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Onderstal, A.M.

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Sander Onderstal

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ter verkrijging van de graad van doctor aan de Katholieke Universiteit Brabant, op gezag van de rector magnificus, prof. dr. F. A. van der Duyn Schouten, in het openbaar te verdedigen ten overstaan van een door het college voor promoties aangewezen commissie in de aula van de Universiteit op vrijdag 31 mei 2002 om 11.15 uur door

ALEXANDER MARINUS ONDERSTAL

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Contents

CHAPTER 1

Introduction

This Ph.D. thesis is a collection of six papers in auction theory. In this Introduction, I will introduce the reader to auction theory. The setup of the Introduction is as follows. In Section 1, I give a verbal definition of an auction, and give some examples of the use of auctions in practice. In Section 2, I will define auction theory, and show its importance. In Section 3, I discuss the auction types that are most commonly studied in auction theory. In Section 4, I summarize insights from auction theory related to bidding behavior in the most commonly studied auction types and discuss the revenue ranking of these auctions. In Section 5, I give a formal definition of an auction, and pay attention to the Revelation Principle and the Revenue Equivalence Theorem, two powerful tools which are closely related to auction theory. Also, I will discuss optimal auctions, i.e., auctions that maximize expected revenue for the seller. In Section 6, I give an overview of the thesis, giving a summary of each paper. Finally, in Section 7, I present a paragraph that summarizes the thesis in a few words.

1. What is an auction, and who uses it?

An auction is a market institution which is used to sell one or several objects according to a set of rules that specify how the object(s) is (are) allocated among bidders, and how much each bidder has to pay depending on submitted bids from herself and the other bidders. The word 'auction' is derived from the Latin word 'augere', which means 'to increase'. This word is somewhat of a misnomer, as in several auction types, the price is not being raised at all.

Auctions have been widely used over thousands of years to sell a remarkable range of commodities (Klemperer, 1999). One of the earliest reports of an auction is by the old Greek historian Herodotus of Halicarnassus, who writes about men bidding for women to become their wives in Babylonia around 500 B.C. (Cassady, 1967).[1] The following is reported about this on the internet.[2]

"In every village once a year all the girls of marriageable age were collected together in one place, while the men stood around them in a circle; an auctioneer then called each one in turn to stand up and offered her for sale, beginning with the best-looking and going on to the second best as soon as the first had been sold for a good price. Marriage was the object of the transaction. The rich men who wanted wives bid against each other for the prettiest girls, while the humbler folk, who had no use for good looks in a wife, were actually paid to take the ugly ones. The money came from the sale of the beauties, who in this way provided dowries for their ugly or misshapen sisters. It was illegal for a man to marry his daughter to anyone he happened to fancy, and no one could take home a girl he had bought without first finding a backer to guarantee his intention of marrying her. In cases of disagreement between husband and wife the law allowed the return of the purchase money. Anyone who wished could come, even from a different village, to buy a wife."

One of the most astonishing auctions in history took place in 193 A.D. when the Praetorian Guard offered the entire Roman Empire for sale in an auction. Didius Julianus was the highest bidder, but he was beheaded two months later when Septimus Severus conquered Rome (Cassady, 1967). In today's terminology, one would say that Julianus was a victim of the winner's curse.

Nowadays, the use of auctions is widespread. There are auctions for art, fish, flowers, oil wells, treasury bills, wine, and many other goods. Also, more abstract commodities are being sold in auctions. In the 1990s, the US government collected tens of billions of dollars in auctions for licenses for second generation mobile telecommunication. In

[1] Some have called Herodotus the Father of History, but, unfortunately, others have called him the Father of Lies (Pipes, 1998-1999). In other words, Herodotus' writings are not universally accepted as being historically sound, so that there may be some doubt whether auctions for women really took place.

[2] http://www.gravittauction.com/history.htm.

the years 2000 and 2001, several European governments followed this example by auctioning licenses for third generation mobile telecommunication, usually referred to as UMTS. Both the British and German governments raised tens of billions of euros. I will come back to the UMTS auctions in Chapter 3.

The Dutch government is also becoming used to auctions as selling and buying mechanisms, for instance in the case of telecommunication. A beauty contest was used, as late as 1996, to assign a license for second generation mobile telecommunication to Libertel. That year, however, seems to have been the turning point. A proposal to change the Telecommunication Law to allow for auctions reached the Dutch parliament in 1996, and the new law was implemented in 1997 (Verberne, 2000). In 1998, DCS-1800 licenses were sold through an auction, and in 2000, the UMTS auction took place (although this auction was not as successful as the English and German auctions in terms of money raised). Also, auctions for licenses for petrol stations and for radio channels are under consideration. In Chapters 2 and 5, the auction for petrol station licenses and the DCS-1800 auction respectively will be used as an illustration for the developed theory.

2. What is auction theory, and why is it important?

Auction theory is a collection of game-theoretic models related to the interaction of bidders in auctions. It is an important theory for two very different reasons. First, for thousands of years, many commodities have been sold in auctions. Therefore, it is important to understand how auctions work, and which auctions perform best, for instance in terms of generating revenues or in terms of efficiency. Second, auction theory is a fundamental tool in economic theory. It provides a price formation model, whereas the widely used Arrow-Debreu model from general equilibrium theory (Arrow and Debreu, 1954) is not explicit in how prices form. Moreover, the theory of monopoly pricing is mathematically the same as the theory of revenue maximizing auctions (Bulow and Roberts, 1989). Also, the insights generated by auction theory can be useful when studying several other phenomena which have structures that resemble auctions, like lobbying contests, queues, and war of attritions (Klemperer, 2000). Reflecting its importance, auction theory has become a substantial field in economic theory.

Auction theory devotes attention to both behavioral issues and design issues. The most important behavioral issues are related to questions like "How much do bidders bid given the auction format?", "Is this auction type efficient?", and "How much revenue does this auction generate?". Design issues are related to questions such as "Which auction format is the most efficient?", and "Which auction type yields the highest expected revenue?". In this thesis, I will discuss both behavioral and design issues.

Auction theorists model an auction as a game, predicting bidding behavior and considering design issues using game theory. The large majority of the models that are used in auction theory, including almost all models in this thesis, are part of noncooperative game theory. For an introduction into noncooperative game theory, and for definitions of the concepts of the theory that are used in this thesis, see Chapters 7-9 of Mas-Colell et al. (1995). Only in rare cases does auction theory use models of cooperative game theory. In this thesis, cooperative game theory is applied only once, namely in Chapter 2, when I study collusion among bidders. For an introduction into cooperative game theory, see Myerson (1991), and Chapter 17 of Mas-Colell et al. (1995).

3. Auction types

The following four auction types are most frequently studied in auction theory: the English auction, the Dutch auction, the first-price sealed-bid auction, and the second-price sealed-bid auction. These auction types are referred to as the standard auctions. In each of the standard auctions, one indivisible object is being offered to the bidders. Sometimes, a reserve price is used, below which the object will not be sold. When I do not explicitly specify a reserve price, I assume that the reserve price is zero.

In the English auction (also known as English open outcry, oral, open, or ascending-bid auction), the price starts at the reserve price, and is raised successively until one bidder remains. This bidder wins the object at the final price. The price can be raised by the auctioneer, or by having bidders call the bids themselves. Auction theorists usually study a version of the English auction called the Japanese auction, in which the price is raised continuously, and bidders announce to quit the auction at a certain price (e.g., by pressing or releasing a button). The English auction is the most famous and most commonly used auction type. Art and wine are sold using this type of auction.

The Dutch auction (descending-bid auction) works in exactly the opposite way from the English auction. The auctioneer begins with a very high price, and successively lowers it, until one bidder announces that she is willing to accept the current price. This bidder wins the object at that price, unless the price is below the reserve price. Flowers are sold this way in the Netherlands.

With the first-price sealed-bid auction (sealed high-bid auction), bidders independently submit sealed bids. The object is sold to the highest bidder at her own price, given that her bid is not below the reserve price. In the US, mineral rights are sold using this auction.

Under the second-price sealed-bid auction (Vickrey auction), bidders independently submit sealed bids. The object is sold to the highest bidder (given that her bid exceeds the reserve price). However, in contrast with the first-price sealed-bid auction, the price the winner pays is not her own bid, but the second highest bid (or the reserve price if it is higher than the second highest bid). Despite its useful theoretical properties, which I will discuss later, this auction format is seldom used in practice (Rothkopf et al., 1990).

Other auction types that attracted the attention of economists are the all-pay auction and the war of attrition. The all-pay auction has the same rules as the first-price sealed-bid auction, with the difference that all bidders must pay their bid, even those who do not win the object. Although the all-pay auction is rarely used as a selling mechanism, there are at least two reasons why economists are interested in it. First, this auction has useful theoretical properties, as it maximizes the expected revenue for the auctioneer if bidders are risk averse (Matthews, 1983), or budget constrained (Laffont and Robert, 1996).

Second, all-pay auctions are used to model several interesting real life phenomena, such as political lobbying (Che and Gale, 1998), political campaigns, research tournaments, and sport tournaments (Moldovanu and Sela, 2001). Efforts of the agents in these models are viewed as their bids. I will come back to this auction type in Chapters 6 and 7.

In the war of attrition, the price starts at the reserve price, and is raised successively until one bidder remains. This bidder wins the object at the final price. Bidders who do not win the object pay the price at which they leave the auction. There are two differences between the war of attrition and the all-pay auction. First, the all-pay auction is a sealed-bid auction, whereas the war of attrition is an ascending auction. Second, in the war of attrition, the highest bidder only pays an amount equal to the second highest bid, and in the all-pay auction, the highest bidder pays his own bid. The war of attrition is used to model several economic phenomena, such as battles to control new technologies, for instance between the CDMA (code division multiple access), the TDMA (time division multiple access), and the GSM (global system for mobile communications) techniques to become the single surviving standard worldwide (Bulow and Klemperer, 1999). In Chapters 5 and 7, I pay some attention to the war of attrition.

4. Equilibrium bidding and revenue comparison

The following questions receive a lot of attention in the literature on auctions: "How much do bidders bid in the standard auctions?", "Are the standard auctions efficient?", and "Which of the four standard auctions yields the highest expected revenue?". In order to answer these, and other, questions, interaction in an auction is usually modelled as a noncooperative game with incomplete information. Unless otherwise indicated, efficiency means that the auction outcome is always such that the bidder who wins the object is the one who attaches the highest value to it.

In the most studied auction model, a seller offers one indivisible object for sale to $n \geq 2$ bidders. I will call this model the *standard auction model.* This model is based on the following set of assumptions.[3]

(A1) *Risk neutrality*: All bidders are risk neutral.

(A2) *Private values:* Bidder i, $i = 1, ..., n$, has value v_i for the object. This number is private information to bidder i, and not known to the other bidders and the seller.

(A3) *Value independence*: The values v_i are independently drawn.

(A4) *No collusion among bidders*: Bidders do not make agreements among themselves in order to achieve the object cheaply. More generally, bidders play according to a Bayesian Nash equilibrium.

(A5) *Symmetry*: The values v_i are drawn from the same smooth distribution function.

(A6) *No budget constraints*: Each bidder is able to fulfill the financial requirements that are induced by her bid.

(A7) *No externalities*: Losers do not receive positive or negative externalities when the object is transferred to the winner of the auction.

(A8) *No financial externalities*: The utility of losing bidders is not effected by how much the winner pays.

At least five different conclusions emerge from the standard auction model with respect to equilibrium bidding.[4] First, the Dutch auction is strategically equivalent with the first-price sealed-bid auction. This implies that the (Bayesian) Nash equilibria of these two auctions must coincide. Second, the English auction and the second-price auction are equivalent in the sense that in both auctions for each bidder it is a weakly dominant strategy to bid her value. Third, all standard auctions have symmetric Bayesian Nash equilibria, which are shown to be unique in the case of two bidders.[5] Fourth, for each standard

[3]For a more detailed discussion on the standard auction model, see for instance McAfee and McMillan (1987).

[4]Some of these conclusions are valid under weaker assumptions.

[5]This result is straightforwardly shown to hold for the English auction and the second-price sealed-bid auction. Fudenberg and Tirole, 1991, show uniqueness of the Bayesian Nash equilibrium of the first-price sealed-bid auction, which is a

auction, there is at least one efficient equilibrium. In this equilibrium, the utility of a bidder is zero when she has the lowest possible value. Fifth, with a positive reserve price, bidders with a value below the reserve price abstain from bidding, and bidders with a value above the reserve price bid according to a bid function that is strictly increasing in their value. For a more detailed discussion on these conclusions, see for instance Milgrom and Weber (1982), and McAfee and McMillan (1987).

When (A1)-(A8) are fulfilled, a remarkable result arises with respect to the seller's expected revenue: It is the same for all standard auctions! Vickrey (1961) is the first to show this result for the simplifying case of a uniform value distribution function on the interval $[0, 1]$. Twenty years later, Myerson (1981), and Riley and Samuelson (1981) generalize Vickrey's result when (A1)-(A8) hold.

Table 1 shows how the ranking of the standard auctions changes when one of the assumptions (A1)-(A7) is relaxed while the other assumptions remain valid. The second-price sealed-bid auction (S) and the first-price sealed-bid auction (F) are ranked in terms of expected revenue. $S \prec F$ ($S \succ F$, $S \succeq F$) means that the second-price sealed-bid auction yields strictly lower (strictly higher, higher) expected revenue than the first-price sealed-bid auction. S ? F implies that the revenue ranking is ambiguous, that is, in some circumstances $S \prec F$ holds, and in other $S \succ F$. I do not discuss the ranking of the other standard auctions.[6] For a discussion of the used models, I refer to the papers mentioned in Table 1. In Chapter 3, I will discuss how relaxing assumption (A8) effects the revenue ranking.

stronger result than uniqueness of the *symmetric* Bayesian Nash equilibrium. As the Dutch auction is strategically equivalent, this result immediately holds for this auctions as well.

[6]However, as mentioned, the Dutch auction and the first-price sealed-bid auction are strategically equivalent, so that in all circumstances they generate the same bidding behavior, and therefore the same revenue. Moreover, in all models mentioned in Table 1, with the exception of the affiliation model by Milgrom and Weber, the English auction and the second-price sealed-bid auction are revenue equivalent as well.

Paper	**Ass.**	**Model**	**Rank.**
Maskin & Riley (1984)	(A1)	Risk aversion	S $\prec$ F
Klemperer (1998)	(A2)	Almost common values	S $\prec$ F
Milgrom & Weber (1982)	(A3)	Affiliation	S $\succ$ F
Graham & Marshall (1987)	(A4)	Collusion	S $\prec$ F
Maskin & Riley (2000)	(A5)	Asymmetry	S ? F
Che & Gale (1998a)	(A6)	Budget constraints	S $\succ$ F
Jehiel et al. (1999)	(A7)	Externalities	S $\succeq$ F

Table 1. Revenue ranking of standard auctions when the assumptions (A1)-(A7) are relaxed.

5. Optimal auctions

Which auction yields the highest expected revenue? In his remarkable paper, published in 1981, Myerson answers this question in an incomplete information model for the case of one indivisible object. In order to do so, he derives two fundamental results, the Revelation Principle, and the Revenue-Equivalence Theorem. In this section, after giving a formal definition of an auction, I will discuss Myerson's results in detail, as these results are used several times throughout the thesis. For the sake of a clear exposition, I will do so in an independent private values model.[7]

5.1. The model. Consider a seller, who wishes to sell one indivisible object to one out of n risk neutral bidders, numbered $1, 2, ..., n$. The seller aims at finding a feasible auction mechanism which gives him the highest possible expected revenue. For simplicity, I assume that the seller does not attach any value to the object.[8] Each bidder i receives a one-dimensional private signal v_i, which represents her value for the object. The v_i's are drawn independently from a distribution

[7] In Chapter 4 of this thesis, I will pay some more attention to revenue-equivalence results and results on revenue maximizing auctions in a more general model which is known in the literature as the independent private signals model.

[8] Myerson assumes that the seller attaches some value to the object, which is commonly known among all bidders.

function F_i. F_i has support on the interval $[\underline{v}_i, \bar{v}_i]$, and continuous density f_i with $f_i(v_i) > 0$, for every $v_i \in [\underline{v}_i, \bar{v}_i]$. I assume that all bidders are serious, i.e., $\underline{v}_i \geq 0$ for all i. In the remainder of the thesis, $\underline{v}_i$ is referred to as bidder i's lowest type.

Define the sets

$$V \equiv [\underline{v}_1, \bar{v}_1] \times ... \times [\underline{v}_n, \bar{v}_n],$$

and

$$V_{-i} \equiv \times_{j \neq i}[\underline{v}_j, \bar{v}_j],$$

with typical elements $\mathbf{v} \equiv (v_1, ..., v_n)$ and $\mathbf{v}_{-i} \equiv (v_1, ..., v_{i-1}, v_{i+1}, ..., v_n)$ respectively. Let

$$g(\mathbf{v}) \equiv \prod_j f_j(v_j)$$

be the joint density of $\mathbf{v}$, and let

$$g_{-i}(\mathbf{v}_{-i}) \equiv \prod_{j \neq i} f_j(v_j)$$

be the joint density of $\mathbf{v}_{-i}$.

In an *auction*, bidders are asked to simultaneously and independently choose a bid. Bidder i chooses a bid $b_i \in B_i$, where B_i is the set of possible bids for bidder i, $i = 1, ..., n$. The auction has the outcome functions

$$\widehat{p} : B_1 \times ... \times B_n \to [0,1]^n$$

with

$$\sum_j \widehat{p}_j(\mathbf{b}) \leq 1,$$

and

$$\widehat{x} : B_1 \times ... \times B_n \to \Re^n.$$

If $\mathbf{b} = (b_1, ..., b_n)$, then $\widehat{p}_i(\mathbf{b})$ is interpreted as the probability that bidder i wins the object, and $\widehat{x}_i(\mathbf{b})$ is the expected payment of bidder i to the seller. I call $\widehat{p}$ the allocation rule, and $\widehat{x}$ the payment rule.

The seller and the bidders are risk neutral and have additively separable utility function for money and the object. Thus, when $\mathbf{b}$ is played, bidder i's utility is given by

$$\hat{U}_i(\mathbf{b}) = v_i \widehat{p}_i(\mathbf{b}) - \widehat{x}_i(\mathbf{b}), \tag{5.1}$$

and the seller's utility is

$$\hat{U}_0(\mathbf{b}) = \sum_j \widehat{x}_j(\mathbf{b}). \tag{5.2}$$

Let $b_1^*, ..., b_n^*$ be the Bayesian Nash equilibrium of the auction, so that

$$b_i^*(v_i) \in \arg\max_{b_i \in B_i} \int\limits_{V_{-i}} \hat{U}_i(b_1^*(v_1), ..., b_{i-1}^*(v_{i-1}), b_i, b_{i+1}^*(v_{i+1}), ..., b_n^*(v_n)) f_{-i}(\mathbf{v}_{-i}) d\mathbf{v}_{-i}$$

for all v_i and i. A *feasible auction mechanism* is an auction together with a description of the strategies the bidders are expected to use, which have the following properties: (1) each bidder expects nonnegative utility, and (2) the strategies form a Bayesian Nash equilibrium of the auction. An *optimal auction* is a feasible auction mechanism that maximizes the seller's expected utility.

A special class of feasible auction mechanisms is the class of *feasible direct revelation mechanisms.* In a feasible direct revelation mechanism, each bidder is asked to announce her value and has an incentive to do so truthfully. More specifically, let (p, x) denote a feasible direct revelation mechanism, with

$$p : V \to [0, 1]^n$$

where

$$\sum_j p_j(\mathbf{v}) \le 1,$$

and

$$x : V \to \Re^n.$$

We interpret $p_i(\mathbf{v})$ as the probability that bidder i wins, and $x_i(\mathbf{v})$ as the expected payments by i to the seller when $\mathbf{v}$ is announced.

Consistently with (5.1), bidder i's utility of (p, x) given $\mathbf{v}$ is given by

$$v_i p_i(\mathbf{v}) - x_i(\mathbf{v}),$$

so that if bidder i knows her value v_i, her expected utility from (p, x) can be written as

$$U_i(p, x, v_i) \equiv \int\limits_{V_{-i}} [v_i p_i(\mathbf{v}) - x_i(\mathbf{v})] g_{-i}(\mathbf{v}_{-i}) d\mathbf{v}_{-i}, \tag{5.3}$$

with $d\mathbf{v}_{-i} \equiv dv_1...dv_{i-1}dv_{i+1}...dv_n$. Throughout the thesis, $U_i(p, x, v_i)$ will be referred to as bidder i's *interim utility*.

There are two types of constraints that must be imposed on (p, x), an *individual rationality constraint* and an *incentive-compatibility constraint.* The individual rationality constraint follows from the assumption that each bidder expects nonnegative expected utility, so that

$$U_i(p, x, v_i) \geq 0, \forall v_i, i. \tag{5.4}$$

The incentive-compatibility constraint is imposed as we demand that each bidder has an incentive to announce her value truthfully. Thus,

$$U_i(p, x, v_i) \geq \int\limits_{V_{-i}} [v_i p_i(\mathbf{v}_{-i}, w_i) - x_i(\mathbf{v}_{-i}, w_i)] g_{-i}(\mathbf{v}_{-i}) d\mathbf{v}_{-i}, \ \forall v_i, w_i, i,$$

where $(\mathbf{v}_{-i}, w_i) = (v_1, ..., v_{i-1}, w_i, v_{i+1}, ..., v_n)$.

In line with (5.2), the seller's expected utility of (p, x) is

$$U_0(p, x) \equiv \int\limits_V \sum_{i=1}^n x_i(\mathbf{v}) g(\mathbf{v}) d\mathbf{v}, \tag{5.5}$$

with $d\mathbf{v} \equiv dv_1...dv_n$.

5.2. Results. When solving the seller's problem, there is no loss of generality in considering feasible direct revelation mechanisms. This follows from the Lemma 1, which is known as the Revelation Principle (see, for instance, Myerson, 1981).

LEMMA 1 (The Revelation Principle). *For any feasible auction mechanism there is a feasible direct revelation mechanism that gives both the seller and the bidders the same expected utility as the given feasible auction mechanism.*

PROOF. Consider a feasible auction mechanism. By definition, this feasible auction mechanism consists of an auction including the strategies played by the bidders which form a Bayesian Nash equilibrium of the auction. Now, consider the following revelation game. First, the seller asks each bidder to announce her value. Then, he determines the bid that each bidder would have chosen in equilibrium of the auction given her announced value, and implements the outcomes that would result in the auction from these bids. As the strategies form an equilibrium of the auction, it is an equilibrium for each bidder to announce her value truthfully in the revelation game. Therefore, the revelation game has the same outcome as the auction, so that both the seller and the bidders obtain the same expected utility as in the feasible auction mechanism. □

Let

$$Q_i(p, v_i) \equiv E_{v_{-i}}\{p_i(\mathbf{v})\}$$

be the conditional probability that bidder i wins the object given her value v_i. Lemma 2 gives a characterization of feasible direct revelation mechanisms (p, x).

LEMMA 2 (Myerson, 1981). *(p, x) is a feasible direct revelation mechanism if and only if*

$$\text{if } w_i \leq v_i \text{ then } Q_i(p, w_i) \leq Q_i(p, v_i),\ \forall w_i, v_i, i, \tag{5.6}$$

$$U_i(p, x, v_i) = U_i(p, x, \underline{v}_i) + \int_{\underline{v}_i}^{v_i} Q_i(p, y_i) dy_i,\ \forall v_i, i, \text{ and} \tag{5.7}$$

$$U_i(p, x, \underline{v}_i) \geq 0,\ \forall i. \tag{5.8}$$

PROOF. Incentive compatibility implies

$$U_i(p, x, w_i) \geq U_i(p, x, v_i) + (w_i - v_i) Q_i(p, v_i), \tag{5.9}$$

so that (p, x) is a feasible direct revelation mechanism if and only if (5.4) and (5.9) hold. With (5.9),

$$(w_i - v_i) Q_i(p, w_i) \leq U_i(p, x, w_i) - U_i(p, x, v_i) \leq (w_i - v_i) Q_i(p, v_i).$$

Then (5.6) follows when $w_i \leq v_i$. Moreover, these inequalities imply

$$\frac{\partial U_i(p, x, v_i)}{\partial v_i} = E_{v_{-i}}\{p_i(\mathbf{v})\} = Q_i(p, v_i), \tag{5.10}$$

at all points where p_i is differentiable in v_i. By integration of (5.10), (5.7) is obtained. Finally, with (5.4) and (5.7), individual rationality is equivalent to (5.8). □

The seller's expected utility is characterized by the following lemma.

LEMMA 3 (Myerson, 1981). *Let (p, x) be a feasible direct revelation mechanism. The seller's expected utility from (p, x) is given by*

$$(5.11)\quad U_0(p,x) = E_{\mathbf{v}}\left\{\sum_{i=1}^{n}\left(v_i - \frac{1-F_i(v_i)}{f_i(v_i)}\right)p_i(\mathbf{v})\right\} - \sum_{i=1}^{n} U_i(p,x,\underline{v}_i).$$

PROOF. With (5.3), (5.5) can be rewritten as

$$(5.12)\quad U_0(p,x) = \sum_{i=1}^{n}\int_V v_i p_i(\mathbf{v})g(\mathbf{v})d\mathbf{v} - \sum_{i=1}^{n}\int_{\underline{v}_i}^{\bar{v}_i} U_i(p,x,v_i)f_i(v_i)dv_i.$$

Taking the expectation of (5.7) over v_i and using integration by parts, I obtain

$$E_{v_i}\{U_i(p,x,v_i)\} = U_i(p,x,\underline{v}_i) + E_{v_i}\left\{\frac{1-F_i(v_i)}{f_i(v_i)}p_i(\mathbf{v})\right\},$$

so that (5.11) follows with (5.12). □

From Lemma 3, interesting insights can be drawn with respect to optimal auctions. Consider the following definition of bidder i's marginal revenue.

$$(5.13)\qquad MR_i(v_i) \equiv v_i - \frac{1-F_i(v_i)}{f_i(v_i)},\ \forall v_i, i.$$

Observe that in (5.11), a key role is played by the marginal revenues. Now, suppose that the seller finds a feasible auction mechanism that (1) assigns the object to the bidder with the highest marginal revenue, provided that the marginal revenue is nonnegative, (2) leaves the object in the hands of the seller if the highest marginal revenue is negative, and (3) gives the lowest types zero expected utility. Then this feasible auction mechanism is optimal.

Such feasible auction mechanisms exists under the following extra restriction.

MR Monotonicity: $MR_i(v_i)$ is strictly increasing in v_i for all v_i, i.

If *MR Monotonicity* does not hold, (5.6) may be violated. See Myerson (1981) for further discussion on the consequences of relaxing this restriction.

When (A1)-(A8) are satisfied, and when *MR Monotonicity* holds, all standard auctions are optimal when the seller imposes the right reserve price. This can be seen as follows. As said, in equilibrium of a standard auction with reserve price, bidders with a value below the reserve price abstain from bidding, and bidders with a value above the reserve price bid according to a bid function that is strictly increasing in their value. If the reserve price is chosen such that the marginal revenue at the reserve price is equal to zero, then all standard auction are optimal as, by *MR Monotonicity,* (1) the object is always assigned to the bidder with the highest nonnegative marginal revenue, (2) the object remains in the hands of the seller in the case that the highest marginal revenue is negative, and (3) the expected utility of the bidder with the lowest value is zero.

Finally, the Revenue Equivalence Theorem immediately follows from Lemmas 2 and 3.

COROLLARY 1 (The Revenue Equivalence Theorem, Myerson, 1981). *Both the seller's and the bidders' expected utility from any feasible auction mechanism is completely determined by the allocation rule* p *and the utilities of the lowest types* $U_i(p, x, \underline{v}_i)$ *for all* i *related to its equivalent feasible direct revelation mechanism* (p, x).

From this corollary, it immediately follows that under (A1)-(A8), all standard auctions yield the same expected utility for the seller and the bidders, provided that all bidders play the efficient Bayesian Nash equilibrium. Efficiency implies that the allocation rule is such that it is always the bidder with the lowest value who wins the object, so that the allocation rule is the same for all standard auctions. Moreover, as said, in the efficient equilibrium of all standard auctions, the utility of the bidder with the lowest type is zero.

5.3. Myersonean World vs. Double Coasean World. Two crucial assumptions in Myerson's model are (1) the seller can prevent resale of the object after the auction, and (2) he can fully commit to not selling the object. The first assumption is made, as the seller may need to misassign the object in the case of asymmetric bidders, i.e., when bidders draw their values from different distribution functions. The second assumption is made, as the seller may optimally withhold the object when only low valued bidders participate, for instance by imposing a reserve price. Both assumptions imply that the seller is not a priori restricted in the allocation rule he aims to implement. When these assumptions hold, I will speak of a *Myersonean World*.

Ausubel and Cramton (1999) argue that sometimes the assumptions of a Myersonean World are not realistic, and study optimal auctions in a setting in which (1) the seller cannot prevent the object changing hands in a perfect resale market,[9] and (2) he cannot commit to keeping the object. I will refer to this setting as a *Double Coasean World*, as the first assumption is related to the Coase Theorem (Coase, 1960), and the second to the Coase Conjecture (Coase, 1972).

In a Double Coasean World, when (A1)-(A8) hold, all standard auctions, without a reserve price, are optimal. To see this, observe that the two assumptions impose two extra restrictions on the seller's problem, namely

$$p_i(\mathbf{v}) > 0 \text{ only if } v_i = \max_j v_j, \ \forall \mathbf{v}, i,$$

and

$$\sum_i p_i(\mathbf{v}) = 1, \ \forall \mathbf{v},$$

respectively. In fact, these restrictions fix $p_i(\mathbf{v})$ (apart from the zero mass events $v_i = v_j$ for some i and j). Then, from Lemma 3, any auction that yields zero utility for the lowest type (from the auction plus resale market) is optimal. Haile (1999) proves that, when (A1)-(A8) hold, equilibrium bidding in the standard auctions does not change when bidders are offered a resale market opportunity after the auction, and that there is no trade in the resale market. Hence, all standard auctions are optimal, as Haile's results imply that the utility of the lowest type is zero in all these auctions.

[9]In a perfect resale market, the object, when being sold in the auction, always ends up in the hands of the bidder with the highest value.

6. Overview of the thesis

This Ph.D. thesis is a collection of six papers in auction theory. I present the thesis in this way, as ultimately, the work of a Ph.D. student is not judged by his thesis, but by the publication in international refereed journals of the papers that are based on the thesis. Setting up the thesis like this implies that each chapter contains a paper that is presented in the form as it will be submitted to the journals, so that each chapter can be read independently from the other chapters. In this section, I give a short summary of the papers.

6.1. Auctions with Network Effects. In Chapter 2, I present my paper Onderstal (2002a), in which I study auctions in an environment with network effects. The analysis in this paper is motivated by the auction for licenses for petrol stations which the Dutch government intends to organize. The government's aim is to increase competition in the petrol market. It seems likely that a standard auction will not lead to an economically efficient outcome, because in that case, competition will be decreased as the largest firms in the market will win all the new licenses. There are two reasons why this is likely. First, a decrease in competition will lead to higher profits, so that there is an incentive for large firms to preempt the market. Second, for a large firm, the willingness-to-pay for a petrol station is higher than for a small firm due to network effects. With network effects, I mean that a large firm is ceteris paribus able to gain more profit per outlet.

In order to analyze the auction for licenses for petrol stations, I construct a complete information model with one seller, and several firms which compete in an oligopolistic market. The seller auctions one license which gives the winner the opportunity to increase its capacity in the market. Total market profit is assumed to be increasing in the size of the firm that wins the license. Moreover, the market is characterized by network effects. I consider two auctions, namely the first-price sealed-bid auction, and a take-it-or-leave-it mechanism. The auctions are evaluated in terms of efficiency, revenue maximization, collusion proofness, and ease of implementation. I assume that efficiency requires the largest firm not to win the license, as when the largest firm increases its capacity, competition in the oligopolistic market decreases. I use the emptiness of the α-core (a concept from cooperative game theory) to measure collusion proofness of the studied auctions. Ease of implementation requires the bidders to have strictly dominant strategies in the auction.

In this model, I consider two different settings. First of all, I consider a benchmark setting in which total market profit does not depend on the winner of the extra capacity. Then, in the case of more than two firms, the largest firm wins the license in the first-price sealed-bid auction in every Nash equilibrium. Also, for each firm, I construct a take-it-or-leave-it mechanism with the property that (1) the firm wins the license, (2) each firm plays a dominant strategy, and (3) the mechanism maximizes revenue. In other words, the seller can choose the firm he prefers as the winner of the license, without having to lose in terms of revenue. Finally, I show that both feasible auction mechanisms are collusion proof.

The second setting is the general model. I show that there is always a Nash equilibrium in which the largest firm wins the license. Also, I construct an example in which in equilibrium, another firm than the largest firm wins. However, this equilibrium does not survive iterative deletion of weakly dominated strategies. I conjecture that the largest firm wins in any Nash equilibrium that survives iterative deletion of weakly dominated strategies. Moreover, I find in the general model a conflict between the targets of efficiency and revenue maximization. Finally, I show that the α-core need not be empty.

6.2. Auctions with Financial Externalities. Chapter 3 is the paper Maasland and Onderstal (2002a), which is about sealed-bid auctions in environments with financial externalities. That is, in this paper we consider an environment in which losers' utilities depend on how much the winner pays. The main target of this paper is to study the effect of relaxing assumption (A8) in the standard auction model. To illustrate the model, we will argue that bidders in the UMTS auctions in Europe faced an environment in which financial externalities may have played an important role.

We derive that the first-price sealed-bid auction has a unique symmetric Bayesian Nash equilibrium. In this equilibrium, larger financial externalities result in lower bids and therefore lead to lower expected revenue. The second-price sealed-bid auction fails to have an equilibrium in weakly dominant strategies, but still has a unique symmetric Bayesian Nash equilibrium. In this auction, the effect of financial externalities on both bids and expected revenue are ambiguous. Moreover, we show that a resale market opportunity does not change equilibrium bidding for both auctions. Finally, with two bidders, the first-price sealed-bid auction yields a strictly lower expected revenue than the second-price sealed-bid auction, so that with financial externalities, Table 1 can be completed with the ranking F $\prec$ S if (A8) is relaxed.

We also perform a study of the effect of a reserve price on equilibrium bidding. Before doing so, we define the concept of a *weakly separating Bayesian Nash equilibrium*, which is a Bayesian Nash equilibrium in which bidders having a type below a certain threshold type submit no bid, and bidders with a type above the threshold type submit a bid according to a bid function that is strictly increasing in their type. For the first-price sealed-bid auction, we find that there is no weakly separating Bayesian Nash equilibrium. However, there is a symmetric Bayesian Nash equilibrium that involves pooling at the reserve price. For the second-price sealed-bid auction, we derive a necessary and sufficient condition for the existence of a weakly separating Bayesian Nash equilibrium.

