

## Tilburg University

### Old Folks and Spoiled Brats

Butler, M.; Harms, P.

*Publication date:*  
2001

[Link to publication in Tilburg University Research Portal](#)

*Citation for published version (APA):*

Butler, M., & Harms, P. (2001). *Old Folks and Spoiled Brats: Why the baby Boomers' Saving Crisis Need Not be that Bad*. (CentER Discussion Paper; Vol. 2001-42). Macroeconomics.

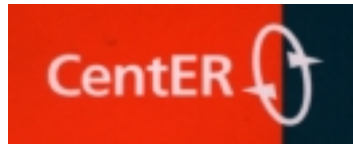
#### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

#### Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.



No. 2001-42

**OLD FOLKS AND SPOILED BRATS: WHY THE BABY  
BOOMERS' SAVING CRISIS NEED NOT BE THAT  
BAD**

By Monika Büttler and Philipp Harms

June 2001

ISSN 0924-7815

**Discussion paper**

# Old folks and spoiled brats: Why the baby boomers' saving crisis need not be that bad.

MONIKA BÜTLER

DEEP Université de Lausanne, CentER Tilburg University & CEPR\*

PHILIPP HARMS

University of Konstanz<sup>†</sup>

June 6, 2001

## Abstract

We study the impact of an anticipated “baby boom” in an overlapping generations economy. The rise of the working population lowers the wage, and the high demand for assets causes a rise in the price of capital which will be reversed when the baby boomers leave the work-force. However, the swings in factor prices are substantially dampened if we allow for more than two generations, endogenous labor supply, and convex capital adjustment costs. This is mainly due to the intertemporal shifts in labor market participation that can be observed if agents work for more than one period. Optimal saving and labor supply decisions of the baby-boomers' preceding and subsequent generations partly offset the impact of the unfavorable demographic shock. Accordingly, the impact of a baby boom on the welfare of different generations crucially depends on the elasticity of labor supply.

---

\*Address: Monika Bütlér, DEEP-HEC, Université de Lausanne, BFSH1, CH-1015 Lausanne, Switzerland; email *Monika.Butler@HEC.unil.ch*, tel + 41 21 692 3451, fax + 41 21 692 3365.

<sup>†</sup>Address: Philipp Harms, University of Konstanz, Dept. of Economics, Fach D 138, D-78457 Konstanz, Germany; email: *philipp.harms@uni-konstanz.de*, tel +49 7531 88 3711, fax +49 7531 88 4119.

## Extended Abstract

The nineties have witnessed an unprecedented increase in share prices in most industrialized countries. A possible explanation is based on the demographic evolution, and in particular the aging of the baby boom generation. We present a 3-generation overlapping generations (OLG) model with endogenous labor supply and a convex capital adjustment cost technology to study the impact of anticipated demographic changes on factor prices, savings and the welfare of different generations. Baby-boomers face a twofold disadvantage: the rise of the working population lowers wages in the first two periods of their lives, and the huge amount of capital accumulated during these periods lowers future returns and thus their retirement income. The stock market boom will be reversed in the future, and the price of capital will undershoot its long-run level before returning to the steady state.

While such a pattern is also borne out by a 2-generation OLG model, we show that using a more sophisticated demographic structure and making labor supply endogenous may substantially reduce the swings in factor prices. This is due to the fact that the simultaneous presence of two generations in the labor market offers considerable scope for intertemporal adjustment: movements in capital returns associated with the demographic shock induce the baby-boomers' preceding generation to save more at young age while later generations will reduce their savings. Moreover, to avoid the anticipated reduction of wages the baby boomers' parents work more intensively in their early working years and retire early, while the baby boomers' offspring concentrate their labor market activities on the second half of their working period. Both effects dampen the upward and downward swings of asset prices and returns. As a consequence, estimated welfare effects relative to a case without baby boomers are much lower if labor supply is assumed elastic.

*Jel-Classification:* E2, E6

*Keywords:* Baby Boom, Asset Prices, Labor Market Adjustments

# 1 Introduction

In most industrialized countries, the 1990s were characterized by a spectacular increase in share prices.<sup>1</sup> Some observers have attributed this evolution to the advent of a “New Economy” and to the technological innovations that enhanced productivity at the end of the past millennium. The recent downturn of the stockmarket, however, has lent some support to a growing number of skeptics who have warned that the bonanza of the past decade might eventually be reversed and end in a resounding crash.

In this paper, we present a model that is able to reproduce the observed bull market of the nineties and that partly supports the skeptics’ view about the future evolution of returns. However, we do not see this development as a result of agents’ “irrational exuberance”, but as an equilibrium outcome that is driven by the changing age structure of industrialized countries. Moreover, we argue that demographic changes are also responsible for the currently widespread early retirement and the late entry into the labor market of the young.

After World War II, most industrialized countries experienced a considerable rejuvenation of their populations. The war had taken its toll on the working-age generations, and high birth rates in the 1950s and early 1960s significantly lowered the average age of US citizens.<sup>2</sup> Forty years later, this picture has changed dramatically: as a result of falling mortality and of the decline in birth rates which began in the 1970s, the average age of the population has started to rise, and while the “baby boom” generation is planning its retirement, it faces a steadily shrinking labor force.

Our paper suggests that the demographic shock of the 1960s and the aging of the baby boomers have contributed to the increase of equity prices in the past decade, and that both the massive capital accumulation of this generation and the shrinking labor force will eventually result in a considerable reduction of returns. This implies that members of the baby boom generation face a twofold disadvantage: not only did the rise of the working population depress wages while they were still on the labor market,<sup>3</sup> but the huge amount of capital accumulated

---

<sup>1</sup>Between December 1989 and December 1999, the Dow Jones industrials index rose from 2753 to 11497 points, which amounts to an average annual return of 15.4 percent.

<sup>2</sup>In most European countries, this phenomenon occurred some years later, with birth rates peaking in the early or mid 1960s.

<sup>3</sup>Empirical evidence that baby boomers have indeed experienced a depressed wage is provided by Welch (1979).

during this period will lower future returns and thus their retirement income.

While such an evolution after a temporary rise in birth rates is also borne out by a simple OLG model with two generations and an exogenous labor supply,<sup>4</sup> the key objective of our paper is to show that several factors dampen the effects of a demographic shock in a more realistic framework. In particular, we focus on the effects of introducing a third generation, endogenizing labor supply, and of allowing for convex capital adjustment costs. We show that these modifications to a standard 2-generation OLG model contribute to smoothing the time paths of factor prices during the demographic transition and may thus partly defuse the looming saving crisis that threatens the retired baby boomers' welfare. The key reason for this result is that agents have sizeable opportunities to intertemporally adjust their consumption and labor supply if they save and work for at least two periods. More specifically, the swings of wages and capital returns that result from the anticipated demographic shock affect the labor supply and saving behavior of the baby boomers' preceding and subsequent generations: while the baby boomers' parents raise their savings and labor supply in anticipation of higher interest rates, their children expand consumption of goods and leisure during their youth and raise their labor supply as soon as the baby boom generation has retired. Under standard assumptions, both forces prevent the marginal productivity of capital and labor from falling too sharply. The intertemporal substitution behavior of the "old folks" and the "spoiled brats" — in particular, their optimal choice of labor supply — thus dampens the upward and downward swings of wages and capital returns. This also implies that welfare comparisons between the different generations are sensitive to the chosen parameterization. The loss of the baby boomer generation relative to the economic well-being of other generations is substantially lower if labor market activities can be shifted across time. Although our setup is too simple to produce quantitatively reliable predictions about the future evolution of factor prices and saving rates, it allows to derive closed form solutions for consumption and labor supply decisions and to describe in a transparent way the forces that drive the time paths of price and quantity variables.

