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Is there a tragedy of a common central bank? A dynamic analysis

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Abstract

A great deal of uncertainty exists about the degree to which a European Central Bank (ECB) can be exploited as common property by undisciplined governments. In a dynamic game between symmetric national governments and a single monetary authority, open-loop strategies in the national setting coincide with strategies in the international setting. Compared to the open-loop outcome, feed-back strategies slow down debt stabilization. Moreover, debt adjustment is quicker with a common ECB than with individual central banks, while inflation as well as fiscal deficits are lower. These results conflict sharply with the notion of the ECB as common property.

Key words: European central bank; Policy coordination; Dynamic games
JEL classification: C73; E58; F33; F42

1. Introduction

A great deal of uncertainty exists about the degree to which a European Central Bank (ECB) can be exploited as common property by undisciplined governments, forcing the monetary authority to monetize outstanding public debt and thus boosting inflation in the whole community. Bovenberg, Kremers, and Masson

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(1991) describe several channels through which monetary unification can reduce fiscal discipline: With a common currency, the cost of borrowing and public debt cannot be internalized through relative monetary adjustments, implying that it is partially shifted to other countries. Moreover, since the elimination of currency risk encourages the residents of other countries to invest in government debt, the pressures for bailouts through budgetary transfers also increase. In addition, as monetary policy can no longer be used to stabilize country-specific shocks, fiscal policy may have to bear a larger burden of stimulating the economy. Employing simulation analysis, Levine (1993) and Levine and Pearlman (1992) show how a common central bank induces excessive fiscal deficits and high government debt, if it cannot commit to low inflation. Moreover, fiscal coordination aggravates fiscal profligacy, although it may help in the presence of fiscal externalities as is shown by Levine and Brochiner (1994).

Much theoretical work has been devoted to analyzing the effect of fiscal and monetary interaction on the evolution of government debt. In an intertemporal setting this gives rise to a number of interesting economic and strategic aspects. Sargent and Wallace (1981) have shown that 'unpleasant' effects can arise if a central bank reacts to fiscal policy and is being constrained by the necessity to finance government debt. Tabellini (1986) demonstrated in a simple strategic setting that, without a specific leadership structure, strategies in the stabilization of government debt can be ranked. In particular, he shows that public debt increases if policy makers cannot commit themselves to an optimal strategy, but instead behave in a subgame-perfect manner. When this idea is projected to a multi-country setting, it gives rise to the various fears connected with a common central bank, whose seigniorage is shared by all countries that are part of the currency union.

In his classic contribution, Hardin (1968) points to exploitation possibilities of common property by characterizing the 'tragedy of the commons' as a social dilemma resulting in the destruction of a common pasture. McMillan (1986) analyzes the optimal extraction of international common property and shows in an extreme example related to international fishing rights how optimal time-consistent behavior leads to the immediate exhaustion of the common resource stock.

We address the question of whether governments view the ECB as common property in their strategic interaction over time. While acknowledging the conventional aspects of fiscal indiscipline, we focus on how monetary unification alters disciplining mechanisms and strategic interactions between monetary and fiscal policies. In a closed economy, a single monetary authority interacts with a single fiscal authority. In a monetary union, in contrast, a single monetary authority is confronted with several fiscal authorities. The implied changes in strategic interactions are intrinsic elements of monetary unification. Therefore, the disciplining effects we uncover will continue to operate in a more complete analysis that would include other relevant channels.

With strategies at the center of our attention, we extend the elegant framework of Tabellini (1986) to a two-country setting. We derive optimal open-loop and feed-back strategies, with and without cooperation between fiscal players, and we compare the outcomes with the case of strategic interaction between governments and their own individual national central banks.

We find that in a symmetric union the open-loop strategies in the national and the international setting coincide. This result holds irrespective of whether or not fiscal authorities cooperate with each other. The absence of exploitation of the common central bank is not necessarily surprising, because policy makers can commit themselves to their optimal strategies.

The more realistic setting, though, involves feed-back strategies. As Tabellini (1986) has shown, in a national setting feed-back strategies slow down stabilization of debt compared to open-loop strategies. We confirm this result in the extended two-country case. However, we find that public debt stabilization occurs quicker with a common central bank than with individual central banks. This conflicts sharply with the notion of the ECB as exploitable common property. The explanation for more fiscal discipline in a monetary union lies in the weakening of the strategic position of the fiscal authorities compared to that of the monetary authorities. This effect vanishes as soon as governments cooperate. Accordingly, the outcome of fiscal cooperation between two symmetric countries in a monetary union coincides with that of separate economies with individual national central banks. Moving to a monetary union thus improves fiscal discipline and monetary stability only as long as governments fail to cooperate.

Thus, from a purely strategic perspective, if one is interested in fiscal discipline, we find arguments in favor of a European Central Bank. The ECB cannot be viewed as common property, but rather as a strategic player, whose power rises if the relative size of individual countries in the monetary union declines as their number increases.

Section 2 formulates the model and describes the general structure of the solutions. Section 3 deals with noncooperative strategies both for the open-loop as well as the feed-back case. Alternative specifications of parameters allow us to compare the performance of a European central bank with that of a national central bank. In Section 4, we consider cooperation between fiscal authorities and analyze both open-loop and feed-back behavior against a common central bank. Section 5 concludes. The main text focuses on the intuition for all propositions and describes the graphical results of numerical simulations. The Appendix contains the mathematical details as well as the proofs.

2. The model

Central to our analysis is the dynamic interaction of monetary and fiscal authorities in determining government debt accumulation, which evolves according to

$$\dot{D}(t) = i(t)D(t) + F(t) - \dot{M}(t),$$

where D denotes the nominal level of government debt and i the nominal interest rate. F is the primary fiscal deficit and $\dot{M}$ stands for the change of the monetary base. Dividing both sides of the budget constraint by the level of nominal income Y and denoting the normalized variables by lower case letters, i.e., $d = D/Y$, $f = F/Y$, and $m = \dot{M}/Y$, one obtains

$$\dot{d}(t) = rd(t) + f(t) - m(t),$$

where r is equal to the difference between the real interest rate and the growth rate of real income. In order to keep the dynamic analysis manageable, we assume that the nominal interest rate adjusts to changes in inflation and that real income grows at a constant rate, implying that r is constant as well.[1]

We assume that the (European) union consists of two countries and a common central bank. Since we explore debt dynamics of two countries, we need two government budget constraints. With only one central bank for both countries, e.g., a European Central Bank (ECB), we must specify how seigniorage is distributed among both countries. We therefore assume that a fraction θ of new money created by the ECB is allocated to country 1, while the remaining fraction, $1-\theta$, goes to country 2. Thus, we can write the two government budget constraints as

$$\dot{d}_1(t) = rd_1(t) + f_1(t) - \frac{\theta}{\omega}m(t),$$

$$\dot{d}_2(t) = rd_2(t) + f_2(t) - \frac{1-\theta}{1-\omega}m(t),$$

where $m(t)$ denotes the change in the monetary base in relation to European GDP and ω is the share of country 1 in European GDP. We focus on countries with equal growth rates, so that the relative size of countries remains constant over time. With integrated financial and goods markets, real interest rates are at the same level across the two countries. Therefore r has the same value for both countries.

If the distribution of seigniorage is determined by the economic size of countries, i.e., $\theta = \omega$, each country receives the same amount of seigniorage from the ECB as from a national central bank, provided that both would implement the same monetary policy. However, other rules may be worth considering.

[1] Note that an endogenous interest rate would imply a nonlinear differential equation as an optimization constraint, which would yield significantly more complicated dynamics.

To save on notation, we write the two constraints above in vector notation:

$$\underbrace{\begin{bmatrix} \dot{d}_1(t) \\ \dot{d}_2(t) \end{bmatrix}}_{\dot{d}(t)} = r \underbrace{\begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix}}_{d(t)} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B_1} f_1(t) + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B_2} f_2(t)$$

$$- \underbrace{\begin{bmatrix} \theta/\omega \\ (1-\theta)/(1-\omega) \end{bmatrix}}_{B_m} m(t). \qquad (1)$$

We assume that policy makers interact strategically in two stages. In the first stage they calculate optimal fiscal and monetary policies in order to satisfy a specified set of economic goals, i.e., targets for real as well as nominal variables. For simplicity, one may assume that all policy makers target the same variables, but have individual preferences regarding their respective weights. Hence, they care differently about the same issues. Policy makers optimize for given dynamic constraints of the international economy, which, however, do not include the government budget constraints. This results in unconstrained optimal strategies which we denote by $\bar{f}_1(t)$, $\bar{f}_2(t)$, and $\bar{m}(t)$.[2] In order to keep matters simple, we will look only at optimal strategies that are constant over time. In the second stage, policy makers again optimize intertemporally, but now targeting their unconstrained strategies while having to satisfy the intertemporal government budget constraints so as to prevent unlimited debt accumulation. Their preferences are thus characterized by political instead of economic objectives.[3]

The fiscal authority of country i ($i = 1,2$) is assumed to minimize

$$V_i(t) = \tfrac{1}{2} \int_t^\infty [(f_i(s) - \bar{f}_i)^2 + (d(s) - \bar{d}_i)' Q_i (d(s) - \bar{d}_i)] \mathrm{e}^{-\delta_f (s-t)} \, \mathrm{d}s, \qquad (2)$$

where the first term in brackets characterizes the objective of targeting an optimal level of fiscal deficits, and the second term shows how much weight the fiscal

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[2] See Pindyck (1977) for a numerical analysis of the interaction between monetary and fiscal policy over time in a dynamic economic model of this type.

[3] This two-stage optimization procedure, which was introduced by Tabellini (1986) is, of course, questionable from a strategic point of view. The main advantage of the two-stage analysis clearly lies in its practicability. Furthermore, taking a complete economic model into consideration often leads to an endogenization of interest rates, which makes the budget constraints (1) nonlinear. Despite this theoretical caveat, the severity of the resulting mistakes for macroeconomic policy evaluation is surely debatable, as the common approach in the traditional literature is to ignore the budget constraint altogether. Also, Tabellini (1986) mentions the problem that the unconstrained optimal policies remain unchanged across regime switches, thus making the analysis vulnerable to the Lucas critique. Again, this theoretical drawback should not be ignored, but rather assessed on the basis of its empirical relevance.

authority attaches to stabilizing government debt around some optimal level $\bar{d}_i$. Concern about government indebtedness could be based on the efficiency loss due to the distributive effects of high debt. Tabellini (1986) refers also to more serious tax distortions, as well as possible crowding out effects on capital accumulation. Hence, debt stabilization can be considered as an independent policy target, even though we abstract from default risk. Moreover, in the context of a European monetary union, debt convergence is dictated also by the Maastricht criteria. With

$$Q_i = \begin{bmatrix} Q_i^{11} & 0 \\ 0 & Q_i^{22} \end{bmatrix},$$

the debt objective becomes $(d - \bar{d}_i)'Q_i(d - \bar{d}_i) = Q_i^{11}(d_1 - \bar{d}_{i1})^2 + Q_i^{22}(d_2 - \bar{d}_{i2})^2$, thus allowing for the possibility that a government cares also about stabilization of the level of debt abroad. We assume that both governments discount time with the same rate δ_f.

