g(n, x, p)$ is given in Figure 1, and the risk-dominant equilibrium is the one with the largest Nash product, i.e. the one for which the product of the deviation losses is largest. Hence, if $n = 2$, then $\bar{\alpha}$ risk-dominates $\bar{\beta}$ if $x^2 > (1 - x)^2$, i.e. if $x > 1/2$, while $\bar{\beta}$ risk-dominates $\bar{\alpha}$ if the reverse (strict) inequality is satisfied. However, as we will see, the various equilibrium selection theories generate different outcomes as soon as the number of players exceeds two. The discussion that follows may also give some insight in the relative merits and drawbacks of the various concepts.

We first discuss the idea of equilibrium selection on the basis of maximal unilateral

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[2] For the special class of 2×2 games, the risk dominance relation is characterized by a convincing set of axioms (Harsanyi and Selten (1988, Sect. 3.9)).

deviation stability that has been put forward in Güth (1990). Suppose that each player i believes that he is the only player who does not know what the unique solution of $g(n, x, p)$ is. All players, however, know that the solution is unique and that it is either $\bar{\alpha}$ or $\bar{\beta}$. Two players i and j may then represent their decision problem by means of a reduced game in which each player i believes that each player $k \notin \{i, j\}$ will take the same choice as player j. Since $g(n, x, p)$ is symmetric, the derived game does not depend on which players i and j are selected from the original player set and is represented by the bimatrix from Figure 1. Assuming that the players i and j consider their decision problem in $g(n, x, p)$ to be equivalent to that in the game of Figure 1 and that the players use risk dominance as the selection criterion in 2×2 games, the players will choose $\bar{\alpha}$ whenever $x > \frac{1}{2}$. Güth (1990) argues that $\bar{\alpha}$ is more stable against unilateral deviations than $\bar{\beta}$ if $x > \frac{1}{2}$. Hence, Güth advocates $\bar{\alpha}$ as the solution of $g(n, x, p)$ if $x > \frac{1}{2}$ and he advocates $\bar{\beta}$ as the solution if $x < \frac{1}{2}$. Clearly, the selection rule proposed by Güth corresponds to choosing that equilibrium with the largest Nash product. This implies, in particular, the somewhat counterintuitive property that the equilibrium selected does not depend on the number of players in the game. Güth himself remarks that selection on the basis of Nash products may not reflect all strategic aspects of the situation and that this might be viewed as a major deficiency of such a selection rule. Nevertheless, the rule may still serve as a benchmark against which other selection rules may be compared.

Next, we discuss selection on the basis of Harsanyi and Selten's (1988) risk-dominance concept. The definition of risk dominance is based on a hypothetical process of expectation formation starting from the initial situation where it is common knowledge that either $\bar{\alpha}$ or $\bar{\beta}$ will be the solution but where players do not yet know this solution. Harsanyi and Selten postulate a process in which players first, on the basis of a preliminary theory, form priors on the strategies played by their opponents. Thereafter, players gradually adapt their prior expectations to final equilibrium expectations by means of the tracing procedure (see Harsanyi and Selten (1988, pp. 207-209)).

According to Harsanyi and Selten, the players' prior beliefs q_i about player i's strategy

should coincide with the prediction of an outside observer who reasons in the following way about the game:

(i) Player i believes that his opponents will either all choose α or that they all choose β; he assigns a subjective probability z_i to the first event and $1 - z_i$ to the second.

(ii) Whatever the value of z_i, player i will choose a best response to his beliefs.

(iii) The beliefs (i.e. the z_i) of different players are independent and they are all uniformly distributed on $[0, 1]$.

