The valuation and hedging of variable rate savings accounts

de Jong, F.C.J.M.; Wielhouwer, J.L.

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Frank de Jong
University of Amsterdam

Jacco L. Wielhouwer
ING Group and CentER, Tilburg University

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Abstract

Variable rate savings accounts have two main features: the client rate is variable and deposits can be invested and withdrawn at any time. The client rate is not always equal to the short rate. However, customer behaviour is sluggish and actions are often performed with a delay. The savings account may therefore have some net value to the bank, and this value typically depends on the interest rate. This paper focuses on measuring the interest rate risk of such savings accounts on a value basis. In order to model customer behaviour we implement a partial adjustment specification for the balance. The interest rate policy of the bank is described in an error-correction model. Based on these models we calculate the duration of the savings accounts and analyze the problem how to hedge these accounts.

Please address correspondence to:
Frank de Jong
Department of Financial Management
University of Amsterdam
Roetersstraat 11, 1018 WB, Amsterdam, the Netherlands
phone: +31-20-5255815
fax: +31-20-5255285
e-mail: fdejong@fee.uva.nl

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1 Introduction

A major part of private savings is deposited in variable rate saving accounts, also known as demand deposits. Typically, deposits can be invested and withdrawn at any time at no cost, which makes demand deposits look similar to a money market account. However, the interest rate paid on demand deposits is often different from the money market rate. On the other hand, depositors do not immediately withdraw their money when rates on alternative investments are higher. Whatever the causes of this behaviour (market imperfections, transaction costs or other), these characteristics imply that the value (from the point of view of the issuing bank) of the demand deposits may be different from the nominal value of the deposits.

In the literature, the valuation of demand deposits is well studied. For example, Hutchison and Pennacchi (1996), Jarrow and Van Deventer (1998) and Selvaggio (1996) provide models for the valuation of such products. The first two papers build on the (extended) Vasicek (1977) model, whereas the latter paper uses a more traditional Net Present Value approach. In all these papers there is little explicit modeling of the dynamic evolution of the savings rate and the balance, and how this evolution depends on changes in the term structure of market interest rates. For example, Jarrow and van Deventer's (1998) model is completely static in the sense that savings rate and balance are linear functions of the current spot rate. In practice, it is well known that deposit rates and balances are rather sluggish and often do not respond immediately to changes in the return on alternative investments, such as the money market rate. Typically, the rate paid to the client (the savings rate) is set by the bank and the balance is determined by client behaviour. The balance depends, among other things, on the deposit rate but also on the return on alternative investments such as the money market rate. Because the paths of the future savings rate and the adjustment of the balance determine the value of the demand deposits, we think a focus on dynamic adjustment patterns is important.

In this paper, we analyze the valuation and hedging of demand deposits with an explicit model for the adjustment of savings rates and balance to changes in the short term interest rate. A recent paper by Janosi, Jarrow and Zullo (JJZ, 1999) presents an empirical analysis of the Jarrow and van Deventer (1998) model. They extend the static theoretical model to a dynamic empirical model, that takes the gradual adjustment of deposit rates and balance into account. Our approach differs

from the JJZ paper in several respects. First, we treat the term structure of interest rates as exogenous and calculate the value of the demand deposit by a simple Net Present Value equation. This approach, suggested by Selvaggio (1996) leads to simple valuation and duration formulas, and is applicable without assuming a particular term structure model. The drawback of this approach is that we have to assume that the risk premium implicit in the discount factor is constant, but this may be a good first approximation because we want to concentrate on the effects of the dynamic adjustment of the deposit rate and balance and not on term structure effects. A second difference between the JJZ model and ours is the modeling of the long run effects of term structure shocks. In our model, there is a long run equilibrium, in which the difference between the deposit rate and the short rate is constant, and the balance of the demand deposit is also constant (possibly around a deterministic trend). Short term deviations from these long run relations are corrected at a constant rate. This model structure is know in the empirical time series literature as an error correction model[1]. This model has some attractive properties, such as the stability of long run effects of shocks.

