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Strategic technology investment under uncertainty*

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Abstract. In this paper the technology investment decision of a firm is analyzed, while competition on the output market is explicitly taken into account. Technology choice is irreversible and the firms face a stochastic innovation process with uncertainty about the speed of arrival of new technologies. The innovation process is exogenous to the firms. For reasons of market saturation and the fact that more modern technologies are invented as time passes, the demand for a given technology decreases over time. This implies that also the sunk cost investment of each technology decreases over time.

The investment decision problem is transformed into a timing game, in which the waiting curve is introduced as a new concept. An algorithm is designed for solving this (more) general timing game. The algorithm is applied to an information technology investment problem. The most likely outcome exhibits diffusion with equal payoffs for the firms.

Key words Investment – Duopoly – Real options – Preemption – War of attrition – Information technology

1 Introduction

In the last two decades information technology has become a very important determinant for economic growth[1]. For the individual firm the technology investment

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[1] Kriebel (1989) notes that roughly 50 procent of new corporate capital expenditures by major U.S. companies is in information technology.

decision is a very complex matter due to the incredible rapid progress of information technology[2]. In former years technology investment decisions were mostly timing problems, in which the optimal time to replace the current technology had to be determined. As a result of the slow technological progress, only one new technology had to be taken into account. For example, at the beginning of this century the technology investment decision of a railway company dealt with the decision when to replace its steam shunters with diesel shunters. Up to the present day most railway companies still work with diesel shunters.

Nowadays, a firm should take into account that the current state of the art in information technology will be old fashioned in a few years. Thus the investment decision problem is no longer only a question of when to adopt a new technology but also a question of which technology should be adopted. Therefore it is important to design a theoretical framework in which several new technologies are considered. This paper wants to contribute to this aim.

Another feature of the last decade is that firms more and more face competition on their output markets. One reason is the abolition of monopolistic markets created by government. In the Netherlands examples are the opening of the markets for telecommunication, railway and power supply. Another reason is the, still ongoing, process of mergers, which due to legislation will not end with a market with only one supplier. The result is that markets with only one supplier and markets with many suppliers seem to disappear. Thus, in its own investment decision, a firm should take into account the investment behavior by its competitors, which is dealt with in this paper.

The existing literature on technology adoption models can be divided into two categories. The models in the first category are decision theoretic models that analyze the technology investment decision of a single firm. In the most advanced models there are multiple new technologies that arrive over time according to a stochastic process. Examples are Balcer and Lippman (1984), Nair (1995), Rajagopalan et al. (1998) and Farzin et al. (1998). The drawback of these models is that they analyze the investment decision of only one firm in isolation, so that the effects of competition are not incorporated.

The second category models are game theoretic models. Two (or more) firms compete on an output market and produce goods using a particular technology. Then, a new and more efficient technology is invented, and the question is at what time the firms should adopt it.

Reinganum (1981) was the first to analyze this kind of model. She considered a duopoly with identical firms, in which there is no uncertainty in the innovation process, and one new technology is considered. The investment expenditure required to adopt the new technology decreases over time and the efficiency improvement

[2] Yorukoglu (1998, p. 552): "Information technology capital has a very high pace of technological improvement. Compared with more traditional types of capital, the efficiency of information technology capital has increased much faster over the last few decades. As an example, consider the market for personal computers. IBM introduced its Pentium PCs in the early 1990s at the same price at which it introduced its 286 PCs in the 1980s. Therefore it took less than a decade for the computing technology to improve on the order of 20 times in terms of both speed and memory capacities, without increasing the cost."

is known. If a firm adopts the new technology before the other one does, it makes substantial profits at the expense of the other firm. On the other hand the investment cost being decreasing over time provides an incentive to wait with investing. Reinganum assumes that the firms precommit themselves to adoption times, so she automatically obtains open-loop equilibria.

Fudenberg and Tirole (1985) proved that in these equilibria the leader (the firm that invests first) earns more than the follower (the firm that invests second). Since precommitment seems not to be very realistic in the strategic setting of a duopoly, Fudenberg and Tirole extend Reinganum's model by relaxing this assumption and by not determining beforehand which firm is the leader. They therefore allow preemption and obtain closed-loop equilibria with rent equalization.

The drawback of the Reinganum-Fudenberg-Tirole model is the assumption that there is only one new technology available for the firms, and, as stated in the beginning of this introduction, present technology investment decisions have to deal with a sequence of new technologies. In this paper we extend the Reinganum-Fudenberg-Tirole model by adding uncertainty to the innovation process and by considering several new technologies. The new technologies are invented at previously unknown points of time[3]. The investment decision problem is solved by introducing the waiting curve as a new concept in timing games.

The remainder of this paper is organized as follows. In Section 2 the investment decision problem of a firm is described. We reformulate the investment decision problem as a timing game, and design an algorithm to solve it in Section 3. In Section 4 we apply the algorithm to an information technology investment problem. Concluding remarks are given in Section 5.

2 Investment decision problem

In this section we describe the model of this paper. A duopoly is considered where both firms maximize their value over an infinite planning horizon. We define T to be the time elapsed since the start of the game. The first assumption is that firms are identical. Each firm has a profit function $\pi(\theta_x, \theta_y)$, where θ_x equals the technology-efficiency parameter of the technology that the firm uses itself and θ_y that of its opponent. The profit function of each firm is non-negative, increasing and concave in its own technology-efficiency parameter and decreasing in its rival technology-efficiency parameter. This for the reason that a firm can make more profits when it produces with a more efficient technology, the fact that the growth of the profits will be limited (due to output market saturation and the fact that production costs are always positive) and a firm will make less profits when its rival uses a more efficient technology. Furthermore it holds that the impact of the technological improvement of the home firm on its profits is decreasing in the technology efficiency parameter

[3] A different framework is considered in the duopoly model by Gaimon (1989). In that paper a continuous stream of new technologies arrives over time, which is known beforehand by the firms.

of the other firm. Therefore, the profit function satisfies the following inequalities:

$$\begin{aligned}
&\text{(i)} \quad \pi(\theta_x, \theta_y) \geq 0, \\
&\text{(ii)} \quad \tfrac{\partial \pi(\theta_x, \theta_y)}{\partial \theta_x} > 0, \\
&\text{(iii)} \quad \tfrac{\partial \pi(\theta_x, \theta_y)}{\partial \theta_y} < 0, \\
&\text{(iv)} \quad \tfrac{\partial^2 \pi(\theta_x, \theta_y)}{\partial \theta_x^2} < 0, \\
&\text{(v)} \quad \tfrac{\partial^2 \pi(\theta_x, \theta_y)}{\partial \theta_x \partial \theta_y} < 0.
\end{aligned} \tag{1}$$

