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Brânzei, R.; Scotti, F.; Tijs, S.H.; Torre, A.

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By R. Brânzei, F. Scotti, S. Tijs and A. Torre

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Discretization of information collecting situations and continuity of compensation rules*

R. Brânzei
Faculty of Computer Science
"Al. I. Cuza" University
11, Carol I Bd., 6600 Iaşi, Romania
e-mail: branzeir@infoiasi.ro

F. Scotti
Dipartimento di Matematica
Università di Pavia
via Ferrata 1, 27100 Pavia, Italy
e-mail: scotti@dimat.unipv.it

S. Tijs[†]
CentER and Department of Econometrics and Operations Research
Tilburg University
P.O. Box 90153, 5000 LE Tilburg, The Netherlands
e-mail: S.H.Tijs@kub.nl

A. Torre
Dipartimento di Matematica
Università di Pavia
via Ferrata 1, 27100 Pavia, Italy
e-mail: atorre@dimat.unipv.it

Abstract

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[†]Corresponding author.

A canonical procedure is described, which associates to each infinite information collecting situation a related information collecting situation with finite state and action spaces, in such a way that the two corresponding IC-games are near to each other. Compensations for informants are then also near to each other in the two IC-situations, if they are based on continuous compensation rules.

Keywords: Bi-monotonic allocation scheme; Cooperative game; Core; Discretization; Information collecting situation

1 Introduction

In this paper information collecting (IC) situations and games (cf. Brânzei et al., 2001 (a, b); Tijs et al., 2001) are central. They model decision-making situations where the outcome of any decision depends on the state of the nature and the decision-maker is imperfectly informed. Collecting information from available agents who are more informed about the situation creates the potential for better decisions. More refined information to support the decision-making process yields, in principle, additional reward which is the source to compensate the informants.

Different procedures for collecting information have given rise to natural compensation rules in the context of cooperative game theory. Relevant for the class of information collecting games are marginal based compensation rules and bi-monotonic allocation schemes.

The state space and the action space in an information collecting situation can be infinite. We consider possibilities of approximating such an IC-situation with a finite IC-situation where the state space and the action space are finite and we relate the original IC-game with the IC-games of finite approximations. The considered approximations turn out to be good in the sense that the corresponding games are nearby. So continuity properties of relevant solutions for the class of IC-games are interesting. We compare relevant solutions such as the core (Gillies, 1953), the set of bi-monotonic allocation schemes (Brânzei et al., 2001*b*) and marginal-based allocation rules (Tijs et al., 2001) of the approximate game with those of the original game, extending results from Lucchetti et al. (1987).

The outline of the paper is as follows. In Section 2 we introduce IC-situations and the corresponding games, look at properties and also at the subclass of IC-games, where a certain concavity condition holds. Section 3 deals with compensation rules and continuity properties of solution concepts which are relevant for the class of IC-games. In Sections 4 and 5 we present procedures for discretizing the state space (going from an infinite state space to a finite one) and for reducing the dimension of the action space, respectively. Section 6 concludes.

2 Information collecting situations and games

Information collecting situations model cooperative behavior of agents when one of them (the decision-maker) is facing uncertainty due to informational deficiencies when making decisions and the others (the informants) can provide additional information about the state of the nature.

In an information collecting (IC) situation the decision-maker has to decide which action to take in order to maximize his expected reward which depends upon both his choice and the true state of the world not precisely known by the agent. An IC situation $\mathcal{C}$ is described by the tuple

$$\mathcal{C} = \left\langle N, n, (\Omega, \mathcal{F}, \mu), \{\mathcal{I}_i\}_{i\in N}, A, r : \Omega \times A \to \Re \right\rangle$$

where agent $n \in N$ is the decision-maker (action taker) who has to choose an action a from infinite action set A and can consult the several informants in $N \setminus \{n\}$. $(\Omega, \mathcal{F}, \mu)$ is a measure space, where Ω is the set of possible states which are relevant to the decision situation, $\mathcal{F}$ is a σ-algebra of subsets of Ω and μ is a probability measure, which describes the prior belief of the decision maker over all the states ω in Ω. The information of each (partially) informed agent in N about the state $\omega \in \Omega$ at hand is represented by the information partition $\mathcal{I}_i$, a finite partition of Ω into $\mathcal{F}$-measurable sets $\mathcal{I}_i(\omega)$ with positive measure. If $\omega \in \Omega$ is the true state, then agent i knows that the event $I_i(\omega)$ happens, where $I_i(\omega)$ is that element (atom) of the partition $\mathcal{I}_i$ of Ω that contains ω. Decision-maker n receives the reward $r(\omega, a)$ if ω turns out to be the true state and he chooses action a. We assume that the reward function is a bounded $\mathcal{F}$-measurable function and that agent n is risk-neutral.