From (i) and (ii), the outside observer concludes that player i chooses α_i if $z_i > 1 - x$, and that he chooses β_i if $z_i < 1 - x$. Hence, using (iii), the outside observer forecasts player i's strategy as

$$q_i = x\alpha + (1 - x)\beta, \tag{3.1}$$

with different q_i being independent. Harsanyi and Selten assume that the mixed strategy vector $q = (q_1, \ldots, q_n)$ describes the players' prior expectations in the game $g(n, x, p)$. Since q is not a Nash equilibrium, this expectation is not self-fulfilling, and, thus, has to be adapted. Adaptation is achieved by using the tracing procedure, i.e. by following a distinguished path in the graph of the correspondence

$$\lambda \to E((1 - \lambda)g(q) + \lambda g(n, x, p)) \quad (\lambda \in [0, 1]) \tag{3.2}$$

from the unique equilibrium of the game $g(q)$ associated with $\lambda = 0$ to an equilibrium of the game $g(n, x, p)$ that corresponds to $\lambda = 1$. (In (3.2), E denotes the set of Nash equilibria and $g(q)$ is the game where each player j computes his payoffs from the matrix $g(n, x, p)$ by assuming that his opponents are committed to use q as in (3.1). Hence, in $g(q)$ a player's optimal strategy does not depend on the strategies of his opponents.) In the special case at hand, tracing is easy: The process comes to an end at the first

iteration, except in degenerate cases. The reason is simply that, since the situation is symmetric, either all players will have α as the unique best response against q in which case $\bar{\alpha}$ is the distinguished equilibrium in (3.2), or they will all have β as the unique best response against q and in this case the result of the tracing procedure is $\bar{\beta}$. (See Harsanyi and Selten (1988, Lemma 4.17.7).)

Write $B_n^p(t)$ for player i's expected payoff associated with β in $g(n, x, p)$ when each of the opponents chooses β with probability t, i.e.

$$B_n^p(t) = \sum_{k=1}^{n} \binom{n-1}{k-1} t^{k-1}(1-t)^{n-k} p(k/n) \tag{3.3}$$

If the players' prior q is as in (3.1), then the expected payoff associated with β is $B_n^p(1-x)$ and each player's best response against q is α if $x > B_n^p(1-x)$, while the best response is β if $x < B_n^p(1-x)$. Hence, if $x > B_n^p(1-x)$ (resp. $x < B_n^p(1-x)$) then the outcome of the tracing procedure is $\bar{\alpha}$ (resp. $\bar{\beta}$) and in this case Harsanyi and Selten say that $\bar{\alpha}$ *risk dominates* $\bar{\beta}$ (resp. that $\bar{\beta}$ risk dominates $\bar{\alpha}$). To derive a more convenient characterization of risk dominance we state the following lemma.

Lemma 3.1 . *(i) $B_n^p(t)$ is nondecreasing in t for any n; (ii) If p is continuous, then $B_n^p(t) \to p(t)$ as $n \to \infty$ for any t.*

Proof. (i) Direct computation shows that $\frac{\partial}{\partial t} B_n^p(t) \geq 0$ since p is non-decreasing.
(ii) By a simple manipulation it is seen that

$$t B_n^p(t) = \sum_{k=0}^{n} \binom{n}{k} t^k (1-t)^{n-k} p(k/n) k/n \tag{3.4}$$

Now for a function f, the polynomial

$$B_n(f; t) = \sum_{k=0}^{n} \binom{n}{k} t^k (1-t)^{n-k} f(k/n)$$

is known as the Bernstein polynomial of order n of f. Hence, (3.4) can be rewritten as $t B_n^p(t) = B_n(f, t)$ where $f(x) = x p(x)$ for $x \in [0, 1]$. By Bernstein's theorem (Klambauer (1975, p. 332)) $B_n(f, \cdot) \to f$ uniformly on $[0, 1]$ as $n \to \infty$ if f is continuous. It follows that $t B_n^p(t) \to t p(t)$ uniformly as $n \to \infty$, so that $B_n^p(t) \to p(t)$ for all $t \in [0, 1]$. $\square$