We analyze a dynamic model with an infinite planning horizon. Risk-neutral firms are considered, which discount the stream of future profits at a constant rate r. Initially each firm produces with a technology designated by $\theta = \theta_0$. As time passes new technologies become available (the i-th technology to arrive with efficiency θ_i). We define T_i to be equal to the point in time at which technology i becomes available. Each firm has the opportunity to adopt one of the new technologies by investing $I(t_i)$, where t_i is the length of the time period passed since the introduction of technology i: $t_i = T - T_i$. We assume the second hand market for these capital goods to be negligible (e.g. information technology products) so that this investment is irreversible (this assumption is extensively motivated in Dixit and Pindyck (1996)). The differences between the technologies are all captured in the different values for θ, so that, without losing anything, investment expenditures $(= I(t_i))$ can be set equal for all technologies. The investment cost $I(\cdot)$ is non-negative, decreasing and convex in time:

$$\begin{aligned}
&\text{(i)} \quad I(t_i) \geq 0, \\
&\text{(ii)} \quad \tfrac{\partial I(t_i)}{\partial t_i} \leq 0, \\
&\text{(iii)} \quad \tfrac{\partial^2 I(t_i)}{\partial t_i^2} \geq 0.
\end{aligned} \tag{2}$$

Such a decrease can be motivated by the fact that better technologies become available as time passes so that the demand for the current technology decreases over time. Another factor can be learning by doing in the production process of the technology supplier.

Furthermore, we assume that the process of technological evolution (innovation supply) is exogenous to the firms. Technologies become more and more efficient over time, and the more efficient a technology the larger the associated parameter θ. However, the arrival process of the new technologies is a stochastic process. We assume that the associated increases in θ are known beforehand, which is imposed to keep the model tractable. But, on the other hand practical examples can be identified where this actually holds. Consider, for instance, the case of micro-chips where the technical parameters and specifications of future designs are known beforehand, but the arrival date is uncertain since the appearance of technology depends on research and development and market factors affecting the introduction of the product (Nair, 1995).

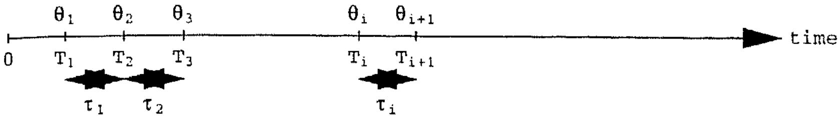

Fig. 1. Arrival times T_i of technologies i with efficiency levels θ_i, where $\tau_i = T_{i+1} - T_i$ is the length of the time interval between the arrival times of technologies i and $i+1$, respectively ($i \in \{1, 2, \dots\}$). T_i is the ex-ante unknown. Further, it holds that $\theta_i > \theta_j$ for $i > j$ and $i, j \in \{1, 2, \dots\}$

For simplicity reasons we restrict ourselves to the case where firms can only make one technology switch. This more or less holds for firms whose financial means are limited.

We denote the newest or best technology available at time T by $B(T)$. To incorporate the uncertainty in the innovation process we assume that $B(T)$ is a Poisson process with rate λ. The new technology i has efficiency level equal to $\theta_i > \theta_{i-1}$, $i = 1, 2, \dots$. The interarrival time τ_i is the time between the invention times of the $(i+1)$-th and i-th technology: $\tau_i = T_{i+1} - T_i$. As a result of the Poisson arrival process the τ_i's are independently and identically distributed according to an exponential distribution with parameter λ. In Figure 1 we illustrate the arrival process of the technologies by presenting a time line.

To summarize the problem we finish this section by describing the strategy space and the relevant payoffs of the two firms. Define $\Theta = \{\theta_0, \theta_1, \dots\}$. Let

$$g : [0, \infty) \to \Theta,$$

be a function that assigns a technology, designated by its efficiency level θ_i ($i \in \{0, 1, 2, \dots\}$) to each point in time u ($u \in [0, \infty)$). Consider the following conditions:

(i) $\forall_{\theta_i \in \Theta} : g(u) = \theta_i \Rightarrow u \geq T_i$,
(ii) $\exists!_{(\theta, t_\theta) \in \Theta \times [0, \infty)} : \forall_{u < t_\theta}\ g(u) = \theta_0$ and $\forall_{u \geq t_\theta}\ g(u) = \theta$.

Now the strategy space for player i ($i \in \{1, 2\}$) is given by the set

$$S_i = \{g : [0, \infty) \to \Theta \,|\, g \text{ satisfies (i) and (ii)}\}.$$

The strategy space of the game is then given by

$$S = S_1 \times S_2.$$

To specify the relevant payoffs, assume that the home firm invests at time A in technology i, while its opponent invests at time B in technology j. If the home firm is the first investor, implying that $A < B$, its payoff is given by

$$\int_{u=0}^{A} \pi(\theta_0, \theta_0) e^{-ru} du + \int_{u=A}^{B} \pi(\theta_i, \theta_0) e^{-ru} du + \int_{u=B}^{\infty} \pi(\theta_i, \theta_j) e^{-ru} du$$
$$-I(A - T_i) e^{-rA},$$

while for $A > B$ the payoff is

$$\int_{u=0}^{B} \pi(\theta_0, \theta_0) e^{-ru} du + \int_{u=B}^{A} \pi(\theta_0, \theta_j) e^{-ru} du + \int_{u=A}^{\infty} \pi(\theta_i, \theta_j) e^{-ru} du$$
$$-I(A - T_i) e^{-rA}.$$

Finally, if both firms invest at the same time, thus $A = B$, the relevant payoff equals

$$\int_{u=0}^{A} \pi(\theta_0, \theta_0) e^{-ru} du + \int_{u=A}^{\infty} \pi(\theta_i, \theta_j) e^{-ru} du - I(A - T_i) e^{-rA}.$$

3 Timing game

We transform the investment decision problem into a two player timing game. In a timing game each player has to decide when to make a single move. The player that moves first is called the leader and the other is the follower. Since firms are identical there seems to be no reason why one of these firms should be given the leader role beforehand. Therefore, we strive at obtaining equilibria where it is not known beforehand which firm will invest first. In the general setting of a timing game the payoff of a player depends on its own date of moving and the other player's date of moving. In case one player has already moved, the problem for the other player is a one person decision problem. A player can react instantaneously to its opponent's action.

