Pollution Abatement Investment When Environmental Regulation Is Uncertain

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Abstract

In a dynamic model of a risk-neutral competitive firm which can lower its pollution emissions per unit of output by building up abatement capital stock, we examine the effect of a higher pollution tax rate on abatement investment both under full certainty and when the timing or the size of the tax increase is uncertain. We show that a higher pollution tax encourages abatement investment *if* it does not exceed a certain threshold rate - a "Laffer-curve" phenomenon. When the size of the tax increase is uncertain, at the time of the tax increase the abatement investment path may shift upward or downward depending on whether the actual tax rate is higher or lower than the firm's expected rate. But, when the time of the tax increase is uncertain, the abatement investment path always jumps upward. Further, the ad hoc practice of raising the discount rate to account for the uncertainty leads to underinvestment in abatement capital. We show how the size of this underinvestment bias varies with the future tax increase. Finally, we show that a credible threat to accelerate the tax increase can induce the firm to undertake more abatement investment.

Keywords: Abatement investment; Pollution tax; Uncertainty

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1. Introduction

A key factor in environmental pollution control is the adoption of more efficient pollution abatement technologies by firms. In turn, the firms' incentive to invest in capital equipment embodying such technologies can be significantly influenced by the specific form of environmental regulation that the government adopts and pursues. In fact, one of the economists' main arguments for preferring the market-based environmental policy instruments (such as tradable pollution permits, emissions taxes and subsidies) over the command-and-control methods (such as performance and technology standards) is that the market-based instruments provide firms with greater incentive to innovate or adopt pollution abatement technology[1]. For example, Millman and Prince's (1989) comparative study of the alternative policy instruments ranked emission taxes superior to standards in encouraging firms to adopt abatement technology. Jung et al (1996) extended this conclusion to the industry level. A similar conclusion also emerged from Jaffe and Stavin's (1995) empirical study of the effect of alternative (price and nonprice-based) instruments on builders's decisions to invest in energy conservation technology as a means of CO_2 emissions control. Likewise, Caswell et al (1990) showed the importance of a drainage effluent charge for adopting more modern water-conserving, and hence less polluting, irrigation technologies.

Although insightful, this literature, however, abstracts from several important aspects of environmental regulation setting and firms' decision making which can greatly affect incentives to invest in abatement technology. First, whereas the literature has often adopted a static framework to analyze the incentive effects of environmental regulations, the decision to invest in abatement technology is inherently a dynamic, and often irreversible, one. Second, there are costs of adjusting abatement capital stock. Third, and perhaps more important, the literature has usually abstracted from environmental policy uncertainties that firms face. Among these are uncertainties about the timing and stringency of environmental regulation. These uncertainties are a common feature of environmental policy making in democratic societies. Under pressure from various interest groups (including environmentalists, industrialists, and the general public), the legislator has to obtain an agreement on the stringency and enactment date of an environmental regulation through a lengthy political bargaining process.

In this paper we allow for these aspects and, by focusing on a specific environmental policy instrument, namely pollution tax, investigate how the uncertainty about the size or timing of a future tax reform may affect a firm's decision to invest in pollution abatement capital. By doing so, we bring together the above cited strand of literature on environmental economics and the strand in public economics literature that explores the effect of tax policy uncertainties on firms' incentive to invest (see Rodrik (1991), Aizenman and Marion (1991), Hassett and Metcalf (1994) and Alvarez et al (1997)

[1] For some of the earlier studies of the firms' incentive to innovate or adopt pollution abatement technology see, e.g., Wenders (1975), Orr (1976), Magat (1978), and Malueg (1989).

among others). Despite taking different technical approaches, these studies generally confirm the conventional wisdom that tax policy uncertainties can adversely affect incentives to invest. For example, using a general density function for the timing of a tax reform and assuming a convex adjustment cost function, Alvarez et al (1997) show that the expectations of a once-and-for-all increase (decrease) in the corporate profits tax rate will have a negative (positive) investment spurt effect and that the uncertainty about the timing of the reform reinforces this effect[2].

In the environmental economics literature, two studies which are close in spirit to our paper are those of Xepapadeas (1992) and Hartl (1992). Xepapadeas considers a dynamic model in which specific inputs (capital and labor) are employed in production and abatement activities. He examines the short and long-run effects of an emission tax and an emission quota on a firm's choice of inputs in the two activities. His model, though insightful, differs from ours both by abstracting from policy uncertainty issues and by assuming that the firm's choice of abatement capital bears no effect on its demand for productive inputs and hence on its output. As a consequence of the latter, he concludes, contrary to a finding of this paper, that an increase in the emission tax *always* leads to a larger stock of abatement capital. Hartl analyzes the investment decision of a firm that has to acquire a certain stock of abatement capital by a future date in order to be in compliance with government pollution standard, but that the firm neither knows the required stock of abatement capital nor the compliance date in advance. He shows that while, in the absence of any uncertainty, the firm's optimal policy is to increase abatement investment rates monotonically over time until the target abatement capital stock is reached (i.e. a delayed investment policy), when the target stock and the compliance date are uncertain, the optimal investment path is U-shaped. Our paper, however, differs also from the Hartl's in several important respects. (i) In contrast to Hartl's model, which ignores the firm's production activity, our model explicitly incorporates this, thereby allowing for the firm's option of reducing pollution emissions by contracting output level. (ii) Whereas Hartl's paper is concerned with minimizing the investment costs of complying with an expected target abatement capital stock at a finite (but uncertain) future date, we consider the case of emissions tax and let the firm choose its desired abatement capital stock, and hence investment path, in order to maximize the present discounted value of the expected cash flow before and after the tax reform (i.e., over infinite horizon). (iii) As another distinctive feature of our model, and a principal source of a firm's incentive to invest in abatement capital, we allow the emissions per unit of output to diminish as the firm accumulates larger stocks of abatement capital. (iv) We analyze the effects of two separate uncertainties on abatement investment: uncertainty about the *size* of a tax increase at a certain future date and

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[2] The reason for the firm to speed up investment on the expectation of a future cut in the tax rate is the assumption of the immediate write off-of the adjustment costs, and thus the firm's incentive to take advantage of the tax shield before the tax rate cut becomes effective.

uncertainty about the *timing* of a known tax increase.

In this setting, a main result of our paper, which has an important implication for the design of emissions taxes is that expectations of a once-and-for-all increase in the future emissions tax rate will result in higher (lower) abatement investment rates depending on whether the expected future tax rate remains below (above) a certain critical level. We show that this "Laffer-curve" phenomenon derives from two opposing effects of a higher pollution tax rate on incentive to invest in abatement capital: a positive tax rate effect and a negative production effect

The paper proceeds as follows. As a foundation for subsequent analysis, in Section 2 we present the basic model which considers the simplest case where the emission tax is constant over the entire planning horizon. Building on this, in section 3 we first examine the deterministic case of a known increase in the tax rate at a know future date, and then proceed to the case where the size of the future tax increase is uncertain but its timing is certain. Subject to the threshold tax rate, we show that the expectation of a higher tax rate motivates the firm to increase its abatement investment rates at all times before the date of the tax increase. But at the time of the tax increase, the investment path can shift upward or downward depending on whether the actual tax rate is higher or lower than the firm's expected rate. Section 4 analyzes the variant where the timing of the tax increase is uncertain. There, we show that, again, subject to the threshold tax rate, the expectation of a higher tax rate at some future date gives the firm an incentive to boost its abatement investment, but that the timing uncertainty causes an unanticipated jump in abatement investment at the time of the tax increase. We show that a credible threat of accelerating the tax increase can further boost the firm's abatement investment. Further, we establish that no certainty-equivalent discount rate exists and that raising the discount rate instead of accounting explicitly for the uncertainty results in underinvestment in abatement capital. We show how the size of this underinvestment bias varies with the future tax increase. Section 5 concludes.

2. Basic Model: Constant Emission Tax Rate

We begin with the analysis of the simplest case where the emission tax rate is known and remains constant over time. The results obtained for this case will considerably facilitate the analysis of uncertain emission tax policy in subsequent sections.[3]

Consider a perfectly competitive firm which uses a variable input v (e.g., labor) to produce a homogenous good according to the simple production function[4]

$$h = h(v), \text{ where } h(0)=0,\ h'(v)>0,\ h''(v)<0 \tag{1}$$

[3] The deterministic analysis in this section somewhat resembles that by Hartle and Kort (1996) for the case of full certainty and investment in both productive and abatement capital stocks.

[4] Time subscripts are supressed when no confusion arises.

The production process generates pollution emissions proportional to output level, i.e.

$$E = \alpha\, h(v) \tag{2}$$

It is assumed that the firm can reduce its emissions per output unit, α, by building up a larger stock of abatement equipment, *K,* although this possibility is subject to diminishing returns*,* i.e.

$$\alpha = \alpha(K) > 0 \text{, where } \alpha'(K) < 0,\ \alpha''(K) > 0 \tag{3}$$

This can be justified by noting simply that a larger stock of abatement capital allows the firm to abate emissions more efficiently. Another justification could be the possibility of “learning by doing” as abatement investment accumulates. It is also both realistic, in view of the second law of thermodynamics, and analytically convenient, to assume that pollution emissions can not be entirely eliminated; so that

$$\lim_{K \to \infty} \alpha(K) = 0 \tag{4}$$

Let $\tau > 0$ be the tax per unit of emissions, which in this section is assumed to be constant over time. Then, the firm’s total pollution tax payment at any point in time is $\tau\, \alpha(K)\, h(v)$.

