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Kumah, F.Y.

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# Common Stochastic Trends in the Current Account

by

F. Y. Kumah\*

## Abstract

Solow residuals are used as proxies for productivity shocks in many empirical studies. Considering the shortcomings of this approach this paper proposes the common trends approach as an alternative. The common trends econometric technique is utilized here in an attempt to identify and analyze the long run effects of country-specific and global productivity shocks on fluctuations in investment and the current account. The theoretical framework utilized provides long run restrictions relevant for identifying global and country-specific productivity shocks. Our estimations yield the following stylized facts. Generally, consistent with theoretical predictions, the long run effects of positive idiosyncratic (country-specific) productivity shocks on the current account are significantly negative. Further, permanent global shocks are impotent (by theoretical restriction) in explaining fluctuations in the current account though very significant in explaining investment fluctuations.

**Key Words:** Current Account, Capital Mobility, Common Trends, Seasonal Integration and Impulse Responses.

**JEL classification:** F32, F41

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*Correspondence address:*

CentER for Economic Research, Tilburg University, P. O. Box 90153, 5000 LE Tilburg, The Netherlands. Tel. # + 31 13 466 8221; Fax. # + 31 13 466 3066; Email address: F.Kumah@kub.nl

## 1 Introduction

Using the intertemporal approach to the current account Glick and Rogoff(1995) - referred to as G&R in the rest of this paper - develop and empirically test specifications for investment and the current account especially with respect to the effects of global and domestic (country-specific) productivity shocks on these using time series data for the G-7 countries. The framework is basically in the tradition of Obstfeld(1986) and Razin(1993) who analyzed the theoretical effects of government spending and productivity shocks. The main departure in G&R (just as in Razin (1993)) is an empirical re-formulation of the problem to distinguish between global and country-specific shocks. Using this framework G&R investigate the relative effects of permanent global and country-specific productivity shocks on investment and the current account.

Earlier attempts at investigating the effects of productivity shocks and/or stochastic trends on economic fluctuations in an open economy use simulations and/or Blanchard-Quah type vector-autoregression methodology: Mendoza(1991) falls into the former categorization whereas Ahmed, Ickes, Wang and Yoo(1993) falls into the latter group. There is also a group of researchers who use Solow residuals as proxies for productivity shocks like G&R and Backus, Kehoe and Kydland(1992) for example. The use of Solow residuals as proxies for productivity shocks is however not without shortcomings. Firstly, as pointed out in Hall(1988) movements in the Solow residuals may not represent exogenous technology shocks - the identifying assumption here is for the Solow residuals to be orthogonal to "variables known to be neither causes of productivity shifts nor to be caused by productivity shifts". G&R do not test for this identifying assumption. Secondly, the use of constructed Solow residuals as productivity shocks has been shown to overestimate the variance of these shocks due to increased variation in the capital stock as the result of varying capacity utilization and other measurement problems [see for instance Eichenbaum(1990)]. Baxter and Crucini(1993) is very eloquent on the possibility of these measurement errors carrying over into obtained empirical results when they criticise the mode of measurement of the Solow residuals by Backus *et al*(1992).

This paper adds to the results in G&R by adopting multivariate cointegration and common

trends techniques that circumvent the measurement problems associated with calculated Solow residuals. The idea here is to decompose the variables of interest directly into permanent and transitory components rather than search for the response of these variables to calculated Solow residuals. Moreso, given that the number of cointegrating vectors is different from zero we can be sure there are certain common factor(s) that explain fluctuations in the variables of interest. We use the common trends technique to identify and interpret these factors as productivity shocks. Our focus here is on the responses of investment and the current account balance to country-specific and global productivity shocks. The specific questions that we seek empirical answers to are the following: i. what are the long run effects of each of these shocks (analyzed separately) on investment and the current account? ii. how does each of our variables of interest (investment and the current account) respond to one standard deviation innovations in the respective shocks? iii. what is the relative effectiveness of the innovations in the respective shocks in explaining fluctuations in investment and the current account? and finally iv. how does our approach compare with that of G&R? In an attempt to address these questions we utilize common trends approach in identifying and estimating the effects of these shocks following the estimation structure as presented in Johansen(1988), King *et al.*(1991), Stock and Watson(1988) and Warne(1990). Our empirical investigations are based on data-sets for Germany and the United States (US) over the period 1974:1 - 1992:4.

Our empirical results indicate that the common trends approach yields results/findings comparable to those of G&R and goes beyond instantaneous least squares estimates by providing us with the possibility of dynamic analyses - which in this context could be implemented using estimated impulse response functions of the effects of innovations in the identified productivity shocks and forecast error decompositions - that are very crucial in empirical investigations of capital mobility and current account fluctuations. It turns out that the estimated productivity shocks are highly persistent and explain almost all variations in our variables of interest at the long run horizon. In fact the estimated impulse response functions attributable to innovations in the transitory components of these shocks - as portrayed by figures 4 and 5 - show that they do not seem relevant in explaining fluctuations in our data-set. Also, long run fluctuations in real output and consumption are better explained by global shocks whereas fluctuations in investment and the current account are better accounted for by idiosyncratic productivity shocks. This evidence is indicative of the fact that the two countries

considered are *very* open economies in which capital is highly mobile and capital flights are carried out in attempts to smoothen consumption - the usual risk-sharing arguments - and hence making consumption highly correlated with global shocks. Given this explanation agents consume a share of the world consumption/output irrespective of domestic idiosyncratic disturbances.

In pursuit of answers to the specific questions raised above we organise the paper as follows. Section 2 presents the theoretical framework used in deriving the reduced-form equations for investment and the current account and distinguishes carefully between the effects of shocks, that could be country-specific or global. We adopt the specification of G&R and Razin(1993). Theoretical implications of the model are derived and discussed in this section. Section 3 inspects the data-set for seasonal integration and tests for the number of cointegrating vectors (or equivalently, the number of common trends), if any. The section after that presents and analyses the estimates of the common trends model the main features of which are summarized in the appendix. The estimates of the impulse response functions are also presented here. Finally, section 5 summarises and concludes the paper.

## 2 Theoretical Framework

In the tradition of the dynamic-optimizing approach<sup>1</sup> to the current account balance fluctuations in the current account are attributable, among other factors, to productivity shocks (which could be transitory or permanent) characterised as global or country-specific and are transmitted through savings-investment decisions of economic agents.

### 2.1 The Investment Decision

Consider an economy producing a single aggregate tradable good,  $Y_t$ , using the Cobb-Douglas production function in (1) below. We assume here that labour is supplied inelastically such that

$$Y_t = Z_t K_t^\alpha \quad (1)$$

where  $Z_t$  denotes the time- $t$  productivity,  $K_t$  is the capital stock and  $\alpha$  is the distributive share of capital. Following G&R we assume that  $Z_t$  represents the time -  $t$  stochastic shock to technology and that it follows a first-order autoregressive process given by

$$Z_t = \rho Z_{t-1} + \epsilon_t \quad \text{where} \quad \rho \in [0,1] \quad (2)$$

where  $\rho$  is a persistence parameter<sup>2</sup> and  $\epsilon_t$  is a white-noise disturbance term. The representative firm maximises the expected value of the discounted sum of profits

$$E \sum_{t=0}^{\infty} R^{-t} \pi_t \quad \text{where} \quad \pi_t = Y_t - \omega_t, \quad \text{and} \quad \omega_t \quad \text{denotes the cost-of-adjustment investment technology}$$

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<sup>1</sup>. See for instance Razin(1993), Obstfeld(1986) and/or G&R for an exposition of this approach.

<sup>2</sup>. We solve the model for  $\rho \in [0,1]$  though our interest is best served with  $\rho = 1$  ( a random-walk specification for the productivity term).

specified as

$$\omega_t = I_t \left[ 1 + \frac{g}{2} \frac{I_t}{K_t} \right] \quad (3)$$

where  $K_{t+1} = K_t + I_t$  and  $g$  is the cost-of-adjustment coefficient. Given this cost-of-adjustment technology there is an incentive for firms to adjust their capital stocks gradually since the cost of changing the capital stock by one unit increases with the speed of adjustment. Using Euler equations the optimal investment rule is characterised as below:

$$E_t R^{-1} \left[ \alpha Z_{t+1} K_{t+1}^{\alpha-1} + \frac{g}{2} \left( \frac{I_{t+1}}{K_{t+1}} \right)^2 + q_{t+1} \right] = q_t \quad (4)$$

where  $E_t$  is the expectations operator based on the current period's information,  $q_t$  is the firm's market-value per unit of capital such that  $q_t = 1 - \frac{gI_t}{2K_t}$  and  $R$  is one plus the world rate of interest. Notice that linearising (4) around the steady-state (where there is equality between the world rate of interest and the marginal productivity of capital: i.e.  $R - 1 = \alpha \bar{Z} \bar{K}^{\alpha}$  where  $\bar{Z}$  and  $\bar{K}$  are the steady state levels of productivity and capital, stock respectively) yields the expression below (where the lag operator,  $L$ , is defined as  $L^k(\chi_{t+1}) = E_t(\chi_{t+1-k})$  for  $k \in [-\infty, +\infty)$ )

$$a_1 [L^0 + \phi_1 L + \phi_2 L^2] k_{t+1} = -b Z_t \quad (5)$$

where  $k_t = K_t - \bar{K}$  denotes the deviation of capital stock from its steady state value,  $\phi_2 = (I/a_1)$  and  $\phi_1 = (a_0/a_1)$ . The parameters  $a_0$  and  $a_1$  are functions of  $R$ ,  $g$  and  $\alpha$ . The polynomial on the left hand side of the above expression can be factorised (see Sargent(1979)) as

$$\begin{aligned} [L^0 + \phi_1 L + \phi_2 L^2] &= (L^0 - \phi_1 L)(L^0 - \phi_2 L) \\ &= -\phi_2 L(L^0 - \phi_2^{-1} L^{-1})(L^0 - \phi_1 L) \end{aligned}$$

We assume  $\phi_1 + \phi_2 < 1$ ,  $\phi_2 - \phi_1 < 1$ , and  $\phi_2 < 1$  so that  $(L^0 - \phi_2^{-1} L^{-1})$  is invertible. [see for

instance Cryer(1989) for the derivation of these conditions]. Multiplying both sides of (5) by the inverse of  $-a_1\phi_2L(L^0 - \phi_2^{-1}L^{-1})$  yields

$$k_t = \lambda_1 k_{t-1} + \lambda_1 b \sum_{i=0}^{\infty} \left(\frac{1}{\lambda_2}\right)^i E_t Z_{t+1+i} \quad (6)$$

where  $\lambda_1 < 1$  and  $\lambda_2 > 1$  are the roots of the quadratic equation  $1 + a_0\lambda + a_1\lambda^2 = 0$ . A simple manipulation of (6) - neglecting the i.i.d term in the AR(1) productivity shock process - yields desired investment as

$$\begin{aligned} I_t &= \lambda_1 I_{t-1} + \lambda_1 b \sum_{i=1}^{\infty} \left(\frac{1}{\lambda_2}\right)^{i-1} [E_t \rho^i Z_{t+i} - E_{t-1} \rho^i Z_{t+i-1}] \\ &= \lambda_1 I_{t-1} + \lambda_1 b \rho \sum_{i=1}^{\infty} \left(\frac{\rho}{\lambda_2}\right)^{i-1} \Delta Z_t \\ &= \lambda_1 I_{t-1} + \frac{\lambda_1 b \rho}{\left(1 - \frac{\rho}{\lambda_2}\right)} \Delta Z_t \quad (7) \\ \Rightarrow \Delta I_t &= (\lambda_1 - 1) I_{t-1} + \frac{\lambda_1 b \rho}{\left(1 - \frac{\rho}{\lambda_2}\right)} \Delta Z_t \end{aligned}$$

The first term of the first line of the above equation captures the effects on current investment of lagged productivity shocks and the second term the revisions in expectations of future productivity shocks. Thus transitory productivity shocks (i.e for  $\rho = 0$ ) have no impact on current investment.