The interest rate sensitivity is quantified in a duration measure. We demonstrate that the duration depends on the adjustment patterns of savings rate and balance. We pay particular attention to the implications of the model for the hedging of interest rate risk on these products. We illustrate how to fund the demand deposits by a mix of long and short instruments that matches the duration of the demand deposit's liabilities.

The paper is organized as follows. First the valuation of the demand deposits is dealt with in section 2. In section 3 the models on the pricing policy and the customer behaviour are presented, and a discrete time version of the model is estimated for the Dutch savings accounts market. Section 4 deals with the duration of this product and section 5 with hedging decisions. The paper is concluded in section 6.

2   Valuation of variable rate savings accounts

The valuation problem of savings accounts and similar products was analyzed by Selvaggio (1996) and Jarrow and Van Deventer (1998). Their approach is to acknowledge that the liability of the bank equals the present value of future cash outflows

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[1]We refer to Davidson et al. (1978) for an introduction to error correction models.

(interest payments and changes in the balance). The present value of these flows not necessarily equals the market value of the money deposited, and therefore the deposits may have some net asset value. Jarrow and Van Deventer (1998) treat the valuation of demand deposits in a no-arbitrage framework and derive the net asset value under a risk-neutral probability measure. However, in our paper we want to implement an empirical model for the savings rate and the balance, and therefore we need a valuation formula based on the *empirical* probability measure. We therefore follow the approach proposed by Selvaggio (1996), who calculates the value of the liabilities as the present value of expected future cash flows, discounted at the risk free rate plus a risk premium or Option Adjusted Spread.

Assume that the appropriate discount rate $R(t)$ can be written as

$$(1) \qquad\qquad\qquad\qquad R(t) = r(t) + \gamma$$

where $r(t)$ is the short rate (say, the money market rate) and γ is the risk premium. The main assumption in this paper is that this risk premium is constant over time and does not depend on the level of the short rate. This assumption is obviously a simplification. Any underlying formal term structure model, such as the Ho and Lee (1984) model, implies risk premia that depend on the short rate. However, the risk premia are typically small and since the focus of the paper is on modeling the dynamic adjustment of savings rates and deposit balance, we ignore the variation in the risk premium and focus on the effect of shocks to the short rate.

The market value of the liabilities then is the expected discounted value of future cash outflows: interest payments $i(t)$ and changes in the balance $D(t)$

$$(2) \qquad\qquad L_D(0) = \mathsf{E}\left[\int_0^\infty e^{-Rs}[i(s)D(s) - D'(s)]ds\right]$$

where for notational clarity, the time variation in the discount rate is suppressed. Working out the integral over $D'(s)$ we find that the value of the liabilities equals

$$(3) \qquad\qquad L_D(0) = \mathsf{E}\left[\int_0^\infty e^{-Rs}[i(s) - R(s)]D(s)ds\right] + D(0)$$

Since the market value of the assets is equal to the initial balance, $D(0)$, the net asset value (i.e., the market value of the savings product from the point of view of the bank) is

$$(4) \qquad V_D(0) = D(0) - L_D(0) = \mathsf{E}\left[\int_0^\infty e^{-Rs}[R(s) - i(s)]D(s)ds\right]$$

For an interpretation of this equation, notice that $R(t) - i(t)$ is the difference between the discount rate and the interest rate paid on the deposit, i.e. the profit margin. The net asset value is simply the present value of future profits (balance times profit margin). Therefore, the net asset value is positive if the savings rate is on average below the discount rate. Obviously, the net asset value is zero if the savings rate always equals the discount rate.

As an example, consider the situation where the savings rate is always equal to the discount rate minus a fixed margin, $i(t) = R(t) - \mu$, and the discount rate is constant over time. Moreover, assume that the balance is constant at the level D^*. In that case, the net asset value of the demand deposits is

$$(5) \qquad\qquad V_D^* = \frac{\mu}{R} D^*$$

Intuitively, this is the value of a perpetuity with coupon rate μ and face value D^*. Figure 1 graphs the net asset value for different values of R and μ. For large profit margins and low discount rates, the net asset value can be a substantial fraction of the total value of the deposits.