6.3. Optimal Auctions with Financial Externalities. Chapter 4 contains the paper Maasland and Onderstal (2002b), in which we construct optimal auctions in environments with financial externalities.

Using the Revelation Principle, we show that the Revenue Equivalence Theorem remains valid. Using this theorem, we derive several results, both for a Double Coasean World and a Myersonean World. In a Double Coasean World, with financial externalities, both the first-price sealed-bid auction and the second-price sealed-bid auction lose their optimality. We define a new auction type, the lowest-price all-pay auction.[10] This auction has a unique symmetric Bayesian Nash equilibrium, which is efficient. With this equilibrium, the lowest-price all-pay auction is optimal.

In a Myersonean World, even with optimal reserve prices, the first-price sealed-bid auction and the second-price sealed-bid auction are not optimal in the case of financial externalities. This is true for two reasons. First, both auctions give the lowest type strictly positive utility because of the payments by others. Second, an optimal auction, if assigning the object to one of the bidders, is required to assign the object to the bidder with the highest marginal revenue. However, the first-price and the second-price sealed-bid auction may not have equilibria with this property (which was shown in Chapter 3). We construct a two-stage mechanism which we show to be optimal. In the first stage of this mechanism, bidders are asked to pay an entry fee, and in the second stage, bidders play the lowest-price all-pay auction with a reserve price.

6.4. The Chopstick Auction. In Chapter 5, I present Onderstal (2002b), in which I consider the exposure problem. The exposure problem occurs in multiple object auctions in which bidders face the risk of winning too few objects when they try to obtain a valuable set of several objects. As an example of a situation where the exposure problem is present, I discuss the DCS-1800 auction in the Netherlands in which licenses for second generation mobile telecommunication channels were sold. Bidders could basically do two things in this auction. They either could try to win a large license, which would give them enough spectrum to operate a profitable network of second generation mobile telecommunication, or they could try to acquire a set of five or

[10]Independently from us, Goeree and Turner (2001) define the same auction type. The name of this auction type is introduced by them.

six small licenses, which together would be sufficient for a profitable network. However, less than five small licenses would be worthless to them. The auction format was such that bidders faced the exposure problem when they decided to bid on the small licenses.

The main body of the paper consists of a game theoretic model of the exposure problem, called the Chopstick Auction. In the Chopstick Auction, three chopsticks are sold. The price, which is the same for each chopstick, is raised continuously. Bidders have the opportunity to step out at each price, until one bidder is left. This bidder receives two valuable chopsticks, and the second highest bidder one worthless chopstick. Each chopstick is sold for the price at which the second highest bidder left the auction, so that the second highest bidder is a victim of the exposure problem.

We analyze the Chopstick Auction with incomplete information and compare it with the second-price sealed-bid auction in which the three chopsticks are sold as one bundle. The targets of the seller are efficiency and revenue. For two risk neutral bidders, the Chopstick Auction has an efficient equilibrium and is revenue equivalent with the second-price sealed-bid auction. However, if bidders are loss averse, then the Chopstick Auction is either inefficient, or raises less revenue than the second-price sealed-bid auction. In the case of three bidders, the Chopstick Auction has no symmetric equilibrium, so that it probably has no efficient equilibrium, in contrast to the second-price sealed-bid auction.

We conclude that avoiding the exposure problem is an important issue in auction design. More specifically, it would have been in the interest of the Dutch government to have chosen another auction format. I suggest three ways for auction designers to avoid the exposure problem. The first, which follows immediately from the theory, is that the auction designer offers large bundles of objects rather than small ones. A second is a withdrawal rule which gives bidders the opportunity to withdraw their bid when an inefficient lock-in is imminent. Third, the seller can avoid the exposure problem by allowing for combinatorial bids.

6.5. The Effectiveness of Caps on Political Lobbying. Chapter 6 is the paper Matejka, Onderstal, and De Waegenaere (2002). In this paper, we analyze a lobby game, in which interest groups submit bids in order to obtain a political prize. Lobbying is modelled as an all-pay auction, in which the bids are restricted to be below a cap imposed by the government. In an interesting study on lobbying, Che and Gale (1998b) show that a cap "may have the perverse effect of increasing aggregate expenditures and lowering total surplus". However, we will argue that their result is an ex post result, whereas an ex ante view is more appropriate.

We assume that the cap is chosen by the government such that it maximizes social welfare. In deciding the optimal cap, the government needs to make a trade-off between the informational benefits lobbying provides, and the social costs that are associated with the fact that the money spent on lobbying cannot be used for other economic activities. The informational benefits arise when interest groups have the opportunity to separate themselves choosing bids that are contingent on the realization of their value. These informational benefits are higher with a higher cap.

We derive several results, both for an incomplete and a complete information setting. While a lower cap may ex post lead to higher lobbying expenditures, we show that ex ante, a lower cap always implies lower expected total lobbying expenditures. Moreover, we show that under plausible assumptions, the incompletely informed government maximizes social welfare by not allowing for any lobbying activities at all.

6.6. Socially Optimal Mechanisms. Chapter 7 contains the paper Onderstal (2002c), in which I construct socially optimal mechanisms. A mechanism is a game in which an indivisible object is allocated to one player out of a set which consists of several players. A mechanism is defined by three elements; (1) an action space, which for each player indicates which actions she can play in the game, (2) an allocation rule, which indicates to which player the object is allocated given the actions chosen by all players, and (3) a payment rule, which defines how much each player has to pay as a function of the played actions. By a socially optimal mechanism I mean a mechanism that maximizes social welfare, which is assumed to be equal to the sum of the players' expected utility (this in contrast to an optimal *auction*, in which the *seller*'s utility is maximized).

In the search for a socially optimal mechanism, instead of calculating social welfare using equilibrium bidding, I use an indirect approach, based on the Revelation Principle. I show that a lottery among the players with the highest expected value for the object maximizes social welfare.

I illustrate the model and the main result with examples from the contest literature. My finding implies that players in a large range of contests have an incentive to collude. For instance, Schmalensee (1976) argues that in markets with a few sellers and differentiated products, competition among firms mainly takes place through promotional expenditures rather than through prices. Competition in these markets has a structure similar to an all-pay auction. My finding suggests that in such markets, competitors optimally agree not to advertise at all. Another interpretation of my result is that interest groups maximize their total utility if they agree not to spend money in lobbying. Moreover, politicians optimally agree among themselves not to spend any money in political campaigns. Finally, my result shows that collusion is profitable in auctions.

7. Conclusion

Summarizing, I would argue that the contribution of this thesis to auction theory is threefold. First of all, I study several new auction models. I introduce new auction types (such as the chopstick auction and the lowest-price all-pay auction), and I investigate standard auction types in non-standard environments (an environment with network externalities, and an environment with financial externalities). Secondly, I illustrate the developed theories with examples of real-life auctions that took place (such as the DCS-1800 auction and the UMTS auctions), or that may take place in the future (the auction for petrol stations along the Dutch highways). Finally, I contribute by paying attention to phenomena that have features in common with auctions (lobbying, advertising, and political campaigns).

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CHAPTER 2

Auctions with Network Effects

1. Introduction

In February 1999, an MDW-study group advised the Dutch government that the market for petrol along the Dutch highways lacks serious competition.[1] The market is characterized by high levels of market concentration with a Herfindahl-Hirschman Index of 3135, and a total market share of the largest four firms equal to 75%. Moreover, the margin on a liter petrol is higher than in surrounding countries as shown in Table 2. The lack of competition in the market worried the study group, and one of the suggestions to the Dutch government was to auction new licenses for petrol stations in order to let new firms enter the market, or give small firms the opportunity to grow.

The Netherlands	Belgium	Germany	France	United Kingdom
0.14	0.14	0.12	0.09	0.05

Table 2. Profit margins on petrol as a fraction of the price, measured in 1996. Source: Coopers & Lybrand (1996). "Investigation on the Price Structure of Euro 95 and Diesel Oil in The Netherlands, Belgium, Germany, France and Great Britain." (In Dutch).

The study group conjectured that a standard auction will not lead to an economically efficient outcome, because competition will be decreased as the largest firms in the market (Shell, Esso, BP and Texaco) will win all the new licenses. There are two reasons why this is likely. First, because of network effects, the willingness-to-pay for a license is higher for a large firm than for a small firm. With network effects we

[1]The report, which is in Dutch, is available on internet at http://www.ez.nl/publicaties/pdfs/11B88.pdf.

mean that a large firm is ceteris paribus able to gain more profit per outlet than a small firm. In the petrol market, the network effects are probably due to loyalty schemes or advertising, which are both more effective for large firms than for small. Second, a decrease in competition will lead to higher profits, so that there is an incentive for large firms to preempt the market, i.e., to buy licenses with the aim of preventing new competitors to enter the market. The economic literature suggests that in standard auctions, the chances for small firms to win capacity are small. For instance, Gilbert and Newbery (1982) show that monopoly persists when the incumbent monopolist and a potential newcomer compete to get a patent. In a related study, Jehiel and Moldovanu (2000a) find that when both incumbents and potential entrants bid for several licenses, all licenses will be sold to incumbents in case the number of incumbents exceeds the number of licenses, or all incumbents acquire a license if the number of licenses exceeds the number of incumbents.[2]

We performed an empirical analysis in order to test for the presence of network effects in the petrol market along the Dutch highways. We modelled the sales $F(n)$ per passing vehicle per petrol station of a firm with n petrol stations located at the Dutch highways with the following expression.

$$F(n) = \alpha + \beta n + \sum_i \gamma_i x_i + \eta.$$

The x_i's are the characteristics of the petrol station, such as location with respect to the nearest city, local competition, and facilities at the site. α, β, and the γ_i's are one-dimensional parameters, and η is a disturbance term for which the standard OLS assumptions are assumed to apply. Using data from petrol stations along the Dutch highway, we

[2]However, Krishna (1993, 1999) finds that newcomers may be able to beat incumbents when licenses are auctioned sequentially.

estimated β to be significantly larger than 0. We concluded that indeed network effects are present in the petrol market.[3,4]

The aim of this paper is to answer two questions for an environment with network effects. First, does the largest firm in the market win a license when the license is sold in the first-price sealed-bid auction? Second, is there a feasible auction mechanism which implements four targets in an environment with network effects, namely (1) the mechanism guarantees an economically efficient outcome, (2) it generates as much revenues as possible for the government, (3) it is not sensitive to collusion, and (4) it is easy to implement?

We will answer these questions in a complete information model, which is given in Section 2. We assume that there is a seller, who desires to sell a license to one firm out of a set of several firms, which compete in an oligopolistic market characterized by network effects. The firm which acquires the license imposes a negative externality on all its competitors by stealing part of their market share. In order to incorporate the network effects, the profit per outlet for a given firm is increasing in its total number of outlets. The size of the "pie" (total market profits) to be divided among the firms depends on which firm gets the license. We assume that the size of the pie is increasing in the size of the winning firm.

In this model, the four targets are formalized as follows. First of all, we do not explicitly calculate a measure for efficiency of the feasible auction mechanism. Instead, we assume that efficiency requires that the largest firm does not win the license. Secondly, the revenue target is straightforward: the feasible auction mechanism should maximize revenue over all feasible auction mechanisms. Thirdly, following Jehiel and Moldovanu (1996), we use the emptiness of the α-core as a measure

[3]The study was performed as part of the project of the Ministry of Finance, and can also be found in the report, which can be found on the internet at http://www.ez.nl/publicaties/pdfs/11B88.pdf.

[4]There are several other empirical studies on network effects. Gandal (1994) observes that network effects exist in the market for computer spreadsheet programs. In a study on the adoption by banks of automated teller machines, Saloner and Shepard (1995) find network effects in the bank sector.

for collusion proofness of the studied feasible auction mechanisms. Finally, a feasible auction mechanism is easy to implement if the bidding firms play a dominant strategy.

In Section 3, we discuss the outcomes of the model in the pure network effects case, i.e., when the size of the pie does not depend on which firm wins the license. With three or more firms, the largest firm wins the license in the first-price sealed-bid auction in any Nash equilibrium, so that the outcome of the auction is not efficient. However, we find that for each firm i, there exist a feasible auction mechanism in which firm i wins the license, and which implements (almost) revenue maximization in strictly dominant strategies. This feasible auction mechanism is a take-it-or-leave-it mechanism, in which the seller, when a firm chooses not to participate, assigns the license to the firm that imposes the worst threat on it (in terms of lost market share). Finally, as the α-core is empty, the take-it-or-leave-it mechanism is collusion proof, so that the four targets are reached.

In Section 4, we study the market when the size of the pie varies with the identity of the winner of the license. There is an equilibrium in which the largest firm wins the license. However, we construct an example in which it is not the largest firm who wins the license in the first-price sealed-bid auction. But, in equilibrium, the second highest bidder submits a bid that exceeds its willingness-to-pay against the winner, and we will show that this equilibrium can be excluded by iterated deletion of weakly dominated strategies. We conjecture that only Nash equilibria exist in which the largest firm wins survive iterated deletion of weakly dominated strategies, which implies that the outcome of the first-price sealed-bid auction is not efficient. Moreover, we show that any feasible auction mechanism that (almost) maximizes revenue, must necessarily assign the license to the largest firm, so that in our model all four targets cannot be reached at once, as there is a conflict between efficiency and revenue maximization. Also, we find that the α-core need not be empty, so that there need not exist a collusion-proof auction mechanism.

2. The model

A seller owns a license for an outlet in an oligopolistic market with n incumbent firms, labeled $1, ..., n$. Let

$$N \equiv \{1, ..., n\}$$

denote the set of firms. We will use i, j, k and l to represent typical firms in N. If the market situation is such that firm j has m_j outlets in the market, $j = 1, ..., n$, then firm i's profit is given by

$$\Pi_i(\mathbf{m}) \equiv S_i(\mathbf{m}) * P(\mathbf{m})$$

with $\mathbf{m} \equiv (m_1, ..., m_n)$ the vector of number of outlets, $P(\mathbf{m})$ total market profits, and $S_i(\mathbf{m})$ firm i's profit share.

We assume that firm i's profit share is given by

$$S_i(\mathbf{m}) \equiv \frac{f(m_i)}{\sum_{l=1}^{n} f(m_l)}, \tag{2.1}$$

where f has the following properties.

$$f(0) = 0, \tag{2.2}$$

$$f(m_i) > f(m_j) \quad \text{if } m_i > m_j, \text{ and} \tag{2.3}$$

$$f(m_i + 1) - f(m_i) > \ f(m_j + 1) - f(m_j) \quad \text{if } m_i > m_j \tag{2.4}$$

Equation (2.2) indicates that a firm with no outlets in the market makes no profit, (2.3) indicates that profit for a firm is increasing in the number of its outlets, and (2.4) is a convexity condition on f.

Equations (2.1)-(2.4) are sufficient to establish network effects in the market in the sense that the profit per outlet for a firm is increasing in the total number of outlets the firm has. Proposition 1 shows that (2.1)-(2.4) imply that profit per outlet is increasing in the number of outlets.

PROPOSITION 1. *Suppose that (2.1)-(2.4) are satisfied. If* $m_i > m_j$, *then* $\Pi_i(\mathbf{m})/m_i > \Pi_j(\mathbf{m})/m_j$.

PROOF. Let $m_i > m_j$. Then the result follows immediately with the following observation.

$$\begin{aligned}
\frac{f(m_i)}{m_i} &= \frac{1}{m_i}\sum_{h=1}^{m_i}[f(h)-f(h-1)] \\
&> \frac{1}{m_i}\sum_{h=1}^{m_j}[f(h)-f(h-1)] + \frac{m_i-m_j}{m_i}[f(m_j)-f(m_j-1)] \\
&> \frac{1}{m_i}\sum_{h=1}^{m_j}[f(h)-f(h-1)] + \frac{m_i-m_j}{m_i m_j}\sum_{h=1}^{m_j}[f(h)-f(h-1)] \\
&= \frac{1}{m_j}\sum_{h=1}^{m_j}[f(h)-f(h-1)] \\
&= \frac{f(m_j)}{m_j}.
\end{aligned}$$

□

The seller plans to sell the license using an auction. Let $\bar{m}_j$ denote the number of outlets firm j has in the market before the license is sold. We assume $\bar{m}_1 > \bar{m}_2 > ... > \bar{m}_n$.[5] Let $\bar{\mathbf{m}} \equiv (\bar{m}_1, ..., \bar{m}_n)$ and $\mathbf{e}_i$ be the vector with the ith entry equal to 1, and the other entries equal to zero. We make the following assumption on P.

$$P(\bar{\mathbf{m}} + \mathbf{e}_i) \geq P(\bar{\mathbf{m}} + \mathbf{e}_j) \text{ if } \bar{m}_i > \bar{m}_j.$$

In words: the larger the firm that wins the license the larger total market profits. Define

$$U_i(j) \equiv \Pi_i(\bar{\mathbf{m}} + \mathbf{e}_j) - \Pi_i(\bar{\mathbf{m}})$$

as the utility of firm i when firm j wins the license. The willingness-to-pay for a firm i depends on which firm is considered by firm i as its opponent. We will say that firm i is willing to pay a specific amount "against" firm j, where the willingness-to-pay is given by the difference in utility for firm i when it gets the license, and when firm j gets the license, i.e., $U_i(i) - U_i(j)$.

[5]This is with some loss of generality. The nongeneric cases can be treated in a completely analogous way, but the case differentiation is more tedious.

In an auction, each firm i simultaneously and independently submits a bid $b_i \in B_i$, where B_i is the set of bids for firm i. In particular, we will study the first-price sealed-bid auction and a take-it-or-leave-it mechanism. In case the auction is the first-price sealed-bid auction, we assume the sets of potential bids B_i to have the form

$$B_i = \{0, \epsilon, 2\epsilon, ...\}$$

with ϵ the smallest money unit,[6] where ϵ is very small relative to all the other parameters. In the take-it-or-leave-it mechanism, B_i has the form

$$B_i = \{\text{“participate”},\ \text{“not participate”}\},$$

which indicates that each firm can either participate or not.

An auction has the following outcome functions

$$\widehat{p} : B_1 \times ... \times B_n \to [0,1]^n$$

with

$$\sum_j \widehat{p}_j(b_1, ..., b_n) \leq 1,$$

and

$$\widehat{x} : B_1 \times ... \times B_n \to \Re^n.$$

If $\mathbf{b} = (b_1, ..., b_n)$, then $\widehat{p}_i(\mathbf{b})$ is interpreted as the probability that firm i gets the license, and $\widehat{x}_i(\mathbf{b})$ is the expected payment of firm i to the seller. For simplicity, we assume that the $\widehat{x}_i$'s are multiples of ϵ. When firm i chooses "not participate" in the take-it-or-leave-it mechanism, $\widehat{p}_i(\mathbf{b}) = \widehat{x}_i(\mathbf{b}) = 0$. We refer to this assumption as the "no-dumping assumption". The firms are risk neutral and have additively separable utility function for money and the allocation of the object, so that if $\mathbf{b}$ is submitted, firm i's utility is given by

$$\sum_{l=1}^{n} \widehat{p}_l(\mathbf{b}) U_i(l) - \widehat{x}_i(\mathbf{b}).$$

[6]Following Jehiel and Moldovanu (1996), we make this assumption in order to avoid problems related to the existence of a Nash equilibrium in pure strategies.

A *strategy* for a firm i is the choice of a bid (or a randomization over several bids) from the set B_i. A *feasible auction mechanism* is an auction including strategies, which form a Nash equilibrium of the auction. An *optimal auction* is a feasible auction mechanism that gives the seller the highest expected revenue. An *almost optimal auction* is a feasible auction mechanism that gives the seller the highest expected revenue minus at most $n\epsilon$. We say that an (almost) optimal auction is *dominant strategy implementable* if each firm plays a strictly dominant strategy. Dominant strategy implementation implies that the auction game is easy to play by a firm, as its optimal bid does not depend on the strategies of the other firms.

Following Jehiel and Moldovanu (1996), we use the concept of α-*core* from cooperative game theory to define collusion proofness of the studied feasible auction mechanisms. The α-core is the core of the α-game, which is a TU game in which the player set consists of the seller and the n firms. The characteristic function is for each coalition defined as the maximal utility the coalition is able to obtain under the assumption that the complement takes the worst action against the coalition.

Formally, let player 0 denote the seller, and let $v : 2^{\{0\}\cup N} \to \Re$ be the characteristic function of the α-game. Let $S \subseteq \{0\} \cup N$. We distinguish two situations, namely $0 \in S$, and $0 \notin S$. In the case $0 \in S$, the complement has no options available, and the best thing the coalition S can do is transfer the license to the firm which maximizes total utility of the firms in S, so that $v(S) \equiv \max_{i \in S} \sum_{j \in S \setminus \{0\}} U_j(i)$. In the case that $0 \notin S$, the worst action the complement can take is assign the license to firm $i \notin S$ that imposes the "worst threat" on the firms in S. Hence, $v(S) \equiv \min_{i \notin S} \sum_{j \in S} U_j(i)$. Then $x \equiv (x_0, x_1, ..., x_n) \in \Re^{n+1}$ is an element of the α-core if and only if

$$\sum_{j \in S} x_j \geq v(S)$$

for all $S \subseteq \{0\} \cup N$, and

$$\sum_{j \in \{0\} \cup N} x_j = v(\{0\} \cup N).$$

Each feasible auction mechanism is called *collusion-proof* if the α-core is empty. We use the emptiness of the α-core as a measure for collusion-proofness as it indicates that no cooperative agreement is stable against a deviation from a coalition. An implicit assumption that we will make throughout the paper is that the seller has complete commitment power, in the sense that he is able commit to any feasible auction mechanism he desires. We have to make this assumption, as the emptiness of the α-core suggests that such commitment is not stable. The strength of the α-core lies in the fact that it is the least sharp core concept, so that if the α-core is empty, other cores are empty as well (Jehiel and Moldovanu, 1996).

3. Constant total market profits

Suppose that total market profit is constant, i.e., total market profit does not depend on the distribution of the outlets over the firms. Without further loss of generally, we assume

$$P \equiv 1.$$

Before we establish equilibrium bidding in the first-price sealed-bid auction, we derive two useful lemmas. Lemma 4 indicates that in the case of three or more firms, each firm gets more utility when a small competitor wins than when a large competitor wins. Lemma 5 shows that firm 1 is always willing to pay more against firm $i \neq 1$ (as its willingness-to-pay is given by $U_1(1) - U_1(i)$), than firm i is willing to pay against firm 1 (as its willingness-to-pay is given by $U_i(i) - U_i(1)$).

LEMMA 4. *Let $n \geq 3$. For all $i, j, k \in N$, $i \neq j \neq k \neq i$, with $\bar{m}_j < \bar{m}_k$,*

$$U_i(j) > U_i(k).$$

PROOF. Let $i, j, k \in N$, $i \neq j \neq k \neq i$, with $\bar{m}_j < \bar{m}_k$. By (2.4),

$$f(\bar{m}_k) + f(\bar{m}_j + 1) < f(\bar{m}_k + 1) + f(\bar{m}_j)$$

so that

$$\frac{f(\bar{m}_i)}{f(\bar{m}_k) + f(\bar{m}_j + 1) + \sum\limits_{l \neq k,j} f(\bar{m}_l)} > \frac{f(\bar{m}_i)}{f(\bar{m}_k + 1) + f(\bar{m}_j) + \sum\limits_{l \neq k,j} f(\bar{m}_l)}$$

which implies

$$\Pi_i(\bar{\mathbf{m}} + \mathbf{e}_j) - \Pi_i(\bar{\mathbf{m}}) > \Pi_i(\bar{\mathbf{m}} + \mathbf{e}_k) - \Pi_i(\bar{\mathbf{m}})$$

which is by definition equivalent to

$$U_i(j) > U_i(k).$$

□

LEMMA 5. *Let $n \geq 3$. For all $i \in N \backslash \{1\}$,*

$$U_i(i) - U_i(1) < U_1(1) - U_1(i).$$

PROOF. Let $i \in N \backslash \{1\}$. By (2.4),

$$f(\bar{m}_1) + f(\bar{m}_i + 1) < f(\bar{m}_1 + 1) + f(\bar{m}_i)$$

or, equivalently, with (2.2) and (2.3)

$$\frac{1}{1 + \sum\limits_{l \neq 1,i} f(\bar{m}_l)/[f(\bar{m}_1) + f(\bar{m}_i + 1)]} < \frac{1}{1 + \sum\limits_{l \neq 1,i} f(\bar{m}_l)/[f(\bar{m}_1 + 1) + f(\bar{m}_i)]}.$$

With some manipulation we obtain

$$\Pi_i(\bar{\mathbf{m}} + \mathbf{e}_i) - \Pi_i(\bar{\mathbf{m}} + \mathbf{e}_1) < \Pi_1(\bar{\mathbf{m}} + \mathbf{e}_1) - \Pi_1(\bar{\mathbf{m}} + \mathbf{e}_i)$$

which is equivalent to

$$U_i(i) - U_i(1) < U_1(1) - U_1(i).$$

□

Proposition 2 shows that when the seller sells the license using the first-price sealed-bid auction, in case of three or more firms, firm 1 wins the license in *any* Nash equilibrium. In the case of two firms, there is an equilibrium in which each firm wins with probability $\frac{1}{2}$. The first part of the proposition follows intuitively from Lemma 5, as in the case that firm 1 and any firm $i \neq 1$ are in direct competition, firm 1 is prepared to pay more for the license than firm i.

PROPOSITION 2. *Suppose $P \equiv 1$. Let $n \geq 3$. Then, in any Nash equilibrium of the first-price sealed-bid auction, firm 1 wins the license. If $n = 2$, then there is a Nash equilibrium in which each firm wins with probability $\frac{1}{2}$.*

PROOF. Let $n \geq 3$. Let $(p_1, ..., p_n)$ denote a Nash equilibrium. We prove the proposition by contradiction. In order not to perform a tedious case differentiation, we suppose the following holds for some $i \in N \backslash \{1\}$, some $j \in N \backslash \{i\}$ and all $l \in N \backslash \{i, j\}$ (other cases proceed in an analogous way).

$$p_i > p_j > p_l.$$

If these strategies are played, another firm than firm 1 wins the auction. These bids constitute a Nash equilibrium if

$$U_i(i) - p_i \geq U_i(j) \tag{3.1}$$

and

$$U_k(i) \geq U_k(k) - p_i \text{ for every } k \in N \backslash \{i\} \tag{3.2}$$

are satisfied. Condition (3.1) indicates that firm i has no incentive to submit a bid strictly lower than p_j. Condition (3.2) indicates that none of the firms other than i is willing to overbid the bid of i.

The contradiction is established, as (3.1) and (3.2) imply

$$\begin{aligned} p_i &\leq U_i(i) - U_i(j) \\ &\leq U_i(i) - U_i(1) \\ &< U_1(1) - U_1(i) \\ &\leq p_i \end{aligned}$$

where the second inequality follows from Lemma 4, and the third inequality from Lemma 5. A similar argument establishes that there is no equilibrium involving mixed strategies, in which a firm other than firm 1 wins the license with strictly positive probability.

For $n = 2$, it is readily checked that a Nash equilibrium is established when both firms submit a bid equal to

$$\frac{f(\bar{m}_1+1)}{f(\bar{m}_1+1)+f(\bar{m}_2)} - \frac{f(\bar{m}_1)}{f(\bar{m}_1)+f(\bar{m}_2+1)}.$$

□

Consider for each $i \in N$, take-it-or-leave-it mechanism M^i, which has the following rules.[7] The firms simultaneously decide whether to participate or not. Let $T \subseteq N$ denote the set of firms who decide to participate, and let

$$w(j) \in \arg\min_{k \neq j} U_j(k)$$

denote the firm that imposes the "worst threat" on firm j. For each $T \subseteq N$, we define the winner of the license and the payments made to/by the seller.

(1) If $|T| \leq n-2$, then the seller keeps the license, and all firms who participate receive ϵ.

(2) If $T = N \backslash \{k\}$, then the winner is $w(k)$. Each firm $j \in T$ is required to pay $U_j(w(k)) - \epsilon$ (where a negative amount indicates that firm j receives money from the seller).

(3) If $T = N$, then the seller gives the license to firm i. Each firm j is required to pay $U_j(i) - U_j(w(j)) - \epsilon$.

Proposition 3 shows that implementing a feasible auction mechanism which results in another firm than firm 1 to win the license does not necessarily lead to a loss in revenue compared to the optimal auction in which firm 1 wins. The intuition behind Proposition 3 is the following. First of all, each take-it-or-leave-it mechanism M^i is defined such that the seller transfers/asks money to/from each participating firm such that the firm's utility is ϵ higher compared to the situation in which it does not participate, so that for each firm participation is a strictly dominant strategy. Secondly, if all firms participate, each firm j pays $U_j(i) - U_j(w(j)) - \epsilon$ to the seller. The sum of the utilities of all

[7]This take-it-or-leave-it mechanism, in a somewhat different form, was introduced by Jehiel et al. (1996) in a situation with negative externalities.

firms from the allocation of the license to firm i is zero, as market profit remains constant. Therefore, $\sum_j U_j(i) = 0$ for all i. As $U_j(w(j))$ does not depend on i, total payments to the seller do not depend on i as well. Finally, $U_j(i) - U_j(w(j))$, is the maximal willingness to pay for firm j given that firm i wins the license, so that $\sum_j \{U_j(i) - U_j(w(j)) - \epsilon\}$ is the highest possible revenue from any mechanism minus $n\epsilon$.

PROPOSITION 3. *Suppose $P \equiv 1$. Then for each $i \in N$, M^i is a feasible auction mechanism that (1) assigns firm i the license, (2) is dominant strategy implementable, and (3) is almost optimal.*

PROOF. We start by showing that for each firm, participation in M^i is a strictly dominant strategy. Consider firm j, and assume that the firms in $N\backslash\{j\}$ play a pure strategy profile such that the set of participating firms is a set $T' \subseteq N\backslash\{j\}$. There are three possible cases.

(1) $T' = N\backslash\{j\}$. If firm j does not participate, its utility is $U_j(w(j))$. If j participates, then its utility equals

$$U_j(i) - [U_j(i) - U_j(w(j)) - \epsilon] = U_j(w(j)) + \epsilon.$$

(2) $T' = N\backslash\{j,k\}$. If firm j does not participate, its utility is 0. If j participates, then the license is allocated to $w(k)$, and j is required to pay $U_j(w(k)) - \epsilon$. Its utility equals

$$U_j(w(k)) - [U_j(w(k)) - \epsilon] = \epsilon.$$

(3) $|T'| < n - 2$. If firm j does not participate, its utility is 0. If j participates, then its utility equals ϵ.

Participation is strictly better for firm j than nonparticipation in each of the three possible cases. Therefore, participation is a strictly dominant strategy.

The seller's revenue of M^i when all firms play their dominant strategy is given by

$$\begin{aligned} R(M^i) &= \sum_{j=1}^{n}[U_j(i) - U_j(w(j)) - \epsilon] \\ &= -\sum_{j=1}^{n} U_j(w(j)) - n\epsilon \\ &\equiv R. \end{aligned}$$

The first equation follows as

$$\begin{aligned}\sum_{j=1}^{n} U_j(i) &= \sum_{j=1}^{n} \Pi_j(\bar{\mathbf{m}}+\mathbf{e}_i) - \sum_{j=1}^{n} \Pi_j(\bar{\mathbf{m}}) \\ &= \sum_{j=1}^{n} S_j(\bar{\mathbf{m}}+\mathbf{e}_i) * P(\bar{\mathbf{m}}+\mathbf{e}_i) - \sum_{j=1}^{n} S_j(\bar{\mathbf{m}}) * P(\bar{\mathbf{m}}) \\ &= \sum_{j=1}^{n} S_j(\bar{\mathbf{m}}+\mathbf{e}_i) - \sum_{j=1}^{n} S_j(\bar{\mathbf{m}}) \\ &= 0\end{aligned}$$

by definition of the S_j's and the assumption $P \equiv 1$.

The highest expected revenue the seller can obtain given that firm i wins the license, is given by

$$\begin{aligned}\sum_{j=1}^{n} [U_j(i) - U_j(w(j))] &= -\sum_{j=1}^{n} U_j(w(j)) \\ &= R + n\epsilon.\end{aligned}$$

This expression follows immediately when taking into account that for each firm j the maximal willingness-to-pay for having firm i win the license is $U_j(i) - U_j(w(j))$. Therefore, the expected revenue from M^i is at most $n\epsilon$ lower than from any other feasible auction mechanism. □

The first-price sealed-bid auction and the take-it-or-leave-it mechanisms M^i are collusion proof. This follows immediately from the following proposition, which shows that the α-core is empty. We prove this proposition by contradiction. Each coalition consisting of the seller and one the firms obtains strictly positive utility when the seller transfers the license to the firm. Moreover, the coalition consisting of all firms, but without the seller, can obtain at most zero utility, as the license cannot be transferred, and the seller has no "threat" available, because by the no-dumping assumption, he cannot transfer the license to one of the firms. These two properties together imply that the grand coalition should get a strictly positive payment. However, the grand coalition gets at most zero utility, as total market profits are assumed to be independent of whom owns the license, so that a contradiction is established.

PROPOSITION 4. *Suppose $P \equiv 1$. Then the α-core is empty.*

PROOF. The characteristic function has the following properties. The best a coalition of the seller and one of the firms can do is transfer the license from the seller to the firm. Therefore, for all $i = 1, ..., n$,

$$v(\{0, i\}) = U_i(i).$$

The coalition of all firms yields zero utility, as the worst threat of the seller against this coalition is to keep the license (in fact, by the no-dumping assumption, the seller has no other options), so that

$$v(N) = 0.$$

Moreover, the grand coalition gets zero utility, as total market profit is 1 regardless who (the seller or one of the firms) owns the license. Therefore

$$v(\{0\} \cup N) = 0.$$

We prove the proposition by contradiction. Suppose that the α-core is not empty. Let $x \equiv (x_0, x_1, ..., x_n)$ be an element of the α-core. The following inequalities must hold.

$$x_0 + x_i \geq v(\{0, i\}) = U_i(i),\ i \in N, \tag{3.3}$$

$$\sum_{i=1}^{n} x_i \geq v(N) = 0, \tag{3.4}$$

and

$$x_0 + \sum_{i=1}^{n} x_i = v(\{0\} \cup N) = 0. \tag{3.5}$$

Equation (3.3) implies, by adding over all $i = 1, ..., n$,

$$nx_0 + \sum_{i=1}^{n} x_i \geq \sum_{i=1}^{n} U_i(i).$$

Then, using (3.4), we get

$$nx_0 + n\sum_{i=1}^{n} x_i \geq \sum_{i=1}^{n} U_i(i)$$

or, equivalently

$$x_0 + \sum_{i=1}^{n} x_i \geq \frac{1}{n} \sum_{i=1}^{n} U_i(i). \tag{3.6}$$

It is easily checked that (2.3) implies that $U_i(i) > 0$ for all $i \in N$. But then (3.6) contradicts (3.5). Therefore, the α-core must be empty. □

4. General total market profits

Assume that the size of the pie may vary with the identity of the winner of the license. We assume

$$P(\bar{\mathbf{m}} + \mathbf{e}_1) > P(\bar{\mathbf{m}} + \mathbf{e}_j) \text{ for all } j \neq 1, \tag{4.1}$$

so that when firm 1 wins the license, total market profits are maximized.[8]

It seems very likely that firm 1 wins the first-price sealed-bid auction, given the fact that firm 1 wins the license in any Nash equilibrium in the case that total market profits are constant. Indeed, according to Proposition 5, there is at least one Nash equilibrium in which firm 1 wins the license.

