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Nijman, T.E.; de Roon, F.A.; Werker, B.J.M.

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# Testing for Spanning with Futures Contracts and Nontraded Assets: A General Approach

Frans A. de Roon, Theo E. Nijman, and Bas J.M. Werker\*<sup>†</sup>

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## Abstract

This paper generalizes the notion of mean-variance spanning as defined in the seminal paper of Huberman & Kandel (1987) in three dimensions. It is shown how regression techniques can be used to test for spanning for more general classes of utility functions, in case some assets are nontraded, and in case some of the assets are zero-investment securities such as futures contracts. We then implement these techniques to test whether a basic set of three international stock indices, the S&P 500, the FAZ (Germany), and the FTSE (UK), span a set of commodity and currency futures contracts. Depending on whether mean-variance, logarithmic, or power utility functions are considered, the hypothesis of spanning can be rejected for most futures contracts considered. If an investor has a position in a nontraded commodity, then the hypothesis of spanning can almost always be rejected for futures contracts on that commodity for all utility functions considered. For currency futures this is only the case for a power utility function that reflects a preference for skewness. Finally, if we explicitly take into account net futures positions of large traders that are known to have predictive power for futures returns, the hypothesis of spanning can be rejected for most futures contracts.

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\*CentER for Economic Research, Department of Econometrics, and Department of Business Administration, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, The Netherlands. Corresponding author: Frans de Roon. E-mail: Roon@KUB.NL

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# 1 Introduction

An important question in financial economics is whether investors will value a richer investment opportunity set that results from adding securities to the assets that are already in their portfolio. Mutual funds, emerging market securities, and futures and options contracts for instance, are useful for an investor if incorporating these securities in his portfolio increases his utility. As outlined in Huberman & Kandel (1987), an investor with a mean-variance utility function is indifferent with respect to holding the additional securities if the Minimum-Variance Frontier (MVF) of the set of assets in an investors portfolio coincides with the MVF of the extended set of these same assets plus the additional securities, in which case there is *mean-variance spanning*. If the two MVF's only have one point in common then there is *intersection*, and only investors for whom the intersection portfolio is optimal need not invest in the additional securities.

It is by now well understood that mean-variance spanning means that the minimum-variance stochastic discount factors, or pricing kernels, that price the initial assets, also price the additional assets correctly (see, e.g., Ferson, Foerster, & Keim, 1993, Bekaert & Urias, 1996, and DeSantis, 1995). The pricing kernel with a given expectation, that prices a set of assets correctly, and that has minimum variance, is linear in the asset returns (Hansen & Jagannathan, 1991). Since the kernel is known to be proportional to the marginal derived utility of wealth of an agent given his optimal portfolio choice, kernels that are linear in the asset returns correspond to mean-variance optimizing behaviour.

The aim of this paper is to generalize the concept of mean-variance spanning to arbitrary classes of utility functions and to show how to test for spanning when futures contracts are considered as well and when there are non-traded assets. An example of a non-traded asset is the position in a foreign currency of an exporter. Other examples are given by a pension fund or insurance company that does not want to trade its liabilities, or a firm that does not want to sell its know-how or its production plants. In general, adding a security or set of securities to a given set of assets may well be beneficial to some investors but not to others, depending on their utility function and the non-traded assets in their current portfolio. Apart from the presence of non-traded assets, we will assume that there are no market frictions such as short sales restrictions or transaction costs. Tests for spanning in economies with such market frictions are discussed in a companion paper

(De Roon, Nijman, & Werker, 1996).

From Huberman & Kandel (1987) it is well known how regression analysis can be used to test for mean-variance spanning. We show that regression techniques can also be used to test for spanning for arbitrary classes of utility functions and to test for spanning in the presence of non-traded assets and when futures contracts are considered. Allowing for non mean-variance utility functions, testing for spanning implies that alternative regression models have to be considered in which restrictions similar to the ones in Huberman & Kandel (1987) have to be tested. For the case in which an investor has a position in a non-traded asset, the payoff of his portfolio will change because of this position. For such investors, regression based tests for spanning can be performed by using returns that are adjusted for the position in the non-traded asset. Finally, for spanning tests the crucial difference between futures contracts and assets is that futures are zero-investment securities. We will show that this implies that the restrictions on the regression coefficients imposed by spanning have to be modified to reflect the zero-investment property of futures contracts.

When applied to a basic set of assets, consisting of the S&P 500, the FAZ (Germany), and the FTSE (UK) indices, it appears that unconditional tests reject the null hypothesis of spanning for many futures contracts, depending on whether we test for mean-variance, logarithmic utility, or power utility spanning. When there is a nonmarketable position in a particular commodity, the null hypothesis of spanning is easily rejected for the futures contract that has the exposure asset as the underlying value for all utility functions considered. When there is an exposure to a foreign currency, spanning can only be rejected for investors with a power utility function that reflects a preference for skewness. Finally, when net futures positions of large traders are used to predict futures returns, conditional tests reject the null hypothesis of spanning for many futures contracts for all utility functions considered.

The plan of this paper is as follows. In Section 2 we will discuss the notion of spanning for arbitrary utility functions and show how to test for spanning. In Section 3 the tests for spanning will be extended to the case where there are futures contracts and nontraded assets. Here we also show how conditional information can be incorporated in tests for spanning. In Section 4 an illustration of the tests is presented for a set of commodity and currency futures contracts, with different kinds of exposures. Finally, Section 5 contains a summary and some concluding remarks.

## 2 Testing for spanning

### 2.1 Spanning for arbitrary classes of utility functions

Suppose that an investor initially considers a set of  $K$  assets, the gross returns of which are given by the vector  $R_{t+1}$ . The set  $R_{t+1}$  may or may not contain a risk free asset. Throughout this section we will take the case where  $R_{t+1}$  consists of non-zero investment securities only. The case of zero-investment securities, such as futures and forward contracts, will be considered in the next section. Assuming that there are no market frictions such as short sales constraints and transaction costs and that the law of one price holds, there exists a stochastic discount factor or pricing kernel,  $M_{t+1}$ , such that:

$$E[R_{t+1}M_{t+1} | I_t] = \iota_K, \quad (1)$$

where  $\iota_K$  is a  $K$ -dimensional vector containing ones, and  $I_t$  is the information set that is known to the investor at time  $t$ .

Recall that  $M_{t+1}$  in (1) can be derived from the first order conditions of a discrete time intertemporal portfolio selection problem. Usually, this optimization program is solved using dynamic programming (see, e.g., Ingersoll, 1987, and Duffie, 1988). The pricing kernel  $M_{t+1}$  is then known to be proportional to the derivative of the derived utility function of wealth (or the value function in the dynamic program), given the agent's optimal portfolio choice. Suppose that the agent subsequently also takes additional securities with gross return  $r_{t+1}$  into account when optimizing his utility. For notational convenience we will assume that  $r_{t+1}$  contains only one element. Spanning occurs if the original first order conditions for the optimal portfolio choice in (1) are also satisfied by the additional security  $r_{t+1}$ , i.e., if:

$$E[r_{t+1}M_{t+1} | I_t] = 1. \quad (2)$$

In other words, spanning occurs for a class of agents if they will not be able to increase their utility by incorporating  $r_{t+1}$  in their portfolio. Since each pricing kernel  $M_{t+1}$  refers to different preferences of an agent, spanning for a given class of preferences occurs if the above reasoning holds for the set of pricing kernels associated with this class of preferences. For instance, mean-variance spanning holds if all pricing kernels that satisfy (1) and that are linear in the returns  $R_{t+1}$ , also satisfy (2) (see also Ferson, Foerster, & Keim, 1993, Bekaert & Urias, 1996, and DeSantis, 1995). Therefore, we explicitly mention a particular set of pricing kernels  $\mathcal{M}$  in the following definition.

