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Is there room for convergence in the E.C.?

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Abstract

In this paper we develop a theoretical framework which makes it possible to analyze several aspects of convergence between E.C. countries. The analysis is done in a dynamic game context, where countries, apart from minimizing individual cost functions, minimize cooperatively a convergence function, which represents the convergence conditions as elaborated in the Maastricht Treaty in 1991. This aspect of convergence is modeled as a dynamic constraint on the individual cost functions. We show that if countries' own welfare is their primary interest (and convergence becomes secondary) the maximum degree of convergence is completely determined by the non-cooperative outcome of the game. The framework is illustrated in a theoretical example. The example shows that costs involved to obtain convergence can differ substantially between countries and that, ultimately, these high costs for some countries will result in non-cooperative behaviour. Furthermore, it is shown that a small deviation from a Pareto optimal solution can increase convergence considerably. An algorithm is devised to obtain solutions of the game which are politically more feasible than the Nash bargaining solution and improve on the non-cooperative solution.

Keywords: Dynamic games; Convergence; Maastricht Treaty

JEL classification: F15

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1. Introduction

The EC governments agreed at the Maastricht meeting in 1991, to start, at the latest in 1999, with a full monetary union. This final step towards EMU sets out, however, that uneven developments in the process of integration are set aside. Therefore, in the Treaty of Maastricht four convergence criteria, for admitting a country to EMU, were designed. The political and economical consequences of these criteria are discussed in various papers (see e.g. Bean, 1992; Buiter et al., 1993). A consequence of the increasing integration process is the strengthened economic interdependence between member countries, which reduces the room for independent policy manoeuvring and increases the importance of cross-border effects. So, the final stages towards EMU involve on the one hand a process of closer convergence, and on the other hand coordination of the macroeconomic policies of the various countries. One should, however, note that convergence and coordination are prerequisites for obtaining a single market but do not necessarily guarantee a successful establishment of it. Now, there is a general consensus amongst the participating countries that convergence and coordination of policies is needed for moving towards EMU. There is these days, however, much less consensus as to how far and how fast this process should take place. The possible long-run significant increases in economic welfare in the Community are much less tangible than the short-term welfare loss effects incurred at various domestic markets. Therefore, a natural reaction one can expect from participating countries is that they strive for convergence in economic variables, but that they are only willing to pay a price (in terms of welfare loss) for it if this price is not too high. These observations suggest to study the convergence problem from a dynamic games point of view. Studies with respect to macroeconomic policy coordination in a dynamic games context appear frequently in economic literature, see, e.g., McKibbin and Sachs (1991), Hughes Hallett (1992). However, the influence of the aspects of convergence, analyzed in a dynamic games setting, on the effects of macroeconomic policy coordination has not been studied before. This motivates the study of this paper.

Starting from the point of view that each country has its own individual welfare loss function it wants to minimize in cooperation with the other countries, we develop a theoretical framework to analyze the trade off between extra welfare loss and more convergence. The analysis will be done in a dynamic games framework. We assume that each policymaker has an individual objective function, he/she wants to minimize and that there is some common sense on a convergence function which they want to minimize simultaneously. In the case of the EMU this convergence function represents the convergence conditions which are specified in the Maastricht Treaty. As Buiter et al. (1993) argue, monetary union requires a *common* rate of inflation of tradable goods which in principle need not be low or even stable. Strictly speaking this means that the countries at this stage do not know with what common rate they will finally end up. This uncertainty in the Maastricht Treaty confuses policymakers (and economists) who, in principle, are searching for shared (fixed) targets. Therefore, it is more logical to incorporate the two criteria of convergence in consumer price inflation and convergence in long-term interest rates in a convergence function (which, finally, will determine the 'optimal' common rate), and to incorporate the budget deficit in a country's own objective function. Under the assumption that all policymakers like to cooperate, we analyze the set of solutions which are obtained by the policymakers when they simultaneously minimize their welfare loss functions and convergence function. In particular we will show various aspects of the game if reducing welfare loss is the primary interest of countries and striving for convergence is of secondary interest.

The organization of the paper is as follows. In Section 2 we introduce the theoretical framework. We consider N countries which cooperatively agree on minimizing a convergence function and, moreover, all have their own individual objective function they like to minimize. The aspect of convergence is modeled as a dynamic constraint on the joint social welfare function. Under the assumption that all of these functions are convex (and some mild regularity conditions) we show the above-mentioned aspects. To help the reader to understand the basics of the presented theory we illustrate the approach in Section 3 by means of a simple theoretical example. In Section 4 we present the conclusions.

2. Incorporating convergence criteria: A theoretical framework

We consider an integrated economy of the European Community with N interdependent economies, where the policymakers in each country face a dynamic economic model which connects the endogenous variables (denoted by y), instrumental variables (denoted by u) and other noncontrollable variables. Each country has control over a set of instruments for economic policy, denoted by u_i. In stacked form $u' = (u'_1, \ldots, u'_N)$. We assume that each policymaker has a convex objective function, which we specify by J_i, which he/she wants to minimize. We denote the set of Pareto optimal solutions in the $(J_1, \ldots, J_N)$-plane by P. The point corresponding to the non-cooperative (Nash) solution, which is used as a bargaining threat-point, will be denoted by $N^c := (J_1^N, \ldots, J_N^N)$. Furthermore we assume that the countries agree to strive for a certain amount of degree of convergence in some of their economic (endogenous and/or instrumental) variables. This agreement will be reflected in a convex convergence function, denoted by C, which is included in the optimization process. It is important to stress that the convergence function differs from the countries' objective functions in a way that the latter contains only variables which belong to its own country whereas the convergence function contains variables of all the countries. Thus minimizing a cost function is something that can, in principle, be done by a country alone whereas minimizing the convergence function has to be done simultaneously.

The decision-making process of the policymakers concerning what strategy to follow, will depend on the following set:

$$\{(J_1(u),\dots,J_N(u),C(u))\mid u\in U\},\tag{1}$$

where we suppose that the strategy-space U is a convex set. The policymakers have to find a cooperative strategy which results in a point in (1) which is acceptable for them all. Now note that whenever two different strategies yield the same individual costs J_i, $i=1,\dots,N$, but different values for the convergence function, only the strategy yielding the lowest value for the convergence function is of interest to all policymakers. So, the set of relevant control strategies consists of:

$$\bar{U}=\{u\in U\mid\forall\bar{u}\in U\ (J_1(u),\dots,J_N(u))=(J_1(\bar{u}),\dots,J_N(\bar{u}))$$
$$\Rightarrow C(u)\leqslant C(\bar{u})\}.$$

This observation makes it possible to consider the decision problem from the following point of view. By varying the strategies over the whole set $\bar{U}$, we obtain the set of all possible objective outcomes in the $(J_1,\dots,J_N)$-plane. To each point in this set is attached a unique value for the convergence function. The problem for the decision makers is now to select cooperatively a point within this set that is acceptable for everyone. Now, as mentioned in the introduction we will assume that minimizing their own cost function is the primary interest of countries and that striving for convergence is of secondary interest. In that case the aspect of convergence acts as a dynamic constraint on joint social welfare. If we, furthermore, refrain from the possibility of side-payments and assume that the axiom of individual rationality holds (see e.g. Petit, 1990; de Zeeuw, 1984) [1], then countries will cooperatively minimize the joint convergence function as long as their individual costs will be lower than their non-cooperative costs. So, the set of possible objective outcomes will then be restricted on the one hand by the non-cooperative Nash threatpoint N^c, and on the other hand by the set of Pareto solutions P. We will call this set in the sequel the negotiation area (see Fig. 1 for an illustration in a two-player context).