The lemma allows us to conclude that there exists a unique $x^* = x^*(n, p)$ for which $x^* = B_n^p(1 - x^*)$. Thus $\bar{\alpha}$ risk dominates $\bar{\beta}$ if $x > x^*$ and $\bar{\beta}$ is risk-dominant if the reverse strict inequality is satisfied. The lemma also implies that, as $n \to \infty$, $x^*(n, p) \to x^*(p)$ where $x^*(p)$ is the unique solution of the equation $x = p(1 - x)$. Hence, we get the intuitive comparative statics result that $x^*(p) \le x^*(p')$ if $p \le p'$. Furthermore, direct computation shows that, if $p(x) = 0$ for $x < 1$ and $p(1) = 1$, then $x^*(n, p) \to 0$ as $n \to \infty$. All in all it seems that the risk dominance notion is more in agreement with the intuition than Güth's selection on the basis of Nash products is.

To conclude this section, we discuss selection on the basis of resistance avoidance as proposed in Güth and Kalkofen (1989). Player i's *resistance* against $\bar{\beta}$ at $\bar{\alpha}$ is defined as the maximum probability that each opponent may assign to β such that, if players randomize independently, player i still prefers α to β. Formally,

$$r_i(\bar{\alpha}, \bar{\beta}) = \max\{z \in [0, 1]; \; x \ge B_n^p(z)\} \tag{3.5}$$

Similarly, player i's resistance against $\bar{\alpha}$ at $\bar{\beta}$ is defined as

$$r_i(\bar{\beta}, \bar{\alpha}) = \max\{z \in [0, 1] : \; B_n^p(1 - z) \ge x\} \tag{3.6}$$

Note that, since the game is symmetric, these resistance values are independent of the player under consideration, so that we may speak of $r(\bar{\alpha}, \bar{\beta}) = r_i(\bar{\alpha}, \bar{\beta})$ as the resistance of $\bar{\alpha}$ against $\bar{\beta}$. Güth and Kalkofen (1989) say that $\bar{\alpha}$ is *resistant-dominant* if

$r(\bar{\alpha},\bar{\beta}) > r(\bar{\beta},\bar{\alpha})$ and that $\bar{\beta}$ is resistant-dominant if the reverse inequality is satisfied. Now Lemma 3.1 implies that $r(\bar{\beta},\bar{\alpha}) = 1 - r(\bar{\alpha},\bar{\beta})$, hence, $\bar{\alpha}$ is resistant-dominant if and only if $r(\bar{\alpha},\bar{\beta}) < \frac{1}{2}$, or equivalently if $x > B_n^p(\frac{1}{2})$. Hence, Güth and Kalkofen pick $\bar{\alpha}$ as the solution of $g(n,x,p)$ whenever each player prefers to play α if he expects all others to randomize equally over both actions, while they choose $\bar{\beta}$ as the solution if a player prefers to play β in this case. Writing $x^{**} = x^{**}(n,p) = B_n^p(\frac{1}{2})$ and using Lemma 3.1 we see that $x^{**} \to p(\frac{1}{2})$ as $n \to \infty$ and that $x^{**} < x^*$ if and only if $x^* < \frac{1}{2}$. Hence, also resistance dominance seems to capture the intuition about the effect of strategic uncertainty in $g(n,x,p)$, but in general the concept may lead to a recommended action that differs from the recommendation obtained from risk dominance considerations. In particular, depending on the shape of the function p, the area where α is resistance-dominant may be either larger of smaller than the area where α is risk-dominant.

The following Proposition summarizes the discussion from this Section.

Proposition 3.1 . *In the game $g(n,x,p)$:*

   (i) $\bar{\alpha}$ is most stable against unilateral deviations (Güth (1990)) if and only if $x > \frac{1}{2}$,

   (ii) $\bar{\alpha}$ is risk-dominant (Harsanyi and Selten (1988)) if and only if $x > x^*$ where x^* is the solution to $x^* = B_n^p(1 - x^*)$, and

   (iii) $\bar{\alpha}$ is resistant-dominant (Güth and Kalkofen (1989)) if and only if $x > B_n^p(\frac{1}{2})$.