**Definition 1** *Let  $\mathcal{M}$  be a set of pricing kernels for the assets  $R_{t+1}$ , i.e.*

$$\forall M_{t+1} \in \mathcal{M} : E[R_{t+1}M_{t+1} | I_t] = \iota_K.$$

Then  $\mathcal{M}$ -spanning of the asset  $r_{t+1}$  by the assets  $R_{t+1}$  holds by definition if the following hypothesis is satisfied:

$$H' : \forall M_{t+1} \in \mathcal{M} : E[r_{t+1}M_{t+1} \mid I_t] = 1.$$

The case where  $\mathcal{M}$  is a singleton is denominated intersection.

This definition clearly generalizes the definition of mean-variance spanning in Huberman & Kandel (1987)<sup>1</sup>.

If the asset set  $R_{t+1}$  does not span the asset  $r_{t+1}$ , then  $\lambda_t = E[r_{t+1}M_{t+1} \mid I_t] - 1$  can be interpreted as a performance measure for the asset  $r_{t+1}$  relative to the set  $R_{t+1}$  (see, e.g., Chen & Knez, 1996, and Glosten & Jagannathan, 1994). If  $\lambda_t > 0$  for a given  $M_{t+1}$ , then an investor with a utility function that corresponds to  $M_{t+1}$  can improve his portfolio performance by taking a long position in the asset  $r_{t+1}$  in addition to his investments in the set  $R_{t+1}$ . For instance, the results in Cumby & Glen (1990) indicate that a sample of fifteen U.S.-based internationally diversified mutual funds do not have added value for investors with either quadratic or power utility functions<sup>2</sup> relative to a broad international equity index and a risk free asset.

One characteristic of the empirical applications of performance evaluation such as in Cumby & Glen (1990) and Chen & Knez (1996), is that the tests are for intersection rather than spanning. This implies that performance is measured with respect to the optimal portfolio of a specific utility function. Our methodology tests for spanning for a prespecified *class* of utility functions and a given set of assets. In order to test for spanning, we need an equivalent formulation of  $H'$  that can be tested easily using regression.

## 2.2 Testing for spanning

To obtain an easily testable equivalent formulation of the spanning hypothesis  $H'$ , we need the following notation. Write  $W = \{w \in \mathbb{R}^k : w' \iota_K = 1\}$ , so that  $W$  consists of those portfolio choices that are valid for an agent in the sense that the portfolio weights of assets sum to one. We can now state

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<sup>1</sup>The definition for spanning used in this paper originates from Huberman & Kandel (1987). Note however, that this definition is slightly different from (although closely related to) another definition that is used in the literature and that can be found, for instance, in Ross (1978). We extend the results of Ross (1978) to the case where  $\mathcal{M}$  is a subset of all monotone concave utility functions.

<sup>2</sup>Cumby & Glen test the performance of the funds using both Jensen's  $\alpha$  relative to a mean-variance efficient portfolio and a positive weighting measure as proposed by Grinblatt & Titman (1989) with weights that are equal to the derivative of a power utility function evaluated at the optimal portfolio choice.

our main result.

**Proposition 1** *Let  $\mathcal{M}$  be a set of kernels that includes at least the minimum second moment pricing kernel  $M_{t+1} = v'_K E_t[R_{t+1}R'_{t+1}]^{-1}R_{t+1}$ <sup>34</sup>. Then the asset  $r_{t+1}$  is  $\mathcal{M}$ -spanned by the assets  $R_{t+1}$  if and only if<sup>5</sup>*

$$\Pi(r_{t+1} \mid [\mathcal{M} \cup \{w'R_{t+1} : w \in W\}]) = w'R_{t+1}, \text{ for some } w \in W. \quad (3)$$

In Appendix A we proof a generalized version of this proposition, in which we also allow for zero-investment securities. The result in Proposition 1 states that  $r_{t+1}$  can be written as a portfolio  $w$  of the assets  $R_{t+1}$  and an error term that is orthogonal to the set  $\mathcal{M}$  of pricing kernels under consideration. This implies that all agents with utility functions corresponding to the class  $\mathcal{M}$  prefer  $w'R_{t+1}$  over  $r_{t+1}$ , since they do not value the return difference  $r_{t+1} - w'R_{t+1}$ . If  $r_{t+1}$  is not  $\mathcal{M}$ -spanned by  $R_{t+1}$ , then  $r_{t+1}$  is of value to some investors because the difference between  $r_{t+1}$  and a portfolio of the initial assets  $R_{t+1}$  covaries with their intertemporal marginal rate of substitution.

The strength of Proposition 1 is that testing  $H'$  is straightforward once the projection in (3) is estimated. If we denote this projection by  $\hat{r}_{t+1}$ , i.e.,

$$\hat{r}_{t+1} = \Pi(r_{t+1} \mid [\mathcal{M} \cup \{w'R_{t+1} : w \in W\}]), \quad (4)$$

then  $\hat{r}_{t+1}$  follows from a regression. After  $\hat{r}_{t+1}$  is estimated, testing  $H'$  can be done by testing the hypothesis

$$H : \hat{r}_{t+1} = w'R_{t+1}, \text{ for some } w \in W \quad (5)$$

In order to estimate the projection in (4) a functional form for the kernels in  $\mathcal{M}$  is needed. It is well known that the pricing kernel is proportional to the marginal utility of consumption, given the optimal portfolio and consumption choice of the agent. The envelope theorem in turn implies that the pricing kernel is also proportional to the marginal derived utility of wealth (see, e.g., Ingersoll, 1987). This allows us to estimate the projection in (4) from a regression of  $r_{t+1}$  on (functions of) the initial asset returns  $R_{t+1}$  only.

The case of mean-variance spanning is discussed extensively in Huberman & Kandel (1987). However, it is useful to put these ideas in our more general

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<sup>3</sup>We use  $E_t[\cdot]$  as shorthand for the conditional expectation  $E_t[\cdot \mid I_t]$ .

<sup>4</sup>Notice that the minimum second moment kernel is automatically included in  $\mathcal{M}$  in all true extensions of mean-variance spanning.

<sup>5</sup>As usual we work in the Hilbert space of square integrable random variables. Projection in this space will be denoted by  $\Pi$ . For any set  $A$  in this Hilbert space, we denote by  $[A]$  the closed linear span of the elements in  $A$ .

context. As stated above, the pricing kernels associated with mean-variance optimizing behaviour are linear in the returns  $R_{t+1}$ . The set  $\mathcal{M}$  therefore coincides with the set of pricing kernels that traces out the volatility bound in Hansen & Jagannathan (1991). More precisely, for investors that choose their portfolio from the assets  $R_{t+1}$  the set  $\mathcal{M}$  is in this case given by all kernels of the form

$$M_{t+1}(v) = v + \alpha'(R_{t+1} - E_t[R_{t+1}]), \quad v \in \mathbb{R}, \quad (6)$$

where

$$\alpha = \text{Var}_t[R_{t+1}]^{-1} \{\iota_K - v E_t[R_{t+1}]\}.$$

Therefore,  $\hat{r}_{t+1}$  equals the projection of  $r_{t+1}$  on all stochastic variables of the form  $\alpha_0 + \alpha'R_{t+1}$ . Consequently, assuming that all expected returns and (co)variances of the returns do not vary over time (this assumption will be relaxed in Section 3.3),  $\hat{r}_{t+1}$  can be estimated by the following regression

$$r_{t+1} = \alpha_0 + \alpha'R_{t+1} + \varepsilon_{t+1}. \quad (7)$$

The hypothesis  $H$  now becomes

$$H : \quad \alpha_0 = 0 \text{ and } \alpha'\iota_K = 1.$$

These linear restrictions are of course identical to the ones given in Huberman & Kandel (1987) and are straightforward to test using a Wald test.

If the set  $R_{t+1}$  also spans  $r_{t+1}$  for investors with non mean-variance (derived) utility function(s)  $U(w^*R_{t+1})$ , where  $w^*$  denotes the optimal portfolio choice for the investor, then the error term  $\varepsilon_{t+1}$  in (7) should be orthogonal to the marginal derived utility  $U'(w^*R_{t+1})$ . To test for both mean-variance spanning and spanning for  $N$  different utility function(s)  $U(w^*R_{t+1})$ , given knowledge of  $w^*$ , the projection of  $r_{t+1}$  can now be estimated by the regression

$$r_{t+1} = \alpha_0 + \alpha'R_{t+1} + \sum_{i=1}^N \gamma_i U_i'(w_i^*R_{t+1}) + \varepsilon_{t+1}, \quad (8)$$

where  $U_i'(w_i^*R_{t+1})$ ,  $i = 1, 2, \dots, N$  are the derivatives of the (non mean-variance) utility functions of interest, i.e., for all utility functions that are in  $\mathcal{M}$ .