PROPOSITION 5. *Suppose (4.1) holds true. There is a Nash equilibrium of the first-price sealed-bid auction in which firm 1 wins the license.*

PROOF. Let p_i be the bid for firm $i \in N$. It is readily checked that the following bids constitute a Nash equilibrium in which firm 1 wins the license.

$$\begin{aligned} p_1 &= U_2(2) - U_2(1), \\ p_2 &= p_1 - \epsilon, \text{ and} \\ p_k &= 0,\ k \in N \backslash \{1, 2\}. \end{aligned}$$

□

[8]This assumption is with some loss of generality, as the non-generic case $P(\bar{\mathbf{m}} + \mathbf{e}_1) = P(\bar{\mathbf{m}} + \mathbf{e}_2)$ is excluded.

In the case of two firms, firm 1 always wins the license. The reason is that the willingness to pay for firm 1 against firm 2 is strictly larger than the willingness to pay for firm 2 against firm 1.

PROPOSITION 6. *Suppose (4.1) holds true, and $n = 2$. Then, in any Nash equilibrium of the first-price sealed-bid auction, firm 1 wins the license.*

PROOF. The proof is by contradiction. Let p_1 and p_2 be the bids of firm 1 and firm 2 in a Nash equilibrium, with $p_2 \geq p_1$. This is an equilibrium only if

$$U_1(2) \geq U_1(1) - p_2,$$

and

$$U_2(2) - p_2 \geq U_2(1).$$

Note that the first (second) condition refers to firm 1 (2) having no incentive to deviate to a bid above p_2 (below p_1). These conditions together imply that an equilibrium in which firm 2 wins only exists if

$$U_1(1) - U_1(2) \leq U_2(2) - U_2(1). \tag{4.2}$$

However, by assumption,

$$P(\bar{m} + e_1) > P(\bar{m} + e_2)$$

so that

$$P(\bar{m}+e_1)(S_1(\bar{m}+e_1)+S_2(\bar{m}+e_1)) > P(\bar{m}+e_2)(S_1(\bar{m}+e_2)+S_2(\bar{m}+e_2))$$

as $S_1 + S_2 = 1$ by definition. Then, with some straightforward manipulation,

$$U_1(1) - U_1(2) > U_2(2) - U_2(1),$$

which contradicts (4.2). □

However, in Example 1, we find an equilibrium of the first-price sealed-bid auction, in which it is not the largest firm that wins the license.

EXAMPLE 1. *Consider the market with three incumbent firms and a potential entrant, with* $\bar{m}_1 = 3$, $\bar{m}_2 = 2$, $\bar{m}_3 = 1$, *and* $\bar{m}_4 = 0$. *The status quo total market profit equals 1, i.e.,*

$$P(\bar{\mathbf{m}}) = 1.$$

When the two largest firms acquire the license, total market profit remains at the initial level, so that

$$\begin{aligned} P(\bar{\mathbf{m}} + \mathbf{e}_1) &= 1, \text{ and} \\ P(\bar{\mathbf{m}} + \mathbf{e}_2) &= 1. \end{aligned}$$

When the smallest incumbent firm gets the license, the "pie" will shrink to $\frac{13}{15}$, *i.e.,*

$$P(\bar{\mathbf{m}} + \mathbf{e}_3) = \frac{13}{15}.$$

If the potential entrant obtains the license, no profit will be made in the market, so that

$$P(\bar{\mathbf{m}} + \mathbf{e}_4) = 0.$$

The other relevant parameters are given by

$$\begin{aligned} f(0) &= 0, \\ f(1) &= 7, \\ f(2) &= 15, \\ f(3) &= 24, \text{ and} \\ f(4) &= 34. \end{aligned}$$

Let $WTP(i,j)$ *denote how much firm* i *is willing to pay for the license in order to prevent firm* j *from winning it. By definition,* $WTP(i,j)$ *can be written as*

$$WTP(i,j) \equiv U_i(i) - U_i(j).$$

We find that

$$\begin{aligned} WTP(1,2) &= \frac{34}{34+15+7} - \frac{24}{24+24+7} \approx 0.17, \\ WTP(2,3) &= \frac{24}{24+24+7} - \frac{13}{15} * \frac{15}{24+15+15} \approx 0.20, \\ WTP(3,2) &= \frac{13}{15} * \frac{15}{24+15+15} - \frac{7}{24+24+7} \approx 0.11, \text{ and} \\ WTP(3,4) &= \frac{13}{15} * \frac{15}{24+15+15} \approx 0.24. \end{aligned}$$

Suppose that firm i bids p_i. Then it is readily verified that the following set of bids establish a Nash equilibrium.

$$\begin{aligned} p_1 &= p_4 = 0, \\ p_2 &= WTP(1,2) + \epsilon, \text{ and} \\ p_3 &= WTP(1,2). \end{aligned}$$

Note that the equilibrium which we derive in Example 1 can be excluded by iterated deletion of weakly dominated strategies. Firm 3 seems to play a rather foolish strategy in the sense that it is bidding much more that it would be willing to bid in order to prevent firm 2 from winning the license, but it does not play a dominated strategy. This follows from the fact that firm 3's willingness-to-pay against firm 4 is higher than its equilibrium bid. However, all firm 4's bids above zero are weakly dominated by a bid of zero, so that firm 3's strategy is deleted in the second iteration of deletion of weakly dominated strategies.

Example 1 and Propositions 2 and 6 together justify the following conjecture.

CONJECTURE 1. *Suppose (4.1) holds true. Then in any Nash equilibrium of the first-price sealed-bid auction that survives iterated deletion of weakly dominated strategies, the largest firm wins the license.*

Proposition 7 shows that, in contrast to the situation with constant total market profits, there is no (almost) optimal auction in which another firm than firm 1 wins the license. The proof follows the following lines. The maximal payment a firm j is willing to make given that firm i wins is given by $U_j(i) - U_j(w(j))$, so that the maximal revenue for the seller, under the restriction that firm i wins the license, equals $\sum_{j=1}^{n}[U_j(i) - U_j(w(j))]$. We show that the highest possible revenue given that firm 1 wins is strictly higher than the highest possible revenue given that any other firm wins. We finish the proof by showing that take-it-or-leave-it mechanism M^1, which is defined above and which assigns the license to firm 1, is almost revenue maximizing.

PROPOSITION 7. *Suppose (4.1) holds true. Only for $i = 1$ there exists a feasible auction mechanism that (1) assigns firm i the license, (2) is an (almost) optimal auction. The maximal expected revenue equals*

$$P(\bar{\mathbf{m}} + \mathbf{e}_1) - \sum_{j=1}^{n} \Pi_j(\bar{\mathbf{m}} + \mathbf{e}_{w(j)}). \tag{4.3}$$

where

$$w(j) \in \arg\min_{k \neq j} U_j(k)$$

denotes the firm that imposes the "worst threat" on firm j.

PROOF. The highest expected revenue the seller can obtain given that firm $i \neq 1$ wins the license, is given by

$$\begin{aligned}
\sum_{j=1}^{n}[U_j(i) - U_j(w(j))] &= \sum_{j=1}^{n}[\Pi_j(\bar{\mathbf{m}} + \mathbf{e}_i) - \Pi_j(\bar{\mathbf{m}} + \mathbf{e}_{w(j)})] \\
&= \sum_{j=1}^{n}[S_j(\bar{\mathbf{m}} + \mathbf{e}_i) * P(\bar{\mathbf{m}} + \mathbf{e}_i) - \Pi_j(\bar{\mathbf{m}} + \mathbf{e}_{w(j)})] \\
&= P(\bar{\mathbf{m}} + \mathbf{e}_i) - \sum_{j=1}^{n} \Pi_j(\bar{\mathbf{m}} + \mathbf{e}_{w(j)}) \\
&< P(\bar{\mathbf{m}} + \mathbf{e}_1) - \sum_{j=1}^{n} \Pi_j(\bar{\mathbf{m}} + \mathbf{e}_{w(j)}) \\
&= \sum_{j=1}^{n}[U_j(1) - U_j(w(j))].
\end{aligned}$$

The last inequality follows by the assumptions on P. Therefore, the optimal auction assigns firm 1 the license. The only thing that is left to check is whether an auction exists which is almost optimal, and assigns firm 1 the license. Consider take-it-or-leave-it mechanism M^1, which is defined in the proof of Proposition 3. In this auction, each firm has a strictly dominant strategy to choose "participate", and if they do so, firm 1 is the winner of the license. The revenue of M^1 is equal to

$$\sum_{j=1}^{n}[U_j(1) - U_j(w(j))] - n\epsilon,$$

so that M^1 is an almost optimal auction in which firm 1 wins the license. □

For generic P, the α-core need not be empty. This is established by Example 2, in which the α-core is not empty, and in which we show how an element in the α-core can be established by a take-it-or-leave-it offer from the seller to firm 1.

EXAMPLE 2. *There are two firms in the market. In the status quo situation, firm 1 has 2 outlets, i.e.,* $m_1 = 2$, *and firm 2 has 1 outlet, i.e.,* $m_2 = 1$. *Suppose that* f *is given by*

$$f(k) = k^2 \text{ for all } k \in \aleph.$$

The status quo total market profit is 1, i.e.,

$$P(2,1) = 1.$$

Suppose that if firm 1 wins the license, total market profit grows to 2, i.e.,

$$P(3,1) = 2.$$

and that total market profit remains unchanged if firm 2 wins the license, i.e.,

$$P(2,2) = 1.$$

With these parameters, we establish the following utilities.

$$\begin{aligned} U_1(1) &= 1, \\ U_2(1) &= 0, \\ U_1(2) &= -\frac{3}{10}, \text{ and} \\ U_2(2) &= \frac{4}{10}. \end{aligned}$$

The characteristic function of the related α-game is then defined as follows

$$\begin{aligned} v(0) = v(2) = v(1,2) &= 0 \\ v(0,1) = v(0,1,2) &= 1 \\ v(1) &= -\frac{3}{10} \\ v(0,2) &= \frac{4}{10}. \end{aligned}$$

The α-core is not empty. It is readily checked that the α-core is given by all vectors $(x_0, x_1, x_2) \in \Re^3$ with

$$\begin{aligned} x_0 + x_1 &= 1, \\ x_0 &\geq \frac{4}{10}, \\ x_1 &\geq 0, \text{ and} \\ x_2 &= 0. \end{aligned}$$

One element of the α-core is $(1-\epsilon, \epsilon, 0)$. This outcome is for instance established with a take-it-or-leave-it offer from the seller to firm 1 to buy the license for a price equal to $1-\epsilon$, where it is in firm 1's interest to accept the offer.

5. Concluding remarks

The analysis has confirmed the conjecture of the MDW-study group advising the Dutch government that a standard auction will lead to an increase of market concentration rather than a decrease the Dutch government aims at, so that the outcome of the auction is inefficient. The inefficiency is caused by the fact that the consumers do not participate in the auction (see also Jehiel and Moldovanu, 2000a). Of course, the issue of increasing market concentration due to auctions of licenses is not restricted to the petrol sector. For instance, participating in the UMTS auctions that took place in Europe in 2000 and 2001 seemed to be much more interesting for incumbents than for entrants (Jehiel and Moldovanu, 2000b; Klemperer, 2001; Van Damme, 2001).

The assumption of a constant pie may not be as unrealistic as it seems. In the Dutch petrol market, the firms developed an interesting way of coordinating on high petrol prices. When Shell, the market leader, decides to change its price, it announces this in the press. Within a few days, all the other firms follow Shell's example. For each firm, this coordination is very profitable as it leads to high prices, and there is no reason to believe that firms will deviate from this coordination in case the market structure changes somewhat. Therefore, independent of which firm wins the license, total market profits will not be affected.

Example 2 may cast some doubt on the usefulness of the α-core as a concept for strategy proofness of feasible auction mechanisms. More specifically, we would argue that an *empty* α-core does indicate collusion-proofness, but a *non-empty* α-core needs not imply that auctions are not collusion proof. In Example 2, none of the standard coalition agreements against the seller seem to work against the proposed take-it-or-leave-it offer of the seller, in which he is able to extract almost the entire surplus of firm 1: It is not in firm 1's interest to reject the take-it-or-leave-it offer; firm 2 is not willing to compensate firm 1 for rejecting the offer; firm 2 is not willing to bribe the seller not to sell the license to firm 1; it is not in the interest of the firms to form a cartel against the seller, in which they agree that firm 1 does not accept the offer; and so forth. In fact, the non-emptiness of the α-core indicates the stability of the agreement between the seller and firm 1. In contrast, such an agreement would not be stable in the case of an empty α-core. In that case we have to make strong assumptions on the commitment of the seller to the auction mechanism he chooses to transfer the license to one of the firms.

A similar reason that the α-core may not be an adequate concept for strategy proofness follows from the discussion in the paper of McAfee and McMillan (1992) on the formation of cartels in auctions. It is easily shown that the α-core is not empty in auction models in which no externalities are assumed. Still, in this setting, McAfee and McMillan assume the possibility of cartels of firms that cooperate against the seller. They rely on the assumption that there is an enforcement mechanism that forces the cartel members not to deviate from the cartel agreement. Deviators may be directly punished, or indirectly through grim-trigger strategies. As preventing collusive behavior is one of the major issues in auction design (Cramton and Schwartz, 1999; Klemperer, 2001), future research should lead to the construction of a better measure for the stability of collusion against the seller.

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CHAPTER 3

Auctions with Financial Externalities

1. Introduction

In this paper, we study sealed-bid auctions with *financial externalities.* Financial externalities arise when losers benefit directly or indirectly from a high price paid by the winner(s). In auction theory, it is generally assumed that losers are indifferent about how much the winner(s) pay(s) in an auction. However, in real life auctions, this assumption may be false. In reality, an auction is not an isolated game, as winners and losers also interact after the auction. Paying a high price in the auction may make a winner a weaker competitor later.

The series of UMTS auctions that took place in Europe offers a concrete example of auctions where losers benefit *indirectly* from a high price paid by the winners. In this context, there are at least three ways how firms that do not acquire a license may benefit from a winning firm paying a high price. First, the share values of winning firms may drop, which makes the winner vulnerable to a hostile take-over by competing firms. For instance, the drop of the share value of the Dutch telecom company KPN with about 95% is partly explained by the huge amount of money the company spent to acquire British, Dutch and German UMTS licences.[1,2] Second, if firms are budget constrained, a high payment in the first auction may give competing firms an advantage in the later auctions. Third, high payments may force the winning firms to cut their budget for investment, which may be favorable for the losers' position in the telecommunications market, as the losing firms are not only competitors of the winning firms in the auction, but in

[1]In the UK, KPN bought part of the TIW license after the auction. In Germany, KPN has a majority share in E-plus.

[2]The other part of the drop is probably explained by the changed sentiment in the market.

the telecommunications market as well. Börgers and Dustmann (2001) argue that financial externalities may have led to seemingly irrational bidding in the British UMTS auction.

Financial externalities occur *directly* when losing bidders get money from the winner(s). For instance, this may happen in the case of bidding rings, in which a member of the ring receives money when she does not win the object (McAfee and McMillan, 1992). Also, partnerships are dissolved using an auction in which losing partners obtain part of the winner's bid (Cramton et al., 1987). Finally, the owner of a large estate may specify in his last will that after his death, the estate is sold to one of the heirs in an auction, where the auction revenue is divided among the losers (Engelbrecht-Wiggans, 1994).[3]

In Section 2, we present a model of bidding in sealed-bid auctions with financial externalities. The first-price sealed-bid auction (FPSB) or the second-price sealed-bid auction (SPSB) is used to sell an indivisible object. We assume an independent private signals model, with private values and common value models as special cases. Financial externalities are exogenously given and modelled by a parameter φ that is inserted in the bidders' utility functions. This is the simplest extension of the independent private signals model which incorporates financial externalities. Despite its admitted simplicity, this model appears to be sufficiently rich to generate interesting insights.

In Section 3, we derive results for FPSB and SPSB without reserve price. We find a unique symmetric and efficient bid equilibrium for each of the two auction types. Equilibrium bids in FPSB decrease as φ increases. An intuition for this result is that larger financial externalities make losing more attractive for the bidders so that they submit lower bids. The effect of financial externalities on the equilibrium bids in SPSB is ambiguous. A possible explanation is that in SPSB, a bidder is not only inclined to bid less the higher is φ (as she gets positive utility from losing), she also has an incentive to bid higher, because, given that she loses, she is able to influence directly the level of payments made by the winner. We also study the effect of a resale market. Haile (1999) shows in an independent private values model that the efficient

[3]More examples can be found in Goeree and Turner, 2001.

equilibria of FPSB and SPSB remain unaffected when the auction is followed by a resale market. We show that this result still holds in our model, and that it extends to any auction which leads to an efficient assignment of the object. Finally, we give a revenue comparison between FPSB and SPSB. We find that in the two-bidder case, SPSB results in a higher expected revenue than FPSB.

In Section 4, we characterize equilibrium bid strategies for the case that a reserve price is imposed in FPSB and SPSB. For simplicity, we assume independent private values and two bidders. In this section, we introduce the concept of a weakly separating Bayesian Nash equilibrium, which is an equilibrium in which all types below a threshold type abstain from bidding, and all types above this type submit a bid according to a strictly increasing bid function. We find that FPSB has no weakly separating Bayesian Nash equilibrium. However, we derive an equilibrium in which bidders with low signals abstain from bidding, bidders with intermediate signals pool at the reserve price, and bidders with high signals submit a bid according to a strictly increasing bid function. For SPSB, we derive a necessary and sufficient condition for the existence of a weakly separating Bayesian Nash equilibrium. For low values of the reserve price, such an equilibrium exists, for high values it does not. If a weakly separating Bayesian Nash equilibrium exists, then all types above the threshold type submit the same bid as in the case of no reserve price.

A closely related paper is Engelbrecht-Wiggans (1994), who considers an auction game in which each bidder receives an equal share α of the revenue. He characterizes equilibrium bid functions for both FPSB and SPSB, and gives a revenue comparison between these two auction types.[4] It is straightforwardly checked that his model is isomorphic to our model. Therefore, the equilibrium bids in our model can directly

[4]Several other papers make use of Engelbrecht-Wiggans' model. Ettinger (2000) extends the model by allowing the revenue shares to differ among the bidders and by introducing reserve prices. Engers and McManus (2000) study *charity auctions*, in which bidders receive a *warm glow* from the auction revenue, so that their utility depends on the auction revenue. Goeree and Turner (2002) compare standard auctions with k-th price all-pay auctions in Engelbrecht-Wiggans' environment. Simultaneously and independently of us, Engers and McManus, and Goeree and Turner derive similar results as Engelbrecht-Wiggans and we with respect to

be derived from the equilibrium bids in Engelbrecht-Wiggans (1994). However, the comparative statics in our model and Engelbrecht-Wiggans' model (the effect of φ respectively α on the equilibrium bids and the seller's revenue) turn out to be different. Engelbrecht-Wiggans shows that the equilibrium bid functions of FPSB and SPSB are increasing in α. In our model, the effect of φ on the equilibrium bids can be both increasing and decreasing. We add to Engelbrecht-Wiggans' analysis that, if attention is restricted to symmetric Bayesian Nash equilibria, these equilibrium bid functions are unique. Also, in addition to Engelbrecht-Wiggans' study, we analyze the effect of a resale market, and of a reserve price on the equilibrium bids.

There are several other papers related to ours. Our companion paper (Maasland and Onderstal, 2002) focuses on optimal auction design in the context of financial externalities. In that paper, we show that in a Double Coasean World, in which the seller cannot prevent a perfect resale market, nor withhold the object, the lowest-price all-pay auction is optimal.[5] Moreover, in a Myersonean World, in which the seller can both prevent resale after the auction and fully commit to not selling the object, we find a two-stage mechanism that is revenue maximizing. In the first stage of this mechanism, bidders are asked whether they accept to pay an entry fee. If and only if all choose to accept, then in the second stage, bidders play the lowest-price all-pay auction with a reserve price.

Jehiel and Moldovanu (1996, 2000), and Jehiel et al. (1996, 1999) study auctions in which losing bidders receive positive or negative allocative externalities from the winner. Since the utility of the bidders is affected by the identity of the winner and not by how much she pays, these externalities are clearly different from financial externalities. Jehiel and Moldovanu (2000) derive equilibrium bid strategies that involve some pooling at the reserve price for SPSB with a reserve

equilibrium bidding in FPSB and SPSB, and the revenue comparison among these two auction types.

[5]In this auction, the bidder that submits the highest bid wins the object, and every bidder pays the lowest submitted bid.

price and positive externalities. This equilibrium structure is similar to the one we found in FPSB.

Benoît and Krishna (2001) study a two-bidder model with complete information in which two objects are sold sequentially. As bidders are budget constrained, a particular bidder's payoff is affected by the price paid by a rival bidder, so that their model can be interpreted as a model with endogenously determined financial externalities.

2. The model

We consider a situation with $n \geq 2$ risk neutral bidders, numbered $1, 2, ..., n$, who bid for one indivisible object. The auction being used is either FPSB or SPSB. Each of these auction types may or may not have a reserve price.

Essentially, we use Milgrom and Weber's (1982) model with independent signals instead of affiliated signals as a starting point. We assume that each bidder i receives a one-dimensional private signal t_i which is drawn, independently from all the other signals, from a cumulative distribution function F. (We also say that bidder i is of type t_i.) F has support on an interval $[\underline{t}, \bar{t}]$, and continuous density f with $f(t_i) > 0$ for every $t_i \in [\underline{t}, \bar{t}]$.

We will let $v_i(\mathbf{t})$ denote the value of the object for bidder i given the vector $\mathbf{t} \equiv (t_1, ..., t_n)$ of all signals. Special cases are private value models ($v_i(\mathbf{t})$ only depends on t_i), and common value models ($v_i(\mathbf{t}) = v_j(\mathbf{t})$ for all $i, j, \mathbf{t}$).

We make the following assumptions on the functions v_i.

Value Differentiability: v_i is differentiable in all its arguments, for all $i, \mathbf{t}$.

Value Monotonicity: $v_i(\mathbf{t}) \geq 0$, $\frac{\partial v_i(\mathbf{t})}{\partial t_i} > 0$, and $\frac{\partial v_i(\mathbf{t})}{\partial t_j} \geq 0$, for all $i, j, \mathbf{t}$.

Symmetry: $F_i = F_j$ for all i, j, and $v_i(..., t_i, ..., t_j, ...) = v_j(..., t_j, ..., t_i, ...)$ for all t_i, t_j, i, j.

Value Differentiability is imposed to make the calculations on the equilibria tractable. *Value Monotonicity* indicates that all bidders are serious, and that bidders' values are strictly increasing in their own signal, and weakly in the signals of the others. *Symmetry* may be crucial for the existence of efficient equilibria in standard auctions.[6] *Value Differentiability, Value Monotonicity,* and *Symmetry* together ensure that the bidder with the highest signal is also the bidder with the highest value, so that these assumptions imply that the seller assigns the object efficiently if and only if the bidder with the highest signal gets it.

We define $F^{[1]}$ and $f^{[1]}$ as the cumulative distribution function and density function respectively of $\max_{j \neq i} t_j$. Also, let us define $v(x, y)$ as the expected value that bidder i assigns to the object, given that her signal is x, and that the highest signal of all the other bidders is equal to y:

$$v(x,y) \equiv E\{v_i(\mathbf{t}) | t_i = x, \max_{j \neq i} t_j = y\}.$$

By *Symmetry*, $F^{[1]}$, $f^{[1]}$, and v do not depend on i.

The bidders are expected utility maximizers. Each bidder is risk neutral, and cares about what other bidders pay in the auction. More specifically, the utility function of bidder i is defined as follows:

$$u_i(j,b) = \begin{cases} v_i - b & \text{if } j = i \\ \varphi b & \text{if } j \neq i, \end{cases}$$

where v_i is the value that i attaches to the object, j is the winner of the object and b is the payment by j. It is a natural assumption to let a bidder's interest in her own payments be larger than her interest in the payments by the other bidders, so that we assume $\varphi \leq 1/(n-1)$.

A specific interpretation of the model is a situation of an auction in which all losing bidders receive an equal share of the auction revenue. In particular, when $\varphi = 1/(n-1)$, the entire auction revenue is divided among all losing bidders, which may be the case in situations of dissolving partnerships, or heirs bidding for a family estate. If $n = 2$ and

[6]Bulow et al. (1999) show that a slight asymmetry in value functions may have dramatic effects on bidding behavior in the English auction in a common value setting, as the bidder with the lower value function faces a strong winner's curse, and therefore bids zero in equilibrium.

$\varphi = 1$, then FPSB and SPSB are special cases of the k-double auction with $k = 0$ and $k = 1$ respectively.[7,8]

3. Zero reserve price

Consider FPSB and SPSB with a zero reserve price.

3.1. First-price sealed-bid auction. The following proposition characterizes the equilibrium bid function for FPSB. To derive equilibrium bidding, we suppose that in equilibrium, all bidders use the same bid function. By a standard argument, this bid function must be strictly increasing and continuous. Let $U(t,s)$ be the utility for a bidder with signal t who behaves as if having signal s, whereas the other bidders play according to the equilibrium bid function. A necessary equilibrium condition is that

$$\frac{\partial U(t,s)}{\partial s} = 0$$

at $s = t$. From this condition, a differential equation can be derived, from which the equilibrium bid function is uniquely determined (at least if we restrict our attention to differentiable bid functions). The auction outcome is efficient. Observe that in the case of private values ($v(x,y)$ only depends on x), the bid function is strictly increasing in n.

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[7]The k-double auction has the following rules. Both bidders submit a bid. The highest bidder wins the object, and pays the loser an amount equal to $kb_L + (1-k)b_W$, where b_L is the loser's bid, b_W the winner's bid, and $k \in [0,1]$.

[8]Cramton et al. (1987) study k-double auctions in a private values environment with symmetric value distributions. It is shown that partners with equal shares may dissolve a partnership efficiently using these auctions. McAfee and McMillan (1992) show that the 0-double auction is a mechanism that allows a bidding ring to allocate the obtained object efficiently among the ring members. Van Damme (1992) shows that k-double auctions may lead to unfair equilibrium outcomes. Angeles de Frutos (2000) and Kittsteiner (2001) generalize the model of Cramton et al. (1987) allowing for asymmetric value distributions and interdependent valuations respectively.

PROPOSITION 8. *The unique symmetric differentiable Bayesian Nash equilibrium of FPSB is characterized by*

(3.1)

$$B_1(\varphi,t) = v(t,t) - \frac{\varphi}{1+\varphi}v(t,t) - \frac{1}{1+\varphi}\int_{\underline{t}}^{t} \frac{dv(y,y)}{dy}\left(\frac{F^{[1]}(y)}{F^{[1]}(t)}\right)^{1+\varphi} dy,$$

where $B_1(\varphi,t)$ is the bid of a bidder with signal t. The outcome of this auction is efficient.

PROOF. A higher type of a bidder cannot submit a lower bid than a lower type of the same bidder. (If the low type gets the same expected surplus from strategies with two different probabilities of being the winner of the object, the high type strictly prefers the strategy with the highest probability of winning, so the high type will not submit a lower bid than the low type.) Also, $B_1(\varphi,t)$ cannot be constant on an interval $[t', t'']$. (By bidding slightly higher, a type t'' can largely improve its probability of winning, while only marginally influencing the payments by her and the other bidders.) Moreover, $B_1(\varphi,t)$ cannot be discontinuous at any t. (Suppose that $B_1(\varphi,t)$ makes a jump from $\underline{b}$ to $\overline{b}$ at t^*. A type just above t^* has an incentive to deviate from $\overline{b}+\delta$ to $\underline{b}$. Doing so, she is able to decrease the auction price, while just slightly affecting its probability of winning the object. As φ is small enough, this type is able to improve its utility.) Hence, a symmetric equilibrium bid function must be strictly increasing and continuous.

Define the utility $U(t,s)$ for a bidder with signal t who misrepresents herself as having signal s, whereas the other bidders report truthfully, if the bid function is indeed strictly increasing. Then,

$$U(t,s) = \int_{\underline{t}}^{s} v(t,y)dF^{[1]}(y) - F^{[1]}(s)B_1(\varphi,s) + \varphi\int_{s}^{\overline{t}} B_1(\varphi,y)dF^{[1]}(y).$$

The first two terms of the RHS of this expression refer to the case that this bidder wins the object. The third term refers to the case that she does not win. Assume that $B_1(\varphi,s)$ is differentiable in s. Maximizing $U(t,s)$ with respect to s and equating s to t gives the FOC of the equilibrium

$$f^{[1]}(t)v(t,t) - f^{[1]}(t)B_1(\varphi,t) - F^{[1]}(t)B_1'(\varphi,t) - \varphi B_1(\varphi,t)f^{[1]}(t) = 0.$$

With some manipulation we get

(3.2)

$$F^{[1]}(t)^{\varphi} f^{[1]}(t)v(t,t) = (1+\varphi)B_1(\varphi,t)f^{[1]}(t)F^{[1]}(t)^{\varphi} + B_1'(\varphi,t)F^{[1]}(t)^{1+\varphi},$$

or, equivalently

$$C_1 + \int_{\underline{t}}^{t} F^{[1]}(y)^{\varphi} f^{[1]}(y)v(y,y)dy = F^{[1]}(t)^{1+\varphi}B_1(\varphi,t),$$

where C_1 is a constant. Substituting $t = \underline{t}$ gives $C_1 = 0$, so that the bid function is given by

$$B_1(\varphi,t) = \frac{1}{F^{[1]}(t)} \int_{\underline{t}}^{t} \left(\frac{F^{[1]}(y)}{F^{[1]}(t)}\right)^{\varphi} f^{[1]}(y)v(y,y)dy. \tag{3.3}$$

It is readily checked that the second order condition $sign\left(\frac{\partial U(t,s)}{\partial s}\right) = sign(t-s)$ is fulfilled. Using integration by parts, (3.3) can be rewritten as (3.1).

From (3.2), we infer that $\frac{\partial B_1(\varphi,t)}{\partial t} > 0$ if and only if $B_1(\varphi,t) < \frac{v(t,t)}{1+\varphi}$, so that indeed $B_1(\varphi,t)$ is strictly increasing in t, as *Value Monotonicity* implies that $\frac{dv(y,y)}{dy} > 0$ for all y. Finally, by *Value Differentiability, Value Monotonicity*, and *Symmetry*, the efficiency of the auction outcome is established. □

Each of the terms of the RHS of (3.1) has an attractive interpretation. The first term is the equilibrium bid for a bidder with type t in SPSB without financial externalities, as in the absence of financial externalities, in SPSB, a bidder will submit a bid equal to her maximal willingness to pay given that her strongest opponent has the same signal as she (Milgrom and Weber, 1982). The second term can be interpreted as the bid shading because of financial externalities. The reason for bid shading is that in the case of financial externalities, the willingness to pay of a bidder with type t bidding against an opponent who has the same signal is given by $\frac{1}{1+\varphi}v(t,t)$. This can be seen as follows. When a bidder wins at a bid of b, her utility is $v(t,t) - b$. When her opponent wins at the same bid, her utility is φb. Equating these utilities results in a bid of $\frac{1}{1+\varphi}v(t,t)$. The third term can be interpreted as the strategic bid shading because in FPSB, a bidder has to pay her own bid rather than the second highest bid which she has to pay in SPSB.

This interpretation of the equilibrium bid function suggests that this function is decreasing in φ, which in fact holds, as Proposition 9 shows. From Proposition 9, it immediately follows that the expected revenue is decreasing in φ.

PROPOSITION 9. *Increasing φ decreases $B_1(\varphi, t)$.*

PROOF. The proof immediately follows from Proposition 8, since $F^{[1]}(y) < F^{[1]}(t)$ for every $y \in [\underline{t}, t)$. □

COROLLARY 2. *Increasing φ decreases the seller's expected revenue.*

3.2. Second-price sealed-bid auction. Equilibrium bids for SPSB are obtained using the same logic as for FPSB. The analysis reveals, just as in situations without financial externalities, uniqueness and efficiency of the equilibrium bid function. Observe that in the case of private values, the bid function does not depend on n.[9]

PROPOSITION 10. *The unique symmetric differentiable Bayesian Nash equilibrium of SPSB is characterized by*

$$\begin{aligned} B_2(\varphi, t) &= v(t,t) - \frac{\varphi}{1+\varphi}v(t,t) + \\ &+ \frac{\varphi}{(1+\varphi)(1+2\varphi)} \int_t^{\bar{t}} \frac{dv(y,y)}{dy} \left(\frac{1-F(y)}{1-F(t)}\right)^{\frac{1+\varphi}{\varphi}} dy \end{aligned} \tag{3.4}$$

where $B_2(\varphi, t)$ is the bid of a bidder with signal t. The outcome of this auction is efficient.

[9]This is actually a quite subtle observation, as n does not appear in the expression for the equilibrium bid. However, in general, $v(t,t)$ depends on n.

PROOF. Following the lines of the proof of Proposition 8 it can be established that a symmetric equilibrium function must be strictly increasing and continuous. The utility for a bidder with signal t acting as if she had signal s is given by

$$U(t,s)=\int_{\underline{t}}^{s}[v(t,y)-B_2(\varphi,y)]dF^{[1]}(y)+\varphi\pi(s)B_2(\varphi,s)+\varphi\int_{y=s}^{\overline{t}}B_2(\varphi,y)d\pi(y),$$

where $\pi(s)$ denotes the probability that there is exactly one opponent with a signal larger than s. The first term of the RHS refers to the case that this bidder wins, the second term to the case that she submits the second highest bid, and the third case to her bid being the third or higher. Assume that $B_2(\varphi,s)$ is differentiable in s. The FOC of the equilibrium is

$$[v(t,t)-B_2(\varphi,t)]f^{[1]}(t)+\varphi\frac{\partial\pi(t)B_2(\varphi,t)}{\partial t}-\varphi B_2(\varphi,t)\pi'(t)=0$$

or, equivalently

$$v(t,t)f^{[1]}(t)=B_2'(\varphi,t)\varphi\pi(t)+B_2(\varphi,t)[(1+\varphi)f^{[1]}(t)]. \tag{3.5}$$

The general solution to the above differential equation is equal to

$$B_2(\varphi,t)(1-F(t))^{\frac{1+\varphi}{\varphi}}=C_2-\int_{\underline{t}}^{t}(1-F(y))^{\frac{1}{\varphi}}v(y,y)f(y)dy,$$

where C_2 is a constant. Substituting $t=\overline{t}$ yields a unique solution for C_2:

$$C_2=\int_{\underline{t}}^{\overline{t}}(1-F(y))^{\frac{1}{\varphi}}v(y,y)f(y)dy.$$

The only possible differentiable bid function that may constitute a symmetric equilibrium is given by

$$B_2(\varphi,t)=\frac{1}{\varphi(1-F(t))}\int_{t}^{\overline{t}}\left(\frac{1-F(y)}{1-F(t)}\right)^{\frac{1}{\varphi}}f(y)v(y,y)dy. \tag{3.6}$$

It is readily checked that the second order condition $sign\left(\frac{\partial U(t,s)}{\partial s}\right)=sign(t-s)$ holds. Using integration by parts on $B_2(\varphi,t)$, we see that (3.6) can also be written as (3.4).

