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Publication date:
1998

Citation for published version (APA):
Kosfeld, M. (1998). *Rumours and Markets*. (CentER Discussion Paper; Vol. 1998-23). Microeconomics.

Rumours and Markets

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Abstract

This paper studies the effects of rumours on markets. We consider a large population of agents who participate in a two-good exchange economy. Agents communicate with their local neighbors which gives rise to the possible spread of a rumour within the population. Since the rumour may affect preferences, the evolution of the rumour has a direct impact on economic variables, such as market demand and market equilibrium prices. If the rumour dies out (long-run) equilibrium prices correspond to fundamental values, while prices differ from fundamentals if the rumour stays present. When rumour effects are strong the market crashes, in the sense that trade breaks down as the ratio of relative prices converges to zero.

Journal of Economic Literature Classification Number: D11, D51

Keywords: rumour, random preference, exchange economy, market equilibrium price

*First draft, June, 1997. This version, March, 1998. I am grateful to Dolf Talman for many valuable comments. I also thank seminar participants in Vienna, Oberwesel, Tilburg, Berlin and Dortmund for helpful and critical discussions. Correspondence to: Department of Econometrics, Tilburg University, P.O.Box 90153, NL-5000 LE Tilburg, The Netherlands, e-mail: M.Kosfeld@kub.nl.

"Fama, ... mobilitate viget virisque adquirit eundo,[1]
... tam ficti pravique tenax quam nuntia veri."

— Vergil, Aeneis, IV.

1 Introduction

Rumours are part of our everyday life. They belong to human attributes since human beings are part of the first society. Sometimes rumours contain confidential information about public figures, in other cases they have hot news concerning important public topics. Rumours can shape the public opinion of a society by affecting and coordinating the individual beliefs of its members.

In consequence rumours may sometimes have a surprising impact also on the existing political system of a society. An example is the political situation in France in the mid 18th century. (Cf. Farge and Revel (1988).) There, the authorities in Paris tried to keep all beggars and vagabonds from the streets. In connection with this also a large number of children were arrested. Immediately, rumours were going round, telling all different kinds of stories. While some of them were quite plausible, saying, e.g., that the children were shipped to the colonies in North America, soon others appeared that said that the children were used for medical experiments, or even that Louis XV was taking a bath in the children's blood. In any case, the result was that the police was no longer able to carry out its original job, i.e. the purge of Paris. The long-run consequences concerning the political system are well-known.

Rumours are also part of economic life. Consider, for instance, financial markets, where traders are highly influenced by news on almost any kind of economic data, unemployment rates, interest rates, growth rates, etc.. At the same time, also industry or firm specific information can have clear effects on respective stock prices. In consequence, every little bit of information is extremely important since it may lead to comparative advantages over other traders. A perfect breeding-ground for rumours.

Other economic examples can be found in relation with consumption goods. Kapferer (1989) presents a rumour that has been spread over Europe for more then ten years. Transmitted via leaflets the rumour accused ten well-known brands of food products of being toxic and producing cancer. Among the brands were such as Coca Cola, Martini, and Schweppes. Consumers were told to boycott these products since they contained additives that despite being authorized would be toxic or carcinogenic. Interestingly enough, one of these "dangerous" additives was nothing else than harmless citric acid, which was named by its code name E 330.

[1] Roughly translated: *The rumour, ... is strong through mobility and gains power through its spread, ... is both seeking deception and a messenger of truth.*

Although rumours are well-known in economic life they are almost absent in economic theory. Only in recent years some authors have started to look at rumours in economics, both from a theoretical and from an empirical point of view. (For an empirical analysis of rumours, see, e.g., the articles of Pound and Zeckhauser (1990) and Zivney *et al.* (1996), who investigate the effect of takeover rumours on the evolution of stock prices. Recent theoretic work is, e.g., Banerjee (1993).) The reason for this is intuitive. While economic theory has always concentrated on rational behavior of individuals, rumours seem to be something very irrational. In some sense, they do not fit into the model. Moreover, the rumour, in principle, appears to be a phenomenon that is hard to capture.

The aim of this paper is to bring together standard microeconomic theory with the intuitive idea of market participants that are influenced by rumours. Our main contribution is to give a theoretical framework for the spread of a rumour through word-of-mouth communication and, to study the effects on market prices when the rumour is present.

The early beginning of a scientific analysis of rumours goes back to the end of World War II, where in the U.S. researchers began to focus on sociological issues of rumours. See e.g. Allport and Postman (1946), Knapp (1944), and Peterson and Gist (1951).[2] Since then people have often expressed the idea that the typical and almost defining characteristic of a rumour is that the information which is spread by the rumour is actually *false*. Therefore, a rumour has often been considered as something that has to be fought. However, as Kapferer (1987) correctly concludes, this idea leads to a logical problem. Why should people ever believe in a rumour if it is well-known and obvious that any information that is spread by rumour will be false? And why should then there be any need for the officials of a country or a company to fight against rumours, since it should be clear that the population has no reason to believe in rumours? Isn't rather the contrary true, that people believe in rumours because sometimes they turn out to be *true*? That sometimes they contain unknown secrets which would never be told by any official media?

The important characteristic of a rumour is not the fact whether it is true or false. Rather it is crucial that it is not possible to tell whether it is true or not. The reason is that a rumour can be both truth and deception. We say therefore that *a rumour is a piece of information that is passed from one person to the other, generally through word-of-mouth communication, without knowing exactly whether the information is true or not.*[3]

Different rumours have common dynamics. The important mechanism is often the same. Imagine somebody who is hearing an inofficial news for the first time. Because he cannot

[2]See Kapferer (1987). I refer to the german translation (1996).

[3]Perhaps a bit different is the situation where the rumour is obviously about something that is completely unrealistic. Then, still the typical characteristics can apply when the basis of reality on which the information is generally founded is replaced by the beauty of the story itself: *"Se non e vero, e bene trovato!"* (Giordano Bruno).

tell whether the news is true, he may both believe it or not. In general, this will depend on his personal attitude towards unknown information, whether he is rather credulous or more doubtful. However, if he hears the story again, this time told by some other person, he may begin to believe that perhaps at least something in it must be true. And if the story is communicated again and again, the likelihood for the rumour to be true seems to be increasing. Thousand voices cannot be mistaken. Thus, the more people know the rumour, the more often one can hear the rumour, and the more likely it is to believe the rumour. This again implies that the more likely one may become also a source of the rumour, which eventually increases again the number of people that get to hear the rumour. So on and so forth. The typical dynamics of a rumour include some kind of recruitment or infection mechanism leading to the fact that once a rumour is sufficiently known it can become self-enforcing. This mechanism will be an important element of the model we discuss in this paper.

Infection dynamics, mainly analyzed in epidemiologic theory, are also part of the economic model that is discussed in Banerjee (1993). There, the author models an agent's decision to believe in the rumour and to pass it on as an individual optimization decision. Precisely, Banerjee uses an individual Bayesian updating approach to obtain a stochastic process that describes the evolution of the rumour in time. The dynamics are then approximated through a system of differential equations that are known from the epidemiological literature. Results are obtained from an analysis of the approximating deterministic system.