The null hypothesis that there is  $\mathcal{M}$ -spanning is now equivalent to

$$H : \quad \alpha'\iota_K = 1, \quad \alpha_0 = \gamma_i = 0, \quad i = 1, 2, \dots, N.$$

As with mean-variance spanning, these restrictions are easy to test using a Wald test.



Given a specific utility function, from the first order conditions

$$E[cU'(w^*R_{t+1})R_{t+1}] = \iota,$$

estimates of  $c$  and  $w^*$ ,  $\hat{c}$  and  $\hat{w}^*$ , can be obtained using, for instance, a GMM-estimator. The parameter  $c$  is a constant of proportionality that is determined by the risk free rate of return (if it exists). Since in empirical applications  $U'(\hat{w}^*R_{t+1})$  in (8) is based on the estimated optimal portfolio weights  $\hat{w}^*$ , this will obviously affect the limit distribution of the regression parameters in (8). In Appendix B the limit distribution of the regression parameters is derived, accounting for the fact that we have to estimate  $c$  and  $w$ .

Several tests for performance evaluation that are known in the literature can also be interpreted in terms of the framework presented here. For instance, Cumby & Glen (1990) test the performance of international mutual funds for a mean-variance investor and for an investor with a power utility function. Since they use prespecified benchmark portfolios that are optimal for a mean-variance investor and a power-utility investor respectively, their tests can be interpreted as tests for intersection for these two utility functions. In terms of the regression in (8) we could (simultaneously) test for mean-variance and power-utility spanning by choosing for  $U(w^*R_{t+1})$  the specific power utility function that is used by Cumby & Glen (1990).

Similarly, Glosten & Jagannathan (1994) propose a performance test where the set  $R_{t+1}$  consists of one asset, an index portfolio, and that is based on a polynomial fit of  $r_{t+1}$  on  $R_{t+1}$ . In terms of (8) this is similar to choosing marginal utilities of the form  $U_i'(R_{t+1}) = R_{t+1}^i$ ,  $i = 2, 3, \dots$ . Our motivation for using a polynomial approach is entirely different from that of Glosten & Jagannathan however. For instance, Glosten & Jagannathan motivate the use of a second order polynomial, i.e. only  $U'(R_{t+1}) = R_{t+1}^2$  is used in (8), to account for market timing, as also suggested by Treynor & Mazuy (1966). In our framework on the other hand, a quadratic term captures a preference for skewness.

In the empirical application in Section 4 we will test for mean-variance spanning and for power-utility spanning. For the power-utility function we will use both a risk aversion coefficient of 0, that corresponds to a logarithmic utility investor, and a risk aversion coefficient of -3, that corresponds to an investor who has a preference for skewness.

## 3 Testing for spanning with futures contracts and nontraded assets

### 3.1 Futures contracts

The main result of the previous section, as stated in Proposition 1, is that spanning of an asset  $r_{t+1}$  by a base set of assets  $R_{t+1}$  is equivalent to stating that the projection of  $r_{t+1}$  on all portfolios of  $R_{t+1}$ ,  $w'R_{t+1}$ , and the relevant class of pricing kernels  $\mathcal{M}$ , gives a portfolio of the base securities. The intuition behind this result is that if  $r_{t+1}$  is spanned by the set  $R_{t+1}$ , then  $r_{t+1}$  can be written as the payoff of a portfolio of the initial assets  $R_{t+1}$  plus an idiosyncratic error term that is orthogonal to the asset returns  $R_{t+1}$  and the pricing kernels in  $\mathcal{M}$ .

The crucial difference between assets and futures contracts in this respect is that futures contracts do not require any initial investment. Whereas the payoff of an asset at time  $t + 1$  is its price  $S_{t+1}$  (ignoring dividends and the like), the payoff of a futures contract is given by the *change* in the futures price,  $F_{t+1} - F_t$ . Whereas asset returns are defined as  $R_{S,t+1} = \frac{S_{t+1}}{S_t}$ , we define a futures return<sup>6</sup> as  $R_{F,t+1} = \frac{F_{t+1} - F_t}{F_t}$ . In case of futures contracts, equation (1) changes to:

$$E[R_{F,t+1}M_{t+1} | I_t] = 0. \quad (9)$$

Denote  $R_{t+1}$  now as the  $K$ -dimensional vector, the first  $K_S$  elements of which are asset returns,  $R_{S,t+1}$ , and the last  $K_F$  elements of which are futures returns,  $R_{F,t+1}$ ,  $K = K_S + K_F$ . These are the  $K$  securities initially considered by the investors. Next, let  $e_K$  be a vector consisting of  $K_S$  ones and  $K_F$  zero's,  $e'_K = (1'_{K_S} \ 0'_{K_F})$ . We can now generalize equations (1) and (9) to:

$$E[R_{t+1}M_{t+1} | I_t] = e_K. \quad (10)$$

Finally, write  $W_e^S = \{w \in \mathbb{R}^K : w'e_K = 1\}$  and  $W_e^F = \{w \in \mathbb{R}^K : w'e_K = 0\}$ . Thus,  $W_e^S$  and  $W_e^F$  define portfolios in which the asset weights must sum to either one ( $w \in W_e^S$ ) or zero ( $w \in W_e^F$ ), but there are no restrictions on the futures positions. Note that the minimum second moment portfolio is now given by  $\alpha = E_t[R_{t+1}R'_{t+1}]^{-1}e_K$ . If  $R_{t+1}$  only contains futures contracts, then  $\alpha = 0$ . As a generalization of Proposition 1 it is now straightforward to show that  $r_{t+1}$  is  $\mathcal{M}$ -spanned by the securities  $R_{t+1}$  if and only if

$$\Pi(r_{t+1} | [\mathcal{M} \cup \{w'R_{t+1} : w \in \widetilde{W}\}]) = w'R_{t+1}, \text{ for some } w \in \widetilde{W}, \quad (11)$$

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<sup>6</sup>Actually, the term futures return itself is a misnomer, for the same reason that futures contracts do not require an investment.

where  $\widetilde{W} = W_e^S$  if  $r_{t+1}$  refers to a non-zero investment security, and  $\widetilde{W} = W_e^F$  if  $r_{t+1}$  refers to a zero-investment security. The proof of this proposition is given in Appendix A. Given this proposition, testing for spanning proceeds in the same way as outlined in Section 2, except that in case  $r_{t+1}$  is a futures contract the restriction that  $w'e_K = 1$  has to be replaced by the restriction  $w'e_K = 0$ .

First of all, note from (11) that spanning only imposes restrictions on the sum of the asset weights, but not on the futures positions. This reflects the fact that assets require a non-zero investment, while futures contracts do not require any investment. Second, if  $r_{t+1}$  refers to a non-zero investment security, spanning requires that the asset weights in  $w$  sum to one, while if  $r_{t+1}$  refers to a zero-investment security like futures contracts, the asset weights in  $w$  must sum to zero. If the return on a futures contract is to be written as the return on a portfolio of assets and futures, then this must be a zero-investment portfolio, since the futures contract itself does not require any investment either. Thus, the difference in the restrictions for futures contracts and assets stems from the fact that futures contracts are zero investment securities.

### 3.2 Nontraded Assets

So far we treated all investors as if they had the same investment opportunity set. However, because investors can have positions in nontraded assets, i.e., they can face different *nonmarketable risks*, they may face different investment opportunity sets. For example, the investment opportunity set of an exporter is affected by his exposure to foreign currency. Similarly, the investment opportunity sets of pension funds and insurance companies are affected by their liabilities. Consequently, when considering additional securities, the initial set of assets may span the extended set for one agent, but not for others. The reason is that the presence of a nontraded asset changes the net portfolio payoff for an investor.

