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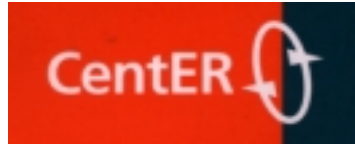
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WHY DOES IT MATTER THAT BELIEFS AND VALUATIONS BE CORRECTLY REPRESENTED?

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Abstract

This paper contains an analysis of a simple principal-agent problem illustrating possible problems that may arise when the prinicpal ascribes to the agent subjective probabilities and utilities that are implied by the subjective expected utility model but do not represent the agent's beliefs and valuations. In particular, it is possible that an incentive contract designed by the principal induces the agent to choose an action that is not in the principal's best interest.

Keywords: subjective probability, moral hazard, state-dependent preferences.

JEL classifications: D81, D82

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1 Introduction

Subjective expected utility theory is founded on the tacit notion that choice among alternative courses of action (acts) is governed by two separate cognitive processes: the assessment of the likelihood of various events, or the formation of beliefs, and the valuation of the consequences associated with those events. Moreover, beliefs are supposed to be coherent enough to allow their representation by a (subjective) probability measure, and the valuation of the consequences sufficiently structured to permit their representation by numerical utilities. Individual preferences on acts are represented by the expected values of the utilities of the consequences of these acts with respect to the subjective probability measure.

Choice-theoretic models of subjective expected utility, including Savage (1954) and Anscombe and Aumann (1963), derive the subjective probabilities and utilities from individuals' preference relations on the set of acts. These, and all similarly conceived, models give rise to equivalent representations of preferences each involving a utility function and a corresponding subjective probability measure. To determine a unique subjective probability the choice-theoretic models invoke the convention, not implied by the axioms, that the utility function is state independent. Put differently, the axiomatic structures of the various choice-theoretic subjective expected utility models require that the preference relations on acts be state independent (for example, Savage's postulate P3 and P4, and Anscombe and Aumann state-independence axiom) but that does not imply that the utility function must be state-independent. In fact, state-independent preferences only require that the utility function representing the valuation of the consequences in different states be affine transformations of one another. Thus the normalization of the utility functions to make them the same across states has no theoretical foundation. Moreover, the subjective probabilities are the normalized multiplicative coefficients of these utility functions. Hence these subjective probabilities are inherently arbitrary (see Schervish, Seidenfeld, and Kadane [1990]; Karni and Schmeidler [1993]; and Karni [1996]). In particular, *it is possible that a decision maker's preference relation on acts satisfy the axioms of subjective expected utility theory and yet be ascribed probabilities that do not represent his beliefs and utilities that do not represent his valuations.*

To remedy this problem and obtain a definition of subjective probability that quantifies the decision-makers' beliefs it is necessary to extend the choice space. One possible extension, due to Karni and Schmeidler (1981), calls for the introduction of a second preference relation over hypothetical lotteries on the set of state-consequence pairs. The new preference relation is linked axiomatically to the preference relation on horse/roulette-lotteries acts in the framework of Anscombe and Aumann (1963). The original intent of Karni and Schmeidler was to model subjective expected utility theory with state-dependent preferences, however, Karni and Mongin (2000) recently noted that the probabilities thus obtained are, in fact, the unique correct representation of decision makers' beliefs. The model of Karni and Schmeidler, as well as the more general expected utility model explored in Karni (2001) and the nonexpected utility theory developed in Grant and Karni (2000), in all of which the subjective probabilities represent of decision-makers' beliefs, rely on the use of objective probabilities on the set of states as a primitive concept. More recently, Karni (2002) developed an axiomatic subjective expected utility model with preferences defined on conditional acts (or, alternatively, preference over actions that delimit the events that might obtain) that leads to a definition of subjective probabilities representing decision makers' beliefs, and utilities that represent their valuations. Like Savage (1954), Karni's theory does not involve the use of objective probabilities as a primitive concept but, unlike Savage, it accommodates state-dependent preferences.