The European Central Bank is concerned about its unconstrained monetary target, as well as the levels of debt of both countries:

$$V_m(t) = \tfrac{1}{2}\int_t^{\infty} [(m(s) - \bar{m})^2 + (d(s) - \bar{d}_m)'Q_m(d(s) - \bar{d}_m)]\mathrm{e}^{-\delta_m(s-t)}\,\mathrm{d}s, \quad (3)$$

where the ECB's concern about the levels of national debt is characterized by the matrix Q_m. The ECB is assumed to care about the deviation of average debt, but not its distribution. Hence, the matrix Q_m can be written as

$$Q_m = \tau \begin{bmatrix} \omega^2 & \omega(1-\omega) \\ \omega(1-\omega) & (1-\omega)^2 \end{bmatrix}. \quad (4)$$

The parameter τ in (4) reflects the central bank's sensitivity to general economic policy in the union.[4] The more independent the ECB is, i.e., the lower the value of τ, the more it is able to shift the burden of debt stabilization to the fiscal authorities. By focusing on the interaction between the central bank and only one of the fiscal authorities ($\theta = \omega = 1$) for a constant value of τ, we can compare the performance of the ECB with that of

[4] Alternatively, the ECB might be concerned about the average deviation of debt in the two countries. In that case, Q_m would have the form

$$Q_m = \tau \begin{bmatrix} \omega & 0 \\ 0 & (1-\omega) \end{bmatrix}.$$

This specification would be more appropriate if one considered monetary cooperation of national central banks.

an individually optimizing national central bank in a noncooperative setting.[5]

With the linear-quadratic optimization problems described above, we propose solutions for the policy strategies, which are linear functions of the levels of debt:

$$f_i = \alpha_i^0 + \alpha_i' d \quad \text{with} \quad \alpha_i' = (\alpha_i^1, \alpha_i^2), \quad i = 1,2, \tag{5}$$

$$m = \alpha_m^0 + \alpha_m' d \quad \text{with} \quad \alpha_m' = (\alpha_m^1, \alpha_m^2). \tag{6}$$

By inserting these reaction functions into the budget constraint (1), one obtains the following dynamic equation for the levels of national debt:

$$\dot{d} = \underbrace{[rI + B_1\alpha_1' + B_2\alpha_2' - B_m\alpha_m']}_{\Gamma} d + \underbrace{[B_1\alpha_1^0 + B_2\alpha_2^0 - B_m\alpha_m^0]}_{c}. \tag{7}$$

With $\dot{d}$ set equal to zero, we arrive at steady-state levels of debt

$$d^* = -\Gamma^{-1}c, \tag{8}$$

so that the general solution for d can be written as

$$d(t) = e^{\Gamma t}(d(0) - d^*) + d^*. \tag{9}$$

We consider different forms of strategic interaction between the three players. Each form yields a different value for Γ and c that determine the dynamic behavior of d, and together with the proposed reaction functions (5) and (6) also that of f_1, f_2, and m.

In order to compare the different types of strategic interaction, we provide numerical simulations for the outcomes of the differential games. We use the same set of parameters for all simulations, unless indicated otherwise. In the two-country case, we assume that both countries are symmetric and of the same size ($\omega = \frac{1}{2}$). Fiscal authorities care only about their own country's debt ($Q_1^{11} = Q_2^{22} = 0.04$ and $Q_1^{22} = Q_2^{11} = 0$). The ECB, in contrast, is concerned about debt in both countries, giving this objective the same weight as the two fiscal players ($\tau = 0.04$). These values generate time paths for national debt that are compatible with empirical estimates of debt adjustment for countries with an objective of stabilization. In accordance with the 3% Maastricht criterion for total budget deficits, we assume that both countries feature the same target for the primary fiscal deficits of 1% of GDP ($\bar{f}_1 = \bar{f}_2 = 0.01$). Monetary policy is aimed at price stability ($\bar{m} = 0$). Initial debt levels are at the approximate current European average of 70% of GDP, while debt targets are set at 50% of GDP ($\bar{d}_1' = \bar{d}_2'$

[5] A comparison with a constant value of τ should be viewed as a benchmark case for a strategic analysis. Despite the fact that differences in central bank independence within the European Union are still considerable, strategic differences between a national and a European central bank are highlighted best if both are assumed to feature the same political objectives.

$= \bar{d}_m' = (0.5, 0.5)$), so that they lie well within Maastricht bounds. All policy makers feature the same discount rate ($\delta_f = \delta_m = 0.1$), and the real interest rate (minus growth of real GDP) remains constant at $r = 0.03$.

3. Noncooperative policies

Without cooperation between any of the authorities in a monetary union, we must consider the optimization problem of each individual player. In this section, we focus on the Nash equilibrium of the differential game described above for both open-loop and feed-back behavior.

3.1. Open-loop strategies

The simplest form of noncooperative behavior is given by the open-loop strategies of each institution. In this case, each player solves his intertemporal optimization problem, being restricted by the time path of government debt while taking the intertemporal strategies of the other players as given. Since the complete time path of policies is fixed at the beginning of the planning period, the open-loop strategies are often referred to as precommitment policies. Thus, their plausibility depends on how well policy makers can adhere to their announced strategies.

The optimization problem of each player is solved by minimizing the current value Hamiltonian composed of the respective preference function and the two budget constraints (1). To illustrate, the Hamiltonian of fiscal authority i ($i = 1,2$) is then

$$H_i = \tfrac{1}{2}[(f_i - \bar{f}_i)^2 + (d - \bar{d}_i)' Q_i (d - \bar{d}_i)] + \lambda_i' \,[rd + B_1 f_1 + B_2 f_2 - B_m m].$$

The Appendix shows how the optimality conditions for this differential game can be derived by using the method of undetermined coefficients. This system of equations is nonlinear, so that in general we are not able to obtain a solution analytically. Moreover, without further restrictions, more than one solution exists. However, following the suggestion of the Delors committee (1989) to impose boundaries on fiscal spending and debt creation, we implement the rather weak restriction that government debt should not grow too fast:

$$\lim_{t \to \infty} \mathrm{e}^{-rt} d(t) = 0.$$

This transversality condition excludes dynamically unstable solutions.

For our benchmark case of two symmetric countries and a European Central Bank with the parameters specified above, a numerical solution for country 1 that satisfies the transversality condition is shown in Fig. 1. We first explore the sensitivity of the solution with respect to θ, the portion of seigniorage that

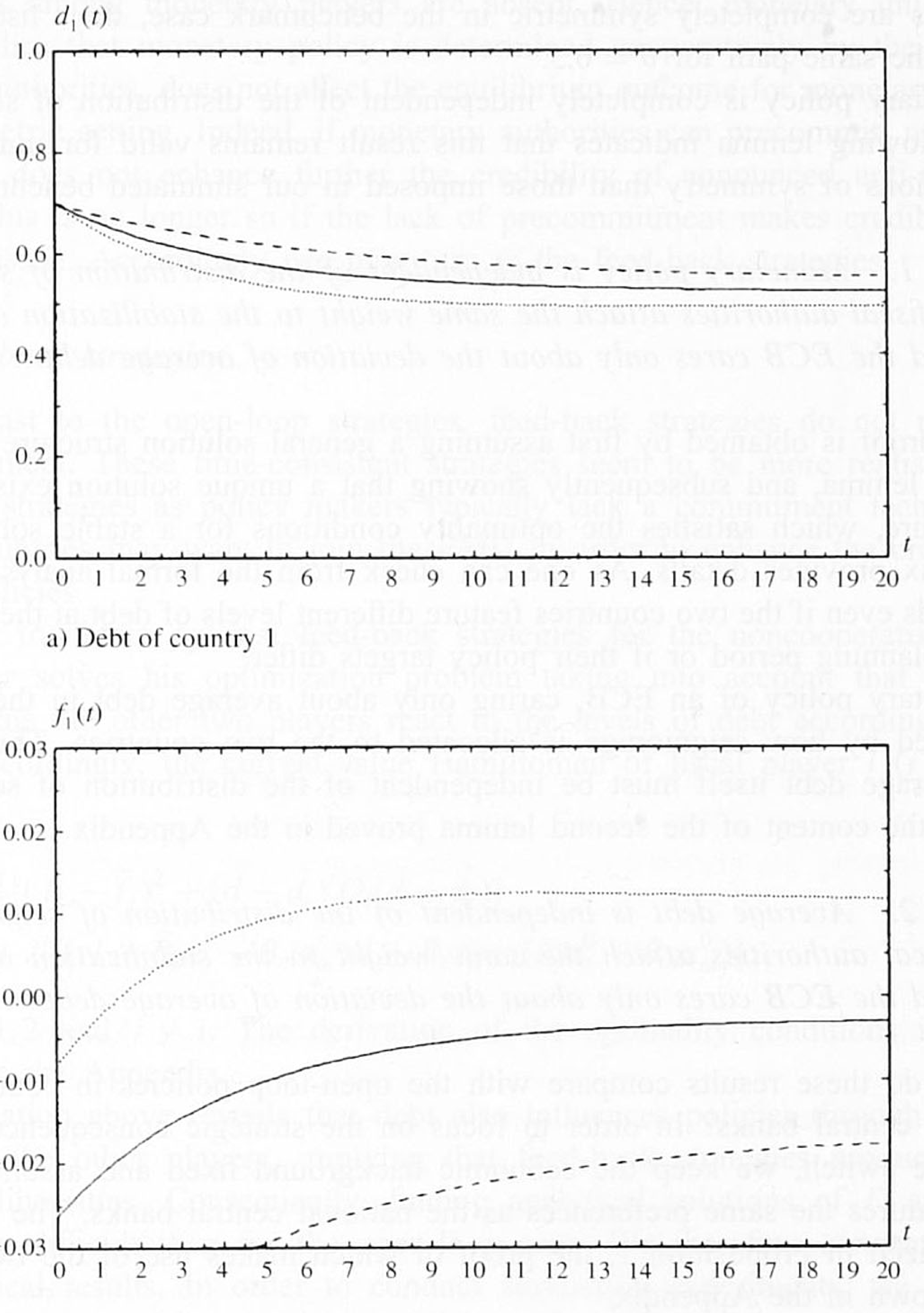

Fig. 1. Open-loop noncooperative solutions for $\theta = 0$ (dashed), 0.5 (solid), and 1.0 (dotted).

country 1 has access to. Fig. 1 shows the simulation results for values of $\theta = 0$, 0.5, and 1.0.

