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Fiscal Policy Interaction in the EMU

by

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Abstract:
This paper studies the interaction of fiscal stabilization policies in the Economic and Monetary Union (EMU). The "Excessive Deficits" Procedure of the Maastricht Treaty and its elaborations in the recent "Stability and Growth Pact" introduce a set of fiscal stringency requirements. Situations might arise where the need for fiscal flexibility and the fiscal stringency requirements will create a conflict and suboptimal macroeconomic policies are implemented. This paper analyses macroeconomic adjustment under non-cooperative and cooperative fiscal policy design in the EMU. In addition it is analyzed how fiscal stringency requirements like the Stability and Growth Pact affect fiscal policy design under EMU.

Keywords: Fiscal Policy Design, EMU, Linear Quadratic Games

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1 Introduction

In the Economic and Monetary Union (EMU), the participating countries loose monetary and exchange rate policies as macroeconomic policy instruments. The EMU implies a considerable change in the design of macroeconomic policies, both at the national and the supranational level. Monetary policy design will be transferred to the European Central Bank (ECB) that will implement the common monetary policy and circulate the common currency, the Euro. The monetary policies of the ECB will be mainly directed at price stability in the EU and maintaining a stable external value of the Euro. In the short and medium term, the burden of fiscal stabilization will primarily rest on the national fiscal authorities given also the small size of the federal EU budget. This situation is likely to increase the need for fiscal policy activism when countries face a recession.

A first important issue regarding fiscal policy design in the EMU concerns the need for fiscal policy coordination. EMU affects both the interaction of fiscal policies and their transmission in the EU economies. Given the high degree of economic interdependence in the EU, important externalities from national fiscal policies exist. Coordination of national fiscal policies enables to internalize these externalities and by that to improve macroeconomic performance compared to non-cooperative fiscal policy design in the EU.

A second important issue concerns the imposed fiscal stringency requirements by the "Excessive Deficits Procedure" of the Maastricht Treaty and its detailed elaboration convened in the "Stability and Growth Pact" (Stability Pact in short) that was signed at the June 1997 Amsterdam summit of the Council of EU ministers. It imposes a set of restrictions on fiscal flexibility under EMU. The Stability Pact has a double role: (i) a preventive role of early warning against excessive budget deficits (budget surveillance), and (ii) a penalizing role for sustained budget shortages. The medium term goal is approximate budget equilibrium or budget surplus. It was motivated by the fear that undisciplined fiscal behaviour is likely to put at risk the low inflation commitment of the ECB since it will be difficult to rule out a monetary bail-out by the ECB under all circumstances. Undisciplined fiscal behaviour may also result in fiscal bail-outs through fiscal transfers in the EU. Finally, excessive deficits could induce upward pressure on interest rates and an appreciation of the Euro. In both cases, pressure on the ECB could arise to ease its monetary policy. In all cases, the burdens associated with individual

fiscal indiscipline will partly be transmitted to the other EU countries.
Situations may arise where the need for greater fiscal flexibility and the greater fiscal stringency will create a conflict and suboptimal macroeconomic policies will be pursued. This paper analyses the interaction of fiscal policies in the EMU and how fiscal stringency criteria like the Stability Pact affect this interaction and macroeconomic adjustment. To do so, we analyse outcomes in two different game-theoretic settings: (i) non-cooperative fiscal policy design and (ii) cooperative fiscal policy design. We also analyse how policies and adjustment are affected in both regimes by externally imposed fiscal stringency measures.
To model the design of fiscal stabilization policies under EMU, we introduce a dynamic two-country model of the EMU that features short term nominal rigidities thus creating scope for active stabilization policies. Our analysis builds on earlier analyses by Turnovsky, Basar and d'Orey [9] and Neck and Dockner [8] who analyze the interaction of the monetary authorities in a similar dynamic two country model. In both papers, the monetary policies of both countries affect short-term output in both the domestic and foreign economies. The interdependencies of both economies, hence, creates a dynamic conflict between both monetary authorities. Output, inflation and exchange rate adjustment and their implications for social welfare are calculated for a number of different modes of strategic interaction: (i) open-loop and feedback Nash equilibria, (ii) fiscal coordination and (iii) open-loop and feedback Stackelberg equilibria. We extend these two-country models into a setting of a monetary union and consider the effects of fiscal policy in such a setting of a monetary union and analyze the effects of fiscal stringency conditions on the outcomes.
The paper is organised as follows: section 2 develops the analytical framework, section 3 analyzes non-cooperative and cooperative fiscal policies under EMU, section 4 presents numerical simulations of the model to illustrate its main characteristics, and the final section concludes.

2 A Dynamic Stabilization Game in the EMU

Consider a situation where EMU has been fully implemented, implying that national currencies have been replaced by a common currency, national central banks by the ECB and that the exchange rate has disappeared as an

adjustment instrument. Capital markets are fully integrated and we abstain from any country-risk premia implying that any interest differential is arbitraged away instantaneously. On the other hand we assume that there is no labour mobility between both EMU parts and that goods and labour market adjust sluggishly. Hence, the model displays Keynesian features in the short-run.
The economic structure of the two-country EMU is given by the following equations,

Table 1 A Stylized Two-Country EMU Model

$$
\begin{aligned}
y(t) &= \delta s(t) - \gamma r(t) + \rho y^*(t) + \eta f(t) && (1)\\
y^*(t) &= -\delta s(t) - \gamma r^*(t) + \rho y(t) + \eta f^*(t) && (2)\\
s(t) &= p^*(t) - p(t) && (3)\\
r(t) &= i^E(t) - \dot{p}(t) && (4)\\
r^*(t) &= i^E(t) - \dot{p}^*(t) && (5)\\
m(t) - p(t) &= \kappa y(t) - \zeta i^E(t) && (6)\\
m^*(t) - p^*(t) &= \kappa y^*(t) - \zeta i^E(t) && (7)\\
\dot{p}(t) &= \xi y(t) && (8)\\
\dot{p}^*(t) &= \xi y^*(t) && (9)
\end{aligned}
$$

in which, y, denotes real output, p, the output price level, i^E, the common nominal interest rate and, r, the real interest rate. s measures competitiveness of country 1 vis-à-vis country 2 as it is defined as the output price differential. f, equals the real fiscal deficit that the fiscal authority sets. m denotes the amount of nominal money balances that the public demands.
Except for the nominal interest rate and the rate of inflation, $\dot{p}$, variables are in logarithms and expressed as deviations from their long-run non-inflationary equilibrium (growth path). The model, therefore, characterizes the business cycles in this two-country EMU. Variables of country 2 are indicated with an asterisk. For simplicity, we assume that both countries are symmetric in their structural model parameters and we ignore the interaction of this two-country EMU with the rest of the world.

(1) is the aggregate demand function having intra-EU competitiveness, the real interest rate, foreign output and the fiscal deficit as arguments. (3) defines the competitiveness of the EMU countries relative to each other. The definition of the real interest rate is given in (4). The demand for real balances of the common currency is given in (6) as a function of output and the common nominal interest rate. We assume for simplicity that its interest targeting policy enables the ECB to have perfect control over the nominal common interest rate, i^E. (8), finally, gives the short run relation between inflation and output, along the Phillips curve. Because of the nominal rigidities, implied by the Phillips curve, output and prices can diverge from their equilibrium values in the short run. In the long run, on the other hand, both economies adjust to a long run equilibrium where output and prices are at their equilibrium values (which have normalized to zero in this analysis).
Both economies are connected by a number of channels through which price and output fluctuations in one part transmit themselves to the other part of the EMU. Output fluctuations in both economies transmit themselves partly to the other EMU country through the import channel. Therefore, the relative openness of both economies, as measured by ρ, implies an important interdependence of both economies. Price fluctuations in the domestic or foreign economy affect intra-EU competitiveness, $s(t)$, and therefore output in both economies. Combining (1)-(9), enables to write output in both countries as a function of competitiveness, the policy instrument of the ECB, $i^E(t)$, and the fiscal deficit set by the two fiscal authorities, $f(t)$ and $f^*(t)$,

$$y(t) = bs(t) - ci^E(t) + af(t) + \frac{\rho}{k}af^*(t) \tag{10}$$

$$y^*(t) = -bs(t) - ci^E(t) + \frac{\rho}{k}af(t) + af^*(t) \tag{11}$$

with $a := \frac{\eta k}{k^2-\rho^2}$, $b := \frac{\delta}{k+\rho}$, $c := \frac{\gamma}{k-\rho}$ and $k := 1 - \gamma\xi$. Substituting (10) and (11) into (8) and (9) yields two first-order linear differential equations in the output price levels. Subtracting them from each other gives the dynamics of intra-EU competitiveness,

$$\dot{s}(t) = \phi_1 f^*(t) - \phi_1 f(t) + \phi_2 s(t) \tag{12}$$

with $\phi_1 := \frac{\xi\eta}{k+\rho}$ and $\phi_2 := -\frac{2\delta\xi}{k+\rho}$.