In this paper we take a simplifying step and model the rumour transmission procedure as a purely mechanical act. This allows us to analyze the resulting stochastic process directly. In particular, we do not need any approximation technique in order to obtain results that concern the evolution of the rumour within the population. Moreover, we restrict communication between agents to local (social) neighborhoods, rather than allowing for global communication as it is done in Banerjee (1993). We believe that this captures in a more intuitive way the transmission dynamics of a rumour.

Other models that are similar in spirit focussing on interaction through word-of-mouth communication or recruitment mechanisms include Ellison and Fudenberg (1995), Kirman (1993), Banerjee (1992), and Bikhchandani *et al.* (1992). The main difference, however, lies in the underlying question that is posed. While the above literature mainly concentrates on the problem of social learning and efficiency: "How does a society of interacting agents aggregate individual information, thereby leading to inefficient or efficient outcomes?", our main question is on the economic effects of word-of-mouth communication as a special kind of interaction within markets: "How do economic variables such as demand and equilibrium prices get affected when preferences of individual agents are a product of rumour-like communication?" With this respect, the work of Kirman (1993) is, perhaps, most closely related.

Mathematically, this paper is very much inspired by the attempt to use recent theoretical concepts from statistical mechanics in order to model the relation between a large number of agents who interact on the so-called *micro-level* and possible resulting structures on the so-called *macro-level*. The main technical tools come from the theory of interacting particle systems. (See Liggett (1985) for an outstanding introduction into this field.) In fact, some of the ideas we present have also been addressed in work of Föllmer (1974) about random economies. There, the author studies the existence and quality of equilibria in economies where individual preferences of agents are both random and depend on the local environment of agents. Our approach coincides with this analysis in the sense that we also consider random preferences with distributions that are conditional on the state of the local neighborhood of an agent. However, our study differs from the one of Föllmer in two main ways. First, we explicitly consider a *dynamic* framework that models the evolution of preferences within time. Equilibria are stationary distributions of the underlying stochastic process. Second, we regard economic agents not as simple statistical objects (the economic counterpart of the molecul in physics), but as individuals who at the same time both are influenced from interaction, here communication, with neighbors and behave individually rational given their conception of the world. The latter is the standard assumption in most of economic theory. The first is, perhaps, more common within sociological work about our human society.

The paper is organized as follows. We start presenting the basic set-up to analyze the spread of a rumour within a population of traders, or consumers, who act within a two-good exchange economy (Sections 2 and 3). Section 4 discusses the assumptions made on the transmission of the rumour which determines the stochastic evolution of the rumour in time. Convergence from arbitrary initial distributions is characterized in Section 5. Section 6 studies the notion of individual and aggregated demand and defines market equilibrium prices for the random economy. In particular we study the impacts on long-run equilibrium prices. Results show that if the rumour dies out long-run equilibrium prices correspond to fundamental values of the economy. However, if the rumour stays present, prices are different from fundamental ones if and only if the rumour affects individual preferences. Notably, when these effects are strong the market may crash, in the sense that trade breaks down as the ratio of relative prices converges to zero. The final Section 7 concludes with a discussion.

2 Structure of the economy

Consider the following economy. There is a countable infinite set of *economic agents*. These are the market participants, i.e. traders or consumers that act within the same market. We consider exactly one market, which is modelled as a two-good exchange economy.

We assume that agents are located on the one-dimensional integer line $\mathbb{Z}$ and identify each agent with his or her location. Typically, agents will be denoted by $x, y, z \in \mathbb{Z}$. We assume furthermore that the economy is endowed with a *neighborhood structure* that looks as follows. For any agent $x \in \mathbb{Z}$, $N(x) = \{x-1,\ x+1\}$ is said to be the set of *(social) neigbors* of agent x. So neighbors are exactly those two agents who are located directly next to an agent, either on the left-hand or on the right-hand side of that agent. Notice that this neighborhood structure has three important features. First, the neighborhood structure is *symmetric*, i.e. every agent is an element of the neighborhood of each of his neighbors. Second, it is *local*, i.e. every agent is socially connected not to the whole rest of the economy but only to a finite subset of other agents. Third, neighborhoods have a considerable *overlap*, i.e. every agent in the economy shares each of his neighbors with another agent who himself is not an element of his neighborhood. Or, to put it differently, every agent is the neighbor of two non-neighboring agents.[4]

We think of neighborhoods as exclusive places of communication. Hence, the model assumes that every agent communicates both directly with two other agents, namely his neighbors, and indirectly, through the channel of overlapping neighborhoods also with the rest of the economy. This assumption seems intuitive for situations of word-of-mouth communication that we have in mind. Rumour-like information often comes to us in a way where one neighbor tells us a story like: “Hey, I ’ve got a neighbor who has a neighbor who again has a neighbor who ... who has heard that ...!”

We want to analyze a two-good exchange economy where agents additionally interact trough communication of a rumour. Within such an economy, at any time every agent is completely characterized by his endowment (portfolio), his preference, and the fact whether he knows the rumour, or not. This is modelled by the function

$$\begin{aligned} \sigma_t : \mathbb{Z} &\rightarrow \mathbb{R}^2_+ \times \mathcal{P} \times \{0,1\} \\ x &\mapsto \sigma_t(x) = (\omega_t(x), \succeq_t(x), \eta_t(x)). \end{aligned} \tag{1}$$

The index t captures time, which we model to be continuous, so $t \in \mathbb{R}^+_0$. We say that $\sigma_t(x)$ gives the *state of agent* x at time t. The first coordinate gives the agents vector of endowments of the two available goods in the economy, $\omega_t(x) = (\omega_{t1}(x), \omega_{t2}(x))$. The second coordinate identifies the preference over vectors of goods, which we assume to be an element of the set of all *admissible* preferences, denoted by $\mathcal{P}$. For the moment, we leave this set unspecified. It will be defined in the next section. The last coordinate is crucial. It determines whether an agent knows the rumour, or not. If $\eta_t(x) = 1$ this stands for the fact that at time t agent x knows the rumour, while $\eta_t(x) = 0$ means that at this time agent x does not know the rumour.

For any $t \geq 0$, σ_t represents the state of the whole population at time t. We say that σ_t gives the current *configuration*. Let $X := \{\sigma | \, \sigma : \mathbb{Z} \rightarrow \mathbb{R}^2_+ \times \mathcal{P} \times \{0,1\}\}$ denote the space of

[4]For example, agents x and $x+2$ both have agent $x+1$ as a neighbor but at the same time are no neighbors themselves.

all possible configurations, thus $\sigma_t \in X$ for any $t \geq 0$. Denote $\Gamma := \{\eta | \eta : \mathbb{Z} \to \{0,1\}\}$ the space of all projections of configurations σ onto the last coordinate, describing the distribution of knowledge of the rumour within the economy. An element $\eta \in \Gamma$ is called a rumour *pattern*.