Panel a shows the time paths of government debt as a fraction of national GDP. The dashed path for $\theta = 0$ shows the slowest adjustment to a higher level of debt, since all seigniorage is flowing to country 2. Without monetary finance, country 1 has to cut its fiscal deficit the most, leading to the lower curve in panel b. If country 1 receives all the seigniorage, the situation is reversed. Since both countries are completely symmetric in the benchmark case, their fiscal deficits follow the same path for $\theta = 0.5$.

Monetary policy is completely independent of the distribution of seigniorage. The following lemma indicates that this result remains valid for much weaker assumptions of symmetry than those imposed in our simulated benchmark case.

Lemma 1. *Monetary policy is independent of the distribution of seigniorage if both fiscal authorities attach the same weight to the stabilization of national debt and the ECB cares only about the deviation of average debt.*

The proof is obtained by first assuming a general solution structure that satisfies the lemma, and subsequently showing that a unique solution exists to such a structure, which satisfies the optimality conditions for a stable solution. The Appendix provides details. As one can check from the formal analysis, this result holds even if the two countries feature different levels of debt at the beginning of the planning period or if their policy targets differ.

Monetary policy of an ECB, caring only about average debt in the union, is unaffected by how seigniorage is allocated to the two countries. This suggests that average debt itself must be independent of the distribution of seigniorage. This is the content of the second lemma proved in the Appendix.

Lemma 2. *Average debt is independent of the distribution of seigniorage, if both fiscal authorities attach the same weight to the stabilization of national debt and the ECB cares only about the deviation of average debt.*

How do these results compare with the open-loop policies in countries with separate central banks? In order to focus on the strategic consequences of such a regime switch, we keep the economic background fixed and assume that the ECB features the same preferences as the national central banks. The results are summarized in Proposition 1, the proof of which makes use of the two lemmas as is shown in the Appendix.

Proposition 1. *A common central bank, caring only about the deviation of average debt, pursues the same monetary policy as a national central bank, if fiscal authorities feature symmetric preferences, while countries feature the same initial levels of debt relative to GDP. Moreover, also government debts and fiscal deficits are the same as in the closed economy, if countries receive a share of ECB seigniorage equal to their share in aggregate GDP (i.e.,* $\theta = \omega$*).*

If policy makers are able to precommit to dynamic strategies, implementing a common central bank will not affect the strategic interaction, as long as countries are symmetric and seigniorage is distributed according to economic size. Intuitively, with open-loop strategies and national central banks, strategic interactions among monetary players are absent. Hence, monetary unification, which implies that monetary policy is determined cooperatively by the various monetary authorities, does not affect the equilibrium outcome for monetary policy in a symmetric setting. Indeed, if monetary authorities can precommit, monetary unification does not enhance further the credibility of announced anti-inflation policies. This is no longer so if the lack of precommitment makes credibility an important issue. Accordingly, we now turn to the feed-back strategies.

3.2. Feed-back strategies

In contrast to the open-loop strategies, feed-back strategies do not presume precommitment. These time-consistent strategies seem to be more realistic than open-loop strategies as policy makers typically lack a commitment technology. Indeed, countries may want to join the EMU in order to enhance the credibility of their policies.

In order to obtain optimal feed-back strategies for the noncooperative case, each player solves his optimization problem taking into account that at each point in time the other two players react to the levels of debt according to (5) or (6). Accordingly, the current-value Hamiltonian of fiscal player i ($i = 1,2$) becomes

$$H_i = \tfrac{1}{2}[(f_i - \bar{f}_i)^2 + (d - \bar{d}_i)' Q_i (d - \bar{d}_i)] + \lambda_i'[(rI + B_j \alpha_j' - B_m \alpha_m')d + B_i f_i + (B_j \alpha_j^0 - B_m \alpha_m^0)],$$

with $j = 1,2$ and $j \neq i$. The derivation of the optimality conditions is again provided in the Appendix.

The equation above reveals that debt also influences policies through the reactions of the other players, implying that feed-back strategies produce additional nonlinearities. Consequently, finding analytical solutions of Γ and c is much more difficult than for the open-loop case. We therefore concentrate on the numerical results. In order to conduct simulation experiments, we use the same benchmark model as in the preceding section. Hence, we employ the same parameters in a different strategic setting.

For the case of $\theta = \omega = 1$, we reproduce in Fig. 2 the results of Tabellini (1986) for the evolution of debt in a single country with its own national central bank.

In the feed-back case, each player explicitly takes into account the reactions of the other players towards changes in public debt. In particular, if players play feed back, they take into account that part of their contributions to stabilizing debt will be undone by changes in the policy instrument controlled by the other player. This countervailing effect reduces the private net benefits of employing the player's own instruments to cut debt accumulation. Accordingly, each player will put less effort in debt stabilization, resulting in higher stocks of debt.

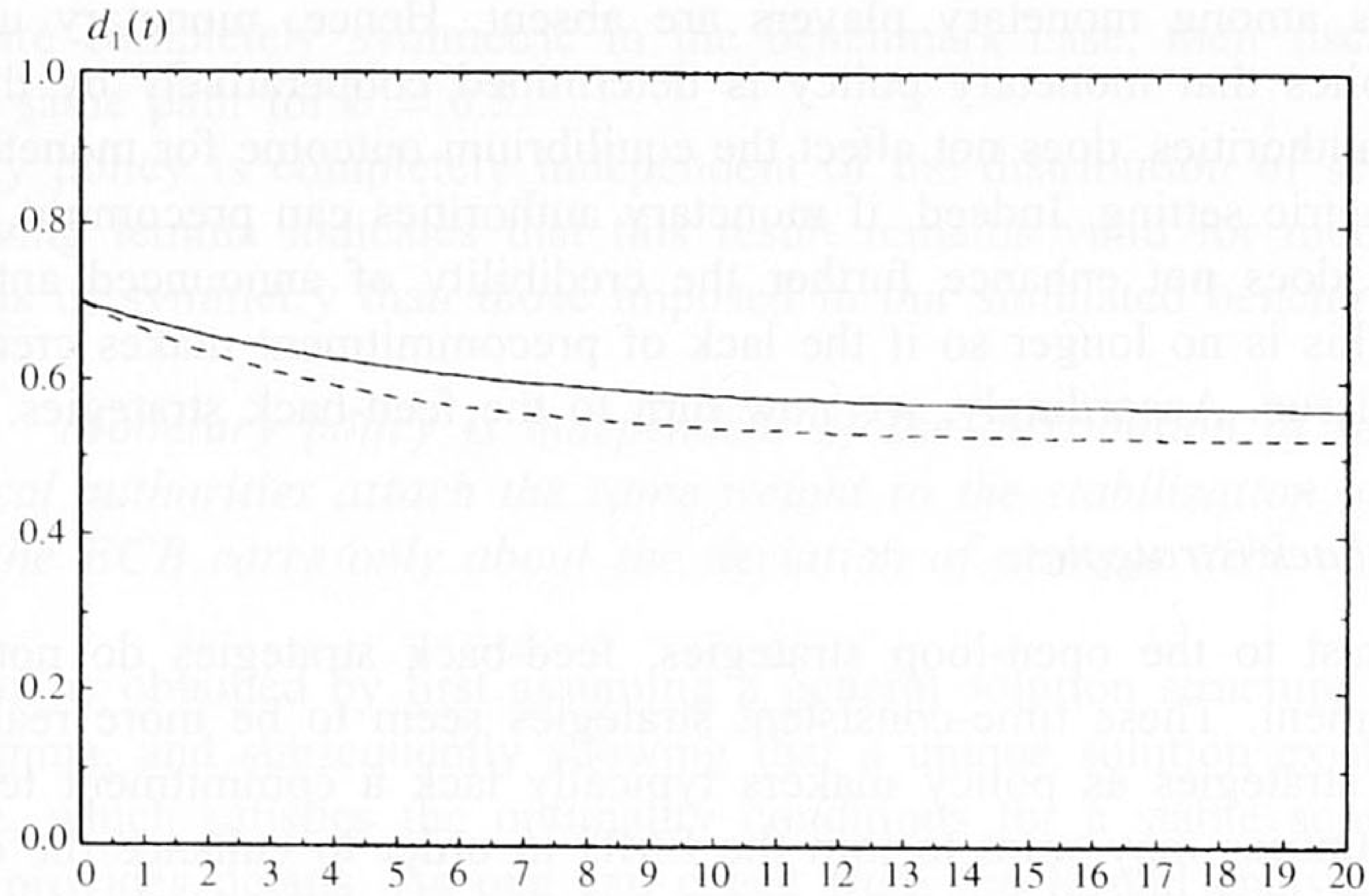

Fig. 2. Open-loop (dashed) vs. feed-back (solid) noncooperative debt for country 1 with a national central bank.

Panels a and b of Fig. 3 show that, in the feed-back case, both the fiscal and the monetary players initially move towards their individual policy targets, thereby leaving the problem of debt stabilization to the other player. This results in a higher level of debt than in the open-loop case. Accordingly, in the long run, feed-back policies must become more accommodating.

We now turn to a comparison of the feed-back strategies in the monetary union with the institutional setting of separate individual central banks. In order to highlight the strategic implications, we switch only the institutional setting while leaving the preferences of the players unchanged.

Fig. 4 shows that, instead of exploiting the ECB, fiscal authorities become more disciplined. In particular, panel a of Fig. 4 reveals that optimal fiscal deficits are lower if fiscal authorities have to deal with a common monetary institution rather than their own national central bank. Moreover, the solid curve in Fig. 4b indicates that the ECB is more successful in reducing inflation than a national central bank, if both central banks feature the same policy preferences.

