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Family Size, Looming Demographic Changes and the Efficiency of Social Security Reform

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Abstract

This paper analyses the effects of ageing and child support in a model with endogenous fertility and Pay-As-You-Go (PAYG) pensions. First, we show that the endogeneity of fertility makes society vulnerable to both pessimistic beliefs and changes in life expectancy. In particular, we show that the private fertility choice may not coincide with the social optimum, due to the existence of two external effects of a child on society as a whole. The market outcome without government intervention is efficient, however, as both externalities exactly cancel out in that case. If the government wants to redistribute towards the old, it cannot replicate the command optimum by merely applying lump-sum transfers, but rather needs a child allowance scheme to effectively alter the number of offspring chosen by households. Finally, we analyse whether a Pareto-improving social security reform is possible. It is shown that a mere reduction of the PAYG-scheme cannot be Pareto-improving, but a combined policy of decreasing the PAYG-tax and introducing child allowances can be.

JEL classification: C61, D10, D84, H55, J13, J14, J18, J26

Keywords: child allowances, ageing, pensions, endogenous fertility, rumours, overlapping generations, social security reform

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1 Introduction

As many societies will experience an ageing population in the near future, concerns about the feasibility of current pension and health care arrangements are growing. Indeed, the fact that most of these old-age facilities are …nanced on a Pay-As-You-Go (PAYG)-basis may eventuate in serious problems when the share of the old in total population is steadily rising. Figure 1 shows that this will be the case in the upcoming decades for most OECD countries. Although extreme cases like Italy and Japan are striking, also the United States will face a considerably aged population, as its relative number of old by 2040 will be 50% higher than its present value.

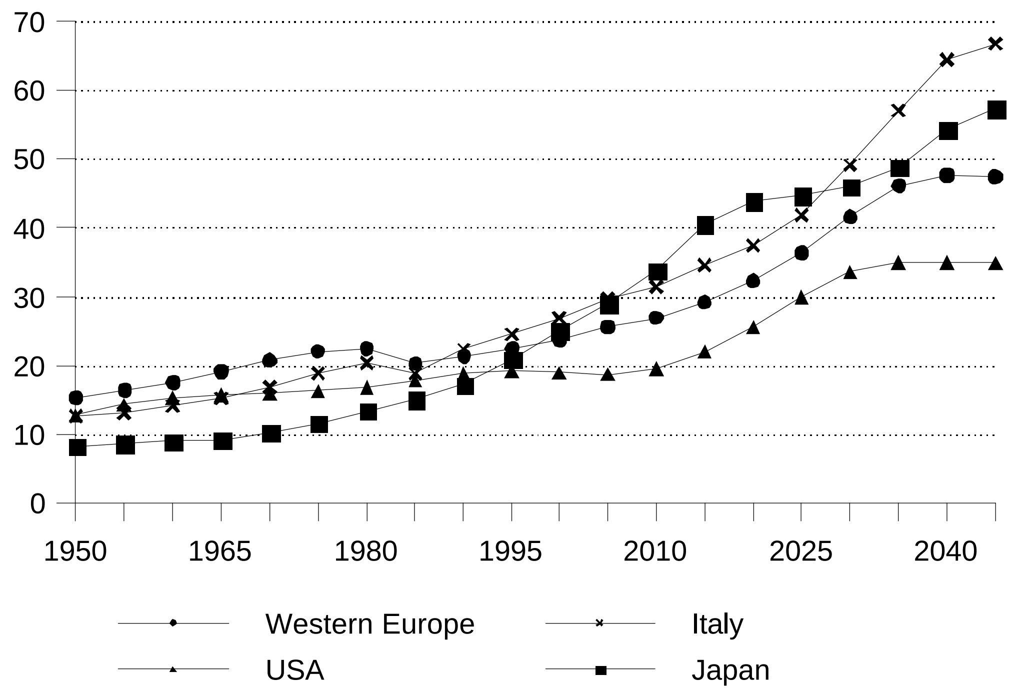

Figure 1: Dependency Ratio Age 65+ (per 100).

Source: World Population Prospects: The 1998 Revision.

The underlying causes of this demographic trend are depicted in Figures 2 and 3. As can be seen from Figure 2, the number of children per woman has fallen dramatically since the 1960s from around 2.5, and is expected to stay at a rather low level of 1.8. Furthermore, people (will) live longer as Figure 3 shows. Due to improved medical treatments and better nutrition, life span is expected to be about 7 years longer in the next …fty years. If the retirement age does not change, this amounts to 60% of the average time that people nowadays spend in retirement.

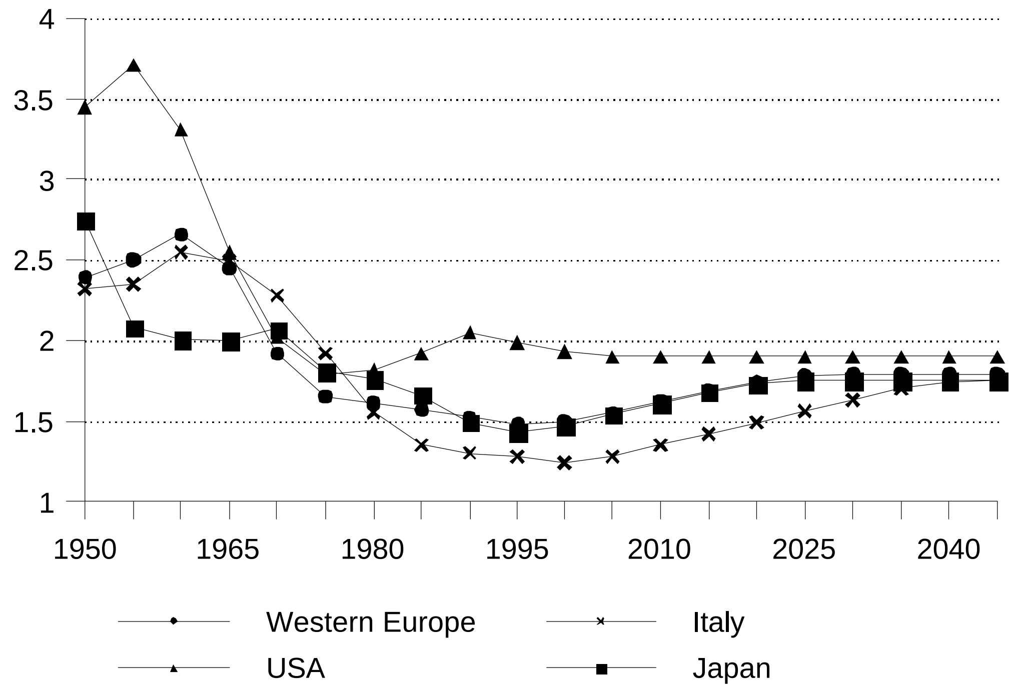

Figure 2: Total Fertility Rate (per woman).

Source: World Population Prospects: The 1998 Revision.

When analysing the economic consequences of such demographic shifts, most economists model population ageing as some exogenous shock by which the economy is hit, thereby often assuming the (drop in the) rate of fertility to be exogenous.[1] This, however, completely passes over the fact that, at least in western countries, people are to a very large extent able to freely choose the number of children they desire. The fertility rate should therefore rather be treated as an endogenous variable, that is, as the result of a rational choice that is in‡uenced by economic constraints and incentives. As Razin and Sadka (1995) put it, once preferences are shaped by cultural, ethnic, sociological and other factors, economics comes into play. Economic

[1] Malthus (1798) was the …rst economist who examined demography and dealt with fertility as an endogenous variable. In his work, though, population growth would approach the biological limit if it were not restricted by the moral restraints and misery that he described. These restrictions, however, cannot explain the recent drop in fertility, as many of the involved economies experienced substantial economic growth over the last decades.

theory can thus help in explaining why the observed decline in the (desired) number of children would occur, and accordingly, many such explanations have been put forward.[2]

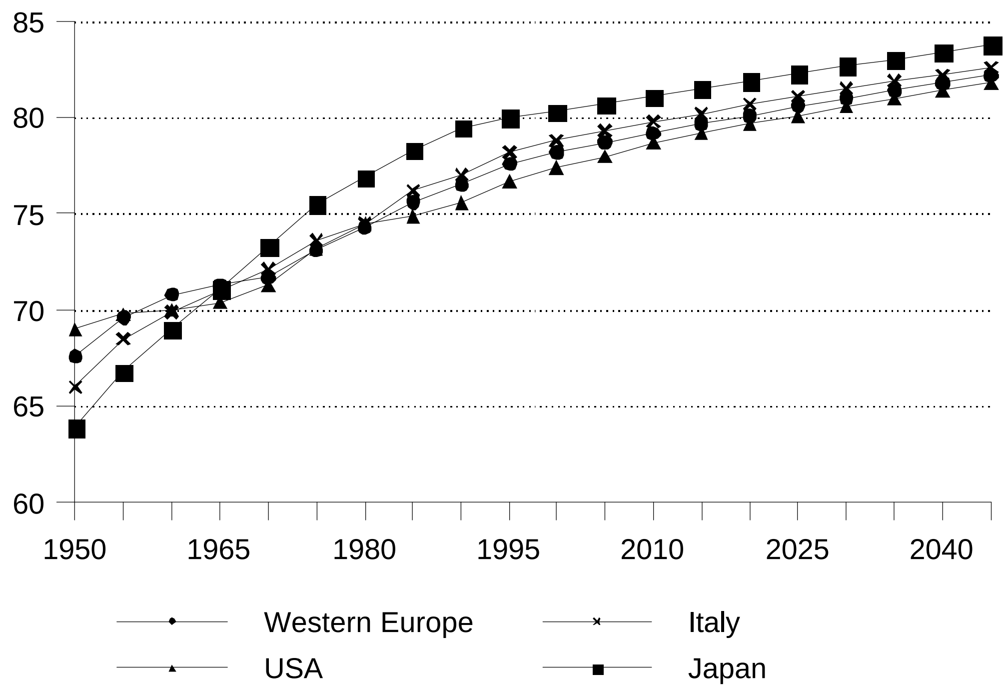

Figure 3: Life Expectancy at Birth (in years).