3 Rumours and preferences

Rumours affect preferences. There exists a link between knowledge of the rumour on the one hand and preferences over commodity bundles on the other. Recall, e.g., the example discussed in Kapferer (1989), where the rumour accused specific brands of food products of being toxic and producing cancer. In this case, the informative element of the rumour affects preferences *directly*. Even great fans of a specific product will cease to consume that product once they believe it to be toxic or to produce cancer.[5] Take sugar and saccherine as an example. Once people believe that saccherine produces cancer, they may easily switch consumption to sugar. Weighing some pounds more, obviously, appears to be less dangerous that facing the risk of cancer. In financial markets, takeover rumours are a prominent example for a link between the rumour and the preferences of market participants. The rumour can affect preferences, e.g., when the takeover is seen as a signal for the value of the firm that is to be taken over. For more information about this issue confirm the articles of Pound and Zeckhauser (1990) and Zivney *et al.* (1996).

Rumours may also affect preferences *indirectly*. As in the example of the vanishing children of Paris, the rumour may concern a topic that is only indirectly related to preferences. While the information of the rumour directly addresses only the possible whereabout of the children it indirectly affects the preference of the people towards the authorities. Economic examples of indirect effects are rumours on macroeconomic data, as unemployment, interest rates, or growth rates.

In any case, whether it is direct or indirect, the effect of a rumour on preferences lies at hand. People that know the rumour can be expected to have different preferences than before. Still, the precise mechanism between rumours and preferences does not seem to be completely understood, yet. This appears to be an open research topic, especially for psychologists and sociologists. However, since in this paper, we are interested not in the mechanism itself but rather in the economic impacts of such rumour-preference effects, we take a pragmatic position and model these effects in a very simple way. Our assumptions are the following.

Corresponding to the fact that there are exactly two goods available in the economy, we assume that there are two different basic opinions: one is in favor of good 1, the other is in favor of good 2. Depending on which opinion applies an agent chooses only that specific good

[5] A usual reply is that people smoke. However, that situation is different, because *all* cigarettes are toxic and produce cancer. Thus, at least the absence of a non-toxic substitute in this case is a special element that may keep people from quitting smoking.

and ignores the other. However, the opinion may change and therefore corresponding behavior is variable instead of being deterministic. Precisely, we assume that with probability θ an agent shares the first opinion and demands only good 1, with probability $1-\theta$ the agent shares the second opinion and demands only good 2, where $0 \leq \theta \leq 1$. Thus, we assume preferences to be random.

Formally, such a random preference looks as follows:

$$\succeq^{\theta} = \begin{cases} \succeq_1 & \text{with prob.} \quad \theta \\ \succeq_2 & \text{with prob.} \quad 1-\theta, \end{cases} \tag{2}$$

where $\succeq_1$ and $\succeq_2$ represent the basic opinions, i.e. for any $\omega, \omega' \in \mathbb{R}^2_+$, $\omega \succeq_1 \omega'$ iff $\omega_1 \geq \omega'_1$ and $\omega \succeq_2 \omega'$ iff $\omega_2 \geq \omega'_2$. The set of admissible preferences is the set of all random preferences of this type,

$$\mathcal{P} := \{\succeq^{\theta} \mid 0 \leq \theta \leq 1\}. \tag{3}$$

Remark: As we will see in section 6, the *expected* demand function of an agent with random preference $\succeq^{\theta} \in \mathcal{P}$ is identical to the demand function obtained from a Cobb-Douglas preference with parameter θ. Thus, the average behavior of an agent in our model meets classical assumptions of economic theory. (Cf. also Hildenbrand (1971).)

In order to model the relation between knowledge of the rumour and preferences, we assume that there exists a preference $\succeq^{\bar{\theta}} \in \mathcal{P}$, which we call the *fundamental* preference, that describes the opinion of participants in the market, before the rumour is introduced. That is, one can think of the situation as being such that there is a share of agents equal to $\bar{\theta}$ who take the first opinion and prefer good 1 over good 2 and a share of agents equal to $1-\bar{\theta}$ who take the opposite position. Thus, the general opinion of the market as a whole can be expressed by the value $\bar{\theta}$. Formally, we model this in the way that every agent has the same random preference $\succeq^{\bar{\theta}}$, if there is no rumour present yet.

Once the rumour is introduced at say, $t = t_0$, it affects preferences of those agents that know the rumour. In our model, at any time $t \geq t_0$ this is the set of agents

$$\{x \in \mathbb{Z} \mid \eta_t(x) = 1\}. \tag{4}$$

We assume that preferences change through a change in its parameter from the value $\bar{\theta}$ to some (other) value θ^*, where $\theta^* \in [0,1]$. (This includes the situation where preferences stay the same, i.e. $\theta^* = \bar{\theta}$.) Thus after the rumour has been introduced the preference of the population can be described as follows. For agent x at time $t \geq t_0$,

$$\succeq_t(x) = \begin{cases} \succeq^{\bar{\theta}} & \text{if} \quad \eta_t(x) = 0 \\ \succeq^{\theta^*} & \text{if} \quad \eta_t(x) = 1. \end{cases} \tag{5}$$

Those agents that do not know the rumour, have the fundamental preference $\succeq^{\bar{\theta}}$, while all agents that know the rumour have the preference $\succeq^{\theta^*}$, where $\succeq^{\theta^*} \in \mathcal{P}$. We hereby assume that the change in preferences is the same for every agent in the economy.

Notice, that these assumptions allow for all rumour-preference effects that have been described above. Within our set-up a specific rumour is completely determined by the pair of parameters $(\bar{\theta}, \theta^*)$. For example, if $\theta^* > \bar{\theta}$ this corresponds to a situation where the rumour is negative about good 2, thus the probability of prefering good 1 over good 2 increases. If $\theta^* < \bar{\theta}$, it is the other way round. Moreover, the difference between $\bar{\theta}$ and θ^* can be seen as a measure of the degree of the effect. If the difference is large, the effect is strong, while if the difference is small, the corresponding effect is weak. The effect is at its extreme in the case where, e.g., $\bar{\theta} = 0$ and $\theta^* = 1$. In this case the rumour produces a complete preference reversal.

4 Communication of the rumour

Having defined the general structure of the economy and the relation between rumours and preferences, we focus in this section on the transmission dynamics of the rumour. In particular, we make assumptions on the communication of the rumour within a given neighborhood. Since the rumour is transmitted via communication within distinct but overlapping neighborhoods, these assumptions will then determine the dynamics of the rumour itself.

First, every agent who knows the rumour directly communicates it only to his two neighbors. This is in line with the idea mentioned above that neighborhoods are the exclusive places of direct communication between agents.

In consequence and *second*, the probability for an agent, who does not know the rumour, to get to know the rumour is zero if none of his two neighbors knows the rumour.

Third, the probability to get to know the rumour, to "believe it" and in consequence to communicate it is strictly greater than zero if at least one neighbor knows the rumour. Moreover, this probability is increasing with the number of neighbors that know the rumour. This assumption is motivated by the idea that people are strongly influenced by majorities. Every neighbor who knows and communicates the rumour increases the subjective probability of an agent that the rumour must be true. If I hear a story once, I can believe it or not, but if I hear it also from a second person there must at least be something in it that is true.