Related to an IC-situation with decision-maker n, we define a cooperative game, the IC-game. The IC-game (N, v) is defined by the player set $N = \{1, 2, \ldots, n\}$ and the characteristic function $v : 2^N \to \Re$, with domain the family 2^N of subsets of N, where $v(S) = 0$ if $n \notin S$ and

$$v(S) = \sum_{I\in\mathcal{I}_S} \sup_{a\in A} \int_I r(\omega, a) d\mu(\omega)$$

for all $S \in 2^N$ with $n \in S$. Here $\mathcal{I}_S = \wedge_{i\in S}\mathcal{I}_i$ is the coarsest partition of Ω that is a refinement of $\mathcal{I}_i$ for each $i \in S$:

$$\mathcal{I}_S = \left\{ \bigcap_{i\in S} I_i \mid I_i \in \mathcal{I}_i, \bigcap_{i\in S} I_i \neq \emptyset \right\}.$$

Notice that alone agent n can attain the expected payoff

$$v(\{n\}) = \sum_{I \in \mathcal{I}_n} \sup_{a \in A} \int_I r(\omega, a) d\mu(\omega).$$

Example 1 *Let* $\mathcal{C} = \left\langle N, n, (\Omega, \mathcal{F}, \mu), \{\mathcal{I}_i\}_{i \in N}, A, r : \Omega \times A \to \Re \right\rangle$ *be the information collecting situation with* $N = \{1, 2, 3\}$, $n = 3$*;* $\Omega = [0, 1]$, $\mathcal{F}$ *is the* σ*-algebra of Borel sets of* $[0, 1]$ *and* μ *is the Lebesgue measure on* $[0, 1]$*;* $\mathcal{I}_1 = \left\{\left[0, \frac{1}{3}\right], \left(\frac{1}{3}, 1\right]\right\}$, $\mathcal{I}_2 = \left\{\left[0, \frac{2}{3}\right], \left(\frac{2}{3}, 1\right]\right\}$, $\mathcal{I}_3 = \{[0, 1]\}$*;* $A = [0, 1]$*;* $r(\omega, a) = 60 - 36|a - \omega|$ *for each* $a, \omega \in [0, 1]$*. Then* $v(\emptyset) = v(\{1\}) = v(\{2\}) = v(\{1, 2\}) = 0$*,*

$$v(\{3\}) = \sup_{a \in A} \int_{[0,1]} r(\omega, a) d\mu(\omega) = \int_{[0,1]} r\left(x, \frac{1}{2}\right) dx =$$

$$60 - 36 \int_0^1 \left|\frac{1}{2} - x\right| dx = 60 - 9 = 51;$$

$$v(\{1, 3\}) = \sup_{a \in A} \int_0^{\frac{1}{3}} r(x, a) dx + \sup_{a \in A} \int_{\frac{1}{3}}^1 r(x, a) dx =$$

$$\int_0^{\frac{1}{3}} r\left(x, \frac{1}{6}\right) dx + \int_{\frac{1}{3}}^1 r\left(x, \frac{2}{3}\right) dx =$$

$$60 - 36 \int_0^{\frac{1}{3}} \left|\frac{1}{6} - x\right| dx - 36 \int_{\frac{1}{3}}^1 \left|\frac{2}{3} - x\right| dx = 55 = v(\{2, 3\}),$$

$$v(\{1, 2, 3\}) = 57 = 60 - 36 \left(\int_0^{\frac{1}{3}} r\left(x, \frac{1}{6}\right) dx + \int_{\frac{1}{3}}^{\frac{2}{3}} r\left(x, \frac{1}{2}\right) dx + \int_{\frac{2}{3}}^1 r\left(x, \frac{5}{6}\right) dx \right).$$

A characterization of IC-games is given in Brânzei et al. (2001 (a, b)) where it is shown that a game is an IC-game with decision-maker n if and only if it is a n-monotonic game in which the decision-maker has veto power, that is $v(S) \leq v(T)$, for all $n \in S \subset T \subset N$, and $v(S) = 0$ for each $S \subset N$ with $n \notin S$. We denote by MV_n the set of such games.

