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Cost Allocation as a Coordination Mechanism*

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Cost Allocation as a Coordination Mechanism

Abstract

This paper shows that cost allocation can endogenously arise as a coordination mechanism in a decentralized firm. This result is derived in a setting with multiple (internally supplied) resources shared by multiple users, which constitutes a departure from previous literature. While standard cost allocation procedures use one allocation base, the optimal cost allocation mechanism derived here can select many allocation bases, therefore providing support for the use of Activity Based Costing. Like the cost allocation itself, the selection of allocation bases also arises endogenously.

Keywords: Coordination, Cost Allocation, Cost tracing, Transfer Pricing

1 Introduction

Why allocate costs? And how? Although these questions have been debated intensively in the literature (and in accounting textbooks), few formal models have attempted to provide an answer to them. Papers analyzing this issue are Zimmerman 1979's seminal paper (cost allocation can act as useful proxy variables for some difficult-to-observe costs that arise from decentralization), Rajan 1992 (eliminating non-truth-telling equilibria in multi-agents games), Magee (providing incentives for efficient resource use), and lately Rogerson 1997 (providing incentives for efficient investment).

In this paper, I argue that cost allocation can endogenously arise as an efficient mechanism to coordinate several divisions of a decentralized firm on the use of internally supplied inputs which are difficult to measure, when communication is difficult or costly. Communication problems is about division managers facing uncertainty, and being unable to communicate the state of nature to the central management once the uncertainty is resolved. Cost allocation then arises whenever there exists common resources which are difficult to measure for the central management. It is indeed necessary for the central management to ensure that, in absence of communication possibilities, divisions' use of the common resource will be coordinated, in that the common resource will be allocated to divisions with "more favorable" state of nature.

If there were a direct measure for every resource used within a firm, then one should never observe cost allocation. Therefore, cost allocation is a consequence of measurability problems within the firm. Such problems can be illustrated by the following statement by Atkinson and Kaplan (1994, 244)

Direct measures may not exist because the cost of obtaining such measures, for example by metering individual production departments or by monitoring the amount of time the house-keeping staff spends in each department, greatly exceeds any potential benefit from using these direct measures.

In this paper, I focus on such measurability issues. In my model, there are two kind of inputs. The use of the first kind of input can be easily measured both by the users' divisions and the top management. I call it the measurable input. The second type of input is measurable by the user divisions, but not by the top management. Put differently, at any time, the user's division knows exactly how much of the input it is using while the top management does not. I call this second type the non measurable input. Therefore, there is a moral hazard problem, but I shall not model it explicitly (I discuss this issue later). Since the measurability assumption is critical to the paper, let me provide some examples.

Horngren Foster and Datar (ninth edition, 515) write that "For example, managers often view the number of machine setups as a driver of indirect manufacturing costs, but some companies do not systematically record this information." Take the case of a machine being commonly used by several departments. While the internal system might not keep records of the machine use by each user, any particular division knows exactly how much it used a given machine, and therefore the amount of energy needed to run the machine.

Although electricity seems a priori easy to meter, its cost is often described in accounting textbooks (including in the above citation by Atkison and Kaplan) as an indirect cost. For example in Horngren, Foster and Datar (p.472), an example of indirect cost is “electricity used to run machine. Electricity metered to firm but not to individual machines”. This is an example in which user divisions will know how much electricity they used, but not top management. Indeed, most machines have a clear and unambiguous technical relation between time of usage and electricity consumed. Since each user knows how much it is using the machine (but not the top management), it also knows how much electricity it uses.

Finally, some of the inputs can be interpreted as quality, which is costly. While the quality of an input can be easily measured by the division using the input, it is not always obvious for top management to measure quality.

To summarize the above examples, many inputs can actually be measured by top management, but at some cost. However, as Horngren, Foster and Datar further note (p. 515), “some firms place a low priority on investments in their internal accounting systems, given that the benefits from such investments are frequently difficult to quantify.”

I analyze a setting with a multidivisional firm in which several upstream departments provide several intermediate goods (or service) to several downstream departments. These departments can be cost or profit centers. To ensure that I get the most general setting possible, I allow for the possibility that the inputs used by a department be complements or substitutes, and also spillovers between upstream divisions . The first relation represents the so-called “correlation” which is used to select cost drivers or allocation bases for allocating costs. For example consider a department producing cell phones by using chips produced by an internal department, labor, and machinery. These inputs are very likely to be complements ¹. The second relation (spillovers) just represent the synergies which can exist between production supports departments (here the upstream divisions). My model therefore allows for one cost center, one profit center and one intermediate good exchanged; one cost center, one profit center and several intermediate goods; several cost centers, one profit center and several intermediate goods etc.

The organization that I study is assumed to be decentralized, due to communication problems I mentioned earlier. I do not model these problems, but rather take it as exogenous. All divisions of the firm face uncertainty, and, for a given division, the realized state of nature is known by this division manager only. If communication was possible or not costly, then the top management would wait until the realization of the uncertainty, and ask all divisions to report their state of nature. Top management would then take centralized production decisions and communicate them to the division managers. However, one problem which might arise then is related to truthful reporting by the division managers. Together with communication problems, superior information at division’s level reinforce the necessity for the firm to be decentralized.

In decentralized firms, top management needs to ensure that the divisions’ de-

¹Labour and machinery can also be substitutes up to some point. However, I believe that there is at least some degree of complementarity. If this is not the case, then the firm would be better off by using the cheapest among the two alternatives.

cisions are coordinated because individual profit maximization will often lead to an outcome which does not maximize global profit. Coordinating managers’s decisions regarding the use of the measurable input is a rather easy task. Indeed, assigning a price to these inputs can induce the managers to behave in the desired way. The problem, however, remains with the inputs the use of which is not measurable. It is indeed not possible for the top management to assign a price to an input the use of which it cannot physically measure. However, top management can take into account the complementarity between measurable and non measurable inputs to coordinate efficiently the use of the non measurable input². What I show in this paper is that if the top management charges a price (or equivalently traces the cost) for measurable inputs, the optimal charge will endogenously allocate the cost of the non measurable inputs through the complementarity relation that exists between measurable and non measurable inputs. Moreover, I show that the cost allocation procedure endogenously selects (many) allocation bases, which confirms the superiority of Activity Based Costing over traditional cost allocation procedures.

This paper is organized as follows. Section 2 briefly relates the paper to existing work. Section 3 introduces the model. Section 4 describes more extensively the firm and the coordinating mechanism used. Moreover a benchmark in which all inputs are measurable is analyzed. In section 5, I derive the endogenous cost allocation mechanism. Section 5 provides an illustration with two inputs being supplied. Finally section 7 concludes.

2 Related literature

Although our paper shares a common goal with with most papers providing a rationale on cost allocation (see first paragraph of introduction), it is more closely related to Zimmerman (1979). Zimmerman shows that “cost allocations appear to proxy for certain hard-to-observe costs that arise when decision-making responsibilities are assigned to and vested in various individuals (i.e., decentralized) within the firm.” Cost allocation is then a tax that induces managers to behave in an appropriate way. This is what happens in our paper, with the difference that cost allocation arises endogenously here, allocation bases are endogenously selected. However, the majority of the literature on cost allocation or transfer pricing (including Zimmerman’s paper) are limited to one resource user and one resource being shared ³ For instance, in Magee (1988), there is one agent using one resource. Although his paper provides a rationale for allocating the cost of the resource to an agent, Magee recognizes himself the limitation of the single agent setting. A notable exception is Rajan (1992), who models one resource, but several users. However, in Rajan, the cost allocation scheme does not arise endogenously but is built. Our paper therefore constitute a departure from previous literature in that it uses the most general model with n users, m resources, with possible spillovers between supplier divisions⁴,

²Efficiency is meant in the second-best sense.

