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SOCIAL INTERACTION AND THE MINORITY-MAJORITY EARNINGS INEQUALITY: WHY BEING A MINORITY HURTS BUT BEING A BIG MINORITY HURTS MORE

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Social Interaction and the Minority-Majority Earnings Inequality: Why Being a Minority Hurts But Being a Big Minority Hurts More*

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Abstract

Empirical findings that minorities typically attain lower economic status than majorities and that relatively larger minorities perform worse than smaller ones pose a challenge to economics. To explain this scale puzzle, I model an economy where the society is bifurcated into two social groups that differ in their size and sociocultural characteristics – the minority and the majority – and individuals form their human capital through social interaction in social networks. I establish that the different social group sizes and the sociocultural differences suffice to generate earnings inequality between the two social groups whenever sociocultural differences hinder social interaction between majority and minority individuals and there are networks effects in human capital acquisition. If there are, in addition, asymmetric information in the labor market and a choice of heterogeneous skills in the economy, minority and majority individuals tend to acquire different (*combinations of*) *skills and the predicted patterns of income inequality comply with the scale puzzle under fairly general conditions. Moreover, in this study I offer an answer why some minorities do better than majorities, why minority individuals tend to spend more time socializing in families than in schools, and why integration may harm minorities.*

Keywords: Human Capital, Income Inequality, Labor Market, Minority, Network Effects, Social Interaction.

JEL Codes: J15, J24, J70, O15.

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1. Introduction

Inequalities in socioeconomic conditions of Black and White Americans, Romany and White Europeans, and other minorities and majorities around the world are persistent and central features of the worldly history.[1] While there are many dimensions of socioeconomic inequality, labor income, as one of the major measures and determinants of socioeconomic inequality, is the principal focus of this study.[2] Two robust empirical findings about the distribution of income across social groups[3] pose a challenge to economic theory. On the one hand, minorities typically earn less income per capita than majorities. On the other hand, minority-majority income disparities vary directly with the relative size of minority populations across habitats. The puzzling feature of this empirical regularity is that while being a member of the smaller social group in a habitat – the minority – is disadvantageous in income terms, given the minority status, minority agents are relatively better off in habitats where they are relatively less plentiful.

This "scale puzzle" has been a major topic of sociologists[4], who have offered several explanations thereof, referring to the social embeddednes of individuals with a special focus on ethnic discrimination. In their studies, however, the economic nature of socioeconomic inequality and human beings per se has been sparsely addressed. Economics of income inequality, on the other hand, has always been at the center of economic research. Economists, habitually adopting the canon of *Homo economicus*, have contributed significantly to our understanding of how economic forces aggravate or alleviate inequality among economic actors.

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[1] Throughout the paper, minority is understood to be a particular racial, ethnic, religious or national group of individuals who share socio-cultural characteristics such as culture, religion, language, history, beliefs, customs, values, and morals that make them distinct from the rest of the population – the majority – in their habitat. While there may be regions where the minority outnumbers the majority, that minorities constitute smaller proportion of population than majorities is a part of most definitions of the minority and I adopt this premise here as well. Although this study explicitly studies the mentioned social groups, it can be straightforwardly extended to nominal social groups defined by gender, language, or the like if they are sufficiently distinct from the rest of the society in terms of their socio-cultural characteristics. The study does not deal with social groups formed on the basis of occupation, wealth, or other ordinal characteristics.

[2] Acknowledging that the dispute over proper measures of socio-economic attainment is perennial, the most promoted measures being individual income, educational attainment, life expectancy, health status, occupational status, or any combination thereof, I focus in this paper on individual labor income and earnings inequality. Nevertheless, some predictions of the theory for educational attainment are discussed as well in the end of the paper.

[3] Social group is understood to be a minority or a majority.

[4] Both the sociological and economic literatures are discussed below.

The embeddedness of individuals in social interaction with its consequences for income distribution has received a much smaller portion of their attention until recently, however.

That economic analysis can improve our understanding of labor income inequality across social groups is natural to expect. After all, the major determinants of this inequality certainly include wages, productivities, and resource allocations. But why does income inequality depend on social group membership and social group sizes in particular? Recalling the definition of minority, at least two fundamental features distinguish minorities from majorities. First, there are sociocultural characteristics that make the two social groups distinct. Behaviorally, social groups may abide by different morals and taboos, speak different languages, have different understanding of what proper behavior is, or have different customs in interpersonal communication. In addition, there often are some observable characteristics of social group membership such as skin color, ethnic names and surnames, or accent. Second, it is indeed the different size of social groups that distinguishes them, as minorities typically constitute smaller proportion of population than majorities.[5]

Can the variation in sizes of social groups coupled with the sociocultural differences explain the patterns of inequality? While the latter by itself cannot, as the differences are in principle completely symmetric[6], the group size differences introduce aggregate asymmetry that may induce income inequality on the individual level. In this paper I argue that it is social interaction in human capital acquisition that links this aggregate asymmetry with income inequality between minority and majority individuals. In particular, I establish that different social group sizes and the sociocultural differences suffice to produce income inequality between the two social groups whenever sociocultural differences hinder social interaction between majority and minority individuals and there are networks effects[7] in human capital acquisition. If there is, in addition, asymmetric information in the labor market and a choice of heterogeneous skills in the economy,

[5] To wit, according to Frisbie and Neidert (1977), the average proportion of Mexican Americans in the population of southwestern Standard Metropolitan Areas of the U.S. was 14.7% (σ=10.3), that of Blacks was 8.1% (σ=6.8).

[6] This symmetry is assumed throughout the paper. Assuming a priori asymmetry of sociocultural differences would be ad hoc and racially prejudiced.

[7] Network effects arise whenever benefits from a good or service, here the service of social network in skill acquisition process, increase in the number of individuals already owning that good or using that service. One consequence of a network effect is that the use of a network service by one individual indirectly benefits others who use it. This side effect in a transaction is known as network externality.

individuals tend to acquire different (combinations of) skills and the predicted patterns of income inequality comply with the scale puzzle under fairly general conditions.

The argument proceeds as follows. First, I discuss empirical evidence about the patterns of income inequality and establish that it validates the existence of the scale puzzle. Second, I discuss alternative theoretical explanations of the scale puzzle found in both sociological and economic literature. Third, I state and explain the main assumptions on which the argument is based. Fourth, I present a formal model and establish its main predictions. Finally, I discuss the relevance of the presented theory and conclude.

2. The Scale Puzzle

A glimpse at the contemporary United States reveals a pattern of income inequality across the major U.S. ethnic minorities that is strikingly consistent with the scale puzzle (c.f. Table 1). In particular, we observe that larger minorities have relatively lower income than smaller ones and that the two largest U.S. minorities are on average significantly poorer than the majority. To wit, the largest minority, the Black minority, attains only about 64% of the income of the Non-Hispanic White majority, the Hispanic minority is between the two other minorities both in terms of relative size and income at about 74% of majority income, and the smallest minority, the Asian and Pacific Islander minority, outperforms the other two minorities. Interestingly, the smallest minority also outperforms the majority, suggesting that the benefits from minority's smaller size can outweigh the detriments of being a minority.

Table 1

Race and Hispanic Origin of Householder	Number of Households (Percent in Total)	Median Income in Dollars (Percent of Non-Hispanic White)
Non-Hispanic White	73.94	100.00
Black	12.18	63.64
Hispanic	9.61	72.49
Asian and Pacific Islander	3.72	115.83

Source: U.S. Census Bureau, Current Population Survey, 2002 and 2001 Annual Demographic Supplements and author's computations.

2.1. Empirical Literature

That minorities typically earn less income than majorities and that minority-majority income disparities vary directly with the relative size of the minority across habitats – the scale puzzle – has been corroborated in a sizeable empirical literature (c.f. Table 2). Blalock (1956) studied the relationship between minority-majority income differentials and percent Black in 88 non-Southern and Southern Standard Metropolitan Areas (SMAs). Using various control variables and data samples, he finds the correlations to range between nonsignificant .19 and highly significant .70, with most correlations in the upper significant part of the range. In another study, Blalock (1957) investigated the same relationship on the random sample of 150 Southern U.S. counties and found the correlation to be .46. In both studies Blalock provides evidence for non-linearity of the relationship, suggesting that very small and very large minorities achieve better economic status than what would the linear relationship predict. Using a different measure of income inequality, Heer (1959) finds a negative correlation -.71 between the ratio of Black median income to White median income and percent Black in 43 Southern U.S. SMAs, thereby corroborating Blalock's findings. Studying 878 non-metropolitan areas of Southern U.S., Brown and Fuguitt (1972) estimate the correlations between measures of racial income disparity and percent Black to range between .16 and .41. Their results are fully supported by an extensive study by Frisbie and Neidert (1977) in 40 southwestern U.S. SMAs, who find the correlations between income disparities and minority labor market percentages to range between .22 to .48 for the Mexican minority and .31 to .43 for the Black minority. Both studies report overrepresentation of the majority in all higher income categories and show that Black income decreases and White income increases with increases in percent Black. Hirschman and Wong (1984) estimate income gaps between minority and majority individuals for Black, Hispanic, Chinese, Japanese, and Filipino men in 1960, 1970, and 1976. They find the gaps to be significant in most cases. The only exception is the Japanese, a relatively small minority, who actually outperformed the Whites in 1976. Suggestive is the pattern that the larger minorities, Black and Hispanic, lag substantially more than the smaller ones, Chinese, Japanese, and Filipino. Finally, Tienda and Lii (1987) test the scale puzzle using microdata samples of Black, Hispanic, Asian and White men from 1980 to confirm that minorities lag behind majorities in income terms and that minority labor market percentages favor the majority while disadvantaging the minorities themselves, thereby validating the scale puzzle.

Table 2

Study	Data	Main Findings	Notes
Blalock (1956)	88 non-Southern and Southern Standard Metropolitan Areas (SMAs), (1950)	Finds a positive correlation .42 between percent Black population and Black-White income differentials. Controlling for subregion, white median income, size of SMA, and percent of employed males in manufacturing, the correlation was reduced to nonsignificant .19. However, for southern SMAs the correlation was .50 and increased to .70 when the same controls were included. When both Southern and non-Southern SMAs were included, the correlation was .67 irrespective of the controls.	Suggests that the marginal Black relative losses due to percent increase are decreasing (non-linearity) in percent Black. Suspects a threshold at about 10% above which correlations significant.
Blalock (1957)	Random sample of 150 Southern US counties having at least 250 non-white households, (1950)	Finds a positive correlation between percent Black population and income (.46) and educational (.68) differentials. The findings were robust with respect to the same controls as in Blalcok (1956)	Finds that in counties with low Black percentage (but not in those with high Black percentage where all results were non-significant) the income and educational gaps are disproportionaly smaller, thereby supporting the non-linearity hypothesis for the low end of the density continuum. No relationship is observed for occupational differentials.
Heer (1959)	43 Southern Standard Metropolitan Areas, Census, (1950)	Finds negative correlation of -.71 between percent Black and the ratio of Black median income to White median income.	
Brown and Fuguitt (1972)	878 non-metropolitan areas, Southern US, PH-5 census, (1960)	Report overrepresentation of the majority in all higher income groups (difference scores range between 31% and 45% when cumulative distributions are compared). Find that the association between percent Black and measures of racial income disparity is positive and ranges between .16 and .41. Moreover, they show that Black income decreases and White income increases with increases in percent Black. White component correlations range between .12 and .21. Black counterparts range between -.09 and -.31.	Positive relationship is observed for occupational differentials.
Frisbie and Neidert (1977)	40 Standard Metropolitan Areas in southwestern U.S. U.S. Bureau of the Census, (1971, 1972)	Report overrepresentation of the majority in all higher income groups (difference scores range between 7% and 20% (Mexican) between 18% and 31% (Black) when cumulative distributions are compared). Finds correlations of .22 to .48 between percent Mexican and the Mexican-Anglo income differential for different income groups. The corresponding values for the Blacks range between 0.31 and 0.43. Controls included: % labor force in manufacturing, % labor force in services, Black median education, 1960-1970 % change in Black population, % Mexican. Uncontrolled correlations similar. Confirms that the majority income is positively correlated with percent minority (correlations between .06 and .21) and that minority income is negatively correlated with percent minority (correlations between -.48 and -.22).	Studies two minorities and a majority in a habitat.
Hirschman and Wong (1984)	Census of the Population, (1960, 1970) and the Survey of Income and Education (1976)	Finds substantial socioeconomic gaps between White majority and Black (1960: -$5700 of yearly income, 1970: -$6000, 1976: -$4400), Hispanic (1960: -$3500, 1970: -$3700, 1976: -$3400), Chinese (1960: -$1000, 1970: -$800, 1976: -$1200), and Filipino (1960: -$3200, 1970: -$3500, 1976: $0) men. Japanese men outperform White men in 1976 (1960: -$900, 1970: $0, 1976: +$1700). Nonsignificant values reported as zeros; all values 1975 $.	Suggestive is the pattern that larger minorities (Black and Hispanic) do worse than smaller ones (Japanese, Chinese, and Filipino). Occupational attainments: strong negative effects of percent Minority for Black, Hispanic, and Filipino, non-significant for Japanese, and positive for Chinese.
Tienda and Lii (1987)	5% A File, men, Public Use Microdata Samples, Census (1980)	Confirms that minorities have lower income than majorities and that minorities lose from increases in their percentages, while the white majority gains. Blacks, Hispanics, and Asians lose 0.7%, 0.2%, and $0.2% of their annual income, respectively, and the majority gains between 0.0-0.5% with every percentage increase of the respective minority density. These results are net of some observable individual characteristics and working time measures.	Minority losses from their percentages most pronounced for educated minority people.

2.2. Theoretical Literature

The scale puzzle has attracted a lot of attention among theoreticians.[8] Two early works, Williams (1947) and Allport (1954), followed by more elaborated studies by Blalock (1967), Noel (1968), and Reich (1971) argue that when social groups are competing over scarce resources, the hostility of superordinate majorities, who control job opportunities, against minorities is increasing in the relative size of the minority as the former fear the greater job competition caused by the larger number of the latter. This argument was extended by Bonacich (1972, 1976), who argues that ethnic antagonism increases in the size of the minority and results in split labor markets that protect the rents appropriated by majorities.

Another strand of literature, represented by Glenn (1964), Spilerman and Miller (1977), and Semyonov et al. (1984), advocates that, given the discrimination-based asymmetry of job status distribution between minority and majority, an increase in the population of subordinate minority workers increases the supply of low-skill labor predominantly supplied by the minority, thereby crowding out majority workers from the low status jobs into more lucrative ones and, as a result, widening the socio-economic gap. That discrimination keeping minority workers subordinate in low status jobs is increasingly worthwhile to employers as the relative size of minority population in a given market increases was advocated by Thurow (1969).

While these sociological studies provide in-depth analyses of associations between social superstructure and socioeconomic differentiation across social groups, they remain superficial about the consequences of changes in ethnic composition and social structure on the economics of the minority-majority inequality, including the determination of relative wages, employment opportunities, and individual incentives. In addition, both strands of literature rely on the assumption that minorities are extensively subordinate relative to majorities with respect to job statuses, economic opportunities, and abilities to maintain or change the subordination. Although such asymmetries may be a plausible premise in a short run analysis, it is less so in the long run perspective, unless we establish that these asymmetries are everlastingly perpetuated. Moreover, the existence of minorities that outperform majorities is either unexplained by these theories or contradicts the job status asymmetry assumption.