Source: World Population Prospects: The 1998 Revision.

According to the so-called new home economics, originated by Leibenstein (1957) and Becker (1960), and which is the approach we will take in this paper, people decide to have a certain number of children simply because they directly enter their utility functions like any other consumption good. Consequently, standard consumer demand theory can be applied to explain the evolution of family size. If o¤spring is not a Gi¤en good, then in line with this, better labour force opportunities of women

[2]Obviously, as Becker (1991) pointed out, the increased availability of contraceptives seems an important explanation of the drop in fertility. Yet one could argue that this availability is merely an induced response to the underlying decrease of the demand for children. It thus does not explain why people actually have these preferences. Furthermore, di¤erent birth control methods already existed before the pill was invented. Besides, "The decline [of fertility] began in the 1950s in countries like the United States and Japan, although the pill is illegal in Japan and was not extensively used in the United States until the 1960s" (Becker, 1991, p. 144).

and higher (real) wage rates increase the opportunity costs of rearing a child, and can therefore account for the decline in fertility.[3] Another explanation may be found in the fact that the number and availability of alternative pleasant commodities and activities has increased. A second motive for rearing children is provided by the old-age security hypothesis, as described by e.g. Bental (1989) and Cigno (1992), which states that people choose children because offspring has the characteristics of a capital good, in that it will provide services to the retired parents. Children thus indirectly enhance utility by serving as insurance against the risk of old-age dependency. They have experienced increasing competition in this respect too, viz. from financial assets since capital markets have become more developed and accessible to many in society.[4] Moreover, modern societies are typically characterized by public insurance against the risk of old-age dependency through public pension schemes and collective health care arrangements. These are nearly all financed on a PAYG-basis, which inevitably implies that part of the benefits of a child mainly reveal itself in a growing tax base and therefore fall to society as a whole. Those benefits thus become imperceptible to the individual (atomistic) parent. Put differently, children nowadays involve positive social externalities. Accordingly, Folbre (1994) typifies offspring as a public (rather than a private) capital good. Because of this, individuals do not consider their progeny as 'investment goods' any more, although they still are for the entire society. So the old-age security hypothesis no longer explains the individual 'demand for children'. On the contrary, people can instead free ride on the public system and neglect the positive social externalities that emanate from their offspring by having just a few children (or none at all) and still being entitled to complete old-age insurance. Considering the fact that raising a child implies high (and, according to many, increasing) parental expenditures for a substantial part of the parents' life, this free-riding behaviour seems quite a plausible explanation for the drop in population growth.

Altogether, family size is typically a decision that impacts the household for a fairly long time. When deciding on the number of children, agents therefore have to form expectations about the financial position during (the rest of) their lives, for that is nearly always uncertain. An important determinant of these expectations is the generosity of the social security system once people are retired. A society with relatively few young citizens, for instance, will be more likely to have low pension benefits. This is indeed what people nowadays fear to occur, urging them to save a lot. Moreover, a PAYG-scheme inherently implies dependency on other generations, whose solidarity is not guaranteed beforehand. Sjoblom (1985) and Kotlikoff et al.

[3] Becker and Lewis (1974) stated that not children per se, but rather their services enter the parents' utility function. These do not only depend on the number of children, but also on their quality. Substitution of quality for quantity (due to a change in the parents' preferences or an increasing price of quality) may then be another explanation for the drop in fertility.

[4] On the other hand, this implies better opportunities to smooth income over life, thus increasing welfare and consequently causing a (positive) income effect on the number of children, which may dominate the negative substitution effect. See e.g. Cigno (1992) and Razin and Sadka (1995).

(1988) demonstrate that a public pension system re‡ects a social contract between old and young generations that can be passed on to successive young generations. Although every generation has an incentive to perform the contract, it is not sure to what extent future generations are willing to contribute to old-age support. As a consequence, the perceived extent to which old-age consumption possibilities are limited plays an important role in the savings decision. Growing pessimism towards the future will induce people to cut back on their expenditures when young, including those for o¤spring if fertility is endogenous and the old-age security hypothesis plays no role in fertility choice. These bad feelings may be rational or totally irrational, based on rumours or generally felt sentiments. Anyway, a declining number of children diminishes the internal rate of a return of the PAYG-scheme, which indeed implies less scope for bounteous old-age support. So the fear of meagre old-age can turn out to be a self-ful…lling prophecy, leading to a 'collapse' of the PAYG-pension system and higher savings, i.e., a (partial) transition to a funded system. The greater the extent to which progeny has the features of a luxurious good, the more amenable society will be to such sentiments. Furthermore, an increase in the expected life span will in‡uence the rate of population growth when fertility is endogenous. If people want to save more because of their increasing longevity, they would do that partly by having fewer children if o¤spring is only considered as a consumption good and the social externalities are neglected. But this only aggravates the ageing problem since one type of ageing leads to another type of ageing and the dependency ratio rises even further.

This paper will assess the sensitivity of social security arrangements in a society where fertility is endogenous, and look at the role the government can play in improving welfare. It is organized as follows. Section 2 presents the basic model. We will consider a small open economy where non-altruistic individuals derive utility from material consumption and the number of their children. At the beginning of their lives, they decide on the allocation of their lifetime income. When old, the government provides a pension bene…t that is …nanced on a PAYG-basis.

Section 3 shows that the endogeneity of fertility makes society vulnerable to both pessimistic beliefs and increasing longevity. Throughout the paper, individuals are assumed to have perfect foresight. Subsection 3.1 is an exception in this respect because there we will assume that individuals are not able to perfectly foresee the future bene…t levels and that they consequently form adaptive expectations. Furthermore, at a certain point in time, some generation is assumed to attach a positive probability to a scenario in which a setback in social security occurs, re‡ecting pessimistic beliefs. This turns out to have a negative impact on both consumption and the number of children, thus narrowing the future tax base. As a result, the actual pension bene…t decreases, making the pessimistic beliefs seemingly come true. In Subsection 3.2 we turn back to perfect foresight and analyse the e¤ects of increasing lifetime. It again appears that individuals desire to save more and thus cut …rst-period spending, including those on o¤spring, because they do not take the positive externalities of their children into account. Contrary to the pessimistic beliefs analyzed in Subsection 3.1, increasing longevity is persistent, and so are its long-run consequences.

Because offspring has the features of a public good, all this raises the question whether the individual fertility choice is also optimal from a societal point of view. Section 4 therefore describes the command optimum. We find that there are two opposite externalities involved. First, an extra child implies a higher future output, and second, an extra child reduces the capital-labour ratio (or per capita debt in a small open economy). Parents who do not take these externalities into account are thus likely to give birth to a suboptimal number of children. We show, however, that the market solution without government intervention is Pareto-efficient as both externalities exactly cancel out. But if the government wants to redistribute between generations, the command optimum cannot be replicated by merely applying lump-sum transfers, but rather needs an additional instrument that induces substitution effects so as to effectively alter the number of children chosen by households. In particular, if the government redistributes from the young to the old, children on balance involve a positive externality and a system of child allowances in addition to the PAYG-transfers makes the achievement of the first-best outcome possible. If, however, the resources are transferred in the opposite direction, i.e., from the old to the young, then the net external effect of offspring is negative and a tax per child is necessary to achieve Pareto-efficiency. A similar conclusion is also drawn by Harford (1998), where children entail a (negative) pollution externality created by their consumption of particular commodities in the future.

Section 5 analyses whether social security reforms can be implemented in a Pareto-improving way, i.e., without utility loss for any generation, if one starts from a situation in which there only exists a PAYG-pension scheme. We first discuss the effects of introducing a system of child allowances that internalizes the positive net externality that offspring generates in this setting. We show that introducing such a system is a Pareto-improvement if the subsidy per child is not too high. Next, we elaborate on the effect of reducing the size of the PAYG-scheme. This is a social security reform that is discussed in many western countries in reaction to the ageing of the population they experience, based on the notion that a lower PAYG-tax would be optimal in the long run when the economy is characterized by dynamic efficiency. It is well known, however, that such a decrease in the size of PAYG-scheme is not Pareto-improving when fertility is exogenous (see Verbon, 1988, and Breyer, 1989), unless there is some externality involved. If abolishing the PAYG-scheme goes along with the elimination of a negative external effect, as for example in Homburg (1990), then a Pareto-improving transition to a funded system is possible. Likewise, such a transition can be achieved if the increase of savings that results when the PAYG-scheme is reduced has positive external effects (see Belan and Pestieau, 1998). One might therefore conjecture that downsizing the PAYG-scheme yields a Pareto-improvement when fertility is endogenous, since it decreases the external effect of the pension system on the number of offspring, thus realizing a positive welfare effect that enables the

Pareto-improving transition to funded pensions that is not possible if fertility were exogenous. Yet, we prove that just like in the models of Verbon (1988) and Breyer (1989) with exogenous fertility, merely reducing the PAYG-tax does not allow for a Pareto-improvement. We also show, however, that downsizing the PAYG-scheme can bring about a Pareto-improvement if it is combined with the introduction of a child allowance scheme. The reason for this is that a higher number of children not only increases PAYG-pension benefits for a given social security tax, but also implies a direct utility gain. The sum of these gains, that fall to all future generations, can outweigh the utility loss suffered by the initial retired for the decrease in their pension benefit. Hence, by appropriate intergenerational redistribution (i.e., after completely compensating these retirees), no generation is worse off and at least one gains from the transition policy.

Section 6 winds up with some concluding remarks.