³In this statement, I do not include the literature on mathematical programming.

⁴For instance, Abdel-khalik and Lusk (1974) found the Hirshleifer (1956)’s assumption-of technological independence between divisions-unrealistic. Although these papers focus on transfer pricing (which, still is related to cost allocation), the criticism holds for organizations in general.

and a possible relation (complementarity or substitutability) between the different resources for the user divisions.

3 Model

Consider a vertically integrated firm composed of $n + m$ divisions. m upstream division (cost or profit centers) produce each an intermediate good (or service), which are used by n downstream divisions (profit centers). The upstream divisions are assumed to be production support departments⁵ like a manufacturing division producing auto engine for the use of assembling divisions, a service department providing engineering/consulting services, a data processing department, or more generally, any division which provides services or an intermediate good to other divisions of the firm. The downstream divisions are all other divisions using the intermediate goods or services to produce the final good and meet external demand, i.e the profit centers.

Let $z \equiv (z^1, \dots, z^m)$ denote the production vector of all intermediate goods. Each of the profit centers uses the inputs and/or services provided by the upstream divisions and turns it into a final good. Let $z_i \equiv (z_i^1, \dots, z_i^m)$ denote the vector of inputs/services used by division i (if division i does not use input j , then $z_i^j = 0$). When it uses the vector of inputs z_i , division i generates a profit $\pi_i(z_i)$, by selling the final good in the market. For the time being, let the profit functions be gross of any charge (from the firm) related to the use of internal resources.

Following Crémer (1980), I assume that the production period is a year and that before the beginning of the year, when making budgets, the cost and benefit functions are not exactly known to the firm. The reason is that after budgeting is made, all divisions need to buy goods (raw material, labor, capital etc..) outside the firm and there is uncertainty related to the price of these goods. Moreover, on the downstream division side, there is demand uncertainty for the final goods⁶. I assume that the cost and profit functions can be approximated as follows:

$$\pi_i(z_i) = \lambda_i + \Gamma_i B_i (z_i - \bar{z}_i)' - \frac{1}{2} (z_i - \bar{z}_i) B_i (z_i - \bar{z}_i)' \quad (1)$$

$$C(z) = \lambda + \Gamma B (z - \bar{z})' + \frac{1}{2} (z - \bar{z}) B (z - \bar{z})' \quad (2)$$

where the symbol \prime denotes the transpose sign. The m -vectors Γ_i and Γ are random vectors representing the uncertainty, and are assumed to be independent. The $m \times m$ -matrices B_i and B are assumed to be symmetric positive definite in order to ensure that the benefit and cost functions are well behaved⁷. The vectors \bar{z}_i and \bar{z}

⁵In particular, they are not headquarters support, advertising, or any division which cost enters the G&A overheads

⁶The firm can buy all these goods in advance, and avoid the uncertainty related to price fluctuations. However, by doing so, it exposes itself to the risk of incurring losses either from over-purchasing (when inputs are perishable, or storage is costly), or from under-purchasing (inability to meet external demand and having to acquire inputs from the more expensive spot market)

⁷Positive definite implies that C is convex and π_i concave. There is no loss of generality in assuming symmetry, which ensures that $B_i^{-1} = (B_i^{-1})'$.

are budgeted targets that I define more precisely in section 4. Finally, the λ_i s and λ are constants, equal to the budgeted profit of division i and budgeted cost of the supplier division respectively (rather than fixed costs)⁸. Without loss of generality, I set $\lambda_i = \lambda = 0 \quad \forall i = 1, \dots, n$.

The diagonal of B_i (respectively B) represents the curvature of division i 's benefit function (respectively the upstream division's total cost function). Using a less abstract language, the j^{th} diagonal element of B_i measures the loss that division i would incur by deviating from the budgeted use of input j by 1 unit. The reason is that by using one more unit of input j than expected, division i would need, for example, to hire more labor in the (more costly) spot market. Therefore, for a given division, the greater the curvature associated with an input j is (measured by the j^{th} diagonal element), the greater will be the loss from diverging from the budgeted quantity of this input. The extra-diagonal elements of the B_i matrices measure the degree of complementarity (when non positive) or substitutability (when non negative) between two inputs. The extra-diagonal elements of the B matrix are a measure of the positive spillovers between upstream divisions and are therefore assumed to be non positive.

In a well-known paper, Weitzman (1974) proves that the functional forms $\pi_i(\cdot)$ and $C(\cdot)$ that I use⁹ can be rigorously defended as approximations to more general functions, under a condition, which in our case is:

ASSUMPTION

$$\mathbf{E} [\Gamma_i B_i] = \mathbf{E} [\Gamma B]. \quad (3)$$

I shall assume through all the paper that condition (3) holds. Therefore, the qualitative results that will be derived in this paper are robust to the specification of the cost and profit functions. The quadratic form simply makes the computations easier.

The firm is assumed to be decentralized, for reasons that I do not model. Following Baldenius and Reichelstein (2002), I assume that, for a given transfer payment (charge) set by the firm, the upstream divisions are required to satisfy the input demand of the downstream divisions. Downstream divisions are assumed to maximize their own divisional profit, that is, the gross profit π_i net of any payment imposed by the firm, by choosing the appropriate combination of inputs. As Baldenius and Reichelstein further note, "this model specification seems reasonably descriptive of most multidivisional firms". The objective of this paper is not to look at optimal mechanisms in general, but rather within the class of mechanisms observed in practice (e.g., cost tracing, transfer pricing). This means that I take the firm structure as given, and see whether I can explain the existence of a phenomena, which is cost allocation.

Because managers maximize divisional profit, I can abstract from moral hazard considerations. The timing of events is as follows:

⁸By replacing z_i and z by their budgeted levels \bar{z}_i and \bar{z} , the last two terms of each function equals zero and it remains only the λ s. Stated differently, if divisions were to produce and use exactly the budgeted amount of inputs, then there is no uncertainty about the profits and benefit functions which are then equal to λ_i and λ respectively.

⁹These functional forms are borrowed from Cr mer (1980).

- Input charges are set at the budgeting stage by central management
- Realization of the state of nature. Only the division's manager observes the realization of his own environment, and not other managers or top management.
- Downstream divisions choose simultaneously, and non cooperatively, the quantity to produce, and hence the quantity of inputs/services to use.

4 Budgeting

4.1 Relevance of the “ideal point” model

In this subsection, I show how the vectors \bar{z}_i and \bar{z} are obtained, and provide an economic interpretation for them. I said that the cost and benefit functions are not known in advance at the budgeting stage. Furthermore, I also assumed that the organization is decentralized.

Now consider the ideal organization in which communication is not a problem. The owners would wait until the uncertainty is resolved, and solve:

$$\begin{cases} \max_{z_1, \dots, z_n} \sum_{i=1}^n \pi_i(z_i) - C(z) \\ \text{subject to} \\ \sum_{i=1}^n z_i = z, \end{cases}$$

where the $\pi_i(\cdot)$ and $C(\cdot)$ are the real functions and not the approximations. Let (z_1^*, \dots, z_n^*) denote the (random) solution of that program.

Lemma 1. *The vectors \bar{z}_i are the expected values of the z_i^* , and $\bar{z} = \sum_{i=1}^n \bar{z}_i$.*

Proof. See appendix A1 □

Therefore, $(\bar{z}_1, \dots, \bar{z}_n)$ can be simply interpreted as a budgeted target. Indeed, budgets are made by top managements (the centralization aspect) based on forecasts made by managers. If I interpret the expected values of the random variables as the forecasts made by division managers¹⁰, then $(\bar{z}_1, \dots, \bar{z}_n)$ is simply the budget target of the firm (this will become even more clear at the end of subsection 4.3). This target is determined by top management by maximizing the total profit of the firm based on forecasts of the cost and benefit functions given by managers.