Obviously, this example describes the value in a static setting. For the interest rate sensitivity of the net asset value, we have to take into account that after a shock in interest rates, the deposit rate and the balance only gradually adjust to their new equilibrium values. In the next section we therefore present a model for the adjustment patterns of interest rate and balance after shocks to the short rate. In the subsequent section we present interest rate sensitivity measures based on these adjustment patterns.

3   Consumer and bank behaviour

The analysis in the previous section shows that the net asset value of demand deposits depends on the specific pattern of expected future savings rates and balances. The main difference between money market accounts and demand deposits is the sluggish adjustment of interest rates and balance to changes in the short rate. In this section we model these adjustment processes. The models highlight the partial adjustment toward the long run equilibrium values of savings rates and balances.

For the expected interest rate paid on demand deposits, we propose the following

error correction specification

(6)
$$di(t) = \kappa[R(t) - \mu - i(t)]dt$$

This equation states that the interest rate adjusts to deviations between the long run value $R(t) - \mu$ and the current rate. Deviations are corrected at speed $\kappa > 0$. In the long run, expected savings rates are a margin μ below the discount rate $R(t)$.

For the balance we propose a partial adjustment specification

(7)
$$dD(t) = -\lambda[D(t) - D^*]dt - \eta[R(t) - \mu - i(t)]dt$$

This specification has two components. First, there is an autonomous convergence to a long run mean D^*. Second, there is an outflow of funds proportional to the excess of the discount rate over the savings rate. There is no immediate adjustment of the savings rate to the interest rate, but this could be included at the cost of notational complexity.

Notice that these equations are deterministic and don't contain an error term or other random shocks. This is not to say that these are absent in practice, but rather we want our models to describe the adjustment after a shock to the exogenous variables (in particular the short rate). In a stochastic environment, these differential equations can be thought of as a description of the expected evolution of interest rates and balance.

Working out the differential equations (6) and (7) gives:

(8a)
$$i(t) = e^{-\kappa t}i(0) + \kappa \int_0^t e^{\kappa(s-t)}[R(s) - \mu]ds$$

(8b)
$$D(t) = D^* + e^{-\lambda t}(D(0) - D^*) - \eta \int_0^t e^{\lambda(s-t)}[R(s) - \mu - i(s)]ds$$

To interpret these equations, let's consider the effects of a once-and-for-all shock to the discount rate. Initially, the discount rate takes the value R, and at time 0 the discount rate jumps to $R(t) = R + \Delta R$ for $t > 0$. Substituting the discount rate jump scenario equation (8a) gives

(9)
$$i(t) = e^{-\kappa t}\{i(0) - [R - \mu]\} + R - \mu + \left(1 - e^{-\kappa t}\right)\Delta R$$

Now assume that initially $i(0) = R - \mu$ and $D(0) = D^*$ are at their equilibrium values. In that case, the future interest rates can also be written as

(10)
$$i(t) = i(0) + \left(1 - e^{-\kappa t}\right)\Delta R$$

which tends to $i(0) + \Delta R$ in the long run. Substituting this in the expression for the balance gives

$$
\begin{aligned}
D(t) &= D(0) - \eta \int_0^t e^{\lambda(s-t)} e^{-\kappa s} ds \Delta R \\
&= D(0) - \eta \left(\frac{e^{-\lambda t} - e^{-\kappa t}}{\kappa - \lambda} \right) \Delta R
\end{aligned}
$$

(11)

which tends to the initial balance $D(0)$ for large t.

These adjustment patterns are illustrated in Figure 2 for an increase in the discount rate by 1%. The parameter values are picked from the empirical estimates to be discussed shortly, and are equal to $\kappa = 0.6$, $\lambda = 0.04$ and $\eta = 4.04$ for the base case. We see that the interest rate doesn't follow the jump in the discount immediately but gradually adjusts to its new equilibrium value. The adjustment of the balance is more complex. Initially, the balance decreases because of withdrawals caused by the relatively low savings rate. But as the interest rate increases, this effect becomes smaller and eventually the autonomous convergence of the balance to its long run level dominates.

