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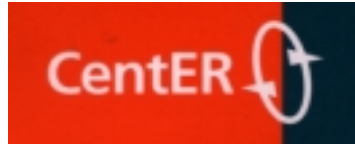
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EFFICIENCY VERSUS RISK DOMINANCE IN AN EVOLUTIONARY MODEL WITH CHEAP TALK

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Discussion paper

Efficiency versus risk dominance in an evolutionary model with cheap talk

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Abstract

We study equilibrium selection by cheap talk in symmetric 2×2 coordination games. Before playing the game, players exchange simultaneously messages coming from a finite set. The messages have no a priori meaning. Individuals from a single population are matched in a round robin fashion to play the game. They update startegies by imitating the currently most succesful individuals. When risk dominance selects a different equilibrium than payoff dominance, the game outcome of a stochastically stable state depends on the number of messages in the message set. For sufficiently many messages, the efficient equilibrium is played. We link the bound on the message set size to the payoff structure of the game.

JEL Classification Number: C720, D830.

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1   Introduction

Introducing payoff irrelevant strategies into a game does not destroy the existing equilibria, therefore cheap talk per se does not select equilibria from the equilibrium set

of the game without communication. It rather creates new equilibria that are payoff equivalent to the existing ones, and supported by different beliefs about the meaning of the messages. Does cheap talk give evolutionary support to the play of the efficient equilibria?

In this paper, we address the evolution of communication via cheap talk in 2×2 symmetric coordination games. We link the evolution of meaningful messages to the payoff structure of the game and extend hereby the equilibrium selection in evolutionary games studied by Kandori, Mailath and Rob [13] (henceforth KMR) in the presence of cheap talk communication.

There have been identified in the literature conditions under which adding cheap talk selects efficient outcomes in evolutionary models. Common interest[1] in two player games has been found sufficient and necessary condition in multi-population models with best response dynamics, or static solution concepts based on the Nash equilibrium concept. Sobel [24] first showed that for finite two-player games with common interest, for every efficient outcome there is a strategy profile supporting this outcome in the evolutionary stable set. Matsui [16] introduced a set valued concept, cyclically stable set[2]. In [17], he shows for the class of 2×2 common interest games in a two population scenario that CSS contains only states yielding the efficient equilibrium.

The multipopulation modelling is a crucial assumption. Schlag [23] shows that with two-population replicator dynamics, common interest is necessary and sufficient for efficient stable outcomes. With one population dynamics, common interest implies that efficient outcomes are stable, but inefficient evolutionary stable strategies may persist.

The most general result for a multipopulation dynamics is presented by Kim and Sobel [15]. Two finite populations play asymmetric $n \times n$ game. The strategy updating process followed by the individuals is rather restrictive. At any occasion, a single individual is given the opportunity to change the current strategy for a strategy that weakly improves upon this strategy. Any better performing strategy is chosen with positive probability. Strategy updates continue even if all strategies perform equally well, in order to avoid lock in suboptimal states.

Under this dynamics, two-sided communication leads to the efficient outcome in games

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[1]Denote by e_i the maximal payoff player i can achieve in the underlying game G, i.e. $e_i = \max_{s_i, s_j} u_i(s_i, s_j)$, where $u_i(s_i, s_j)$ is the payoff to player i when he chooses strategy s_i and the other player chooses strategy s_j. Game G is a game of common interest if $u_i(s_i, s_j) = e_i \Leftrightarrow u_j(s_j, s_i) = e_j$.

[2]A collection of strategies forms a cyclically stable set (CSS) if it is a set closed under the best response dynamics, and any two members of CSS are mutually accessible by a path generated by the best response dynamics in CSS.

with common interest. For general games, all efficient outcomes recur infinitely often. One-sided communication always leads to the preferred outcome of the player sending the message, if it is part of a strict Nash equilibrium.

Kim and Sobel [15] consider an environment without mistakes or mutations, and therefore cannot address two important aspects: effect of noise in communication, and the role of simultaneous strategy adjustments. The first issue is handled by Bhaskar [3], the second by Blume [4].

Bhaskar [3] considers noisy communication in finite, possibly asymmetric games, played as a truly asymmetric contest. With a strictly positive probability, the message received by a player does not coincide with the message sent by the other player. The exchanged messages are thus not common knowledge, and successful strategies have to be resistant to the noise. When allowing countably infinitely many messages, a noise robust Nash equilibrium, a limit of Nash equilibria in games with the noise level converging to zero, is efficient.

Blume [4] assumes that members of two populations update strategies simultaneously according to a best response dynamics with incomplete sampling from the currently used strategies. With one-sided communication, the author finds an upper bound on the message set size so that communication is effective in selecting the preferred outcome of the sender population as the only outcome in the set closed under best responses. If the risk measure of the preferred equilibrium increases, the minimal required number of messages increases as well. In $2 \pounds 2$ symmetric games, the appropriate risk measure is Harsanyi and Selten's risk dominance.

With both-sided communication, Blume finds no upper bound on message set size connected to the risk of the preferred equilibrium of senders, so that the efficient outcome is stable under the evolutionary best response dynamics. When the symmetry of the messages is broken, the efficiency result is recovered. In particular, Blume assumes that for any strict Nash equilibrium in the underlying game, there is one message exogenously linked to it. If players exchange the messages linked to that equilibrium, an individual linking the message to the equilibrium receives a small additional payoff boost. Hence, a priori information content is assigned to particular messages in an equilibrium. The payoff boosts exclude the possibility of an unrestricted drift. Once the efficient equilibrium is played. players are locked in using the message connected to that equilibrium. Efficiency is achieved disregarding the message space size and risk.

We introduce a few important relaxations compared to Kim and Sobel [15] and Blume [4]. Players are allowed to update their strategies simultaneously. Strategies performing

worse in the current population than the present strategies can enter the population via mutations. We study two-sided communication, but do not restrict the meaning of the messages. While Blume [4] uses the assumption that players "recognize" meaningful communication if the messages form a strict Nash equilibrium, we let the players who behave according to this assumption compete with players who ignore the messages and players who use di¤erent messages to "recognize" the same equilibrium. Hence, the communicating individuals we consider behave as if the messages had some pre-speci...ed a priori information content, and we study under which conditions these individuals survive the pressure of the imitation dynamics. There is no payo¤ boost connected to any message/strategy combination.

The adaptation dynamics is driven by imitation. Moreover, we choose a single-population evolutionary dynamics, in order to be able to confront selection criteria based on risk with the presence of cheap talk communication.

The time runs in discrete steps, and in one period, an individual is randomly re-matched in a round robing fashion to play the game against all other individuals in the population. Each individual is preprogrammed to play a ...xed strategy during one time period. At the end of any period, all individuals imitate the most successful strategies, that earned the highest average payo¤ in that period. Occasionally, an individual makes a mistake and chooses another strategy.

The one shot game consists of a communication stage and actual strategy choice in the coordination game. In the communication stage, both players send simultaneously a message from the message set. A strategy of an individual is the message sent in the communication stage, and a map from the exchanged message pair to the action set of the $2 \pounds 2$ coordination game.

We restrict the set of strategies to fully "communicating" and "not communicating" strategies. A strategy is a fully communicating strategy if it assigns probability one to one message in the communication stage, and it conditions the action taken in the underlying game on the sent and received messages. In particular, a communicating strategy assigns in the underlying game probability one to the action corresponding to the payo¤ dominant equilibrium whenever the message received in the communication stage is the same as the message sent. Otherwise, a communicating strategy assigns in underlying game probability one to the action corresponding to the payo¤ dominated equilibrium.

Hence, a communicating strategy, prescribing to sent message m in the communication stage, leads to behavior that can be interpreted in the following way. Individual using

such a strategy behaves as if the message m indicates the intention to play the payoff dominant equilibrium and all individuals in the population share the belief that message m indicates the intention to play the payoff dominant equilibrium.

A strategy is not communicating if it assigns a positive probability to any message in the communication stage, and it does not condition the action taken in the underlying game on the sent and received messages. This amounts to "babbling" in the communication stage.

We assume that an individual cannot recognize the strategy of the individual he is matched to. The babbling of the noncommunicating individuals therefore creates noise for the "communicating" individuals. The "as if" type of behavior of an individual using a communicating strategy when matched to an individual using a noncommunicating strategy may lead to an out-of-equilibrium play.

We show that under these conditions, the stochastically stable states do not always lead to the play of the risk dominant equilibrium, as it is in the model without communication. For coordination games where the risk dominant equilibrium is not payoff dominant, there are games where the message set with two messages is large enough for the communicating player type to turn the population dynamics towards the play of the efficient equilibrium in the long run, but also games where the number of messages must be considerably higher to achieve this effect. We are able to link the noise in the communication stage, represented by the number of messages in the message set, and riskiness of the payoff dominant equilibrium to the efficient equilibrium play in the long run. In particular, we show that when the message space is small, the result of KMR prevails and the inefficient equilibrium will be played in the long run if it is risk dominant. Otherwise, efficient equilibrium play will be the long run outcome. Increasing the number of messages makes the survival of communicating strategies more likely. This is in contrast to Kim and Sobel's [15] results based on gradual evolutionary dynamics, where there is no connection between the message set size and stability of communication as soon as the message set contains at least two messages, or Blume's [4] result where messages are assigned meaning exogenously.

In order to assess the importance of the uniform babbling assumption, we also investigate a model restricting the message set to two messages such that one message is more informative in the sense of being used with lower probability by the noncommunicating individuals.

The remainder of the paper is organized as follows. In Section 2 we present the model and the solution concept. In Section 3, we analyze communication when the message

	A	B
A	$(a; a)$	$(b; c)$
B	$(c; b)$	$(d; d)$

Figure 1: The underlying game G

set consists of at least two messages and noncommunicating players babble uniformly. We identify the set of long run outcomes of the imitation dynamics depending on the number of messages in the message set. In Section 4, we restrict the message set to two messages and consider noncommunicating strategies that use one of the messages with a smaller probability than the other message. Hence, we introduce exogenously some asymmetry into the message set. Both in Section 3 and Section 4, we investigate under what conditions the efficient equilibrium is played by the population in the long run. Section 5 concludes. The proofs are given in the Appendix.

2 The model

Consider a finite population consisting of N individuals. N is even, to avoid an unmatched individual at any moment. Time runs in discrete steps (periods). In every period, any individual in the population is sequentially anonymously matched to any other individual in the population to play the symmetric two-player game G in Figure 1: So, in one period, an individual plays the game G to every other individual in the population exactly once. In each of the games an individual earns payoff according to the payoff matrix in Figure 1, and the relevant evolutionary fitness of the individual is the average payoff earned in one period. Before the begin of the next period, all individuals update their strategies simultaneously and independently by imitating currently the most successful strategy.

We focus on games with strictly positive payoffs satisfying $a > c; d > b; a > d$; i.e. games with two strict Nash equilibria, one of which, $(A; A)$, payoff dominates the other equilibrium, $(B; B)$. Payoff dominance of $(A; A)$ and the assumption that $(B; B)$ is an equilibrium implies $a > d > b$; i.e. $a > b$: If, additionally, $(A; A)$ is dominated in risk by $(B; B)$ as defined by Harsanyi and Selten [12], then $a_i\ c < d_i\ b$: For games where this inequality is satisfied, moreover, $0 < a_i\ d < c_i\ b$ implies here $c > b$.

The underlying game G is preceded by a communication stage. Two matched individuals send simultaneously to each other a message from a finite set M with cardinality m: Each player in a match observes the message sent by the other player. Then both players

simultaneously choose an action in the underlying game G: We denote the game with a communication stage by G^M: The messages in G^M are cheap talk because the identity of the message that is sent in the communication stage does not a¤ect per se the payo¤s players earn in the underlying game.

Each individual is programmed to play a particular strategy in G^M: A strategy in G^M is a message sent in the communication stage, and a mapping f from the product set of the messages sent and the messages received to the strategy set of G; $f : M £ M ! fA; Bg$: Individual's strategy is his type. We restrict the set of feasible types to a subset X of the set of all feasible types.

In general, we consider two classes of types - communicating and noncommunicating.

A noncommunicating type sends any available message with positive probability and chooses a ...xed action in the underlying game. We consider ...rst uniform babbling where all available messages are sent with equal probability. Later, restricting the message set to two messages, we consider also a restriction where one message is sent by the noncommunicating types with a lower probability than the other message.