Remark. To complicate matters not unnecessarily we do not address here the issue of information patterns and period of commitment (Basar and Olsder, 1982). For explaining our ideas it is sufficient (and most convenient) to use an open-loop information structure and binding commitments, which fixes the 'negotiation area'

[1] This axiom states that policymakers, if they behave rationally, will never accept an outcome for their individual object function which is worse than the one a policymaker can obtain by acting independently (which is represented by the non-cooperative outcome N^c).

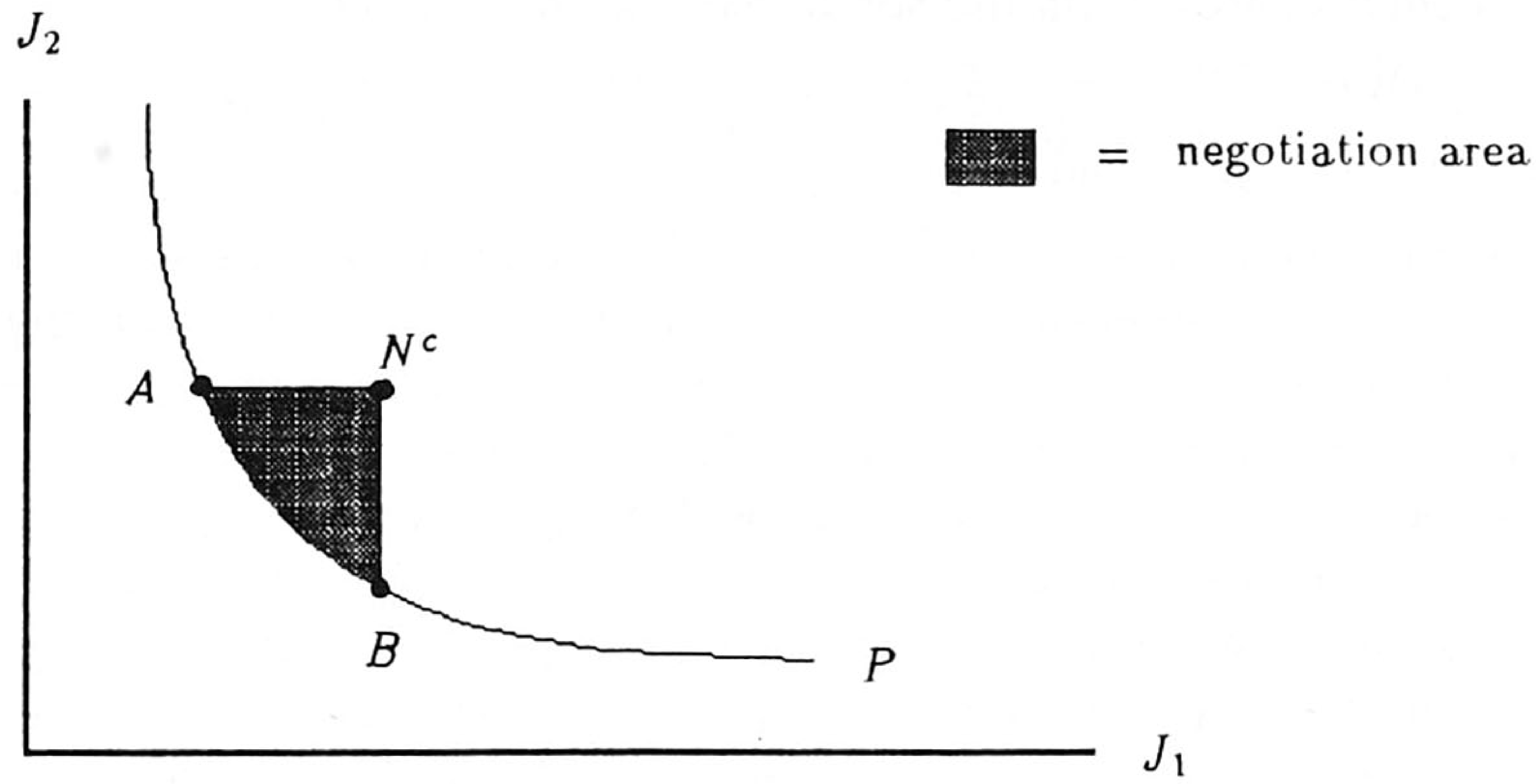

Fig. 1. Representation of the negotiation area in a two-player context.

throughout the entire planning period. In the closed-loop case, we have to take account of multiple (Nash threatpoint) equilibria and if we also take account of the possibility of renegotiation our 'negotiation area' would vary over time. In the case of multiple equilibria, in the literature various kind of arbitration schemes and algorithms have been proposed to discriminate between these equilibria. An overview of the literature can be found in de Zeeuw and van der Ploeg (1991) and Hughes Hallett (1991). We use the convergence criteria as an arbitration scheme to discriminate between the cooperative points. However, when introducing such an arbitration scheme points outside the negotiation set (i.e., the Pareto optimal solutions between A and B in Fig. 1) become interesting too. This is what we will investigate in the sequel.

The basic question is now of course, how we can determine all strategies $u \in \overline{U}$ which yield outcomes in the negotiation area. The solution we will present has a number of nice properties. First of all it attaches to a point in the negotiation area a unique control strategy that can be obtained by minimizing a strict convex combination of the individual object functions and the convergence function. Secondly, we will show that this control strategy is parametrized by N parameters and that this parametrization is a continuous function of its parameters.

The solution is motivated by our assumption that each policymaker is primarily interested in minimizing his own objective function in a cooperative setting and that convergence plays a minor role. We model this aspect by rewriting the convex combination of individual cost and convergence cost in a special way. Consider

$$\overline{\alpha}_1 J_1 + \ldots + \overline{\alpha}_N J_N + \overline{\alpha}_{N+1} C, \quad \text{with} \quad \sum_{i=1}^{N+1} \overline{\alpha}_i = 1.$$

This is equivalent with (in the non-trivial case $\bar{\alpha}_{N+1} \neq 1$)

$$(1-\lambda)(\alpha_1 J_1 + \ldots + \alpha_N J_N) + \lambda C,$$

$$\text{where} \quad \lambda = \bar{\alpha}_{N+1} \text{ and } \alpha_i = \bar{\alpha}_i/(1-\bar{\alpha}_{N+1}),$$

which has the nice property that $\sum_{i=1}^{N}\alpha_i = 1$. If we minimize this second convex combination of the individual object functions and the convergence function, then we have the property that $\lambda = 0$ resembles the case that countries completely ignore the convergence goal (and because $\sum_{i=1}^{N}\alpha_i = 1$ we find the Pareto optimal solutions), and that $\lambda = 1$ corresponds with the case that countries only pay attention to their mutual convergence interests. We will show (under some smoothness conditions) that the set of cooperative optimal strategies corresponding with these adapted object functions for each of the N countries, can be parametrized by the $N-1$ parameters $\alpha_1, \ldots, \alpha_{N-1}$ and λ, and that this parametrization is a continuous differentiable function of all these parameters. By varying these parameters, in particular λ, it is then possible to analyze the trade off between the costs individual countries have to pay and more convergence. First, we present a preliminary result. The next theorem shows that if one considers a certain convex combination of all object functionals J_i, $i = 1, \ldots, N$ and C, the optimal strategy minimizing this combination will be a continuous differentiable function of N out of $N+1$ parameters.