## 2.2 Consumption

The representative agent chooses a consumption path that maximizes the lifetime utility

$$E_t \left[ \sum_{i=1}^{\infty} \beta^i U(C_{t+i}) \right], \quad \text{where } U(c) = C - \frac{1}{2}C^2 \quad (8)$$

subject to the intertemporal budget constraint

$$F_t = Y_t - C_t + RF_{t-1} \quad (9)$$

where  $\beta$  and  $F$  denote the subjective discount factor and the stock of foreign assets respectively. Assuming for simplicity that  $\beta R \approx 1$ , the solution to this optimisation problem can be expressed compactly as

$$C_t = \delta W_t, \quad \text{for } \delta = \frac{R - 1}{R} \quad (10)$$

where  $W_t$  denotes the permanent net (investment) income at time  $t$  consisting of the expected discounted flow of current and future income and initial foreign assets:

$$W_t = E_t \left[ \sum_{i=0}^{\infty} R^{-i} Y_{t+i} \right] + RF_{t-1} \quad (11)$$

It is clear that the induced optimal investment path and hence the realised current and future productivity shocks are the main driving forces behind consumption spending according to this model. To ascertain this we linearise the production function around the steady state yielding

$$Y_t = d_0 + d_K K_t + d_Z Z_t \quad (12)$$

which is then substituted together with (2) and (7) into the wealth term in (11) and (10)

yielding the closed-form solution for optimal consumption as dependent on past and current productivity shocks and the level of initial foreign asset holdings. Specifically, after the necessary substitutions have been made the first difference of (12) can be expressed as

$$\begin{aligned}\Delta Y_t &= d_K \lambda_1 I_{t-1} + \frac{d_K \lambda_1 b \rho}{(1 - \frac{\rho}{\lambda_2})} \Delta Z_t + d_K \Delta Z_t \\ &= d_K \lambda_1 I_{t-1} + \psi \Delta Z_t\end{aligned}\tag{13}$$

Notice here that  $\psi > 0$  (for  $\lambda_2 > \rho$ ) indicating that positive technology shocks have positive effects on output. Some further algebraic manipulations then give us the closed-form solution of the first difference of consumption

$$\begin{aligned}\Delta C_t &= \delta \Delta W_t \\ &= \delta E_t \left[ \sum_{j=0}^{\infty} R^{-j} \Delta Y_{t+j} \right] + \delta (R - 1) \Delta F_{t-1} \\ &= \frac{\delta d_K \lambda_1}{(1 - \frac{1}{R})} I_{t-1} + \frac{\delta \psi}{(1 - \frac{1}{R})} \Delta Z_t + \delta R \Delta F_{t-1}\end{aligned}\tag{14}$$

Armed with equations (7), (13) and (14) we derive the equilibrium expression for the change in the current account using the national income accounting identity

$$\Delta ca_t = \Delta Y_t - \Delta I_t - \Delta C_t + (R - 1) \Delta F_{t-1}\tag{15}$$

Substituting equations (7), (13) and (14) for the terms on the right hand side of the above expression yields the first difference of the current account as a function of changes in foreign asset holdings and productivity shocks as expressed below:

Thus, if the coefficient in front of  $\Delta Z_t$  is negative, permanent country-specific productivity-raising shocks must worsen the current account balance. The explanation for this is simply

$$\begin{aligned}
\Delta ca_t &= d_K \lambda_1 I_{t-1} + \psi \Delta Z_t - (\lambda_1 - 1) I_{t-1} - \frac{\lambda_1 b \rho}{(1 - \frac{\rho}{\lambda_2})} \Delta Z_t \\
&\quad - \frac{\delta d_K \lambda_1}{(1 - \frac{1}{R})} I_{t-1} - \frac{\delta \psi}{(1 - \frac{1}{R})} \Delta Z_t + [(R - 1) - \delta R] \Delta F_{t-1} \\
&= \left[ \lambda_1 \left( d_K - \frac{d_K \delta}{(1 - \frac{1}{R})} - 1 \right) + 1 \right] I_{t-1} + [(R - 1) - \delta R] \Delta F_{t-1} \\
&\quad + \left[ \psi - \frac{\lambda_1 b \rho}{(1 - \frac{\rho}{\lambda_2})} - \frac{\delta \psi}{(1 - \frac{1}{R})} \right] \Delta Z_t
\end{aligned} \tag{$$

the fact that these shocks, as we discussed above do not cause only investment spending to rise but also do cause consumption spending to rise at least by as much as the rise in output emanating from these same shocks. These are the issues that we investigate in the empirical section of the paper.<sup>3</sup>

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<sup>3</sup>. Those interested in the technical details of the econometric methods used here can turn to appendix for a brief exposition on common trends and related issues. Otherwise the reader can move on to the next section without losing the thread of the discussion.

### 3.3 Preliminary Data Analyses

#### 3.3.1 *The Data*

The data used in the empirical analysis include seasonally adjusted quarterly data on real gross national product ( $y_t$ ), private consumption ( $c_t$ ), gross investment ( $inv_t$ ) and the current account balance ( $ca_t$ ) obtained from International Financial Statistics (IFS) database as published by the IMF. The choice of the variables in the data-set is guided by the main variables of the intertemporal approach to the current account as presented in the theoretical section above. The other relevant variables - the world interest rate and net foreign asset holdings - are assumed to be already captured by the current account which by definition is the sum of net exports and interest earnings on net foreign asset holdings (i.e. the product of the world interest rate and the net foreign asset holdings). Since the quarterly current account series are reported/expressed in US dollars we convert them into deutsche marks (in the case of Germany) by multiplying by the average market exchange rates for the respective quarters. Following G&R investment is defined as gross fixed capital formation plus changes in (inventory) stocks. Consumption is defined as private consumption expenditures.

#### 3.3.2 *Seasonal Integration*

It is quite advantageous when dealing with seasonal data to start by examining the set of plots as depicted in Figures 1a and 1b for Germany and the US respectively. The first row of each figure shows the level of the series (in the first column) and plots the first quarter values of the series,  $q_1$ , the second quarter values,  $q_2$ , and so on (in the second column). Thus the quarterly series are graphed in yearly terms. According to Bowswijk and Franses(1991) the plots of these  $q_i$  series will be parallel to each other if the seasonal movements in the data are constant (and hence can be satisfactorily modelled using dummies) whereas for a varying seasonal movement (which is better modelled by a stochastic model) they are non-parallel. The last column of the first row as well as the second row show plots of transformations of the original series based on the transformation

$$\begin{aligned}
(1 - L^4) &= (1 - L)(1 + L + L^2 + L^3) = (1 - L)(1 + L)(1 + L^2) \\
&= (1 - L)(1 + L)(1 - iL)(1 + iL)
\end{aligned}$$

where  $L$  is the lag operator and  $i^2 = -1$ . Hence if the transformation above renders the quarterly series stationary then the quarterly seasonal unit root process has four roots of modulus unity: one at the zero frequency (which can be removed using the transformation  $(1 - L)$ ), one at the two-quarter or half-yearly frequency (which can be removed using the transformation  $(1 + L)$ ) and a pair of complex conjugate roots at the four-quarter or yearly frequency as captured by  $(1 - iL)(1 + iL)$ . The figures titled "NO ROOTS", "ZERO-FREQUENCY ROOT", "SEMI-ANNUAL ROOT" and "ANNUAL ROOT" depict these respective transformations. It does seem reasonable, judging from these plots, to conclude that generally speaking seasonality in the data can be satisfactorily modelled using seasonal dummies.

This hypothesis of constant seasonal movements as opposed to that of a varying seasonal pattern in the data is formally testable using the testing strategy proposed by Hylleberg *et al.* (1990) - the HEGY procedure. The test procedure requires OLS estimation of the equation

$$\Delta_4 X_t = \pi_1 z_{1t-1} + \pi_2 z_{2t-1} + \pi_3 z_{3t-2} + \pi_4 z_{3t-1} + \epsilon_t \quad (17)$$

and the estimated value of the  $\pi$ s used to draw inferences. In the above equation  $X_t$  is the original series,  $z_{1t} = (1 + L + L^2 + L^3)X_t$ ,  $z_{2t} = -(1 - L + L^2 - L^3)X_t$  and  $z_{3t} = -(1 - L^2)X_t$ . Lags of the dependent variable,  $\Delta_4 X_t$  could be added to capture autocorrelation in the error term. To test the null hypothesis of a unit root at the zero-frequency we simply test  $\pi_1 = 0$ ; to test for a root of  $-1$  (half-yearly frequency) we test  $\pi_2 = 0$  and finally to test for roots of  $\pm i$  (annual frequency) we perform the joint test  $\pi_3 = \pi_4 = 0$ . If none of the null hypotheses above can be accepted then the original series is stationary. Critical values of these null hypotheses against their respective alternatives  $\pi_1 < 0$ ,  $\pi_2 > 0$  and  $\pi_3 \cup \pi_4 \neq 0$  are taken from Hylleberg *et al.* (1990). The results of this HEGY testing strategy, applied to the data set, are presented in Table 1 below. An intercept term, three seasonal dummies and a linear trend are

included in all the regressions performed except in the case of the  $ca_t$ , where an additional test is conducted with no trend included in the regressions since (from a pragmatic point of view it may be more appropriate to regard the  $ca_t$  series as non-trending otherwise current account imbalances will be self-sustaining. However - as the results show - the inclusion, or otherwise, of the trend term in this case (as well as in the case of the other variables) does not yield any qualitative differences in the results.

