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# **ECONOMIC HEDGING PORTFOLIOS**

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# Economic Hedging Portfolios\*

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#### Abstract

In this paper we study portfolios that investors hold to hedge economic risks. Using a model of state-dependent utility, we show that agents' economic hedging portfolios can be obtained by an intuitively appealing, risk aversion-weighted approximate replication of the economic risk variables using the investment opportunity set, as opposed to the unweighted hedging demand obtained in the traditional mean-variance framework. We find that agents across a broad range of levels of risk aversion are willing to pay significant compensations for hedges against inflation risk, real interest-rate risk, and dividend-yield risk. Furthermore, our results show that all economic risk variables we consider require significant, often risk aversion-dependent hedging adjustments with respect to one or more securities. Moreover, we analyze investors' speculative positions and find that hedges against economic risks may potentially explain the anomalies found in stock markets as well as the term and default premiums in bond markets.

JEL classification: G10, G11.

Keywords: Economic risks, risk aversion, state-dependent utility.

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# 1 Introduction

The purpose of this paper is to estimate and interpret the composition of hedging portfolios that investors hold on account of various economic risks. Furthermore, the paper estimates and tests the significance of the hedging costs associated with these economic hedging portfolios.

We use a model of state-dependent preferences to show that economic hedging portfolios can be obtained as combinations of traded assets which mimic as far as possible the economic risk variables to which investors are exposed. The weights in these mimicking portfolios turn out to be a function of the level of risk aversion of investors. The weighting scheme implies that the composition of economic hedging portfolios is investor-specific, as is the associated premium investors pay—at least, if the risk variables under consideration cannot be perfectly replicated. This will, of course, typically be the case, as we generally observe an incomplete securities market, which makes it impossible to hedge all sources of risk perfectly.

Portfolios and premiums associated with economic risks have been studied by several authors in various contexts. For example, Breeden, Gibbons, and Litzenberger (1989) test the consumption-based CAPM using a portfolio that has maximum correlation with consumption growth. Vassalou (2002) constructs a mimicking portfolio to proxy news related to future GDP growth to explain the cross-section of equity returns. Balduzzi and Kallal (1997) tighten the variance bounds of Hansen and Jagannathan (1991) using hedging portfolios for various economic risk variables. And Balduzzi and Robotti (2001) use the minimum-variance kernel of Hansen and Jagannathan to estimate economic risk premiums.

In all of these papers, the mimicking portfolios are constructed by means of an ordinary least squares projection of the risk variables on a set of security returns. As a consequence, portfolio weights and hedging costs are identical for all agents in these studies. In this paper, however, hedging is achieved by a *weighted* least squares projection of the risk variables on the security returns, in which the weights depend on investors' appetite for risk, making the composition of hedging portfolios and the implied cost of hedging individual specific.

We derive these risk aversion-weighted hedging portfolios from a model of state-dependent preferences, in which economic risk variables enter the investor's utility function in addition to the return on financial wealth. In this framework, we define an investor's economic hedging portfolio as the difference between the expected utility maximizing investment portfolio and a portfolio constructed on the basis of the return on financial wealth only, i.e., in the absence of economic risk exposures. Using a linear approximation of the investor's first order optimality conditions, we show that the resulting hedging portfolio weights are in fact approximately equal to the regression coefficients in a weighted least squares regression of the economic risk variable on the available asset returns, in which the weights are proportional to the second derivative of the utility function.<sup>1</sup> The implied hedging cost is then the compensation investors are willing to pay for investing in a hedged position instead of a zero-exposure portfolio, in terms of expected return forgone.

Our approach is related to the literature on nonmarketable risks. Nonmarketable risks arise from positions in nontraded claims such as human capital (Mayers, 1972) and commodities (Stoll, 1979). As is well-known from mean-variance investment analysis with nonmarketable risks, an investor's optimal portfolio holdings can be split up into speculative demand (i.e., the standard Markowitz portfolio choice) and hedging demand due to the nonmarketable risks to which the investor is exposed. This hedging demand is an ordinary least squares projection of the nontraded risk onto the traded security returns. In fact, a more general utility framework would produce a non-orthogonal projection similar to the one in our state-dependent utility approach.

In the empirical analysis, we focus on economic risk variables that have been found to command risk premiums in empirical studies of multi-beta and of multi-factor models. We consider the inflation rate, real interest rate, term spread, default spread, dividend yield, and consumption growth. Similar variables have been used by, for instance, Chen, Roll, and Ross (1986), Burmeister and McElroy (1988), Ferson and Harvey (1991), Campbell (1996), and Balduzzi and Kallal (1997). The possibilities for hedging these economic risks will, of course, depend on which traded assets are included in the analysis. We focus on a number of equity and bond factors of which it is well-known that they induce significant risk premiums. We include the Fama-French-Carhart factors in our set of securities to represent the stock market, and we use a two-factor model to represent the bond market. Using these priced risk factors, of several of which it is as yet not clear how they are related to economic fundamentals, allows us to explore the possibility that they are induced by an underlying hedging demand for economic risks.

We find that several stock-market and bond-market portfolios provide

<sup>&</sup>lt;sup>1</sup>Similar ideas have been applied by DeRoon, Nijman, and Werker (2003) in the context of currency hedging for international stock portfolios.

hedges for economic risks for a wide range of levels of risk aversion. In particular, inflation risk and real interest-rate risk can be partially hedged using corporate bonds; the term factor provides a good hedge for termstructure risk; and default risk can be partially hedged using bond portfolios. Bond portfolios, in combination with the equity market and momentum portfolios, also provide a good hedge against dividend-yield risk. Finally, the size portfolio appears to be useful for hedging consumption-growth risk.

Not all hedging instruments are equally useful in every situation, however. For some levels of risk aversion, portfolio adjustments are required in a particular security, while for other levels, no such adjustments are needed. For instance, a relatively risk tolerant investor may hedge against inflation risk by taking a short position in the momentum portfolio, while this is not true for relatively risk averse agents. Hence, introducing a risk aversiondependent weighting scheme in the hedging problem can indeed lead to different hedging instruments being important for different types of investors, which is in contrast with what the more restrictive mean-variance analysis predicts.

Furthermore, we find that both inflation risk, real interest risk, and dividend-yield risk imply statistically and economically significant hedging costs, while there is no evidence of a compensation for hedging default risk, consumption-growth risk, or term-structure risk.

Finally, using a decomposition of investment portfolios into speculative and hedging demand, we find that deviations from two-fund separation, i.e. investments in only the risk-free asset and the market portfolio, can be attributed to hedges against economic risks. Our results show that the size factor can be attributed to hedges against consumption-growth risk; that the term factor in bond markets is related to hedges against real interest-rate, term-structure, default, and dividend-yield risk; and that the default factor in bond markets is related to hedges against default, dividend-yield, and consumption-growth risk. However, a complete explanation of anomalies remains elusive, as we find that part of investors' demand for assets is due to speculative motives.

The structure of the remainder of the paper is as follows. Section 2 describes the model and its implications for investors' hedging demand due to economic risks as well as the associated risk premiums. In Section 3 we discuss the data on securities returns and economic risk variables which are used in Section 4 to estimate and test the significance of risk premiums and hedging portfolios associated with economic risks. Furthermore, we investigate whether hedging motives can explain the premiums on the Fama-French portfolios. Section 5 concludes.

# 2 Hedging Economic Risks

Assume that K risky securities are traded, and a risk-free one. Let  $R_t$  denote the K-vector of gross returns on the risky securities from date t-1 to date t, and let  $R_{f,t-1}$  be the gross risk-free rate of return from date t-1 to date t. Under the law of one price, there exist stochastic discount factors or pricing kernels  $M_t$  that satisfy

$$E_{t-1}\left[M_t R_t\right] = \iota_K \tag{1}$$

and

$$E_{t-1}[M_t] = \frac{1}{R_{f,t-1}},$$
(2)

with  $\iota_K$  being a K-vector of ones, and where  $E_{t-1}$  denotes the conditional expectation given all information up to date t-1; see, e.g., Cochrane (2001). If, furthermore, there are no arbitrage opportunities, then there is at least one such pricing kernel which is strictly positive almost surely.