A communicating type sends a unique message as a signal to play the e¢cient equilibrium. For any message available in the message set, there is one type who sends that message in the communication stage. The communicating types also di¤er from the noncommunicating types in the action choice in the underlying game. A communicating type chooses action A in the underlying game if the message he received coincides with the message he sent, otherwise he chooses action B:

The population state is described by a vector $z \in Z \subset N^{jXj}$ with components z_x denoting the number of individuals of type $x \in X$ in the population. $z_x \, \, 0$ for all $x \in X$ and $\sum_{x \in X} z_x = N$: An individual of type x earns in one period of the game G^M a payo¤ $\frac{1}{4}_x(z)$. It depends on the current population state z and on the individual's type x, but not on individual's name. Let us denote by $\frac{1}{4}(x; y)$ the payo¤ earned by type x; if he is matched to type y; in the one-shot game G^M: Then $\frac{1}{4}_x(z)$ can be written as

$$\frac{1}{4}_x(z) = \frac{1}{N \, \text{\i} \, 1} \left(\sum_{y \in X} z_y \frac{1}{4}(x; y) \, \text{\i} \, \frac{1}{4}(x; x) \right) :$$

The payo¤ function $\frac{1}{4}_x(z)$ summarizes how well type $x \in X$ is performing in state z: We now postulate a dynamic process on the set of population states z; derived from individuals' behavior.

2.1   Imitation dynamics

We assume that the individual behavior is driven by imitation of types that currently perform best in the population. We index by t the population state in period t: Denote by $W(z_t) \mu X$ the set of types who earn in period t the highest average payo¤s, $W(z_t)$ ´ $\arg \max_{y2X} \frac{1}{4}_y(z_t)$:

The imitation dynamics is such that if $z_{t;x} > 0$ and $x 2 W(z_t)$ then $z_{t+1;x}$ ¸ 0 otherwise $z_{t+1;x} = 0$: In other words, if there is a unique type in the population at time t which achieves a maximal average payo¤ in the population; then it is the only type that will be present in the population in time period $t + 1$: If several types achieve maximal average payo¤ in time t; then any population composition where all or at least one of these types are present is achieved in time $t + 1$ with positive probability. We will not specify the imitation dynamics in detail in this case. Later we show that the speci...cation does not a¤ect our conclusions because nonmonomorphic states are never the long run outcome of the imitation dynamics.

The imitation dynamics generates a Markov chain on the ...nite state space Z and we let $P = (p_{zz^0})_{z;z^02Z}$ denote the transition matrix where p_{zz^0} is the probability that the state z^0 is reached from the initial state z via imitation dynamics:

2.2   Solution concept

We introduce now some concepts and results we use to solve for the long run equilibria of our model.

De...**nition 1** The vector ¸ = (¸₁;::::; ¸jZj), ¸ᵢ 2 [0; 1] and $\overset{P}{_i}$ ¸ᵢ = 1 is a stationary distribution over the states in Z of the process P if ¸P = ¸:

A state that is assigned probability 1 in a stationary distribution is called a stationary state. Trivially, all monomorphic states where $z_x = N$ for some $x 2 X$ are stationary under the imitation dynamics.

In general, the Markov process derived from the imitation dynamics can have several stationary distributions and the initial state of the population determines which stationary distribution will be reached in the long run. The intriguing observation in the literature on stochastic evolutionary dynamics is that if the individuals' behavior is perturbed by allowing mistakes or experimentation then the long run behavior does not depend on the initial conditions.

We take this approach to guarantee path independence of the long run outcomes and perturb the process P in the following way. Each time period, an individual updates his current type to a new type by imitating the currently most successful type(s) with probability $1 - \varepsilon$; and updates to any type with probability $\varepsilon > 0$. Let this perturbed dynamics be described by a transition matrix $P^\varepsilon = (p^\varepsilon_{zz'})_{z;z' \in Z}$: Now, $p^\varepsilon_{zz'} > 0$ for any $z; z'$: The Markov process associated with P^ε is irreducible. It is a standard result in the theory of Markov processes[3] that P^ε has a unique stationary distribution μ^ε being

   i) stable: for any $\mu = (\mu_1;::::; \mu_{|Z|})$; $\mu_i \in [0; 1]$ and $\sum_i \mu_i = 1$; $\mu P^t \to \mu^\varepsilon$ as $t \to 1$

   ii) ergodic: for any initial state z; $\frac{1}{T} \sum_{t=1}^{T} z_t \to \mu^\varepsilon_z$ almost surely as $T \to 1$

Foster and Young [10] introduce the concept of stochastic stability.

De...nition 2 The stochastically stable distribution μ^* of the process associated with the transition matrix P^ε; is $\mu^* = \lim_{\varepsilon \to 0} \mu^\varepsilon$:

Ultimately, we look for the states that will be observed in the long run with positive probability, which are the stochastically stable states.

De...nition 3 The set of stochastically stable states SSS is given by SSS = $\{z \in Z | \mu^*_z > 0\}$.

To characterize the limit distribution μ^* of the perturbed dynamics we apply the method of directed graphs on the state space and mutation counting method by Freidlin and Wentzell [11], and introduced into economics in KMR [13] and Young [26]. Now, we introduce some preliminaries of the tools applied later.

De...nition 4 For $z \in Z$; a z-tree T is a set of ordered pairs $(z'; z'')$, $z'; z'' \in Z$; such that $\forall z_i \in Z; z_i \neq z$; there is only one pair $(z'; z'') \in T$ such that $z' = z_i$, and from every state $x \in Z; x \neq z$; there is a sequence of pairs $(z_0; z_1); (z_1; z_2); ::::; (z_{k-1}; z_k)$ such that $z_0 = x$ and $z_k = z$:

A z-tree is a tree on the state space Z with the root at the state $z \in Z$; and a unique directed path without cycles from any state $z' \in Z; z' \neq z$, to the root z:
Denote by T_z the set of all z-trees: To every state z, we assign a measure of the transition probability along all trees of this state. This number, denoted by q^ε_z; is the sum of

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[3] A classical reference to introduction into theory of Markov processes is the text by Karlin and Taylor [14].

transition probabilities along all z-trees in T_z. The transition probability along a z-tree is the product of the transition probabilities associated with the pairs of states creating the tree, i.e. $q_z^\varepsilon = \prod_{T \in T_z} \prod_{(z';z'') \in T} p_{z';z''}^\varepsilon$:

Individuals update their types independently, therefore this expression is a polynomial in ε: For a tree $T \in T_z$, we denote by $c_z(T)$ the lowest order of the polynomial summarizing the cost of tree T, and by c_z the order of a z- tree with the lowest order polynomial, i.e. $c_z = \min_{T \in T_z} c_z(T)$: Also, let a_z be the coefficient of the term ε^{c_z} in a minimum cost z-tree.

Freidlin and Wentzell [11] prove the following.

Lemma 5 Let be given a Markov chain with a set of states Z and transition probabilities $p_{zz'}$ for $z; z' \in Z$, and assume that every state can be reached from any other state in a finite number of steps. Then the stationary distribution of the chain is the vector

$$\left\{ \frac{q_z^\varepsilon}{\sum_{z' \in Z} q_{z'}^\varepsilon} \right\}_{z \in Z}.$$

The following theorem (Theorem 1 in KMR [13], p. 42) utilizes the polynomial form of q_z^ε to describe the set of stochastically stable states.

Theorem 6 The limit distribution μ^* exists and is unique. In particular, $\mu_z^* = \frac{a_z}{\sum_{x \in Z} a_x}$; $z \in Z$; and the set of stochastically stable states is given by $\arg\min_{x \in Z} c_x$:

Stochastically stable states are the states with the lowest order in the polynomial q_z^ε. We show that c_z is the minimal number of mutations needed to construct a minimum cost tree of state z:

First, let us consider a transition probability $p_{zz'}$ between two states $z; z' \in Z$ under the perturbed dynamics.

Lemma 7 $p_{zz'}$ is a polynomial in ε of order $c_{zz'} = \sum_{x \in W(z)} Z_x^{z'}$:

Proof. Denote by $c_{zz'}$ the minimal number of individuals that have to update their type via mutation so that z' can be reached from z under the perturbed imitation dynamics. These are all the individuals of types present in the state z' that would not be imitated under the state z; i.e. $c_{zz'} = \sum_{x \in W(z)} Z_x^{z'}$.

All individuals in the population update their type according to the perturbed imitation dynamics independently, therefore any transition from state z to z' that requires k mutations will be realized with probability $(1 - \varepsilon)^{N-k} \varepsilon^k = (1 - \varepsilon + \cdots + (-1)^{N-k} \varepsilon^{N-k}) \varepsilon^k$: Denote by $B_{zz'}(k)$ the number of mutually exclusive events that require k mutations so

that state z^0 is reached from state z: Then, $p''_{zz^0} = \sum_{k=c_{zz^0}}^{N} B_{zz^0}(k) (1 - \varepsilon)^{N-k} \varepsilon^k$: The leading term of p''_{zz^0} is of order c_{zz^0} with coefficient $B_{zz^0}(c_{zz^0})$: ∎

The coefficient $B_{zz^0}(c_{zz^0})$ is the product of two combinatorial expressions, one quantifying the number of times we can sample c_{zz^0} individuals from the population described by z so that state z^0 can be reached performing necessary mutations by the sampled individuals, and another quantifying the number of times we can allocate these mutating individuals to the types so that ...nal state z^0 is reached.

Consequently, q''_z as a product of transitions $p_{z^0z^{00}}$ along a z-tree is polynomial in ε with the smallest power to the mutation probability ε equal to the sum of mutations needed to achieve transitions between pairs of states in some z-tree T; and so $c_z(T) = \sum_{(z^0;z^{00})2T} c_{z^0z^{00}}$: The lowest order of a z-tree T is the total mutation cost of this tree: The state(s) with a tree that uses the smallest number of mutations among all minimum cost trees are the stochastically stable states. In this way the search for the set of stochastically stable states reduces to the search of states for which we can build a tree using the smallest number of mutations to achieve transitions between the pairs of states forming arcs of the tree.

We now prove two lemmas that simplify this task for our imitation dynamics. Both proofs are based on a tree cutting procedure which is often used in the literature to provide counterexamples to minimal cost trees, see Young [26] and Levine and Pesendorfer [18]. If we "cut" a tree at a certain point by eliminating one arc, the set of nodes of the tree is divided into two subsets, and the structure induced on each of these subsets by the pairs of nodes as in the original tree is a tree as well. A new tree can now be created by adding an arc from the root of the original tree to a new root of the new tree created by this cutting procedure. The proofs can be found in Appendix 3.

Lemma 8 Denote the set of stationary states under the unperturbed dynamics P by F: Then, $z 2 SSS$ implies $z 2 F$:

According to this lemma, when looking for states with the cost of minimum cost tree that is smallest among all states we may focus on trees constructed on the restricted set of states F that are stationary under the unperturbed dynamics. A cost of transition between any two states in F is given by the number of individuals that have to mutate so that the transition takes place.

We can also eliminate from among the candidates for stochastically stable states those stationary states which are not monomorphic. The following proposition states that the mutation cost of a minimum cost tree for some state $z 2 Z$ is minimized at a monomorphic state. The proof is again based on a tree cutting procedure.

Proposition 9 If $z \in Z$ is such that for some $x \in X$, $z_x \in (0, 1)$, then there exists $z' \neq z$, $z' \in Z$ so that $c_{z'} < c_z$.

The least costly way to incorporate a stationary mixed population state into a minimum cost tree of a monomorphic stationary state is to introduce sufficient number of mutations to achieve this mixed state from one of the monomorphic states, and then a single mutation leads to another monomorphic state. Travelling back and forth between monomorphic states via the mixed states is the cheapest way to incorporate the mixed states into the minimum cost tree at an additional cost of one mutation.

Moreover, any monomorphic state tree has to include the same nodes corresponding to the mixed stationary states (if any), hence these additional mutations costs add up to the same number in any minimum cost tree of a monomorphic state. We can therefore simplify the search for stochastically stable states by constructing minimum cost on trees on the set of monomorphic states only.

3   Uniform babbling and a finite number of messages

Let us now assume that there is a finite number of messages, $m \geq 2$. For each message, there is one communicating type who sends this message in the communication stage, and chooses the action corresponding to the efficient equilibrium after receiving this message. Otherwise, the type chooses the action corresponding to the other strict Nash equilibrium. So, a communicating type behaves as if the population members shared information content of messages, and exactly the message he sends has the meaning "let's play the efficient equilibrium".

There are two noncommunicating types, programmed to choose a fixed action, A or B, in the coordination game in Figure 1. These noncommunicating type send each message with equal probability $\frac{1}{m}$. Therefore we call this form of "noncommunication" uniform babbling.

We show in this case that the set of outcomes implied by the stochastically stable states is not always robust to the introduction of cheap talk. When there are sufficiently many messages in the message set, then the outcome of the Kandori, Mailath and Rob's model without communication, the play of the risk dominant equilibrium, does not coincide with the outcome we derive, the play of the efficient equilibrium. The efficiency, however, is not achieved if the message set is small. The size of the message set is measured relative to a number that is derived from the payoff structure of the game. These results hold under the assumption that the population size is large, as specified later.

The set of types $X \equiv \{A, B, C_1, ..., C_m\}$. The noncommunicating types are denoted by the action they choose in the underlying game G as A and B; and the m communicating types are denoted by C_i where index i denotes the i-th message in the set M. Let us denote an arbitrary communicating state by M: The state of the world at any time is the population composition $z = (z_A, z_B, z_{C_1}, ..., z_{C_m})$; where z_x is the number of individuals of type $x \in X$: Z is the set of all feasible population states, $Z = \{z \in N^{m+2} | z_A + z_B + \sum_{i=1}^{m} z_{C_i} = N\}$. We refer to a monomorphic state with $z_x = N$ for some $x \in X$ as state x. If $x \in \{A, B\}$; the state is a noncommunicating state, and if $x \in \{C_1, ..., C_m\}$, the state is a communicating state.