**TABLE 1: RESULTS OF THE HEGY TESTS**

**A. Germany**

Series	Augm.	$t(\pi_1)$	$t(\pi_2)$	$F(\pi_3 \cap \pi_4)$	BP(30)	ARCH(1)	ARCH(4)
$y_t$	0	-3.486	-3.314*	5.700*	0.998	0.793	0.707
$c_t$	0	-0.596	-4.954*	36.033*	0.109	0.847	0.718
$inv_t$	0	-1.926	-3.561*	58.373*	0.707	0.875	0.995
$ca_t$	0 [0]	-2.509 [-1.732]	-3.036* [-3.024*]	43.176* [41.080*]	0.476 [0.495]	0.961 [0.969]	0.852 [0.902]

**B. US**

Series	Augm.	$t(\pi_1)$	$t(\pi_2)$	$F(\pi_3 \cap \pi_4)$	BP(30)	ARCH(1)	ARCH(4)
$y_t$	0	-2.701	-4.969*	27.177*	0.382	0.966	0.999
$c_t$	0	-2.217	-4.716*	34.059*	0.879	0.886	0.822
$inv_t$	0	-2.499	-6.178*	22.641*	0.967	0.757	0.754
$ca_t$	1 [1]	-0.672 [-1.527]	-0.710 [-0.724]	0.885 [0.989]	0.963 [0.963]	0.642 [0.632]	0.229 [0.207]

**Notes:** Augm. (=Augmentation) depicts the number of lags of the dependent variable included in the regression to attain i.i.d. residuals. P.Values appear under each of the columns labelled 'BP(30)', 'ARCH(1)', and 'ARCH(4)'. 'BP(30)' is the Box-Pierce test for residual autocorrelation based on 30 correlations whereas 'ARCH(k)' tests for autoregressive conditional heterogeneity, at lag k, in the residuals. Rejection of the null hypotheses at the 5% and 10% significant levels are indicated with '\*' and '\*\*' respectively. The critical values are taken from Hylleberg et al (1990) p. 226-227.

From the results of the HEGY test as presented in the table above we confirm that the seasonality in the data set can be satisfactorily modelled using seasonal dummies and further that, though not unanimously, the hypotheses of the existence of unit roots at the semi-annual and annual frequencies have been rejected at the 5% significance level. Hence we infer that the variables seem to be characterised by stochastic non-stationarities that can be removed through first order differencing. Having analyzed the stationarity characteristics of the data-set we proceed to test for the existence of cointegration among the variables in the data-set using the so-called *Johansen Procedure*.

### 3.3 Cointegration Tests

The empirical framework we have chosen in analysing the theoretical conclusions of the intertemporal approach to the current account balance requires the existence of cointegration among the variables of interest (see Appendix A). We utilise the Johansen approach in testing for the number of cointegration vectors in the data-set. A set of variables,  $X_t$ , is said to be cointegrated of order  $(d,b)$  - denoted  $CI(d,b)$  - if  $X_t$  is integrated of order  $d$  and there exists a vector  $\beta$ , such that  $\beta'X_t$  is integrated of order  $(d - b)$ <sup>4</sup>. The most common test for cointegration is the Engle and Granger(1987) two-step procedure which performs the tests in a univariate setup. Recent developments in the literature include the Johansen procedure (see Johansen(1988) and Johansen and Juselius(1990))<sup>5</sup>.

Following Johansen and Juselius(1990) we obtain the results as summarised in Table 3 below. Prior to the implementation of the Johansen procedure we need to establish an appropriate/optimal lag length/order,  $p$ , for the underlying VARX. To obtain  $p$  we use multivariate lag order tests - the Akaike Information Criterion(AIC), the Swartz Criterion(SIC) and the Hannan and Quinn Criterion (HQ) - the results of which are presented in Table 2 below. The information criteria, as usual, are not unanimous as to the optimal lag length<sup>6</sup>. However guided by the statistical performance of the residuals obtained through the Johansen procedure (see Table 2 below<sup>7</sup>) we are convinced that an optimum lag of 2 is suitable for Germany and  $p=5$  adequately captures the dynamic structure of the data-set of the US.

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<sup>4</sup>. A variable is said to be integrated of order  $z$  - denoted  $I(z)$  if the said variable becomes covariance stationary after differencing  $z$  times. See Cryer(1986) for a further definition of the concept of stationarity.

<sup>5</sup>. These studies examine the question of cointegration and provide not only an estimation methodology but also explicit procedures for testing for the number of cointegrating vectors as well as for restrictions suggested by economic theory - in a multivariate setting.

<sup>6</sup>. Paulsen(1984) has shown that both the Swartz Bayesian criterion (SIC) and the law-of-iterated-logarithm Criterion (LIL) of Hanna and Quinn are weakly consistent in the presence of unit roots whereas the Akaike Information Criterion (AIC) asymptotically over-estimates the optimal lag length lag.

<sup>7</sup>. The statistical performance of residuals obtained using other lag lengths not only yield insufficient evidence for the rejection of the null hypothesis of no cointegration vectors (in some cases) but also leave much to be desired. In estimating the cointegrating vectors we allow for trend in the data.

**Table 2: Tests for Optimal Lag Lengths**

	Tests	Number of lags							
		1	2	3	4	5	6	7	8
Germany	AIC	20.94	20.34	20.82	19.75	20.36	19.91	20.58	19.59*
	SIC	22.97*	24.41	25.92	26.87	28.52	30.09	31.79	32.85
	HQ	21.64	21.73	21.91	21.52*	21.83	22.08	22.44	22.15
US	AIC	19.01	19.38	18.85	19.25	18.77*	19.49	18.94	19.26
	SIC	21.14*	22.64	24.24	25.77	27.42	29.27	30.85	32.31
	HQ	19.74*	19.84	20.05	20.18	20.42	20.88	21.06	21.12

*Notes: All statistics are calculated using multivariate methods. The starred numbers in each row indicate the minimum value attained (and hence correspondingly the optimum lag selected) by the respective information criteria.*

For  $p=2$  for Germany (and  $p=5$  for the US) for we perform the *trace* and *lambda max* tests using the Johansen Procedure. From the results below we infer that it is reasonable, using the *trace* test, to accept the null of two cointegrating vectors at the 10% significance level for both countries. Univariate residual diagnostic tests are performed using the Ljung-Box test for autocorrelated residuals, ARCH test for autoregression and conditional heteroscedasticity, the skewness and excess-kurtosis statistics, and the Jacque-Bera test for normality. The test results as depicted in table 4 below reveal no indications of misspecification error in the estimated VARX model based on the existence of two cointegrating vectors. However the likelihood ratio test results indicate that investment and the current account balance series are stationary at the 5% significance level - a finding that seems contrary to the results of the seasonal integration results presented in table 1 above. Considering that the former test deals mainly with the *residuals* of the said variable (within a multivariate context) whereas the HEGY tests - just like all the other tests for unitroots - tests the *variable itself* directly we proceed with the paper based on the results of the HEGY tests.



**Table 3: Initial Results of Cointegration Analyses****A. The Maximum Eigenvalue ( $\lambda_{max}$ ) Criteria.**

Hypothesis		Test Statistics		Critical Values	
$H_0$	$H_1$	Germany	US	90%	95%
$r=0$	$r=1$	28.67	26.14	24.73	27.07
$r \leq 1$	$r=2$	20.14	15.48	18.60	20.97
$r \leq 2$	$r=3$	6.43	9.91	12.07	14.07
$r \leq 3$	$r=4$	2.80	3.51	2.69	3.76

**B. The Trace Criteria.**

Hypothesis		Test Statistics		Critical Values	
$H_0$	$H_1$	Germany	US	90%	95%
$r=0$	$r \leq 1$	58.05	55.04	43.95	47.21
$r \leq 1$	$r \leq 2$	29.38	28.90	26.79	29.68
$r \leq 2$	$r \leq 3$	9.23	13.32	13.33	15.41
$r \leq 3$	$r=4$	2.80	3.51	2.69	3.76

*Notes:* An intercept and three seasonal dummy variables are included in each estimated equation. The reported critical values are taken from Osterwald-Lenum (1990).

**c. Likelihood Ratio (LR) Tests for Stationarity (Given  $r=2$ ).**

Null Hypothesis	$y_t \sim I(0)$	$c_t \sim I(0)$	$inv_t \sim I(0)$	$ca_t \sim I(0)$
Germany	16.009 (0.0009)	14.169 (0.0008)	5.854 (0.054)	4.301 (0.116)
USA	6.509 (0.039)	6.308 (0.043)	5.417 (0.067)	5.672 (0.059)

*Notes:* The test statistics here have asymptotic  $\chi(k)$  distribution where  $k$  is the number of common trends. the numbers in parentheses are estimated significance levels.

**d. Estimated Cointegration Vectors<sup>8</sup>**

$$\beta^{Germany} = \begin{bmatrix} 0.07 & -0.17 & 0.33 & 1 \\ -0.09 & 0.16 & 0.02 & 1 \end{bmatrix} \quad \beta^{US} = \begin{bmatrix} -0.25 & 0.27 & 0.62 & 1 \\ -0.54 & 0.71 & 0.40 & 1 \end{bmatrix}$$

<sup>8</sup>. We do not try to identify and interpret the cointegrating relationships here since we are not interested in these equilibrium relationships as such. Note that the estimates of the Common Trends model are not sensitive to the mode of normalisation of the estimated cointegration vectors.

