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Intertemporal and Spacial Location of Disposal Facilities*

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Abstract

Optimal capacity and location of a sequence of landfills are studied, and the interactions between both decisions are pointed out. The decision capacity has some spatial implications, because it affects the feasible region for the rest of landfills, and some temporal implications, because the capacity determines the lifetime of the landfill and hence the instant of time where next landfills will need to be constructed. Some general mathematical properties of the solution are provided and interpreted from an economic point of view. The resulting problem turns out to be no convex and therefore it can not be solved by conventional optimization techniques. Some global optimization methods are used to solve the problem in a particular case, in order to illustrate the behavior of the solution depending on parameter values.

Keywords: Landfilling, Optimal Capacity, Optimal Location, Global Optimization.

1 Introduction

The increasing generation of municipal solid waste has become an important issue from a social, economic and environmental point of view point, and the optimal management of this waste constitutes an important technical challenge for regional and local policy makers[1]. The location of treatment or disposal facilities are among the main decisions that need to be made concerning waste management (see Highfill et.al 1994, Kunreuther and Easterling 1996, Swallow et.al. 1992, Quah and K. 2002).

From the viewpoint of a resource manager or policy maker, the decision of locating a landfill implies selecting a specific piece of land, among the available possibilities, which will be devoted to waste disposal for some time. Since a landfill will typically be in use for quite a long period, that decision will have some associated (economic and environmental) temporal costs and consequences about the future availability of land close to the landfill. Therefore, it is crucial to perform a careful design of landfills, and specifically, to make optimal capacity and location decisions.

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[1]See Quadrio-Curzio et al 1994, Beede and Bloom 1995, Porter 2002 or Fullerton and Kinnaman 2002 for a revision of different aspects about waste management.

As a matter of fact, the operation of disposal facilities, concerning their location and capacity, has changed dramatically during the last 20 years for both economic and environmental reasons. The location of landfills has been typically moving further away from cities because of the growing price of land in densely populated urban areas and the increased concern for the effects of dumps on our health and the environment. Regarding capacity, at the start of the 1970's, there were 20,000 landfills in the United States, but by the end of the 1980s only 6,000 and by 1998 barely 2,000 (U.S. EPA 1988; Repa 2000). Small landfills closed and big landfills grew in number and size. By the end of the 1980s, a few hundred landfills handled half of all the municipal solid waste generated in the United States.

Some papers in the literature on economics and operations research have studied the optimal location or the optimal capacity of landfills, but to the best of our knowledge, no one has studied both decisions at the same time. We present a model where the decisions on capacity and location of landfills are jointly made and show how these decisions interact with each other. This approach allows us to take explicit account of the space constraints, which are one the main real problems which waste managers are faced with, and also to measure the cost of space in every part of the feasible region.

The location of facilities has been thoroughly studied in the literature (see, for example, Kuhn, 1967; Love et al, 1988; Francis et al, 1992; Wesolowsky, 1993, Drezner, 1994 and Drezner et al, 2002). The so-called Fermat or Weber Problem problem consists of finding a point (for example, the location of a disposal facility or landfill) which minimizes the sum of weighted distances from itself to a number of fixed points (say, cities). Weiszfeld (1936) provided an iterative procedure to find the solution to the Weber problem and showed that this solution is on the convex envelope of the fixed points. If there are some constraints concerning the region where the facility can be feasibly located (i.e., some forbidden regions), then we are faced with a so-called Constrained Weber Problem. This is obviously the case when dealing with landfill location. Forbidden regions can refer to military areas, protected regions, such as ecological parks or, of course, inhabited areas. A central result for the problem with forbidden regions is the boundary theorem due to Aneja and Parlar (1994) and Hamacher and Nickel (1995), which states that if the feasible region is a connected set, then the Constrained Weber Problem has an optimal solution on the boundary of the feasible set. Hansen, Peeters and Thisse (1981) showed that the solution necessarily lies in the visible boundary of the set of restrictions, as projected from the unconstrained solution.

In the economics literature, (the capacity of) landfills have been sometimes rationalized as a particular kind of natural resource. As noted in Ready and Ready (1995), landfills can be viewed as depletable and replaceable resources. Unlike other natural resources, whose depletion is irreversible, once a landfill is full it can be replaced at some cost, by constructing a new one. The new landfill will also be depleted and so on. There are at least two additional important features related to landfill management that make it different from standard natural resource problems. First, the building of landfills is characterized by high setup costs -given by the tasks of building and preparing the new landfills to be used, as well as closing the full ones- as compared to the operating costs, which are basically given by the transportation and processing of residuals. Second, unlike other resources (whose initial stock is given by nature), the capacity of a landfill can be chosen by the decision maker who is responsible for waste management.

Deciding the capacity of a landfill has some relevance for the setup costs and also for the switching time of a sequence of landfills. On the one hand, the smaller the capacity of the landfill to be constructed, the smaller the construction cost but, on the other hand, the lifetime of such a landfill will be shorter as well, so that the construction of a new landfill will have to be undertaken sooner. This conflict between present and future costs gives rise to a dynamic decision problem implying that a planning time horizon has to be divided into several subintervals, the length of which is endogenously determined. As a consequence, the capacity of a landfill should not be decided just by considering its own associated costs, but also the costs linked to the following ones. The sequential nature of the use of landfills is also recognized in a number of papers, like Jacobs and Everett (1992), Ready and Ready (1995), Huhtala (1997), Gaudet, Moreaux and Salant (2001) and André and Cerdá (2001, 2004). In all these papers, except André and Cerdá (2001, 2003), landfill capacity is a given and therefore the problem of obtaining the optimal capacity is not explicitly considered. André and Cerdá (2001, 2004) study the optimal capacity of a sequence of landfills from a dynamic point of view and provide the so-called Optimal Capacity Condition, which determines the optimal balance between present and future costs when determining such a sequence of capacities. However, they do not study the optimal location of landfills.

In practice, both location and capacity of landfills are relevant for landfill management and there are some important interactions between both decisions. The main idea is that the capacity decision has some spatial and some temporal implications. Spatial, because the larger a landfill, the smaller the remaining feasible region, and therefore the location of future landfills is affected by the capacity of the current one. Temporal, because the capacity determines the lifetime of the landfill and hence the instant of time where next landfills will need to be constructed. As a consequence, an optimal design of a sequence of landfills requires the joint determination of both the capacity and location of the whole sequence.

This joint problem is modeled in this paper within an intertemporal setting, and some of its basic mathematical and economic properties are discussed. In section 2 we present the problem and discuss some of its basic features. We show that the problem is non-convex in nature, so that conventional optimization techniques are not suitable to address it. In section 3 we analyze some basic mathematical and economic properties of the solution. Specifically, we state the first order conditions and interpret them form an economic point of view. This conditions make explicit the interaction between capacity and location and provide a measure for the value of land depending on its scarcity around every landfill. Furthermore, we derive some results concerning the optimal number of landfills and the possibility of obtaining the counterintuitive result of an optimal excess capacity for the whole sequence of landfills. Finally, we discuss the optimal order of landfills and show that the model is consistent with the fact that, as time goes on, landfills are typically constructed further away from cities. Despite the valuable insight that the first order conditions provide, the non-convex structure of the problem prevents us from finding the solution just by solving these conditions, so that some numeric global optimization technique is needed. In section 4 we discussed some techniques that can be suitable for solving the problem and use them to solve a specific numerical example. Some sensitivity analysis exercises are performed in order to get some further insights about the effect of different parameters of the model. Specifically, we show that a larger fixed construction cost makes it optimal to reduce the number of landfills and make the sequence of capacities more decreasing and the opposite happens for the marginal construction cost. An increment in the transportation cost leads to reduce the capacity of the first and the last landfills, while an increment in the size of the planning horizon causes a stair-shape increment for the number of landfills and a sawtooth shape for the average capacity. Section 5 summarizes the conclusions and offer some guidelines for interesting extensions and future research.

2 Formulation of the problem

Assume that there are m cities indexed by $j = 1, \ldots, m$, located at different points of the map $P_j \equiv (p_{j1}, p_{j2})$. At time t, every city generates an instantaneous amount of waste equal to $q_j(t)$. A planner has to take the following actions in order to manage, with the smallest possible cost, the waste produced in a time horizon $[0, \tau]$:

1. At instant $t = 0$, to construct a landfill, with arbitrary capacity Y_0, located at a point $R_0 \equiv (r_{01}, r_{02}) \in \Omega$, being Ω a bounded feasible region. The construction cost depends on Y_0, according to the increasing, convex and twice differentiable cost function $C(Y_0)$.[2] Although the simplest approach consists of rationalizing $C(Y_0)$ as being purely economic costs, it could also be constructed to measure an aggregation of economic and environmental costs, by using a suitable valuation method for the latter. The same consideration applies to the operating costs.

2. While the first landfill is being used, to pay the instantaneous waste operating costs, that are mainly determined by the transportation costs from all the cities to the landfill, which equal $\phi \sum_{j=1}^{m} q_j(t)\, d(P_j, R_0\}$, where $d(P_j, R_0)$ represents the distance between P_j and R_0 and ϕ is a parameter which measures transportation cost per unit of waste and distance. In the standard Weber location problem, the distance from the facility to each city j is weighted by some coefficient w_j. When the facility to be located is a landfill, the weights are given, in a natural way, by the amount of waste generated by each city. The general setting is compatible with any type of distance, but for the sake of clarity, we will focus on the Euclidean distance. Moreover, we assume for simplicity that, although the landfills can be located just in some specific feasible region, there are not forbidden regions concerning the transportation of residuals. The parameter ϕ can be tailored to account for the average difficulty to travel across the whole region under study.