Let  $W_t$  be the wealth invested in assets by an investor, excluding nontraded assets. The fraction of wealth invested in asset  $j$  is given by  $w_{Sj}$ , and  $w_S$  is a vector containing all  $w_{Sj}$ . Notice that  $w'_S \iota = 1$ . Besides investing in assets, an investor can also take a position in futures contract  $k$ , which is also expressed as a fraction of  $W_t$ . The vector  $w_F$  similarly contains all the futures positions of the agent. Finally, the agent may have a position in a nontraded asset with a size  $q_X$  that yields a return  $R_{X,t+1}$ . The size of the position is also expressed as a fraction of  $W_t$ , implying that  $\iota'w_S + q_X$  will not be equal to one if  $q_X \neq 0$ . Thus, the total return on his invested wealth

for the investor is given by:

$$R_{W,t+1} = w'_S R_{S,t+1} + w'_F R_{F,t+1} + q_X R_{X,t+1}. \quad (12)$$

Of course, a similar expression arises when the additional security  $r_{t+1}$  is included.

Notice that the asset weights  $w_{Sj}$  must sum to one. Therefore, an equivalent way of writing the total return in (12) is:

$$R_{W,t+1} = w'_S (R_{S,t+1} + q_X R_{X,t+1}) + w'_F R_{F,t+1} = w'_S \tilde{R}_{S,t+1} + w'_F R_{F,t+1}, \quad (13)$$

where  $\tilde{R}_{S,t+1}$  are the returns adjusted for the position in the nontraded asset. Since there is only a restriction on the asset weights and not on the futures positions, only the asset weights must be adjusted for the position in the nontraded asset. Denote  $\tilde{R}_{t+1}$  as the total adjusted return vector,  $\tilde{R}'_{t+1} = (\tilde{R}'_{S,t+1} R'_{F,t+1})$ . To see the implications of the presence of nontraded assets for spanning, observe that one valid stochastic discount factor is the intertemporal marginal rate of substitution of agent  $i$ . Since agent  $i$  will choose his portfolio taking into account the nontraded asset, his interest will be in the adjusted returns,  $\tilde{R}_{t+1}$ , rather than the normal returns,  $R_{t+1}$ . It's easy to see that this implies that the following should hold:

$$E[\tilde{R}_{t+1} \tilde{M}_{t+1} | I_t] = e_K, \quad (14)$$

where  $\tilde{M}_{t+1}$  indicates the stochastic discount factor that prices the adjusted asset returns.

It is now straightforward to test for spanning taking into account the nontraded assets.  $\mathcal{M}$ -spanning of  $r_{t+1}$  by the securities  $R_{t+1}$  occurs if and only if

$$\prod(\tilde{r}_{t+1} | [\mathcal{M} \cup \{w' \tilde{R}_{t+1} : w \in W\}]) = w' \tilde{R}_{t+1}, \text{ for some } w \in W. \quad (15)$$

All tests described in Section 2 are still valid, provided that we replace the asset returns  $R_{S,t+1}$  and  $r_{S,t+1}$  by adjusted returns,  $\tilde{R}_{S,t+1}$  and  $\tilde{r}_{S,t+1}$ , while the futures returns remain unchanged.

### 3.3 Testing for spanning using conditioning information

So far we assumed that expected returns, (co)variances, and all relevant expected moments are constant over time. Especially in the futures markets literature however, there is substantial evidence of return predictability.

For instance, there is ample evidence that futures returns can be predicted from the net positions of large traders, known as hedging pressure (see e.g. Carter, Rauser, & Schmitz, 1983, Chang, 1985, and Bessembinder, 1992). Also, Fama & French (1987) present evidence that commodity returns can be predicted from the observed spread between the futures and the spot price. Similarly, Glen & Jorion (1993) show that the efficiency of international asset portfolios significantly improves if it is taken into account that expected currency returns depend on the forward premium. Finally, there is substantial evidence that stock and bond returns can be predicted using instruments like lagged returns, dividend yields, short term interest rates, and default premiums (see, e.g., Ferson, 1995).

If expected returns are dependent on conditioning information at time  $t$ , then the parameters  $\alpha_0$  and  $\alpha$  in equation (7) should also be dependent on that information. In this case, there may be spanning in one period, but not in other periods, because of a change in economic conditions.

Suppose that expected returns are linearly dependent on a vector of variables  $x_t$  that are in the investor's information set at time  $t$ , i.e.,  $x_t \in I_t$ . We will still assume that the (co)variances of the returns are constant. The extension to time-varying covariances is straightforward however. It can then easily be shown that tests for mean-variance spanning can be based on the following regression:

$$r_{t+1} = \alpha_0 + \beta'x_t + \alpha'R_{t+1} + \varepsilon_{t+1}. \quad (16)$$

In this case spanning occurs for arbitrary values of  $x_t$  if and only if  $\alpha_0 = \beta = 0$  and  $\alpha'\iota_K = 1$ . In case  $\alpha'\iota_K = 1$  and  $\beta \neq 0$ , it follows that spanning occurs for  $\alpha_0 = -\beta'x_t$ . This implies that we can test whether there is spanning under certain economic conditions, i.e., for specific values of  $x_t$ . A slightly different way to incorporate conditional information in tests for mean-variance spanning can be found, e.g., in Harvey (1995) and Bekaert & Urias (1996).

This way of using conditioning information to allow for return predictability readily extends to the tests for spanning for arbitrary classes of utility functions. We will again assume that only expected returns are dependent on conditioning information  $x_t$  that is known at time  $t$ . The (co)variances and all other relevant moments are assumed to be constant. As in the unconditional case, if there is also spanning of  $r_{t+1}$  by  $R_{t+1}$  for investors with non mean-variance (derived) utility functions  $U(w^*R_{t+1})$ , then the error term  $\varepsilon_{t+1}$  in (16) should be orthogonal to the marginal derived utility  $U'(w^*R_{t+1})$ . It is now again straightforward to show that testing for  $\mathcal{M}$ -spanning can be

based on the regression

$$r_{t+1} = \alpha_0 + \beta'x_t + \alpha'R_{t+1} + \sum_{i=1}^N \gamma_i U_i(w_i^{*'}R_{t+1}) + \varepsilon_{t+1}. \quad (17)$$

If there is spanning regardless of the value of  $x_t$ , i.e., regardless of the economic conditions, then  $\alpha_0 = \beta = \gamma = 0$  and  $\alpha' \iota_K = 1$ . If  $\alpha' \iota_K = 1$  and  $\gamma = 0$ , then there is only spanning if  $\alpha_0 = -\beta'x_t$ , as in the mean-variance case.

## 4 Empirical results for commodity and currency futures

In this section we illustrate the analysis in the previous sections for a number of commodity and currency futures contracts. We test whether a base set of three international stock indices spans the extended set of these same portfolios plus a number of futures contracts.

[Please insert Table 1]

We use semi-monthly data from January 1984 until December 1993 to construct monthly holding returns for the S&P 500, the FTSE (UK) and the FAZ (Germany), as well as for a number of commodity and currency futures. The returns on the FTSE and the FAZ used here, are unhedged dollar returns, so we take the perspective of a US-investor. The three indices used here allow a US-investor to form a well-diversified asset portfolio. Summary statistics for monthly holding returns on the three indices and the futures contracts are presented in Table 1. The data for the FTSE and the FAZ are obtained from Datastream, while all other data are obtained from the Futures Industry Institute. Because semi-monthly observations of monthly holding returns create an overlapping samples problem, consistent estimates of the relevant covariance matrices are calculated as in Newey & West (1987). Returns for the futures contracts are always for the nearest-to-delivery contract, excluding observations in the delivery month.

### 4.1 Unconditional tests without nontraded assets

[Please insert Table 2]

Table 2 reports results of tests whether there is spanning for several utility functions, assuming that all relevant moments of the returns are time-invariant. The first column presents results for the null hypothesis that there

is mean-variance spanning of the futures contracts by the three international stock indices. The Wald test-statistics show that this null hypothesis is only rejected for live cattle futures. This suggests that in a market without frictions and using no conditioning information, most futures contracts considered here do not have added value for a US-investor with a mean-variance utility function.

