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**A NOTE ON THE RELATION BETWEEN
INCOME AND WELFARE**

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A note on the relation between income and welfare*

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Abstract

This notes shows how intertemporal and cross-section welfare are related in a general class of stochastic continuous time models. In the steady state intertemporal welfare is shown to be proportional to cross-sectional income. This result holds for economies where each agent maximizes his own expected discounted utility. That is, we do not assume that aggregate utility is maximized. We provide an application to search in the labor market and one to pollution externalities.

Keywords: intertemporal welfare, search models, green accounting.

JEL code: D60.

1. Introduction

Welfare at an aggregate economic level can be thought of as the sum of incomes in the economy, or as the expected discounted value of future income flows. Both in theory and practice the distinction between essentially static concepts of income and inherently intertemporal welfare concepts plays a role. Since future levels of income are unobservable, operational national accounting systems are necessarily based on static cross-sectional income measures. Also in theoretical models,

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income is often easier to calculate than intertemporal measures of welfare. Yet, in a dynamic world with changes of income levels and agents moving between sectors over time, intertemporal welfare measures seem more relevant. This paper investigates the link between the two concepts.

Three basic questions motivate this note. First, can we construct a measure of intertemporal welfare based on observable contemporaneous variables alone? This would help us in using national accounts to measure intertemporal welfare. Second, can we exploit the relationship between income and welfare to analyse intertemporal welfare in theoretical models? Avoiding the explicit calculation of intertemporal welfare may simplify the analysis. Third, is it allowed to infer from changes in current income how welfare has changed? This would help us for example to evaluate the desirability of economic growth and economic policies.

It should be noted that, in principle, there is no relationship between the flow on a certain moment and the sum of flows over time. That is, if income tomorrow is higher than today, we cannot expect welfare to be higher tomorrow than today. The basic reason is of course that we need to know all relevant flows to calculate welfare, instead of just two of them (that of today and tomorrow). To take an extreme case, if the day after tomorrow an earthquake destroys everything, welfare tomorrow will be lower than welfare from the point of today, despite the fact that income today was lower.

However, in certain specific cases, something can be said about the connection between static and dynamic concepts of welfare. The extensive literature on Green Accounting (summarized in Aronsson and Lofgren 1998) has focussed on cases in which agents face the same trade-off between the different actions they can undertake, no matter at which moment in time they have to decide. For example, investing a certain amount starting from a certain level of the relevant capital stocks yields the same future returns, no matter in which period we face this investment decision. (Technological progress that arises exogenously over time would destroy this property). Weitzman (1976) showed that in a first-best optimal economy with representative consumers, Net National Income is directly proportional to the present value of future utility. Later papers show how Net National Income has to be augmented, using observable variables, to reflect intertemporal welfare, if the environment or natural resources play a role in the economic system (see, e.g. Vellinga and Withagen 1996).

We also focus on similar special cases and explore the connection between income levels and welfare. We derive a relationship between the two that holds for both distorted economies and economies in which welfare is maximal (first-

best economies). Doing so, we use a different method than the green accounting literature which concentrates on the latter type of economies. This literature shows the equivalence between income, welfare and the current value Hamiltonian for welfare maximization. It has few things to say about distorted economies except from the statement that the Hamiltonian for these economies should be augmented by a complicated term to find the welfare index. The reason is that the Hamiltonian is set up for welfare maximization while welfare maximization does not take place in a distorted economy. By contrast, we do not rely on the Hamiltonian but directly look at welfare and find a simpler and more intuitive welfare index for distorted economies that is in fact the natural counterpart to that for the first-best economy.

We find that income and welfare are related in an easily interpretable way. Our result can be directly applied to labour market models in the spirit of e.g. Mortensen and Pissarides (1998), as shown in the next section. However, increases in income cannot be unambiguously related to increases in welfare, even not if the economy is on a first-best growth path. The reason is that a parameter change might not only affect income and hence welfare, but might also change the factor of proportionality between income and welfare.

The next section gives a simple example to illustrate the analysis here. Section 3 outlines the general model and proves the proportionality claim. In Section 4, we relate our results to the green accounting literature.

2. Simple Example

This section illustrates our analysis using a simple version of the Mortensen Pissarides framework. This type of model is often used now in labor economics. For a survey, see Mortensen and Pissarides (1998).