The intuition behind the additional fiscal discipline in a monetary union is as follows. In a monetary union, each fiscal player acting individually exerts less impact on monetary policy than in an economy with national policy making. Accordingly, in setting their policy instruments in a monetary union, the fiscal players take into account a smaller countervailing response of the monetary authority compared to the case when they would interact with a national central bank. In particular, reducing the fiscal deficit in order to cut the stock of public debt causes the monetary authority to decrease money growth by a smaller amount, thereby producing a larger net reduction of public debt. Hence, in a monetary union, governments perceive higher net benefits from cutting fiscal deficits

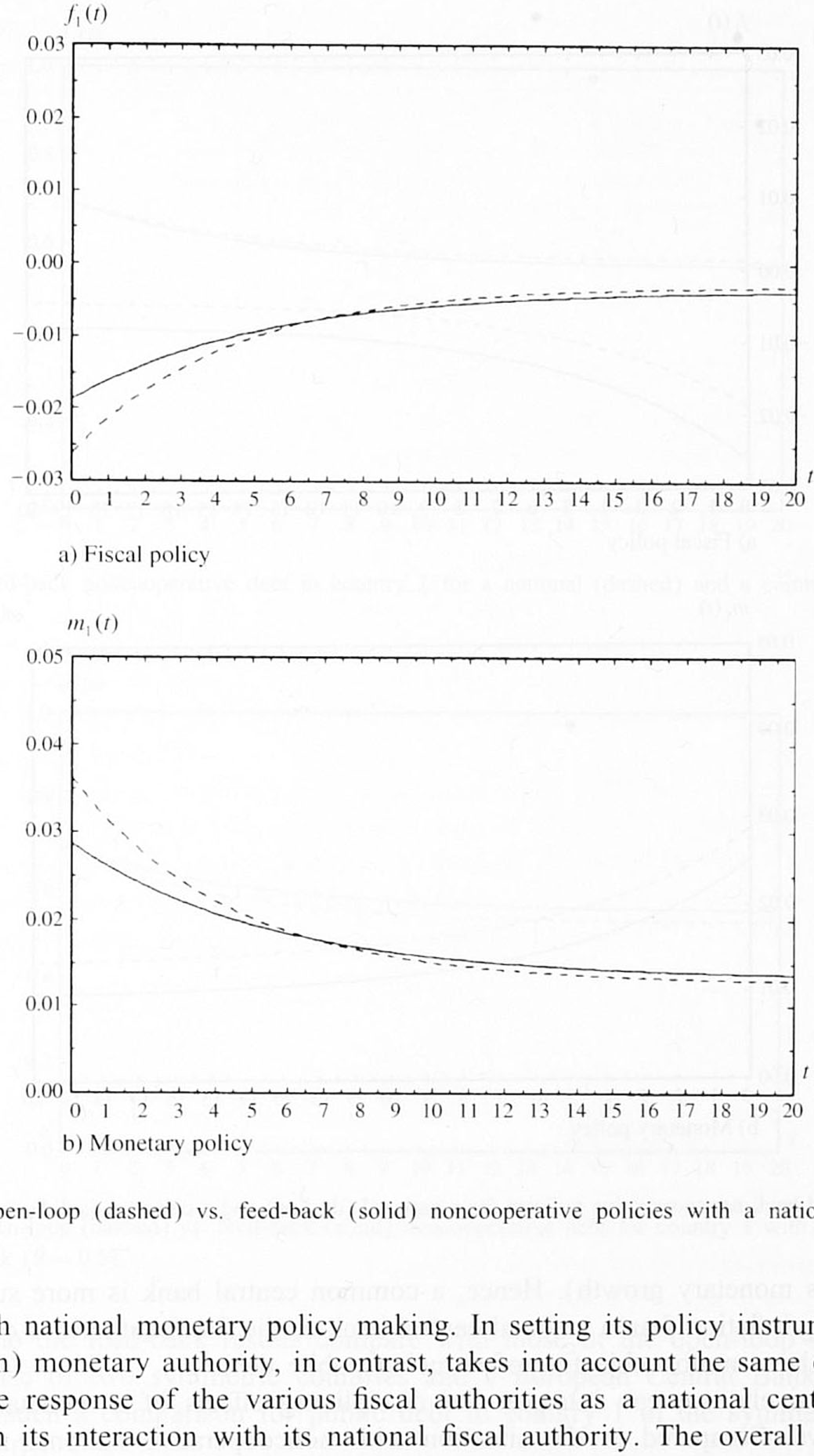

Fig. 3. Open-loop (dashed) vs. feed-back (solid) noncooperative policies with a national CB in country 1.

than with national monetary policy making. In setting its policy instrument, the (common) monetary authority, in contrast, takes into account the same offsetting aggregate response of the various fiscal authorities as a national central bank would in its interaction with its national fiscal authority. The overall effect of the weaker strategic position of the fiscal authorities in a monetary union is to reduce debt accumulation (see Fig. 5) and primary fiscal deficits (because the fiscal authorities perceive higher benefits from debt stabilization) and decrease monetary growth (because the lower stocks of debt induce the central bank to

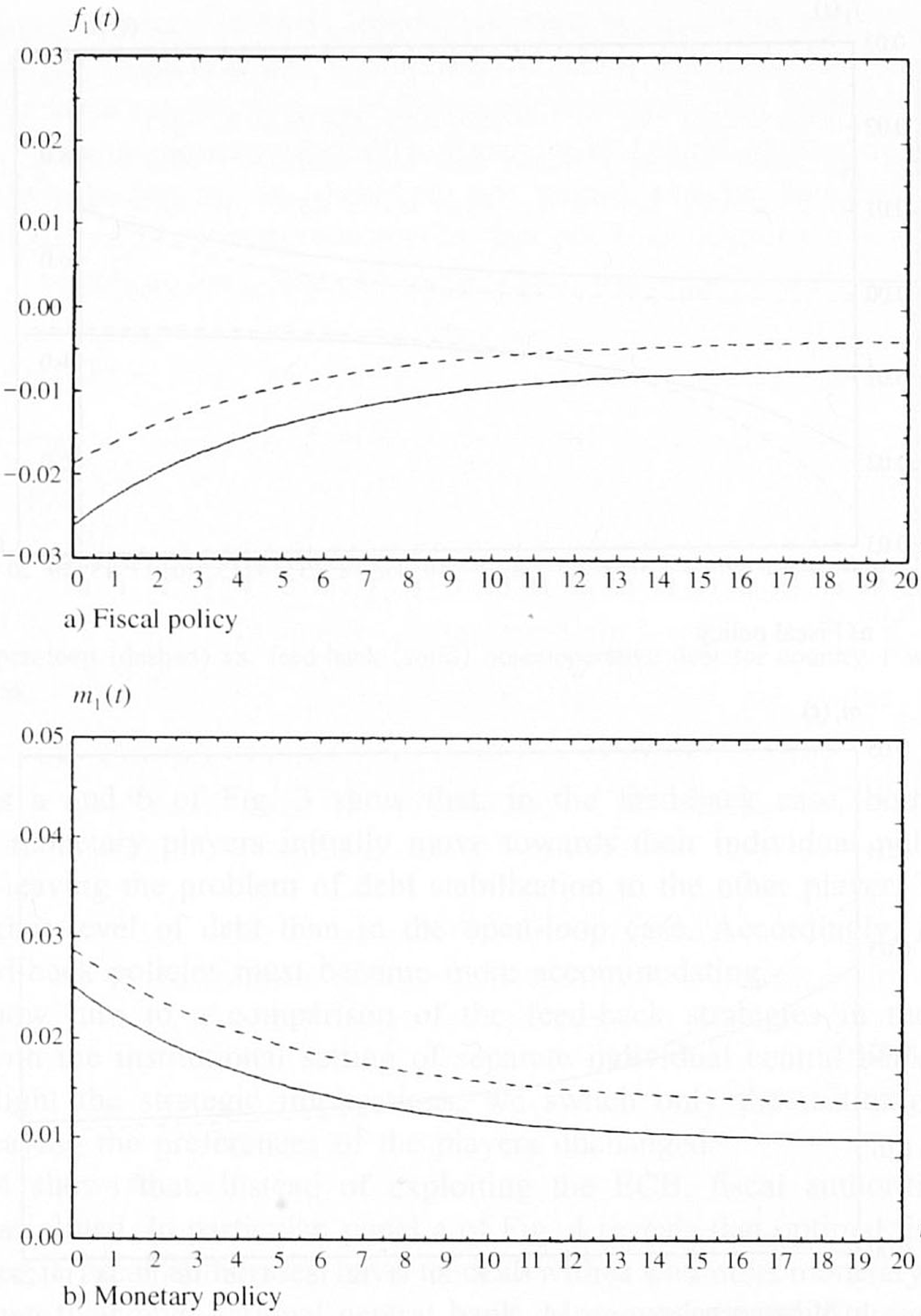

Fig. 4. Feed-back noncooperative policies for a national (dashed) and a common (solid) central bank.

reduce its monetary growth). Hence, a common central bank is more successful in fighting inflation than a national central bank as the fiscal authorities contribute more to the common objective of reducing public debt.

Alternatively, we can interpret the disciplining effects of a monetary union as follows. Compared to the situation with noncooperative national monetary policies, monetary unification in fact implies that monetary policies are set co-operatively. The implicit cooperation of the monetary authorities strengthens the strategic position of these authorities compared to that of the fiscal players, which continue to set their policies noncooperatively.