Notice that the vector of budgeted marginal costs (MC^B) is given by

$$MC = \bar{\Gamma}B - (z_B - \bar{z})B$$

where z_B is the budgeted vector of total input produced, and $\bar{\Gamma}$ is the expected value of Γ . Given that the budgeted target \bar{z}_i is such that $\sum_{i=1}^n \bar{z}_i = \bar{z} = z_B$, the vector

¹⁰Most budgets are based on expected (standard) values rather than any other values. Transfer prices, cost tracing and cost allocations are also based on expected values.

of budgeted marginal costs (MC^B) therefore simplifies to

$$MC^B = \bar{\Gamma}B$$

This remark is important to keep in mind for what will follow.

4.2 The coordination mechanism

Since resources are allocated on a budgeted basis (the \bar{z}_i 's are based on $\bar{\Gamma}_i$ and $\bar{\Gamma}$), what happens if the realization of the Γ_i 's and that of Γ differ from $\bar{\Gamma}_i$ and $\bar{\Gamma}$, or simply put, if the forecasts are not correct. In that case, there is necessarily a need for reallocating resources, especially when they are limited. The crucial question is then the following: given that communication is not feasible, how can top management ensure that ex-post (after the uncertainty is resolved), resources will be used by the divisions having a more favorable state of nature? There are several ways of achieving this and I shall discuss the two which seems the most relevant.

An obvious way is to organize a second round of budgeting after that the random events are realized. However, Zimmerman (2003, p.603) notes that “most firms are very reluctant to change standards during the year. Once a standard is set at the beginning of the year, it is rarely revised during the year.”

A second way is simply to set ex-ante a mechanism that will reallocate resources to their best use and hence ensure that individual maximization coincides with global maximization. Obviously, this is a second-best solution.

In this paper, I choose the second coordination way. Moreover, since the firm is decentralized, the owners, who seek to maximize $\mathbf{E} \left(\sum_{i=1}^n \pi_i - C(z) \right)$, need to make the divisions' maximization consistent with global maximization. The owners' instruments consist in a vector of transfer payments by each profit center, which I assume to be linear in prices charged to the only measurable inputs. These linear charges are set ex-ante (see footnote 10). The economic interpretation behind these payments is transfer pricing, or cost tracing. Although the literature and most accounting textbooks use these two terms in a way which suggests that they are not related, tracing a cost to a department is usually equivalent to charging the department a transfer price for the resources that caused the cost. There are however some exceptions. Zimmerman (2003, p.211) notes that “most firms use cost allocations as a method of charging internal users for goods or services received from another part of the organization” and that “These cost allocations are nothing more than transfer prices”. Lambert (2001, p.67) also views cost allocations as “a special case of transfer pricing”.

Each profit center's manager thus maximizes the gross benefit π_i , net of the payment imposed by the owners. Note however that a direct per unit rate can be applied only to an input for which top management has an accurate measure. Therefore, the inputs which are not accurately measurable will not be directly charged. But I shall show that the optimal charge for the measurable inputs does in fact endogenously determine the optimal indirect rate for inputs which are not measurable.

The reason for which I have chosen linear transfers is not innocent. Firstly, it

can represent transfer prices which are often linear¹¹. Secondly, given that our cost functions are valid approximation to any cost function, including linear costs, cost tracing at variable cost is equivalent to charging each input its variable cost, hence the linear charge. Therefore my assumption of linear payments are fairly realistic.

At this point, it is worth noticing that the term “cost allocation” does not always have the same meaning. Sometimes cost allocation refers only to the allocation of indirect costs. It can also refer both to cost tracing of direct costs and allocation of indirect costs. Finally, allocation can sometimes mean tracing, while the term apportionment is then used to describe the allocation of indirect costs. In this paper, I will use the term cost tracing or transfer pricing to refer to the charge of measurable inputs (which cause direct costs), and use the term cost allocation to refer to the (indirect) charge of the non measurable inputs (which cause indirect costs). Moreover, I analyze only variable manufacturing overheads. My model therefore does not explain the allocation General and Administrative costs, as well as fixed costs allocations.

4.3 Decentralizing the budget target in a perfect world

The previous section described the budgeting process and the way top management centrally determines budgeted targets. However, in decentralized firms (as it is the case in our model), once budgets targets are set, some mechanisms are put in place to achieve these targets. Indeed, managers of profit centers are usually not asked to produce a given level of output, but rather, to maximize their profits at a given internal charge (the coordination mechanism) determined optimally by the central level¹². These mechanisms can be designed in such a way that when decentralized units maximize their profits, the (expected) resulting outcome coincides with the budgeted target. In this subsection, I look into the optimal¹³ coordination mechanism when the use of all inputs is perfectly measurable. Such setting will provide a useful benchmark for the problem that I analyze.

The optimal input charges are obtained by solving backward:

1. given the vector of input charges set ex-ante, each division maximizes its ex-post accounting profit by choosing the optimal level of inputs to purchase from the supplier division.
2. Anticipating the reaction of the divisions to the input charges, the organization sets them to maximize expected global profit.

¹¹Baldenius, Edlin and Reichelstein (1999) report that, when the supplying division has also access to external markets, intracompany transfers are often subject to discounts. However, the authors use a constant discount which they describe as “the most common alternative”, therefore keeping the linearity of transfer prices.

¹²If the charge is a transfer price, then my model focuses on centrally determined transfer prices. In particular, it excludes negotiated transfer pricing. Charges that relate to cost allocation or cost tracing are always centrally determined.

¹³Whenever I say optimal mechanism, it should be understood within the class of linear transfer payments.

Let $p \equiv (p_1, \dots, p_m)$ denote the vectors of input charges set by the firm at the budgeting stage. Division i chooses z_i by solving

$$\max_{z_i} \Gamma_i B_i (z_i - \bar{z}_i)' - \frac{1}{2} (z_i - \bar{z}_i) B_i (z_i - \bar{z}_i)' - p z_i'.$$

The first order conditions of this program is

$$\Gamma_i B_i - (z_i - \bar{z}_i) B_i = p, \quad i = 1, \dots, n$$

or equivalently

$$z_i = \bar{z}_i + \Gamma_i - p B_i^{-1}, \quad i = 1, \dots, n. \quad (4)$$

Equation (4) describes division i 's ex-post reaction to the vector of input charges set ex-ante. The top management takes it into account to set the optimal prices. Given that the upstream divisions are required to satisfy the demand of the downstream divisions, I need not specify a reaction function for the supplier divisions to the vector of input charges. The following result is obtained.

Proposition 1. *If the use of all inputs were perfectly measurable, then the optimal coordination mechanism would imply budgeted marginal cost pricing. There is no cost allocation, but instead pure transfer pricing or cost tracing, as there are no overheads.*

Proof. If we set quantities before uncertainty is resolved, the first-best is obtained by z_i 's such that $\mathbf{E}[\pi_i'(z_i)] = \mathbf{E}[C'(z)]$, where $'$ represents the derivative. This implies

$$\mathbf{E}(\Gamma_i B_i) - \mathbf{E}(z_i - \bar{z}_i) B_i - \mathbf{E}(\Gamma B) - \left(\sum_{i=1}^n \mathbf{E}(z_i - \bar{z}_i) \right) B = 0 \quad i = 1, \dots, n,$$

which is a necessary and sufficient condition for optimality. Given condition (3), it is satisfied if $\mathbf{E}(z_i - \bar{z}_i) = 0$. From Division i 's first order condition, i.e. equation (4), it can be seen that if one sets p equal to the vector of budgeted marginal cost $\bar{\Gamma} B$ ($= \bar{\Gamma}_i B_i$ by (3)), then division i will choose z_i such that

$$\begin{aligned} z_i - \bar{z}_i &= \Gamma_i - p^* B_i^{-1} \\ &= \Gamma_i - \bar{\Gamma}_i B_i B_i^{-1} \\ &= \Gamma_i - \bar{\Gamma}_i. \end{aligned} \quad (5)$$

By taking expectation on both sides, I get $\mathbf{E}(z_i) = \bar{z}_i$. \square

Again, to prove that the \bar{z}_i are to be interpreted as the budgeted target, assume that the forecasts provided by managers at the budgeted stage are perfectly accurate (or that there is no uncertainty), i.e. that $\Gamma_i = \bar{\Gamma}_i$. Given that there are no measurability issues, the budgeted plan and the actual realization should perfectly coincide. From equality (5), it is clear that when $\Gamma_i = \bar{\Gamma}_i$, then $z_i = \bar{z}_i$, i.e. division i produce the budgeted target. Referring to Hirshleifer (1956), there is no surprise that the vector of optimal charges which best coordinate divisions is simply the vector of budgeted marginal costs.