For each coalition $S \subset N$ and $i \in S$, the marginal contribution of player i in the game $\langle N, v\rangle$ is defined by

$$M_i(S, v) = v(S) - v\left(S \setminus \{i\}\right).$$

A game $\langle N, v\rangle$ in MV_n is a so-called *big-boss* game if it satisfies

$$v(N) - v(S) \geq \sum_{i \in N \setminus S} M_i(N, v)$$

for all $S \subset N$ with $n \in S$.

Interesting are those IC-games with the n-concavity property. An IC-game (N, v) is called n-concave if

$$M_i(S, v) = v(S) - v(S \setminus \{i\}) \geq v(T) - v(T \setminus \{i\}) = M_i(T, v)$$

for all $i \in N \setminus \{n\}$, and for all $S \subset T \subset N$ with $\{i, n\} \subset S$.

We denote this subclass of IC-games by MV_nC_n. It is shown in Brânzei et al. (2001*b*) that a non-negative IC-game with decision maker n is n-concave if and only if it is a total big-boss game (cf. Muto et al., 1988).

A game $\langle N, v\rangle$ in MV_n is called a *total big-boss* game if it satisfies

$$v(T) - v(S) \geq \sum_{i \in T \setminus S} M_i(T, v)$$

for all $S \subset T \subset N$ with $n \in S$.

3 Compensation rules and continuity considerations

If the decision-maker n works together with the agents in $N \setminus \{n\}$ to improve his reward, then the question 'How to compensate the informants?'

arises. The introduced IC-game opens the possibility to consider compensation schemes which correspond to solution concepts for cooperative games.

In particular, we focus on marginal based allocation rules, where the compensation of each informant is based on the marginal contribution induced by his information to the decision-maker's reward and on bi-monotonic allocation schemes which take the decision-maker's veto power into account. Both of these compensation solutions are related with the core of an IC-game, which is always non-empty.

Recall that the *core* of the game $\langle N, v\rangle$ (Gillies) is defined by

$$C(v) = \left\{ x \in \Re^N \mid \sum_{i\in S} x_i \geq v(S) \text{ for each } S \subset N,\ \sum_{i\in N} x_i = v(N) \right\}.$$

The continuity properties of the core for cooperative games are discussed in Lucchetti et al. (1987). In that paper and also here the linear space of n-person games is endowed with a metric d defined by

$$d(v,w) = \|v-w\|_\infty = \max_{S\in 2^N} |v(S) - w(S)|.$$

In order to study continuity properties of relevant solution concepts for IC-games, we give some definitions and introduce some useful notations.

Denote by P_n the set $\{S \subset N \mid n \in S\}$ of coalitions containing the decision-maker n.

An allocation scheme $[b_{S,i}]_{S\in P_n, i\in S}$ is a *bi-mas* of v if

$$(b_{S,i})_{i\in S} \in C(S,v),\ b_{S,n} \leq b_{T,n} \text{ and } b_{S,i} \geq b_{T,i}$$

for all $S,T \in P_n$ with $S \subset T$ and all $i \in S \setminus \{n\}$. Here $C(S,v)$ denotes the core of the subgame (S,v).

Example 2 *Take the information collecting situation of Example 1. Note* *that $v \in MV_3$ and that also v is 3-convex. The allocation scheme*

	1	2	3
$\{3\}$	–	–	51
$\{1,3\}$	2	–	53
$\{2,3\}$	–	2	53
N	1	1	55

is a bi-monotonic allocation scheme corresponding to the τ-value (cf. Tijs, 1981), where each of the informants 1 and 2 obtains half of his marginal contribution in the subgames to which he belongs.

An imputation x is *bi-mas extendable* if there is a bi-mas $[b_{S,i}]_{S\in P_n, i\in S}$, such that $x_j = b_{N,j}$ for all $j \in N$.

We denote by BM_n^N the set of all games (N,v) which have at least one bi-mas, and where n is a veto-player.