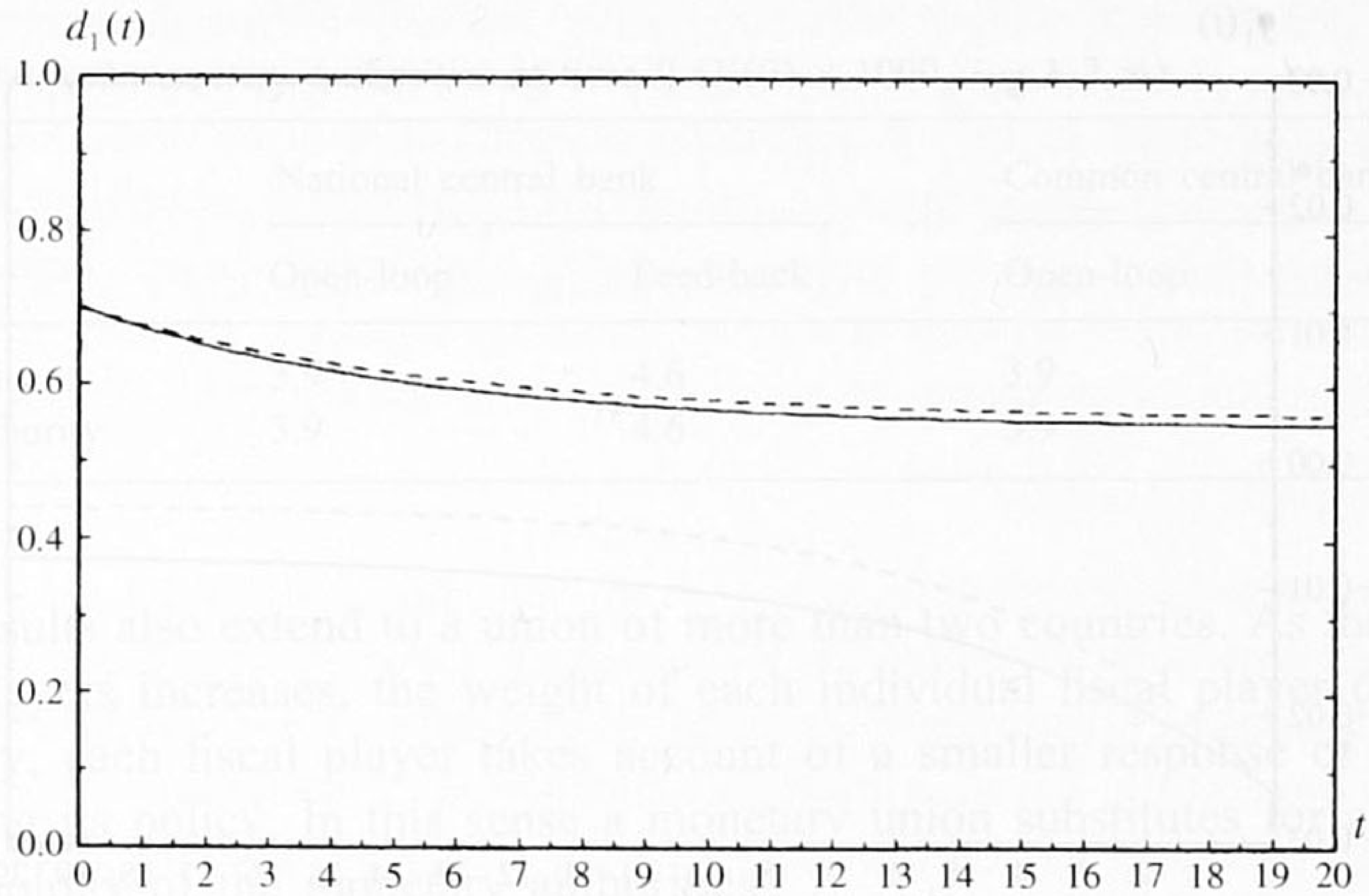

Fig. 5. Feed-back noncooperative debt in country 1 for a national (dashed) and a common (solid) central bank.

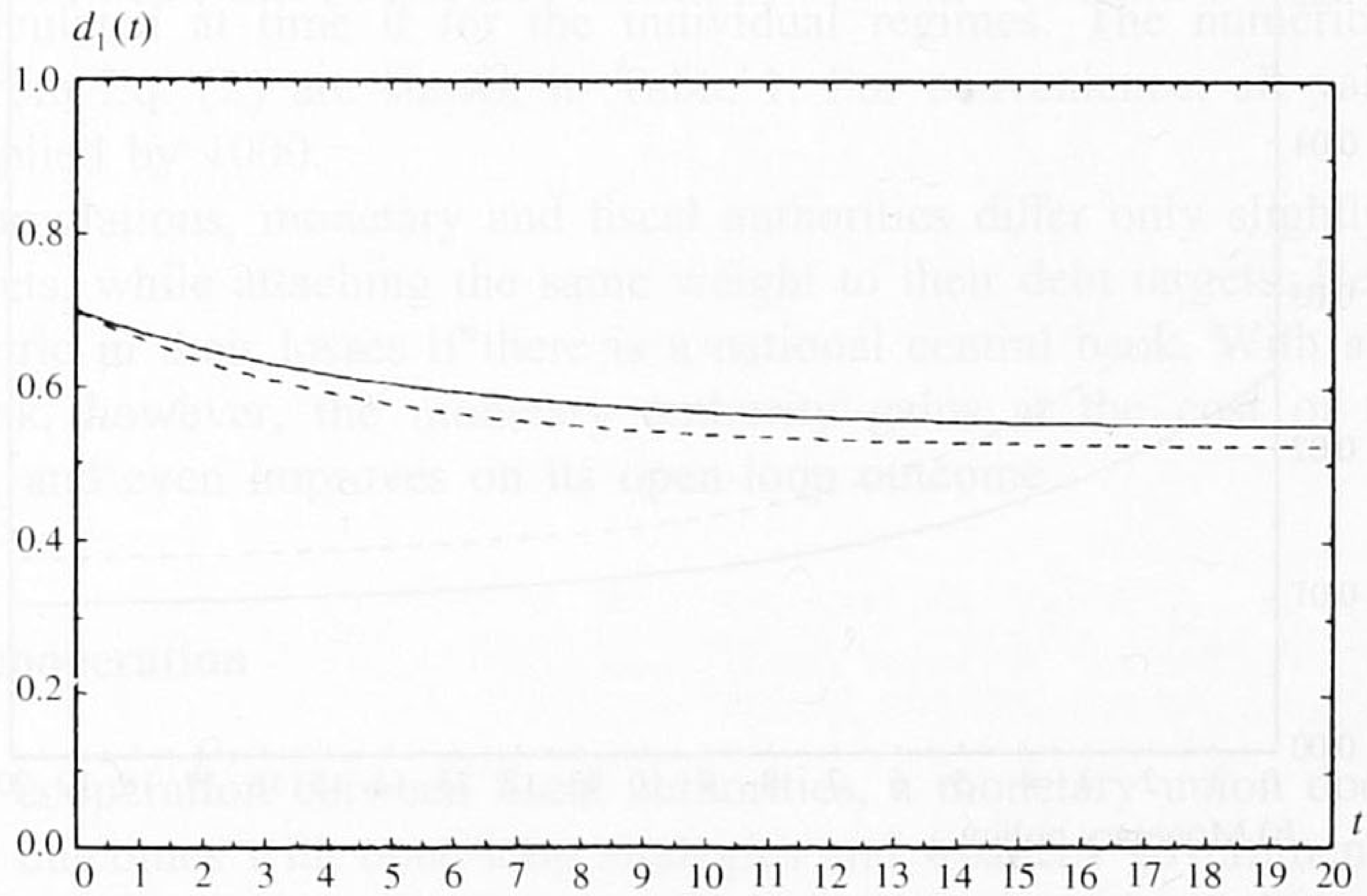

Fig. 6. Open-loop (dashed) vs. feed-back (solid) noncooperative debt for country 1 with a common central bank ($\theta = 0.5$).

How do the feed-back results compare with those of the open-loop strategies in the case of two symmetric countries and a European Central Bank? Fig. 6 contains such a comparison for public debt in country 1 in the symmetric case with $\theta = 0.5$.

As in Fig. 2, the feed-back case, depicted by the solid line, still leads to a slower adjustment of debt than the open-loop case. Accordingly, the results of Tabellini (1986), derived in a single-country setting, concerning the comparison of debt dynamics across open-loop and feed-back strategies appear to generalize

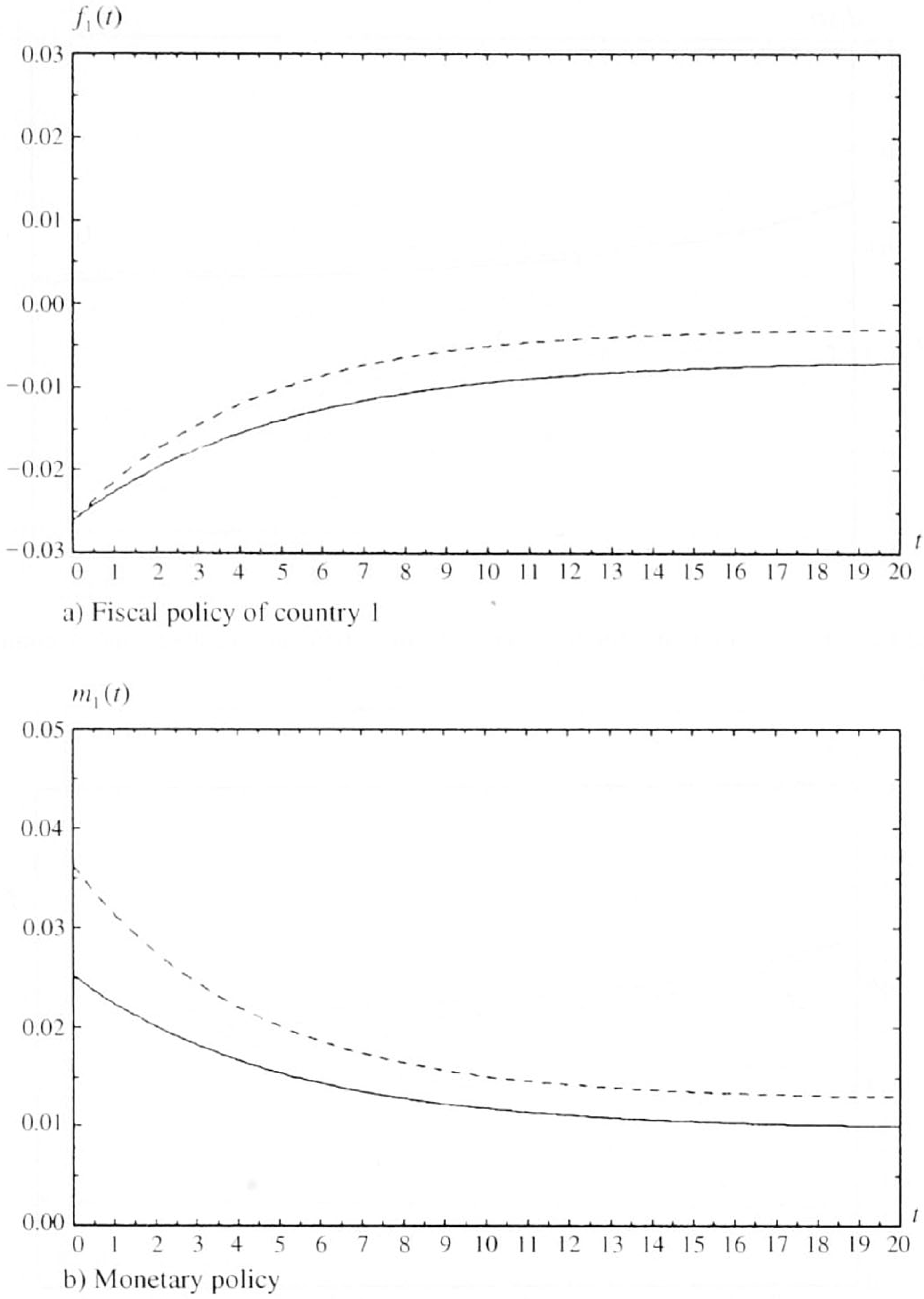

Fig. 7. Open-loop (dashed) vs. feed-back (solid) noncooperative policies with a common central bank.

to a two-country setting. However, if we compare the time paths of policy variables for the open-loop and feed-back case, we find them to be different than with national monetary policy making. In contrast with Fig. 3a, Fig. 7a shows that, in a monetary union, governments deviate further from their fiscal targets, whereas Fig. 7b reveals that the ECB moves closer to its monetary target with feed-back behavior. Intuitively, the fiscal player is forced to do more adjustment in a monetary union as the stronger monetary player does less adjustment under subgame-perfect strategies.