The equations also highlight the effects of the model parameters on the adjustment of interest rates and balance to a shock in the discount rate (say, due to a shock in the short rate). The effect of η is obvious, it increases the impact of an interest rate shock. The effect of the mean-reversion parameters κ and λ is more complicated. A higher value of λ speeds up the adjustment of the balance itself, but doesn't affect the interest rates. With a lower value of κ, both the adjustment of the interest rate and the balance are slower. The effect of the balance is a result of the dependence of the balance on the interest rate. These effects are illustrated in Figure 2, where the dashed line gives the adjustment pattern for a lower value of κ, and the dotted line the pattern with a higher value of λ.

We now present some indicative estimates of the model parameters. This exercise is not meant to be a thorough empirical investigation of the adjustment pattern but merely serves as an illustration of the model. In order to translate the continuous time parameters to a discrete time setting, we use the following approximate[2] discretization of the continuous time model

(12a) $\qquad \Delta i_t = \kappa[R_{t-1} - \mu - i_{t-1}]\Delta t$

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[2] This approximation is quite accurate. For example, the exact mean reversion parameter for the interest rate equation is $1 - \exp(-\kappa \Delta t)$, which for small values of κ or Δt is close to $\kappa \Delta t$.

(12b) $$\Delta D_t = -\lambda (D_{t-1} - D^*) \Delta t - \eta [R_t - \mu - i_t] \Delta t$$

The discount rate is not directly observed in the data. Since a savings deposit shares characteristics of both a money market account and a long term deposit, its required rate of return or discount rate is proxied by a weighted average of money market rate (r_t) and long term bond yields (y_t).[3] We treat the weight δ as an unknown parameter which is estimated form the data. This leads to the following empirical model

(13a) $$\Delta i_t = \alpha_0 + \alpha_1 \Delta r_t + \alpha_2 [i_{t-1} - \{\delta r_{t-1} + (1-\delta) y_{t-1}\}] + e_{1,t}$$

(13b) $$D_t = \beta_0 + \beta_1 D_{t-1} + \beta_2 [i_{t-1} - \{\delta r_{t-1} + (1-\delta) y_{t-1}\}] + e_{2,t}$$

This model is slightly more general than the theoretical model because it contains a direct effect of the short rate on the savings rate. This effect turned out to be so important empirically that we included it in the empirical model. The parameters of the continuous time model can be solved from the following equations (with $\Delta t = 0.25$ for quarterly data)

$$\kappa = -\alpha_2 / \Delta t$$
$$\lambda = (1 - \beta_1) / \Delta t$$
$$\eta = \beta_2 / \Delta t$$

In fact, the long run deposit level and the average spread of the discount rate over the weighted average of short and long interest rates could be unraveled from the constant terms of the model. These are not very accurately estimated however and we refrain from drawing inferences about these parameters from the estimates.

We use quarterly data on interest rates and deposits from the Dutch savings account market. The sample period is 1982-IV to 1997-II, spanning 15 years which is slightly longer than the samples of Hutchison and Pennacchi (1996) or JJZ. To remove deterministic trends in the total savings volume, we define the balance D_t as the fraction of demand deposits to total savings. As the deposit rate, the client rate of one of the price-setters in the Dutch market is chosen. The following empirical estimates[4] are obtained using least squares:

(14a) $$\Delta i_t = -0.19 + 0.18 \Delta r_t - 0.15 [i_{t-1} - \{\delta r_{t-1} + (1-\delta) y_{t-1}\}] + e_{1,t}$$

---

[3] An alternative but equivalent way to justify this proxy is to assume that the risk premium of the savings deposit is a fraction of the risk premium on long term bonds.