To complete the proof, we must show that $B_2(\varphi, t)$ is indeed increasing in t. From (3.6), it follows that

$$B_2(\varphi, t) > \frac{v(t,t) \int_t^{\bar{t}} (1 - F(y))^{\frac{1}{\varphi}} f(y)dy}{\varphi(1 - F(t))^{\frac{1+\varphi}{\varphi}}} = \frac{v(t,t)}{1+\varphi}.$$

As (3.5) implies that $B_2'(\varphi, t) > 0$ if and only if $B_2(\varphi, t) > \frac{v(t,t)}{1+\varphi}$, $B_2(\varphi, t)$ is indeed strictly increasing in t. Then, by *Value Differentiability, Value Monotonicity*, and *Symmetry*, it follows that the outcome of the auction is efficient. □

Each term of the RHS of (3.4) has its attractive interpretation. From the discussion of FPSB it follows that the first term is the bid in SPSB in the absence of financial externalities. The second term is the bid shading due to positive externalities from the payment of the winning bidder. The third term increases the bid due to the fact that each bidder is willing to drive up the final price, as it is the second highest bid that is paid by the winner.

In contrast to FPSB, the effect of an increase in φ on the equilibrium bids in SPSB is dependent on a bidder's type. From (3.4), it is clear that the equilibrium bid of the highest type is decreasing in φ. The reason is that as this bidder does not have a type above her, she does not have an incentive to drive up the price. However, the effect of φ on the equilibrium bids of the other types is not clear. The effect of the second term of the RHS of (3.4) (without the minus sign) may be larger as well as smaller than the third term. The following example illustrates how equilibrium bidding is affected when φ is varied.

EXAMPLE 3. ***(Effect of φ on equilibrium bidding)*** *Let $F(t) = t$ (uniform distribution), $v(t,t) = t$ (independent private values) for all $t \in [0,1]$. The equilibrium bid function is given by*

$$B_2(\varphi, t) = \frac{\varphi}{(1+\varphi)(1+2\varphi)} + \frac{1}{1+2\varphi}t,\ t \in [0,1].$$

As B_2 is a continuous function in both φ and t, the following can be derived. First, there is a strictly positive mass of types close to zero for which the effect of φ is ambiguous in the sense that for φ close to 0, *an increase in φ leads to higher bids and for φ close to 1, an increase in φ leads to lower bids. This follows from the following observations.*

$$\frac{\partial B_2(0,0)}{\partial \varphi} = 1 > 0,$$

and

$$\frac{\partial B_2(1,0)}{\partial \varphi} = -\frac{1}{36} < 0.$$

Intuitively, if φ is large enough, $B_2(\varphi, t)$ decreases as for each bidder, losing becomes more interesting due to higher financial externalities. Second, the equilibrium bids of types close to 1 are decreasing in φ. This follows from the fact that $B_2(\varphi, 1) = \frac{1}{1+\varphi}$.

Also, the effect of φ on the expected revenue may be ambiguous. This follows from Example 4, in which the expected revenue is increasing if φ is small, and decreasing if φ is large.

EXAMPLE 4. ***(Effect of φ on the expected revenue)*** *Let $F(t) = t$ (uniform distribution), $v(t,t) = t$ (independent private values) with $t \in [0,1]$ and $n = 2$ (two bidders). The expected revenue is equal to the expectation of $B_2(\varphi, t^{(2)})$ with respect to the second highest signal $t^{(2)}$, which is given by*

$$E_{t^{(2)}}\{B_2(\varphi, t^{(2)})\} = \frac{1+4\varphi}{3(1+\varphi)(1+2\varphi)}.$$

This continuous function is increasing for φ close to 0 and decreasing for φ close to 1, as

$$\frac{\partial E_{t^{(2)}}\{B_2(0, t^{(2)})\}}{\partial \varphi} = \frac{1}{3} > 0,$$

and

$$\frac{\partial E_{t^{(2)}}\{B_2(1, t^{(2)})\}}{\partial \varphi} = -\frac{11}{108} < 0.$$

3.3. Resale market. The presence of a resale market does not have any effect on equilibrium behavior. In order to obtain this result, the following assumptions are made. First, trade is voluntary. None of the bidders can be forced to be involved in an exchange if she is made worse off by it. Thus, trade only takes place if it is mutual profitable for the bidders. Second, the participants in the resale market are the same as in the auction. There are no third parties involved.

We assume the following conditions for trade to occur in the resale market. Let bidder i be the winner of the object in the auction, and bidder j be another bidder, who desires to buy the object from bidder i in the resale market. Let $\tilde{p}$ be the price of the object in the resale market. As trade is voluntary, none of the bidders may be worse off by the trade. For bidder i, the following condition for trade must be fulfilled:

$$\tilde{p} + \varphi\tilde{p} \geq v_i. \tag{3.7}$$

In words, bidder i prefers receiving a price of $\tilde{p}$, which also yields her a financial externality of $\varphi\tilde{p}$, to keeping the object, which gives her a value of v_i. For bidder j a similar condition holds:

$$v_j - \tilde{p} - \varphi\tilde{p} \geq 0, \tag{3.8}$$

which is equivalent to

$$\tilde{p} + \varphi\tilde{p} \leq v_j. \tag{3.9}$$

Note that, $\varphi\widetilde{p}$ in (3.8) is a correction factor. This can be seen as follows. Without trade, the utility of bidder j is φp, where p is the price paid by bidder i in the auction. With trade in the resale market, bidder i has paid $p - \widetilde{p}$. This would give bidder j a utility increase of $\varphi(p - \widetilde{p})$ due to financial externalities. Therefore, bidder j loses an extra $\varphi\widetilde{p}$, if she decides to buy the object in the resale market. Observe that, (3.7) and (3.9) exclude inefficient trade (trade from a bidder with a high value to one with a low value). Moreover, for both bidders the maximal gains from trade are $v_j - v_i$.

Proposition 11 shows that equilibrium bidding is not affected by the presence of a resale market if the equilibrium of the auction without resale market leads to an efficient outcome. We prove this proposition by assuming that all bidders, apart from bidder i, bid "as usual", i.e., they bid in the auction as if there were no resale market. Then we calculate bidder i's utility both for the case when she submits a lower bid than "usual", and for the case that she submits a higher bid. In both cases, we separately calculate bidder i's utility from the auction, and the maximal utility she can obtain in the resale market, which is the difference between her value, and the highest value among the other bidders. Adding these, we show that bidder i has no incentive to deviate from bidding "as usual".

PROPOSITION 11. *A Bayesian Nash equilibrium of an auction (without resale market) which leads to an efficient assignment of the object, is also a Bayesian Nash equilibrium when the same auction is followed by a resale market where the same bidders participate. In equilibrium, no trade will take place in the resale market.*

PROOF. To prove that "bidding as usual" is still an equilibrium, suppose that all bidders, apart from bidder i, bid as usual. Then it should be a best response for bidder i to bid as usual as well. Let $U(t,s)$ be the expected surplus for bidder i from the auction plus the resale market, when she has signal t, but behaves as if she has type s.

Suppose for the moment that all bidders play the efficient Bayesian Nash equilibrium of the auction game without resale market. Then, by *Value Differentiability, Value Monotonicity,* and *Symmetry,* the bidder with the highest signal wins the auction. Moreover, by the assumption of voluntary trade (which exclude inefficient trade), no trade will take place after the auction. From the Revenue Equivalence Theorem,[10] $U(t,t)$ is given by

$$U(t,t) = U(\underline{t},\underline{t}) + \int_{\underline{t}}^{t} \int_{\underline{t}}^{x} \frac{\partial v(x,y)}{\partial x} dF^{[1]}(y)dx.$$

When we change the order of integration and integrate the inner integral we get

$$U(t,t) = U(\underline{t},\underline{t}) + \int_{\underline{t}}^{t} [v(t,y) - v(y,y)]dF^{[1]}(y). \tag{3.10}$$

[10]See Maasland and Onderstal (2002) for this result in the context of financial externalities.

The utility $\widetilde{U}(t,s)$ from the auction alone for bidder i who has type t, but represents herself as if she has type s is given by

$$\widetilde{U}(t,s) = \widetilde{U}(s,s) + \int_{\underline{t}}^{t} [v(t,y) - v(s,y)] dF^{[1]}(y). \tag{3.11}$$

Suppose that bidder i misrepresents herself as having a signal $s > t$. Trade will only take place when bidder i wins the auction, and there is another bidder j who has a higher valuation for the object. The gains from trade for bidder i from the resale market are at most the absolute difference between her value and the value of bidder j, which is, by *Value Differentiability, Value Monotonicity,* and *Symmetry,* the bidder with the highest signal. Let y be bidder j's signal, then bidder j's value is at most $v(y,y)$. Bidder i's value is given by $v(t,y)$. Then, with (3.10) and (3.11),

$$\begin{aligned}
U(t,s) - U(t,t) \leq\ & \widetilde{U}(s,s) + \int_{\underline{t}}^{s} [v(t,y) - v(s,y)] dF^{[1]}(y) + \\
& + \int_{t}^{s} [v(y,y) - v(t,y)] dF^{[1]}(y) - U(t,t) \\
=\ & \int_{\underline{t}}^{s} [v(s,y) - v(y,y)] dF^{[1]}(y) - \int_{\underline{t}}^{t} [v(t,y) - v(y,y)] dF^{[1]}(y) \\
& + \int_{\underline{t}}^{s} [v(t,y) - v(s,y)] dF^{[1]}(y) + \int_{t}^{s} [v(y,y) - v(t,y)] dF^{[1]}(y) \\
=\ & 0.
\end{aligned}$$

So, bidder i cannot gain from deviating to a higher signal.

Suppose instead that bidder i deviates to a lower signal. Then, similarly,

$$\begin{aligned} U(t,s)-U(t,t) &\le \widetilde{U}(s,s)+\int_{\underline{t}}^{s}[v(t,y)-v(s,y)]dF^{[1]}(y)+ \\ &\quad +\int_{s}^{t}[v(t,y)-v(y,t)]dF^{[1]}(y)-U(t,t) \\ &\le \int_{\underline{t}}^{s}[v(s,y)-v(y,y)]dF^{[1]}(y)-\int_{\underline{t}}^{t}[v(t,y)-v(y,y)]dF^{[1]}(y) \\ &\quad +\int_{\underline{t}}^{s}[v(t,y)-v(s,y)]dF^{[1]}(y)+\int_{s}^{t}[v(t,y)-v(y,y)]dF^{[1]}(y) \\ &= 0. \end{aligned}$$

Hence a deviation to a lower type is not profitable. So, indeed it is a best response for bidder i to bid as usual.

As the equilibrium of the auction is efficient, it is always the bidder with the highest value who obtains the object after the auction. As inefficient trade is excluded in the resale market, no trade will take place there. □

A corollary of the above result is that the equilibrium bids in FPSB and SPSB do not change when resale market opportunities are introduced. This immediately follows from the fact that both auctions have efficient equilibria, as was shown in Propositions 8 and 10. Moreover, no trade will take place in the resale market.

3.4. Revenue comparison for $n = 2$. For the tractable case of two bidders, if $0 < \varphi < 1$, SPSB generates a strictly higher expected revenue than FPSB.[11] This revenue ranking result is obtained by proving that the utility of the lowest type is strictly higher for FPSB than

[11] Engelbrecht-Wiggans (1994) claims the same result for n bidders, but his proof is not correct, even not for the case of two bidders.

for SPSB. This short-cut immediately follows from the Revenue Equivalence Theorem (Myerson, 1981) which remains valid in case financial externalities are introduced (Maasland and Onderstal, 2002). According to the Revenue Equivalence Theorem, two auctions which are both efficient, and yield zero utility for the lowest type, yield the same expected revenue. For $\varphi = 1$, both auctions are revenue equivalent, which follows as the utility of the lowest type is the same for both auctions.

To obtain the proof of Proposition 12, the following lemma appears to be useful.

LEMMA 6. *For every $y \in (0,1)$ and $\varphi \in (0,1)$, the following inequality is satisfied:*

$$y + \frac{\varphi}{1+\varphi}(1-y)^{\frac{1+\varphi}{\varphi}} - \frac{1}{1+\varphi}y^{1+\varphi} - \frac{\varphi}{1+\varphi} < 0.$$

If $\varphi = 1$, then for every $y \in (0,1)$,

$$y + \frac{\varphi}{1+\varphi}(1-y)^{\frac{1+\varphi}{\varphi}} - \frac{1}{1+\varphi}y^{1+\varphi} - \frac{\varphi}{1+\varphi} = 0.$$

PROOF. See the Appendix. □

PROPOSITION 12. *For $\varphi < 1$ and $n = 2$, SPSB generates a strictly higher expected revenue than FPSB. For $\varphi = 1$ and $n = 2$, FPSB and SPSB are revenue equivalent.*

PROOF. Let $U_1(\underline{t})$ and $U_2(\underline{t})$ be the equilibrium utility of the lowest type in FPSB and SPSB respectively. As the outcome of both auctions is efficient, a bidder with type $\underline{t}$ loses the auction with probability 1, and gets financial externalities as the other bidder has to pay. So, $U_1(\underline{t})$ and $U_2(\underline{t})$ are respectively given by

$$U_1(\underline{t}) = \varphi \int_{\underline{t}}^{\bar{t}} B_1(\varphi, t) dF(t)$$

and

$$U_2(\underline{t}) = \varphi B_2(\varphi, \underline{t}).$$

Applying integration by parts twice on the expression for $\int_{\underline{t}}^{\bar{t}} B_1(\varphi,t)dF(t)$, we obtain

$$\begin{aligned}
\varphi\int_{\underline{t}}^{\bar{t}} B_1(\varphi,t)dF(t) &= \varphi\int_{\underline{t}}^{\bar{t}} \frac{1}{F(t)}\int_{\underline{t}}^{t}\left(\frac{F(y)}{F(t)}\right)^{\varphi} f(y)v(y,y)dydF(t) \\
&= \int_{\underline{t}}^{\bar{t}} (1-F(t)^{\varphi})v(t,t)dF(t) \\
&= \frac{\varphi}{1+\varphi}v(\bar{t},\bar{t}) - \int_{\underline{t}}^{\bar{t}}\left(F(t)-\frac{1}{1+\varphi}F(t)^{1+\varphi}\right)\frac{dv(t,t)}{dt}dt.
\end{aligned}$$

Manipulating $B_2(\varphi,\underline{t})$, we find

$$\begin{aligned}
\varphi B_2(\varphi,\underline{t}) &= \int_{\underline{t}}^{\bar{t}} (1-F(t))^{\frac{1}{\varphi}} f(t)v(t,t)dt \\
&= \frac{\varphi}{1+\varphi}v(\underline{t},\underline{t}) + \frac{\varphi}{1+\varphi}\int_{\underline{t}}^{\bar{t}} (1-F(t))^{\frac{1+\varphi}{\varphi}}\frac{dv(t,t)}{dt}dt.
\end{aligned}$$

Then

$$\begin{aligned}
U_1(\underline{t})-U_2(\underline{t}) &= \varphi\int_{\underline{t}}^{\bar{t}} B_1(\varphi,t)dF(t) - \varphi B_2(\varphi,\underline{t}) \\
&= \int_{\underline{t}}^{\bar{t}}\left(\frac{F(t)^{1+\varphi}}{(1+\varphi)} - F(t) - \frac{\varphi}{1+\varphi}(1-F(t))^{\frac{1+\varphi}{\varphi}} + \frac{\varphi}{1+\varphi}\right)\frac{dv(t,t)}{dt}dt.
\end{aligned}$$

When we apply Lemma 6 with $y = F(t)$ to the difference between $U_1(\underline{t})$ and $U_2(\underline{t})$, we find for $\varphi \in (0,1)$ that the utility of the lowest type is strictly higher for FPSB than for SPSB. For $\varphi = 1$, by Lemma 6, $U_1(\underline{t}) - U_2(\underline{t}) = 0$. □

4. Positive reserve price

Consider FPSB and SPSB with a reserve price $R > 0$. In order to keep the model tractable, we assume that the standard independent private values model holds, i.e., $v_i(\mathbf{t}) = t_i$ for all i, $\mathbf{t}$. Also, we restrict our attention to the case of two bidders.

This section mainly focuses on the existence of *weakly separating Bayesian Nash equilibria*, for which the following definition applies.

DEFINITION 1. *A weakly separating Bayesian Nash equilibrium is a Bayesian Nash equilibrium in which all types below a threshold type abstain from bidding, and all types above this type submit a bid according to a strictly increasing bid function.*

4.1. First-price sealed-bid auction. In contrast to a situation without financial externalities, there exists no weakly separating Bayesian Nash equilibrium for FPSB. Proposition 13 shows that, if such an equilibrium would exist, R must be the threshold type. The equilibrium bid function can be constructed analogous to the equilibrium bid function for FPSB without reserve price. But then a contradiction is established, as a bidder with type R turns out to submit a bid below the reserve price.

PROPOSITION 13. *Let $v_i(\mathbf{t}) = t_i$ for all i, $\mathbf{t}$, and $n = 2$. There exists no weakly separating Bayesian Nash equilibrium of FPSB if $R > 0$.*

PROOF. The proof is by contradiction. Suppose for the moment that a weakly separating equilibrium does exist. Then it is easily derived that all bidders with a type below R abstain from bidding, and all types above R submit a bid according to a strictly increasing bid function, which we denote by h. Using similar arguments as in the proof of Proposition 8, it can be established that $h'(t) \geq 0$ if and only if $h(t) \leq \frac{t}{1+\varphi}$. Hence, for $t = R$, it holds that $h(R) \leq \frac{R}{1+\varphi}$. In other words, in a weakly separating equilibrium, a bidder with type R submits a bid strictly below R. This contradicts the fact that all submitted bids should exceed the reserve price R. □

However, there is a symmetric equilibrium that involves pooling at the reserve price. Proposition 14 describes a Bayesian Nash equilibrium in which bidders with a type below a threshold type L do not bid, bidders with a type t above a threshold type H bid $g^R(t)$, where g^R is a strictly increasing function, and types in the interval $[L, H]$ submit a bid equal to R. More specifically, let

$$H = (1 + \varphi)R,$$

L the unique solution to

$$\frac{\varphi[F(H) - F(L))]R}{F(H) + F(L)} = L - R,$$

and

$$g^R(t) = F^{[1]}(t)^{-1-\varphi} \int_{(1+\varphi)R}^{t} F^{[1]}(y)^{\varphi} f^{[1]}(y) y dy + F^{[1]}((1+\varphi)R)^{1+\varphi} R.$$

This is an equilibrium, as L turns out to be indifferent between abstaining from bidding, and submitting a bid equal to the reserve price, and H turns out to be indifferent between bidding R (and therefore pool with all types in the interval $[L, H]$), and bidding marginally higher than R, and g^R is derived from the same differential equation as the bid function for FPSB without reserve price.

PROPOSITION 14. *Assume independent private values and two bidders. Let $B_1^R(\varphi, t)$, the bid of a bidder with value t, be given by*

$$B_1^R(\varphi, t) = \begin{cases} g^R(t) & \text{if } t > H \\ R & \text{if } L \leq t \leq H \\ \text{"no bid"} & \text{if } t < L. \end{cases}$$

Then $B_1^R(\varphi, t)$ constitutes a symmetric Bayesian Nash equilibrium of FPSB if $R > 0$.[12]

PROOF. Assume that threshold types L and H exist such that in equilibrium all types $t < L$ abstain from bidding, all types $t \in [L, H]$

[12]Note that $B_1^R(\varphi, t)$ is continuous at H. This must be the case in equilibrium. Suppose, on the contrary, that the bid function has a jump at H. Then a bidder with a type slightly higher than H has an incentive to deviate from the bid strategy to a bid of just above R.

bid R, and all types $t > H$ bid according to a strictly increasing bid function g^R.

A type L is indifferent between not bidding and bidding R. The utility of abstaining from bidding is equal to

$$\varphi \int_H^{\bar{t}} g^R(t)dF(t) + \varphi R[F(H) - F(L)].$$

The utility when bidding R is equal to

$$\varphi \int_H^{\bar{t}} g^R(t)dF(t) + \frac{1}{2}\left[F(H) - F(L)\right]\{\varphi R + (L - R)\} + F(L)(L - R).$$

Equating both expressions yields

$$\frac{\varphi[F(H) - F(L)]R}{F(H) + F(L)} = L - R. \tag{4.1}$$

L is uniquely determined from (4.1) as the LHS of (4.1) is strictly decreasing in L and the RHS of (4.1) is strictly increasing in L for $L \geq 0$.

A type H is indifferent between bidding R and bidding an infinitesimal δ above R. The utility when bidding R is equal to

$$\varphi \int_H^{\bar{t}} g^R(t)dF(t) + \frac{1}{2}\left[F(H) - F(L)\right]\{\varphi R + (H - R)\} + F(L)(H - R).$$

The utility when bidding $R + \delta$ when δ converges to 0 is equal to

$$\varphi \int_H^{\bar{t}} g^R(t)dF(t) + [F(H) - F(L)](H - R) + F(L)(H - R).$$

Equating both expressions, and some manipulation yields

$$H = (1 + \varphi)R.$$

In order to complete the proof, we need to check whether types have no incentive to deviate from the proposed equilibrium. We only check if a type $t > H$ has no incentive to mimic another type $t' > H$, as by a standard argument, other deviations are not profitable. Incentive compatibility of types $t > H$ implies that g^R should follow from the same differential equation as derived in the proof of Proposition 8 with the boundary condition $g^R(H) = R$. Analogous to the proof of Proposition 8, it can be established that $g^R(t)$ is strictly increasing for $t \geq H$ if and only if $g^R(t) < \frac{t}{1+\varphi}$. Now, for $t > H$,

$$\begin{aligned} g^R(t) &= F^{[1]}(t)^{-1-\varphi} \int_H^t F^{[1]}(y)^{\varphi} f^{[1]}(y) y dy + F^{[1]}(H)^{1+\varphi} R \\ &< F^{[1]}(t)^{-1-\varphi} \int_H^t F^{[1]}(y)^{\varphi} f^{[1]}(y) t dy + F^{[1]}(H)^{1+\varphi} R \\ &= \frac{t}{1+\varphi} - \left[\frac{t}{1+\varphi} - R\right] \left[\frac{F^{[1]}(H)}{F^{[1]}(t)}\right]^{1+\varphi} \\ &\leq \frac{t}{1+\varphi}. \end{aligned}$$

□

To get an intuition why pooling at R occurs in equilibrium, consider a situation in which $R \geq \frac{\bar{t}}{1+\varphi}$. The threshold level H, above which bidders bid according to a strictly increasing bid function, lies above $\bar{t}$, so that bidders either abstain from bidding, or bid R. Why is this an equilibrium? Suppose that one of the two bidders submits a bid $b \geq R$. Then the other bidder prefers losing to winning. This can be seen as follows. If she loses, then her utility is

$$\varphi b \geq \varphi R \geq \frac{\varphi \bar{t}}{1+\varphi} \geq \frac{\varphi t}{1+\varphi},$$

whereas winning gives her a utility of at most

$$t - R \leq t - \frac{\bar{t}}{1+\varphi} \leq t - \frac{t}{1+\varphi} = \frac{\varphi t}{1+\varphi}.$$

Low types are then willing to lose the opportunity of getting the object by abstaining from bidding. High types bid R, assuring themselves the object if the other bidder does not bid, but also making sure that if the other bids, to lose as often as possible.

4.2. Second-price sealed-bid auction. In contrast to FPSB, SPSB sometimes has a (weakly) separating Bayesian Nash equilibrium when a reserve price is imposed. This observation follows trivially when the reserve price is smaller than the lowest submitted equilibrium bid, which is strictly positive according to Proposition 10. However, also in nontrivial cases weakly separating Bayesian Nash equilibria exist. Proposition 15 gives a necessary and sufficient condition for the existence of a weakly separating Bayesian Nash equilibrium. If the equilibrium exists, types up to a threshold type $\widehat{t}$ abstain from bidding, and types above $\widehat{t}$ submit the same bid as in the case of no reserve price.

PROPOSITION 15. *Assume independent private values and two bidders. Let $R \in [B_2(\varphi, \underline{t}), B_2(\varphi, \overline{t})]$. SPSB with a reserve price R has a weakly separating Bayesian Nash equilibrium if and only if $B_2(\varphi, R) \geq R$. If an equilibrium exists, then it is given by*

$$B_2^R(\varphi, t) = \begin{cases} B_2(\varphi, t) & \text{if } t \geq \widehat{t} \\ \text{“no bid”} & \text{if } t < \widehat{t} \end{cases}$$

where $B_2^R(\varphi, t)$ is the bid of a bidder with value t, and where $\widehat{t}$ is the unique solution to

$$\varphi(B_2(\varphi, \widehat{t}) - R)(1 - F(\widehat{t})) = F(\widehat{t})[R - \widehat{t}].$$

PROOF. Suppose there is an R for which a weakly separating equilibrium exists. Suppose that an indifference type $\widehat{t}$ exists, such that

$$B_2^R(\varphi, t) = \begin{cases} B_2(\varphi, t) & \text{if } t \geq \widehat{t} \\ \text{“no bid”} & \text{if } t < \widehat{t} \end{cases}$$

is an equilibrium, where B_2 is the equilibrium bid function in the case of $R = 0$. $\widehat{t}$ is indifferent between submitting no bid, and submitting a bid equal to $B_2(\varphi, \widehat{t})$. Hence, $\widehat{t}$ follows from the following equation

$$(1 - F(\widehat{t}))\varphi R = F(\widehat{t})(\widehat{t} - R) + (1 - F(\widehat{t}))\varphi B_2(\varphi, \widehat{t}),$$

which is equivalent to

$$\varphi R = \frac{F(\widehat{t})}{(1 - F(\widehat{t}))}(\widehat{t} - R) + \varphi B_2(\varphi, \widehat{t}). \tag{4.2}$$

For $t \geq \widehat{t}$, $B_2^R(\varphi, t)$ follows from the same differential equation as derived in the proof of Proposition 10 with the same boundary condition $B_2^R(\varphi, \overline{t}) = \frac{\overline{t}}{1+\varphi}$, so that indeed $B_2^R(\varphi, t) = B_2(\varphi, t)$ for all R and $t \geq \widehat{t}$.

A weakly separating equilibrium exists if and only if $B_2(\varphi, \widehat{t}) \geq R$, as all bids should be above R. We will show now that $B_2(\varphi, \widehat{t}) \geq R$ is equivalent to the condition $B_2(\varphi, R) \geq R$, which completes the proof.

Define $\widetilde{t}$ such that

$$B_2(\varphi, \widetilde{t}) = R. \tag{4.3}$$

As $B_2(\varphi, t)$ is strictly increasing in t, $\widetilde{t}$ is uniquely determined. Consider the function h with

$$h(t) \equiv \frac{F(t)}{1 - F(t)}(t - R) + \varphi B_2(\varphi, t)$$

for all t. Note that h is a strictly increasing function, with

$$h(\widehat{t}) = \varphi R,$$

(which follows from (4.2)), and

$$h(\widetilde{t}) = \frac{F(\widetilde{t})}{1 - F(\widetilde{t})}(\widetilde{t} - R) + \varphi R. \tag{4.4}$$

Now, with (4.3), as B_2 is strictly increasing,

$$B_2(\varphi, R) \geq R \iff B_2(\varphi, \widetilde{t}) = R \leq B_2(\varphi, R) \iff \widetilde{t} \leq R.$$

Moreover, with (4.4), as h is strictly increasing,

$$\widetilde{t} \leq R \iff h(\widetilde{t}) \leq \varphi R = h(\widehat{t}) \iff \widetilde{t} \leq \widehat{t}.$$

Finally, as B_2 is strictly increasing, and from (4.3),

$$\widetilde{t} \leq \widehat{t} \iff B_2(\varphi, \widehat{t}) \geq R.$$

□

An intuition for the condition $B_2(\varphi, R) \geq R$ being necessary is the following. In a weakly separating Bayesian Nash equilibrium, a bidder with type R is always prepared to submit a bid of at least R. To see this, observe that for this bidder, in a weakly separating Bayesian Nash equilibrium, a bid equal to R yields the same revenue as abstaining from bidding. However, in equilibrium, each type that submits a bid, does so according to the equilibrium bid function for the situation with no reserve price. This implies that if $B_2(\varphi, R) < R$, a bidder with type R would submit a bid below the reserve price, which is not possible, so that a contradiction is established.

The intuition for the condition being sufficient is as follows. In a weakly separating Bayesian Nash equilibrium, each bidder who submits a bid, submits a bid as if there were no reserve price. Then, for the existence of a weakly separating equilibrium, it remains to be checked that $B_2(\varphi, \widehat{t}) \geq R$. If $B_2(\varphi, R) \geq R$, then there is a type $\widetilde{t} \leq R$ for which $B_2(\varphi, \widetilde{t}) = R$. As a reserve price does not affect equilibrium bidding of types that submit a bid, it follows that if type $\widetilde{t}$ would submit a bid in equilibrium, she would submit a bid equal to R. However, type R is indifferent between bidding R and not submitting a bid, so that $\widetilde{t}$ prefers not to submit a bid. Therefore, $\widehat{t}$ must exceed $\widetilde{t}$, so that indeed $B_2(\varphi, \widehat{t}) \geq B_2(\varphi, \widetilde{t}) = R$.

The necessary and sufficient condition $B_2(\varphi, R) \geq R$ implies that for small R a weakly separating Bayesian Nash equilibrium exists, but not for large R. As said, the existence of such an equilibrium is trivial in the case of small R. However, for R close to $\bar{t}$, $B_2(\varphi, R) < R$, as, by Proposition 3.4, $B_2(\varphi, \bar{t}) < \bar{t}$.

5. Concluding remarks

We have studied auctions in which losing bidders obtain financial externalities from the winning bidder. We have derived bidding equilibria for FPSB and SPSB, and have established that the presence of a resale market does not affect equilibrium behavior. Also, we have shown that in the two-bidder case SPSB dominates FPSB in terms of expected auction revenue if $\varphi < 1$ and that both auctions are revenue equivalent if $\varphi = 1$. Moreover, we have studied equilibrium bidding for FPSB and SPSB when a reserve price is imposed. We have observed pooling at the reserve price for FPSB. For SPSB, we found a necessary and sufficient condition for the existence of a weakly separating Bayesian Nash equilibrium.

An interesting possibility for future research is to investigate what the effects are of asymmetric financial externalities in a private values environment. For instance, one may examine what happens in case only one of the bidders imposes a financial externality on the other bidders. Bulow et al. (1999) consider a situation in which two bidders bid for a common value object, and one of the bidders receives a fraction of the auction revenue. The bidder without toehold in the auction revenue faces a strong winner's curse, and therefore bids zero in equilibrium, even if the toehold of the other bidder in the auction revenue is infinitesimally small. Although the authors restrict their attention to a common value environment, their analysis shows that asymmetric financial externalities may have dramatic effects on the auction revenue.

Motivated by the observation that in SPSB, low signal bidders increase their bids when φ is increased (for φ not too large), also a model with asymmetries in the valuation function may be fruitful to study. One may imagine that with one bidder with a low value, and one bidder with a high value, the price in SPSB may be higher with financial externalities than without financial externalities, as the bidder with the low value has an incentive to push up the price when φ is strictly positive.

6. Appendix

Proof of Lemma 6. Define $\psi(y) \equiv y + \frac{\varphi}{1+\varphi}(1-y)^{\frac{1+\varphi}{\varphi}} - (\frac{1}{1+\varphi})y^{1+\varphi} - \frac{\varphi}{1+\varphi}$. The first and second order derivatives of ψ are respectively given by

$$\begin{aligned} \psi'(y) &= 1-(1-y)^{\frac{1}{\varphi}} - y^{\varphi}, \text{ and} \\ \psi''(y) &= \frac{1}{\varphi}(1-y)^{\frac{1}{\varphi}-1} - \varphi y^{\varphi-1}. \end{aligned}$$

Observe that

$$\begin{aligned}\psi(0) &= \psi(1) = 0,\\ \psi'(0) &= \psi'(1) = 0,\\ \lim_{y \downarrow 0} \psi''(y) &= -\infty,\\ \psi''(1) &= -\varphi < 0.\end{aligned}$$

Hence, if y is close to 0, $\psi(y)$ must be below zero and concave, and similarly for y close to 1, $\psi(y)$ is negative and concave. Suppose now that, in contrast to what is stated in the lemma, $\psi(y) > 0$ for some $y \in (0,1)$. As ψ and all its derivatives are continuous functions on the interval $(0,1)$, $\psi''(y)$ must change sign at least four times, or, equivalently, $\psi''(y) = 0$ for at least four values of y in $(0,1)$. Define $\mu \equiv \frac{1}{\varphi}$, $\nu \equiv \varphi - 1$, and $\xi \equiv \frac{1}{\varphi} - 1$. Note that $\frac{\xi}{\nu} < 0$. Then,

$$\begin{aligned}\psi''(y) &= \mu(1-y)^{\xi} - \varphi y^{\nu} = 0 \Longrightarrow\\ \frac{\varphi}{\mu} &= \frac{y^{\nu}}{(1-y)^{\xi}} = \left(\frac{y}{(1-y)^{\frac{\xi}{\nu}}}\right)^{\nu} \Longrightarrow\\ \frac{\varphi^{\frac{1}{\nu}}}{\mu}(1-y)^{\frac{\xi}{\nu}} &= y.\end{aligned}$$

The last expression has at most *two* solutions in the interval $[0,1]$, as the left hand side is strictly convex in y, and the right hand side is a linear function in y. A contradiction is established, so the first part of the lemma must be true. The second part is trivial. ■

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CHAPTER 4

Optimal Auctions with Financial Externalities

1. Introduction

We will consider the problem of a seller who wishes to sell one indivisible object in an optimal auction in an environment with *financial externalities.* An optimal auction is a feasible auction mechanism that maximizes the seller's expected revenue. To get an idea about the environment, imagine that two firms bid for a license to increase their capacity in the market in which they compete. When financial markets are assumed not to work perfectly, the winner is able to invest less, the higher the price it pays in the auction. This is an advantage to the losing firm, so that the losing firm's utility depends on the payments made in the auction by the winner. Throughout the paper, we will refer to the effect of other bidders' payments on a bidder's utility as a financial externality.[1] Especially in high stake auctions, like the UMTS auctions that took place in Europe in 2000 and 2001, financial externalities may influence bidding behavior (Maasland and Onderstal, 2002; Börgers and Dustmann, 2001).