Theorem 2.1. Suppose U is a convex set, $J_i(u)$, $i = 1, \ldots, N$ and $C(u)$ are strictly convex functionals which are twice continuously differentiable in $u \in U$, and u^* is an interior point of U. Consider

$$J(u, \alpha_1, \ldots, \alpha_N, \lambda) := (1-\lambda)\left(\sum_{i=1}^{N}\alpha_i J_i(u)\right) + \lambda C(u)$$

for $u \in U$, $\lambda \in [0, 1]$ and $\alpha_i \in [0, 1]$ for $i = 1, \ldots, N$, with $\sum_{i=1}^{N}\alpha_i = 1$. Let

$$u^* := \arg\min_u J(u, \alpha_1, \ldots, \alpha_N, \lambda)$$

Then, for every $\lambda \in [0, 1]$ and $\alpha_i \in [0, 1]$ for $i = 1, \ldots, N$, with $\sum_{i=1}^{N}\alpha_i = 1$, u^* is uniquely determined as a function of the parameters $\alpha_1, \ldots, \alpha_{N-1}$, λ, i.e. $u^* = u^*(\alpha_1, \ldots, \alpha_{N-1}, \lambda)$. Moreover, this function u^* is a continuously differentiable function in $(\alpha_1, \ldots, \alpha_{N-1}, \lambda) \in [0, 1] \times \ldots \times [0, 1]$, with $\sum_{i=1}^{N-1}\alpha_i \leqslant 1$.

Proof. Let $\bar{\alpha} := (\bar{\alpha}_1, \ldots, \bar{\alpha}_N, \bar{\lambda}) \in [0, 1] \times \ldots \times [0, 1]$ be fixed numbers, with $\sum_{i=1}^{N} \bar{\alpha}_i = 1$. The strictly convex properties of $J_1, \ldots, J_N, C$ imply that the function $J(u)$ is strictly convex in $u \in U$. So, J has a unique global minimum on U at $u^*(\bar{\alpha})$ if and only if

$$F(\bar{\alpha}_1, \ldots, \bar{\alpha}N, \bar{\lambda}, u) := \frac{\partial J(u)}{\partial u} = (1-\bar{\lambda})\bar{\alpha}_1 \frac{\partial J_1(u)}{\partial u} + \ldots + (1-\bar{\lambda})\bar{\alpha}_N \frac{\partial J_N(u)}{\partial u} + \bar{\lambda}\frac{\partial C(u)}{\partial u} = 0$$

evaluated at the point $u = u^*(\bar{\alpha})$. Note that, since J is by assumption twice continuous differentiable, the functional F is continuous differentiable in $(\bar{\alpha}_1, \dots, \bar{\alpha}_{N-1}, \bar{\lambda}, u) \in [0, 1] \times \dots \times [0, 1] \times U$. Furthermore, since $J_1, \dots, J_N, C$ are strictly convex functionals in u, we have that

$$\forall \bar{\alpha} \in [0, 1] \times \dots \times [0, 1], \det \frac{\partial F(\bar{\alpha}, u^*(\bar{\alpha}))}{\partial u} \neq 0.$$

Applying the implicit function theorem yields then that there is a unique continuously differentiable function, say f, such that for all $\alpha := (\alpha_1, \dots, \alpha_N, \lambda) \in [0, 1] \times \dots \times [0, 1]$, $F(\alpha, f(\alpha)) = 0$, with $f(\alpha) = u^*(\bar{\alpha})$. So, $u^* := u^*(\alpha_1, \dots, \alpha_N, \lambda) := f(\alpha_1, \dots, \alpha_N, \lambda)$ is a continuous differentiable function in $\alpha \in [0, 1] \times \dots \times [0, 1]$. Using the fact that $\Sigma_{i=1}^N \alpha_i = 1$ gives $u^* = u^*(\alpha_1, \dots, \alpha_{N-1}, \lambda)$. □

Remark. In the sequel we will use the notation $(\alpha_1, \dots, \alpha_{N-1}) \in [0, 1] \times \dots \times [0, 1]$, but, by doing so, we implicitly assume that the α_i, $i = 1, \dots, N-1$ satisfy the constraint $\Sigma_{i=1}^{N-1} \alpha_i \leq 1$.

Using the previous result we show now that the set of control strategies defined in Theorem 2.1., parametrized by

$$\tilde{U} := \{u^*(\alpha_1, \dots, \alpha_{N-1}, \lambda) \mid (\alpha_1, \dots, \alpha_{N-1}, \lambda) \in [0, 1] \times \dots \times [0, 1]\}$$

has the advertised properties. Formally the result reads as follows:

Theorem 2.2. There exists a bijective mapping between the set of unique points

$$\{u^*(\alpha_1, \dots, \alpha_{N-1}, \lambda) \mid (\alpha_1, \dots, \alpha_{N-1}, \lambda) \in [0, 1] \times \dots \times [0, 1]\}$$

and the set

$$\{(J_1(u^*), \dots, J_N(u^*), C(u^*)) \mid (\alpha_1, \dots, \alpha_{N-1}, \lambda) \in [0, 1] \times \dots \times [0, 1]\}.$$

Furthermore $J_1(u^*), \dots, J_N(u^*)$, $C(u^*)$ *are continuous functions in* $(\alpha_1, \dots, \alpha_{N-1}, \lambda) \in [0, 1] \times \dots \times [0, 1]$.

Proof. Because $(1 - \lambda)(\Sigma_{i=1}^N \alpha_i) + \lambda = 1$, with $\lambda \in [0, 1]$ and $\alpha_i \in [0, 1]$, for $i = 1, \dots, N$, the unique solution u^* of $J(u)$ is a Pareto solution for the objective function $J(u)$ which represents a game with $N + 1$ players, where each player minimizes the objective function represented by J_i for player i, $(i = 1, \dots, N)$ and C for player $N + 1$. According to, e.g., de Zeeuw (1984) there is a bijective mapping between the Pareto solutions for $J_1, \dots, J_N$, C and the optimal solution for J. The set of Pareto solutions can be found by varying the parameters $(\alpha_1, \dots, \alpha_N, \lambda)$ between $[0, 1] \times \dots \times [0, 1]$ with $\Sigma_{i=1}^N \alpha_i = 1$.

Because u^* is a continuous function in $(\alpha_1, \ldots, \alpha_{N-1}, \lambda) \in [0, 1] \times \ldots \times [0, 1]$ it is straightforward that $J_1(u^*(\alpha_1, \ldots, \alpha_{N-1}, \lambda)), \ldots, J_N(u^*(\alpha_1, \ldots, \alpha_{N-1}, \lambda)), C(u^*(\alpha_1, \ldots, \alpha_{N-1}, \lambda))$ are continuous functions in $(\alpha_1, \ldots, \alpha_{N-1}, \lambda) \in [0, 1] \times \ldots \times [0, 1]$. □

Using the theorem, the set of control strategies $\tilde{U}$ gives us the following subset of (1):

$$\{(J_1(u^*), \ldots, J_N(u^*), C(u^*)) \mid u^* \in \tilde{U}\}. \tag{2}$$

To see that this reduction of the set in (1) still contains all the interesting points, we analyze the set in (2) in combination with J more specifically. We have that:

(i) the set in (2) contains the whole set of points $(J_1, \ldots, J_N)$ which belong to the Pareto optimal solutions P. To find these solutions we substitute $\lambda = 0$ in $\tilde{U}$ and fill in the resulting control strategies in (2).