Having modeled the economies of both EMU countries and derived the adjustment dynamics of output and prices over time, we still need to determine the fiscal policies and their dynamic adjustment over time as a consequence of the different modes of interaction of these macroeconomic policymakers. In order to do so, we need to specify the objective functions of the players. The objectives are optimized subject to the dynamics of s in (12). We assume that the players have quadratic objective functions. The dynamic strategic interaction of the policymakers in that case reduces to a linear-quadratic (LQ) differential game.
In particular, both fiscal authorities seek to minimize the following intertemporal loss functions that are assumed to be quadratic in the rate of inflation, output and fiscal deficits,

$$J^F = \frac{1}{2}\int_0^\infty \{\alpha \dot{p}^2(t) + \beta y^2(t) + \chi f^2(t)\} e^{-\theta t} dt \tag{13}$$

$$J^{F*} = \frac{1}{2}\int_0^\infty \{\alpha \dot{p}^{*2}(t) + \beta y^{*2}(t) + \chi f^{*2}(t)\} e^{-\theta t} dt \tag{14}$$

Future losses are discounted at a rate θ. The costs of price and output fluctuations are standard in most analyses of macroeconomic policy design. The assumption that the fiscal authorities also value budget balance reflects the notion that high deficits, while beneficial to stimulate output, are not costless: they to some extent crowd out private investment and lead to debt accumulation. Deficits in the loss function also features the possibility that excessive deficits in the EMU will be subject to sanctions, as proposed in the Stability Pact. Therefore, countries prefer low fiscal deficits to high fiscal deficits. In case where $\chi \to \infty$, (cyclical) budget balance becomes the sole objective of the fiscal authority and fiscal activism is reduced accordingly. On the other hand, $\chi \to 0$, implies that fiscal stringency is minimal and that the fiscal authorities have maximal fiscal flexibility under EMU.
We consider the dynamic stabilisation game in the context of a situation of an initial disequilibrium in intra-EU competitiveness, implying that $s(0) \neq 0$ and that the EU countries experience an asynchronous business cycle. We analyze how fiscal policies adjust over time as a result of the dynamic interaction between the macroeconomic policymakers in the EMU and how this macroeconomic performance is affected. In this dynamic interaction we focus on the different adjustment patterns that arise under non-cooperative

and cooperative fiscal policy design in the EMU and how these patterns are affected by different degrees of fiscal stringency.

3 Non-cooperative versus Cooperative Fiscal Policies in the EMU

3.1 The non-cooperative case

We first analyse the design of fiscal policy in the EMU if the fiscal authorities implement non-cooperative fiscal policy strategies. In a Nash equilibrium setting the players implement their optimal strategies simultaneously. The optimization problems of the fiscal players in that case can be written as,

$$\begin{aligned} \min_{u_i(t)} J_i &= \frac{1}{2}\int_0^\infty \{[x(t)^T\ u_1^T(t)\ u_2^T(t)]^T F_i \begin{bmatrix} x(t) \\ u_1(t) \\ u_2(t) \end{bmatrix}\} dt \\ \text{s.t. } \dot{x}(t) &= Ax(t) + B_1u_1(t) + B_2u_2(t), \\ x(0) &= x_0,\ i = 1, 2 \end{aligned} \tag{15}$$

in which

$$x(t) := e^{-\frac{1}{2}\theta t}\begin{bmatrix} s(t) \\ i^E(t) \end{bmatrix},\ u_1(t) := e^{-\frac{1}{2}\theta t}\begin{bmatrix} f_1(t) \\ 0 \end{bmatrix},\ u_2(t) := e^{-\frac{1}{2}\theta t}\begin{bmatrix} f_2(t) \\ 0 \end{bmatrix}.$$

The system parameters are

$$A := \begin{bmatrix} \phi_2 - \frac{1}{2}\theta & 0 \\ 0 & -\frac{1}{2}\theta \end{bmatrix},\ B_1 := \begin{bmatrix} -\phi_1 \\ 0 \end{bmatrix},\ \text{and } B_2 := \begin{bmatrix} \phi_1 \\ 0 \end{bmatrix}$$

and F_i is a positive semi-definite matrix that can be factorized as,

$$F_i =: \begin{bmatrix} Q_i & P_i & L_i \\ P_i^T & R_{1i} & N_i \\ L_i^T & N_i^T & R_{2i} \end{bmatrix}$$

in which Q_i, P_i, L_i, N_i and R_{ii}, $(i = 1, 2)$, represent submatrices that are given in Appendix I.

Using the symbolic computational program Mathematica, it is shown in Appendix I that (depending on the sign of the λ_i's, see (22)) either the game has no solution or the closed-loop system dynamics satisfy the relationship

$$\dot{x}(t) = \begin{bmatrix} -\lambda_i & 0 \\ 0 & -\frac{1}{2}\theta \end{bmatrix} x(t). \tag{16}$$

where $i = 1 \vee 2$. In other words if the game has a solution then, in principle, two different adjustment schemes of the closed-loop system towards its long-term equilibrium can occur. Assuming that the parameter k is positive and denoting $\frac{\rho}{k}$ by r, $\frac{\mu(\theta k+2\delta\xi)\eta^2\theta}{(\rho-k)(4\delta\xi+\theta(k+\rho))^2}$ by x_1 and $\frac{\rho-k}{k}\mu\left(\frac{\eta k}{k^2-\rho^2}\right)^2$ by x_2, the next table illustrates the possibilities

Table 2

# equilibria	parametervalues
1	$r \leq 1$
0	$x_1 < \chi < x_2$, $r > 1$
1	$\chi \leq \min(x_1, x_2)$, $r > 1$
1	$\chi \geq \max(x_1, x_2)$, $r > 1$
2	$x_2 < \chi < x_1$, $r > 1$

Given our model we expect, usually, that $k > \rho$ will hold. That is $r \leq 1$ and therefore the closed-loop adjustment scheme will be uniquely determined. In the following figure 1 we illustrated the situations that can occur in case $k < \rho$

figure 1

In particular note that if ρ is much larger than k the situation occurs that the game permits 2 different types of adjustment schemes for the closed-loop system if χ is chosen "appropriately". Which adjustment scheme actually

will occur under these circumstances depends on additional requirements which are imposed on the outcome of the game. A natural choice seems to select that outcome of the game that increases the adjustment scheme for the closed-loop system towards its long-term equilibrium most. For, under such an adjustment scheme also unanticipated shocks to the system are dealt with best. Furthermore, this equilibrium seems to be a natural candidate that may be Pareto efficient (that is both players infer lower cost by playing this equilibrium). However, given the fact that we expect this to be a rare situation we do not elaborate this subject here.
Finally note that the state variable s in the closed-loop system (16) does not depend directly on the value of i^E. This variable i^E has only an indirect influence on the closed-loop dynamics of the model, that is via the parameters in the cost functionals.

3.2 The cooperative case

The various interdependencies and spillovers between the two countries are not internalized if countries decide upon fiscal policies in a non-cooperative manner. Therefore, it is important to compare fiscal policies and macroeconomic outcomes under non-cooperative equilibria with outcomes under cooperation. The importance of surveillance and coordination of macroeconomic policies in the EU is stressed in the Maastricht Treaty which requires member states to regard their macroeconomic policies as a "matter of common concern" and to coordinate these within the Council of Ministers. In these ECOFIN meetings coordination and surveillance of macroeconomic policies has now been institutionalized.
Under cooperation fiscal policies are directed at minimizing a joined loss function, J^C, rather than at minimizing the individual national loss functions,

$$J^C = J^F + \omega J^{F*} \tag{17}$$

where ω is the Pareto constant that measures the relative weight attached to the losses of both players. One could assume that it is the outcome of an earlier bargaining problem that the two players have solved to determine the relative weights of the individual objectives in the cooperative design of fiscal policies. In that case the Nash-bargaining solution could be considered as the most natural solution to such a bargaining solution of the cooperative

decision making process.
We can rewrite the cooperative decision making problem in the standard format introduced earlier when analysing the Nash open-loop case as,

$$\begin{aligned} \min J^C &= \frac{1}{2}\int_0^\infty \{[x(t)^T\ u_1^T(t)\ u_2^T(t)]^T W \begin{bmatrix} x(t) \\ u_1(t) \\ u_2(t) \end{bmatrix}\}dt \\ \text{s.t. } \dot{x}(t) &= Ax(t) + B_1u_1(t) + B_2u_2(t), \\ x(0) &= x_0,\ i = 1, 2 \end{aligned} \tag{18}$$

where the positive definite matrix W is partioned as,

$$W := \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix}$$

in which Q, R and S represent 2x2 sub matrices that are given in Appendix II. Proceeding as before we use the Hamiltonian approach to calculate the optimal strategy (see Appendix II). After some lengthy calculations we find the following closed-loop system:

$$\dot{x}(t) = \begin{bmatrix} -\lambda & v \\ 0 & -\frac{1}{2}\theta \end{bmatrix} x(t), \tag{19}$$

where λ is the positive square root that follows directly from (25) in Appendix II and v is a (in general non-zero) parameter that depends on the system parameters. Note that, different from the non-cooperative case, the variable i^E now has a direct impact on the closed-loop dynamics of the system.
Taking a closer look at λ as a function of the relative weight parameter ω, we see that it can be written as:

$$\lambda = \sqrt{\frac{\nu_1\omega + \nu_2(1+\omega)^2 - \nu_3(1+\omega^2)}{\nu_4\omega + \nu_5(1+\omega)^2}},$$

where ν_i are positive constants (see Appendix V, table 6). Differentiation of this expression w.r.t. ω yields:

$$\lambda'(\omega) = \frac{1}{2\sqrt{\lambda}} \frac{(1-\omega^2)(\nu_1\nu_5 - \nu_2\nu_4 + \nu_3\nu_4 + 2\nu_5\nu_3)}{(\nu_4\omega + \nu_5(1+\omega)^2)^2}.$$

So, we conclude that, ceteris paribus, λ is maximized for $\omega = 1$ in case $\nu := \nu_1\nu_5 - \nu_2\nu_4 + \nu_3\nu_4 + 2\nu_5\nu_3$ is positive, and that λ is minimal for $\omega = 1$ in case $\nu < 0$. In Appendix III we show that $\nu < 0$ if and only if $(2ar(\phi_2 - \frac{1}{2}\eta) - b\phi_1)(a^2\mu(r^2-1) - \chi) - 2a^2b\phi_1\mu r(r+1) > 0$.
In figure 2, below, we illustrated the behaviour of λ as a function of the coordination parameter ω.