4   Global Payoff Uncertainty

In this section we will show that the equilibrium selection problem in a Stag Hunt Game can be solved using the approach in Carlsson and Van Damme (1990) which is based on the idea that the payoff parameters of a game can only be observed with some noise. To be specific, assume that all data of the Stag Hunt Game $g(n,x,p)$ are common knowledge, except for the payoff x associated with the safe action α. Each player i will receive a signal x_i that provides an unbiased estimate of x, but the signals are noisy so the

true value of x will not be common knowledge. It should be noted that in Carlsson and Van Damme (1990) attention is restricted to 2×2 games and players are allowed to be imperfectly informed about more than one (possibly even all) parameters. In our earlier paper we derive a justification of the risk dominance selection criterion for 2×2 games, by showing that, under some rather weak conditions, considerations of iterated dominance will force the players to play the risk-dominant equilibrium for every possible observation as the noise vanishes. It is quite remarkable that in the 2×2 games this result is to a large extent independent of the players' prior beliefs and of the distributions of the observation errors. We will not seek the same degree of generality here, but confine ourselves to analyzing the special case where the prior on x is uniform and the players' observation errors are identically and independently distributed. As we will see, the results obtained differ from any of those discussed in the previous section.

Let X be a random variable that is uniformly distributed on an interval that strictly contains $[0,1]$ and let $(E_i)_{i=1}^{n}$ be an n-tuple of mutually independent, identically distributed random variables, each having zero mean. The E_i are assumed to be independent of X, to allow a density and to have support within $[-1,1]$. For $\varepsilon > 0$, write $X_i^\varepsilon = X_i + \varepsilon E_i$. As our model for the situation where each player i observes the true value of x in $g(n,x,p)$ only with some slight noise, we will consider the incomplete information game $g^\varepsilon(n,p)$ described by the following rules:

1. A realization $(x, x_1, \ldots, x_n)$ of $(X, X_1^\varepsilon, \ldots, X_n^\varepsilon)$ is drawn,

2. player i is informed about x_i and chooses between α and β,

3. each player i receives payoffs as determined by $g(n,x,p)$ and the choices in 2.

We now address the question of which choice of player i is rational in $g^\varepsilon(n,p)$ at the observation x_i. Note that player i will certainly choose α if $x_i > 1$: Since the expected value from choosing α at x_i is $E(X|X_i^\varepsilon = x_i) = x_i$, player i knows that α is strictly dominant at each such observation. For the same reason β will be chosen when $x_i <$ 0. The following proposition shows that considerations of iterated dominance allow

player i to solve his decision problem for all observations but one. Let p^* denote the expected value from choosing β when the number of opponents choosing β is uniform on $\{0, \ldots, n-1\}$:

$$p^* := \sum_{k=1}^{n} p(k/n)/n.$$

Then we have

Proposition 4.1 . *In any strategy that survives iterative elimination of strictly dominated strategies in $g^{\varepsilon}(n,p)$, player i chooses α if he observes $x_i > p^*$ and β if he observes $x_i < p^*$.*

Proof. Assume that α has already been shown to be iteratively dominant for each player j at each observation $x_j > \bar{x}$. (By the above such an $\bar{x}$ exists in $[0,1]$.) Now let us assume player i observes $x_i = \bar{x}$ and let us derive an upper bound for his expected payoff from playing β provided that no opponent uses an iteratively dominated strategy. Thus any opponent j can choose β only if $x_j \leq \bar{x}$. Let s be the strategy vector in which each player chooses β if $x_j \leq \bar{x}$ and α if $x_j > \bar{x}$. Since the function p is non-decreasing player i's expected payoff from β cannot exceed the payoff that β yields against s as long as the opponents do not play dominated strategies. To compute the expected payoff of β against s we have to know, for each $k \in \{0, 1, \ldots, n-1\}$, the probability that exactly k opponents make observations that do not exceed $\bar{x}$, i.e. we have to know

$$P(X_j^{\varepsilon} < \bar{x} \text{ for exactly } k \text{ opponents } j | X_i^{\varepsilon} = \bar{x}) \qquad (4.1)$$