(ii) the set in (2) contains the points where C is minimal. To find these points we substitute $\lambda = 1$ in $\tilde{U}$ and fill in the resulting strategies in (2).

Furthermore, from Theorem 2.2, we have that the set of points in (2) form a continuous surface in the $(J_1, \ldots, J_N, C)$-plane, which indicates that we have parametrized all the interesting points between (i) and (ii) as well. These points can be found by varying λ between 0 and 1.

From now on we will skip the u^* in the notation and describe the set in (2) as

$$\{(J_1, \ldots, J_N, C) \mid (\alpha_1, \ldots, \alpha_{N-1}, \lambda) \in [0, 1] \times \ldots \times [0, 1]\}. \tag{3}$$

We will now define some sets of interesting points. A projection of the set in (3), on the $(J_1, \ldots, J_N)$-plane is

$$S := \{(J_1, \ldots, J_N) \mid (\alpha_1, \ldots, \alpha_{N-1}, \lambda) \in [0,1] \times \ldots \times [0,1]\}.$$

The subset of S:

$$P := \{(J_1, \ldots, J_N) \mid (\alpha_1, \ldots, \alpha_{N-1}, 0) \in [0, 1] \times \ldots \times [0, 1]\}$$

represents the set of Pareto solutions. Iso-convergence lines, i.e. lines with the same degree of convergence, are defined as follows:

$$I_\gamma := \{(J_1, \ldots, J_N) \mid C(\alpha_1, \ldots, \alpha_{N-1}, \lambda) = \gamma,\ (J_1, \ldots, J_N) \in S,\ \gamma \in \mathbb{R}^+\}.$$

Note that a small value of γ corresponds with much convergence (and vice versa). The negotiation area is defined by:

$$\mathcal{N} := \{(J_1, \ldots, J_N) \mid J_1 \leqslant J_1^N, \ldots, J_N \leqslant J_N^N,\ (J_1, \ldots, J_N) \in S\}.$$

Using the axiom of individual rationality it is clear that policymakers will not agree to a certain degree of convergence, denoted by γ, if $I_\gamma \cap \mathcal{N} = \emptyset$. Moreover, the largest degree of convergence policymakers are willing to accept is given by

$$\gamma^* := \min_\gamma \{\gamma \mid I_\gamma \cap \mathcal{N}\} \neq \emptyset.$$

So, in general policymakers should set their degree of convergence with care because if this degree is set too ambitious policymakers are not willing to cooperate anymore. In the next theorem we will prove that, if $\mathscr{N} \subset S$, the point in (3), which after projection on the $(J_1, \ldots, J_N)$-plane is an element of the negotiation area and yields maximum degree of convergence coincides, after projection, with N^c. This (unique) point in (3) will in the sequel be denoted by $C^{\max}$.

Theorem 2.3. If $\mathscr{N} \subset S$ then the point in the negotiation area $\mathscr{N}$ for which convergence is maximal equals N^c.

Proof. According to Theorem 2.2, there is a bijective relationship between $\tilde{U}$ and the set of Pareto solutions which correspond to a game of $N+1$ players, where player i, $(i = 1, \ldots, N)$, minimizes an objective function represented by J_i, and player $N+1$ minimizes C. Suppose that $u \in \tilde{U}$ yields a point in S which lies in the negotiation area $\mathscr{N}$ which differs from N^c but for which convergence is maximal. Since u yields a point $(J_1(u), \ldots, J_N(u))$ in the negotiation area it satisfies the property that $J_i(u) \leqslant J_i^N$. Because the strategy u corresponds with a point in S that differs from N^c, there is an $i \in 1, \ldots, N$ for which $J_i(u) < J_i^N$. Making use of the angular shape of $\mathscr{N}$ and the assumption $\mathscr{N} \subset S$, it is now always possible to find a strategy $v \in \tilde{U}$ which corresponds with a point in S which lies on the boundary of $\mathscr{N}$ and for which $J_1(v) = J_1(u), \ldots, J_i(v) = J_i^N, \ldots, J_N(v) = J_N(u)$. Comparing these two points in the negotiation area, we have that of all the J_j-values $(j = 1, \ldots, N)$, only the J_i-value of the two points differ. Due to the fact that u and v are both Pareto optimal solutions it follows from the definition of Pareto optimality that the convergence value in both points differs as well. This observation implies that $C(v) < C(u)$. The fact that strategy v corresponds with a lower convergence value than u, violates the assumption that u corresponds with a point in $\mathscr{N}$, for which convergence is maximal. □

It is important to indicate here that the non-cooperative strategy which results in the point $N^c \in S$ in general differs from the cooperative strategy which results in the point $C^{\max}$. In general, if $\mathscr{N} \subset S$, the gains in convergence policymakers will receive by playing cooperatively will be at most $\gamma^* - C(u_{N^c})$, where u_{N^c} represents the non-cooperative strategy which yields N^c. If $\mathscr{N} \not\subset S$, the threatpoint is not guaranteed to fall within S. In that case our approach will not work, because we can not calculate all the points within the negotiation area. However, it is our experience that $\mathscr{N} \subset S$ will apply in most applications. [2]

[2] Counter examples can be constructed by introducing erratic convergence functions or specifying dynamics for which the Pareto solutions and the Nash threatpoint are situated very far from each other.

3. An illustrative example

We consider a theoretic example in a (discrete time) deterministic linear quadratic difference game framework with two players (countries). The dynamic behaviours of player 1 and player 2 are described by

$$y_1(t) = y_1(t-1) + u_1(t) + 0.3y_2(t-1), \quad y_1(0) = 1,$$
$$y_2(t) = y_2(t-1) + u_2(t) + 0.6y_1(t-1), \quad y_2(0) = 0,$$

where, for $i = 1, 2$, $y_i(t) \in \mathbb{R}$ is the target variable and $u_i(t) \in \mathbb{R}$ is the instrumental variable. From the interaction terms ($0.3y_2(t-1)$ for player 1 and $0.6y_1(t-1)$ for player 2) follows that each player faces a different dynamical structure. Player 2 is more influenced by player 1 than vice versa. Each player makes his plans for the future. We assume that each player has a planning period of 2 and chooses his desired paths for the future, as follows:

$$\text{desired paths}\begin{cases}\text{player 1:} & y_1^*(1) = 2, \quad y_1^*(2) = 3\\ \text{player 2:} & y_2^*(1) = 1.5, \quad y_2^*(2) = 3.\end{cases}$$

These desired paths reflect the policymakers' own wishes of the future and are obtained independently from each other. In this example the players have different preferences but, as can be seen from the ideal paths, both players are striving for convergence of their target variables in period 2. It is of course not necessary to choose desired paths which converge but by doing so we will be able to demonstrate the fact that Pareto optimal solutions do not coincide with convergence solutions, even if policymakers strive for convergence in their desired values. We represent the cost functions J_1, J_2 for every individual player by

$$J_1 = 0.5\left((y_1(1) - 2)^2 + (y_1(2) - 3)^2 + u_1(1)^2 + u_1(2)^2\right),$$
$$J_2 = 0.5\left((y_2(1) - 1.5)^2 + (y_2(2) - 3)^2 + u_2(1)^2 + u_2(2)^2\right).$$

Each player wants to play a strategy, during his planning period, which minimizes his costs. So the control problem for every individual player ($i = 1, 2$) is

$$\min_{u_i(1),\, u_i(2)} J_i.$$

Because the target variable (and indirectly the instrumental variable) of each player is directly related to those of the other player, the control problem of each player depends on the actions undertaken by the other player. This gives rise to various solution concepts. From the non-cooperative solutions we will just consider the open loop Nash solution, which we denote by N^c. The cooperative solutions are represented by the set of Pareto solutions which can be found by solving

$$\min_u \alpha J_1 + (1-\alpha) J_2$$

for $\alpha \in [0, 1]$, where $u := (u_1(1), u_1(2), u_2(1), u_2(2))$.