It is well-known that stochastic discount factors or pricing kernels can be thought of as investors' marginal utility. Consider a risk-averse investor who maximizes the expected utility of the gross return on his wealth,  $R_{W,t}$ , by choosing his investments in the K + 1 available securities according to

$$\max_{w} E_{t-1} \left[ u(R_{W,t}) \right]$$
  
s.t.  $R_{W,t} = R_{f,t-1} + w^{\top} R_t^e$ , (3)

where  $R_t^e \equiv R_t - R_{f,t-1}\iota_K$  is the *K*-vector of excess returns on the risky securities. Note that the *w*'s need not sum to one. The first order conditions of problem (3) imply that a valid stochastic discount factor is

$$\frac{u'(R_{f,t-1} + w_0^{\top} R_t^e)}{R_{f,t-1} E_{t-1} [u'(R_{f,t-1} + w_0^{\top} R_t^e)]},$$
(4)

with marginal utility being evaluated at the optimal portfolio choice  $w_0$ . Note that positive marginal utility implies the absence of arbitrage opportunities.

We extend this simple portfolio problem by allowing for state-dependent utility, in which sources of risk other than the uncertain security returns may affect the investor's utility. Typically, these sources of risk are investorspecific. In principle, they can be anything from human capital and illiquid equity to health risk and the weather. In this paper, however, we focus on a set important (macro)economic risk variables such as inflation, the term spread, and consumption growth, following, for example, Chen, Roll, and Ross (1986), Ferson and Harvey (1991), Campbell (1996), and Balduzzi and Kallal (1997).

To be more precise, let  $y_t$  be an economic risk variable, and write an investor's state-dependent utility as  $\mathcal{U}(R_{W,t}; y_t)$ . Hence, the investor's utility does not only depend on the return on his invested wealth, but also on the realization of the economic risk variable. We assume that the risk variable enters the individual's utility function linearly:

$$\mathcal{U}(R_{W,t}; y_t) = u(R_{W,t} - qy_t),\tag{5}$$

where q is a parameter reflecting the extent to which the investor cares about the economic risk under consideration. Relation (5) can also be interpreted as a linearization of  $\mathcal{U}(R_{W,t}; y_t)$ , with  $q = -\mathcal{U}_y(R_{W,t}; y_t)/\mathcal{U}_R(R_{W,t}; y_t)$ .

To motivate this specification, consider the rate of inflation of the investor's consumption as an economic risk variable, and assume, for now, that q equals unity. Then the argument of the utility function can be interpreted as the individual's real return on wealth (taking  $R_W$  to be the nominal return on wealth). Depending on the investor's inclination to look at real returns rather than nominal returns, parameter q may assume other values. In particular, q = 0 may be interpreted as the investor being prone to money illusion.

More generally, any economic risk may affect the individual's utility of wealth. For instance, an interest rate shock can have an effect on the investor's utility of wealth, perhaps through his positions in non-tradable assets, such as a mortgage. Similarly, default risk can affect utility as bankruptcy jeopardizes one's labor income. Furthermore, a change in dividend yield may cause one's investment opportunity set to shift (dividendyield risk), as well as an unanticipated fall in consumption growth (business cycle risk).

We will refer to q as the individual's *exposure* to the economic risk, by analogy with the literature on non-marketed securities mentioned in the introduction. Note that in case of zero exposure, the utility function reduces to the one considered in (3). In case of non-zero exposure, however, the economic risk will affect the investor's portfolio choice, and, hence, give rise to hedging demand.

The portfolio choice problem now becomes:

$$\max_{w} E_{t-1} \left[ u(R_{W,t} - qy_t) \right]$$
  
s.t.  $R_{W,t} = R_{f,t-1} + w^{\top} R_t^e$ , (6)

and the corresponding first order conditions read

$$E_{t-1}[u'(R_{f,t-1} + w_1^\top R_t^e - qy_t)R_t^e] = 0_K,$$
(7)

where  $w_1$  denotes the vector of optimal portfolio weights. We take a first order Taylor series approximation around the optimal portfolio in case of no exposure, i.e., around  $w_1 = w_0$  and q = 0, to obtain

$$\begin{aligned}
0_{K} &= E_{t-1}[u'(R_{f,t-1} + w_{1}^{\top}R_{t}^{e} - qy_{t})R_{t}^{e}] \\
&\approx E_{t-1}[u'(R_{f,t-1} + w_{0}^{\top}R_{t}^{e})R_{t}^{e}] \\
&+ E_{t-1}[u''(R_{f,t-1} + w_{0}^{\top}R_{t}^{e})R_{t}^{e}((w_{1} - w_{0})^{\top}R_{t}^{e} - qy_{t})] \\
&= 0_{K} - E_{t-1}[R_{t}^{e}\Omega_{t}R_{t}^{e^{\top}}](w_{1} - w_{0}) + E_{t-1}[R_{t}^{e}\Omega_{t}y_{t}]q, \end{aligned}$$
(8)

where  $\Omega_t \equiv -u''(R_{f,t-1} + w_0^{\top} R_t^e) > 0$ , and the last equality follows from the first order conditions of problem (3). Hence, the difference in optimal portfolio weights per unit of exposure is

$$\frac{w_1 - w_0}{q} \approx E_{t-1} [R_t^e \Omega_t R_t^{e^{\top}}]^{-1} E_{t-1} [R_t^e \Omega_t y_t].$$
(9)

Formula (9) tells us how an individual's investment portfolio should be reallocated on account of his exposure to economic risk; some assets will require additional investment, while others will require less. Hence, this portfolio of incremental (dis)investments constitutes the investor's hedging demand associated with the economic risk variable under consideration. Accordingly, we refer to (9) as an investor's *economic hedging portfolio*.<sup>2</sup>

To further elaborate on this hedging interpretation, note that the expression on the right-hand side of equation (9) is equal to the vector of regression coefficients in a weighted least squares regression of the economic risk variable on the excess returns  $R_t^e$ , in which the weight is given by the negative of the second derivative of the utility function evaluated at the zero-exposure optimum:

$$y_t = \delta^\top R_t^e + \varepsilon_t, \tag{10}$$

where  $E_{t-1}[R_t^e \Omega_t \varepsilon_t] = 0_K$  and  $\delta = (w_1 - w_0)/q$ . This regression is, in effect, an approximate replication of the economic risk variable using the set of traded securities; the investor hedges his exposure to the economic risk by taking an offsetting position in a portfolio that mimics the economic risk variable best. By investing in this economic hedging portfolio, the investor essentially minimizes the weighted expected squared hedging error  $\varepsilon_t$ :

$$\min_{s} E_{t-1}[\Omega_t \varepsilon_t^2]. \tag{11}$$

<sup>&</sup>lt;sup>2</sup>Note that this economic hedging portfolio does not have the interpretation of a "pure" hedge as in Anderson and Danthine (1981), in the sense that it minimizes the variance of the return on wealth. In our more general expected utility framework we cannot speak of such a pure hedge, as other moments of the distribution matter as well.

Weighting by the concavity of the utility function implies that for utility functions with an upward sloping second derivative (like, for instance, power utility), large negative returns on wealth get a large weight, while large positive returns get a small weight. This makes sense intuitively, since riskaverse investors will want their hedge against economic risk to work best when wealth is low, whereas the quality of the hedge is less important to the investor when wealth is high.

It is well-known that in a traditional mean-variance framework, hedging demand is independent of the level of risk aversion. Hence, for mean-variance investors, the weight is constant, and the hedging problem reduces to an ordinary least squares projection. Ergo, in this special case, heterogeneity of risk preferences is not an issue. Many theoretical papers, including Mayers (1972), Stoll (1979), Anderson and Danthine (1980), Anderson and Danthine (1981), and Hirshleifer (1989), effectively adopt this restrictive assumption. Moreover, Balduzzi and Kallal (1997) and Balduzzi and Robotti (2001) also make use of unweighted hedging.<sup>3</sup> However, weighted hedging is important for non-mean variance utility, as our results show.

Given the above analysis, it is natural to define the *implied hedging cost* associated with the economic risk variable as the expected excess return on the corresponding economic hedging portfolio:

$$\lambda_{t-1} \equiv \delta^{\top} E_{t-1}[R_t^e]. \tag{12}$$

The implied hedging cost is the expected return an investor with preferences described by u is willing to give up to hold a position that is hedged against economic risk. Equivalently, it is the required compensation for an investor providing the hedge in terms of additional expected return.