3. When the capacity of the first landfill is exhausted, which happens at time T_1, implicitly determined by the condition $\int_0^{T_1} Q(t)\, dt = Y_0$, where $Q(t) \equiv \sum_{j=1}^{m} q_j(t)$, the planner has to close it and to construct a new one, with capacity Y_1, at another location $R_1 \equiv (r_{11}, r_{12})$. Note that the feasible region is now smaller than the original one, because the new landfill can not be constructed too close to the first one. In fact, there is a safety region around each landfill, because of sanitary, legal and environmental reasons. Furthermore, the larger the landfill capacity, the more potential risks, so a wider safety region is needed. We model this limitation by imposing the following constraint:

$$d(R_0, R_1) \geq \beta (Y_0 + Y_1)$$

β being a known parameter. The construction costs are given by $C(Y_1)$. The new landfill will last until time T_2, which is given by $\int_{T_1}^{T_2} Q(t)\, dt = Y_1$.

[2] As noted in André and Cerdá (2004), $C(Y)$ can be thought of as measuring the (discounted) aggregation of both construction and closure costs. If $G_1(Y)$ denotes the construction cost and $G_2(Y)$ the closure cost of a landfill built at time $t = 0$ with capacity Y, the present value of the aggregation of both costs is given by

$$G(Y, T) \equiv G_1(Y) + e^{-\delta T} G_2(Y)$$

but, once Y is decided and Q being exogenous, T can be expressed as a function $T(Y)$, so that $G(Y, T)$ collapses to a function depending only on Y and the parameters of the model:

$$G(Y, T) \equiv G_1(Y) + e^{-\delta T} G_2(Y) = G_1(Y) + e^{-\delta T(Y)} G_2(Y) \equiv C(Y)$$

4. From T_1 to T_2, he or she has also to pay the instantaneous transportation costs associated to the waste produced in this period, given by $\phi \sum_{j=1}^{m} q_j(t)\, d(P_j, R_1)$.

And so on, until the last landfill, denoted by $K-1$, K being a decision variable. In general, a landfill constructed at time T_i, located at $R_i \equiv (r_{i1}, r_{i2})$, with capacity Y_i will last until T_{i+1}, implicitly defined by the equation $\int_{T_i}^{T_{i+1}} Q(t)\, dt = Y_i$. The construction and instantaneous transportation costs associated to such a landfill are given by $C(Y_i)$ and $\phi \sum_{j=1}^{m} q_j(t)\, d(P_j, R_i)$ respectively. The location has to meet the following constraints:

$$d(R_i, R_k) \geq \beta(Y_i + Y_k), \quad k = 1, \dots, i-1$$

The planner's problem consists of finding a number of landfills K, a sequence of capacities $Y \equiv \{Y_0, Y_1, ..., Y_{K-1}\}$, a sequence of switching times $T \equiv \{T_1, ..., T_K\}$ and a sequence of locations $R \equiv \{R_0, R_1, ..., R_{K-1}\}$ in order to minimize the function

$$H(K,Y,T,R) = \sum_{i=0}^{K-1} e^{-\delta T_i} \left[C(Y_i) + \int_{T_i}^{T_{i+1}} e^{-\delta(t-T_i)} \left(\phi \sum_{j=1}^{m} q_j(t)\, d(P_j, R_i) \right) dt \right] \tag{1}$$

subject to the following constraints:

$$\begin{aligned} T_0 &= 0,\, T_K \geq \tau, \\ \int_{T_i}^{T_{i+1}} Q(t)\, dt &= Y_i, \qquad i = 0,1,2,...,K-1, \\ V_0 &\leq Y_i \leq V_1, \\ R_i &\subset \Omega \\ d(R_i, R_k) &\geq \beta(Y_i + Y_k), \quad i \neq k \end{aligned} \tag{2}$$

where δ is the discount rate, while V_0 and V_1 represent some minimum and maximum capacity constraints, which can be given by legal or technical reasons. The constraint $T_K \geq \tau$ accounts for the fact that the overall capacity of the whole sequence needs to be large enough to meet the waste requirements in the time horizon $[0, \tau]$. We discuss below the rationale for considering this condition with inequality instead of strict equality. To keep the analysis as simple as possible $q_j(t)$ is assumed to be constant across time[3]: $q_j(t) \equiv q_j\ \forall j$, $Q(t) \equiv Q$, therefore, from (2), we have

$$T_{i+1} = T_i + \frac{Y_i}{Q}, \qquad i = 0, \dots, T_{K-1}. \tag{3}$$

Note that, substituting (3) for (2), problem (1) can be viewed as a discrete time, finite horizon optimal control problem with free horizon, where T_i plays the role of state variable and R_i, Y_i are control variables. Nevertheless, we show now that it is also possible to address (1) as a static problem. For that purpose, use (3) recursively to obtain

$$T_i = \frac{\sum_{l=0}^{i-1} Y_l}{Q} \qquad i = 0, \dots, T_{K-1}$$

[3] André and Cerdá (2001) study a case where the total flow of waste changes with time according to some specific dynamic law of motion $Q(t)$. Then, if we substitute the expression for $Q(t)$ and solve the integral $\int_{T_i}^{T_{i+1}} \left(\sum_{j=1}^{m} Q_j(t) \right) dt$, we get a condition of the type $Y_i = F(T_i, T_{i+1})$ or, solving for T_{i+1}, a condition of the type $T_{i+1} = \Phi(T_i, Y_i)$, that can be regarded as the state equation of an discrete-time optimal control problem where T_i is the state variable and R_i, Y_i are control variables. This makes the solution procedure quite more complicated, so we stick to the simpler case with constant waste generation, to focus ourselves on the new issues arising from the interaction between capacity and location decisions.

which can be substituted in the objective function to eliminate T_i and obtain the following alternative expression:

$$J(K,Y,R) \equiv C(Y_0) + \frac{TC_0}{\delta}\left(1 - e^{-\delta\frac{Y_0}{Q}}\right) + \sum_{i=1}^{K-1} e^{-\frac{\delta}{Q}\sum_{\ell=0}^{i-1} Y_\ell}\left[C(Y_i) + \frac{TC_i}{\delta}\left(1 - e^{-\delta\frac{Y_i}{Q}}\right)\right]$$

where $TC_i \equiv \phi \sum_{j=1}^{m} q_j d(P_j, R_i)$ denotes total instantaneous transportation costs associated to landfill i. Note that problem (1) involves deciding the optimal value of a discrete variable (K) and some continuous variables (R and Y). A possible way to solve it consists of finding the solution for all possible values of K, and choosing that which provides the minimum total cost. For every possible value of K, we have the following problem:

$$\begin{aligned} \mathcal{J}_K \equiv \min_{\{Y,R\}} \quad & J(K,Y,R) \\ s.t. \quad & \\ & \textstyle\sum_{i=0}^{K-1} Y_i \geq \tau Q \\ & V_0 \leq Y_i \leq V_1, \\ & R_i \subset \Omega \\ & d(R_i, R_k) \geq \beta(Y_i + Y_k), \quad i \neq k \end{aligned} \tag{4}$$

where $\mathcal{J}_K$ represents the optimal value of the objective function when K landfills are constructed.

The last set of constraints in problem (4) are crucial and they imply that there is an important interaction between capacity and location decisions, as illustrated in figure 1. Assume that the shadowed area represents the feasible set Ω. Consider a solution with two landfills ($K = 2$) located at $R_0 = (r_{01}, r_{02})$ and $R_1 = (r_{11}, r_{12})$, with capacities Y_0 and Y_1 respectively. The white circles around R_0 and R_1 represent the safety regions. Suppose that Y_0 increases while R_0 and Y_1 remain unchanged. It is clear that R_1 becomes unfeasible as a location for the second landfill. In the problem as a whole, the capacity and location of a landfill affect the feasible capacities and locations for the rest of landfills. Figure 1 also illustrates the non-convex nature of the problem. Even if Ω is a convex set, once any landfill is located, the *remaining* feasible set $\{\Omega - B_{\beta Y_0}(R_0)\}$, where $B_{\beta Y_0}(R_0)$ denotes the ball centered at R_0 with radius βY_0, is non-convex. We can also conclude that the feasible set is non-convex by noting that $d(R_i, R_k)$ is a convex function when we use the Euclidean distance or any metric of the type $d(R_i, R_k) = [(R_{i1} - R_{k1})^p + (R_{i2} - R_{k2})^p]^{\frac{1}{p}}$ with $p > 1$[4]. The non-convex nature of the problem prevents us from solving it by conventional optimization methods.