**Table 4: Univariate Residual Analysis****A. Germany** (Given  $r = 2$ )

Residual Diagnostics					
Equation	L-B(17)	ARCH(2)	Skew.	Ex-Kurt.	J-B.Norm.
$\Delta y$	29.186	0.075	-0.619	1.328	9.605
$\Delta C$	24.331	0.838	-0.074	0.449	0.651
$\Delta inv$	16.369	2.753	-0.703	0.568	6.699
$\Delta ca$	22.052	0.114	1.508	6.291	141.965

**B. US** (Given  $r = 2$ )

Residual Diagnostics					
Equation	L-B(16)	ARCH(5)	Skew.	Ex-Kurt.	J-B.Norm.
$\Delta y$	22.648	4.687	0.034	0.710	1.419
$\Delta C$	16.212	3.702	-0.225	0.232	0.713
$\Delta inv$	12.075	1.309	-0.090	0.336	0.404
$\Delta ca$	6.631	3.572	1.083	2.953	37.442

Notes: The entries under the Ljung-Box (L-B(17)) are test statistics for autocorrelation and have  $\chi^2(16)$  distribution. Under the column labelled ARCH(1) are statistics for testing autoregression and conditional heteroschedsticity and have  $\chi^2(1)$  distributions. The next two columns are statistics for testing skewness and excess kurtosis respectively. They have  $\chi^2(1)$  distributions. The next column entries are statistics for the Jarque-Bera test for normality (with a  $\chi^2(2)$  distribution) based on the Skewness and Excess Kurtosis statistics

#### 4 The Estimated Common Trends Model

From the preliminary data analyses and the results from implementing the Johansen procedure as presented for both countries in the previous sections we infer that i). tests for stationarity are strongly rejected for all of the series and hence the data-set is characterised by stochastic non-stationarities that can be removed by first-difference transformations; and ii). the data-set is characterised by two cointegrating vectors as represented table 3 above - implying the long run peculiarities of the data-set are driven by two common stochastic trends. It seems therefore most appropriate to conclude that an error-correction model with two cointegrating vectors is a reasonably congruent representation of the data-set in the cases of both countries. Deducing from the theoretical framework presented in section 2 above we postulate that the productivity shocks contain both country-specific and global components. We indicate these shocks as  $\tau_{Dt}$  and  $\tau_{Gt}$  respectively. The exact identification (and estimation) of these trends requires some restriction(s) on the  $A$  matrix of the common trends model as described in the appendix and reproduced in the equation below.

$$X_t = X_0 + A\tau_t + C^*(L)e_t \quad ( 18 )$$

where  $\tau_t = \mu + \tau_{t-1} + \varphi_t$  and  $X_t = [ y_t, c_t, inv_t, ca_t ]'$ . Since we have  $k (= n - r) = 2$  common trends - where  $r$  indicates the number of cointegration vectors in the  $n$ -variable data-set - the  $A$  matrix above must necessarily contain  $n \times k (= 8)$  elements. From the requirements already derived [see the Appendix] as  $\beta'A = 0$  (yielding  $r \times k$  restrictions/equations) and  $C(I)\Xi C(I)' = AA'$  (which yields  $k(k+1)/2$  restrictions) we obtain a total of  $rk + k(k+1)/2 (= 7)$  equations. Hence to exactly identify the  $A$  matrix we require only one additional restriction. The most suitable candidate in this case (and that is also consistent with the theoretical model presented/discussed above is that which states that global shocks, because they affect all countries, have no permanent effects on the current accounts of these countries<sup>9</sup>. To choose the structure of the  $A$  matrix to suit/effect this particular restriction note that for estimation purposes  $A = A_0\Pi$  where  $A_0$  is an  $n \times k$  initial impact matrix and  $\Pi$  is  $k \times k$  lower triangular. We find, in this particular paper, the choice of the initial impact matrix,  $A_0$ , of the form

$$A_0 = \begin{bmatrix} 0 & 1 \\ 1 & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \\ \alpha_{41} & 0 \end{bmatrix}$$

very suitable since it produces the desired structure of  $A (= A_0\Pi)$  with the embedded proposition of no long-run effect of global productivity shocks on the current account balance given that  $\tau_t = [\tau_{Dt} \ \tau_{Gt}]'$ <sup>10</sup>.

Having discussed the necessary restrictions for distinguishing between country-specific and

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<sup>9</sup>. Notice here that given that we have two stochastic trends in the data-set for each country, the imposition of this theoretical a priori restriction implies the imposition of another fundamental theoretical restriction/implication of the intertemporal model - that which states that a permanent country-specific shock induces a rise in the current account deficit in excess of a corresponding rise in investment.

<sup>10</sup>. Given the interrelatedness of shocks across countries the most appropriate way to conceptualise these shocks is to think of them as the outcome of a number of unidentified system-wide shocks (having effects across country borders) rather than as a set of country-specific shocks. However, as we do here, the imposition of restrictions/implications derived a priori from economic theory may provide acceptably convincing means of identifying the effects of a specific type of shocks to any particular country under consideration.

global shocks we turn next to the quantitative effects that the data assigns each of these shocks. Using a  $\mathbf{VAR}(p)$  with  $p=2$  in the case of Germany and  $p=5$  in the case of the US the estimates of the

CT model are as follows.

$$\begin{aligned}
 \text{Germany:} \quad \begin{bmatrix} y_t \\ C_t \\ Inv_t \\ Ca_t \end{bmatrix} &= X_0 + \begin{bmatrix} 6.317 & 30.047^* \\ (12.895) & (10.906) \\ 7.391 & 16.526^* \\ (7.493) & (5.998) \\ 4.594^* & 2.103^* \\ (1.471) & (0.763) \\ -0.705^* & 0 \\ (0.135) & ( - ) \end{bmatrix} \begin{bmatrix} \tau_{Dt} \\ \tau_{Gt} \end{bmatrix} + C^*(L)e_t \\
 \\
 \text{US:} \quad \begin{bmatrix} y_t \\ C_t \\ Inv_t \\ Ca_t \end{bmatrix} &= X_0 + \begin{bmatrix} -4.032 & 43.006^* \\ (2.610) & (16.174) \\ 0.869 & 30.534^* \\ (0.871) & (11.484) \\ 7.059 & 4.309^* \\ (4.547) & (1.617) \\ -5.635^* & 0 \\ (2.713) & ( - ) \end{bmatrix} \begin{bmatrix} \tau_{Dt} \\ \tau_{Gt} \end{bmatrix} + C^*(L)e_t
 \end{aligned}
 \tag{ 19 }$$

The asymptotic standard errors in parenthesis (where \* indicates statistical significance at the 5% level) are obtained under the assumption of normality (see Theorem 2.3 on page 23-25 of Warne(1990)). The coefficients measure the long-run effects of the respective stochastic trends on the corresponding elements of the  $X_t$ -vector.

In what follows we discuss the empirical results seeking answers to the following specific questions asked in the introduction: i) what are the long run effects of each of these shocks (analyzed separately) on investment and the current account? ii) how does each of our variables of interest (investment and the current account) respond to one standard deviation innovations in the respective shocks? iii) what is the relative performance of the innovations in the respective shocks in explaining fluctuations in investment and the current account? and finally iv) how does our approach compare with that of G&R? The first question is attempted

using the results from the estimated common trends model. More specifically the answer to this question is based on the estimates of the  $A$  matrix as reported for both countries in (19) above. Forecast error variance decompositions, as reported in table 5, aid us in tackling the third question whereas the second question is addressed using estimated impulse response functions as reported in figures 2 through 5 for both countries. The conclusion has some stuff on the final question.

In conformity with the predictions of the theory the current account balance is significantly affected by permanent domestic(country-specific) shocks. Thus whereas a positive domestic (country-specific) shock significantly worsens the current account of the US by as much as 5.64 billion US dollars the corresponding figure in the case of Germany is 0.71 billion deutsche marks as shown by the estimates of the  $A$  matrix in (19) above. Permanent global shocks (by restriction derived from the theory and imposed as an identifying assumption) do not have any long run effects whatsoever on the current account balance. This result is depicted by the zero coefficient in the estimated  $A$  matrix for each country. These shocks have however very significant positive long run effects on investment in both countries. More specifically, whereas permanent global shocks significantly increase investment by as much as 4.31 billion US dollars in the US the corresponding effect of these shocks in Germany is an estimated increase of 2.10 billion deutsche marks in investment. These results are very consistent with our theoretical predictions as outlined in section 2. Permanent domestic shocks do have positive effects on investment in both countries.

Plots of the impulse response functions of each of the elements of  $\mathbf{X}_t = [y_t, c_t, inv_t, ca_t]'$  to a one standard deviation innovation in the permanent stochastic trends and their transitory counterparts are reported in figures 2, 3, 4 and 5 where the responses over 40 quarters are plotted with their 95% confidence bounds. As reported in figure 2 responses of these variables to the permanent domestic shocks are not significant at any of the horizons even though there is a clear indication of the direction of the effects especially as regards investment and the current account in both countries. Investment responds positively to these shocks whereas the response of the current account is negative over the entire horizon of the impulse response functions in both countries. Permanent innovations in the global trend however lead generally to significantly positive responses in output, consumption and investment in the two countries over the entire horizon of the estimated impulse response functions. The current account, by

restriction as imposed on the A matrix before estimation, does not respond to shocks to these permanent global shocks except during the first three quarters in Germany when the effect is positive but moves rapidly to zero. We turn now to discussing the effects of the transitory shocks as reported in figures 4 and 5. Except for their effects on investment during the first five quarters (following a one standard deviation shock) in the case of Germany transitory (domestic and global) shocks do not have any significant influences on the endogenous variables. Even in the case of Germany the effects of innovations in transitory domestic and global shocks on consumption and investment respectively are but *transitory* and move rapidly to null within five quarters. This indicates that the productivity shocks - domestic/country-specific and global - can be considered as following a random walk processes (ie. the absolute value of the autoregressive parameter in equation (2) on page 68 is not significantly different from one - i.e.  $|p| \approx 1$ )<sup>11</sup>. This result is confirmed by the forecast error variance decompositions (the results of which are reported in table 5 below) showing that all fluctuations in the respective variables are exclusively explained by permanent shocks.

Having discussed the dynamic responses of each variable with respect to innovations in the two stochastic trends we move on next to examine the relative significance of these innovations in explaining long run fluctuations in investments and the current account balance. To do this we present the results of our forecast error variance decompositions showing the importance of each shock in explaining the variability of the forecast error in respect of each variable in the long run<sup>12</sup>. Permanent shocks to domestic (country-specific) trends explain between 73% and 83% of total long run fluctuations in investment and are solely responsible (by restriction as required by the theory) for all variations in the current account.

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<sup>11</sup>. This result showed up in G&R as well. Using standard univariate unit root tests G&R could not reject this  $\rho = 1$  hypothesis for any of the countries considered. Hence all their subsequent empirical results assumed  $\rho = 1$  - that is to say the shocks are exclusively permanent.

<sup>12</sup>. Forecast error variance decompositions for different time horizons are not presented here since they indicate proportions of variations in the respective variables within the specified short run horizons (horizons on which our theoretical model gives very little information, if any) explained by innovations in the respective trends.

**Table 5: Long-Run Forecast Error Variance Decompositions**

	Domestic Shocks ( $\tau_D$ )		Global Shocks ( $\tau_G$ )	
	Germany	US	Germany	US
Output ( $y_t$ )	0.042 (0.162)	0.009 (0.113)	0.958* (0.162)	0.991* (0.133)
Consumption ( $c_t$ )	0.167 (0.279)	0.001 (0.035)	0.833* (0.279)	0.999* (0.282)
Investment ( $inv_t$ )	0.827* (0.129)	0.729* (0.282)	0.173 (0.129)	0.271 (0.282)
Current Acc. ( $ca_t$ )	1.000 (-)	1.000 (-)	0.000 (-)	0.000 (-)

*Notes:* The estimated standard errors are based on Theorem 2.4 of Warne(1990). Very insignificant or zero standard errors are indicated by (-) in the table. A gain, a \* indicates statistical significance at the 5% level.