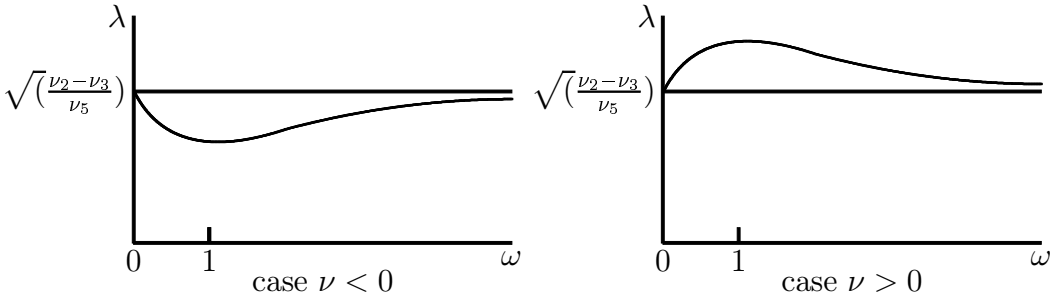


figure 2

¿From this figure we see that s converges as fast as possible to zero in the cooperative game if both cost-functionals have an equal weight in case $\nu > 0$. So, under these parameter conditions both players have an incentive to cooperate, since cooperation increases the adjustment speed of the closed-loop system dynamics towards its long-term equilibrium. On the other hand in case $\nu < 0$, s converges as fast as possible to zero in case either $\omega = 0$ or $\omega = \infty$. So, one might expect that cooperation under these parameter conditions will be much more difficult to achieve.

3.3 The effect of fiscal stringency conditions

The impact of fiscal stringency is measured by the model parameter χ.
In section 3.1 we analyzed already the consequences of fiscal stringency on the number of non-cooperative equilibria. We saw that if the model parameter r is smaller than one, fiscal stringency has no impact on the number of equilibria. There is always a unique equilibrium. However, in case $r > 1$ fiscal stringency does have an impact. If fiscal stringency conditions are either

rather weak or very strong, again a unique equilibrium will occur, whereas if fiscal stringency is in between two bounds either two or no equilibrium can occur.
In section 3.2 we showed that in case the sign of the parameter ν is negative, one may expect that cooperation will be difficult to achieve. In fact this happens if and only if $(2ar(\phi_2 - \frac{1}{2}\eta) - b\phi_1)(a^2\mu(r^2-1) - \chi) - 2a^2b\phi_1\mu r(r+1) > 0$ or, stated differently in terms of the fiscal stringency measure χ, $\chi > \frac{a^2\mu(r+1)(2ar(r-1)+b\phi_1(r-\frac{1}{3}))}{b\phi_1 - 2ar(\phi_2 - \frac{1}{2}\theta)}$.
In other words there is always a threshold after which, if fiscal stringency is increased even more, the realisation of a cooperative equilibrium will be very unlikely. Note that in case $r < \frac{1}{3}$, irrespective of the other parameter values, always $\nu < 0$ holds. So, if $3\rho < 1 - \gamma\chi$, the realisation of a cooperative equilibrium between both countries will probably not occur.
Next, we analyze the impact of fiscal stringency conditions on the closed-loop dynamics of the system under both scenarios. In table 3 we show the impact of χ on the closed-loop dynamics of the model under the assumption that $r < 1$. Details of the calculations are given in Appendix IV.

Table 3

χ	Non-Cooperative	Cooperative
0	$\lambda = \frac{1}{2}\theta$	$\lambda = \frac{1}{2}\theta$
:	increasing	increasing/(decreasing)
large	$\lambda = -a_{uc}$	$\lambda = -a_{uc}$

Here $a_{uc} := \phi_2 - \frac{1}{2}\theta$.
The table should be interpreted as follows. If χ increases, the corresponding λ for the non-cooperative case increases (monotonically) from $\frac{1}{2}\theta$ to $-a_{uc}$. For the cooperative case two different situations can occur depending, again, on the sign of a certain parameter σ. The value of this parameter is $\sigma := -b\phi_1(1+\omega)^2 + 2aa_{uc}(r\omega^2 - 2\omega + r)$. If $\sigma > 0$, λ will increase (monotonically) in the cooperative case too. In case $\sigma < 0$, λ will first increase towards its maximum value (larger than $-a_{uc}$) and then decrease to $-a_{uc}$. We illustrated the situation in figure 3.

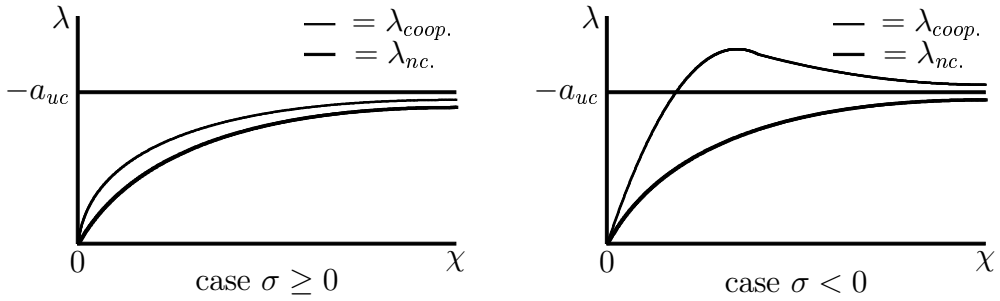


figure 3

¿From table 3 and figure 3 we see that the adjustment speed of s towards its long-term equilibrium grows in case fiscal deficits are taken more seriously, at least in the non-cooperative case. In the cooperative case it may happen that this phenomenon also occurs and that this convergence speed will even be larger than that in the non-cooperative case. In that case there exists, however, a threshold after which this convergence speed does not increase anymore (though it remains above that of the non-cooperative case). In case fiscal deficits are taken strongly into account, implying that χ is large, the impact on the convergence speed of s towards zero is almost the same in both scenario's. Note that this is also the case if in both scenario's fiscal deficits are (almost) neglected.

3.4 Consequences of a European federal transfer system

It is well-known (see e.g. Weber [12], Bayoumi and Eichengreen [1], Bayoumi and Prassad [2] and Christodoulakis et al. [3]) that asymmetric macroeconomic shocks are fairly important in most countries of the European Union. Furthermore, Decressin et al. show in [4] that labour mobility is considerably smaller in the EU than in the US. Therefore, a system of fiscal transfers (EFTS) that aims at stabilising aymmetric shocks in the EMU has been advocated by van der Ploeg [10] and has been elaborated further by e.g. Italianer et al. [6] and von Hagen [11].
In this section we will include such an automatic stabilization rule into our model and analyse its consequences.

To that end we define net government expenditures as follows

$$\begin{aligned} g &:= f - z \\ g^* &:= f^* - z, \end{aligned}$$

where $z := \epsilon(y - y^*)$ is the net transfer from one country to the other country. The output equations (1), (2) then become

$$\begin{aligned} y(t) &= \delta s(t) - \gamma r(t) + \rho y^*(t) + \eta g(t) \\ y^*(t) &= -\delta s(t) - \gamma r^*(t) + \rho y(t) + \eta g^*(t) \end{aligned}$$

After some elementary calculations we have that this model can be rewritten into the previous framework, with the following redefinition of parameters [1]:

$$\begin{aligned} a &:= \frac{\eta X}{X^2 - Z^2} \\ b &:= \frac{\delta}{X + Z} \\ k &:= \frac{\rho X}{Z} \\ \phi_1 &:= \frac{\xi\eta}{X + Z} \\ \phi_2 &:= \frac{-2\delta\xi}{X + Z}, \end{aligned}$$

$X := 1 - \gamma\xi + \eta\epsilon$ and $Z := \rho + \eta\epsilon$.
Using these parameter redefinitions all results obtained in the previous sections can be applied now. In particular, if we recalculate the eigenvalue λ for the cooperative case we obtain the following result:

Table 4

χ	old model	model with EFTS
0	$\frac{1}{2}\theta$	$\frac{1}{2}\theta$
∞	$\frac{2\delta\xi}{1-\gamma\xi+\rho} + \frac{1}{2}\theta$	$\frac{2\delta\xi}{1-\gamma\xi+\rho+2\eta\epsilon} + \frac{1}{2}\theta$

[1] we like to thank Maurice Peek for elaborating all details here

¿From this table we see that the ultimate adjustment speed of the closed-loop dynamics towards its long-term equilibrium decreases in the EFTS model. Due to the automatic transfer, output of both countries (that is deviations w.r.t. their long-term discounted equilibrium) is more close to eachother. So, less fiscal deficits are needed to minimize additional output deviations. Convergence of the output price differential, $s()$, is not modeled as an explicit objective of the players. As a consequence this variable converges less fast to zero. This is, obviously, a less desirable property since it implies that unanticipated shocks will have a more serious short-term impact on the model. An interesting subject, which we will not elaborate here, is whether the welfare cost will change in this EFTS model compared to the previous model.

4 Numerical Simulations with the Model

A numerical example is very useful to illustrate the main aspects of the analysis in the preceeding section. For the model parameters we take the following values

Table 5
Model Parameters

$\delta = 0.3$	$\zeta = 1$	$\theta = 0.15$
$\gamma = 0.4$	$\xi = 0.25$	$\omega = 1$
$\rho = 0.4$	$\alpha = 2$	$i^E = 0$
$\eta = 1$	$\beta = 5$	$s(0) = 0.05$
$\kappa = 1$	$\chi = 2,5$	

Figure 4 graphs the adjustment dynamics that result in the non-cooperative (solid line) and cooperative case (dotted line).
Compared with uncoordinated fiscal policies, fiscal policy coordination leads to less expansionary fiscal policies in country 1 (panel (a)) and to more expansionary fiscal policies in country 2 (panel (b)). This implies a more flat output profile in both countries under fiscal coordination (panel (b) and (d)).

Under fiscal policy coordination, adjustment of intra-EU competitiveness (panel (e)) is faster under non-cooperative than under cooperative fiscal policies.
A stricter interpretation of the Maastricht restrictions on fiscal deficits reduces fiscal activism in the recession, leading to a longer and therefore more costly adjustment process. To analyse the effects of a higher degree of fiscal stringency on fiscal policies and macroeconomic adjustment in the EMU, we compare outcomes in two cases: (i) $\chi = 0$ (solid line) and (ii) $\chi = 5$ (dotted line). Figures 6 and 7 compare both cases in the non-cooperative and cooperative case, respectively.