Fourth, agents can forget the rumour. Due to other objects that may appear in everyday communication the importance or relevance of a rumour may decrease. In consequence, agents that have heard the rumour may stop thinking of it and forget about it after a while. The probability to forget the rumour is constant and independent from the state of neighbors. The intuition is that imperfect recall comes from individual bounded rationality rather than from non-existing communication between neighbors. In particular, the fact that neighbors confirm the rumour has no influence on forgetting.

Fifth, communication does not depend on the first two elements of agents' states, i.e. endowments and preferences. Of course, people also have preferences on talking about different rumours. Some rumours are more likely to be communicated than others. However, since we assume that preferences in our model concern only preferences over commodity bundles, these should be independent from the attitude towards communicating the rumour itself. In other words, we do not model the preferences that capture the talk about the rumour itself as part of the preference relations in $\mathcal{P}$. It is done through assumptions on the communication directly.[6]

Formally, we model the transmission of the rumour as a continuous-time Markov process. This is done by defining so-called flip rates $c(x, \sigma_t)$, where $x \in \mathbb{Z}$ and $\sigma_t \in X$. They determine the probability that given a configuration σ_t agent x changes his rumour-specific state in such a way that for small $s \downarrow 0$ it holds that

$$P[\eta_{t+s}(x) \neq \eta_t(x)] = c(x, \sigma_t) \cdot s + o(s). \tag{6}$$

The following definition of flip rates captures the five assumptions made above.

Definition 4.1 *For $x \in \mathbb{Z}$, $\sigma_t \in X$, flip rates are defined as follows:*

$$c(x, \sigma_t) := \begin{cases} \lambda \sum\limits_{y \in N(x)} \eta_t(y) & \text{if} \quad \eta_t(x) = 0 \\ 1 & \text{if} \quad \eta_t(x) = 1, \end{cases} \tag{7}$$

where λ is some constant, $\lambda \geq 0$.

Notice first that, in accordance to assumption five, flip rates depend only on the projection of σ_t onto its third coordinate, i.e. η_t. Moreover, they also depend only on the restriction of the configuration to the neighborhood of an agent. This corresponds to the first assumption. The second assumption relates to the fact that for $\eta_t(x) = 0$, $c(x, \sigma_t) = 0$ if and only if $\eta_t(y) = 0$ for every $y \in N(x)$. Assumption three concerns the part of the definition that holds for $\eta_t(x) = 0$: $c(x, \sigma_t)$ is increasing with every neighbor y such that $\eta_t(y) = 1$. The fourth assumption is obviously captured in the part of the definition that holds for $\eta_t(x) = 1$.

In this model the parameter λ is constant and exogenous. It determines the degree of social interaction and communication between neighbors. In this sense, it also captures the preference of agents to talk about the rumour, as described in assumption five. If $\lambda = 0$, this implies that there is no communication between neighbors, a situation we shall not be interested in during the following. If $\lambda > 0$, the value directly determines the probability for an agent who does

[6]Notice that assumption five is not in contradiction with the assumption made before that rumours affect preferences over commodity bundles.

not know the rumour yet to get himself infected by talking to one of his neighbors. As we can see from equation (7), the higher the value of λ the higher this probability is. Thus, λ measures also the social closeness between neighbors. Another interpretation is that λ is related to the relevance of the rumour. High values correspond to a great importance of the rumour to individual agents. This idea is clearly connected to the first one, since hot issues are generally communicated more intensively.

The fact that the rate for forgetting the rumour is exactly equal to one is simply due to normalization and has no qualitative effects on the results. It rather allows us to control the relation between the probability to forget the rumour and the probability to get infected by the rumour via the single parameter λ.

Following standard theory on interacting particle systems, flip rates as in Definition 4.1 define a unique continuous time Markov process $\{\sigma_t\}_{t\geq 0}$ on the state space of all possible configurations X. Since the dynamics of the process follow those of the well-known contact process, results concerning the evolution of the rumour follow from general results on spin systems and from specific results on the contact process (Liggett (1985)).

5 Evolution of the rumour

In this section we focus on the evolution of the rumour itself, i.e. the (projected) process $\{\eta_t\}_{t\geq 0}$. Let $\mathcal{M}$ denote the set of probability measures on the space $\Gamma = \{\eta | \, \eta : \mathbb{Z} \to \{0,1\}\}$, provided with the topology of weak convergence. We assume that the rumour is introduced at time $t = 0$ in the way that the inital pattern is described by some probability distribution $\mu \in \mathcal{M}$. The objective is therefore to characterize convergence of the process η_t for arbitrary initial distributions.

The first observation is the following. For any parameter value λ the Dirac-measure $\nu_0 := \delta_\emptyset$, i.e. the measure that puts total mass on the rumour pattern $\eta \equiv 0$, signifying that nobody in the economy knows the rumour, is an invariant distribution for the rumour.[7] This means that once nobody in the economy knows the rumour the rumour will be gone for ever in the future. There is no spontaneous source for the rumour except (possibly) at the beginning.

The second and crucial observation is that — in spite of its simplicity — the rumour exhibits a *phase transition*.[8] This means that there exists a critical value λ^* such that for any $\lambda < \lambda^*$, ν_0

[7] Note that there exists a one to one correspondence between the space Γ and $\Delta = \{T| \; T \subset \mathbb{Z}\}$ via the mapping $\chi : \Gamma \to \Delta$, $\chi(\eta) := \{x| \; \eta(x) = 1\}$. Thus, for any set T, the Dirac-measure δ_T puts probability one on the rumour pattern, where exactly all agents $x \in T$ know the rumour and nobody else.

[8] The phenomenon of phase transitions has already been studied in other economic applications. See, e.g., Föllmer (1974) for an early economic analysis of phase transitions in the situation of a random exchange economy. Other examples include Allen (1982) and Durlauf (1993). Allen investigates the effects of innovation in an economy that is described by a Markov random field. Durlauf works in a growth-theoretic context. He uses the random field approach in order to describe the possible dynamics of an economic take-off as an effect

is the unique invariant distribution and also the unique limiting distribution, independent on initial conditions. In this case the rumour will always disappear. However, for any $\lambda > \lambda^*$, the set of invariant measures is equal to the convex set that is spanned by the extreme measures ν_0 and ν_λ, where the latter is the limiting distribution obtained from starting with the intitial pattern $\eta \equiv 1$. Hence, λ^* is the critical value where the ergodicity of the rumour breaks down. Obviously, the interesting case — also from an economic point of view — is the second nonergodic one.

The following proposition characterizes the possibilities for the stochastic evolution of the rumour in both the ergodic and the nonergodic case. Proofs are given in the appendix. The subsequent definition then explains what we mean by saying, the rumour disappears, or, the rumour stays persistently present.