Global shocks explain 17% (27%) of total long run variations in investment in Germany (the US) and domestic shocks are accountable for 83% (73%) of long run variations in the same variable in the respective countries. Further, variations in output and consumption are explained solely by global shocks - domestic shocks explain rather low and statistically insignificant proportions of variations in both variables. More specifically, given our identifying assumption (that global shocks have no long run effects on the current account) global shocks tend to explain the bulk of fluctuations in income and consumption whereas domestic shocks explain between 70% and 80% of fluctuations in investment. What is the likely explanation for this? It may be due to the fact that the adjustment costs associated with investment have their origins in domestic structural/institutional arrangements that are more responsive to domestic shocks than they are to global shocks.

### 3.5 Summary and Conclusion

Using the common trends approach this paper identifies and analyses the relative effects of idiosyncratic and global stochastic shocks that influence investment and the current account. To be able to identify these stochastic shocks we use the intertemporal approach to the current account in the tradition of G&R, Obstfeld(1986) and Razin(1993). The intertemporal approach acts as the background theory for explicit specification and analyses of the two shocks - domestic (country-specific) and global - and provides the long run restriction(s) that is(are) used not only in identifying crucial matrices during the estimation of the common trends

model but also in explaining the results of our impulse response experiments and variance decompositions.

Our empirical estimates yield the following stylized facts. Generally the long run effects of domestic (country-specific) productivity shocks on the current account are significantly negative as the theoretical model predicts. The estimates of the A matrix of our common trends model however reveal that investment responds positively to both permanent innovations; responding more to domestic (country-specific) shocks than to global shocks. Nevertheless the apparent puzzle that the current account responds, in absolute terms, by much less than investment to domestic (country-specific) productivity shocks - as pointed out by G&R - seems to persist even with the approach adopted here. In our view the solution to this apparent puzzle requires a research strategy that explicitly models, estimates the parameters of the cost-of-adjustment investment technology and examines relative responses of investment and the current account to the two productivity shocks for given parameter values of the cost of adjustment of investment. Despite this the common trends technique used in this paper adequately describes fluctuations in the data-set and yields results that are consistent not only with the intertemporal model of the current account as adopted here but also with most aspects of earlier empirical work on the subject. Hence it is not unreasonable to conclude that the common trends approach is a potential alternative to estimations based on calculated Solow residuals and gives the researcher more information on the dynamics of the effect of innovations in productivity shocks - a dynamics so crucial in empirical investigations on capital mobility and the related issues of current account fluctuations.



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## APPENDIX: Cointegration, Error Correction And Common Trends

This appendix reviews the common trends statistical model used to characterize the nature of the shocks and the responses of investment and the current account balance to these. Consider any  $n \times 1$  dimensional macroeconomic time series  $X_t$  ( $= [y_p, c_p, inv_p, ca_t]'$ ) characterised by the cointegrated Wold vector moving average representation

$$(I - L) X_t = C(L)[\delta + \varepsilon_t] \quad (A.1)$$

where  $C(L) = I_n + C_1L + C_2L^2 + \dots$  (an invertible lag polynomial),  $\varepsilon_t$  is white-noise with a zero mean vector (i.e.  $E[\varepsilon_t] = \mathbf{0}$ ) and a positive definite covariance matrix,  $\Xi = E[\varepsilon_t \varepsilon_t']$ , and  $L$  is the lag operator such  $L^i \chi_t = \chi_{t-i}$ . Under fairly general conditions it is possible (using matrix algebra) to find another polynomial  $C^*(L)$  such that

$$C(L) = C(1) + (1 - L)C^*(L), \quad \text{where} \quad C^*(L) = \sum_{j=i+1}^{\infty} C_j \quad (A.2)$$

We do know from the Granger Representation Theorem (GRT) that:

i)  $C(1)$  is of rank  $n - r$  where  $n$  is the number of variables in  $X_t$  and  $r$  is the number of cointegrating vectors.

ii) there exists an ARMA representation

$$\Pi(L)X_t = d(L)\varepsilon_t \quad (A.3)$$

where  $\Pi(0) = I_n$ , rank of  $\Pi(1)$  is  $r$ , and  $d(L)$  is a scalar lag polynomial.

iii) there exists  $n \times r$  matrices  $\beta$  and  $\alpha$ , of rank  $r$  such that  $\beta'C(1) = \mathbf{0}$ ,  $C(1)\alpha = \mathbf{0}$ , and  $\Pi(1) = \alpha\beta'$

iv) there exists an Error Correction Representation (EC) with  $Z_t = \beta'X_t$ , an  $r \times 1$  vector of stationary variables:

$$D(L)(I - L)X_t = \delta + \alpha Z_{t-1} + \varepsilon_t \quad (A.4)$$

with  $D(0) = I_n$ , an  $n \times n$  identity matrix.

Utilizing (A.2) and the usual convention of letting  $\varepsilon_s$  be zero (for  $s \leq 0$ ) and  $X_0$  representing

the non-random initial value of  $X_t$ , then by recursive substitution<sup>13</sup> of (A.1) we obtain,

$$X_t = X_0 + \delta t + C(I)(I + L + L^2 + \dots + L^{t-1})\epsilon_t + C^*(L)\epsilon_t \quad (\text{A.5})$$

Since  $\delta$  is one of the vectors in the nullspace of  $C(I)$ , it can be expressed as a linear combination of the columns of  $C(I)$ , say  $\delta = C(I)\mathbf{v}$  where  $\mathbf{v}$  is an  $n \times 1$  vector. Substituting this expression into (A.5) above we obtain

$$X_t = X_0 + C(I)[t\mathbf{v} + \sum_{i=0}^{t-1} \epsilon_{t-i}] + C^*(L)\epsilon_t$$

Denoting the expression in the square brackets (the random-walk component) by  $\tau_t = \mu + \tau_{t-1} + \varphi_t$ , we obtain, after some manipulations, the expression

$$\begin{aligned} X_t &= X_0 + A\tau_t + C^*(L)\epsilon_t \\ &= X_0 + A[\mu + \tau_{t-1} + \varphi_t] + C^*(L)\epsilon_t \end{aligned} \quad (\text{A.6})$$

where  $C(I)\epsilon_t = A\varphi_t$ ,  $C(I)\delta = A\mu$ , and hence (for  $E[\varphi_t\varphi_t'] = \Phi$ )  $C(I)\Xi C(I)' = A\Phi A'$ . Assuming, for ease of interpretation, that  $E[\varphi_t] = \mathbf{0}$  and  $E[\varphi_t\varphi_t'] = \Phi = I_{n-r}$ ,  $C(I)\Xi C(I)' = A\Phi A' = AA'$ . Further, pre-multiplying (A.6) by  $\beta'$  reveals that for the structure of the  $A$  matrix to be consistent with the cointegrating vectors we require that  $\beta'A = \mathbf{0}$ . Equation (A.6) is the multivariate version of the Beveridge-Nelson decomposition<sup>14</sup> of the cointegrated vector moving average representation, (A.1), expressing  $X_t$  as a linear combination of  $n-r$  linearly independent stochastic trends (the common trends that have permanent effects on  $X_t$ ) and transitory components,  $C^*(L)\epsilon_t$ , which are stationary.  $X_0$  contains the initial values of  $X_t$ .

$$\tau_t = \mu + \tau_{t-1} + \varphi_t \quad (\text{A.7})$$

is a  $k \times 1$  vector of non-stationary variables (with structural white-noise shock vector,  $\varphi_t$ , with the variance-covariance matrix,  $\Phi$ ) that drive the system,  $A$  is an  $n \times k$  matrix of coefficients

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<sup>13</sup>. See Stock and Watson(1988) for the procedure applied here.

<sup>14</sup>. See Beveridge, Stephen & Charles R. Nelson (1981) and/or Stock and Watson (1988) for the details of this decomposition.

to be characterized (by the aid of the underlying theory) and estimated. It measures the long-run impact from the  $k$  common stochastic trends in  $\tau_t$ . The transitory part of the model is described by the  $C^*(L)$  polynomial. The shocks to the trends,  $\phi_t$ , can also influence the short-run behaviour of the variables of interest.

### *Impulse Responses and Variance Decompositions*

On the basis of the analysis of Campbell and Shiller(1988), Warne(1990) shows that in the case of cointegration the impulse response functions and variance decompositions can be obtained by inverting a particular restricted VAR model of the form

$$H(L)y_t = \delta^* + \epsilon_t^* \quad (A.8)$$

To show the relationship between the variables and the parameters of the EC model and this RVAR define the following matrices:

$$M^* = \begin{bmatrix} S_k \\ \beta' \end{bmatrix}, \quad \alpha^* = [ \mathbf{0} \quad \alpha ], \quad D^*(L) = \begin{bmatrix} I_k & \mathbf{0} \\ \mathbf{0} & (1-L)I_r \end{bmatrix}, \quad \text{and} \quad D_{\perp}^*(L) = \begin{bmatrix} (1-L)I_k & \mathbf{0} \\ \mathbf{0} & I_r \end{bmatrix}$$

where  $S_k$  is a  $k \times n$  matrix chosen such that its rows are linearly independent of those of  $\beta'$ , the rank of  $M^*$  is  $n$  and  $\alpha^*$  is an  $n \times n$  matrix. Given these definitions we can further derive the following relationships

$$\begin{aligned} \delta + \alpha \beta' X_{t-1} + \epsilon_t &= D(L)D^*(L)D_{\perp}^*(L)X_t \\ &= D(L)M^{*-1}D^*(L)D_{\perp}^*(L)M^*X_t \end{aligned} \quad (A.9)$$

Further, define an  $n \times 1$  dimensional  $I(0)$  matrix of time series vectors denoted  $y_t$  such that  $y_t = D_{\perp}^*(L)M^*X_t$  so that we can re-write (A.9) as  $\delta + \alpha \beta' X_{t-1} + \epsilon_t = D(L)M^{*-1}D^*(L)y_t$ . Thus, for  $H(L) = M^*[D(L)M^{*-1}D^*(L) + \alpha^*L]$  where  $H(0) = M^{*-1}$  (and hence  $M^*H(0) = I_n$ ) we obtain the RVAR model given that  $\alpha \beta' X_{t-1} = [0 \quad \alpha][\Delta S_k X_{t-1} \quad \beta' X_{t-1}]'$ ,  $\delta^* = M^*\delta$  and  $\epsilon_t^* = M^*\epsilon_t$ . Once we estimate the above equation - (A.8) - we can also easily estimate the Wold Moving