Four payoff curves are important in our timing game. Each payoff curve is a function of time t, which is the time passed since the last technology has become available for the firms: $t = T - T_{B(T)}$. Let the leader move at time t. Then the value of the follower, which is the outcome of the one person's decision problem, is denoted by $F(t)$. The value of the leader is given by $L(t)$, in which the optimal action of the follower is included. In case of a simultaneous move at time t the value of a player is denoted by $M(t)$. Since simultaneous moving is always possible for the follower, it holds that

$$\forall_{t \geq 0} : F(t) \geq M(t). \tag{3}$$

The fourth curve is called the waiting curve, which is a new concept within the area of timing games. Here, the waiting curve is used to transform the investment decision problem under consideration into a timing game. The waiting curve represents the expected payoff of a firm if both firms do not move (at least) until the next arrival of a new technology and act optimally afterwards. This implies that we need to know the equilibrium outcome of the game that starts after the arrival of the next technology. As a result we have to consider a finite number of new technologies. This assumption is not too strict due to discounting. In order to find the right number of new technologies to take into account in the model, the following algorithm can be used:

Step 0 Solve the model with one technology.
Step 1 Add one extra technology to the model and solve the model.
Step 2 If the results of the last two models are very different go to step 1, otherwise the right number of technologies has been found.

A model with n new technologies is solved as follows. Start with solving the timing game that starts after the arrival of the n-th technology. This game is a classical timing game, since it contains no waiting curve. The equilibrium outcomes of this game are used to construct the waiting curve for the game that starts at some time during the interval $[T_{n-1}, T_n)$. Solve this game and use the equilibrium outcomes to construct the waiting curve for the game that starts somewhere at the time interval $[T_{n-2}, T_{n-1})$. This procedure goes on until the game that starts at time $T_1 = 0$ is solved.

This section describes the construction of the four payoff curves. In Subsection 3.1 we derive the value of a firm given each firm's strategy. After that we determine the leader, follower and joint-moving curves using this value function in Subsection 3.2. In Subsection 3.3 possible equilibria of timing games without waiting curve are explained. The waiting curve is constructed in Subsection 3.4. In Subsection 3.5 we explain the implication of adding the waiting curve for the possible equilibria of timing games. Finally, in Subsection 3.6 the algorithm for solving the investment decision problem with a finite number of new technologies is summarized.

3.1 Value function

In this investment decision problem firms not only have to decide when to adopt a technology, but also which technology to adopt. Define $V(S, i, T, j)$ as the expected value (at time T) of a firm that itself adopts technology i at time $S \geq T$, while its rival adopts technology j at time T. Of course, it must be true that $T \geq T_j$ and $S \geq T_i$. The expected value of the firm equals

$$V(S, i, T, j) = E\left[\int_{u=0}^{S-T} \pi(\theta_0, \theta_j) e^{-ru} du + \int_{u=S-T}^{\infty} \pi(\theta_i, \theta_j) e^{-ru} du - I(S - T_i) e^{-r(S-T)}\right]. \tag{4}$$

The expected value of the firm's opponent is equal to

$$V(T, j, S, i) = E\left[\int_{u=0}^{S-T} \pi(\theta_i, \theta_0) e^{-ru} du + \int_{u=S-T}^{\infty} \pi(\theta_i, \theta_j) e^{-ru} du - I(T - T_j)\right]. \tag{5}$$

Rewriting (4) gives

$$V(S,i,T,j) = \frac{\pi(\theta_0,\theta_j)}{r}\left(1 - E\left[e^{-r(S-T)}\right]\right) \\ +\frac{\pi(\theta_i,\theta_j)}{r}E\left[e^{-r(S-T)}\right] - I(S-T_i)\,E\left[e^{-r(S-T)}\right]. \tag{6}$$

Equation (5) can be written as follows

$$V(T,j,S,i) = \frac{\pi(\theta_i,\theta_0)}{r}\left(1 - E\left[e^{-r(S-T)}\right]\right) \\ +\frac{\pi(\theta_i,\theta_j)}{r}E\left[e^{-r(S-T)}\right] - I(T-T_j). \tag{7}$$

For determining the value functions (6) and (7) there is one thing left to derive: an expression for $E\left[e^{-r(S-T)}\right]$. Please remember that $B(T)$ is the number of the most efficient technology that is available at time T. We distinguish two cases: in the first case the second investor wants to invest in an already existing technology, while in the second case this firm plans to invest in a technology that does not exist yet. In the first case, it holds that $B(T) \geq i$, and therefore the value of S is known for sure at time T:

$$E\left[e^{-r(S-T)}\middle| B(T) \geq i\right] = e^{-r(S-T)}.$$

Now consider the second case where $B(T) < i$. Define s as the time between invention and adoption of technology i: $s = S - T_i$, then

$$E\left[e^{-r(S-T)}\middle| B(T) < i\right] = e^{-rs}E\left[e^{-r(T_i-T)}\middle| B(T) < i\right].$$

We denote the number of technologies that arrive during a time interval $[T, T+U)$ by $R(U)$. Thus, it holds that $R(U) = B(T+U) - B(T)$. Due to the fact that B is a Poisson process with rate λ, the stochastic variable $R(U)$ is distributed according to a Poisson distribution with parameter λU. Now it is not hard to see that

$$\Pr(T_i - T \leq U|\, B(T) < i) = \sum_{k=i-B(T)}^{\infty} \Pr(R(U) = k) \tag{8}$$

Using equation (8) we derive the following expression[4]:

[4] Assume that the stochastic variable X is distributed over the interval $[0,\infty)$ according to some distribution with distribution function $F(x) := \Pr(X \leq x)$, and that the function $f(x)$ is continuous and differentiable in x.
Then $E[f(X)] = \int_{x=0}^{\infty} f(x)\,dF(x) = \int_{x=0}^{\infty}\left[\int_{t=0}^{x} f'(t)\,dt + f(0)\right]dF(x) = \int_{t=0}^{\infty}\left[\int_{x=t}^{\infty} f'(t)\,dF(x)\right]dt + f(0)\int_{x=0}^{\infty} dF(x) = \int_{t=0}^{\infty} f'(t)(1-F(t))\,dt + f(0)$.

$$
\begin{aligned}
&E\left[e^{-r(T_i-T)}\Big| B(T)<i\right] \\
&=1-r\int_{u=0}^{\infty} e^{-ru}\left(1-\Pr\left(T_i-T\leq u|\, B(T)<i\right)\right) du \\
&=1-r\int_{u=0}^{\infty} e^{-ru}\left(1-\sum_{k=i-B(T)}^{\infty}\Pr\left(R(u)=k\right)\right) du \\
&=1-r\int_{u=0}^{\infty} e^{-ru}\sum_{k=0}^{i-B(T)-1}\Pr\left(R(u)=k\right) du \\
&=1-r\sum_{k=0}^{i-B(T)-1}\int_{u=0}^{\infty} e^{-ru}e^{-\lambda u}\frac{(\lambda u)^k}{k!}du \\
&=1-r\sum_{k=0}^{i-B(T)-1}\frac{\lambda^k}{(r+\lambda)^{k+1}}\int_{u=0}^{\infty}\frac{(r+\lambda)^{k+1}e^{-(r+\lambda)u}u^k}{k!}du \\
&=1-\frac{r}{(r+\lambda)}\frac{1-\left(\frac{\lambda}{r+\lambda}\right)^{i-B(T)}}{1-\left(\frac{\lambda}{r+\lambda}\right)} \\
&=\left(\frac{\lambda}{r+\lambda}\right)^{i-B(T)}. \qquad (9)
\end{aligned}
$$

