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Normal's deconvolution and the independence of sample mean and variance (problem 03.4.1)

Abadir, K.M.; Magnus, J.R.

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PROBLEMS AND SOLUTIONS

PROBLEMS

03.4.1. Normal's Deconvolution and the Independence of Sample Mean and Variance

Karim M. Abadir

University of York, UK

Jan R. Magnus

Tilburg University, The Netherlands

- (a) Let x_1 and x_2 be independent variates having m.g.f.s $m_1(t_1)$ and $m_2(t_2)$, respectively, and define $y := x_1 + x_2$. Prove that y is normal if and only if x_1 and x_2 are both normal. Is the existence of m.g.f.s necessary for this result?
- (b) Let $\mathbf{x} := (x_1, \dots, x_n)'$ be a vector of independent (but not necessarily identically distributed) components, where $2 \leq n < \infty$. Define $\bar{x} := (1/n) \sum_{i=1}^n x_i$ and $z := \sum_{i=1}^n (x_i - \bar{x})^2$. It is well known that if $\mathbf{x} \sim N(\boldsymbol{\mu}, \sigma^2 \mathbf{I}_n)$, then $\bar{x} \sim N(\mu, \sigma^2/n)$ independently from $z/\sigma^2 \sim \chi^2(n-1)$. For $n \geq 3$, prove that if $\bar{x} \sim N(\mu, \sigma^2/n)$ and $z/\sigma^2 \sim \chi^2(n-1)$, then $\mathbf{x} \sim N(\boldsymbol{\mu}, \sigma^2 \mathbf{I}_n)$.
- (c) Why is the last statement in (b) not necessarily true for $n = 2$? What additional conditions are needed to make it hold for $n = 2$?

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