Balduzzi and Kallal (1997) and Balduzzi and Robotti (2001) refer to the implied hedging cost as an economic risk premium. The term risk premium, however, suggests the existence of an equilibrium price of economic risk that is the same for all agents. Clearly, the implied hedging cost does not have an equilibrium interpretation, since the underlying economic risk is typically not traded. Rather, the implied hedging cost is a compensation for economic risk that is required by an individual investor. For this reason we avoid the use of the term risk premium.<sup>4</sup>

 $<sup>^{3}</sup>$ Anderson and Danthine (1981) do mention the possibility of a general expected utility formulation, but they do not explore the issue further. Neither do they examine the empirical implications of weighted hedging.

 $<sup>^{4}</sup>$ Balduzzi and Kallal (1997) and Balduzzi and Robotti (2001) do recognize that the implied hedging cost depends on (marginal) utility and, hence, the selected pricing kernel.

In Section 4 we examine the implied hedging costs associated with several economic risk variables using investments in both stocks and bonds. Furthermore, we analyze the composition of the underlying hedging portfolios.

# **3** Description of the Data

This section describes the data used in the empirical analysis. The data is at a monthly frequency, and the period considered is August 1960 through December 2001, giving a total of 497 months.

### 3.1 Securities Returns

The set of traded securities we consider includes the three factor portfolios of Fama and French (1992)—market, size, and book-to-market value—as well as the momentum portfolio of Carhart (1997). These factors have been found to explain the premiums on stocks. Furthermore, following Fama and French (1993), we include two bond-market factors: a term factor (the difference between a long-term government bond return and the one-month T-bill rate) and a default factor (the difference between the return on a portfolio of long-term corporate bonds and a long-term government bond return). The one-month T-bill rate is used as a proxy for the risk-free rate.

The market (RM-RF), size (SMB), book-to-market value (HML), and momentum (UMD) portfolios are from Kenneth French' data library.<sup>5</sup> The bond factors (TERM and DEF) are constructed using long-term government and corporate bond series from Ibbotson and Associates, and the riskfree rate (RF) which is also from French.

Table I reports summary statistics for the securities data.<sup>6</sup> These are

In fact, Balduzzi and Kallal (1997) analyze the *bounds* on economic risk premiums, for given levels of the pricing-kernel variance. Moreover, they compare these bounds to the kernel of a representative consumer with power utility. Balduzzi and Robotti (2001) use a very specific kernel (the minimum-variance kernel of Hansen and Jagannathan), which leads to premiums that are equal for all agents.

<sup>&</sup>lt;sup>5</sup>These are acronyms for "small minus big" (*SMB*), "high minus low" (*HML*), and "up minus down" (*UMD*).

<sup>&</sup>lt;sup>6</sup>The risky securities we consider are all zero-cost portfolios, but some of them are financed at the risk-free rate, while others are financed using other short positions. Nevertheless, we can take  $R^e$  to be equal to the selected vector of excess returns, and the analysis of Section 2 continues to apply. The only difference is in the interpretation of the portfolio weights. In particular, the fraction of wealth invested in the risk-free rate is one minus the fractions invested in RM-RF and TERM, and the fraction of wealth invested in the long-term government bond is equal to the difference between the portfolio weights

very much in line with the results reported by other authors. The assets considered cover a fairly wide range of average returns. The market risk premium was about 49 basis points per month on average in our sample period, which corresponds to about 6 percent annually. Only the risk-free rate exhibits strong positive autocorrelation; the risky returns are typically not very autocorrelated. The size portfolio and the term factor are positively correlated with the market portfolio, while book-to-market value has a sizeable negative correlation with the market. The bond-market factors, DEF and TERM, are strongly negatively correlated, which is due to the fact that they are constructed using the same long-term government bond.

#### 3.2 Economic Risk Variables

We consider six (macro)economic risk variables that have also been used in previous studies. See, for example, Chen, Roll, and Ross (1986), Burmeister and McElroy (1988), Ferson and Harvey (1991), Campbell (1996), Balduzzi and Kallal (1997), and Balduzzi and Robotti (2001). They are:

- 1. Inflation (INF): The monthly net rate of inflation.
- 2. Real interest (RI): The monthly real net return on a one-month T-bill.
- 3. Term spread (TS): The yield spread between a long- and a short-term government bond.
- 4. *Default spread* (*DS*): The difference in yields between corporate bonds rated Baa by Moody's Investor Service and Aaa corporate bonds.
- 5. *Dividend yield (DIV)*: The monthly dividend yield on the S&P 500 composite.
- 6. Consumption growth (CG): Monthly real per-capita consumption growth of du-rables, nondurables, and services.

The monthly inflation rate is provided by Ibbotson and Associates and is computed as the relative change of the consumer price index for all urban consumers. The monthly real interest rate is the CRSP one-month T-bill rate deflated by *INF*, the inflation rate. The default spread and the term spread are constructed using government bond-yield series (10-year and 1year) and corporate bond-yield series (Baa and Aaa) obtained from the

invested in TERM and DEF.

Federal Reserve Statistical Release. The dividend yield and consumption growth series are obtained from Datastream.

Summary statistics for the six economic risk variables are provided in Table II. Note that the risk variables are much less volatile than the security returns, and that they are typically highly autocorrelated. Only consumption growth is negatively autocorrelated at the first lag, which is consistent with previous research (e.g., Balduzzi and Kallal (1997)). A clear pattern emerges from the correlation matrix of the risk variables. In particular, note the high negative correlation between the inflation rate and the real riskfree rate, which is not surprising given that the real risk-free rate is equal to the nominal risk-free rate less the rate of inflation, and the nominal riskfree rate is relatively constant over our sample period. Also note the strong positive correlations between the dividend yield on the one hand, and the default spread and the inflation rate on the other, as well as the negative correlation between the term spread and the inflation rate.

# 4 Hedging Portfolios and Implied Hedging Costs

In this section, we compute and interpret the hedging portfolios and implied hedging costs associated with the six economic risk variables under scrutiny using the available set of traded assets. As a first step in the analysis, we estimate a vector autoregressive (VAR) model for the "raw" economic risk variables, and use the residuals as our actual economic risk variables, as in Campbell (1996). The reason for this is that we are only interested in hedging the unanticipated components of economic risks (shocks); any anticipated part can be hedged trivially using the risk-free asset.

Table III reports the coefficients in a first-order VAR, as well as the standard deviations and the correlations of innovations to the system. Many variables enter significantly with either positive or negative signs in the forecasting equations. In particular, the regression coefficients on the dependent variables' own lags are all highly significant due to the substantial autocorrelation in the economic variables. The autocorrelation is most pronounced in the term spread, the default spread, and dividend yield, explaining the high  $R^2$  in those regressions. The innovations in inflation and the real interest rate are highly negatively correlated, while the correlations of the other innovations are on average less than 10 percent.

The economic hedging portfolios and their corresponding hedging costs are estimated in two steps. In the first step, we use the generalized method of moments (GMM) of Hansen (1982) to estimate the optimal zero-exposure portfolio weights for investors in a standard constant relative risk aversion or power utility framework. The power utility function is given by  $u(x) = x^{1-\gamma}/(1-\gamma)$ , where  $\gamma > 0$  is the parameter of risk aversion which we allow to vary. These zero-exposure portfolio weights are subsequently used in the second step in the weighted least squares regression to obtain estimates of the hedging portfolios and the implied hedging costs. This procedure involves an errors-in-variables problem and requires an adjustment of the standard errors. The econometric details are given in the appendix.

#### 4.1 Implied Hedging Costs

The hedging costs associated with the economic risk variables are in Panel A of Table IV. These hedging costs are measured in units of risk, or Sharpe ratios, as in Balduzzi and Kallal (1997); that is, the vector-autoregressive residuals are scaled by their standard deviations, so that they can be compared to each other. To get an idea of the order of magnitude of these implied hedging costs, or their economic significance, consider that the monthly market Sharpe ratio is about 0.11 for the period under scrutiny.

Our results show that investors are willing to pay for inflation shocks and innovations in dividend yield only. This holds for all types of agents, with levels of risk aversion ranging from  $\gamma = 1$  to  $\gamma = 20$ . The estimated hedging costs for inflation and dividend yield are both statistically and economically significant. Both are negative, indicating that investors must forgo expected return if they want to hedge long exposures to these economic risks. Note that the implied hedging cost for inflation decreases as we consider more risk averse investors. This result may at first sight seem counterintuitive, however the magnitude of the hedging cost is in fact not determined by the level of risk aversion directly, but rather by its effect on the weight,  $\Omega$ , in the hedging problem. Neither size nor sign of this effect can be predicted without examination of the data. An increase in the level of risk aversion, which makes the investor put more weight on low returns than on high returns, apparently decreases the slope in the hedging regression (in absolute value) and thus reduces the associated hedging cost for the case of inflation risk.<sup>7</sup> Contrary to inflation, the hedging cost associated with dividend yield seems to be independent of people's attitudes toward risk. We find no evidence for significant risk compensations for other economic risk variables.