The fact that the prior distribution of X is uniform allows us to conclude that the probability in (4.1) is independent of $\bar{x}$, at least as long as $\bar{x}$ is at least ε inside the support of X. (The formal proof is by direct computation, also see Carlsson and Van Damme

(1990). The intuition, however, is obvious: If the prior is uniform, such observations do not give new information.) The fact that the probability in (4.1) is independent of $\bar{x}$ allows us to conclude that this probability must be equal to the a priori probability that E_i is the $(k+1)$th smallest among the errors. Hence, the probability in (4.1) is equal to

$$P(E_j < E_i \text{ for exactly } k \text{ opponents } j) \tag{4.2}$$

Obviously, this probability is the same for any player. On the other hand, ties between different E_j are zero probability events so we know that exactly one player's error will be the $(k+1)$th smallest. Therefore the probability in (4.2) must equal $1/n$. As a consequence, as long as the opponents do not play (iteratively) dominated strategies, the expected payoff of β at $\bar{x}$ cannot exceed p^*. Since the expected payoff of α at $\bar{x}$ is $E(X|X_i^\varepsilon = \bar{x}) = \bar{x}$, it follows that the range where α is strictly (iterative) dominant for each player i may be extended below $\bar{x}$ if $\bar{x} > p^*$. This shows the first part of the proposition. The proof of the second part is completely analogous. □

It is remarkable that the result described in Proposition 4.1 does not depend on the scale parameter ε of the observation errors. In particular, the result remains valid for observation errors that are infinitesimally small. However, as ε tends to zero, the players' observations become perfectly correlated and in the limit they coincide with the value of x that actually prevails in the game. Hence, if one accepts the game $g^\varepsilon(n,p)$ with infinitesimal ε as an accurate model of the situation in which $g(n,x,p)$ has to be played but x can only be observed with a small amount of noise, then one will also have to accept that the game should be played as described in Proposition 4.1. Consequently, adding some noise allows us to solve the equilibrium selection problem. Comparing the switching point $\bar{x} = \bar{x}(n,p)$ in Proposition 4.1 with the cut-off levels x^* and x^{**} obtained in the previous section we see that they generally will differ. Hence, although all the approaches that have been discussed yield the switching point $x = \frac{1}{2}$ if $n = 2$, they typically lead to different answers if $n > 2$.

5 Discussion

5.1 Equilibrium Selection

The foundations of equilibrium selection theory were laid in the seminal papers of Nash on bargaining (Nash (1950, 1953)). Nash initiated what is nowadays called the Nash program, noting that cooperative games may be reformulated as noncooperative ones, by modelling explicitly the bargaining process through which agreements may be reached. Nash proposed that every bargaining game be solved by selecting one of the equilibria of its noncooperative representation as the solution and, he suggested unanimity games as noncooperative models of bargaining situations. In such games, players simultaneously propose an outcome and an outcome is implemented if and only if it has been proposed by all players; all other cases result in the status quo. Such a game indeed has many (strict) equilibria and, thus, Nash was forced to address the equilibrium selection problem. He proposed to select as the solution that equilibrium for which the product of the deviation losses is largest and he offered an axiomatic as well as a noncooperative justification for this selection rule. The latter is based on a perturbation argument: The unanimity game is 'smoothed' by introducing some uncertainty (about the size of the pie that is to be divided) and it is shown that only one equilibrium of the original game is a necessary limit of equilibria of the smoothed games as the amount of smoothing approaches zero. Hence, Nash writes

> "Thus the equilibrium points do not lead immediately to a solution of the game. But if we discriminate between them by studying their relative stabilities we can escape from this troublesome nonuniqueness" (Nash (1953, p. 132)

The approaches to equilibrium selection that were discussed in the previous sections were all inspired by Nash's ideas and they can be viewed as attempts to extend the solution obtained by Nash beyond the class of unanimity games. [3] For example, Harsanyi and Selten write

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[3]Nash also considered only 2-person games.