Proposition 5.1 *There exists a critical value λ^* such that $\forall \lambda < \lambda^*$, ν_0 is the unique invariant and limiting distribution for the rumour. If $\lambda > \lambda^*$, the set of invariant distributions is equal to the nondegenerate convex set*

$$\mathcal{I} = \{\nu \mid \nu = \alpha\nu_0 + (1-\alpha)\nu_\lambda,\ 0 \le \alpha \le 1\}. \tag{8}$$

The measure ν_λ is obtained as the limit

$$\nu_\lambda = \lim_{t\to\infty} \mu_t, \tag{9}$$

where μ_t denotes the distribution of the rumour at time t when $\mu_0 = \delta_{\underline{1}}$ is the Dirac-measure that puts full mass on the pattern $\eta \equiv 1$.

Results of Liggett (1985) put an upper bound of 2 on the value of λ^*. This indicates already that the nonergodic regime is not only the more interesting one but, if we allow λ to take any finite value, is also the one that is more likely. For $\lambda > \lambda^*$, the two distributions ν_0 and ν_λ are orthogonal (see the proof of Proposition 5.1), i.e. in particular $\nu_\lambda(\eta \equiv 0) = 0$. Therefore, ν_λ ensures that with probability one at least some agent in the economy knows the rumour. Hence, convergence to the distribution ν_λ corresponds to saying that the rumour will be persistently present in the economy, while convergence to the distribution ν_0 corresponds to saying that the rumour disappears.

of impulses coming from leading technologies in an economy. For a short and rather informal introduction into the economic sense of phase transitions see also Hors (1995).

Definition 5.2 *For given λ let μ_t be the distribution of the rumour at time t, starting with initial distribution $\mu \in \mathcal{M}$. Thus, in particular $\mu_0 = \mu$. The rumour* **disappears** *if*

$$\lim_{t\to\infty} \mu_t = \nu_0, \tag{10}$$

or equivalently,

$$\lim_{t\to\infty} \mu_t(\eta \equiv 0) = 1. \tag{11}$$

If $\lambda > \lambda^*$, the rumour is **persistently present** *if*

$$\lim_{t\to\infty} \mu_t = \nu_\lambda. \tag{12}$$

In this case it follows that

$$\lim_{t\to\infty} \mu_t(\eta \equiv 0) = 0, \tag{13}$$

and

$$\lim_{t\to\infty} \mu_t\Big(\eta(x) = 1\Big) > 0, \tag{14}$$

for any $x \in \mathbb{Z}$.

Passing from the ergodic to the nonergodic regime the evolution of the process undergoes an abrupt change. While in the first regime it is sure that the rumour will disappear for any initial distribution, in the second regime the evolution of the process is much more ambiguous. Since the set of invariant distributions $\mathcal{I}$ is no longer a singleton but the whole "interval" between ν_0 and ν_λ, in principle any measure $\nu \in \mathcal{I}$ is a candidate for the limiting distribution of the process starting with some initial distribution μ. Proposition 5.4 clarifies this convergence for arbitrary initial distributions. A special class of initial distributions for which convergence can be determined very easily is the class of translation invariant distributions.

Definition 5.3 *A probability measure $\mu \in \mathcal{M}$ is* **translation invariant** *if for any $(x_1, \ldots, x_k)$, $x_j \in \mathbb{Z}$, any values $(i_1, \ldots, i_k)$, $i_j \in \{0, 1\}$, and any $z \in \mathbb{Z}$*

$$\mu\Big(\eta(z + x_1) = i_1, \ldots, \eta(z + x_k) = i_k\Big) = \mu\Big(\eta(x_1) = i_1, \ldots, \eta(x_k) = i_k\Big), \tag{15}$$

i.e. probabilities do not depend on z.

Denote τ the stopping time for the rumour to enter the state $\eta \equiv 0$. Let P^μ denote the probability measure on the canonical path space of the rumour starting with initial distribution

μ.[9]

Proposition 5.4 *Consider the case where $\lambda > \lambda^*$. Let $\mu \in \mathcal{M}$ be any arbitrary initial distribution ($\mu_0 = \mu$). Then*

$$\nu = \lim_{t\to\infty} \mu_t = \alpha\nu_0 + (1-\alpha)\nu_\lambda, \tag{16}$$

where

$$\alpha = P^\mu[\tau < \infty]. \tag{17}$$

Thus, α equals the probability for the rumour to disappear in finite time. If μ is translation invariant,

$$\alpha = \mu(\eta \equiv 0). \tag{18}$$

For translation invariant distributions the value of α that determines the mixture between ν_0 and ν_λ is explicitly given by the probability for the initial pattern to be $\eta \equiv 0$, signifying that nobody in the economy hears the rumour at the beginning. Once we can ensure this probability to be zero we obtain weak convergence to the distribution ν_λ, thus we know that the rumour will be persistently present in the economy for any time in the future. At the same time we see that the rumour dies out if and only if $\mu(\eta \equiv 0) = 1$, i.e. at the beginning there is simply nobody who knows the rumour.

This result has a nice consequence. Assume that at the beginning everybody in the economy has the same chance to hear the rumour. In order to model this, consider the initial distribution to be determined as follows. A random process assigns to each agent independently the value 0 or 1. Let this process be binomially distributed with parameter ϵ. Hence, μ is the Bernoulli product measure with $\mu\big(\eta(x) = 1\big) = \epsilon$, for every agent $x \in \mathbb{Z}$. In other words, at the beginning everybody hears the rumour with the same probability ϵ. In this situation $\mu(\eta \equiv 0) = 0$ if and only if $\epsilon > 0$. Thus, if we know that the probability for every agent in the economy to hear the rumour is strictly positive, we can conclude that in consequence the rumour will never die out but will be persistently present, even if ϵ is arbitrarily small. Only if $\epsilon = 0$ the rumour will (trivially) die out since it will not even be known at the beginning.

The assumption for μ to be translation invariant can be very restrictive. As we have just seen, this can mean that everybody has in fact the same access to relevant information, an

[9]In general a continuous time Markov process with state space Γ is given by a collection of probability measures $\{P^\eta\}_{\eta\in\Gamma}$ on the canonical path space of the process. Each P^η determines the stochastic evolution of the process starting with initial configuration η. In consequence,

$$P^\mu = \int_\Gamma P^\eta \, \mu(d\eta).$$

assumption that can perhaps be hard to be argued in many economic situations. However, since in general condition (17) applies, it is possible to specify the value of α for every initial distribution. Particularly, also for Dirac-distributions δ_η, with η being some arbitrary pattern. The next proposition determines the value of α in this case.

Proposition 5.5 *If the initial distribution is $\mu = \delta_\eta$, with $\eta \in \Gamma$ being some pattern,*

$$P^{\delta_\eta}[\tau < \infty] = 0 \;\Leftrightarrow\; \eta \text{ contains infinitely many agents that know the rumour,} \tag{19}$$

$$\Leftrightarrow\; |\{x \,|\, \eta(x) = 1\}| = \infty. \tag{20}$$

Thus, in this case, the only requirement for the rumour to be persistently present is that the set of agents that get to know the rumour at the very beginning is infinitely large, while it is irrelevant how dense the set is. The final corollary summarizes.