[4] The unconstrained estimate of the parameter of the lagged balance, β_1, is slightly bigger than 1, but not very accurate. To obtain a stable model we need a positive value of λ, which implies that β_1 should be less than 1. We therefore fixed the parameter of the lagged balance (β_1) at 0.99, which is less that one standard deviation away from the unconstrained estimate.

(14b) $D_t = 2.21 + 0.99 D_{t-1} + 1.01 \left[i_{t-1} - \{ \delta r_{t-1} + (1 - \delta) y_{t-1} \} \right] + e_{2,t}$

The estimate of δ is around 0.2. These estimates imply the following annualized values for the continuous time parameters: $\kappa = 0.60$, $\lambda = 0.04$, and $\eta = 4.04$.

4 Duration

The previous section showed that the savings rate and the balance of demand deposits are related to interest rate changes. Therefore, the interest rate sensitivity of savings deposits will be different from the interest rate sensitivity of a money market account (which has a duration of zero). In this section, we study the sensitivity of the net asset value of a demand deposit to once-and-for-all parallel changes in the yield curve. This is close to a traditional duration analysis, see e.g. Bierwag (1987), but of course we take into account the dependence of future cash flows on the interest rate changes.

Straightforward differentiation of the net asset value with respect to the discount rate gives

$$
\begin{aligned}
\frac{\partial V_D(0)}{\partial R} =\ & \mathsf{E}\left[-\int_0^\infty s e^{-Rs} [R - i(s)] D(s) ds \right. \\
& \left. + \int_0^\infty e^{-Rs} \frac{\partial [R - i(s)]}{\partial R} D(s) ds + \int_0^\infty e^{-Rs} [R - i(s)] \frac{\partial D(s)}{\partial R} ds \right]
\end{aligned}
$$

(15)

The three components of this expression can be interpreted as follows:

1. the interest rate sensitivity of the expected discounted profits;

2. the change in the margin on the expected future balances;

3. the expected margin times increases or decreases in the balance of the deposit.

Notice that if the future balances do not change as a result of the interest rate change, and if the margin is constant, only the first term (the sensitivity of the present value of the profits) remains. The second and third term are specific for the demand deposits with their slow adjustment of client rate and balances, and are therefore the most interesting for our analysis. We shall now discuss the duration of the accounts given the specific model for the evolution of interest rates and balances.

For the model presented in the previous section we can derive the following partial derivatives with respect to an interest rate change

(16a) $$\frac{\partial i(t)}{\partial R} = 1 - e^{-\kappa t}$$

(16b)
$$\frac{\partial D(t)}{\partial R} = -\eta\left(\frac{e^{-\lambda t} - e^{-\kappa t}}{\kappa - \lambda}\right)$$

The long run derivative of the savings rate is one, but in the short run the effect is less than one. If $\eta > 0$ and $\kappa > \lambda$ (which is clearly the case empirically), the partial derivative of the balance is negative, and converges to zero in the long run. Substituting these partial derivatives into equation (15) gives after some calculation

(17)
$$\frac{\partial V_D(0)}{\partial R} = -\frac{\mu}{R^2}D(0) + \frac{1}{R+\kappa}D(0) - \frac{\eta\mu}{\kappa - \lambda}\left(\frac{1}{R+\lambda} - \frac{1}{R+\kappa}\right)$$

With an increase in the discount rate, the first term reflects the loss of value of the (perpetual) profit margin, the second term the discounted value of the interest payments not made on the original balance during the time the savings rate is below the discount rate, and the third term the discounted value of the profit foregone on the balance outflows.

We can transform this change of value to a duration measure if we assume that initially, the net asset value equals $V_D(0) = D(0)\mu/R$ as in the example above.