The abatement capital stock can be increased by undertaking investments, I, in abatement equipment. To reflect investment adjustment costs (e.g., costs of equipment installation and personnel training), and the fact that such costs increase rapidly as the stock build up is accelerated, the total investment cost, *C(I)*, is assumed to be a convex, increasing function of the investment level, i.e.

$$C(0) = 0,\ C'(I) > 0,\ C''(I) > 0 \tag{5}$$

It is also assumed that the investment in abatement equipment is irreversible, so that $I \geq 0$. Without new investment, the capital stock is assumed to depreciate at the constant proportional rate of δ. The firm takes as given and fixed the market prices of output, *p,* and the variable input, *w.* For analytical convenience, we assume that production remains profitable even in the absence of abatement capital stock, so that

$$\{p - \tau\alpha(0)\}\, h'(0) > w \tag{6}$$

which, by (1) and (3), implies that $p - \tau\alpha(K) > 0$ for all $K \geq 0$, i.e., the price always exceeds the emissions tax per unit of *output*. The risk-neutral firm chooses the level of its variable input, *v*, and abatement investment, *I*, to maximize the present value of its cash flow over an infinite planning horizon. Formally,

$$\max_{I,v} \int_0^\infty e^{-rt} \{p\, h(v) - w\, v - \tau\, \alpha(K)\, h(v) - C(I)\}\, dt \tag{7}$$

$$\text{s.t.} \quad \dot{K} = I - \delta K, \ with \quad K(0) = K_0\ (given) \tag{8}$$

where *r* is the firm’s constant discount rate.

Maximization of (7) w.r.t. *v* yields the optimal *v* as an implicit function of *K*, i.e., $v = v(K)$, such that

$$\{p - \tau\alpha(K)\}\, h'(v) = w \tag{9}$$

that is, for a given K, the variable input is employed up to the level where its after-tax marginal product revenue equals its marginal (=unit) cost. From (9) we obtain

$$v'(K) = \frac{\tau\ \alpha'(K)\ h'(v)}{\{p - \tau\ \alpha(K)\} h''(v)} > 0 \qquad (10)$$

Intuitively, by reducing the emissions cost per unit of output, a larger K makes production more profitable and thus increases the employment of the variable input. Substituting $v(K)$ from (9), we can rewrite the objective function (7) as

$$\underset{I}{Max} \int_0^{\infty} \{p\ h(v(K)) - w\ v(K) - \tau\ \alpha(K)\ h(v(K)) - C(I)\}\ e^{-rt}\ dt \qquad (11)$$

The current-value Hamiltonian of the optimal control problem ((11) and (8)) is

$$H = \{p - \tau\ \alpha(K)\}\ h(v(K)) - w\ v(K) - C(I) + \lambda\ (I - \delta K) \qquad (12)$$

where λ is the shadow price of abatement capital. The necessary conditions for an optimal policy, using (9), are:

$$\lambda = C'(I), \qquad (13)$$

$$\dot{\lambda} = (r + \delta)\ \lambda + \tau\ \alpha'(K)\ h(v(K)) \qquad (14)$$

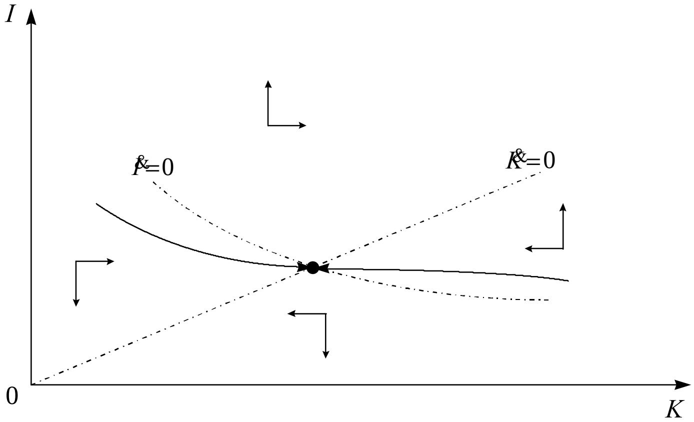
with the straightforward economic interpretation that along the optimal investment policy, the shadow price of abatement investment must always equal its marginal cost (equation (13)), and that the competitive return on a unit of abatement capital should be the same whether accrued in the form of net capital gains $[\dot{\lambda} - (r+\delta)\lambda]$ or reduced emissions tax payments (equation(14)). We can obtain further insight into the characteristics of the optimal investment policy by analyzing its phase diagram in the *(K, I)* plane. The $\dot{K}=0$ isocline is the positively-sloped straight line $I=*K$. To derive a differential equation for I, differentiate (13) with respect to t, substitute in (14), and eliminate λ to get:

$$\dot{I} = \frac{1}{C''(I)} \{(r + \delta)\ C'(I) + \tau\ \alpha'(K)\ h(v(K))\} \qquad (15)$$

Using (10), the slope of the $\dot{I}=0$ isocline is, from (15)

$$\left.\frac{dI}{dK}\right|_{\dot{I}=0} = \frac{-\tau\{p\ h''\ \alpha''\ h - \tau\ \alpha\ \alpha''\ h\ h'' + \tau(\alpha')^2\ (h')^2\}}{(r + \delta)\ C''\{p - \tau\ \alpha\}\ h''} \qquad (16)$$

from which it can readily be verified that the $\dot{I}=0$ isocline is globally decreasing for $\tau < \tau^*$, where τ^* is given by



$$\frac{p\ h''\ \alpha''\ h}{\chi''\ h\ h'' - (\alpha')^2\ (h}$$ (17)

Thus, as long as $\tau < \tau^*$, the system converges to a unique saddle-point steady state $(\dot{K}=\dot{I}=\dot{\lambda}=0)$. When $J > J^*$, it can readily be shown that the $\dot{I}=0$ isocline is positively sloped and convex in K, so that it cuts the straight line $I=*K$ from below, implying an unstable steady-state [5]. To focus on the effects of uncertainties about future emissions tax rate on the firm's investment behavior, in the analysis that follows we concentrate on the case where $J < J^*$, with a unique saddle point steady-state, K^*, which (from (15) and $I=*K$) is given by

$$-\ \tau\ \alpha'(K^*)\ h(v(K^*)) = (r + \delta)\ C'(\delta K^*) \qquad (18)$$

Figure 1 presents the phase diagram of the optimal solution for this case.

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[5] Note that the marginal return on abatement capital is the emissions tax reduction, -J "N (K) hN(v). It can be verified that for $\tau > \tau^*$, the derivative of this term with respect to K is positive, implying that the total return on abatement capital is a convex function of K.

Figure 1. Optimal solution when J is constant over time

Further, we can determine the optimal investment path and therefore characterize its behavior towards the steady-state equilibrium. To do this, simply solve the differential equation (14) and use (13), to obtain[6]

$$C'(I(t)) = \lambda = -\int_t^{\infty} \tau \ \alpha'(K(s)) \ h(v(K(s))) \ e^{-(r+\delta)(s-t)} \ ds \qquad (19)$$

This has a simple economic interpretation. Correcting the discount rate for the capital depreciation rate, the expression on the RHS of (19) presents the shadow price of abatement capital, which in turn is the discounted value of the stream of benefits (in terms of reductions in emission tax payments) resulting from an additional unit of abatement investment at time *t*. Along the optimal investment path, this shadow price should always equal the marginal cost of investment. As Figure 1 shows, when the firm's initial abatement stock K_0 is small, the optimal investment policy starts from a relatively high investment rate *(I>* * K_0) and monotonically falls over time until it reaches its steady-state level *(I^*=* * K^*). The reverse holds for the optimal policy when K_0 is relatively large.

3. An Increase of Unknown Size in the Future Emissions Tax Rate

In this section we consider the case where the firm believes that at a known future date, *T,* the government will raise the emissions tax rate, but it is uncertain about the exact magnitude of the tax increase. We, however, begin with the simpler case where the tax rate after the increase is known beforehand, and then use the result for this case to facilitate the analysis for the case of uncertainty. Let us suppose that at time *T* the tax rate will increase from its current level of J_L to J_H $(>J_L)$ and remains at J_H thereafter. The problem facing the firm is

$$\max_{I,v} \int_0^{\infty} e^{-rt} \left\{p\,h(v) - w\ v - C(I) - \tau(t)\ \alpha(K)\ h(v)\right\} dt \qquad (20)$$

$$s.t. \qquad \dot{K} = I - \delta\, K, \quad K(0) = K_0 \qquad (21)$$

$$\tau(t) = \tau_L \ \text{for} \ t \le T \qquad (22)$$

$$\tau(t) = \tau_H \ \text{for} \ t > T \qquad (23)$$

From the analysis in the previous section we know that the optimal level of the variable input *v*

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[6] In deriving equation (19) we have used the transversality condition $\lim_{t\to\infty} \lambda\, e^{-rt} = 0$.

can be determined by solving a static optimization problem, yielding $v = v(K)$. For the present problem, the relevant tax rate for calculating the optimal v is J_L for $t\#T$ and J_H for $t>T$, and the optimal v is determined from $v = v_{\tau_L}(K)$ and $v = v_{\tau_H}(K)$ respectively for $t\#T$ and $t>T$, where, as in (9), $v_{\tau_L}(K)$ satisfies

$$\{p - \tau_L \, \alpha(K)\} \, h'(v_{\tau_L}) = w \tag{24}$$

and $v_{\tau_H}(K)$ satisfies

$$\{p - \tau_H \, \alpha(K)\} \, h'\left(v_{\tau_H}\right) = w \tag{25}$$

Comparing (24) with (25) and recalling that $h0<0$, we have, for *every K*,

$$v_{\tau_L}(K) > v_{\tau_H}(K) \tag{26}$$

which is to be expected since a higher emissions tax rate makes production less profitable and hence reduces the employment of the variable input.