Our paper is related to a number of previous contributions that investigate the link between demographic factors and returns on investment. Abel (2000) discusses the impact of an *unanticipated* fertility shock on capital accumulation

---

<sup>4</sup>See, for example, Bohn (1999).

and factor prices in a OLG model with a tax–financed social security system. As in our paper the adjustment cost technology for converting consumption goods into capital goods may be convex, and comprises as extreme cases the Neoclassical model as well as the Lucas–tree (1978) model.<sup>5</sup> However, Abel limits his attention to a two-generation framework with exogenous labor supply, excluding most of the dampening forces described above, and due to a log–specification of utility, saving effects can only be observed in the presence of a social security system.<sup>6</sup> Constantinides et al. (1998) discuss the impact of borrowing constraints (which may bind for the low–income young generation) on asset prices, using a stochastic three–generation OLG model. While both our and their paper stress the fact that cohorts differ in their propensity to save and in their risk aversion, Constantinides et al. (1998) cut the link to the supply side of the economy by assuming an exogenous stream of incomes. Finally, Ríos–Rull (2001) investigates the quantitative implications of population aging for a number of different assumptions of fertility patterns in a multi–generation OLG model. He clearly demonstrates the impact of a large generation on the aggregate saving rate, but does not consider the labor supply patterns of the different generations involved.

The remainder of our paper is set up as follows: Section 2 introduces the structure of our model and the assumptions we make. Equilibrium dynamics for a three–generation setup are discussed in section 3. Section 4 presents the results of numerical simulations and interprets the time paths of factor prices, labor supply, and consumption, as well as the welfare effects of the demographic shock for different generations. Section 5 summarizes and concludes.

## 2 The Model

We consider a closed economy that is populated by agents who live for  $J$  periods, have perfect foresight, and leave no bequests. All members of a generation are identical, but generations may vary in size, and we denote the number of agents born in period  $t$  by  $N_t$ . The supply side of the economy consists of two competitive sectors: The first produces a homogenous consumption good using labor and capital, while the second employs present capital and the consumption good to

---

<sup>5</sup>Magill & Quinzii (1999) use a variant of this model in which consumption goods can be transformed into capital goods, but not vice-versa.

<sup>6</sup>Abel uses a log–specification to derive closed form solutions under a stochastic fertility pattern.

produce physical capital which will be used in the following period.

Agents have two ways to transfer income across periods: In period  $t$ , they can purchase real bonds which entitle to receive a fixed interest payment  $R_{t+1}$  in the next period. Alternatively, they can buy physical capital which is produced in the capital goods sector. While we use the price of the consumption good as the numeraire, the price of a unit of capital in period  $t$  is  $q_t$ . In the first “subperiod” of period  $t+1$ , capital purchased in period  $t$  is used in the production of consumption goods, and capital owners receive a rental price  $r_{t+1}^C$ . In the second subperiod, the existing physical capital stock is used up in the production of new capital goods, and capital owners are reimbursed by receiving a rental price  $r_{t+1}^K$ . Since a unit of capital can first be used in the production of consumption goods and then as an input in the construction of next period’s capital stock, the gross return on physical capital can be determined by combining the rental prices in the two sectors.

## 2.1 Production and factor prices

### 2.1.1 The consumption goods sector

In the consumption goods sector, a large number of perfectly competitive firms use the following technology:

$$Y_{f,t} = K_{f,t}^\alpha L_{f,t}^{1-\alpha}. \quad (1)$$

In (1),  $Y_{f,t}$  is the output produced by firm  $f$  in period  $t$ , while  $K_{f,t}$  and  $L_{f,t}$  represent the physical capital and effective labor employed by firm  $f$  in period  $t$ , respectively. Perfect competition on the markets for labor and capital implies that the real wage (in terms of consumption good units) and the rental price of capital are given by

$$w_t = (1 - \alpha)K_t^\alpha L_t^{-\alpha} \quad (2)$$

and

$$r_t^C = \alpha K_t^{\alpha-1} L_t^{1-\alpha}. \quad (3)$$

We have removed the firm subscript  $f$  in equations (2) and (3) to denote aggregate values. Aggregate effective labor supply is given by

$$L_t = \sum_{i=0}^{J-1} N_{t-i} e_i l_t^{t-i}, \quad (4)$$



where  $e_i$  denotes age- $i$  labor productivity, and  $l_i^{t-i}$  is the time  $t$  labor supply of an agent born in  $(t-i)$ . The latter will be determined by the agent's optimal allocation of resources.

### 2.1.2 The capital goods sector

In period  $t$ , the representative firm in the capital goods sector employs consumption goods and capital to produce physical capital which can be used in period  $t+1$ . Its technology is given by

$$K_{f,t+1} = I_{f,t}^\phi K_{f,t}^{1-\phi}. \quad (5)$$

In (5),  $K_{f,t+1}$  represents the amount of capital goods produced by firm  $f$  in period  $t$ , while  $I_{f,t}$  is the amount of consumption goods and  $K_{f,t}$  the amount of capital goods used in the production process. This specification, which goes back to Basu (1987) and which is also used by Abel (2000), allows to capture the notion that there are adjustment costs in installing new capital. While our framework is a simple general equilibrium OLG model with costless adjustment if we set  $\phi = 1$ , the production of capital goods for the next period depends on the current capital stock if  $0 < \phi < 1$ , and in the extreme case of  $\phi = 0$ , the capital stock is fixed at a constant level.<sup>7</sup>

Since the technology in the capital goods sector exhibits constant returns to scale, we can determine the price of capital in period  $t$ ,  $q_t$  (or Tobin's  $q$ ), by deriving the unit cost function for a representative firm, which has to equal the output price under perfect competition.<sup>8</sup> We thus get, using (5),

$$q_t \equiv \left( \frac{dK_{t+1}}{dI_t} \right)^{-1} = \frac{1}{\phi} \left( \frac{K_{t+1}}{K_t} \right)^{\frac{1-\phi}{\phi}}. \quad (6)$$

Profit maximization, moreover, implies that the period  $t$  rental price of capital in the capital goods sector is the marginal product of capital in the investment technology,  $(1-\phi)I_t^\phi K_t^{-\phi} = (1-\phi)K_{t+1}/K_t$ , multiplied by the price of capital  $q_t$ , i.e.,

$$r_t^K \equiv q_t \frac{\partial K_{t+1}}{\partial K_t} = \frac{1-\phi}{\phi} \left( \frac{K_{t+1}}{K_t} \right)^{\frac{1}{\phi}}. \quad (7)$$

---

<sup>7</sup>Note, however, that it is not possible to consider this boundary case by simply setting  $\phi = 0$  in all subsequent expressions. Instead, this case requires a separate analysis which we will provide in appendix A.

<sup>8</sup>In doing this, we use the fact that the price of consumption goods is one.

Equations (6) and (7) show that the current price of capital goods and the rental rate in the capital goods sector increase in the growth rate of the aggregate capital stock between periods  $t$  and  $t + 1$ .

## 2.2 Households

In every period of their lives, agents are endowed with one unit of time which they can allocate to either labor or leisure. Labor productivity is age-dependent and denoted by  $e_j$ . Note that  $j$  measures the number of periods since birth, i.e., agents start working at age 0. The wage rate from working in the perfectly competitive consumption goods sector is proportional to productivity, i.e., equal to  $e_j w_t$  for an age- $j$  agent at time  $t$ . We assume that  $e$  is strictly positive during the first  $J^w$  periods of life and 0 afterwards. This implies that an agent's labor supply  $l_t$  drops to zero once he has reached age  $J^w$ . Hence, in period  $t$ , agents born in periods  $t$  to  $(t - (J - 1))$  are alive, but only agents born in periods  $t$  to  $(t - (J^w - 1))$  supply a strictly positive amount of effective labor.