Let V_u (V_e) denote the value to a worker of being unemployed (employed). An employed worker receives a wage w . An unemployed worker receives unemployment insurance equal to $b\bar{w}$ where $b \in [0, 1)$ is the replacement rate in terms of the economy wide average wage \bar{w} .

Agents are represented on the unit interval $[0, 1]$. Let $u \in (0, 1)$ denote the fraction of unemployed agents and $v > 0$ the number of vacancies posted by firms. Then each period $m(u, v) > 0$ of the unemployed agents and vacancies are matched. The matchingsfunction satisfies $\frac{\partial m(u, v)}{\partial u}, \frac{\partial m(u, v)}{\partial v} > 0$ and $\frac{\partial^2 m(u, v)}{\partial u^2}, \frac{\partial^2 m(u, v)}{\partial v^2} < 0$. Further, $m(\cdot, \cdot)$ is homogenous of degree 1 in u and v .

The probability that an unemployed worker is matched with a vacancy equals

$\frac{m(u,v)}{u} = m(1, \theta) \equiv m(\theta)$ where $\theta \equiv \frac{v}{u}$ equals the labor market tightness. Finally, assume that with an exogenous probability δ a match between a worker and firm is dissolved and the worker becomes unemployed.

Then in steady state it is the case that

$$\begin{aligned}\rho V_u &= b\bar{w} + m(\theta)(V_e - V_u) \\ \rho V_e &= w + \delta(V_u - V_e)\end{aligned}$$

where $\rho > 0$ denotes the discount rate.

Turning to the demand side of the labor market, let J_e denote the value for a firm of employing an agent. J_v denotes the value of creating a vacancy. A firm matched with a worker produces a flow of y units of output and pays the worker a wage w . Further, posting a vacancy entails a flow cost of c units of output y . The probability that a vacancy is matched with a worker equals $\frac{m(u,v)}{v} = \frac{m(\theta)}{\theta}$. When a match is dissolved, the firm posts a vacancy again. Hence, in steady state we find

$$\begin{aligned}\rho J_v &= -cy + \frac{m(\theta)}{\theta}(J_e - J_v) \\ \rho J_e &= y - w + \delta(J_v - J_e)\end{aligned}$$

We assume that there is free entry into vacancy creation and hence $J_v = 0$. In these models, it is often assumed that the wage is determined by the Nash bargaining function. That is,

$$w = \arg \max_{w'} (V_e - V_u)^\beta (J_e - J_v)^{1-\beta}$$

where β ($1 - \beta$) denotes the bargaining power of the employee (employer) and V_u (J_v) is the fall back position of the employee (employer) in case bargaining breaks down and the match is dissolved. Using the equations above, the wage is determined by

$$w = \arg \max_{w'} \left(\frac{w'}{\rho + \delta} - \frac{\rho}{\rho + \delta} V_u \right)^\beta \left(\frac{y - w'}{\rho + \delta} \right)^{1-\beta}$$

where $\frac{\partial V_u}{\partial w} = 0$ because the replacement rate b is in terms of the economy average wage $\bar{w} \equiv \frac{\int_a w_a}{1-u}$ (where the integration is over all employed agents) on which the effect of the agent's wage w_a is negligible. In equilibrium all wages are the same and hence $\bar{w} = w$. Solving for the wage yields

$$w = \beta y + (1 - \beta)\rho V_u$$

Thus the higher a worker's bargaining power β the bigger the share he gets from the production surplus y .

For given value of θ , unemployment u changes over time according to

$$\dot{u} = \delta(1 - u) - m(\theta)u$$

where $\delta(1 - u)$ equals the flow from employment into unemployment and $m(\theta)u$ equals the flow from unemployment into employment. Hence in steady state where θ is constant over time and $\dot{u} = 0$, unemployment equals

$$u = \frac{\delta}{\delta + m(\theta)}$$

Summarizing, in steady state the private outcome $(w^0, \theta^0, u^0, v^0, V_u^0, V_e^0, J_e^0, J_v^0)$ is determined by¹

$$\begin{aligned} \frac{y - w^0}{\rho + \delta} &= \frac{c\theta^0}{m(\theta^0)} \\ w^0 &= \beta y + (1 - \beta)\rho V_u^0 \\ \rho V_u^0 &= bw^0 + m(\theta^0)(V_e^0 - V_u^0) \\ \rho V_e^0 &= w^0 + \delta(V_u^0 - V_e^0) \\ \rho J_e^0 &= y - w^0 + \delta(J_v^0 - J_e^0) \\ J_v^0 &= 0 \\ u^0 &= \frac{\delta}{\delta + m(\theta^0)} \\ v^0 &= \theta^0 u^0 \end{aligned}$$