[Insert Figures 6 and 7]

A higher degree of fiscal stringency reduces fiscal policy activism (panel (a) and (c)) in both countries both under non-cooperative and cooperative fiscal policy design implying larger short-run output fluctuations (panel (b) and (c)). The adjustment speed of the system dynamics increases when the degree of fiscal stringency is increased. In our example, the effects from fiscal stringency are stronger in the case of policy coordination, entailing larger welfare losses. This seems to hint at a more general problem: the introduction of restrictions on fiscal policy design is likely to be more inefficient and therefore costly in the case where unrestricted fiscal policy is designed in a more efficient manner as it is under coordination.
In the case of fiscal policy coordination, the weighting parameter ω - that can also be interpreted as the relative bargaining strength of country 2 in the cooperative decision making process - plays an important role as it determines how much weight is attributed to the preferences of both countries in policy design. In Figure 8 the effect of reducing ω from 1 (solid lines) to 0.5 (dashed lines) is displayed. Note that we have assumed again that $\chi = 2.5$.

[Insert Figure 8]

With fiscal policies being more oriented to the objectives of country 1 we see less policy activism in country 1 (panel (a)). This in particular has a negative effect on output volatility in country 2 (panel (d)). The adjustment speed of the state variable $s(t)$ is slightly higher when ω is reduced to 0.5.

5 Conclusions

This paper has analyzed the design of fiscal policies under EMU. Under EMU, countries loose monetary and exchange rate policies as macroeconomic stabilisation tools. Therefore, the entire burden of stabilisation is shifted to fiscal policy adjustment. Non-cooperative and cooperative optimal fiscal policies were considered in a two country model with sluggish output and price adjustment in the short run. It was shown how fiscal stabilization policies were directed at stabilisation of the business cycle fluctuations. In addition, the effects of a set of externally imposed constraints on fiscal flexibility, such as those involved in the Stability Pact, have been studied. The fiscal stringency criteria reduce the degree of fiscal policy activism and by that the degree of effective stabilisation of output and prices in the EMU. In that perspective, these constraints are causing suboptimal macroeconomic policies.

References

[1] Bayoumi, T. and B. Eichengreen (1993), Shocking aspects of European monetary integration, in F. Torres and F. Giavazzi (eds.), Adjustment and Growth in the EMU, Cambridge University Press

[2] Bayoumi, T. and E. Prassad (1995), Currency unions, economic fluctuations and adjustment: some empirical evidence, CEPR Discussion paper no.1172

[3] Christodoulakis, N., S. Dimelis and T. Kollintzas (1995), Comparisons of business cycles in the EC: ideosyncracies and regularities, Economica, vol. 62, 1-27

[4] Decressin, J. and A. Fatás (1995), Regional labour market dynamics in Europe, European Economic Review, vol. 39, 1627-1655

[5] Engwerda, J.C., B. van Aarle and J.E.J. Plasmans, (1997), The (in)finite horizon open-loop Nash LQ game: an application to EMU, CentER DP no.9784, Tilburg University, The Netherlands, to appear in Annals of Operations Research

[6] Italianer, A. and M. van Heukelen (1993), Proposals for Community stabilization mechanisms: some historical applications, European Economy, special edition no. 5, The Economics of Community Public Finance

[7] Lancaster, P. and L. Rodman (1995), Algebraic Riccati Equations, Clarendon Press, Oxford

[8] Neck, R. and E. Dockner (1995), Commitment and Coordination in a Dynamic Game Model of International Economic Policy-Making, *Open Economies Review*, vol. 6, 5-28

[9] Turnovsky, S., T. Basar and V. d'Orey (1988), Dynamic Strategic Monetary Policies and Coordination in Interdependent Economies, *American Economic Review*, vol. 78, 341-361

[10] Van der Ploeg, F. (1991), Macroeconomic policy coordination issues during the various phases of economic and monetary integration in Europe, European Economy, special edition no.1, The Economics of EMU.

[11] Von Hagen, J. and G. Hammond (1995), Regional insurance against asymmetric shocks. An empirical study for the European Community, CEPR Discussion Paper no.1170

[12] Weber, A. (1991), EMU and asymmetries and adjustment problems in the EMS. Some empirical evidence, European Economy, special edition no.1, The Economics of EMU.

6 Appendix

I. The noncooperative case

¿From our model the next values for the matrices follow:

$$A = \begin{bmatrix} \phi_2 - \frac{1}{2}\theta & 0 \\ 0 & -\frac{1}{2}\theta \end{bmatrix};\ B_1 = \begin{bmatrix} -\phi_1 \\ 0 \end{bmatrix};\ B_2 = \begin{bmatrix} \phi_1 \\ 0 \end{bmatrix};\ Q_1 = \mu \begin{bmatrix} b^2 & -bc \\ -bc & c^2 \end{bmatrix};$$

$$P_1 = \mu \begin{bmatrix} ab \\ -ac \end{bmatrix};\ L_1 = rP_1;\ R_{11} = \mu a^2 + \chi;\ N_1 = r\mu a^2;\ R_{21} = r^2\mu a^2$$

and $\mathrm{Q_2} = \mu \begin{bmatrix} b^2 & bc \\ bc & c^2 \end{bmatrix};\ \mathrm{P_2} = \mathrm{r}\mu \begin{bmatrix} -ab \\ -ac \end{bmatrix};\ \mathrm{L_2} = \frac{1}{\mathrm{r}}\mathrm{P_2};\ \mathrm{R_{12}} = \mathrm{r^2}\mu\mathrm{a^2};\ \mathrm{N_2} = \mathrm{r}\mu\mathrm{a^2};\ \mathrm{R_{22}} = \mu\mathrm{a^2} +$

Assuming that the matrix

$$G := \begin{bmatrix} R_{11} & N_1 \\ N_2^T & R_{22} \end{bmatrix} = \begin{bmatrix} \mu a^2 + \chi & r\mu a^2 \\ r\mu a^2 & \mu a^2 + \chi \end{bmatrix}$$

is invertible we recall from Engwerda et al [5] the following result. Consider the case that we restrict ourselves to consider only control functions which yield finite cost and which, moreover, permit a feedback synthesis. Then, if both (A, B_1) and (A, B_2) are stabilizable and Q_i is positive definite w.r.t. the controllability subspace $< A, B_i >$, the infinite-planning horizon two-player linear quadratic differential game has for every initial state an open-loop Nash equilibrium strategy if and only if there exist K_1 and K_2 that are solutions of the algebraic Riccati equations (see below) (ARE) satisfying the additional constraint that the eigenvalues of $A_{cl} := A - (B_1 B_2)G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \end{pmatrix}$ are all situated in the left half complex plane. In that case, the strategy

$$\begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} = -G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \end{pmatrix} \Phi(t,0)x_0,$$

where $\Phi(t,0)$ satisfies the transition equation $\dot{\Phi}(t,0) = A_{cl}\Phi(t,0);\ \Phi(0,0) = I$, is an open-loop Nash equilibrium strategy. Furthermore the corresponding

cost are $x_0^T M_i x_0$, where M_i is the unique positive semi-definite solution of the Lyapunov equation

$$A_{cl}^T M_i + M_i A_{cl} + (I - G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \end{pmatrix}) F_i \begin{pmatrix} I \\ (-G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \end{pmatrix})^T \end{pmatrix} = 0.$$

The with this problem associated set of algebraic Riccati equations, (ARE), are:

$$\begin{aligned} \begin{pmatrix} 0 \\ 0 \end{pmatrix} &= (\begin{pmatrix} -A^T & 0 \\ 0 & -A^T \end{pmatrix} + \begin{pmatrix} P_1 & L_1 \\ P_2 & L_2 \end{pmatrix} G^{-1} \begin{pmatrix} B_1^T & 0 \\ 0 & B_2^T \end{pmatrix}) \begin{pmatrix} K_1 \\ K_2 \end{pmatrix} \\ &+ \begin{pmatrix} K_1 \\ K_2 \end{pmatrix} \left(-A + (B_1 B_2) G^{-1} \begin{pmatrix} P_1^T \\ L_2^T \end{pmatrix} \right) + \\ &\begin{pmatrix} K_1 \\ K_2 \end{pmatrix} ((B_1 B_2) G^{-1} \begin{pmatrix} B_1^T & 0 \\ 0 & B_2^T \end{pmatrix}) \begin{pmatrix} K_1 \\ K_2 \end{pmatrix} + \\ &\left(\begin{pmatrix} P_1 & L_1 \\ P_2 & L_2 \end{pmatrix} G^{-1} \begin{pmatrix} P_1^T \\ L_2^T \end{pmatrix} - \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \right). \end{aligned} \tag{20}$$

To calculate (both theoretically and numerically) the optimal policies in the open-loop Nash equilibrium we use the Hamiltonian approach.
In Engwerda et al [5] the next algorithm is provided to calculate all equilibria for this infinite planning horizon game.

Algorithm 1:

- Step 1: Calculate the Hamiltonian matrix $M :=$

$$\begin{pmatrix} -A + (B_1 B_2) G^{-1} \begin{pmatrix} P_1^T \\ L_2^T \end{pmatrix} & (B_1 B_2) G^{-1} \begin{pmatrix} B_1^T \\ 0 \end{pmatrix} & (B_1 B_2) G^{-1} \begin{pmatrix} 0 \\ B_2^T \end{pmatrix} \\ Q_1 - (P_1 L_1) G^{-1} \begin{pmatrix} P_1^T \\ L_2^T \end{pmatrix} & A^T - (P_1 L_1) G^{-1} \begin{pmatrix} B_1^T \\ 0 \end{pmatrix} & -(P_1 L_1) G^{-1} \begin{pmatrix} 0 \\ B_2^T \end{pmatrix} \\ Q_2 - (P_2 L_2) G^{-1} \begin{pmatrix} P_1^T \\ L_2^T \end{pmatrix} & -(P_2 L_2) G^{-1} \begin{pmatrix} B_1^T \\ 0 \end{pmatrix} & A^T - (P_2 L_2) G^{-1} \begin{pmatrix} 0 \\ B_2^T \end{pmatrix} \end{pmatrix} \tag{21}$$

- Step 2: Calculate the spectrum of matrix M. If the number of positive eigenvalues (counted with algebraic multiplicities) is less than n, goto Step 5.