(18)
$$Dur = -\frac{\partial V_D(0)}{\partial R}\frac{1}{V_D(0)} = \frac{1}{R} - \frac{1}{\mu}\frac{R}{R+\kappa} + \frac{\eta}{D(0)}\frac{1}{\kappa - \lambda}\left(\frac{R}{R+\lambda} - \frac{R}{R+\kappa}\right)$$

The first term reflects the duration of a perpetuity, and is determined by the present value of the profits in the steady state. The second term reflects the value of the lower interest rates paid on the existing balance, and is always negative. The third term is the duration of the profits on the additional balance outflows, and is positive under the assumption $\kappa > \lambda$. Especially when the margin μ is thin and the net asset value is low, the second term may dominate the other terms, leading to a negative duration for the net asset value of a demand deposit. In that case, an increase in the short rate (and hence the discount rate) will increase the net asset value because for some time the interest rate paid to the clients is lower than the interest rate earned on the assets deposited.

As an illustration Figure 3 shows the durations as a function of the discount rate R and the margin μ (the other parameters are put equal to the estimates of the previous section). We see that the duration is typically positive, except for low values of μ, and declines with the level of the discount rate. Most of this effect is due to the duration of the discounted profit margin, $1/R$. Leaving out this term, we find the 'extra' duration of the net asset value induced by the sluggish adjustment pattern. Figure 4 shows these measures. Interestingly, for relatively big profit margins μ the 'extra' duration is positive, but for thin margins it is typically negative.

5 Hedging

In this section we consider the problem of hedging the net asset value. We start from the balance sheet of the savings bank

$$
\begin{array}{c|c}
D & L_D \\
  & V_D
\end{array}
$$

where D is the total amount of money deposited, L_D the value of the liabilities and V_D the net asset value. In the previous section we considered the duration of the net asset value under the implicit assumption that the deposits were invested in market instruments whose value does not depend on the interest rate, e.g. a money market account. For a hedging analysis it is more natural to take the interest rate sensitivity of the liabilities L_D as given, as this is determined by consumer behavior. From the balance sheet equality we obtain

$$
(19) \qquad \frac{\partial L_D}{\partial R} = -\frac{\partial V_D}{\partial R} = Dur \cdot V_D
$$

where Dur is the duration of the *unhedged* net asset value, calculated in the previous section.

Given the duration of the liabilities of the variable rate savings accounts, one can hedge the net asset value by immunization. For simplicity we assume the money deposited can be invested in two instruments, Long Investments (LI) and Short Investments (SI)). The balance sheet of the bank then becomes

$$
\begin{array}{c|c}
V_{LI} & L_D \\
V_{SI} & V_D
\end{array}
$$

We now consider the construction of an investment portfolio where the interest rate risk on the net asset value is fully hedged, i.e. the net asset value V_D is not sensitive to the interest rate. From the balance sheet we see that this requires

$$
(20) \qquad \frac{\partial V_{SI}}{\partial R} + \frac{\partial V_{LI}}{\partial R} = \frac{\partial L_D}{\partial R}
$$

which is equivalent to

$$
(21) \qquad -(V_{SI} Dur_{SI} + V_{LI} Dur_{LI}) = Dur \cdot V_D
$$

Of course, the solution to this equation, and hence the composition of the investment portfolio, depends on the durations of the short and long investments. As a

simple example, consider the case where the short instrument has zero duration. In that case the investment in the long instrument should be

$$(22) \qquad V_{LI} = \frac{-Dur}{Dur_{LI}} V_D$$

As an illustration, Figure 5 graphs the required position in long (10 year maturity) bonds in the hedge portfolio for different value of R and μ. As seen before, the duration of variable rate savings accounts may be negative, in particular when the profit margin μ is fairly small. In that case the bank can hedge the accounts by taking a long position in long investments. But if Dur is positive, which happens for example when the profit margin μ is fairly high, one should take a short position in the long asset. Alternatively, if one does not like to take short positions in bonds, one could use derivative instruments such as caps, which typically have a negative duration, or forward contracts.

6 Conclusion

This paper focuses on the valuation and interest rate sensitivity of variable rate savings accounts. The duration can be split in three different effects:

- the duration of the expected discounted profits;

- the change in margin on expected future balances due to a change in interest rate;

- the expected margin times increases or decreases in the balance of the deposit.