At this point, it is important to say something about contingent budgets. If managers' forecasts are inaccurate, then we are in a second-best situation. Indeed, the optimal charge is calculated based on the budgeted plan which ultimately will differ from actual realization. To avoid this situation, the firm could set contingent targets, instead of the single target \bar{z}_i based on forecasts, and again first-best would always obtain. However, to the best of my knowledge, contingent budgets is very little (if not never) reported or discussed in any accounting textbook.

5 Cost allocation

I now get back to the central assumption that the use of some inputs is not easily measurable. In the previous section, it is said that in absence of uncertainty, the actual input use will coincide with the budgeted plan. Does this always hold? The answer turns out to be that it does *only* in the case all inputs are easily measurable. When the use of some inputs is difficult to measure, variances will occur even in absence of uncertainty, due to the moral hazard problem about the use of the common non measurable inputs. What I shall prove in this section is that when the measurable inputs are (exogenously) charged to the upstream divisions, then allocation of the non measurable input's cost endogenously arises.

I model the measurability problem by assuming that only the use of the k first inputs ($k < m$) are measurable by top management. The $(m - k)$ last inputs are therefore assumed to be not measurable at all. Such assumption might seem a bit extreme at first glance. However, it is not. Firstly, it reflects more the likelihood that division managers can better measure their use of some inputs than does top management. Secondly, since I use linear charges to coordinate (regulate) the use of inputs, these charges can apply only if the input is perfectly and unambiguously measurable. Therefore, when an input is imperfectly measurable, no matter if top management can observe a partial signal or not about its use, linear charges (such as a price) cannot apply to that input. More complex mechanisms can be used to take into account the signal, if exists. However, such complex mechanisms are little observed within firms, and this is the reason for which I look at simpler and more widely observed mechanisms such as transfer pricing, cost tracing or, as I shall prove soon, cost allocation.

The objective is now to search for the k -vector of prices p^{**} , for the *only measurable* inputs, which maximize total expected profit. The prices of all non-measurable goods are set equal to zero since they are freely available inputs. The vector of prices take here the following form

$$\begin{aligned} p &\equiv (p_1, p_2, \dots, p_k, 0, \dots, 0) \\ &\equiv p_1(1, 0, \dots, 0) + p_2(0, 1, 0, \dots, 0) + \dots + p_k(0, \dots, 1, 0, \dots, 0) \\ &\equiv \sum_{j=1}^k p_j e_j \end{aligned}$$

where e_j is the vector of length m which j^{th} component is equal to 1, all other components being equal to zero.

The first-order condition of division i 's program can be expressed as

$$\Gamma_i B_i - (z_i - \bar{z}_i) B_i = \sum_{j=1}^k p_j e_j, \quad i = 1, \dots, n$$

and its optimal level of input given by

$$(z_i - \bar{z}_i) = \Gamma_i - \sum_{j=1}^k p_j e_j B_i^{-1}, \quad i = 1, \dots, n. \quad (6)$$

As said in the previous subsection, the organization takes into account equation (6) into the total profit function, and maximize its expected value with respect to p^{**} . The following lemma provides the central result of this paper.

Lemma 2. *The optimal charge for the measurable is given by $p^{**} = p_o^* + p_u^* H$, where p_o^* is the k -vector of budgeted marginal costs for the measurable inputs, p_u^* is the $(m - k)$ -vector of budgeted marginal cost for the non measurable inputs, and H is an $(m - k) \times k$ matrix.*

Proof. See appendix A2 □

Lemma 2 states that the optimal charge for each measurable can be expressed as its budgeted marginal cost plus a linear combination of the budgeted marginal cost of all non measurables. For instance, assume that there are five upstream divisions providing five inputs/services to n downstream divisions. Assume that only inputs 1 and 2 are measurable. According to lemma 2, the optimal charge for these inputs is given by

$$p_1 = c_1 + \alpha c_3 + \beta c_4 + \epsilon c_5$$

and

$$p_2 = c_2 + \nu c_3 + \sigma c_4 + \zeta c_5$$

where c_i is the budgeted marginal cost of input i and is given by the i^{th} element of the vector $\bar{\Gamma} B$. The coefficients $\alpha, \beta, \epsilon, \nu, \sigma$ and ζ are the elements of the H matrix, and are functions of the parameters of all the B_i 's and B matrices. In general, it is very hard to determine the sign of these coefficients. However, in the next section, I prove in a two-inputs case that they are positive (negative) whenever all inputs are complements (substitutes), and provide the intuition for the conjecture that this result holds with more than two inputs. When some inputs are complements and others are substitute, then the overhead rates can be either positive or negative.

One could argue that, given the fact that some inputs are not measurable, the optimal charges derived in lemma 2 are the true budgeted marginal costs (adjusted for the non measurable inputs). However, they are not, because the budgeted allocation (and hence the budgeted marginal costs) are centrally determined, based on the forecasts given by division managers (see subsection 4.1). Therefore, the budgeted marginal costs are calculated assuming that all inputs are measurable. However, top management is not so naive, and anticipates the overuse of the non measurable resources which it can regulate through an appropriate charge. I can now state the following proposition, which is a consequence of lemma 2:

Proposition 2. *The optimal coordination mechanism implies transfer pricing (or cost tracing) for the measurable plus cost allocation for the non measurable.*

The above proposition states that there is cost allocation, in the sense that an element of the cost of non measurable inputs (marginal cost) is charged to divisions, based on their use of the measurable inputs. Although this definition of cost allocation is a priori different from standard definitions of cost allocation, the results obtained here allow us to gain useful insights about the emergence and practice of cost allocation. Moreover, in what follows, I will show that my definition of cost allocation is consistent with variable cost allocation. Finally, notice that such a definition of cost allocation does not stand alone in the literature. For instance, in Magee (1988) the cost allocation consists in including the level of resource use in the user's compensation contract, instead of basing the contract on the outcome alone, as is standard in moral hazard problems. Again, although Magee's definition of cost allocation is not the accounting textbooks' standard definition, his paper provides useful insights about the existence of cost allocation.

Now, returning to the example of 5 inputs being provided, the following points can be emphasized

Firstly, despite the fact that the use of inputs 3 and 4 and 5 are not measurable and cannot be directly charged, profit centers are nevertheless indirectly charged for the cost of these inputs. At this point, it is very difficult to say how much of that cost divisions are charged. But I come back on this in my second point. The optimal charge for the measurable inputs includes a direct charge for the use of these inputs (transfer pricing or cost tracing at budgeted cost), *and* an indirect charge for the non measurable use of inputs 3, 4 and 5 (cost allocation at budgeted cost). For each unit of a measurable input, a profit center will therefore have to pay a rate for the use of the non measurable input, equal to a constant times the budgeted marginal cost of the non measurable input. This rate can be interpreted as the overhead rate. However, notice that there is not one single overhead rate for the allocation of a given non measurable resource. There should be as many overhead rates as allocation bases. If for instance, input 1 and input 2 present the same degree of complementarity with input 3, then this would mean that the proportion of input 3 to both inputs 1 and 2 is the same. In this case, a single overhead rate can be defined, equal to $(\alpha + \nu)c_3$. This rate can then be indifferently applied to input 1 or input 2.