[4] For example, assume that $\beta = 1$, $K = 2$ and we have two feasible solutions given by

$$\begin{array}{llll} Y_0 = 2 & R_0 = (0,0) & Y_1 = 3 & R_1 = (0,5) \\ Y_0' = 3 & R_0' = (0,1) & Y_1' = 2 & R_1' = (4,4) \end{array}$$

which satisfy $d(R_0, R_1) = d(R_0', R_1') = Y_0 + Y_1 = Y_0' + Y_1'$. If we construct the following linear convex combination of both solutions

$$\begin{array}{ll} Y_0'' = 0.5Y_0 + 0.5Y_0' = 2.5 & R_0'' = 0.5R_0 + 0.5R_0' = (0, 0.5) \\ Y_1'' = 0.5Y_1 + 0.5Y_1' = 2.5 & R_1'' = 0.5R_1 + 0.5R_1' = (2, 4.5) \end{array}$$

such a combination turns out to be unfeasible, given that $d(R_0'', R_1'') < Y_0'' + Y_1''$.

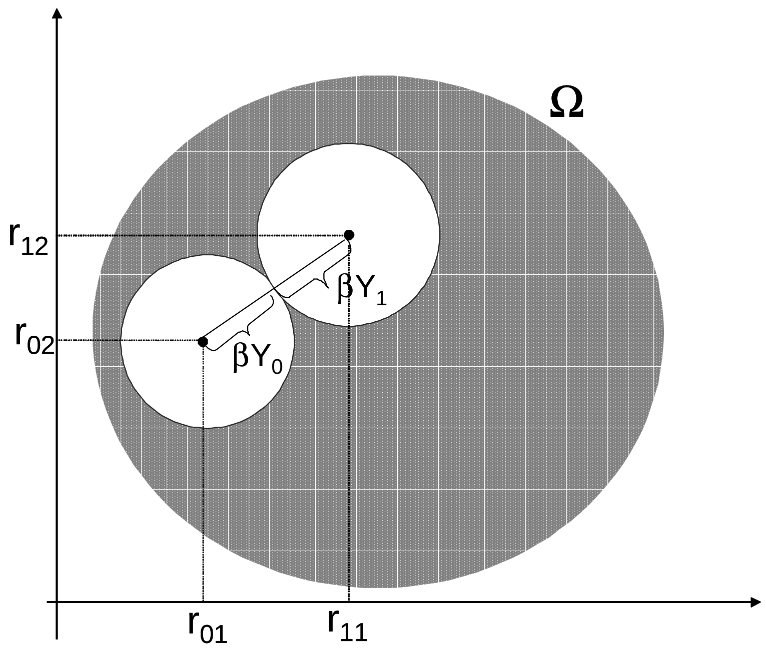

Figure 1: Interaction between capacity and location

In the optimal solution, the condition $d(R_i, R_k) \geq \beta(Y_i + Y_k)$ may be binding for some pairs of landfills and not binding for other. When this conditions holds with equality for two landfills i and k we say that landfills i and k are 'as-close-as-possible' (as illustrated in figure 1).

3 Basic properties of the solution and economic interpretation

3.1 Optimality conditions

For any value of K, we can construct the Lagrangian function

$$\mathcal{L} = J(K, Y, R) + \mu\left(\tau Q - \sum_{i=0}^{K-1} Y_i\right) + \sum_{i=0}^{K-1} \alpha_{0i}(V_0 - Y_i) + \sum_{i=0}^{K-1} \alpha_{1i}(Y_i - V_1) + \sum_{\substack{i,k=0 \\ i \neq k}}^{K-1} \lambda_{ik}\left[\beta(Y_i + Y_k) - D_{ik}\right]$$

where μ, α_{0i}, α_{1i} and λ_{ik} are the multipliers associated to the constraints of the problem and, for the sake of brevity, we denote as $D_{ik} \equiv d(R_i, R_k)$ the distance between landfills i and k. Note that all the constraints are linearly independent and so the Kuhn-Tucker conditions apply although we should keep in mind that, given the non-convexity of the problem, several local minima may exist so that a global minimum can not be obtained just by solving the necessary conditions. Nevertheless, the study of these conditions provide some useful insight into the mathematical and economic properties of the solution. Given a value of K, the Kunh-Tucker conditions for problem (4) are

$$\begin{gathered} C'(Y_0) + e^{-\frac{\delta}{Q}Y_0}\frac{TC_0}{Q} + \beta\sum_{i=1}^{K-1}\lambda_{0i} = \\ \frac{\delta}{Q}\sum_{i=1}^{K-1} e^{-\frac{\delta}{Q}\sum_{\ell=0}^{i-1}Y_\ell}\left[C(Y_i) + \left(1 - e^{-\frac{\delta}{Q}Y_i}\right)\frac{TC_i}{\delta}\right] + \mu + \alpha_{00} - \alpha_{10} \end{gathered} \tag{5}$$

$$e^{-\frac{\delta}{Q}\sum_{\ell=0}^{h-1}Y_\ell}\left(C'(Y_h)+e^{-\frac{\delta}{Q}Y_h}\,\frac{TC_h}{Q}\right)-\frac{\delta}{Q}\left\{\sum_{i=h+1}^{K-1}e^{-\frac{\delta}{Q}\sum_{\ell=0}^{i-1}Y_\ell}\left[C(Y_i)+\left(1-e^{-\frac{\delta}{Q}Y_i}\right)\frac{TC_i}{\delta}\right]\right\}$$
$$=\mu+\alpha_{0h}-\alpha_{1h}-\beta\sum_{i\neq h}\lambda_{hi} \qquad h=1,\ldots K-2 \tag{6}$$

$$e^{-\frac{\delta}{Q}\sum_{\ell=0}^{K-2}Y_\ell}\left(C'(Y_{K-1})+e^{-\frac{\delta}{Q}Y_{K-1}}\,\frac{TC_{K-1}}{Q}\right)+\beta\sum_{i=0}^{K-2}\lambda_{K-1,i}=\mu+\alpha_{0K-1}-\alpha_{1,K-1} \tag{7}$$

$$-\frac{\phi}{\delta}e^{-\frac{\delta}{Q}\sum_{\ell=0}^{h-1}Y_\ell}\left(1-e^{-\frac{\delta}{Q}Y_h}\right)\sum_{j=1}^{m}q_j\,\frac{p_{j1}-r_{h1}}{d_{jh}}=\sum_{i\neq h}\lambda_{ih}\frac{r_{i1}-r_{h1}}{D_{ih}} \qquad h=0,\ldots K-1 \tag{8}$$

$$-\frac{\phi}{\delta}e^{-\frac{\delta}{Q}\sum_{\ell=0}^{h-1}Y_\ell}\left(1-e^{-\frac{\delta}{Q}Y_h}\right)\sum_{j=1}^{m}q_j\,\frac{p_{j2}-r_{h2}}{d_{jh}}=\sum_{i\neq h}\lambda_{ih}\frac{r_{i2}-r_{h2}}{D_{ih}} \qquad h=0,\ldots K-1 \tag{9}$$

$$\mu\left(\tau Q-\sum_{i=0}^{K-1}Y_i\right) = 0 \tag{10}$$
$$\alpha_{0i}\left(V_0-Y_i\right) = 0 \quad i=0,\ldots,K-1 \tag{11}$$
$$\alpha_{1i}\left(Y_i-V_1\right) = 0 \quad i=0,\ldots,K-1 \tag{12}$$
$$\lambda_{ik}\left[\beta\left(Y_i+Y_k\right)-D_{ik}\right] = 0 \quad i,k=0,\ldots,K-1;\ i\neq k \tag{13}$$
$$\mu,\alpha_{0i},\alpha_{1i},\lambda_{ik} \geq 0 \quad i,k=0,\ldots,K-1;\ i\neq k \tag{14}$$

where $d_{jh}\equiv d\left(P_j,R_h\right)$ denotes the distance between city j and landfill h.

Consider the economic interpretation of condition (5), related to Y_0: when the capacity of the first landfill marginally increases,some marginal costs and some marginal gains follow. Equation (5) states the equalization of both marginal costs and marginal gains in the solution. The first term is the marginal construction costs of the first landfill, as measured by the first derivative of C evaluated at Y_0. The second marginal costs comes from the fact that the first landfill will have a longer lifetime and hence the transportation costs TC_0 will have to be paid for a longer period. Both of these effects are discussed in André and Cerdá (2001, 2004), but in the present problem there is a third possible source of marginal cost from increasing Y_0, which comes from the interaction between capacity and location. Assume that landfills 0 and k, for some $k\neq 0$, are as-close-and-possible. Then, an increment in Y_0 leads to increase the safety region of this landfill and therefore to reduce the available space to locate landfill k. Landfill k will have to move to a different location, possibly augmenting its transportation cost. The term $\beta\lambda_{0k}$ measures such a marginal cost increment and it can be interpreted, from an economic point of view, as the shadow price of land (or space) between landfills 0 and k. Observe that, given the multi-location structure of the problem, the shadow price of land varies across different regions in the map. Note also that, if landfills 0 and k are not as-close-as-possible, then from condition (13) we know that $\lambda_{0k}=0$ and the third effect does not show up. The total marginal cost linked to the third effect for landfill 0 (or, alternatively, the total marginal cost of a unit of space around landfill 0) is given by $\Lambda_0 \equiv \beta \sum_{i \in A_0} \lambda_{0i}$, where

$$A_0 = \{i = 1, \ldots, K-1 \ / \ d(R_i, R_0) \ = \ \beta (Y_i + Y_0)\}$$

is the set of landfills that are as-close-as-possible to landfill 0 and we can define, in the same way, Λ_i and A_i for any $i = 1, \ldots K-1$. The marginal gain of increasing Y_0 (first term of the right-hand-side of (5)) comes from the fact that a longer lifetime of the first landfill defers all the (construction and transportation) costs of future landfills. Given the time preference, as measured by the discount rate δ, this results in a smaller discounted cost. Concerning the rest of terms in the equation, the multiplier μ is common for all landfills and it measures the marginal impact on the construction costs from an additional unit on total waste, τQ. When an excess capacity exist, i.e. $\tau Q > \sum_{i=0}^{K-1} Y_i$, (10) implies that this marginal impact equals zero, because marginally increasing the total amount of waste does not involve any additional construction cost. The multipliers α_{00} and α_{01} account for the possibility of the minimum and maximum capacity constraints being binding. For all the landfills, either $\alpha_{h0} = 0$ or $\alpha_{h1} = 0$, or both hold. In an interior solution (concerning capacity) we have $\alpha_{00} = \alpha_{01} = 0$. The same interpretation applies for conditions (6) linked to Y_h, $h = 1, \ldots, K-2$, and condition (7), linked to Y_{K-1}. In the latter case, note that the effect of deferring future costs is not present, because *future does not exist.*