2 The Economy

In this section we present a two-period overlapping generations model of a small open economy that will be applied throughout the paper. We first briefly describe the production side of the economy, then we introduce the households. Furthermore, we describe the social security system superimposed on the economy. The section concludes with the first-order conditions that follow from optimizing household behaviour.

Production

Consider a small open economy that consists of a large number of identical agents, who only differ in age. Production is described by a standard neoclassical constant-returns-to-scale production function,

$$y_t = f(k_t); \tag{1}$$

where k_t and y_t stand for the amount of capital and the output per young individual in period t, respectively. Perfect competition among producers gives the usual equilibrium conditions on the production factor markets, $r_t = \frac{df(k_t)}{dk_t} \equiv f'(k_t)$ and $w_t = f(k_t) - f'(k_t)k_t$, where r_t is the interest rate and w_t denotes the corresponding wage rate in period t. Under the assumption of a small open economy, the interest rate is exogenously determined at the world capital markets ($r_t = r; \forall t$) and consequently both the capital-labour ratio and the wage rate are constant[5]

$$k = f'^{-1}(r); \tag{2.a}$$

$$w = f(k) - f'(k)k: \tag{2.b}$$

[5] The assumption of a small open economy is made for analytical convenience and does not qualitatively affect the results in the paper.

Households

The economy is populated by non-altruistic individuals who live for two periods, such that in each period, both a young and an old generation are alive. Apart from age, individuals are identical. Every young individual faces a probability of ε to grow old, so $1 - \varepsilon$ is the fraction of young that dies after one period of life. When young, the agent inelastically supplies one unit of labour. Part of her gross wage income is taxed away by the government via an exogenous lump-sum tax (τ); the remainder is either spent on consumption (both material consumption (c^y) and offspring[6] (n)) or is saved for old age (s). That is, we assume children to be viewed as normal goods, which can be 'bought' at an exogenous price p. This price reflects the cost of raising a child in terms of material consumption goods.[7] Consequently, at any time t, first-period consumption is restricted by the following budget constraint,

$$c_t^y + pn_t = w - \tau - s_t. \tag{3}$$

In the second period of her life, the individual is retired and entitled to a public pension benefit. She derives utility from old-age (material) consumption (c^o)[8], which is financed from the return on first-period savings and the pension benefit (η). Savings are invested in annuities or through an actuarially fair pension fund. As only a fraction ε_{t+1} of young savers born at time t survives to period $t+1$, the assets of those who deceased fall to the surviving contemporaries. The total return on the loans is therefore $\frac{1+r}{\varepsilon_{t+1}}$. The second-period budget constraint can thus be written as

$$c_{t+1}^o = \frac{1+r}{\varepsilon_{t+1}} s_t + \eta_{t+1}. \tag{4}$$

As for the social security scheme, a young individual does not know the level of the future public pension benefit when deciding how much to save. She therefore has to form expectations on the future generosity of the scheme. Throughout the paper we assume agents to form rational expectations. As we abstract from factual uncertainty, this boils down to perfect foresight. Subsection 3.1 is an exception in this respect. There we will assume that expectations are non-rational, at least in the short-run, and liable to (irrational) rumours.

Young people derive utility from their children (n_t) and material consumption (c_t^y), whereas the retired value material consumption (c_{t+1}^o) only, as mentioned above.

[6]For simplicity we neglect gender differences as well as the difference between an individual and a household. Furthermore, we assume that every individual is able to freely choose a number of children from the set of nonnegative real numbers. The quality of children is constant and equal for all children.

[7]See e.g. Van Praag and Warnaar (1997) for a survey on the measurement of the cost of children. These costs include food, clothing, housing, child care and education. Longman (1998) estimated that these costs can amount to $200,000 (corrected for inflation and not taking the foregone wage income into account). For another estimation of the cost of children, see e.g. Bourguignon (1999).

[8]If she derived utility from children too, the main results still hold.

If we assume utility to be additively separable over time, the expected lifetime utility of a representative agent born at any time t is given by

$$E_t U(c_t^y, n_t, c_{t+1}^o) = u(c_t^y, n_t) + \beta \pi_{t+1} E_t v(c_{t+1}^o), \qquad (5)$$

where β is the private discount rate, u and v stand for the strictly concave and twice continuously differentiable felicity functions when young and old, respectively, and E_t denotes the mathematical expectation over $v(c_{t+1}^o)$ conditional on the information set at time t. Notice that individuals are assumed to be non-altruistic: they do not care about the welfare of others, and in particular, the quality of their offspring's life is not an argument in their utility function.

The social security system

The government runs the social security system as a PAYG-scheme with, at any time t, the following (balanced) budget constraint,

$$\pi_t \theta_t = n_{t-1}\tau, \qquad (6)$$

where n_{t-1} is the number of children per individual at time $t-1$, which is by definition the population growth rate (i.e., $N_t = n_{t-1}N_{t-1}$, with N_t the number of young individuals at time t). However, individual agents act atomistically and do not take this budget constraint into account when deciding on the number of children. So we abstract from the possibility that an individual household considers offspring as an 'investment good' that yields a return in the form of a transfer when old. This 'old-age security hypothesis' played an important role in traditional economies (and still does in many developing countries) where the transfers from the young to the old took place within the family (see e.g. Cigno, 1992). However, in modern western economies, intergenerational transfers for old age are institutionalized in public pension systems. This socializes the investment aspect of children and introduces the possibility for an individual household to free ride on the system by rearing few children (or none at all) and still being entitled to a full pension benefit (see e.g. Folbre, 1994). Consequently, the decision on the family size is completely determined by the utility that parents directly derive from their offspring. To assure a determinate steady state, we assume that $p > \frac{\tau}{1+r}$, that is, the costs of raising children are larger than the (implicit social) return on a child, which equals the present value of its contribution to the PAYG-scheme.[9]

Individuals maximize expected lifetime utility (5) subject to their single-period budget constraints (3) and (4). Given the properties of the felicity functions, the optimal

[9] If this were not the case, the number of children would grow infinitely large and the steady state of the model would be indeterminate.

levels of material consumption (c_t^{y*}) and offspring (n_t^*) are uniquely determined by the following first-order conditions

$$u_c(c_t^{y*}; n_t^*) = \beta(1+r)E_t v_c(c_{t+1}^{o*}); \tag{7.a}$$

$$u_n(c_t^{y*}; n_t^*) = p\, u_c(c_t^{y*}; n_t^*); \tag{7.b}$$

where $u_c \equiv \frac{\partial u(\cdot)}{\partial c^y}$, $u_n \equiv \frac{\partial u(\cdot)}{\partial n}$, $v_c \equiv \frac{\partial v(\cdot)}{\partial c^o}$, and $c_{t+1}^{o*} = \frac{1+r}{\varepsilon_{t+1}} s_t^* + \eta_{t+1}$ with $s_t^* = w - \tau - c_t^{y*} - pn_t^*$.

Equation (7.a) is the standard condition that households equate the marginal rate of substitution between current and future consumption to the rate of interest. According to equation (7.b), households choose the number of children and consumption in their first period of life such that the marginal rate of substitution between a child and current consumption equals the marginal cost of rearing an extra child.

3 Endogenous Fertility and Societal Vulnerability

In this section we show that in the presence of a PAYG-social security scheme the endogeneity of fertility makes society vulnerable to certain shocks. Subsection 3.1 analyzes the economic consequences of pessimistic beliefs. In Subsection 3.2 we investigate the effects of a higher life expectancy. In both cases it appears that the individuals' decisions do not coincide with those that would maximize social welfare.

3.1 Rumours

In this subsection we analyse the effects of a change in the agents' beliefs. In particular, we investigate the effects of a stronger belief that the PAYG-scheme will less generous than it currently is. To capture these beliefs, we assume some sort of adaptive expectations, that is, individuals estimate the benefit level to be equal to its actual current level with probability $(1-\xi_t)$, and they attach a probability of ξ_t to a scenario in which their future pension benefits will be Δ below the current level.[10] The level of the public pension benefit perceived by an individual born at time t can thus be described as

$$E_t\eta_{t+1} = \begin{cases} \eta_t - \Delta & \text{with probability } \xi_t \\ \eta_t & \text{with probability } (1-\xi_t) \end{cases}; \tag{8}$$

A positive value of ξ_t could be due to factual uncertainty with respect to the level of the future pension benefit. It could, for instance, reflect uncertainty with respect

[10] This way of modeling beliefs is similar to Galor and Stark (1990), who analyse the impact of the probability of return migration on migrants' savings. Another way to model adaptive expectations would be to assume that the expected benefit level at time t + 1 equals the actual level (η_t) minus some term reflecting the forecast error made one period earlier ($\eta_t - E_{t-1}\eta_t$). This does not change the results.

to the degree of solidarity displayed by future young generations. As a matter of fact, PAYG-pension schemes can be regarded as implicit social contracts (see e.g. Sjoblom, 1985, and Kotlikoff et al., 1988). Such contracts may be renounced by future generations with a positive probability. In this case, there will always be a difference between the ex ante rational expectation of the pension benefit and its ex post realization. In the same vein, one could think of ξ_t as the objective probability that ageing will occur because ageing implies that the internal rate of return of the PAYG-scheme declines. In the sequel we abstract from factual uncertainty, however, and we start from an equilibrium in which all exogenous factors are constant and $\xi_t = 0; \forall t$. In that case, the adaptive expectations boil down to perfect foresight, i.e., $E_t \pi_{t+1} = \pi_t, \forall t$.

Starting from this perfect-foresight equilibrium, we analyse the effects of a temporary increase in ξ_t.[11] So we assume that expectations become irrational for a while. The reason for this may be that people suffer from what Pigou (1920) called a 'faulty telescopic faculty' and misjudge the future benefit level.[12] They may also base their expectations on rumours, that is, attach a positive subjective probability to a scenario that would occur with zero probability. For example, the subjective probability that ageing saps the social security benefits may be positive, even though there is no objective reason to anticipate this.