My second point will now focus on how much cost is allocated. As said before, it is difficult to say in general how much of the total cost of the non measurable resource is allocated. However, there is one case in which this can be easily done. This case corresponds to variable costing. In the section describing the model, I said that the cost function is a general approximation to any cost function. Take the case of a linear cost function. In this case, marginal cost equals variable costs. For a level z_i^1 and z_i^2 of inputs 1 and 2 used by division i , the allocated cost of input 3 to division i is $c_3(z_i^1\alpha + z_i^2\nu)$, and the total allocated cost is $c_3 \sum_{i=1}^n (z_i^1\alpha + z_i^2\nu)$. If the total amount of input 3 used¹⁴ is z^3 , then the common cost of input 3 is c_3z^3 .

¹⁴Remember that the total amount of a non measurable resource is assumed to be known. What is unknown is the repartition of this resource across divisions which use them.

Depending on whether z^3 is greater, equal or smaller than $\sum_{i=1}^n (z_i^1 \alpha + z_i^2 \nu)$, I can conclude that there is under-allocation, full allocation or over-allocation.

Finally, our model endogenously select allocation bases. Indeed, our results show that several allocation bases should be selected, instead of just one, as it is the case with Activity Based Costing (ABC). Although ABC has been claimed to be superior to traditional allocation systems, there is (to the best of my knowledge) no analytical result supporting this claim. Our results provide such support. In our example, both inputs 1 and inputs 2 should serve as an allocation base for allocating the cost of, say, input 3 as long as α and ν are non nil. In the next section, I show in an example with two inputs that α (respectively ν) are nil only if input 1 (respectively input 2) has a zero degree of complementarity with input 3. On the other hand, if there is perfect complementarity, then first-best would obviously be obtained¹⁵.

The reason for selecting many allocation bases is quite intuitive. Assume for instance that both input 1 and input 2 have a positive relation with input 3, but with different degrees of complementarity. If input 1 alone is selected as the allocation base, then the overhead rate will not optimally take into account the relation between these three inputs. It will be a compromise between regulating optimally the use of input 3 by affecting the use of input 1, and regulating optimally the use of input 3 by affecting the use of input 2. This means that too much distortion will be imposed on input 1. By selecting both variables as allocation bases, this distortion can be distributed, in a balanced way, across the two allocation bases.

6 An illustration with two upstream divisions

So far, the generality of our model does not allow us to say something about the “overhead rates”, i.e. the rates indirectly charged to the non measurable resources. This section intends to fill this gap through an example with two upstream divisions only. Much of the intuition derived here can be extended to the general setting. Assume that the input provided by division 1 is measurable, while the input provided by division 2 is non measurable. For instance, division 1 can be the personnel department supplying labor force to other departments, labour hours being measurable, while division 2 is the power supply department. The parameters of the benefit and cost functions are

$$\Gamma_i = (\gamma_i^1, \gamma_i^2) \quad \Gamma = (\gamma^1, \gamma^2) \quad B_i = \begin{pmatrix} b_i^1 & \delta_i \\ \delta_i & b_i^2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} b^1 & \delta \\ \delta & b^2 \end{pmatrix}.$$

I then have

$$B_i^{-1} = \frac{1}{\Delta_i} \begin{pmatrix} b_i^2 & -\delta_i \\ -\delta_i & b_i^1 \end{pmatrix},$$

where $\Delta_i = b_i^1 b_i^2 - (\delta_i)^2$ is the determinant of the B_i matrix.

¹⁵In the case of perfect complementarity, the use of non measurable inputs can be accurately inferred from the use of measurable inputs.

The optimal overhead rate is given by (see Appendix A3)

$$\frac{-b^1 \sum_{i=1}^n \frac{\delta_i}{\Delta_i} \sum_{i=1}^n \frac{b_i^2}{\Delta_i} + \delta \sum_{i=1}^n \frac{b_i^2}{\Delta_i} \sum_{i=1}^n \frac{b_i^1}{\Delta_i} + \delta \left(\sum_{i=1}^n \frac{\delta_i}{\Delta_i} \right)^2 - b^2 \sum_{i=1}^n \frac{\delta_i}{\Delta_i} \sum_{i=1}^n \frac{b_i^1}{\Delta_i} - \sum_{i=1}^n \frac{\delta_i}{\Delta_i}}{b^1 \left(\sum_{i=1}^n \frac{b_i^2}{\Delta_i} \right)^2 - 2\delta \sum_{i=1}^n \frac{b_i^2}{\Delta_i} \sum_{i=1}^n \frac{\delta_i}{\Delta_i} + b^2 \left(\sum_{i=1}^n \frac{\delta_i}{\Delta_i} \right)^2 + \sum_{i=1}^n \frac{b_i^2}{\Delta_i}} (\bar{\gamma}^2 b^2)$$

Notice how lengthy is the formula of the overhead rate with only two upstream divisions. If there are two downstream divisions, then each term is doubled. However, even in general (with $n + m$ divisions), the overhead rate is easily computable by any simple statistical program. All what is needed is some algebraic transformations on the B_i s and B matrices. It is very difficult to say something about the overhead rate in general. But there are some cases in which more can be said.

Absence of spillovers between upstream divisions

In this case, $\delta = 0$ and the overhead rate simplifies to

$$OHR = \frac{-\sum_{i=1}^n \frac{\delta_i}{\Delta_i} \left(b^1 \sum_{i=1}^n \frac{b_i^2}{\Delta_i} + b^2 \sum_{i=1}^n \frac{b_i^1}{\Delta_i} + 1 \right)}{b^1 \left(\sum_{i=1}^n \frac{b_i^2}{\Delta_i} \right)^2 + b^2 \left(\sum_{i=1}^n \frac{\delta_i}{\Delta_i} \right)^2 + \sum_{i=1}^n \frac{b_i^2}{\Delta_i}} (\bar{\gamma}^2 b^2)$$

Notice that the overhead rate is positive if inputs are complements ($\delta_i < 0$) and negative if inputs are substitutes ($\delta_i > 0$). Indeed, given the assumption that the B_i matrices are symmetric and positive definite, the following properties hold: $\Delta_i > 0$, $b_i^1 > 0$, $b_i^2 > 0$. The denominator is then always positive, and the term in brackets in the numerator is also positive.

Notice also that whenever there is no relation between input 1 and input 2 (i.e. $\delta_i = 0$), then the overhead rate is equal to zero, meaning that no cost should be allocated¹⁶. Any cost allocation would simply be arbitrary.

Because overhead rates are positive in practice, I focus only on the case of complementary inputs ($\delta_i < 0$) for the rest of this subsection (see the next subsection for an interpretation of negative overheads)¹⁷. To understand how cost allocation operates, it is necessary to see what happens when no cost allocation occurs. Assume therefore that there is only cost tracing, i.e. that input 1 is charged its budgeted marginal cost c_1 (denote also by c_2 the budgeted marginal cost of input 2). Using equation (6), I can determine the amount of inputs 1 and 2 to be used by division i

$$\begin{pmatrix} z_i^1 - \bar{z}_i^1 \\ z_i^2 - \bar{z}_i^2 \end{pmatrix} = \begin{pmatrix} \gamma_i^1 \\ \gamma_i^2 \end{pmatrix} - \frac{1}{\Delta_i} (c_1, 0) \begin{pmatrix} b_i^2 & -\delta_i \\ -\delta_i & b_i^1 \end{pmatrix}$$

¹⁶In the next subsection, I show that, surprisingly, this result relies on the fact that there are no spillovers between upstream divisions, i.e., $\delta = 0$.