Concerning the optimality conditions for r_{h1} and r_{h2}, (8) and (9), note that changing the location of a landfill h (while keeping other variables unchanged) has two effects: the first one is related to the distance from landfill h to the different cities and hence the (discounted) transportation cost from each one. For example, an increment in r_{h1} increases (decreases) the distance between landfill h and city j if $r_{h1} > r_{j1}$ ($r_{h1} < r_{j1}$). A change in the location of a landfill necessarily affects the distance between such a landfill and every city, so that, this effect needs to be added up across all cities. Observe that when the location of landfill h changes, its distance from some cities may increase while from other can decrease. So, some of the terms in the sum $\sum_{j=1}^{m} q_j \frac{p_{j2} - r_{h2}}{d_{jh}}$ in (8), and the equivalent one in (9), may be positive and other negative. The second effect has to do with the relative position of landfills. Suppose that landfills h and i are as-close-as-possible, then a marginal change in the location of landfill h will require a movement in the location of landfill i (as far as landfill h moves "towards" landfill i), and hence to change the (transportation) cost associated to landfill i. To evaluate the economic effect of a marginal movement of landfill h, with respect to landfill i, we have to multiply by the shadow price λ_{ih}. Obviously, this effect does not show up for those landfills that are not as-close-as-possible to landfill h (displaying $\lambda_{ih} = 0$).

If we manipulate the first order condition (6) for the capacity of two consecutive landfills Y_h, Y_{h+1}, and assume that the minimum and maximum capacity constraints are not binding (so that $\alpha_{0h} = \alpha_{1h} = \alpha_{0,h+1} = \alpha_{1,h+1} = 0$), then we obtain the following non linear first order difference equation relating Y_h and Y_{h+1}, which can be considered as a generalization of the so-called Optimal Capacity Condition presented by André and Cerdá (2001, 2004):

$$C'(Y_h) = e^{-\frac{\delta}{Q} Y_h} \left[C'(Y_{h+1}) + \frac{\delta}{Q} C(Y_{h+1}) + \frac{(TC_{h+1} - TC_h)}{Q} \right] + e^{\frac{\delta}{Q} \sum_{\ell=0}^{h-1} Y_\ell} \left[\Lambda_{h+1} - \Lambda_h \right] \qquad (15)$$

Equation (15) can be interpreted as a no-arbitrage condition stating that no benefit can be made by transferring some capacity from landfill h to landfill $h+1$ or vice versa. If we disregard, for a moment, the last term (assuming that $\Lambda_{h+1} = \Lambda_h$) and moreover assume that there is no time preference, and therefore $\delta = 0$, we get the following simpler condition:

$$C'(Y_h) + \frac{TC_h}{Q} = C'(Y_{h+1}) + \frac{TC_{h+1}}{Q}$$

meaning that the marginal cost of a unit of capacity in landfill h (as measured by the marginal construction cost plus the transport cost per unit of waste) needs to be equal to that of landfill $h+1$. If there is a positive time preference, we have to discount the marginal cost in period $h+1$ and take into account that, when the capacity (and hence the lifetime) of landfill h increases, the whole building cost of landfill $h+1$ will be delayed implying a smaller discounted cost (second term in square brackets in (15)). Finally, the last term accounts for the different value of space around landfill h and around landfill $h+1$. Given that, by construction, Λ_h measures the value of space in present value at time $t = 0$, it is necessary to multiply by $e^{\delta T_h} = e^{\frac{\delta}{Q}\sum_{\ell=0}^{h-1} Y_\ell}$ to make the comparison in present value at time $t = T_h$.

Assume that problem (4) has a fully interior solution, meaning that all the "$\geq$" and "$\leq$" constraints hold with strict inequality. Now, consider that, starting from an optimal solution, we exogenously vary the capacities of the landfills and compute the optimal movements of the locations. Given the continuity of the problem for a given value of K, the optimal locations will vary continuously. In a similar way, suppose that, starting from an optimal solution for problem (1), -once the optimal value of K has been found- we perform some "small" change in the value of any of the parameters of the problem (δ, ϕ, Q, τ). Then, given the continuous nature of the problem, the solution (both the capacities and the locations) should vary continuously. Nevertheless, since the solution is not typically interior, in most cases changing the value of the parameters results in jump effects on the solution. Moreover, for some -large enough- parameter changes, the optimal value of K can change. K being a discrete variable, this will also result in a jump, in such a way that the solution of the problem typically turns out to be a piece-wise continuous function of the parameters. We illustrate this feature in section 4.

3.2 Discussion about the number of landfills

Note that the minimum capacity constraint, together with the fact that Ω is a bounded set, guarantee that K is a finite number. Define $\tau_0 \equiv \frac{V_0}{Q}$ and $\tau_1 \equiv \frac{V_1}{Q}$, so that $\frac{\tau}{\tau_0} \equiv \tau\frac{Q}{V_0}$ and $\frac{\tau}{\tau_1} \equiv \tau\frac{Q}{V_1}$. Using these definitions, the following propositions determines the minimum and maximum value of K in the solution of the problem.

Proposition 1 *The number of optimal landfills in problem (1) is bounded by:*

$$\Psi\left(\frac{\tau}{\tau_1}\right) \leq K \leq \Psi\left(\frac{\tau}{\tau_0}\right)$$

where

$$\Psi(x) \equiv \begin{cases} x & \textit{if } x \textit{ is an integer} \\ Int(x+1) & \textit{otherwise} \end{cases}$$

where the operator Int denotes the integer part of the argument.

Proof. See section 6.1 ■

If condition $\sum_{i=0}^{K-1} Y_i \geq \tau Q$ in problem (4) is replaced with $\sum_{i=0}^{K-1} Y_i = \tau Q$ (so that excess capacity is ruled out), then the second part of proposition 1 changes into $K \leq Int\left(\frac{\tau}{\tau_0}\right)$. To see this, note that the minimum capacity constraint implies that, when K landfills are constructed, $KV_0 \leq \tau Q$ holds, and using the definition of τ_0, we get $K \leq \dfrac{\tau}{\tau_0}$. But if $\dfrac{\tau}{\tau_0}$ is not an integer, $K = Int\,(x+1)$ is not feasible because $\sum_{i=0}^{K-1} Y_i \geq KV_0 > \tau Q$, so the maximum feasible number of landfills is $Int\left(\frac{\tau}{\tau_0}\right)$.

Using proposition 1 we can perform the following analysis of the solution depending on the value of τ. Pick up an integer value $\ell \in \left[\frac{V_1}{V_0} - 1, \frac{V_1}{V_0}\right]$ so that $\tau_1 \in [\ell\tau_0, (\ell+1)\tau_0]$. Split the range of possible values for τ in subintervals of the type $[n\tau_1,\ (n+1)\,\tau_1]$ for $n = 0, 1, \ldots$ and subintervals of the type $[m\tau_0,\ (m+1)\,\tau_0]$ for $m = 0, 1, \ldots$ As, by definition, $\tau_1 > \tau_0$, within any interval of the type $[n\tau_1,\ (n+1)\,\tau_1]$, several subintervals of the type $[m\tau_0,\ (m+1)\,\tau_0]$ may be contained. We come up with the following possibilities:

1. Assume $\tau \in [0, \tau_1]$,

If $\tau \in [0, \tau_0]$, then $\tau Q \leq V_0$. We trivially obtain that the solution is $Y_0 = V_0$ and consequently we have $K = 1$.

If $\tau \in [j\tau_0, (j+1)\,\tau_0]$, for $j = 1, \ldots, \ell - 1$, the number of landfills is bounded by $1 \leq K \leq j + 1$.

If $\tau \in [\ell\tau_0, \tau_1]$, we have $1 \leq K \leq \ell + 1$.

2. If $\tau \in [\tau_1, 2\tau_1]$, then $K = 1$ is ruled out and we always have $K \geq 2$.

If $\tau \in [\tau_1, (\ell+1)\,\tau_0]$, then the number of landfills is bounded by $2 \leq K \leq \ell + 1$.