The analysis of this change in expectations will be based on a linearized version of the small open economy presented in the previous section. Let

$$\xi_t = \lambda h_t; \qquad \forall t; \tag{9}$$

where h_t describes the time pattern of a perturbation of the steady state of ξ - which is equal to zero - and λ measures the magnitude of this perturbation. The effects of a marginal change in the beliefs can then be traced by differentiation of the first-order conditions (7.a) and (7.b) with respect to λ. Without loss of generality in the remainder of this subsection, we consider the following particular shock to the beliefs: pessimism occurs at time $t = 0$ and lasts for only one period, i.e., $h_0 > 0$; $h_1 = h_2 = \cdots = 0$. We find the following relation between a marginal change in the probability ξ and the optimal number of offspring,

$$\frac{\partial n_t^*}{\partial \lambda} = \phi \frac{\partial n_{t-1}^*}{\partial \lambda} + \psi h_t; \quad t = 0, 1, \ldots; \tag{10}$$

with $0 < \phi < 1$ and $\psi < 0$. For a description of the coefficients of the first-order difference equation (10) it is referred to Appendix A.

[11] We only allow for temporary shocks to avoid the problems which might occur if beliefs turn out to be inconsistent in the long run

[12] Feldstein (1985) also referred to this lack of perfect foresight when analysing the optimal level of pension benefits in a society consisting of both 'life-cyclers' and 'myopes'. In this respect, ξ could be interpreted as the fraction of population that has pessimistic feelings about the future level of social security benefits.

From (10) it directly follows that increasing pessimism ($h_0 > 0$) will decrease family size instantaneously $\left(\frac{\partial n_0^*}{\partial \lambda} < 0\right)$ and therefore also material consumption when young $\left(\frac{\partial c_0^{y*}}{\partial \lambda} < 0\right)$. The pessimistic expectation of a less generous pension scheme initially inclines people to save more,

$$\frac{\partial s_0^*}{\partial \lambda} = -\frac{\partial c_0^{y*}}{\partial \lambda} - p\frac{\partial n_0^*}{\partial \lambda} > 0; \tag{11}$$

so spreading rumours acts as a stimulus for funded social security. Moreover, the rumour that ageing saps the return on premiums for PAYG-pension schemes appears to be self-fulfilling in the short run, i.e., at any given level of the social security tax (τ) it holds that

$$\frac{\partial \eta_1^*}{\partial \lambda} = \frac{\tau}{\varepsilon}\frac{\partial n_0^*}{\partial \lambda} < 0. \tag{12}$$

The pension benefit that the first generation actually receives (η_1) may indeed equal their expectation ($\eta_{-1} - \xi\Delta$).[13] Figure 4 visualizes the evolution of the change of the fertility rate over time.

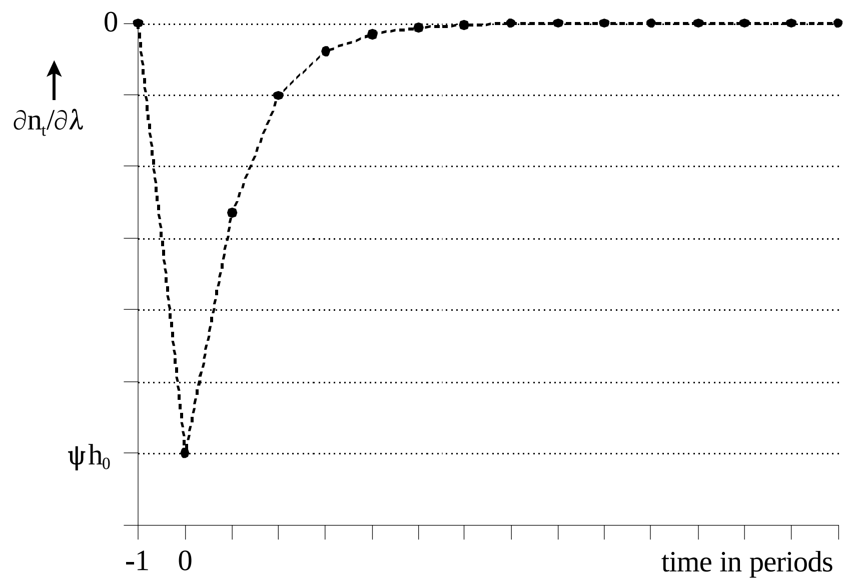

Figure 4: The Evolution of the Change in Family Size.

As difference equation (10) is stable ($0 < \phi < 1$), the initial decline in the number of offspring will be followed by a continuous increase and the economy will gradually

[13] This is the case if $\tau dn_0 = -\lambda h_0 \Delta$, i.e., if $\tau\psi = -\Delta$ when $\lambda = 1$.

recover its initial steady $\left(\frac{\partial c^{y*}}{\partial \lambda} = \frac{\partial n^{*}}{\partial \lambda} = 0\right)$.[14] This increase is due to the fact that during this transition period the expected benefits always lag behind their realizations, so any subsequent generation will adapt its expectations with respect to the benefit level upwards. Even if the expectation of the first generation indeed comes true, and is thereby equal to the benefit that their children expect to receive, family size nevertheless rises. This is so because the first generation takes two different future scenarios into account when deciding on their family size, whereas subsequent generations only anticipate one future state (albeit adaptively). They thus face no uncertainty. Because individuals are assumed to be risk averse, this means that the generation without uncertainty decides to have more children than the initial generation if their expected benefits equal. As this implies an increasing PAYG-benefit, the expected benefit also increases and the number of children gradually rises. Bad beliefs thus only cause temporary effects; in the long run, expectations have to coincide with realizations. This means rational expectations, bringing us back to the original steady state.

The following proposition summarizes the analysis above.

Proposition 1 If the members of one generation perceive their future pension benefits to be less generous, the level of savings initially increases and the number of children declines. Consequently, rumours appear to be a self-fulfilling prophecy to the members of this generation. However, the economy recovers its original equilibrium, so there are no long run effects and pessimism is not persistent.

Adaptive expectations and the endogeneity of fertility thus have important consequences for the dynamics of the economy.

Endogenous versus exogenous population growth

Comparing the previous results to those in case fertility were exogenous, it can be seen that first-period material consumption decreases to a lesser degree if fertility is endogenous, because in that case people will also economise on children when expected lifetime income declines.[15] Furthermore, the increase in savings with an exogenous rate of population growth is smaller than the increase in case of endogenous fertility, i.e.,

$$0 < \left(\frac{\partial s_0^*}{\partial \lambda}\right)_{n \text{ exogenous}} < \left(\frac{\partial s_0^*}{\partial \lambda}\right)_{n \text{ endogenous}} : \qquad (13)$$

This is due to the fact that the costs of rearing children are only incurred in the first period of one's life, so pessimistic feelings about the future decrease first-period

[14] Whenever time subscripts are omitted we refer to the initial steady state value of the respective variable.

[15] Notice that it only makes sense to compare the effects of a rumour if the initial (steady) states are the same. The assumptions to guarantee this are given in Appendix A.

expenditures to a greater extent when fertility is endogenous. Moreover, an exogenous fertility rate implies that the level of the PAYG-pensions does not change, and accordingly, neither do expectations about the future levels. As, due to the small-open-economy assumption, factor prices are fixed too, the economy would therefore immediately converge to the old steady state (i.e., in our setup within one period), so savings are also higher in case of endogenous fertility compared to exogenous population growth in subsequent periods; they would only equal in the long run.

The next subsection looks at a higher life expectancy, a demographic change that is permanent both in its occurrence and, as it appears, in its economic consequences.

3.2 Increasing longevity

The ageing of the population that is expected in the next decades for most industrialized countries is not only brought about by a decrease in the fertility rate, but also by an increasing life span. Because people are expected to live longer and the retirement age does not rise (or even decreases), a higher fraction of lifetime consumption will have to be backed by pension income, such as savings and benefits. This demographic change is likely to be permanent, so unless the retirement age rises, its consequences are permanent as well. This subsection explores the relation between increasing longevity, the number of children and the size of the PAYG-pension scheme in the small-open economy described in Section 2.

Again assuming that the only reason for people to give birth to children is the fact that they directly enter their first-period utility function, the consumption levels and number of children that they choose are uniquely determined by the first-order conditions (7.a) and (7.b). Taking the individual budget constraints into account, comparative statics of these equations (as described in Appendix B) result in $\frac{\partial s^*}{\partial \varepsilon} > 0$ and $\frac{\partial n^*}{\partial \varepsilon} < 0$, as verbalized in the following proposition.

Proposition 2 If the (expected) life span increases, savings increase and the number of children declines. This causes the dependency ratio to rise even further. Consequently, the PAYG-pension scheme deteriorates both in the short and in the long run.

As people live longer and the retirement age does not change, old-age consumption will get a higher weight in the decision on the intertemporal allocation of lifetime income. Although the return on annuities decreases if ε gets higher, the expected interest rate does not change. So increasing longevity merely acts like a decline in the rate of time preference. People do not consider the fact that their child increases next period's output because these gains not only fall to them, but to society as a whole. They thus fail to recognize the positive impact of a child on second period utility. If they did so, a higher life expectancy might have a positive impact on family size. This, however, is not the case. On the contrary, a longer life span causes both consumption when young and offspring to decrease instantaneously to the new steady state values.[16] This implies a lower PAYG-benefit, which intensifies the increase in savings. Increasing longevity therefore implies that a larger part of old-age income is financed through a funded scheme.

The problem of ageing is thus exacerbated by individual fertility decisions and public pension arrangements. In contrast to the effects of rumours, that turned out to be temporary, increasing longevity has lasting consequences for the economy. Agents apparently take decisions that are not optimal for society as a whole, thus eventually putting themselves at a disadvantage. The next sections therefore explore the socially optimal number of children and what the appropriate government action might be.