¹⁷Notice that this is also the most realistic case. If only two inputs are internally provided, and these inputs are substitutes, then it would make sense for the firm to produce the cheapest among the two alternatives.

From equation (3), I know that the vector of budgeted marginal cost is

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \bar{\Gamma}B = \bar{\Gamma}_i B_i = \begin{pmatrix} \bar{\gamma}_i^1 b_i^1 + \bar{\gamma}_i^2 \delta_i \\ \bar{\gamma}_i^1 \delta + \bar{\gamma}_i^2 b_i^2 \end{pmatrix}$$

Therefore,

$$\begin{aligned} z_i^1 - \bar{z}_i^1 &= \gamma_i^1 - \frac{1}{\Delta_i} (b_i^2 (\bar{\gamma}_i^1 b_i^1 + \bar{\gamma}_i^2 \delta_i)) \\ &= \gamma_i^1 - \frac{1}{\Delta_i} (\bar{\gamma}_i^1 b_i^1 b_i^2 - (\bar{\gamma}_i^1 (\delta_i)^2 - \bar{\gamma}_i^1 (\delta_i)^2) + \bar{\gamma}_i^2 \delta_i b_i^2) \\ &= \gamma_i^1 - \frac{1}{\Delta_i} (\bar{\gamma}_i^1 (b_i^1 b_i^2 - (\delta_i)^2) + \delta_i (\bar{\gamma}_i^1 \delta_i + \bar{\gamma}_i^2 b_i^2)) \end{aligned}$$

Putting both terms on the same denominator, and simplifying, I get

$$z_i^1 - \bar{z}_i^1 = \gamma_i^1 - \bar{\gamma}_i^1 - \frac{\delta_i}{\Delta_i} c_2$$

and similarly,

$$z_i^2 - \bar{z}_i^2 = \gamma_i^2 - \bar{\gamma}_i^2 + \frac{b_i^1}{\Delta_i} c_1$$

By taking expectation on both sides, I get

$$\mathbf{E}(z_i^1) = \bar{z}_i^1 - \frac{\delta_i}{\Delta_i} c_2 \quad (7)$$

$$\mathbf{E}(z_i^2) = \bar{z}_i^2 + \frac{b_i^1}{\Delta_i} c_1 \quad (8)$$

Therefore, $\mathbf{E}(z_i^2) > \bar{z}_i^2$. Because I focus on the case $\delta_i < 0$, I also have $\mathbf{E}(z_i^1) > \bar{z}_i^1$. This remark calls for the following proposition

Proposition 3. *Assume that input 1 is selected as the basis for allocating the cost of input 2. This will imply a decrease in the use of input 1 by profit centers. Such a distortion is actually desirable for the firm.*

Proposition 3 addresses an issue widely discussed in accounting textbooks: production distortions created by selecting a particular allocation base. The most common case occurs when machine hours is selected as an allocation base, leading division's managers to use less machine, or substitute machine with labor. Such distortions are described as harmful to the organization. Our results say that such downward distortion are actually desirable, and if there are some substitution possibilities, they actually do not matter.

Let me first examine the case in which there are no substitution possibilities for the allocation base (input 1). Not allocating the cost of input 2 has only negative consequences, in the sense that too much of input 2 will be used. This is one negative consequence. Moreover, given that there is a positive relation between input 1 and input 2, an overuse of input 2 will subsequently lead to too much of input 1 being used. This is another negative consequence of not allocating the cost of input 2. For

instance, in our example, if electricity cost is not allocated, then electricity will tend to be overused. If, however, one assumes that whenever a machine is running, it is because an employee is running the machine¹⁸, then a high use of the machine power also means a high use of labor. Therefore, it is desirable to reduce the amount of labor used by selecting labour hours as the variable to which, using Zimmerman's terminology, the "tax" should be applied. It is by taxing the allocation base that the firm will discourage its overuse, and because of the complementarity, also discourage the overuse of the non measurable resource.

Now, consider that there are possible substitutes for labour, for example machine hours. One can then argue that labor will be reduced, but more cost are imposed on the firm because more machine hours will then be used. Our general cost allocation scheme says that this substitution will be costly to the division's manager. Indeed, if there exists a substitute, this means that our example is extended to three inputs: machine, labor and energy, with energy still being the non measurable input. In this case, machine hours and energy are still complements because labor and machine hours are substitutes. Therefore, our general cost allocation rule calls for both labour and machine hours to be selected as basis for allocating the cost of energy. This means that because machine hours will also be taxed, it will not be overused.

Absence of complementarity or substitutability

This case is interesting only from a theoretical aspect, because it is not very relevant empirically. Indeed, it is hard to imagine that all inputs used are independent. The results that I will get will confirm this.

One should expect that whenever there is no relation between inputs ($\delta_i = 0 \quad \forall i$), then any cost allocation would be arbitrary. Surprisingly, this is not true in general and holds only when there are no spillovers between upstream departments. Indeed, substituting δ_i by zero yields the following overhead rate

$$\frac{\delta \sum_{i=1}^n \frac{b_i^2}{\Delta_i} \sum_{i=1}^n \frac{b_i^1}{\Delta_i}}{b^1 \left(\sum_{i=1}^n \frac{b_i^2}{\Delta_i} \right)^2 + \sum_{i=1}^n \frac{b_i^2}{\Delta_i}} (\bar{\gamma}^2 b^2)$$

which has the same sign as δ and is therefore non positive.

A negative overhead rate means that instead of being taxed, the allocation bases are subsidized. The intuition for this result is the following. Given that there is no relation between measurable and non measurable inputs, any cost allocation would have a negative result. Indeed, taxing the allocation base will have no impact on the use of the non measurable input, which in expectation will always be overused (see equation (8)) and will only distort the allocation base itself away from its desired level; Remember that not taxing input 1 results in the desired budgeted use (see equation (7) in which $\delta_i = 0$). This therefore rules out cost allocation (positive overhead rate).

Now consider the case in which no cost allocation occur, but also no subsidy is offered. In this case, If only cost tracing occur, then because (in expectation) the

¹⁸This excludes for instance the case in which a machine is running without performing any task.

use of input 1 will be the budgeted level while the use of input 2 will be higher than the budgeted level, the firm incurs a loss because of the positive spillovers. Indeed, these spillovers are higher when both inputs are used (and therefore produced) intensively. To gain on spillovers, the firm must encourage the use of the allocation base, which it does by subsidizing it. Again, although interesting in theory, such practice is however not observed in reality, and is due to the unrealistic assumption of independence between inputs.

7 Conclusion

The mechanisms of cost tracing, transfer pricing and cost allocation are very much related. Our results have several empirical implications. Firstly, whenever cost tracing or transfer pricing occur, and that some resources are difficult to measure, then cost allocation will endogenously occur. Secondly, while most cost allocation procedures select one allocation base, our optimal mechanism can select several allocation bases: all those that present some degree of complementarity with the resource the cost of which has to be (endogenously) allocated. Since allocation bases need to be measurable, the prevalence of single allocation bases suggest that the cost of making many variables perfectly measurable outweighs the benefit from having several allocation bases. This might explain why ABC has not been adopted (or been dropped) by many firms. Finally, the (criticized) production distortions that arise with the allocation base (decrease or substitution) is shown to be actually desirable for the firm. All these reasons can explain the still predominant use of cost allocation, despite the criticisms related to this practice.