[8] I dismiss all theories based on any kind of genetic predispositions as empirically unfounded and invidious.

Economists have addressed the issue of ethnic inequality from two closely related perspectives. First, they study how discrimination drives a wedge between the labor market rewards of ethnic groups with the same levels of human capital and other production factors. Second, they aim to explain the quantitative differences in individual human capital across social groups. From the first perspective, following Becker (1957) and materializing in Welch (1967) and Arrow (1972a, 1972b, 1973), discrimination-driven economic inequality is viewed as a demand driven phenomenon arising due to the so-called "taste for discrimination" of actors on the labor market. There are, however, at least three arguments that undermine the "taste for discrimination" premise. First, this premise, although it does not contradict the rational choice theory, is ad hoc, with respect to any social inequality argument. Second, it contradicts the economic theory. If discrimination is purely race based, the discriminating employers either make losses (if they pay more than the marginal product) or they are competed away by nondiscriminating employers (if they pay less) in competitive labor market. Third, because those employers that discriminate more are economically less successful than the less discriminating ones[9], the Darwinian evolution would eliminate them and in the limit spare only the nondiscriminating ones. Moreover, the assumption that it is the majority who controls most employment opportunities is inconsistent with another Becker's assumption that there is perfect competition on the demand side of the labor market.[10]

From the second perspective, early works addressing the supply side of income inequality include Becker and Tomes (1979) and Loury (1981), who argue that intergenerational transfers of ability to acquire human capital result in human capital variation across families and thereby generate income inequality. Shifting the focus from the family to the neighborhood, Benabou (1993, 1996) and Durlauf (1994, 1996) explain how persistent income stratification may arise as the consequence of social and geographical stratification due to local public goods or neighborhood externalities and the resulting uneven access to education across neighborhoods. More recent studies in this vein include Steele (1992), Akerlof (1997), and Lundberg and Starz (1998), who explicitly account for the role of social interaction in human capital distribution and suggest that it is the social or psychological (dynamic) externalities in segregated neighborhoods

[9] This is because discriminating agents deviate from economically most efficient choices to satisfy their preference for discrimination.
[10] c.f. Darity (1982, pp. 72-75) and Arrow (1998) for a discussion.

or workplaces that promote social and economic inequalities. For example, Lundberg and Starz (1998) explain how initial disadvantages of minorities such as those engendered by past discrimination can be sustained indefinitely. While explaining variation of income across families and neighborhoods, these studies do not, however, provide a systematic explanation of minority-majority income inequality and the scale puzzle in particular.

Another approach to ethnic inequality, including the recent works of Lundberg and Starz (2002) and Coate and Loury (1993), who build on the groundbreaking ideas of Phelps (1972), Arrow (1972a, 1972b, 1973), and Aigner and Cain (1977), combines the two economic perspectives studying the consequences of asymmetric information between suppliers and employers of human capital. Statistical discrimination is a form of asymmetric information in the labor market arising when important individual characteristics of the employee are not observable by the other party of the contract, the employer, but there are some observable individual characteristics, such as race, that are used by the employers as a proxy for the unobservable characteristics, the power of these proxies being based on past experience or statistical investigation. This approach either assumes external asymmetry with respect to risk of employing employees from different social groups[11], or asymmetry of prior beliefs of employers regarding abilities of employees of different races that are, in turn, substantiated through employees reaction to the asymmetric incentives to acquire human capital.[12] While the risk asymmetry hypothesis is rather ad hoc, theories based on self-fulfilled prior belief asymmetry perhaps offer a better account of the economic history of racial inequality. Yet, asymmetric prior beliefs are often difficult to rationalize and remain ad hoc with respect to the social inequality argument.[13]

Remarkable is the study of Lundberg and Starz (2002) who, in the search framework, provide an explanation of ethnic inequality without assuming any prior asymmetry across individuals. These authors assume imperfect information about the return to transactions with particular individuals.

[11] Such as higher variance of skill distribution of black workers or better predictive powers of screening tests for white workers, c.f. Darity (1982).
[12] Alternatively, a prior asymmetry on the supply side is assumed to justify itself through employers' beliefs and the reaction of employees thereto.
[13] Immigration of unskilled workers can rationalize a priori initial asymmetry on the supply side. For example, Topel (1997) argues that human capital differences arise as the consequence of immigration of unskilled minority workers, who decrease average human capital in the minority population. In the statistical discrimination context, such transitory asymmetries may become permanent.

Race arising as a signal of these returns, different group sizes are shown to suffice to engender the asymmetry between the returns to learning and the preferential treatment of the more populous majority agents on the labor market and may lead to exclusion of minority agents form employment opportunities. Two issues arise, however. First, in odds with the empirical evidence, the model predicts that larger minorities are *less* likely to be excluded and disadvantaged. Second, it does not explain why some minorities outperform majorities.

2.3. Contribution

In this paper, I provide a novel theoretical explanation of the link between the relative size of minority population and its relative economic achievement. In contrast to the aforementioned sociological studies, the economics of income inequality is thoroughly analyzed in a general equilibrium model. In comparison to the economic studies discussed above, I systematically explain, focusing on the social and institutional organization of the economy, why inter-group asymmetry of benefits from social interaction arises despite the prior homogeneity of individuals. I establish that, to explain that asymmetry, no ad hoc “taste for discrimination”, prior beliefs assumption, neither any assumption asymmetric with respect to individual characteristics is necessary. In addition, rather than studying neighborhood, social, and psychological externalities given the (initial) patterns of segregation, in this study the mutually dependent degree of segregation and benefits from social interaction are simultaneously determined. Moreover, no a priori disadvantage of the minority, such as past discrimination, is necessary to explain the minority-majority income inequality. Importantly, as contrasted to Lundberg and Starz (2002), I shift the focus from the role of organization of social interaction in job allocation on its role in human capital acquisition. Moreover, I highlight the role of the variation of price per unit of efficient labor in determining income inequality between social groups. Finally, within this study, I explain why some minorities attain higher economic status than majorities and I argue that minorities not only typically lag behind majorities in terms of income per capita but also that they lag more when they are relatively bigger, thereby explaining the scale puzzle.

3. *The Determination of Minority-Majority Earnings Inequality*

Variation of labor income across individuals reflects variation of the value of their efficient labor – the composite of time and skills – on the labor market. The value of efficient labor can be divided into price and quantity components. Of these, the quantitative variance of efficient labor has received most attention by students of inequality, such as those mentioned above, who habitually assume that the price of efficient labor is invariant across individuals due to homogeneity of efficient labor. While this assumption may be appropriate in many contexts, I argue that it is inappropriate in the context of ethnic inequality. The reason is that, as I establish below, sociocultural differences that hinder social interaction between minority and majority agents and asymmetric social group sizes, through network effects in skill acquisition, tend to expose members of different ethnic groups to different incentive structures in skill acquisition and thus motivate them to acquire different (combinations of) skills. If skills qualitatively vary across individuals, that is, if individuals have different skills, the prices of their efficient labor vary as well, because individual efficient labors are no longer perfect substitutes. As a result of the imperfect substitutability, two important effects emerge. Given the quantity of individual's efficient labor, labor market earnings of an individual decrease in the *aggregate* quantity of efficient labor of her type in the labor market due to the textbook economic law that, ceteris paribus, more abundant goods sell at lower price. On the other hand, however, her labor market earnings directly increase in the quantity of her *individual* efficient labor, given the unit price and thus the aggregate quantity thereof. To establish the existence of these effects and to determine their relative strengths, it is important to understand the social embeddedness of human capital acquisition and its consequences.

3.1. Social Networks and Human Capital Acquisition

Since the works of Becker (1962), Mincer (1958), and Schultz (1961), economists have typically conceptualized human capital – the collection of skills that constitute efficient labor – as a rival and excludable production factor, similar in its production and use to physical capital. While this simplification facilitated development of many insightful theories, it neglected an important feature of human capital that, as Lucas (1988, p.19) points out, "human capital accumulation is a *social* activity, involving *groups* of people in a way that has no counterpart in the accumulation of physical capital."[14] Social embeddedness of human capital acquisition, as conceptualized in the pioneering works of Rees and Schultz (1972), Loury (1977), Bourdieu (1986), Coleman (1988, 1990), and Granovetter (1973, 1974, 1985), is central to my argument. According to these authors, human capital is acquired in social interaction along social ties that constitute social networks – established structures of social ties among actors in a social system. A number of scholars, such as Dore (1983), Powell (1990), and Uzzi (1997) have stressed the role of network forms of organization in learning. Two main mechanisms through which network organizations foster learning found in the literature are, first, channeling interpersonal exchange of information, e.g. Root (1988), Hamel (1991) and Kogut (1988), and, second, facilitating synthesis of pieces of information that were previously located in distinct nodes, e.g. Powel and Brantley (1992). A sizeable literature focuses on the role of social networks in social learning. Ellison and Fudenberg (1995) study a model in which communicational structure determines the sets of agents with whom individuals share experience. Valente (1995), Feick and Price (1987), Gladwell (2000), and Foster and Rosensweig (1995) substantiate such approach and observe that social learning is a significant vehicle of human capital accumulation. These authors document that colleagues, friends, or neighbors share information about their discoveries, experiment outcomes, or search results. Conley and Udry (2002), Munshi (2002), and Bandiera and Rasul (2001) empirically corroborate the premise that social networks significantly affect individual adoption of new technologies. In line with these concepts, I conceptualize individual human capital to be the composite of individual skills, capabilities, and expertise[15] that are acquired through time consuming learning that involves social interaction in social networks and that, coupled with working time, constitute individual efficient labor valued by the labor market.

That the benefits from social interaction are increasing in the number of people involved and the pieces of information to exchange and synthesize they hold – the size of social network – is a straightforward corollary of the aforementioned arguments about the role of social interaction and social networks in social learning. It leads to the first essential assumption about the effects of social interaction on human capital acquisition on which the main argument of this paper is

[14] Italics are in the original.
[15] Below all referred to as "skills".

based. Namely, I assume that skill acquisition process exhibits network effects that positively depend on the size of the network in which the particular skill is acquired. In other words, while an agent may be able to acquire a skill alone, that is, in a network of zero measure, the efficiency of her learning is increasing in the number of people that acquire the same skill, engaging in social interaction with this agent in the skill-specific social network.[16] Because most social interactions in social networks do not involve pecuniary remuneration for the benefits that one's membership in a network creates for the other members of this network, I assume that network effects are external.[17]

There are several arguments that social interaction is prevalent in skill acquisition. Technically, there are two extreme modes of learning processes. In the one, agents acquire skills in social interaction with people who facilitate their skill acquisition. In the other one, an individual acquires skills without any social interaction using a perfect blueprint how to acquire a particular skill. First, I argue that it is highly unlikely that all skills are acquired without any social interaction. Any theory assuming the existence of a complete set of blueprints for every skill available to everyone such that social interaction is unnecessary is exceedingly unrealistic. Even if one allowed for the existence of such a complete set, it would be problematical to rule out any benefits from social interaction in learning and the resulting involvement therein. Furthermore, to rule out indirect social interaction it is necessary to assume away any dependence of the quality and availability of blueprints on the number of people that use them. Second, strengthening the claim, I contend that skills are mostly acquired in social interaction. Examples of interactive skill acquisition include learning in families, friendship circles, schools, workplaces, sports clubs, student societies, religious circles, kinships, and many others. All these examples support the premise that people engage in and benefit from social interaction in the process of skill acquisition. Certainly, people also read textbooks, cookbooks, articles, watch television, listen to radio and browse the Internet, where typically no direct social interaction is involved. However,

[16] A doubt may arise from the possibility that the benefits from social interaction are decreasing in the number of interacting people whenever the size of the network exceeds certain threshold; that the congestion effect may arise. While I agree that this is theoretically possible, rational agents would never voluntarily engage in interactive learning that would exceed such threshold, even if the physical network size would. Nonetheless, the existence of such threshold level does not undermine the main insights of this paper.

[17] In fact, that network effects are external can be seen as the consequence of perfect competition among actual or potential network members that eliminates, given the zero costs of entry in networks, any pecuniary compensation for the incremental network effect an entrant produces to the benefit of other network members.

the degree of indirect social interaction in these cases is tremendous. An obvious example is the Internet that is, in fact, one of the prime examples of industries that generate strong network effects in the literature on network industries. Taking another one, cookbooks, one has to realize the positive effects the cook obtains from a given cookbook whenever there are many other cooks who use the same recipes and create a large demand for these books, thereby providing the publisher with strong incentives to increase availability, quality, and variety of the recipes in the cookbook. In addition, very often a cookbook is a collection of recipes that are shared and improved by the readers of some magazine that also publishes the cookbook. Thus, it is in fact difficult to find examples of learning that do not involve social interaction.

It is natural to expect that benefits from social interaction not only depend on the number of individuals one interacts with but also on who these individuals are. In the context of minority-majority social interaction, sociocultural differences between minorities and majorities are likely to determine the quality of social interaction in any network. To operationalize these sociocultural differences, in line with Poole (1927), I define social distance to be the measure of the subjective and objective dissimilarities between social groups that hinders social interaction between the members of these social groups. Thus, in contrast to Akerlof (1997), who studies endogenous social distance between homogeneous agents, I consider social distance between members of different social groups to be a predetermined variable that reflects the defining distinctiveness of social groups. The natural corollary of the definition of social distance above is that agent's ability to benefit from social interaction in a given network negatively depends on her social distance to the other members of this network. Based on this argument, the second essential assumption of this paper is that individual benefits from network effects are decreasing in interpersonal social distance.

3.2. Skill Heterogeneity

To establish that minority and majority efficient labors are imperfect substitutes, an additional argument instituting heterogeneity of skills in the economy is necessary. Given the omnipresent segregation of social institutions, I assume that social structure consists of institutionally exclusive and inclusive social networks among which agents choose to participate and acquire skills.[18] I argue, in turn, that such institutional environment engenders variation of skills in the economy. Specifically, while inclusive networks permit any membership, any given exclusive network only permits memberships from one social group and thus exclusive social networks are segregated. Inclusive social networks may be integrated as well as segregated, depending on the composition of their members. The distinction made in this paper is that exclusiveness (inclusiveness) is understood as exogenous institutional constraint on network membership while segregation (integration) as endogenous variable concerning equilibrium organization of social interaction. The prime examples of typically exclusive networks include families, kinships, ghetto social networks, religious groups, expatriate communities, radical groups, some exclusive discotheques and clubs, and ethnically or religiously exclusive schools. The list of inclusive networks includes most schools, student societies, and workplaces, academic communities, and television viewers and Internet users.