Apart from a single risk exposure, agents may very well be exposed to

 $<sup>^{7}</sup>$ Note that there is also no reason why the implied hedging cost should increase with risk aversion. In fact, in a mean-variance framework, the hedging cost is independent of the investor's risk aversion.

several economic risks simultaneously. This implies that hedging portfolios are constructed to hedge for multiple risks. The resulting hedging costs are linear combinations of the hedging costs in Panel A of Table IV. These then constitute the price for simultaneously hedging for several economic risks.

An alternative way of analyzing risk premiums is to look at an innovation in isolation, disregarding innovations in other risk variables. To achieve this, we follow Campbell (1996) and Balduzzi and Kallal (1997) by orthogonalizing the VAR-residuals using a Cholesky decomposition of their variancecovariance matrix. The first innovation, the one in the rate of inflation, is unaffected by this procedure; the other innovations are. The orthogonalized innovation in the real interest rate is equal to the part of the original real interest rate innovation orthogonal to the innovation in inflation; the orthogonalized innovation in term-structure risk is equal to the part of the original innovation in term-structure risk orthogonal to the innovations in inflation and the real interest rate; et cetera. The variables are ordered in such a way that the orthogonalized innovation in the real interest rate is a change in the real interest rate that is not caused by a change in the inflation rate. Hence, it amounts to a shock in the nominal rate.

The hedging costs related to the orthogonalized economic innovations are in Panel B of Table IV. We find that both inflation risk and dividend yield still require economically and statistically significant hedging costs for a broad range of levels of risk aversion. In addition, shocks to the real interest rate that are unrelated to inflation surprises also require a negative and significant hedging cost for all types of investors considered. This cost is quite sizeable for relatively risk tolerant agents, but gets smaller for higher levels of risk aversion.

#### 4.2 Economic Hedging Portfolios

Table V reports the hedging portfolios underlying the hedging costs associated with each of the economic risk variables. Several securities provide hedges for economic risks. For instance, a significant long position in the default portfolio is required to hedge against inflation risk. That is, investors prone to inflation risk should reduce their investment in government bonds and buy corporate bonds. This is because when inflation is higher than anticipated, the return on the default factor is high. This result holds for a broad range of levels of risk aversion.

Observe, however, that a hedge against inflation requires other portfolio adjustments as well. For instance, the momentum portfolio appears to be a useful hedging instrument for relatively risk tolerant agents, while the market portfolio provides inflation protection for relatively risk averse investors. This shows that differences in risk aversion can have such an effect on the weighting in the hedging problem, that some securities turn out to be good hedges for certain types of agents, while others do well for other types of agents.

Note furthermore, that the total (dis)investment in the hedging portfolios depends on the investor's exposure to inflation risk. Naturally, if the exposure is zero, no hedging is needed, while in case of a non-zero exposure, some portfolio adjustments are required. Table V shows the hedging portfolios for an investor with unit exposure to innovations in the economic risk variables. Hence, an investor with relative risk aversion of one who cares about real returns instead of nominal returns, i.e., an investor with  $\gamma = 1$ and q = 1, should increase his investment in the default portfolio by almost 5 percentage points. For investors with  $\gamma = 1$  and q = .5, the adjustment is half this amount.

For investors who face real interest-rate risk, a short position in the default portfolio is required. Hence, an exposure to the real interest rate can be offset by a disinvestment in corporate bonds and an investment in government bonds, as the return on the default portfolio is low. Moreover, risk averse investors ( $\gamma = 1$ ) should increase their position in the momentum factor.

Furthermore, the term portfolio provides a good hedge for term-structure risk across all levels of risk aversion. That is, when there is a shock in the interest rate differential, investors with an exposure to term-structure risk ought to use the term factor as a hedging instrument. For instance, investors whose portfolio is adversely affected by a high long-term interest rate and a low short-term interest rate, perhaps due to a mortgage loan and a savings account, can hedge the risk of a high interest rate differential by increasing their investment in long-term bonds and decreasing their investment in Tbills, because the excess return on long-term bonds is expected to be higher at such times.

As for hedges against default risk, most investors (if they face an exposure to this economic risk) seem to be best off taking long positions in the term portfolio and the default portfolio. Hence, corporate bonds appear to perform best when the risk of default is high, that is, if the yield spread rises.

For all levels of risk aversion considered, the hedging portfolio associated with dividend yield requires significant short positions in the market and momentum portfolios as well as long positions in the term and default portfolios. Note that an unanticipated increase in the dividend yield usually coincides with an unexpected drop in stock prices. Hence, investors can offset such a price drop by selling the market and momentum portfolios and buying relatively cheap bonds.

Finally, hedging consumption-growth risk calls for a disinvestment in the size portfolio. Hence, if an investor fears an unexpected drop in consumption growth, he had better avoid small company stocks, and increase his investment in large company stocks.

Note that while it is true that the magnitude and significance of hedging positions typically dies down with increasing risk aversion, many individual security positions remain economically and statistically significant even for high levels of risk aversion. It is also interesting to note that most hedging portfolio weights have the same sign for different risk-aversion levels. Only rarely do the weights change sign, and when they do, statistical significance disappears.

#### 4.3 Speculative versus Hedging Demand

We can extend our model by not only considering investors' hedging demand, but also their speculative demand for assets, analogously to the meanvariance case, as studied in, e.g., Anderson and Danthine (1980; 1981). In case of mean-variance investors, one may break down investors' total demand for assets into a pure speculation component, which is equivalent to the position of an investor with no exposure to exogenous risk, and a pure hedge component, which is equal to the position of an infinitely risk-averse investor. In our more general setting, we cannot make this distinction, as investors' hedges against economic risk will in general depend on the concavity of their utility functions, and hence not be "pure" in the sense of Anderson and Danthine (1981). Nevertheless, we can separate investors' demand for assets due to speculative motives, and their demand for assets due to hedging, with both components being risk-aversion dependent.

For reasons that will become clear shortly, we define speculative demand as the set of (dis)investments in the available assets relative to a position in just the risk-free asset and the market portfolio. Hence, we look at the portfolio choice problem from the point of view of an agent who invests according to the premise of two-fund separation which follows from the capital asset pricing model. Moreover, we consider the possibility that this investor has an exposure to one or more economic risks. The economic hedging demand that is induced by this exposure may potentially shed light on the Fama-French anomalies, to the extent that the Fama-French premiums may in fact be explained by hedges against economic risk.

As suggested before, let us consider an agent who invests only in the risk-free asset and the market portfolio. Write the vector of available risky excess returns,  $R^e$ , as

$$R^e = \left[ \begin{array}{c} R^e_m \\ R^e_F \end{array} \right],$$

where  $R_m^e$  denotes the excess return on the market portfolio and  $R_F^e$  are the returns on the (other) Fama-French portfolios. Consider the optimal portfolio choice in case of an exposure q to economic risk with unrestricted investment opportunities (i.e., the investor may choose all available traded assets), and the optimal portfolio choice in case of no exposure with the restricted investment opportunity set (the CAPM investor):

$$w_1 = \arg \max_w E[u(R_f + w^{\top}R^e - qy)]$$
  
$$w_{m,0} = \arg \max_{w_m} E[u(R_f + w_m R_m^e)].$$

The corresponding first-order conditions are

$$0_K = E[u'(R_f + w_1^{\top} R^e - qy)R^e] 0 = E[u'(R_f + w_0^{\top} R^e)R_m^e],$$

where  $w_0 \equiv (w_{m,0}, 0_{K-1}^{\top})^{\top}$ . A first-order Taylor expansion around  $w_1 = w_0$  and q = 0 implies:

$$w_{1} - w_{0} \approx E[R^{e}\Omega_{0}R^{e^{\top}}]^{-1}E[u'(R_{f} + w_{0}^{\top}R^{e})R^{e}] + E[R^{e}\Omega_{0}R^{e^{\top}}]^{-1}E[R^{e}\Omega_{0}y]q, \qquad (13)$$

where  $\Omega_0 = -u''(R_f + w_0^{\top}R^e)$ . Note that the weight  $\Omega_0$  will be different from the weight obtained in Section 2, since the restricted model will imply a different optimal zero-exposure portfolio choice than the unrestricted model. The first term on the right-hand side of (13) can now be interpreted as speculative demand, and the second term can be interpreted as an economic hedging component. Note that speculative demand depends on the term  $E[u'(R_f + w_0^{\top}R^e)R^e]$ , which is proportional to the generalized Jensen measure  $E[M_0R^e]$ , where  $M_0 \equiv u'(R_f + w_0^{\top}R^e)/R_f E[u'(R_f + w_0^{\top}R^e)]$  is the stochastic discount factor of an investor restricted to the market portfolio and the risk-free asset. It measures the attractiveness of new investments relative to a set of reference assets (in this case the market portfolio and the risk-free asset); a positive value indicates that the investor can improve his expected utility by going long in the new investment, whereas a negative value implies a short position. See, for instance, Glosten and Jagannathan (1994) and Chen and Knez (1996). Since the market portfolio is our point of reference, the first element of this vector is equal to zero.