Myerson (1981) initiates research on optimal auctions in an environment without financial externalities.[2,3] He derives three important results. The first is the celebrated Revenue-Equivalence Theorem, which states that the expected utility of both the bidders and the seller is

[1]In our companion paper (Maasland and Onderstal, 2002), we study equilibrium behavior in first-price and second-price sealed-bid auctions in an environment with financial externalities. Theories of equilibrium bidding in related environments can be found in Engelbrecht-Wiggans (1994), and in Bulow et al. (1999).

[2]Independently and simultaneously, Riley and Samuelson (1981) derive similar results.

[3]Myerson (1981) was followed by, among others, Engelbrecht-Wiggans (1988), Cremer and McLean (1985, 1988), Maskin and Riley (1989), McAfee and Reny (1992), and Bulow and Klemperer (1996).

completely determined by the allocation rule of the feasible auction mechanism and the utilities of the lowest types. We refer to this result as the *Weak Revenue-Equivalence Theorem.* Second, with symmetric bidders, all standard auctions yield the same expected revenue. We refer to this result as the *First Strong Revenue-Equivalence Theorem.* Third, with symmetric bidders, all standard auctions are optimal when the seller imposes the same, optimal reserve price. We refer to this result as the *Second Strong Revenue-Equivalence Theorem.*[4]

With asymmetric bidders, under a regularity condition, Myerson shows that the optimal auction assigns the object to the bidder with the highest marginal revenue, provided that the highest marginal revenue is nonnegative. In case all bidders have a negative marginal revenue, the seller keeps the object. Moreover, the utilities of the lowest types in an optimal auction are equal to zero. For this result, Myerson assumes that (1) the seller can prevent resale of the object after the auction, and (2) he can fully commit to not selling the object. The first assumption is made, as the seller may need to misassign the object, i.e., assign it to a bidder who does not have the highest value for it. The second assumption is made, as the seller may optimally withhold the object when only low valued bidders participate. When these assumptions hold, we will speak of a *Myersonean World.*

Ausubel and Cramton (1999) argue that sometimes the assumption of a Myersonean World is not realistic, and study optimal auctions in a setting in which (1) the seller cannot prevent the object changing hands in a perfect resale market,[5] and (2) he cannot commit to keeping the object. We will refer to this setting as a *Double Coasean World,* as the first assumption is related to the Coase Theorem (Coase, 1960), and the second to the Coase Conjecture (Coase, 1972). Haile (1999) proves that, with symmetric bidders, equilibrium bidding in standard auctions does not change when bidders are offered a resale market opportunity after the auction. With this result, the *Third Strong Revenue-Equivalence*

[4]Myerson does not mention this result explicitly, but it follows from his study. Riley and Samuelson (1981) explicitly derive the result in an independent private values model.

[5]In a perfect resale market, the object, when being sold in the auction, always ends up in the hands of the bidder with the highest value.

Theorem can be derived: In a Double Coasean World, with symmetric bidders, all standard auctions (without reserve price) are optimal.

In this paper, we modify Myerson's model by allowing for financial externalities, given by an exogenous parameter φ. We assume a model with independent private signals. The model has independent private values models and pure common value models as special cases. With symmetric bidders, this model is a special case of the affiliated private signals model of Milgrom and Weber (1982). We will show that with financial externalities, the Weak Revenue-Equivalence Theorem remains valid. Also the conditions for optimality remain the same as in Myerson.

Our companion paper (Maasland and Onderstal, 2002) shows that the strong revenue-equivalence results are not valid when bidders are confronted with financial externalities. The First Strong Revenue-Equivalence Theorem does not hold as in the case of two bidders, the first-price sealed-bid auction yields less expected revenue than the second-price sealed-bid auction. The driving force behind this result is that the expected utility of the lowest type in the first-price auction is higher than the expected utility of the lowest type in the second-price auction. The Second Strong Revenue-Equivalence Theorem does not hold for two reasons. First, a standard auction with reserve price gives the lowest type strictly positive expected utility because of the payments by others. Second, the first-price sealed-bid auction and the second-price sealed-bid auction may not have equilibria in which active bidders submit bids according to a function that is strictly increasing in their type, so that the winner of the object is not always the bidder with the highest marginal revenue. The Third Strong Revenue-Equivalence Theorem fails to hold as in both the first-price and the second-price sealed-bid auction, the lowest type gets a strictly positive expected utility.

In the remainder of the paper, we will show the optimality of the *lowest-price all-pay auction* when we take a symmetry assumption. (Goeree and Turner, 2002, derive a similar result in a related environment.) In Section 4, we solve the seller's problem in a Double Coasean World. We start with this setting, as it is more straightforward to find an optimal auction here than in a Myersonean World. We derive that the lowest-price all-pay auction is optimal, as the lowest type gets zero expected utility. In Section 5, we find a two-stage feasible auction mechanism which solves the seller's problem in a Myersonean World. In the first stage, all bidders pay an entry fee, in order to make sure that the lowest type gets zero expected utility. If at least one of the bidders indicate not to be willing to accept the entry fee, the seller keeps the object, and no payments are made. Otherwise, in the second stage, the lowest-price all-pay auction with a reserve price is played. The optimality of the lowest-price all-pay auction with a reserve price follows from the observation that, if it assigns the object, it always assigns the object to the bidder with the highest marginal revenue. In both worlds, in an optimal auction, the highest possible expected revenue is strictly increasing in φ, and a bidder's expected utility is independent of φ.

2. The model

Consider a seller, who wishes to sell one indivisible object to one out of n risk neutral bidders, numbered $1, 2, ..., n$. The seller aims at finding a feasible auction mechanism which gives him the highest possible expected revenue. We assume that the seller does not attach any value to the object. Each bidder i receives a one-dimensional private signal t_i. (We also say that "bidder i is of type t_i".) t_i is drawn, independently from all the other private signals, from a distribution function F_i. F_i has support on the interval $[\underline{t}_i, \bar{t}_i]$, and continuous density f_i with $f_i(t_i) > 0$, for every $t_i \in [\underline{t}_i, \bar{t}_i]$. Define the sets

$$T \equiv [\underline{t}_1, \bar{t}_1] \times ... \times [\underline{t}_n, \bar{t}_n],$$

and

$$T_{-i} \equiv \times_{j \neq i} [\underline{t}_j, \bar{t}_j],$$

with typical elements $\mathbf{t} \equiv (t_1, ..., t_n)$ and $\mathbf{t}_{-i} \equiv (t_1, ..., t_{i-1}, t_{i+1}, ..., t_n)$ respectively. Let

$$g(\mathbf{t}) \equiv \prod_j f_j(t_j)$$

be the joint density of $\mathbf{t}$, and let

$$g_{-i}(\mathbf{t}_{-i}) \equiv \prod_{j \neq i} f_j(t_j)$$

be the joint density of $\mathbf{t}_{-i}$.

The value of the object for a bidder is defined as a function of her own signal, and the signals of all the other bidders. Denote by $v_i(\mathbf{t})$ the value for bidder i given that the vector of types is $\mathbf{t}$. We make the following assumptions on the functions v_i.[6]

Value Differentiability: v_i is differentiable in all its arguments, for all $i, \mathbf{t}$.

Value Monotonicity: $v_i(\mathbf{t}) \geq 0$, $\frac{\partial v_i(\mathbf{t})}{\partial t_i} > 0$, and $\frac{\partial v_i(\mathbf{t})}{\partial t_j} \geq 0$, for all $i, j, \mathbf{t}$.

Value Differentiability ensures the existence of each bidder's marginal revenue (which will be defined later). *Value Monotonicity* indicates that all bidders are serious, and that bidders' values are strictly increasing in their own signal, and weakly increasing in the signals of the others.

In Sections 4 and 5, we make the following extra assumption.

Symmetry: $F_i = F_j$ for all i, j, and $v_i(..., t_i, ..., t_j, ...) = v_j(..., t_j, ..., t_i, ...)$ for all t_i, t_j, i, j.

Symmetry may be crucial for the existence of efficient equilibria in standard auctions.[7] *Value Differentiability, Value Monotonicity*, and *Symmetry* together ensure that the bidder with the highest signal is

[6]Myerson (1981) uses the following value functions:

$$v_i(\mathbf{t}) \equiv t_i + \sum_{j \neq i} e_j(t_j),$$

where e_j is the revision effect function related to bidder j, with $e_j : [\underline{t}_j, \bar{t}_j] \rightarrow \Re$. These value functions are not necessarily included in our model.

[7]Klemperer (1998) shows that a slight asymmetry in value functions may have dramatic effects on bidding behavior in the English auction in a(n almost) common value setting. Although efficiency is not an issue with (almost) common values, the result shows the importance of symmetry in value functions. Maskin and Riley

also the bidder with the highest value, so that these assumptions imply that the seller assigns the object efficiently if and only if the bidder with the highest signal gets it.

When *Symmetry* holds, let $F \equiv F_1 = ... = F_n$, $f \equiv f_1 = ... = f_n$, $\underline{t} \equiv \underline{t}_1 = ... = \underline{t}_n$, and $\overline{t} \equiv \overline{t}_1 = ... = \overline{t}_n$. Also, we will let $F^{[1]}$ and $f^{[1]}$ ($F^{[n-1]}$ and $f^{[n-1]}$) denote the cumulative distribution function and density function of $\max_{j \neq 1} t_j$ ($\min_{j \neq 1} t_j$). Finally, let us define $v(y, z)$ as the expected value that bidder i assigns to the object, given that her signal is y, and that the highest signal of all the other bidders is equal to z.

$$v(y, z) \equiv E_{\mathbf{t}_{-i}}[v_i(\mathbf{t})| \; t_i = y, \; \max_{j \neq i} t_j = z].$$

With *Symmetry*, this model is a special case of the affiliated signals model of Milgrom and Weber (1982).

Throughout the paper, we use the following definition of bidder i's marginal revenue.

$$MR_i(\mathbf{t}) \equiv v_i(\mathbf{t}) - \frac{1 - F_i(t_i)}{f_i(t_i)} \frac{\partial v_i(\mathbf{t})}{\partial t_i}.$$

This expression can be derived, like in Bulow and Roberts (1989) (for independent private values) and Bulow and Klemperer (1996) (for independent private signals), from the monopolist's problem in third-degree price discrimination. This can be done by constructing bidder i's demand curve from her value function and signal distribution function, and differentiate the related monopolist's profit function with respect to quantity. When *Symmetry* is satisfied, let $MR(\mathbf{t}) \equiv MR_1(\mathbf{t}) = ... = MR_n(\mathbf{t})$. We make the following assumption on MR_i.

MR Monotonicity: $MR_i(\mathbf{t})$ is strictly increasing in t_i for all $i, \mathbf{t}$.

(2000) study the effect of asymmetric distributions on bidding behavior in the first-price and the second-price sealed-bid auction, and show that the equilibrium of the first-price auction is inefficient.

MR Monotonicity is equivalent to the assumption made in standard micro-economic theory that the monopolist's demand curve is downward-sloping.

The bidders are risk-neutral expected utility maximizers. In order to incorporate the financial externalities, we insert an exogenous nonnegative parameter φ into the bidders' utility functions. This parameter indicates each bidder's interest in the others' payments. The utility of bidder i is

$$v_i - x_i + \varphi \sum_{j \neq i} x_j$$

when i wins the object, and

$$-x_i + \varphi \sum_{j \neq i} x_j$$

when i does not win the object, with x_j the payment by bidder j to the seller. We assume $\varphi \in [0, \frac{1}{n-1})$.[8]

3. Weak revenue equivalence

Using the Revelation Principle of Myerson (1981), we may assume, without loss of generality, that the seller only considers feasible auction mechanisms in the class of feasible direct revelation mechanisms.[9] Let (p, x) be a feasible direct revelation mechanism, with

$$p : T \to [0, 1]^n,$$

where

$$\sum_j p_j(\mathbf{t}) \leq 1,$$

and

$$x : T \to \Re^n.$$

[8] In case $\varphi \in [0, \frac{1}{n-1})$, a bidder's interest in his own payments is larger than his interest in the payments by the other bidders. In footnote 13, we will discuss the consequences of allowing φ to be larger than $\frac{1}{n-1}$.

[9] A feasible direct revelation mechanism is an auction mechanism in which each bidder is asked to announce his type, which satisfies individual rationality conditions, incentive compatibility conditions, and straightforward restrictions on the allocation rule.

We interpret $p_i(\mathbf{t})$ as the probability that bidder i wins, and $x_i(\mathbf{t})$ as the expected payments by i to the seller when $\mathbf{t}$ is announced.

Bidder i's utility of (p, x) given $\mathbf{t}$ is given by

$$v_i(\mathbf{t})p_i(\mathbf{t}) - x_i(\mathbf{t}) + \varphi \sum_{j \neq i} x_j(\mathbf{t}),$$

so that bidder i's interim utility of (p, x) can be written as

$$U_i(p, x, t_i) \equiv \int_{T_{-i}} [v_i(\mathbf{t})p_i(\mathbf{t}) - x_i(\mathbf{t}) + \varphi \sum_{j \neq i} x_j(\mathbf{t})] g_{-i}(\mathbf{t}_{-i}) d\mathbf{t}_{-i}, \tag{3.1}$$

with $d\mathbf{t}_{-i} \equiv dt_1 ... dt_{i-1} dt_{i+1} ... dt_n$.

Similarly, the seller's expected utility of (p, x) is

$$U_0(p, x) \equiv \int_T \sum_{i=1}^{n} x_i(\mathbf{t}) g(\mathbf{t}) d\mathbf{t},$$

with $d\mathbf{t} \equiv dt_1 ... dt_n$.

The following two lemmas will be used to solve the seller's problem.

LEMMA 7. *Let (p, x) be a feasible direct revelation mechanism. Then the interim utility of (p, x) for bidder i is given by*

$$U_i(p, x, t_i) = U_i(p, x, \underline{t}_i) + \int_{\underline{t}_i}^{t_i} w_i(s_i) ds_i, \tag{3.2}$$

with

$$w_i(t_i) \equiv E_{t_{-i}}\{p_i(\mathbf{t}) \frac{\partial v_i(\mathbf{t})}{\partial t_i}\}.$$

PROOF. Incentive compatibility implies

$$U_i(p, x, s_i) \geq U_i(p, x, t_i) + E_{t_{-i}}\{p_i(\mathbf{t})(v_i(s_i, \mathbf{t}_{-i}) - v_i(\mathbf{t}))\}$$

for all s_i, t_i and $\mathbf{t}_{-i}$, or, equivalently

$$\frac{\partial U_i(p, x, t_i)}{\partial t_i} = E_{t_{-i}}\{p_i(\mathbf{t}) \frac{\partial v_i(\mathbf{t})}{\partial t_i}\} = w_i(t_i), \tag{3.3}$$

at all points where p_i is differentiable in t_i (by *Value Differentiability*, v_i is differentiable at any t_i). By integration of (3.3), we get (3.2). □

LEMMA 8. *Let (p, x) be a feasible direct revelation mechanism. The seller's expected revenue from (p, x) is given by*

$$U_0(p,x) = \frac{E_{\mathbf{t}}\{\sum_{i=1}^{n} MR_i(\mathbf{t})p_i(\mathbf{t})\} - \sum_{i=1}^{n} U_i(p,x,\underline{t}_i)}{1-\varphi(n-1)}. \tag{3.4}$$

PROOF. Define

$$X_i \equiv \int_T x_i(\mathbf{t})g(\mathbf{t})d\mathbf{t}, \tag{3.5}$$

$$V_i \equiv \int_T v_i(\mathbf{t})p_i(\mathbf{t})g(\mathbf{t})d\mathbf{t}, \tag{3.6}$$

and

$$Y_i \equiv \int_{T_i} U_i(p,x,t_i)f_i(t_i)dt_i. \tag{3.7}$$

By (3.1), we have, for all i,

$$Y_i = V_i - X_i + \varphi \sum_{j \neq i} X_j. \tag{3.8}$$

Adding the equations in (3.8) over i and rearranging terms implies that the seller's expected revenue from a feasible direct revelation mechanism (p, x) is given by

$$U_0(p,x) = \sum_{i=1}^{n} X_i = \frac{\sum_{i=1}^{n} V_i - \sum_{i=1}^{n} Y_i}{1-\varphi(n-1)}. \tag{3.9}$$

Taking the expectation of (3.2) over t_i and using integration by parts, we obtain

$$E_{t_i}\{U_i(p,x,t_i)\} = U_i(p,x,\underline{t}_i) + E_{t_i}\{\frac{1-F_i(t_i)}{f_i(t_i)} w_i(t_i)\},$$

with

$$w_i(t_i) \equiv E_{t_{-i}}\{p_i(\mathbf{t})\frac{\partial v_i(\mathbf{t})}{\partial t_i}\},$$

so that (3.4) follows with (3.9) and (3.5)-(3.7). □

From Lemmas 7 and 8, it immediately follows that the Weak Revenue-Equivalence Theorem remains valid with financial externalities.

COROLLARY 3. *Both the seller's and the bidders' expected utility from any feasible auction mechanism is completely determined by the probability function p and the utilities of the lowest types* $U_i(p, x, \underline{t}_i)$ *for all i related to its equivalent feasible direct revelation mechanism* (p, x).

Observe from Lemmas 7 and 8 respectively that, provided that the expected utility of the lowest type remains zero when φ is varied, a bidder's interim utility does not depend on φ, whereas the seller's expected revenue is increasing in φ. An intuition for the first observation is the following. Suppose that bidders, instead of receiving financial externalities, obtain a fraction φ of what the other bidders pay in the auction. Then Myerson (1981) shows that the interim utility of a bidder does not depend on φ. From a bidder's perspective, these two situations are equivalent, and the observation follows immediately. The intuition for the second observation follows from the first. Fix the payments of all bidders. Then a bidder's expected utility increases with φ. Therefore, to make sure that a bidder's interim utility does not depend on φ, her expected payment must increase as well.

From Lemma 8, interesting insights can be drawn with respect to optimal auctions. Observe that in the expression for the seller's expected revenue, a key role is played by the marginal revenues of the bidders. Suppose that the seller finds a feasible auction mechanism that assigns the object to the bidder with the highest marginal revenue, provided that the marginal revenue is nonnegative, and that leaves the object in the hands of the seller if the highest marginal revenue is negative. Suppose also that this feasible auction mechanism gives the lowest types zero expected utility. Then, under *MR Monotonicity*,[10] with the individual rationality constraints $U_i(p, x, \underline{t}_i) \geq 0$, this feasible auction mechanism is optimal. In Section 5, we will discuss this observation in more detail, and we will show how the seller can construct an optimal auction in an environment with financial externalities.

[10]This assumption is needed for incentive compatibility considerations. See Myerson (1981) for a further discussion on the consequences of relaxing this assumption.

4. The Double Coasean World

For the remainder of the analysis, we assume that *Symmetry* holds. Consider the *lowest-price all-pay auction*, which has the following rules. All bidders simultaneously and independently announce a bid to the seller. The bidder who announces the highest bid gets the object, with ties being broken among the highest bidders with equal probability. Each bidder has to pay the lowest submitted bid. We will show now that in a Double Coasean World, the lowest-price all-pay auction is optimal.

Recall that a Double Coasean World is a situation in which (1) the seller cannot prevent a perfect resale market, and (2) the seller cannot withhold the object. These assumptions impose two extra restrictions on the seller's problem, namely

$$\text{for all } \mathbf{t} \text{ and } i,\ p_i(\mathbf{t}) > 0 \text{ only if } t_i = \max_j t_j \tag{4.1}$$

and

$$\text{for all } \mathbf{t},\ \sum_i p_i(\mathbf{t}) = 1 \tag{4.2}$$

respectively. In fact, these restrictions fix $p_i(\mathbf{t})$ (apart from the zero mass events $t_i = t_j$ for some i and j) in such a way that the object is always assigned to the bidder with the highest signal.

As restrictions (4.1) and (4.2) fix the allocation rule p, by Lemma 8, a sufficient condition for the optimality of a feasible auction mechanism is that the lowest types expect zero utility (from the auction plus resale market). The lowest-price all-pay auction is a natural candidate to fulfill this requirement. To see this, suppose that in equilibrium, the auction is efficient, and that a bidder with the lowest type considers to bid b. Then, as the equilibrium is efficient, all the other bidders have to pay b. The expected utility of the lowest type equals $-b+(n-1)\varphi b$, which is strictly negative for all $b > 0$ when $\varphi \in [0, \frac{1}{n-1})$. Therefore, the lowest type prefers to bid zero, so that she obtains zero expected utility, as when she is present, each bidder pays zero in the auction.

Proposition 16 characterizes the symmetric equilibrium for the lowest-price all-pay auction. By a standard argument, the equilibrium bid function must be strictly increasing and continuous. Let $U(t, s)$ be the utility for a bidder with signal t who behaves as if having signal s, whereas the other bidders play according to the equilibrium bid function. A necessary equilibrium condition is that

$$\frac{\partial U(t,s)}{\partial s} = 0$$

at $s = t$. From this condition, a differential equation can be derived, from which the equilibrium bid function is uniquely determined (at least if we restrict our attention to differentiable bid functions). Observe that indeed the lowest type bids zero, that the equilibrium is efficient, and that bids increase with φ.[11]

PROPOSITION 16. *Suppose that all bidders submit a bid according to the following bid function.*

$$B(t) = \frac{1}{(1-\varphi(n-1))} \int_{\underline{t}}^{t} \frac{v(y,y) f^{[1]}(y)}{1 - F^{[n-1]}(y)} dy.$$

Then B constitutes the unique symmetric differentiable Bayesian Nash equilibrium of the lowest-price all-pay auction. The outcome of this auction is efficient.

PROOF. The following observations imply that a symmetric equilibrium bid function must be strictly increasing and continuous. First, a higher type of a bidder cannot submit a lower bid than a lower type of the same bidder. (If the low type gets the same expected surplus from strategies with two different probabilities of being the winner of the object, the high type strictly prefers the strategy with the highest probability of winning, so that the high type will not submit a lower bid than the low type.) Second, $B(t)$ cannot be constant on an interval $[t', t'']$. (By bidding slightly higher, t'' can largely improve its probability of winning, while only marginally influencing the payments by her

[11] In case of a uniform signal distribution on the interval $[0, 1]$, independent private values, and two bidders, the unique symmetric differentiable Bayesian Nash equilibrium of the lowest-price all-pay auction is established by

$$B(t) = \frac{1}{1-\varphi}[-t - \log(1-t)].$$

and the other bidders.) Third, $B(t)$ cannot be discontinuous at any t. (Suppose that $B(t)$ makes a jump from $\underline{b}$ to $\overline{b}$ at t^*. A type just above t^* has an incentive to deviate to $\underline{b}$. Doing so, she is able to substantially decrease the expected auction price, while just slightly decreasing the probability of winning the object.)

We proceed assuming a strictly increasing and differentiable equilibrium bid function. The probability of having the lowest bid for a bidder with signal t is equal to $1 - F^{[n-1]}(t)$. If x is the auction price, then, in terms of utility, each bidder loses $(1 - \varphi(n-1))x$. Define

$$\tilde{B}(s) \equiv (1 - \varphi(n-1))B(s),$$

and $U(t, s)$ as the expected utility of a bidder with type t who misrepresents herself as type s given that the other bidders report truthfully. Then,

$$U(t,s) = \int_{\underline{t}}^{s} v(t,y)dF^{[1]}(y) - \left[1 - F^{[n-1]}(s)\right] \tilde{B}(s) - \int_{\underline{t}}^{s} \tilde{B}(y)dF^{[n-1]}(y).$$

The first term of the RHS refers to the value of the object when the highest bid is submitted. The second term refers to the payments made in case the lowest bid is submitted, and the third term refers to the expected payments in case another bidder submits a lower bid. The FOC of the equilibrium is given by

$$v(t,t)f^{[1]}(t) + f^{[n-1]}(t)\tilde{B}(t) - \left[1 - F^{[n-1]}(t)\right] \tilde{B}'(t) - f^{[n-1]}\tilde{B}(t) = 0. \tag{4.3}$$

With some manipulation, we get

$$\tilde{B}(t) = \tilde{B}(\underline{t}) + \int_{\underline{t}}^{t} \frac{v(y,y)f^{[1]}(y)}{1 - F^{[n-1]}(y)} dy$$

or, equivalently

$$B(t) = \frac{B(\underline{t})}{1 - \varphi(n-1)} + \frac{1}{1 - \varphi(n-1)} \int_{\underline{t}}^{t} \frac{v(y,y)f^{[1]}(y)}{1 - F^{[n-1]}(y)} dy.$$

The only best response of a bidder with signal $\underline{t}$, given that the outcome of the auction is efficient, is to bid zero, so that $B(\underline{t}) = 0$. The SOC is fulfilled, as

$$sign\left(\frac{\partial U(t,s)}{\partial s}\right) = sign\left(\frac{\partial U(t,s)}{\partial s} - \frac{\partial U(s,s)}{\partial s}\right) = sign(v(t,s) - v(s,s)) = sign(t-s).$$

An immediate consequence of the fact that $v(y,y) > 0$ for all $y > \underline{t}$ (by *Value Monotonicity*) is that the bid function $B(t)$ is strictly increasing in t, which is the assumption we started with. □

In Proposition 17, we establish that the presence of a perfect resale market has no influence on equilibrium behavior. This result follows from our companion paper, where we derive that any Bayesian Nash equilibrium of any auction (without resale market) which leads to an efficient assignment of the object, is also a Bayesian Nash equilibrium when the same auction is followed by a resale market where the same bidders participate. As B constitutes an efficient Bayesian Nash equilibrium, the proposition must be true.

PROPOSITION 17. *The bid function B described in Proposition 16 establishes a Bayesian Nash equilibrium of the lowest-price all-pay auction when this auction is followed by a (perfect) resale market with the same bidders participating.*

The optimality of the lowest-price all-pay auction immediately follows.[12]

PROPOSITION 18. *Consider a Double Coasean World. Suppose that in the lowest-price all-pay auction, bidders play according to the equilibrium bid function given in Proposition 16. Then the lowest-price all-pay auction is optimal.*

[12] In the light of Myerson and Satterthwaite (1983), the assumption of a *perfect* resale market seems rather strong. However, if *MR Monotonicity* holds, the assumption of a perfect resale market can be relaxed to allow for any type of resale market. In our companion paper we show that auctions with efficient equilibria still have an equilibrium with efficient bidding in case of a resale market. Therefore, when *MR Monotonicity* is satisfied, Lemma 8 implies that every efficient auction with zero utility for the lowest type (which includes the lowest-price all-pay auction) is optimal under the restriction that the seller cannot keep the object.

PROOF. The equilibrium bid function of the lowest-price all-pay auction given in Proposition 16 is an efficient Bayesian Nash equilibrium, in which the expected utility of the lowest type is zero. By Proposition 17, this is still an equilibrium when the auction is followed by a resale market, so that the expected utility of the lowest type remains zero. Then, by Lemma 8, with restrictions (4.1) and (4.2), the lowest-price all-pay auction is optimal. □

COROLLARY 4. *Consider a Double Coasean World. Then the highest possible expected revenue is strictly increasing in φ. In an optimal auction, a bidder's expected utility is independent of φ.*

PROOF. Follows immediately from Lemmas 7 and 8, Propositions 16-18, and the fact that the lowest-price all-pay auction is efficient with zero expected utility for the lowest type. □

5. The Myersonean World

Consider a Myersonean World. As said, Lemma 8 implies that a feasible auction mechanism is optimal when it yields zero expected utility for the lowest type, leaves the object in the hands of the seller when all marginal revenues are negative, and assigns the object to the bidder with the highest marginal revenue otherwise. Consider two-stage auction mechanism Γ. In the first stage of Γ, the bidders are asked whether or not to participate. If at least one of the bidders refuses to participate, the game ends, and the seller keeps the object. Otherwise, each bidder pays the seller the same entree fee, which we denote by Φ. Then the bidders enter the second stage, and play the lowest-price all-pay auction with reserve price R. Each bidder follows the strategy to choose "participate" in the first stage, and to play according to a Bayesian Nash equilibrium in the second stage.

The lowest-price all-pay auction with a reserve price R has the following rules. Each bidder either submits a bid of at least R, or abstains from bidding. If all bidders abstain, the object remains in the hands of the seller, otherwise it will be sold to the bidder with the highest bid. In the case of ties, the winner is chosen from the highest bidders with equal probability. All bidders who submitted a bid pay the auction price, which is equal to the lowest submitted bid in case all bidders submit a bid, and equal to R otherwise.

Proposition 19 shows that the lowest-price all-pay auction has an equilibrium in which, up to a threshold type $\hat{t}$, bidders do not submit a bid, and all bidders with a type $t \geq \hat{t}$ bid $h(t,\hat{t})$, with

$$h(t,\hat{t}) \equiv R + \frac{1}{(1-\varphi(n-1))} \int_{\hat{t}}^{t} \frac{v(y,y) f^{[1]}(y)}{1-F^{[n-1]}(y)} dy.$$

We derive h using the same differential equation as for the lowest-price all-pay auction without a reserve price, with boundary condition $h(\hat{t},\hat{t}) = R$. Observe that $h(t,\hat{t})$ is strictly increasing in both t and φ. In equilibrium, a type $\hat{t}$ is indifferent between bidding R and submitting no bid.

PROPOSITION 19. *Let $B^R(t)$, the bid of a bidder with signal t, be given by*

$$B^R(t) \equiv \begin{cases} h(t,\hat{t}) & \text{for } t \geq \hat{t} \\ \text{“no bid”} & \text{for } t < \hat{t}, \end{cases}$$

where $\hat{t}$ is the unique solution to

$$\int_{\underline{t}}^{\hat{t}} v(\hat{t},y) dF^{[1]}(y) = R. \tag{5.1}$$

Then B^R constitutes a symmetric Bayesian Nash equilibrium of the lowest-price all-pay auction with a reserve price R.

PROOF. Assume that a threshold type $\hat{t}$ exists such that in equilibrium, all types $t < \hat{t}$ abstain from bidding, and all types $t \geq \hat{t}$ bid according to h. It is straightforwardly verified that $h(\cdot,\hat{t})$ satisfies (4.3) with the boundary condition $h(\hat{t},\hat{t}) = R$. In equilibrium, $\hat{t}$ must be indifferent between not bidding and bidding R. Hence

$$\varphi RN(R) = -R + \varphi RN(R) + \int_{\underline{t}}^{\hat{t}} v(\hat{t},y) dF^{[1]}(y), \tag{5.2}$$

where $N(R)$ is the expected number of the other bidders who submit a bid. (5.2) is equivalent to (5.1). Since v is strictly increasing in its first argument (by *Value Monotonicity*), (5.1) has a unique solution for $\widehat{t}$. It is then standard to check that no type has an incentive to deviate from the equilibrium. □

Proposition 20 shows that when *MR Monotonicity* is satisfied, Γ is optimal if the entry fee is given by (5.3).[13] In an optimal auction, the seller's revenue is strictly increasing in φ, and a bidder's expected utility does not depend on φ.

PROPOSITION 20. *Consider a Myersonean World. Suppose that MR Monotonicity is satisfied. Let the entry fee in* Γ *be given by*

$$\Phi \equiv \frac{\underline{u}}{1-\varphi(n-1)}, \tag{5.3}$$

where $\underline{u}$ *is the expected utility of the lowest type in the lowest-price all-pay auction when the equilibrium of Proposition 19 is played. Also, suppose that the reserve price* R *is such that for the threshold type* $\widehat{t}$ $MR(\widehat{t}) = 0$ *holds, that all bidders choose "participate" in equilibrium, and that bidders play according to the equilibrium given in Proposition 19. Then* Γ *is optimal.*

PROOF. According to the equilibrium defined in Proposition 19, all types above $\widehat{t}$ submit a bid according to a strictly increasing bid function. All types below $\widehat{t}$ abstain from bidding. Let p^* be the allocation rule of the feasible direct revelation mechanism related to the lowest-price all-pay auction with the specified reserve price and the given equilibrium. Then, by *MR Monotonicity*, p^* maximizes $E_{\mathbf{t}}\{\sum_{i=1}^{n} MR_i(\mathbf{t})p_i(\mathbf{t})\}$ over all feasible direct revelation mechanisms (p, x). Moreover, by definition of Φ, the expected utility of bidder i's lowest type equals zero

[13] The assumption $\varphi \in [0, \frac{1}{n-1})$ is crucial for Proposition 20. If $\varphi > \frac{1}{n-1}$, the seller can establish an arbitrarily high revenue by a take-it-or-leave-it offer to all bidders, in which he asks an arbitrarily high entry fee $\tilde{\Phi}$. The take-it-or-leave-it offer is such that only if every bidder accepts to pay the fee, the seller collects the payments. It is a dominant strategy for every bidder to accept the mechanism, since participation gives a utility larger than zero ($\tilde{\Phi}[-1+\varphi(n-1)] > 0$).

over both stages of Γ, as

$$\underline{u} + \varphi \sum_{j \neq i} \Phi - \Phi = 0.$$

The given strategies constitute a Bayesian Nash equilibrium, and when these are played, Γ maximizes (3.4). Therefore, Γ is optimal. □

COROLLARY 5. *Consider a Myersonean World. Suppose that MR Monotonicity is satisfied. Then the highest possible expected revenue is strictly increasing in φ. In an optimal auction, a bidder's expected utility is independent of φ.*

PROOF. Follows immediately from Lemmas 7 and 8, and Propositions 19 and 20. □

6. Concluding remarks

In this paper, we have investigated optimal auctions with financial externalities. We have established the optimality of the lowest-price all-pay auction in this environment. In a Double Coasean World, the lowest-price all-pay auction itself is optimal. In a Myersonean World, we have found an optimal two-stage auction mechanism in which each bidder pays an entry fee, and plays the lowest-price all-pay auction with a reserve price.

Goeree and Turner (2001) study optimal auctions in an environment that is related to ours. In Goeree and Turner's model, bidders receive (potentially different) shares of the seller's revenue. The seller's net revenue is optimized under the restriction that the seller cannot withhold the object. Goeree and Turner define an auction, called the *all-pay-all auction*, in which each bidder's payment is a weighted sum of all bids, which depends on all bidders' shares in the seller's revenue. Goeree and Turner show that with symmetric bidders, the all-pay-all auction is optimal. Moreover, with equal shares, Goeree and Turner prove the optimality of the lowest-price all-pay auction in their environment.

So far it's unclear whether there exists an auction in our environment (perhaps having the same structure as the all-pay-all auction), which is optimal when we allow for asymmetric financial externalities. A major advantage of the lowest-price all-pay auction over the all-pay-all auction is that the rules of the lowest-price all-pay auction are context independent, in contrast to the rules of the all-pay-all auction. The rules of both auction games do not depend on the distribution function F or the value functions v. However, the rules of the all-pay-all auction do depend on the bidders' shares of the seller's revenue, whereas the rules of the lowest-price all-pay auction do not.