These developments raises the question: other than for philosophical reasons, why is it important to represent decision makers' of beliefs and valuations correctly? Karni (1996) argued that such a representation is desirable since it renders the decision-makers' observed choice behavior and their verbal exchange of information consistent. Karni (2002) showed, in the context of a simple principal-agent problem, that if the principal ascribes to the agent probabilities, implied by the choice-theoretic subjective expected utility model, that misrepresent the agent's beliefs, and designs an incentive-contract based on these probabilities, the principal runs the risk of inducing the agent to choose an action that is not in the principal's best interest.

Our purpose, in this paper, is to explore this issue further. In particular, we intend to examine the role of ascribing the agent the correct utility function. To do this we show that if the principal

ascribes incorrect utilities and/or subjective probabilities to the agent, he may fail to induce the agent to act in a way that serves the best interest of the principal. In other words, we show that a contract designed, on the basis of ascribed probabilities and utilities implied by choice-theoretic subjective expected utility model, to motivate the agent to choose one action motivates him, instead, to choose another action that is less desirable for the principal.

2   The Envious Agent Problem

In classical economic theory self-interest seeking behavior is portrayed strictly as a quest to improve the individual's material well-being. This narrow view of human nature has recently been challenged and the possibility of incorporating emotions into the theory of choice is explored (see, for example, a survey by Elster [1998] and discussions by Loewenstein [2000], Romer [2000]). The interest in broadening the psychological basis underlaying the conduct of economic agents is due, in part, to experimental evidence indicating a tendency of individuals to cooperate in situations in which maximization of material self-interest alone would imply non-cooperative behavior (see Camerer [1997], Berg, Dickhaut, and McCabe [1995]). Against this backdrop, we consider next a principal-agent relation that may be influenced by envy. Specifically, we analyze a principal-agent problem in which the agent's preferences incorporate envy, and yet are representable by a subjective expected utility functional. In other words, the agent's choice behavior is consistent with the principal ascribing to him subjective probabilities and utilities implied by the choice-theoretic expected utility model. We show that the failure of the principal to detect the presence of envy results in a contract, based on the principal's ascribed utilities and probabilities, that motivates the agent to act in a way that is not in the principal's best interest.

2.1   An advertising campaign

Consider the following principal-agent problem. A producer (the principal) engages an advertising agency to promote an event (e.g., a rock concert). The revenue is a random variable that depends on the state of nature which, in this instance, represents the state of demand and on the advertising

campaign. Specifically, suppose that there are three states of nature $S = \{L, M, H\}$, where L signifies low demand, M signifies moderate demand, and H signifies high demand. The agent must choose between an advertising campaign, a^n, that would reach a narrow potential audience, and an advertising campaign, a^ℓ, that would reach a wide potential audience. Assume that if he chooses a^n then, if the demand is high the producer will sell the concert-hall capacity and attain the high level of revenue, r^H, if the demand is moderate he will sell half of the concert-hall capacity and attain a revenue of r^M, and if the demand is low he will sell only 15 percent of the concert-hall capacity and attain low revenue level, r^L. If the agent chooses an advertising campaign to reach a wide audience, namely, a^ℓ, he can boost the demand to the point of ensuring himself of selling at least half of the concert-hall capacity. In other words, he can prevent the situation in which only 15 percent of the capacity is sold, and will either sell half the concert-hall capacity or the entire concert-hall capacity. Assume that the nature of the advertising campaign (effort and cost invested to reach the potential audience) is private information of the agent.