If $\tau \in [(\ell + j)\,\tau_0, (\ell+2)\,\tau_0]$, for $j = 1, \ldots, \ell - 1$, we have $1 \leq K \leq j + 1$.

and so on. Summing up:

A. If we have $\tau \in [m\tau_0,\ (m+1)\,\tau_0]$ and $[m\tau_0,\ (m+1)\,\tau_0] \subset [(n-1)\,\tau_1,\ n\tau_1]$, or alternatively $\tau \in [m\tau_0,\ (n-1)\,\tau_1]$ and $[(n-1)\,\tau_1,\ (m+1)\,\tau_0] \subset [m_0\tau_0,\ (m+1)\,\tau_0]$, then the number of landfills in the solution is bounded by $n \leq K \leq m + 1$.

B. If we have $\tau \in [m\tau_0,\ (n-1)\,\tau_1]$ and $[m\tau_0,\ (n-1)\,\tau_1] \subset [m\tau_0,\ (m+1)\,\tau_0]$, then $n - 1 \leq K \leq m + 1$.

Define the indirect cost function for K landfills as $\mathcal{J}_K(\Theta) \equiv \min\limits_{\{Y,R\}} J\,(K, Y, R, \Theta)$, where Θ denotes the set of parameters of the problem, including τ, ϕ, δ, β, Q, V_0, V_1 and other possible parameters included in the cost function. The following proposition states the impact of any of these parameters on the indirect cost function.

Proposition 2 *$\mathcal{J}_K(\Theta)$ is non-decreasing in τ, ϕ, β, Q, V_0 and non-increasing in δ and V_1.*

Proof. See section 6.2 ■

3.3 Discussion about excess capacity

Note that inequality $T_K \geq \tau$ in problem (1), or alternatively $\sum_{i=0}^{K-1} Y_i \geq \tau Q$ in problem (4), explicitly recognizes the possibility of an excess capacity, in such a way that when time τ is reached there is some capacity (of the last landfill[5]) that remains unexhausted. Since construction cost is increasing with capacity, it seems unreasonable that a rational decision maker could be willing to incur such an

[5] It is immediate to conlude that it is never optimal to under-exhaust any landfill $h = 0, \ldots, K-2$, because total discounted cost could be reduced just by exhausting landfill h and so delaying the costs of the whole following sequence $h+1, \ldots, K-1$.

excess capacity. André and Cerdá (2004, proposition 1) show that this counter-intuitive may arise when dealing with landfill construction and provide a necessary and sufficient condition for it to show up, in a context were all the landfills are constrained to have the same capacity. The presence of an (optimal) excess capacity is even more surprising in this model that in the one by André and Cerdá (2004), for two reasons: first, because we are jointly modelling the capacity and location decisions, the scarcity of space implies an additional cost, in terms of wasted space, coming from constructed but unexhausted capacity. Secondly, unlike the case studied in André and Cerdá (2004), in this paper the capacity of landfills is assumed to be variable, so it is possible, in general, to increase the capacity of the first landfill(s) in order to delay future costs, and then decrease that of the last one(s) to avoid an excess capacity and reduce total discounted cost.

Nevertheless, as we will show below, this result can also show up in this case. Specifically, there is a particular situation where some excess capacity can arise in a natural way; namely, when the lower capacity constraint is binding for all, or at least for some landfills. The simplest case is that in which $\tau < \tau_0$, implying $Q\tau < V_0$, so that, even the capacity of the smallest feasible landfill is too large to meet the requirements in the planning horizon. In this case, the solution implies $K = 1$ and $Y_0 = V_0 > \tau Q$. Assume now $K = 2$. If we set $\tilde{Y}_0 = Q\tau - V_0$, $\tilde{Y}_1 = V_0$, and choose locations R_0, R_1 consistent with $\tilde{Y}_0$ and $\tilde{Y}_1$, then we have a feasible solution without excess capacity. Now assume that the derivative of the Lagrangians with respect to Y_0, evaluated at $\tilde{Y}_0$, $\tilde{Y}_1$ is negative, what happens is the following condition holds,

$$\begin{aligned} C'(\tilde{Y}_0) + e^{-\frac{\delta}{Q}(\tilde{Y}_0)}\frac{TC_0}{Q} + \beta\lambda_{01} < \\ \frac{\delta}{Q}e^{-\frac{\delta}{Q}(\tilde{Y}_0)}\left[C(\tilde{Y}_1) + \left(1 - e^{-\frac{\delta}{Q}\tilde{Y}_1}\right)\frac{TC_1}{\delta}\right] + \mu + \alpha_{00} - \alpha_{10} \end{aligned} \tag{16}$$

Then, the total discounted cost would be reduced by increasing Y_0. That would result in an overall excess capacity that could not be eliminate by reducing Y_1, which is already at its minimum possible value. The main idea behind (16) is that, if the marginal cost is "low enough", a small increment of the capacity of the first landfill would result in a small cost increment that could be overcompensated because discounted cost reduces as a consequence of postponing the construction of future landfills (in the example, just the second one).

As a numerical illustration, consider the following example. The construction cost function is given by $C(Y) = 2100 + 10Y$. There are 5 cities located at the points (0,0); (1,0); (1,1); (0,1); (2,2), which produce the following amounts of waste: 3; 4; 2; 1; 3, so that $Q = 13$. The feasible region is the rectangle defined by the extreme points $(2, 3)$ and $(17, 16)$. The rest of parameter values are $\beta = 0.01$, $\delta = 0.05$, $\tau = 63$, $\phi = 1$, $V_0 = 90$, $V_1 = 400$. If we solve the problem for $K = 9$ landfills, we obtain the optimal sequence of locations $R^* = \{(9.8, 5.5), (4.1\ 5.1), (4.9, 10.0), (10.9, 11.1), (14.5, 6.8), (6.8, 3.9), (16.1, 3.9), (14.9, 10.0), (6.6, 6.6)\}$ and the optimal sequence of capacities $Y^* = \{247, 208, 288, 324, 236, 90, 90, 90, 90\}$. Note that $\sum_{i=0}^{8} Y_i^* = 1665 > \tau Q = 819$, so that there is an excess capacity equal to 846. The discounted cost of this solution equals 10383. Assume we try to improve the solution by reducing the capacity of some landfills. The capacities $Y_5, \ldots, Y_8$ can not be reduced because they are already set equal to the lower bound. We set $Y_0 = 99$, $Y_1 = \cdots = Y_9 = 90$, while keeping unchanged the locations of all the landfills. In the new solution there is no excess capacity; nevertheless the discounted cost is 15204, which is larger than that of $\{R^*, Y^*\}$[6].

[6] As a matter of fact, the locations of the alternative proposed solution are not optimal and the cost can be reduced by

Note that, in the solution $\{R^*, Y^*\}$, the capacity of the last landfills is set equal to the lower bound. This is what typically happens when there is an excess capacity. The idea is that, for the last landfill, there is not gain in increasing capacity, because future costs can not be delayed, as future does not exist. The following lemma shows that, if some excess capacity exists, the minimum capacity constraint binds at least for the last landfill.

Lemma 1 *In the solution to problem (4), if* $\sum_{i=0}^{K-1} Y_i > \tau Q$*, then it must be the case that* $Y_{K-1} = V_0$*.*

Proof. From condition (10), we know that $\mu = 0$. The left-hand side of (7) is always positive, so it is immediate to conclude that $\alpha_{1,K-1} = 0$, $\alpha_{0K-1} > 0$, and then $Y_{K-1} = V_0$ follows from (11) ■

3.4 Discussion about location and optimal order of landfills

Once the optimal number of landfills K^*, the optimal capacities $Y^* \equiv \left(Y_0^*, ..., Y_{K-1}^*\right)$, and the optimal locations $R^* \equiv \left(R_0^*, ..., R_{K-1}^*\right)$ have been determined, since $\{K^*, Y^*, R^*\}$ is feasible by definition, any solution $\left\{K^*, \tilde{Y}, \tilde{R}\right\}$, where $\tilde{Y}$ is a permutation of the elements of Y^* and $\tilde{R}$ is the associated permutation of the elements of R^*, would yield a feasible (although not necessarily optimal) solution. So, it is relevant to study the optimal order in which landfills should be used, once we know their capacity and location.

Since landfill space can be understood as a natural resource, different landfills can also be conceptualized as several deposits of a natural exhaustible resource. A classic result by Herfindahl (1967) states that, in a situation where several deposits of a natural resource exist, the deposits have to be exploited in an increasing order of marginal extraction costs. André and Cerdá (2001) show that the Herfindahl's result holds for the problem of landfill construction in the sense that, if the only difference among the various places available for building landfills is the attached management (or transportation) cost per unit of waste, then it is optimal to make use of such places beginning from the lowest cost one and following in the order of increasing unit cost. In the case of a single city or waste generating center, this result implies that the distance to the city should be increasing across the sequence of landfills (i.e. landfill i is closer to the city than landfill j, for any $j > i$). The equivalent result in our model would imply the weighted distance, or equivalently the instantaneous transportation costs TC_i, to be increasing in i.

Nevertheless, in Herfindahl (1967) both the location and capacity of the resource deposits are given and in André and Cerdá (2001), although different landfills are assumed to have different transportation costs, the location decision is not explicitly modelled as space constraints are not taken into account. Proposition 3 shows that, when location and capacity decisions are jointly made, what matters to determine the optimal order of landfills is the aggregation of both construction and transportation costs. Nevertheless, using corollary 1, we also show that, if the capacity of landfills is not *very increasing*, then we also get the result that TC_i turns out to be increasing in i.