We find significant speculative demand for the various assets relative to the CAPM portfolio. The market portfolio, the book-to-market value portfolio, and the momentum portfolio all require significant additional investments on account of speculative motives. As expected, the size of this speculative demand decreases with risk aversion, but the effect remains statistically significant. Hence, we conclude that the CAPM does not hold. We do not find significant speculative demand for the size portfolio and the two bond portfolios.

Table VI reveals that speculative motives are not the only reason for people to diverge from the CAPM. There is significant hedging demand for almost all assets by various agents. For instance, investors with a unit exposure to inflation risk require a short position of about three quarters of a percent in the market portfolio. This result is quite robust across different levels of risk aversion. In fact, most hedging positions are, contrary to the hedging portfolios in Table V, independent of risk aversion.

Note that only the market portfolio provides a good hedge in case of an inflation-risk exposure, whereas in the unrestricted case, the momentum and default portfolios do, too. Apparently, the weighting scheme implied by the CAPM makes these assets less useful as hedging instruments than they are for the unrestricted investor. Different weighting schemes are also the reason for the differences we find in the hedging portfolios associated with real interest-rate risk. For the CAPM case, a long position in the term portfolio is required; in the unrestricted case, a hedge is obtained by shorting the default portfolio (and for relatively risk tolerant investors, going long in the momentum portfolio). For the other risks, we find that by and large the same assets turn up as useful hedging instruments as in the unrestricted case. Hence, restricting the investment opportunity set does not have an important effect on the hedging portfolios in those cases. An exception is consumption-growth risk, in which case, in addition to a long position in the size portfolio, a long position in the default portfolio is called for. The only asset which does not show up as a useful hedging instrument in Table VI is the momentum portfolio.

The results in Table VI have interesting implications for the *raison d'être* of the Fama-French risk premiums. While we find that some of investors' demand for assets is due to speculative motives, part of the reason why they deviate from the CAPM is attributable to economic risks. In particular, we find no significant speculative demand for the size portfolio, the term port-

folio, and the default portfolio. Therefore, investments in these portfolios can be entirely explained by hedging. Our results suggest that the size premium is related to hedges against consumption-growth risk; that the term premium in bond markets is caused by hedging against real interest-rate, term-structure, default, and dividend-yield risk; and that the default premium in bond markets is related to hedging against default, dividend-yield, and consumption-growth risk. The anomalies in the investments for which we do find significant speculative demand can still be partly explained by economic risks. Only the momentum portfolio seems to be unrelated to any of the economic risks included in the analysis.

# 5 Conclusion

In this paper we estimate and interpret the composition of portfolios that investors hold to hedge various economic risks. We also consider the implied hedging costs associated with these economic hedging portfolios for various types of agents. We wish to stress that these hedging portfolios are individual specific. Using a model of state-dependent utility, we show that agents' economic hedging portfolios can be obtained by a risk aversion-weighted least squares regression of the economic risk variables onto the available risky security returns, as opposed to the unweighted hedging demand one obtains in the traditional mean-variance framework.

We find that agents across a broad range of levels of risk aversion are willing to pay (or demand) significant compensations for hedges against three sources of economic risk: inflation risk, real interest-rate risk, and dividend-yield risk. Furthermore, our results show that all economic risk variables we consider require a significant hedging adjustment with respect to one or more traded securities. Some of these securities prove to be useful hedging instruments across different types of investors, whereas others only serve as hedges for particular levels of risk aversion, which demonstrates the empirical relevance of risk aversion-weighted hedging.

Furthermore, we contribute to the discussion on asset pricing anomalies by examining whether the Fama-French premiums can be attributed to economic hedging motives. While we cannot conclude that book-to-market and momentum anomalies are (solely) due to reasons of economic hedging, we do find that the size effect found in stock markets as well as the term and default premiums found in bond markets, may potentially be explained by hedges against economic risk, most notably by hedges against real interestrate risk, default risk, term-structure risk, and consumption-growth risk.

# Appendix: Econometric Issues

This appendix discusses the estimators of several key parameters in the paper and their limiting distributions. They are w, the zero-exposure portfolio weights;  $\delta$ , the hedging portfolio weights;  $\lambda$ , the implied hedging cost; and  $\alpha$ , the speculative demand for risky assets.

Let  $R_f$  denote the risk-free rate, and let  $R^e$  be the K-vector of excess returns. k of the K risky assets are basis assets,  $k \leq K$ . Let  $R_b^e$  denote the excess returns on these basis assets. In Section 4.3 we take the excess return on the market portfolio as the basis asset.

#### 1. Zero-exposure portfolio weights

Using standard GMM notation, the pricing errors are

$$e(\theta) = cu' \left( R_f + w^{\top} R_b^e \right) \begin{bmatrix} R_f \\ R_b^e \end{bmatrix} - \begin{bmatrix} 1 \\ 0_k \end{bmatrix},$$

where  $\theta = (c, w^{\top})^{\top}$ . The moment conditions read  $0_{k+1} = E[e(\theta)] \equiv g(\theta)$ . The GMM-estimator is then given by  $0_{k+1} = E_T[e(\hat{\theta})] \equiv g_T(\hat{\theta})$ , where  $E_T$  denotes the sample average. Note that

$$\sqrt{T}(\hat{\theta} - \theta) \simeq -\left(\frac{\partial g_T(\theta)}{\partial \theta^{\top}}\right)^{-1} \sqrt{T} E_T[e(\theta)].$$

Hence, under standard regularity conditions, the limiting distribution of  $\hat{\theta}$  is given by

$$\sqrt{T}(\hat{\theta} - \theta) \longrightarrow N\left(0, \left(\frac{\partial g(\theta)}{\partial \theta^{\top}}\right)^{-1} A\left(\frac{\partial g^{\top}(\theta)}{\partial \theta}\right)^{-1}\right),$$

where  $A = var [e(\theta)]$ .

#### 2. Hedging portfolio weights

Let  $\Omega(\theta) = -u''(R_f + w^{\top}R_b^e)$ , and  $\varepsilon(\theta) = y - \delta(\theta)^{\top}R^e$ , where

$$\delta(\theta) = E \left[ R^{e} \Omega\left(\theta\right) R^{e\top} \right]^{-1} E \left[ R^{e} \Omega\left(\theta\right) y \right].$$

Estimate  $\Omega(\theta)$  by the plug-in estimator  $\Omega(\hat{\theta}) \equiv \hat{\Omega}$ , and  $\delta = \delta(\theta)$  by

$$\hat{\delta} = \hat{\delta}(\hat{\theta}) = E_T [R^e \hat{\Omega} R^{e^\top}]^{-1} E_T [R^e \hat{\Omega} y]$$

Note that

$$\hat{\Omega} \simeq \Omega(\theta) + \frac{\partial \Omega(\theta)}{\partial \theta^{\top}} (\hat{\theta} - \theta)$$
  
$$\simeq \Omega(\theta) - \frac{\partial \Omega(\theta)}{\partial \theta^{\top}} \left( \frac{\partial g_T(\theta)}{\partial \theta^{\top}} \right)^{-1} E_T[e(\theta)].$$

Hence,

$$\sqrt{T}(\hat{\delta} - \delta) \simeq E_T [R^e \Omega(\theta) R^{e^\top}]^{-1} \sqrt{T} E_T [\zeta(\theta)],$$

where

$$\zeta(\theta) = R^e \Omega(\theta) \varepsilon(\theta) - E \left[ R^e \frac{\partial \Omega(\theta)}{\partial \theta^{\top}} \varepsilon(\theta) \right]^{-1} \left( \frac{\partial g(\theta)}{\partial \theta^{\top}} \right)^{-1} e(\theta),$$

and so the limiting distribution of  $\hat{\delta}$  is

$$\sqrt{T}(\hat{\delta} - \delta) \longrightarrow N\left(0, E[R^e \Omega(\theta) R^{e^{\top}}]^{-1} B E[R^e \Omega(\theta) R^{e^{\top}}]^{-1}\right),$$

where  $B = var[\zeta(\theta)]$ .