With the help of equation (9) we derive that

$$
E\left[e^{-r(S-T)}\right]=\begin{cases} e^{-r(S-T)} & S<T_{B(T)+1}, \\ e^{-rs}\left(\frac{\lambda}{r+\lambda}\right)^{i-B(T)} & S\geq T_{B(T)+1}. \end{cases} \qquad (10)
$$

3.2 Leader, follower and joint-moving curves

At each point of time T the leader can choose to immediately invest in a technology j from the finite set $\{1,2,\ldots,B(T)\}$. Given an adoption strategy of the leader (T,j) the optimal reaction of the follower can be calculated in two steps.

In the first step, derive for each technology i the optimal adoption date S_i^* for the follower. Since the follower's payoff depends on the technology the other firm uses, S_i^* will be a function of T and j. Therefore,

$$
S_i^*(T,j)=\arg\max_{u\geq\max(T_i,T)} V(u,i,T,j). \qquad (11)
$$

In order to be more specific about $S_i^*(T,j)$ consider the following scenario: the leader has already adopted technology j and technology i has just been invented. The follower can either adopt technology i right away or delay adoption. Let $W_i^F(j)$ denote the optimal waiting time for the follower, that is the length of the time period between invention and optimal adoption of technology i. Solving the maximization problem (11), in which $V(\cdot,\cdot,\cdot,\cdot)$ is given by (6), yields that $W_i^F(j)=0$ if

$$\pi(\theta_i,\theta_j)-\pi(\theta_0,\theta_j)\geq rI(0)-I'(0), \tag{12}$$

and that $W_i^F(j)$ is implicitly determined by

$$\pi(\theta_i,\theta_j)-\pi(\theta_0,\theta_j)-rI\left(W_i^F(j)\right)+I'\left(W_i^F(j)\right)=0, \tag{13}$$

otherwise. Equation (13) states that the marginal costs, $rI\left(W_i^F(j)\right)-I'\left(W_i^F(j)\right)$, and the marginal benefits, $\pi(\theta_i,\theta_j)-\pi(\theta_0,\theta_j)$, are balanced at time $W_i^F(j)$. The marginal costs are equal to the opportunity costs of the investment, $rI(t)$, and the costs resulting from the fact that the firm invests right away so that it does not take advantage from I being decreasing over time. If at time 0 the marginal benefits exceed the marginal costs (cf. equation (12)) the firm should adopt immediately so that $W_i^F(j)=0$.

Using the definition of $W_i^F(j)$[5], we extend this particular scenario to the general case and conclude that the optimal adoption time $S_i^*(t,j)$ is equal to

$$S_i^*(T,j)=\begin{cases}T & \text{if } T\geq T_i+W_i^F(j),\\ T_i+W_i^F(j) & \text{if } T<T_i+W_i^F(j).\end{cases} \tag{14}$$

In the second step, use (14) to determine the technology i^* that maximizes the follower's payoff, given that the leader invests at time T in technology j:

$$i^*(T,j)=\arg\max_k V\left(S_k^*(T,j),k,T,j\right). \tag{15}$$

The leader, having read the previous lines, takes into account the follower's investment behavior in choosing at time T the technology $j^*(T)$ that results in the largest payoff:

$$j^*(T)=\arg\max_{k\in\{1,2,\dots,B(T)\}} V\left(T,k,S^*_{i^*(T,k)}(T,k),i^*(T,k)\right). \tag{16}$$

The process described above results in the following value functions for the timing game that starts at time T_k:

$$\begin{aligned}L(t)=L(T-T_k)&=\left(1-e^{-r(T-T_k)}\right)\frac{\pi(\theta_0,\theta_0)}{r} \\ &+e^{-r(T-T_k)}V\left(T,j^*(T),S^*_{i^*(T,j^*(T))}(T,j^*(T)),i^*(T,j^*(T))\right),\end{aligned} \tag{17}$$

$$\begin{aligned}F(t)=F(T-T_k)&=\left(1-e^{-r(T-T_k)}\right)\frac{\pi(\theta_0,\theta_0)}{r} \\ &+e^{-r(T-T_k)}V\left(S^*_{i^*(T,j^*(T))}(T,j^*(T)),i^*(T,j^*(T)),T,j^*(T)\right),\end{aligned} \tag{18}$$

$$\begin{aligned}M(t)=M(T-T_k)&=\left(1-e^{-r(T-T_k)}\right)\frac{\pi(\theta_0,\theta_0)}{r} \\ &+e^{-r(T-T_k)}V\left(T,j^*(T),T,j^*(T)\right).\end{aligned} \tag{19}$$

[5] Since $\pi(\theta_i,\theta_j)$ is increasing in θ_i, we know that there exists a $\hat{i}(j)$ such that $W_i^F(j)=0$ for all $i\geq\hat{i}(j)$.

3.3 Equilibria for timing games without waiting curve

In this subsection possible equilibria for classical timing games, i.e. timing games without waiting curves, are presented. In our model with n new technologies, the game that starts after time T_n is a classical timing game.

Classical timing games can be divided in two classes. The first class consists of the so-called preemption games and the elements of the second class are called wars of attrition. Preemption games are characterized by the fact that there exists a point of time where there is a first mover advantage:

$$\exists_{t \geq 0} : L(t) > F(t). \tag{20}$$

In a war of attrition the follower's payoff exceeds the leader's payoff at all times:

$$\forall_{t \geq 0} : F(t) > L(t). \tag{21}$$

In general a (classical) timing game can be split up into a finite number of subgames, where each subgame is a preemption game or a war of attrition. Due to the definitions of preemption games and wars of attrition, the split up points will be the points at which the function $L(t) - F(t)$ changes its sign. The equilibrium of a general timing game is found by first solving the last subgame, using the resulting value functions of the equilibrium of this subgame in the second last subgame and so forth and so on.

Since we analyze identical firms we are especially interested in equilibria with symmetric strategies. For identical and rational firms there is no reason why they should act differently. An example and a rigorous treatment of a preemption game can be found in Fudenberg and Tirole (1985). Hendricks et al. (1988) analyze wars of attrition in detail and equilibrium strategies can be found in that paper.