When the population state is z; the payoff to the player type $x \in X$ is equal to:

$$\pi_A(z) = \left[a(z_A - 1) + bz_B + \sum_{k=1}^{n} \frac{a + (m - 1)b}{m} z_{Ck} \right] \frac{1}{N - 1}$$

$$\pi_B(z) = \left[cz_A + d(z_B - 1) + \sum_{k=1}^{n} \frac{c + (m - 1)d}{m} z_{Ck} \right] \frac{1}{N - 1}$$

$$\pi_{C_i}(z) = \left[\frac{a + (m - 1)c}{m} z_A + \frac{b + (m - 1)d}{m} z_B + (z_{C_k} - 1)a + \sum_{k \neq i; k=1}^{n} z_{Ck} d \right] \frac{1}{N - 1}:$$

For illustration, we now ...rst consider the stochastically stable states in populations where the type set is restricted to two types, i.e. the dynamics operates on a face of the state space allowing all player types in X.

To ...nd the state with the minimal minimum cost tree under the imitation dynamics when only two types are considered we have to identify which of the two monomorphic states can be easier reached via mutations from the other monomorphic state.

For any two states $x, y \in X$; let c_{xy} be the number of mutations needed to reach state y from state x under the perturbed imitation dynamics: c_{xy} can be easily calculated as the minimal k for $0 < k < N$; such that $\pi_y(z(k)) \geq \pi_x(z(k))$ where $z(k)$ is a state such that $z_y(k) = k$ and $z_x(k) = N - k$: It is the minimal number of individuals needed to change type x to type y in a population originally composed only of type x player types so that the type y is imitated by all players in the next period.

Two "noncommunicating" types A and B : Kandori, Mailath and Rob [13] show for the best response dynamics and uniform matching that if the population consists only of noncommunicating player types A and B. The monomorphic state in which all players play the risk dominant equilibrium strategy is the only stochastically stable state. For the class of games considered here, the same obtains under

the imitation dynamics. We find that $c_{AB} = dN \frac{a-c}{a-c+d-b}e$ and $c_{BA} = dN \frac{d-b}{a-c+d-b}e$; so that $c_{AB} > c_{BA}$ iff $a-c > d-b$: If $a-c < d-b$; $(A; A)$ is the payoff dominant equilibrium but it is dominated in risk by the equilibrium $(B; B)$: In this case, $c_{AB} < c_{BA}$. It follows that state B is the stochastically stable state rather than state A when the payoff dominant equilibrium $(A; A)$ is not risk dominant.

Two "communicating" types M and M^0 : In a population consisting of two communicating types, individuals always arrive at equilibrium outcomes. When two individuals of the same type are matched, they coordinate on the payoff dominant equilibrium, and earn payoff a: When two types using different messages are matched, they earn payoff $d < a$. As soon as one communicating type is more likely to be matched with an individual of his own type than with the other type, this type earns a higher payoff. So, $c_{MM^0} = c_{M^0M} = d\frac{N}{2}e$, and both communicating types are stochastically stable.

The "noncommunicating" type A and a "communicating" type M : The transition costs between the states A and M are $c_{AM} = dN \frac{a-c}{a-c+a-b}e$ and $c_{MA} = dN \frac{a-b}{a-c+a-b}e$, so that $c_{AM} > c_{MA}$ iff $c < b$: The message set size does not play a role here. Individuals of type A and M earn the same payoff, a; always when matched their own type, and $\frac{1}{m}$ times when matched with the other type. The payoffs of type A and M differ in matches where they are matched together when the noncommunicating type A sends a message that is not the message used by the communicating type, what happens with probability $1 - \frac{1}{m}$: Then they arrive at a disequilibrium outcome. The stochastic stability of the noncommunicating versus communicating type depends on who is punished at this disequilibrium outcome. If $c < b$; then out of equilibrium choosing action B is relatively worse than choosing action A: In this case, it is harder to disrupt the monomorphic state A than the monomorphic state M: The communicating types are not stochastically stable, and their responsiveness to the messages exchanged in the presence of types unconditionally choosing the action of the efficient equilibrium is harming their survival chances. On the other hand, if $c > b$; choosing B is rewarded at the disequilibrium outcome relatively to choosing A, and it is harder to disrupt the monomorphic state M than the monomorphic state A:

The "noncommunicating" type B and "a communicating" type M : This leads to a handshake model in the spirit of Robson's model [20]. There is one type who ignores the message sent and received in the communication stage, and al-

ways plays the inefficient strategy. The other type conditions own behavior on the messages, and plays the efficient strategy if receiving a proper message, the one he sends. Robson assumes that the communicating types can recognize the player type they are matched to by the message received, and hence play an efficient strategy only if matched to own type. Unlike in Robson's model, in our model the communicating player cannot distinguish the type of the individual he is matched to with probability 1: Type B matched to his own type earns a payoff d; while the communicating type matched to his own type earns a payoff $a > d$. When an individual of a communicating type is matched to an individual of type B; $\frac{m-1}{m}$ times they receive the same payoff d and $\frac{1}{m}$ times the disequilibrium payoffs b and c, respectively: Hence, both the equilibrium premium of choosing A; $a - d$; and the disequilibrium premium of choosing A; $b - c$; which may be negative; will play a role at determining which type will be more successful in the population. We calculate $c_{MB} = dN\left[\frac{m(a-d)+d-c}{m(a-d)+d-c+d-b} - \frac{m(a-d)}{m(a-d)+2d-b-c}\right]$ and $c_{BM} = dN\left[\frac{d-b}{m(a-d)+2d-b-c} + \frac{m(a-d)}{m(a-d)+2d-b-c}\right]$; where $m(a-d) + 2d - b - c > 0$ for any $m > 1$ as $m > 1 > \frac{b+c-2d}{a-d}$ due to the assumption that $(A;A)$ and $(B;B)$ are both Nash equilibria, i.e. $b - d < 0 < a - c$: Adding and substracting the term $\frac{m(a-d)}{m(a-d)+2d-b-c}$ in transition costs between state B and M does not affect the set of stochastically stable states if N is large enough. The role of the size of the message set is evident from the dependence of transition costs between states M and B on m: As the number of available messages increases, the mutation cost of transition from the communicating state M to the state B converges to $N - 1$ and vice versa the cost of leaving the state B to state M can be made very "cheap" in terms of mutations, $c_{MB} > c_{BM}$, $m > \frac{c-b}{a-d}$: So, the communicating state will be stochastically stable and the efficient equilibrium will be the outcome of the perturbed imitation dynamics in the long run, when the mutation probability converges to zero, if there are sufficiently many messages in the message set. Moreover when $c < b$; there are always sufficiently many messages. The condition on the message set size can be binding only if $c > b$:

As we have shown, some transition costs between the monomorphic states depend on the number of messages. Hence, we may expect that also the minimum cost trees for states on the set of all states will depend on m: Intuitively, increasing the message space size makes the type B less successful in the presence of communicating player types. The more messages has the type B to randomize from, the lower is the probability that he "hits" the message interpreted by the communicating player type as a signal to play
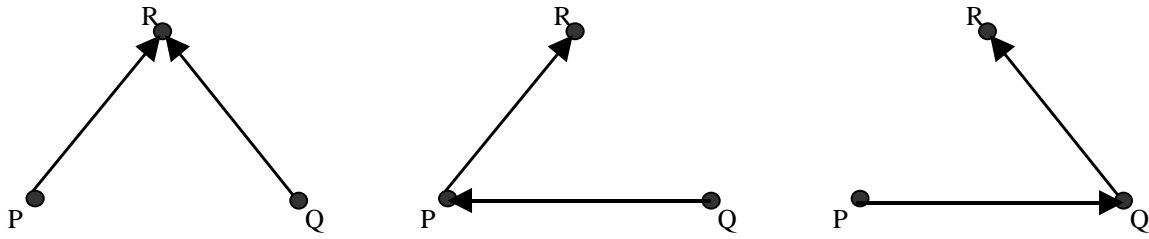
Figure 2: All R-trees on the set of nodes P,Q,R.

strategy A; i.e. the lower is the noise.

To ...nd the minimum cost tree for each state, we can use the symmetry of the problem with respect to the communicating states. The transition from or to any of the communicating states to or from any of the noncommunicating states does not depend on the index of the communicating state. Thus we will ...nd the minimum cost trees for the state space consisting of three monomorphic states A, B, M where M is one of the communicating monomorphic states and extend the resulting minimum cost trees by the m_i 1 edges such that each edge starts in one (yet unconnected) communicating state and ends either in another communicating state (in a way so that cycles do no arise), or in the state A; or in the state B; depending whether c_{MM^0}; c_{MA} or c_{MB}; respectively, is the smallest among these three transition costs. Due to the fact that the transition costs involving the communicating states are independent of the name of the communicating state, we may construct many trees with the same cost that di¤er only in the permutation of the order in which the communicating states are connected to the tree. All such trees will have the same cost.

The extension procedure preserves the tree structure of the tree built on the state space fA; B; Mg; and creates a tree at the state space with m communicating states connecting them at minimal cost an existing tree. Consequently, it is a minimum cost tree.

There are three directed rooted trees on the state space consisting of three states, see Figure 2. In this ...gure, R is the state which is at the root of the tree, and P and Q; are the remaining states.

From the payo¤ function $\frac{1}{4}_x(z)$ in the evolutionary game, we can calculate explicitly the mutation costs of transition between any two monomorphic states:

$$^2 \; c_{AB} = dN \frac{a_i \, c}{a_i \, c + d_i \, b} \; i \; \frac{a_i \, d}{a_i \, c + d_i \, b} e, \; c_{BA} = dN \frac{d_i \, b}{a_i \, c + d_i \, b} + \frac{a_i \, d}{a_i \, c + d_i \, b} e$$

$$^2 \; c_{MM^0} = d \frac{N}{2} e$$

[2] $c_{MA} = \lceil N \frac{a-b}{2a-b-c} \rceil$ and $c_{AM} = \lceil N \frac{a-c}{2a-b-c} \rceil$, where $2a-b-c > 0$ as $a-c > 0 > b-a$. The last inequality follows from the assumption that (A,A) payoff dominates (B,B); i.e. $a > d > b$ hence $a > b$.

[2] $c_{MB} = \lceil N \frac{m(a-d)+d-c}{m(a-d)+d-c+d-b} - \frac{m(a-d)}{m(a-d)+2d-b-c} \rceil$ and $c_{BM} = \lceil N \frac{d-b}{m(a-d)+2d-b-c} + \frac{m(a-d)}{m(a-d)+2d-b-c} \rceil$ where $m(a-d) + 2d - b - c > 0$ for any $m > 1$ as $m > 1 > \frac{b+c-2d}{a-d}$ due to the assumption that (A,A) and (B,B) are both Nash equilibria, i.e. $b-d < 0 < a-c$.

Let $c(x)$ be the cost of the minimum cost tree of a monomorphic state $x \in X$; i.e. the total number of mutations needed for transition along the minimum cost tree of the state x. The comparisons of the transition costs between monomorphic states in terms of mutations can be found in lemmas A1.1 to A1.7 in Appendix 1. We refer to these lemmas in the proofs of Propositions 10, 11 and 12.

We concentrate here on the results that are not driven by a small population size and asymmetries resulting in the matching protocol: in a population consisting of types x and y; type y is more likely to meet type x than types x himself, but this difference is of order $\frac{1}{N-1}$ so that as N increases, the matching asymmetry resulting payoffs diminish. More precisely, we assume, $N > \max\{\frac{2d-b-c}{d-b}, \frac{2a-b-c}{a-c}, \frac{2m(a-d)}{m(a-d)-(c-b)}, \frac{m(2a-b-c)}{m(a-b)+b-c}\} \equiv \hat{N}$. For any population size $N \geq \hat{N}$, we consider two cases: $c > b$ and $c < b$.

The first proposition uncovers the subclass of games where the result of KMR holds even in the presence of the communication stage. In these games, communication is not effective in selecting the efficient outcome.

Proposition 10 Consider the class of games G where $c > b$: If the message set is "small", i.e. M consists of $2 \leq m < \frac{c-b}{a-d}$ messages, and the noncommunicating players babble uniformly, then the limit distribution $\mu^* = (0,1,0,\ldots,0)$ where $\mu_z^* = 1$ for z such that $z_B = N$. Hence, the state B is the unique state in the set of stochastically stable states and the inefficient equilibrium play is the long run outcome of the perturbed imitation dynamics.