- Step 3: Calculate all 2-dimensional M invariant subspaces $\mathcal{K}$ for which $Re(\lambda) > 0$ for all $\lambda \in \sigma(M|_{\mathcal{K}})$. Calculate 3 2x2 matrices X, Y and Z such that $Im \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \mathcal{K}$. Consider only those $\mathcal{K}$ for which X is invertible. If this set is empty, goto Step 5.

- Step 4: Let K be an arbitrary element of the set determined in Step 3. Denote $\mathrm{K}_1 := YX^{-1}$ *and* $K_2 := ZX^{-1}$. $\begin{pmatrix} u_1^*(t) \\ u_2^*(t) \end{pmatrix} = -G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \end{pmatrix} \Phi(t,$ *loop Nash equilibrium strategy. The spectrum of the corresponding closed−loop matrix* $A-(B_1\ B_2)G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \end{pmatrix}$ *equals* $\sigma(-M|_K)$. *If the set determin*

- Step 5: End of algorithm.

Denoting $(1-r)\mu a^2 + \chi$ by α_1, $(1+r)\mu a^2 + \chi$ by α_2 and $\phi_2 - \frac{1}{2}\theta$ by a_{uc} elementary calculations yield

$$M = \begin{bmatrix} -a_{uc} - \frac{2\mu ab\phi_1}{\alpha_1} & 0 & \frac{\phi_1^2}{\alpha_1} & 0 & \frac{\phi_1^2}{\alpha_1} & 0 \\ 0 & \frac{1}{2}\theta & 0 & 0 & 0 & 0 \\ \frac{\mu b^2\chi}{\alpha_1} & -\frac{\mu bc\chi}{\alpha_2} & a_{uc} + \frac{\mu ab\phi_1((1-r^2)a^2\mu+\chi)}{\alpha_1\alpha_2} & 0 & -\frac{r\mu ab\phi_1\chi}{\alpha_1\alpha_2} & 0 \\ -\frac{\mu bc\chi}{\alpha_1} & \frac{\mu c^2\chi}{\alpha_2} & -\frac{\mu ac\phi_1((1-r^2)a^2\mu+\chi)}{\alpha_1\alpha_2} & -\frac{1}{2}\theta & \frac{r\mu ac\phi_1\chi}{\alpha_1\alpha_2} & 0 \\ \frac{\mu b^2\chi}{\alpha_1} & \frac{\mu bc\chi}{\alpha_2} & -\frac{r\mu ab\phi_1\chi}{\alpha_1\alpha_2} & 0 & a_{uc} + \frac{\mu ab\phi_1((1-r^2)a^2\mu+\chi)}{\alpha_1\alpha_2} & 0 \\ \frac{\mu bc\chi}{\alpha_1} & \frac{\mu c^2\chi}{\alpha_2} & -\frac{r\mu ac\phi_1\chi}{\alpha_1\alpha_2} & 0 & \frac{\mu ac\phi_1((1-r^2)a^2\mu+\chi)}{\alpha_1\alpha_2} & -\frac{1}{2}\theta \end{bmatrix}$$

The eigenvalues of this matrix are: $\{-\frac{1}{2}\theta, -\frac{1}{2}\theta, \frac{1}{2}\theta, p := -\frac{1}{2}\theta + \phi_2 + \frac{\mu ab\phi_1}{\alpha_1\alpha_2}((1+r)\chi + (1-r^2)a^2\mu), \lambda_1, \lambda_2\}$, where

$$\lambda_{1,2} = \frac{1}{2}\{\frac{-(1+r)\mu ab\phi_1}{\alpha_1} \pm \sqrt{(\frac{-(1+r)\mu ab\phi_1}{\alpha_1})^2 - \frac{4\alpha_3}{\alpha_1^2}}, \} \qquad (22)$$

in which $\alpha_3 := -\{(a_{uc}\alpha_1^2 + 2\mu ab\phi_1\alpha_1)(a_{uc} + \frac{1}{\alpha_1}\mu ab\phi_1(1-r)) + 2\phi_1^2\mu b^2\chi\}$.

Note that the square root term always exists as a real number, since this term can be rewritten as the sum of two positive numbers:

$$\frac{1}{\alpha_1^2}(\{(-3+r)\mu ab\phi_1 - 2a_{uc}((1-r)\mu a^2+\chi)\}^2 + 8\mu\chi b^2\phi_1^2)$$

It is easily verified that the first two entries of the eigenvector corresponding to the eigenvalue p are zero. Moreover, the first entry of the eigenvector corresponding to the eigenvalue $\frac{1}{2}\theta$ is always zero as is the second entry of the eigenvector corresponding to λ_i, $i = 1, 2$. From this immediately follows that the model has at most 2 different equilibria. Moreover, by calculating the exact structure of the eigenvalues corresponding to the eigenvalues $\frac{1}{2}\theta$ and λ_i, and using the above computational algorithm the closed-loop structure can be determined, as summarized in (16).
Some elementary rewriting shows that α_3 can be rewritten as

$$-\frac{1}{4}\frac{\alpha_1}{(k+\rho)^2}\{(4\delta\xi+\theta(k+\rho))^2\chi + \frac{\mu\eta^2\theta(\theta k+2\delta\xi)}{k-\rho}\}.$$

It is now easily verified that if $k > \rho$ the parameters a and α_1 are positive and α_3 is, consequently, negative. So, M has exactly 2 positive eigenvalues. In case $k < \rho$, then $a < 0$. So there will be exactly one equilibrium if either $\alpha_1 < 0$ and $(4\delta\xi+\theta(k+\rho))^2\chi < \frac{\mu\eta^2\theta(\theta k+2\delta\xi)}{\rho-k}$ or $\alpha_1 > 0$ and $(4\delta\xi+\theta(k+\rho))^2\chi > \frac{\mu\eta^2\theta(\theta k+2\delta\xi)}{\rho-k}$. Denoting $(4\delta\xi+\theta(k+\rho))^2\chi$ by y_1 and $\frac{\mu\eta^2\theta(\theta k+2\delta\xi)}{\rho-k}$ by y_2, it is moreover easily verified that there exists no equilibrium in case $\alpha_1 < 0$ and $y_1 > y_2$, and that there are two equilibria in case $\alpha_1 > 0$ and $y_1 < y_2$. Using the definition of α_1 and denoting $\frac{\mu(\theta k+2\delta\xi)\eta^2\theta}{(\rho-k)(4\delta\xi+\theta(k+\rho))^2}$ by x_1 and $\frac{\rho-k}{k}\mu\left(\frac{\eta k}{k^2-\rho^2}\right)^2$ by x_2 we can rewrite these conditions in terms of inequalities that should be satisfied by the design parameter χ. That is, there is one equilibrium if either $\chi < \min(x_1, x_2)$ or $\chi > \max(x_1, x_2)$; there is no equilibrium if $x_1 < \chi < x_2$; and there are 2 equilibria if $x_2 < \chi < x_1$. We summarized these results in table 2. □

II. The cooperative case

¿From our model the next values for the matrices follow:

$$A = \begin{bmatrix} \phi_2 - \frac{1}{2}\theta & 0 \\ 0 & -\frac{1}{2}\theta \end{bmatrix};\ B := [B_1\ B_2] = \begin{bmatrix} -\phi_1 & \phi_1 \\ 0 & 0 \end{bmatrix};\ \text{and}$$

$$W = \begin{bmatrix} (1+\omega)\mu b^2 & (\omega-1)\mu bc & (1-\omega r)\mu ab & (r-\omega)\mu ab \\ (-1+\omega)\mu bc & (1+\omega)\mu c^2 & (-1-\omega r)\mu ac & (-r-\omega)\mu ac \\ (1-\omega r)\mu ab & (-1-\omega r)\mu ac & (1+\omega r^2)\mu a^2+\chi & r(1+\omega)\mu a^2 \\ (r-\omega)\mu ab & (-r-\omega)\mu ac & r(1+\omega)\mu a^2 & (r^2+\omega)\mu a^2+\omega\chi \end{bmatrix}.$$

Factorization of W immediately yields then the following parametervalues for the matrices Q, S and R:

$$Q = \begin{bmatrix} (1+\omega)\mu b^2 & (\omega-1)\mu bc \\ (-1+\omega)\mu bc & (1+\omega)\mu c^2 \end{bmatrix};$$

$$S = \begin{bmatrix} (1-\omega r)\mu ab & (r-\omega)\mu ab \\ (-1-\omega r)\mu ac & (-r-\omega)\mu ac \end{bmatrix};\ \text{and}\ R = \begin{bmatrix} (1+\omega r^2)\mu a^2+\chi & r(1+\omega)\mu a^2 \\ r(1+\omega)\mu a^2 & (r^2+\omega)\mu a^2+\omega\chi \end{bmatrix}.$$