Corollary 5.6 *Suppose $\lambda > \lambda^*$. Given any initial distribution μ, the rumour will be persistently present if and only if*

$$\int_X P^\eta[\tau < \infty]\; \mu(d\eta) = 0. \tag{21}$$

If μ is translation invariant this condition reduces to

$$\mu(\emptyset) = 0. \tag{22}$$

If $\mu = \delta_\eta$ the condition reduces to η containing infinitely many agents that know the rumour.

6 Market equilibrium prices

In the previous section we have seen that the evolution of the rumour is nonergodic if the parameter λ that measures the intensity of communication between neighbors is sufficiently large. This nonergodicity has an immediate consequence on economic variables as market demand and market equilibrium prices.

Assume, for the moment, the state of the economy to be given by some concrete configuration σ. Thus, we freeze dynamics and analyze the economy from a static viewpoint. Since preferences are random, denote $\zeta(\sigma(x), p)$ the *expected excess demand function* of agent x, given his state $\sigma(x)$ and some price vector $p = (p_1, p_2)$. In view of our assumptions on the set of admissible preferences, we can write expected excess demand as follows.

$$\zeta(\sigma(x), p) = \zeta((\omega(x), \succeq^\theta, \eta(x)), p) \tag{23}$$

$$= \left(\theta \frac{p_2 \omega_2(x)}{p_1} - (1-\theta)\omega_1(x), -\theta\omega_2(x) + (1-\theta)\frac{p_1\omega_1(x)}{p_2}\right), \tag{24}$$

where, $\theta = \bar{\theta}$ if $\eta(x) = 0$, and $\theta = \theta^*$ if $\eta(x) = 1$. As explained in section 3, $\bar{\theta}$ determines the fundamental preference of agents that do not know the rumour, while θ^* identifies the change in preferences for those agents that know the rumour.

Note that for any random preference $\succeq^{\theta}$, expected excess demand equals just the excess demand of an agent with Cobb-Douglas utility function $u(\omega) = \omega_1^{\theta}\omega_2^{1-\theta}$. On average, an agent spends θ of his wealth on good 1 and $1 - \theta$ on good 2.

In our model a rumour shapes the population of market participants in a stochastic manner. Thus, we have to analyze the market as a random environment which is distributed according to some probability measure on the space of configurations X. What is an appropriate equilibrium condition for a market equilibirum price vector in such a random economy? Föllmer (1974) studies the notion of equilibrium prices in an economy that is modelled as a random field. Since our economy can be seen as a dynamic analogon to this approach, we may consider his definition of a market equilibrium price.

Let ρ be a probability measure on the space X, that describes the random configuration σ of the economy.

Definition 6.1 *A price vector p is a* **market equilibrium price vector** *for the economy that is described by the measure ρ if*

$$\lim_{n\to\infty} \frac{1}{|B_n|} \sum_{x\in B_n} \zeta(\sigma(x), p) = (0,0), \qquad \rho - a.s. \tag{25}$$

where $B_n = \{-n/2, \ldots, n/2\}$ and n even.

Market equilibrium prices are prices where per capita excess demand is zero, ρ-a.s..

Let us now reconsider the dynamic set-up. From Definition 6.1 we know that for any time t, a market equilibrium price vector p_t for the economy that at time t is described by the distribution ρ_t must satisfy the condition

$$\lim_{n\to\infty} \frac{1}{|B_n|} \sum_{x\in B_n} \zeta(\sigma(x), p_t) = (0,0). \qquad \rho_t - a.s. \tag{26}$$

Be μ_t the marginal distribution of ρ_t, describing the distribution of the rumour within the economy. From section 5 we know that, taking the limit $t \to \infty$, the distribution μ_t converges to an invariant distribution of the rumour. If $\lambda < \lambda^*$, the unique invariant distribution is ν_0. The rumour always disappears. If $\lambda > \lambda^*$, the rumour stays persistently present for a large class of initial distributions, i.e. we obtain $\lim_{t\to\infty} \mu_t = \nu_\lambda$. Moreover, convergence to equilibrium ν_0 if $\lambda < \lambda^*$, and to ν_λ if $\lambda > \lambda^*$, can be shown to be exponentially rapid (cf. Liggett (1985)). Hence, both of these limiting distributions may serve as an approximation for the distribution of the rumour when t is large, that is when the rumour has been around in the population for a sufficient amount of time. We use this approximation in order to calculate so-called *long-run equilibrium prices.*

Up to now we have not made any restricting assumptions on individual endowments of agents. However, in order to be able to calculate equilibrium prices, at this point we have to make the assumption that, at least in the long run, all agents have the same endowment $\omega(x) \equiv \omega$. In consequence, for any fixed parameter pair $(\bar{\theta}, \theta^*)$ with corresponding preferences of agents, the spaces X and Γ are isomorph. Hence probability measures ρ on X and μ on Γ are equivalent and we can apply Definition 6.1 directly to distributions μ, i.e. in particular to ν_0 and ν_λ.

The crucial fact we can exploit is the observation that for distributions $\nu \in \{\nu_0, \nu_\lambda\}$ the following holds,

$$\lim_{n\to\infty} \frac{1}{|B_n|} \sum_{x\in B_n} \zeta(\sigma(x), p) = E^\nu[\,\zeta(\sigma(0), p)\,], \qquad \nu - a.s. \tag{27}$$

with 0 denoting the agent that is located at the origin. This follows from ergodic theory since both measures are translation invariant and also extreme within the set of translation invariant measures (cf. Liggett (1985)). Thus, in a sense we are in a situation where the agent at location zero is in fact a *representative agent* for the economy. Note, however, that this relation breaks down as soon as we consider any other invariant distribution for the rumour, that is a real mixture of both extreme measures ν_0 and ν_λ. In that case we are left alone with equation (25) again.

In consequence, applying Definition 6.1 to distributions $\nu \in \{\nu_0, \nu_\lambda\}$, the condition for long-run equilibrium prices reduces to a much simpler one.

Definition 6.2 *A price vector p is a* **long-run equilibrium price vector** *for the distribution $\nu \in \{\nu_0, \nu_\lambda\}$ if the following condition holds,*

$$E^\nu[\,\zeta(\sigma(0), p)\,] = (0, 0). \tag{28}$$

For notational convenience, denote for a measure $\nu \in \{\nu_0, \nu_\lambda\}$,

$$\nu(1) := \nu\big(\eta(0) = 1\big), \tag{29}$$

and

$$\nu(0) := \nu\big(\eta(0) = 0\big). \tag{30}$$

The next proposition gives the condition for long-run equilibrium prices assuming endowments to be strictly positive.