Proof. The restriction on the message set size $2 \leq m < \frac{c-b}{a-d}$ implies that the equilibrium (A,A) is the payoff dominant equilibrium, but not the risk dominant equilibrium. For $c > b$ and $m < \frac{c-b}{a-d}$, the minimum cost trees of the monomorphic states have the following costs, see Appendix 1: $c(A) = c_{BA} + mc_{MB}$; $c(B) = c_{AM} + mc_{MB}$; $c(M) = c_{AM} + c_{BM} + (m-1)mc_{MB}$. Now $c_{MB} < c_{BM}$; see (A1.1) in Appendix 1, therefore $c(B) < c(M)$. From $m(a-d) < c-b$ and $m \geq 2$ it follows that $a-c < d-b$. In

this case, $c_{AM} < c_{BA}$ because $(a_i c)^2_i (d_i b)(a_i b) < 0$; see (A1.5) in Appendix 1, therefore $c(B) < c(A)$: ∎

The second proposition states when communication via cheap talk leads to the e¢cient outcome in games for which the payo¤ dominant equilibrium is not risk dominant.

Proposition 11 Consider the class of games G where $c > b$: If the message set is "large", i.e. M consists of $m > \frac{c_i b}{a_i d}$ messages, and the noncommunicating players babble uniformly, then the limit distribution $,^{¤} = (0; ::::; 0; \frac{1}{m}; ::::; \frac{1}{m}; ::::; 0)$ where $,^{¤}_z = \frac{1}{m}$ for all $z \, 2 \, Z$ such that $z_x = N$ for $x \, 2 \, fC_1; ::::; C_mg$: Hence, all monomorphic communicating states C_i; $i = 1; 2; ::::; m$; are in the set of stochastically stable states and the e¢cient equilibrium play is the long run outcome of the perturbed imitation dynamics. Any of the messages attains the meaning of signalling the e¢cient equilibrium with the same probability $\frac{1}{m}$:

Proof. The restriction on the message set size $m > \frac{c_i b}{a_i d}$ allows games where $(A; A)$ is a payo¤ dominant and a risk dominant Nash equilibrium, or games where $(A; A)$ is payo¤ dominant, but not risk dominant equilibrium. For $c > b$ and $m > \frac{c_i b}{a_i d}$; the minimum cost trees of the monomorphic states have the following costs, see Appendix 1: $c(A) = c_{BM} + c_{MA} + (m_i 1)c_{MM}$; $c(B) = minf c_{AB} + c_{MA} + (m_i 1)c_{MM}$; $c_{AM} + c_{MB} + (m_i 1)c_{MM}g$; $c(M) = c_{AM} + c_{BM} + (m_i 1)c_{MM}$: Now, $c_{AM} < c_{MA}$; see (A1.2) in Appendix 1, therefore $c(M) < c(A)$: Moreover $c_{AM} + c_{BM} < c_{AB} + c_{MA}$ because $c_{AM} < c_{AB}$; see (A1.6) in Appendix 1, $c_{BM} < c_{MA}$; see (A1.7) in Appendix 1, and $c_{BM} < c_{MB}$, see (A1.1) in Appendix 1, therefore $c(M) < c(B)$. ∎

Finally, the noisy environment drives out the communicating player types in games where $(A; A)$ is the payo¤ and risk dominant equilibrium and choosing action A is not punished at the disequilibrium outcomes, i.e. $b > c$.

Proposition 12 Consider the class of games G where $c < b$: If the noncommunicating players babble uniformly; then the limit distribution $,^{¤} = (1; 0; ::::; 0)$ where $,^{¤}_z = 1$ for $z \, 2 \, Z$ such that $z_A = N$. Hence, the state A is the unique state in the set of stochastically stable states and the e¢cient equilibrium play is the long run outcome of the perturbed imitation dynamics.

Proof. The restriction on the payo¤ structure of the game G; $c < b$; implies that $(A; A)$ is a payo¤ and risk dominant Nash equilibrium of the underlying game. For $c < b$; the minimum cost trees of the monomorphic states have the following costs, see Appendix 1: $c(A) = c_{BM} + mc_{MA}$; $c(B) = c_{AB} + mc_{MA}$; $c(M) = c_{AM} + c_{BM} + (m_i 1)c_{MA}$:

It holds, $c_{MA} < c_{AM}$; see (A1.2) in Appendix 1, therefore $c(A) < c(M)$: Moreover, $c_{AM} + c_{BM} < c_{AB} + c_{MA}$ as $c_{AM} < c_{AB}$; see (A1.6) in Appendix 1, and $c_{BM} < c_{MA}$, see (A1.7) in Appendix 1, therefore $c(M) < c(B)$: ∎

We ...nd that if $c < b$; any message set with m ¸ 2 is "large enough" so that e₵cient outcomes are connected to the stochastically stable states. If $c > b$, the outcome of the stochastically stable states depends on the size of message space m. The relevant bound on the message set depends on how does the coordination premium of the e₵cient equilibrium, a ¡ d; compare to the payo¤s the players achieve at a disequilibrium outcome, c ¡ b. The more messages there are, the lower the coordination premium has to be so that the e₵cient equilibrium is the long run outcome of the dynamics.

4   Nonuniform babbling with two messages

Now, there are two messages in the message set and the noncommunicating types are not babbling "uniformly". They are more likely to send one of the available messages than the other. This introduces some exogenous di¤erentiation into the message set. The message that is sent less often by the noncommunicating types serves as a better signal for a communicating player type to identify when he is matched to his own player type. A limit case has been previously studies in the literature by Robson [20] who considers that there is one message that is not sent by the noncommunicating types at all. The communicating players can then use this message as a "secret handshake" to recognize each other, and coordinate always on the e₵cient equilibrium when playing to own type. Robson ...nds that the "communicating" types fare always weakly better than any noncommunicating type, and the e₵cient equilibrium is observed. Does this result obtain even when communicating players are not sending an exclusive message, but rather a message that is used with a low, but positive probability by the noncommunicating type?

A message is more reliable if the probability with which it is sent by noncommunicating types is lower. At one side of the modelling spectrum is Robson's model where the reliability of the mutant's message is one.

We might expect that the presence of messages with high reliability leads to the e₵cient outcome (A; A) in stochastically stable states - and this is indeed the case. Nevertheless, we will show that it is not always the case that only the communicating types using the more reliable message are stochastically stable. There is no selection for communicating types when the payo¤ dominant equilibrium is risk dominant. And, when risk
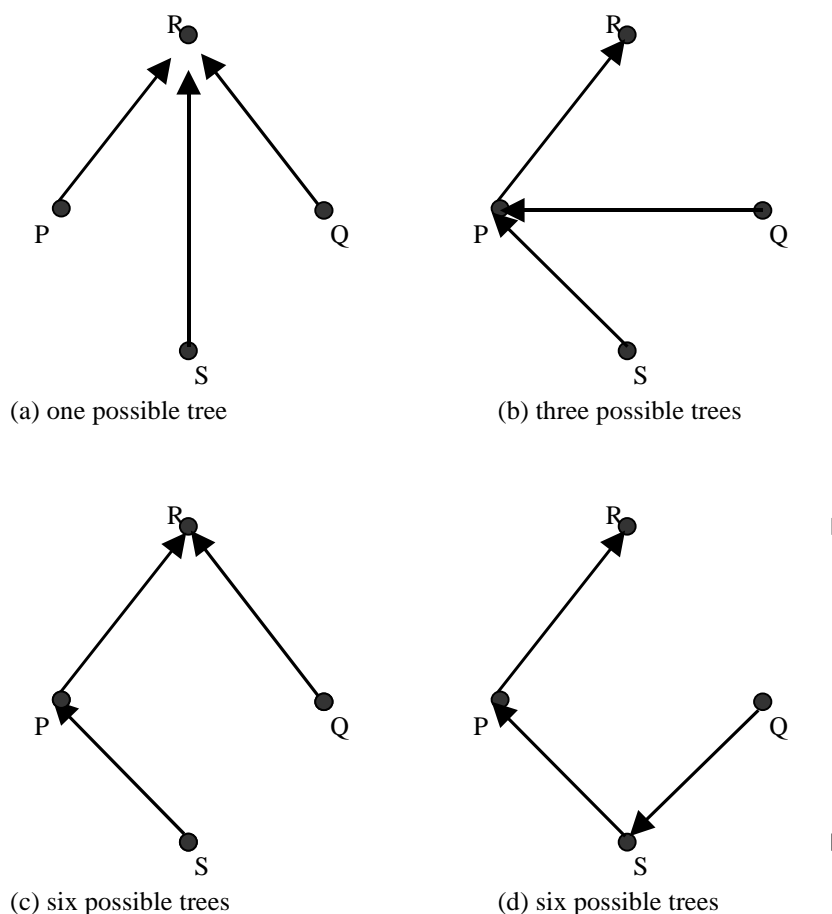
(a) one possible tree

(b) three possible trees

(c) six possible trees

(d) six possible trees

Figure 3: All R-trees on the set of nodes P,Q,R,S.

dominance selects a di¤erent equilibrium than payo¤ dominance, the reliability of one of the messages has to be high enough, so that the set of stochastically stable states no longer contains all the communicating player types and only the type using the more reliable message is stochastically stable. These observations are stated below in three propositions.

Let us now denote the communicating player type using the more reliable message sent by the noncommunicating types with probability $p \in [0; \frac{1}{2})$ by L; and the communicating player type using the other message by H:

We will look for minimum cost trees for each monomorphic state $x \in X = \{A; B; L; H\}$: There are sixteen directed rooted trees on the state space consisting of four states, see Figure 3. In this ...gure, R is the state which is at the root of the tree, and $P; Q;$ and S are the remaining three states in any order. We consider all combinations in trees (b) and (c) and all permutations in tree (d); to generate all the trees.

When the population state is z; the payoff to the player type $x \in X$ is equal to:

$$\pi_A(z) = [a(z_A - 1) + bz_B + (ap + b(1 - p))z_L + (bp + a(1 - p))z_H]\frac{1}{N - 1}$$

$$\pi_B(z) = [cz_A + d(z_B - 1) + (cp + d(1 - p))z_L + (dp + c(1 - p))z_H]\frac{1}{N - 1}$$

$$\pi_L(z) = [(ap + c(1 - p))z_A + (bp + d(1 - p))z_B + a(z_L - 1) + dz_H]\frac{1}{N - 1}$$

$$\pi_H(z) = [(cp + a(1 - p))z_A + (dp + b(1 - p))z_B + dz_L + a(z_H - 1)]\frac{1}{N - 1}:$$

From the payoff function $\pi_x(z)$ in the evolutionary game, we can calculate explicitly the mutation costs of transition between any two monomorphic states:

- $c_{AB} = \left\lceil N \frac{a-c}{a-c+d-b} - \frac{a-d}{a-c+d-b}\right\rceil$, $c_{BA} = \left\lceil N \frac{d-b}{a-c+d-b} + \frac{a-d}{a-c+d-b}\right\rceil$

- $c_{LH} = c_{HL} = \left\lceil d\frac{N}{2}\right\rceil$

- $c_{LA} = c_{HA} = \left\lceil N \frac{a-b}{2a-b-c}\right\rceil$ and $c_{AL} = c_{AH} = \left\lceil N \frac{a-c}{2a-b-c}\right\rceil$, where $2a - b - c > 0$ as $a - c > 0 > b - a$: The last inequality follows from the assumption that (A, A) payoff dominates (B, B); i.e. $a > d > b$ hence $a > b$:

- $c_{LB} = \left\lceil N \frac{a-d+p(d-c)}{a-d+p(2d-b-c)} - \frac{a-d}{a-d+p(2d-b-c)}\right\rceil$ and $c_{BL} = \left\lceil N \frac{p(d-b)}{a-d+p(2d-b-c)} + \frac{a-d}{a-d+p(2d-b-c)}\right\rceil$.
  As the reliability of the message L increases to 1; i.e. p decreases to 0; c_{BL} converges to 0; i.e. it becomes very "cheap" in terms of mutations to leave state B for state L:

- $c_{HB} = \left\lceil N \frac{a-c-p(d-c)}{a-c+d-b-p(2d-b-c)} - \frac{a-d}{a-c+d-b-p(2d-b-c)}\right\rceil$ and $c_{BH} = \left\lceil N \frac{(1-p)(d-b)}{a-c+d-b-p(2d-b-c)} + \frac{a-d}{a-c+d-b-p(2d-b-c)}\right\rceil$. As the reliability of the message L increases to 1; i.e. p decreases to 0; c_{BH} converges to c_{BA} and c_{HB} converges to c_{AB}.

$c(x)$ is the cost of the minimum cost tree of a monomorphic state $x \in X$; which it is the total number of mutations needed for transition along the minimum cost tree of the state x: The comparisons of the transition costs between monomorphic states in terms of mutations can be found in Lemmas (A2.1) to (A1.10) in Appendix 2. We refer to them in the Propositions 13, 14 and 15.