If we use a utilitarian welfare function, steady state welfare can be defined as

$$W^0 \equiv u^0 V_u^0 + (1 - u^0)(V_e^0 + J_e^0) + v^0 J_v^0$$

A weighted average of value functions can be quite a complicated expression. The point of this paper is to show that

$$\begin{aligned} W^0 &= \frac{1}{\rho} (u^0 b w^0 + (1 - u^0)(w^0 + y - w^0) - v^0 c) \\ &= \frac{1}{\rho} (u^0 b w^0 + (1 - u^0)y - v^0 c) \end{aligned}$$

¹In fact, it is straightforward to solve for these variables explicitly, but we do not need that here.

which is a weighted average of per period (static) pay offs.

Finally, Mortensen and Pissarides (1995) analyze a version of this model with technological progress, where $y_t = ye^{gt}$. In that case, the steady state solution to the model is

$$\begin{aligned}
\theta(t) &= \theta^0 \\
w(t) &= w^0 e^{gt} \\
V_u(t) &= V_u^0 e^{gt} \\
V_e(t) &= V_e^0 e^{gt} \\
J_e(t) &= J_e^0 e^{gt} \\
J_v(t) &= 0 \\
u(t) &= u^0 \\
v(t) &= v^0
\end{aligned}$$

Welfare at time t can now be defined as

$$W(t) \equiv u(t)V_u(t) + (1 - u(t))(V_e(t) + J_e(t)) + v(t)J_v(t) + \dot{W}(t)$$

where $\dot{W}(t) = \frac{dW(t)}{dt}$. We show that in steady state (defined precisely below), welfare at time $t = 0$ equals

$$W(0) = \frac{1}{\rho - g} \left(u^0 b w^0 + (1 - u^0) y - v^0 c \right)$$

So, again, we find in steady state a simple relation between intertemporal welfare and per period (static) pay offs. The next section shows this result to be true for a more general model.

3. The General Model

Consider a group of agents $a \in [0, 1]$ each of which can at each moment in time be in one of S states $s \in \{1, 2, \dots, S\}$. In state s at time t agent a can choose an action from his action space X_s , that is $x_{a;t} \in X_s$. Together with the actions $x_{-a;t}$ of the other agents, this action affects his pay off in state s , $p_s(x_{a;t}, x_{-a;t}; \phi) \geq 0$, and his transition rate from state s to state s' , $m_{s,s'}(x_{a;t}, x_{-a;t})$. Pay offs (but not transition rates) are directly affected by parameter ϕ . One can think of ϕ as a tax parameter. By imposing taxes, governments directly affect pay offs

but only indirectly affect transition rates via actions (x_a, x_{-a}) . The model is in continuous time, hence $m_{s,s'}(x_a; t, x_{-a}; t) \geq 0$ are Poisson arrival rates. The following assumption defines states in such a way that agents within the same state are homogenous² in terms of the consequences of their actions.

Assumption: If, at time t , two agents a' and a'' are in the same state \bar{s} then $\frac{\partial p_s(x_a; t, x_{-a}; t; \phi)}{\partial x_{a'; t}} \equiv \frac{\partial p_s(x_a; t, x_{-a}; t; \phi)}{\partial x_{a''; t}}$ and $\frac{\partial m_{s,s'}(x_a; t, x_{-a}; t)}{\partial x_{a'; t}} \equiv \frac{\partial m_{s,s'}(x_a; t, x_{-a}; t)}{\partial x_{a''; t}}$ for each $a \neq a', a''$ and each s and s' .

The assumption implies that we focus on the symmetric equilibrium where all agents in the same state s choose (with probability 1) the same action, denoted x_s . The value function $V_{s;t}$ for an agent a in state s at time t can now be written as

$$\rho V_{s;t} = \max_{x_a \in X_s} \{p_s(x_a, x_{-a}; t; \phi) + \sum_{s' \neq s} m_{s,s'}(x_a, x_{-a}; t)(V_{s';t} - V_{s;t}) + \dot{V}_{s;t}\} \quad (3.1)$$

where $\rho > 0$ denotes the discount factor and $\dot{V}_{s;t} = \frac{dV_{s;t}}{dt}$. Thus the expected present value of being in state s at time t is the sum of three terms: the direct pay off received in state s , the sum of transition probabilities from state s to state s' in which case the agent will receive $V_{s';t}$ instead of $V_{s;t}$ and the change in the value $V_{s;t}$ over time.