However, before playing the game both players want to be sure that there will be some degree of convergence of their target variables. In this example we assume that both players want to converge to the average of their target variables. We take as a measure for the degree of convergence the following convergence function:

$$C = \sum_{i=1}^{2} (y_i(1) - \bar{y}(1))^2 + 4(y_i(2) - \bar{y}(2))^2,$$

where $\bar{y}(t) := 0.5(y_1(t) + y_2(t))$ for $t = 1, 2$. So, both players agree that they want to minimize the variance of their target variables in each period. Moreover, minimizing the variance in period 2 is given more weight than minimizing the variance in period 1, which is represented by the weights of 1 in period 1 and 4 in period 2. These weights indicate that both players find it more important that there is convergence at the end of the planning period than during the planning period. [3]

Now, together, the players have to take a decision about the strategy they are going to follow. In order to choose a strategy they have to weigh out all possible strategies. So, ultimately they have to find a strategy which is 'optimal' in some sense. In the next subsection we demonstrate the solution concepts developed in Section 2 and analyze the space of interesting outcomes. After that we give a proposal to determine a feasible degree of convergence, γ, for both players.

3.1. Analysis of the possible outcomes

As stressed in Section 2, the decision about what strategy to follow, will depend upon the following set:

$$\{(J_1(u), J_2(u), C(u)) \mid u \in \mathbb{R}^4\}. \tag{4}$$

Because J_1, J_2, C are strictly convex functions which are twice differentiable in u, the set $\bar{U}$ can be found by solving the following problem:

Let $\alpha, \lambda \in [0, 1]$, and

$$J(u) := (1-\lambda)(\alpha J_1 + (1-\alpha) J_2) + \lambda C.$$

Find now for every α, $\lambda \in [0, 1]$:

$$u^* := \arg\min_u J(u).$$

[3] The above-mentioned convergence criterium is in our two-player case the same as minimizing the quadratic sum of the differences between the target values: $C = 0.5(y_1(1) - y_2(1))^2 + 2(y_1(2) - y_2(2))^2$. Furthermore, note, that for convenience's sake we did not include control strategies in the convergence function which would make the problem indefinite. If one wants to use our approach for practical purposes, one should realize that scaling of the object functions and convergence function might be necessary.

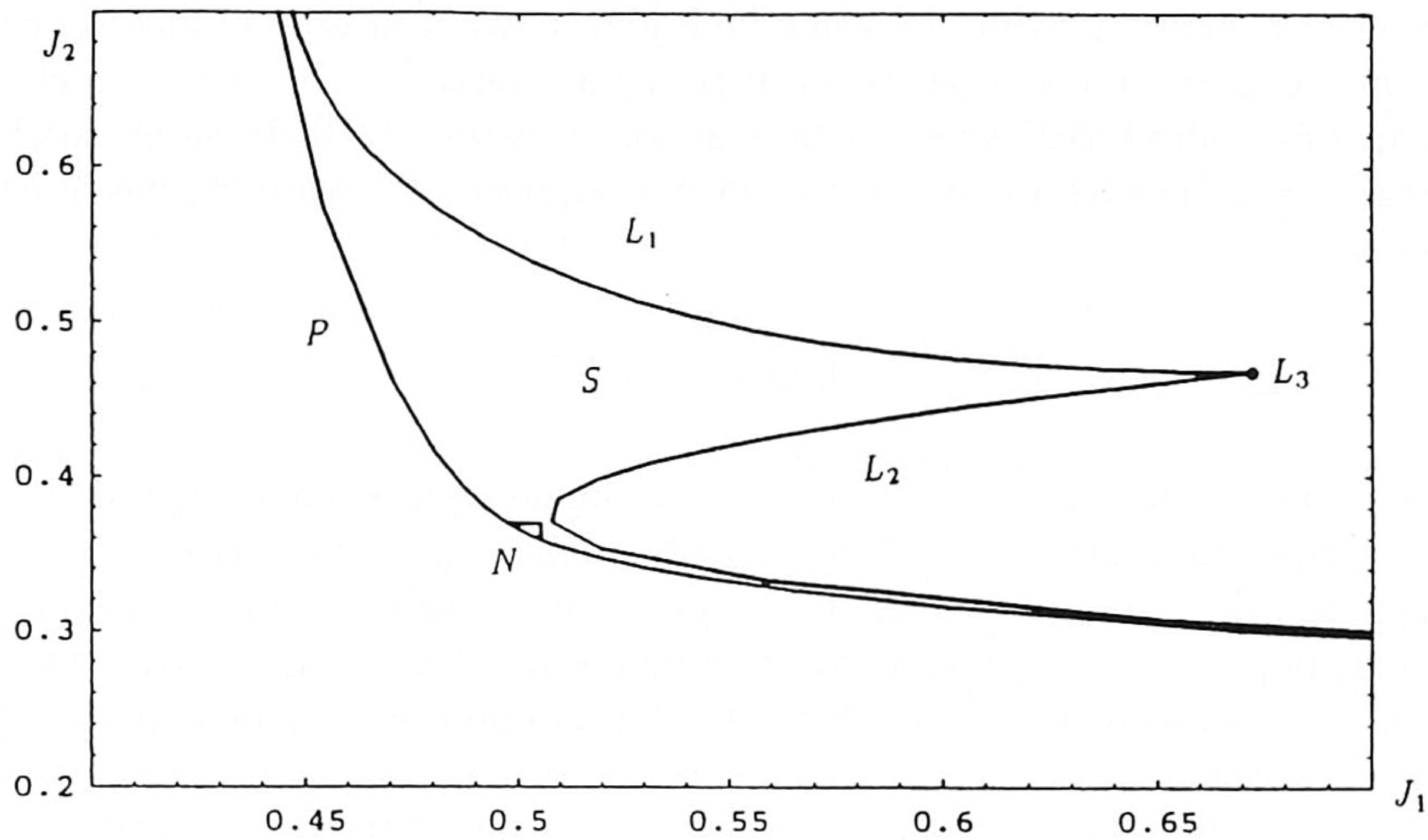

Fig. 2. The parametrized area S, the leftmost curve represents the Pareto solutions P, the small triangle on this curve represents the negotiation area $\mathcal{N}$.

From Section 2, the set of control strategies U is given by

$$\{u^*(\alpha, \lambda) \mid (\alpha, \lambda) \in [0, 1] \times [0, 1]\}.$$

Substituting these control strategies in (4) gives the following set (compare with (3)):

$$\{(J_1(\alpha, \lambda), J_2(\alpha, \lambda), C(\alpha, \lambda)) \mid (\alpha, \lambda) \in [0, 1] \times [0, 1]\}. \quad (5)$$

In the sequel we will analyze this set of points for the given example.