4 The First-Best Solution

In this section we first derive the command optimum, i.e., the level of savings and the family size that would be chosen by a social-welfare maximizing dictator. Subsequently, we compare the social optimum with the private optimum that was derived above and analyse what instruments a government needs in order to be able to replicate this command optimum in a market setting.

Consider a social planner whose objective function consists of the lifetime utilities of all current and future generations,

$$W_t = \sum_{i=t}^{\infty} \alpha^{i-t} U_{i-1}; \tag{14}$$

where $\alpha < 1$ is the social discount factor, i.e., the factor at which the planner discounts lifetime utility of future generations. Notice that we assume the weight of a generation's utility in the social welfare function to be independent of the size of this generation. In other words, we apply a Millian rather than a Benthamite social welfare function. The reason for this is that there is no well-defined optimum for a Benthamite social welfare function when fertility is endogenous as this implies that the weights in the welfare function are a choice variable. For analytical convenience, we assume the utility function of a representative individual to be additively separable and the felicity functions to be logarithmic, $U_i = \log(c_i^y) + \gamma \log(n_i) + \beta \varepsilon_{i+1} \log(c_{i+1}^o)$.

The social planner is each period restricted by the small open economy's resource

[16] Due to the small-open-economy assumption factor prices are fixed and the economy converges to its new steady state within one period.

constraint, which can be expressed in per capita terms as[17]

$$f(k_t) + k_t + n_t d_{t+1} = c_t^y + p n_t + \frac{\varepsilon_t c_t^o}{n_{t-1}} + n_t k_{t+1} + (1 + r) d_t; \tag{15}$$

with d_t denoting the per capita foreign debt of the country.[18] Because of perfect capital mobility, the capital-labour ratio is constant at $k = f'^{-1}(r)$.

Maximizing (14) at time t subject to the resource constraint (15) results in the following first-order conditions for the centrally planned optimum,

$$\frac{c_{t+1}^{y**}}{c_t^{y**}} = \frac{\alpha(1 + r)}{n_t^{**}}; \tag{16.a}$$

$$\frac{c_{t+1}^{o**}}{c_t^{y**}} = \beta(1 + r); \tag{16.b}$$

$$\frac{c_t^{y**}}{n_t^{**}} = \frac{p + k - d_{t+1}^{**}}{\gamma + \beta\varepsilon_{t+1}}; \tag{16.c}$$

where x^{**} denotes the socially optimal value of variable x. Conditions (16.a) and (16.b) describe the optimal inter- and intragenerational allocation, respectively. Notice that, in the steady state, condition (16.a) boils down to $n^{**} = \alpha(1 + r)$. This means that the socially optimal number of children is uniquely determined by the social discount factor and the exogenous world interest rate. Furthermore, condition (16.b) is equivalent to equation (7.a), implying that the dictator respects the individual's savings decision. Condition (16.c) gives the optimal relationship between the number of children and young-age consumption. This equation differs from the one the private sector uses to choose family size. The equivalent of equation (7.b) for the logarithmic utility specification used here is

$$\frac{c_t^{y*}}{n_t^{*}} = \frac{p}{\gamma}: \tag{7.b'}$$

Comparing (16.c) with (7.b') indicates the two reasons why the choice of individual households deviates from the social optimum:

- the dependency-ratio effect, as captured by the $\beta\varepsilon$-term in (16.c) which the private sector does not take into account: a higher number of children implies that in the future, total output increases, which is shared with the same number of pensioners, so the PAYG-benefits could increase; private agents effectively discount future consumption at the rate $\beta\varepsilon$;

[17] This equation can be read as follows. If x_t denotes (per capita) net exports, the (per capita) current account can be written as $x_t - r d_t$ and the (per capita) capital account is given by $n_t d_{t+1} - d_t$. Equilibrium of the balance of payments requires these two to sum up to zero, implying $n_t d_{t+1} = (1 + r) d_t - x_t$. Notice that in case of a closed economy $x_t = d_t \equiv 0$ and k_t replaces d_t as one of the social planner's instruments.

[18] We assume that the small open economy can borrow any amount at the world interest rate.

- the capital-dilution effect: the private sector does not take the term $k - d$ in equation (16.c) into account (which equals savings at time t). In a small open economy, the capital-labour ratio is given at k. Part of the capital stock is financed by national savings (s), the remainder by foreigners (d). If the number of children increases, a higher capital stock is needed in order to keep per capita production at its former level. This means that savings should be higher, unless the total capital stock is financed with foreign debt (i.e., $k = d$). However, individual non-altruistic households do not take into account that an extra child should induce them to save more. In fact, the true price of an extra child is not just p, but also the extra capital stock that the child needs to be endowed with in order to be as productive (in the future) as the other children. From this point of view, people decide to have too many children, because they do not realize that an extra child implies - ceteris paribus - a lower capital-labour ratio, so less production per capita.

So the number of children in a market economy may be too low or too high as compared to the command optimum, depending on whether the dependency-ratio effect dominates or is dominated by the capital-dilution effect. Finally, we can conclude that fertility is optimal if the dependency-ratio effect happens to exactly offset the capital-dilution effect.

It is well known that in case of exogenous fertility, the government is able to replicate the first-best solution in a market setting provided it has a system of lump-sum intergenerational transfers at its disposal (see Blanchard and Fischer, 1989, Chapter 3). The analysis above shows, however, that in general, the government needs an additional instrument in order to be able to do so when the number of offspring is also a choice variable. If the dependency-ratio effect exceeds the capital-dilution effect, the government is in want of an additional instrument in order to stimulate households to have a larger number of offspring (in the opposite case it would need an instrument to decrease population growth). Obviously, this could be accomplished by lump-sum financed child allowance (or a lump-sum rebated tax on children). Since offspring can be considered a normal good, a decrease (increase) of its price due to a subsidy (tax) per child would induce people to have more (fewer) children. Empirical evidence suggests that child allowances indeed have a significant (though moderate) positive effect on fertility choice (see e.g. Blanchet and Ekert-Jaffé, 1994). In Appendix C we show that the government can realize the first-best solution in a market economy if it is able to use a combination of lump-sum financed child allowances and PAYG-transfers. In particular, the following proposition is proven.

Proposition 3 The government is able to replicate the command optimum in a market if it has lump-sum intergenerational transfers and an instrument that affects the price of children at its disposal. In particular, the government can realize the first-best outcome in a market

- without intervention if $\alpha = \frac{n^*}{1+r}$, ii using both child allowances and PAYG-transfers to the old if $\alpha > \frac{n^*}{1+r}$,

iii using both a tax on children and lump-sum transfers from the old to the young if $\alpha < \frac{n^*}{1+r}$.

The first part of Proposition 3 defines the government discount factor for which the market solution coincides with the command optimum. So for this social discount factor, the government does not redistribute between generations. Moreover, the dependency-ratio effect and the capital-dilution effect exactly cancel out, implying that the government does not need an instrument to affect the number of offspring. This means that the market solution without government intervention is Pareto-efficient. The remainder of the proposition describes a result that is remarkable for two reasons. First, it states that (starting from $\alpha = \frac{n^*}{1+r}$) an increase in α, implying that the government attaches a relatively lower weight to the utility of the current old as compared to that of the current young, leads to the introduction of PAYG-pensions. Second, it follows from Proposition 3 that the optimal level of child allowances is positive if and only if it is optimal to have a PAYG-pension system. So it appears that the net external effect of children is positive, that is, the number of offspring without child allowances is too low, iff there is a PAYG-pension system. The reason for this is of course that the existence of a PAYG-pension system implies that part of the social benefits of having a child reveal itself in a growing tax base and are imperceptible to the individual parent. If, on the other hand, the government redistributes from the older to younger generations, the capital-dilution effect dominates the dependency-ratio effect, so that it is optimal to introduce a tax on offspring.

5 Child Allowances as a Pareto-Improving Device to Reform Social Security

Naturally, the first-best solution derived in the previous section is Pareto-efficient. This does not imply, however, that introducing the optimal instruments in a market economy (where these instruments previously did not exist) leads to a Pareto-improvement. In particular, it is well-known that if the economy is dynamically efficient, the introduction of intergenerational redistribution through a lump-sum financed PAYG-scheme harms future generations as in that case savings yield a higher return than the PAYG-scheme ($1 + r > n$). Yet, many western countries have extensive intergenerational redistribution schemes of this type. The reason for this may be that they were introduced in times of dynamic inefficiency or that they were decided upon through a political process that did not take account of the adverse welfare effects on future generations (see e.g. Browning, 1975). In this section we study the possibilities of Pareto-improving social security reforms if one starts from a situation in which there exists a PAYG-pension scheme ($\tau > 0$) but no child allowance.

We first discuss the effects of introducing a child allowance scheme that internalizes the positive net externality that offspring generates in this setting. We show that introducing (or extending) a child allowance scheme is a Pareto-improvement if the subsidy per child is not too high. Next, we elaborate on the effect of reducing the size of the existing PAYG-scheme. This kind of social-security reform is considered in many countries, arguing that in view of the population ageing that decreases the implicit rate of return of PAYG-schemes, a transition from unfunded to funded pension schemes is desired. But once it is installed, a PAYG-scheme cannot be abandoned without harming at least one generation. Without additional policy, the current and future young gain from a transition to a funded system that yields a higher rate of return, whereas current elderly and retired suffer a loss. It is often thought that such a transition can be turned into a Pareto-improvement using an adequate debt policy. The idea is that the loss suffered by the current elderly can be compensated through an increase in public debt which can subsequently be redeemed using part of the gains of future generations. However, Verbon (1988) and Breyer (1989) have shown that this is not possible when fertility is exogenous: the gains of all future generations are exactly equal to what is needed for the compensation of the current elderly, so no generation can gain from the transition.[19] In this section we also analyse the possibility of a Pareto-improving transition from unfunded to funded pensions when fertility is endogenous. We generalize the Verbon-Breyer result to this case, i.e., we show that - abstaining from distortionary taxes and endogenous growth - a policy that merely reduces the PAYG-tax cannot bring about a Pareto-improvement. Subsequently, we study whether it is possible to have a Pareto-improving transition when it is combined with the introduction of a child-support system. We show that in a dynamically efficient economy a policy of abandoning the PAYG-pension scheme in combination with the introduction of a child allowance scheme may indeed be a Pareto-improvement.