Note that in our case matrix R is invertible. Furthermore, (A, B) is stabilizable and (Q, A) is detectable.
¿From Lancaster and Rodman ([7], chapter 16) we recall that the optimal policies that result, equal,

$$\begin{bmatrix} u_1^*(t) \\ u_2^*(t) \end{bmatrix} = -R^{-1} \begin{bmatrix} S^T + B^T K \end{bmatrix} x(t) \tag{23}$$

where K is the unique positive semi-definite solution of the algebraic Riccati equation

$$KBR^{-1}B^TK - K(A - BR^{-1}S^T) - (A - BR^{-1}S^T)^TK - (Q - SR^{-1}S^T) = 0. \tag{24}$$

The corresponding minimal cost are $x_0^T K x_0$.
To calculate the optimal policy for the cooperative game one can proceed now similarly as in algorithm 1 of Appendix I (see e.g. Lancaster and Rodman ([7], chapter 7). The only differences are that M must be replaced by

$$H := \begin{bmatrix} -(A - BR^{-1}S^T) & BR^{-1}B^T \\ Q - SR^{-1}S^T & (A - BR^{-1}S^T)^T \end{bmatrix};$$

step 3 yields a unique solution; and that $K := YX^{-1}$ is obtained similarly as in step 4 by considering the decomposition $Im \begin{pmatrix} X \\ Y \end{pmatrix} = \mathcal{K}$.
Substitution of the above mentioned parameter values into H yields after some tedious manipulations, the following eigenvalues for this Hamiltonian: $\{\frac{1}{2}\theta, -\frac{1}{2}\theta, \pm\lambda\}$, where:

$$\begin{aligned}\lambda^2 &= \frac{1}{4(\omega(\chi+\mu a^2(1-r^2))^2 + r^2\mu a^2\chi(1+\omega)^2)}\{(a\mu)^2\omega\{2(1-r^2)aa_{uc} \\ &\quad +4(1+r)b\phi_1\}^2 + \mu\chi\{-2(1+\omega^2)aa_{uc}(2(1-r^2)aa_{uc}+4(1+r)b\phi_1 \\ &\quad +(1+\omega)^2(2aa_{uc}+2b\phi_1)^2\} + 4\omega\chi^2 a_{uc}^2\}.\end{aligned} \tag{25}$$

By calculating the eigenvectors corresponding to the eigenvalues $\frac{1}{2}\theta$ and λ, and using algorithm 1 the closed-loop structure (19) results. □

III. A detailed study of the parameter ν

By definition $\nu = \nu_1\nu_5 - \nu_2\nu_4 + \nu_3\nu_4 + 2\nu_5\nu_3$. This can be rewritten as

$$\begin{aligned}\nu &= (\nu_1+2\nu_3)\nu_5 + (\nu_3-\nu_2)\nu_4 \\ &= 4[2a\mu(1-r^2)aa_{uc} + 4a\mu(1+r)b\phi_1 + 2\chi a_{uc}]^2\mu\chi r^2a^2 - \\ &\quad 16\mu\chi(b\phi_1 - raa_{uc})^2(\chi+\mu a^2(1-r^2))^2 \\ &= 16\mu\chi\{a^2r^2[a_{uc}(\chi+\mu a^2(1-r^2)) + 2a\mu b\phi_1(1+r)]^2 - \\ &\quad (b\phi_1 - raa_{uc})^2(\chi+\mu a^2(1-r^2))^2\} \\ &= -16\mu\chi b\phi_1(\chi+a^2\mu(1+r)^2)[(2ara_{uc}-b\phi_1)(a^2\mu(r^2-1)-\chi) - 2a^2b\phi_1\mu r(r+1)]\end{aligned}$$

The last equality can be verified, e.g., by straightforward expansion of both sides of the equation and then comparing terms.
Since $16\mu\chi b\phi_1(\chi + a^2\mu(1+r)^2) > 0$, the conclusions concerning the sign of ν follow directly.

IV. Sensitivity analysis of the closed-loop eigenvalues w.r.t. χ

By substituting $\chi = 0$ and $\chi = \infty$ into the $\lambda's$ one obtains the figures as mentioned in table 3.
To analyse the intermediate behaviour we consider the derivative of both $\lambda's$ w.r.t. χ. First consider the non-cooperative case under the assumption that $r < 1$. Then the appropriate λ is $\lambda = \frac{1}{2\alpha_1}\{-c_1 + \sqrt{c_1^2 - 4\alpha_3}\}$, where $c_1 := (1+r)\mu ab\phi_1$ (see 22). For analysis purposes we rewrite α_1 as $q_1 + \chi$ and α_3 as $-\frac{1}{4}\alpha_1(p_1\chi + p_2)$ (with $q_1 := (1-r)\mu a^2$, $p_1 := \frac{4\delta\xi+\theta(k+\rho))^2}{(k+\rho)^2}$ and $p_2 := \frac{\mu\theta\eta^2(\theta k+2\delta\xi)}{(k+\rho)^2(k-\rho)}$).
Next, we rewrite λ as

$$\begin{aligned}\lambda &= \frac{1}{2\alpha_1}\frac{-4\alpha_3}{c_1 + \sqrt{c_1^2 - 4\alpha_3}} \\ &= \frac{1}{2}\frac{p_1\chi + p_2}{c_1 + \sqrt{c_1^2 - 4\alpha_3}}\end{aligned}$$

So,

$$\begin{aligned}2\frac{d\lambda}{d\chi} &= \frac{p_1(c_1 + \sqrt{c_1^2 - 4\alpha_3}) - \frac{1}{2\sqrt{c_1^2-4\alpha_3}}(p_1\alpha_1 + p_1\chi + p_2)(p_1\chi + p_2)}{\left(c_1 + \sqrt{c_1^2 - 4\alpha_3}\right)^2} \\ &= \frac{p_1c_1\sqrt{c_1^2 - 4\alpha_3} + p_1(c_1^2 - 4\alpha_3) - \frac{1}{2}(p_1\alpha_1 + p_1\chi + p_2)(p_1\chi + p_2)}{\sqrt{c_1^2 - 4\alpha_3}\left(c_1 + \sqrt{c_1^2 - 4\alpha_3}\right)^2} \\ &= \frac{p_1c_1(\sqrt{c_1^2 - 4\alpha_3} + c_1) - \frac{1}{2}(p_1\chi + p_2)^2 + \frac{1}{2}p_1\alpha_1(p_1\chi + p_2)}{\sqrt{c_1^2 - 4\alpha_3}\left(c_1 + \sqrt{c_1^2 - 4\alpha_3}\right)^2} \\ &= \frac{p_1c_1(\sqrt{c_1^2 - 4\alpha_3} + c_1) + \frac{1}{2}(p_1\chi + p_2)(p_1\alpha_1 - p_1\chi - p_2)}{\sqrt{c_1^2 - 4\alpha_3}\left(c_1 + \sqrt{c_1^2 - 4\alpha_3}\right)^2} \\ &= \frac{p_1c_1(\sqrt{c_1^2 - 4\alpha_3} + c_1) + \frac{1}{2}(p_1\chi + p_2)(p_1q_1 - p_2)}{\sqrt{c_1^2 - 4\alpha_3}\left(c_1 + \sqrt{c_1^2 - 4\alpha_3}\right)^2}\end{aligned}$$

¿From this it is clear that $\frac{d\lambda}{d\chi} > 0$ if we can show that $p_1q_1 - p_2 > 0$. Substitution of the modelparameters into this expression (see table 6) yields (note that by assumption $r < 1$, i.e. $k > \rho$)

$$sgn(p_1q_1 - p_2) = sgn\{\frac{(4\delta\xi + \theta(k+\rho))^2}{(k+\rho)^2}\frac{k-\rho}{k}\mu\frac{(k\eta)^2}{(k^2-\rho^2)^2} - \frac{\mu\theta\eta^2(\theta k + 2\delta\xi)}{(k+\rho)^2(k-\rho)}\}$$

$$
\begin{aligned}
&= sgn\{\frac{k}{(k+\rho)^2}(4\delta\xi + \theta(k+\rho))^2 - (\theta k + 2\delta\xi)\theta\} \\
&= sgn\{\frac{k}{(k+\rho)^2}(8\delta\xi\theta(k+\rho) + 16\delta^2\xi^2) - 2\delta\xi\theta\}
\end{aligned}
$$

Next, we show that this last expression is always positive. Thereto we first note that since $k > \rho$, we have $2k > k + \rho$. Therefore, $\frac{k}{(k+\rho)}8\delta\xi\theta - 2\delta\xi\theta > \frac{1}{2}8\delta\xi\theta - 2\delta\xi\theta > 0$. Using this inequality, the claim is obvious now. Which proves the positiveness of $\frac{d\lambda}{d\chi}$ for the non-cooperative case.
Next, we consider the cooperative case. Some elementary analysis shows that in that case the corresponding λ (see 25) can be rewritten as

$$
\lambda = \sqrt{\frac{d_1\chi^2 + d_2\chi + d_3}{d_4\chi^2 + d_5\chi + d_6}},
$$

where $d_i,\ i = 1, .., 6$ are pointed out in table 6.
Differentiation w.r.t. χ yields:

$$
\frac{d\lambda}{d\chi} = \frac{1}{2\sqrt{\lambda}}\frac{e_1\chi^2 + 2e_2\chi + e_3}{(d_4\chi^2 + d_5\chi + d_6)^2},
$$

where $e_i,\ i = 1, 2, 3$ are simple expressions in d_i (see either table 6 or below). To analyze this derivative we first consider the sign of the parameters e_2 and e_3. By definition we have that

$$
\begin{aligned}
e_2 &= d_1d_6 - d_4d_3 \\
&= 64a^2\mu^2\omega^2b\phi_1(1+r)^2(-b\phi_1 + aa_{uc}(r-1))
\end{aligned}
$$

Furthermore, by first substituting the appropriate modelparameters into d_i and next comparing terms on both sides of equality signs we obtain

$$
\begin{aligned}
e_3 &= d_2d_6 - d_3d_5 \\
&= 16\omega a^4b\phi_1\mu^3(r+1)^3[b\phi_1(1 + 2r\omega + \omega^2) - 3b\phi_1(r\omega^2 + 2\omega + r) + \\
&\quad 2aa_{uc}(r-1)(r\omega^2 + r + 2\omega)] \\
&= 16\omega a^4b\phi_1\mu^3(r+1)^3[b\phi_1(1-r)(1+\omega)^2 + 2(aa_{uc}(r-1) - b\phi_1)(r\omega^2 + r + 2\omega)].
\end{aligned}
$$

¿From the above expressions we see that both e_2 and e_3 are positive if we can show that $(aa_{uc}(r-1) - b\phi_1) > 0$. Using the definition of these parameters