To model the situation described above let $\mathbf{B} = \{a^\ell, a^n\}$ denote the set of feasible actions. The effects of the alternative advertising campaigns are expressed by the mapping $F : \mathbf{B} \rightarrow \mathcal{E}$, where $\mathcal{E}$ is the set of events (that is, subsets of the set S). Thus the advertising campaign a^ℓ corresponds to the event $F\left(a^\ell\right) = \{M, H\}$ and a^n corresponds to the universal event $F(a^n) = \{L, M, H\}$. Note that, once the agent chooses an action, say $a \in \mathbf{B}$, the elements of the set $F(a)$ correspond to Savage's definition of the state of the world (nature), namely, "a description of the world, leaving no relevant aspect undescribed," (Savage 1954, p. 9) since, given a, the state of demand alone determines the revenue, which is the only relevant aspect of nature. The fact that the set of states depends on the action means that our framework is that of preferences on conditional acts (see Luce and Krantz [1971], Fishburn [1973], and the adaptation in Karni [2002]).

Assume that both the principal and the agent are expected utility-maximizing Bayesian decision makers whose preferences are state independent. This terminology merits some elaboration. First, a subjective expected utility maximizing decision maker is Bayesian if he updates his prior subjective probabilities using Bayes rule. While the choice-theoretic subjective expected utility model does not imply this particular updating rule, it is, nevertheless, consistent with it. Sub-

jective expected utility models in which Bayesian updating is implied require the extension of the analytical framework. For example, Ghirartato (2002) uses conditional preferences on acts and Karni (2002) uses preferences on conditional acts to obtain subjective expected utility representations of Bayesian decision makers' preferences. Second, as noted above, the framework that we use is that of preferences on conditional acts. This means that for every $a \in \mathbf{B}$, the set of states is $F(a)$, and the corresponding (conditional) acts are functions from $F(a)$ to the set of consequences. A decision maker is a subjective expected utility maximizer if his preferences on conditional acts are representable by a subjective expected utility functional. The axiomatic foundations of subjective expected utility theory of Bayesian decision making, which is the theory used here, is developed in Karni (2002).

Suppose that both the principal and the agent believe that if the narrow advertising campaign is launched then the three states are equally likely to obtain, but these beliefs are not common knowledge. In other words, the beliefs of both parties are represented by the uniform probability distribution $\pi^H = \pi^M = \pi^L$, where π^s denotes the subjective probability of state $s \in S$, but the principal does not know this and must infer the agent's probabilities from his observed choice-behavior (e.g., his observed response when presented with a proper scoring rule such as described in Savage [1971]). Assume that the principal is risk neutral and her utility function is state independent (that is, the principal's utility function is the identity function) and that the agent is risk averse and that his valuations of the payoff, w, are depicted by state-dependent utility functions $u_H(w) = \beta^H \sqrt{w} + \alpha^H$, $u_M(w) = \beta_M \sqrt{w} + \alpha_M$, and $u_L(w) = \beta_L \sqrt{w} + \alpha_L$. Without loss of generality let $\beta^H = 1$ and $\alpha^H = 0$.

On the basis of the agent's choice behavior, the principal ascribes to him subjective probabilities, $p = \{p^H, p^M, p^L\}$ and a state-independent utility function, $u(w)$, implied by the choice-theoretic subjective expected utility model. In other words, as far as the principal is concerned, the agent's preferences are represented by:

$$p^H u(w_H) + p^M u(w_M) + p^L u(w_L), \tag{1}$$

where $u(w) = \sqrt{w}$, $p^H = \pi^H/(\pi^H + \pi^M \beta^M + \pi^L \beta^L)$, $p^M = \pi^M \beta^M/(\pi^H + \pi^M \beta^M + \pi^L \beta^L)$, and $p^L = \pi^L \beta^L/(\pi^H + \pi^M \beta^M + \pi^L \beta^L)$. Moreover, the principal ascribes to the agent the probability

$p(\{H, M\}) = p^H + p^L$ for the event $\{H, M\}$.