Table 1
Losses of fiscal and monetary authorities at time 0 ($V_i(0) \times 1000$, $i = 1, 2, m$)

	National central bank		Common central bank	
	Open-loop	Feed-back	Open-loop	Feed-back
Fiscal authority	3.9	4.6	3.9	5.2
Monetary authority	3.9	4.6	3.9	3.6

These results also extend to a union of more than two countries. As the number of fiscal players increases, the weight of each individual fiscal player decreases. Accordingly, each fiscal player takes account of a smaller response of the ECB when setting its policy. In this sense a monetary union substitutes for a commitment technology of the monetary authorities.

Our conclusions conflict sharply with the notion of the ECB as common property. Indeed, if there is any exploitation, it involves the ECB taking advantage of its stronger strategic position. This is also confirmed by the losses of both players calculated at time 0 for the individual regimes. The numerical values obtained from Eq. (2) are shown in Table 1. For convenience, all values have been multiplied by 1000.

In our simulations, monetary and fiscal authorities differ only slightly in their policy targets, while attaching the same weight to their debt targets. Hence, they are symmetric in their losses if there is a national central bank. With a common central bank, however, the monetary authority gains at the cost of the fiscal authorities, and even improves on its open-loop outcome.

4. Fiscal cooperation

Without cooperation between fiscal authorities, a monetary union does not affect policy outcomes with open-loop strategies and weakens governments' strategic positions with feed-back strategies. Therefore the question arises: How are strategic positions altered when two fiscal authorities cooperate against a common central bank?[6] In order to address this question, we specify the cooperative fiscal objective function as the weighted sum of the fiscal player's cost functions:

$$V_f(t) = \tfrac{1}{2}\int_t^{\infty} [\chi(f_1(s) - \bar{f}_1)^2 + (1-\chi)(f_2(s) - \bar{f}_2)^2 + (d(s) - \bar{d}_f)' Q_f (d(s) - \bar{d}_f)] e^{-\delta_f (s-t)} ds,$$

[6] In the context of the European Monetary Union, ECOFIN may be a natural forum for this partial coordination to occur.

where χ denotes the weight of cooperative welfare to fiscal authority 1. We assume symmetric debt targets, so that $\bar{d}_f = \bar{d}_1 = \bar{d}_2$, and the matrix weighting these targets is given by $Q_f = \chi Q_1 + (1-\chi)Q_2$.

We consider two cases: the open-loop strategies, where players can commit to an optimal policy, and the feed-back strategies, which imply subgame-perfect behavior.

4.1. Open-loop strategies

Commitment is typically regarded as an unrealistic form of strategic policy interaction. However, it serves as a benchmark for the more realistic feed-back strategies. Moreover, the open-loop concept becomes more attractive for the case of fiscal cooperation, because cooperation requires commitment of governments. If fiscal authorities can commit to a joint policy, they are less likely to defect from their optimal plans. Alternatively, if policy makers are able to commit to a specific policy, they are more likely to cooperate.

The current-value Hamiltonian for two cooperating fiscal players amounts to

$$H_f = \tfrac{1}{2}[\chi(f_1-\bar{f}_1)^2 + (1-\chi)(f_2-\bar{f}_2)^2 + (d-\bar{d}_f)'Q_f(d-\bar{d}_f)]$$
$$+\lambda_f'[rd + B_1 f_1 + B_2 f_2 - B_m m],$$

while the Hamiltonian for the ECB is the same as in Section 3.1.

The analytical derivation of the optimality conditions is provided in the Appendix. For the case of symmetric countries, numerical simulation reproduces Fig. 1, irrespective of the value of χ. This yields the following proposition:

Proposition 2. If policy makers are able to commit to optimal (open-loop) strategies, cooperation between fiscal authorities exerts no additional effect on the outcome.

The proof can be found in the Appendix. Proposition 2 states that economic performance in a monetary union is the same with or without fiscal cooperation. This increases the relevance of the noncooperative open-loop equilibrium even though the noncooperative equilibrium may not be an attractive solution concept if agents have access to a commitment technology. Together with Proposition 1, Proposition 2 implies that the cooperative outcome in a monetary union also coincides with that of a separate economy with a national central bank.

4.2. Feed-back strategies

Fiscal authorities can also be assumed to cooperate but nevertheless pursue a subgame-perfect strategy against the central bank. If the ECB does the same, its feed-back behavior is taken into account when fiscal players jointly optimize.

The Appendix shows that this only affects the optimality condition involving the costate vector and derives the necessary equations for simulation.

If we continue to assume symmetric countries, the most interesting numerical simulations involve a symmetric distribution of seigniorage ($\theta = 0.5$). The dynamics for debt, fiscal deficits and seigniorage turn out to coincide with those of the feed-back case for a closed economy, which is illustrated by the solid curves in Figs. 2 and 3. This results in the following proposition, which is proved in the Appendix.

Proposition 3. *Symmetric cooperation ($\chi = 0.5$) between the fiscal authorities of two identical countries that play feed-back strategies against a common central bank yields the same outcome as separate governments playing feed-back strategies against individual national central banks.*

Fiscal cooperation clearly strengthens the strategic position of governments, thus allowing them to pursue higher fiscal deficits at the cost of higher inflation as well as higher debt with a slower rate of convergence to the long-run target. The most interesting and appealing result, though, is that fiscal cooperation in the monetary union produces the same outcome as fiscal behavior in the closed economy. Intuitively, both monetary and fiscal authorities cooperate, thereby producing the same strategic situation as in a closed economy with national monetary and fiscal policy makers. Moreover, both institutional settings induce less fiscal discipline than the monetary union in which fiscal authorities do not cooperate.

5. Conclusions

Is a European Central Bank common exploitable property, which undisciplined governments abuse to monetize their outstanding national debts? To answer this question, we employed a dynamic framework, which captures the interaction of monetary and fiscal policy through the government budget constraint.

In a differential game with two governments and alternatively one European Central Bank or two national central banks, we analyzed the dynamics of national debts, primary fiscal deficits, and money creation. By parameterizing all economic influences we focused on the strategic issues that emerge when the financing of national debts becomes interrelated through the introduction of a common monetary institution.

If policy makers are able to commit to their optimal strategies, we showed that switching from a national to a common central bank does not affect policies in a symmetric monetary union. However, if policy makers behave in a subgame-perfect manner, which is often considered to be the more realistic case, the introduction of a monetary union has noticeable strategic consequences. In particular, the relative strategic position of the common central bank strengthens as governments lose power due to their diminished relative economic size. Consequently, fiscal authorities absorb a larger share of stabilizing public debt. Hence, fiscal policy is more disciplined than in separate countries with individual central banks.

Fiscal authorities regain the power they wielded with national central banks if they cooperate with each other. Compared to the noncooperative equilibrium, debt stabilization slows down while inflation and fiscal deficits rise. Fiscal cooperation thus results in less discipline.

If only for strategic reasons, our analysis suggests that the fear of an exploitable common central bank is unfounded. On the contrary, a monetary union strengthens the position of the central bank. As long as fiscal authorities do not cooperate, the strong monetary authority is in a better position to enforce fiscal discipline.

The two-country case can be extended in various directions. For example, the ECB could be concerned about the average deviation of debt rather than the deviation of average debt. With Q_m specified accordingly, the monetary authority reacts to the distribution of debt, with the logical consequence that monetary policy is affected by the distribution of seigniorage. However, if seigniorage is distributed according to economic size (i.e., $\theta = \omega$), all other results characterized by Propositions 1–3 remain valid.

Appendix

A.1. Optimality conditions for noncooperative open-loop strategies

The Hamiltonian of fiscal authority i ($i = 1, 2$) yields the following first-order conditions:

$$f_i = \bar{f}_i - B_i' \lambda_i = 0, \tag{A.1}$$

$$\dot{\lambda}_i = (\delta_f - r)\lambda_i - Q_i(d - \bar{d}_i). \tag{A.2}$$

The first optimality condition (A.1) shows f_i to be a linear function of λ_i. With our proposed linear policy functions (see Eq. (5)), λ_i must also be a linear function of d:

$$\lambda_i = k_i + K_i d, \tag{A.3}$$

where k_i is a 2×1 vector and K_i is a 2×2 matrix. Differentiation of (A.3) with respect to time yields $\dot{\lambda}_i = K_i \dot{d}$. This implies that the second optimality condition (A.2) can be written as

$$K_i \dot{d} = [(\delta_f - r)K_i - Q_i]d + (\delta_f - r)k_i + Q_i \bar{d}_i.$$

By comparing this equation with (7) and applying the method of undetermined coefficients, we obtain the two equations

$$(\delta_f - r)K_i - Q_i = K_i\Gamma, \tag{A.4}$$

$$(\delta_f - r)k_i + Q_i\bar{d}_i = K_i c. \tag{A.5}$$

For a given matrix Γ, Eq. (A.4) determines the matrix K_i, which can be substituted into (A.5) to obtain k_i. By substituting (A.3) into (A.1), we can solve for the feed-back coefficients of f_i given by $\alpha_i^0 = \bar{f}_i - B_i'k_i$ and $\alpha_i' = -B_i'K_i$ (see Eq. (5)). Note, however, that the system is still underdetermined because we do not yet have a solution for Γ.

We now turn to the solution of the Hamiltonian for the ECB. This, together with the assumption

$$\lambda_m = k_m + K_m d,$$

yields the equation needed to solve for the matrix K_m,

$$(\delta_m - r)K_m - Q_m = K_m\Gamma. \tag{A.6}$$

The equation for k_m is given by

$$(\delta_m - r)k_m + Q_m\bar{d}_m = K_m c. \tag{A.7}$$

Armed with these results, we can determine $\alpha_m^0 = \bar{m} + B_m'k_m$ and $\alpha_m' = B_m'K_m$.

In order to close the system of equations, we insert the results for α_1', α_2', and α_m' into (7), and again use the method of undetermined coefficients to obtain

$$\Gamma = rI - B_1B_1'K_1 - B_2B_2'K_2 - B_mB_m'K_m. \tag{A.8}$$

With (A.4), (A.6), and (A.8) we now have four equations to determine all the matrices K_1, K_2, K_m, and Γ. If we insert the results of α_1^0, α_2^0, and α_m^0 in (7), the method of undetermined coefficients provides a solution for the constant term

$$c = B_1\bar{f}_1 + B_2\bar{f}_2 - B_m\bar{m} - B_1B_1'k_1 - B_2B_2'k_2 - B_mB_m'k_m, \tag{A.9}$$

so that with (A.5), (A.7), and (A.8) we also have four equations to solve for k_1, k_2, k_m, and c.

Proof of Lemma 1

In the lemma we assume that both fiscal authorities put the same weight on the stabilization of national debt (i.e., $Q_1^{11} = Q_2^{22} = q$ and $Q_1^{22} = Q_2^{11} = 0$), while the debt preferences of the ECB are characterized by (4).