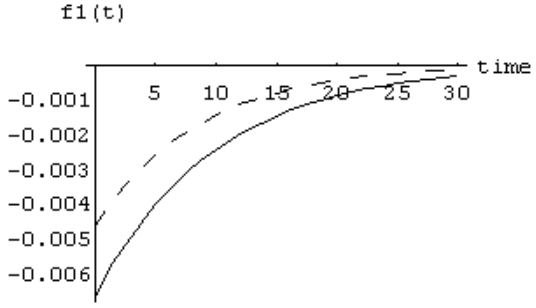
it is easily verified that $(aa_{uc}(r-1)-b\phi_1) = \frac{\frac{1}{2}\theta\eta}{k+\rho}$, from which the above inequality follows. So, both $e_2 > 0$ and $e_3 > 0$.
Finally, we consider e_1. Some elementary rewriting shows:

$$\begin{aligned} e_1 &= d_1 d_5 - d_2 d_4 \\ &= 16\omega\mu b\phi_1(-b\phi_1(1+\omega)^2 + 2aa_{uc}(r\omega^2 - 2\omega + r)) \end{aligned}$$

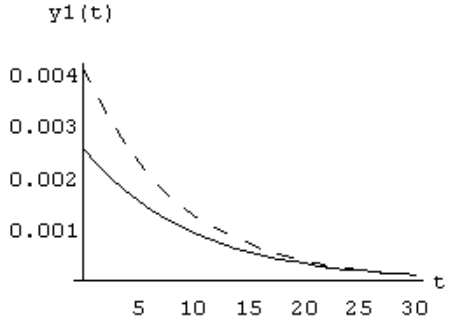
So, denoting $-b\phi_1(1+\omega)^2 + 2aa_{uc}(r\omega^2 - 2\omega + r)$ by σ, we have that $e_1 = 16\omega\mu b\phi_1\sigma$.
Note that the sign of the derivative is completely determined by the sign of $e_1\chi^2 + 2e_2\chi + e_3$. Using the above derived information concerning the signs of $e_i,\ i = 1,2,3$ it is clear that if $\sigma > 0$, $\frac{d\lambda}{\chi} > 0$ for all $\chi > 0$, and that if $\sigma < 0$, $\frac{d\lambda}{\chi}$ will be positive for small χ and becomes negative if χ is large. From this the conclusions w.r.t. the behaviour of λ as a function of χ summarized in table 3 and figure 3, respectively, are obvious then. □

V. Table 6: List of parameters

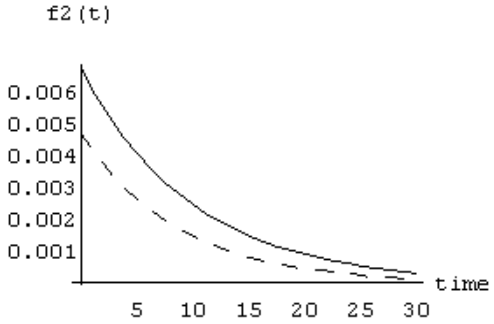
name	value
a	$\frac{k\eta}{k^2-\rho^2}$
a_{uc}	$\phi_2 - \frac{1}{2}\theta$
α_1	$(1-r)\mu a^2 + \chi$
α_2	$(1+r)\mu a^2 + \chi$
α_3	$-((a_{uc}\alpha_1^2 + 2\mu ab\phi_1\alpha_1)\ (a_{uc} + \frac{1}{\alpha_1}\mu ab\phi_1(1-r))\ + 2\phi_1^2\mu b^2\chi)$
b	$\frac{\delta}{k+\rho}$
c	$\frac{\gamma}{k-\rho}$
d_1	$4\omega a_{uc}^2$
d_2	$\mu\{-2(1+\omega^2)aa_{uc}(2(1-r^2)aa_{uc} + 4(1+r)b\phi_1) + (1+\omega)^2(2aa_{uc} + 2b\phi_1)^2\}$
d_3	$(a\mu)^2\omega\{2(1-r^2)aa_{uc} + 4(1+r)b\phi_1\}^2$
d_4	4ω
d_5	$8\omega\mu a^2(1-r^2) + 4r^2\mu a^2(1+\omega)^2$
d_6	$4\omega\mu^2 a^4(1-r^2)^2$
e_1	$d_1d_5 - d_2d_4$
e_2	$d_1d_6 - d_4d_3$
e_3	$d_2d_6 - d_3d_5$
k	$1 - \gamma\xi$
μ	$\alpha\xi^2 + \beta$
ν_1	$(a\mu)^2\{2(1-r^2)aa_{uc} + 4(1+r)b\phi_1\}^2 + 4\chi^2 a_{uc}^2$
ν_2	$\mu\chi(2aa_{uc} + 2b\phi_1)^2$
ν_3	$2\mu\chi aa_{uc}(2(1-r^2)aa_{uc} + 4(1+r)b\phi_1)$
ν_4	$4(\chi + \mu a^2(1-r^2))^2$
ν_5	$4\mu\chi r^2 a^2$
ν	$\nu_1\nu_5 - \nu_2\nu_4 + \nu_3\nu_4 + 2\nu_5\nu_3$
p_1	$\frac{(4\delta\xi + \theta(k+\rho))^2}{(k+\rho)^2}$
p_2	$\frac{\mu\theta\eta^2(\theta k + 2\delta\xi)}{(k+\rho)^2(k-\rho)}$
ϕ_1	$\frac{\xi\eta}{k+\rho}$
ϕ_2	$\frac{-2\delta\xi}{k+\rho}$
q_1	$(1-r)\mu a^2$
r	$\frac{\rho}{k}$
σ	$-b\phi_1(1+\omega)^2 + 2aa_{uc}(r\omega^2 - 2\omega + r)$


f1(t)
time
-0.001
-0.002
-0.003
-0.004
-0.005
-0.006
5
10
15
20
25
30

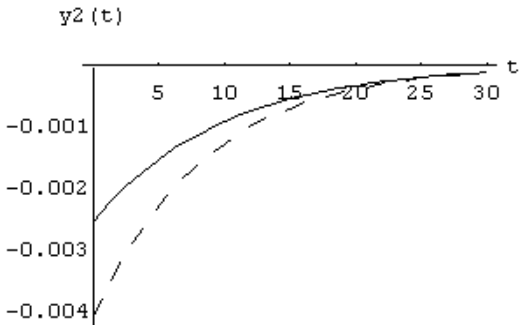
(a)


y1(t)
0.004
0.003
0.002
0.001
t
5
10
15
20
25
30

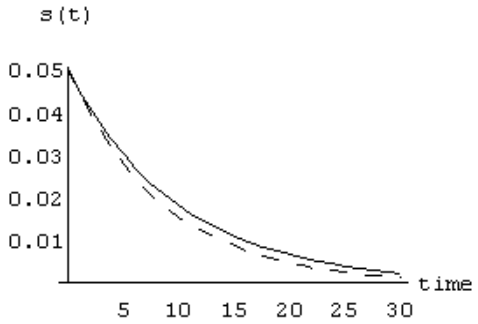
(b)


f2(t)
0.006
0.005
0.004
0.003
0.002
0.001
time
5
10
15
20
25
30

(c)

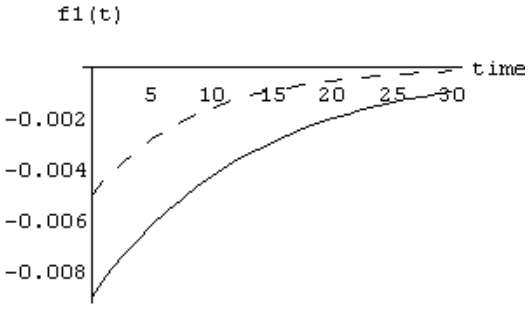


(d)

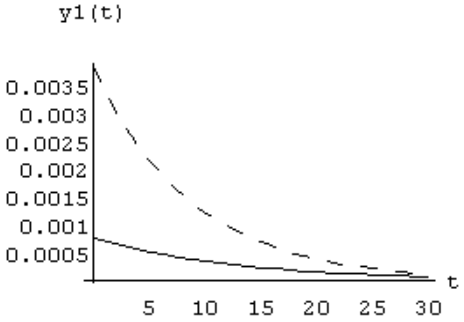


(e)

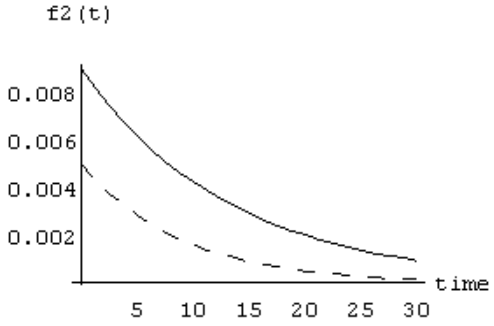
Figure 4
Non-Cooperative (solid) and Cooperative (dashed) Fiscal Policies


f1(t)
time
5
10
15
20
25
30
-0.002
-0.004
-0.006
-0.008

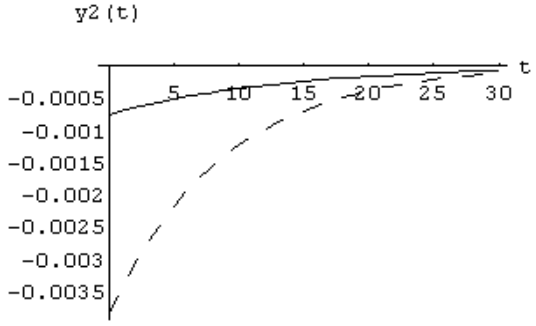
(a)


y1(t)
0.0035
0.003
0.0025
0.002
0.0015
0.001
0.0005
t
5
10
15
20
25
30

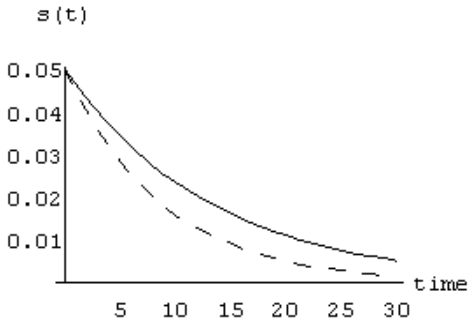
(b)


f2(t)
0.008
0.006
0.004
0.002
time
5
10
15
20
25
30

(c)