Jehiel et al. (1996) study optimal auctions in environments with allocative externalities, i.e., environments in which a loser's utility depends on the identity of the winner (not on how much she pays). They derive the optimality of a feasible auction mechanism which is similar to two-stage feasible auction mechanism Γ. In the first stage of this feasible auction mechanism, bidders are asked whether to participate or not. In the second stage, depending on which bidders participate, the object remains in the hands of the seller, or is allocated to one of the bidders. Each participating bidder receives (pays) money from (to) the seller.

It remains an open question whether the lowest-price all-pay auction performs well in practice. The auction seems to be very sensitive for collusion. Moreover, apart from the efficient equilibrium, the lowest-price all-pay auction also has highly inefficient equilibria in the case of two bidders. It is easily verified that there is an equilibrium in which one bidder submits a very high bid, and the other bids zero.[14] An experimental study may put some light on this matter.

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[14]In case of three bidders there is no equilibrium in which one bidder bids very high and the other bidders bid zero, because in such an equilibrium one of the low bidders has an incentive to overbid the high bidder.

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CHAPTER 5

The Chopstick Auction

1. Introduction

In February 1998, the Dutch government auctioned licenses for second generation mobile telecommunication. Two big lots and sixteen small lots were sold using a variant of the ascending multiple object auction format that had been used by the FCC to sell PCS licenses in the US.[1] The big lots (A and B) consisted of 75 DCS-1800 channels each, 15 small lots (1,...,15) consisted of 12 or 13 DCS-1800 channels each, and 1 small lot (16) contained 22 channels.[2] The Dutch government decided to split the spectrum in such small fractions in order to give bidders enough flexibility to get an optimal division of the channels. Incumbents bidders (KPN[3] and Libertel[4]) were not allowed to bid on lots A and B, and newcomers (Telfort,[5] Dutchtone,[6] TeleDanmark, Orange/Veba[7] and Airtouch[8]) were allowed to bid on all lots. A newcomer was believed to need one big lot or at least five small lots in order to operate a feasible network for mobile telecommunication.

[1]See, McMillan (1994), Cramton (1995, 1998), McAfee and McMillan (1996), and Milgrom (2000) for descriptions and discussions of these auctions.

[2]One channel is equivalent to 0.2 MHz.

[3]KPN used to be the state monopolist of telecommunication and mail in The Netherlands.

[4]Libertel was at that time a consortium of Vodaphone and the Dutch bank ING.

[5]Telfort was at that time a consortium of British Telecom and Dutch Railways.

[6]DutchTone was bidding under the name of Federa, a consortium of Deutsche Telekom, France Telecom and two Dutch banks. After the auction, Deutsche Telekom withdrew from the consortium.

[7]Orange and Veba are mobile telecom operators in the UK and Germany respectively.

[8]Airtouch is a US baby bell.

A set of less than five small lots would, if we neglect the possibility of resale, be of no value to a newcomer.[9]

The lots were sold according to the following rules.[10] There is a sequence of rounds, in which bidders submit bids on lots. For each lot, the minimum bid in round 1 equals 0. For the following rounds, the minimum bid on each lot is equal to the current highest bid on this lot plus a small bid increment of at most 10% of the current highest bid. Each bidder is eligible to bid in round $t+1$ if either she submits at least one bid in round t, or she is overbid in round t on at least one of the lots she currently has the highest bid on. When eligible to bid, a bidder is allowed to submit bids on all lots.[11] The only exception to this rule is that a bidder is not allowed to be active on both lot A and lot B. At the beginning of round $t+1$, each bidder receives information about bidding activity in round t, and information that is relevant for the current round. The auction ends when in a certain round no bids are submitted. Each lot is allocated to the bidder with the current highest bid for a price equal to this bid.

The Dutch rules differ from the ones used for the US auctions in at least four ways. First, bidders are not allowed to withdraw their bid when an inefficient lock-in is imminent. Second, there is no activity rule which forces bidders to remain active on a given fraction of the number of channels they desire to obtain. Third, there is no common knowledge about who has the highest bid on which lots in a round. Fourth, the auction is asymmetric in the sense that incumbents are restricted on the lots they are allowed to bid on. In this paper, we will discuss the effect of the first difference on the outcome of the auction. For discussion of the effects of the other differences, see Van Damme (2000).

[9]Resale was in fact possible, but only to a party that did not actively participate in the auction, and then only with the approval of the Minister of Economic Affairs.

[10]See Van Damme (2000) for a more detailed description of the auction rules.

[11]In fact, in the Dutch DCS-1800 auction, a bidder was only allowed to bid on lots for which he paid a relatively small deposit before the start of the auction. All bidder had paid the deposit for all lots they were eligible to bid on.

In contrast to the outcome of the American auctions,[12] the outcome of the Dutch DCS-1800 auction was probably not efficient. From Table 3, it can be seen that identical objects[13] were sold for substantial different prices.[14] In other words, the Law of One Price was not satisfied, which should have been the case in a perfect market, and which indicates inefficiency. Another indicator of inefficiency was the fact that there was resale of channels after the auction. Ben[15] was authorized by the Dutch authorities to acquire the licences bought by TeleDanmark and Orange/Veba in the auction.

Lot	**C**	**Winner**	**P**	**P/C**
A	75	Dutchtone	600	8
B	75	Telfort	545	7.3
1	13	Libertel	40.4	3.1
2	12	KPN	40.2	3.4
3	13	Orange/Veba	38	2.9
4	12	Telfort	40.5	3.4
5	13	KPN	43	3.3
6	12	TeleDanmark	41.1	3.4
7	13	KPN	40.4	3.1
8	12	KPN	39.1	3.3
9	13	Orange/Veba	46.5	3.6
10	12	TeleDanmark	41.25	3.4
11	13	KPN	42.98	3.3
12	12	TeleDanmark	39.9	3.3
13	13	KPN	39.9	3.1
14	12	KPN	40.5	3.4
15	13	Libertel	45.5	3.5
16	22	TeleDanmark	71.5	3.3

[12]See Cramton (1998).

[13]In fact, there were small differences. Some of the frequencies could not be used for regions near the Belgium and/or the German border, and the A and B lots also included some GSM frequencies. Van Damme (2000) argues that the effects of these heterogoneities on the value per frequency are not large enough to explain the large price differences.

[14]The incumbents, KPN and Libertel, seem to have profited from the auction design. They paid a lower price for their frequencies than two of their upcoming competitors, even though they were limited in the sense that they were not allowed to bid for the large lots.

[15]Ben was at the time a joint venture of Belgacom (70%) and TeleDanmark (30%).

Table 3. Summary of the outcome of the DCS-1800 auction. **P** stands for the final price of the lot in millions of Dutch guilders, **C** for the number of channels the lot consisted of, and **P/C** for the price (in millions of Dutch guilders) paid per channel.

We conjecture, following Van Damme (2000), that the auction format that was used in the Netherlands leads to lower bids and to a less efficient auction outcome than the American auction format, as the Dutch auction format confronts bidders with the *exposure problem*, whereas the American format does not, as bidders are allowed to withdraw their bid. As a consequence of the fact that a bid could not be withdrawn in the Dutch DCS-1800 auction, the money a bidder spent on the small lots on which she had the highest bid should be regarded as sunk costs. Bidders then rather play the war of attrition than a standard auction game. Being aware of the possibility of losing money on the small lots, bidders were very active on the large ones, and bid hardly on the small ones. Such bidding behavior probably leads to inefficient outcomes and a low revenue. In the literature on multiple object auctions, this problem is referred to as the exposure problem, as bidders are *exposed* to the risk of paying more for an object than what it is worth to them.[16]

Motivated by the Dutch DCS-1800 auction, and with the aim to get a better understanding of the exposure problem, we will study a stylized model of a multiple object auction in which the exposure problem is present. The model is defined in Section 2. A seller simultaneously sells three chopsticks in an auction, which we will call the Chopstick Auction (CSA). In CSA, the price, which is the same for each object, is raised continuously. Bidders have the opportunity to step out at each price, until one bidder is left. The outcome of CSA is such that the second highest bidder gets one chopstick for the auction price p, and the highest bidder gets two chopsticks for a price of $2p$. We assume that bidders' marginal values are zero on the first chopstick, positive on second, and zero on the third. Bidders are incompletely informed about the other bidders' marginal value for the second chopstick. As

[16]See Bykowsky et al. (1998), and Milgrom (2000).

the second highest bidder wins a worthless chopstick for a positive price, bidders face the exposure problem in CSA.

We will investigate whether an auction with an exposure problem is less efficient and/or yield less revenue than an auction in which the exposure problem is not present. In order to do so, in Sections 3 and 4, we will compare CSA with the second-price sealed-bid auction, in which the three chopsticks are sold as one bundle (SPSB). From standard auction theory, we learn that SPSB has an efficient equilibrium (in dominant strategies), in which each bidder submits a bid equal to her value for the chopsticks. In the case of two risk neutral bidders with identical value distribution functions, we show that CSA has a unique symmetric Bayesian-Nash equilibrium. CSA is efficient, and revenue equivalent with SPSB. However, in the case of loss averse bidders, SPSB has either a more efficient outcome or a higher expected auction revenue than CSA. With three bidders, under general assumptions on the value distributions and the utility functions of the bidders, we derive an impossibility result: CSA has no symmetric equilibrium. We conjecture that this result implies that CSA has no efficient equilibrium, so that the seller, when aiming at efficiency, is better off by replacing CSA with SPSB.

From these findings, we conclude that the Dutch government could have improved the DCS-1800 auction by designing an auction in which bidders do not suffer from the exposure problem. There are at least three ways for auction designers to prevent auctions from suffering from the exposure problem. The first is that the auction designer offers large packages of objects rather than small ones. Specifically, the Dutch government may have obtained a better auction outcome in the DCS-1800 auction by not splitting up the spectrum in such small lots, despite the fact that in that case, the bidders are not given the opportunity to "let the market decide" on the division of the spectrum.[17] In the model, we have shown that the seller is weakly better off if he auctions the chopsticks as one bundle rather than as three different objects.

[17] "Letting the market decide" has another important drawback, namely that the realized market structure may be very concentrated, which is a concern when thinking about consumer surplus (Jehiel and Moldovanu, 2000; Klemperer, 2001).

A second way to get rid of the exposure problem is a withdrawal rule. Such a rule gives bidders the opportunity to withdraw their bid when an inefficient lock-in is imminent. In the FCC auctions in the US, and also in the DCS-1800 auction and the UMTS auction in Germany,[18] a withdrawal rule was implemented. After the auction, bidders were allowed to withdraw their bid on certain licenses. A withdrawing bidder had to pay a penalty in case the final price of the license was lower than her bid. It is questionable if such a withdrawal rule completely solves the exposure problem, as bidders still face a considerable risk of having to pay the penalty. In our model, the driving force behind the results is that the losing bidder has to pay money, so that these results remain valid with the introduction of a withdrawal rule with penalty.

A third way to get around the exposure problem is to allow for combinatorial bids. In our model, rational bidders will only submit strictly positive bids on bundles of two or three chopsticks. If the payments rules are such that the winning bidder pays the highest bid of its opponents on a bundle of two, the auction reduces to the second-price sealed-bid auction, which we have shown to be (weakly) better than the Chopstick Auction. However, allowing for combinatorial bids may lead to other problems. Bykowsky et al. (1998) identify the *threshold problem* in such auctions, which states that small bidders may have to solve complicated coordination problems in order to be able to overbid large bidders. Another problem is that in the case of a large number of objects, determining the winning combination may be computationally intractable. In fact, Rothkopf et al. (1998) show the winner determination problem to be NP-hard. Also, bidding in the case of combinatorial bids is complicated for the bidders. For instance, in the Dutch DCS-1800 auction, a bidder has the possibility to submit bids on all $2^{18} - 1 \approx 250,000$ possible combinations of licenses!

Several papers of Robert Rosenthal and co-authors are closely related to ours. Krishna and Rosenthal (1996), and Rosenthal and Wang (1996) analyze multiple object auctions with two types of bidders, namely "local" bidders who are interested in only one object, and

[18]See Jehiel and Moldovanu (2000) for a theoretical analysis of the German UMTS auction.

"global" bidders who try to acquire several. The global bidders, in competition with the local ones, face the exposure problem when attempting to realize synergies between the objects. The equilibrium outcome of the auction is typically not efficient. Szentes and Rosenthal (2001a, 2001b) construct equilibria in the first-price sealed-bid, the second-price sealed-bid, and the all-pay version of CSA with complete information. The value of a bundle of chopsticks is the same for each bidder. In equilibrium, these auctions are efficient. The most important different between Rosenthal's studies and ours is that all mentioned papers consider one shot auctions, whereas CSA is an ascending auction, as are the PCS auctions in the US and the DCS-1800 auction in the Netherlands.

Some other papers in the literature on multiple object auctions are related to ours as well. Bykowsky et al. (1998) gives an illustrative example in which in equilibrium the auction outcome is either inefficient, or at least one of the bidders ends up paying more for the purchased items than they are worth to her. Ausubel and Cramton (1998) stress the importance of efficiency of the auction outcome in terms of revenues for the seller in auctions of perfectly divisible objects. Ausubel and Cramton (1999) show that efficiency of the auction outcome is necessary for revenue maximization when the auction is followed by a perfect resale market and when the seller cannot commit to not selling some objects. Milgrom (2000) constructs an example in which, in contrast to our results, the seller realizes a less efficient outcome when using larger packages (but gets a higher revenue). Klemperer (2001) lists issues that are of practical importance in the design of multiple object auctions. The results derived in this paper indicate that the warning "avoid the exposure problem" should be added to this list.

2. The model

Consider a situation with n bidders, $n \in \{2,3\}$, labeled $1,...,n$, who wish to eat Chinese food. However, none of the bidders has anything to eat with. Suppose that a seller sells 3 chopsticks in the Chopstick Auction (CSA) which has the following rules. The price starts at zero, and is continuously raised. Bidders have the opportunity to quit the auction at any price they desire. The seller informs all remaining bidders when one of the bidders quit. The auction ends when one bidder is left, who wins two chopsticks, and pays two times the price at which the second highest bidder quits. The second highest bidder wins one chopstick and pays the price at which she quits.[19] We will compare CSA with the second-price sealed bid auction in which the three chopsticks are sold as one bundle (SPSB).

The value $V_i(s)$ bidder i attaches to owning s chopsticks is given by

$$V_i(s) = \begin{cases} v_i & s = 2, 3 \\ 0 & s = 0, 1, \end{cases} \tag{2.1}$$

where v_i is a private signal of bidder i. In words, a bidder attaches a value of v_i to winning two chopsticks, and no value to winning only one chopstick or to winning a third one. We assume that v_i is drawn independently from the other signals from the interval $[\underline{v}, \overline{v}]$, with $0 \leq \underline{v} < \overline{v}$, according to a strictly increasing, continuous probability distribution $F_i(.)$ with density $f_i(.) \equiv F_i'(.)$. Sometimes we will take the simplifying assumption that bidders draw their signal from the same distribution.

Each bidder is an expected utility maximizer. The utility for bidder i who buys a set of chopsticks which gives her value V_i for a price of p_i is given by $U_i(V_i - p_i)$. For every i, U_i is assumed to be a continuous function which is strictly increasing, with $U_i(0) = 0$. For the sake of tractability, we assume in Section 3 that the bidders are either risk neutral (i.e., $U_i(x) = x$ for all x) or loss averse, which will be defined later. In Section 4, we use general utility functions.

[19]In this auction, ties are broken as follows. In case of two (remaining) bidders, when a tie takes place at a price of p, a fair coin is tossed. If tails comes up, the bidder with the lowest label wins two chopsticks for a price of $2p$, and the other bidder wins one chopstick for a price of p. If heads come up, the outcome is reversed. When the auction is played by three bidders, in the first stage, either two or three bidders may decide to step out at the same price of p. In either case, the game ends immediately. When two bidders step out, then the third bidder gets two chopsticks for a price of $2p$. With 50-50 probability, one of the other bidders is awarded one chopstick for a price of p. When all three bidders decide to step out at p, the bidders' labels are randomly ordened in such a way that each ordering is equally likely. The first bidder then gets two chopsticks for a price of $2p$, and the second one for a price of p. The third neither gets nor pays anything.

In CSA, there is one winner, the bidder who wins both chopsticks, and one "real" loser, which is the bidder who buys one worthless chopstick for a positive price. Table 4 shows the effect of the quitting order on the utility levels of the bidders in the case of three bidders, when the price of a chopstick is equal to p. From Table 4, it becomes clear that CSA can also be seen as an English auction, in which the winner pays the bid of the second highest bidder, and the second highest bidder pays half of her own bid.

Bidder	**Quits**	**# Chopsticks won**	**Payment**	**Utility**
i	First	0	0	$U_i(0) = 0$
j	Second	1	p	$U_j(-p)$
k	Third	2	$2p$	$U_k(v_k - 2p)$

Table 4. Possible outcomes of CSA.

We assume that the seller aims at reaching the following two goals.

Efficiency: Generate an efficient outcome, i.e., the bidder with the highest signal obtains two chopsticks;

Revenue: Given that *Efficiency* is fulfilled, maximize expected auction revenue.

3. Two bidders

Consider CSA with two bidders. The game ends immediately when one of the bidders indicates to quit. Therefore, the strategy of a bidder is a bid in the interval $[0, \infty)$ for each realization of her signal.

3.1. Risk neutral bidders. In order to keep the model tractable, we restrict our attention to identical distributions, i.e., both bidders draw their signal from the same distribution $F \equiv F_1 = F_2$.

Proposition 21 gives equilibrium bidding in CSA when both bidders are risk neutral. By a standard argument, this bid function must be strictly increasing and continuous. Let $U(v, w)$ be the utility for a bidder with signal v who behaves as if she has signal w, whereas the other bidders play according to the equilibrium bid function. A necessary equilibrium condition is that

$$\frac{\partial U(v,w)}{\partial w} = 0$$

at $w = v$. From this condition, a differential equation is derived, from which the equilibrium bid function is uniquely determined.

PROPOSITION 21. *Let $n = 2$. Suppose both bidders are risk neutral, and draw their signals from the same distribution function F. Let $B(v)$, the bid of a bidder with signal v, be given by*

$$B(v) = (1 - F(v)) \int_{\underline{v}}^{v} \frac{f(x)x}{(1 - F(x))^2} dx.$$

Then B is the unique symmetric Bayesian Nash equilibrium of CSA. The outcome of the auction is efficient.

PROOF. The following observations imply that a symmetric equilibrium bid function must be strictly increasing. First, a higher-value type of a bidder cannot exit before a lower-value type of the same bidder would exit. (Suppose the lower type is indifferent between two different strategies, giving her two different probabilities of being the ultimate winner of two chopsticks. The higher type then strictly prefers the strategy with the higher probability to win. Therefore, she will never quit earlier than the lower type.) Furthermore, there is no range in which the bid function is flat. (Suppose there is the bid function is flat at a price level of p. Then each bidder being in the range of signals that bid p exits the auction with positive probability at p. But if this is the case, then each bidder strictly prefers staying just a bit longer.)

Let $\tilde{B}$ be a symmetric and strictly increasing equilibrium bid function. If the other bidder bids according to $\tilde{B}$, the expected utility of a bidder with signal v who bids as if she has signal w is given by

$$U(v,w) = -(1 - F(w))\tilde{B}(w) + \int_{\underline{v}}^{w} f(x)(v - 2\tilde{B}(x))dx.$$

The first (second) term of the RHS refers to the case that the bidder makes the second highest (highest) bid.

The FOC of the equilibrium is

$$\frac{\partial U(v,w)}{\partial w} = -(1-F(w))\tilde{B}'(w) - f(w)\tilde{B}(w) + vf(w) = 0 \tag{3.1}$$

at $w = v$. Rearranging terms we find

$$\frac{(1-F(v))\tilde{B}'(v) + f(v)\tilde{B}(v)}{(1-F(v))^2} = \frac{f(v)v}{(1-F(v))^2},$$

which is equivalent to

$$\frac{\tilde{B}(v)}{1-F(v)} = \int_{\underline{v}}^{v} \frac{f(x)x}{(1-F(x))^2}dx + C,$$

for some C. C must be zero (C must be at least zero, otherwise a bidder with signal $\underline{v}$ submits a negative bid; if C is larger than zero, a bidder with signal $\underline{v}$ submits a strictly positive bid. As $\tilde{B}$ is (by assumption) strictly increasing, this bidder submits the lowest bid with probability one, and has to buy one chopstick for a positive price. Clearly, she is strictly better off by bidding zero.) Also the SOC is fulfilled as $sign(\frac{\partial U(v,w)}{\partial w}) = sign(v-w)$. It is readily checked that B is a solution.

What remains to be checked it that B is strictly increasing. From (3.1), B is strictly increasing if and only if $B(v) < v$ for (almost) all v. This is true, as

$$\begin{aligned} B(v) &= (1-F(v))\left[\int_{\underline{v}}^{v} \frac{f(x)x}{(1-F(x))^2}dx\right] \\ &= v - \underline{v} - (1-F(v))\int_{\underline{v}}^{v} \frac{1}{(1-F(x))}dx \\ &< v. \end{aligned}$$

As B is strictly increasing, CSA is efficient.

The uniqueness of the equilibrium follows with the Revenue Equivalence Theorem which states that the expected payment made by any bidder given her signal is unique by the efficiency of the outcome and the utility of the lowest type. As the equilibrium bid function is strictly increasing, and the utility of the lowest type is always zero in an efficient equilibrium, B is the unique equilibrium bid function. □

Using CSA, the seller reaches both his goals *Efficiency* and *Revenue*. By the Revenue Equivalence Theorem (Myerson, 1981), CSA yields the same expected revenue as any other efficient auction mechanism in which the bidder with the lowest signal obtains zero expected utility. This follows from the fact that CSA is an auction of a single object, namely a set of two chopsticks, which is allocated efficiently according to Proposition 21. Therefore, there is no efficient auction that can improve the revenues for the seller in comparison with CSA, so that the seller reaches both *Efficiency* and *Revenue*. More specifically, the seller is indifferent between using CSA and SPSB to sell the three chopsticks.

COROLLARY 6. *Let $n = 2$. Suppose both bidders are risk neutral, and draw their signals from the same distribution function. When the seller uses either CSA or SPSB, then his goals Efficiency and Revenue are fulfilled.*

3.2. Loss averse bidders. What is the effect on the outcome of CSA when bidders are loss averse rather than risk neutral? We model loss aversion in the following simplified way. Bidder i is called θ_i-loss averse if her utility function $U_i(.)$ is given by

$$\begin{aligned} U_i(x_i) &= x_i \text{ for all } x_i \geq 0 \\ U_i(x_i) &= \theta_i x_i \text{ for all } x_i < 0, \end{aligned}$$

where $\theta_i > 1$ is the loss aversion parameter for bidder i. The interpretation of θ_i-loss aversion is the following. If a θ_i-loss averse bidder loses x_i units, then she perceives this as if she were a risk neutral bidder losing $\theta_i x_i$ units. More specifically, the realized utility u_i from CSA for bidder i having signal v_i, who buys $s \in \{1, 2\}$ chopsticks in CSA at a price of p per chopstick, is given by

$$u_i(v_i, s, p) = \begin{cases} v_i - 2p & \text{if } s = 2 \text{ and } v_i \geq 2p, \\ \theta_i(v_i - 2p) & \text{if } s = 2 \text{ and } v_i < 2p, \\ -\theta_i p & \text{if } s = 1. \end{cases}$$

Proposition 22 establishes that the seller strictly prefers SPSB over CSA. As SPSB has en efficient outcome, this auction fulfills *Efficiency*.

There are two possibilities that have to be checked. If is CSA not efficient, then the Proposition is immediately established, as the targets of efficiency and revenue maximization are lexicographically ordened. If CSA is efficient, then it remains to be checked that SPSB yields strictly more revenue than CSA. Using the Envelope Theorem, we show that the expected utility for each bidder i given her signal v_i is higher than in SPSB, which implies that expected payments in CSA are lower than in SPSB.

PROPOSITION 22. *Suppose that each bidder i is θ_i-loss averse. The seller who aims at fulfilling the criteria Efficiency and Revenue is strictly better off replacing CSA with SPSB.*

PROOF. As SPSB has an equilibrium in weakly dominated strategies, in which each bidders bids her value for the bundle of three chopsticks, the outcome of SPSB is efficient, so that *Efficiency* is fulfilled. Myerson (1981) shows that for this auction, the interim utility for bidder i having signal v_i is given by

$$U_i^{SPSB}(v_i) = \int_{\underline{v}}^{v_i} P_i(x)dx,$$

with $P_i(x)$ the probability that x is the highest signal.

Let (p_i, δ_i) denote the outcome of CSA for bidder i, where p_i is her payment, $\delta_i = 1$ if she wins two chopsticks and $\delta_i = 0$ if she wins 0 or 1 chopstick. Let $d_i(p_i, v_i, \delta_i)$ be the difference between the realized *value* in CSA (i.e., $\delta_i v_i$) and the realized *utility* level for bidder i having signal v_i and loss aversion parameter θ_i if the auction outcome is (p_i, δ_i). Hence,

$$d_i(p_i, v_i, \delta_i) = \begin{cases} \theta_i p_i & \text{if } \delta_i = 0, \\ p_i & \text{if } \delta_i = 1 \text{ and } p_i \leq v_i, \\ \theta_i p_i - (\theta_i - 1)v_i & \text{if } \delta_i = 1 \text{ and } p_i > v_i. \end{cases}$$

Call $d_i(p_i, v_i, \delta_i)$ the *subjective costs for bidder i*. Observe that $d_i(p_i, v_i, \delta_i) \geq p_i$ (the subjective costs are higher than the actual payments), and that $d_i(p_i, v_i, \delta_i)$ is (weakly) decreasing in v_i. Let $D_i(w, v_i)$ denote the expected value of $d_i(p_i, v_i, \delta_i)$ for bidder i with signal v_i, who bids as if she has signal w, while all the other bidders play according to their Bayesian-Nash equilibrium strategy.

Suppose that for CSA, *Efficiency* holds (otherwise SPSB is already better). Then the equilibrium probability for a bidder with signal v_i to win in the auction is given by $P_i(v_i)$. Given the equilibrium strategies of the other bidders, a bidder optimally announces her true signal v_i, maximizing

$$U_i(w, v_i) \equiv P_i(w)v_i - D_i(w, v_i)$$

with respect to w. Let

$$U_i^{CSA}(v_i) \equiv U_i(v_i, v_i).$$

By the Envelope Theorem,

$$\frac{d\tilde{U}_i(v_i)}{dv_i} = P_i(v_i) - D_i^2(v_i, v_i), \tag{3.2}$$

where $D_i^2(v_i, v_i)$ denotes the derivative of $D_i(v_i, v_i)$ with respect to its second argument. By definition, $D_i^2(v_i, v_i) \leq 0$. Integrating (3.2), and by individual rationality, we have

$$U_i^{CSA}(v_i) = \int_{\underline{v}}^{v_i} \{P_i(x) - D_i^2(x, x)\}dx + \tilde{U}_i(\underline{v}) \geq \int_{\underline{v}}^{v_i} P_i(x)dx = U_i^{SPSB}(v_i)$$

so that the interim utility of bidder i in CSA is (weakly) higher than in SPSB. This implies that the expected subjective costs in the CSA of bidder i are (weakly) lower than the expected payments in SPSB. Efficiency implies that there is always a bidder who buys one chopstick for a strictly positive price, so that the expected *payments* to the seller are strictly lower than the expected *subjective costs*. Therefore, the expected revenue of CSA is strictly lower than the expected revenue of SPSB. □

Proposition 22 is intuitive in the light of the Revenue Equivalence Theorem. In CSA, loss averse bidders bid the same as risk neutral bidders who pay $\theta_i b$ when their bid b is the second highest bid. By the Revenue Equivalence Theorem, in the case of efficiency, the expected payment of risk neutral bidders to the seller does not depend on θ_i. Loss averse bidders, however, only pay their bid b rather than $\theta_i b$ in the case they lose, so that they pay less than their risk neutral equivalents.

Therefore, the seller is better off if he chooses an efficient auction in which the bidder cannot incur losses, such as SPSB.[20]

4. Three bidders

In the case of three bidders, CSA consists of two stages. In stage 1, each bidder decides at which point to leave the auction. At some point in time, one of the bidders leaves the auction, and the two remaining bidders enter stage 2. In stage 2, both remaining bidders have to make a decision about how long to stay, given the price at which the other bidder left.

A symmetric Bayesian Nash equilibrium is a Bayesian Nash equilibrium, in which bidders with the same value play the same strategy. Proposition 23 establishes that a symmetric Bayesian Nash equilibrium cannot exist. We prove this by contradiction. Suppose that a symmetric equilibrium exists. Then, by a standard argument, in both stages, a bidder with a low value must step out earlier than a bidder with a high value. Let bidders 2 and 3 step out according to the same strictly increasing bid function in stage 1. Then bidder 1 prefers not to bid according to this bid function. Intuitively, this can be seen as follows. Suppose that the price approaches the bid at which the other bidders would step out given that they have the same value as bidder 1. Bidder 1 knows that if one of the other bidders steps out earlier than her, then there is a high probability that she enters stage 2 having the lowest value. As also the bid function in the second stage is strictly increasing in value, with high probability, bidder 1 is the second highest bidder. In that case, she wins only one chopstick for a positive price, so that she makes a loss. Therefore, bidder 1 prefers to deviate to a lower bid, which is a contradiction to the assumption that the equilibrium is symmetric.

[20]In Proposition 22, CSA can be replaced by any auction in which the losing bidder has to pay, such as the all-pay auction. This is true, as the only property of the Chopstick Auction that is used in the proof is the fact that the losing bidder has to pay. Also, this result holds in the case of three or more bidders.

PROPOSITION 23. *Let $n = 3$. Then CSA has no symmetric Bayesian Nash equilibrium.*

PROOF. Suppose, in contrast, that a symmetric equilibrium exists. Then, for both stages, the equilibrium bid function must be strictly increasing. It must be weakly increasing by the same argument as used in the proof of Proposition 21. Also, no pooling can occur in equilibrium. (Suppose instead that there is some pooling at a price p. Then at least one of the two following situations occur. Either the bidder at the lower end of the interval of bidders who bid p makes a loss at p, so that she is better off by deviating to a lower price. Or the winner at the upper end of the interval gets a strictly positive expected utility, but then she can strictly improve by bidding slightly higher.)

However, in the first stage, bidder 1 prefers to deviate if both other bidders submit bids according to a strictly increasing bid function. Let B_1 denote the equilibrium bid function in the first stage. Suppose that the auction reaches a price $B_1(v_1 - \epsilon)$ before anybody quits. Then, bidder 1 gets zero utility when she quits at this point. In the event that one and only one of the other bidders has a signal in the interval $[v_1 - \epsilon, v_1]$, bidder 1 has the second highest signal, and she will win 1 (worthless) chopstick for a price of at least $B_1(v_1 - \epsilon)$. In the event that both bidders have a signal in the interval $[v_1 - \epsilon, v_1]$, bidder 1 wins both chopsticks. The first event happens with a probability which is of the order ϵ, and the second event with a probability of the order ϵ^2, so that bidder 1 strictly prefers not to bid $B_1(v_1)$, but to step out earlier. Therefore, a symmetric equilibrium does not exist. □

In the case that all bidders are risk neutral, and draw their signals from the same distribution, Proposition 23 can also be derived with the Revenue Equivalence Theorem. Suppose a symmetric equilibrium exists. It is shown in the proof of Proposition 23 that this equilibrium is necessarily efficient, implying that the expected payment by the bidders with the two highest signals in stage 2 is equal to the expected payment in SPSB, namely the expectation of the second-highest signal, $v_{(2)}$. When two bidders enter stage 2, they are already sure that they have to pay at least the price reached in stage 1, so that this payment can be considered as sunk costs. Stage 2, with the bidders who have the two highest signals, is also revenue equivalent with SPSB with these two bidders, so that the expected payment by the two highest bidders *above the sunk costs* is again given by the expectation of the second highest value, i.e., $v_{(2)}$. Hence, the costs the bidders commit themselves to in stage 1 should be equal to 0. This implies that an efficient equilibrium will be characterized by an immediate drop-out of the bidder with the lowest signal. Therefore, in stage 1, equilibrium bids should be equal to 0. But this cannot happen in equilibrium, as any bidder is better off by waiting a bit longer.

An asymmetric equilibrium of CSA is easily found, namely when one bidder decides to always stay in the auction, no matter what the other bidders do, and the other bidders step out immediately.[21] If these strategies are played, the auction outcome will be very inefficient, and the revenue will be zero. However, this type of equilibrium involves a dominated strategy, so that it is very unlikely to be played.

The impossibility result of Proposition 23 suggests that the seller is better off when he replaces CSA with SPSB. The nonexistence of symmetric equilibria indicates that CSA probably has no efficient equilibrium either. This conjecture is based on the following consideration. Asymmetry implies that if three bidders have the same type, one of them steps out strictly earlier than the other two. Assuming continuous bid functions, this implies that the bidder who steps out first, also steps out earlier than the other two bidders when they have slightly lower values, so that the outcome is inefficient. This reasoning justifies the conjecture that SPSB is strictly better according to the seller's goals.

CONJECTURE 2. *Let $n = 3$. The seller who aims at fulfilling the criteria Efficiency and Revenue is strictly better off replacing CSA with SPSB.*

[21] Also the second-price sealed-bid auction has such equilibria.

5. Concluding remarks

In this paper, we have studied the exposure problem in multiple object auctions. We have found in all the investigated settings that a seller who aims at efficiency and high auction revenues (weakly) prefers to sell the three chopsticks as one package in the second-price sealed-bid auction (in which the exposure problem is not present) over selling them using the Chopstick Auction (in which bidders face an exposure problem). We conclude that avoiding the exposure problem is an important issue in auction design.

The results for the Chopstick Auction can be straightforwardly generalized to allow for $L \geq 3$ objects being sold to $n \geq 3$ bidders who need $M \geq 2$ objects. Let $W \equiv \lfloor \frac{L}{M} \rfloor < n$ be the maximal number of "winners" in the auction. Assume there is a strictly positive number S of superfluous objects, i.e., $S \equiv L - M\lfloor \frac{L}{M} \rfloor > 0$. The outcome of the auction is such that the W highest bidders get the M objects they need, and $(n+1)$th highest bidder has to buy and pay for the S superfluous lots, which are of no value to her. For the case $W = n+1$ results analogous to Propositions 21 and 22 can be derived using similar arguments. If $W > n+1$, i.e., if there is more than one bidder who does not win in the auction, then, analogous to Proposition 23, the auction has no symmetric equilibria.

Loss aversion, which we assumed for Proposition 22, seems to be a reasonable assumption for bidders in the Dutch DCS-1800 auction. In this auction, the bidders are "agents" trying to win valuable licenses for their "principals", the shareholders of the firms they represent. For the agents, leaving the auction with an expensive, but worthless, set of channels has more impact on the negative side (as they may lose their jobs), than has winning a valuable set on the positive side.