The equilibrium outcome of the timing game that starts after time T_n depends on the interarrival time τ_{n-1}. We denote the (expected) equilibrium outcome of the game that starts after time T_n by $\Omega_n(\tau_{n-1})$. If the game has more than one equilibrium, we use the most reasonable[6] equilibrium in the calculations.

3.4 Waiting curve

In general, the equilibrium outcome of the game that starts in the interval $[T_{k+1}, T_{k+2})$ is denoted by $\Omega_{k+1}(\tau_k)$. Using this notation, the waiting curve for a game that starts in the interval $[T_k, T_{k+1})$ equals

$$W(t) = W(T - T_k) = \left(1 - e^{-r(T-T_k)}\right) \frac{\pi(\theta_0, \theta_0)}{r} + e^{-r(T-T_k)} \int_{\tau_k=0}^{\infty} \left[\int_{u=0}^{\tau_k} \pi(\theta_0, \theta_0) e^{-ru} du + e^{-r\tau_k} \Omega_{k+1}(\tau_k) \right] \lambda e^{-\lambda \tau_k} d\tau_k. \tag{22}$$

[6] The most reasonable equilibrium is defined as the equilibrium under which the player's payoffs are maximal (the Pareto optimal equilibrium, cf. Fudenberg and Tirole (1985)).

The first part represents the profits made by the firm on the time interval $[T_k, T]$. The second part resembles the expected payoff of the firm from time T onwards conditioned on the interarrival time τ_k, where $\Omega_{k+1}(\tau_k)$ represents the (expected) outcome of the game that arises at the moment that the $(k+1)$-th technology is invented.

The waiting curve represents the option to invest in some future technology that is not invented yet. As such it is not equal to the option value of waiting since it does not take into account the increased profitability over time (due to the decreasing investment costs) of the already existing technologies.

3.5 Equilibria for timing games with waiting curve

The equilibria of a timing game with waiting curve are found in two steps. In the first step the timing game is split up into subgames. This is done as is described in Subsection 3.3, i.e. the split up points are the points at which the function $L(t) - F(t)$ changes its sign. In the second step the subgames are solved. The last subgame is solved as first, then the second last subgame, and so forth and so on.

The first class of subgames with waiting curve are those in which the leader curve exceeds the waiting curve for all points in time. The implication is that for the leader investing dominates waiting. Consequently, the equilibria of such a subgame are given by the equilibria of the corresponding subgame without waiting curve.

In a subgame for which the waiting curve exceeds the leader curve for all t: $W(t) > L(t)$, none of the firms is going to invest as first. This for the reason that waiting gives them a higher expected value. Therefore, the equilibrium outcome for both firms is waiting.

If in a subgame the leader curve exceeds the waiting curve for some but not all points in time, then for at least one firm investing is better than waiting for those points in time. There are two cases: (i) the subgame without the waiting curve is a preemption game, and (ii) the subgame without the waiting curve is a war of attrition.

In the first case (preemption game) the equilibria of the subgame with waiting curve are given by the equilibria of the subgame without waiting curve. This is a direct result of the fact that the equilibria in the subgame without waiting curve are Nash equilibria, i.e. one firm can not improve his expected value by deviating from the equilibrium strategy.

Contrary, in the second case (attrition game) the firm that is leader can increase its profit by waiting with investing at the points in time where the waiting curve exceeds the leader curve. The equilibrium strategies for the part of the subgame where the leader curve exceeds the waiting curve are given by the equilibrium strategies of the subgame without waiting curve. At the points in time of the other part of the subgame none of the firms will invest.

3.6 Solution procedure

In this subsection the solution procedure is summarized. In the first step the classical timing game that starts at time T_n is solved. This gives the equilibrium outcome function $\Omega_n(\tau_{n-1})$. Using this equilibrium outcome function we construct the waiting curve (22) and solve the timing game that starts at a point in time on the interval $[T_{n-1}, T_n)$. The resulting outcomes are incorporated in the function $\Omega_{n-1}(\tau_{n-2})$ which is again used to construct the waiting curve for the timing game that starts somewhere during the time interval $[T_{n-2}, T_{n-1})$. This process is repeated until the game that starts at T_1 is solved.

Combining the equilibrium strategies of each step gives the optimal investment strategy of the firm. The ex-ante probabilities of each equilibrium outcome can be derived using the calculations of each step. After each realization of an interarrival time these probabilities must be updated.

4 The information technology investment problem

In this section we apply the algorithm of the previous section to a specific information technology investment problem. Information technology products are heavily dependent on micro-chips. The memory and arithmetic power of micro-chips develop in an exponential way over time. This was firstly recognized by one of the Intel-founders Gordon Moore in 1964, who found out that the amount of information on a piece of silicium doubles every year. This statement is called Moore's law. Nowadays, Moore's law still applies although the doubling time has risen to two to three years. In our calculations it is assumed that on average every three years a new generation of chips arrives: $\lambda = \frac{1}{3}$. A new generation of chips is a generation that is twice as efficient as the preceding generation. After applying the algorithm stated in the beginning of Section 3, it turned out that we need to take four generations of chips into account. After normalizing the technology parameter of the current technology to one, this gives rise to the following θ scheme:

$$\theta_0 = 1, \tag{23}$$

$$\theta_{i+1} = 2\theta_i,\ i \in \{0, 1, 2, 3\}, \tag{24}$$

so that

$$\theta_i = 2^i,\ i \in \{0, 1, 2, 3, 4\}. \tag{25}$$

Due to the rapid innovation process, prices of information technology products go down quickly. Therefore, we assume that

$$I(t_i) = I_0 \exp(-\alpha t_i), \tag{26}$$

where

$$I_0 = 50, \tag{27}$$

$$\alpha = 1. \tag{28}$$

Formulas (26)-(28) hold for every technology i we consider. Hence, the technologies only differ in their technology parameter. The strange thing with micro-electronics is that their fast efficiency improvement does not impress consumers. As an illustration, consider a telephone in which a certain amount of telephone numbers can be stored. A new generation of chips doubles this amount, but most likely this will not be a reason for the customers to sell their old telephone and buy a new one. Another example is that a new generation of personal computers will not double the research output of a scientist. Therefore, a manager of Philips, Theo Claassen, has argued that utility is a logarithmic function of technology, in the sense that utility increases with one unit in case technology power becomes ten times as large[7]. For this reason we assume that profit increases with the technology-efficiency parameter in a logarithmic way with base 10 (cf. (25)):

$$\pi\left(\theta_i, \theta_j\right) = \frac{^{10}\log\left(2\theta_i^2\right)}{^{10}\log\left(2\theta_j\right)} = \frac{2i+1}{j+1}. \tag{29}$$

It is easily seen that this specification satisfies the requirements in (1). This is that the profit is positive, it is concavely increasing in the efficiency parameter of its own technology, while profit as well as marginal profit is decreasing in the efficiency parameter of the technology the other firm uses. The discount rate equals $r = 0.05$.