For that purpose, define the total discounted cost of landfill i as $TD_i \equiv C(Y_i) + \frac{TC_i}{\delta}\left(1 - e^{-\delta \frac{Y_i}{Q}}\right)$. Then the following result holds.

Proposition 3 *In the optimal solution for problem (1),* $TD_i \leq TD_j$ *holds for any pair of landfills* i*,* j*, such that* $i \leq j$*.*

relocating the landfills. The optimal solution without excess capacity (imposing the condition $\sum_{i=0}^{K-1} Y_i = \tau Q$), consists of the locations are $R = \{(4.8, 5.8), (2.9\ 3.9), (2.9, 5.7), (2.9, 7.5), (4.7, 3.9), (6.7, 5.7), (6.5, 3.9), (8.1, 4.6), (9.8, 3.9)\}$ and the optimal capacities are those proposed in the main text. Nevertheless, with this combination, the discounted cost is 12767, which is still larger than that of $\{R^*, Y^*\}$.

Proof. See section 6.3 ■

Using the definition of TD_i, and the fact that $C(Y)$ is increasing, the following corollary follows immediately from proposition 3.

Corollary 1 *In the optimal solution for problem (1), if* $Y_i \geq Y_j$ *holds for any pair of landfills* $i \neq j$*, then* $TC_i \leq TC_j$.

From corollary 1, and using a continuity argument, we can conclude that, for any $j > i$ if Y_j is not *much larger* than Y_i, then the weighted distance of landfill j is larger than that of landfill i. This result is consistent with the evidence that, as time goes on, landfills are normally constructed further away from cities. We obtain this result in most of the empirical exercises performed in the next section.

4 Empirical methodology and results

The complex structure of the problem, together with its non-convex nature, prevents us from obtaining an analytical solution, so that some numerical optimization method is needed to obtain an operational solution. Specifically, to overcome the difficulty arising from the possibility of having different local minima, the right approach is that of using some global optimization technique. We briefly review some of the basic features about the global optimization approach and present an empirical example which allows us to get some further insight about the behavior of the solution.

4.1 Global Optimization

Many problems of continuous location theory are expressed as global optimization problems (see Hansen, 1995). A global optimization problem is specified in the form

$$\begin{aligned} (GOP): \min \quad & f(x) \\ s.t. \quad & x \in \mathcal{C} \end{aligned} \tag{17}$$

where $\mathcal{C} \subset \mathbb{R}^n$ is a compact set and $f : \mathcal{C} \to \mathbb{R}$ is a continuous function defined on $\mathcal{C}$. The theorem of Weiesrstrass assure that, under these assumptions, a minimum and a maximum for f exist in $\mathcal{C}$. Points $x \in \mathcal{C}$ are called feasible, and a solution of (17) is a feasible point $\hat{x} \in C$ such that

$$f(\hat{x}) = \min_{x \in \mathcal{C}} f(x) \tag{18}$$

A local minimizer only satisfies $f(\hat{x}) \leq f(x)$ for all $x \in C$ in some neighborhood of $\hat{x}$. Since every global minimizer is a local minimizer, the solutions for (17) are the local minimizers with smallest objective function value. The main difficulties in global optimization stem from the fact that there are generally many local minimizers but only one, or a few of them are global minimizers, and that the feasible region may be disconnected. Among the most well-known global optimization methods are branch and bound, interval methods, constraint satisfaction techniques, genetics algorithms and radial basis function algorithms. Stochastic methods and genetic algorithms use only function values but their rate of convergence is slow. Deterministic methods like branch and bound, assume that one can compute a lower bound of f on a subset $\mathcal{H}$, what can be done if we know a Lipschitz constant on f. A basic reference on most aspects of global optimization is the handbook of Global optimization by Horst and Pardalos (1995) ((**?**)). It is also possible to see the state of the art in COCONUT (2001).

In our case, we solve a global optimization problem for every possible value of K and choose that which provides the minimum value of the objective function. We use the optimization environment Tomlab, which implements several global optimization methods in Matlab language (see Holsmtröm, 1999). Specifically we use as benchmark glcCluster solver, implementing an extended version of the routine DIRECT, which is a modification of the standard Lipschitzian approach that eliminates the need to specify a Lipschitz constant (see Jones, 2001). The results are also checked using the radial basis function routine glcSolver (see Gutmann, 2001[7]) and genetic algorithms.

4.2 Numerical Illustration

We now construct a numerical example which allows us to analyze the behavior of the solution and perform some sensitivity analysis. Assume that the construction cost function is of the linea type $C(Y) = a + bY$. Parameter a represents some fixed cost and parameter b measures marginal cost, indicating how total construction cost increases with capacity. There are five cities located at points $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$ and $(2,2)$, which produce the amounts of waste 3, 4, 2, 1 and 3 respectively, so that the total amount Q equals 13. The feasible region is given by the rectangle defined by the extreme points $(2,3)$, $(17,16)$. To guarantee that all the landfills and also their safety regions fall within the feasible region, we add to the optimization problem the constraints $2 + \beta Y_i \leq r_{i1} \leq 17 - \beta Y_i$, $3 + \beta Y_i \leq r_{i2} \leq 16 - \beta Y_i$, for $i = 0, \ldots K - 1$. We set the following values for the parameters:

$$\begin{array}{llll} a = 1000 & V_0 = 90 & \beta = 0.01 & \delta = 0.05 \\ b = 10 & V_1 = 400 & \phi = 1 & \tau = 56 \end{array} \tag{19}$$

From proposition (1), we know that the number of landfills is bounded by $2 \leq K \leq 9$. We solve numerically the global optimization problem and obtain that the optimal number of landfills is $K^* = 3$, the optimal locations are $R^* = \{(3.97,4.97), (8.54,5.65), (4.66,9.55)\}$ and the optimal capacities $Y^* = \{197.15,$ $265.09, 265.72\}$. The solution is illustrated in figure 2. The left panel displays the location of landfills. The triangles represent cities and the squares represent landfills. The order of landfills is indicated by numbers 1, 2, 3. The feasible region is delimited by the white rectangle. The right panel shows the sequence of capacities. Observe that there is no excess capacity and the sequence of capacities is increasing but, as we will show below, no general result can be drawn about this sequence being increasing or decreasing. The instantaneous transportation costs associated of landfills are given by $TC_1 = 68.49$, $TC_2 = 118,36$,

[7] Gutmann (2001) show that for most types of radial basis functions, convergence can be achieved without further assumptions on the objective function. The rbfSolver use radial basis function to define a utility function. The goal is to compute a response surface that interpolates the objective function in given points, and to choose the global minimizer of the surface. Gutmann finds a response surface proposed by Jones (1996) and using radial basis functions as interpolants he find that the uniqueness of an interpolant is achieved under very mild conditions on the location of the interpolation points.

$TC_3 = 125.12$, so that they are increasing as predicted by proposition 3 and corollary 1.

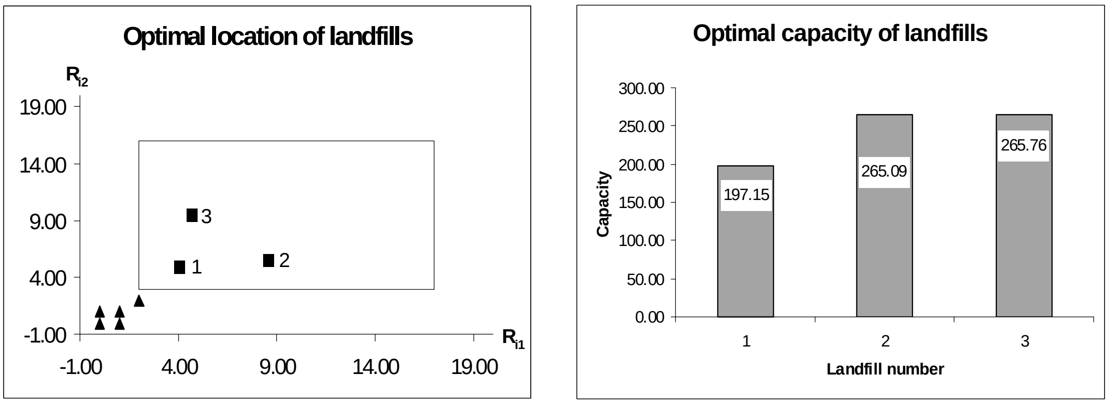

Figure 2: Optimal location and capacity of landfills in the benchmark example

We performed now some sensitivity analysis exercises starting from the benchmark parameter values given in (19) and show now the most interesting results. First, note that the location an the order of landfills is primarily determined by the position of cities. As an illustration, assume that the location of the second city is moved from the point (1, 0) to the point (1, 20), while keeping unchanged the rest of the setting. The new solution, illustrated in figure 3, is given by $K^* = 3$; $R^* = \{(4.20,5.20), (4.85,10.22), (8.41,6.59)\}$; $Y^* = \{220.32, 285.41, 222.27\}$. Note that the locations of the landfills are very similar to those in the benchmark case, but the order is different. Now the second landfill is that one closer to the city at (1,20) to minimize total discounted cost. Observe also that, in this case, the sequence of capacity is not monotonically increasing or decreasing, but it displays a inverted-U shape. This feature depends on the specific combination of the parameter values and no general statement can be made. If, starting from this set of parameter values, we increase the weight (i.e., the amount of waste) associated to the city located at (1, 20), the locations of all landfills progressively move north to be closer to this point.