# 3. Implied hedging cost

The implied hedging cost,  $\lambda$ , is defined as

$$\lambda = \lambda \left( \theta \right) = E[R^{e^{\top}} \delta \left( \theta \right)] = E[R^{e}]^{\top} \delta.$$

It can be estimated by

$$\hat{\lambda} = \hat{\lambda}(\hat{\theta}) = E_T[R^e]^\top \hat{\delta}.$$

Notice

$$\begin{split} \sqrt{T}(\hat{\lambda} - \lambda) &= \sqrt{T} \left( E_T[R^e]^\top \hat{\delta} - E[R^e]^\top \delta \right) \\ &= \sqrt{T} \left( \hat{\delta}^\top (E_T[R^e] - E[R^e]) + E[R^e]^\top (\hat{\delta} - \delta) \right) \\ &= \begin{bmatrix} \hat{\delta} \\ E[R^e] \end{bmatrix}^\top \sqrt{T} \begin{bmatrix} E_T[R^e] - E[R^e] \\ \hat{\delta} - \delta \end{bmatrix} \\ &\simeq \begin{bmatrix} \hat{\delta} \\ E[R^e] \end{bmatrix}^\top \sqrt{T} \begin{bmatrix} E_T[R^e] - E[R^e] \\ E_T[R^e\Omega(\theta) R^{e^\top}]^{-1} E_T[\zeta(\theta)] \end{bmatrix} \\ &= \begin{bmatrix} \hat{\delta} \\ E[R^e] \end{bmatrix}^\top \begin{bmatrix} I_K & 0 \\ 0 & E_T[R^e\Omega(\theta) R^{e^\top}]^{-1} \end{bmatrix} \times \\ &\sqrt{T} \begin{bmatrix} E_T[R^e - E[R^e]] \\ E_T[\zeta(\theta)] \end{bmatrix}. \end{split}$$

Hence, the limiting distribution of  $\hat{\lambda}$  is given by

$$\sqrt{T}(\hat{\lambda} - \lambda) \longrightarrow N(0, a^{\top} H C H a),$$

where

$$a = \begin{bmatrix} \delta \\ E[R^e] \end{bmatrix}$$
$$H = \begin{bmatrix} I_K & 0 \\ 0 & E[R^e \Omega(\theta) R^{e^{\top}}]^{-1} \end{bmatrix}$$
$$C = var \begin{bmatrix} R^e \\ \zeta(\theta) \end{bmatrix}.$$

# 4. Speculative demand

Speculative demand is defined as

$$\alpha = \alpha \left( \theta \right) = E[R^e \Omega \left( \theta \right) R^{e^{\top}}]^{-1} E[u'(R_f + w^{\top} R_b^e) R^e],$$

and it is estimated by

$$\hat{\alpha} = \hat{\alpha}(\hat{\theta}) = E_T [R^e \hat{\Omega} R^{e^\top}]^{-1} E_T [u'(R_f + \hat{w}^\top R_b^e) R^e].$$

Using a first-order Taylor approximation, we obtain

$$u'(R_f + \hat{w}^{\top} R_b^e) \simeq u'(R_f + w^{\top} R_b^e) + u''(R_f + w^{\top} R_b^e) R_b^{e^{\top}}(\hat{w} - w)$$

Let  $\phi = \phi(\theta) = E[u'(R_f + w^\top R_b^e)R^e]$ , and  $\hat{\phi} = \hat{\phi}(\hat{\theta}) = E_T[u'(R_f + \hat{w}^\top R_b^e)R^e]$ . Then

$$\begin{split} \sqrt{T}(\hat{\phi} - \phi) &= \sqrt{T} \left( E_T[u'(R_f + \hat{w}^\top R_b^e)R^e] - E[u'(R_f + w^\top R_b^e)R^e] \right) \\ &\simeq \sqrt{T} \left( E_T[u'(R_f + w^\top R_b^e)R^e] - E[u'(R_f + w^\top R_b^e)R^e] \right) \\ &+ E_T[R^e u''(R_f + w^\top R_b^e)R_b^{e^\top}] \sqrt{T}(\hat{w} - w) \\ &= \left[ I_K \quad 0 \quad -E_T[R^e \Omega R_b^{e^\top}] \right] \sqrt{T} \left[ \begin{array}{c} E_T[\eta(\theta)] \\ \hat{\theta} - \theta \end{array} \right], \end{split}$$

where 
$$\eta = \eta(\theta) = u'(R_f + w^{\top}R_b^e)R^e - E[u'(R_f + w^{\top}R_b^e)R^e]$$
. Hence,  
 $\sqrt{T}(\hat{\alpha} - \alpha) \simeq E[R^e\Omega(\theta)R^{e^{\top}}]^{-1} \begin{bmatrix} I_K & 0 & -E[R^e\Omega(\theta)R_b^{e^{\top}}] \end{bmatrix} \times \begin{bmatrix} I_K & 0 \\ 0 & -\left(\frac{\partial g_T(\theta)}{\partial \theta^{\top}}\right)^{-1} \end{bmatrix} \sqrt{T} \begin{bmatrix} E_T[\eta(\theta)] \\ E_T[e(\theta)] \end{bmatrix}$ 
 $\longrightarrow N\left(0, PQRDR^{\top}Q^{\top}P\right)$ 

where

$$P = E[R^{e}\Omega(\theta) R^{e^{\top}}]^{-1}$$

$$Q = \begin{bmatrix} I_{K} & 0 & -E[R^{e}\Omega(\theta)R_{b}^{e^{\top}}] \end{bmatrix}$$

$$R = \begin{bmatrix} I_{K} & 0 \\ 0 & -\left(\frac{\partial g(\theta)}{\partial \theta^{\top}}\right)^{-1} \end{bmatrix}$$

$$D = var \begin{bmatrix} \eta(\theta) \\ e(\theta) \end{bmatrix}.$$

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#### Table I: Summary Statistics for Security Returns

The sample period is August 1960 through December 2001. Mean returns and standard deviations are in percentage points per month. RM-RF is the return on the market portfolio in excess of the risk-free rate, SMB is the return on the size portfolio, HML is the return on the book-to-market value portfolio, UMD is return on the momentum portfolio, TERM is the return on a long-term government bond in excess of the risk-free rate, DEF is the return on a long-term corporate bond less the return on a long-term government bond, and RF is the risk-free rate. Corr<sub>t</sub> is the autocorrelation at lag t.

Panel A: Means, Standard Deviations, and Autocorrelations									
Variable	Mean	Std. Dev.	$\operatorname{Corr}_1$	$\operatorname{Corr}_2$	$\operatorname{Corr}_3$	$\operatorname{Corr}_6$	$\operatorname{Corr}_{12}$		
RM– $RF$	0.491	4.453	0.061	-0.054	-0.003	-0.032	0.010		
SMB	0.160	3.217	0.075	0.033	-0.105	0.080	0.136		
HML	0.446	2.934	0.125	0.067	0.043	0.063	0.028		
UMD	0.879	3.874	-0.025	-0.056	-0.030	0.088	0.114		
TERM	0.125	2.765	0.060	0.001	-0.107	0.041	-0.017		
DEF	0.014	1.192	-0.170	-0.078	-0.020	-0.034	-0.024		
RF	0.485	0.216	0.945	0.909	0.885	0.825	0.714		
		Panel B:	Correlat	ions Mat	rix				
Variable	RM– $RF$	SMB	HML	UMD	TERM	DEF	RF		
RM– $RF$	1	0.301	-0.422	-0.026	0.278	0.082	-0.106		
SMB		1	-0.298	0.005	-0.091	0.151	-0.046		
HML			1	-0.161	0.002	0.024	0.043		
UMD				1	0.041	-0.191	-0.012		
TERM					1	-0.473	0.022		
DEF						1	-0.059		
RF							1		

#### Table II: Summary Statistics for Economic Risk Variables

The sample period is August 1960 through December 2001. Means and standard deviations are in percentage points per month. *INF* is the monthly net rate of inflation, RI is the monthly real net risk-free rate, TS is the yield spread between long- and short-term government bonds, DS is the yield spread between Baa and Aaa corporate bonds, DIV is dividend yield on the S&P 500 composite, and CG is monthly real per-capita consumption growth of durables, nondurables, and services. Corr<sub>t</sub> is the autocorrelation at lag t.