Let

$$E_n = \left\{(S,i) \in \left(2^N \setminus \{\emptyset\}\right) \times N \mid n, i \in S\right\}.$$

Then consider the multifunction

$$BIMAS : BM_n^N \to \Re^{E_n}.$$

BIMAS is the non-empty-valued multifunction which assigns to each $v \in BM_n^N$ the set of all bi-monotonic allocation schemes $[b_{S,i}]_{S\in P_n, i\in S}$ of v, where for each $S \in P_n$ the 'row' $(b_{S,i})_{i\in S}$ is a core element of the subgame (S,v) with extra conditions of the form $b_{S,i} \geq b_{T,i}$ for each $i \in N \setminus \{n\}$ and $b_{S,n} \leq b_{T,n}$, which are linear inequalities.

By the related stability theorems in this field (Walkup and Wets, 1969; Lucchetti et al., 1987) we obtain

Theorem 1 *The multifunction BIMAS is upper and lower semicontinuous.*

It is shown in Brânzei et al. (2001*b*) that any core element of a game $v \in MV_nC_n$ is extendable to a bi-mas. In total big-boss games the use of the nucleolus (cf. Schmeidler, 1969) as an allocation mechanism generates a bi-mas (see Voorneveld et al., 2000) and the nucleolus coincides here with the τ-value (cf. Tijs, 1981).

Now we concentrate on marginal based allocation rules, where the reward compensation of each informant is a fraction of his marginal contribution.

In a marginal based allocation rule informant i receives $\alpha_i M_i(N,v)$ with $\alpha_i \in [0,1]$ and the decision-maker is given the remainder

$$v(N) - \sum_{i \in N \setminus \{n\}} \alpha_i M_i(N,v).$$

For every fixed $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_{n-1}) \in [0,1]^{N \setminus \{n\}}$, we define the marginal allocation rule

$$\Psi^\alpha : MV_nC_n \to \Re^n,$$

$$\Psi^\alpha(v) = \left(\alpha_1 M_1(N,v), \ldots, \alpha_{n-1} M_{n-1}(N,v), v(N) - \sum_{i \in N \setminus \{n\}} \alpha_i M_i(N,v) \right).$$

Theorem 2 *For every fixed α, the function Ψ^α is $(2n-1)$-Lipschitz.*

Proof. Let v and w belong to MV_nC_n :

$$\alpha_i \left| M_i(N,v) - M_i(N,w) \right| \leq \alpha_i \left| v(N) - w(N) \right| + \alpha_i \left| v(N \setminus \{i\}) - w(N \setminus \{i\}) \right| \leq$$

$$2 \left\| v - w \right\|_\infty$$

for every $i \in N \setminus \{n\}$ and

$$\left| v(N) - \sum_{i \in N \setminus \{n\}} \alpha_i M_i(N,v) - \left(w(N) - \sum_{i \in N \setminus \{n\}} \alpha_i M_i(N,w) \right) \right| \leq$$

$$|v(N) - w(N)| + 2\alpha_1 \|v - w\|_\infty + \ldots + 2\alpha_{n-1} \|v - w\|_\infty \leq (2n-1) \|v - w\|_\infty .$$

So

$$\|\Psi^\alpha(v) - \Psi^\alpha(w)\|_\infty \leq (2n-1) \|v - w\|_\infty .$$

■

It is shown (Tijs et al., 2001) that any core element of a game $v \in MV_n$ is a marginal based allocation rule and that the set of marginal based allocation rules for a game v in MV_n coincides with the core of the game if and only if v is a big-boss game.

4 The canonical discretization of an IC-situation

In this section we describe a natural way to discretize the infinite state space Ω of an IC-situation

$$\mathcal{C} = \left\langle N, n, (\Omega, \mathcal{F}, \mu), \{\mathcal{I}_i\}_{i \in N}, A, r : \Omega \times A \to \Re \right\rangle$$

and to arrive at a strategic equivalent IC-situation

$$\mathcal{C}' = \left\langle N, n, (\Omega', \mu'), \{\mathcal{I}'_i\}_{i \in N}, A, r' : \Omega' \times A \to \Re \right\rangle$$

where the state space Ω' is finite. The idea behind the discretization is simple. Each of the players in the IC-situation $\mathcal{C}$ is provided with a finite information partition $\mathcal{I}_i$ of Ω. The common refinement $\mathcal{I}_N = \wedge_{i \in N} \mathcal{I}_i$ is also a finite partition of Ω. Each atom of this common refinement consists of points which are neither distinguishable by agents nor by coalitions who pool their information. So it is natural to replace such an atom by one point and then these points generate Ω'. The probability measure, the information partitions and the reward function can then be adjusted, leading to the canonical discretization.