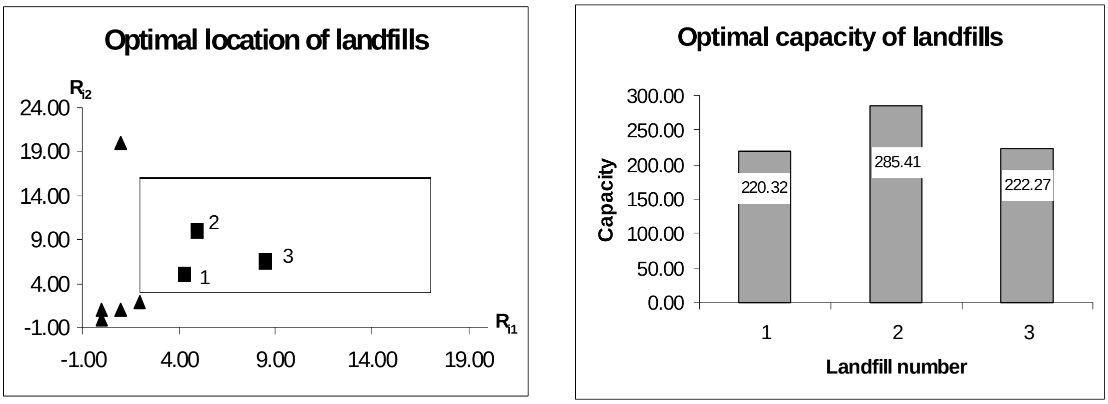

Figure 3: Optimal location and capacity after changing the location of a city

An increment in parameter a makes the construction of any landfill more expensive irrespective of its capacity. As a consequence, when a increases enough, it becomes optimal to reduce the number of landfills (and therefore to increase their average capacity) to avoid incurring many times a large fixed cost. See left panel of figure 4. Another interesting result is that increasing a makes the sequence of capacities to be more decreasing, i.e., the capacity of the first landfills becomes larger and that of the last landfills become smaller. The economic interpretation for this is that, as the set-up cost becomes higher, it pays more to increase the capacity of the first landfill(s) -recall that a is a fixed costs, so it does not affect to impact of capacity on total cost- in order to postpone the construction of future landfills. See right panel of figure 4. To appreciate the effect on the sequence of capacities, we just show a range of a for which K^* is constant (in this case, $K^* = 3$).

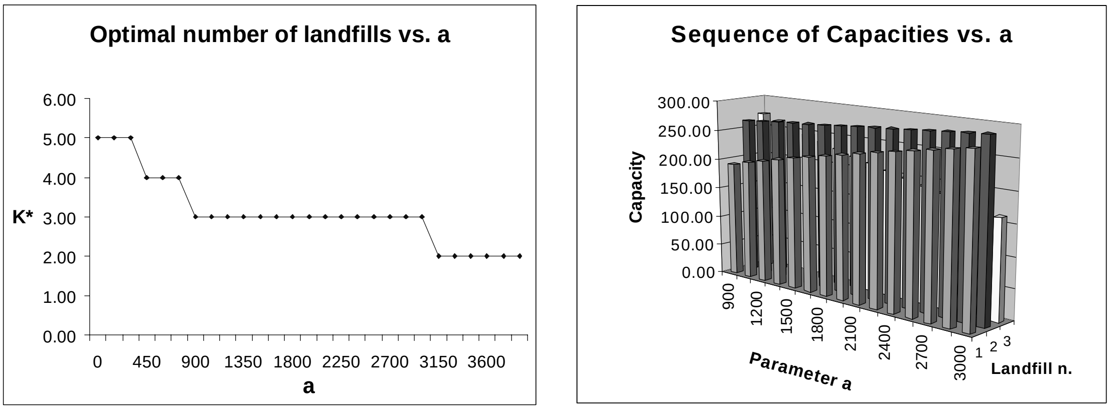

Figure 4: Impact of parameter a on the solution

This exercises also gives us the opportunity to observe empirically the interaction between capacity and location. When a increases, it pays to build more landfills with a larger capacity. But as the capacity of a landfill increases, so does the safety region around it. Take the value $a = 3300$. The optimal solution implies buying two landfills with capacities $Y^* = \{328, 400\}$ located at $R^* = \{(5.32, 6.32); (12.57, 6.96)\}$, as illustrated in figure 5. Note that there is a trade-off when increasing the capacity of the first landfill. On the one hand, the construction of the second landfill will need to be undertaken later (reducing discounted cost). On the other hand, the safety region around landfill 1 increases, so that the location of landfill 2 is displaced further away from the cities, implying larger transportation costs.

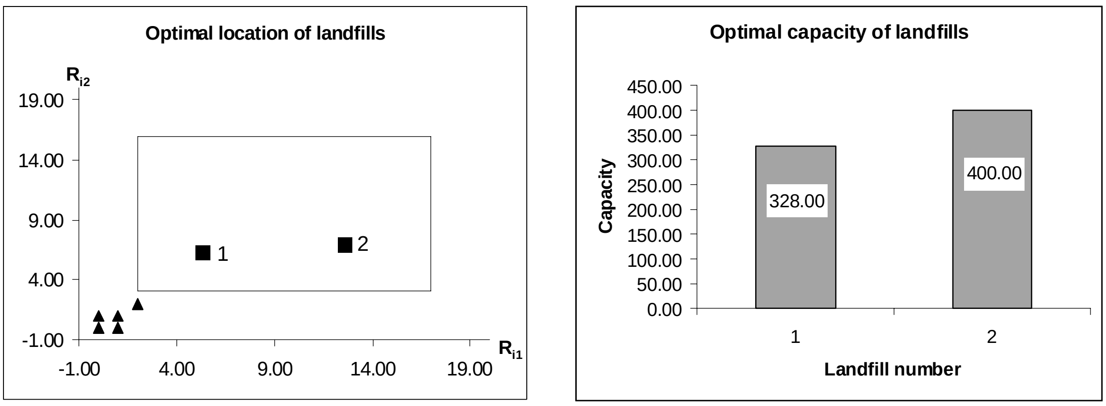

Figure 5: Solution with $a = 3300$

Parameter b, which measures marginal construction cost, has the contrary effect to that of a. As b increases, it becomes more costly to construct large landfills, so that, it becomes profitable to built many small landfills. As a consequence, K^* is increasing with b and the average capacity, $\sum_{i=0}^{K-1} Y_i/K$ is decreasing with b. As landfills become smaller, so do their safety regions and therefore, they also become closer among them and to the waste-generating cities. It is also interesting to note that, as b increases, the capacity of the first landfills tends to decrease and the capacity of the last landfills tends to increase. The reason is that now a larger capacity implies a larger construction cost, and that effect is more important for the initial landfills given the time preference (see illustration in figure 6). As we could expect, the optimal value of the objective function increases with a and b, and the impact of the rest of parameters on the objective function are those predicted in proposition 2.

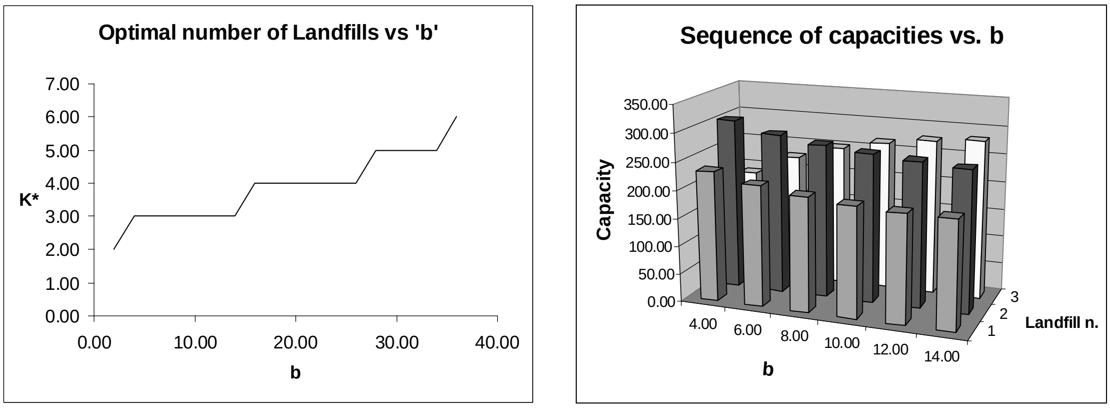

Figure 6: Impact of parameter b on the solution

Figure 7 illustrates the impact of parameter τ on the optimal number of landfills K^* (left panel) and the average capacity $\bar{Y} = \sum_{i=0}^{K-1} Y_i/K$ (right panel)[8]. An increment in parameter τ implies a larger overall amount of waste to be landfilled, which is given by $Q\tau$. As a consequence, feasibility requires either increasing the number or the capacity of landfills. As shown in the figure, "small" increments of τ lead to increase the average individual capacity and keep K^* unchanged, up to a point that the increase of τ is large enough to cause a new landfill to be profitable, allowing a reduction in average capacity. Henceforth, K^*, as a function of τ, has a stair shape and $\bar{Y}$, as a function of τ, has a sawtooth shape.

[8] In order to keep the problem being feasible, while we performed this exercise, the feasible region was enlarged to the rectangle $[2, 50] \times [3, 50]$.