Proposition 6.3 *Assuming $\omega_1, \omega_2 > 0$ and $(\bar{\theta}, \theta^*) \neq (0,0)$, long-run equilibrium prices are given by the condition*

$$\frac{p_2}{p_1} = \frac{\nu(0)(1-\bar{\theta}) + \nu(1)(1-\theta^*)}{\nu(0)\bar{\theta} + \nu(1)\theta^*} \frac{\omega_1}{\omega_2}. \tag{31}$$

For a price vector to constitute a long-run equilibrium price vector, the ratio of individual prices has to equal the ratio of endowments times weigthed shares of different opinions in the market, where weights equal the probability for the representative agent to know the rumour or not.

We start analyzing the situation when the rumour disappears, $(\nu = \nu_0)$.

If the long-run distribution equals ν_0, all terms involving θ^* in equation (31) disappear, since $\nu_0(0) = 1$ and $\nu_0(1) = 0$. Hence, the unique long-run equilibrium price is the *fundamental price* vector for goods 1 and 2, that corresponds to fundamental preferences:

$$\frac{p_2}{p_1} = \frac{1-\bar{\theta}}{\bar{\theta}} \frac{\omega_1}{\omega_2}. \tag{32}$$

If the rumour stays persistently present $(\nu = \nu_\lambda)$, the situation looks different. In this case, from equation (31) we obtain a unique long-run equilibrium price vector $p(\lambda)$ which differs from the fundamental price vector if and only if θ^* differs from $\bar{\theta}$, thus iff the rumour has any effect on preferences.

Assume $\theta^* > \bar{\theta}$, hence the rumour is negative about good 2; the probability for preferring good 1 over good 2 increases. Since $\nu_\lambda(1) = 1 - \nu_\lambda(0)$, we obtain

$$\frac{p_2(\lambda)}{p_1(\lambda)} = \frac{\nu_\lambda(0)(1-\bar{\theta}) + \nu_\lambda(1)(1-\theta^*)}{\nu_\lambda(0)\bar{\theta} + \nu_\lambda(1)\theta^*} \frac{\omega_1}{\omega_2} \tag{33}$$

$$< \frac{1-\bar{\theta}}{\bar{\theta}} \frac{\omega_1}{\omega_2}. \tag{34}$$

Thus, also the relative price of good 1 increases with respect to the fundamental one. By the same argument, the relevant price of good 2 increases when the rumour is negative about good 1, $(\theta^* < \bar{\theta})$.

Assume we increase the parameter λ (keeping other parameters fixed), signifying the degree of communication to become stronger. The interpretation is that either, people discuss new information more extensively, or the rumour is about a more relevant or more important issue. However, of course, no interpretation has to exclude the other.

Using a result from Liggett (1985), which says that for any $\lambda \geq 2$,

$$\nu_\lambda(1) \geq \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{1}{2\lambda}}, \tag{35}$$

we obtain that $\lim_{\lambda\to\infty} \nu_\lambda(1) = 1$. Thus, increasing λ to infinity, the condition for long-run equilibrium prices becomes

$$\lim_{\lambda\to\infty} \frac{p_2(\lambda)}{p_1(\lambda)} = \frac{1-\theta^*}{\theta^*} \frac{\omega_1}{\omega_2}. \tag{36}$$

The latter can be seen as convergence to the *rumour price*, in contrast to the fundamental price above. Intensifying the communication increases the probability for agents to eventually believe in the rumour, thereby shifting preferences from fundamental values to rumour values which, finally, increases the demand in the good that is "preferred" by the rumour.

Before we conclude, imagine a situation where the rumour effects are strong, i.e. the parameter θ^* differs sufficiently from the fundamental equivalent $\bar{\theta}$. Without loss of generality, consider the situation to be such that $0 < \bar{\theta} < \frac{1}{2}$ while $\theta^* \approx 1$. Before appearance of the rumour the general opinion in the market is slightly in favour of good 2, while all agents that know the rumour tend to prefer good 1 with a high probability. In order to make it clear, assume that agents knowing the rumour prefer good 1 over good 2 with probability one, i.e. $\theta^* = 1$. From equation (36) we see immediately that, if we increase the communication parameter λ to infinity, the ratio of corresponding long-run equilibrium prices converges to zero:

$$\lim_{\lambda\to\infty} \frac{p_2(\lambda)}{p_1(\lambda)} = 0. \tag{37}$$

Since the intensity of communication of the rumour has no limit the probability for agents to eventually believe in the rumour increases to one. Consequently, because of the effects on preferences, this leads to an infinite increase of demand in the good that is "preferred" by the rumour, letting the relative price of that good increase to infinity and/or the relative price of the other good decrease down to zero. In other words, the market crashes! While the economy was stable before appearance of the rumour, in the sense that there exists a unique vector of relative prices that clears the market, trade eventually breaks down when the rumour is spreading through the population.

7 Discussion

The analysis in this paper has focused on rumours as an information transmission procedure between agents that act in a two-good exchange economy. The aim has been to look for reasonable dynamics that model the evolution of a rumour and that can be used to answer questions concerning the impact of rumours on economic issues, such as market demand and, especially, long-run equilibrium prices. Results have shown that if the rumour disappears, long-run equilibrium prices correspond to fundamental values of the economy. If the rumour stays present, which happens for a large class of initial distributions, long-run prices are different from fundamental ones if and only if the rumour affects individual preferences. In particular, when these effects are strong relative prices may differ from corresponding fundamentals by an infinite degree, leading to a crash of the market.

Still, there are a lot of issues that have to be addressed in order to obtain a true understanding of the relation between rumours and markets. One important object is certainly the parameter λ that determines the probability for uninformed agents to get to know the rumour from neighbors. While in this model λ was modelled exogenously, an obvious extension is to endogenize that parameter and derive it from other variables. With this respect, a crucial variable will be the (subjective) *value* of the information that is transmitted. This again could be linked to prices of some commodities or expected returns of an investment opportunity, depending on the situation one wants to analyze.

In any case, future models will have to link the rumour and the economic process not just via a mechanical act but via true decision procedures undertaken by the individual agents. Here, Banerjee (1993) represents a promising attempt, using a Bayesian updating approach. Other ideas may come from information theory, decision theory, or game theory.

Due to technical difficulty we have not yet been able to calculate equilibrium prices for an economy that is described by a distribution different from ν_0 and ν_λ, although the generality of our model allows for a definition of such a price. In particular, our main results concern the effects on long-run equilibrium prices where agents do not trade before dust in the economy settles down. Although this approach serves as an approximation for the general situation, a richer model will connect the rumour with trading behavior more directly. Then, it will be interesting not only to look for impacts of the rumour on trade but also vice versa. It seems to be quite obvious that trading may have direct implications on the evolution of a rumour, if the rumour itself affetcs some of the related economic issues.

An interesting additional topic is to build an economic theory of the rumour itself. Such a theory shall regard the rumour as an economic good that can be traded between agents. Questions are then the following ones: How does a market of rumours look like? What is the price of a rumour? Is there something like an equilibrium (price) for rumours? Clearly, answers to these questions will be highly connected to the ones mentioned above. Therefore results in this direction can also be very helpful in order to analyze questions as we have addressed in this paper.