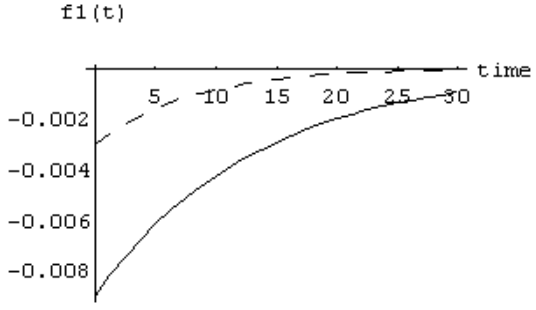


(d)

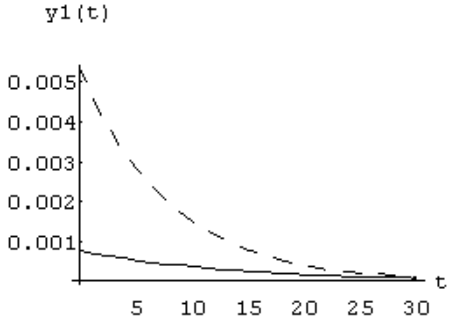


(e)

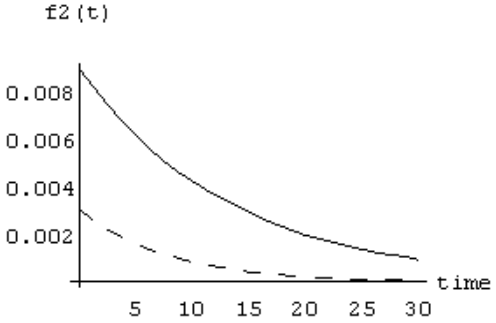
Figure 5
Non-Cooperative Fiscal Policies: χ=0 (solid) vs. χ=5 (dashed)


f1(t)
time
5
10
15
20
25
30
-0.002
-0.004
-0.006
-0.008

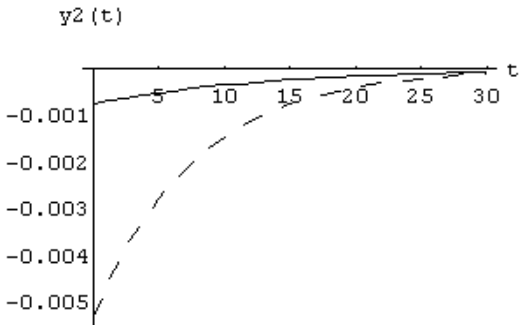
(a)


y1(t)
0.005
0.004
0.003
0.002
0.001
t
5
10
15
20
25
30

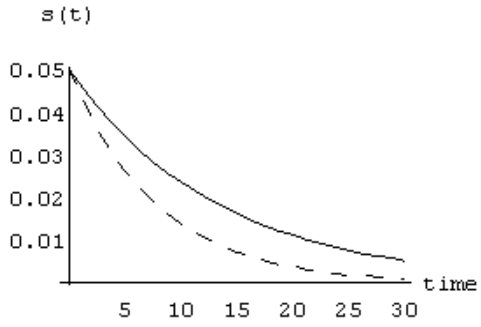
(b)


f2(t)
0.008
0.006
0.004
0.002
time
5
10
15
20
25
30

(c)

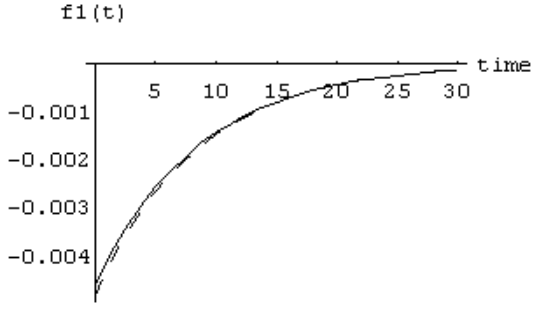


(d)

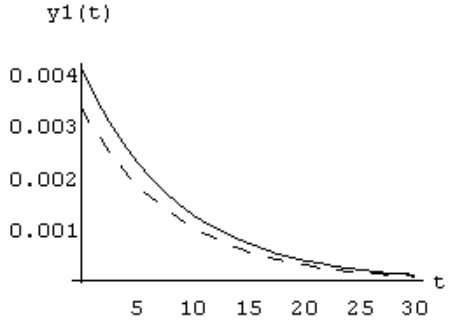


(e)

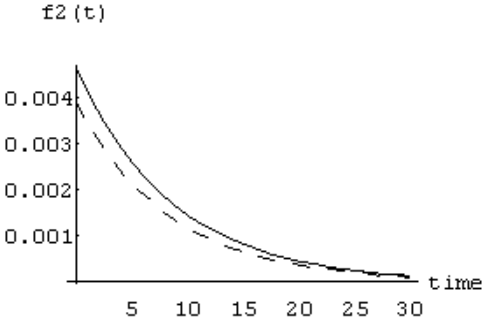
Figure 6
Cooperative Fiscal Policies: χ=0 (solid) vs. χ=5 (dashed)


f1(t)
time
5
10
15
20
25
30
-0.001
-0.002
-0.003
-0.004

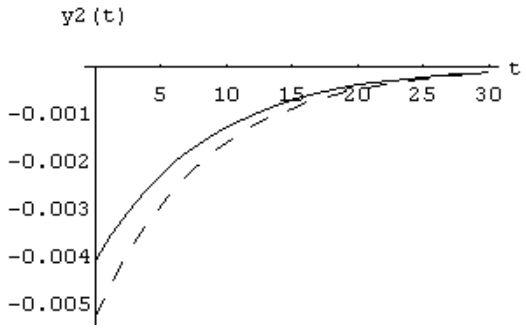
(a)


y1(t)
0.004
0.003
0.002
0.001
t
5
10
15
20
25
30

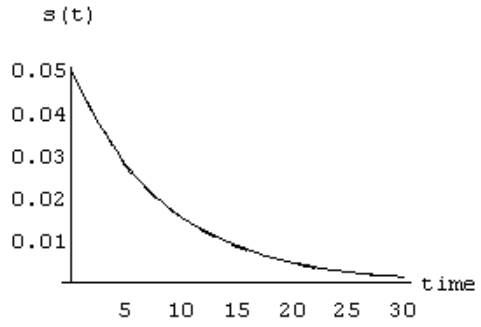
(b)


f2(t)
0.004
0.003
0.002
0.001
time
5
10
15
20
25
30

(c)

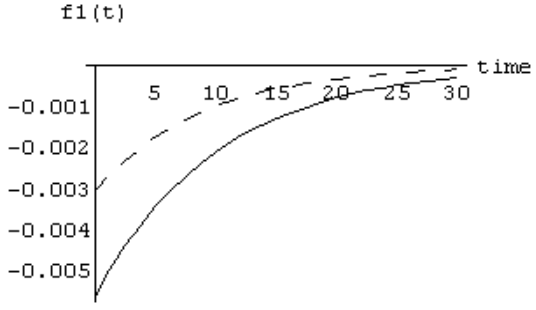


(d)

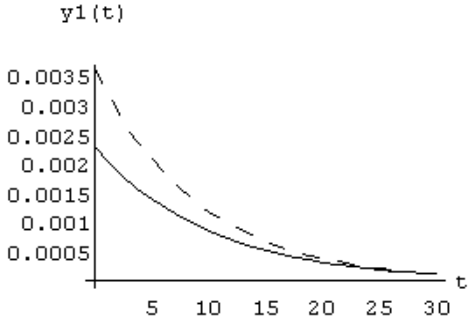


(e)

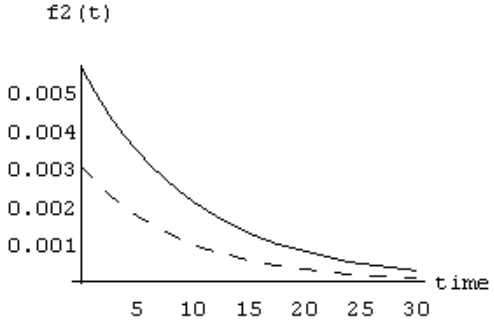
Figure 7
Cooperative Fiscal Policies, ω=1 (solid) vs. ω=0.5 (dashed)


f1(t)
time
5
10
15
20
25
30
-0.001
-0.002
-0.003
-0.004
-0.005

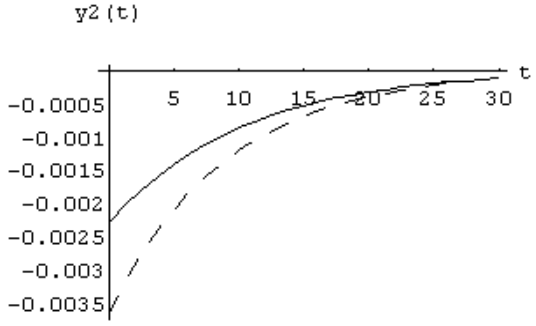
(a)


y1(t)
0.0035
0.003
0.0025
0.002
0.0015
0.001
0.0005
t
5
10
15
20
25
30

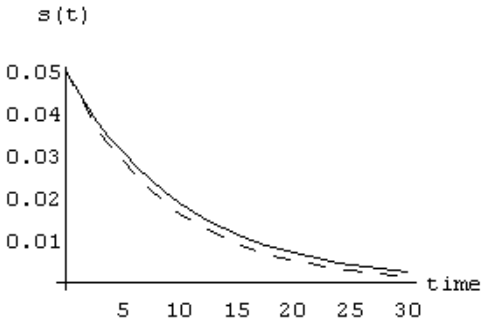
(b)


f2(t)
0.005
0.004
0.003
0.002
0.001
time
5
10
15
20
25
30

(c)



(d)



(e)

Figure 8
Non-Cooperative (solid) and Cooperative (dashed) Fiscal Policies with ξ=0.3