5.1 Introducing child allowances

Suppose that at some time $t = 0$ a child allowance (φ) is introduced in an economy with a PAYG-pension financed by a lump-sum tax $\tau > 0$: Because we assume that the child allowance scheme is financed by a lump-sum tax θ on the young generation, the welfare of the old generation at time $t = 0$ is not affected. Moreover, the effect on the utility of the current young and all future generations is identical. It therefore suffices to check whether the current young gain in order to assess if the introduction

[19] This conclusion hinges on the assumptions with respect to the form of the production function and the absence of tax distortions. Homburg (1990), for instance, demonstrated that if the PAYG-tax distorts the labour-leisure choice of individuals, and private contributions to a funded scheme do not, a transition can bring about a Pareto-improvement. Furthermore, Belan et al. (1998) and Corneo and Marquardt (2000) described such a transition policy in an endogenous growth model. Because the aggregate capital stock then exerts a positive externality on the productivity of workers, a higher savings rate due to a shift to a more funded scheme causes efficiency gains that can be high enough for the transition policy to be Pareto-improving.

of a child allowance scheme is Pareto-improving.

Using equations (C.2) and (C.4) and the fact that $\theta_t = n_t\varphi$, we can write the young's life-time utility as:

$$U_0 = \log\left(\left[\frac{(p-\varphi)(1+r)(w-\tau)}{p(1+r)(1+\beta+\gamma)-\varphi(1+\beta)(1+r)-\gamma\tau}\right]^{1+\beta+\gamma}\left[\frac{\gamma}{p}\right]^{\gamma}(1+r)^{\beta}\beta^{\beta}\right). \quad (17)$$

When we compare lifetime utility in the initial situation without a child allowance ($\varphi = 0$) with that in case there is a child allowance scheme ($\varphi > 0$), we see that utility rises due to the introduction of such an allowance if

$$\left[\frac{p}{p-\varphi}\right]^{\frac{1+\beta}{1+\beta+\gamma}}[p(1+r)(1+\beta+\gamma)-\varphi(1+\beta)(1+r)-\gamma\tau] < p(1+r)(1+\beta+\gamma)-\gamma\tau. \quad (18)$$

Notice that both sides of (18) are equal for $\varphi = 0$. Furthermore, it can easily be checked that the derivative of the left-hand side of (18) with respect to φ in the initial situation ($\varphi = 0$) is negative if the PAYG-tax is positive ($\tau > 0$). Finally, it is evident that the left-hand side of (18) exceeds its right-hand side if φ tends to p. This proves the following proposition.

Proposition 4 Introducing a child allowance in an economy with a PAYG-pension scheme is Pareto-improving provided the subsidy on children is not too high compared to the costs of raising a child.

The intuition behind this result is that a higher number of children implies utility gains through two channels. First, children per se raise the parents' utility and second, a higher number of children has a positive effect on the level of PAYG-pension benefits of the parents' generation. In contrast to the former, the latter effect is not taken into account by parents when deciding on their number of offspring. Consequently, the number of children they choose is too low and utility can be raised by introducing a small subsidy per child that (partly) internalizes this external effect. If, however, the subsidy per child is relatively large as compared to the costs of raising a child, then the effective price of children is too low and the number of children exceeds the optimal number so that utility may fall.

5.2 Merely reducing the PAYG-scheme

As was demonstrated in Section 4, when family size is a choice variable individuals in general suboptimally decide on the number of offspring as children involve externalities. This emanates from the fact that the future contribution of a child to the PAYG-scheme falls to society as a whole, and not exclusively to the individual parent. This might incite one to conjecture that downsizing the PAYG-scheme leads to a decrease in this external effect, yielding a positive welfare effect that enables the

Pareto-improving transition from unfunded to funded pensions that is not possible if fertility were exogenous. Yet, just like in the models of Verbon (1988) and Breyer (1989) with exogenous fertility, merely reducing the PAYG-tax does not allow for a Pareto-improvement. This leads to the following proposition.

Proposition 5 Reducing the PAYG-tax in combination with a debt policy that compensates the first generation of pensioners is not a Pareto-improvement.

Proof Suppose that at time t the PAYG-tax is cut by $\mu \leq \tau$. If N_t^o denotes the number of old at that time, an increase in public debt of $N_t^o \mu n_{t-1}$ is required to completely compensate them, which must be redeemed by the current young and future generations. If the government imposes on these generations a constant lump-sum tax of z when young, the number of children chosen by these generations will be constant and equal to n. The present value of the tax revenues then equals $N_t^o n_{t-1} z \left(\frac{1+r}{1+r-n}\right)$. Hence, the intertemporal budget constraint of the government is satisfied if $z = \frac{\mu(1+r-n)}{1+r}$. Assuming that the interest rate r exceeds the new constant rate of population growth n, the current and all future generations of young gain $\frac{\mu(1+r-n)}{1+r}$ from investing in a funded rather than an unfunded pension scheme. However, this gain is exactly offset by the lump-sum tax z. So lifetime income of the current and future young generations is not affected by the transition. Because prices are not affected either, this implies that the number of children as well as utility remain unchanged, i.e., $n = n_{t-1}$. As the utility of the current generation of old is not affected by construction, it follows that no generation gains from the transition. Hence, it cannot be a Pareto-improvement. ■

This proposition generalizes the Verbon-Breyer result to a model with endogenous fertility. The intuition behind this result is that a reduction in the size of the transfer scheme in combination with a debt policy does not decrease the external effect of having children. It is true that the external effect due to the existence of the PAYG-scheme disappears if this scheme is abandoned as was conjectured above. But when public debt is increased to compensate the loss incurred by the current generation of pensioners, this introduces a new external effect of equal size. What the policy merely does is replacing the implicit debt of the PAYG-scheme by an explicit public debt. This, however, does not alter the fact that an additional child enlarges the tax base for this (implicit or explicit) debt.

5.3 Replacing pensions by child allowances

From Proposition 5 it follows that the ageing problem cannot be alleviated by a transition to a more funded scheme, unless at least one generation's welfare decreases. Obviously, in order to allow for a Pareto-improvement, the external effect of children should be internalized so that offspring increases. The following proposition shows that decreasing the PAYG-scheme and using the proceeds to …nance child allowances, such that the cost of child rearing decreases, may indeed lead to a Pareto-improvement. The proof is given in Appendix D.

Proposition 6 Reducing the PAYG-tax in combination with a debt policy that compensates the …rst generation of pensioners can be Pareto-improving if it is accompanied by the introduction of a child allowance scheme so that the total tax is not a¤ected.

The intuition behind this result is, as noted above, that the parents' utility rises both because they give birth to a larger number of children, which directly increases their utility, and because they receive a higher PAYG-pension bene…t when they are old. Depending on the speci…c parameter values, these utility gains can exceed the loss from taxing current young and future generations in order to compensate the initial retired for the decrease in their pension bene…t. In Appendix D we derive an explicit condition for this to be true. Numerical simulation examples learn that especially the rate of interest and the PAYG-tax are important for this condition to hold, whereas the weight of children in the utility function (°) is not of great signi…cance for this result. Figure 5 shows the maximal values of the child subsidy for which, at a particular size of the PAYG-scheme, a Pareto-improvement is still possible, in case the interest rate equals 2 and 5, respectively.[20]

[20] The parameter values underlying this …gure are ¯ = ° = " = 1, p = 0:1. Furthermore, a Cobb-Douglas production function is assumed with a production elasticity of 0.5.

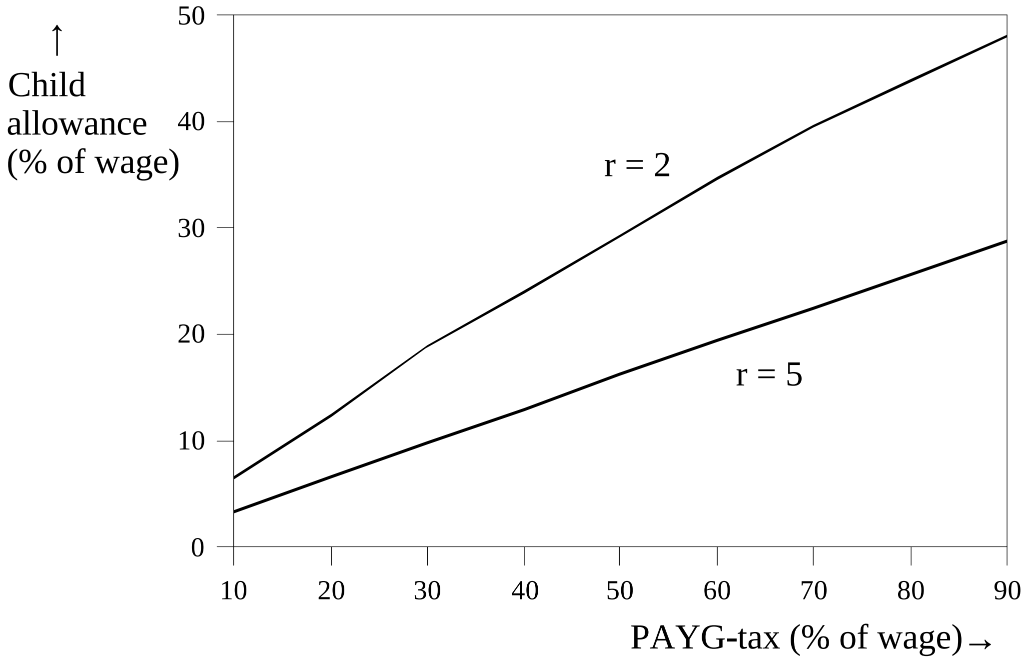

Figure 5: Maximal Child Allowance for Pareto-Improvement.