In Table 1 all possible outcomes and the probabilities are summarized. For the derivations we refer to the Appendix. As can be inferred from Table 1, probabilities are assigned to outcomes. This is the case, because it depends on the realizations of the arrival times of technologies 2, 3, and 4 which outcome will actually occur[8]. In the Appendix it is explained which realizations result in what outcome.

Table 1. Equilibria and ex-ante probabilities at time $T_1 = 0$. Type "P" is preemption game and type "WA" is war of attrition. The leader adoption times are defined as follows: $t_{24}^P := \min\{t|V(t,2,w_4^F(2),4) = V(w_4^F(2),4,t,2).\} = 1.81706$, $t_{34}^P := \min\{t|V(t,3,w_4^F(3),4) = V(w_4^F(3),4,t,3).\} = 0.727495$, $t_{23}^L(\tau_2) := \min\{t|V(t,2,w_4^F(2),4) = V(t,3,w_4^F(3),4).\}$, $t_4^P := \min\{t|V(t,4,w_4^F(4),4) = V(w_4^F(4),4,t,4).\} = 0.734579$ and $t_{34}^L(\tau_3) := \min\{t|V(t,3,w_4^F(3),4) = V(t,4,w_4^F(4),4).\}$

Probability	Type	Equilibrium			
		Leader		Follower	
		Technology	Time	Technology	Time
0.54570	P	2	$T_2 + t_{24}^P$	4	$T_4 + w_4^F(2)$
0.26711	P	3	$T_3 + t_{34}^P$	4	$T_4 + w_4^F(3)$
0.086802	WA	2	$T_3 + t_{23}^L(\tau_2)$	4	$T_4 + w_4^F(2)$
0.099576	P	4	$T_4 + t_4^P$	4	$T_4 + w_4^F(4)$
0.00081337	WA	3	$T_4 + t_{34}^L(\tau_3)$	4	$T_4 + w_4^F(3)$

From Table 1 it can be concluded that in the most likely outcome one firm will invest first and adopt technology 2, while the other firm will adopt technology 4 at a later point in time. In Figure 2 the use of different technologies over time is depicted. From this figure it can be obtained that beforehand it is difficult to say

[7] This was stated in the Dutch magazine Elsevier (January 24, 1998).

[8] For instance, technology 4 will dominate technology 3, when the realized interarrival time is small. In this case it is always suboptimal to invest in technology 3.

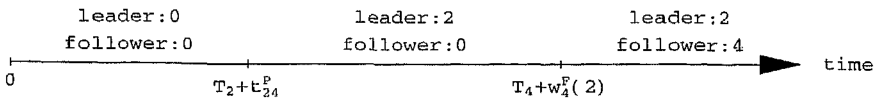

Fig. 2. Technology use over time by both firms

which firm is better off. This is because on the interval $\left[T_2 + t_{24}^P, T_4 + w_4^F(2)\right)$ the leader's payoff will be higher since this firm produces with a more modern technology than its opponent. But, on the other hand, at $T_4 + w_4^F(2)$ the follower replaces its old fashioned technology by the newest technology 4. This implies that from this moment on the follower's payoff is higher than the payoff of the leader.

It turns out that in the preemption equilibria listed in Table 1 (first, second and fourth row) rent equalization occurs in expectation (cf. Fudenberg and Tirole, 1985). Hence, both firms have an equal expected performance, so that the payoff surplus of the leader in the earlier period is exactly outweighed by the payoff surplus of the follower in the last period (see Figure 2).

From Table 1 it can also be obtained that with probability 0.087615 a second mover advantage occurs. In this case the payoff surplus of the last period more than outweighs the efficiency advantage of the leader in the earlier period so that the follower is better off.

With probability 0.90042 the leader adopts another technology than the follower. The follower is expected to adopt technology 4 in all equilibria. Joint adoption does not occur as an equilibrium outcome.

We did not add an extra new technology to the model, because the probability that both firms adopt technology 4 is less than 0.10. Hence, with a probability of more than 90% one firm invests in another technology than the last one. For this reason we choose not to analyze the game with one technology more.

5 Conclusion

We analyzed a framework in which consecutive generations of new technologies arrive over time, and a firm has to make its optimal technology investment decision. Competition on the output market is taken into account. As time passes more efficient technologies arrive according to a stochastic arrival process. The investment cost of a particular technology drops over time.

Introducing the waiting curve as a new concept, the investment decision problem was converted into a timing game. The timing game changes every time a new technology enters the market. We designed an algorithm that can be used to solve this game.

The algorithm is applied to an information technology investment problem with four new technologies. The most likely outcome exhibits diffusion, one firm adopts technology 2 early and the other technology 4 later on, while the expected payoffs of the first and second investor are the same. With a probability of more than 90% the expected payoffs of the firms are equal. In the other cases the firm that invests as second does better than the firm that invests as first. Thus the temporary gain

Table 2. Equilibria and type of subgames starting at time T_4 as function of τ_3. Type "P" is preemption game and type "WA" is war of attrition. The leader adoption times are defined as follows: $t_4^P := \min\{t|V(t,4,w_4^F(4),4) = V(w_4^F(4),4,t,4).\} = 0.734579$, $t_{34}^L(\tau_3) := \min\{t|V(t,3,w_4^F(3),4) = V(t,4,w_4^F(4),4).\}$, $S_{34}^L(\tau_3) := \arg\max_{t\in[0,t_{34}^L(\tau_3))} V(t,3,w_4^F(3),4)$ and $t_{34}^P(\tau_3) := \min\{t|V(t,3,w_4^F(3),4) = V(w_4^F(3),4,t,4)\}$

τ_3 region	Type	Equilibrium			
		Leader		Follower	
		Technology	Time	Technology	Time
$\tau_3 \in [0, 0.800591)$	P	4	$T_4 + t_4^P$	4	$T_4 + w_4^F(4)$
$\tau_3 \in [0.800591, 1.17938)$	WA	3	$T_4 + t_{34}^L(\tau_3)$	4	$T_4 + w_4^F(3)$
$\tau_3 \in [1.17938, 1.87931]$	WA	3	$T_4 + S_{34}^L(\tau_3)$	4	$T_4 + w_4^F(3)$
$\tau_3 \in (1.87931, 1.89322)$	P	3	$T_4 + t_{34}^P(\tau_3)$	4	$T_4 + w_4^F(3)$
$\tau_3 \in [1.89322, \infty)$	P	3	T_4	4	$T_4 + w_4^F(3)$

of market share by the leader does not make up for the market share gain of the follower.