The second and third column of Table 2 show tests for the hypothesis that, besides mean-variance spanning, there is also spanning for a logarithmic utility and power-utility investor respectively. As outlined in Section 2, the parameters for the kernels that correspond to these utility functions are estimated using a GMM-estimator. The Wald test-statistics in the second column are for the hypothesis that the three stock indices span the futures contracts for both mean-variance investors and investors with a utility function  $U(W_{t+1}) = \log(W_{t+1})$ . The spanning hypothesis can now be rejected for wheat and soybean oil futures and, again, for live cattle futures. Apparently, wheat and soybean oil futures do not have added value for investors with a mean-variance utility function, but they do for investors with a logarithmic utility function. For all other futures contracts the spanning hypothesis can not be rejected.

Similar results can be reported for the third column of Table 2. Here the null hypothesis is that the futures contracts are spanned by the three stock indices for investors with a mean-variance utility function and for investors with a power utility function  $U(W_{t+1}) = \frac{1}{\gamma}(W_{t+1})^\gamma$ , where the risk aversion coefficient  $\gamma = -3$ , reflecting a preference for skewness. The main difference with mean-variance and log utility spanning is that now spanning can be rejected for the currency futures. Apparently, US-investors with a preference for skewness would like to hedge their currency exposure that arises from their investments abroad.

Finally, the fourth column in Table 2 shows test statistics for the null hypothesis that the three international stock indices span the futures contracts for investors with either a logarithmic or power ( $\gamma = -3$ ) utility function, but not for investors with a mean-variance utility function. Except for corn futures the hypothesis of spanning can now be rejected for all commodity futures, but it can not be rejected for the currency futures.

Summarizing, depending on the utility functions of interest, most of the futures contracts considered here appear to have added value for a US-investor who invests in the three stock indices.

## 4.2 Unconditional tests in case there are nontraded assets

As outlined in Section 3, if an investor has a position in a nontraded asset, this will change his investment opportunity set. Therefore, in this section we will test whether the set of three international stock indices spans the futures contracts for investors with mean-variance utility functions as well as for investors with logarithmic utility or power utility ( $\gamma = -3$ ) functions when there are nontraded assets. We consider the case of agents that have a nonmarketable position in one of the assets underlying the futures contracts considered here. The size of the position is assumed to be 25% of the wealth invested in the three stock indices (or, equivalently, 20% of total wealth).

[Please insert Table 3]

The Wald test-statistics in Table 3 are for the null hypothesis that there is mean-variance spanning. The results show that whenever there is an exposure in a commodity, adding a futures contract on that same commodity almost always adds value for mean-variance investors. Of course this is what can be expected a priori. Somewhat surprisingly, the hypothesis of mean-variance spanning can not be rejected for the currency futures, even though there is a 25% exposure to the foreign currency. The explanation for this result may be that the returns on the international stock indices already contain a currency component which allows mean-variance investors to choose their portfolio in such a way that adding currency futures is not useful.

Table 3 also shows that the five agricultural futures, wheat, corn, soybeans, soybean meal, and soybean oil, are related, in the sense that if there is an exposure in one of the five agriculturals, spanning is usually rejected for most of the agricultural futures contracts. The same is true for live cattle and live hogs. Clearly, a position in a nontraded commodity changes the investment opportunity set in such a way that the MVF of the adjusted stock indices with a futures contract added is no longer spanned by the adjusted stock indices only. Almost all investors with a mean-variance utility function can benefit from adding futures contracts to their portfolio according to their position in nontraded assets.

[Please insert Table 4]

[Please insert Table 5]

[Please insert Table 6]

We also test whether there is spanning for investors with logarithmic utility and power utility functions. Table 4 presents Wald test-statistics for the



hypothesis that there is both mean-variance and logarithmic utility spanning, Table 5 presents test-statistics for the hypothesis that there is mean-variance and power utility spanning ( $\gamma = -3$ ), and Table 6 presents test-statistics for the hypothesis that there is logarithmic and power-utility spanning, but not mean-variance spanning. Most of the results are similar to the results for mean-variance spanning only: For the commodity futures the spanning hypothesis is almost always rejected whenever there is a nonmarketable position in the commodity underlying the futures contract. Also, a nonmarketable position in an agricultural commodity usually implies that the spanning hypothesis is rejected for most agricultural futures contracts, and again the same is true for live cattle and live hogs.

As with the results in Table 2 where there is no exposure, the main difference is that the test for mean-variance and power utility spanning in Table 5 rejects the spanning hypothesis for the currency futures, whatever the nonmarketable position is. Thus, unlike investors with a mean-variance or logarithmic utility function, investors with a power utility function ( $\gamma = -3$ ), showing a preference for skewness, can benefit from adding currency futures to their portfolio, while we can not draw this conclusion for the other utility functions, even though there is a 25% exposure to the foreign currencies.

### 4.3 Conditional tests of spanning

As indicated in Section 3.3, there is ample evidence that futures returns can be predicted from the net positions of large hedgers in the futures markets, known as hedging pressure. These positions are reported by the Commodity Futures Trading Commission (CFTC). In this section we will test whether the three international stock indices span the futures contracts in our sample, if we use hedging pressure variables to predict futures returns.

In order to use this kind of conditioning information, we construct a hedging pressure variable  $x_{i,t}$  for commodity or currency  $i$  as the difference between the positions of large hedgers that are short in futures contract  $i$  at time  $t$ , and the positions of large hedgers that are long in futures contract  $i$  at time  $t$ , divided by the total position of these hedgers in contract  $i$ . Thus, the hedging pressure,  $x_{i,t}$ , is always in the range between -1 and +1 and represents the net position of large hedgers as a fraction of the total position of large hedgers. Using this variable, the models in equations (16) and (17) are estimated. Because data on hedging pressure are in our dataset since January 1986 only, the empirical tests that use the hedging pressure variable are for the period from January 1986 until December 1993.

[Please insert Table 7]

Table 7 shows the results of the spanning tests with conditioning information. The first three columns show test-statistics for the hypothesis that there is mean-variance spanning for three different values of  $x_{i,t}$ : -0.50, 0.00, +0.50. For instance, the first column with  $x_{i,t}$  gives the Wald test-statistic for the null hypothesis whether there is spanning if the hedging pressure variable is -0.50, i.e., if 75% of the positions of hedgers are short positions and 25% of the positions are long positions. The first three columns show that for three commodity futures and for the three currency futures, mean-variance spanning can be rejected convincingly, given the appropriate economic conditions. Especially for currency futures this is in sharp contrast with the results in the previous tables. Note that for currency futures mean-variance spanning is rejected when the hedging pressure variable is either +0.50 or -0.50, i.e., when hedgers are either predominantly on the long or the short side of the market, but not when the hedging pressure variable is 0.00, i.e., when the positions of hedgers are spread evenly over the long and short side of the market.

Columns 4, 5, and 6 of Table 7 show similar tests for the hypothesis that there is spanning for both mean-variance and logarithmic utility investors and columns 7, 8, and 9 show tests for the hypothesis that there is spanning for both mean-variance and power utility investors. Finally, the last three columns show the tests for the hypothesis that there is for all classes of utility functions considered: mean-variance, logarithmic utility, and power utility. Note that because our conditional tests require that we include an intercept and  $x_t$  in the regression, our test procedure automatically tests for mean-variance spanning besides the other utility functions included. The results for these tests confirm the findings for mean-variance spanning. The major difference occurs in the columns 4, 5, and 6, and the last three columns, that show that when logarithmic utility functions are included spanning can also be rejected for wheat and soybean futures, which is not the case for the other utility functions.