2.2 Principal-agent problems

Let the state-contingent revenue $\mathbf{r} = (r^H, r^M, r^L)$ satisfy $r^H > r^M > r^L$. A contract, $\mathbf{w}$, is a point in $\mathbb{R}^3_+$ representing the agent's state contingent pay. We assume that contracts requiring the agent to pay the principal in some states are not enforceable.[1] Then, given his perception of the agent's subjective probabilities and utilities, the principal's problem, as seen by the principal, may be stated as follows:

Choose $(a^*, \mathbf{w}^*) \in \mathbf{B} \times \mathbb{R}^3_+$ so as to maximize

$$\sum_{s \in S} \pi_P(s \mid F(a^*))(r_s - w_s^*) \tag{2}$$

subject to the participation constraint:

$$\sum_{s \in S} p_A(s \mid F(a^*))\sqrt{w_s^*} + v(a^*) \geq v_0, \tag{3}$$

and the incentive compatibility constraints: for all $a \in \mathbf{B}$

$$\sum_{s \in S} p_A(s \mid F(a^*))\sqrt{w_s^*} + v(a^*) \geq \sum_{s \in S} p_A(s \mid F(a))\sqrt{w_s^*} + v(a), \tag{4}$$

where the subscripts P denotes the conditional subjective probabilities of the principal and the subscript A denotes the conditional subjective probabilities *ascribed to the agent* by the principal; v_0 is the outside option available to the agent in case he reject the contract; and $v(a)$ represents the disutility (cost) to the agent of taking the action a. Let $v_0 = 0$ and, in view of the relative time and effort and financial costs required to mount the two different advertising campaigns, assume that $0 > v(a^n) > v(a^\ell)$. Note that $p_A(s \mid F(a)) = p^s / p(F(a))$ for all $a \in \mathbf{B}$.

The agent's problem may be stated as follows:

Given $\mathbf{w}^*$ choose $a \in \mathbf{B}$ so as to maximize

$$U(a; \mathbf{w}^*) = \sum_{s \in S} \pi_A(s \mid F(a))\left(\beta^s\sqrt{w_s^*} + \alpha^s\right) + v(a) \tag{5}$$

and implement the optimal action a^* if $U(a^*; \mathbf{w}^*) \geq v_0$. Otherwise reject the contract.

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[1] See section 3 below for a discussion of this assumption.

It is obvious that the principal's perception of the agent's motives is different from the agent's true motives. We turn next to examine some potential implications of misconstruing the agent's motives. Consistent with our depiction of the problem, we let $r^H = \$200$, $r^M = \$100$, $r^L = \$30$, $v(a^n) = -1$, and $v(a^\ell) = -3$. To illustrate the potential pitfalls of misconstrued assignment of utilities and probabilities to the agent, we analyze two specific cases in which the agent is envious of the principal.

2.3 Case I

Let the agent's envy affect the marginal utility of his income. More specifically, suppose that increase in the principal's income reduces the agent's marginal utility of his income uniformly. To capture this trait of the agent's attitudes and at the same time preserve the preference structure, we let $\alpha^M = \alpha^L = 0$, $\beta^M = 1.5$, and $\beta^L = 2.5$. The assumption $\beta^L > \beta^M > \beta^H$ is given the interpretation that, for any given level of w, the agent's utility and his marginal utility of income are higher the lower is the principal's income. This malevolent attitude cannot be detected by observing the agent's choice behavior.

Observe next that the principal would like to implement the action a^ℓ. To see this, note that from the principal's point of view, to induce the agent to choose a^ℓ the participation constraint is:

$$\frac{p^H}{p^H + p^M}\sqrt{w^H} + \frac{p^M}{p^H + p^M}\sqrt{w^M} - 3 \geq 0, \tag{6}$$

and the incentive compatibility constraint is reduced to:

$$\frac{p^H}{p^H + p^M}\sqrt{w^H} + \frac{p^M}{p^H + p^M}\sqrt{w^M} - 3 \geq p^H\sqrt{w^H} + p^M\sqrt{w^M} + p^L\sqrt{w^L} - 1. \tag{7}$$