One possible extension of this model is to relax the assumption that firms are allowed to make only one technology switch. We believe that this model can be solved in the same fashion: use the waiting curve concept to convert the game to a timing game with multiple actions and solve that game following the work by Simon (1987).

Another interesting extension is to make the number of active firms on the output market endogenous. If the active firms make positive profits it may be interesting for a new firm to enter the market. How does the threat of entering change the technology adoption behavior of the existing firms? Will they try to prevent firms to enter the market by adopting new technologies sooner?

A Appendix

From equations (12), (13), (26) and (29) we derive that

$$W_i^F(j) = \begin{cases} \frac{1}{\alpha}\log\left(\frac{(r+\alpha)I_0(j+1)}{2i}\right) & \text{if } i < \frac{1}{2}(j+1)(r+\alpha)I_0, \\ 0 & \text{else.} \end{cases} \tag{30}$$

The expected equilibrium outcomes for the subgames starting right at the invention times T_4, T_3 and T_2 are summarized in Tables 2-4[9].

In Tables 3 and 4 the equilibrium outcomes are conditional on the next technology not arriving too early. That is the next technology does not arrive before the time at which the leader changes technologies according to the table.

If technology 4 arrives shortly after technology 3 (see first line of Table 2), technology 4 dominates technology 3 and both firms will adopt technology 4. If

[9] The derivation of these outcomes can be obtained from the authors upon request.

Table 3. Equilibria and type of subgames starting at time T_3 as function of τ_2. Type "P" is preemption game and type "WA" is war of attrition. The leader adoption times are defined as follows: $t_{34}^P := \min\{t | V(t,3,w_4^F(3),4) = V(w_4^F(3),4,t,3).\} = 0.727495$, $t_{23}^L(\tau_2) := \min\{t | V(t,2,w_4^F(2),4) = V(t,3,w_4^F(3),4).\}$ and $S_{24}^L(\tau_2) := \arg\max_{t\in[0,t_{24}^L(\tau_2)]} V(t,2,w_4^F(2),4)$

τ_2 region	Type	Equilibrium			
		Leader		Follower	
		Technology	Time	Technology	Time
$[0, 1.24843)$	P	3	$T_3 + t_{34}^P$	4	$T_4 + w_4^F(3)$
$[1.24843, 2.94586)$	WA	2	$T_3 + t_{23}^L(\tau_2)$	4	$T_4 + w_4^F(2)$
$[2.94586, 3.95758)$	WA	2	$T_3 + S_{24}^L(\tau_2)$	4	$T_4 + w_4^F(2)$
$[3.95758, \infty)$	WA	2	T_3	4	$T_4 + w_4^F(2)$

Table 4. Equilibria and type of subgames starting at time T_2 as function of τ_1. Type "P" is preemption game and type "WA" is war of attrition. The leader adoption time is defined as follows: $,4) = V(w_4^F(2),4,t,2).\} = 1.81706$

τ_1 region	Type	Equilibrium			
		Leader		Follower	
		Technology	Time	Technology	Time
$[0, \infty)$	P	2	$T_2 + t_{24}^P$	4	$T_4 + w_4^F(2)$

it takes a little longer before technology 4 becomes available, technology 3 is the most attractive technology for the leader to adopt. In the second and third τ_3 region the follower's value is higher than the leader's value. To explain this second mover advantage, consider the second line of Table 2. The value of the gain of market share of the follower during the time interval $[T_4 + w_4^F(3), \infty)$ outweighs the value of the gain of market share of the leader during the interval $[T_4 + t_{34}^L(\tau_3), T_4 + w_4^F(3))$. A late arrival of technology 4 makes technology 3 attractive enough for direct adoption, see last line of Table 2. Tables 3 and 4 should be interpreted in the same way.

We now analyze the game at the moment where technologies 2, 3 and 4 are not invented yet in a more elaborate way. Using the outcome function $\Omega_2(\tau_1)$ we construct the waiting curve for the game that starts at time T_1 (cf. (22)), which is the invention time of the first technology:

$$W(t) = \frac{\pi(\theta_0,\theta_0)}{r}\left(1 - e^{-rt}\right) + \int_{\tau_1=0}^{\infty}\left[\int_{u=0}^{\tau_1} \pi(\theta_0,\theta_0)\, e^{-ru}du + e^{-r\tau_1}\Omega_2(\tau_1)\right]\lambda e^{-\lambda\tau_1}d\tau_1. \tag{31}$$

The leader, follower and joint-moving curves are derived with the equations presented in Section 3. In Figure 3 the four curves are plotted.

From Figure 3 the following ordering of the curves is derived: $F(t) > W(t) > L(t) > M(t)$ for all $t \in [T_1, T_2)$. This implies that each firm likes the other to

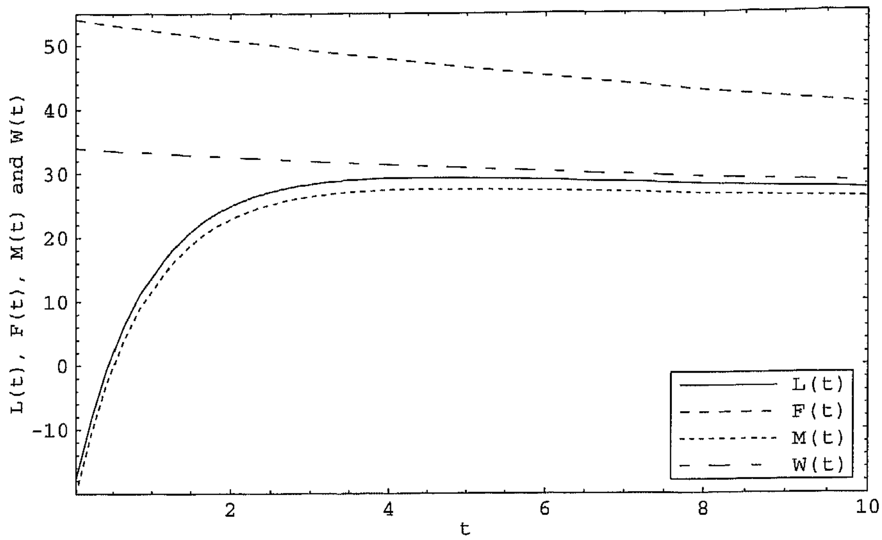

Fig. 3. $T_1 = 0$

invest as first and does not want to invest as first itself. Thus waiting is the optimal strategy for the firms in the game that starts in the interval $[T_1, T_2)$.

Then at time T_2 the game starts where technologies 1 and 2 are present, but the remaining technologies 3 and 4 are not invented yet. From Table 4 we derive that one firm will adopt technology 2 at time $T_2 + t_{24}^P$ and the other technology 4 if the third technology does not arrive before time $T_2 + t_{24}^P$. With probability

$$\Pr\left(\tau_2 \geq t_{24}^P\right) = e^{-\lambda t_{24}^P} = 0.54570,$$

this is the case.