To simplify notation, let $p_{s;t} \equiv p_s(x_a; t, x_{-a}; t; \phi)$ denote the pay offs and $m_{s,s';t} \equiv m_{s,s'}(x_a; t, x_{-a}; t)$ the transition rates in state s at time t that follow from the Bellman equation (3.1) in a symmetric equilibrium.

Definition 3.1. Define *intertemporal welfare at time t* as the weighted average of the value functions $V_{s;t}$ in the states at time t with the weights equal to the proportion of agents in each state s at time t , $n_{s;t}$

$$W_t \equiv \sum_{s=1}^S n_{s;t} V_{s;t}$$

That is, we use a utilitarian welfare function. In other words, the welfare function does not introduce distributive considerations in addition to the ones implicit in the pay off function $p_{s;t}$. In particular, agents may be risk averse in the sense that the pay off function $p_{s;t}$ is a concave function of the income received in state s at time t .

We now consider in turn (i) how welfare is related to pay offs in general, (ii) how they are related in the steady state and (iii) how changes in steady state welfare and pay offs as a result of a change in ϕ are related.

²Below we argue that this approach can be extended to the case of heterogenous agents.

3.1. The proportionality between Welfare and Income

In the following proposition we state how welfare and pay-offs are related:

Proposition 3.2. $\rho W_t = \sum_{s=1}^S n_{s;t} p_{s;t} + \frac{dW_t}{dt}$.

Proof. It turns out that the proposition is most easily proved using matrix notation. Define a matrix A_t as $a_{ii;t} = -\sum_{j \neq i} m_{i,j;t}$ and $a_{ij;t} = m_{i,j;t}$ $i \neq j$. Then one can see that $\sum_{j=1}^S a_{ij;t} = 0$. It follows that over time the number of agents in the states evolves as $\dot{\mathbf{n}}_t = A_t^T \mathbf{n}_t$, where $\mathbf{n}_t = (n_{1;t}, n_{2;t}, \dots, n_{S;t})^T$. Let \mathbf{V}_t denote the vector of values $\mathbf{V}_t = (V_{1;t}, V_{2;t}, \dots, V_{S;t})^T$ and \mathbf{P}_t the vector of per period pay offs $\mathbf{P}_t = (p_{1;t}, p_{2;t}, \dots, p_{S;t})^T$ and I the identity matrix. Then equation (3.1) can be written as

$$(\rho I - A_t) \mathbf{V}_t = \mathbf{P}_t + \dot{\mathbf{V}}_t \quad (3.2)$$

With this notation we want to prove that

$$\mathbf{n}_t^T \rho \mathbf{V}_t = \mathbf{n}_t^T \mathbf{P}_t + \frac{d(\mathbf{n}_t^T \mathbf{V}_t)}{dt}$$

or equivalently

$$\mathbf{n}_t^T \rho \mathbf{V}_t = \mathbf{n}_t^T \mathbf{P}_t + \mathbf{n}_t^T \dot{\mathbf{V}}_t + \dot{\mathbf{n}}_t^T \mathbf{V}_t$$

Using equation (3.2) and $\dot{\mathbf{n}}_t = A_t^T \mathbf{n}_t$ this can be written as

$$\mathbf{n}_t^T (\rho \mathbf{V}_t - (\rho I - A_t) \mathbf{V}_t) - (A_t^T \mathbf{n}_t)^T \mathbf{V}_t = 0$$

which indeed holds. ■

This proposition has a clear interpretation. The right hand side of the equality in the proposition represents income, consisting of a consumption term (the sum of pay-offs over all individuals) and an investment term (increase in intertemporal welfare). The left hand side is a term proportional to intertemporal welfare. Accordingly, if income is measured appropriately, it is equivalent to the stream of interest from intertemporal welfare, the interest rate being ρ .

Clearly, in models with $\frac{dW_t}{dt} = \frac{d(\sum_{s=1}^S n_{s;t} V_{s;t})}{dt} = 0$ there is a straightforward relation between intertemporal welfare and cross-section pay offs. In the example of section 2 without growth $\frac{d(\sum_{s=1}^S n_{s;t} V_{s;t})}{dt} = 0$ holds in the steady state. The next subsection shows that for more general cases the relationship is still tractible.