## 5 Summary and conclusions

In this paper we generalize the notion of mean-variance spanning as defined in the seminal paper of Huberman & Kandel (1987) in three dimensions. First of all we show how regression techniques can be used to test for spanning for more general classes of utility functions. It is shown that in projecting a security's return on a specific class of kernels and portfolios of the initial set of securities, spanning implies that the projection yields a portfolio of these securities. Second, we show how to test for spanning in case of zero-

investment securities like futures, forwards, and swaps. If zero-investment securities are considered, then spanning implies restrictions on the coefficients in the spanning regression that reflect the zero-investment property. Finally, we show how to test for spanning in case there are nontraded assets. If an investor has a position in a nontraded asset, then this changes his investment opportunity set. In spanning tests this can be incorporated by using returns that are adjusted for the return on the nontraded asset.

We test whether three international stock indices, i.e., the S&P 500, the FAZ (Germany), and the FTSE (UK), span a set of commodity futures and currency futures. If it is assumed that all relevant moments of monthly holding returns are constant, and that there are no market frictions like short selling constraints and transaction costs, then we can reject the hypothesis that there is spanning for most futures contracts, but whether or not the spanning hypothesis is rejected depends on the specific utility functions of interest. If an investor has a nonmarketable position in a commodity underlying one of the futures contracts, then spanning can almost always be rejected for the futures contract on that same commodity for all utility functions considered. Moreover, a nonmarketable position in one agricultural commodity usually implies that the hypothesis of spanning is rejected for most of the agricultural futures contracts. If there is an exposure to a foreign currency, then spanning can only be rejected for investors with power utility functions that reflect a preference for skewness. Finally, allowing expected returns to depend on the net positions of large hedgers in the futures market, spanning can be rejected for many futures contracts for all utility functions considered.

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## A Proof of Proposition 1

In this appendix we show the proof of proposition 1 allowing for zero-investment securities. Let  $w \in \widetilde{W}$ , where  $\widetilde{W} = W_e^S$  if  $r_{t+1}$  refers to a non-zero investment security and  $\widetilde{W} = W_e^F$  if  $r_{t+1}$  refers to a zero-investment security. Also let  $e_r$  be 1 if  $r_{t+1}$  refers to a non-zero investment security and 0 if  $r_{t+1}$  refers to a zero-investment security.

**Proof.** First (sufficiency) assume that (3) holds for  $w \in \widetilde{W}$ . Fix any  $M_{t+1} \in \mathcal{M}$ . Then

$$E_t[r_{t+1}M_{t+1}] = E_t[w'R_{t+1}M_{t+1}] = w'e = e_r.$$

Second (necessity), assume that  $H'$  holds. We will show that the projection in (3) equals

$$w'_0 R_{t+1} + \Pi(r_{t+1} - w'_0 R_{t+1} \mid \{(w - w_0)'R_{t+1} : w \in \widetilde{W}\}), \quad (18)$$

with  $w_0 = E_t[R_{t+1}R'_{t+1}]^{-1}e/e'E_t[R_{t+1}R'_{t+1}]e$ . Note that  $\{(w - w_0)'R_{t+1} : w \in \widetilde{W}\}$  defines a linear and finite dimensional (hence closed) subspace. Denote by  $\bar{w}$  that value of  $w$  that yields the projection in (18), i.e.  $\bar{w}$  solves

$$E_t[(r_{t+1} - w'_0 R_{t+1} - (\bar{w} - w_0)'R_{t+1})(w - w_0)'R_{t+1}] = 0, \quad (19)$$

for all  $w \in \widetilde{W}$ . Now we obviously have that

$$w'_0 R_{t+1} + (\bar{w} - w_0)'R_{t+1} = \bar{w}'R_{t+1} \in \{w'R_{t+1} : w \in \widetilde{W}\}.$$

Furthermore, if  $M_{t+1} \in \mathcal{M}$ , then  $H'$  implies that

$$E_t[(r_{t+1} - \bar{w}'R_{t+1})M_{t+1}] = e_r - \bar{w}'e = 0.$$

In particular, this holds for  $M_{t+1}^0 = e'E_t[R_{t+1}R'_{t+1}]^{-1}e * w'_0 R_{t+1}$ , so that

$$E_t[(r_{t+1} - \bar{w}'R_{t+1})w'_0 R_{t+1}] = 0.$$

This implies for all  $w \in \widetilde{W}$  that

$$E_t[(r_{t+1} - \bar{w}'R_{t+1})w'R_{t+1}] = E_t[(r_{t+1} - \bar{w}'R_{t+1})(w - w_0)'R_{t+1}] = 0,$$

by (19). ■

## B Derivation of the limit distribution of the OLS-estimates in (8)

In this appendix we will derive (under sufficient regularity conditions) the limit distribution for the OLS-estimates of the regression parameters in (??). Recall that we can test for spanning for a certain utility function  $U$  by testing whether in the regression

$$r_{t+1} = \alpha' R_{t+1} + \gamma U'(R'_{t+1} \varphi) + \varepsilon_{t+1}, \quad (20)$$

$\alpha' \iota = 1$  and  $\gamma = 0$ . For simplicity, we consider the case where there is only one utility function and we have reparametrized  $cU'(w^* R_{t+1})$  as  $U'(\varphi' R_{t+1})$ . In empirical applications the parameters  $\varphi$  have to be estimated. From

$$E_t[R_{t+1} U'(R'_{t+1} \varphi)] = \iota,$$

estimates of  $\varphi$  can be obtained with GMM using the sample moments:

$$g(\varphi, R) \equiv \frac{1}{T} \sum_{t=1}^T g_t(\varphi, R_t) = \frac{1}{T} \sum_{t=1}^T U'(R'_t \varphi) R_t - \iota, \quad (21)$$

where  $g_t(\varphi, R_t) = U'(R'_t \varphi) R_t - \iota$ . Denote

$$G(\varphi, R) = \partial g(\varphi, R) / \partial \varphi = \frac{1}{T} \sum_{t=1}^T U''(R'_t \varphi) R_t R'_t.$$

Then we have that

$$\sqrt{T}(\hat{\varphi} - \varphi) \approx -G(\varphi, R)^{-1} \sqrt{T} g(\varphi, R). \quad (22)$$

Denoting the limiting covariance matrix of  $\sqrt{T} g(\varphi, R)$  as  $S_{gg}$ , and the probability limit of  $G(\varphi, R)$  as  $A$ , then the limit distribution of  $\hat{\varphi}$  is given by:

$$\sqrt{T}(\hat{\varphi} - \varphi) \xrightarrow{L} N(0, A^{-1} S_{gg} A^{-1'}).$$

As a next step, rewrite (??) as

$$\begin{aligned} r_{t+1} &= \alpha' R_{t+1} + \gamma U'(R'_{t+1} \hat{\varphi}) + \varepsilon_{t+1} + \gamma \{U'(R'_{t+1} \varphi) - U'(R'_{t+1} \hat{\varphi})\} \\ &= \alpha' R_{t+1} + \gamma U'(R'_{t+1} \hat{\varphi}) + \varepsilon_{t+1} + u_{t+1}, \end{aligned}$$

which defines  $u_{t+1}$ . Defining  $x_t \equiv (R'_t U'(R'_t \varphi))'$  and  $\hat{x}_t \equiv (R'_t U'(R'_t \hat{\varphi}))'$ , the estimates  $\hat{\alpha}$  and  $\hat{\gamma}$  satisfy:

$$\sqrt{T} \begin{pmatrix} \hat{\alpha} - \alpha \\ \hat{\gamma} - \gamma \end{pmatrix} = \left[ \frac{1}{T} \sum_{t=1}^T \hat{x}_t \hat{x}'_t \right]^{-1} \frac{1}{\sqrt{T}} \sum_{t=1}^T (\hat{x}_t (\varepsilon_t + u_t)). \quad (23)$$

Since under the null hypothesis that there is spanning  $\gamma = 0$ , the error term  $u_t$  equals 0, and hence does not affect the limit distribution. Using a linear expansion we obtain for the last factor in (??) that

$$\frac{1}{\sqrt{T}} \sum_{t=1}^T \hat{x}_t \varepsilon_t \approx \frac{1}{\sqrt{T}} \sum_{t=1}^T x_t \varepsilon_t + \frac{1}{T} \sum_{t=1}^T \begin{pmatrix} 0 \\ U''(R'_t \varphi) R'_t \varepsilon_t \end{pmatrix} \sqrt{T} (\hat{\varphi} - \varphi). \quad (24)$$