Are the skills available in exclusive networks different from those available in inclusive ones? The examples listed above suggest that the typical exclusive and inclusive social networks are predisposed to be different with respect to their objectives and functions. Clearly, these reflect the various social roles of an agent in the society. For example the parental role of an agent is carried out in the family while her economic role is performed in her workplace community that have very different objectives and functions. Moreover, nearly tautologically, purposefully exclusive networks have different objectives or functions than inclusive networks; otherwise any costs associated with exclusion, such as forgone network benefits, are wasteful and thus the efforts to exclude irrational. It follows that skills that reflect the functions or objectives of networks in which they are acquired are different across exclusive and inclusive networks. Another source of differentiation between skills acquired in exclusive and inclusive networks lies in the defining characteristics of social groups such as religion, culture, or ethnic history, as these are likely to be reflected more intensely in exclusive social networks than in inclusive ones. For example, in a religiously divided society, the skills and abilities that individuals acquire in their exclusive religious association are more likely to reflect religious concern than those that they

[18] There is an enormous literature on social structure and ethnic segregation. Recent contributions include Massey and Denton (1993) and Farley and Frey (1994). Schelling (1978, ch. 4) argues that mild preferences to reside with one's own social group suffice to produce severe residential segregation. Ethnic segregation has been documented by Farley and Frey (1994), Glaeser and Vigdor (2001), Reardon, Yun, and Eitle (2000).

acquire in inclusive associations, such as sports clubs, universities, or pubs where religion plays lesser if any role.[19] From a different viewpoint, Annen (2003) studies how social networks respond to the complexity of their environment and changes in communication technologies by changing their degree of exclusiveness. Because skills learned in environments with different complexity and communication technologies are likely to be different, the skills in exclusive and inclusive networks are prone to be different. Based on these arguments, I posit that skills acquired in exclusive networks are generally different from those acquired in inclusive networks.[20] For the sake of brevity, I let "inclusive" and "exclusive" denote the respective social networks and skills.

It is worthwhile to realize that none of the aforementioned essential assumptions by itself engenders any asymmetry between members of different social groups whatsoever. However, the network effects and social distance assumptions taken together render the benefits that a given individual derives from social interaction in a given network dependent on the number and origin of the members of this network and the origin of this given individual. Thus, for instance, the network effect benefits of a minority individual in a given social network increase in the share of minority members in this network, given the size of the network, and these benefits are also different from those that a majority individual enjoys in the same network. If then, for example, all social networks are of the same size and composition such that there is more majority than minority members in every network, network effects and social distances disadvantage minority individuals. These two assumptions indeed suffice to establish that minorities tend to lag behind majorities, as minority individuals tend to enjoy smaller network benefits in human capital acquisition and thus acquire a lesser measure thereof.

Assuming institutional heterogeneity of social networks institutionalizes heterogeneity of skills available in the economy and constitutes the ground for imperfect substitutability of effective labors of minority and majority individuals. First, as shown below, if exclusive and inclusive skills are good substitutes, individuals specialize and network effects and social distances may coordinate individuals such that skills differ across social groups, resulting in imperfect

[19] It is worthwhile to note that no a priori variation of skills across social groups is assumed here.
[20] This difference will be operationalized below. Note that no asymmetry between the market values of different skills is imposed here.

substitutability of minority and majority efficient labors. Second, if exclusive and inclusive skills are poor substitutes and individuals acquire both types of skills each, minority individuals tend to acquire different combination of skills than majority individuals. To outline the argument, reasonably, asymmetric social group sizes are reflected in compositions of social networks. In particular, on average, it is natural to expect proportional representation of social groups in inclusive networks and thus overrepresentation of majority relative to minority members in these networks. One could also argue that, due to the greater geographical density of majority population, the exclusive networks of majority are easier to form and maintain than those of minority and therefore are larger. These asymmetries, through network effects and social distances, cause the relative incentives to engage in exclusive and inclusive social networks to differ between members of different social groups. As a consequence, members of different social groups acquire different combinations of exclusive and inclusive skills and thus supply imperfectly substitutable efficient labors in this case as well.

3.3. Individual Income Effect vs. Inter-Group Substitution Effect

Based on the arguments above, the three assumptions above engender the inter-group substitution and individual income effects[21] that link social group sizes and social group inequality and thereby determine equilibrium income distribution. The individual income effect directly rewards individuals for supplying more efficient labor, given the unit price thereof. Ceteris paribus, members of social groups with a higher efficiency of skill acquisition process acquire more skills and supply more efficient labor. Because the efficiency of skill acquisition is a function of network effects that in turn depend on group sizes, the individual income effect links group sizes and individual incomes. It is natural to expect that, due to social distances, individuals benefit from a larger relative size of their own social group. Therefore, the individual income effect favors members of relatively larger social groups.

Through prices, on the other hand, the inter-group substitution effect rewards members of social groups that on aggregate produce less group-specific efficient labor, either because they are small or because they have low per capita production of efficient labor. As a consequence, concerning the inter-group substitution effect, small social groups benefit from their smaller size

[21] The choice of this terminology will become clear below.

directly, through the small number of suppliers of their particular kind of efficient labor, as well as indirectly, through their lower efficiency in producing efficient labor, as discussed above, and the resulting lower per capita supply thereof. Summarizing, while the inter-group substitution effect is decreasing in the relative size of a given social group, the individual income effect is increasing in its size. Thus, as the relative size of minority increases, it benefits from the individual income effect while being hurt by the diminishing benefits from the inter-group substitution effect. In the analysis below I formally demonstrate that these two effects tend to produce the pattern of inequality between social groups that is consistent with the scale puzzle and establish the other important results of this paper.

4. The Model

4.1. Demand for Efficient Labor

In this section, I study the demand side of the labor market where the society is bifurcated into two social groups – the minority I and the majority J – and clarify the extent to which it accounts for the inter-group substitution and individual income effects. Let i and j denote the respective elements and I and J the respective measures of the continua of minority and majority agents. As argued above, $I < J$, in other words, the minority is less numerous than the majority. I assume that all individuals are identical with respect to their preferences and endowments, group membership excepting. Individual preferences are represented by the utility function $u(\cdot)$ defined on individual consumptions of the consumption good, C_k, where $k \in \{i, j\}$, such that the utility function satisfies $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Let the consumption good be produced by combining individual efficient labors of minority agents, H_i, and majority agents, H_j, in a perfectly competitive industry according to the constant elasticity of substitution (CES) aggregate production function

$$C = \left(\left(\int_0^I H_i di\right)^{(\rho-1)/\rho} + \left(\int_0^J H_j dj\right)^{(\rho-1)/\rho}\right)^{\rho/(\rho-1)} \tag{1}$$

with the elasticity parameter $\rho > 1$.[22]

[22] Proposition 4 below reveals that production function (1) can be seen as a harmless simplification of a more general production technology with many kinds of efficient labor in the context of this paper.

As suggested by Arrow (1998) and Loury (1998), I adopt the statistical discrimination hypothesis about the informational structure of the labor market. In particular, I assume that while employers observe the aforementioned observable characteristics of social group membership of individuals[23] and the measure of efficient labor they supply, and thus can observe H_i and H_j, they are not able to directly observe the marginal product of efficient labor supplied by an individual or to decompose individual efficient labor into skills and labor times when these skills are utilized.[24] From experience or statistical investigation, however, they understand that social group membership predicts the marginal product of individual efficient labor. By corollary, employees from the same social group are not distinguishable with respect to their type of efficient labor, as they do not perceptibly differ, and they always receive the same wage for a unit of their efficient labor.

Applying the representative agent hypothesis group-wise and, given the infinitesimal measure of any individual, and taking all prices as given on the individual level, production function (1) gives rise to individual demands for efficient labors

$$\begin{aligned} H_i &= P_C^{\rho} W_i^{-\rho} C / I \\ H_j &= P_C^{\rho} W_j^{-\rho} C / J \end{aligned}, \qquad (2)$$

where P_C is the price of consumption good C and W_i and W_j are the prices per unit of efficient labors of the minority and majority agents, respectively. As a result of the homogeneity of degree one of the CES production function, the sector does not generate any profits in the equilibrium and we can derive that $P_C = \left(W_i^{1-\rho} + W_j^{1-\rho}\right)^{1/(1-\rho)}$. Combining the demands for H_i and H_j, I obtain the relative demand for individual efficient labors

$$w = \left(\frac{I}{1-I} h\right)^{\frac{-1}{\rho}}, \qquad (3)$$

where I adopt the notational convention that lower case letters denote either minority-to-majority or exclusive-to-inclusive ratios of the respective upper case lettered variables. Furthermore, I adopt a convenient normalization $I+J=1$. Equation (3) plainly reveals the inter-group substitution effect that, given a finite ρ, the relative wage is decreasing in the relative size of the social group

[23] E.g. skin color, group-specific name, or accent.
[24] Thus, employers cannot remunerate employees skill-wise.

and its relative supply of efficient labor. Premultiplying by h and defining $\Omega_k \equiv H_k W_k$ to be both the total labor income and the only income of individual k, I derive the following expression for relative per capita income of minority individuals.

$$\omega \equiv hw = h\left(\frac{I}{1-I}h\right)^{\frac{-1}{\rho}} = \left(\frac{I}{1-I}\right)^{\frac{-1}{\rho}} h^{\frac{\rho-1}{\rho}} \qquad (4)$$

Equation (4) is the main result from studying the demand side of the economy. It accounts for the inter-group substitution effect and brings to light the individual income effect. In particular, while the relative wage w is driven by the inter-group substitution effect and is decreasing in the relative per capita supply of efficient labor, h, relative income of the minority individual, ω, depends, through the individual income effect, positively on h, given w. In fact, given I, the individual income effect overwhelms the negative dependence of w on h and individuals benefit from increasing their efficient labor. Markedly, the individual income effect and the inter-group substitution effects correspond to the standard income and substitution effect inherent in the CES function, which substantiates the adopted terminology.

Three conditions about the relative supply of efficient labor and in particular about the network effects and social distances under which equation (4) produces a pattern of income inequality consistent with the scale puzzle for a range of values of I can be identified. First, the individual income effect should favor the larger social groups so that $h < 1$. Second, the relative weight of the inter-group substitution effect in income determination should be small enough (elasticity of substitution ρ should be large enough), so that the inter-group substitution effect does not outweigh the individual income effect through relative wage and $\omega \equiv hw < 1$. Third, the marginal individual income effect should be smaller than the marginal inter-group substitution effect, so that the ω schedule is decreasing in I or, in other words, that a marginal increase in I decreases minority's relative wage w more than it increases its relative efficient labor h.[25] Below, I will

[25] Generally, if there exists a range of I for which $h(I)/h'(I) > I(1+I)(\rho-1)$, $h(I)<1$, and ρ is large enough such that the inter-group substitution effect is relatively weak and dominated by the individual income effect so that $\omega<1$, then variation in I produces a pattern of $\omega(I)$ consistent with the scale puzzle ($\omega<1$ and $\omega'(I)<0$) in this range of I. One example of such function is $h = (1+I)/(1+(1-I))$, where linear network effects in skill acquisition and infinite social distances are assumed. For example for $\rho=6$ the scale puzzle is reproduced in the range of I between 4 and 17 percent under this specification of h.

show how the model satisfies these conditions when taking into account the organization of the supply of efficient labor.

Equation (4) directly shows that we need imperfect substitutability of minority and majority efficient labors to generate the inter-group substitution effect. If minority and majority efficient labors are perfect substitutes and thus $\rho \to \infty$, equation (4) boils down to $\omega = h$ and the inter-group substitution effect is non-operative as wages per unit of efficient labor are equal for all individuals. As a result, as established below, network effects and social distances still cause the minority to lag but the lag is diminishing in the relative size of minority, contrary to the scale puzzle. Therefore, an additional fundamental argument has to be made to explain the scale puzzle. In particular, it is essential to establish that minority and majority efficient labors are imperfect substitutes, that is, that ρ is finite. In the following section I analyze supply of efficient labor and show how the supplies of minority and majority efficient labor depend on the sizes of social groups through network effects and social distances. Most importantly, I establish that, under certain circumstances, network effects and social distances determine the organization of supply of efficient labor such that it justifies a finite ρ, that is, that minority and majority individuals choose different (combinations of) skills to acquire.

4.2. Supply of Efficient Labor

Individuals are each endowed with one unit of time that they divide between acquisition and utilization of a variety of skills that increase the efficiency of individuals' efficient labor. Denoting exclusive and inclusive skills and network types $m \in \{x, n\}$, respectively, I assume the constant elasticity of substitution technology of producing efficient labor[26]

$$H_k = \left(\left(S_{k,x} \left(T_{k,x} - L_{k,x} \right) \right)^{(\varepsilon-1)/\varepsilon} + \left(S_{k,n} \left(T_{k,n} - L_{k,n} \right) \right)^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)} \tag{5}$$

[26] In the literature, typically, efficient labor is the product of time and human capital. While it is still the case here that the components of efficient labor are products of time and skills that constitute human capital, I account for the heterogeneity of skills and conceptualize efficient labor to be the CES composite of products of time and skills.

and the decreasing-returns-to-scale skill acquisition technology

$$S_{k,m} = L_{k,m}^{\phi}\left(1 + N_{k,m}\right), \tag{6}$$

where $S_{k,m}$ is the measure of skills of type m of agent k, $T_{k,m}$ is the corresponding total time invested in acquiring and utilizing skills, and $L_{k,m}$ is the corresponding time spent on acquiring the particular skills. The parameter $\varepsilon > 0$ denotes the elasticity of substitution between the composites of different skills and time in production of individual efficient labor. $N_{k,m}$ is the network effect in network m enjoyed by member k and $\phi \in (0,1]$ is the measure of decreasing returns in skill acquisition.

In general, there can be more skills of any given type in the economy. For the sake of tractability, I assume that skills differ only along the one dimension discussed above, that is, whether they are exclusive or inclusive. Given this assumption, in Proposition 3 below it is established that in any stable equilibrium there are no agents involved in more than one social network of any given type, exclusive or inclusive. In this sense, equation (5) is a harmless simplification of a more general technology with an arbitrary number of exclusive and inclusive skills acquired in the corresponding social networks.

Given the difference of exclusive and inclusive skills, from the production technology (5) we see that the qualitative properties of individual efficient labor are determined by the combination of skills that constitute efficient labor. I operationalize this qualitative variation of efficient labor such that efficient labors that consist of different (combinations of) skills are imperfect substitutes on the labor market. If, for example, the skills of one agent are predominantly exclusive and the skills of the other agent are predominantly inclusive, the elasticity of substitution between the efficient labors of these two agents is finite.

Having explained the structure of the economy, individuals maximize their utility, taking their resource constraints, available technologies, network effects, wages, and the price level as given. From the properties of the utility function it follows that the agents' problem boils down to

$$H_k^* \equiv \underset{L_{k,m},T_{k,m}}{Max} \left|H_k\right| \tag{7}$$

subject to (5), (6), and the resource constraints $T_{k,m} \geq 0$, $L_{k,m} \geq 0$, and $T_{k,x} + T_{k,n} \leq 1$.

Proposition 1

In the economy where individuals solve the problem specified in (7), given the total time spent on skill *m* by agent *k*, $T_{k,m}$, individuals divide their time between acquisition and utilization of skills according to the rule

$$L_{k,m} = \frac{\phi}{1+\phi} T_{k,m} . \tag{8}$$

Proof in Appendix 1.