3.2. Welfare and income in the steady state

To further explore the implications of the proposition above, consider the following – fairly general – type of steady state:

Definition 3.3. A steady state (\mathbf{x}, \mathbf{n}) is defined as a situation in which $\dot{\mathbf{n}}_t = \dot{\mathbf{x}}_t = 0$ and $\frac{dW_t}{dt} = f(\mathbf{x}, \mathbf{n}, \phi)W_t$ for some function $f(\cdot) < \rho$.

This definition generalises the concept used in most matching models (in fact all models discussed in Mortensen and Pissarides (1998)) where $f(\cdot) \equiv 0$. In the model of section 2 with growth, it is the case that $f(\cdot)$ equals the exogenous growth rate g . The condition that $f(\cdot) < \rho$ is needed for stability reasons.

Corollary 3.4. In the steady state (\mathbf{x}, \mathbf{n}) , intertemporal welfare W_t at time t equals

$$W_t = \frac{\sum_{s=1}^S n_s p_{s;t}}{\rho - f(\mathbf{x}, \mathbf{n}, \phi)} \quad (3.3)$$

Proof. Equation (3.3) follows immediately from proposition 2.2 above and the definition for the steady state. ■

The interpretation is that in the steady state welfare is proportional to income defined as the sum of pay-offs over all individuals. Hence, to calculate steady state intertemporal welfare, only current variables need to be known.

Note that we allow for externalities in the sense that the actions of one individual might affect the pay-offs of other agents, that is $\frac{\partial p_{s;t}(x_a, x_{-a}, \phi)}{\partial x_{a'}} \neq 0$ for $a \neq a'$. Hence, while the proposition shows how to appropriately measure actual welfare, nothing need to be (nor can be) said about the maximal welfare level at this stage. In contrast, most results in the "green accounting" literature only apply to first best economies, as argued below.

In practice, it is sometimes possible to use equation (3.3) directly to find W_t . For this the function $f(\cdot)$ needs to be identified³. This is straightforward in models which have the property that the proportional rise in pay offs is the same for all states. That is, the growth rate of pay offs equals $\frac{\dot{p}_{s;t+\tau}}{p_{s;t+\tau}} = f(\mathbf{x}, \mathbf{n}, \phi)$. Indeed this is a necessary condition for a steady state to exist, if \mathbf{x} in equation (3.1) influences the transition probabilities $m_{s,s'}$. Since W_t is a weighted average of

³Trivially, W_t can be solved directly in cases where $f(\cdot) = 0$.

value functions, which are themselves some weighted average of per period pay offs, W_t can be written as

$$W_t = e^{-\rho t} \int_0^\infty e^{-\rho\tau} \omega_{s;t+\tau} p_{s;t+\tau} d\tau$$

for some weights $\omega_{s;t+\tau}$, which depend on $n_{s;t+\tau}$, the transition probabilities $m_{s,s'}$ and the discount rate ρ . Now it is clear that the steady state $\dot{\mathbf{x}} = \dot{\mathbf{n}} = 0$ implies

$$W_t = e^{-\rho t} \int_0^\infty e^{-\rho\tau} e^{f(\mathbf{x}, \mathbf{n}, \phi)} d\tau \sum_{s=1}^S \omega_s p_s$$

Hence in such models the function $f(\cdot)$ can be identified and W_t can then be found using equation (3.3).

Furthermore note that the model easily allows for heterogeneity. Consider Z types of agents each of which can be in one of S_z states. For example a type I agent has a higher transition probability from state s to s' than a type II agent. Similarly, pay-offs may differ among agents. We now have to reformulate the model by defining enough states s (by stacking the vectors for n , p , and V and constructing a partitioned matrix for the transition probabilities). In particular, there are $\sum_{z \in Z} S_z$ states and agents can move between states within their own type but not between types.

3.3. Does a rise in income mean higher welfare?

Finally, consider the case where ϕ is changed to examine whether cross-section and intertemporal steady state welfare move in the same direction. The motivation for this proposition is that the effect of a change in ϕ on cross-section pay offs may be more easily determined (either because it can be more readily observed in practice or because it is analitically simpler to derive) than the effect of ϕ on intertemporal welfare. However, it is not always true that a rise in income due to a change in ϕ implies a rise in welfare. In the private outcome there is no reason to suppose that this is the case. But even in the social optimum this is not necessarily the case.