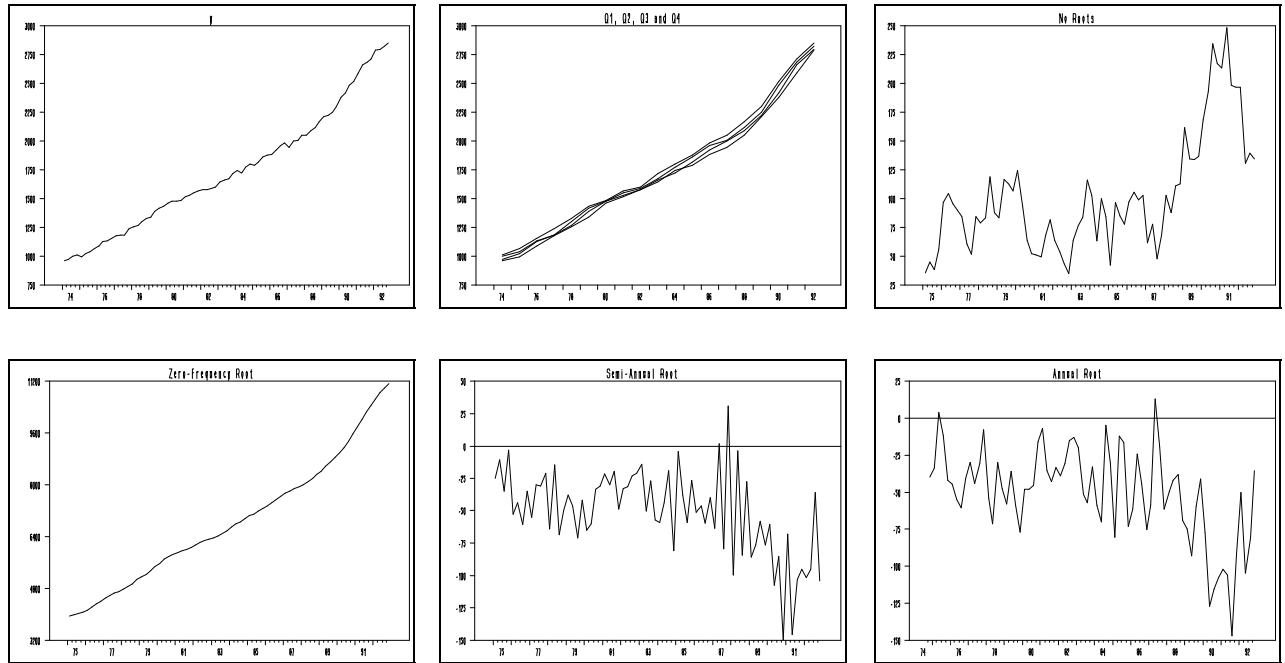
Average Representation (A.1) and hence the impulse response functions as well as the variance decompositions. However to calculate the impulse responses and the variance decompositions associated with the shocks to the common trends we need some additional restrictions on the model so far specified.

Let  $\eta_t = [\phi_t', \psi_t']' = F\varepsilon_t$  be a matrix of shocks where  $F = [F_k', F_r']'$ ,  $\phi_t$  may be considered as an  $n$ -dimensional vector of  $k$  permanent shocks and  $\psi_t$  as an  $r \times 1$  vector of transitory shocks. Further, define  $R(\lambda) = C(\lambda)F'$ . Then given the specification/definitions under (A.6) above we observe that  $\eta_t = (A'A)^{-1}A'C(I)\varepsilon_t = F\varepsilon_t$  and hence the Wold vector moving average representation can be re-written as

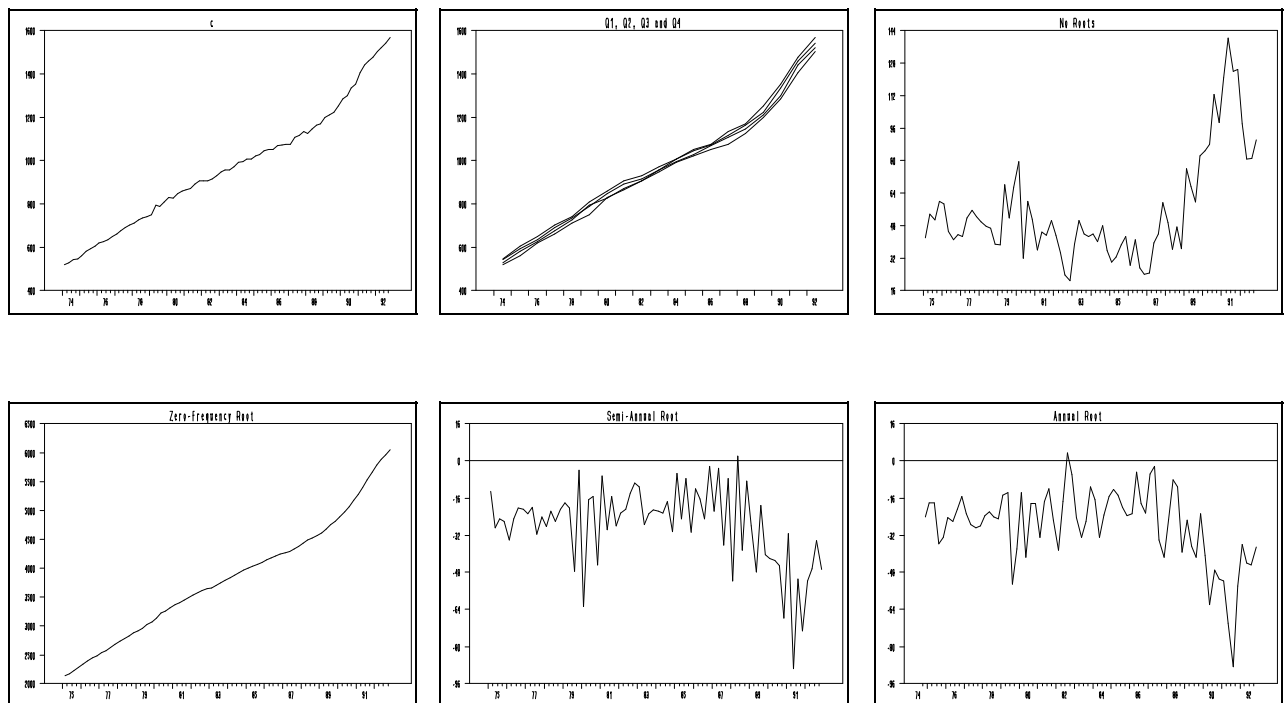
$$(1 - L)X_t = C(1)\delta + R(L)\eta_t \quad (\text{A.11})$$

Given the  $F_k$  matrix as defined above, Warne(1990) has shown that  $F_r = Q_r^{-1}\alpha\Xi^{-1}$  where  $Q_r$  is an  $r \times r$  invertible matrix satisfying the assumption of the independence of permanent and transitory shocks (ie.  $E[\phi_t\psi_t'] = (A'A)^{-1}A'C(I)\Xi F_r' = 0$  since, as shown by Engle and Granger(1987),  $C(I)\alpha = 0$ ) and is chosen such that the covariance matrix of the transitory shocks is diagonal. In practice the identification of the transitory shocks ( $\psi_t$ ) requires  $r(r - 1)/2$  zero-restrictions on their contemporaneous effects on the endogenous variables,  $X_t$ .

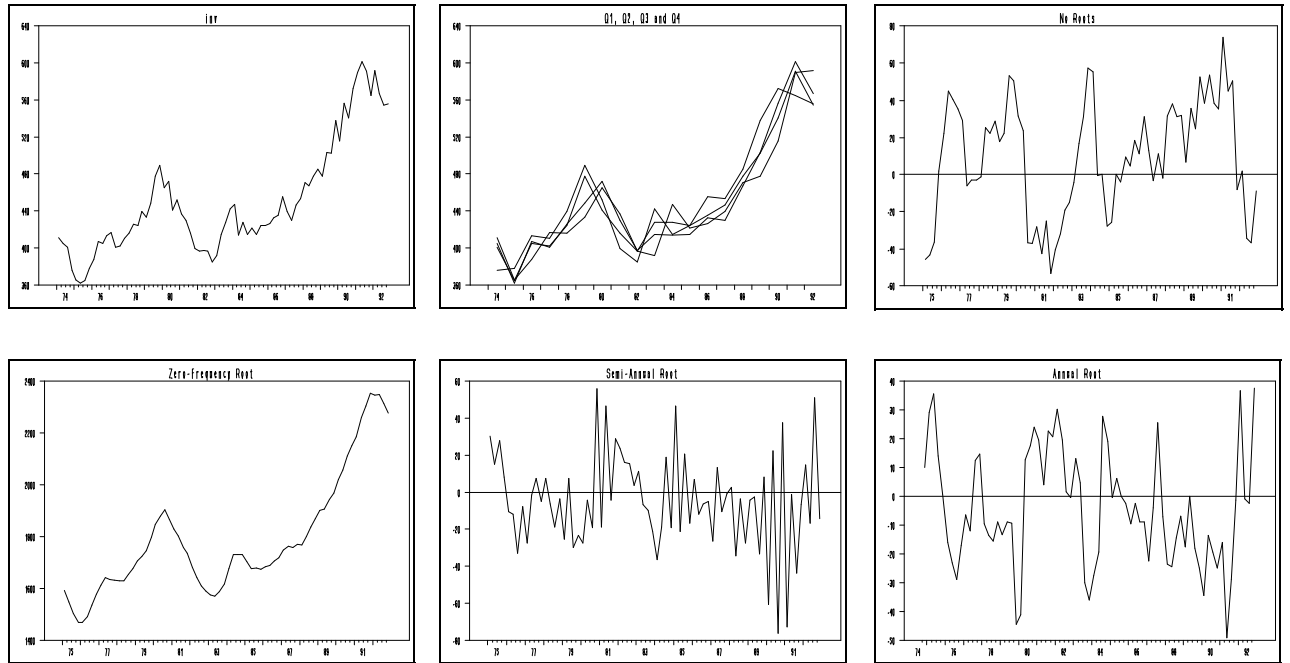
**Figure 1a: Seasonal Analyses of German GDP,  $y_t$**