Proposition 1 says that agent *k* spends a fixed share $\phi/(1+\phi)$ of the time $T_{k,m}$ that he allocates to skill *m* on acquisition of this skill. The rest of this time, $1/(1+\phi)T_{k,m}$, or $T_{k,m}$ - $L_{k,m}$, is spent on utilizing this skill. To save on notation in what follows, define

$$\tilde{N}_{k,m} \equiv \left(\left(\frac{\phi}{1+\phi} \right)^{\phi} \left(1 - \frac{\phi}{1+\phi} \right) (1 + N_{k,m}) \right)^{(\varepsilon-1)/\varepsilon} .$$

Proposition 2

In the economy where agents solve the problem specified in (7), network effects and wages are taken as given by individuals, wages are invariant within any social group, and $\varepsilon \geq (\phi+1)/\phi$, a maximum arises as a corner solution where all the time available to an individual is spent on acquisition and utilization of the one skill whose acquisition is most efficient. In particular,

$$\begin{aligned} \tilde{N}_{k,n} \geq \tilde{N}_{k,x} &\Rightarrow H_k^* = \tilde{N}_{k,n}^{\varepsilon/(\varepsilon-1)} \\ \tilde{N}_{k,n} \leq \tilde{N}_{k,x} &\Rightarrow H_k^* = \tilde{N}_{k,x}^{\varepsilon/(\varepsilon-1)} \end{aligned} . \tag{9}$$

On the other hand, whenever $\varepsilon < (\phi+1)/\phi$, the interior solution

$$H_k^* = \left(\tilde{N}_{k,x} T_{k,x}^{(\phi+1)\frac{(\varepsilon-1)}{\varepsilon}} + \tilde{N}_{k,n} T_{k,n}^{(\phi+1)\frac{(\varepsilon-1)}{\varepsilon}} \right)^{\varepsilon/(\varepsilon-1)} \tag{10}$$

is the maximum, where the equilibrium time allocation in the interior solution is

$T_{k,x} = \dfrac{t_k}{1+t_k}$ and $T_{k,n} = \dfrac{1}{1+t_k}$, and the optimal time allocation is governed by the arbitrage condition

$$t_k \equiv \frac{T_{k,x}}{T_{k,n}} = \left(\frac{1+N_{k,x}}{1+N_{k,n}}\right)^{\frac{\varepsilon-1}{1+\phi-\varepsilon\phi}}. \tag{11}$$

Proof in Appendix 2.

Corresponding to interior and corner solutions in Proposition 2, I define two classes of regimes – specialization and diversification. In particular, I call diversification the class of regimes in which every agent acquires both types of skills, exclusive and inclusive. Formally, these are regimes in which $T_{k,m}>0$ for all k and m. Similarly, the class of regimes when agents specialize in either exclusive or inclusive skills is called specialization. These are regimes in which $T_{k,m}=0$ for some m. From Proposition 2, specialization regimes prevail if and only if $\varepsilon \geq (\phi+1)/\phi$ and diversification regimes prevail if and only if $\varepsilon < (\phi+1)/\phi$.[27]

Given the results for H_k^* in Proposition 2, the relative supply of efficient labor is, under specialization,

$$h = \frac{1+N_{i,m}}{1+N_{j,m}} \tag{12}$$

and, under diversification,

$$h = \frac{\left(\dfrac{1}{1+t_i}\right)^{\phi+1}\left(\left(t_i^{\phi+1}\left(1+N_{i,x}\right)\right)^{(\varepsilon-1)/\varepsilon} + \left(1+N_{i,n}\right)^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)}}{\left(\dfrac{1}{1+t_j}\right)^{\phi+1}\left(\left(t_j^{\phi+1}\left(1+N_{j,x}\right)\right)^{(\varepsilon-1)/\varepsilon} + \left(1+N_{j,n}\right)^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)}}. \tag{13}$$

As we see, the relative supply of efficient labor is fully determined by network effects and time allocation in skill acquisition. In particular, it does not depend on wages. The reason is that

[27] In general, there may be dual regimes in which agents of one social group diversify while agents of the other social group specialize. Because the choice to diversify or specialize entirely depends on parameters ϵ and ϕ of the model and these are assumed to be the same for every agent in the economy, I disregard these cases here. The rationale for this approach is that the emphasis in this paper is put on the question how network effects engender heterogeneity of human capitals of minority and majority and how this heterogeneity translates into income inequality when individual characteristics are the same for both social groups.

individuals take wages as given and skill acquisition does not involve any pecuniary exchange. To determine t_k and $N_{k,m}$, we need to specify network effects and investigate the allocation of individual involvements across networks under the various equilibrium regimes of skill acquisition.

4.3. Network Effects and the Equilibrium Regimes of Skill Acquisition

In the skill acquisition technology, (6), network effects play a pivotal role in determining the efficiency of acquiring skills in a social network and thus the equilibrium allocation of individual involvements across social networks. In line with the arguments above, network effects that any given agent *k* enjoys in network *m* depend on the extent of social interaction in this network, which is measured by the total time agents spend in this network.[28] Consistently with the assumption about social distance, agents benefit more from interaction with ethnically similar agents as compared to ethnically distant agents. This effects is captured by the social distance parameter $\delta>0$. For the sake of the clarity of exposition, I posit that the one-dimensional social distance parameter completely represents the multidimensional dissimilarities between the minority and the majority. Based on these premises, I assume network effect specification

$$\begin{aligned} N_{i,m}(I_m, L_{i,m}, J_m, L_{j,m}, \delta) &= \left(I_m L_{i,m} + \frac{1}{1+\delta} J_m L_{j,m} \right)^{\gamma} \\ N_{j,m}(I_m, L_{i,m}, J_m, L_{j,m}, \delta) &= \left(\frac{1}{1+\delta} I_m L_{i,m} + J_m L_{j,m} \right)^{\gamma} \end{aligned}, \qquad (14)$$

where the parameter $\gamma\in(0,1]$ captures decreasing returns to involvement of agents in a social network and I_m and J_m are the numbers of, respectively, minority and majority members in network *m*. These numbers depend on the organization of skill acquisition as discussed below. Alternatively, when convenient, I will assume that network effects do not depend on the time spent rather on the number of agents only such that

$$\begin{aligned} N_{i,m}(I_m, J_m, \delta) &= \left(I_m + \frac{1}{1+\delta} J_m \right)^{\gamma} \\ N_{j,m}(I_m, J_m, \delta) &= \left(\frac{1}{1+\delta} I_m + J_m \right)^{\gamma} \end{aligned}. \qquad (15)$$

[28] Due to infinitesimal measure of an agent, I disregard the distinction between the total time spent in network *m* and the time that agents other than the given agent *k* spend in this network.

Throughout the paper I assume that agents take network effects as given, given the infinitesimal measure of any individual. Having specified network effects, we can state the following propositions about stable equilibrium regimes of skill acquisition:

Proposition 3

In any stable equilibrium, no agent is involved in more than one network of any given type, exclusive or inclusive.

Proof:

Given that skills of any given type are perfectly substitutable on the individual level, as they do not differ, individuals pick those social networks of the given type that offer the largest network effects and thus are the most efficient. Assume there is an equilibrium with an individual violating the proposition thus involved in two networks of a given type. It must then be that the network effects in these two networks are the same for this individual; otherwise she would pick the one that is more efficient. Such equilibrium is unstable, however. Given that network effects increase in agents' involvements, any marginal deviation in allocation of agents causes the network effects between the two networks to differ and, as a consequence, the agent to abandon the less efficient network. Obviously, the reaction of the other individuals to such marginal deviation does not stabilize the equilibrium, as network effects are increasing in individual involvements. This completes the proof.

Proposition 4

In any stable equilibrium, all agents of the same social group choose the same combination of skills to acquire.

Proof

Given the statistical discrimination on the labor market, individuals take the wage for the unit of their efficient labor as given with respect to their choice of skills. Therefore, individuals pick that combination of social networks and thus skills that is the most efficient in production of efficient labor. To prove Proposition 4 by contradiction, assume there is an equilibrium with two individuals from a given social group that are involved in two different combinations of social networks. Because the two individuals are free to choose between networks, in this equilibrium it must be that the efficiencies of these two combinations of social networks for the two individuals in production of efficient labor are the same. Such equilibrium is, however, unstable. Any marginal deviation from the equilibrium agent involvements across these two different combinations of networks causes their efficiencies to differ. As a result, the agent involved in the less efficient set of social networks switches to the more efficient one. As above, the reaction of the other individuals to the marginal deviations does not stabilize the equilibrium. This completes the proof.

Consequently, given statistical discrimination in the labor market, network effects and social distances in skill acquisition coordinate individuals such that at most two different efficient labors are supplied – minority and majority specific. In this sense, the production function in equation (1) can be seen as a harmless simplification of a more general production technology with an arbitrary number of kinds of efficient labor. The next section studies equilibrium regimes of skill acquisition under specialization.

4.3.1. Specialization

Given Proposition 3 and Proposition 4, in the class of specialization regimes that arise whenever $\varepsilon \geq (\phi + 1)/\phi$ all agents of a given social group choose exactly one and the same network to join and skill to acquire under specialization. Accordingly, I consider the five possible allocations of social groups across networks under specialization and test their stability. I adopt the Nash concept of equilibrium where agents choose social networks (skills) freely and the equilibrium arises as the state where no agent has incentives to deviate.

Proposition 5

In the class of specialization regimes the following equilibria are always stable in the Nash sense:

1. Both social groups specialize in their exclusive skills (EE)
2. Both social groups join the same inclusive network and specialize in inclusive skills (II)
3. The minority specializes in exclusive and the majority in inclusive skills (IE)

The following allocations

4. The minority specializes in inclusive and the majority in exclusive skills (EI)
5. Social groups specialize in inclusive skills, acquiring them in two non-connected inclusive networks, each composed of members of only one social group (IIS)

are stable if and only if

$$I \geq \left((1+\delta)^{\frac{1}{\gamma}} + 1 \right)^{-1}. \tag{16}$$

Proof in Appendix 3.

The equilibria in which minority and majority agents choose different types of skills, the EI and IE equilibria, substantiate imperfect substitutability of minority and majority efficient labor.

Based on this analysis, the EI equilibrium offers an explanation of the often-observed lesser involvement of minorities in (inclusive) formal educational institutions. In particular, if the EI equilibrium prevails in the labor market, due to network effects, social distances, and the failure to coordinate, minority individuals prefer staying in their (exclusive) kinship and family networks to individually deviating to formal schooling.

Relative Income under Specialization

In this section I turn to the income ratio in specialization regimes. Plugging the result from equation (12) into equation (4), the relative minority-majority income is

$$\omega(I) = \left(\frac{I}{1-I} \right)^{\frac{-1}{\rho}} \left(\frac{1+N_{i,m}}{1+N_{j,m}} \right)^{\frac{\rho-1}{\rho}} .$$

Specifying the network effects according (14), in the EI and IE equilibria we obtain the following expressions for relative incomes

$$\omega^r(I)=\left(\frac{I}{1-I}\right)^{\frac{-1}{\rho}}\left(\frac{1+I^\gamma\left(\frac{\phi}{1+\phi}\right)^\gamma}{1+(1-I)^\gamma\left(\frac{\phi}{1+\phi}\right)^\gamma}\right)^{\frac{\rho-1}{\rho}} \quad for\ r\in\{IE,EI\}, \tag{17}$$

where r denotes the equilibrium regime. Because, recalling the assumption about the perfect substitutability of skills of the same type, exclusive or inclusive, in the EE and IIS equilibria the elasticity of substitution between the two kinds of efficient labors ρ is infinite and the relative wealth is

$$\omega^r(I)=\frac{1+I^\gamma\left(\frac{\phi}{1+\phi}\right)^\gamma}{1+(1-I)^\gamma\left(\frac{\phi}{1+\phi}\right)^\gamma}\ for\ r\in\{EE,IIS\}. \tag{18}$$

In the II equilibrium, where agents acquire the same skill, there are both minority and majority individuals in the integrated network. Taking this into account and noting that ρ is again infinite,

$$\omega^{II}(I)=\frac{1+\left(I+\frac{1}{1+\delta}(1-I)\right)^\gamma\left(\frac{\phi}{1+\phi}\right)^\gamma}{1+\left(\frac{1}{1+\delta}I+(1-I)\right)^\gamma\left(\frac{\phi}{1+\phi}\right)^\gamma}. \tag{19}$$

From equations (18) and (19) it is clear that minority individuals are poorer than majority individuals under the EE, II, and IIS regimes due to the lower network effects they benefit from. In particular, $\omega^r(I)<1\ for\ r\in\{EE,II,IIS\}$. One needs to realize that this relationship is purely determined by the individual income effect that rewards social groups for their size. In particular, it can be shown that these equilibria predict that larger minorities are better off and thus conflict with the scale puzzle.

On the other hand, the EI and IE equilibria give rise to $\rho<\infty$ as agents acquire different skills, one social group inclusive and the other one exclusive skills. Given the result in equation (17), I state one of the two central propositions of this paper.

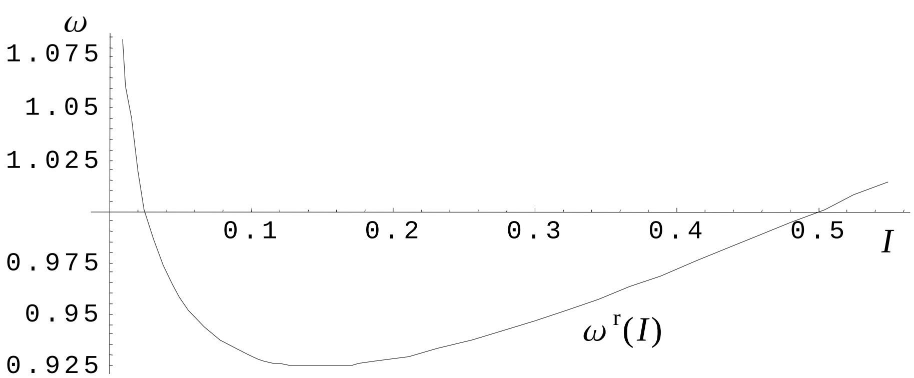
Proposition 6

There are stable specialization regimes of societal organization that give rise to patterns of inequality consistent with the scale puzzle for a range of minority relative sizes *I*. In particular, in the EI and IE regimes where minority and majority efficient labors are imperfect substitutes or, technically, ρ is finite, equation (17) produces a U-shaped relationship between ω and I such that there is a downward sloping segment of the $\omega(I)$ curve that is below one whenever ρ is large enough.

Proof in Appendix 4.

The shape of the $\omega^r(I)$ curve where $r \in \{EI, IE\}$ is depicted in Figure 1 for the case $\phi=0.9$, $\gamma=0.8$,and $\rho=10$.