Remark. Computing the outcomes for $\lambda = 1$, $\alpha = 0$, $\alpha = 1$ gives some difficulties because in that case we have a singular system of equations. However, we are not particularly interested in those situations so we used in our calculations values which are close to these points.

A projection of the surface in (5), on the (J_1, J_2)-plane is drawn in Fig. 2. This set of points is denoted by S, like in Section 2.

$$S = \{(J_1(\alpha, \lambda), J_2(\alpha, \lambda)) \mid (\alpha, \lambda) \in [0, 1] \times [0, 1]\}.$$

The black lines in Fig. 2 represent the edges of S. One of these edges is the set of Pareto solutions, which is given by the left black line. It is obtained by computing for various α:

$$P = \{(J_1(\alpha, 0), J_2(\alpha, 0)) \mid \alpha \in [0, 1]\}.$$

Points on the upper part of the Pareto line correspond with a high value of α and points on the lower part to a low value of α. The edge L_1 in Fig. 2 is obtained

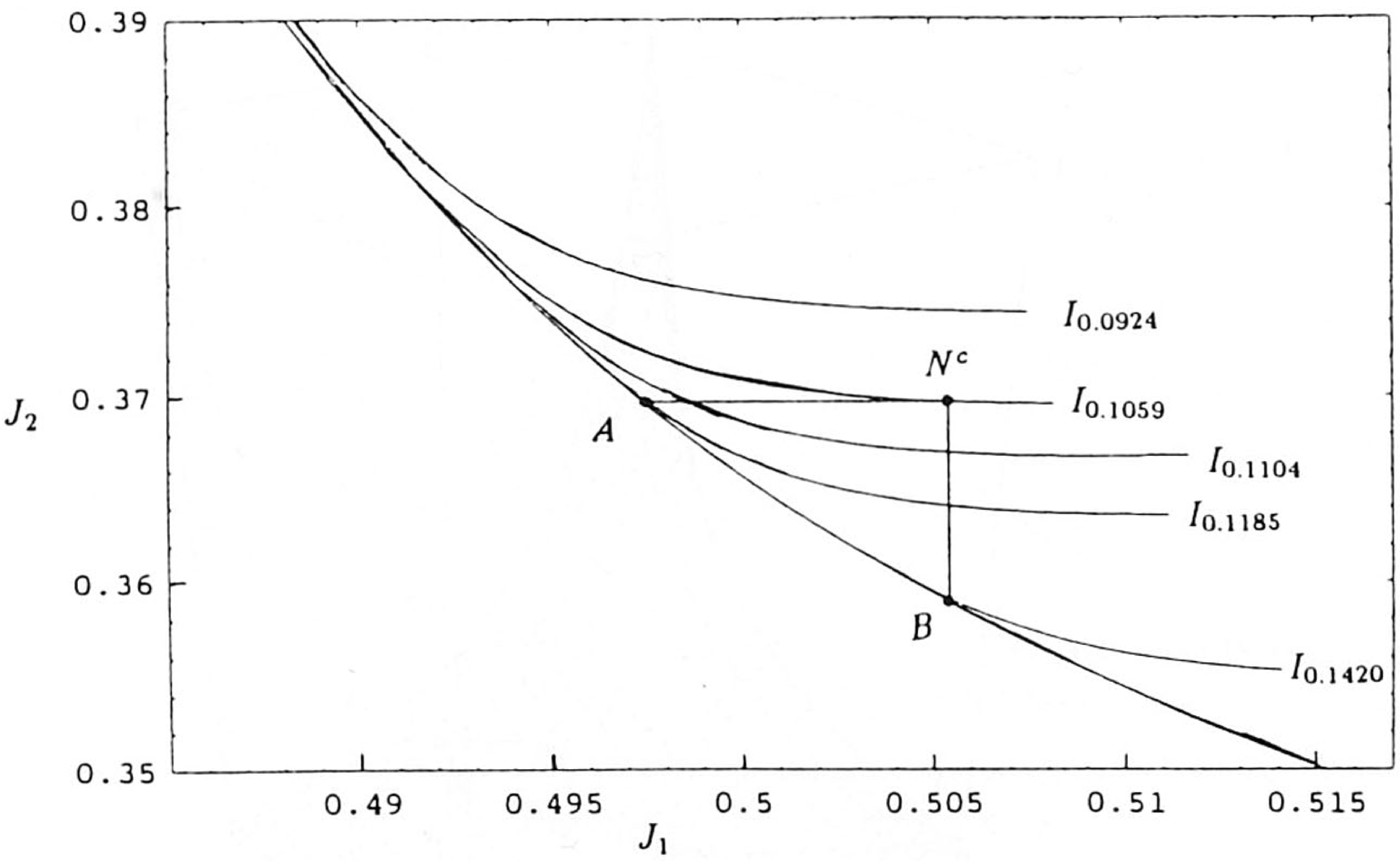

Fig. 3. Zooming in on Fig. 2 around the negotiation area. Iso-convergence lines are drawn.

by computing for various $\lambda \in [0, 1]$: $(J_1(1, \lambda), J_2(1, \lambda))$ and the edge L_2 by computing for various $\lambda \in [0, 1]$: $(J_1(0, \lambda), J_2(0, \lambda))$. The edge in the figure which corresponds to $(J_1(\alpha, 1), J_2(\alpha, 1))$ for $\alpha \in [0, 1]$ is reduced to one point in the figure. We denoted this point by L_3. The small triangle on the Pareto line denotes the negotiation area $\mathscr{N}$ as defined in Section 2. Note that the negotiation area $\mathscr{N}$ is completely covered by S. Zooming in on Fig. 2 around the negotiation area. Iso-convergence lines are drawn. Zooming in on Fig. 2, around the negotiation area $\mathscr{N}$, gives us Fig. 3. Specific information about the points A, B, N^c, and $C^{\max}$ are given in Table 1. In Fig. 3 we draw some iso-convergence lines, as defined in Section 2. In the figure for each iso-convergence line the corresponding convergence value is given. The degree of convergence on the Pareto line increases from B to A. As proven in Section 2 and visible in the figure, the point with the largest degree of convergence in the negotiation area $C^{\max}$ lies on the edge of the negotiation area and coincides exactly with the N^c point which

Table 1
Characteristics of some interesting points

	J_1	J_2	C	α	λ
Cooperation					
C^m	0.505	0.370	0.1059	0.080	0.270
A	0.497	0.370	0.1258	0.625	0
B	0.505	0.359	0.1483	0.523	0
Non-cooperation					
N^c	0.505	0.370	0.1365	–	–