Proposition 3.5. *In a steady state social optimum, a change in ϕ moves steady state intertemporal welfare and cross- section pay offs in the same direction if*

$$\text{sign} \left(\sum_s n_s \frac{\partial p_s}{\partial \phi} \right) = \text{sign} \left(\frac{\partial f(\mathbf{x}, \mathbf{n}, \phi)}{\partial \phi} \right)$$

Proof. In social optimum $\frac{dW_t}{dx} = 0$ (envelope theorem), hence

$$\frac{dW_t}{d\phi} = \frac{1}{\rho - f(\mathbf{x}, \mathbf{n}, \phi)} \left(\sum n_s \frac{\partial p_{s;t}}{\partial \phi} + \frac{\sum n_s p_{s;t}}{\rho - f(\mathbf{x}, \mathbf{n}, \phi)} \frac{\partial f(\mathbf{x}, \mathbf{n}, \phi)}{\partial \phi} \right)$$

Thus, a sufficient condition for $sign\left(\frac{dW_t}{d\phi}\right) = sign\left(\sum_s n_s \frac{\partial p_s}{\partial \phi}\right)$ is that $sign\left(\sum_s n_s \frac{\partial p_s}{\partial \phi}\right) = sign\left(\frac{\partial f(\mathbf{x}, \mathbf{n}, \phi)}{\partial \phi}\right)$. ■

The condition in the proposition has the following interpretation. A change in ϕ moves steady state intertemporal welfare in the same direction as cross-section pay offs if ϕ changes per period pay offs and the growth rate $f(\cdot)$ in the same direction. There is, however, no reason to expect the condition to hold a priori. Hence, whenever we observe a rise in income (pay-offs), there is no reason to expect a rise in intertemporal welfare despite the proportionality between welfare and income. This is obvious in an imperfect market economy with uninternalized externalities, because then $\frac{dW_t}{dx} = 0$ no longer holds. The proposition shows that even in the social optimum this caveat applies. A change in parameters might simultaneously change pay-offs and the factor of proportionality between welfare and income.

4. Green Accounting

Green accounting literature studies how income-related indexes can be constructed to measure welfare (see Aronsson and Lofgren (1998) for a clear survey). The central result of this literature, which goes back to Weitzman (1976), is that in an economy that follows the first-best optimal path and experiences no technological change, welfare is proportional to the current value Hamiltonian which can be interpreted as net national income. The useful implication of this result is that in such an economy observable market data can be used to construct the intertemporal welfare index. However, once technological change and uninternalized externalities play a role, the central result breaks down: welfare is proportional to the current value Hamiltonian augmented by terms representing future technological changes or future external effects. Hence, in this more realistic case, the suggested construction of the welfare index requires knowledge of future variables. In addition, the current value Hamiltonian no longer corresponds to net national income data, since in the presence of externalities, market prices no longer reflect the shadow prices of the maximization problem for which the Hamiltonian is formulated. Intuitively, the Hamiltonian reflects maximization of utility which does not take place in the economy under consideration because of externalities.

This section shows, first, that our approach solves for these two problems, although we can only do so in the steady state. The key to our solution is that we do not start from the first-best economy and try to correct for externalities and growth, but that we directly start from the distorted economy. In particular, equation (3.3) defines the welfare index that can be applied directly to distorted economies (as well as to first-best economies). Second, we apply our last proposition to show that even if green accounting is possible, that is even if we can construct welfare measures from observable data, the resulting figures are in general misleading when welfare comparisons are made, essentially because the factor of proportionality between income and welfare might change as a result of shifts in policy.⁴

Consider a model where increases in pollution adversely affect productivity and/or utility by reducing payoffs in the S states where agents are active. In particular, assume that all payoffs depend directly on agents' actions \mathbf{x} , to be interpreted as polluting inputs, and on an aggregate index of environmental quality Q :

$$p_s = Q\pi_s(x_a, x_{-a}) \quad (4.1)$$

Environmental quality deteriorates by the flow of pollution which is related to total use of polluting inputs in each state $n_s x_s$:

$$\dot{Q} = -\gamma g(\mathbf{n}^T \mathbf{x})Q \quad (4.2)$$

where γ captures the overall extent of damage from economic activity. Individual agents exert a negligible influence on environmental quality and take Q as given, which gives rise to a pollution externality.