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8 Appendix

Appendix A1: proof of lemma 1

With slight adjustments, this proof is borrowed from Crémer (1980). The optimal solution (z_1^*, \dots, z_n^*) satisfy the following first order conditions

$$\Gamma_i B_i - (z_i^* - \bar{z}_i) B_i - \Gamma B - \left(\sum_{i=1}^n (z_i^* - \bar{z}_i) \right) B = 0 \quad i = 1, \dots, n$$

Taking the expectation, I get

$$\mathbf{E}(\Gamma_i B_i) - \mathbf{E}(z_i^* - \bar{z}_i) B_i - \mathbf{E}(\Gamma B) - \left(\sum_{i=1}^n \mathbf{E}(z_i^* - \bar{z}_i) \right) B = 0 \quad i = 1, \dots, n \quad (9)$$

Given that $\mathbf{E}[\Gamma_i B_i] = \mathbf{E}[\Gamma B]$, by post multiplying by B_i^{-1} , equation (9) becomes

$$\mathbf{E}(z_i^* - \bar{z}_i) + \left(\sum_{i=1}^n \mathbf{E}(z_i - \bar{z}_i^*) \right) B B_i^{-1} = 0 \quad i = 1, \dots, n \quad (10)$$

Summing over i , it comes

$$\left(\sum_{i=1}^n \mathbf{E}(z_i^* - \bar{z}_i) \right) (I_m + B B_n^{-1}) = 0$$

where I_m is the identity matrix of \mathbf{R}^m , and $B_n^{-1} = \sum_{i=1}^n B_i^{-1}$. Given the assumed properties of B and B_i , the matrix $(I_m + B B_n^{-1})$ is regular, and therefore invertible. It follows that

$$\left(\sum_{i=1}^n \mathbf{E}(z_i^* - \bar{z}_i) \right) = 0$$

Substituting in (10), I get

$$\mathbf{E}(z_i - \bar{z}_i) = 0 \quad i = 1, \dots, n$$

which ultimately means that $\bar{z}_i = \mathbf{E}(z_i^*)$

Appendix A2: proof of lemma 2

For notational simplicity, denote by $p \equiv \sum_{j=1}^k p_j e_j$.

$$\begin{aligned}
\Pi &= \sum_{i=1}^n \pi_i(z_i) - C(z) \\
&= \sum_{i=1}^n \left[\Gamma_i B_i (\Gamma_i - p B_i^{-1})' - \frac{1}{2} (\Gamma_i - p B_i^{-1}) B_i (\Gamma_i - p B_i^{-1})' \right] \\
&\quad - \Gamma B \left(\sum_{i=1}^n (\Gamma_i - p B_i^{-1}) \right)' - \frac{1}{2} \left(\sum_{i=1}^n (\Gamma_i - p B_i^{-1}) \right) B \left(\sum_{i=1}^n (\Gamma_i - p B_i^{-1}) \right)' \\
&= \sum_{i=1}^n \Gamma_i B_i \Gamma_i' - \sum_{i=1}^n \Gamma_i B_i B_i^{-1} p' - \frac{1}{2} \left[\sum_{i=1}^n \Gamma_i B_i \Gamma_i' - \sum_{i=1}^n \Gamma_i B_i B_i^{-1} p' - \sum_{i=1}^n p B_i^{-1} B_i \Gamma_i' + \sum_{i=1}^n p B_i^{-1} B_i B_i^{-1} p' \right] \\
&\quad - \Gamma B \sum_{i=1}^n \Gamma_i' + \Gamma B \left(\sum_{i=1}^n B_i^{-1} \right)' p' \\
&\quad - \frac{1}{2} \left[\left(\sum_{i=1}^n \Gamma_i \right) B \left(\sum_{i=1}^n \Gamma_i \right)' - \left(\sum_{i=1}^n \Gamma_i \right) B \left(\sum_{i=1}^n B_i^{-1} \right)' p' - p \left(\sum_{i=1}^n B_i^{-1} \right) B \left(\sum_{i=1}^n \Gamma_i \right)' + p \left(\sum_{i=1}^n B_i^{-1} \right) B \left(\sum_{i=1}^n B_i^{-1} \right)' p' \right]
\end{aligned}$$

Since B_i is symmetric positive definite, I have that $B_i^{-1'} = B_i^{-1}$. Notice that

$$\left(\sum_{i=1}^n \Gamma_i \right) B \left(\sum_{i=1}^n B_i^{-1} \right)' p' = \left(p \left(\sum_{i=1}^n B_i^{-1} \right)' B' \left(\sum_{i=1}^n \Gamma_i \right)' \right)' = \left(p \left(\sum_{i=1}^n B_i^{-1} \right) B \left(\sum_{i=1}^n \Gamma_i \right)' \right)',$$

as B is also symmetric. Notice also that since each term of the profit function is a real number, it is equal to its transpose. These remarks allow to write the profit in a simpler way

$$\begin{aligned}
\Pi &= \sum_{i=1}^n \Gamma_i B_i \Gamma_i' - \sum_{i=1}^n \Gamma_i p' - \frac{1}{2} \left[\sum_{i=1}^n \Gamma_i B_i \Gamma_i' - 2 \sum_{i=1}^n \Gamma_i p' + \sum_{i=1}^n p B_i^{-1} p' \right] - \Gamma B \sum_{i=1}^n \Gamma_i' \\
&\quad + \Gamma B \left(\sum_{i=1}^n B_i^{-1} \right)' p' - \frac{1}{2} \left[\left(\sum_{i=1}^n \Gamma_i \right) B \left(\sum_{i=1}^n \Gamma_i \right)' - 2 \left(\sum_{i=1}^n \Gamma_i \right) B \left(\sum_{i=1}^n B_i^{-1} \right)' p' + p \left(\sum_{i=1}^n B_i^{-1} \right) B \left(\sum_{i=1}^n B_i^{-1} \right)' p' \right] \\
&= \frac{1}{2} \sum_{i=1}^n \Gamma_i B_i \Gamma_i' - \frac{1}{2} p \left(\sum_{i=1}^n B_i^{-1} \right)' p' - \Gamma B \left(\sum_{i=1}^n \Gamma_i \right)' + \Gamma B \left(\sum_{i=1}^n B_i^{-1} \right)' p' \\
&\quad - \frac{1}{2} \left[\left(\sum_{i=1}^n \Gamma_i \right) B \left(\sum_{i=1}^n \Gamma_i \right)' - 2 \left(\sum_{i=1}^n \Gamma_i \right) B \left(\sum_{i=1}^n B_i^{-1} \right)' p' + p \left(\sum_{i=1}^n B_i^{-1} \right) B \left(\sum_{i=1}^n B_i^{-1} \right)' p' \right]
\end{aligned}$$

Substituting p by its value, the expected value of the total firm's profit is (recall that the Γ_i and Γ are independent)

$$\begin{aligned}
\mathbf{E}(\Pi) &= -\frac{1}{2} E \left(\sum_{i=1}^n \Gamma_i B_i \Gamma_i' \right) - \frac{1}{2} \sum_{j=1}^k p_j e_j B_n^{-1} \left(\sum_{j=1}^k p_j e_j \right)' - \bar{\Gamma} B \bar{\Gamma}' + \bar{\Gamma} B B_n^{-1} \left(\sum_{j=1}^k p_j e_j \right)' \\
&\quad - \frac{1}{2} \left[E(\Gamma_n B \Gamma_n') - 2 \bar{\Gamma}_n B B_n^{-1} \left(\sum_{j=1}^k p_j e_j \right)' + \sum_{j=1}^k p_j e_j B_n^{-1} B B_n^{-1} \left(\sum_{j=1}^k p_j e_j \right)' \right]
\end{aligned}$$

where

$$\begin{cases} \Gamma_n = \sum_{i=1}^n \Gamma_i \\ \bar{\Gamma}_n = E(\Gamma_n) \\ \bar{\Gamma} = E(\Gamma) \\ B_n^{-1} = \sum_{i=1}^n B_i^{-1} \end{cases}$$