We concentrate here on the results that are not driven by a small population size and asymmetries resulting in the matching protocol and assume in the remainder of this subsection $N > \max\left\{\frac{2d-b-c}{d-b}; \frac{2a-b-c}{a-c}; \frac{2(a-d)}{a-c-(d-b)+p(c-b)}; \frac{(a-d)(2a-b-c)}{(a-c)(a-d)-p(c-b)(a-c+d-b)}\right\} \equiv \overline{N}:$

Proposition 13 Assume that the message set consists of two messages and the non-communicating player types send one of the messages with probability $p < \frac{1}{2}$: Consider the class of games G where $c > b$: If (i) (A; A) is both risk and payoff dominant Nash equilibrium of the underlying game G, or if (ii) (A; A) is payoff dominant and (B; B) is a risk dominant Nash equilibrium of the underlying game G; and the reliability of the message sent with probability $p < \frac{1}{2}$ by the noncommunicating player types is low, i.e. $p > \frac{d_i\ b_i\ (a_i\ c)}{c_i\ b}$; then the limit distribution $\mu^* = (0; ....; \mu_z^*; \mu_w^*; ....; 0)$ where $\mu_z^* > 0$; $\mu_w^* > 0$; $\mu_z^* + \mu_w^* = 1$; for $z; w \in Z$ such that $z_L = N$ and $w_H = N$: Hence, both states L and H are in the set of stochastically stable states and the efficient equilibrium play is the long run outcome of the perturbed imitation dynamics;

Proof. The minimum cost trees for states A; B; L; H have the following costs, see Appendix 2 : $c(L) = c(H) = c_{AL} + c_{BL} + c_{LH}$; $c(A) = \min\{c_{LA} + c_{BL} + c_{HB}; c_{BA} + c_{LH} + c_{HB}\}$; $c(B) = \min\{c_{AB} + c_{HA} + c_{LH}; c_{AH} + c_{HB} + c_{LH}\}$:
Neither of the two candidates for minimum cost B-trees has the minimal cost. On one hand, $c_{AH} + c_{HB} + c_{LH} > c_{AL} + c_{BL} + c_{LH}$ because $c_{HB} > c_{LH}$; see (A2.2), and $c_{LH} > c_{BL}$; see (A2.3); and on the other hand, $c_{AB} + c_{HA} + c_{LH} > c_{AL} + c_{BL} + c_{LH}$ because $c_{AB} > c_{AL}$; see (A2.5), $c_{LH} > c_{BL}$; see (A2.3), and $c_{HA} > c_{LH}$; see (A2.4). Hence, $c(B) > c(L)$:
Also, neither of the two candidates for minimum cost A-tree has minimal cost. On one hand, $c_{BA} + c_{LH} + c_{HB} > c_{AL} + c_{BL} + c_{LH}$ because $c_{BA} > c_{BL}$; see (A2.6) and $c_{LH} > c_{AL}$; see (A2.4) and $c_{HB} > c_{LH}$; see (A2.2); and on the other hand, $c_{LA} + c_{BL} + c_{HB} > c_{AL} + c_{BL} + c_{LH}$ because $c_{LA} > c_{AL}$ see (A2.4) and $c_{HB} > c_{LH}$ see (A2.2). Hence, $c(A) > c(L)$: The states with the minimal cost tree among all states are in this case the states H and L: ∎

Proposition 14 Assume that the message set consists of two messages and the non-communicating player types send one of the messages with probability $p < \frac{1}{2}$: Consider the class of games G where $c > b$: If (A; A) is payoff dominant equilibrium and (B; B) is risk dominant Nash equilibrium of the underlying game G; and the reliability of the message sent with probability $p < \frac{1}{2}$ by the noncommunicating player types is high, i.e. $p < \frac{d_i\ b_i\ (a_i\ c)}{c_i\ b}$; then the limit distribution $\mu^* = (0; ....; \mu_z^*; ....; 0)$ where $\mu_z^* = 1$ for $z \in Z$ such that $z_L = N$: Hence, then the state L is the unique state in the set of stochastically stable states and the efficient equilibrium play is the long run outcome of the perturbed imitation dynamics.

Proof. The minimum cost trees for states A; B; L; H have the following costs, see

Appendix 2: $c(L) = c_{HB} + c_{AH} + c_{BL}$; $c(H) = c_{AL} + c_{BL} + c_{LH}$; $c(A) = \min\{c_{LA} + c_{BL} + c_{HB}; c_{BA} + c_{LH} + c_{HB}\}$; $c(B) = \min\{c_{AB} + c_{HA} + c_{LH}; c_{AH} + c_{HB} + c_{LH}\}$:

Neither of the two candidates for minimum cost B-trees has minimal cost. On one hand, $c_{LA} + c_{BL} + c_{HB} > c_{HB} + c_{AH} + c_{BL}$ because $c_{LA} > c_{AH}$; see (A2.4); on other hand, $c_{BA} + c_{LH} + c_{HB} > c_{HB} + c_{AH} + c_{BL}$ because $c_{BA} > c_{BL}$; see (A2.6), and $c_{LH} > c_{AH}$; see (A2.4).

Also, neither of the two candidates for minimum cost A-trees has the minimal cost. On one hand, $c_{AB} + c_{HA} + c_{LH} > c_{AL} + c_{BL} + c_{LH}$ because $c_{AB} > c_{AL}$; see (A2.5), $c_{HA} > c_{LH}$; see (A2.4), and $c_{LH} > c_{BL}$; see (A2.3); and $c_{AH} + c_{HB} + c_{LH} > c_{HB} + c_{AH} + c_{BL}$ because $c_{LH} > c_{BL}$; see (A2.3).

Finally, $c(L) < c(H)$ because $c_{HB} < c_{LH}$; see (A2.2). State L has the minimal cost tree among all states. ∎

Proposition 15 Assume that the message set consists of two messages and the non-communicating player types send one of the messages with probability $p < \frac{1}{2}$: Consider the class of games G where $b > c$; then the limit distribution $\mu^\ast = (1; 0; ...; 0)$ where $\mu_z^\ast = 1$ for $z \in Z$ such that $z_A = N$. Hence, the state A is the unique state in the set of stochastically stable states and the efficient equilibrium play is the long run outcome of the perturbed imitation dynamics.

Proof. The condition $b > c$ implies that $(A; A)$ is payoff and risk dominant Nash equilibrium of the underlying game G: The minimum cost trees of the monomorphic states have the following costs, see Appendix 2 : $c(L) = c(H) = c_{AH} + c_{BL} + c_{LA}$; $c(A) = c_{LA} + c_{BL} + c_{HA}$; $c(B) = c_{LA} + c_{AB} + c_{HA}$:

Now, $c_{BL} < c_{BA}$; see (A2.6) in Appendix 2; and $c_{BA} < c_{AB}$ when $(A; A)$ is a payoff and risk dominant Nash equilibrium, hence $c(A) < c(B)$. Also, $c_{LA} < c_{AL}$ for $b > c$; hence $c(L) = c(H) > c(A)$: The minimal cost A-tree has the minimum cost of minimum trees of all states. ∎

We find that with two exogenously differentiated messages , communication always leads to efficient outcomes. If one of the messages is sufficiently reliable, then in the long run, we will observe with probability one that only individuals who use this message survive the evolutionary pressures. A unique message is selected by the dynamics as a signal of intention to play the efficient equilibrium.

5 Conclusion

In this paper, we study the evolution of communication in a model where messages have no exogenous meaning. The underlying game is a symmetric 2×2 coordination game and we assume that individuals of one population are anonymously and randomly matched in a round robin fashion to play the game. Most of the models considering evolution of communication via cheap talk avoid the own-population effects by assuming the roles to the players are assigned exogenously. Our assumption of single population complements the information on the role of the message set size and riskiness of the efficient equilibrium outcome to achieve efficiency via cheap talk communication.

We assume that there is a finite message set and the communication is noisy. Before the game playing stage, both players send simultaneously a message from a finite set of available messages. The noise is endogenously generated by allowing the presence of individuals who randomize among all messages in the message set. All individuals update their strategies simultaneously according to an imitation dynamics, imitating the player type with the highest average payoff.

We show that with uniform babbling, if risk dominance is not in conflict with payoff dominance, the efficient outcomes will be observed in the long run. When risk dominance selects a different equilibrium than payoff dominance, and the noncommunicating players are babbling uniformly sending any of the available messages with equal probability, then the efficient outcome depends on the number of messages available in the message set, i.e. on the level of noise generated by the noncommunicating player types. The higher is the number of messages, the lower is the noise. If the message set is large enough, $m > \frac{a-d}{c-b}$, the efficient equilibrium will be played in the long run, and any of the messages may attain the meaning of signalling the efficient equilibrium in the long run. Here, $a-d$ is the coordination premium of the efficient equilibrium, and $c-b$ is the disequilibrium premium from choosing the action corresponding to the risk dominant equilibrium. If the message set is small, $m < \frac{a-d}{c-b}$, the risk dominant equilibrium will be observed in the long run, and messages do not attain meaning.

In the first case, the efficient equilibrium is the outcome of the dynamics in the long run, as suggested by Robson's model without noise, and in the second case, the inefficient equilibrium is the outcome of the dynamics in the long run, as suggested KMR's model without communication. We connect these two results by assigning to them a level of noise in the population, given by the number of messages available to the noncommunicating players. The more messages, the lower is the noise in the population.

Also in the experimental literature, there is evidence collected that e¤ectivity of communication in coordination games depends on the payo¤ structure of the game. For example Battalio, Samuelson and van Huyck [1] ...nd that subjects are more likely to coordinate on the e¢ cient equilibrium in a 2x2 game if the coordination premium, compared to the other equilibrium, is higher[4].

Then we consider a message set containing only two messages and introduce exogenously asymmetry into the message set. We assume that one of the messages is sent by the noncommunicating player types with a lower probability than the other message. If the messages are di¤erentiated in this way, e¢ ciency is always achieved. If payo¤ dominance does not con‡ict with risk dominance in selecting the e¢ cient equilibrium in the underlying game, the noncommunicating player type playing this equilibrium survives. In case of a con‡ict between these equilibrium selection criteria, the e¢ cient equilibrium will be played in any case. If the probability with which one of the messages is sent by the noncommunicating player types is small enough, then the asymmetry introduced into the message set selects a unique communicating player type in the long run - the player type using the message set infrequently by the noncommunicating types. In this case, there is a unique message that attains meaning in the long run, and it is the message assigned to the more reliable message. Otherwise, both communicating player types will be observed in the long run, and any message can attain meaning in the long run.

Robson and Vega-Redondo [21] show that the results of KMR are also sensitive to the matching protocol. If the uniform random matching is replaced by a true random matching, the payo¤ function of every strategy becomes a function of the realized matching. They show that this dynamics converges relatively fast to a Pareto e¢ cient equilibrium rather than selecting the risk dominant equilibrium. They moreover extend this result, the selection of e¢ cient equilibrium, to the common interest games. The nature of interaction in the population matters as well for the outcomes of the dynamic process. Ellison [9] shows that with local interaction, the speed of convergence can be considerably higher. These observations could be relevant for a model with cheap talk for example when considering the spreading of "di¤erent languages" in a population, i.e. the assignment of meanings to messages in a game when interactions take place locally, and may be considered to extend the present model.

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[4]For more experimental studies on communication via cheap talk in coordination games, see e.g. Burton, Loomes and Sefton [5], Charness [6], Clark, Kay and Sefton [7], Cooper et. al.[8], Rankin, van Huyck and Battalio [19].

6 Appendix 1: m messages and uniform babbling

The mutation costs of transition between any two monomorphic states, A, B, M and M', where M and M' are two distinct communicating states, are as follows:

- $c_{AB} = \lceil N \frac{a-c}{a-c+d-b} - \frac{a-d}{a-c+d-b} \rceil$, $c_{BA} = \lceil N \frac{d-b}{a-c+d-b} + \frac{a-d}{a-c+d-b} \rceil.$

- $c_{MM'} = \lceil \frac{N}{2} \rceil.$

- $c_{MA} = \lceil N \frac{a-b}{2a-b-c} \rceil$, $c_{AM} = \lceil N \frac{a-c}{2a-b-c} \rceil.$

- $c_{MB} = \lceil N \frac{m(a-d)+d-c}{m(a-d)+d-c+d-b} - \frac{m(a-d)}{m(a-d)+2d-b-c} \rceil$, $c_{BM} = \lceil N \frac{d-b}{m(a-d)+2d-b-c} + \frac{m(a-d)}{m(a-d)+2d-b-c} \rceil.$

These transition costs are compared in the following lemmas.

Lemma A1.1: Suppose $N > \frac{2m(a-d)}{m(a-d)-(c-b)}$. If $c < b$, or $c > b$ and $m > \frac{c-b}{a-d}$, then $c_{BM} < c_{MM'} < c_{MB}$. If $c > b$ and $m < \frac{c-b}{a-d}$, then $c_{MB} < c_{MM'} < c_{BM}$.