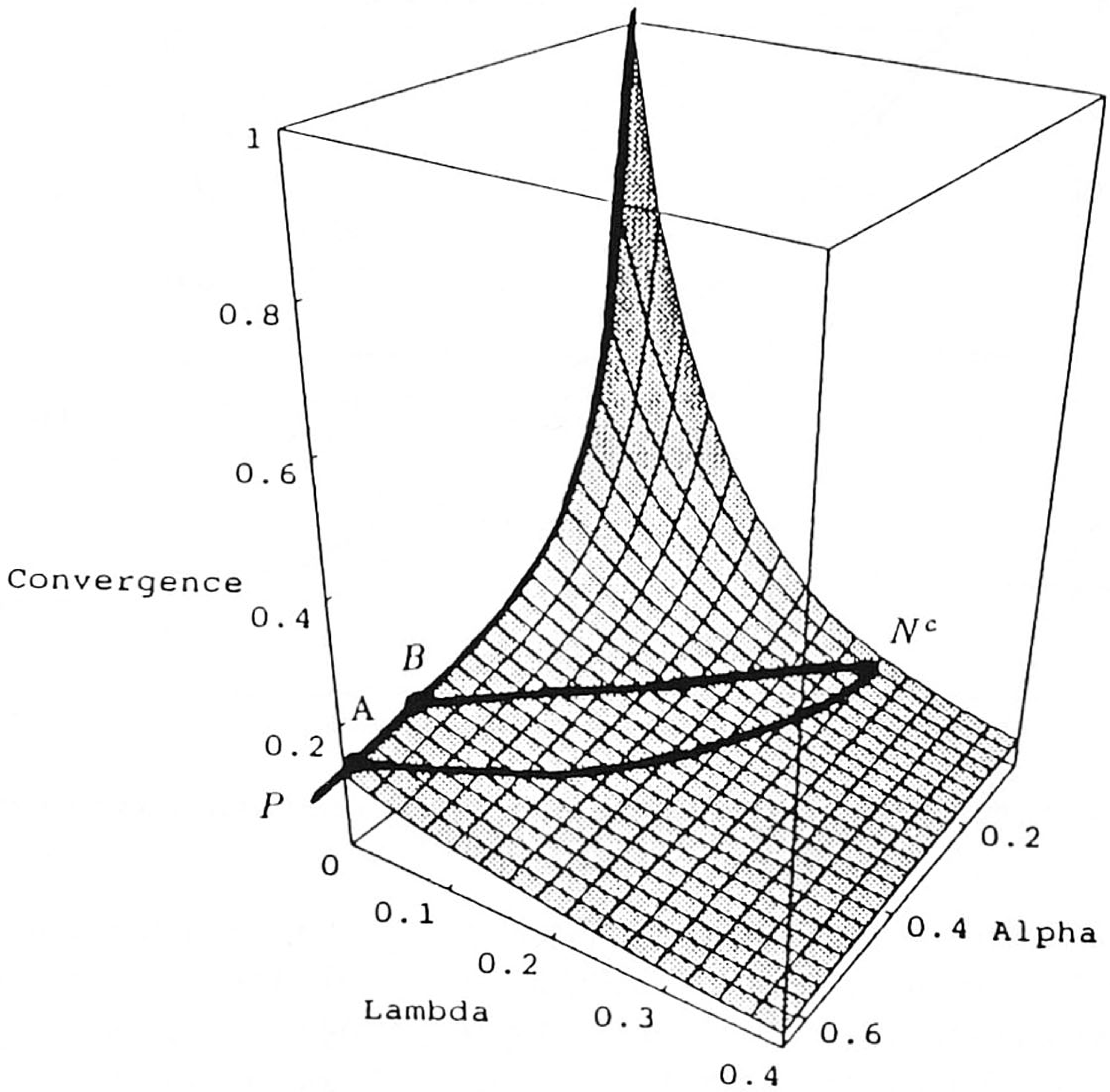

Fig. 4. A three-dimensional plot, where for each ($0.05 \leqslant \alpha \leqslant 0.65$) and ($0 \leqslant \lambda \leqslant 0.4$) the corresponding convergence outcome is plotted. The curve on the back ($\lambda = 0$) represents a subset of the Pareto solutions P. The interior of the curve drawn on the surface represents the negotiation area $\mathcal{N}$.

belongs to the iso-convergence line $I_{0.1059}$. So, the γ^*, as defined in Section 2, equals 0.1059. Furthermore, the gains in convergence are 0.306.

To get an idea of the degree of convergence of some points in (3) we plotted Fig. 4. This figure shows a three-dimensional plot of the following surface:

$$\{C(\alpha, \lambda) \mid (\alpha, \lambda) \in [0.05, 0.65] \times [0, 0.4]\}.$$

Looking at this figure we get an indication of which values of α, λ belong to the negotiation area $\mathcal{N}$, which is drawn on the surface in Fig. 4. The corresponding α, λ-values for A, B, and $C^{\max}$ are given in Table 1. As one notes, the convergence value declines (so converges increases) as λ increases.

3.2. An approach to determine a reliable degree of convergence

In this section we present an algorithm to determine a feasible degree of γ. We have already noted that, without any other agreements between the players, a degree of convergence which has no corresponding outcome in the negotiation area is unlikely to happen. The question remains, however, which degree of