Substituting (??) into (??) gives:

$$\frac{1}{\sqrt{T}} \sum_{t=1}^T \hat{x}_t \varepsilon_t \approx \frac{1}{\sqrt{T}} \sum_{t=1}^T x_t \varepsilon_t - \frac{1}{T} \sum_{t=1}^T \begin{pmatrix} 0 \\ U_t'' R'_t \varepsilon_t \end{pmatrix} G(\varphi, R)^{-1} \sqrt{T} g(\varphi, R).$$

If we denote the limit distribution of this term with  $N(0, V)$ , then the limit distribution of the regression parameters is given by:

$$\sqrt{T} \begin{pmatrix} \hat{\alpha} - \alpha \\ \hat{\gamma} - \gamma \end{pmatrix} \xrightarrow{L} N(0, Q),$$

where an estimate of  $Q$  can be obtained from

$$\hat{Q} = \left[ \frac{1}{T} \sum_{t=1}^T \hat{x}_t \hat{x}'_t \right]^{-1} \hat{V} \left[ \frac{1}{T} \sum_{t=1}^T \hat{x}_t \hat{x}'_t \right]^{-1}.$$

In our applications we estimated  $V$  by  $\hat{V}$  using the Newey-West approach.





Table 1: The table contains summary statistics for semi-monthly observations of net monthly holding period returns,  $(P_{t+1} - P_t)/P_t$ , over the period from January 1984 until December 1993. The total number of observations in this period is 240. Average returns, standard deviations, median, minimum, and maximum are all in percentages.

<b>Summary statistics of net monthly returns</b>							
	Average	Median	Std.dev.	Skew.	Kurt.-3	Max.	Min.
<i>Basic assets</i>							
S&P 500	0.59	1.28	6.51	-0.05	1.23	18.6	-22.8
FAZ (Germany)	1.31	0.70	5.91	-0.06	1.30	17.8	-22.5
FTSE (UK)	1.17	0.80	4.23	-0.73	4.82	17.1	-22.1
<i>Futures contracts</i>							
wheat	0.34	0.65	5.48	0.20	0.99	25.1	-15.3
corn	-0.39	-0.79	6.45	2.18	13.86	45.8	-18.6
soybeans	-0.34	-0.45	5.72	0.72	3.47	26.4	-21.6
soybean meal	0.06	-0.80	6.50	1.31	4.34	36.0	-16.6
soybean oil	-0.14	-0.99	7.36	0.50	0.62	24.5	-18.4
live cattle	0.78	0.78	3.71	-0.07	0.69	14.5	-9.9
live hogs	1.07	0.75	6.18	0.31	0.07	21.6	-15.4
German Mark	0.42	0.21	3.69	-0.01	-0.15	11.8	-10.5
British Pound	0.42	0.22	3.98	0.24	1.66	18.4	-11.3
Japanese Yen	0.55	-0.07	3.43	0.45	0.43	12.1	-7.5

Table 2: The results in the table are for the null hypothesis that a basic set of assets spans a futures contract. The column "mean-variance" is for the null hypothesis that there is mean-variance spanning, "log" is for the null hypothesis that there is both mean-variance and log-utility spanning, "power" is for the null hypothesis that there is both mean-variance and power utility spanning, where the power utility function has a risk aversion coefficient, and "log+power" is for the null hypothesis that there is spanning for both logarithmic and power utility functions. \* indicates that the null hypothesis is rejected at the 5 %-significance level, \*\* indicates rejection at the 1 %-significance level. The initial set of assets are the S&P 500, the FTSE (UK) and the FAZ (Germany). Results are based on semi-monthly observations of monthly holding period returns from January 1984 until December 1993, resulting in a total of 240 observations. All test statistics are based on a Newey-West covariance matrix with one lag.

<b>Tests for unconditional spanning</b>				
	mean-variance	log	power	log+power
wheat	4.95	11.19*	6.17	10.97*
corn	1.66	1.65	3.49	2.50
soybeans	0.74	7.37	0.97	9.30*
soybean meal	0.73	3.51	5.81	8.67*
soybean oil	0.15	11.07*	2.93	10.32*
live cattle	8.47 *	9.18 *	19.48**	11.32*
live hogs	5.20	6.26	5.66	9.43*
German Mark	0.75	2.33	60.54**	1.70
British Pound	0.77	4.46	41.95**	0.43
Japanese Yen	2.23	2.33	25.92**	4.32

Table 3: The figures in the table are Wald test statistics for the null hypothesis that a basic set of assets consisting of the S&P 500, the FAZ (Germany), and the FTSE (UK) span futures contracts for an investor with a mean-variance utility function and who has a nonmarketable position in an asset. The nonmarketable position is equal to 25% of invested wealth. The column heading indicates the asset in which there is a position. \* indicates that the null hypothesis is rejected at the 5%-significance level, \*\* indicates rejection at the 1%-significance level. The initial set of assets are the S&P 500, the FTSE (UK) and the FAZ (Germany). Results are based on semi-monthly observations of monthly holding period returns from January 1984 until December 1993, resulting in a total of 240 observations. All test statistics are based on a Newey-West covariance matrix with one lag.

<b>Tests for mean-variance spanning with nontraded assets</b>											
	wc	cn	sy	sm	bo	lc	lh	dm	bp	jy	
wheat	22.86**	13.51**	11.65**	8.54*	17.23**	6.57	5.31	5.79	4.45	6.52*	
corn	7.74*	15.77**	10.42**	8.04*	14.03**	1.12	1.33	0.77	1.12	0.88	
soybeans	2.19	6.16*	8.51*	8.53*	9.13*	0.96	0.72	0.95	0.82	0.91	
soybean meal	0.08	1.05	1.94	4.10	0.70	0.64	0.93	0.62	0.61	0.53	
soybean oil	1.35	7.77*	7.48*	1.47	16.60**	0.65	0.39	0.36	0.16	0.50	
live cattle	9.52*	6.83*	7.56*	7.33*	7.15*	25.58**	16.33**	8.61*	7.85*	8.78*	
live hogs	4.92	4.60	5.25	5.07	5.64	10.09**	24.21**	5.43	5.21	5.77	
German Mark	0.58	0.30	0.63	1.05	0.31	0.87	0.63	4.28	2.93	2.50	
British Pound	0.39	0.81	0.86	1.37	0.49	0.80	0.52	3.22	4.37	2.06	
Japanese Yen	2.23	2.38	2.08	2.61	2.26	2.64	2.64	4.13	3.54	6.79*	

Table 4: The figures in the table are Wald test statistics for the null hypothesis that a basic set of assets consisting of the S&P 500, the FAZ (Germany), and the FTSE (UK) span futures contracts for an investor with a mean-variance utility function or a log-utility function and who has a nonmarketable position in an asset. The nonmarketable position is equal to 25% of invested wealth. The column heading indicates the asset in which there is a position. \* indicates that the null hypothesis is rejected at the 5 %-significance level, \*\* indicates rejection at the 1 %-significance level. The initial set of assets are the S&P 500, the FTSE (UK) and the FAZ (Germany). Results are based on semi-monthly observations of monthly holding period returns from January 1984 until December 1993, resulting in a total of 240 observations. All test statistics are based on a Newey-West covariance matrix with one lag.