In order to realize their optimal consumption and leisure path, agents save a portion of their income. Since we assume that there is no public pension system, all retirement consumption has to be financed out of private savings.<sup>9</sup> We assume that preferences are additively separable and that the instantaneous utility function displays a constant intertemporal elasticity of substitution (CIES). The discount factor is  $\beta$ , and the relative weights of consumption and leisure in an agent's utility are  $\theta$  and  $(1 - \theta)$ , respectively. Hence, an agent born in period  $t$  maximizes

$$U_t = \sum_{j=0}^{J-1} \beta^j \frac{[(c_{t+j}^t)^\theta (1 - l_{t+j}^t)^{1-\theta}]^{1-\sigma} - 1}{1 - \sigma} \quad (8)$$

subject to the budget constraints ( $\forall 0 \leq j \leq J - 1$ )

$$c_{t+j}^t = e_j l_{t+j}^t w_{t+j} + (r_{t+j}^C + r_{t+j}^K) K_{t+j}^t + R_{t+j} B_{t+j}^t - q_{t+j} K_{t+j+1}^t - B_{t+j+1}^t, \quad (9)$$

---

<sup>9</sup>Social security could easily be added into our framework. A setup without social security system can demonstrate that the (implicitly) low return on savings for the baby boomers cannot be fully attributed to the existence of a pay-as-you-go public pension system.

where  $c_{t+j}^t$  and  $l_{t+j}^t$  denote consumption and labor supply of a representative member of generation  $t$  in period  $t + j$ .  $K_{t+j+1}^t$  is the amount of physical capital purchased by a member of generation  $t$  at the end of period  $t + j$  (and used productively in period  $t + j + 1$ ), and  $B_{t+j+1}^t$  is the amount of real bonds purchased in period  $t + j$ . It follows from individual rationality and from our assumption that agents do not leave bequests that  $K_t^t = B_t^t = 0$  and that  $K_{t+J}^t = B_{t+J}^t = 0$ .

### 2.3 Equilibrium on the asset market

In equilibrium, the return on bonds has to be equal to the return on physical capital, i.e.,

$$R_{t+1} = \frac{r_{t+1}^C + r_{t+1}^K}{q_t}. \quad (10)$$

The no-arbitrage condition (10) has a straightforward interpretation: the return on bonds is equal to the sum of the rental rates in the consumption and the capital goods sector divided by the price of physical capital in the preceding period. If (10) is satisfied, the bond market is redundant and agents are willing to spend their entire savings on purchasing physical capital. However, it will be helpful to use the bond rate of return when deriving agents' saving functions.

Let  $s_{t+j}^t = q_{t+j}K_{t+j+1}^t + B_{t+j+1}^t$  denote the total savings of a generation- $t$  member in period  $t + j$ . Since bonds are in zero net supply, aggregate savings in period  $t$  have to equal the total value of capital goods produced in that period, i.e.,

$$\sum_{i=0}^{J-1} N_{t-i} s_t^{t-i} = q_t K_{t+1} = \frac{1}{\phi} K_t^{(\phi-1)/\phi} K_{t+1}^{1/\phi}. \quad (11)$$

The equation on the right follows from the price of a unit of capital  $q_t$  given in (6).

## 3 Equilibrium and dynamics with 3 generations

Having specified our model in a fairly general way so far, we will now drastically simplify matters by focusing on an economy with three living generations, two of them working (i.e., productivity in the third period of life,  $e_2$ , is 0). Moreover,

log–utility ( $\sigma = 1$ ) is assumed in this section. These simplifications will allow us to derive transparent closed form solutions and to identify the mechanisms that drive the evolution of factor prices and savings. We will start by describing the dynamics and the steady state for an economy with inelastic labor supply ( $\theta = 1$ ). In the following subsection we will then describe saving rates and labor supply in a setting with endogenous labor market participation.

### 3.1 Inelastic labor supply: Dynamics and steady state

It is easy to show that the saving functions of a generation– $t$  member in the first and second period of his life are given by

$$s_t^t = \frac{1}{1 + \beta(1 + \beta)} \left[ \beta(1 + \beta)e_0w_t - \frac{e_1w_{t+1}}{R_{t+1}} \right], \quad (12)$$

$$s_{t+1}^t = \frac{\beta^2}{1 + \beta(1 + \beta)} [R_{t+1}e_0w_t + e_1w_{t+1}]. \quad (13)$$

Note that, unlike in a 2–period setting with log–utility, aggregate savings depend on future returns and wage rates, since members of the young generation adjust their consumption to the anticipated time path of factor prices.<sup>10</sup> It follows from (11), (12), and (13) that the evolution of the capital stock is implicitly determined by

$$K_{t+1} = \left\{ \phi \left( \frac{N_t}{1 + \beta(1 + \beta)} \left[ \beta(1 + \beta)e_0w_t - \frac{e_1w_{t+1}}{R_{t+1}} \right] \right) \right.$$

---

<sup>10</sup>If  $\sigma \neq 1$ , savings for both the young and the middle–aged agents depend on future factor prices:

$$s_t^t = \left( \frac{1}{1 + \mu_t^t R_{t+1}} \right) [e_0w_t - \mu_t^t e_1w_{t+1}],$$

$$s_{t+1}^t = \left( \frac{1}{1 + \mu_t^t R_{t+1}} \right) \left( \frac{1}{1 + \mu_{t+1}^t} \right) [R_{t+1}e_0w_t + e_1w_{t+1}],$$

where  $\mu_t^t$  and  $\mu_{t+1}^t$  are defined as

$$\mu_t^t = \left\{ \beta R_{t+1} \left[ \left( \frac{\mu_{t+1}^t}{1 + \mu_{t+1}^t} \right)^{1-\sigma} + \beta \left( \frac{R_{t+2}}{1 + \mu_{t+1}^t} \right)^{1-\sigma} \right] \right\}^{-\frac{1}{\sigma}},$$

$$\mu_{t+1}^t = \left\{ \beta (R_{t+2})^{1-\sigma} \right\}^{-\frac{1}{\sigma}}.$$

$$+ \frac{N_{t-1}\beta^2}{1 + \beta(1 + \beta)} [R_t e_0 w_{t-1} + e_1 w_t] \Big) \Big\}^\phi K_t^{1-\phi}, \quad (14)$$

where  $w_t$  is given by (2), and  $R_t$  can be derived from (10), (3), (6), and (7). Given the evolution of the capital stock, we can derive the time paths of the factor prices and of all other endogenous variables.

### 3.1.1 The steady state

The steady state level of capital per unit of effective labor  $k \equiv \frac{K}{L}$  can be derived by using equations (10)– (11) and (14):

$$k^{1-\alpha} = \frac{\phi(1-\alpha)}{(e_0 + e_1)(1 + \beta(1 + \beta))} \times \left[ \beta e_0 + \beta^2 (e_0 (2 + \alpha\phi k^{\alpha-1} - \phi) + e_1) - \frac{e_1}{(1 - \phi) + \alpha\phi k^{\alpha-1}} \right] \quad (15)$$

**Lemma 1** *The steady-state value of  $k$  that is implicitly defined by equation (15) is unique and strictly positive.*

**Proof:** Apparently,  $k$  cannot be negative since  $k^{1-\alpha}$  is not defined for negative values of  $k$ . By comparing the LHS and the RHS it is also apparent that  $k = 0$  does not solve (15). Finally, since the LHS is strictly increasing in  $k$ , while the RHS is strictly decreasing, there is one point of intersection which defines the steady-state value of  $k$ .