Proof. $c_{BM} < c_{MM'}$, $N \frac{d-b}{m(a-d)+2d-b-c} + \frac{m(a-d)}{m(a-d)+2d-b-c} < \frac{N}{2}$.
This reduces to $N(c-b-m(a-d)) < -2m(a-d)$. Hence $c > b$ and $\frac{c-b}{a-d} > m$ implies $c_{MM'} < c_{BM}$, while $\frac{c-b}{a-d} < m$, and $N > \frac{2m(a-d)}{m(a-d)-(c-b)}$ implies $c_{BM} < c_{MM'}$. Moreover, $c_{BM} = N - c_{MB}$, hence $c_{BM} < c_{MM'} \Rightarrow c_{MB} > c_{MM'}$. ∎

Lemma A1.2: $c_{AM} < c_{MM} < c_{MA}$, $c > b$.

Proof. $c_{AM} < c_{MM}$, $N \frac{a-c}{2a-b-c} < \frac{N}{2}$. This reduces to $2(a-c) < 2a-b-c$. Moreover, $c_{MA} = N - c_{AM}$, i.e. $c_{AM} < c_{MM}$, $c_{MM} < c_{MA}$. ∎

Lemma A1.3: Suppose $N > \frac{2d-b-c}{d-b}$. Then $c_{AB} < c_{MB}$ and $c_{BM} < c_{BA}$.

Proof. $c_{AB} < c_{MB}$, $N(\frac{a-c}{a-c+d-b} - \frac{m(a-d)+d-c}{m(a-d)+d-c+d-b}) < \frac{a-d}{a-c+d-b} - \frac{m(a-d)}{m(a-d)+2d-b-c}$.
This reduces to $N(d-b)(a-d)(1-m) < (a-d)(2d-b-c)(1-m)$, where $a-d > 0$ and $1-m < 0$. ∎

Lemma A1.4: If $b > c$ then $c_{MA} < c_{MB}$.

Proof. If $b > c$ then $c_{MA} < c_{MM'}$ by Lemma (A1.2) and $c_{MM'} < c_{MB}$ by Lemma (A1.1). ∎

Lemma A1.5: Suppose $(a-c)^2 - (d-b)(a-b) < 0$. Then $c_{AM} < c_{BA}$.

Proof. $c_{BA} < c_{AM}$, $N((a-c)^2 - (d-b)(a-b)) > (d-b)(a-b)$: ∎

Lemma A1.6: Suppose $N > \frac{2a-b-c}{a-c}$: Then $c_{AM} < c_{AB}$ and $c_{BA} < c_{MA}$:

Proof. $c_{AM} < c_{AB}$, $N(a-c)\left(\frac{1}{2a-b-c} - \frac{1}{a-c+d-b}\right) < -\frac{a-d}{a-c+d-b}$
This reduces to $N(a-c)(d-a) < (d-a)(2a-b-c)$ where $d-a < 0$: ∎

Lemma A1.7: Suppose $N > \frac{m(2a-b-c)}{m(a-b)+b-c}$: Then $c_{BM} < c_{MA}$ and $c_{AM} < c_{MB}$:

Proof. $c_{BM} < c_{MA}$, $N\frac{d-b}{m(a-d)+2d-b-c} + \frac{m(a-d)}{m(a-d)+2d-b-c} < N\frac{a-b}{2a-b-c}$:
This reduces to $N((d-b)(a-c) - (a-b)(d-c) - (a-b)m(a-d)) < -m(a-d)(2a-b-c)$; i.e. to
$N((a-d)(c-b) - (a-b)m(a-d)) < -m(a-d)(2a-b-c)$ where $a-d > 0$; so that the condition is equivalent to $N((a-b)m - (c-b)) > m(2a-b-c)$: Similarly to prove $c_{AM} < c_{MB}$: ∎

We identify now minimum cost trees on the set of states A; B; and m replicas of the communicating state M. First, we find the way to connect $m-1$ communicating states at a minimum cost to a state A; B; or a communicating state M: Then we find minimum cost tree on the set of three remaining states $\{A; B; M\}$; where M is one of the communicating states. This then forms a tree on the set of states A; B; and m replicas of the communicating state M: Let $c(x)$ denote the cost of such minimum cost $x-$tree:

Lemma A1.8: Suppose that for any m; $N > \frac{m(a-d)(2d-b-c)}{m(a-d)-(c-b)}$. (a) For $c > b$ and $m < \frac{c-b}{a-d}$; $c_{MB} < c_{MM} < c_{MA}$; hence a minimum cost tree for any state x on the set of states A; B and m replicas of the communicating state M will contain $m-1$ arcs starting in one of the $m-1$ communicating states other than M and ending in state B: (b) For $c > b$ and $m > \frac{c-b}{a-d}$; $c_{MM} < c_{MB}$; hence a minimum cost tree for any state x on the set of states A; B and m replicas of the communicating state M will contain $m-1$ arcs starting in a one of the $m-1$ communicating states other than M and ending in another communicating state in such a way that cycles are not created: (c) For $c < b$; $c_{MA} < c_{MM}$, and $c_{MA} < c_{MB}$; a minimum cost tree for any state x on the set of states A; B and m replicas of the communicating state M will contain $m-1$ arcs starting in one of the $m-1$ communicating states other than M and ending in state A:

Proof. (a) If $c > b$ then $c_{MM'} < c_{MA}$; see (A1.2). Moreover, if $m < \frac{c-b}{a-d}$; $c_{MB} < c_{MM'}$; see (A1.1), therefore $c_{MB} < c_{MM'} < c_{MA}$. (b) If $c > b$ and $m > \frac{c-b}{a-d}$; $c_{MM'} < c_{MA}$;

see (A1.2), and $c_{MM^0} < c_{MB}$; see (A1.1). (c) For $c < b$; $c_{MA} < c_{MM^0}$; see (A1.2), and $c_{MA} < c_{MB}$; see (A1.4). ∎

Denote by M a generic element of the set of monomorphic communicating states $\{C_1; ....; C_m\}$: In the previous lemma, we determined the way $m-1$ of these states will be connected to a minimum cost tree of any state as a root. Now we combine this information with information in which way the remaining three states will form a tree to find a minimum cost tree for each state on the set of monomorphic states $A; B;$ and $C_1; ....; C_m$:

6.1 Minimum cost A-tree on the set of monomorphic states $A; B;$ and $C_1; ....; C_m$

There are three $A-$ trees on the set $\{A; B; M\}$ with costs $c_1^A = c_{MA} + c_{BA}$; $c_2^A = c_{MA} + c_{BM}$; $c_3^A = c_{BA} + c_{MB}$:

Lemma A1.A: The cost of a minimum cost A-tree $c(A)$ is as follows: (a) for $c > b$ and $m < \frac{c_i b}{a_i d}$; $c(A) = mc_{MB} + c_{BA}$; (b) for $c > b$ and $m > \frac{c_i b}{a_i d}$; $c(A) = c_{MA} + c_{BM} + (m-1)c_{MM^0}$; (c) for $c < b$; $c(A) = mc_{MA} + c_{BM}$:

Proof. (a) $c_1^A > c_3^A$ because $c_{MA} > c_{MB}$; see A1.8, and $c_2^A > c_3^A$ because $c_{MA} > c_{BA}$; see A1.6, and $c_{BM} > c_{MB}$; see A1.x. Additional $m-1$ arcs each with cost c_{MB} are added by Lemma A1.8. (b) $c_3^A > c_1^A$ because $c_{MB} > c_{MA}$; see A1.8, and $c_1^A > c_2^A$ because $c_{BA} > c_{BM}$; see A1.3. Additional $m-1$ arcs with cost c_{MM^0} are added by Lemma A1.8. (c) $c_3^A > c_1^A$ because $c_{MB} > c_{MA}$; see A1.8, and $c_1^A > c_2^A$ because $c_{BA} > c_{BM}$ see A1.3. Additional $m-1$ arcs with cost c_{MA} are added by Lemma A1.8. ∎

6.2 Minimum cost B-tree on the set of monomorphic states $A; B;$ and $C_1; ....; C_m$

There are three $B-$ trees on the set $\{A; B; M\}$ with costs $c_1^B = c_{AB} + c_{MA}$; $c_2^B = c_{AB} + c_{MB}$; $c_3^B = c_{AM} + c_{MB}$:

Lemma A1.B: The cost of a minimum cost B-tree $c(B)$ is as follows: (a) for $c > b$ and $m < \frac{c_i b}{a_i d}$; $c(B) = c_{AM} + mc_{MB}$; (b) for $c > b$ and $m > \frac{c_i b}{a_i d}$; $c(B) = \min\{c_1^B + (m-1)c_{MM^0}; c_3^B + (m-1)c_{MM^0}\}$; (c) for $c < b$; $c(B) = mc_{MA} + c_{AB}$:

Proof. (a) $c_1^B > c_2^B$ because $c_{MA} > c_{MB}$; see A1.8, and $c_2^B > c_3^B$ because $c_{AB} > c_{AM}$; see A1.6. Additional $m-1$ arcs each with cost c_{MB} are added by Lemma A1.8. (b)

$c_2^B > c_1^B$ because $c_{MB} > c_{MA}$, see A1.8. Additional $m - 1$ arcs with cost $c_{MM'}$ are added by Lemma A1.8. (c) $c_2^B > c_1^B$ because $c_{MB} > c_{MA}$; see A1.8, and $c_3^B > c_1^B$ because $c_{MB} > c_{AB}$; see A1.3, and $c_{AM} > c_{MA}$; see A1.2. Additional $m - 1$ arcs with cost c_{MA} are added by Lemma A1.8. ∎

6.3 Minimum cost M-tree on the set of monomorphic states $A, B,$ and $C_1, ..., C_m$

There are three $M-$ trees on the set $\{A, B, M\}$ with costs $c_1^M = c_{AM} + c_{BA}$; $c_2^M = c_{AM} + c_{BM}$; $c_3^M = c_{AB} + c_{BM}$:

Lemma A1.M: The cost of a minimum cost M-tree $c(M)$ is as follows: (a) for $c > b$ and $m < \frac{c - b}{a - d}$; $c(M) = c_{AM} + c_{BM} + (m - 1)c_{MB}$; (b) for $c > b$ and $m > \frac{c - b}{a - d}$; $c(M) = c_{AM} + c_{BM} + (m - 1)c_{MA}$; (c) for $c < b$; $c(M) = c_{AM} + c_{BM} + (m - 1)c_{MM'}$:

Proof. $c_3^M > c_2^M$ because $c_{AB} > c_{AM}$; see A1.6, and $c_1^M > c_2^M$ because $c_{BA} > c_{BM}$; see A1.3. The minimum cost trees then obtain by referring to Lemma A1.8. ∎

7 Appendix 2: Two messages and nonuniform babbling

The mutation costs of transition between any two monomorphic states A, B, L and H are as follows:

- $c_{AB} = dN \frac{a - c}{a - c + d - b} - \frac{a - d}{a - c + d - b}e$, $c_{BA} = dN \frac{d - b}{a - c + d - b} + \frac{a - d}{a - c + d - b}e$:

- $c_{LH} = c_{HL} = d\frac{N}{2}e$:

- $c_{LA} = c_{HA} = dN \frac{a - b}{2a - b - c}e$, $c_{AL} = c_{AH} = dN \frac{a - c}{2a - b - c}e$.

- $c_{LB} = d\frac{N(a - d + p(d - c)) - (a - d)}{a - d + p(2d - b - c)}e$, $c_{BL} = d\frac{Np(d - b) + a - d}{a - d + p(2d - b - c)}e$.

- $c_{HB} = d\frac{N(a - c - p(d - c)) - (a - d)}{a - c + d - b - p(2d - b - c)}e$, $c_{BH} = d\frac{N(1 - p)(d - b) + a - d}{a - c + d - b - p(2d - b - c)}e$.

For each monomorphic state $x \in X$ there are sixteen trees on the set of monomorphic states $\{A, B, L, H\}$; with state x as a root. Some of these trees have the same cost due to the symmetries $c_{LH} = c_{HL}$, $c_{LA} = c_{HA}$; and $c_{AL} = c_{AH}$: We denote by $\mathbb{T}(z)$ the set of minimum cost trees of state z:

Transition costs between monomorphic states A, B, L and H are compared in the following lemmas.