To characterize the optimal investment policy, we note that for the period $[T,4)$ the firm needs to solve the same optimal control problem as in Section 2, but now with $\tau = \tau_H$. So, the results of Section 2 carry over here. For the period $(0,T]$, we note that in the phase diagram of Figure 1 the $\dot{K}=0$ isocline remains the same $(I=\delta K)$ as before, but the $\dot{I}=0$ isocline changes according to the following proposition:

Proposition *1*: *In the (I, K)-plane the* $\dot{I}=0$ *isocline shifts upwards when the emissions tax rate is raised provided that the increased tax rate satisfies the inequality :*

$$\tau_H \le \min\left(\tau^*, \hat{\tau}\right) \tag{27}$$

where τ^* *and* $\hat{\tau}$ *are given by*

$$\tau^* = \min_K \left[\left\{ \frac{p\, h''\, \alpha''\, h}{\alpha\, \alpha''\, h''\, h - (\alpha')^2\, (h')^2} \bigg|_{v=v_{\tau_L}(K)} \right\}, \left\{ \frac{p\, h''\, \alpha''\, h}{\alpha\, \alpha''\, h\, h'' - (\alpha')^2\, (h')^2} \bigg|_{v=v_{\tau_H}(K)} \right\} \right] \tag{28}$$

$$\hat{\tau} = \min_K \left[\{\hat{\tau}(K) | v=v_{\tau_L}(K)\} , \{\hat{\tau}(K) | v=v_{\tau_H}(K)\} \right] \tag{29}$$

and where $\hat{\tau}(K)$ *is given by*

$$\hat{\tau}(K) = \frac{p h'' h}{\alpha \, \{h'' h - (h')^2\}} \tag{30}$$

Proof. See Appendix A

Remark *1*. In the special case where production function is of constant elasticity, *i.e.* *h(v)*= v^a (*0<"<1*), and the emissions-output ratio declines exponentially, *i.e.* *"(K)="*$_0$ $e^{-b(K-K_0)}$, where *"*$_0$*="(K*$_0$*)>0* , we obtain

$$\tau^* = \hat{\tau} = \frac{p(1-a)}{\alpha_0} \qquad (31)$$

and

$$\hat{\tau}(K) = \frac{p(1-a)}{\alpha(K)} = \frac{p(1-a)}{\alpha_0} e^{b(K-K_0)} \qquad (32)$$

So that, as long as *J*$_H$ ≤ *p(1-a)/"*$_0$, there exists a unique saddle point equilibrium and an upward shift of the $\dot{I}=0$ isocline associated with an increase in the tax rate.

To provide intuition for condition (27), we recall from the analysis of the previous section that for a unique saddle-point steady state to exist the (increased) tax rate should not exceed the critical value of *J** . Economic interpretation of $\hat{\tau}$ is facilitated by examining how, for any given *K,* total emission tax payment, $\tau\ \alpha(K)\ h(v)$, varies with the level of the tax rate. We thus have

$$\frac{d(\tau\ \alpha(K)\ h(v))}{d\ \tau} = \alpha\ h + \tau\ \alpha\ h'\ \frac{dv}{d\tau} \qquad (33)$$

From (9) we also have

$$\frac{dv}{d\tau} = \frac{\alpha h'}{(p-\tau\alpha)h''} < 0 \qquad (34)$$

implying that a higher emission tax rate will, *ceteris paribus*, reduce the demand for the variable input, as should be expected. Substituting (34) into (33) gives

$$\frac{d(\tau\alpha h)}{d\tau} = \alpha h + \frac{\tau\alpha^2(h')^2}{(p-\tau\alpha)h''} \qquad (35)$$

According to (35) an increase in the emission tax rate has two opposing effects. On the one hand, as the first term on the RHS of (35) indicates, for a given output level, it increases the firm’s emission tax payment. We may term this positive effect as the marginal *tax rate effect*. On the other hand, as the second term shows, a higher emission tax rate reduces the firm’s profits and thus leads to a reduction in its output, and hence emissions, level. The consequent reduction in emissions in turn reduces the firm’s emissions tax payment- of course as long as the tax rate is not raised to so high a level as to render production unprofitable (i.e., as long as *p-J">0*). We may term this negative effect as the marginal *production effect* (see also Carraro and Soubeyran (1995)). Thus, whether the firm's emissions tax payment increases or not depends on which of these two effects dominates. It is noted from the second term in (35) that, as long as production remains profitable, the negative production effect of a higher tax rate will be larger the larger is the tax rate.[7] In fact, it is easy to verify from (35) that emission tax payment increases with τ as long as $\tau<\hat{\tau}(K)$, where $\hat{\tau}(K)$ is given by (30). For $\tau>\hat{\tau}(K)$, the production effect dominates and therefore emission tax payment decreases with τ. Thus, $\hat{\tau}(K)$ is the tax rate at which the Laffer curve attains its maximum.

Remark 2. In the special case of Remark 1, when $h(v)= v^a$ $(0<"<1)$ and $"(K)="_0\, e^{-b(K-K_0)}$, the Laffer curve function has the following form and properties. Using (9), the emission tax payment, $'$, is

$$' = J\,"(K)\;\; v^{a} = J\,"_0\;\; e^{-b(K-K_0)}\,[a\,(p - J\,"\,)/\,w]^{\,a/1-a}$$

from which, and using (34), we obtain

$$\partial\,'\,/MJ = "\,v^{a}\,[1- J"\,a/\,(1-a)(\,p- J")]$$

and

$$M^{2}\,'\,/MJ^{2} = -\,"^{2}\,a\,v^{a}\,[\,2\,(1-a)\,p - "\,J]/\,(1-a)^{2}\,(p-"\,J)^{2}$$

It is then easy to check that for $\hat{\tau}=\dfrac{(1-a)p}{\alpha}$ we have $M'\,/MJ=0$ and $M^{2}\,'\,/MJ^{2}<0$, implying that æ a O $(0, 1)$ and any given $K=\bar{K}>0$, and hence $"\,(\bar{K})>0$, the Laffer curve attains its maximum,

$$'_{max}= (1-a)\;(a^{2}/w)^{\,a/1-a}\,p^{\,1/1-a} \;\text{ at }\; \hat{\tau}=\frac{(1-a)p}{\alpha}.$$

It is worth noting that (I) the condition $J_H \leq \hat{\tau} = p(1-a)/"$ for a tax increase to lead to a larger steady-state abatement capital stock, and hence, investment rate is more likely to hold the smaller is the firm's output elasticity, a, and/or the cleaner is the firm initially (i.e. the smaller is $"_0$), for in either case the negative production effect of a tax increase will be relatively small; (ii) whereas $\hat{\tau}$ depends on $"$, and hence on K, the value of $'_{max}$ is independent of K; (iii) the Laffer curve will be concave throughout if a #½, but for a >½ will have an inflection point at $\tau = 2(1-a)/"$. For a = ½, the Laffer curve is a

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[7] It is remembered that while in the short run too high an emissions tax rate may render some firms uncompetative and hence drive them out of the market, in the long run it is passed onto consumers and the equilibrium will be restored.

parabola given by ' = "(pJ-"J^2)/2w, with $\hat{\tau}$ = p/2" and '$_{max}$ = $p^2/8w$. Figure 2 depicts the Laffer curve in the special case for various values of *a* and the parameter values of *p=1.5* , *w=1*, and *"=0.75* .