Panel A: Means, Standard Deviations, and Autocorrelations										
Variable	Mean	Std. Dev.	$\operatorname{Corr}_1$	$\operatorname{Corr}_2$	$\operatorname{Corr}_3$	$\operatorname{Corr}_6$	$Corr_{12}$			
INF	0.361	0.318	0.642	0.548	0.510	0.469	0.521			
RI	0.124	0.274	0.481	0.362	0.351	0.310	0.436			
TS	0.062	0.086	0.959	0.894	0.836	0.704	0.501			
DS	0.082	0.037	0.971	0.932	0.903	0.826	0.679			
DIV	0.288	0.091	0.997	0.994	0.990	0.968	0.905			
CG	0.207	0.571	-0.204	0.012	0.019	0.051	-0.005			
		Panel B:	Correlat	tions Mat	trix					
Variable		INF	RI	TS	DS	DIV	CG			
INF		1	-0.743	-0.422	0.234	0.414	-0.225			
RI			1	0.118	0.206	-0.014	0.157			
TS				1	0.100	-0.091	0.107			
DS					1	0.660	-0.034			
DIV						1	-0.030			
CG							1			

Table III: **First-Order VAR of the Economic Risk Variables** Regression coefficients of a first-order vector autoregression of the economic risk variables. The sample period is August 1960 through December 2001. Standard errors are in parentheses. Acronyms are defined in Table II. Standard deviations of the innovations are in percentage points per month.

Panel A: Regression Coefficients										
Depend.			Regres	sors						
Variable	INF	RI	TS	DS	DIV	CG	$R^2$			
INF	0.723	0.253	-0.521	-0.707	0.542	0.049	0.777			
	(0.086)	(0.086)	(0.165)	(0.465)	(0.165)	(0.019)				
RI	0.146	0.588	0.323	0.839	-0.411	-0.047	0.390			
	(0.089)	(0.089)	(0.170)	(0.480)	(0.170)	(0.019)				
TS	0.018	0.024	0.973	0.118	-0.058	-0.001	0.950			
	(0.009)	(0.009)	(0.017)	(0.048)	(0.017)	(0.002)				
DS	0.011	0.011	-0.003	0.927	0.002	-0.002	0.991			
	(0.003)	(0.003)	(0.006)	(0.017)	(0.006)	(0.001)				
DIV	0.004	0.002	-0.003	-0.020	1.000	-0.001	1.000			
	(0.002)	(0.002)	(0.005)	(0.013)	(0.005)	(0.001)				
CG	-0.675	-0.446	0.207	0.515	0.608	-0.255	0.198			
	(0.207)	(0.205)	(0.394)	(1.114)	(0.394)	(0.044)				
Panel B	: Standard I	Deviations	and Corr	elation M	atrix of V.	AR Innov	ations			
Variable	Std. Dev.	INF	RI	TS	DS	DIV	CG			
INF	0.228	1	-0.957	-0.056	-0.077	0.124	-0.157			
RI	0.235		1	-0.058	0.081	-0.126	0.153			
TS	0.024			1	0.182	0.063	-0.063			
DS	0.008				1	0.174	-0.038			
DIV	0.007					1	-0.023			
CG	0.545						1			

### ${\rm Table \ IV: \ Implied \ Hedging \ Costs}$

Implied hedging costs associated with innovations in risk variables, measured in units of risk (i.e. standard deviation). The sample period is August 1960 through December 2001. Standard errors are in parentheses. Acronyms are defined in Tables I and II.

	Panel A: First-Order VAR Innovations										
Risk aver.	INF	RI	TS	DS	DIV	CG					
$\gamma = 1$	-0.0669**	0.0321	0.0856	0.0037	-0.0356**	0.0409					
	(0.0286)	(0.0261)	(0.0789)	(0.0343)	(0.0141)	(0.0260)					
$\gamma = 2$	-0.0587**	0.0286	0.0622	0.0083	$-0.0354^{***}$	0.0354					
	(0.0251)	(0.0235)	(0.0566)	(0.0266)	(0.0134)	(0.0225)					
$\gamma = 5$	-0.0487**	0.0227	0.0507	0.0075	-0.0348***	0.0285					
	(0.0243)	(0.0233)	(0.0484)	(0.0255)	(0.0135)	(0.0222)					
$\gamma = 20$	$-0.0425^{*}$	0.0193	0.0413	0.0072	$-0.0341^{**}$	0.0237					
	(0.0246)	(0.0235)	(0.0427)	(0.0242)	(0.0137)	(0.0223)					
	Panel B:	Orthogonaliz	zed First-C	Order VAR In	nnovations						
Risk aver.	INF	RI	TS	DS	DIV	CG					
$\gamma = 1$	-0.0669**	-0.1104**	0.0426	-0.0075	-0.0323*	0.0351					
	(0.0286)	(0.0461)	(0.0694)	(0.0468)	(0.0171)	(0.0272)					
$\gamma = 2$	-0.0587**	-0.0956***	0.0239	0.0014	-0.0330**	0.0297					
	(0.0251)	(0.0347)	(0.0504)	(0.0342)	(0.0153)	(0.0233)					
$\gamma = 5$	-0.0487**	-0.0828***	0.0174	0.0023	-0.0331**	0.0236					
	(0.0243)	(0.0299)	(0.0433)	(0.0312)	(0.0151)	(0.0224)					
$\gamma = 20$	-0.0425*	-0.0738***	0.0114	0.0036	-0.0328**	0.0192					
	(0.0246)	(0.0267)	(0.0387)	(0.0287)	(0.0152)	(0.0221)					

\*\*\*/\*\*/\* indicates significance at the 1/5/10% level.