Figure 1



This analysis demonstrates that the EI and IE segregation regimes produce labor income distribution consistent with the scale puzzle for empirically relevant range of values of *I* whenever the elasticity of substitution between minority and majority efficient labors is sufficiently large.[29] Moreover, I find support for the hypothesis that the relationship between the relative minority-majority income and minority percentage is convex, as suggested by Blalock (1956, 1957). In particular, under the conditions in Proposition 6, $\omega^r(I)$ is above one for $I \rightarrow \infty$,

[29] For example, for parametric values $\phi=1$, $\gamma=1$,and $\rho=10$ the range of *I* where the $\omega(I)$ curve is downward sloping and below one is from 3.2% to 16.3%. If $\phi=0.8$, $\gamma=0.7$,and $\rho=8.5$, the predicted range is 6.8% to 20.2%. Note that if the true relationship is convex as in Figure 1, the most widely tested linear relationship between ω and I would exhibit the scale puzzle for much wider ranges of *I*.

below one for some $I \in (0, 0.5)$, and one for I=0.5.[30] Because the $\omega^r(I)$ is a smooth and continuous function of I, there must exist a convex segment thereof on the range $I \in (0, 0.5)$. From Figure 1 it is also apparent that the theory predicts that small minorities receive larger labor income than majorities in IE and EI regimes.

Proposition 7

Under the EI and IE regimes there exists $\bar{I}$ small enough such that $\omega(\bar{I})=1$ and a minority of size $I \in (0, \bar{I})$ earns higher per capita labor income than the majority.[31]

Proof in Appendix 4.

Accordingly, the analysis above explains why some minorities outperform majorities on the labor market. In particular, it is the scarcity of minority specific human capital that, if the minority is small enough, drives the unit price of minority efficient labor so high as to overweigh the disadvantage of minority individuals from their lower human capital caused by their lesser efficiency in human capital acquisition such that minority earnings are higher than those of the majority. In the following sections I investigate whether the scale puzzle can be theoretically explained if ε is relatively small such that diversification arises.

4.3.2. Diversification

As argued above, diversification regimes arise in the equilibrium if and only if the two types of skills are complements or poor substitutes, such that $\varepsilon < (\phi + 1)/\phi$. Recalling that in diversification regimes all agents acquire both exclusive and inclusive skills, besides the optimality condition in equation (8), the arbitrage condition in equation (11) holds as well. Because all agents of a given type choose the same set of networks and thus skills to acquire, as we know from Proposition 4, two different equilibrium regimes can arise. In the D regime both social groups acquire their exclusive skills in their group-specific social networks and their inclusive skills in one integrated inclusive social network. In the DS regime, on the other hand,

[30] C.f. the proof of Proposition 6.

[31] While such IE equilibrium is always stable, the EI equilibrium is stable if $I \geq \left((1+\delta)^{\frac{1}{\gamma}} + 1 \right)^{-1}$ for $I \in (0, \bar{I})$.

the inclusive skills are obtained in two non-connected minority- and majority-only (segregated) inclusive social networks.

Proposition 8

The D regime of diversification is always stable. The DS regime is stable if and only if the inequality (16) holds.

Proof in Appendix 5.

An example of the D regime is the situation in which white and black (exclusive) families are segregated but (inclusive) workplaces are integrated. On the other hand, the DS equilibrium corresponds to the situation in which workplaces are also segregated because people prefer working with the members of their own social group to individually deviating to the workplaces where the other social group prevails, although workplaces are inclusive and do not institutionally prohibit integration.[32] In this sense I call the D regime integrated and the DS regime segregated. Note, however, that there is a degree of segregation in the D regime as well, as the exclusive networks are segregated. I discuss the D and DS equilibria one by one in the following sections.

In the D equilibrium, from Proposition 2 we know that $T_{k,x}^{D} = \dfrac{t_k^D}{1+t_k^D}$ and $T_{k,n}^{D} = \dfrac{1}{1+t_k^D}$. For expositional convenience, I adopt here the network effects specification from equation (15), that is, I assume that network effects depend on the number of network members only. This network effect specification and the fact that all agents join all permissible networks under the diversification regime result in the following specifications of relative times spent in any social networks:

[32] Certainly, it may also be that workplaces are institutionally exclusive and the exclusion therein is institutional.

$$t_i^D = \left(\frac{1+I^{\gamma}}{1+\left(I + \frac{1}{1+\delta}(1-I) \right)^{\gamma}} \right)^{\frac{\varepsilon-1}{1+\phi-\varepsilon\phi}} \text{ and } t_j^D = \left(\frac{1+(1-I)^{\gamma}}{1+\left(\frac{1}{1+\delta} I + (1-I) \right)^{\gamma}} \right)^{\frac{\varepsilon-1}{1+\phi-\varepsilon\phi}} \qquad (20)$$

Recalling that $\varepsilon < (\phi+1)/\phi$ under the D regime, from the results above we observe that all agents spend more time in exclusive networks than in inclusive ones whenever the two types of skills are complements. This result arises as the consequence of the complementarity that forces agents to compensate for the lower efficiency of exclusive networks by the longer times spent in exclusive networks. On the other hand, if the skills are substitutes and the D regime arises, all agents spend more time in inclusive networks. Finally, if the technology of combining skills is Cobb-Douglas and $\varepsilon = 1$, individuals spend equal shares of their times in exclusive and inclusive networks[33]. Formally, noting that, from equation (8), $t_k = l_k$,

$\varepsilon < 1 \Rightarrow t_k^D = l_k^D > 1$
$\varepsilon > 1 \Rightarrow t_k^D = l_k^D < 1$.[34]
$\varepsilon = 1 \Rightarrow t_k^D = l_k^D = 1$

Proposition 9

Under the D regime, minority individuals spend relatively more time in exclusive networks than majority individuals whenever $\varepsilon < 1$ such that complementarity of exclusive and inclusive skills prevails. Formally, $\varepsilon < 1 \Rightarrow t_i^D / t_j^D = l_i^D / l_j^D > 1$. The opposite is true if $\varepsilon > 1$, that is, $\varepsilon > 1 \Rightarrow t_i^D / t_j^D = l_i^D / l_j^D < 1$. Finally, if $\varepsilon = 1$, then $t_i^D / t_j^D = l_i^D / l_j^D = 1$.

Proof in Appendix 6.

[33] Certainly, this is the consequence of the assumption that the acquisition of exclusive and inclusive skills is equally efficient, as are the skills themselves in formation of efficient labor. Cobb-Douglass case is not the focus here, as the allocation of time is arbitrary in this case and is fully determined by the efficiencies of skill acquisition process, thus the parameters of the model.

[34] The proof is omitted, it is straightforward to observe in the specifications of t_i^D and t_j^D above that the terms in parenthesis are less than one, as *I*<0.5 and social distance is positive.

These results stem from the relatively smaller efficiency of the internal networks of the minority as compared to those of the majority. As a result, if skills are complements, as compared to the majority, minority individuals spend more time in their exclusive networks in order to compensate for this handicap. This finding reveals that the often-observed lesser involvement of minorities in formal educational institutions, as compared to the majority population, can be explained under the D regime of diversification as well. If the skills that minority students obtain at home through family socializing are complementary to those that they obtain in schools, these students spend relatively less time in schools than their majority counterparts. The opposite result holds whenever skills are substitutes and diversification prevails. The most important insight, however, is that minority and majority individuals choose different combinations of skills in the equilibrium.

Proposition 10

Social groups of different sizes choose different skill compositions in the D regime of diversification. In particular,

$$\frac{s_i^D}{s_j^D} \equiv \frac{S_{i,x}^D}{S_{i,n}^D} \Big/ \frac{S_{j,x}^D}{S_{j,n}^D} < 1.$$

Proof in Appendix 6.

Thus, even though minority agents under some circumstances spend more time in their exclusive networks in the D regime, they unambiguously acquire relatively *less* exclusive skills than majority individuals. The essential result here is that skill composition is different across social groups under the D regime of diversification. Therefore, the elasticity of substitution between efficient labors of minority and majority individuals is finite in the D regime.[35]

[35] Note that it is sufficient that minority and majority individuals have different relative incentives to spend time in exclusive and inclusive networks to obtain this result. Considering workplace inclusive networks and family exclusive networks, to establish the main results of the paper it suffices that network effects in minority and majority exclusive networks – families of approximately similar sizes – are the same for the respective individuals and there is less minority than majority individuals in the average inclusive network – the workplace. The analysis in the main text where the sizes of exclusive networks differ depend on group sizes better reflect the case where exclusive networks stand for ghettos, neighborhoods, and the like.

Turning to the diversification regime when minority and majority agents join two disconnected inclusive networks, the DS regime, it is obvious that for any individual inclusive and exclusive networks she is involved in provide the same network effects, as they are of the same size and composition. Therefore, she distributes her time such that she spends half of her time in the exclusive network and the other half in the inclusive network. In effect, in the DS regime both minority and majority individuals have the same skill compositions, their efficient labors are perfectly substitutable on the labor market, and they receive the same wage per unit of their efficient labor.

Relative Income under Diversification

Knowing the time allocation in the D regime, I now return to the income ratio. Plugging the relative supply of efficient labor (13) into (4), the relative income under the D regime is[36]

$$\omega^D(I)=\left(\frac{I}{1-I}\right)^{\frac{-1}{\rho}}\left(\frac{\left(\frac{1}{1+t_i^D}\right)^{\phi+1}\left(\left(\left(t_i^D\right)^{\phi+1}\left(1+N_{i,x}^D\right)\right)^{(\varepsilon-1)/\varepsilon}+\left(1+N_{i,n}^D\right)^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)}}{\left(\frac{1}{1+t_j^D}\right)^{\phi+1}\left(\left(\left(t_j^D\right)^{\phi+1}\left(1+N_{j,x}^D\right)\right)^{(\varepsilon-1)/\varepsilon}+\left(1+N_{j,n}^D\right)^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)}}\right)^{\frac{\rho-1}{\rho}}, \quad (21)$$

where, realizing that all individuals join all permissible networks,

$$N_{i,x}^D=I^\gamma,\; N_{i,n}^D=\left(I+\frac{1}{1+\delta}(1-I)\right)^\gamma,\; N_{j,x}^D=(1-I)^\gamma,\; N_{j,n}^D=\left(\frac{1}{1+\delta}I+(1-I)\right)^\gamma$$

and

$$t_i^D=\left(\frac{1+I^\gamma}{1+\left(I+\frac{1}{1+\delta}(1-I)\right)^\gamma}\right)^{\frac{\varepsilon-1}{1+\phi-\varepsilon\phi}} \text{ and } t_j^D=\left(\frac{1+(1-I)^\gamma}{1+\left(\frac{1}{1+\delta}I+(1-I)\right)^\gamma}\right)^{\frac{\varepsilon-1}{1+\phi-\varepsilon\phi}}.$$

[36] The full version of this equation is in Appendix 6.

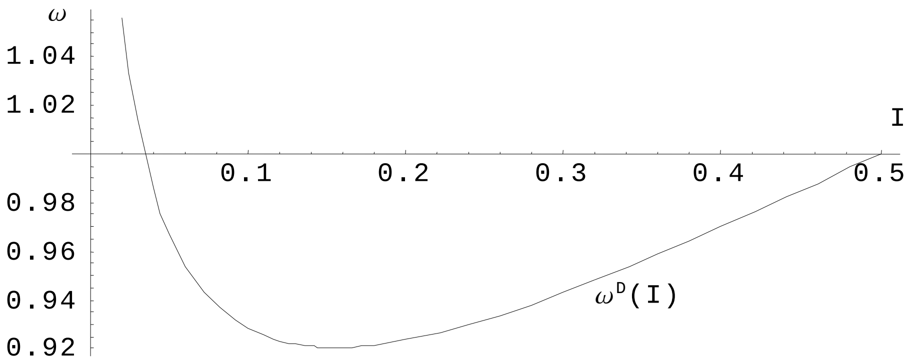
Proposition 11

The D regime of societal organization gives rise to patterns of inequality consistent with the scale puzzle for a range of minority relative sizes I whenever the finite ρ is large enough and $\varepsilon \neq 1$. In particular, for such ρ, equation (21) produces a U-shaped relationship between ω and I such that there is a downward sloping segment of the $\omega(I)$ curve that is below one.

Proof in Appendix 8.

The shape of the $\omega^D(I)$ schedule under the D regime of diversification is depicted for the parametric values ϵ=0.5, ϕ=0.8, γ=0.7, ρ=7, δ=20 in Figure 2.

Figure 2



In effect, in the D regime the theoretically predicted pattern of income inequality is consistent with the scale puzzle for a range of minority relative sizes, similarly as in the specialization regimes EI and IE. Similarly to the argument about convexity of the $\omega(I)$ relationship under the EI and IE regimes, whenever ρ is large enough and finite, $\omega^D(I)$ also exhibits (segments of) convexity. Moreover, small minorities tend to outperform majorities.

Proposition 12

Under the D regime there exists $\bar{I}$ small enough such that $\omega(\bar{I})=1$ and a minority of size $I \in (0, \bar{I})$ earns higher per capita labor income than the majority.[37]

Proof:

Similarly to the proof of Proposition 7, from the first step of the proof in Appendix 4 it is clear that $\omega(I)$ from equation (21) attains values higher than 1 for I small enough. This completes the proof.

In the DS regime, however, minority and majority agents choose the same skill composition and, therefore, the elasticity of substitution between their efficient labors ρ is infinite. Using this result and the facts that individuals divide their time evenly between acquisition and utilization of skills and that network effects are I^{γ} for minority individuals and $(1-I)^{\gamma}$ for majority individuals in any network they join, we obtain that

$$\omega^{DS}(I)=\frac{1+I^{\gamma}}{1+(1-I)^{\gamma}}\,. \tag{22}$$

Clearly, $\omega^{DS}(I)=\dfrac{1+I^{\gamma}}{1+(1-I)^{\gamma}}<1$, or, in words, in the DS regime minority individuals are always poorer than majority individuals. In conflict with the scale puzzle, equation (22) predicts that larger minorities lag less.

4.4. The Predictions of The Equilibrium Regimes and The Scale Puzzle

Having analyzed the patterns of predicted income inequality under the different equilibrium regimes, it is worthwhile to summarize which equilibria are congruent with the scale puzzle and the other empirical regularities. It has been shown that the EE, II, IIS, and DS equilibria predict that minorities lag under any parametric conditions. These equilibria, however, conflict with two empirical observations. First, the observation that some minorities outperform majorities conflicts with the predictions of these equilibria. Second, in conflict with the scale puzzle, according to these equilibria larger minorities lag behind majorities less than smaller ones. The

[37] We know that the D regime is stable for any *I*.

EI, IE, and D equilibria, on the other hand are congruent with the scale puzzle whenever the elasticity of substitution between minority and majority efficient labors is high enough. In particular, under this condition, there is a range of minority relative size where minorities lag and the more so the relatively larger they are. In addition, these equilibria predict that small enough minorities outperform majorities.

It is worthwhile to note that I have adopted a conservative approach throughout the paper, assuming that the elasticity of substitution between efficient labors composed of external skills acquired in different social networks, minority and majority specific, is infinite. One could argue, however, that the sociocultural difference between minorities and majorities suffice to produce imperfect substitutability of such efficient labors on the labor market. If this is the case, the EE and DS equilibria behave similarly to the EI, IE, and D equilibria and in the same way agree with the scale puzzle and the other empirical observations as well.