## 3.2 Elastic labor supply: Saving rates and labor supply

As before, let  $s_{t+j}^t = q_{t+j} K_{t+j+1}^t + B_{t+j+1}^t$  be the total savings of a generation- $t$  member in period  $t + j$ . For notational simplicity we define  $\xi$  and  $\zeta$  as

$$\begin{aligned} \xi &\equiv \beta(1 + \theta\beta) \\ \zeta &\equiv \frac{1}{1 + \beta\theta} \left[ (1 + \beta)\theta + \frac{1 - \theta}{1 + \beta(1 + \theta\beta)} \right]. \end{aligned}$$

Note that  $\zeta = 1$  for inelastic labor supply (i.e.,  $\theta = 1$ ). Individual maximization over savings and labor supply yields the following expressions

$$s_t^t = \frac{\xi}{1+\xi} e_0 w_t - \frac{1}{1+\xi} \frac{e_1 w_{t+1}}{R_{t+1}} \quad (16)$$

$$s_{t+1}^t = \frac{\theta \beta^2}{1+\xi} (e_0 w_t R_{t+1} + e_1 w_{t+1}) \quad (17)$$

$$l_t^t = \theta + (1-\theta) \left[ \frac{\xi}{1+\xi} - \frac{1}{1+\xi} \frac{e_1 w_{t+1}}{e_0 w_t R_{t+1}} \right] \quad (18)$$

$$l_{t+1}^t = \zeta - \frac{\beta(1-\theta)}{1+\xi} \frac{e_0 w_t R_{t+1}}{e_1 w_{t+1}} \quad (19)$$

Young agents save more in a period with a high labor income and reduce their savings if they anticipate higher wages and lower returns. Moreover, equation (18) shows that the labor supply of the young generation crucially depends on future capital returns and wages. Growing wages and low future returns induce young agents to consume more leisure, while declining wages and high anticipated returns boost their labor supply. This picture is mirrored by the labor supply of the middle-aged generation. Note, finally, that (16) and (17) coincide with (12) and (13) if we set  $\theta = 1$  (i.e., leisure has no value in the utility function), and that the young generation's saving and labor supply response to changes in future factor prices is reinforced if we lower  $\theta$ .

Due to their large number, the baby boomer generation suffers from depressed wages in both working periods, as well as from a low return on their savings upon retirement. This income effect reduces the level of both consumption and savings, but since there is little scope for intertemporal substitution, their saving *profile* can be expected to be similar to that of a generation not affected by the boom. Adjacent generations, on the other hand, have substantial possibilities to adjust their consumption and labor supply, and this may dampen the effects of a one-time demographic shock: for the baby boomers' parents (the "old folks") the prospect of higher returns provides an incentive to save more, especially in the first period of their lives. They work hard when young, but retire early as wage rates are low in their second working period. Members of this generation clearly benefit from the stock market boom, but they also generate some benefits for the subsequent generation: due to their higher savings, the capital stock will be higher when the baby boomers enter the work force, and the old folks' reduced labor supply at middle-age further dampens the drop in the wage rate associated with the baby boom.

The baby boomers' offspring (the "spoiled brats"), on the other hand, inherits

a large capital stock. Unlike their grand–parents, members of this generation do not find it profitable to work much in the first period of their lives as they anticipate a sharp increase of wage rates once the baby boom generation has retired. Moreover, young–age savings are low for this generation since the large capital stock accumulated by their parents depresses future returns. The high young–age consumption of the “spoiled brats” (reducing the future capital stock and thus increasing returns when baby boomers retire) combined with their low labor supply further dampens movements of factor prices.

For a non–unity elasticity of intertemporal substitution  $1/\sigma$ , and for positive capital adjustment costs (i.e.,  $\phi < 1$ ) closed form solutions cannot be derived. The lower the former, the less inclined people are to substitute consumption and leisure across time. Capital adjustment costs, on the other hand, should lead to greater swings of the wage rate and should thus reinforce the substitution of leisure – in particular during the baby boomers’ active years when wages drop to very low levels. While it is clear that the baby boomers suffer from both a low wage and a low return on their investment, the impact on the welfare of adjacent generations is *a priori* unclear: both parents and children share one period of depressed wages with the boomers, but also benefit from a higher return on their savings (parents) or a high wage in their middle–age (children).

## 4 Simulations

In the previous section we have identified the mechanisms that drive the dynamics of our model. We will now explore the impact of different parameter choices on factor prices and agents’ optimal saving and labor supply decisions. Of course, this computational exercise does not deliver quantitatively realistic predictions on saving rates and factor prices. However, it is extremely helpful in checking the intuition developed above and in identifying the effects of different parameters and their interaction. We will especially focus on consumption and labor supply patterns of the baby boomers as well as their parents and offspring. Table 1 summarizes the parameters used for the different simulations as depicted in Figures 1–4. For all presented combinations of parameters the relative welfare of different generations — expressed as a consumption equivalent variation relative to the steady state without a baby boom — are depicted in Figure 5.

<i>Parameter</i>	<i>benchmark</i>	<i>alternatives</i>
size of a normal generation	1	
size of baby-boom generation (born in 4)	2	
$\alpha$ capital share	1/3	
$\phi$ investment technology (1 = neoclassical)	1	0.6
$\beta$ discount factor	1	
$\theta$ consumption/leisure trade-off	1	0.5
$\sigma$ 1 / elasticity of intert. subst.	1	2
$e_0$ productivity of young	1	
$e_1$ productivity of middle-aged	1	

Table 1: Parameter values for simulations.

We assume that in period 3, agents learn about a “baby boom” – i.e. the advent of a generation twice as large as normal – to take place in period 4, and that it is common knowledge that the size of cohorts will return to its steady state value in period 5. Implicitly we therefore also assume a baby bust following the baby boom generation. While this is clearly a rather extreme assumption, fertility data of most industrialized countries are not inconsistent with the assumed path. Figures 1–4 plot the evolution of factor prices and labor supply as well as saving and consumption paths for the three generations that are alive at each point in time. To make these plots comparable, all steady state variables are normalized to one. In all graphs labor supply, savings, and consumption of baby boomers, their parents, and their children are indicated by the symbols  $\circ$ ,  $\square$ , and  $*$ , respectively.

### Large swings in factor prices in the benchmark case

Figure 1 refers to the benchmark parameterization with  $\sigma = 1$  (logarithmic utility),  $\theta = 1$  (constant labor supply), and  $\phi = 1$  (costless adjustment of the capital stock). While Tobin’s  $q$  and labor supply are constant by definition, the wage and the interest rate exhibit considerable variation, moving in opposite directions.<sup>11</sup> In period 4, the large increase in labor supplied by the baby boom generation props up the interest rate while the wage rate decreases sharply. Once the baby boomers have left the labor market (period 6), both the huge capital

<sup>11</sup>Note that, with  $\phi = 1$ , the existing capital stock is not used in the production of new capital goods, and the interest rate is given by the return on capital in the consumption goods sector.



stock accumulated by this generation and the lower labor supply contribute to raising the wage and depressing the interest rate. In period 10, both factor prices have returned to the steady state. These time paths convey the essence of the baby boomers’ “saving crisis”: members of this generation earn low wages when they are young and face low interest rates when they have retired. However, as we will see shortly, the effect of these factor price movements on agents’ saving behavior may heavily depend on our assumptions about preferences and technology.

The saving behavior of the young generations in the third panel of Figure 1 reflects the anticipated evolution of  $w/R$ : the low wage and high interest rate in period 4 induce the young of period 3 to raise their savings (see equation 16). Conversely, the baby boomers’ children reduce their savings in period 5, anticipating a high wage and a low interest rate in period 6. The consumption paths in the fourth panel of Figure 1 mirror each generation’s saving behavior, which, in turn, depends on factor prices: while the generation born in period 3 chooses a consumption path that is rising over the life cycle, the young who are born in period 5 consume less at old age than in their youth. The explanation of this result is straightforward: for generation 3, high returns in the future, that is, in periods 4 and 5, provide an incentive to substitute old-age consumption for young-age consumption. On the other hand, members of generation 5 anticipate low returns in periods 6 and 7 and therefore have an incentive to enjoy a high consumption level during their youth. Notably, both forces contribute to dampening the swings of factor prices. Finally, the baby boomer generation’s consumption path has a hump shape, which reflects the low wage in the first and the low interest rate in the last period of this generation’s life cycle.

### **Elastic labor supply dampens swings in factor prices**

Dropping the assumption of a constant labor supply by setting  $\theta = 0.5$  does not alter the qualitative properties of our results. However, it is obvious from Figure 2 that the volatility of factor prices is substantially reduced, while movements in the young generation’s savings are reinforced. Equations (16) and (18) offer the key to understanding this result: lowering  $\theta$  reinforces the young generation’s saving response to the anticipated decrease of  $w/R$  in period 3 and provides an incentive to raise the labor supply in the same period (see the second panel of Figure 2). On the other hand, the young in period 5 anticipate that  $w/R$  will rise in period 6 and therefore reduce both labor supply and savings. Hence, the high

consumption and low labor supply of the “spoiled brats” who are raised by the baby boom generation is nothing but an optimizing reaction to the anticipated time path of factor prices. The fourth panel in Figure 2 reveals that, in spite of an endogenous labor supply, the consumption paths of the three generations have the same features as in the benchmark case. However, the baby boomer’s consumption is now less volatile than with exogenous labor supply, which is due to the fact that the endogenous labor supply response further dampens the swings of factor prices. Consumption paths of the young and the old generations in the fourth panel very much mirror the movements of the wage and the interest rate, respectively.