Lemma A2.1: Suppose $N > \frac{2d-b-c}{d-b}$ and $p \in [0, 1/2)$: Then $c_{BL} < c_{BH} < c_{BA}$ and $c_{AB} < c_{HB} < c_{LB}$:

Proof. $c_{BL} < c_{BH}$, $\frac{Np(d-b)+a-d}{a-d+p(2d-b-c)} < \frac{N(1-p)(d-b)+a-d}{a-c+d-b-p(2d-b-c)}$:

This reduces to $(1-2p)(a-d)(c+b-2d+N(d-b)) > 0$:

$c_{BH} < c_{BA}$, $\frac{N(1-p)(d-b)+a-d}{a-c+d-b-p(2d-b-c)} < N\frac{d-b}{a-c+d-b} + \frac{a-d}{a-c+d-b}$:

This reduces to $(2d-b-c)(a-d) < N(a-d)(d-b)$: ■

Lemma A2.2: Suppose for any $p \in [0, 1/2)$; $N > \frac{2(a-d)}{a-c-(d-b)+p(c-b)}$: If $a-c > d-b$, or $a-c < d-b$ and $p > \frac{d-b-(a-c)}{c-b}$; then $c_{BH} < c_{HL} < c_{HB}$, otherwise $c_{HB} < c_{HL} < c_{BH}$:

Proof. $c_{BH} < c_{HB}$, $\frac{N(1-p)(d-b)+a-d}{a-c+d-b-p(2d-b-c)} < \frac{N(a-c-p(d-c))-(a-d)}{a-c+d-b-p(2d-b-c)}$:

This reduces to $2(a-d) < N(a-c-(d-b)+p(c-b))$: ■

Lemma A2.3: Suppose $N > \frac{2(a-d)}{a-d-p(c-b)}$: If $c > b$; or $c < b$ and $p > \frac{a-d}{b-c}$, then $c_{BL} < c_{HL} < c_{LB}$; otherwise $c_{LB} < c_{HL} < c_{BL}$:

Proof. $c_{LB} < c_{BL}$, $\frac{N(a-d+p(d-c))-(a-d)}{a-d+p(2d-b-c)} < \frac{Np(d-b)+a-d}{a-d+p(2d-b-c)}$:

This reduces to $N(a-d-p(c-b)) < 2(a-d)$: The comparison for $c_{HL} = \frac{N}{2}$ follows from $c_{LB} = N - c_{BL}$: ■

Lemma A2.4: $c_{AL} < c_{HL} < c_{LA}$, $c > b$:

Proof. $c_{AL} < c_{HL}$, $N\frac{a-c}{2a-b-c} < \frac{N}{2}$: ■

Lemma A2.5: Suppose $N > \frac{2a-b-c}{a-c}$: Then $c_{AL} < c_{AB}$ and $c_{BA} < c_{LA}$:

Proof. $c_{AL} < c_{AB}$, $N\frac{a-c}{2a-b-c} < N\frac{a-c}{a-c+d-b} - \frac{a-d}{a-c+d-b}$:

This reduces to $N(a-c)(a-d) > (a-d)(2a-b-c)$; where $a > d$: ■

Lemma A2.6: Suppose $N > \frac{(a-d)(2a-b-c)}{((a-c)(a-d)-p(c-b)(a-c+d-b))}$: If $b > c$; then $c_{AL} < c_{BL}$ and $c_{LB} < c_{LA}$:

Proof. $c_{AL} < c_{BL}$, $N\frac{a-c}{2a-b-c} < \frac{Np(d-b)+a-d}{a-d+p(2d-b-c)}$:

This reduces to $N((a-c)(a-d)-p(c-b)(a-c+d-b)) > (a-d)(2a-b-c)$: ■

Lemma A2.7: Suppose $N > \max\{\frac{2d-b-c}{d-b}, \frac{2d-b-c}{d-b}\}$: If $b > c$, then $c_{HB} > c_{LA}$:

Proof. By Lemma (A2.1) $c_{HB} > c_{AB}$, and by Lemma (A2.5), $c_{AB} > c_{AL}$. By Lemma (A2.4), if $b > c$ then $c_{AL} > c_{LA}$, hence $c_{HB} > c_{LA}$: ■

7.1 Minimum cost A-tree on $\{A, B, L, H\}$

There are sixteen A_i trees t_i^A with the following costs c_i^A; $i = 1, \ldots, 16$:

- $c_1^A = c_{LA} + c_{BA} + c_{HL}$: $c_1^A = c_2^A > c_3^A = c_4^A$ because $c_{BL} < c_{BA}$ see (A2.6), therefore $t_1^A \notin \bar{T}(A)$:

- $c_2^A = c_{HA} + c_{BA} + c_{LH}$: $c_2^A = c_1^A > c_3^A = c_4^A$ because $c_{BL} < c_{BA}$ see (A2.6), therefore $t_2^A \notin \bar{T}(A)$.

- $c_3^A = c_{HA} + c_{LH} + c_{BL}$: $c_3^A = c_4^A$:

- $c_4^A = c_{LA} + c_{HL} + c_{BL}$: $c_4^A = c_3^A$:

- $c_5^A = c_{HA} + c_{LH} + c_{BH}$: $c_5^A = c_6^A > c_3^A = c_4^A$ because $c_{BH} > c_{BL}$ see (A2.1), therefore $t_5^A \notin \bar{T}(A)$:

- $c_6^A = c_{LA} + c_{HL} + c_{BH}$: $c_6^A = c_5^A > c_3^A = c_4^A$ because $c_{BH} > c_{BL}$ see (A2.1), therefore $t_6^A \notin \bar{T}(A)$:

- $c_7^A = c_{LA} + c_{BA} + c_{HB}$: $c_7^A > c_9^A$ because $c_{BA} > c_{BL}$ see (A2.6), therefore $t_7^A \notin \bar{T}(A)$:

- $c_8^A = c_{HA} + c_{BA} + c_{LB}$: $c_8^A > c_7^A$ because $c_{LB} > c_{HB}$ see (A2.1), therefore $t_8^A \notin \bar{T}(A)$:

- $c_9^A = c_{LA} + c_{BL} + c_{HB}$:

- $c_{10}^A = c_{BA} + c_{LH} + c_{HB}$:

- $c_{11}^A = c_{BA} + c_{LB} + c_{HL}$: $c_{11}^A > c_{10}^A$ because $c_{LB} > c_{HB}$ see (A2.1), therefore $t_{11}^A \notin \bar{T}(A)$:

- $c_{12}^A = c_{HA} + c_{BH} + c_{LB}$: $c_{12}^A > c_9^A$ because $c_{BH} > c_{BL}$ and $c_{LB} > c_{HB}$ see (A2.1), therefore $t_{12}^A \notin \bar{T}(A)$:

- $c_{13}^A = c_{BA} + c_{HB} + c_{LB}$:

- $c_{14}^A = c_{BL} + c_{LA} + c_{HA}$:

- $c_{15}^A = c_{BH} + c_{LA} + c_{HA}$: $c_{15}^A > c_{14}^A$ because $c_{BH} > c_{BL}$ see (A2.1), therefore $t_{15}^A \notin \bar{T}(A)$:

- $c_{16}^A = c_{BA} + c_{LA} + c_{HA}$: $c_{16}^A > c_{15}^A$ because $c_{BA} > c_{BH}$ see (A2.1), therefore $t_{16}^A \notin \bar{T}(A)$:

Now, for $b > c$, we ...nd that $c_{10}^A > c_{13}^A$ because $c_{LH} > c_{LB}$ see (A2.3), therefore $t_{10}^A \notin \bar{T}(A)$; $c_3^A = c_4^A > c_{14}^A$ because $c_{LA} < c_{LH}$ see (A2.4), therefore $t_3^A \notin \bar{T}(A)$ and $t_4^A \notin \bar{T}(A)$; and $c_{13}^A > c_{11}^A$ because $c_{HB} > c_{HL}$ see (A2.2), therefore $t_{13}^A \notin \bar{T}(A)$: Finally, $c_9^A > c_{14}^A$ because $c_{HA} < c_{HB}$ see (A2.7), therefore $t_9^A \notin \bar{T}(A)$. We conclude that for $b > c$; $c(A) = c_{14}^A$: For $c > b$; $c_3^A = c_4^A > c_{10}^A$ because $c_{HA} > c_{BA}$ see (A2.5) and $c_{BL} > c_{HB}$ as $c_{LB} > c_{BL} > c_{BH}$ see (A2.3) and (A2.1), therefore $t_3^A \notin \bar{T}(A)$ and $t_4^A \notin \bar{T}(A)$: The two remaining candidate A-trees for a minimum cost tree of state A are t_{10}^A and t_9^A: Finally, $c(A) = \min\{t_9^A; t_{10}^A\}$:

7.2   Minimum cost B-tree on $\{A; B; L; H\}$

There are sixteen B_i trees t_i^B with the following costs c_i^B; $i = 1; ....; 16$:

² $c_1^B = c_{AB} + c_{HA} + c_{LA}$: if $b < c$; $c_1^B > c_3^B$ because $c_{LA} > c_{LH}$ see (A2.4), therefore if $b < c$; $t_1^B \notin \bar{T}(B)$:

² $c_2^B = c_{AB} + c_{HA} + c_{LB}$:

² $c_3^B = c_{AB} + c_{HA} + c_{LH}$: $c_3^B = c_4^B$; if $b > c$; $c_3^B > c_1^B$ because $c_{LA} < c_{LH}$ see (A2.4), therefore if $b > c$; $t_3^B \notin \bar{T}(B)$:

² $c_4^B = c_{AB} + c_{HL} + c_{LA}$: $c_3^B = c_4^B$; if $b > c$, $t_4^B \notin \bar{T}(B)$:

² $c_5^B = c_{AH} + c_{HB} + c_{LH}$: $c_5^B = c_6^B$; $c_5^B > c_{11}^B$ because $c_{LA} < c_{LH}$ see (A2.4), therefore if $b > c$; $t_5^B \notin \bar{T}(B)$.

² $c_6^B = c_{AL} + c_{HB} + c_{LH}$: $c_6^B = c_5^B$; if $b > c$, $t_6^B \notin \bar{T}(B)$:

² $c_7^B = c_{AL} + c_{HL} + c_{LB}$: $c_7^B = c_8^B$; $c_7^B > c_6^B$ because $c_{LB} > c_{HB}$ see (A2.1), therefore $t_7^B \notin \bar{T}(B)$:

² $c_8^B = c_{AH} + c_{HL} + c_{LB}$: $c_8^B = c_7^B$; $t_8^B \notin \bar{T}(B)$:

² $c_9^B = c_{AL} + c_{HB} + c_{LB}$: $c_9^B = c_{10}^B$:

² $c_{10}^B = c_{AH} + c_{HB} + c_{LB}$: $c_{10}^B = c_9^B$:

² $c_{11}^B = c_{AH} + c_{HB} + c_{LA}$: $c_{11}^B > c_5^B$ because $c_{LA} > c_{LH}$ see (A2.4), therefore if $b < c$, $t_{11}^B \notin \bar{T}(B)$:

² $c_{12}^B = c_{AL} + c_{HA} + c_{LB}$: $c_{12}^B > c_{11}^B$ because $c_{LB} > c_{HB}$ see (A2.1), therefore $t_{12}^B \notin \bar{T}(B)$.

² $c_{13}^B = c_{AB} + c_{HL} + c_{LB} : c_{13}^B > c_8^B$ because $c_{AB} > c_{AH}$ see (A2.5), therefore t_{13}^B $\notin \mathbb{T}(B)$:

² $c_{14}^B = c_{AB} + c_{HB} + c_{LH} : c_{14}^B > c_5^B$ because $c_{AB} > c_{AH}$ see (A2.5), therefore t_{14}^B $\notin \mathbb{T}(B)$:

² $c_{15}^B = c_{AB} + c_{HB} + c_{LB} : c_{15}^B > c_{10}^B$ because $c_{AB} > c_{AH}$ see (A2.5), therefore t_{15}^B $\notin \mathbb{T}(B)$:

² $c_{16}^B = c_{AB} + c_{HB} + c_{LA} : c_{16}^B > c_{11}^B$ because $c_{AB} > c_{AH}$ see (A2.5), therefore t_{16}^B $\notin \mathbb{T}(B)$.

For $b > c$; $c_{LA} < c_{HL} < c_{AL}$ see (A2.4), hence $c_{LA} < c_{LB}$ for $b > c$; now $c_9^B > c_2^B$ and $c_{10}^B > c_2^B$ because $c_{LA} < c_{AL}$ see (A2.4) and $c_{AB} < c_{HB}$ see (A2.1), therefore $t_9^B \notin \mathbb{T}(B)$ and $t_{10}^B \notin \mathbb{T}(B)$; and similarly $c_1^B > c_{11}^B$ therefore $t_{11}^B \notin \mathbb{T}(B)$; and $c_2^B > c_1^B$ because $c_{LB} > c_{LA}$ therefore $t_2^B \notin \mathbb{T}(B)$: Consequently, if $b > c$; $\mathbb{T}(B) = \{t_1^B\}$:

For $b < c$; the remaining candidate B-trees for a minimum cost tree of state B are either trees t_3^B and t_4^B, or trees t_5^B and t_6^B: Finally, $c(B) = \min\{t_3^B; t_5^B\}$

7.3 Minimum cost L-tree on $\{A; B; L; H\}$

There are sixteen L_i trees t_i^L with the following costs c_i^L; $i = 1; ::::; 16$:

² $c_1^L = c_{BL} + c_{AH} + c_{HB} : c_1^L = c_2^L$:

² $c_2^L = c_{BL} + c_{AL} + c_{HB} : c_1^L = c_2^L$:

² $c_3^L = c_{HL} + c_{AH} + c_{BL} : c_3^L = c_4^L$; if $b > c$; $c_3^L > c_6^L$ because $c_{HL} > c_{HA}$ see (A2.4), therefore if $b > c$; $t_3^L \notin \mathbb{T}(L)$.