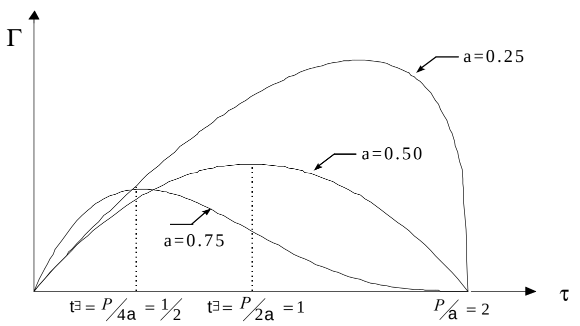


Figure 2. The Laffer curve for different values of a in the special case

In (33) - (35) capital stock is held constant. When the tax rate increases from τ_L to τ_H, and $\tau_H<\hat{\tau}$, the total emission tax payment goes up. This in turn raises the pay off from pollution abatement and hence gives the firm more incentive to invest in abatement capital stock, thus Proposition 1. On the other hand, if $\tau_L>\hat{\tau}(K)$ and τ increases from τ_L to τ_H, then the production effect dominates for this level of K . In this case, the firm reduces production to such an extent that the emission tax payment decreases too. This, of course, has a negative effect on abatement investment.[8] If $\tau_L<\hat{\tau}(K)<\tau_H<\tau^*$, then whether the emission tax payment increases or decreases depends

[8] It is important to note that $\hat{\tau}(K)$ changes with *K*. So, while initially $\hat{\tau}(K)<\tau_L$, it may very well be the case that at a later point in time $\hat{\tau}(K)>\tau_L$ so that then the tax effect will dominate the production effect. Since this will have a positive effect on subsequent abatement investments, here nothing defenite can be concluded about the effect of a higher tax rate on abatement investment.

on the shape of the Laffer curve and the size of the tax rate increase.[9]

Interestingly, from (35) it is seen that generally the negative production effect of a higher emissions tax always dominates when the production function *h(v)* is almost linear, i.e. when *h0(v)* is close to zero in magnitude (see also (31) where in the special case $\hat{\tau}\to 0$ as *a61*). In such cases, output is sufficiently elastic so that, faced with a higher emissions tax rate, the firm finds it more profitable to reduce emissions by reducing output rather than investing in abatement capital. The reverse is true when output is very inelastic (i.e. the marginal productivity of the variable input is very low, *hN(v)*→0, or, for the special case, *a60*). This theoretical insight has a valuable implication for the design of environmental policy. Namely, when the primary goal of environmental policy is to encourage firms to invest in pollution abatement capital, then raising the emission tax rate to achieve this is, *ceteris paribus*, more likely to succeed if firms' output elasticity is low and/or the tax rate is not raised to an unduly high level. On the other hand, if the output elasticity is large and the government is particularly concerned about the level of employment, and hence production, and at the same time is under pressure from environmentalist groups to control pollution, then combining a relatively small pollution tax rate with a subsidy on abatement investment may be a more desirable policy than resorting to a large increase in emission tax rate.

From Proposition 1 we conclude that the $\dot{I}=0$ isocline shifts upwards under condition (27). Consequently, for T sufficiently large, the optimal trajectory starts out near the saddle point path for the case where $\tau(t)=\tau_L$, $\forall\ t\geq 0$. After a while it deviates from this path because it is known that at time $t=T$ the higher tax rate τ_H becomes effective. Therefore, the firm will invest more in abatement capital than it would when $\tau(t)=\tau_L$, $\forall\ t\geq 0$. At time T, the optimal trajectory passes into the saddle point path for the case where $\tau(t)=\tau_H$, $\forall\ t\geq 0$. For a small initial value of K, the optimal trajectory is depicted in Figure 3.a.

[9] It should be noted that in general the regulator's optimal tax rate differes from (and most likely exceeds) $\hat{\tau}(K)$.

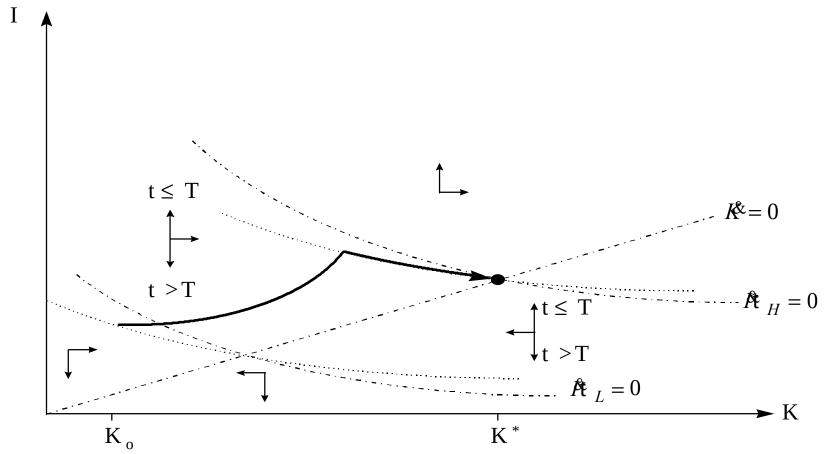


Figure 3.a. Optimal trajectory when K_0 is small and the emissions tax rate satisfies condition (27)

It is important to note that the shape of the optimal trajectory, and hence of the time path of optimal investment, depends on the initial value of abatement capital stock. For example, for a sufficiently large K_0 ($>K^*$) , it starts on the lower saddle point path corresponding to J_L , with K decreasing initially and then increasing as T becomes nearer. Again, at time T, the trajectory will pass into the higher saddle point path corresponding to J_H . The time paths of optimal investment corresponding to different values of K_0 are presented in Figure 3.b.

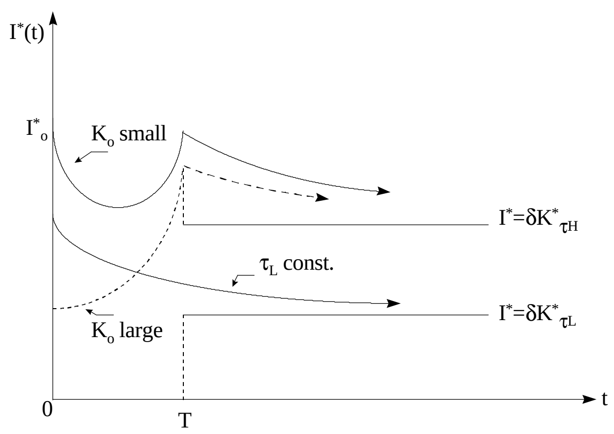


Figure 3.b. The time paths of optimal investment for different values of K_0

Analogous to the case analyzed in section 2, the investment optimality condition that the marginal cost of investment should equal the shadow price of abatement investment, is: for $t\#T$,

$$C'(I(t)) = -\int_t^T \tau_L \ \alpha'(K(s)) \ h(v_{\tau_L}(K(s))) \ e^{-(r+\delta)(s-t)} \ ds - \int_T^\infty \tau_H \ \alpha'(K(s)) \ h(v_{\tau_H}(K(s))) \ e^{-(r+\delta)(s-t)} \ ds \tag{36}$$

and for *t>T*,

$$C'(I(t)) = -\int_t^\infty \tau_H \ \alpha'(K(s)) \ h(v_{\tau_H}(K(s))) \ e^{-(r+\delta)(s-t)} \ ds \tag{37}$$

Two points can be seen immediately from equation (36). First, in determining its current investment rate, the firm already takes into account the future increase in the tax rate (i.e., it adjusts its shadow price of abatement capital) even though the tax rate will remain at its low level until time *T*. Second, the optimal trajectory approaches the saddle point path of the high tax rate as time *T* draws nearer. This implies that the sooner the emissions tax rate is increased (i.e. the smaller is *T*), the higher will be the shadow price of abatement capital, and therefore the greater will be the optimal abatement investment rates at any time before *T*. In other words, a quicker emissions tax increase shifts up the time path of abatement investment before time *T*.

Bearing this analysis in mind, let us now examine the situation where the firm knows that the current low tax rate J_L will be raised at time *T*, but does not know precisely by how much. We assume that the only information that the firm has about the size of the increase, *u,* is that it is uniformly distributed over the interval ($0 \ , \ 2 \ (J_H - J_L)$), so that

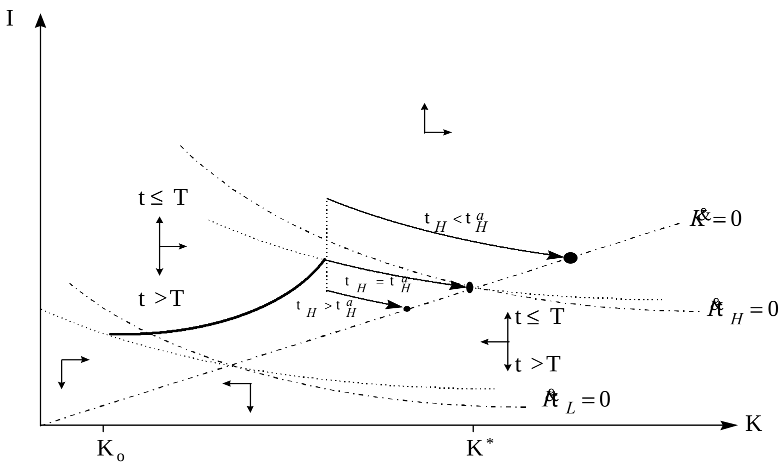
$$\tau(t) \ = \ \tau_L \ + \ u \quad , \ \forall t>T \tag{38}$$

which implies that

$$E(\tau(t)) \ = \ \tau_H \ , \ \forall t>T \tag{39}$$

Since we assume that the firm's objective is to maximize the present value of its cash flow, then after time *T* the firm's optimal investment policy will be the same as in the case where J_H is known because in determining its investment rate it takes into account that the expected tax rate after time *T* is also J_H. At time *T*, the actual new tax rate becomes known to the firm. So, the effect of uncertainty on the firm’s abatement investment path depends on whether the actual new tax rate is higher or lower than the firm’s expected rate, J_H . If the actual new tax rate exceeds J_H , then, still under the assumption that the new tax rate satisfies (27), the firm will react with an upward jump in the optimal investment path. This jump is from the saddle point path for $E(J(t)) = J_H$ to the saddle point path corresponding to the higher actual new tax rate, and is upward due to Proposition 1. In this case, for all $t>T$, the optimal investment rate will be higher than when the firm knows with certainty that the future tax rate is going to be J_H. The economic intuition is simple: since the emissions tax rate turns out to be higher than initially expected, so does the pay offs from pollution reductions, and therefore the incentive for the firm to adjust the time path of its abatement investment upwards. The opposite will be the case if the actual new tax rate turns out to be lower than the firm's expected rate. In that case, for $t>T$, the firm adjusts its abatement investment rates downward. Figure 4 presents the optimal trajectories for these cases, where J^a_H denotes the actual new tax rate and J_H is the firm's expected new tax rate.