### **Elastic labor supply dominates intertemporal substitution ...**

Figure 3 demonstrates that choosing a lower intertemporal elasticity of substitution by setting  $\sigma = 2$  has almost no additional effect on the evolution of factor prices and labor supply. At first glance this is surprising, since we would expect that reducing agents’ willingness to exploit movements in relative factor prices should enhance the volatility of the wage and the interest rate. In fact, this pattern emerges if we set  $\sigma = 2$  while keeping the labor supply constant (i.e.  $\theta = 1$ ).<sup>12</sup> However, making labor supply endogenous by setting  $\theta = 0.5$  substantially dampens these movements, and in the end we are left with pictures that roughly equal the one in Figure 2. The last panel of Figure 3 demonstrates that setting  $\sigma = 2$  changes the consumption paths of the baby boomers’ parents and children: instead of monotonically increasing and decreasing, these paths now have a U-shape and a hump-shape, respectively. This is due to the fact that a lower intertemporal elasticity of substitution reinforces the income effect of interest rate changes. Hence, members of generation 3 raise their young age consumption in anticipation of higher returns while members of generation 5 reduce their consumption as a reaction to the lower returns in period 6.

### **... and capital adjustment costs**

In Figure 4 we consider the effect of capital adjustment costs by setting  $\phi = 0.6$ . This, of course, introduces a range of new effects into the model: not surprisingly, introducing adjustment costs reduces the volatility of the capital stock and thus dampens swings in the return on capital. Moreover, there is now

---

<sup>12</sup>The plots depicting this case are available on request.

a role for Tobin's  $q$  since the price of capital goods is not automatically equal to the price of consumption goods. Our specification implies that, with  $\phi < 1$ , the rental rate in the consumption goods sector is augmented by the returns earned in the capital goods sector, and that the magnitude of these additional returns depends on the current as well as on the future capital stock.

As the first panel of Figure 4 demonstrates, Tobin's  $q$  closely follows the evolution of the interest rate, but with a somewhat greater volatility, since it is not directly affected by (dampening) movements of employment. While the labor supply of the young does not differ by much from Figure 3, the second panel of Figure 4 reveals that, with costly adjustment, employment of the middle-aged exhibits much greater volatility. The reason is that, with  $\phi < 1$ , nonlabor income represents a greater part of total income, and that swings in the return on capital thus have a stronger effect on the labor supply of the middle-aged. The consumption patterns of the three generations further change as a result of introducing adjustment costs. The consumption of the baby boomers is now monotonically rising over the life cycle, generation 3 has a U-shaped, and generation 5 a hump-shaped consumption path. This observation can be explained with the help of equation (7), which shows that the rental price in the capital goods sector is increasing in the growth rate of the aggregate capital stock. Hence, members of the baby boom generation and their parents benefit from the huge capital accumulation that is taking place in periods 5 and 6, while the baby boomers' children receive a substantially lower income from leasing the existing capital stock to the capital goods sector.<sup>13</sup> Since the intertemporal elasticity of substitution is smaller than one, this effect further raises the young-age consumption of generation 3 and depresses the young-age consumption of generation 5.

### **Welfare measures depend crucially on the chosen parameters**

Per capita welfare comparisons for all four parameter combinations are shown in Figure 5. Welfare effects are measured as a consumption equivalent variation (i.e., as a percentage of life-time consumption) relative to an artificial steady state without the demographic shock.<sup>14</sup> The three generations we are mainly

---

<sup>13</sup>Plots that depict the evolution of the aggregate capital stock for different parameter values are available on request.

<sup>14</sup>More precisely, we ask the following question: What fraction of life-time consumption would we have to add to the no-baby-boom case in order to achieve the same utility as for the generation of interest during the baby boom? For the specification of utility (8) used in our

concerned with are again marked with their respective symbols. Not surprisingly, the baby boomers themselves suffer the most from being born into a large generation. However, the relative welfare depends crucially on the weight of leisure in the utility function. If agents can substitute labor supply across time, baby boomers are clearly better off than in an inelastic setup despite the fact that they themselves have little scope for intertemporal adjustments. The positive effect is mainly due to the fact, that their parents and children substitute labor away from the two low-wage periods when the boomers are in the work force. The impact of elastic labor supply on parents and the boomers' offspring is ambiguous: As before, shifting working hours away from periods of depressed wages is welfare increasing. However, the general equilibrium effect of these shifts on factor prices, reduces this advantage to a certain degree. For the baby boomers' parents elastic labor supply leads to a lower wage in their first working period, and a lower return to savings relative to the inelastic case. For their grandchildren — the baby boomers' children — the countervailing effects stem from a lower wage rate in their second working period and lower returns in period 7. Note that both the baby boomers' grandparents (the generation born in period 2) and their grand-children (born in period 6) enjoy a positive welfare effect regardless of the parameterization of the model. The former experience a windfall gain due to an increased return on their savings. The latter, on the other hand, inherit a relatively high capital stock leading to relatively high wages in the aftermath of the baby boom.

Let us summarize: Figures 1–4 show that endogenizing the labor supply and allowing for convex capital adjustment costs substantially dampens the swings of factor prices after a temporary demographic shock and may modify the consumption profiles of different generations. The effect of adjustment costs is driven by the fact that existing capital gets a higher reward if further growth of the capital stock is anticipated. Endogenous labor supply, on the other hand, creates the main channel through which the optimizing behavior of the baby boomers' analysis, this fraction  $\gamma$  can be calculated as follows:

$$\begin{aligned}\gamma &= \left( \frac{U}{U_{\text{no-boom}}} \right)^{\frac{1}{\theta(1-\sigma)}} - 1, & \text{if } \sigma \neq 1; \\ \gamma &= \exp \left\{ \frac{U - U_{\text{no-boom}}}{\theta \sum_{i=1}^J \beta^{i-1}} \right\} - 1, & \text{if } \sigma = 1;\end{aligned}$$

$U$  and  $U_{\text{no-boom}}$  denote life-time utility for the generation of interest with and without the demographic shock, respectively.

parents and children partially offsets the effects of the demographic shock. Figure 5 shows that welfare effects depend crucially on the used parameterization, notably the elasticity of labor supply. While it is clear that these estimates must not be taken at face value, they are a clear indication that welfare effects may be grossly over-estimated if intertemporal channels of substitution are not taken into account.

## 5 Conclusions

The huge baby boom and the subsequent baby bust in most industrialized countries have obvious consequences on capital accumulation and factor prices. In this paper, we have shown that the passage of a large generation creates substantial swings in wages and capital returns — in particular a stock-market boom during the baby boomers’ working years and a subsequent dramatic decline in returns.

The economic impact of this large generation, however, is dampened by the responses of both the baby boomers’ parents and children. These intertemporal substitution effects are especially pronounced if labor supply is endogenous and if agents work for at least two periods.

Anticipating the demographic shock, the pre-boomers (the “old folks”) save more in their early working years as they can expect a higher return on their savings. The boomers’ massive capital accumulation, on the other hand, induces later generations (the “spoiled brats”) to reduce their savings. Both reactions reduce the swings of factor prices generated by the baby boom. These offsetting effects are further reinforced by the fact that the baby boomers’ parents choose to work less when middle-aged, while the baby boomers’ children do the exact opposite: enjoy leisure while young and work harder when middle-aged. In this light, the tendency to retire early (parents) or postpone entry into the labor market (offspring) may be interpreted as optimal responses to current demographics that contribute to defusing the baby boomers’ looming saving crisis.