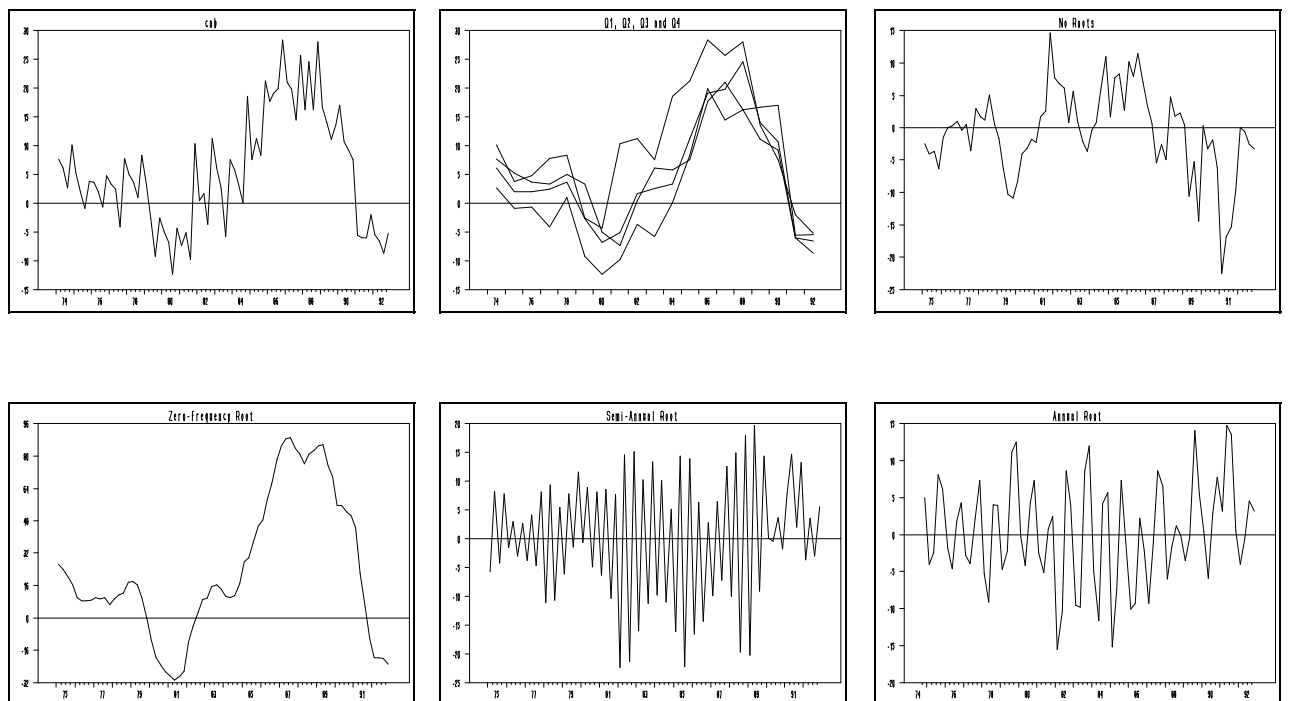
**Figure 1a (cont.): Seasonal Analyses of German Private Consumption,  $c_t$**



**Figure 1a (cont): Seasonal Analyses of German Gross Investment,  $inv_t$ .**

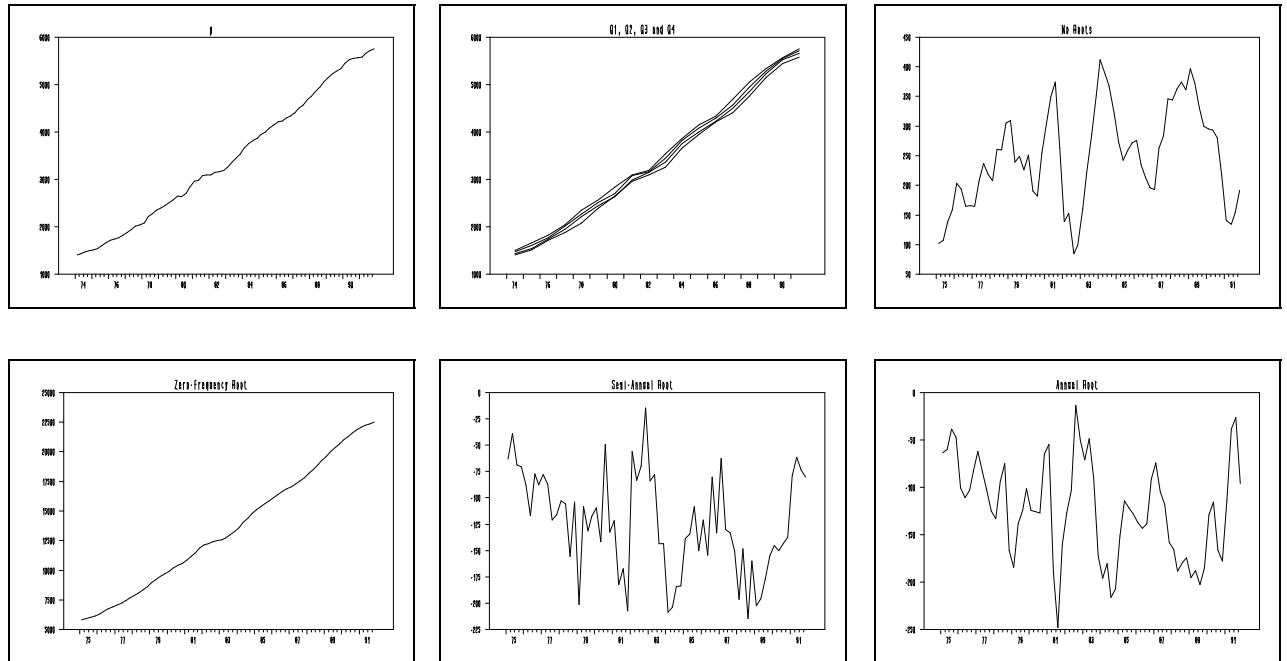


**Figure 1a (cont): Seasonal Analyses of German Current Account Balance,  $ca_t$ .**

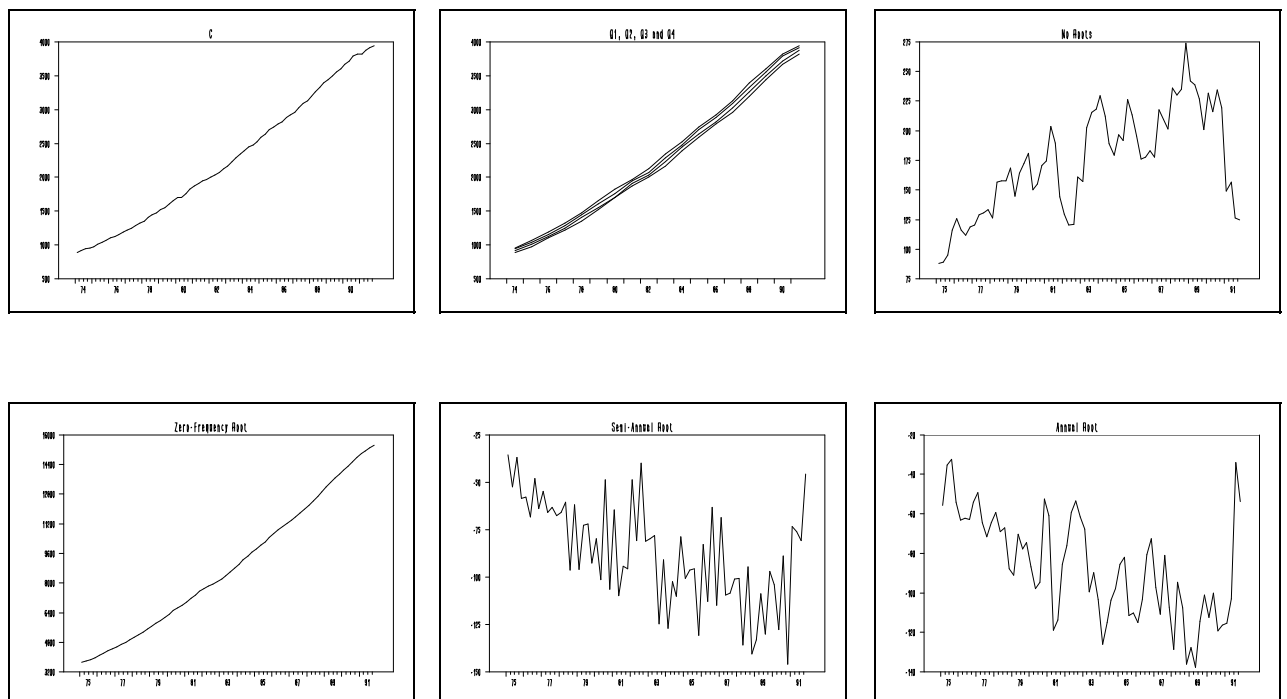




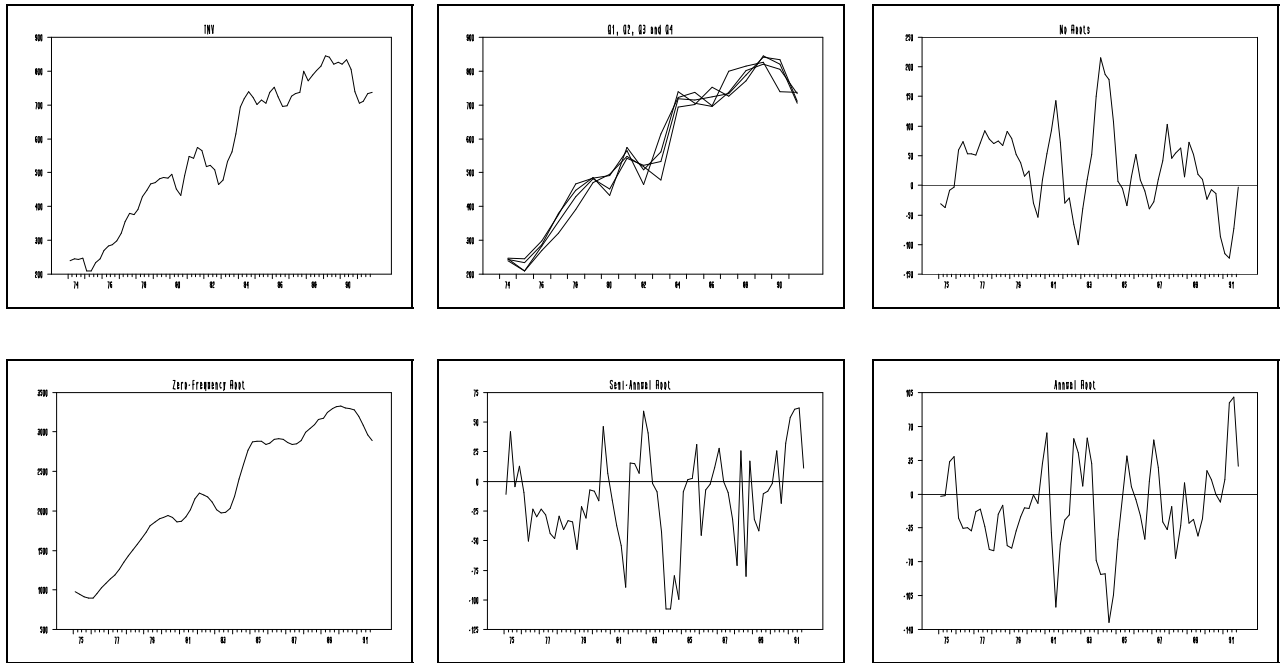
**Figure 1b: Seasonal Analyses of US GDP,  $y_t$ .**