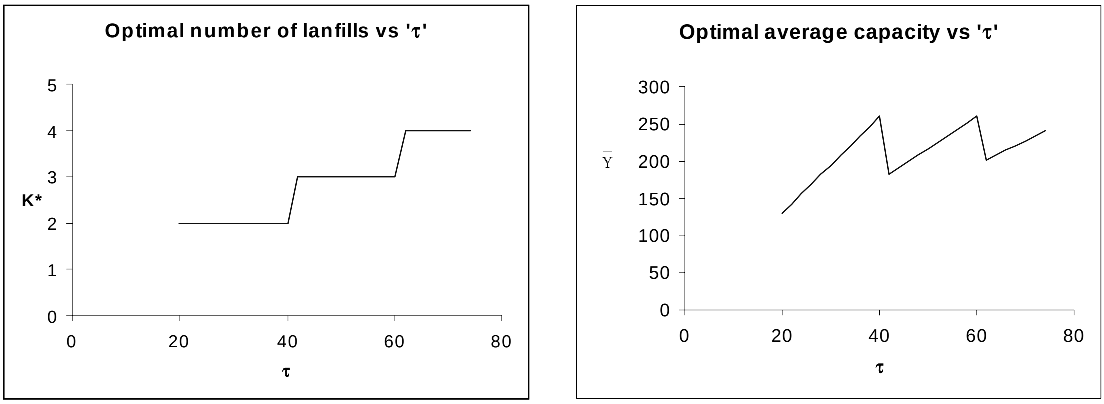

Figure 7: Impact of parameter τ on the solution

To save some space, we just offer a brief summary of the results concerning the rest of the parameters in (19). Increasing the discount rate δ makes the optimal number of landfills to increase and the sequence of capacities to be more decreasing, since the costs associated to the first landfill become more important for the objective function. Concerning parameter ϕ, after performing a sensitivity analysis for the range $\phi \in [0.5, 20]$, we obtain, first, that it does not show any effect on the number (and hence, on the average capacity) of landfills. If keeping the rest of parameters at their benchmark values given in (19), the optimal number is always $K^* = 3$. Interestingly, as ϕ increases, the capacity of the first and the third landfill increase, while the capacity of the second decreases, and the locations adjust accordingly. The effect of parameter β is rather predictable, as it makes the safety regions around landfills to increase and so they become more distant one from another. This reduces the feasible region and the problem may ultimately become infeasible if β gets large enough. Changes in parameters V_0 and V_1 only become relevant when the lower and upper capacity limits are binding, and the effect is the trivial one in capacity (i.e., if the lower capacity constraint is binding for some landfills and V_0 increases, then the capacity of such landfills have to increase and so on), and the locations optimally adjust to this changes.

5 Conclusions and further research

We have presented a sequential model to study the joint determination of the optimal capacity and location of landfills and shown how these decisions interact with each other. Summing up, the capacity decision has some spatial implications because the capacity of a landfill affects the feasible region for the rest of landfills, and also temporal implications, because the capacity determines the lifetime of the landfill and hence the instant of time where next landfills will need to be constructed. We have shown that this structure gives rise to a non-convex problem which can not be solved with traditional methods.

From the first order conditions we get, as a by-product, a measure of the value of land which varies across different areas, from one landfill to another. We also get the Optimal Capacity Condition, which establishes the impossibility to reduce cost by transferring capacity from one landfill to another.

Despite the fact that construction cost depends positively on the capacity of landfills, under some circumstances it may be optimal to setup an excess capacity if the marginal construction cost is overcompensated by the reduction in total discounted cost achieved by deferring the construction of future landfills. This result implies that the lower capacity constraint binds for at least the last landfill of the sequence.

Given a feasible sequence of locations and capacities, any permutation is also feasible. Optimality requires that landfills are used in such an order that the total discounted cost is increasing. This, in turn, implies that, if the sequence of capacities is not *very increasing*, then landfills are used in increasing order of distance from the cities, as it usually happens in practice.

We have illustrated the use of global optimization methods to find the solution in a specific example with a linear construction cost function. From the sensitivity analysis performed we know that a larger fixed construction costs results in optimally deceasing the number of landfills and making the sequence of capacities to be more decreasing. If marginal construction cost increases, the optimal number of landfills increases and the optimal sequence of capacities becomes more decreasing. When the time horizon varies, the optimal number of landfills behaves as a stair-shape function and the average capacity displays a sawtooth shape.

Some interesting lines of further research and extensions for this paper are the following. First, we observe that there is an increasing interest for recycling so that it is interesting to study the joint decision of landfilling and recycling. Apart from the setting of disposal (and perhaps recycling) facilities, societies have to decide which proportion of waste should be devoted to each treatment method. Obviously, this decision interacts with those of capacity and location of waste facilities. Moreover, the flow of waste may not be constant, as it has been historically the case in practice. Taking this fact into account introduces a new dynamic element in the problem.

When deciding the location of landfills we have only included a generic cost function, which can be suitably interpreted to measure purely economic costs. As a matter of fact, there are some important social and environmental costs associated to waste management that could be explicitly addressed by means of a multicriteria approach.

Finally, in order to calculate transportation costs, some distance measure is needed. We have restricted ourselves to the standard Euclidean distance, but this is not necessary the best measure in practice. For example, the Manhattan distance could be more suitable for cities and the different mathematical properties of this distance can result in different properties of the solution.

6 Appendix

6.1 Proof of proposition 1

If K landfills are constructed, given the maximum capacity constraint, we have $KV_1 \geq \sum_{i=0}^{K-1} Y_i$ and using the feasibility constraint $\sum_{i=0}^{K-1} Y_i \geq \tau Q$, we get $KV_1 \geq \tau Q$ or, using the definition of τ_1, $K \geq \dfrac{\tau}{\tau_1}$. If $\dfrac{\tau}{\tau_1}$ is an integer, $\Psi\left(\dfrac{\tau}{\tau_1}\right) = \dfrac{\tau}{\tau_1}$ and he have proved the first part of the proposition. If $\dfrac{\tau}{\tau_1}$ is not an integer, note that $K = Int\left(\dfrac{\tau}{\tau_1}\right)$ violates the feasibility constraint, so $K = Int\left(\dfrac{\tau}{\tau_1}+1\right)$ is the smallest feasible value of K. To prove the second part, suppose we have a solution given by $\{Y^*, R^*\} \equiv \left\{Y_0^*, \ldots, Y_{K-1}^*; R_0^*, \ldots, R_{K-1}^*\right\}$ where $K > \Psi\left(\dfrac{\tau}{\tau_0}\right)$. Given that both K and $\Psi\left(\dfrac{\tau}{\tau_0}\right)$ are integers by definition, we have $K \geq \Psi\left(\frac{\tau}{\tau_0}\right)+1$, and given the lower bound for the capacity and the definitions of Ψ and τ_0, we have the following chain of inequalities

$$\sum_{i=0}^{K-2} Y_i \geq (K-1)\,V_0 \geq \Psi\left(\frac{\tau}{\tau_0}\right) V_0 \geq \frac{\tau}{\tau_0} V_0 \equiv Q\tau$$

implying that the combination $\{Y^{*\prime}, R^{*\prime}\} \equiv \left\{Y_0^*,\dots,Y_{K-2}^*; R_0^*,\dots,R_{K-2}^*\right\}$ is a feasible solution and has a strictly smaller discounted cost than that of $\{Y^*, R^*\}$, so that Y^* can not be the cost-minimizing solution.

6.2 Proof of proposition 2

Let $S_\Theta^* \equiv \{Y^*, R^*/\Theta\}$ denote the solution for problem (4) given the value of Θ. If, starting from Θ, τ or Q decrease then S_Θ^* is still feasible (although not necessarily optimal), so $\mathcal{J}_K$ can not increase. Symmetrically, when τ or Q increase, $\mathcal{J}_K$ can not decrease. ϕ and δ do not affect the feasible set, so that, after a change in ϕ or δ, S_Θ^* is still feasible. Consequently, just by computing the derivative of the objective function with respect to these parameters, we see that, if ϕ decreases or δ increases, $\mathcal{J}_K$ can not increase and vice-versa. A decrease (increase) in V_0 or a increase (decrease) in V_1 does not affect directly the objective function, but it increases (decreases) the size of the feasible set, so that $\mathcal{J}_K$ can not decrease (increase).

6.3 Proof of proposition 3

First, note that the objective function of problem (1) can be expressed as $J(K, Y, R) \equiv \sum_{i=1}^{K-1} \gamma_i TD_i$, where $\gamma_i \equiv e^{-\delta T_i}$ and $\gamma_i > \gamma_j, \forall i > j$. Assume the optimal solution si given by $\{K^*, Y^*, R^*\}$, such that $TD_i > TD_{i+1}$ for some $i = 0, \dots, K-2$. Then consider the alternative solution $\left\{K^*, \tilde{Y}, \tilde{R}\right\}$ where $\tilde{Y}$ is constructed by shifting the positions of landfills i and $i+1$ and $\tilde{R}$ is the associated permutation of the elements of R^*, while keeping the rest of element of $\{K^*, Y^*, R^*\}$ unchanged. It is immediate to show that $\left\{K^*, \tilde{Y}, \tilde{R}\right\}$ is feasible and provides a smaller discounted cost than $\{K^*, Y^*, R^*\}$, therefore $\{K^*, Y^*, R^*\}$ can not be the optimal solution for (1).

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