## References

- Abel, Andrew B.:** “The Effects of a Baby Boom on Stock Prices and Capital Accumulation in the Presence of Social Security,” Mimeo, University of Pennsylvania, (2000).
- Basu, Parantap:** “An Adjustment Cost Model of Asset Pricing” *International Economic Review*, 28: 609 – 621, (1987).
- Bohn, Henning:** “Will Social Security and Medicare Remain Viable as the U.S. Population is Aging” *Carnegie–Rochester Conference Series on Public Policy*, 50: 1–53, (1999).
- Constantinides, George M. & Donaldson, John B. & Mehra, Rajnish:** “Junior Can’t Borrow: A New Perspective on the Equity Premium Puzzle,” NBER working paper 6617 (1998).
- Lucas, Robert E.:** “Asset Prices in an Exchange Economy,” *Econometrica*, 46: 1429–1445, (1978).
- Magill, Michael & Quinzii, Martine:** “The Stock Market in the Overlapping Generations Model with Production,” Mimeo, University of Southern California and University of California, Davis, (1999).
- Ríos–Rull, José–V́ctor :** “Population Changes and Capital Accumulation: The Aging of the Baby Boom,” *Advances in Macroeconomics* 1(1). (2001).
- Welch, Finis:** “Effects of Cohort Size on Earnings: the Baby Boom Babies’ Financial Bust,” *Journal of Political Economy*, 87(5): S65–S97, (1979).

## A Constant capital stock

The boundary case of a constant capital stock cannot be analyzed by simply setting  $\phi = 0$  in all expressions derived so far. Instead, we have to slightly modify our framework. As an alternative to buying real bonds, people may still purchase physical capital that is rented to consumption goods firms in the following period. However, instead of being used up in the production of capital goods later on, the full amount of physical capital is returned to its owners and can be either sold or stored. Hence, buying physical capital amounts to purchasing shares of consumption goods firms which entitle the owner to receive a dividend and which can be sold in later periods.

We define  $V_{t+j} = q_{t+j}K_{t+j+1}$  and  $x_{t+j}^t = K_{t+j+1}^t/K_{t+j+1}$ . These expressions have a straightforward interpretation: in period  $t + j$ , a representative member of generation  $t$  purchases real bonds and a share  $x_{t+j}^t$  of all consumption goods firms, whose aggregate value is  $V_{t+j}$ . In addition, we define the *dividends* of all consumption goods firms as  $d_{t+j}^C = r_{t+j}^C K_{t+j}$  and replace the rental price of capital in the capital goods sector ( $r_{t+j}^K$ ) by  $q_{t+j}$ . By taking into account that  $K_{t+j+1} = K_{t+j}$ , we thus get  $(r_{t+j}^C + r_{t+j}^K)K_{t+j}^t = x_{t+j-1}^t(d_{t+j}^C + V_{t+j})$ , and we can rewrite (9) as

$$c_{t+j}^t = l_{t+j}^t e_j w_{t+j} + x_{t+j-1}^t (d_{t+j}^C + V_{t+j}) + R_{t+j} B_{t+j}^t - x_{t+j}^t V_{t+j} - B_{t+j+1}^t. \quad (20)$$

As agents are born without assets,  $x_{t-1}^t$  and  $B_t^t$  are zero. Moreover, it is not optimal to die with positive amounts of assets, hence  $x_{t+J-1}^t = B_{t+J}^t = 0$ .

With a constant capital stock, the no-arbitrage condition in (10) is replaced by

$$R_{t+1} = \frac{d_{t+1}^C + V_{t+1}}{V_t}. \quad (21)$$

The sum of dividends and the resale value divided by the initial price of consumption goods firms has to equal the return on bonds.

Finally, since the shares of the aggregate firm value purchased by individual agents have to add up to one in every period, that is  $\sum_{i=0}^{J-1} N_{t-i} x_t^{t-i} = 1$ , and since aggregate bond holdings are zero, the equilibrium condition (11) becomes

$$\sum_{i=0}^{J-1} N_{t-i} s_t^{t-i} = V_t. \quad (22)$$

We define the price-earnings ratio in period  $t$  as  $\psi_t \equiv V_t/d_t^C$ . The steady-state value of  $\psi_t$  is implicitly given by

$$\psi = \frac{(1 - \alpha)}{(e_0 + e_1)\alpha(1 + \beta(1 + \beta))} \times \left[ e_0\beta(1 + \beta) - e_1\frac{\psi}{1 + \psi} + \beta^2 \left( e_0\frac{1 + \psi}{\psi} + e_1 \right) \right] \quad (23)$$

Following the lines of Lemma 1, it is easy to show that this steady-state value is unique and strictly positive.



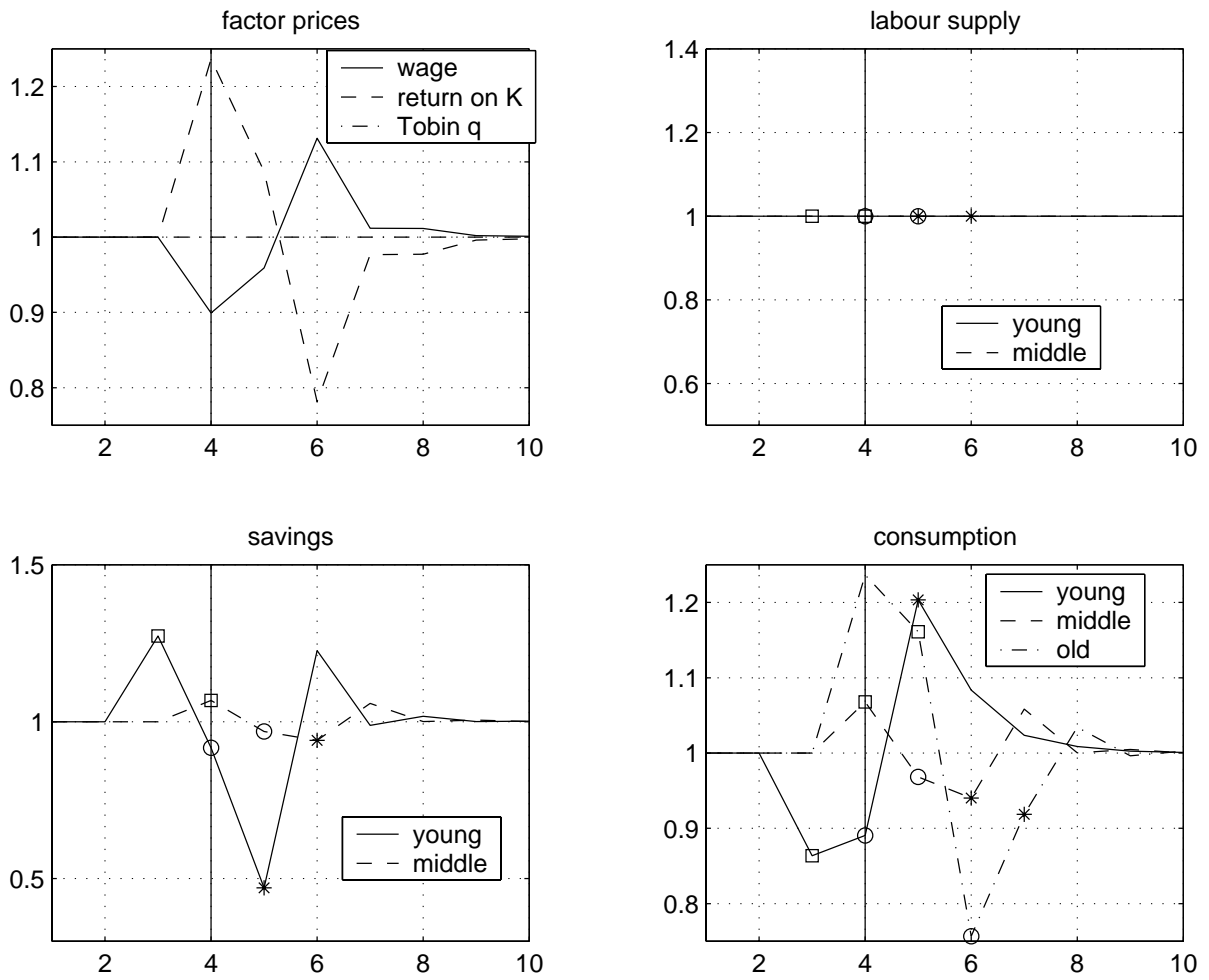


Figure 1: *Benchmark case,  $\sigma = 1$ ,  $\theta = 1$ , and  $\phi = 1$ . Symbols used on trajectories denote baby boomers =  $\circ$ , their parents =  $\square$ , and the boomers' children =  $*$ .*