In the following we describe these constituents of $\mathcal{C}'$ in terms of $\mathcal{C}$. Each atom $I(J, K, \ldots)$ in the common refinement $\mathcal{I}_N$ of $\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n$ is considered as one point of Ω' and we denote this point by i $(j, k, \ldots)$. Hence, $\Omega' = \{i \mid I \in \mathcal{I}_N\}$ is a finite set. The probability measure μ on Ω induces canonically the measure μ' on Ω', where each subset B of Ω' is measurable and the measure $\mu(B)$ is given by $\sum_{k=1}^{m} \mu(I_k)$, if B consists of the points $i_1, i_2, \ldots, i_k$ corresponding to the atoms $I_1, I_2, \ldots, I_k$ in $\mathcal{I}_N$.

Note that for each of the partitions $\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n$ (and also for $\mathcal{I}_S = \wedge_{i \in S} \mathcal{I}_i$ for each $S \in 2^N \setminus \{\emptyset\}$) the atoms are finite unions of elements in $\mathcal{I}_N$. So the atoms in $\mathcal{I}_i$ $(i \in N)$ correspond to disjoint subsets of Ω', which form a partition of Ω', that we denote by $\mathcal{I}'_i$.

The reward $r'(i, a)$ corresponding to action $a \in A$ and state $i \in \Omega'$ is defined by

$$r'(i, a) = \mu(I)^{-1} \int_I r(\omega, a) d\mu(\omega)$$

where I is the atom in $\mathcal{I}_N$ corresponding to i.

So, we have defined the canonical discretization $\mathcal{C}'$ of $\mathcal{C}$. Note that there is also a one to one relation between the atoms of $\mathcal{I}_S$ and $\mathcal{I}'_S = \wedge_{i \in S} \mathcal{I}'_i$. We will denote in the following the atom in $\mathcal{I}'_S$ which corresponds to the atom J in $\mathcal{I}_S$ by $\overline{J}$. The question arises: What is the relation between the corresponding IC-games v' and v? The answer is simple.

Theorem 3 *The IC-games v' and v corresponding to $\mathcal{C}'$ and $\mathcal{C}$ coincide.*

Proof. Take $S \in 2^N \setminus \{\emptyset\}$. We have to show that $v'(S) = v(S)$. Using the definition of $v'(S)$ and the fact that (Ω', μ') is a finite measure space we obtain

$$v'(S) = \sum_{\overline{J} \in \mathcal{I}'_S} \sup_{a \in A} \int_{\overline{J}} r'(i, a) d\mu'(i) =$$

$$\sum_{\overline{J}\in\mathcal{I}'_S}\sup_{a\in A}\sum_{i\in\overline{J}}\mu'(\{i\})r'(i,a). \tag{1}$$

Now using the definition of μ' and r'we obtain

$$\sum_{i\in\overline{J}}\mu'(\{i\})r'(i,a)=\sum_{I\subset J,I\in\mathcal{I}_N}\mu(I)\left(\mu(I)^{-1}\right)\int_I r(\omega,a)d\mu(\omega)=$$

$$\int_J r(\omega,a)d\mu(\omega). \tag{2}$$

Combining (1) and (2) we obtain

$$v'(S)=\sum_{J\in\mathcal{I}_S}\sup_{a\in A}\int_J r(\omega,a)d\mu(\omega)=v(S).$$

■

Example 3 *Take the information collecting situation of Example 1. Then* $\mathcal{I}_N=\{P,Q,R\}$ *with* $P=\left[0,\frac{1}{3}\right],Q=\left(\frac{1}{3},\frac{2}{3}\right]$ *and* $R=\left(\frac{2}{3},1\right]$. *In the canonical discretization*

$$\mathcal{C}'=\left\langle N,n,(\Omega',\mu'),\{\mathcal{I}'_i\}_{i\in N},A,r':\Omega'\times A\rightarrow\Re\right\rangle$$

of $\mathcal{C}$ *we have*

$\Omega'=\{p,q,r\}$, *where* p *is the point corresponding to* P, *etc*

$\mu'(\{p\})=\mu'(\{q\})=\mu'(\{r\})=\frac{1}{3},\mu'(\{p,q\})=\mu'(\{p,r\})=\mu'(\{q,r\})=\frac{2}{3},\mu'(\{p,q,r\})=1.$

$\mathcal{I}'_1=\{p,\{q,r\}\},\mathcal{I}'_2=\{\{p,q\},r\},\mathcal{I}'_3=\{\{p,q,r\}\}.$

$A=[0,1]$

$r'(p,a)=60-108\int_0^{\frac{1}{3}}|a-x|\,dx$ *for each* $a\in A$,

etc

In the next section we go one step further and we will describe a method to find for an IC-situation with a finite state space a related appropriate IC-situation with the same state space and also with a finite action space.