4.5. The Historicity of Prevailing Equilibria

Technically, which particular regime prevails in the labor market is indeterminate up to the elasticity of substitution between skills in production efficient labor ε, the degree of decreasing returns to scale in skill acquisition ϕ, and the stability of the EI, IIS, and DS equilibria. First, the parameters ε and ϕ determine whether specialization or diversification regimes prevail, as depicted in Proposition 2. Second, if the economy starts in the EI, IIS, or DS allocation and the social distance or minority diminishes enough, the economy tips into the II equilibrium in the former two cases and into the D equilibrium in the last case. The reason is that minority agents switch from their segregated network, exclusive or inclusive, respectively, to the inclusive network of the majority as this gives higher benefits to any minority individual. In this sense, which equilibria prevail in practice is largely historically determined.

Can anything be said about which equilibria are more likely to prevail? To answer this question fully is beyond the scope of this paper, yet some cautious predictions can be made. If, for example, minority entered the larger majority dominated society in small groups or individually, one could argue that the II equilibrium is more likely than the EI and IIS equilibria and that the D equilibrium is more likely than the DS equilibrium, as the stability condition for the latter equilibria could hardly hold in this case as the effective *I* was small. If we then, for example, believe that exclusive and inclusive skills are good substitutes and there is a moderate degree of decreasing returns in skill acquisition such that $\varepsilon \geq (\phi+1)/\phi$, under the aforementioned mode of entry the II equilibrium that contradicts the scale puzzle is likely to prevail. If, however, skills are complements or poor substitutes and the condition above is violated, the D equilibrium, which generates the scale puzzle prediction, is likely to prevail. If, on the other hand, the minority entered the labor market in large communities, the EI and DS equilibria were not likely to tip into the corresponding integrated equilibria and are in this sense more likely to prevail. The degree of coordination is also a key factor in determining the prevailing equilibria. If, for example, the minority community is divided or very heterogeneous, coordination is less likely and for example the EI equilibrium is less likely to topple into the II equilibrium. Detailed empirical investigation is necessary to investigate these issues in a greater depth. Certainly, available empirical evidence described above points at either of the EI, IE, or D equilibria that predict patterns of income inequality consistent with the scale puzzle.

5. *The Roles of Integration and Exclusion*

In this paper integration has a distinct role in determining relative income of minorities that challenges the habitual belief that integration necessarily leads to greater equality of social groups. While it is true that both minority and majority individuals benefit from integration through the increased network effects that integration brings about, integration also hurts minority individuals whenever it obliterates the inter-group substitution effect that benefits the smaller social group. In other words, it is possible that integration *decreases* the relative income of minority individuals, if the obliteration of the inter-group substitution effect offsets the relatively larger benefits that minority agents receive from integration through the individual income effect.[38] Proposition 13 below states that this is possible in the case of integration from the EI or IE regime into the II regime. It is worthwhile to note that, in contrast to the specialization regimes, integration produces imperfect substitutability under the diversification

[38] Minority individuals benefit from integration relatively more than majority individuals do, as they gain access to social interaction with the larger pool of majority individuals as compared to the access to the smaller group of minority individuals gained by majority individuals.

regimes and thus benefits minorities in terms of both individual income and inter-group substitution effects.

Proposition 13

There exist parametric values and I such that integration from EI or IE equilibrium into the II equilibrium hurts minority individuals such that their relative income ω decreases with integration, that is, $\omega^{EI}(I)=\omega^{IE}(I)>\omega^{II}(I)$. Moreover, if $\delta>0$ and ρ is large enough, there exists I such that minority is poorer than majority under EI and IE regimes and this inequality increases with integration, or, $1>\omega^{EI}(I)=\omega^{IE}(I)>\omega^{II}(I)$, resulting in larger inequality.

Proof:

Because $\omega^{II}(I)<1$ and there always exists I such that $\omega^{EI}(I)=\omega^{IE}(I)>1$ (c.f. Appendix 4), it must be that $\omega^{EI}(I)=\omega^{IE}(I)>\omega^{II}(I)$ for some I. Moreover, we know that if ρ is large enough there exists I such that $\omega^{EI}(I)=\omega^{IE}(I)<1$. We also know that in this case there also exists I such that $\omega^{EI}(I)=\omega^{IE}(I)=1$ and the functions $\omega^{EI}(I)$ and $\omega^{EI}(I)$ are continuous. Because $\omega^{II}(I)<1$ for all I whenever $\delta>0$, there exists I such that $1>\omega^{EI}(I)=\omega^{IE}(I)>\omega^{II}(I)$. This completes the proof.

Exclusion in exclusive networks has insofar been accepted as an exogenous institutional constraint on agents' behavior. Although it is fully symmetric across social groups and is supported by overwhelming evidence, it is sensible to put this constraint under scrutiny, as it prevents agents from individually benefiting from integration in all except the II equilibria. In particular, it is informative to ask whether the explanation of the scale puzzle developed in this paper remains valid, or, in other words, whether the EI, IE and D equilibria remain stable, if agents in exclusive networks permit inclusion of individuals from the other social group. The answer is yes, if the excluded agents individually choose not to join exclusive networks of the other social group even when allowed to do so. This is the case whenever social distance is large enough such that the inequality (16) holds.[39] Under this condition, taking the D equilibrium as the original state, while minority and majority individuals are integrated in their workplaces, they remain segregated in their neighborhoods even if the institutional barriers to integrate are removed. In this sense, exclusive behavior is not necessary to sustain differentiation of individuals' efficient labors across social groups. In particular, if there exist two different (types of) skills and a degree of segregation in skill acquisition prevails, as in the EI, IE, and D equilibria, network effects and social distances are sufficient to perpetuate this segregation whenever the inequality above holds. It is this segregation resulting in skill differentiation across social groups that gives rise to both the inter-group substitution and individual income effects as described above and thus generates patterns of income inequality consistent with the scale puzzle. This generalizes the argument of the paper to societies without institutional exclusion.

6. Conclusions

This paper provides theoretical explanation of the persistent patterns of the income inequality between minorities and majorities. Namely, I have shown that network effects and social distances in skill acquisition directly favor the members of larger social groups through the individual income effect, and that, coupled with asymmetric information in the labor market, suffice to sustain equilibria in which minority and majority individuals choose different skills or combinations of skills and thus supply imperfectly substitutable efficient labors on the labor market. In consequence, the prices of minority and majority efficient labors follow the textbook economic law that scarcer goods sell at higher price than the more abundant ones and thus network effects and social distances indirectly, through the inter-group substitution effect, favor smaller social groups who on aggregate supply less efficient labor, ceteris paribus. I have established above that the individual income and inter-group substitution effects explain the scale puzzle for a wide range of parameters. Importantly, neither discrimination on the labor market, nor asymmetry of individual characteristics across social groups is necessary to obtain this result. Moreover, elimination of exclusion does not necessarily lead to integration and the segregation equilibria that predict the scale puzzle are sustained whenever the size of minority or the social

[39] The proof is omitted at this point and is obvious from the proof of Proposition 5 in Appendix 3.

distance between social groups is large enough. An important result of this paper is that integration may increase as well as decrease the relative income of minority as compared to majority individuals. In this sense, although there certainly are benefits of integration for both minority and majority individuals, integration is not necessarily a universal remedy against inequality across social groups. The analysis also offers an explanation why minorities tend to spend more time in family and neighborhood socializing than in socializing in schools and other integrated social institutions. As shown in the section about diversification, under some circumstances minority agents need to compensate for the low efficiency of their segregated networks by spending relatively more time in these networks than in integrated networks as compared to majority individuals. If this is the case, increasing the efficiency of integrated social networks such as schools may drive minorities out of these institutions. Nevertheless, minority agents unambiguously acquire relatively less exclusive skills than the majority individuals under the integrated diversification regime. The analysis also predicts that small minorities may benefit from the scarcity of their efficient labor to such a degree that it outweighs their disadvantage in acquiring human capital and, as a result, they outperform the majority in income terms. In addition, as suggested in the literature, I find some support for the hypothesis that the relationship between minority-majority relative income and minority relative size exhibits convexity. Finally, I have clarified the role of social organization of skill acquisition in explaining the scale puzzle.

This paper opens several potential areas of future research. First, the analysis of which particular regime prevails in which labor market is limited in this paper and thus no predictions are drawn for particular minorities. Second, it is worthwhile to investigate the relative wealths of an arbitrary number of social groups in one habitat. Finally, an empirical analysis explicitly testing and disentangling the scarcity and efficiency effects and the parameters of the model is desirable.

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Appendices

Appendix 1

Take the allocation of time $\{T_{k,x}, T_{k,n}\}$ as given and rewrite the agents' problem (7) as follows.

$$\begin{aligned}
&\underset{L_{k,m}}{Max}\left|H_k\right| \\
&\quad s.t. \\
&H_k = \left(\left(S_{k,x}\left(T_{k,x}-L_{k,x}\right)\right)^{(\varepsilon-1)/\varepsilon} + \left(S_{k,n}\left(T_{k,n}-L_{k,n}\right)\right)^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)} \\
&S_{k,m} = L_{k,m}^{\phi}\left(1+N_{k,m}\right) \\
&L_{k,m} \geq 0
\end{aligned}$$

Plugging the technological constraints into the objective function and deriving the first order conditions with respect to $L_{k,m}$, I find that the optimality conditions on time distribution between acquisition and utilization of skills are $L_{k,m} = \frac{\phi}{1+\phi}T_{k,m}$. The sufficiency conditions are also satisfied, as the objective function is concave at the optimal allocation. This completes the proof.

Appendix 2

Substitute for $L_{k,m}$ in the agent's problem (7) using the optimality conditions from (8). In addition, substitute for $S_{k,m}$ from the skill acquisition technology and use the definition of $\tilde{N}_{k,m}$. Consequently, the agent's problem is:

$$\begin{aligned}
&\underset{T_{k,x},T_{k,n}}{Max}\left|\left(\tilde{N}_{k,x}T_{k,x}^{(\phi+1)\frac{(\varepsilon-1)}{\varepsilon}} + \tilde{N}_{k,n}T_{k,n}^{(\phi+1)\frac{(\varepsilon-1)}{\varepsilon}}\right)^{\varepsilon/(\varepsilon-1)}\right| \\
&\quad s.t. \\
&T_{k,x} + T_{k,n} \leq 1 \\
&T_{k,m} \geq 0
\end{aligned}$$

Note that if $\varepsilon = (\phi+1)/\phi > 1$ the problem boils down to

$$\begin{aligned}
&\underset{T_{k,x},T_{k,n}}{Max}\left|\left(\tilde{N}_{k,x}T_{k,x} + \tilde{N}_{k,n}T_{k,n}\right)^{\varepsilon/(\varepsilon-1)}\right| \\
&\quad s.t. \\
&T_{k,x} + T_{k,n} \leq 1 \\
&T_{k,m} \geq 0
\end{aligned}$$

and, obviously, the maximum is the corner solution with the higher $\tilde{N}_{k,m}$ and thus $\tilde{N}_{k,m}^{\varepsilon/(\varepsilon-1)}$.

Now assume $\varepsilon \neq (\phi+1)/\phi$. Form the Kuhn-Tucker Lagrangian $L(\bullet)=\left(\tilde{N}_{k,x}T_{k,x}^{(\phi+1)\frac{(\varepsilon-1)}{\varepsilon}}+\tilde{N}_{k,n}T_{k,n}^{(\phi+1)\frac{(\varepsilon-1)}{\varepsilon}}\right)^{\varepsilon/(\varepsilon-1)}-\lambda(T_{k,x}+T_{k,n}-1)$ to obtain the first order Kuhn-Tucker conditions for a maximum

$$
\begin{aligned}
&\tilde{N}_{k,x}(1+\phi)T_{k,x}^{\frac{\phi\varepsilon-\phi-1}{\varepsilon}}H_k-\lambda \leq 0\\
&\tilde{N}_{k,n}(1+\phi)T_{k,n}^{\frac{\phi\varepsilon-\phi-1}{\varepsilon}}H_k-\lambda \leq 0\\
&T_{k,x}\left(\tilde{N}_{k,x}(1+\phi)T_{k,x}^{\frac{\phi\varepsilon-\phi-1}{\varepsilon}}H_k-\lambda\right)=0\\
&T_{k,n}\left(\tilde{N}_{k,n}(1+\phi)T_{k,n}^{\frac{\phi\varepsilon-\phi-1}{\varepsilon}}H_k-\lambda\right)=0\\
&1-T_{k,x}-T_{k,n}\geq 0\\
&\lambda(1-T_{k,x}-T_{k,n})=0
\end{aligned}
.
$$

First realize that $H_k \geq 0$, $H_k^* > 0$, and that both H_k and H_k^* are finite, for any admissible parametric values on the constrained domain. The finiteness follows from the limited time resources and the fact that, for any admissible parametric values, the production technology of efficient labor does not permit infinite output with limited resources. Moreover, it is always possible to allocate some resources to production of efficient labor such that it is positive and thus $H_k^* > 0$. Now, to satisfy the first two Kuhn-Tucker conditions realize that $\lambda > 0$, otherwise both $T_{k,x}$ and $T_{k,n}$ would have to be zero implying $H_k^* = 0$, which is inadmissible. Therefore, the time constraint is binding.

Now use the Kuhn-Tucker conditions to study the corner solution $T_{k,x} = 0$ and $T_{k,n} = 1$. Realize that $\left(\tilde{N}_{k,n}(1+\phi)T_{k,n}^{\frac{\phi\varepsilon-\phi-1}{\varepsilon}}H_k-\lambda=0 \wedge \{T_{k,x},T_{k,n}\}=\{0,1\}\right) \Rightarrow \tilde{N}_{k,n}(1+\phi)H_k=\lambda>0$. Because both $\tilde{N}_{k,n}$ and H_k are finite, λ is finite as well. Now, substituting for λ,

$\tilde{N}_{k,x}(1+\phi)T_{k,x}^{\frac{\phi\varepsilon-\phi-1}{\varepsilon}}H_k-\lambda\le 0 \Leftrightarrow \tilde{N}_{k,x}T_{k,x}^{\frac{\phi\varepsilon-\phi-1}{\varepsilon}}-\tilde{N}_{k,n}\le 0$. Because $T_{k,x}$=0 and $\tilde{N}_{k,x}T_{k,x}^{\frac{\phi\varepsilon-\phi-1}{\varepsilon}}-\tilde{N}_{k,n}\le 0$, it follows that $\tilde{N}_{k,x}T_{k,x}^{\frac{\phi\varepsilon-\phi-1}{\varepsilon}}$ is finite (and well defined). This is the case whenever the exponent is larger than zero, that is, whenever $\varepsilon>(\phi+1)/\phi$. This condition is thus the necessary condition for the studied corner solution to be the maximum. By symmetry, the same necessary condition must hold for the corner solution $T_{k,n}$ = 0 then $T_{k,x}$ = 1 to be the maximum.