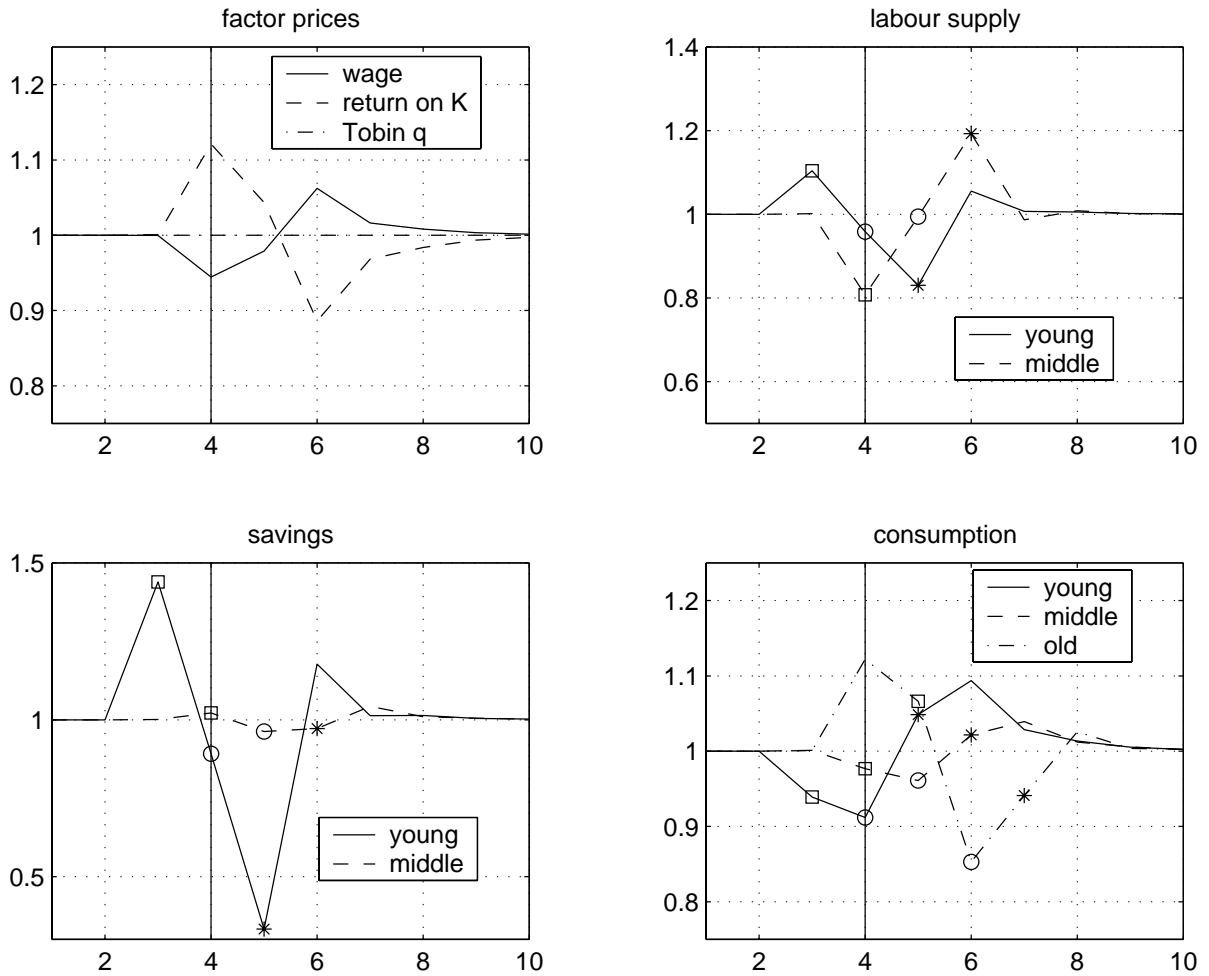


Figure 2: *Endogenous labor supply*,  $\sigma = 1$ ,  $\theta = 0.5$ , and  $\phi = 1$ . Symbols used on trajectories denote baby boomers =  $\circ$ , their parents =  $\square$ , and the boomers' children =  $*$ .

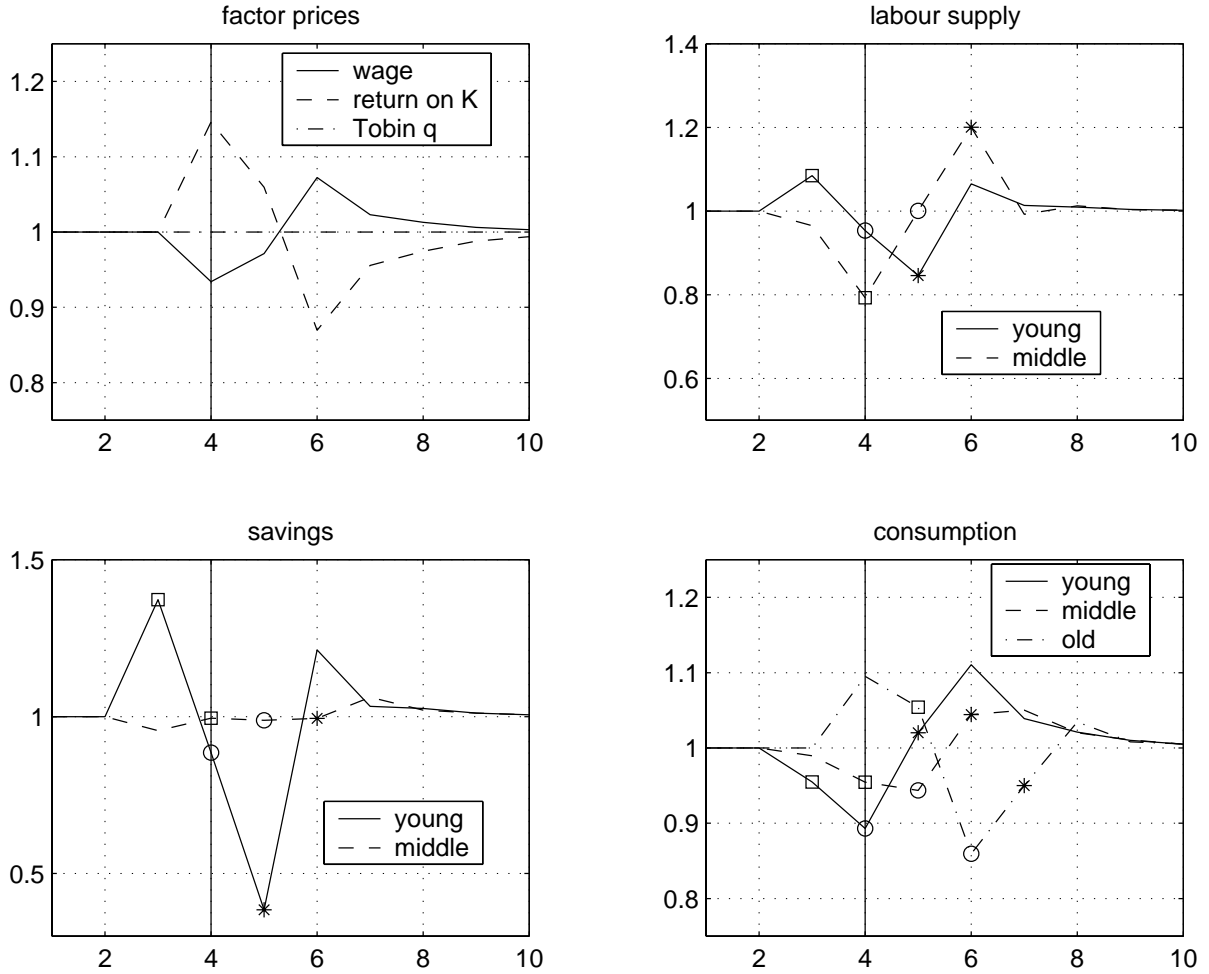


Figure 3: *Smaller degree of intertemporal substitution and endogenous labor supply  $\sigma = 2$ ,  $\theta = 0.5$ , and  $\phi = 1$ . Symbols used on trajectories denote baby boomers =  $\circ$ , their parents =  $\square$ , and the boomers' children =  $*$ .*