With probability

$$\Pr\left(\tau_2 < t_{24}^P\right) = 1 - e^{-\lambda t_{24}^P} = 0.45430,$$

technology 3 arrives before time $T_2 + t_{24}^P$. Now, there are two cases. In case A, τ_2 is smaller than the boundary 1.24843 (see Table 3), which occurs with probability

$$\Pr\left(\tau_2 < 1.24843\right) = 1 - e^{-\lambda \cdot 1.24843} = 0.34041,$$

and in case B, $1.24843 < \tau_2 < t_{24}^P$, which occurs with probability

$$\Pr\left(1.24843 \leq \tau_2 < t_{24}^P\right) = e^{-\lambda \cdot 1.24843} - e^{-\lambda t_{24}^P} = 0.11389.$$

Table 3 states that, in case A, the outcome will be adoption of technology 3 at time $T_3 + t_{34}^P$ if technology 4 does not arrive before that time. This outcome occurs with the following probability:

$$\Pr\left(\tau_2 < t_{24}^P \text{ and } \tau_3 \geq t_{34}^P\right) = \Pr\left(\tau_2 < t_{24}^P\right) \Pr\left(\tau_3 \geq t_{34}^P\right)$$
$$= \left(1 - e^{-\lambda t_{24}^P}\right) e^{-\lambda t_{34}^P}$$
$$= 0.26711.$$

Technology 4 arrives before time $T_3 + t_{34}^P$, with probability

$$\Pr\left(\tau_2 < t_{24}^P \text{ and } \tau_3 < t_{34}^P\right) = \Pr\left(\tau_2 < t_{24}^P\right)\Pr\left(\tau_3 < t_{34}^P\right)$$
$$= \left(1 - e^{-\lambda t_{24}^P}\right)\left(1 - e^{-\lambda t_{34}^P}\right)$$
$$= 0.073303.$$

In this case the outcome will be a preemption equilibrium in which one firm adopts technology 4 at time $T_4 + t_4^P$ and the other firm technology 4 at time $T_4 + w_4^F(4)$. Here it is important to note that $t_{34}^P = 0.727495$ is smaller than the first τ_3 boundary 0.800591. Hence, with probability one the outcomes listed on the lines 2-5 of Table 2 will not occur here.

Case B is a little more complicated. The outcome exhibits adoption of technology 3 at time $T_3 + t_{23}^L(\tau_2)$ by one firm (the other firm adopts technology 4) if technology 4 arrives after time $T_3 + t_{23}^L(\tau_2)$, which happens with probability:

$$\Pr\left(1.24843 \le \tau_2 < t_{24}^P \text{ and } \tau_3 \ge t_{34}^L(\tau_2)\right)$$
$$= \int_{\tau_2=1.24843}^{t_{24}^P} \Pr\left(\tau_3 \ge t_{34}^L(\tau_2)\right)\lambda e^{-\lambda\tau_2}d\tau_2$$
$$= \int_{\tau_2=1.24843}^{t_{24}^P} e^{-\lambda t_{34}^L(\tau_2)}\lambda e^{-\lambda\tau_2}d\tau_2$$
$$= 0.086802.$$

Otherwise the outcome is of the preemption type (first line of Table 2) if $\tau_3 < 0.800591$ or a war of attrition (second line of Table 2) if $\tau_3 \ge 0.800591$. The probability that the preemption equilibrium occurs is equal to

$$\Pr\left(1.24843 \le \tau_2 < t_{24}^P \text{ and } \tau_3 < t_{34}^L(\tau_2) \text{ and } \tau_3 < 0.800591\right)$$
$$= \int_{\tau_2=1.24843}^{t_{24}^P} \Pr\left(\tau_3 < \min\left(t_{34}^L(\tau_2), 0.800591\right)\right)\lambda e^{-\lambda\tau_2}d\tau_2$$
$$= 0.026273.$$

With probability

$$\Pr\left(1.24843 \le \tau_2 < t_{24}^P \text{ and } \tau_3 < t_{34}^L(\tau_2) \text{ and } \tau_3 \ge 0.800591\right)$$
$$= \int_{\tau_2=1.24843}^{t_{24}^P} \Pr\left(0.800591 \le \tau_3 < t_{34}^L(\tau_2)\right)\lambda e^{-\lambda\tau_2}d\tau_2$$
$$= \int_{\tau_2=1.24843}^{t_{24}^P} \left(e^{-\lambda\cdot 0.800591} - e^{-\lambda t_{34}^L(\tau_2)}\right)1_{\left\{0.800591 \le t_{34}^L(\tau_2)\right\}}\lambda e^{-\lambda\tau_2}d\tau_2$$
$$= 0.00081337,$$

the war of attrition will happen. Here the leader adopts technology 3 and the follower invests in technology 4. So, on the longer term the follower produces with the more efficient technology which here leads to a higher payoff.

The analysis above implies that only the first two lines of Tables 2 and 3 matter. This for the reason that one of the firms adopts an existing technology, if a new technology arrives too late.

References

Balcer Y, Lippman SA (1984) Technological expectations and adoption of improved technology. Journal of Economic Theory 34 : 292–318

Dixit AK, Pindyck RS (1996) Investment under uncertainty, 2nd printing. Princeton University Press, Princeton, NJ

Farzin YH, Huisman KJM, Kort PM (1998) Optimal timing of technology adoption. Journal of Economic Dynamics and Control 22 : 779–799

Fudenberg D, Tirole J (1985) Preemption and rent equalization in the adoption of new technology. The Review of Economic Studies 52 : 383–401

Gaimon C (1989) Dynamic game results of the acquisition of new technology. Operations Research 37 : 410–425

Hendricks K, Weiss A, Wilson C (1988) The war of attrition in continuous time with complete information. International Economic Review 29 : 663–680

Kriebel CH (1989) Understanding the strategic investment in information technology. In: Laudon KC, Turner JA (eds) Information technology and management strategy. Prentice-Hall, Englewood Cliffs, NJ

Nair SK (1995) Modeling strategic investment decisions under sequential technological change. Management Science 41 : 282–297

Rajagopalan S, Singh MR, Morton TE (1998) Capacity expansion and replacement in growing markets with uncertain technological breakthroughs. Management Science 44 : 12–30

Reinganum JF (1981) On the diffusion of new technology: A game theoretic approach. The Review of Economic Studies 48 : 395–405

Simon LK (1987) A multistage duel in continuous time. Working Paper 8757, University of California at Berkeley, Berkeley, CA

Yorukoglu M (1998) The information technology productivity paradox. Review of Economic Dynamics 1 : 551–592