<b>Tests for mean-variance and logarithmic utility spanning with nontraded assets</b>												
	wc	cn	sy	sm	bo	lc	lh	dm	bp	jy		
wheat	29.22**	17.55**	15.90**	12.01**	20.92**	12.04**	9.57*	10.34*	9.20*	11.35**		
corn	8.24*	17.73**	10.70*	8.40*	14.88**	1.38	1.37	0.79	1.16	0.93		
soybeans	6.39	8.68*	10.51*	11.46**	10.52*	6.24	5.61	7.12	6.77	7.00		
soybean meal	0.77	1.40	2.05	4.09	1.34	2.32	2.24	3.44	3.44	3.06		
soybean oil	11.29**	16.88**	17.53**	12.32**	22.23**	10.07*	8.93*	9.08*	8.19*	9.85*		
live cattle	9.99*	7.14	7.94*	7.60	7.81	26.86**	17.40**	9.13*	8.41*	9.18*		
live hogs	6.55	5.67	6.14	6.13	6.61	10.90*	25.73**	5.94	5.70	6.50		
German Mark	2.78	1.36	1.94	2.23	1.56	2.28	2.70	6.61	4.75	4.48		
British Pound	4.45	3.72	3.99	3.87	3.98	3.92	4.01	6.18	7.63	5.54		
Japanese Yen	2.44	2.42	2.13	2.65	2.31	2.70	3.01	4.18	3.59	7.09		

Table 5: The figures in the table are Wald test statistics for the null hypothesis that a basic set of assets consisting of the S&P 500, the FAZ (Germany), and the FTSE (UK) span futures contracts for an investor with a mean-variance utility function or apower utility function with risk aversion coefficient -3, and who has a nonmarketable position in an asset. The nonmarketable position is equal to 25% of invested wealth. The column heading indicates the asset in which there is a position.\* indicates that the null hypothesis is rejected at the 5 %-significance level, \*\* indicates rejection at the 1 %-significance level. The initial set of assets are the S&P 500, the FTSE (UK) and the FAZ (Germany). Results are based on semi-monthly observations of monthly holding period returns from January 1984 until December 1993, resulting in a total of 240 observations. All test statistics are based on a Newey-West covariance matrix with one lag.

<b>Tests for mean-variance and power utility spanning with nontraded assets</b>											
	wc	cn	sy	sm	bo	lc	lh	dm	bp	jy	
wheat	22.31**	14.67**	11.27**	8.85*	16.02**	6.98	6.75	6.65	6.19	7.05	
corn	8.46*	21.44**	12.83**	8.98*	18.92**	1.13	1.98	2.02	2.12	1.59	
soybeans	2.52	8.30*	13.21*	10.01*	15.78*	1.16	0.72	1.64	1.39	1.39	
soybean meal	6.88	8.52*	11.59**	15.37**	7.04	5.05	15.35**	2.31	2.27	3.04	
soybean oil	2.60	7.38	6.59	3.42	23.25**	4.39	2.70	3.59	2.79	4.05	
live cattle	15.38**	9.90*	13.34**	10.31*	14.38**	33.42**	32.84**	13.46*	12.84*	13.10*	
live hogs	4.91	4.72	5.57	5.16	5.46	9.92*	28.51**	5.89	5.46	5.96	
German Mark	53.11**	18.58 **	21.11**	14.78**	22.99**	28.52**	80.73**	38.79**	34.05**	33.88**	
British Pound	36.10**	12.40 **	18.47**	12.85**	22.36**	23.88**	47.45**	28.19**	34.72**	30.28**	
Japanese Yen	23.73**	18.58 **	15.54**	10.61**	15.41**	18.37**	26.18**	17.26**	17.87**	28.92**	

Table 6: The numbers in the table are Wald test statistics for the null hypothesis that a basic set of assets consisting of the S&P 500, the FAZ (Germany), and the FTSE (UK) span futures contracts for an investor with a log-utility function or a power-utility function with risk aversion coefficient -3 and who has a nonmarketable position in an asset. The nonmarketable position is equal to 25% of invested wealth. The column heading indicates the asset in which there is a position. \* indicates that the null hypothesis is rejected at the 5 %-significance level, \*\* indicates rejection at the 1 %-significance level. The initial set of assets are the S&P 500, the FTSE (UK) and the FAZ (Germany). Results are based on semi-monthly observations of monthly holding period returns from January 1984 until December 1993, resulting in a total of 240 observations. All test statistics are based on a Newey-West covariance matrix with one lag.

<b>Tests for logarithmic and power utility spanning with nontraded assets</b>											
	wc	cn	sy	sm	bo	lc	lh	dm	bp	jy	
wheat	36.26**	19.79**	17.90**	12.74**	23.42**	12.13**	9.58*	10.30*	8.89*	11.28*	
corn	6.42	12.47**	8.46*	6.78	10.48*	1.61	2.05	1.75	2.07	1.71	
soybeans	8.64*	11.72**	15.42**	14.02**	15.25**	8.46*	10.15*	7.99*	7.85*	8.33*	
soybean meal	6.15	6.45	7.80	9.83*	6.17	8.48*	14.58**	7.30	7.10	7.41	
soybean oil	10.71*	18.54**	19.36**	9.58*	29.33**	6.39	4.74	7.76	8.09*	7.46	
live cattle	11.93**	7.29	8.80*	8.07*	8.19*	31.80**	24.77**	10.03*	8.95*	10.35*	
live hogs	7.58	7.28	9.10*	8.49*	8.84*	13.00**	18.43**	8.36*	7.52	8.93*	
German Mark	1.84	1.21	1.38	1.51	1.26	2.23	3.57	5.94	4.48	3.80	
British Pound	0.42	0.33	0.42	0.58	0.24	0.71	1.56	2.70	4.11	1.53	
Japanese Yen	5.33	6.02	4.27	3.91	5.81	4.94	6.95	5.75	5.28	8.76	

Table 7: The figures in the table are Wald test statistics for the null hypothesis that a basic set of assets consisting of the S&P 500, the FAZ (Germany), and the FTSE (UK) span futures contracts for investors with a mean-variance utility function, a mean variance and a log-utility function, a mean-variance and a power utility function with risk aversion coefficient 3, or a mean-variance, log, and power-utility function. The tests allow for return predictability based on hedging pressure as conditioning information. \* indicates that the null hypothesis is rejected at the 5 %-significance level, \*\* indicates rejection at the 1 %-significance level. The initial set of assets are the S&P 500, the FTSE (UK) and the FAZ (Germany). Results are based on semi-monthly observations of monthly holding period returns from January 1986 until December 1993, resulting in a total of 192 observations. All test statistics are based on a Newey-West covariance matrix with one lag.

	<b>Tests for conditional spanning</b>								
	<b>mean-variance</b>		<b>mv + logarithmic utility</b>		<b>mv + power utility</b>				
	hedging pressure	hedging pressure	hedging pressure	hedging pressure	hedging pressure	hedging pressure			
wheat	3.69	4.01	3.13	10.71*	10.83*	9.66*	5.15	5.70	4.78
corn	2.63	3.13	3.25	2.78	3.24	3.36	3.25	3.75	3.25
soybeans	1.07	0.78	1.28	7.44	8.12*	7.39	1.35	1.20	1.40
soybean meal	4.69	0.87	4.33	7.07	4.21	6.80	12.48**	6.71	12.01**
soybean oil	13.68**	10.27**	21.03**	25.45**	34.00**	39.15**	16.21**	18.41**	12.01**
live cattle	8.12*	6.10	4.14	8.29*	6.27	4.33	29.09**	29.56**	28.54**
live hogs	11.19**	8.89*	0.16	14.56**	11.40**	1.38	13.47**	10.66*	3.56
German Mark	21.72**	0.31	26.12**	22.27**	0.37	26.18**	102.25**	42.69**	74.32**
British Pound	19.35**	0.96	8.00*	19.51**	1.63	8.87*	69.15**	42.12**	46.40**
Japanese Yen	33.30**	0.18	19.47**	33.17**	0.49	19.30**	76.35**	21.75**	46.22**



Table 7 (Continued)

	Tests for conditional spanning		
	mv	log	power utility
	hedging pressure		
	-0.50	0.00	+0.50
wheat	13.43**	13.05**	12.46**
corn	3.99	4.42	4.53
soybeans	9.15	9.78*	8.78
soybean meal	24.95**	21.46**	24.89**
soybean oil	26.98**	28.03**	42.52**
live cattle	37.39**	38.26**	37.70**
live hogs	31.05**	24.41**	17.69**
German Mark	257.67**	100.01**	121.96**
British Pound	156.32**	93.97**	86.67**
Japanese Yen	70.17**	30.34**	51.30**