The first element is the standard duration for products without embedded options. The second and third term are non-standard (for example, they are zero for a money market account) and arise due to the variable interest rate and the option of the clients to withdraw and invest in the deposit at any time. The duration therefore crucially depends on the rapidness of the adjustment of the savings rate to the interest rate changes and on the reactions of the clients. These reactions will principally be determined by the clients interest rate sensitivity and by the market efficiency. The models are estimated for the Dutch savings account market. Duration curves are given for different margins.

When hedging the savings deposits, one can construct a portfolio with the same duration as the variable rate savings accounts. However, when one does not want to

go short into a certain asset class, one might need to include derivatives (for example caps) to hedge these products, since it is possible to have negative durations. The intuition is that an interest rate increase might lead to a flight of clients to money market accounts. So buy 'insurance' when money market accounts are less attractive, which result in profits when interest rates spike up (the insurance pays out). The gain due to the caps in an increasing interest rate environment then offsets the loss in the demand deposits. Hedging in this way certainly smoothens the results on these products. Of course this can be achieved by going short in long assets as well.

For future research it might be interesting to analyze the second order effects. Then multiple immunization can be achieved with a portfolio with three asset classes. Finally, it is possible to make the discount rate a function of a number of interest rates with different maturities. This will of course increase the complexity of the model but allows for the calculation of key-rate durations.

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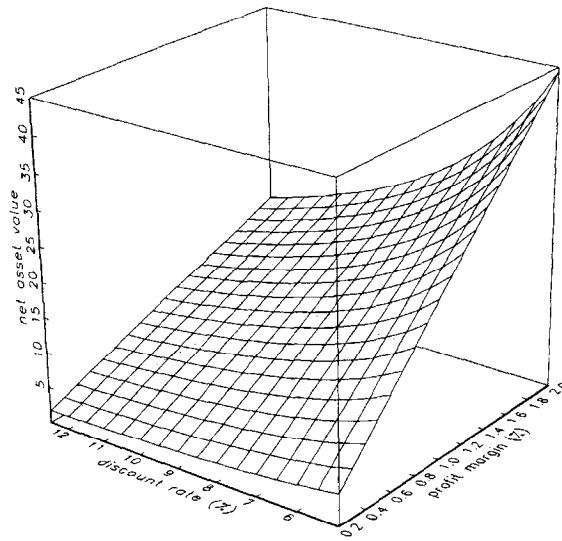
Figure 1: **Net asset value**

This figure shows the net asset value of a deposit of 100, as a function of the discount rate R and the profit margin μ.
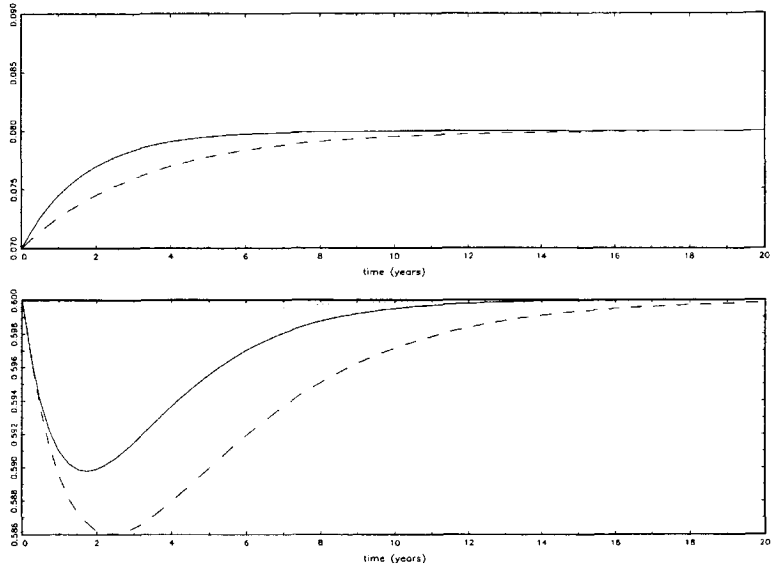
Figure 2: **Adjustment of interest rate and balance of demand deposits**