Risk aversion	Security	INF	RI	TS	DS	DIV	CG
$\gamma = 1$	RM– $RF$	-0.56 (0.42)	-0.13 (0.40)	0.16 (0.12)	-0.01 (0.02)	$-0.03^{***}(0.01)$	$1.64^*$ (0.86)
	SMB	-0.18 (0.45)	-0.11 (0.45)	$0.24^*$ (0.14)	-0.04 (0.03)	0.00 (0.01)	$4.52^{***}$ (1.35)
	HML	-0.94 (0.66)	0.35 (0.63)	$0.27^*$ (0.15)	-0.03 (0.03)	$-0.03^{**}$ (0.01)	2.39 (1.70
	UMD	-0.94*** (0.30)	$0.77^{**}$ (0.35)	$-0.10^{*}$ (0.06)	$0.02^*$ (0.01)	0.00 (0.00)	-0.25 (0.48
	TERM	-0.22 (0.82)	0.37 (0.91)	$0.42^{***}(0.15)$	0.05 (0.03)	$0.04^{**}$ (0.02)	-0.78 (1.32
	DEF	4.76*** (1.61)	-3.61* (1.97)	0.07 (0.47)	0.17** (0.08)	$0.04^{*}$ (0.02)	-4.14 (3.22)
$\gamma = 2$	RM– $RF$	$-0.72^{**}$ (0.35)	0.16 (0.33)	0.10 (0.08)	0.00 (0.02)	-0.03*** (0.01)	$1.30^{*}$ (0.74)
	SMB	-0.19 (0.45)	-0.03 (0.42)	$0.14^*$ (0.09)	$-0.03^*$ (0.02)	0.00 (0.01)	$4.21^{***}$ (1.11)
	HML	-0.90 (0.57)	0.46 (0.54)	$0.17^*$ (0.10)	-0.01 (0.02)	$-0.03^{**}$ (0.01)	2.03 (1.34)
	UMD	$-0.65^{**}$ (0.31)	0.42 (0.35)	-0.05 (0.04)	0.01 (0.01)	0.00 (0.00)	-0.16 (0.42
	TERM	-0.24 (0.59)	0.45 (0.64)	$0.30^{***}(0.09)$	$0.08^{***}$ (0.03)	$0.03^{**}$ (0.01)	-1.05 (1.17
	DEF	$3.52^{***}$ (1.16)	$-2.95^{**}$ (1.37)	0.27 (0.26)	$0.14^{***}$ (0.05)	$0.05^{**}$ (0.02)	-0.49 (2.53
$\gamma = 5$	RM– $RF$	$-0.77^{**}$ (0.34)	0.28 (0.34)	0.08 (0.06)	0.00 (0.01)	-0.02*** (0.01)	1.12 (0.74
	SMB	-0.22 (0.43)	0.04 (0.41)	0.10 (0.07)	$-0.02^*$ (0.01)	0.00 (0.01)	$3.91^{***}$ (1.02)
	HML	-0.81 (0.54)	0.45 (0.53)	0.13 (0.08)	-0.01 (0.02)	$-0.03^{**}$ (0.01)	1.81 (1.25)
	UMD	-0.38 (0.36)	0.18 (0.39)	-0.03 (0.05)	0.00 (0.01)	0.00 (0.01)	-0.33 (0.52
	TERM	-0.28 (0.56)	0.50 (0.58)	$0.25^{***}$ (0.07)	$0.09^{***}(0.03)$	$0.03^{**}$ (0.01)	-1.16 (1.15
	DEF	$2.77^{***}$ (1.05)	$-2.41^{**}$ (1.20)	0.30 (0.21)	$0.14^{***} (0.05)$	$0.06^{**}$ (0.02)	1.14 (2.47
$\gamma = 20$	RM– $RF$	$-0.77^{**}$ (0.34)	0.32 (0.34)	0.07 (0.06)	0.00 (0.01)	-0.02*** (0.01)	0.98 (0.74
	SMB	-0.22 (0.43)	0.06 (0.41)	0.08 (0.06)	-0.02 (0.01)	0.00 (0.01)	$3.75^{***}$ (0.98
	HML	-0.73 (0.53)	0.41 (0.52)	0.11 (0.07)	0.00 (0.02)	$-0.03^{**}$ (0.01)	1.62 (1.22)
	UMD	-0.25 (0.39)	0.07 (0.41)	-0.03 (0.05)	0.00 (0.01)	0.00 (0.01)	-0.44 (0.57
	TERM	-0.30 (0.55)	0.51 (0.57)	$0.23^{***}(0.07)$	$0.09^{***}(0.03)$	$0.03^{**}$ (0.01)	-1.10 (1.15
	DEF	$2.40^{**}$ (1.05)	$-2.13^{*}$ (1.17)	0.31 (0.19)	$0.13^{***}$ (0.05)	$0.06^{**}$ (0.02)	1.90 (2.47)

# Table V: Economic Hedging Portfolios

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Hedging portfolio weights (in percentage points) due to a unit exposure to innovations in economic risk variables. The sample period is August 1960 through December 2001. Standard errors are in parentheses. Acronyms are defined in Tables I and II.

\*\*\*/\*\*/\* indicates significance at the 1/5/10% level.

period is August	1960 through December 2001			. Standard errors are in parentheses.			Acronyms are defined in Tables I and II.						
Risk aversion	Security	i	INF		RI	-	TS	- 1	DS	L	PIV	C	G
$\gamma = 1$	RM– $RF$	-0.73**	(0.30)	0.36	(0.31)	0.05	(0.04)	0.00	(0.01)	-0.03***	(0.01)	0.96	(0.65)
	SMB	-0.11	(0.37)	-0.11	(0.36)	0.05	(0.04)	-0.01	(0.01)	0.00	(0.01)	$2.86^{***}$	(0.82)
	HML	-0.54	(0.40)	0.23	(0.40)	$0.09^{*}$	(0.05)	-0.01	(0.02)	-0.02**	(0.01)	$1.46^{*}$	(0.86)
	UMD	0.38	(0.32)	-0.48	(0.33)	-0.04	(0.04)	-0.01	(0.01)	0.00	(0.01)	-0.92	(0.63)
	TERM	-0.69	(0.49)	$1.02^{*}$	(0.53)	$0.17^{**}$	(0.07)	$0.10^{***}$	(0.03)	$0.03^{*}$	(0.02)	-1.00	(0.97)
	DEF	0.86	(1.04)	-0.55	(1.12)	0.20	(0.15)	$0.15^{***}$	(0.05)	$0.07^{**}$	(0.03)	$5.67^{**}$	(2.55)
$\gamma = 2$	RM– $RF$	-0.75**	(0.30)	0.40	(0.31)	0.05	(0.04)	-0.01	(0.01)	-0.02***	(0.01)	0.90	(0.68)
1	SMB	-0.12	(0.38)	-0.09	(0.36)	0.05	(0.04)	-0.01	(0.01)	0.00	(0.01)	2.83***	(0.80)
	HML	-0.52	(0.40)	0.23	(0.41)	0.09*	(0.05)	-0.01	(0.02)	-0.02**	(0.01)	1.35	(0.86)
	UMD	0.35	(0.32)	-0.41	(0.33)	-0.04	(0.04)	-0.01	(0.01)	0.00	(0.01)	-0.90	(0.62)
	TERM	-0.64	(0.49)	$0.93^{*}$	(0.53)	0.17**	(0.07)	0.11***	(0.03)	0.03*	(0.02)	-1.02	(1.01)
	DEF	0.87	(1.05)	-0.61	(1.13)	0.22	(0.15)	0.16***	(0.05)	0.06**	(0.03)	5.63**	(2.47)
$\gamma = 5$	BM-BF	-0 74**	(0.30)	0.41	(0.31)	0.05	(0, 04)	-0.01	(0, 01)	-0.02**	(0, 01)	0.85	(0.69)
1 0	SMR	-0.12	(0.38)	-0.08	(0.37)	0.05	(0.01)	-0.01	(0.01)	0.02	(0.01)	0.00 9 89***	(0.00)
	HML	-0.49	(0.00)	0.00	(0.01)	0.00*	(0.01)	-0.01	(0.01)	-0.02**	(0.01)	1.02	(0.86)
	IIMD	0.40	(0.30)	-0.38	(0.11) (0.33)	-0.04	(0.00)	-0.01	(0.02)	0.02	(0.01)	-0.89	(0.00)
	TERM	-0.62	(0.02) (0.49)	0.80*	(0.53)	0.01	(0.01)	0.11***	(0.01)	0.00	(0.01)	-1.02	(0.02) (1.02)
	DEF	0.02 0.87	(1.06)	-0.64	(0.00) (1.14)	0.23	(0.01)	0.16***	(0.03)	0.02 $0.06^{**}$	(0.02)	$5.56^{**}$	(2.43)
			()	0.0 -	()	0.20	(012-)	0.20	(010-)	0.00	(0.00)	0.000	()
$\gamma = 20$	RM– $RF$	-0.74**	(0.30)	0.42	(0.31)	0.05	(0.04)	-0.01	(0.01)	-0.02**	(0.01)	0.80	(0.70)
	SMB	-0.11	(0.38)	-0.09	(0.37)	0.04	(0.04)	-0.01	(0.01)	0.00	(0.01)	2.81***	(0.80)
	HML	-0.47	(0.40)	0.21	(0.41)	$0.09^{*}$	(0.04)	-0.01	(0.02)	-0.02**	(0.01)	1.20	(0.86)
	UMD	0.32	(0.32)	-0.37	(0.33)	-0.05	(0.03)	-0.01	(0.01)	0.00	(0.01)	-0.91	(0.62)
	TERM	-0.60	(0.49)	0.86	(0.52)	0.16**	(0.07)	0.11***	(0.03)	0.02	(0.01)	-0.94	(1.03)
	DEF	0.84	(1.05)	-0.64	(1.13)	0.24*	(0.14)	0.15***	(0.04)	0.06**	(0.03)	$5.56^{**}$	(2.42)
			. ,										. ,

## Table VI: CAPM Economic Hedging Portfolios

CAPM hedging portfolio weights (in percentage points) due to a unit exposure to innovations in economic risk variables. The sample period is August 1960 through December 2001. Standard errors are in parentheses. Acronyms are defined in Tables I and II.

\*\*\*/\*\*/\* indicates significance at the 1/5/10% level.

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