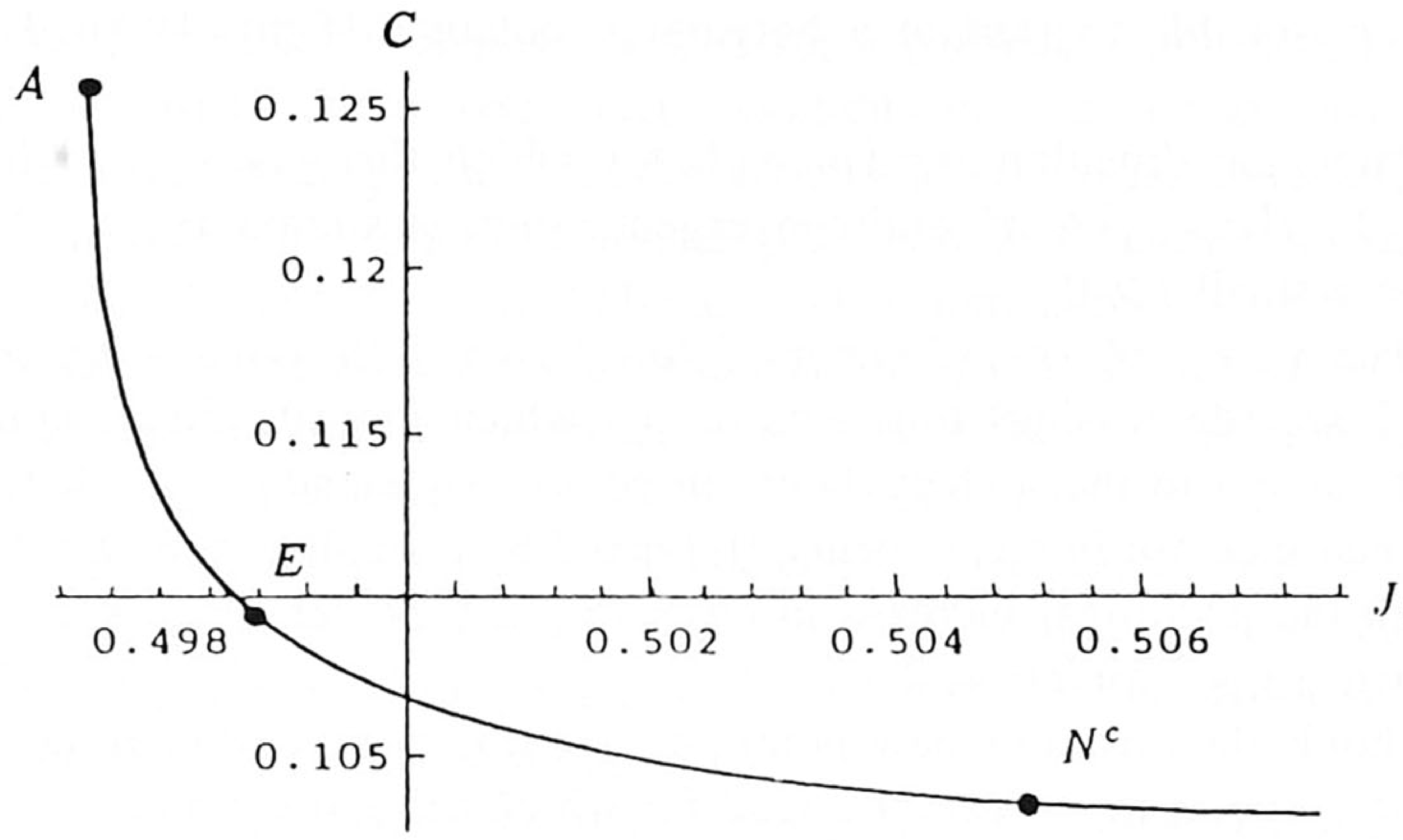

Fig. 5. For the edge of the negotiation area, from A to N^c, the convergence value is plotted. Point E is the point where the derivative of the tangent of the curve is -1.

convergence within this negotiation area ultimately will be selected by the players. In fact without making any further assumptions on the negotiation process, every point in the negotiation area is possible. One way to come to a unique point within the negotiation area is by axiomatizing the negotiation game. We shall not elaborate this subject here, since for the moment we are more interested in qualitative rather than quantitative statements. All we will do is sketch how a feasible degree of γ can be determined, using some heuristic arguments. First we will give an example and then we will present two algorithms which illustrate the approach in general.

In Fig. 5 the convergence value is plotted against the costs of player 1, along the line A to N^c, where the costs of player 2 remain constant. Starting at point A and moving towards N^c, the convergence value declines rapidly. This continues until the point where $(J_1, C) = (0.4987, 0.1098)$. After that point the derivative of the slope of the curve gets larger than -1. In Fig. 5 we denoted this point by E. From that point on, towards N^c, the costs increase more rapidly than the degree of convergence. If player 1 has to choose an outcome on the line in Fig. 5, he will start in point A where his costs are minimal. From there onward, if player 1 wants to increase convergence, he will have to weigh out costs against convergence. For instance, if player 1 starts in A and moves towards N^c and accepts only points where the slope $\partial C/\partial J_1 \leqslant -1$, the result will be the outcome E.

The general idea expressed in the above example is that players accept an increase in convergence only if the corresponding costs stay within a prespecified region. So, a sketch of a numerical approach for determining a feasible degree of γ would be the following:

(1) Because minimizing their own welfare loss is of primary interest of the players we start from a point $(\bar{J}_1, \bar{J}_2)$ on the Pareto line between A, B. It

seems reasonable to start at a bargaining outcome (Petit, 1990; de Zeeuw, 1984).

(2) Determine the direction $v = (v_1, v_2)$, for which there is a $t > 0$ for which $(\bar{J}_1, \bar{J}_2) + t(v_1, v_2) \in \mathcal{N}$, and convergence increases maximal.

(3) Choose a small $t > 0$.

(4) Calculate $\chi_c = -\partial C/\partial v$. Check if $\partial J_i/\partial v < \chi_i(\chi_c)$, for $i = 1, 2$ where $\chi_i(\chi_c)$, $i = 1, 2$ are (decreasing) functions of χ_c which indicate the weight players want to assign to the tradeoff between convergence and costs. That is, if the additional increase in convergence (reflected by a smaller value for C) equals χ_c then the additional increase in costs for each player separately should be less than $\chi_i(\chi_c)$ for $i = 1, 2$.

(5) If (4) holds then use this new point as a starting point and start again in (2). Stop, if no point in $\mathcal{N}$ can be found for which (2) and (4) hold.

A drawback of this approach is that it is timeconsuming, even for small models. The reason is that the functions J_1, J_2, C are parametrized in α and λ and therefore calculating 'simple looking' expressions like $\partial C/\partial v$ or $\partial J_i/\partial v$ for $i = 1, 2$, or finding a direction v in step (2), take a lot of time.

A good alternative which is strongly related with the previous algorithm, but is easier to compute, is the following algorithm:

(1) Start in some feasible point between A, B. With this point there corresponds an uniquely determined α.

(2) Fix α.

(3) Increase λ from 0 to 1 by using a stepsize of, for instance, 0.01. Check if the point stays in the negotiation area $\mathcal{N}$.

(4) Check for every λ whether $-\partial C/\partial\lambda > \partial J_1/\partial\lambda$ and $-\partial C/\partial\lambda > \partial J_2/\partial\lambda$.

(5) Stop if no λ can be found for which (3) and (4) holds.

The conditions in step (4) of the algorithm can be compared with the conditions in step (4) of the previous algorithm. These conditions state that if for each player separately costs rise less than convergence falls when λ increases by one unit both players are willing to accept more convergence (as long as they stay within the negotiation area). Note that for our convenience we took $\chi_c = \chi_1(\chi_c) = \chi_2(\chi_c)$.

4. Conclusions

In this paper we presented a theoretical approach how to deal with the issue of convergence between E.C. countries. Based on the assumption that the primary interest of the countries is minimizing their own individual welfare loss, we considered the question how cooperative strategies yielding maximal convergence can be determined. We showed that for a large class of problems, i.e. problems where the individual costfunctions and convergence function are twice differentiable and convex, a parametrization for a large set of cooperative strategies can be determined. Using this approach a number of interesting questions can be considered.

For instance whether it is possible that for a particular time horizon the E.C. countries can satisfy the convergence conditions in such a way that for every country the corresponding costs are acceptable, and how these costs differ among countries. In Section 3 we showed in a simple theoretical example how to analyze such questions. The next step should be to use the same approach on more realistic dynamic (macro)econometric country models, or just on a part of these models where the interaction between countries is most essential, e.g. the monetary sector. In dealing with that problem countries should realize that

(1) it must be clear where one should converge to (van der Ploeg, 1990). Should they converge to the lowest, the highest or the average rates of their target/ instrumental variables? In our approach this means that countries should agree on a common convergence function C.
(2) the preferences of countries should be finetuned. It is clear that if preferences differ strongly among countries, convergence will be a very tough issue. In the dynamical game approach this can be analyzed with the desired paths and choice of weights for the target/ instrumental variables. The theoretical example was chosen in such a way that in the last period of the planning horizon the countries, at least, strive for convergence, which was implemented by choosing equal values for the corresponding desired paths.
(3) the time-horizon, necessary for reaching the convergence conditions within a limited period, plays a crucial role too. This aspect is strongly related to the determination of the degree of convergence. We expect that for a short planning period the costs for convergence can be very costly and this may ultimately result in non-cooperative behaviour of some countries. This subject remains, however, a topic for future research.
(4) costs for convergence differ among countries. The example in the paper gives a way how to determine these costs for any given degree of convergence. In general these differences will depend on the economical structures of the participating countries. The theoretical example gives already an indication that these costs could be much higher for countries which have less influence in the Community.

The approach designed here for analysing convergence can be used for many other problems as well. If players in a dynamic game have common objectives, apart from their usual costfunctions, the approach can be used as long as we take twice differentiable convex functions. If we stay in a multicountry setting, common objectives appear in e.g. environmental issues and trade issues.

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