Since the agent is risk averse, the least costly contract that satisfies the participation constraint is to set $w^H = w^M = w^*$ and $w^L = 0$. Substitute these in equations (6) and (7) together with the implied values of the probabilities to obtain the participation constraint is

$$\sqrt{w^*} - 3 \geq 0. \tag{8}$$

and the incentive compatibility constraint:

$$\sqrt{w^*} - 3 \geq \frac{1}{2}\sqrt{w^*} - 1. \tag{9}$$

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The incentive compatibility constraint is binding, and it implies that $w^* = 16$. Substituting this into the participation constraint, it is easy to verify that it is satisfied. Hence the optimal contract is $\mathbf{w}^* = (16, 16, 0)$. The principal believes that, if he accepts the contract $\mathbf{w}^*$, the agent's best response is to choose a^ℓ. If he does, the principal's subjective expected utility is

$$\frac{1}{2}200 + \frac{1}{2}100 - 16 = 134.$$

The alternative contract that would implement a^n need only satisfy the participation constraint, namely,

$$\sum_{s \in S} p^s \sqrt{w_s} - 1 \geq 0. \tag{10}$$

Because the agent is risk averse the cheapest way to meet the constraint (10) is by setting $w^s = \bar{w}$ for all $s \in S$. The participation constraint requires that $\bar{w} = 1$. The expected utility of the principal under a^n is

$$\frac{1}{3}200 + \frac{1}{3}100 + \frac{1}{3}30 - 1 = 109.$$

Hence the principal's perceived best interest is to implement a^ℓ. It is easy to verify that no other contract that implements a^ℓ is less costly. Thus the solution of the principal's problem is $\left(a^\ell, \mathbf{w}^*\right)$.

Consider next the agent's choices among the action-acts pair $\left(a^i, \mathbf{w}^*\right), a^i \in \mathbf{B}$. According to the agent's beliefs, the probabilities of the event $F\left(a^\ell\right)$ is $\pi^A\left(\{H, M\}\right) = 2/3$. Let $U\left(a^i, \mathbf{w}^*\right)$ be the agent's subjective expected utility corresponding to the action-acts pair $\left(a^i, \mathbf{w}^*\right)$. Then,

$$U\left(a^n; \mathbf{w}^*\right) = \frac{1}{3}\sqrt{w^*} + \frac{1}{3}\beta^M\sqrt{w^*} - 1 = 2\frac{1}{3} > 2 = \frac{1}{2}\sqrt{w^*} + \frac{1}{2}\beta^M\sqrt{w^*} - 3 = U\left(a^\ell; \mathbf{w}^*\right), \tag{11}$$

Expression (11) implies that, given the contract $\mathbf{w}^*$ which he accepts, the agent chooses the action a^n contrary to the wishes of the principal. *Because she misconstrued the agent's subjective probabilities and utilities, the principal designed a contract that induced the agent to choose an action that is not in the principal's best interest.*

2.4 Case II

Envy may manifest itself by affecting the level of the agent's utility without, at the same time affect his marginal utilities. In this case the principal ascribes to the agent probabilities that accurately reflects his beliefs. Yet, by misunderstanding the agent's motives the principal still fails to induce

him to choose the desirable action. To analyze this situation we let $\alpha^H = 0, \alpha^M = 1, \alpha^L = 2$, and $\beta^H = \beta^M = \beta^L = 1$. In this case the agent's envy is captured by values of the additive constants. Specifically, $\alpha^H < \alpha^M < \alpha^L$ is interpreted to mean that, for any given level of w, the agent's utility is higher the lower is the principal's income.

As before, the principal's problem is to design a contract $\mathbf{w}^*$ that will implement a^ℓ. Because the agent is risk averse and the principal is risk neutral, the optimal contract requires that $w^L = 0$ and $w^M = w^H = w^*$. Unlike the previous case, this time the probabilities that the principal ascribes to the agent agrees with the agent's own probabilities .