Figure 4. Comparison of the optimal trajectories when J_H is certain and when it is not



4. Unknown Timing of the Tax Increase

In this section we analyze the case where the firm knows with certainty that the emissions tax rate will be increased from J_L to J_H, but does not know beforehand the precise time T at which this will occur.[10] Again, v and I are the firm's choice variables. As in the previous section, the optimal choice of v depends only on the current tax rate; so, it is given by (24) and (25) respectively before and after the tax increase.

In determining its optimal path of I, the firm takes into account that while currently the low emissions tax rate, J_L, is in force, there is a positive probability that at any future date this rate will increase to J_H, thereby increasing the pay off from abatement investments. To see how the firm should optimally adjust its investment decisions for the timing uncertainty, let generally $T(T)$ be the firm's

[10] This problem resembles those of optimal saving under uncertain lifetime (Yaari(1965)) and optimal resource extraction in presence of uncertainty about the date of discovery of a backstop substitute (Dasguptal and Heal (1974)), or of expropriation of an open-ended foreign concession (Heal (1975)), or of a resource embargo (Farzin (1982)).

subjective probability density function of the random date, *T0(0, 4)*, when the tax increase will be enacted. Let also *PVB(T)* denote the present value of the cash flow when the tax increase occurs at a date *T*, i.e.

$$PV\pi(T) = \int_0^T \pi_{\tau_L}(K)\, e^{-rt} dt + \int_T^\infty \pi_{\tau_H}(K)\, e^{-rt} dt \qquad (40)$$

where $\pi_{\tau_L}(K)$ and $\pi_{\tau_H}(K)$ are the cash flows before and after the tax increase, i.e.

$$\pi_{\tau_L}(K) \equiv p\ h\ (v_{\tau_L}(K))\ -\ w\ v_{\tau_L}(K)\ -\ \tau_L\ \alpha(K)\ h\ (v_{\tau_L}(K))\ -\ C(I) \qquad (41.a)$$

$$\pi_{\tau_H}(K) \equiv p\ h\ (v_{\tau_H}(K))\ -\ w\ v_{\tau_H}(K)\ -\ \tau_H\ \alpha(K)\ h\ (v_{\tau_H}(K))\ -\ C(I) \qquad (41.b)$$

Then, the expected present value of profits is

$$E(PV\pi) = \int_0^\infty \omega(T)\, pv\pi(T)\, dT = \int_0^\infty \omega(T) [\int_0^T \pi_{\tau_L}(K)\, e^{-rt}\, dt + \int_T^\infty \pi_{\tau_H}(K)\, e^{-rt}]\, dT \qquad (42)$$

Defining by $\Omega(t) \equiv \int_t^\infty \omega(s)\, ds$ the probability that the tax increase occurs *after* a date *t0[0, 4),* we have

T(t) = - $\dot{\Omega}(t)$. Using this and integrating by parts, we can rewrite (42) as

$$E(PV\pi) = \int_0^\infty [\Omega(t)\ \pi_{\tau_L}(K) + (1 - \Omega(t))\ \pi_{\tau_H}(K)]\, e^{-rt}\, dt \qquad (43)$$

So, as long as the tax increase remains uncertain, the firm’s problem will be to choose the abatement investment path, *I*, to

$$\max_{I} \quad E(PV\pi) = \int_0^\infty [\Omega(t)\ \pi_{\tau_L}(K) + (1 - \Omega(t))\ \pi_{\tau_H}(K)]\, e^{-rt}\, dt \qquad (44)$$

$$s.t. \quad (8)$$

The current-value Hamiltonian of (44) is

$$H = \Omega(t)\, \pi_{\tau_L}(K) + (1-\Omega(t))\, \pi_{\tau_H}(K) + \lambda_U (I - \delta K)$$

giving the necessary optimality conditions

$$\lambda_u = C'(I) \tag{45.a}$$

$$\dot{\lambda}_u = (r+\delta)\ \lambda_u + \Omega(t)\ \tau_L\ \alpha'(K)\ h(v_{\tau_L}(K)) + (1-\Omega(t))\ \tau_H\ \alpha'(K)\ h(v_{\tau_H}(K)) \tag{45.b}$$

It is important to note that the investment policy characterized by (45.a) and (45.b) is a conditional optimal policy, conditional upon the tax increase not having been yet effected. Denoting by $K_u^*(t)$ the optimal steady-state abatement capital stock conditional upon the tax increase not having been enacted before a date $t0[0, 4)$, we can set $\dot{\lambda}_u = \dot{I} = \dot{K} = 0$ and use (45.a), (45.b) and (8) to obtain $K_u^*(t)$

from

$$C'(\delta K_u^*) = -\ [\ \Omega(t)\ \tau_L\ \alpha'(K_u^*)\ h(v_{\tau_L}(K_u^*)) + (1-\Omega(t))\ \tau_H\ \alpha'(K_u^*)\ h(v_{\tau_H}(K_u^*))\ ]/(r+\delta) \tag{46}$$

Also, solving (45.b) for λ_u, using the transversality condition $\lim_{t\to\infty} \lambda_u\ K_u^*\ e^{-rt} = 0$ and (45.a), gives

$$C'(I(t)) = \lambda_u = -\int_t^{\infty} e^{-(r+\delta)(s-t)} \left\{ \Omega(s)\ \alpha'(K(s))\ \tau_L\, h(v_{\tau_L}(K(s))) + (1-\Omega(s))\ \alpha'(K(s))\ \tau_H\, h(v_{\tau_H}(K(s))) \right\} ds \tag{47}$$

This gives the optimal investment policy which is to be followed as long as the timing of the tax increase remains uncertain, i.e. until the date when the emission tax is raised. Notice that the uncertainty about the timing of the tax increase calls for the shadow price of the abatement capital to be appropriately adjusted from its value in the certainty case. Specifically, it requires that the tax savings flow under full certainty case be replaced by the *expected* flow presented by the bracketed term on the RHS of (47), where here the probability weights are respectively $\Omega(t) \equiv \int_t^{\infty} \omega(s)\, ds$, the probability that the tax increase does not occur before t, and $1\text{-}S(t)$, the probability that it does.

We are now in a position to state the following Lemma.

Lemma: *If J_H satisfies condition (27), then for any time interval [0,t) during which the tax increase remains uncertain, the optimal abatement stock just before the higher tax rate becomes effective (i.e.,*

$K_u^*(t) \equiv \lim_{s \rightarrow^- t} K_u(s)$ *) increases with t and asymptotically approaches the steady-state level* K_H^* *which would prevail under the certainty case with* $J = J_H$ *. Correspondingly, the optimal abatement investment rate at that time (i.e.,* $I_u^*(t) \equiv \lim_{s \rightarrow^- t} I_u(s)$ *) also increases with t and asymptotically approaches the steady-state rate* δK_H^* *of the certainty case.*

Proof. See Appendix B

The economic interpretation of the Lemma is straightforward. The case where the tax rate is known to remain constant at the low rate, $J_{L,}$ corresponds to the extreme situation where *S(t)=1* for all *t0[0, 4)* , *i.e.*, when it is certain that a tax increase never occurs in future. In this case, (46) reduces to (18) with $J = J_L$ and the corresponding steady-state abatement stock of K_L^*. Similarly, the case where the tax rate is known to remain constant at the high rate, J_H, corresponds to *S(t)=0* for all *t0[0, 4)*, and the corresponding steady-state abatement stock, K_H^*, is the solution to (18) for $J = J_{H}$,, where $K_H^* > K_L^*$. We also noted previously that when the timing of the tax increase, *T*, is known with certainty, the steady-state abatement stock makes rises from K_L^* to K_H^*. In the present situation, however, there is a probability of 1-*S(t)>0* that before any time *t* the tax rate will be raised to J_H, and hence that the future payoff from abatement investment will be higher. So, depending on its belief about this probability, the firm optimally adjusts its investment policy during the period *[0, t)* so that by the end of the uncertainty period, *i.e.*, just before the tax rate is actually increased, the optimal abatement capital stock, $K_u^*(t)$, will have reached a level between K_L^* and K_H^*. Since the probability of the tax increase rises with *t*, it is then clear that the optimal abatement stock at the end of uncertainty period will be larger the greater is *t*.