If $T_{k,x}$ > 0 and $T_{k,n}$ > 0, in the interior solution, the first two Kuhn-Tucker conditions are equalities and we obtain the following result, where the definitions of $\tilde{N}_{k,m}$ are used.

$$\tilde{N}_{k,x}(1+\phi)T_{k,x}^{\frac{\phi\varepsilon-\phi-1}{\varepsilon}}H_k-\lambda=0\wedge\tilde{N}_{k,n}(1+\phi)T_{k,n}^{\frac{\phi\varepsilon-\phi-1}{\varepsilon}}H_k-\lambda=0\Rightarrow$$

$$1=\frac{\lambda}{\lambda}=\frac{\tilde{N}_{k,x}T_{k,x}^{\frac{\phi\varepsilon-\phi-1}{\varepsilon}}}{\tilde{N}_{k,n}T_{k,n}^{\frac{\phi\varepsilon-\phi-1}{\varepsilon}}}=\frac{\tilde{N}_{k,x}}{\tilde{N}_{k,n}}\left(\frac{T_{k,x}}{T_{k,n}}\right)^{\frac{\phi\varepsilon-\phi-1}{\varepsilon}}=\left(\frac{1+N_{k,x}}{1+N_{k,n}}\right)^{(\varepsilon-1)/\varepsilon}\left(\frac{T_{k,x}}{T_{k,n}}\right)^{\frac{\phi\varepsilon-\phi-1}{\varepsilon}}$$

It follows that $\frac{T_{k,x}}{T_{k,n}}=\left(\frac{1+N_{k,x}}{1+N_{k,n}}\right)^{\frac{\varepsilon-1}{1+\phi-\varepsilon\phi}}$. Using this and the time constraint we obtain $T_{k,x}=\frac{t_k}{1+t_k}$ and $T_{k,n}=\frac{1}{1+t_k}$.

There are three possible candidates for the maximum, two corner solutions and one interior solution. Given the results above, evaluating the objective function at each of these candidates, the values of H_k at the candidate time allocations are

$$H_k\Big|_{T_{k,x}=\frac{t_k}{1+t_k},T_{k,n}=\frac{1}{1+t_k}}\left(\frac{t_k}{1+t_k},\frac{1}{1+t_k}\right)=\left(\tilde{N}_{k,x}\left(\frac{t_k}{1+t_k}\right)^{(\phi+1)\frac{(\varepsilon-1)}{\varepsilon}}+\tilde{N}_{k,n}\left(\frac{1}{1+t_k}\right)^{(\phi+1)\frac{(\varepsilon-1)}{\varepsilon}}\right)^{\varepsilon/(\varepsilon-1)}$$

$$H_k\Big|_{T_{k,x}=0,T_{k,n}=1}=\left(\tilde{N}_{k,x}T_{k,x}^{(\phi+1)\frac{(\varepsilon-1)}{\varepsilon}}+\tilde{N}_nT_{k,n}^{(\phi+1)\frac{(\varepsilon-1)}{\varepsilon}}\right)^{\varepsilon/(\varepsilon-1)}=\tilde{N}_{k,n}^{\varepsilon/(\varepsilon-1)}$$

$$H_k\Big|_{T_{k,x}=1,T_{k,n}=0}=\left(\tilde{N}_{k,x}T_{k,x}^{(\phi+1)\frac{(\varepsilon-1)}{\varepsilon}}+\tilde{N}_{k,n}T_{k,n}^{(\phi+1)\frac{(\varepsilon-1)}{\varepsilon}}\right)^{\varepsilon/(\varepsilon-1)}=\tilde{N}_{k,x}^{\varepsilon/(\varepsilon-1)}$$

Now I verify the sufficient conditions under each of these candidates is the maximum. Note that from the Kuhn-Tucker first order conditions the time constraint is binding and therefore we can rewrite agent's problem in the following way

$$\underset{T_{k,x}}{Max}\left|\left(\tilde{N}_{k,x}T_{k,x}^{(\phi+1)\frac{(\varepsilon-1)}{\varepsilon}}+\tilde{N}_{k,n}\left(1-T_{k,x}\right)^{(\phi+1)\frac{(\varepsilon-1)}{\varepsilon}}\right)^{\varepsilon/(\varepsilon-1)}\right|$$
$$s.t.$$
$$1\geq T_{k,x}\geq 0$$

Consider the new objective function on the constrained domain. If it is convex for any $T_{k,x}>0$ from the constrained domain, that is if its second derivate is positive at any permissible $T_{k,x}>0$, then the interior candidate cannot be the maximum. Now I prove that the objective function is indeed convex for any $T_{k,x}>0$ whenever $\varepsilon>(\phi+1)/\phi$. Differentiate the objective function to obtain

$$\frac{\partial^2\left(\left(\tilde{N}_{k,x}T_{k,x}^{(\phi+1)\frac{(\varepsilon-1)}{\varepsilon}}+\tilde{N}_{k,n}\left(1-T_{k,x}\right)^{(\phi+1)\frac{(\varepsilon-1)}{\varepsilon}}\right)^{\varepsilon/(\varepsilon-1)}\right)}{\partial T_{k,x}^2}=$$

$$\frac{(1+\phi)H_k\left(-\tilde{N}_{k,x}\tilde{N}_{k,n}(\phi\varepsilon-\phi-1)\left(1-T_{k,x}\right)^{\frac{1+\phi}{\varepsilon}}\left(T_{k,x}\left(1-T_{k,x}\right)\right)^{\phi}T_{k,x}^{\frac{1+\phi}{\varepsilon}}-\varepsilon\phi\left(1-T_{k,x}\right)T_{k,x}\left(\tilde{N}_{k,x}T_{k,x}^{\phi}\left(1-T_{k,x}\right)^{\frac{\phi+1}{\varepsilon}}-\tilde{N}_{k,n}\left(1-T_{k,x}\right)^{\phi}T_{k,x}^{\frac{\phi+1}{\varepsilon}}\right)^2\right)}{-\varepsilon\left(1-T_{k,x}\right)T_{k,x}\left(\tilde{N}_{k,x}T_{k,x}^{\phi+1}\left(1-T_{k,x}\right)^{\frac{\phi+1}{\varepsilon}}+\tilde{N}_{k,n}\left(1-T_{k,x}\right)^{\phi+1}T_{k,x}^{\frac{\phi+1}{\varepsilon}}\right)^2}$$

Now it is straightforward to see that

$$Sign\left\{\frac{\partial^2 H_k}{\partial T_{k,x}^2}\right\} = Sign\left\{\tilde{N}_{k,x}\tilde{N}_{k,n}(\phi\varepsilon-\phi-1)(1-T_{k,x})^{\frac{1+\phi}{\varepsilon}}\left(T_{k,x}(1-T_{k,x})\right)^{\phi}T_{k,x}^{\frac{1+\phi}{\varepsilon}}+\varepsilon\phi(1-T_{k,x})T_{k,x}\left(\tilde{N}_{k,x}T_{k,x}^{\phi}(1-T_{k,x})^{\frac{\phi+1}{\varepsilon}}-\tilde{N}_{k,n}(1-T_{k,x})^{\phi}T_{k,x}^{\frac{\phi+1}{\varepsilon}}\right)^2\right\}$$

and that the term on the right hand side has a positive sign for all $T_{k,x}>0$ whenever $\phi\varepsilon-\phi-1>0 \Leftrightarrow \varepsilon>(\phi+1)/\phi$. Thus, the interior solution cannot be the maximum if $\varepsilon>(\phi+1)/\phi$. As argued above, the objective function is certainly continuous and bounded on the constrained domain. Therefore, there must exist a maximum. Having excluded the interior candidate, the corner solution with the higher $\tilde{N}_{k,m}$ and thus $\tilde{N}_{k,m}^{\varepsilon/(\varepsilon-1)}$ is the maximum on the constrained domain.

From the Kuhn-Tucker conditions we know that if $\varepsilon<(\phi+1)/\phi$, none of the corner candidates can be the maximum. As above, because the objective function is continuous and bounded on the constrained domain, there must exist a maximum. Being the only remaining possibility, the interior solution is the maximum and $H_k^* = H_k\left(\frac{t_k}{1+t_k},\frac{1}{1+t_k}\right)$ whenever $\varepsilon<(\phi+1)/\phi$.

For the implication in (9), note that $\varepsilon/(\varepsilon-1)>0$. This completes the proof[40].

Appendix 3

Recall that, due to statistical discrimination, employers differentiate wages per unit of efficient labor across social groups but not within groups and that individuals cannot change their group membership. Therefore, agents pick those networks to acquire skills that they are allowed to join and that offer the largest network effects and thus are the most efficient. Applying (8) to network effect specification in (14), the network effects are:

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[40] A simpler way to determine which of the three candidates is the maximum is possible, noting that $(\phi+1)\frac{(\varepsilon-1)}{\varepsilon}$ and $\frac{\varepsilon}{(\varepsilon-1)}$ fully determine the properties of the maximization problem. The lengthier and more formal approach was adopted here.

$$N_{i,m}^{r}(I_{m}^{r}L_{i,m}, J_{m}^{r}L_{j,m}, \delta) = \left(I_{m}^{r} + \frac{1}{1+\delta}J_{m}^{r}\right)^{\gamma}\left(\frac{\phi}{1+\phi}\right)^{\gamma}$$

$$N_{j,m}^{r}(I_{m}^{r}L_{i,m}, J_{m}^{r}L_{j,m}, \delta) = \left(\frac{1}{1+\delta}I_{m}^{r} + J_{m}^{r}\right)^{\gamma}\left(\frac{\phi}{1+\phi}\right)^{\gamma}$$

where the superscript $r \in \{EE, II, EI, IE\}$ denotes the prevailing regime. The network effects in the EE equilibrium are

$$N_{i,x}^{EE} = I^{\gamma}\left(\frac{\phi}{1+\phi}\right)^{\gamma}$$

$$N_{j,x}^{EE} = J^{\gamma}\left(\frac{\phi}{1+\phi}\right)^{\gamma}$$

$$N_{k,n}^{EE} = 0$$

Because network effects in exclusive networks are always larger than the zero network effects that a single agent would face upon deviation to the inclusive network of zero size, the EE equilibrium is always stable. Consider now the II equilibrium. The network effects in inclusive networks are

$$N_{i,n}^{II} = \left(I + \frac{1}{1+\delta}J\right)^{\gamma}\left(\frac{\phi}{1+\phi}\right)^{\gamma}$$

$$N_{j,n}^{II} = \left(\frac{1}{1+\delta}I + J\right)^{\gamma}\left(\frac{\phi}{1+\phi}\right)^{\gamma}$$

$$N_{k,x}^{II} = 0$$

and again deviation from the II equilibrium is unbeneficial because $N_{i,n}^{II} > N_{i,x}^{II} \wedge N_{j,n}^{II} > N_{j,x}^{II}$. Therefore, the II equilibrium is always stable. Now investigate the IE equilibrium. Here the stability conditions are

$$N_{i,n}^{IE} \geq N_{i,x}^{IE} \Leftrightarrow I \geq 0$$

$$N_{j,x}^{IE} \geq N_{j,n}^{IE} \Leftrightarrow J^{\gamma} \geq \frac{1}{1+\delta}I^{\gamma}.$$

Both of these conditions are always satisfied and, consequently, the IE equilibrium is always stable. Considering the EI regime, formally, it must be that in the EI equilibrium,

$$N_{i,x}^{EI} \geq N_{i,n}^{EI} \Leftrightarrow I^{\gamma} \geq \frac{1}{1+\delta}J^{\gamma}$$

$$N_{j,n}^{EI} \geq N_{j,x}^{EI} \Leftrightarrow J^{\gamma} \geq 0$$

The first condition holds if the minority (or social distance) is large enough. Obviously, the second condition holds always. Now consider the IIS regime. In any stable IIS regime it must be that agents prefer staying in the inclusive networks occupied by their own social group, enjoying network effects $N_{i,n}^{IIS}=I^{\gamma}\left(\frac{\phi}{1+\phi}\right)^{\gamma}$ and $N_{j,n}^{IIS}=J^{\gamma}\left(\frac{\phi}{1+\phi}\right)^{\gamma}$ to deviating to the inclusive social network of the other social group and obtaining network benefits $\frac{1}{1+\delta}J^{\gamma}\left(\frac{\phi}{1+\phi}\right)^{\gamma}$ and $\frac{1}{1+\delta}I^{\gamma}\left(\frac{\phi}{1+\phi}\right)^{\gamma}$, respectively. These conditions hold if and only if $I^{\gamma}\geq\frac{1}{1+\delta}J^{\gamma}$ and $J^{\gamma}\geq\frac{1}{1+\delta}I^{\gamma}$, of which the second is always satisfied. Note that

$$I^{\gamma}\geq\frac{1}{1+\delta}J^{\gamma}\Leftrightarrow\frac{I}{1-I}\geq\left(\frac{1}{1+\delta}\right)^{\frac{1}{\gamma}}\Leftrightarrow I\geq\left((1+\delta)^{\frac{1}{\gamma}}+1\right)^{-1}.$$

This completes the proof.

Appendix 4

Recall I+J=1, $I\in[0, 0.5]$. Realize that the $\omega(I)$ curve is continuous for all admissible parametric values and I>0. At I=0 it is continuous from the right. The rest of the proof consists of three steps. First, we show that the $\omega(I)$ is larger than one at $I\rightarrow 0$ whenever ρ is finite, that is, the $\omega(I)$ curve starts above one from the left.

$$\lim_{I\rightarrow 0}\left(\left(\frac{I}{1-I}\right)^{\frac{-1}{\rho}}\left(\frac{1+I^{\gamma}\left(\frac{\phi}{1+\phi}\right)^{\gamma}}{1+(1-I)^{\gamma}\left(\frac{\phi}{1+\phi}\right)^{\gamma}}\right)^{\frac{\rho-1}{\rho}}\right)=\infty$$

The second step demonstrates that $\omega(I)$=1 at I=0.5, that is, equally large social groups are equally wealthy.

$$\omega(0.5)=\left(\frac{0.5}{1-0.5}\right)^{\frac{-1}{\rho}}\left(\frac{1+0.5^{\gamma}\left(\frac{\phi}{1+\phi}\right)^{\gamma}}{1+(1-0.5)^{\gamma}\left(\frac{\phi}{1+\phi}\right)^{\gamma}}\right)^{\frac{\rho-1}{\rho}}=1$$

In the last step it is shown that for some parametric values the $\omega(I)$ curve is upward sloping at I=0.5.

$$\left.\frac{\partial(\omega(I))}{\partial I}\right|_{I=0.5}>0\Leftrightarrow\left(\frac{\phi}{1+\phi}\right)^{\gamma}(\rho\gamma-\gamma-1)>2^{\gamma}$$

Therefore, the continuous $\omega(I)$ curve starts above one and reaches the point $\omega(0.5)=1$ from below if the parametric condition holds. As a result, there must be a segment of the $\omega(I)$ curve that is downward sloping and where $\omega(I)<1$. There always exists $\rho^*>0$ such that the parametric condition above is satisfied for all $\rho>\rho^*$. In particular, $\rho^*=\left(\left(2\frac{\phi+1}{\phi}\right)^{\gamma}+\gamma+1\right)\Big/\gamma$.