"Our attempts to define risk dominance in a satisfactory way have been guided by the idea that it is desirable to reproduce the result of Nash's cooperative bargaining theory with fixed threats. The Nash property is not an unintended by-product of our theory." (Harsanyi and Selten (1988, p. 215))

Although all the solutions considered in this paper indeed reproduce the outcome proposed by Nash for the special class of unanimity games, the example of the Stag Hunt shows that they no longer coincide outside this restricted class. The discussion in the previous sections has made clear that different outcomes result because the approaches differ in their assumptions on what players believe about the amount of correlation in the beliefs and/or strategies of their opponents. The main difference between the equilibrium selection approaches discussed in Section 3 and the payoff uncertainty approach in Section 4 is that the former rely on more or less ad hoc thought processes to model the players' reasoning about the game, while the latter — in the spirit of the Nash program — is based on a fully specified noncooperative game.

In connection with the payoff uncertainty approach, several important questions remain to be answered. In particular, one would like to know how robust the result is with respect to the distributional assumptions and to the parametrization of the underlying class of games. Naturally, one would also like to know whether the approach can be extended to other classes of games. At present we are not able to answer these questions in a satisfactory way. As far as robustness is concerned we can point to Carlsson and Van Damme (1990), in which we show that, for the special class of 2×2 games, the results obtained are independent both of distributional assumptions and of which parameters are allowed to vary, at least as long as the ex ante uncertainty is sufficiently large so that players consider the various types of dominance solvable games to be possible. Of course, it is still an open question whether this independence result generalizes to other classes of games.

5.2 The Stag Hunt

It should be clear that the result of Proposition 4.1 is driven by the fact that the areas where $x > 1$ (resp. $x < 0$) exert a remote influence on any x that is inside the interval $(0, 1)$. In the context considered by Rousseau, x might be viewed as a measure of the number of hares in the forest. The model described in Section 4 then assumes that each hunter, by looking around, can make an unbiased estimate x_i of x, and the intuitive argument underlying the proof of Proposition 4.1 is as follows. (For simplicity, assume $p(x) = 0$ if $x < 1$ and $p(1) = 1$, so that the stag hunt is successful only if all hunters cooperate.) If $x_i > 1$, then hunter i thinks that there are so many hares around that it is simply not worthwhile to continue the stag hunt. If $x_i < 1$, but $x_i \approx 1$ player i believes that it is very likely that some other hunter j thinks that hunting the stag is not worthwhile and, hence, decides not to cooperate, so that player i concludes that staghunting is not worthwhile for himself either. This reasoning process may continue to very low values of x_i: As long as player i thinks that some player j may think that some player k may think that ..... some player l does not cooperate in the stag hunt, player i will not cooperate himself. Presenting the argument in this way makes clear that what is driving the result is the lack of common knowledge (Aumann (1976)) in the perturbed game. Although in the game $g^\varepsilon(n, p)$ player i has very precise *knowledge* about X if he observes x_i (i.e. he knows that $X \in [x_i - \varepsilon, x_i + \varepsilon]$) there is no *common knowledge* about X among the players, except for the prior distribution of this random variable. This lack of common knowledge forces the players to take a global perspective in solving the perturbed game: in order to know what to do at the observation x_i one should also know what to do at observations that are far away from x_i. This is why the regions $x_i > 1$ and $x_i < 0$ exert a remote influence. [4]