It is important to note that along the optimal policy, the conditional steady-state $K_u^*(t)$ may never realize because the tax increase may occur before $K_u^*(t)$ is reached. An immediate implication of the Lemma is that

Corollary: *When* J_H *satisfies condition (27), then the optimal abatement investment path associated with an uncertainty period (0,t) will be higher the larger is the value of t, (i.e. the longer is the period of uncertainty).*

Proof: See Appendix C.

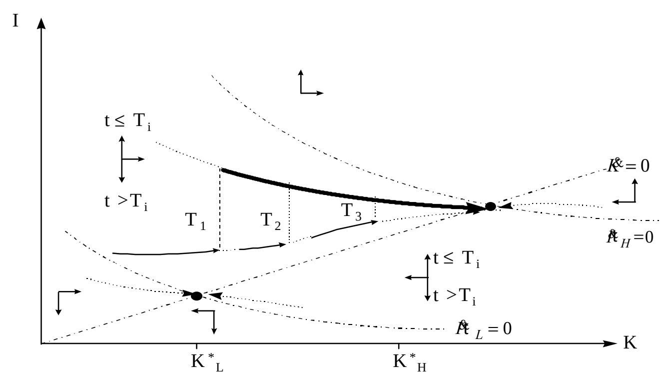
We can also use the Lemma, to characterizes the behavior of the optimal abatement investment path at the moment of the tax increase and thereafter when J_H becomes effective.

Proposition 2: *Provided that* J_H *satisfies condition (27), at the time when the tax rate is increased, the optimal abatement investment path jumps upward to the optimal saddle point path associated with the high tax rate* J_H *, and then asymptotically approaches the steady-state level* $\hat{I}_u^* = \delta K_H^*$.

Proof. See Appendix D.

We can now completely characterize the optimal investment strategy. During the period of uncertainty, i.e. before the higher tax rate becomes effective, the optimal investment path is governed by equation (47). Then, at the moment of the tax increase, it jumps up to the saddle-point path for in the full certainty case, and thereafter it follows that path (governed by equation (37)) towards its steady-state level. Figure 5 displays the optimal trajectory when T is uncertain and $T_3 > T_2 > T_1$ are three likely realization of it.

Figure 5. Optimal trajectory when T_i is uncertain and K_0 is small



Several points about the optimal investment strategy deserve emphasizing. First, as long as the higher tax rate has not become effective, the increasing conditional probability of its occurrence motivates the firm to raise its abatement investment rates over time and above the rates that would otherwise prevail. Second, whereas in the case when the timing of the tax increase is known with certainty, the optimal investment path rises smoothly to the saddle point path associated with J_H , in the present case it makes a *discontinuous jump* to the saddle point path at the moment of the tax increase. The reason for this difference is that, contrary to the certainty case, here the firm is unable to perfectly anticipate the date of the tax increase and consequently fails to adjust its abatement investment path so that its investment level reaches that of the J_H-saddle point path exactly at the time when the tax increase occurs.[11] Of course, the timing uncertainty imposes a sudden increase in the firm's investment adjustment cost. And, by the Corollary and convexity of the adjustment cost, this increase in the adjustment cost will be larger if the tax increase occurs sooner. In determining its optimal investment strategy, and based on its calculation of the conditional probability of the tax increase at any future date, the firm optimally balances this adjustment cost increase against a larger present value of payoffs (reduced pollution tax payments) associated with higher abatement investment rates which are undertaken sooner. Stated differently, the firm optimally trades off the lower conditional probability of the tax increase against the larger present shadow price of abatement capital. Thus, the sooner the likely tax increase occurs the larger is the investment adjustment at the time of the tax increase.

An interesting question is: Does there exist a certainty-equivalent discount rate that the firm can use to evaluate the shadow price of its abatement investment and then disregard the uncertainty in its future payoffs stream stemming from the uncertainty about the time of the tax increase? And if not, in what direction would such an erroneous practice bias the abatement investment path? The following proposition provides insight into these questions.

Proposition 3*: There exists no certainty-equivalent discount rate for the problem of uncertain date of tax increase, and raising the discount rate in leu of accounting for the uncertainty explicitly leads to underinvestment in abatement capital, where, all else being equal, the magnitude of the bias will increase (decrease) with the size of the future tax rate,J_H,depending on whether* $\tau_H < (>) \,\hat{\tau}(K)$.

Proof. See Appendix E.

The nonexistence of a certainty-equivalent discount rate in the present case is in accord with the well known result in the public economics literature. The underivestment bias arises because raising the discount rate to account for the risk of a future tax increase not only places particularly heavy penalties on the abatement investment payoffs in the distant future, here it also ignores the possibility that a higher emission tax rate increases the investment payoffs if $\tau_H < \hat{\tau}(K)$. It is then intuitively clear why the

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[11] In models where there is no uncertainty about the timing of environmental regulation, the convexity of investment adjustment cost would normally preclude a discontinuous jump in the optimal investment path (see, e.g., Xepapadeas (1992)).

magnitude of the bias increases with J_H as long as $\tau_H<\hat{\tau}(K)$ but diminishes with it when $\tau_H>\hat{\tau}(K)$. In the former case, a higher future tax rate raises the payoffs from abatement investment, thus warranting higher *optimal* investment rates, whereas it lowers the payoffs in the latter case, thereby implying optimal investment rates that would be lower than those for the case of full certainty with $\tau = \tau_L$ and the discount rate of $(r+*)$. A policy implication of Proposition is that where the erroneous practice of raising the discount rate to handle future risk is common among firms and where the regulator wishes to encourage pollution abatement, then subsidizing the cost of abatement capital to correct for the firm's undervaluation of the shadow price of abatement investment may be a justified second-best policy.

Another question of interest is: When the regulator cannot credibly commit to a certain date for the tax increase, how would a threat of imposing the higher tax rate sooner affect the firm's investment strategy? It is plausible to suppose that any such credible threat would be reflected in a revision of the firm's probability density distribution (i.e., a leftward shift in $T(s)$) such that greater probabilities are attached to the occurrence of the tax increase in earlier periods. More precisely, we can define a larger threat of facing the higher tax rate at time *t* as an increase in the *conditional* probability of the tax increase occurring at any date during *[0, t)* given that it has not occurred before. The revision in the firm's probability density distribution, and hence the implied increase in the conditional probability of the tax increase, may be caused, for example, by observation of increased pressure from environmental lobbyists to speed up the legislation of tougher environmental regulations. Or, it may be indicated, for example, by the urgency felt by the government to raise more revenue through higher emissions tax rates. Whatever the reason underlying the firm's revision of its probability density function, the following proposition shows the effect on its abatement investment decision.

Proposition 4*: *As long as the timing of the tax increase remains uncertain, a larger threat of imposing the higher tax rate at time t, as indicated by a higher conditional probability of the tax increase occurring at any date during [0, t) given that it has not occurred before, will lead to higher abatement investment rates and hence larger abatement capital stocks.

Proof. As in the proof of Proposition 3 (see Appendix E), we have $\Omega(t) = e^{-\int_0^t \psi(s)ds}$. So, for any $s0[0, t)$, an increase in $R(s)$ reduces $S(t)$. Next, noting that $J\,h(v(K))$ is increasing in J for a given K, it follows from (47) that $M8_u/MS(t) < 0$ and hence $MI(t)/MS(t) < 0$ for all *t,* and from (46) that $MK_u^*/MS(t) < 0$ for all *t*. ‖

Figure 6 portrays the optimal investment trajectories for three different conditional probability distributions, R_1, R_2, and R_3, representing respectively increasing degrees of threat of a future tax increase.

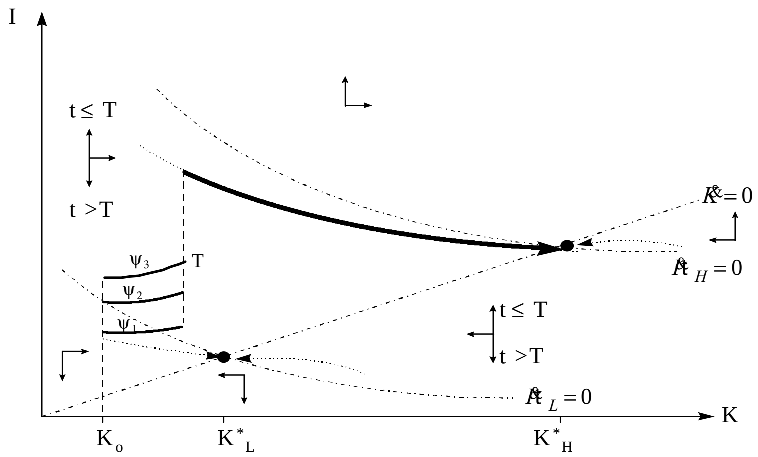

Figure 6. Optimal trajectories for different degrees of the threat of the tax increase

A policy implication of Proposition 4 is that when, for whatever reason, the regulator can not credibly commit to a specific implementation date, one way to motivate polluters to increase their abatement investments would be to take measures that firms would perceive as a credible threat of an accelerated tax increase .