Appendix

Proof of Proposition 5.1: The first observation is that the rumour is *attractive*, i.e. flip rates are such that every agent is more likely to flip if he generally "disagrees" with his neighborhood than if he generally "agrees" with it. In this sense agents can be considered as attracting each other. By Theorem 2.3, chapter III of Liggett (1985), this implies that the following limiting distributions do exist and are invariant:

$$\nu_0 = \lim_{t\to\infty} \delta_{\emptyset} S(t), \quad (38)$$
$$\nu_\lambda = \lim_{t\to\infty} \delta_{\mathbb{Z}} S(t), \quad (39)$$

where $S(t)$ denotes the semigroup of the rumour. Since we do not want to go to deeply into the mathematics of interacting particle systems we refer to the book of Liggett (1985) to get an understanding of the theory of semigroups and continuous time Markov processes. In this context it will perhaps be sufficient to understand that for any initial distribution μ the distribution of the rumour $\{\eta_t^\mu\}_{t\geq 0}$ at time t, here denoted as μ_t, is given by

$$\mu_t = \mu S(t). \quad (40)$$

Intuitively speaking, the semigroup is the continuous time analogue to the transition matrix of a discrete time Markov process.

Both distributions are extreme points in the convex set of all invariant distributions, $\mathcal{I}$. Using another result from Liggett (Theorem 3.13, chapter III) it follows then that these two measures are the only extreme points in this set, hence

$$\mathcal{I} = \{\nu \mid \nu = \alpha\nu_0 + (1-\alpha)\nu_\lambda,\ 0 \leq \alpha \leq 1\}. \quad (41)$$

Liggett shows that this is generally true for every particle system on $\mathbb{Z}$ that fulfils the conditions (i) attraction, (i) nearest neighbor interaction (flip rates depend only on the states of neighbors, which are those that are located directly on the left-hand or on the right-hand side), and the more technical one (iii) $c(x,\eta) + c(x,\eta_x) > 0$ whenever $\eta(x-1) \neq \eta(x+1)$, where η_x is defined as

$$\eta_x(z) := \begin{cases} \eta(z) & : \ z \neq x \\ 1-\eta(z) & : \ z = x. \end{cases} \quad (42)$$

Since the rumour meets even the condition that $c(x,\eta)+c(x,\eta_x) \geq 1$ for any x and any η these conditions are obviously fulfilled.

The ergodic case is the one where the two measures ν_0 and ν_λ coincide. Consider the function

$$\rho(\lambda) := \nu_\lambda\big(\eta(x) = 1\big),$$

which does not depend on x since ν_λ is translation invariant. The situation where the two measures coincide is then simply the one where $\rho(\lambda) = 0$. By a result of Liggett (Corollary 1.7, chapter III) it can be shown that $\rho(\lambda)$ is a non-decreasing function of λ. Thus it suffices to look for the critical value λ^* as

$$\lambda^* = \inf\{\lambda \geq 0 \mid \rho(\lambda) > 0\} \quad (43)$$
$$= \sup\{\lambda \geq 0 \mid \nu_0 = \nu_\lambda\}. \quad (44)$$

Clearly, this does not imply that $\lambda^* < \infty$. However, Liggett proves that $1.18 \leq \lambda^* \leq 2$. For our purpose this is far enough and concludes the proof of Propositions 5.1. □

Proof of Proposition 5.4: The claim for translation invariant distributions follows from the general claim of the proposition. In order to see this, assume that μ is translation invariant. In consequence, $\mu(\eta) = 0$, for every η with $|\eta| := |\{x \in \mathbb{Z} : \ \eta(x) = 1\}| < \infty$. At the same time $|\eta| = \infty$ implies that

$$P^{\eta}[\tau = \infty] = 1, \tag{45}$$

which proves Proposition 5.5. This follows from self-duality of the rumour. By this it is understood that for any η and any $A \in Y := \{T \subset \mathbb{Z} : \ |T| < \infty\}$ the following holds:

$$P^{\eta}[\eta_t \cap A \neq \emptyset] = P^{A}[\eta \cap A_t \neq \emptyset], \tag{46}$$

where A_t is the finite version of the rumour. (Recall the one to one correspondence between spaces Γ and $\Delta = \{T | \ T \subset \mathbb{Z}\}$, via the mapping $\chi : \Gamma \to \Delta, \ \ \chi(\eta) := \{x | \ \eta(x) = 1\}$.) Thus, it is particularly true that

$$\begin{aligned} P^{\mathbb{Z}}[\eta_t \cap A \neq \emptyset] &= P^{A}[\mathbb{Z} \cap A_t \neq \emptyset] &(47)\\ &= P^{A}[A_t \neq \emptyset]. &(48) \end{aligned}$$

Taking the limit $t \uparrow \infty$, this leads to

$$\nu_\lambda\Big(\eta_t(x) = 1 \text{ for some } x \in A\Big) = P^{A}[\tau = \infty].$$

With $|A| = n+1$ implying $P^{A}[\tau = \infty] \geq P^{B_n}[\tau = \infty]$, where $B_n = \{-n/2, \ldots, n/2\}$, the claim then follows from taking the limit $n \uparrow \infty$ and the fact that $\nu_\lambda(\emptyset) = 0$. (Replace A by η.)

The general case is technically rather complex and fully given in Liggett (1985), chapter VI, pp. 284-287. Therefore we omit a reproduction at this point. Just in order to mention the idea, since we know already that $P^{\eta}[\tau < \infty] = 0$ whenever $|\eta| = \infty$, it suffices to show that

$$\lim_{t \to \infty} \int f(\eta_t) \, dP^{\eta} = f(\emptyset) P^{\eta}[\tau < \infty] + P^{\eta}[\tau = \infty] \int_X f \ \nu_\lambda(d\eta), \tag{49}$$

for every $\eta \in Y$ and $f \in C(X)$, where $C(X)$ is the set of real-valued bounded continuous functions on X. This is done first in the case $\eta = \{0\}$, which is then used to proof the claim for general $\eta \in Y$. □

Proofs of Proposition 5.5 and Corollary 5.6: Proposition 5.5 has been proven in the proof of Proposition 5.4. Corollary 5.6 is an immediate consequence of the results before.

Proof of Proposition 6.3: Recall the expression of expected excess demand given in equation (24). Then

$$E^{\nu}[\,\zeta(\sigma(0), p)\,] = (0, 0) \tag{50}$$

is equivalent to

$$\begin{aligned}
&\nu(0)\left(\bar\theta\frac{p_2\omega_2}{p_1}-(1-\bar\theta)\omega_1,-\bar\theta\omega_2+(1-\bar\theta)\frac{p_1\omega_1}{p_2}\right)\\
+\;&\nu(1)\left(\theta^*\frac{p_2\omega_2}{p_1}-(1-\theta^*)\omega_1,-\theta^*\omega_2+(1-\theta^*)\frac{p_1\omega_1}{p_2}\right) \;=\; (0,0).
\end{aligned} \tag{51}$$

Remember that $\omega_i(x)\equiv\omega_i$ for every agent $x\in\mathbb{Z}$, thus in particular for agent 0. Rearranging this equation leads to equation (31), concluding the proof. □

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