As can be seen from the …gure, a high interest rate implies a smaller set of possible Pareto-improving child allowances. Indeed, a high interest rate means a lower present value of future gains, making a Pareto-improvement less likely. Furthermore, it may even be that the pension bene…t ultimately increases if the number of children is getting much higher than before the policy change. This happens when the initial PAYG-scheme is relatively large.[21]

6 Conclusion

Both a smaller family size and increasing longevity imply serious challenges for modern western societies with extensive social security programmes in this new century. How to best deal with these is a question that cannot be answered unambiguously, as many studies have already pointed out. Still, most economists analyse the economic consequences of population ageing without paying much attention to the underlying

[21] At an interest rate of 2, for instance, a child subsidy of 5% of the wage rate implies an eventual decrease of the public pension bene…ts of almost 17% if the PAYG-tax is 10%, whereas the bene…t increases by 2% if the tax equals 50%.

causes. These stem to a large extent from the fact that in modern societies, fertility is the endogenous source of population growth, i.e., family size is something people can decide upon, besides savings and material consumption. Moreover, the existence of PAYG-financed social security arrangements implies that the investment element is completely socialized. Fertility choice is therefore only based on the direct utility that a household gets from its progeny. Furthermore, factors that impact the household's (expected) lifetime income have consequences for the rate of population growth, and subsequently for the social security arrangements that are financed on a PAYG-basis.

This paper does take the endogeneity of fertility into account. By considering an economy where the investment aspect of a child is institutionalized via a public PAYG-pension scheme, it is shown that this endogeneity causes the economy to be seriously sensitive to shocks like upcoming pessimistic beliefs and increasing longevity that influence intertemporal decisions. Both tend to depress population growth and thereby reduce the generosity of the social security system. These results are driven by the fact that in an economy with social security, part of the benefits that progeny brings about do not fall exclusively to the individual parent, but to all in society. Apparently, children are to some extent a public good and consequently involve positive social externalities. Because individuals typically do not take these into account when deciding on their family size, the rate of population growth may deviate from its socially optimal value. We show, however, that the market outcome without government intervention is efficient. That is, the number of offspring is socially optimal if the government does not redistribute between generations. On the other hand, in a society with a PAYG-pension scheme, the number of offspring will be too low if there is no child allowance. In that case, an appropriate child subsidy is required to replicate the command optimum. So pensions and child allowances are like Siamese twins: you should see neither of them or both together, but never one without the other.

Of course, it is questionable whether the implementation of the optimal policy is politically feasible, as it may harm some generations. In particular, in many societies with an extensive PAYG-scheme the general feeling is that, in view of ageing, the existing pension scheme is too generous, but downsizing the system is difficult because this comes at the cost of lower utility for at least one generation. We have shown that there may be a solution to this problem. If initially, as a consequence of the PAYG-scheme, the number of children is too low, then it can be a Pareto-improvement to replace part of the public pension scheme by a child-allowance scheme, thereby assuring political feasibility. Our model thus proves the assertion that a Pareto-improving cut in PAYG-pension benefits is impossible, to be no longer tenable, even in the absence of endogenous growth and distortionary taxation. This suggests a simple policy advise for countries that see their extensive PAYG-scheme endangered by ageing: these countries should not only decrease the size of their pension schemes, but at the same time introduce a child allowance scheme so as to broaden the tax base for the remaining smaller PAYG-scheme.

Appendix

A Appendix to Subsection 3.1

Linearizing first-order condition (7.b) with respect to λ gives the following relation between the change in the number of children and the change in first-period material consumption,

$$\frac{\partial c_t^*}{\partial \lambda} = \kappa \frac{\partial n_t^*}{\partial \lambda}; \quad t = 0, 1, \ldots; \tag{A.1}$$

with $\kappa = \frac{p u_{cn} - u_{nn}}{u_{cn} - p u_{cc}}$; $u_{cn} = u_{nc} = \frac{\partial^2 u(c^{y*}, n^*)}{\partial n \partial c^y}$; $u_{nn} = \frac{\partial^2 u(c^{y*}, n^*)}{\partial n^2}$ and $u_{cc} = \frac{\partial^2 u(c^{y*}, n^*)}{\partial (c^y)^2}$. Using (A.1), linearizing first-order condition (7.a) gives difference equation (10),

$$\frac{\partial n_t^*}{\partial \lambda} = \phi \frac{\partial n_{t-1}^*}{\partial \lambda} + \psi h_t; \quad t = 0, 1, \ldots; \tag{A.2}$$

with

$$\phi = \frac{\hat{v}_{cc} \beta (1 + r) \tau / \varepsilon}{\kappa u_{cc} + u_{cn} + \hat{v}_{cc} \beta (1 + r)^2 (\kappa + p) / \varepsilon};$$

$$\psi = \frac{\beta (1 + r)(\bar{v}_c - \hat{v}_c)}{\kappa u_{cc} + u_{cn} + \hat{v}_{cc} \beta (1 + r)^2 (\kappa + p) / \varepsilon};$$

where $\hat{v}_{cc} = \frac{\partial^2 v(\hat{c}^{o*})}{\partial (\hat{c}^o)^2}$; $\bar{v}_c = \frac{\partial v(\bar{c}^{o*})}{\partial \bar{c}^o}$; $\hat{v}_c = \frac{\partial v(\hat{c}^{o*})}{\partial \hat{c}^o}$; $\bar{c}^{o*} = \frac{1+r}{\varepsilon} s^* + \eta - \Delta$; and $\hat{c}^{o*} = \frac{1+r}{\varepsilon} s^* + \eta$.

If $p > \frac{\tau}{1+r}$, the felicity functions are strictly concave and (material) consumption when young and children are weak complements ($u_{cn} \gtrsim 0$), then it holds that $\kappa > 0$; i.e., there is a positive relation between the marginal change in first-period material consumption and the change in family size. Moreover, it also holds that $|\phi| < 1$ and $\psi < 0$. So the first-order difference equation (A.2) is locally stable and $\frac{\partial c_0^{y*}}{\partial \lambda}, \frac{\partial n_0^*}{\partial \lambda} < 0$.

If children do not enter the utility function, it is straightforward to derive the following relation

$$\frac{\partial c_t^{y*}}{\partial \lambda} = \frac{\beta (1 + r)(\bar{v}_c - \hat{v}_c)}{u_{cc} + \hat{v}_{cc} \beta (1 + r)^2 / \varepsilon} h_t; \quad t = 0, 1, \ldots; \tag{A.3}$$

with $u_{cc} = \frac{\partial^2 u(c^{y*})}{\partial (c^y)^2}$. Assuming that the exogenous rate of population growth (n) equals n^*, that $u_{cn} = u_{nc} = 0$ (additive separability), and emending first-period endowment for the costs of raising the optimal number of children, i.e.,

$$(w)_{n \text{ endogenous}} - (w)_{n \text{ exogenous}} = p n^*;$$

the level of material consumption, both when young and old, is equal in both cases. Moreover, consumption when young initially declines to a greater extent and the initial increase in savings is less if the number of children is exogenous. This can be seen as follows. In case of endogenous fertility the initial marginal change in savings is given by equation (11), i.e.,

$$\left(\frac{\partial s_0^*}{\partial \theta}\right)_{n \text{ endogenous}} = -\frac{\partial c_0^{y*}}{\partial \theta} - p\frac{\partial n_0^*}{\partial \theta} = \left(\frac{-u_{nn} - p^2 u_{cc}}{p u_{cc}}\right)\frac{\partial n_0^*}{\partial \theta} \tag{A.4}$$
$$= \left(\frac{-u_{nn} - p^2 u_{cc}}{p u_{cc}}\right)\frac{\beta(1+r)(v_c - \hat{v}_c)}{u_{nn}/p + \hat{v}_{cc}\beta(1+r)^2(p + \frac{u_{nn}}{p u_{cc}})/\varepsilon} h_0.$$

In case of exogenous fertility the initial marginal change in savings is given by the negative of (A.3) for $t = 0$, i.e.,

$$\left(\frac{\partial s_0^*}{\partial \theta}\right)_{n \text{ exogenous}} = -\frac{\partial c_0^{y*}}{\partial \theta} = -\frac{\beta(1+r)(v_c - \hat{v}_c)}{u_{cc} + \hat{v}_{cc}\beta(1+r)^2/\varepsilon} h_0. \tag{A.5}$$

Comparing (A.4) and (A.5) it follows that it is always the case that $\left(\frac{\partial s_0^*}{\partial \theta}\right)_{n \text{ exogenous}} < \left(\frac{\partial s_0^*}{\partial \theta}\right)_{n \text{ endogenous}}$ as $\frac{u_{nn}}{u_{nn} + p^2 u_{cc}} < 1$.

B Appendix to Subsection 3.2

Taking the government budget constraint into account, comparative statics of the first-order conditions (7.a) and (7.b) with respect to ε yield

$$\begin{bmatrix} \frac{\partial c^{y*}}{\partial \varepsilon} \\ \frac{\partial n^*}{\partial \varepsilon} \end{bmatrix} = \frac{B}{|B|}\begin{bmatrix} \pi \\ 0 \end{bmatrix},$$

where the matrix $B = \{b_{ij}\}$, $i, j = 1, 2$, with

$$\begin{aligned} b_{11} &= u_{nn} - p u_{cn}, \\ b_{12} &= -\frac{\beta(1+r)^2}{\varepsilon} v_{cc}\left(p - \frac{\tau}{1+r}\right) - u_{cn}, \\ b_{21} &= p u_{cc} - u_{nc}, \\ b_{22} &= u_{cc} + \frac{\beta(1+r)^2}{\varepsilon} v_{cc}, \end{aligned}$$

$v_{cc} = \frac{\partial^2 v(c^{o*})}{\partial (c^o)^2}$ and $\pi = -\frac{\beta(w - c^{y*} - pn^* - \tau)(1+r)^2}{\varepsilon^2} v_{cc} > 0$.