Remark 3: In the special case where $T(t)$ is a Poisson distribution, i.e. $T(t) = \mu e^{-\mu t}$, then $S(t)=e^{-\mu t}$, and the conditional probability will be constant , $R(t) = \mu$. For this special case matters become simpler in that (i) in proposition 3, the adjusted discount rate will take the simple form of raising the discount rate r by a constant amount, equal to μ, so that it becomes $(r+\mu)$; (ii) in Proposition 4, a threat of accelerating the date of the tax increase will correspond to a larger value for μ and hence a pivoting of the Poisson distribution so that $T(t)$ will be larger for all $t \in [0, 1/\mu]$.

5. Conclusion

One of the main reasons why economists have often favored the taxation of emissions as a pollution control instrument has been the argument that emissions taxes encourage firms to invest in more efficient pollution abating technologies. In this paper, we have subjected this argument to a close scrutiny, taking explicitly into account that in the real world firms face uncertainties about the timing and size of a pollution tax increase. In a simple dynamic model where a risk neutral competitive firm can reduce its emissions per unit of output by building up its stock of abatement capital, we first showed that under full certainty, a higher tax rate can encourage abatement investment only if it does not exceed a certain threshold level. In our model, this "Laffer-curve" effect derives from two opposing effects of a higher pollution tax: a positive marginal tax effect and a negative marginal production effect. Only if the tax rate is not raised above a threshold level will the former effect dominate.

Subject to this condition, we have shown that when the time of the tax increase is certain but its magnitude is not, the expectation of a higher tax rate motivates the firm to increase its abatement investment rates at all times before the date of the tax increase. But at the time of the tax increase, the investment path can shift upward or downward depending on whether the actual tax rate is higher or lower than the firm's expected rate. In the opposite case where the time of the tax increase is uncertain, we have shown that, again, subject to the threshold tax rate, the expectation of the higher tax rate at some future date gives the firm an incentive to boost its abatement investment rates.But, in that case the uncertainty causes an unanticipated jump in abatement investment at the time of the tax increase. More interestingly perhaps, we have shown that a credible threat to accelerate the tax increase can further boost the firm's abatement investment. A further result of the paper has been to show the nonexistence of a certainty-equivalent discount rate and the direction and size of error resulting from raising the discount rate to account for the uncertainty in the timing of a future tax increase.

Although the theoretical insights obtained in the present paper may apply more generally to situations in which the timing or the extent of a policy reform (or regulation) is uncertain, the model we have analyzed has several limitations. We have assumed that the probability of the tax increase, conditional on its not having occurred previously, rises with time and eventually approaches unity. Alternatively, it would be plausible to suppose that if the tax increase has not occurred after the lapse of a certain time interval, then the firm may develop a disbelief about the threat of the tax increase and therefore lower its conditional probability of it occurring later on (as, for example, when the firm's probability density function is log normal). It would then be interesting to examine how the firm's optimal abatement investment strategy may change under such a situation.

We have also assumed that the size of the tax increase is given exogenously to the firm. A more sophisticated model would allow polluting firms to invest in a lobbying process which may lead to a smaller tax increase than would otherwise be mandated. A question that then arises is whether such an endogenized emissions tax would undermine the incentive to invest in pollution abatement capital. Finally, it is sometimes argued that investing in abatement capital can render beneficial spillover effects in the form of increased productivity of other inputs used in production processes (e.g., Porter (1995). It could be insightful to see how the results of the present paper would be affected in an extended model that incorporates such externalities.

Appendix

Appendix A: Proof of Proposition 1

On the $\dot{I} = 0$ isocline it holds that (cf. equation (15))

$$(r+\delta)\ C'(I) = -\tau\ \alpha'(K)\ h(v(K)) \tag{A.1}$$

Keeping *I* constant in (A.1) and differentiating the right-hand side of (A.1) totally, gives

$$-\tau\ \frac{d\ \alpha'\ h}{dK}\ d\ K - \alpha' \left\{h + \tau\ h'\ \frac{dv}{d\ \tau}\right\} d\ \tau = 0 \tag{A.2}$$

Let us first examine

$$\frac{d\ \alpha'(K)\ h(v(K))}{dK} = \alpha''\ h + \alpha'\ h'\ v'(K) \tag{A.3}$$

Substituting (10) into (A.3) gives

$$\frac{d\ \alpha'(K)\ h(v(K))}{dK} = \frac{p\ \alpha''\ h\ h'' - \tau\ \{\alpha\ \alpha''\ h\ h'' - (\alpha')^2 (h')^2\}}{(p - \tau\ \alpha)\ h''} \tag{A.4}$$

Furthermore, from (9) we obtain

$$\frac{dv}{d\tau} = \frac{\alpha\ h'}{(p - \tau\ \alpha)\ h''} \tag{A.5}$$

Substitution of (A.4) and (A.5) into (A.2) ultimately leads to

$$\frac{dK}{d\ \tau} = \frac{-\ \alpha' \left[p\ h''\ h - \tau\ \alpha\ \{h''\ h - (h')^2\}\right]}{\tau \left[p\ \alpha''\ h\ h'' - \tau\ \{\alpha\ \alpha''\ h\ h'' - (\alpha')^2\ (h')^2\}\right]} \tag{A.6}$$

From (28)- (30) we now obtain $\frac{dK}{d\ \tau} > 0$ if $\tau < \min\,(\hat{\tau},\ \tau^*)$, implying in turn that the $\dot{I}=0$ isocline shifts upwards when the emissions tax increases.

Appendix B: *Proof of the Lemma*

Let $\hat{K}_u^* \equiv \lim_{t\to\infty} K_u^*$ and $\hat{I}_u^* \equiv \lim_{t\to\infty} I_u^*$ denote the steady-state levels of the abatement stock and investment respectively. Since *S(t)60,* and hence *1-S(t)61*, as *t64*, it follows from (45.a) and (45.b) that

$$\lim_{t\to\infty} \lambda_u = \lim_{t\to\infty} C'(\delta K_u^*) = C'(\delta \hat{K}_u^*) = -\ \tau_H\ \alpha'(\hat{K}_u^*)\ h(v_{\tau_H}(\hat{K}_u^*))\,/(r+\delta) \tag{B.1}$$

which in turn implies that $\lim_{t\to\infty} I_u^* = \hat{I}_u^* = \delta \hat{K}_u^*$, and $\lim_{t\to\infty} \dot{\lambda}_u = 0$.

Further, from (18), the steady-state abatement stock in the full certainty case with $J=J_H$ is

$$C'(\delta K_H^*) = -\tau_H \, \alpha'(K_H^*) \, h(v_{\tau_H}(K_H^*))/(r+\delta) \tag{B.2}$$

Comparing (B.1) with (B.2) and recalling from (A.4) that - "N(K) h(v(K) is decreasing in K for $J_H < J^*$, it follows that $\hat{K}_u^* = K_H^*$ and $\hat{I}_u^* = \delta K_H^*$ .Next, we show that $K_u^*(t)$ increases with t monotonically. Differentiating () with respect to t yields,

$$\dot{K}_u^* = \frac{-\dot{\Omega}(t)[\tau_L \, \alpha'(K_u^*) \, h(v_{\tau_L}(K_u^*) - \tau_H \, \alpha'(K_u^*) \, h(v_{\tau_H}(K_u^*)]}{\delta(r+\delta)\, C'' + \Omega(t)\dfrac{d}{dK_u^*}[\tau_L \, \alpha'(K_u^*) \, h(v_{\tau_L}(K_u^*)] + (1-\Omega(t))\dfrac{d}{dK_u^*}[\tau_H \, \alpha'(K_u^*) \, h(v_{\tau_H}(K_u^*)]}$$

Recalling that $\dot{\Omega}<0$, $\alpha'<0$, and noting that for $\tau_L<\tau_H<\hat{\tau}$ one has $\frac{d}{d\tau}[\tau \, h(v_\tau(K)]>0$ (see also (29) and (30)), it then follows from condition (27) that the numerator is positive. Further, for $\tau_L<\tau_H<\tau^*$ one has from (A.4) that $\frac{d}{dK}[\alpha'(K)\, h(v(K)]>0$, implying, together with C0>0 and condition (27), that the denominator is also positive, and hence, $\dot{K}_u^*(t)>0$. Recalling that along the $\dot{K}=0$ isocline $I_u^*(t)=$ * $K_u^*(t)$, it follows that $\dot{I}_u^*(t) = \delta \dot{K}_u^*(t) > 0$. ‖

Appendix C: Proof of Corollary

Since, by the Lemma, $K_u^*(t)$ increases with t, and the $\dot{K}=0$ isocline ($I_u^*(t)=\delta\, K_u^*(t)$) is upward sloping, it must be that ($I_u^*(t)$, $K_u^*(t)$), the intersection point of the two isoclines, rises with t. So, to show that the optimal investment path shifts up with t, it suffices to show that the $\dot{I}=0$ isocline is monotonic, or, more specifically, that $\partial I_u/\partial K|_{\dot{I}=0}<0$. Now, using (45.a) and (45.b), the $\dot{I}=0$ isocline for this uncertainty case is given by

$$C'(I_u) = -[\,\Omega\,\tau_L\;\alpha'(K)h(v_{\tau_L}(K)) + (1-\Omega)\tau_H\;\alpha'(K)h(v_{\tau_H}(K))\,]/(r+\delta) \qquad \text{(C.1)}$$