Exogenous productivity shocks cause agents to move from one state to another. That is, we assume that the states represent productivity states (for instance, for given x_a and x_{-a} it is the case that $\pi_s(x_a, x_{-a})$ is increasing in s) and that productivity shocks, modelled by $m_{i,j;l}$, are exogenous and constant. We will show that even with exogenous transition probabilities there is no reason to believe that cross-section pay offs and steady state intertemporal welfare move in the same direction.

In the steady state $\dot{\mathbf{n}} = \mathbf{0}, \dot{\mathbf{x}} = \mathbf{0}, \dot{p}_s/p_s = \dot{Q}/Q = -\gamma g$. Hence, the value associated to each state declines at rate $\dot{Q}/Q = -\gamma g$, i.e. $\dot{\mathbf{V}} = -\gamma g \mathbf{V}$. From the

⁴To apply our approach to practical problems, we need valuation techniques to find appropriate shadow prices (e.g. Peskin and Peskin 1976), that is, to translate flows observed in the market into payoffs, and we need to know the $f(\cdot)$ function.

definition of welfare, we find $\dot{W} = \dot{\mathbf{n}}^T \mathbf{V} + \mathbf{n}^T \dot{\mathbf{V}} = -\gamma g W$. Hence, the steady state satisfies our definition of the steady state with $f(\cdot) = -\gamma g(\mathbf{n}\mathbf{x})$ and from the Corollary we may directly write:

$$W = \frac{1}{\rho + \gamma g(\mathbf{n}\mathbf{x})} \sum_{s=1}^S n_s p_s$$

Intertemporal income is proportional to current income $\sum_{s=1}^S n_s p_s$, but the factor of proportionality is smaller the larger the adverse effect of pollution is (i.e. the larger γg is).⁵

Now consider what happens to income and welfare if γ increases, that is if pollution becomes more damaging. First, consider the distorted market economy where pollution (learning) externalities are not internalized. Agents choose x so as to maximize pay offs, taking Q as given. Hence, a change in γ does not affect x and pay offs remain unaffected. So is the steady state distribution over sectors \mathbf{n} which depends on exogenous parameters $m_{i,j}$ only. Hence, income is unaffected. However, welfare declines, see the formula above. We may conclude that in the market economy welfare and income diverge, which comes at no surprise given the presence of externalities.

Second, consider the first-best economy. Optimal choice of polluting inputs requires that the marginal contribution of polluting inputs to pay offs relative to their marginal contribution to environmental degradation should be equal across states:

$$\frac{n_s (\partial \pi_s / \partial x_s)}{(\partial g / \partial x_s)} = \frac{n_{s'} (\partial \pi_{s'} / \partial x_{s'})}{(\partial g / \partial x_{s'})}$$

A change in γ does not affect the first-best choice of actions x . Hence income in the first-best economy is also unaffected by an increase in γ , while welfare falls due to such an increase. Hence, also in the first-best economy, welfare and income diverge.

⁵An alternative interpretation is that Q represents total factor productivity and that equation (4.2) represents learning-by-doing, that is cumulative production determines total factor productivity. The parameter γ is a spillover parameter that links sectoral activity to economy-wide learning. Individual agents do not internalize and take Q as given. In this interpretation, $-\gamma g$ represents the rate of (sectoral-neutral) technological progress. Accordingly, the welfare to income ratio $\frac{1}{\rho - (-\gamma g)}$ becomes larger if technological progress accrues more rapidly.

5. Conclusions

We have explored how welfare and income are related in a class of stochastic models. We have found a useful formula relating welfare and income in the steady state. This formula allows easy calculations of steady state welfare in certain types of models that are frequently used in the literature, such as Frederiksson and Holmlund (1999), Ljungqvist and Sargent (1995) and these surveyed by Mortensen and Pissarides (1998).

We have also pointed out some conclusions for green accounting. Although welfare and income are related in the steady state, it seems unlikely that we may exploit this knowledge in practice to infer changes in welfare from changes in national income as recorded in national accounts. We have pointed out that even in our simple set-up unobservable parameter changes cause welfare and income to diverge even in a first-best economy. This result is likely to carry over to more complex settings, notably settings in which capital accumulation matters (which was ignored in our analysis), and to an analysis of the relationship between income and welfare outside the steady state.

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