In order for monetary policy to be independent of the distribution of seigniorage, α_m^0 and α_m must be independent of θ. We have also assumed that monetary policy reacts to the weighted average of debt in the union. With $\alpha_m^0 = \bar{m} + B_m' k_m$ and $\alpha_m' = B_m' K_m$, the vector k_m and the matrix K_m must have the structure

$$k_m = \kappa_0 \begin{bmatrix} \omega \\ 1-\omega \end{bmatrix}, \tag{A.10}$$

$$K_m = \kappa \begin{bmatrix} \omega^2 & \omega(1-\omega) \\ \omega(1-\omega) & (1-\omega)^2 \end{bmatrix}. \tag{A.11}$$

Panel a in Fig. 1 shows that the debt of a country adjusts quicker to a lower steady-state level, the higher that country's share of seigniorage is. This implies that Γ is dependent on θ. The influence of θ must, however, cancel out in Eq. (A.6) in order to maintain the structure of K_m in (A.11). This is guaranteed if Γ is of the form

$$\Gamma = \gamma_0 I + \gamma \begin{bmatrix} \theta & \theta\dfrac{1-\omega}{\omega} \\ (1-\theta)\dfrac{\omega}{1-\omega} & 1-\theta \end{bmatrix}. \tag{A.12}$$

After substituting K_m and Γ into (A.6), we obtain

$$\kappa = \frac{\tau}{\delta_m - r - \gamma_0 - \gamma}, \tag{A.13}$$

and, from (A.8), we can derive

$$\begin{bmatrix} K_1^{11} & K_1^{12} \\ K_2^{21} & K_2^{22} \end{bmatrix} = (r-\gamma_0)I - (\gamma+\kappa) \begin{bmatrix} \theta & \theta\dfrac{1-\omega}{\omega} \\ (1-\theta)\dfrac{\omega}{1-\omega} & 1-\theta \end{bmatrix}. \tag{A.14}$$

Since we have assumed symmetric matrices Q_1 and Q_2, we can use the last result with either version of (A.4) in order to derive the two equations

$$q = (\delta_f - r - \gamma_0)(r - \gamma_0), \tag{A.15}$$

$$(\delta_f - r - \gamma_0 - \gamma)\kappa = \gamma(\gamma + 2\gamma_0 - \delta_f). \tag{A.16}$$

From the other equations of (A.4) we find $K_1^{21} = K_1^{22} = K_2^{11} = K_2^{12} = 0$. With (A.13), (A.15), and (A.16), we now have three nonlinear equations which determine the three parameters κ, γ_0, and γ, which are all independent of θ.

From (A.15) we obtain

$$\gamma_0 = \tfrac{1}{2}\left[\delta_f \pm \sqrt{\delta_f^2 + 4[q - r(\delta_f - r)]}\right], \tag{A.17}$$

which is one of the eigenvalues of Γ. Stability of the dynamic system requires the trace of Γ to be negative and its determinant to be positive. Thus, according to (A.12), the conditions $2\gamma_0 + \gamma < 0$ and $\gamma_0(\gamma_0 + \gamma) > 0$ must hold. Note that our parameter specifications allow us to ignore complex solutions.

If we make use of our assumption that fiscal authorities discount the future at the same rate as the ECB (i.e., $\delta_f = \delta_m = \delta$), we can use (A.13) in order to rewrite (A.16) as

$$\tau = \gamma(\gamma + 2\gamma_0 - \delta).$$

Thus, only a negative value of γ satisfies the stability condition:

$$\gamma = -\tfrac{1}{2}\left[(2\gamma_0 - \delta) + \sqrt{(2\gamma_0 - \delta)^2 + 4\tau}\right] < 0. \tag{A.18}$$

By rearranging (A.18) and inserting (A.17), we obtain

$$\gamma_0 + \gamma = \tfrac{1}{2}\left[\delta - \sqrt{\delta^2 + 4[q - r(\delta - r)] + 4\tau}\right] < 0,$$

which is the second eigenvalue of Γ. Hence, in order to ensure stability, we must choose the negative solution for γ_0 in Eq. (A.17) above. Eq. (A.13) implies $\kappa > 0$, if $\delta_m > r$, as we have assumed.

Thus, if we postulate stability of the dynamic system, the time paths of all variables are unique, as there is only one stable solution to the above equations.

Now we apply the assumed structure of k_m in (A.10) to Eq. (A.7), and obtain

$$(\delta_m - r)\kappa_0 + \tau[\omega\bar{d}_{m1} + (1 - \omega)\bar{d}_{m2}] = \kappa[\omega c_1 + (1 - \omega)c_2]. \tag{A.19}$$

Eq. (A.9) can be separated into the two equations

$$c_1 = \bar{f}_1 - \frac{\theta}{\omega}(\bar{m} + \kappa_0) - k_{11}, \tag{A.20}$$

$$c_2 = \bar{f}_2 - \frac{1 - \theta}{1 - \omega}(\bar{m} + \kappa_0) - k_{22}, \tag{A.21}$$

which together yield the weighted average

$$[\omega c_1 + (1 - \omega)c_2] = [\omega\bar{f}_1 + (1 - \omega)\bar{f}_2] - (\bar{m} + \kappa_0) - [\omega k_{11} + (1 - \omega)k_{22}].$$

We can also form the weighted sum of terms in the two equations contained in (A.5) in order to obtain

$$[\omega k_{11} + (1 - \omega)k_{22}] = \frac{r - \gamma_0 - \gamma - \kappa}{\delta_f - r}[\omega c_1 + (1 - \omega)c_2] - \frac{q}{\delta_f - r}[\omega\bar{d}_{11} + (1 - \omega)\,\bar{d}_{22}].$$

If this result is substituted into the preceding equation, we can derive

$$[\delta_f - \gamma_0 - \gamma - \kappa][\omega c_1 + (1-\omega)c_2]$$
$$= (\delta_f - r)[\omega \bar{f}_1 + (1-\omega)\bar{f}_2] - (\delta_f - r)(\bar{m} + \kappa_0) + q[\omega \bar{d}_{11} + (1-\omega)\bar{d}_{22}]. \tag{A.22}$$

Eqs. (A.19) and (A.22) yield unique solutions for κ_0 and $[\omega c_1 + (1-\omega)c_2]$ which are independent of θ.

If we solve (A.20) for k_{11}, substitute this into Eq. (A.5) for fiscal player 1, and use the solution of K_1 from (A.14), we obtain

$$(\delta_f - r)\bar{f}_1 + q\bar{d}_{11}$$
$$= (\delta_f - \gamma_0)c_1 + \frac{\theta}{\omega}\{(\delta_f - r)(\bar{m} + \kappa_0) - (\gamma + \kappa)[\omega c_1 + (1-\omega)c_2]\},$$

which we can rearrange to solve for c_1. Analogously, we can derive a solution for c_2 by using (A.21). This shows that the vector c has the structure

$$c = \begin{bmatrix} c_{01} \\ c_{02} \end{bmatrix} + \bar{c} \begin{bmatrix} \dfrac{\theta}{\omega} \\ \dfrac{1-\theta}{1-\omega} \end{bmatrix}, \tag{A.23}$$

where c_{01}, c_{02}, and $\bar{c}$ are parameters that are independent of θ. Finally, by substituting these results into (A.20) and (A.21), we can derive the solutions for k_{11} and k_{22},

$$\begin{bmatrix} k_{11} \\ k_{22} \end{bmatrix} = \begin{bmatrix} k_{01} \\ k_{02} \end{bmatrix} + \bar{k} \begin{bmatrix} \dfrac{\theta}{\omega} \\ \dfrac{1-\theta}{1-\omega} \end{bmatrix},$$

with the parameters k_{01}, k_{02}, and $\bar{k}$ independent of θ. From Eq. (A.5) we also obtain the last two coefficients, $k_{12} = k_{21} = 0$. □

Proof of Lemma 2

We derive the average level of debt with the help of the general solution given in Eq. (9). In order to determine the steady-state value of debt, we use the solution for Γ in (A.12) of Lemma 1 to compute the inverse,

$$\Gamma^{-1} = \frac{1}{\gamma_0 + \gamma} I + \frac{\gamma}{\gamma_0(\gamma_0 + \gamma)} \begin{bmatrix} 1-\theta & -\theta \dfrac{1-\omega}{\omega} \\ -(1-\theta)\dfrac{\omega}{1-\omega} & \theta \end{bmatrix}.$$

If this result is substituted into (8) together with c taken from (A.23), we obtain

$$d^* = -\Gamma^{-1}c = -\frac{1}{\gamma_0+\gamma}c - \frac{\gamma}{\gamma_0(\gamma_0+\gamma)}\begin{bmatrix} (1-\theta)c_{01} - \theta\frac{1-\omega}{\omega}c_{02} \\ -(1-\theta)\frac{\omega}{1-\omega}c_{01} + \theta c_{02} \end{bmatrix}.$$

Next, we write the adjustment matrix Γ in Eq. (A.12) as $\Gamma = X\Lambda X^{-1}$, with Λ as the diagonal matrix of eigenvalues,

$$\Lambda = \begin{bmatrix} \gamma_0 & 0 \\ 0 & \gamma_0+\gamma \end{bmatrix},$$

and X as the matrix of eigenvectors,

$$X = \begin{bmatrix} 1 & 1 \\ -\frac{\omega}{1-\omega} & \frac{\omega(1-\theta)}{(1-\omega)\theta} \end{bmatrix}.$$

By using this result in Eq. (9), we can write the weighted average of debt as

$$\begin{aligned}&[\omega d_1(t) + (1-\omega)d_2(t)] \\ &= (\omega, 1-\omega)\mathrm{e}^{\Gamma t}(d(0)-d^*) + (\omega, 1-\omega)d^* \\ &= [\omega d_1(0) + (1-\omega)d_2(0)]\mathrm{e}^{(\gamma_0+\gamma)t} - \frac{\omega c_1 + (1-\omega)c_2}{\gamma_0+\gamma}[1-\mathrm{e}^{(\gamma_0+\gamma)t}]. \quad \text{(A.24)}\end{aligned}$$

From the proof of Proposition 1, we know that γ_0, γ, and $[\omega c_1 + (1-\omega)c_2]$ are independent of θ. Thus, the average level of debt will also be independent of θ. □