In the introduction of this paper, we have argued that in the presence of the exposure problem, bidders rather play the war of attrition than a standard auction game. In fact, Bulow and Klemperer (1999) found a result analogous to Proposition 23 for the *generalized war of attrition*. The generalized war of attrition is a game in which n bidders are bidding for $m(<n)$ prizes in a multiple object button auction. In

this auction, bidders drop out while the price is rising, until m bidders are left. Those bidders win a prize, and pay the current price. Each bidder who drops out earlier, pays her bid plus c times the difference between the final price and her bid. In the limit ($c \to 0$) of the unique efficient equilibrium, all but the $m+1$ bidders with the highest signals drop out immediately. However, this cannot be an equilibrium in the game with $c = 0$, as bidders have an incentive to deviate, and bid just above 0. Therefore, the generalized war of attrition has no symmetric equilibrium.

Several issues related to our model need further investigation. For instance, the effect of all remaining bidders being informed when one of the bidders quit is not well understood. More specifically, does the Chopstick Auction have symmetric equilibria if bidders would not observe each other leave the auction? Moreover, we have assumed that a bidder does not acquire any value when she wins only one chopstick. A question that may be interesting for further research is how the analysis would change if the marginal value of the first and the third chopstick are strictly positive. Finally, the impossibility result in the case of three bidders is not very informative about equilibrium bidding. A further study is needed to get a better understanding about how bidders behave in the Chopstick Auction in the case of three bidders.

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CHAPTER 6

The Effectiveness of Caps on Political Lobbying

1. Introduction

Lobbying has become an established practice in modern democracies. Its role in society is an intriguing phenomenon, and it has received a lot of attention from game theorists. Tullock (1980) views lobbying as an all-pay auction, in which interest groups submit "bids" in order to win a political prize. The literature that follows Tullock focuses mainly on the social costs of lobbying, which are associated with the fact that the money spent on lobbying cannot be used for other economic activities. Therefore, this branch of the literature devotes much attention to the calculation of total lobbying expenditures by the interest groups (Baye et al., 1993, 1996; Amann and Leiniger, 1995, 1996; Krishna and Morgan, 1997). Another stream of work focuses on the social benefits of lobbying, which arise when interest groups have the opportunity to separate themselves choosing bids that are contingent on policy relevant private information. This stream of work views lobbying as a signaling game, in which interest groups submit informative signals to the government (Potters and Van Winden, 1992; Lohmann, 1993; Lagerlöf, 1997).

In this study, we combine the two views of the literature on lobbying by making a trade-off between social costs and social benefits of lobbying. We do so, taking Tullock's all-pay auction model, and investigating the effect of a cap on the amount of money interest groups are allowed to spend on lobbying. We assume that the cap is chosen by the government with the target of maximizing social welfare. In deciding the optimal cap, the government needs to make a trade-off between the informational benefits lobbying provides, and the social costs. The trade-off turns out to be non-trivial, as both total lobbying expenditures and informational benefits are higher with a higher cap.

We will focus on the following two questions. "What effect would a cap on lobbying expenditures have on their total?" and "Should there be legislation to introduce such a cap in order to increase social welfare?" While the latter question is not answered yet by the economic literature, the former one is addressed in Che and Gale (1998). Their findings challenge the intuitively appealing expectation that a cap on lobbying expenditures decreases their total. They show that a cap "may have the perverse effect of increasing aggregate expenditures and lowering total surplus".

Before answering these questions, we need to emphasize the importance of distinction between the *ex ante* and *ex post* effect of a cap. The distinction is important as it allows us to model the legislative role of the government. New legislation, once introduced, regulates all lobbying activities for a long period of time. As a result, when taking a legislative initiative, the government cannot predict the exact effect of a proposed cap. It is therefore appropriate to model the government's decision on a cap as an *ex ante* choice, i.e., a decision made before the government learns the realizations of the interest groups' values. In contrast, the "perverse effect" described by Che and Gale holds *ex post*, i.e., after the interest groups' values are realized.

Depending on the situation, the interest groups may be or may not be better informed than the government about the characteristics of other interest groups. In this paper, we will investigate the effect of a cap in two different settings. In an incomplete information setting, we assume that each interest group is privately informed about its own value for the prize. The government and the other interest groups only know the distribution function this value is drawn from. In a complete information setting, we assume, following Che and Gale, that the interest groups commonly know each others' values. However, the government is only aware of the value distribution function.

Our contribution is threefold. First, in the case of incompletely informed interest groups, we derive equilibrium bidding in the case that the interest groups are confronted with a cap. Second, we show that the ex ante expected lobbying expenditures decrease by imposing a cap. Thus, legislators need not be overly concerned about the "perverse effect" of a cap, in contrast to what the result of Che and Gale suggests. Third, we point out that the government should optimally ban lobbying by imposing a prohibitive cap. Although a high cap generates information benefits by allowing the government to choose the socially optimal action more often, we show that these benefits do not outweigh the expected social costs.

Two other papers are closely related to ours. McAfee and McMillan (1992) show that weak cartels optimally let all cartel members submit zero bids in the first-price sealed-bid auction. The proof of this result follows the same logic as the proof of the optimality of a prohibitive cap in the incomplete information setting. Also, for the incomplete information setting, Gavious et al. (2001) simultaneously and independently develop alternative proofs for the results on equilibrium bidding and ex ante total lobbying expenditures in the all-pay auction with caps.

We proceed as follows. In Section 2, we outline the structure of our model. In Section 3, we derive the results about the effect of a cap in the incomplete information setting. In Section 4, we show that these results hold in the complete information setting. In Section 5 we conclude with some critical remarks on the results, and with an indication for some directions for further research.

2. The model

Consider the following lobby game. There is a government, G, which owns a political prize,[1] and n interest groups, numbered $1, ..., n$. Let

$$N \equiv \{1, ..., n\}$$

denote the set of all interest groups. We will use i and k to represent typical interest groups in N. Interest groups participate in the all-pay

[1]The prize could for instance be a license to operate in a certain market, a building contract, or the right to organize an important event.

auction, in which they submit *bids*[2] in order to obtain the prize. We will let b_i denote the bid submitted by interest group i. G restricts b_i to be contained in the interval $[0, c]$, where c denotes a *cap*. The interest group that submits the highest bid wins, but each interest group has to pay its bid. In case of ties, the winner is chosen among the interest groups with the highest bid with equal probabilities.

Each interest group i learns its private value v_i of the prize. The v_i's are drawn, independently from each other, from a distribution function F. F has support on the interval $[0, 1]$, and has a continuous density function f with $f(v_i) > 0$, for every $v_i \in [0, 1]$. We consider two information structures. In the *incomplete information setting*, each interest group only knows its own value, and not the values of the other interest groups. In the *complete information setting*, the values of all interest groups are commonly known among the interest groups. In both settings, G is incompletely informed, and only knows F.

We assume that interest groups are risk neutral expected utility maximizers. Let $u_i(k, v_i, b_i)$ be the utility of interest group i when its value is v_i, its bid is b_i and interest group k wins the prize. Then, interest group i's utility is given by

$$u_i(k, v_i, b_i) \equiv \begin{cases} v_i - b_i & \text{if } k = i \\ -b_i & \text{otherwise.} \end{cases} \tag{2.1}$$

G chooses c that maximizes *ex ante* social welfare among the interest groups. Let $SW(k, v_1, ..., v_n, b_1, ..., b_n)$ denote *ex post* social welfare given that interest group k wins, given the values $v_1, ..., v_n$, and the bids $b_1, ..., b_n$. Ex post social welfare is defined as the sum of interest groups' utilities, so that

$$SW(k, v_1, ..., v_n, b_1, ..., b_n) \equiv \sum_{i=1}^{n} u_i(k, v_i, b_i) = v_k - \sum_{i=1}^{n} b_i. \tag{2.2}$$

Ex ante social welfare is the expectation of ex post social welfare over the values and the played strategies. We assume that interest groups play a Bayesian Nash equilibrium.

[2]We use the terminology from the literature on all-pay auctions and refer to the amount paid by an interest groups as its bid. Direct bribes, writing research reports, or hiring lobbyists are instances of bids. We use the term *total lobbying expeditures* for the sum of all bids.

3. Incomplete information

Consider the incomplete information setting. Before we establish our main results, we derive two useful lemmas and a corollary. Define the differentiable functions $C : [0, 1] \to \Re$ and $D : [0, 1] \to \Re$ with

$$C(y) \equiv \int_0^y [zf(z) + F(z) - 1]F(z)^{n-1}dz + \frac{y}{n} * [1 - F(y)^n]$$

and

$$D(y) \equiv \frac{y}{n} * \frac{1 - F(y)^n}{1 - F(y)} - \int_0^y F(z)^{n-1}dz$$

for all $y \in [0, 1]$.

LEMMA 9. *C is strictly increasing.*

PROOF. See the Appendix. □

LEMMA 10. *D is strictly increasing.*

PROOF. See the Appendix. □

COROLLARY 7. *If $c \leq 1 - \int_0^1 F(z)^{n-1}dz$, then there is a unique ξ for which $D(\xi) = c$.*

PROOF. See the Appendix. □

Let $v^*(c)$ be the unique solution to $D(v^*(c)) = c$ if $c \leq 1 - \int_0^1 F(z)^{n-1}dz$, and let $v^*(c) = 1$ otherwise. Proposition 24 shows that in equilibrium, the strategy of interest groups with a value below the threshold value $v^*(c)$ is not affected by the cap. Interest groups with a value above $v^*(c)$ submit a bid equal to c. This equilibrium is derived using an indirect approach based on the Revenue Equivalence Theorem (Myerson, 1981), which states that an interest group's interim utility (i.e., its utility as a function of its private value) is entirely determined by the function that assigns a probability that the interest group wins the prize given each possible realization of its value (provided that the utility of an interest groups is zero when is has the lowest possible value). As the bid function (3.1) determines this probability function, the interim utility for each interest group is fixed. In order to prove that (3.1) is an equilibrium, we show that the interim utility of each interest group is compatible with (3.1).

PROPOSITION 24. *Consider the lobby game with incomplete information. Let*

$$B(v_i, c) = \begin{cases} \int_0^{v_i} [F(v_i)^{n-1} - F(z)^{n-1}]dz & \text{if } v_i \in [0, v^*(c)] \\ c & \text{if } v_i \in (v^*(c), 1], \end{cases} \tag{3.1}$$

where $v^*(c)$ follows uniquely from $D(v^*(c)) = c$ if $c \leq 1 - \int_0^1 F(z)^{n-1}dz$, and $v^*(c) = 1$ otherwise. Then B constitutes a symmetric Nash equilibrium of the lobby game.[3]

PROOF. By Corollary 7, $v^*(c)$ is uniquely determined if $c \leq 1 - \int_0^1 F(z)^{n-1}dz$. Myerson (1981) shows that in equilibrium, the interim utility $\pi_i(v_i)$ of interest group i when having value v_i is given by

$$\pi_i(v_i) = \pi_i(0) + \int_0^{v_i} Q_i(w_i)dw_i, \text{ for all } v_i \in [0,1] \text{ and } i \in N, \tag{3.2}$$

where $Q_i(w_i)$ is the conditional probability that interest group i wins the prize, given that it has value w_i.

The proposed bid function implies that

$$Q_i(p, w_i) = F(w_i)^{n-1} \text{ if } w_i \in [0, v^*(c)], \tag{3.3}$$

as $B(w_i, c)$ is strictly increasing in w_i for all $w_i \in [0, v^*(c)]$, and

$$Q_i(w_i) = \bar{Q} = \frac{1 - F(v^*(c))^n}{n(1 - F(v^*(c)))} \text{ if } w_i \in (v^*(c), 1]. \tag{3.4}$$

[3]For an alternative proof, derived simultaneously and independently, see Gavious et al. (2001). The result can also be derived indirectly from Laffont and Robert (1996).

The last expression follows from the ex ante probability (i.e., before the interest groups know their value) that a given interest group wins, which is given by

$$\frac{1}{n} = (1-F(v^*(c)))\bar{Q}+\int_0^{v^*(c)} F(v_i)^{n-1}dF(v_i) = (1-F(v^*(c)))\bar{Q}+\frac{1}{n}F(v^*(c))^n.$$

It remains to be checked if B is compatible with (3.2). As $\pi_i(0) = 0$, with (3.3) and (3.4), (3.2) can be rewritten as

$$\text{(3.5)} \quad \pi_i(v_i) = \int_0^{v_i} F(w_i)^{n-1}dw_i, \text{ if } v_i \in [0, v^*(c)], \text{ and}$$

$$\text{(3.6)} \quad \pi_i(v_i) = \int_0^{v^*(c)} F(w_i)^{n-1}dw_i + \int_{v^*(c)}^{v_i} \bar{Q}dw_i, \text{ if } v_i \in (v^*(c), 1]$$

for all $i \in N$. Moreover, the expected utility of interest group i can be expressed as follows

$$\text{(3.7)} \quad \pi_i(v_i) = F(v_i)^{n-1}v_i - b(v_i, c) \text{ if } v_i \in [0, v^*(c)], \text{ and}$$

$$\text{(3.8)} \quad \pi_i(v_i) = \bar{Q}v_i - b(v_i, c) \text{ if } v_i \in (v^*(c), 1],$$

where $b(v_i, c)$ is the bid made by an interest group with value v_i when the cap equals c. It is readily verified that the proposed bid function B is a solution to (3.5)-(3.8). Therefore, B constitutes a Bayesian Nash equilibrium. □

Proposition 24 implies that imposing a lower cap can ex post lead to higher lobbying expenditures. This can be seen as follows. It is readily verified that the equilibrium bid function makes a jump upwards at threshold value $v^*(c)$. Now, take $v_1, ..., v_n$ and c such that $v_2, ..., v_n < v^*(c)$, and $v_1 = v^*(c)$. As v^* is the inverse function of D, by Lemma 10, v^* is strictly increasing in c. Therefore, when c is marginally decreased, interest group 1 will change its bid to c, which is higher than its original bid, whereas the bids of the other bidders remain unchanged, so that total lobbying expenditures increase.

Thus, there are two opposing effects of a decrease in c. On the one hand, it lowers the bids of interest groups with high values, which on the other hand induces interest groups with intermediate values to increase their bid to c so as to pool with the high types and to increase their probability of winning the prize. Depending on the specific values, the second effect sometimes dominates the first.

Proposition 25 shows that the "ex post" result does not hold "ex ante". Let ex ante expected total lobbying expenditures be the expected sum of interest groups' equilibrium bids, where the expectation is taken over the values of the interest groups. The proof follows by calculating the sum of the equilibrium bids given by Proposition 24 as a function of c, and by showing that the resulting function is strictly increasing in c.

PROPOSITION 25. *Consider the lobby game with incomplete information. Suppose that c is strictly decreased. Then ex ante expected total lobbying expenditures are strictly decreased as well.*

PROOF. Let $\tilde{L}^a(c)$ denote the expected ex ante total lobbying expenditures as a function of c. Then

$$\begin{aligned}
\frac{1}{n}\tilde{L}^a(c) &= \frac{1}{n}\sum_{i=1}^{n}\int_0^1 B(v_i,c)f(v_i)dv_i \\
&= \int_0^{v^*(c)}\left[zF(z)^{n-1}-\int_0^z F(y)^{n-1}dy\right]f(z)dz+[1-F(v^*(c))]c \\
&= \int_0^{v^*(c)} zF(z)^{n-1}f(z)dz - F(v^*(c))\int_0^{v^*(c)}F(z)^{n-1}dz+\int_0^{v^*(c)}F(z)^n dz \\
&\quad +\frac{v^*(c)}{n}*[1-F(v^*(c))^n]-[1-F(v^*(c))]\int_0^{v^*(c)}F(z)^{n-1}dz \\
&= \int_0^{v^*(c)}[zf(z)+F(z)-1]F(z)^{n-1}dz+\frac{v^*(c)}{n}*[1-F(v^*(c))^n] \\
&= C(v^*(c)).
\end{aligned}$$

Now, as v^* is the inverse function of D, by Lemma 10, v^* is strictly increasing in c. Then, by Lemma 9, $\tilde{L}^a(c)$ is strictly increasing in c. □

Proposition 25 implies that if total lobbying expenditures were the only part of social welfare, then a lower cap would always be preferred to a higher one. However, social welfare as defined in (2.2) is also an increasing function of the winner's private value. As $v^*(c)$ is strictly increasing in c, a lower cap leads to more bidders pooling at the cap, so that the probability that the winner is the interest group with the highest value decreases. Therefore, a lower cap implies less informational benefits.

The trade-off between social costs and social benefits is non-trivial. In order to make the trade-off, we make the simplifying assumption that $\frac{1-F}{f}$ is a strictly decreasing function, which is the case for several standard distributions such as the uniform distribution. Suppose that G is not restricted in letting the interest groups play the all-pay auction, but that it has a much broader class of *feasible mechanisms* to choose from.

We start by defining a *mechanism*. In a mechanism, interest groups are asked to simultaneously and independently choose an action. Interest group i chooses an action $a_i \in A_i$, where A_i is the set of actions for interest group i. The mechanism has the following outcome functions

$$\widehat{p} : A_1 \times ... \times A_n \to \Re^n,$$

and

$$\widehat{x} : A_1 \times ... \times A_n \to \Re^n.$$

If $\mathbf{a} = (a_1, ..., a_n)$, then $\widehat{p}_i(\mathbf{a})$ is interpreted as the probability that interest group i gets the prize and $\widehat{x}_i(\mathbf{a})$ is the expected expenditures for interest group i. Interest group i's utility when $\mathbf{a}$ is played is, consistently with (2.1), given by

$$\hat{U}_i(\mathbf{a}) = v_i \widehat{p}_i(\mathbf{a}) - \widehat{x}_i(\mathbf{a}).$$

Let a strategy be a function $\widehat{b}_i : [0,1] \to A_i$ such that $\widehat{b}_i(v_i)$ is the action interest group i plays when it has value v_i. A feasible mechanism is a mechanism including strategies, which have the following properties: (1) each interest group expects nonnegative utility, and (2) the strategies form a Bayesian Nash equilibrium of the mechanism. A *socially optimal auction* is a feasible mechanism that maximizes ex ante social welfare.

By the Revelation Principle (Myerson, 1981), we may assume, without loss of generality, that G only considers *feasible direct revelation mechanisms*, which is are feasible mechanisms in which each interest group is asked to announce its value, in which it has an incentive to participate (individual rationality) and in which it has an incentive to announce its value honestly (incentive compatibility). Let (p, x) be a feasible direct revelation mechanism, with

$$p : V \to [0, 1]^n$$

having

$$\sum_j p_j(\mathbf{v}) \leq 1,$$

and

$$x : V \to \Re^n.$$

We interpret $p_i(\mathbf{v})$ as the probability that interest group i wins, and $x_i(\mathbf{v})$ as the expected payment by i when $\mathbf{v} \equiv (v_1, ..., v_n)$ is announced.

Let

$$Q_i(p, v_i) \equiv E_{\mathbf{v}_3} \; \{p_i(\mathbf{v})\}$$

be the conditional probability that interest group i wins given its value v_i, and

$$U_i(p, x, v_i) \equiv v_i Q_i(p, v_i) - E_{\mathbf{v}_3} \; \{x_i(\mathbf{v})\}$$

be interest group i's interim utility from (p, x), with $\mathbf{v}_{3\,i} \equiv (v_1, ..., v_{i3\,1}, v_{i+1}, ..., v_n)$. Myerson (1981) shows that individual rationality and incentive compatibility are equivalent to

$$\text{(3.9)} \qquad \text{if } w_i \;\leq\; v_i \text{ then } Q_i(p, w_i) \leq Q_i(p, v_i), \; \forall w_i, v_i, i,$$

$$\text{(3.10)} \quad U_i(p, x, v_i) \;=\; U_i(p, x, 0) + \int_0^v Q_i(p, y_i) dy_i, \; \forall v_i, i, \text{ and}$$

$$\text{(3.11)} \quad U_i(p, x, 0) \;\geq\; 0, \; \forall i.$$

Ex ante social welfare from (p, x) is given by

$$\tilde{S}(p,x) = \sum_{i=1}^{n} \int_{0}^{1} U_i(p,x,v_i) f(v_i) dv_i.$$

Then,

$$\begin{aligned}
\tilde{S}(p,x) &= \sum_{i=1}^{n} \int_{0}^{1} \left(U_i(p,x,0) + \int_{0}^{v_i} Q_i(p,y_i) dy_i \right) f(v_i) dv_i \\
&= \sum_{i=1}^{n} U_i(p,x,0) + \int_{0}^{1} \frac{(1-F(v_i))}{f(v_i)} Q_i(p,v_i) f(v_i) dv_i \\
&\leq \sum_{i=1}^{n} U_i(p,x,0) + \int_{0}^{1} (1-F(v_i)) dv_i * \int_{0}^{1} Q_i(p,v_i) f(v_i) dv_i \\
&= \sum_{i=1}^{n} U_i(p,x,0) + E\{v_i\} \int_{0}^{1} Q_i(p,v_i) f(v_i) dv_i \\
&= E\{v_i\}. \qquad (3.12)
\end{aligned}$$

The first equality follows from (3.10), and the second by integration by parts. The first inequality follows from a theorem from Statistics which tells that the expectation of a product is less or equal than the product of the expectations in case the first term of the product is strictly decreasing, and the second term is increasing (McAfee and McMillan, 1992). Here, $\frac{1-F}{f}$ is strictly decreasing (by assumption), and Q_i is increasing v_i (by (3.9)). The other manipulations are straightforward.

Consider a feasible direct revelation mechanism $(\tilde{p}, \tilde{x})$ with

$$\begin{aligned}
\tilde{p}_i(\mathbf{v}) &= \frac{1}{n}, \text{ and} \\
\tilde{x}_i(\mathbf{v}) &= 0,
\end{aligned}$$

for all i. Basically, $(\tilde{p}, \tilde{x})$ is a lottery in which each interest group has the same probability of winning. The expected social welfare among the interest groups is then expected value generated by the lottery, so that

$$\tilde{S}(\tilde{p}, \tilde{x}) = E\{v_i\}.$$

With (3.12) it follows that $(\tilde{p}, \tilde{x})$ is a socially optimal mechanism, as

$$\tilde{S}(\tilde{p}, \tilde{x}) \geq \tilde{S}(p, x)$$

for all feasible direct revelation mechanisms (p, x). $(\tilde{p}, \tilde{x})$ is straightforwardly implemented with $c = 0$.

PROPOSITION 26. *Consider the lobby game with incomplete information. If* $\frac{1-F}{f}$ *is strictly decreasing, then* $c = 0$ *maximizes ex ante social welfare.*

An intuition behind Proposition 26 is the following. Observe in the second line of the chain (3.12) that player i, if winning the object, adds $\frac{1-F(v_i)}{f(v_i)}$ to social welfare. As, by assumption, $\frac{1-F}{f}$ is a strictly decreasing function, G prefers a low type of interest group i to win more often than a high type. However, (3.9) requires the probability for interest group i to win the object to be (weakly) increasing in v_i. Hence, the best G can do is make the probability that a low type wins equal to the probability that a high type wins. G can do this optimally by implementing a cap equal to zero.

4. Complete information

Consider the complete information setting with two interest groups. For completeness, we first report the finding by Che and Gale (1998) showing that ex post lobbying expenditure may increase as a result of a decrease in c. Let $v_h \equiv \max\{v_1, v_2\}$ and $v_l \equiv \min\{v_1, v_2\}$ and let $L^p(c, v_h, v_l)$ be the ex post expected total lobbying expenditures by both interest groups, given the cap c, v_h and v_l. We speak of *expected* total lobbying expenditures as in equilibrium, interest groups play mixed strategies (Che and Gale, 1998).

PROPOSITION 27. *Consider the lobby game with complete information. Let* $n = 2$. *Then generically, there is a unique Nash equilibrium,*[4]

[4]For the zero mass event $c = v_l/2$, there a continuum of equilibria, which results in total lobbying expenditures in the interval $[v_l(v_h+v_l)/2v_h, 2c]$. See Che and Gale (1998).

in which $L^p(c, v_h, v_l)$ *is given by*

$$L^p(c, v_h, v_l) = \begin{cases} v_l(v_h + v_l)/2v_h & \text{if } c > v_l/2 \\ 2c & \text{if } c < v_l/2. \end{cases} \tag{4.1}$$

If $c \in \left(\frac{v_l(v_h+v_l)}{4v_l}, \frac{v_l}{2}\right)$, *then* $L^p(c, v_h, v_l) > L^p(\infty, v_h, v_l)$

PROOF. See Che and Gale (1998). □

Note that for a non-zero mass of realizations of c, v_h, and v_l, $L^p(c, v_h, v_l) > L^p(\infty, v_h, v_l)$, which implies that there is a substantial set of cases in which a *decrease* in c results in an *increase* in total lobbying expenditures. The intuition behind this result is that a decrease in the cap limits the interest group with the highest value, so that the interest group with the lowest value is willing to bid more aggressively, which in certain cases leads to an increase in total lobbying expenditures.

Assume that each interest group draws its value from a uniform distribution on the interval $[0, 1]$. We calculate ex ante expected total lobbying expenditures taking the expectation of (4.1) with respect to v_l and v_h. Proposition 28 shows that ex ante total lobbying expenditures are always increasing in the cap.

PROPOSITION 28. *Consider the lobby game with complete information. If* $n = 2$ *and* $v_i \sim U[0, 1]$, *then the ex ante expected total lobbying expenditures are strictly increasing in* c *for all* $c \in [0, \frac{1}{2}]$.

PROOF. Let $L^a(c)$ denote the ex ante expected total lobbying expenditures as a function of c. Then,

$$\begin{aligned} L^a(c) &\equiv 2\int_0^1 \int_{v_l}^1 L^p(c, v_h, v_l)dv_h dv_l \\ &= 2\int_0^{2c} \int_{v_l}^1 \frac{v_l(v_h + v_l)}{2v_h} dv_h dv_l + 2c(1 - 2c)^2. \end{aligned}$$

The expression is multiplied by 2 as the role of the interest group with the higher and the lower value is interchanged with probability $\frac{1}{2}$.

Taking the first derivative of L^a w.r.t. c yields

$$\begin{aligned}\frac{\partial L^a(c)}{\partial c} &= 4\int_{2c}^{1} \frac{2c(v_h+2c)}{2v_h} dv_h + 2 - 16c + 24c^2 \\ &= 2 - 12c + 16c^2 - 8c^2\log(2c).\end{aligned}$$

As $\log(z) < z - 1$ for all $z \in (0,1)$, it holds for all $c \in (0, \frac{1}{2})$ that

$$\frac{\partial L^a(c)}{\partial c} > 2 - 12c + 16c^2 - 8c^2(2c-1) = 2(1-2c)^3 > 0. \tag{4.2}$$

Therefore, as $L^a(c)$ is a continuous function of c, $L^a(c)$ is strictly increasing in c. □

Proposition 29 shows that the $c = 0$ result of the incomplete information setting has its parallel in the complete information setting. This result follows from Che and Gale (1998), who show that for $c > \frac{1}{2}v_l$, expected utility for the bidder with the highest value is $v_h - v_l$, and expected utility for the bidder with the lowest value equals 0. Hence, in this case, social welfare equals $v_h - v_l$. If $c < \frac{1}{2}v_l$, both bidders bid c, so that ex post social welfare is given by $\frac{1}{2}(v_l + v_h) - 2c$. Taking the expectation of ex post social welfare with respect to v_h and v_l, ex ante social welfare is determined. Straightforward calculations reveal that ex ante social welfare is maximized at $c = 0$.

PROPOSITION 29. *Consider the lobby game with complete information. If $n = 2$ and $v_i \sim U[0,1]$, then $c = 0$ maximizes ex ante social welfare.*

PROOF. If $v_l > 2c$, both interest groups bid c, so that ex post social welfare is given by $\frac{v_1+v_2}{2} - 2c$, and if $v_l < 2c$, expected utility for the interest groups is $v_h - v_l$ and 0 respectively for the high and the low value interest group (Che and Gale, 1998). Let $S(c)$ denote ex ante social welfare as a function of the imposed cap c. Then

$$S(c) = \int_{2c}^{1}\int_{2c}^{1} \left[\frac{v_1+v_2}{2} - 2c\right] dv_1 dv_2 + 2\int_{0}^{2c}\int_{v_2}^{1} (v_1 - v_2) dv_1 dv_2.$$

The first term of the RHS refers to the case that $v_l > 2c$. The second term of the RHS applies to $v_l < 2c$. Calculating the integrals we find

$$S(c) = \frac{1}{2} - c + 2c^2 - \frac{4}{3}c^3.$$

The first order derivative of S is then given by

$$\begin{aligned}\frac{dS(c)}{dc} &= -(1-2c)^2 \\ &\leq 0\end{aligned}$$

so that $S(c)$ is maximized at $c = 0$. □

5. Concluding remarks

Our results encourage governments to introduce caps on lobbying. We have found for both the incomplete and the complete information setting that although introducing caps on lobbying may ex post lead to an increase in total lobbying expenditures, this effect is reversed for ex ante expected total lobbying expenditures. Moreover, making the trade-off between social costs and social benefits of lobbying, we have shown that it is optimal for a benevolent government to completely ban lobbying.

This conclusion, however, relies heavily on at least three debatable assumptions. By far the strongest, and therefore most serious assumption, is that interest groups play a Bayesian Nash equilibrium. This assumption is probably not valid in many real-life cases of political lobbying, as often, interest groups cannot be viewed as a single entity, but are poorly organized lobbies instead that suffer seriously from free-riding problems. Second, our results are built on the assumption of a benevolent government which maximizes social welfare, which at first sight seems to be strong as well. However, also a self-interested government may rationally aim at maximal social welfare, so that its probability of being re-elected is maximized. Finally, we have limited the action space of the government to the choice of a cap on lobbying expenditures. We implicitly assume that the government is not able to implement other, probably more efficient mechanisms such as auctions, for instance because the constitution precludes this.[5]

There are several interesting directions for future research. For instance, the analysis was simplified by the assumption of independence (the interest groups' values are drawn independently) and symmetry (the values are drawn from the same distribution). The assumption of independence is not valid when there are external factors which influence the interest groups' values equally. For instance, the value for a license to operate in a certain market depends on consumer's demand, which effects the values for the different interest groups in the same direction. In this respect, extensions to models with affiliated values, interdependent values, or multidimensional signals may provide additional insights. In Onderstal (2002), the model with incomplete information is extended to allow for interest group specific distribution functions. Onderstal shows that a cap of zero is still optimal, provided that interest groups with low ex ante values (i.e., expected values) are not allowed to participate in the lobby game.

6. Appendix

PROOF OF LEMMA 9. The first and the second order derivatives of C have the following properties.

$$C'(y) = \frac{(n-1)F(y)^n - nF(y)^{n-1} + 1}{n}$$

for all $y \in [0, 1]$, so that

$$C'(1) = 0.$$

$$C''(y) = (n-1)f(y)F(y)^{n-2}(F(y) - 1) < 0$$

for all $y \in [0, 1)$. It immediately follows that $C'(y) > 0$ for all $y < 1$.

[5]See Moore (1992) and Palfrey (1992) for a survey of the literature on the implementation of efficient mechanisms in environments with complete and incomplete information respectively.

Proof of Lemma 10. We deduce for all $y \in [0,1)$,

$$\begin{aligned} D'(y) &= \frac{[1-F(y)][1-F(y)^n - yf(y)nF(y)^{n-1}] + f(y)y[1-F(y)^n]}{n[1-F(y)]^2} - F(y)^{n-1} \\ &= \frac{1-F(y)+yf(y)}{[1-F(y)]^2} * \frac{(n-1)F(y)^n - nF(y)^{n-1}+1}{n} \\ &= \frac{1-F(y)+yf(y)}{[1-F(y)]^2} * C'(y) \\ &> 0, \end{aligned}$$

where the inequality follows from Lemma 9. Therefore, D is strictly increasing.

Proof of Corollary 7. As $D(0) = 0$, $\lim_{y\uparrow 1} D(y) = 1 - \int_0^1 F(z)^{n-1}dz$, and D differentiable and strictly increasing (by Lemma 10), ξ is uniquely determined.

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CHAPTER 7

Socially Optimal Mechanisms

1. Introduction

Consider the problem of a social planner, who wishes to allocate an indivisible object to one out of a group of players, and who is only incompletely informed about the value of the object for the players. The social planner aims at finding a socially optimal mechanism, i.e., a mechanism which maximizes social welfare among the players.

Typical examples of a mechanisms in which players compete for an indivisible object are contests. A contest is a situation in which players compete with one another by expending irreversible effort to win a prize (Baik, 1998). Examples of economic situations that are modelled as contests include political lobbying, beauty contests, procurement, R&D races, job applications, advertising, and queues for tickets. These situations are usually modelled as the all-pay auction or the war of attrition, and in some cases as the first-price sealed-bid auction. Also an auction, in which bidders expand money in order to obtain an indivisible object, can be seen as a special example of a contest. In this paper, we will illustrate our model and our main result in the context of contests.

We consider a model with incomplete information. Before the mechanism is played, each player receives a private signal for the value of the object. These signals are independently drawn from idiosyncratic distribution functions (i.e., the distribution functions may differ from one player to the other), which are assumed to be commonly known (i.e., by all players and the social planner). We will make several plausible assumptions on the smoothness and shape of the distribution functions. We assume that the players are expected utility maximizers, and interact in a mechanism according to a Bayesian Nash equilibrium.

We assume that the social planner aims at finding a socially optimal mechanism, i.e., a feasible mechanism that maximizes social welfare among the players. The social planner is aware of the stochastic structure of the players' values but he does not know the individual values for the players. Social welfare among the players is defined as the ex ante expected sum of the players' utilities. Therefore, a socially optimal mechanism is an ex ante incentive efficient decision rule in the sense of Holmström and Myerson (1983).

We will show that the social planner maximizes social welfare when he assigns the object to one of the players with the highest expected value for the object using a lottery. Instead of calculating social welfare using equilibrium bidding, we will use an indirect approach to prove this result, based on the Revelation Principle (Myerson, 1981). Our finding implies that the social planner optimally bases his decision on the allocation of the object solely on his limited information on the players' values of the object, and not on information revealed by the players having the opportunity to separate themselves choosing actions that are contingent on the realization of their value. To put it differently, players optimally agree among themselves not to spend efforts in a contest. A similar result, derived in a complete information setting, is found by Huck, et al. (2000), who show that collusion among players is profitable when the discriminatory power of a contest is sufficiently high.

Our result has several interesting real-life interpretations. For instance, it implies that firms optimally agree among themselves not too spend money in advertising. Schmalensee (1976) argues that in markets with a few sellers and differentiated products, competition among firms mainly takes place through promotional expenditures rather than through prices.[1] A typical example of such a market is the market for cars. The firms can be seen as being engaged in a contest, in which efforts (in the form of tv-commercials, mailings, glamorous shop outlets, etc.) are spend in order to win (a share of) the object in the form of (part of) total market profits. In this light, it is surprising to observe that in the past, the tobacco industry lobbied against the prohibition

[1]See also Huck et al. (2000).

of advertising cigarettes. A legal prohibition would have implied that tobacco firms were better off, as they would not lose so much money in advertising.