If $p > \frac{\tau}{1+r}$ and the felicity functions are strictly concave and additively separable ($u_{cn} = u_{nc} = 0$), then b_{11}, b_{21}, $b_{22} < 0$ and $b_{12} > 0$, so $|B| > 0$. Consequently, $\frac{\partial c^{y*}}{\partial \varepsilon}$, $\frac{\partial n^*}{\partial \varepsilon} < 0$.

C Appendix to Section 4

We extend the model presented in Section 2 by introducing a child allowance scheme: a subsidy of $\varphi < p$ per child is paid to the household which is financed by a lump-sum tax θ. The households' first-period budget constraint now looks like

$$c_t^y + (p - \varphi)n_t = w - \tau - \theta_t - s_t. \tag{C.1}$$

The households' first-order conditions then become

$$c_{t+1}^o = (1+r)\beta c_t^y; \tag{C.2.a}$$
$$(p - \varphi)n_t = \gamma c_t^y. \tag{C.2.b}$$

From these equations we can derive the steady state for this model with PAYG-pensions and child allowance,

$$c^{y*} = \frac{(p-\varphi)(w-\tau-\theta)}{(p-\varphi)(1+\beta+\gamma) - \frac{\gamma\tau}{1+r}}; \qquad n^* = \frac{\gamma(w-\tau-\theta)}{(p-\varphi)(1+\beta+\gamma) - \frac{\gamma\tau}{1+r}}. \tag{C.3}$$

Bearing in mind that $n\varphi = \theta$, (C.3) can be written as

$$c^{y*} = \frac{w - \tau - \theta\left(\frac{p(1+r)-\tau}{p(1+r)}\right)}{1+\beta+\gamma - \frac{\gamma\tau}{p(1+r)}}; \qquad n^* = \frac{\gamma(w-\tau) + \theta(1+\beta)}{(1+\beta+\gamma)p - \frac{\gamma\tau}{1+r}}. \tag{C.4}$$

The steady-state command optimum can be derived from the equations in Section 4,

$$c^{y**} = \frac{w - (1+r)p}{1+\beta+\gamma-\frac{\gamma}{\alpha}}; \qquad n^{**} = \alpha(1+r). \tag{C.5}$$

When we compare the conditions (C.2.a) and (C.2.b) to the first-order conditions for the command optimum (16.a)-(16.c) we see that these coincide if

$$\theta = \theta^{**} \equiv pn^{**} - \gamma c^{y**}; \tag{C.6.a}$$
$$\tau = \tau^{**} \equiv w - (1+\beta+\gamma)c^{y**}. \tag{C.6.b}$$

Moreover, it follows from equations (C.3), (C.5), (C.6.a) and (C.6.b) that the steady state of the market economy coincides with the command optimum (i.e., $\theta = \theta^{**} = 0$; $\tau = \tau^{**} = 0$) if $\alpha = \frac{n^*|_{\theta=\tau=0}}{1+r}$. Finally, equation (C.5) implies that $\frac{\partial c^{y**}}{\partial\alpha} < 0$ and $\frac{\partial n^{**}}{\partial\alpha} > 0$, from which it follows that $\frac{\partial\theta^{**}}{\partial\alpha} > 0$ and $\frac{\partial\tau^{**}}{\partial\alpha} > 0$. This proves Proposition 3.

D Appendix to Section 5

Suppose the economy is in a steady state characterized by $\tau = \bar{\tau} > 0$ and $\varphi = \bar{\varphi} = 0$ when we introduce a subsidy $\varphi < p$ per child financed by the PAYG-social security tax. If N_t^o denotes the number of old at that time, an increase in public debt of $N_t^o n_{t-1} \varphi n_t$ is required to prevent their public pension benefits to decrease. This debt must be redeemed by the current young and future generations. If the government imposes on these generations a constant lump-sum tax of z when young, the number of children chosen by these generations will be constant and equal to $\bar{n}$. The present value of the tax revenues then equals $N_t^o n_{t-1} z \left(\frac{1+r}{1+r-\bar{n}}\right)$. Hence, the intertemporal budget constraint of the government is satisfied if $z = z_0 \equiv \frac{\varphi \bar{n}(1+r-\bar{n})}{1+r}$.

The lifetime budget constraint of the current young and future generations reads

$$c^o_{t+1} = (1+r)\left[w - \tau - (p-\varphi)n_t - c^y_t - z\right] + n_t(\tau - \varphi n_{t+1}). \qquad \text{(D.1)}$$

Because we consider a small open economy with perfect capital mobility, the new steady state is reached after one period. Combining the first-order conditions (C.2.a) and (C.2.b) with (D.1), the steady state material consumption when young equals

$$\bar{c}^y = \frac{-b + \sqrt{b^2 + 4a(1+r)(w - \tau - \bar{z})}}{2a},$$

where $a = \frac{\gamma^2 \varphi}{(p-\varphi)^2}$ and $b = (1+r)\left(1 + \beta + \gamma - \frac{\gamma\tau}{(1+r)(p-\varphi)}\right)$. Comparing utility levels with and without the child benefit scheme, it can be calculated that the level of z at which future generations are equally well off is

$$\bar{z} = w - \tau - \frac{x(xa+b)}{1+r},$$

with $x = \frac{p(1+r)(w-\tau)}{p(1+r)(1+\beta+\gamma)-\gamma\tau}\left(\frac{p-\varphi}{p}\right)^{\gamma/(1+\beta+\gamma)}$. A Pareto-improvement is possible if and only if the tax that leaves the utility level of future generations unchanged exceeds the additional tax that is necessary to redeem the initial government debt, that is, if $\bar{z} > z_0 = \frac{\varphi\bar{n}(1+r-\bar{n})}{1+r}$.

References

[1] Becker, G.S. (1961) "An Economic Analysis of Fertility" in NBER, Demographic and Economic Change in Developed Countries Princeton: Princeton University Press.

[2] Becker, G.S. (1991) A Treatise on the Family (revised and enlarged edition) Cambridge Mass.: Harvard University Press.

[3] Becker, G.S. and G.H. Lewis (1973) "On the Interaction between the Quantity and Quality of Children" Journal of Political Economy 81: 279-88.

[4] Belan, P., M. Philippe and P. Pestieau (1998) "Pareto-Improving Social Security Reform" Geneva Papers on Risk and Insurance Theory 23: 119-25.

[5] Bental, B. (1989) "The Old Age Security Hypothesis and Optimal Population Growth" Journal of Population Economics 1: 285-301.

[6] Blanchard, O.J. and S. Fischer (1989) Lectures on Macroeconomics Cambridge, Mass.: MIT Press.

[7] Blanchet, D. and O. Ekert-Ja¤é (1994) "The Demographic Impact of Family Bene…ts: Evidence from a Micro-Model and from Macro-Data" in J. Ermisch and N. Ogawa (eds.) The Family, the Market and the State in Ageing Societies Oxford: Clarendon Press.

[8] Bourguignon, F. (1999) "The Cost of Children: May the Collective Approach to Household Behavior Help?" Journal of Population Economics 12: 503-21.

[9] Breyer, F. (1989) "On the Intergenerational Pareto E¢ciency of Pay-as-You-Go Financed Pension Systems" Journal of Institutional and Theoretical Economics 145: 643-58.

[10] Browning, E.K. (1975) "Why the Social Insurance Budget is Too Large in a Democracy" Economic Inquiry 13: 373-88.

[11] Cigno, A. (1992) "Children and Pensions" Journal of Population Economics 5: 175-83.

[12] Corneo, G. and M. Marquardt (2000) "Public Pensions, Unemployment Insurance, and Growth" Journal of Public Economics 75: 293-311.

[13] Feldstein, M.S. (1985) "The Optimal Level of Social Security Bene…ts" Quarterly Journal of Economics 100: 303-20.

[14] Folbre, N. (1994) "Children as Public Goods" American Economic Review 84: 86-90.

[15] Galor, O. and O. Stark (1990) "Migrants' Savings, the Probability of Return Migration and Migrants' Performance" International Economic Review 31: 463-67.

[16] Harford, J.D. (1998) "The Ultimate Externality" American Economic Review 88: 260-65.

[17] Homburg, S. (1990) "The E¢ciency of Unfunded Pension Schemes" Journal of Institutional and Theoretical Economics 146: 630-47.

[18] Kotliko¤, L.J., T. Persson and L.E.O. Svensson (1988) "Social Contracts as Assets: A Possible Solution to the Time-Consistency Problem" American Economic Review 78: 662-77.

[19] Leibenstein, H.M. (1957) Economic Backwardness and Economic Growth New York: Wiley.

[20] Longman, P.J. (1998) "The Cost of Children" U.S. News March 30th.

[21] Malthus, T.P. (1798) An Essay on the Principle of Population and a Summary View of the Principle of Population, reprint, Baltimore: Penguin, 1970.

[22] Pigou, A.C. (1920) The Economics of Welfare London: MacMillan Press.

[23] Praag, B. van and M. Warnaar (1997) "The Cost of Children and the Use of Demographic Variables in Consumer Demand" in M. Rosenzweig and O. Stark (eds.) Handbook of Population and Family Economics Amsterdam: Elsevier Science.

[24] Razin, A. and E. Sadka (1995) Population Economics Cambridge, Mass.: MIT Press.

[25] Verbon, H.A.A. (1988) "Conversion Policies for Public Pensions Plans in a Small Open Economy" in B. Gustafsson and N. Anders Klevmarken (eds.) The Political Economy of Social Security Amsterdam: Elsevier Science.