Notice that B_n^{-1} is the harmonic mean of the B_i 's. Maximizing $\mathbf{E}(\Pi)$ with respect to p_j yields:

$$\sum_{j=1}^k p_j e_j (B_n^{-1} + B_n^{-1} B B_n^{-1}) e_j' = (\bar{\Gamma} + \bar{\Gamma}_n) B B_n^{-1} e_j' \quad j = 1, \dots, k. \quad (11)$$

In order to simplify the above equality, I shall use condition (3). Post-multiplying both sides of equation (3) by B_i^{-1} , I get

$$\bar{\Gamma}_i = \bar{\Gamma} B B_i^{-1} \quad i = 1, \dots, n.$$

Summing over i , I obtain

$$\bar{\Gamma}_n = \bar{\Gamma} B B_n^{-1}. \quad (12)$$

Using equation (12), I can rewrite (11) as

$$\sum_{j=1}^k p_j e_j (B_n^{-1} + B_n^{-1} B B_n^{-1}) e_j' = (\bar{\Gamma} + \bar{\Gamma} B B_n^{-1}) B B_n^{-1} e_j' \quad j = 1, \dots, k.$$

Re-arranging the terms, I get

$$\sum_{j=1}^k p_j e_j (B_n^{-1} + B_n^{-1} B B_n^{-1}) e_j' = \bar{\Gamma} B (B_n^{-1} + B_n^{-1} B B_n^{-1}) e_j' \quad j = 1, \dots, k.$$

Let $M = B_n^{-1} + B_n^{-1} B B_n^{-1}$. The above equation becomes

$$\sum_{j=1}^k p_j e_j M e_j' = \bar{\Gamma} B M e_j' \quad j = 1, \dots, k. \quad (13)$$

Denote by M_j the j^{th} row of M . Denote also by p^{**} the k -vector of prices I search for, by p_o^* the k -vector of first best prices of the observable inputs, by p_u^* the $(m - k)$ -vector of first best prices of the unobservable inputs. The subscripts o and u refer to *observable*¹⁹ and *unobservable* respectively. I have

$$\begin{aligned} M e_j' &= M_j = (M_j^o | M_j^u) \\ \sum_{j=1}^k p_j^{**} e_j &= (p^{**} | 0_{m-k}) \\ \bar{\Gamma} B &= p^* = (p_o^* | p_u^*) \end{aligned}$$

where the vertical line is used to cut the vector (or matrix) into two vectors (or matrices). M_j^o is then a vector of length k , M_j^u is of length $m - k$ and 0_{m-k} is the zero vector of

¹⁹I use the word “observable” instead of “measurable” to avoid using the subscript m which already denotes the number of inputs.

length $m - k$. With these notations, equation (13) can be re-expressed as

$$\begin{aligned} (p^{**}|0_{m-k})(M_j^o|M_j^u)' &= (p_o^*|p_u^*)(M_j^o|M_j^u)' \quad j = 1, \dots, k \\ \Leftrightarrow p^{**}M_j^{o'} &= p_o^*M_j^{o'} + p_u^*M_j^{u'} \quad j = 1, \dots, k \end{aligned}$$

Written in matricial form, those k equations become

$$p^{**}M_o = p_o^*M_o + p_u^*M_u$$

where M_o is therefore the matrix which components are the k first rows and columns of M and M_u the matrix which components are the $m - k$ last rows and k first columns of M . Post-multiplying both sides by M_o^{-1} yields

$$p^{**} = p_o^* + p_u^*M_uM_o^{-1}$$

Appendix A3 I have

$$B_n^{-1} = \begin{pmatrix} \sum_{i=1}^n \frac{b_i^2}{\Delta_i} & -\sum_{i=1}^n \frac{\delta_i}{\Delta_i} \\ -\sum_{i=1}^n \frac{\delta_i}{\Delta_i} & \sum_{i=1}^n \frac{b_i^1}{\Delta_i} \end{pmatrix},$$

and

$$\begin{aligned} B_n^{-1}B &= \begin{pmatrix} b^1 \sum_{i=1}^n \frac{b_i^2}{\Delta_i} - \delta \sum_{i=1}^n \frac{\delta_i}{\Delta_i} & \delta \sum_{i=1}^n \frac{b_i^2}{\Delta_i} - b^2 \sum_{i=1}^n \frac{\delta_i}{\Delta_i} \\ -b^1 \sum_{i=1}^n \frac{\delta_i}{\Delta_i} + \delta \sum_{i=1}^n \frac{b_i^1}{\Delta_i} & -\delta \sum_{i=1}^n \frac{\delta_i}{\Delta_i} + b^2 \sum_{i=1}^n \frac{b_i^1}{\Delta_i} \end{pmatrix}. \\ B_n^{-1}BB_n^{-1} &= \begin{pmatrix} b^1 \left(\sum_{i=1}^n \frac{b_i^2}{\Delta_i} \right)^2 - 2\delta \sum_{i=1}^n \frac{b_i^2}{\Delta_i} \sum_{i=1}^n \frac{\delta_i}{\Delta_i} + b^2 \left(\sum_{i=1}^n \frac{\delta_i}{\Delta_i} \right)^2 & \epsilon \\ -b^1 \sum_{i=1}^n \frac{\delta_i}{\Delta_i} \sum_{i=1}^n \frac{b_i^2}{\Delta_i} + \delta \sum_{i=1}^n \frac{b_i^2}{\Delta_i} \sum_{i=1}^n \frac{b_i^1}{\Delta_i} + \delta \left(\sum_{i=1}^n \frac{\delta_i}{\Delta_i} \right)^2 - b^2 \sum_{i=1}^n \frac{\delta_i}{\Delta_i} \sum_{i=1}^n \frac{b_i^1}{\Delta_i} & \sigma \end{pmatrix} \end{aligned}$$

I denoted σ and ϵ the other elements of the M matrix, because I do not need those values.

In this particular case, I then have

$$\begin{aligned} M_o &= b^1 \left(\sum_{i=1}^n \frac{b_i^2}{\Delta_i} \right)^2 - 2\delta \sum_{i=1}^n \frac{b_i^2}{\Delta_i} \sum_{i=1}^n \frac{\delta_i}{\Delta_i} + b^2 \left(\sum_{i=1}^n \frac{\delta_i}{\Delta_i} \right)^2 + \sum_{i=1}^n \frac{b_i^2}{\Delta_i} \\ M_u &= -b^1 \sum_{i=1}^n \frac{\delta_i}{\Delta_i} \sum_{i=1}^n \frac{b_i^2}{\Delta_i} + \delta \sum_{i=1}^n \frac{b_i^2}{\Delta_i} \sum_{i=1}^n \frac{b_i^1}{\Delta_i} + \delta \left(\sum_{i=1}^n \frac{\delta_i}{\Delta_i} \right)^2 - b^2 \sum_{i=1}^n \frac{\delta_i}{\Delta_i} \sum_{i=1}^n \frac{b_i^1}{\Delta_i} - \sum_{i=1}^n \frac{\delta_i}{\Delta_i} \end{aligned}$$

Applying proposition 2 yields

$$p_1^{**} = p_1^* + \frac{M_u}{M_o} p_2^*. \quad (14)$$

Given the assumption that the B_i matrices are symmetric and positive definite, the following properties hold: $\Delta_i > 0$, $b_i^1 > 0$, $b_i^2 > 0$. Therefore, M_o is always positive. Moreover, since inputs are complements, $\delta_i > 0$ and therefore M_u is also positive.