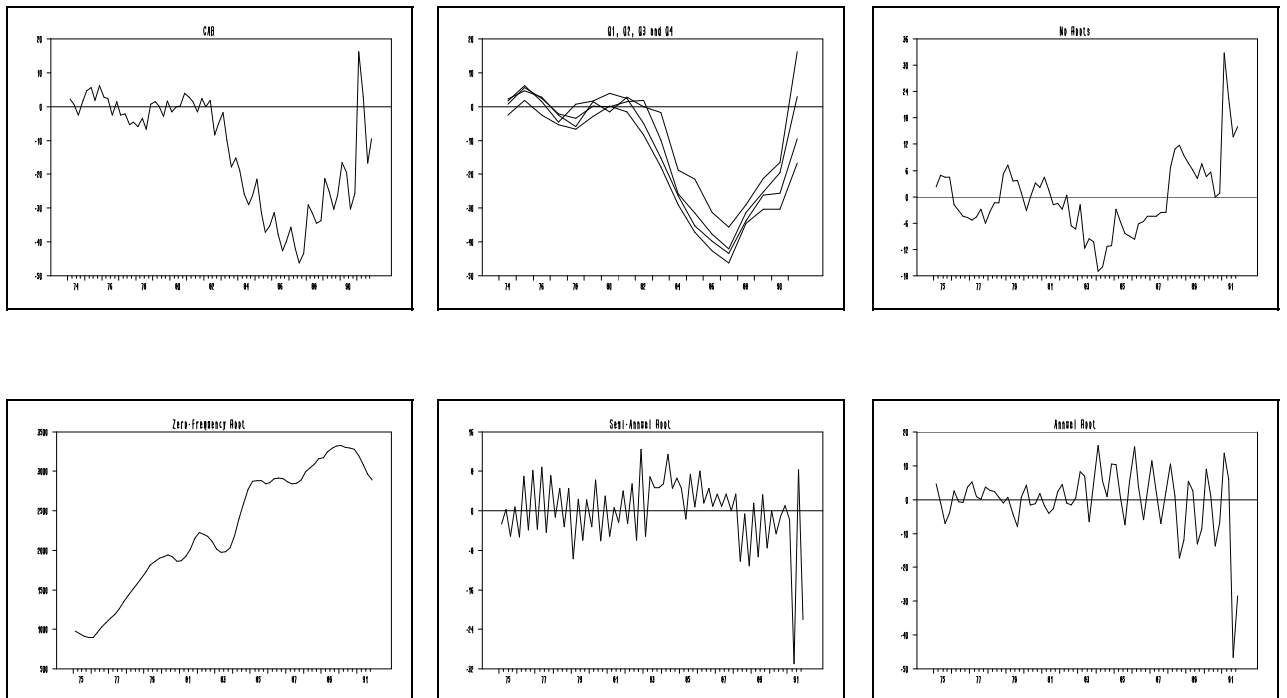
**Figure 1b (Cont.): Seasonal Analyses of US Private Consumption,  $c_t$ .**



**Figure 1b (Cont.): Seasonal Analyses of US Gross Investment,  $inv_t$ .**



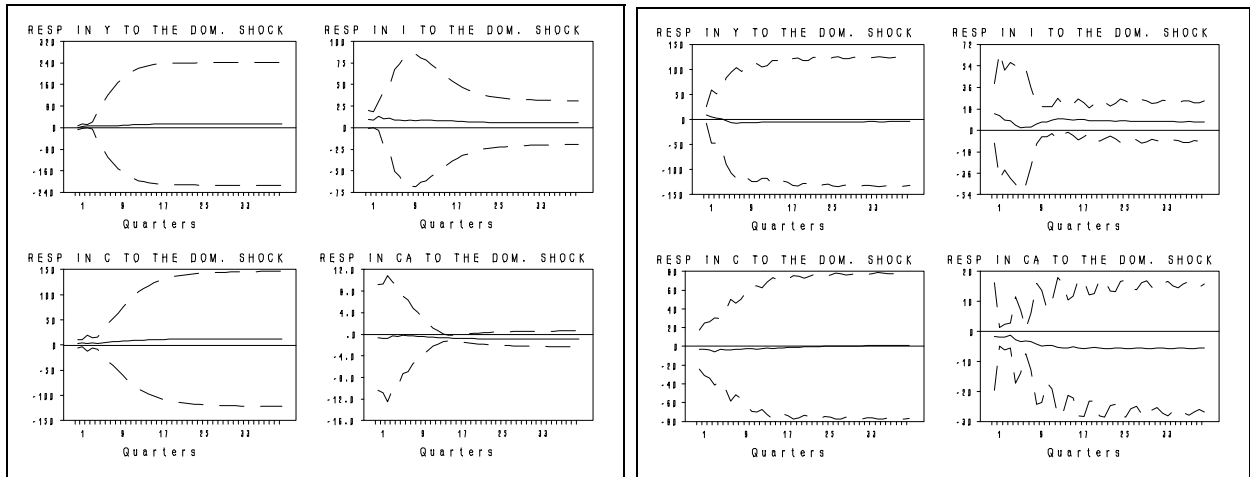
**Figure 1b (Cont.): Seasonal Analyses of US Current Account Balance,  $ca_t$ .**



**Figure 2: Impulse response functions (with 95% Confidence Intervals) from a one Standard deviation shock to the permanent domestic (country-specific) productivity trend ( $\tau_{D,t}$ ).**

**A. Germany**

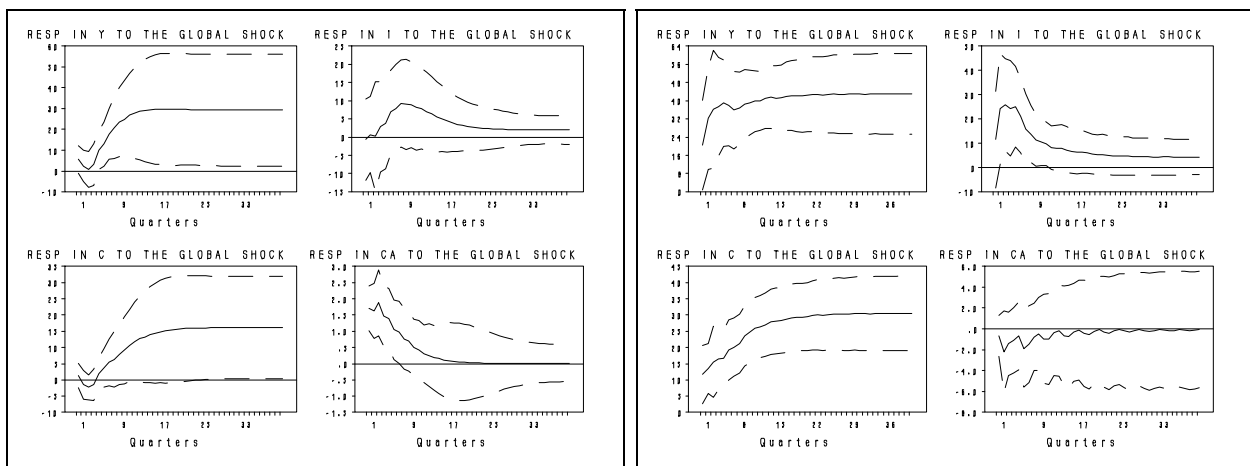
**B. US.**



**Figure 3: Impulse response functions (with 95% Confidence Intervals) from a one Standard deviation shock to the permanent global trend ( $\tau_{G,t}$ ).**

**A. Germany**

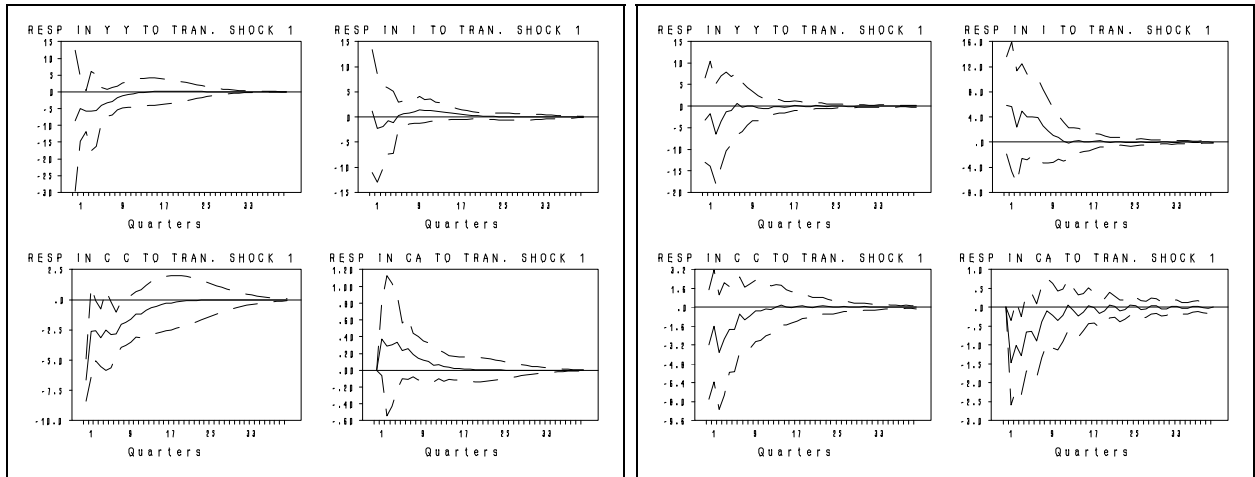
**B. US**



**Figure 4: Impulse response functions (with 95% Confidence Intervals) from a one Standard deviation shock to the transitory domestic (country-specific) productivity trend.**

**A. Germany**

**B. US**



**Figure 5: Impulse response functions (with 95% Confidence Intervals) from a one Standard deviation shock to the transitory global productivity trend.**

**A. Germany**

**B. US**