From the principal's viewpoint the participation constraint is,

$$\sqrt{w^*} - 3 \geq 0,$$

and the incentive compatibility constraint is,

$$\sqrt{w^*} - 3 \geq \frac{2}{3}\sqrt{w^*} - 1.$$

The incentive compatibility constraint is binding and the solution is $w^* = 36$. The principal's perceived expected utility under $\left(a^\ell; \mathbf{w}^*\right)$ is 114. If she tries to implement a^n then she design a contract $w^H = w^M = w^L = \bar{w}$ that satisfies the participation constraint $\sqrt{\bar{w}} - 1 \geq 0$. Thus $\bar{w} = 1$ and the principal's expected utility under $(a^n; \bar{w})$ is 109.

Next consider the problem from the viewpoint of the agent.

$$U\left(a^n; \mathbf{w}^*\right) = \frac{1}{3}\sqrt{w^*} + \frac{1}{3}\left(\sqrt{w^*} + \alpha^M\right) + \frac{1}{3}\alpha^L - 1 > \frac{1}{2}\sqrt{w^*} + \frac{1}{2}\left(\sqrt{w^*} + \alpha^M\right) - 3 = U\left(a^\ell; \mathbf{w}^*\right).$$

Under the assumed values of the parameters and the contract $\mathbf{w}^*$ we have:

$$U\left(a^n; \mathbf{w}^*\right) = 4 > 3\frac{1}{2} = U\left(a^\ell; \mathbf{w}^*\right).$$

Moreover, $U\left(a^n; \mathbf{w}^*\right) > 0$ implies that the participation constraint is satisfied. Hence, the agent chooses a^n, which again is an action that was not in the principal's best interest.

3   Concluding Remarks

The analysis in this paper illustrates and underscores the possible pitfalls of employing subjective expected utility theory to the analysis of principal-agent problems. The source of difficulty is

that the agent's preferences may admit alternative equivalent representations involving distinct subjective probabilities and state-dependent utility functions. If one's only concern is with individual decisions and is willing to assume state-independent preferences, then nothing essential is lost by imposing the convention that the utility functions are state-independent and defining subjective probabilities consistent with this convention. Decision makers' beliefs, namely, a binary relation on the set of events depicting the notion of "more likely to obtain," are *defined* by the probabilities. In other words, if the only application of the theory is to individual decision making then it is not necessary to separate utility and true probability, since only the product of the two matters. Our analysis shows that this is no longer the case if the model is to be applied to the richer context of the principle-agent theory, in which individuals face the need to infer the true probabilities and utilities. We analyzed a simple example but the reader will recognize that the issue pervades the entire principal-agent literature.

Our analysis imposes the restriction that contracts stipulating a payment by the agent to the principal in some events are not enforceable. It is well known that if such payments could be enforced it would be possible to penalize the agent to coerce him to avoid taking certain actions that may be detected, ex post. In our example, if a large penalty could be imposed in case the revenue r^L is realized it would be possible to force the agent to avoid the action a^n for fear of being detected and penalized after the fact. In the literature on principal-agent problem this issue is dealt with by assuming that all the conditional probability distributions have the same support (see Salanié [1997]). To justify our approach we note that, as a matter of fact, it may be impossible, in some situations, to enforce the required penalty. On the theoretical level we note that the traditional analyses of the principal-agent problem (e.g., Holmstrom [1979], Shavell [1979]) suppress the explicit consideration of the states of nature and focus instead on the conditional probabilities of the random variables representing the payoff to the principal. This approach conceals the fact that, if the principal and the agent are Bayesian subjective expected utility maximizers then the probability distributions conditional on the agent's actions must be derived from some priors, in which case it is impossible that they all have exactly the same support. In other words, if the agent is Bayesian, then the conditional probabilities representing his posterior

beliefs are obtained from his prior probability by increasing, proportionally, the probability mass on a subset of the original probability space. Hence, exluding trivial cases, the essential support of the posterior is a proper subset of that of the prior.

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