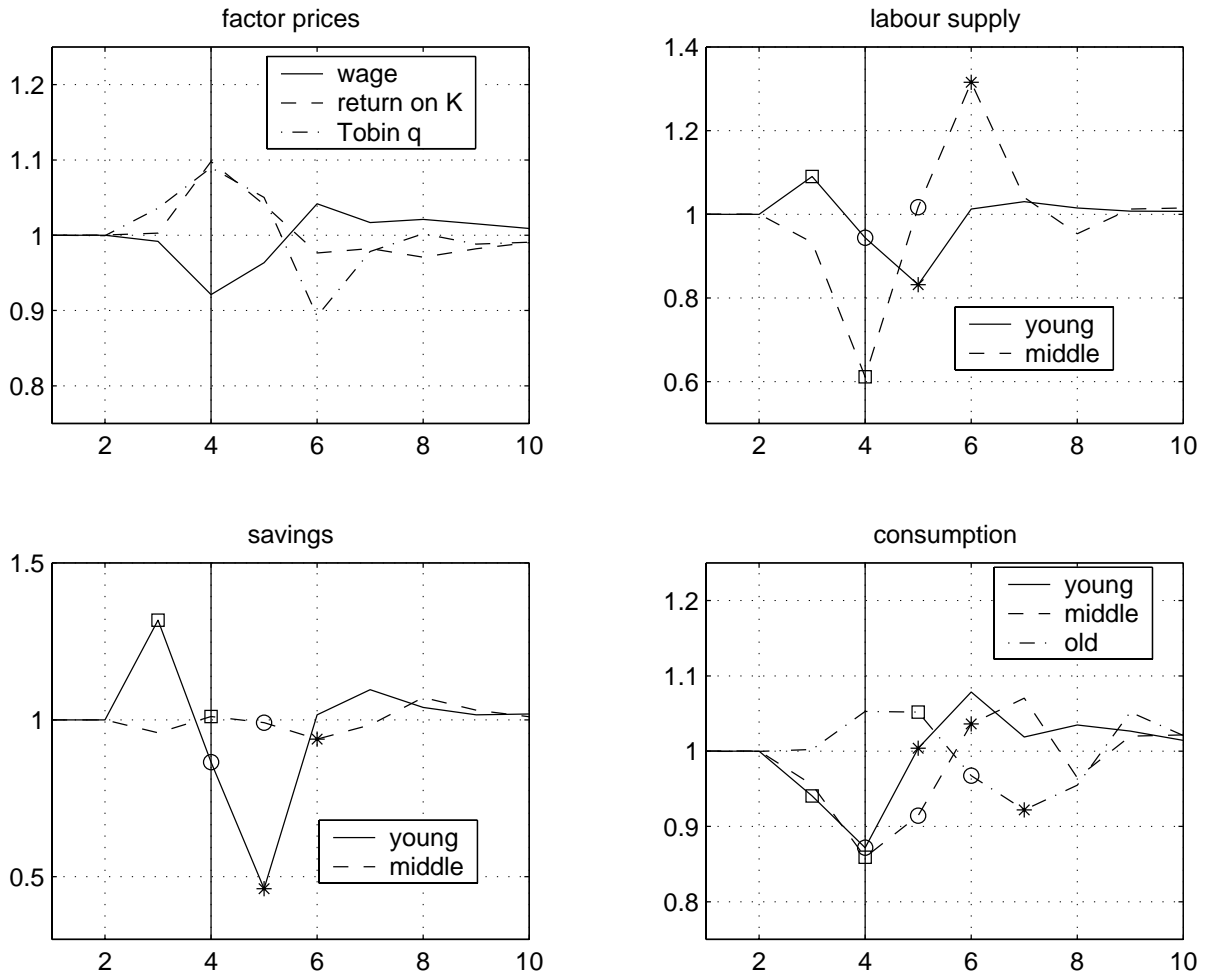


Figure 4: *The Full Monty*,  $\sigma = 2$ ,  $\theta = 0.5$ , and  $\phi = 0.6$ . Symbols used on trajectories denote baby boomers = o, their parents =  $\square$ , and the boomers' children = \*.

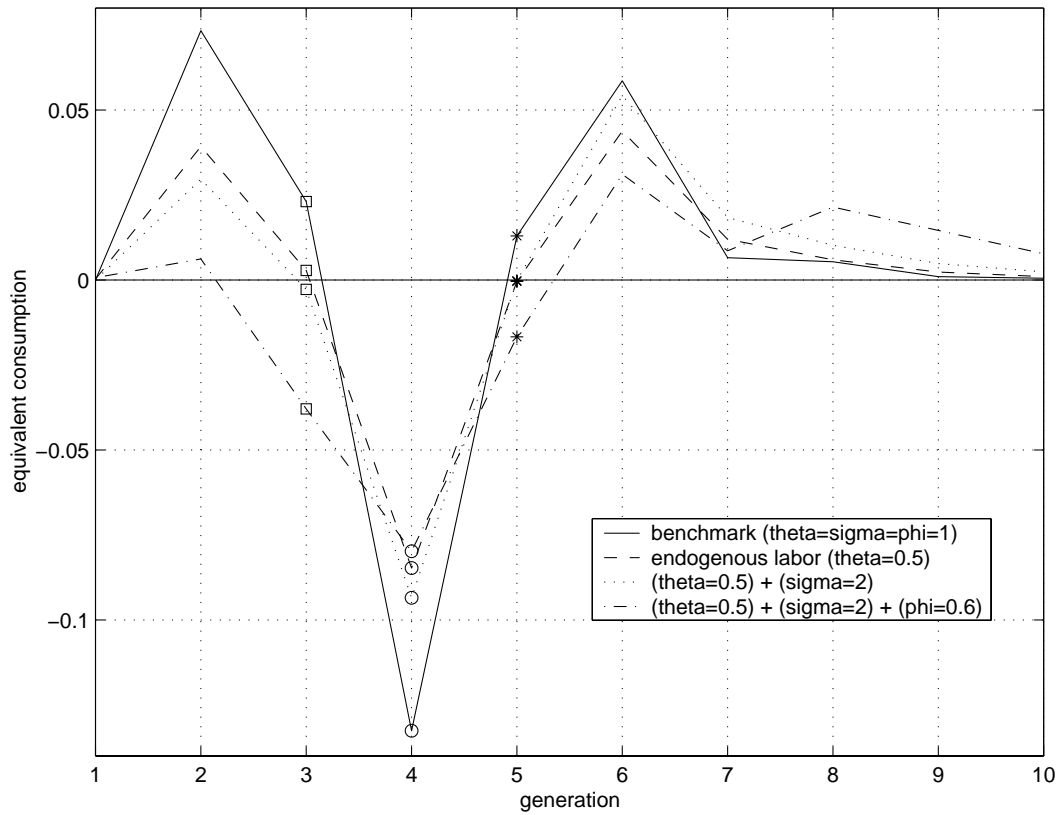


Figure 5: *Lifetime utility of different generations, expressed as a consumption equivalent difference to the long-run equilibrium. Generation 4 are the baby boomers = o, generation 3 their parents = □, and generation 5 their children = \*.*