Having these results and recalling the continuity of $\omega(I)$, there exists $\bar{I}$ such that $\omega(\bar{I})=1$ and for $I\in[0,\bar{I})$ minority individuals have larger income than majority individuals, that is, $\omega(I)>1$ in this range.

This completes the proof.

Appendix 5

Recall that, agents always pick those networks to acquire skills that they are allowed to join and that offer the largest network effects and thus are the most efficient. The only possibility for an individual to deviate in the equilibrium where both social groups acquire inclusive skills in one inclusive social network is to form his own inclusive social network. Because such network would offer zero network benefits, as compared to substantial network benefits in the integrated inclusive network, such deviation is never profitable and therefore the equilibrium is stable.

If, on the other hand, inclusive skills are acquired in two segregated inclusive networks, for this regime to be stable it must be that all individuals prefer staying in the inclusive networks occupied by their own social group, enjoying network effects $N_{i,n}^{DS}=I^{\gamma}$ and $N_{j,n}^{DS}=J^{\gamma}$ to deviating to the inclusive social network of the other social group and obtaining network benefits

$\frac{1}{1+\delta}J^{\gamma}$ and $\frac{1}{1+\delta}I^{\gamma}$, respectively. These conditions hold if and only if $I^{\gamma} \geq \frac{1}{1+\delta}J^{\gamma}$ and $J^{\gamma} \geq \frac{1}{1+\delta}I^{\gamma}$, of which the second is always satisfied. Note that $I^{\gamma} \geq \frac{1}{1+\delta}J^{\gamma} \Leftrightarrow \frac{I}{1-I} \geq \left(\frac{1}{1+\delta}\right)^{\frac{1}{\gamma}} \Leftrightarrow I \geq \left((1+\delta)^{\frac{1}{\gamma}}+1\right)^{-1}$. This completes the proof.

Appendix 6

Here I prove the inequality in Proposition 10 and the preceding inequalities.

$$I < \frac{1}{2} \Leftrightarrow \left(\frac{1}{1+\delta}\frac{I}{(1-I)}+1\right)^{\gamma} < \left(1+\frac{1}{1+\delta}\frac{(1-I)}{I}\right)^{\gamma} \Leftrightarrow \frac{1+\left(\frac{1}{1+\delta}I+(1-I)\right)^{\gamma}}{1+\left(I+\frac{1}{1+\delta}(1-I)\right)^{\gamma}} =$$

$$= \frac{1+(1-I)^{\gamma}\left(\frac{1}{1+\delta}\frac{I}{(1-I)}+1\right)^{\gamma}}{1+I^{\gamma}\left(1+\frac{1}{1+\delta}\frac{(1-I)}{I}\right)^{\gamma}} < \frac{1+(1-I)^{\gamma}}{1+I^{\gamma}} \Leftrightarrow \frac{1+I^{\gamma}}{1+(1-I)^{\gamma}}\frac{1+\left(\frac{1}{1+\delta}I+(1-I)\right)^{\gamma}}{1+\left(I+\frac{1}{1+\delta}(1-I)\right)^{\gamma}} < 1 \Leftrightarrow$$

1) $$\left(\frac{1+I^{\gamma}}{1+(1-I)^{\gamma}}\frac{1+\left(\frac{1}{1+\delta}I+(1-I)\right)^{\gamma}}{1+\left(I+\frac{1}{1+\delta}(1-I)\right)^{\gamma}}\right)^{\frac{\varepsilon-1}{1+\phi-\varepsilon\phi}} = \frac{t_i^D}{t_j^D} = \frac{l_i^D}{l_j^D} > 1 \Leftrightarrow \frac{\varepsilon-1}{1+\phi-\varepsilon\phi} < 1 \Leftarrow \varepsilon < 1 \wedge \varepsilon < (1+\phi)/\phi$$

2) $$\frac{t_i^D}{t_j^D} = \frac{l_i^D}{l_j^D} < 1 \Leftrightarrow \frac{\varepsilon-1}{1+\phi-\varepsilon\phi} > 1 \Leftarrow \varepsilon > 1 \wedge \varepsilon < (1+\phi)/\phi$$

3) $$\frac{t_i^D}{t_j^D} = \frac{l_i^D}{l_j^D} = 1 \Leftrightarrow \frac{\varepsilon-1}{1+\phi-\varepsilon\phi} = 0 \Leftarrow \varepsilon = 1 \wedge \varepsilon < (1+\phi)/\phi$$

4) $$\left(\frac{1+I^{\gamma}}{1+(1-I)^{\gamma}}\frac{1+\left(\frac{1}{1+\delta}I+(1-I)\right)^{\gamma}}{1+\left(I+\frac{1}{1+\delta}(1-I)\right)^{\gamma}}\right)^{\frac{1}{1+\phi-\varepsilon\phi}} = \frac{S_{i,x}^D}{S_{i,n}^D} \Big/ \frac{S_{j,x}^D}{S_{j,n}^D} < 1 \quad \forall \varepsilon < (1+\phi)/\phi$$

This completes the proof.

Appendix 7

$\omega^P(I)=$

$$\left(\frac{I}{1-I}\right)^{\frac{-1}{\rho}}\left(\frac{\left(\frac{1}{1+\left(\frac{1+I^{\gamma}}{1+\left(I+\frac{1}{1+\delta}(1-I)\right)^{\gamma}}\right)^{\frac{\varepsilon-1}{1+\phi-\varepsilon\phi}}}\right)^{\phi+1}\left(\left(\left(\left(\frac{1+I^{\gamma}}{1+\left(I+\frac{1}{1+\delta}(1-I)\right)^{\gamma}}\right)^{\frac{\varepsilon-1}{1+\phi-\varepsilon\phi}}\right)^{\phi+1}\left(1+I^{\gamma}\right)\right)^{(\varepsilon-1)/\varepsilon}+\left(1+\left(I+\frac{1}{1+\delta}(1-I)\right)^{\gamma}\right)^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)}}{\left(\frac{1}{1+\left(\frac{1+(1-I)^{\gamma}}{1+\left(\frac{1}{1+\delta}I+(1-I)\right)^{\gamma}}\right)^{\frac{\varepsilon-1}{1+\phi-\varepsilon\phi}}}\right)^{\phi+1}\left(\left(\left(\left(\frac{1+(1-I)^{\gamma}}{1+\left(\frac{1}{1+\delta}I+(1-I)\right)^{\gamma}}\right)^{\frac{\varepsilon-1}{1+\phi-\varepsilon\phi}}\right)^{\phi+1}\left(1+(1-I)^{\gamma}\right)\right)^{(\varepsilon-1)/\varepsilon}+\left(1+\left(\frac{1}{1+\delta}I+(1-I)\right)^{\gamma}\right)^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)}}\right)^{\frac{\rho-1}{\rho}}$$

Appendix 8

To prove this proposition, I state and prove the following lemma:

Lemma 1:

There is always $I>0$ such that $h(I)<1$; $\epsilon\neq1$.

Proof (superscript D omitted):

1) $H_i(I)\big|_{I\to0.5}=H_j(I)\big|_{I\to0.5}$, by the symmetry in this case.

2) $H_i(I)\big|_{I\to0}<H_j(I)\big|_{I\to0}$ and both well defined if $\epsilon\neq1$

3) H_i and H_j are continuous in I on the domain.

If 1), 2), and 3) then there exists I*>0 such that $H_i(I^*) < H_j(I^*)$ and thus $h(I^*) < 1$.

Steps 1, 3, and the final implication are straightforward. Step 2 needs a detailed proof:

$$H_i(I)\big|_{I\to 0} = \left(1+\left(\frac{1}{1+\left(\frac{1}{1+\delta}\right)^{\gamma}}\right)^{\frac{1-\varepsilon}{\phi\varepsilon-\phi-1}}\right)^{-1-\phi}\left(\left(\left(\left(\frac{1}{1+\left(\frac{1}{1+\delta}\right)^{\gamma}}\right)^{\frac{1-\varepsilon}{\phi\varepsilon-\phi-1}}\right)^{1+\phi}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(1+\left(\frac{1}{1+\delta}\right)^{\gamma}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

After some algebra we get $H_i(I)\big|_{I\to 0} = \left(1+\left(\frac{1}{1+\delta}\right)^{\gamma}\right)\left(\left(1+\left(\frac{1}{1+\delta}\right)^{\gamma}\right)^{\frac{\varepsilon-1}{\phi\varepsilon-\phi-1}}+1\right)^{\frac{1}{\varepsilon-1}-\phi}$. Now consider $H_j(I)\big|_{I\to 0}$. It is easy to see that $H_j(I)\big|_{I\to 0} = 2^{\frac{\varepsilon}{\varepsilon-1}-\phi}$. Moreover, $0 < \left(1+\left(\frac{1}{1+\delta}\right)^{\gamma}\right) \le 2$. Therefore

$$H_i(I)\big|_{I\to 0} = \left(1+\left(\frac{1}{1+\delta}\right)^{\gamma}\right)\left(\left(1+\left(\frac{1}{1+\delta}\right)^{\gamma}\right)^{\frac{\varepsilon-1}{\phi\varepsilon-\phi-1}}+1\right)^{\frac{1}{\varepsilon-1}-\phi} \le 2\left(\left(1+\left(\frac{1}{1+\delta}\right)^{\gamma}\right)^{\frac{\varepsilon-1}{\phi\varepsilon-\phi-1}}+1\right)^{\frac{1}{\varepsilon-1}-\phi}.$$

Having this result, dividing both sides of equation by 2, we obtain

Lemma 2:

$$\left(\left(1+\left(\frac{1}{1+\delta}\right)^{\gamma}\right)^{\frac{\varepsilon-1}{\phi\varepsilon-\phi-1}}+1\right)^{\frac{1}{\varepsilon-1}-\phi} < 2^{\frac{1}{\varepsilon-1}-\phi} \Rightarrow H_i(I)\big|_{I\to 0} < H_j(I)\big|_{I\to 0}$$

I prove that the left hand inequality indeed holds and then the whole implication. Realize that $\varepsilon < \frac{\phi+1}{\phi} \Leftrightarrow \left((\varepsilon>1)\wedge\left(\frac{1}{\varepsilon-1}-\phi>0\right)\right)\vee\left((\varepsilon<1)\wedge\left(\frac{1}{\varepsilon-1}-\phi<0\right)\right)\vee \varepsilon=1.$

Now consider all the possibilities, except for ϵ=1, which is excluded by assumption. Realize also that $1 \le \left(1+\left(\frac{1}{1+\delta}\right)^{\gamma}\right) \le 2$.

Consider Case A when $(\varepsilon > 1) \wedge \left(\frac{1}{\varepsilon-1} - \phi > 0\right)$. Due to the second inequality and noting that the term in the brackets in Claim 1 is larger or equal to one, we can take away the powers without changing the inequality in Claim 1. Rewrite that inequality to obtain $\left(1+\left(\frac{1}{1+\delta}\right)^{\gamma}\right)^{\frac{\varepsilon-1}{\phi\varepsilon-\phi-1}} < 1$. Now realize that $\frac{\varepsilon-1}{\phi\varepsilon-\phi-1} < 0$. Because $1+\left(\frac{1}{1+\delta}\right)^{\gamma} \ge 1$ the left hand side inequality in Lemma 2 holds and it must be that $H_i(I)\big|_{I\to 0} \le H_j(I)\big|_{I\to 0}$. If δ finite, $1+\left(\frac{1}{1+\delta}\right)^{\gamma} > 1$, and $H_i(I)\big|_{I\to 0} < H_j(I)\big|_{I\to 0}$. If $\delta \to \infty$, we obtain $H_i(I)\big|_{I\to 0} < H_j(I)\big|_{I\to 0}$ directly from the inequality $H_i(I)\big|_{I\to 0,\delta\to\infty} = 2^{\frac{\varepsilon}{\varepsilon-1}-\phi-1} < 2^{\frac{\varepsilon}{\varepsilon-1}-\phi} = H_j(I)\big|_{I\to 0,\delta\to\infty}$. Thus $H_i(I)\big|_{I\to 0} < H_j(I)\big|_{I\to 0}$ in Case A.

Consider now Case B where $(\varepsilon < 1) \wedge \left(\frac{1}{\varepsilon-1} - \phi < 0\right)$. As in Case A, we can take away the powers but the direction of the inequality must be changed because of the powers are now negative. Thus, we can rewrite the inequality in Claim 1 to obtain $\left(1+\left(\frac{1}{1+\delta}\right)^{\gamma}\right)^{\frac{\varepsilon-1}{\phi\varepsilon-\phi-1}} > 1$. Now realize that $\frac{\varepsilon-1}{\phi\varepsilon-\phi-1} > 0$. This and that $1+\left(\frac{1}{1+\delta}\right)^{\gamma} \ge 1$ prove that the inequality in Lemma 2 holds and that $H_i(I)\big|_{I\to 0} < H_j(I)\big|_{I\to 0}$ in Case B as well. Therefore, Cases A and Case B together prove that the $H_i(I)\big|_{I\to 0} < H_j(I)\big|_{I\to 0}$ and $h(I)\big|_{I\to 0} < 1$ for all admissible parametric values.

This proves Lemma 1. Hence, there exists $I^*>0$ such that $H_i(I^*) < H_j(I^*)$ and thus $h(I^*) < 1$ under the D regime and we can proceed with the proof of Proposition 11.

Recalling that $\omega^D(I) = \left(\frac{I}{1-I} h^D(I)\right)^{\frac{-1}{\rho}} h^D(I)$, take the limit of $\omega^D(I)$ as ρ approaches infinity. $\underset{\rho\to\infty}{Lim}\left(\omega^D(I)\right) = \underset{\rho\to\infty}{Lim}\left(\left(\frac{I}{1-I} h^D(I)\right)^{\frac{-1}{\rho}} h^D(I)\right) = h^D(I)$ As we see, this limit is equal to $h^D(I)$ which, from Lemma 1, is less than 1 for some I^*. It is because as ρ grows, the inter-group substitution effect disappears and the term $\left(\frac{I}{1-I} h^D(I)\right)^{\frac{-1}{\rho}}$ approaches 1. Thus, there is a point I^* at which the individual income effect dominates the inter-group substitution effect for ρ very large and $\omega^D(I^*) < 1$. In fact, it suffices that $\rho > \frac{Log\left(\frac{I}{(1-I)} h^D(I)\right)}{Log\left(h^D(I)\right)}$ at $I=I^*$ to obtain that $\omega^D(I^*) < 1$. Note that the inter-group substitution effect diminishes and approaches the lower bound zero while the individual income effect is constant as ρ increases.

Similarly to the first two steps of the proof of Proposition 6, $\omega^D(I)$ approaches infinity as I approaches zero and is one as I=0.5. Adding the fact that for ρ large enough there is I such that $\omega^D(I) < 1$ suffices to ensure that the $\omega^D(I)$ curve must at some point cross the $\omega^D(I) = 1$ line from above and that there must be a segment of this curve that lies below the $\omega^D(I) = 1$ line and is downward sloping. If ϵ=1, equation (21) boils down to $\omega^D(I) = \left(\frac{I}{1-I}\right)^{\frac{-1}{\rho}}$. Such function conflicts with the prediction that minorities tend to earn less than majorities. This completes the proof.