This figure shows the adjustment of interest rate (top panel) and balance (bottom panel) to a 1% shock in the short rate. The solid line is the base case. The dashed line is for a smaller value of κ, the dotted line for a larger value of λ. The scale of the horizontal axis is years.
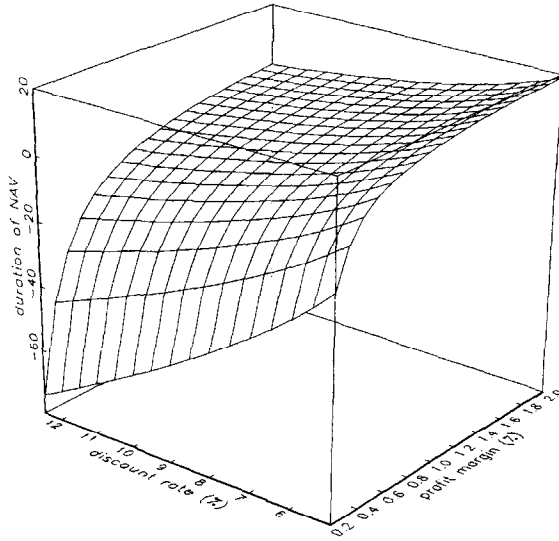
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Figure 3: **Duration of savings deposits**

This figure shows the duration (in years) of savings deposits as a function of the discount rate (R) and the profit margin μ.
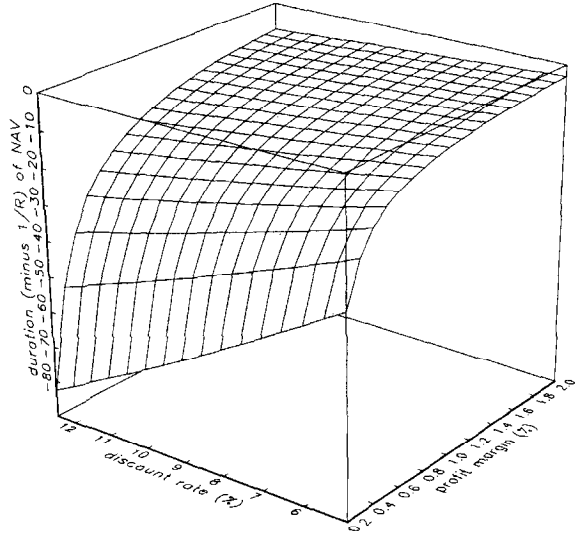
Figure 4: **Duration of savings deposits (excluding profit margin)**

This figure shows the extra duration (in years) of savings deposits, in excess of the duration of a perpetuity $(1/R)$, as a function of the discount rate (R) and the profit margin μ.
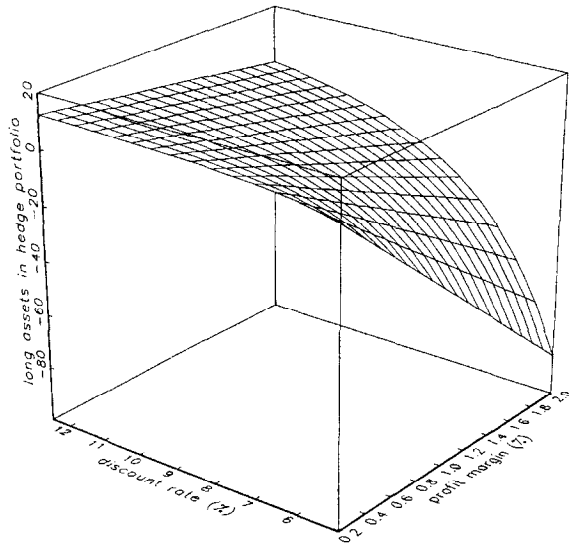
Figure 5: **Hedge portfolio**

This figure shows the position in long bonds (duration 10 years) in the hedge portfolio of a 100 deposit, as a function of the discount rate R and the profit margin μ.