5 A reduction procedure for the action space

Let $\mathcal{C} = \left\langle N, n, (\Omega, \mu), \{\mathcal{I}_i\}_{i \in N}, A, r : \Omega \times A \to \Re \right\rangle$ be an IC-situation with Ω finite. Take $\varepsilon > 0$. The objective is to find an IC-situation

$$\mathcal{C}^\varepsilon = \left\langle N, n, (\Omega, \mu), \{\mathcal{I}_i\}_{i \in N}, A(\varepsilon), r : \Omega \times A(\varepsilon) \to \Re \right\rangle,$$

where $A(\varepsilon)$ is a finite subset of A, such that the distance of the two corresponding games v and v^ε does not exceed ε. Since Ω is finite, also the family 2^Ω of non-empty subsets of Ω is finite. For each $J \in 2^\Omega$ we take an action $a(\varepsilon, J) \in A$ with the property that

$$\int_J r(\omega, a(\varepsilon, J)) d\mu(\omega) \geq \sup_{a \in A} \int_J r(\omega, a) d\mu(\omega) - \varepsilon\mu(J). \tag{3}$$

Define $A(\varepsilon) = \left\{ a(\varepsilon, J) \mid J \in 2^\Omega \right\}$. Then $A(\varepsilon) \subset A$ and for the corresponding $\mathcal{C}^\varepsilon$ and its IC-game v^ε we have

Theorem 4 $d(v, v^\varepsilon) \leq \varepsilon$.

Proof. Take $S \in 2^N \setminus \{\emptyset\}$.Then

(i) $v(S) = \sum_{J \in \mathcal{I}_S} \sup_{a \in A} \int_J r(\omega, a) d\mu(\omega) \geq$
$\sum_{J \in \mathcal{I}_S} \max_{a \in A(\varepsilon)} \int_J r(\omega, a) d\mu(\omega) = v^\varepsilon(S)$.
(ii) In view of (3)
$v(S) = \sum_{J \in \mathcal{I}_S} \sup_{a \in A} \int_J r(\omega, a) d\mu(\omega) \leq$
$\sum_{J \in \mathcal{I}_S} \left(\int_J r(\omega, a(\varepsilon, J)) d\mu(\omega) + \varepsilon\mu(J)\right) \leq$
$\varepsilon + \sum_{J \in \mathcal{I}_S} \max_{a \in A(\varepsilon)} \int_J r(\omega, a) d\mu(\omega) = \varepsilon + v^\varepsilon(S)$.
Hence, $d(v, v^\varepsilon) = \max_{S \in 2^N \setminus \{\emptyset\}} |v^\varepsilon(S) - v(S)| \leq \varepsilon$. ■

Example 4 *Take the information collecting situation $\mathcal{C}'$ from Example 3. Then $\Omega' = \{p, q, r\}$ and optimal actions corresponding to the subsets $\{p\}, \{q\}$,*

$\{r\}, \{p,q\}, \{p,r\}, \{q,r\}$, and $\{p,q,r\}$ of Ω' are $\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}$ and $\frac{1}{2}$, respectively. So the information collecting situation $\mathcal{C}''$, obtained from $\mathcal{C}'$ by replacing $A = [0,1]$, by $A'' = \left\{\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}\right\}$ is an information collecting situation for which the corresponding game v'' coincides with v'.

6 Concluding remarks

Combining the procedures in Sections 4 and 5 we can find for each IC-situation $\mathcal{C}$ with infinite state and action spaces a related finite approximation $\mathcal{C}''$ such that the two involved IC-games are as near to each other as one wants. The continuity properties of the different compensation schemes guarantee then that the corresponding compensations in the two games are also near to each other. A similar discretization procedure can be developed for information sharing games, introduced in Slikker et al. (2000). In this context it is interesting to note that the multifunction PMAS, which assigns to a game the set of its population monotonic allocation schemes (cf. Sprumont, 1990) is upper and lower continuous by the same argument used for the multifunction BIMAS.

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