² $c_4^L = c_{HL} + c_{AL} + c_{BL} : c_4^L = c_3^L$, if $b > c$; $t_4^L \notin \mathbb{T}(L)$.

² $c_5^L = c_{AL} + c_{HA} + c_{BH} : c_5^L > c_6^L$ because $c_{BL} < c_{BH}$ see (A2.1), therefore $t_5^L \notin \mathbb{T}(L)$:

² $c_6^L = c_{BL} + c_{AL} + c_{HA} :$ if $b < c$; $c_6^L > c_4^L$ because $c_{HA} > c_{HL}$ see (A2.4), therefore if $b < c$; $t_6^L \notin \mathbb{T}(L)$:

² $c_7^L = c_{AL} + c_{BA} + c_{HB} : c_7^L > c_2^L$ because $c_{BL} < c_{BA}$ see (A2.6), therefore $t_7^L \notin \mathbb{T}(L)$:

² $c_8^L = c_{HL} + c_{AH} + c_{BA} : c_8^L = c_9^L$; $c_8^L > c_4^L$ because $c_{BL} < c_{BA}$ see (A2.6), therefore $t_8^L \notin \mathbb{T}(L)$:

- $c_9^L = c_{HL} + c_{AL} + c_{BA} : c_9^L = c_8^L$, therefore $t_9^L \notin \bar{T}(L)$:

- $c_{10}^L = c_{HL} + c_{AL} + c_{BH} : c_{10}^L = c_{11}^L; c_{10}^L > c_4^L$ because $c_{BH} > c_{BL}$ see (A2.1), therefore $t_{10}^L \notin \bar{T}(L)$:

- $c_{11}^L = c_{HL} + c_{AH} + c_{BH} : c_{11}^L = c_{10}^L$, therefore $t_{11}^L \notin \bar{T}(L)$.

- $c_{12}^L = c_{HL} + c_{AB} + c_{BH} : c_{12}^L > c_{13}^L$ because $c_{BH} > c_{BL}$ see (A2.1), therefore $t_{12}^L \notin \bar{T}(L)$.

- $c_{13}^L = c_{HL} + c_{AB} + c_{BL} : c_{13}^L > c_3^L$ because $c_{AB} > c_{AL}$ see (A2.5), therefore $t_{13}^L \notin \bar{T}(L)$

- $c_{14}^L = c_{BL} + c_{AB} + c_{HB} : c_{14}^L > c_2^L$ because $c_{AB} > c_{AL}$ see (A2.5), therefore $t_{14}^L \notin \bar{T}(L)$:

- $c_{15}^L = c_{BL} + c_{AB} + c_{HA} : c_{15}^L > c_6^L$ because $c_{AB} > c_{AL}$ see (A2.5), therefore $t_{15}^L \notin \bar{T}(L)$:

- $c_{16}^L = c_{AL} + c_{HA} + c_{BA} : c_{16}^L > c_5^L$ because $c_{BA} > c_{BH}$ see (A2.1), therefore $t_{16}^L \notin \bar{T}(L)$:

For $b > c$; $c_6^L > c_1^L$ because $c_{HB} > c_{HA}$ see (A2.7), therefore $t_1^L \notin \bar{T}(L)$ and $t_2^L \notin \bar{T}(L)$: $\bar{T}(L) = \{t_6^L\}$.

For $b < c$; the minimum cost trees candidates are t_1^L and t_2^L or t_3^L and t_4^L: $c_1^L < c$ if $c_{HB} < c_{HL}$ see (A2.2). For $a_j c > d_j b$; and for $a_j c < d_j b$ and $p < \frac{d_j b_j (a_j c)}{c_j b}$, $\bar{T}(L) = \{t_1^L; t_2^L\}$: For $a_j c < d_j b$ and $p > \frac{d_j b_j (a_j c)}{c_j b}$; $\bar{T}(L) = \{t_3^L; t_4^L\}$:

7.4 Minimum cost H_j tree :

There are sixteen H_j trees t_i^H with the following costs c_i^H; $i = 1; ...; 16$:

- $c_1^H = c_{AH} + c_{BH} + c_{LB} : c_1^H = c_2^H$:

- $c_2^H = c_{AL} + c_{BH} + c_{LB} : c_2^H = c_1^H$:

- $c_3^H = c_{AH} + c_{BL} + c_{LA}$:

- $c_4^H = c_{AH} + c_{BL} + c_{LH} : c_4^H = c_5^H$:

- $c_5^H = c_{AL} + c_{BL} + c_{LH} : c_5^H = c_4^H$:

² $c_6^H = c_{AL} + c_{BA} + c_{LH} : c_6^H = c_7^H; c_6^H > c_9^H$ because $c_{BA} > c_{BH}$ see (A2.1), therefore $t_6^H \notin \bar{T}(H)$:

² $c_7^H = c_{AH} + c_{BA} + c_{LH} : c_7^H = c_6^H, c_7^H > c_9^H$, therefore $t_7^H \notin \bar{T}(H)$

² $c_8^H = c_{AL} + c_{BH} + c_{LH} : c_8^H = c_9^H; c_8^H > c_5^H$ because $c_{BH} > c_{BL}$ see (A2.1), therefore $t_8^H \notin \bar{T}(H)$:

² $c_9^H = c_{AH} + c_{BH} + c_{LH} : c_9^H = c_8^H; c_9^H > c_5^H$, therefore $c_9^H \notin \bar{T}(H)$:

² $c_{10}^H = c_{AH} + c_{BH} + c_{LA} : c_{10}^H > c_3^H$ because $c_{BH} > c_{BL}$ see (A2.1), therefore $t_{10}^H \notin \bar{T}(H)$:

² $c_{11}^H = c_{BH} + c_{AB} + c_{LH} : c_{11}^H > c_{12}^H$ because $c_{BH} > c_{BL}$ see (A2.1), therefore $t_{11}^H \notin \bar{T}(H)$:

² $c_{12}^H = c_{BL} + c_{AB} + c_{LH} : c_{12}^H > c_5^H$ because $c_{AB} > c_{AL}$ see (A2.5), therefore $t_{12}^H \notin \bar{T}(H)$:

² $c_{13}^H = c_{BH} + c_{AB} + c_{LB} : c_{13}^H > c_1^H$ because $c_{AB} > c_{AL}$ see (A2.5), therefore $t_{13}^H \notin \bar{T}(H)$:

² $c_{14}^H = c_{BH} + c_{AB} + c_{LA} : c_{14}^H > c_{10}^H$ because $c_{AB} > c_{AL}$ see (A2.5), therefore $t_{14}^H \notin \bar{T}(H)$:

² $c_{15}^H = c_{AH} + c_{BA} + c_{LB} : c_{15}^H > c_1^H$ because $c_{BA} > c_{BH}$ see (A2.1), therefore $t_{15}^H \notin \bar{T}(H)$:

² $c_{16}^H = c_{AH} + c_{BA} + c_{LA} : c_{16}^H > c_3^H$ because $c_{BA} > c_{BL}$ (A2.6), therefore $t_{16}^H \notin \bar{T}(H)$:

For $b > c$; $c_4^H = c_5^H > c_3^H$ because $c_{LA} < c_{LH}$ see (A2.4), therefore $t_4^H \notin \bar{T}(H)$ and $t_5^H \notin \bar{T}(H)$. Also, $c_1^H = c_2^H > c_3^H$, because $c_{BH} > c_{BL}$ see (A2.1) and $c_{LB} > c_{LA}$ see (A2.6), therefore $t_1^H \notin \bar{T}(H)$; $t_2^H \notin \bar{T}(H)$. Finally, $\bar{T}(H) = \{t_3^H\}$ if $b > c$:

For $b < c$; $c_1^H = c_2^H > c_3^H$ because $c_{LH} < c_{LA}$ see (A2.5), therefore $t_1^H \notin \bar{T}(H)$ and $t_2^H \notin \bar{T}(H)$. And $c_3^H > c_4^H$ because $c_{LH} < c_{LA}$ see (A2.4), therefore $t_3^H \notin \bar{T}(H)$. Finally, $\bar{T}(H) = \{t_4^H; t_5^H\}$ if $b < c$:

8   Appendix 3

Lemma 8: We ...rst need some notation and a way how to decompose mutation cost of a tree. We work with a tree which is oriented to the root, i.e. every state di¤erent

from the root has a unique successor and possibly several predecessors. For a tree T on A, denote by $L_T \subseteq Z$ states that are only a start to an arc but not an end of an arc in T. This is the set of leaves of the tree T. For a state z', denote by $\text{succ}_T(z')$ the unique state such that $(z', \text{succ}_T(z')) \in T$ and by $\text{pred}_T(z')$ a set of states that satisfy $\text{pred}_T(z') = \{y \in Z \mid y = \text{succ}^k(x), k = 1, 2, \ldots, \text{ for } x \in L_T \text{ such that there is } \hat{k} < 1$ satisfying $\text{succ}^{\hat{k}}(x) = z'\}$, where $\text{succ}^k(x)$ is the successor function applied k-times in sequence. If $z' \in L_T$, $\text{pred}_T(z') = \emptyset$.

If T is z-tree, for any $z' \in Z, z' \neq z$, it can be written as $T = T_{\text{pred}_T(z')} \cup T_{Z \mid \text{pred}_T(z')} \cup \{(z', \text{succ}_T(z'))\}$, where $T_{\text{pred}_T(z')}$ is a z'-tree constrained to nodes being states in the set $\text{pred}_T(z')$ such that for any $(y, y') \in T_{\text{pred}_T(z')}$ it holds $(y, y') \in T$, and $T_{Z \mid \text{pred}_T(z')}$ is a z-tree constrained to the nodes being states in the set Z but not in the set $\text{pred}_T(z')$ such that for any $(y, y') \in T_{Z \mid \text{pred}_T(z')}$ it holds $(y, y') \in T$.

Denote by $c(T')$ the cost of some tree T', i.e. the sum of mutations needed to achieve transitions from the leaves to the root between the state pairs forming the arcs of the tree T'. Then for any $z' \in Z$, the cost of tree T, denoted by $c(T)$, can be decomposed as $c(T) = c(T_{\text{pred}_T(z')}) + c(T_{Z \mid \text{pred}_T(z')}) + c_{z' \text{succ}_T(z')}$, where $c_{z' \text{succ}_T(z')}$ is the cost of transition from state z' to its successor. We use this decomposition to prove Lemma 8.

Proof. Suppose $z \in SSS$ but z is not stationary under P. Then, there is a monomorphic state $z' \in Z$ such that $p_{zz'} > 0$. This follows from the properties of the imitation dynamics: if z is not stationary, then it is composed of at least two types $x, y \in X$, $z_x > 0$ and $z_y > 0$, such that $\mu_x(z) \geq \mu_{x'}(z)$ for all $x' \in X$ with $z_{x'} > 0$, and $\mu_x(z) > \mu_y(z)$. The state z' with $z'_x = N$ is trivially a stationary state, and $p_{zz'} > 0$ by assumption that any individual updates his own type to any currently best performing type with positive probability. The cost of the minimum cost z-tree T can be decomposed as $c(T) = c(T_{\text{pred}_T(z')}) + c(T_{Z \mid \text{pred}_T(z')}) + c_{z' \text{succ}_T(z')}$ with $c_{z' \text{succ}_T(z')} \geq 1$ because z' is a monomorphic state, hence any other state can be reached from it only via mutations.

Now consider a z'-tree T' constructed on the state space Z such that $T' = T_{\text{pred}_T(z')} \cup T_{Z \mid \text{pred}_T(z')} \cup \{(z, z')\}$. Now, $c(T') = c(T_{\text{pred}_T(z')}) + c(T_{Z \mid \text{pred}_T(z')}) + c_{zz'}$ with $c_{zz'} = 0$ since z is not stationary under the imitation dynamics and z' is the state reached without mutations from state z. So, we have shown that there is another state z' such that $c_{z'} < c_z$, and a state that is not stationary cannot be in SSS. ∎

Proposition 9.

Proof. Suppose $z \in SSS$ but z is not monomorphic. Then, there is some monomorphic state $z' \neq z$, $z' \in Z$ such that $p_{zz'} = 0$. Hence $c_{zz'} = 0$, and there are no mutations needed to reach the state z' from the state z. Let T be a minimum cost tree of state z:

Then $c(T) = c(T_{pred_T(z^0)}) + c(T_{Zjpred_T(z^0)}) + c_{z^0succ_T(z^0)}$: Since z^0 is a monomorphic state, $c_{z^0succ_T(z^0)}$ ، 1:

Now consider a z^0-tree T^0 constructed on the state space Z such that $T^0 = T_{pred_T(z^0)}$ [$T_{Zjpred_T(z^0)}$ [f$(z; z^0)$g: Now, $c(T^0) = c(T_{pred_T(z^0)}) + c(T_{Zjpred_T(z^0)}) + c_{zz^0}$ with $c_{zz^0} = 0$ since z^0 is reached with positive probability from z via imitation dynamics. So, we have shown that there is another state z^0 such that $c_{z^0} < c_z$; and a state that is not monomorphic cannot be in SSS: ∎

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