Differentiating (C.1) with respect to K , we obtain

$$\partial I_u/\partial K\big|_{\dot{I}=0} = -\;\{\Omega\tau_L\;\frac{\partial}{\partial K}[\alpha'(K)h(v_{\tau_L}(K))\,] + (1-\Omega)\tau_H\;\frac{\partial}{\partial K}[\;\alpha'(K)h(v_{\tau_H}(K))\,]\}/[(r+\delta)C'']\qquad \text{(C.2)}$$

where time subscripts are omitted for notational ease . Recalling from (A.4) that for $J < J^*$, $\frac{\partial}{\partial K}[\alpha'(K)h(v(K))]>0$, and that $C0>0$, it follows that $\partial I_u/\partial K\big|_{\dot{I}=0}<0$. ‖

Appendix D: Proof of Proposition 2

Let $T\in(0,\infty)$ be the time at which the tax rate is raised to J_H,. Then from (37) we have

$$\lim_{t\to T^+} C'(I(t)) = -\int_T^\infty e^{-(r+\delta)(s-t)}\;\;\alpha'(K(s))\,\tau_H\;h(v_{\tau_H}(K(s)))\;ds \qquad \text{(D.1)}$$

Also, from (47) we have

$$\lim_{t\to^- T} C'(I(t)) = -\int_T^\infty e^{-(r+\delta)(s-t)}\,\alpha'(K(s)\Big\{\Omega(s)\;\;\tau_L\,h(v_{\tau_L}(K(s))) + (1-\Omega(s))\,\tau_H\,h(v_{\tau_H}(K(s)))\Big\}ds \qquad \text{(D.2)}$$

Writing the bracketed term in the integrand of (D.2) as $\tau_H\,h(v_{\tau_H}(K(s))) + \Omega(s)[\,\tau_L\,h(v_{\tau_L}(K(s)))\;-\;\tau_H\,h(v_{\tau_H}(K(s)))]$, recalling that for $\tau<\hat{\tau}$ the function that $J\;h(v_J\,(K))$ is increasing in J, and that $''N(K)<0$, it follows,, by comparing (D.1) and (D.2), that $\lim_{t\to T^+} C'(I(t)) > \lim_{t\to^- T} C'(I(t))$. Since $C0(I)>0$, it then follows that and $\lim_{t\to T^+} I(t) > \lim_{t\to^- T} I(t)$ ‖

Appendix E: Proof of Proposition 3

Writing $R(t)\,/\,T(t)/S(t)$ for the conditional probability of the tax being increased at t, we have $\dot{\Omega}/\Omega = -\omega(t)/\Omega(t) = -\psi(t)$, which gives $\Omega(t) = e^{-\int_0^t \psi(s)ds}$. Substituting this in (47) and denoting by $I_u(t)$ the optimal investment during the uncertainty period, we get

$$
\begin{aligned}
C'(I_u(t)) = \lambda_u = &-\int_t^{\infty} e^{-(r+\delta)(s-t)}\ \alpha'(K(s))\ \tau_H\, h(v_{\tau_H}(K(s)))]\, ds \\
&-\int_t^{\infty} e^{-[(r+\delta)(s-t)+\int_0^s \psi(\theta)\, d\theta]}\ \alpha'(K(s))[(\tau_L\, h(v_{\tau_L}(K(s))) - \tau_H\, h(v_{\tau_H}(K(s)))]\, ds
\end{aligned}
\tag{E.1}
$$

Next, denote by $\bar{I}_{\tau_L}(t)$ the investment path when the firm continues to operate under the certainty case with $J = J_L$ but accounts for the risk of a future tax increase by raising the discount rate by an amount equal to $\int_0^t \psi(s)\, ds$. From (19) we then have

$$
C'(\bar{I}_{\tau_L}(t)) = -\int_t^{\infty} e^{-[(r+\delta)(s-t)+\int_0^s \psi(\theta)\, d\theta]}\ \alpha'(K(s))\,(\tau_L\, h(v_{\tau_L}(K(s)))\, ds \tag{E.2}
$$

Subtracting (E.2) from (E.1) and simplifying yields

$$
C'(I_u(t)) - C'(\bar{I}_{\tau_L}(t)) = -\int_t^{\infty} e^{-[(r+\delta)(s-t)+\int_0^s \psi(\theta)\, d\theta]}\ \alpha'(K(s))\ \tau_H\, h(v_{\tau_H}(K(s))[e^{\int_0^s \psi(\theta)\, d\theta} - 1]\, ds \tag{E.3}
$$

which, upon recalling ''N(K)<0 and CO>0, implies that $\bar{I}_{\tau_L}(t) < I_u(t)$ during the uncertainty period.

Finally, using (A.5), it is readily verified that

$$
\frac{\partial}{\partial \tau_H}[\tau_H\, h(v_{\tau_H}(K))] \gtrless 0 \quad as \quad \tau_H \lessgtr \frac{hh''P}{\alpha\,(hh'' - h'^2)} = \hat{\tau}(K) \tag{E.4}
$$

which together with (E.3) implies that $\frac{\partial}{\partial \tau_H}[C'(I_u(t)) - C'(\bar{I}_{\tau_L}(t))] \gtrless 0 \quad as \quad \tau_H \lessgtr \hat{\tau}(K)$. ‖

References

Aizenman, J. and N. Marion (1991), "Policy Uncertainty, Persistence and Growth", NBER Working Paper Series No. 3848.

Alvarez, L H. R., Kannianen, V., and J. Södersten (1997), "Tax Policy Uncertainty and the Corporation-Theory of Tax-Induced Investment Spurts", Working Papers Series No. 1997:5, Department of Economics, Uppsala University, Sweden.

Carraro, C. and A. Soubeyran (1995) "R&D Cooperation, Innovation Spillovers and Firm's Location in a Model of Environmental Policy", paper presented at the EU Network Meeting on "Environmental Policy and Technological Innovation", Venice, Italy.

Caswell, M., Lichtenberg, E. and D. Zilberman, (1990), "The Effects of Pricing Policies on Water Conservation and Drainage", *American Journal of Agricultural Economics*, November, 883-890.

Dasgupta, P. and G. M. Heal (1974), "The Optimal Depletion of Exhaustible Resources", *Review of Economic Studies*, Symposium, pp. 3-28.

Farzin, Y. H. (1982), "Efficient Resource Extraction and Pricing Policy under the Threat of an Embargo", mimeo, Development Research Department, World Bank, Washington D.C.

Hartl, R.F., (1992) "Optimal Acquisition of Pollution Control Equipment Under Uncertainty", *Management Science*, 38, 609-622.

Hartl, R. F. and P. M. Kort, (1996), "Capital Accumulation of a Firm Facing an Emissions Tax", *Journal of Economics*, 63, 1-23.

Hasseett, K. and G. E. Metcalf, (1994), "Investment with Uncertain Tax Policy: Does Random Tax Policy Discourage Investment?", NBER Working Papers Series No. 4780.

Heal, G. M. (1975), "Economic Aspects of Natural Resource Depletion", in *The Economics of Natural Resource Depletion,* D.W. Pearce and J. Rose (eds.), Macmillan Press, London.

Jaffe, A. and R.N. Stavins, (1995), "Dynamic Incentives of Environmental Regulations: The Effects of Alternative Policy Instruments on Technology Diffusion", *Journal of Environmental Economics and Management*, 29, S-43 - S-63.

Jung, C., Krutilla, K. and R. Boyd, (1996), "Incentives for Advance Pollution Abatement Technology at the Industry Level: An Evaluation of Policy Alternatives", *Journal of Environmental Economics and Management*, 30, 95-111.

Magat, W. A., (1978), "Pollution Control and Technological Advance: A Model of the Firm", *Journal of Environmental Economics and Management*, 5, 1-28.

Malueg, D. A., (1989), "Emission Credit Trading and the Incentive to Adopt New Pollution Abatement Technology", *Journal of Environmental Economics and Management*, 16, 52-57.

Millman, S. R. and R. Prince, (1989), "Firm Incentives to Promote Technological Innovation in Pollution Control", *Journal of Environmental Economics and Management*, 17, 247-265.

Orr, L., (1976), "Incentive for Innovation as the Basis for Effluent Charge Strategy", *American Economic Review*, 56, 441-447.

Porter, M.E. and C. van der Lind, (1995), "Toward a New Conception of the Environment-Competitiveness Relationship", *Journal of Economic perspectives*, Vol.9, No. 4, 97-118.

Rodrik, D. (1991), "Policy Uncertainty and Private Investment in Developing Countries", *Journal of Development Economics*, 36, 229-242

Wenders, J. T., (1975), "Methods of Pollution Control and the Rate of Change in Pollution Abatement Technology", *Water Resources Research*, 11, No.3, 393-396.

Xepapadeas, A. (1992), "Environmental Policy, Adjustment Costs and Behavior of the Firm", *Journal of Environmental Economics and Management*, 23, 258-275.

Yaari, M.E. (1965), "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer", *Review of Economic Studies,* 32, 137-158.