In all honesty we are compelled to say that Rousseau attributes much less rationality to the players than we do in the proof of Proposition 4.1. He writes

"Voilà comment les hommes purent insensiblement acquérir quelque idée

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[4] A similar action from a distance occurs in Rubinstein's (1989) electronic mail game.

grossière des engagements mutuels, et de l'avantage de les remplir, mais seulement autant que pouvait l'exiger l'intérêt présent et sensible; car la prévoyance n'était rien pour eux; et, loin de s'occuper d'un avenir éloigné, ils ne songeaient pas même au lendemain. S'agissait-il de prendre un cerf, chacun sentait bien qu'il devait pour cela garder fidèlement son poste; mais si un lièvre venait à passer à la portée de l'un d'eux, il ne faut pas douter qu'il ne le poursuivît sans scrupule, et qu'ayant atteint sa proie il ne se souciât fort peu de faire manquer la leur à ses compagnons." (Rousseau (1971, p. 229)).

The above quotation also suggests a quite different perturbation of the game $g(n, x, p)$, viz. the probability of being confronted with a hare may be independent across hunters. For the sake of simplicity assume that the stag hunt is successful only if all players cooperate ($p(x) = 0$ if $x < 1$, $p(1) = 1$), let the hunter's utility of consuming his part of the stag be normalized to 1, interpret $\underline{x} \in (0, 1)$ as the disutility of the effort spent in hunting and let $\bar{x} - \underline{x}$ (with $\bar{x} > 1$) be the utility of consuming a hare. Then, if each hunter encounters a hare with probability ε, and if the probabilities associated with different players are independent, the situation may be represented by the following game $g^\varepsilon(n, \underline{x}, \bar{x})$.

1. For each i, a realization x_i of X_i is drawn, where X_i takes the value $\underline{x}$ with probability $1 - \varepsilon$ and the value $\bar{x}$ with probability ε and where $(X_i)_{i=1}^n$ are mutually independent;

2. player i learns x_i and chooses between α and β,

3. if player i chooses α his payoff is x_i; if he chooses β his payoff is 1 if all opponents also choose β, otherwise it is 0.

The analysis of this game yields a result that is completely different from that of the game $g^\varepsilon(n, p)$ discussed in the previous section for, in $g^\varepsilon(n, \underline{x}, \bar{x})$ cooperation may be feasible even for high values of $\underline{x}$. Specifically, as long as

$$(1 - \varepsilon)^{n-1} \geq \underline{x} \tag{5.1}$$

the game $g^\varepsilon(n, \underline{x}, \bar{x})$ has an equilibrium where each player i chooses β if $x_i = \underline{x}$. (Note that this game also has an equilibrium where each player i always, i.e. for each realization of X_i, chooses β.) Whether or not condition (5.1) can be satisfied depends on whether or not ε is small in relation to n, so it matters in which order limits are taken. For fixed ε, condition (5.1) will not be satisfied for n sufficiently large, but if ε is infinitesimally small, then (5.1) is satisfied for all n.

The game $g^\varepsilon(n, \underline{x}, \bar{x})$ is a special case of a game with independent randomly disturbed payoffs as considered originally in Harsanyi (1973). Harsanyi's main result is that *each* equilibrium (be it pure or mixed) of a "generic" unperturbed game can be approximated by pure equilibria of games in which there is slight payoff randomness. Hence, Harsanyi's approach offers a justification for the set of all equilibria, and does not enable us to select particular equilibria. The aim of this paper has been to show that selection becomes possible if players' payoffs (or at least players' observations, see Carlsson and Van Damme (1990) are correlated. An obvious question, therefore, is which approximation method (correlated or independent) is most appropriate. In our view this question cannot be answered in the abstract, but must be related to specific contexts. We believe that both the independent and the correlated case yield valuable insights concerning the stability and the selection of equilibria.

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