Another interpretation of our result is that lobbies optimally agree among themselves not to spend money in lobbying. Baye et al. (1993) observe that the justice system in the Western World precludes governments to sell political favors by efficient mechanisms like auctions, so that governments are forced to make use of the wasteful institution of lobbying to acquire information from interest groups. They argue that a lobby game has the same structure as the all-pay auction. Our result implies that players of an all-pay auction optimally agree to collude in such a way that the player with the highest ex ante value wins the object.

Analogously, political parties optimally agree among themselves not to spend any effort in political campaigns, and limit the voters' choice to the set consisting of the strongest parties. Especially in the US, presidential candidates spend huge amounts of money in their political campaigns (which is spent on tv-advertising, tours around the country, fancy internet sites, etc.). Therefore, a political campaign can be seen as a contest, so that our finding implies that colluding parties agree to spend no money in their campaign, and limit the choice set of the voters.

Finally, our result shows that colluding bidders in auctions optimally agree among themselves to let only the strongest bidders participate in the auction, in which they submit a bid of zero, so that the winner of the object is chosen at random. McAfee and McMillan (1992) already show this for the case of symmetric bidders in a study on weak cartels in the first-price sealed-bid auction.

All these interpretations rely on the assumption that colluding players are able to exclude "entrants" from the contest. We will not discuss this assumption is further detail, and leave it as it is. In our Conclusion, we will focus on two examples in which a social planner can restrict the set of participants in the contest.

Several papers in the economic literature are related to ours. Some of these papers focus on equilibrium behavior in specific mechanisms.

For instance, Baye et al. (1996) derive equilibrium bidding in the all-pay auction with complete information. Amann and Leiniger (1996), and Krishna and Morgan (1997) derive equilibrium bidding in the all-pay auction with incomplete information in models with asymmetric value distributions and affiliated values respectively. Equilibria of the war of attrition are derived by Krishna and Morgan (1997), and Bulow and Klemperer (1999) for models with incomplete information. Equilibrium bidding in auctions is extensively studied in the auction literature, see e.g., Vickrey (1961), and Milgrom and Weber (1982).

Other papers in the economic literature concentrate on mechanism design. Usually, the mechanism designer is assumed to maximize total bids or total efforts by the players, his instruments being reserve prices, exclusion of players, caps, the mechanism format, etc. All standard auctions with the right reserve price maximize expected revenue for the seller of an indivisible object in the case of symmetric risk neutral bidders (Myerson, 1981; Riley and Samuelson, 1981). Baye et al. (1996) show that expected total bids may increase when a subset of players is excluded from participation in the all-pay auction in a complete information model. Che and Gale (1998) show that a cap may lead to higher total bids in the all-pay auction. Matejka et al. (2002), and Gavious et al. (2001) argue that this result is an ex post result, and show that ex ante (i.e., before the social planner knows the values), a tighter cap leads to lower bids. The all-pay auction maximizes the expected revenue for the seller of an indivisible object if bidders are risk averse (Matthews, 1983), or budget constrained (Laffont and Robert, 1996).

Sometimes, like in our model, the mechanism designer has other aims than effort maximization. For instance, Van Damme (1992) considers fair and efficient mechanisms in a model with incomplete information. He shows that some classical division methods, which turn out to be fair in complete information settings, are not fair anymore in the case of incomplete information, and constructs mechanisms that do guarantee fair and efficient outcomes. Maskin (2000) derives mechanisms that are constrained efficient in the case of budget constrained bidders.

2. The model

Consider a situation in which one indivisible object has to be allocated to one player out of a group of n, numbered $1, 2, ..., n$. We assume that there is a social planner who aims at finding a mechanism for allocating the object, such that social welfare among the players is maximized. We let N represent the set of players, so that

$$N \equiv \{1, ..., n\}.$$

Each player i receives a one-dimensional private signal v_i about the value of the object. For each i, v_i is drawn, independently from all the other private signals, from a distribution function F_i. F_i has support on the interval $[\underline{v}_i, \bar{v}_i]$, and continuous density f_i with $f_i(v_i) > 0$, for every $v_i \in [\underline{v}_i, \bar{v}_i]$. Define the sets

$$V \equiv [\underline{v}_1, \bar{v}_1] \times ... \times [\underline{v}_n, \bar{v}_n],$$

and

$$V_{-i} \equiv \times_{j \neq i} [\underline{v}_j, \bar{v}_j],$$

with typical elements $\mathbf{v} \equiv (v_1, ..., v_n)$, and $\mathbf{v}_{-i} \equiv (v_1, ..., v_{i-1}, v_{i+1}, ..., v_n)$ respectively. Let

$$f(\mathbf{v}) \equiv \prod_{j \in N} f_j(v_j)$$

be the joint density of $\mathbf{v}$, and

$$f_{-i}(\mathbf{v}_{-i}) \equiv \prod_{j \neq i} f_j(v_j)$$

the joint density of $\mathbf{v}_{-i}$. Let $EV^{\max}$ be the highest expected value, i.e.,

$$EV^{\max} \equiv \max_{i \in N} E\{v_i\}.$$

We make the assumption that for each player i, $\frac{1-F_i}{f_i}$ is a strictly decreasing function. Several common distributions, including the uniform distribution, satisfy this condition.

The social planner's problem is to select a mechanism to maximize social welfare among the players. A mechanism is a game specifying a set of allowed actions for each player, the allocation rule, and the payment rule. Formally, a mechanism is a tuple $\mu =< A, q, y >$, where

$$A = A_1 \times ... \times A_n, \text{ with } A_i \text{ player } i\text{'s set of possible actions,}$$

$$q = (q_1, ..., q_n), \text{ with } q_i : A \to [0, 1] \text{ for all } i, \text{ and} \sum_i q_i(\mathbf{a}) \leq 1 \text{ for all } \mathbf{a} \in A$$

and

$$y = (y_1, ..., y_n), \text{ with } y_i : A \to \Re \text{ for all } i.$$

We will refer to q as the *allocation rule* of μ, with $q_i(\mathbf{a})$ the probability that player i gets the object in the case $\mathbf{a} \in A$ occurs. We call y the *payment rule* of μ, where $y_i(\mathbf{a})$ denotes the monetary transfer by player i if $\mathbf{a} \in A$ is chosen.

We assume that players are expected utility maximizers and have a utility function that is additively separable in money and the object. Thus, player i's utility is given by

$$u_i(\mathbf{a}) = v_i q_i(\mathbf{a}) - y_i(\mathbf{a})$$

when $\mathbf{a} \in A$ is chosen. We assume that players play according to a Bayesian Nash equilibrium of the mechanism μ.

A feasible mechanism $\tilde{\mu}$ is a mechanism μ including strategies that form a Bayesian Nash equilibrium of μ. Let $a_1^*, ..., a_n^*$ be the Bayesian Nash equilibrium of μ, so that

$$a_i^*(v_i) \in \arg\max_{a_i \in A_i} \int_{V_{-i}} u_i(a_1^*(v_1), ..., a_{i-1}^*(v_{i-1}), a_i, a_{i+1}^*(v_{i+1}), ..., a_n^*(v_n)) f_{-i}(\mathbf{v}_{-i}) d\mathbf{v}_{-i}$$

for all v_i and i.

Let $SWF(\tilde{\mu})$ denote social welfare for the feasible mechanism $\tilde{\mu}$. Social welfare is assumed to be the sum of players' utility in equilibrium, so that

$$SWF(\tilde{\mu}) = \sum_{i \in N} \int_V u_i(a_1^*(v_1), ..., a_n^*(v_n)) f(\mathbf{v}) d\mathbf{v}. \tag{2.1}$$

A feasible mechanism $\tilde{\nu}$ is said to be a *socially optimal mechanism* if it maximizes social welfare over all feasible mechanisms. Formally, if M is the set of all feasible mechanisms, than $\tilde{\nu} \in M$ is socially optimal if

$$SWF(\tilde{\nu}) \geq SWF(\tilde{\mu})$$

for all $\tilde{\mu} \in M$.

Before we discuss our main result, we give several examples of sets A_i of possible actions, allocation rules q, and payment rules p that are used in the contest literature. Usually in the this literature, A_i is a subset of $\Re$. Typical sets A_i are

$$A_i = \emptyset$$

when player i is excluded from the contest,

$$A_i = [0, \infty)$$

when no further restriction is placed on player i's possible actions,

$$A_i = [0, c_i]$$

when player i is not allowed to submit a bid above a cap c_i, and

$$A_i = [r_i, \infty)$$

when player i should meet a certain reserve price r_i.

In the contest literature, the functions q_i are referred to as the contest success functions. Typical examples of contest success functions are the following. The *logit form contest success function* is characterized by

$$q_i(\mathbf{a}) = \frac{g_i(a_i)}{\sum_{j \in N} g_j(a_j)}$$

where $g_1, ..., g_n$ are strictly increasing functions. Tullock (1980) uses this contest success function, making the special assumption that $g_i(a_i) = (a_i)^\alpha$ for all $i \in N$ with $\alpha > 0$. Skaperdas (1996) provides an axiomatic underpinning of this special type of logit form contest success functions, and Fullerton and McAfee (1999) give a further microeconomic support. The *difference form contest success function* is only defined for the case of two players. The function is given by

$$\begin{aligned} q_1(a_1, a_2) &= h(\beta a_1 - a_2) \\ q_2(a_1, a_2) &= 1 - h(\beta a_1 - a_2) \end{aligned}$$

where β is an ability parameter, and h an strictly increasing function, with $h(0) = \frac{1}{2}$, and $h(-y) = 1 - h(y)$. See Baik (1998) for a more extensive discussion on difference form contest success functions. The *perfectly discriminatory contest success function* is given by

$$q_i(\mathbf{a}) = \begin{cases} \frac{1}{m} & \text{if } a_i = \max_j a_j \text{ with } m = \#\{k | a_k = \max_j a_j\}, \\ 0 & \text{otherwise.} \end{cases}$$

This is the contest success function most commonly used in the literature (Baye et al., 1996; Che and Gale, 1998; Bulow and Klemperer, 1998).

Several payment rules are studied in the contest literature. For instance, in the case of the all-pay auction,

$$y_i(\mathbf{a}) = a_i,$$

in the case of the war of attrition,

$$y_i(\mathbf{a}) = \begin{cases} a_i & \text{if } a_i < \max_j a_j, \\ a_k & \text{if } a_i = \max_j a_j, \text{ with } a_k = \max_{j \neq i} a_j, \end{cases}$$

and in the case of the first-price sealed-bid auction,

$$y_i(\mathbf{a}) = \begin{cases} \frac{a_i}{m} & \text{if } a_i = \max_j a_j \text{ with } m = \#\{k | a_k = \max_j a_j\}, \\ 0 & \text{otherwise.} \end{cases}$$

Baye et al. (1998) define a parameterized class of contests, which includes the above examples as special cases. They restrict themselves to two players. In the class, the expected payments for player i are given by

$$y_i(\mathbf{a}) = \begin{cases} \gamma a_i + \delta a_j & \text{if } a_i > a_j, \\ \zeta a_i + \eta a_j & \text{if } a_i < a_j, \\ \frac{1}{2}(\gamma a_i + \delta a_j + \zeta a_i + \eta a_j) & \text{if } a_i = a_j, \end{cases}$$

where a_j is the action of the other player, with γ, δ, ζ, and η one-dimensional parameters.[2]

3. A socially optimal mechanism

By the Revelation Principle (Myerson, 1981), we may assume, without loss of generality, that the social planner only considers feasible direct revelation mechanisms. A feasible direct revelation mechanism is a feasible mechanism, in which each player is asked to announce her

[2] The all-pay auction has $\gamma = \zeta = 1$, and $\delta = \eta = 0$. For the war of attrition, $\delta = \zeta = 1$, and $\gamma = \eta = 0$. In the first-price sealed-bid auction, $\gamma = 1$, and $\delta = \zeta = \eta = 0$.

value, in which she has an incentive to participate (individual rationality) and in which she has an incentive to announce her value honestly (incentive compatibility).

Let (p, x) be a feasible direct revelation mechanism, with

$$p : V \to [0, 1]^n$$

having

$$\sum_j p_j(\mathbf{v}) \leq 1,$$

and

$$x : V \to \Re^n.$$

We interpret $p_i(\mathbf{v})$ as the probability that player i wins, and $x_i(\mathbf{v})$ as the expected payments by i, when $\mathbf{v}$ is announced.

Let

$$Q_i(p, v_i) \equiv E_{\mathbf{v}_{-i}}\{p_i(\mathbf{v})\}$$

be the conditional probability that player i wins given her value v_i, and

$$U_i(p, x, v_i) \equiv v_i Q_i(p, v_i) - E_{\mathbf{v}_{-i}}\{x_i(\mathbf{v})\}$$

be player i's interim utility from the feasible direct revelation mechanism. Myerson (1981) shows that individual rationality and incentive compatibility are equivalent to

$$\text{(3.1)} \qquad \text{if } w_i \leq v_i \text{ then } Q_i(p, w_i) \leq Q_i(p, v_i),\ \forall v_i,\ i,$$

$$\text{(3.2)} \qquad U_i(p, x, v_i) = U_i(p, x, \underline{v}_i) + \int_{\underline{v}_i}^{v_i} Q_i(p, y_i) dy_i,\ \forall v_i,\ i, \text{ and}$$

$$\text{(3.3)} \qquad U_i(p, x, \underline{v}_i) \geq 0,\ \forall i.$$

Consistently with (2.1), social welfare from (p, x) is given by

$$SW(p, x) = \sum_{i \in N} \int_{\underline{v}_i}^{\bar{v}_i} U_i(p, x, v_i) f_i(v_i) dv_i.$$

The following result is the key to our main finding.

LEMMA 11. *For each feasible direct revelation mechanism* (p, x), $SW(p, x) \leq EV^{\max}$.

PROOF. Let (p, x) be a feasible direct revelation mechanism. Then,

$$
\begin{aligned}
SW(p,x) &= \sum_{i\in N} \int_{\underline{v}_i}^{\bar{v}_i} \left(U_i(p,x,\underline{v}_i) + \int_{\underline{v}_i}^{v_i} Q_i(p,y_i)dy_i \right) f(v_i)dv_i \\
&= \sum_{i\in N} U_i(p,x,\underline{v}_i) + \int_{\underline{v}_i}^{\bar{v}_i} \frac{(1-F(v_i))}{f(v_i)} Q_i(p,v_i)f(v_i)dv_i \qquad (3.4)\\
&\leq \sum_{i\in N} U_i(p,x,\underline{v}_i) + \int_{\underline{v}_i}^{\bar{v}_i} (1-F(v_i))dv_i * \int_{\underline{v}_i}^{\bar{v}_i} Q_i(p,v_i)f(v_i)dv_i \\
&= \sum_{i\in N} U_i(p,x,\underline{v}_i) + (E\{v_i\} - \underline{v}_i) \int_{\underline{v}_i}^{\bar{v}_i} Q_i(p,v_i)f(v_i)dv_i \\
&\leq \sum_{i\in N} U_i(p,x,\underline{v}_i) + (EV^{\max} - \underline{v}_i) \int_{\underline{v}_i}^{\bar{v}_i} Q_i(p,v_i)f(v_i)dv_i \\
&\leq EV^{\max} \sum_{i\in N} \int_{\underline{v}_i}^{\bar{v}_i} Q_i(p,v_i)f(v_i)dv_i \\
&= EV^{\max}.
\end{aligned}
$$

The first equality in the above chain follows with (3.2), and we get the second equality using integration by parts. The first inequality follows from a theorem from Statistics which tells that the expectation of a product is less or equal than the product of the expectations in case the first term of the product is strictly decreasing in the variable over which the expectation is taken, and the second term is increasing in this variable (McAfee and McMillan, 1992). In this case, $\frac{1-F_i(v_i)}{f_i(v_i)}$ is strictly decreasing in v_i (by assumption), and Q_i is increasing in v_i (by (3.1)). The other manipulations are straightforward. □

Now, consider the feasible direct revelation mechanism $(\tilde{p}, \tilde{x})$ with

$$\begin{aligned} \tilde{p}_i(\mathbf{v}) &> 0 \text{ only if } E\{v_i\} = EV^{\max}, \forall i \in N, \\ \tilde{p}_i(\mathbf{v}) &= 0 \text{ if } E\{v_i\} < EV^{\max}, \forall i \in N, \\ \sum_{i \in N} \tilde{p}_i(\mathbf{v}) &= 1, \text{ and} \\ \tilde{x}_i(\mathbf{v}) &= 0, \forall i \in N. \end{aligned}$$

Observe that $(\tilde{p}, \tilde{x})$ is a lottery in which only the players with the highest expected value participate. The expected social welfare among the players is then expected value generated by the lottery, so that

$$SW(\tilde{p}, \tilde{x}) = EV^{\max}.$$

Then it immediately follows from Lemma 11 that $(\tilde{p}, \tilde{x})$ is socially optimal, as

$$SW(\tilde{p}, \tilde{x}) \geq SW(p, x)$$

for all feasible direct revelation mechanisms (p, x).

PROPOSITION 30. *$(\tilde{p}, \tilde{x})$ is socially optimal.*

An intuition behind Proposition 30 is the following. Observe in the second equality in the chain (3.4) that player i, if winning the object, adds $\frac{1-F_i(v_i)}{f_i(v_i)}$ to social welfare. As, by assumption, $\frac{1-F_i}{f_i}$ is a strictly decreasing function, the social planner prefers a low type of player i to win more often than a high type. However, (3.1) requires the probability for player i to win the object to be (weakly) increasing in v_i. Hence, the best the social planner can do is make the probability that a low type wins equal to the probability that a high type wins. He can do this optimally using a lottery among the players with the highest expected value for the object.

The socially optimal mechanism can be straightforwardly implemented in a contests. A contest in which only the players with the highest expected value can obtain the object, and in which no payments are made, is socially optimal. Usually this can be done by only allowing players to participate who have the highest expected value for the object, and by allowing these players to only submit a bid equal to zero.

4. Concluding remarks

In this paper, we have investigated socially optimal mechanisms in a situation with incomplete information. We have shown that optimally, the social planner should always assign the object to one of the players with the highest expected value for it. Interestingly, the social planner only needs very limited information on the value distributions of the players in order to be able to implement the socially optimal mechanism. He only needs to know which player attaches the highest ex ante value to the object.

The analysis was simplified by the assumptions on the distribution functions ($\frac{1-F_i}{f_i}$ is strictly increasing for each player i) and the values (each player's value for the object only depends on her own signal, and not on the signals of the other players). Further research should be devoted to relaxing these assumptions. A model with interdependent values will probably lead to the same conclusion. Namely, in the extreme case of a common values model (each player attaches the same value to the object), efficiency is of no concern to the social planner, so that in terms of social welfare, there is no need for the players to separate themselves in the contest. Therefore, in this case, a lottery is always optimal, even without the assumption on the distribution functions. It seems likely that that in models in between pure private values and pure common values, such as interdependent values models, a less strong assumption on the distribution functions is needed to come to the same conclusion.

Also the interpretation of our result needs further investigation. In the Introduction, we have restricted our attention to collusion among players in several contest-like situations. However, other interpretations seem to be possible as well. For instance, consider markets in which competition only takes place through advertising. Is seems to make sense to argue that a social planner who aims at maximizing total (consumer plus producer) surplus should be merely concerned about producer surplus, as consumer surplus is fixed due to the lack of price competition. Then our finding suggests that optimally, no competition should take place at all in such markets, so that consumers will decide which products to buy based on their own, limited information. Further research on this matter is needed, however, for instance to check if "lack of price competition" is a sensible concept.

Another extreme interpretation of our result, which needs further investigation as well, is that political lobbying should be prohibited, and that governments should base their political decisions on their own limited information. When we follow Baye et al. (1993) in viewing lobbying as the all-pay auction, then our finding implies that when a government is interested in social welfare among the interest groups (as it is benevolent, or seeks to be re-elected), it should optimally ban lobby completely, and allow only the interest groups which it expects to have the highest value for the political favor to win it.[3]

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[3]See also Matejka et al. (2002).

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Samenvatting (Summary in Dutch)

Sinds de Oudheid gebruikt de mensheid veilingen voor het verkopen van een groot scala aan goederen. De eerste schriftelijke melding van een veiling wordt toegeschreven aan de oud-Griekse historicus Herodotus. Herodotus beschreef hoe in Babylon 500 jaar voor het begin van onze jaartelling huwbare vrouwen werden verkocht aan de hoogstbiedende man. Tegenwoordig worden veilingen onder andere gebruikt bij de verkoop van bederfelijke waar als vis, bloemen en groenten, duurzame goederen als kunst, huizen en wijn, en abstracte objecten als UMTS frequenties, staatsobligaties en het recht om reizigers te vervoeren over spoorlijnen.

Gezien het veelvuldig voorkomen van veilingen als verkoopmechanisme, besteedt de economische wetenschap ruim aandacht aan veilingen. Sinds het baanbrekende werk van Vickrey in 1961 zijn er vele papers in de economische literatuur verschenen die het bieden in veilingen wiskundig modelleren. Samen vormen deze modellen de veilingtheorie. De modellen uit de veilingtheorie zijn speltheoretisch van aard, en pogen een aantal interessante vragen te beantwoorden. Gedragsvragen die veilingtheorie aan de orde stelt zijn: "Wat zal het bod zijn van een bieder in een bepaalde veiling?", "Hoe efficiënt is een bepaald veilingtype?" en "Hoeveel geld zal deze veiling naar verwachting genereren?" Ontwerpvragen waar veilingtheorie aandacht aan besteedt zijn "Welke veiling is het meest efficiënt?" en "Van welke veiling kan de verkoper de hoogste opbrengst verwachten?".

De veilingtheorie onderscheidt vier standaardtypen van veilingen wanneer er sprake is van de verkoop van één, ondeelbaar object. In de Engelse veiling (English auction) wordt de prijs van het object stap voor stap verhoogd, totdat er slechts één bieder overblijft die aangeeft bereid te zijn de prijs te betalen. Deze bieder wint het object voor de prijs waarbij de laatste bieder is uitgestapt. De Nederlandse veiling (Dutch auction) werkt precies andersom. Daar wordt de prijs eerst op een hoog niveau gezet, en vervolgens verlaagd, totdat één van de bieders aangeeft het object voor deze prijs te willen kopen. Deze bieder wint het object dan voor die prijs. In de eerste-prijs gesloten-bod veiling (first-price sealed-bid auction) doen bieders onafhankelijk van elkaar een bod op het object, waarbij de bieder die het hoogste bod doet het object wint voor een prijs gelijk aan zijn eigen bod. De tweede-prijs gesloten-bod veiling (second-price sealed-bid auction) werkt hetzelfde als de eerste-prijs gesloten-bod veiling, met het verschil dat de winnaar niet zijn eigen bod betaalt, maar het bod van de tweede hoogste bieder.

Een belangwekkend resultaat uit de veilingtheorie is de opbrengst-equivalentie stelling (revenue-equivalence theorem). Deze stelling claimt dat de verkoper, onder bepaalde omstandigheden, van elk standaard-type veiling precies evenveel opbrengst mag verwachten.

Naast voor het beantwoorden van bovenstaande gedrags- en ontwerpvragen over veilingen, blijkt veilingtheorie verrassenderwijs nog veel breder toepasbaar bij het bestuderen van economische vraagstukken. Zo argumenteren economisch wetenschappers dat politiek lobbyen veel wegheeft van een iedereen-betaalt veiling (all-pay auction). In deze veiling is het de hoogste bieder die het geveilde object wint, maar moeten, in tegenstelling tot wat het geval is in de standaard veilingtypen, alle bieders hun bod betalen. De iedereen-betaalt veiling wordt beschouwd als een geschikt model voor lobby, omdat belangengroepen veel geld spenderen in lobby om het "object", in dit geval een politiek voordeeltje, te winnen, en dit geld ook kwijt zijn als ze het politieke voordeeltje niet in handen krijgen.

Een ander voorbeeld van een economische fenomeen dat veel wegheeft van een veiling is de strijd tussen bedrijven om hun nieuwe technologie te laten overleven als standaardtechnologie. Een dergelijk strijd wordt gemodelleerd als een uithoudingsoorlog (war-of-attrition), wat op abstract niveau kan worden gezien als een veiling. In een uithoudingsoorlog blijven de spelers net zo lang verwikkeld in een dure oorlog, totdat slechts één van hen overleeft. Deze speler wint dan het object (het recht om winst te maken met zijn technologie), en net als in de iedereen-betaalt veiling betalen ook hier de verliezers hun bod (de kosten die ze maken zolang ze in de strijd zijn).

In dit proefschrift geef ik op een drietal wijzen een bijdrage aan de veilingtheorie. In de eerste plaats bestudeer ik een aantal nieuwe veilingmodellen. Zo introduceer ik een aantal nieuwe veilingtypen, en onderzoek ik veilingen in situaties die afwijken van de standaard veilingmodellen. Ten tweede illustreer ik de ontwikkelde theoriën met voorbeelden van veilingen die in de praktijk hebben plaatsgevonden, of die in de toekomst plaats kunnen vinden. De derde bijdrage van dit proefschrift ligt op een ruimer economisch gebied. Ik besteed aandacht aan lobbyen, reclame en politieke campagnes.

Ik heb het proefschrift opgedeeld in zeven hoofdstukken. Het eerste hoofdstuk is een introductie tot het proefschrift, en de volgende hoofdstukken worden gevormd door zes papers, waarbij elk hoofdstuk één paper is. Deze papers presenteer ik volstrekt onafhankelijk van elkaar in vrijwel dezelfde vorm als waarin ik (met mijn co-auteurs) ze aan internationale vaktijdschriften heb aangeboden ter publicatie. Deze manier van het presenteren van een promotieonderzoek is tegenwoordig standaard in de economische wetenschap. De belangrijkste reden om een proefschrift op deze wijze op te zetten (en niet als een meer samenhangend boek) is dat het grootste deel van de erkenning van een promotieonderzoek door collegawetenschappers plaatsvindt via publicaties in de internationale vakliteratuur die voortvloeien uit het proefschrift, en niet via het proefschrift zelf. Ik concludeer deze samenvatting met een korte omschrijving van de zeven hoofdstukken.

In Hoofdstuk 1, de introductie, ga ik in op de geschiedenis en de praktijk van het veilen, de belangrijkste veilingtypen, het belang van veilingtheorie, en de meest aansprekende resultaten die de veilingtheorie afleidt. In dit hoofdstuk geef ik ook een korte samenvatting van de zes papers.

In Hoofdstuk 2 leg ik de relatie tussen een veiling en een consumentenmarkt. Ik bestudeer een veilingmodel, waarbij de bieders bedrijven zijn die opereren op een productmarkt. Deze markt wordt gekenmerkt door netwerkeffecten, d.w.z. hoe groter het aantal filialen van een bedrijf, hoe hoger de winst *per filiaal* voor dat bedrijf. In een veiling wordt een licentie voor een extra filiaal verkocht. Ik bekijk twee situaties. In de ene situatie hangt de totale marktwinst niet af van welk bedrijf de licentie wint. In de andere situatie is de totale winst in de markt groter naarmate het winnende bedrijf groter is. Daarbij onderzoek ik onder andere het biedgedrag in de eerste-prijs gesloten-bod veiling, welk mechanisme opbrengstmaximaliserend is, en of de bieders mogelijkheden hebben tot samenspanning. Ik gebruik de veiling van benzinestations in Nederland die momenteel door de Nederlandse overheid wordt bestudeerd als toepassing van het model.

In Hoofdstuk 3 bestudeer ik biedgedrag wanneer de verliezers in een veiling financiële externaliteiten genieten van de winnaar. In de standaard veilingmodellen wordt aangenomen dat de verliezers van de veiling indifferent zijn over hoeveel de winnaar betaald heeft. Er zijn echter praktische situaties denkbaar waarbij dat niet het geval hoeft te zijn. Bijvoorbeeld bij de UMTS veilingen in Europa waren bieders vermoedelijk bijzonder geïnteresseerd in hoeveel een tegenstander betaalde in een van de veilingen, vanwege de invloed op zijn financiële positie. Dit kan wat van belang kan zijn als bieders elkaar in andere markten weer ontmoeten. Ik spreek in dit soort situaties van financiële externaliteiten. In dit hoofdstuk onderzoek ik of bepaalde resultaten uit de standaard veilingtheorie nog wel van toepassing zijn wanneer financiële externaliteiten een rol spelen. Zo kijk ik naar het biedgedrag in de eerste- en de tweede-prijs gesloten-bod veiling, en concludeer ik dat de opbrengst-equivalentie stelling niet meer geldig is in deze nieuwe omgeving.

In Hoofdstuk 4 beschouw ik veilingen die de verwachte opbrengst voor de verkoper maximaliseren in het geval van financiële externaliteiten. In het standaard veilingmodel is elke standaardveiling opbrengstmaximaliserend wanneer de verkoper de juiste bodemprijs (reserve price) stelt, waar beneden bieders geen geldig bod kunnen uitbrengen. In het geval van financiële externaliteiten verandert dit resultaat cruciaal. De veilingen die in het standaard model optimaal blijken te zijn, zijn niet meer optimaal wanneer bieders financiële externaliteiten op elkaar uitoefenenen. Ik maak twee modificaties om tot een optimale veiling te komen: (1) de bieders moeten voorafgaand aan de veiling een entreeprijs (entry fee) betalen, en (2) de veiling is geen standaardveiling met bodemprijs, maar de *laagste-prijs iedereen-betaalt veiling* (lowest-price all-pay auction) met bodemprijs. Ook kijk ik naar een omgeving die afwijkt van het standaardmodel, waarin de verkoper het object *moet* verkopen (hij kan dus geen bodemprijs zetten), en waarin hij niet kan voorkomen dat het object van eigenaar wisselt na afloop van de veiling. In deze omgeving blijkt de laagste-prijs iedereen-betaalt veiling (zonder bodemprijs) optimaal te zijn.

In Hoofdstuk 5 kijk ik naar het blootstellingsprobleem (exposure problem) in multi-object veilingen. In een multi-object veiling worden simultaan meerdere goederen verkocht. Wanneer deze goederen complementair zijn in de ogen van de bieders, kan een dergelijke veiling voor hen riskant zijn. Ze lopen immers het risico te weinig objecten in de veiling te winnen, zodat de complementariteit niet gerealiseerd kan worden. Dit wordt in de veilingliteratuur het blootstellingsprobleem genoemd, vanwege de "blootstelling" van bieders aan een dergelijk risico. In een veilingmodel kijk ik naar een zeer eenvoudige situatie waar dit probleem speelt, namelijk wanneer een verkoper drie Chinese stokjes veilt. Ik neem aan dat één stokje niets waard is voor de bieders, dat bieders een strikt positieve waarde toekennen aan twee stokjes, en dat een derde stokje eveneens waardeloos is. Ik vergelijk twee veilingvormen: (1) de Chinese-stokjes veiling (Chopstick auction), waarin de drie stokjes individueel worden geveild, en (2) de tweede-prijs gesloten-bod veiling waarin de drie stokjes als één bundel onder de hamer komen. Ik concludeer dat het veilen van de bundel beter is dan veilen van de losse stokjes.

In Hoofdstuk 6 besteed ik aandacht aan een lobbyspel dat gemodelleerd is als een iedereen-betaalt veiling. Ik voeg in dit hoofdstuk drie dingen toe aan de huidige economische literatuur. In de eerste plaats leid ik biedgedrag af wanneer bieders in de iedereen-betaalt veiling worden geconfronteerd met een plafond waarboven geen bod mag worden gedaan. Hoe hoger het plafond, hoe groter de kans dat de winnaar van de veiling diegene is die het meest voor het object over heeft. Ten tweede bereken ik hoeveel de bieders naar verwachting betalen als functie van de hoogte van het plafond. Deze functie blijkt strikt stijgend te zijn, d.w.z. hoe hoger het plafond, hoe méér bieders betalen in de veiling. De derde bijdrage is een trade-off tussen de kosten die de bieders maken gegeven de hoogte van het plafond, en de informatiebaten. Naast biedkosten zijn er immers ook informatiebaten te verwachten in de veiling, al naar gelang deze de potentie heeft de beider met de hoogste waarde te selecteren. Als ik aanneem dat het totale sociale nut van de veiling gegeven wordt door de verwachte som van het nut dat de bieders realiseren, dan blijkt dat in het optimum, het plafond op nul moet worden gesteld. Ik concludeer vervolgens dat het sociale nut gemaximaliseerd wordt als lobbyen verboden wordt.

In Hoofdstuk 7 onderzoek ik sociaal optimale mechanismen. Een mechanisme is een spel waarin een ondeelbaar object wordt gealloceerd aan één speler uit een verzameling spelers die bestaat uit één verkoper en een aantal potentiële kopers (de bieders). Een mechanisme wordt beschreven door drie elementen: (1) een actieruimte, die voor elke bieder aangeeft welke acties zij in het spel kan ondernemen, (2) een allocatieregel, die aangeeft hoe het goed gealloceerd wordt gegeven de gespeelde acties van de bieders, en (3) een betalingsregel, die vastlegt hoeveel elke bieder moet betalen als een functie van de gespeelde acties. In dit hoofdstuk construeer ik een sociaal optimaal mechanisme, d.w.z. een mechanisme dat de verwachten som van het nut van de spelers maximaliseert. Ik neem aan dat iedere speler alleen over haar eigen waarde voor het object volledig geïnformeerd is. De verkoper en de andere bieders zijn slechts op de hoogte van de verdeling waaruit deze waarde is getrokken. Ik laat vervolgens zien dat een mechanisme dat het object toewijst aan één van de bieders die naar verwachting de hoogste waarde aan het object toekent, optimaal is. Ik suggereer toepassingen van dit resultaat in lobby, reclame, politieke campagnes, en veilingen.

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SANDER ONDERSTAL graduated from Tilburg University in 1997 with a specialization in Operations Research. He carried out his Ph.D. research at the CentER for Economic Research (Tilburg University), and at University College London within the framework of the European Network for Training in Economic Research. Currently, he works at CPB Netherlands Bureau for Economic Policy Analysis as an advisor for the Dutch and the European government.

This Ph.D. thesis is a collection of six papers in auction theory, with several economic applications, both to real life auctions and to other economic phenomena. In the introduction to the thesis, Onderstal argues why auction theory is an important branch of economic theory, and discusses several interesting results that emerge from auction theory. The first paper is about situations in which the outcome of an auction determines the market structure of a consumer market. The Dutch petrol market is used as an illustration for this model. The second and the third papers, both motivated by the UMTS auctions that took place in Europe, consider auctions in which losing bidders obtain financial externalities from the winner. The fourth paper deals with the exposure problem in auctions, and is applied to the Dutch DCS-1800 auction. The fifth paper interprets political lobbying as an 'all-pay auction', and considers situations in which the government maximizes social welfare by completely banning lobbying. Finally, the sixth paper constructs mechanisms that are optimal from the bidders' point of view, with applications to lobbying, advertising, political campaigns, and auctions.

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