Proof of Proposition 1

We impose the assumption of complete symmetry in the open-loop solutions for Γ and c, and set $\theta = \omega$. This results in identical levels of debt, which follow the same time path as average debt in (A.24) of Lemma 2:

$$d_i(t) = d_i(0)\mathrm{e}^{(\gamma_0+\gamma)t} - \frac{c_i}{\gamma_0+\gamma}[1-\mathrm{e}^{(\gamma_0+\gamma)t}], \qquad i = 1,2,$$

where $c_1 = c_2$ and $d_1(0) = d_2(0)$. For symmetric countries, all parameters in the above equation are independent of ω, as one can verify from the proof of Lemma 1. Thus, the dynamic equation holds for all values of ω and, in particular, for $\omega = 1$, which characterizes the closed economy. If we use this result to calculate the optimal fiscal deficits, we see that the time paths for the separate and united countries are identical. Finally, since the equation for individual debt is identical to the equation of average debt, regardless of the value of θ, the open-loop policies of the national and the common central bank are identical. □

A.2. Optimality conditions for noncooperative feed-back strategies

The current-value Hamiltonian of fiscal player i ($i = 1,2$) yields the two first-order conditions

$$f_i = \bar{f}_i - B_i'\lambda_i, \tag{A.25}$$

$$\dot{\lambda}_i = [(\delta_f - r)I - \alpha_j B_j' + \alpha_m B_m']\lambda_i - Q_i(d - \bar{d}_i). \tag{A.26}$$

For the costate vector λ_i we again introduce a linear feedback equation of the form (A.3). This allows us to write (A.26) as

$$\begin{aligned} K_i\dot{d} &= \{[(\delta_f - r)I - \alpha_j B_j' + \alpha_m B_m']K_i - Q_i\}d \\ &\quad +[(\delta_f - r)I - \alpha_j B_j' + \alpha_m B_m']k_i + Q_i\bar{d}_i. \end{aligned}$$

By comparing this equation with (7) and applying the method of undetermined coefficients, we arrive at the two sets of equations

$$[(\delta_f - r)I - \alpha_j B_j' + \alpha_m B_m']K_i - Q_i = K_i\Gamma, \tag{A.27}$$

$$[(\delta_f - r)I - \alpha_j B_j' + \alpha_m B_m']k_i + Q_i\bar{d}_i = K_i c. \tag{A.28}$$

The coefficients α_i^0 and α_i ($i = 1,2$) can be derived as in the previous section.

The analysis for the ECB is analogous and yields the solution for the other feed-back coefficients. Accordingly, we can rewrite (A.27) with the help of (A.8) as

$$[\delta_f I - \Gamma' - K_i' B_i B_i']K_i - Q_i = K_i\Gamma, \tag{A.29}$$

and (A.28) as

$$[\delta_f I - \Gamma' - K_i' B_i B_i']k_i + Q_i\bar{d}_i = K_i c. \tag{A.30}$$

In determining its optimal strategy, the ECB takes the feed-back policies of both fiscal authorities into consideration. This yields the following equations:

$$[\delta_m I - \Gamma' - K_m' B_m B_m']K_m - Q_m = K_m\Gamma, \tag{A.31}$$

$$[\delta_m I - \Gamma' - K_m' B_m B_m']k_m + Q_m\bar{d}_m = K_m c. \tag{A.32}$$

As in Section 3.1, (A.8) and (A.9) close the system of equations, but now yielding different feed-back coefficients α_j^0 and α_j ($j = 1,2,m$).

A.3. Optimality conditions for cooperative open-loop strategies

Optimization of the Hamiltonian in Section 4.1 yields the three optimality conditions

$$f_1 = \bar{f}_1 - \frac{1}{\chi} B_1' \lambda_f, \tag{A.33}$$

$$f_2 = \bar{f}_2 - \frac{1}{1-\chi} B_2' \lambda_f, \tag{A.34}$$

$$\dot{\lambda}_f = (\delta_f - r)\lambda_f - Q_f(d - \bar{d}_f). \tag{A.35}$$

Specifying the costate vector as a linear function of debt,

$$\lambda_f = k_f + K_f d, \tag{A.36}$$

we follow the same analytical procedure as above and find the two following equations:

$$(\delta_f - r)K_f - Q_f = K_f \Gamma, \tag{A.37}$$

$$(\delta_f - r)k_f + Q_f \bar{d}_f = K_f c. \tag{A.38}$$

By inserting (A.36) into the two optimality conditions (A.33) and (A.34) and comparing with (5), we find the feed-back coefficients for the two cooperating fiscal authorities:

$$\alpha_1^0 = \bar{f}_1 - \frac{1}{\chi} B_1' k_f, \qquad \alpha_1' = -\frac{1}{\chi} B_1' K_f,$$
$$\alpha_2^0 = \bar{f}_2 - \frac{1}{1-\chi} B_2' k_f, \quad \alpha_2' = -\frac{1}{1-\chi} B_2' K_f.$$

The optimization problem for the ECB is the same as above, and if all results are inserted in (7), the system is closed by the two equations

$$\Gamma = rI - \frac{1}{\chi} B_1 B_1' K_f - \frac{1}{1-\chi} B_2 B_2' K_f - B_m B_m' K_m, \tag{A.39}$$

$$c = B_1 \bar{f}_1 + B_2 \bar{f}_2 - B_m \bar{m} - \frac{1}{\chi} B_1 B_1' k_f - \frac{1}{1-\chi} B_2 B_2' k_f - B_m B_m' k_m. \tag{A.40}$$

Proof of Proposition 2

If open-loop outcomes are the same with or without fiscal cooperation, then this should be revealed in the matrices K_f, K_m, and Γ, as well as the vectors k_f, k_m, and c.

If K_m and Γ have the same values with or without cooperation, the fiscal authorities' optimality condition (A.37) is satisfied if

$$K_f = \chi K_1 + (1-\chi)K_2,$$

because then Eq. (A.37) can be written as

$$(\delta_f - r)[\chi K_1 + (1-\chi)K_2] - [\chi Q_1 + (1-\chi)Q_2] = [\chi K_1 + (1-\chi)K_2]\Gamma.$$

If K_1 and K_2 satisfy the noncooperative optimality conditions (A.4), they will also satisfy the equation above.

From the proof of Lemma 1 we know that $K_1^{21} = K_1^{22} = K_2^{11} = K_2^{12} = 0$. This implies

$$\frac{1}{\chi}B_1 B_1' K_f + \frac{1}{1-\chi}B_2 B_2' K_f = B_1 B_1' K_1 + B_2 B_2' K_2,$$

which shows that K_1 and K_2 also satisfy Eq. (A.39).

With analogous reasoning and the fact that $k_{12} = k_{21} = 0$ for the open-loop noncooperative case, k_1 and k_2 will satisfy (A.38) and (A.40) if

$$k_f = \chi k_1 + (1-\chi)k_2,$$

and k_m and c are the same with or without cooperation. □

A.4. Optimality conditions for cooperative feed-back strategies

Cooperative feed-back behavior changes only the optimality condition related to the costate vector to

$$\dot{\lambda}_f = [(\delta_f - r)I + \alpha_m B_m']\lambda_f - Q_f(d - \bar{d}_f),$$

while the other two optimality conditions are the same as (A.33) and (A.34). Through the method of undetermined coefficients, we obtain the two equations

$$[(\delta_f - r)I + K_m' B_m B_m']K_f - Q_f = K_f\Gamma, \tag{A.41}$$

$$[(\delta_f - r)I + K_m' B_m B_m']k_f + Q_f\bar{d}_f = K_f c. \tag{A.42}$$

The optimization procedure for the ECB is the same as in Section 3.2, leading to the optimality conditions (A.31) and (A.32), and the system is again closed by Eqs. (A.39) and (A.40).

Proof of Proposition 3

We assume identical countries that are of the same size ($\omega = 0.5$), an equal distribution of seigniorage ($\theta = 0.5$), and symmetric gains from cooperation ($\chi = 0.5$). In this case the matrices K_f, K_m, and Γ are symmetric, so that

$K_f^{11} = K_f^{22}$, $K_f^{12} = K_f^{21}$, $\Gamma^{11} = \Gamma^{22}$, and $\Gamma^{12} = \Gamma^{21}$. And since the ECB does not need to distinguish between countries, $K_m^{11} = K_m^{22} = K_m^{12} = K_m^{21}$.

With the help of (A.39) we can write first fiscal optimality condition (A.41) as

$$[\delta_f I - \Gamma' - 2K_f' B_1 B_1' - 2K_f' B_2 B_2'] K_f - K_f \Gamma = Q_f.$$

This equation is composed of 2 × 2 matrices that give rise to the four separate equations

$$\delta_f K_f^{11} - \Gamma^{11} K_f^{11} - \Gamma^{21} K_f^{21} - 2K_f^{11^2} - 2K_f^{21^2} - \Gamma^{11} K_f^{11} - \Gamma^{21} K_f^{12} = \tfrac{1}{2} q,$$

$$\delta_f K_f^{12} - \Gamma^{11} K_f^{12} - \Gamma^{21} K_f^{22} - 2K_f^{11} K_f^{12} - 2K_f^{21} K_f^{22} - \Gamma^{12} K_f^{11} - \Gamma^{22} K_f^{12} = 0,$$

$$\delta_f K_f^{21} - \Gamma^{12} K_f^{11} - \Gamma^{22} K_f^{21} - 2K_f^{11} K_f^{12} - 2K_f^{21} K_f^{22} - \Gamma^{11} K_f^{21} - \Gamma^{21} K_f^{22} = 0,$$

$$\delta_f K_f^{22} - \Gamma^{12} K_f^{12} - \Gamma^{22} K_f^{22} - 2K_f^{12^2} - 2K_f^{22^2} - \Gamma^{12} K_f^{21} - \Gamma^{22} K_f^{22} = \tfrac{1}{2} q.$$

Summation over the first two equations yields

$$\delta_f (K_f^{11} + K_f^{12}) - \Gamma^{11}(K_f^{11} + K_f^{12}) - \Gamma^{21}(K_f^{21} + K_f^{22}) - 2K_f^{11}(K_f^{11} + K_f^{12})$$
$$-2K_f^{21}(K_f^{21} + K_f^{22}) - (\Gamma^{11} + \Gamma^{12}) K_f^{11} - (\Gamma^{21} + \Gamma^{22}) K_f^{12} = \tfrac{1}{2} q. \qquad \text{(A.43)}$$

Given the symmetry of K_f, we can define a scalar parameter $\tilde{K}_f$ which is defined by

$$\tfrac{1}{2}\tilde{K}_f = K_f^{11} + K_f^{12} = K_f^{21} + K_f^{22} = K_f^{11} + K_f^{21} = K_f^{12} + K_f^{22}.$$

Analogously, the symmetry of Γ implies that we can define a scalar parameter $\tilde{\Gamma}$ by

$$\tilde{\Gamma} = \Gamma^{11} + \Gamma^{12} = \Gamma^{21} + \Gamma^{22} = \Gamma^{11} + \Gamma^{21} = \Gamma^{12} + \Gamma^{22}.$$

If we substitute $\tilde{K}_f$ and $\tilde{\Gamma}$ into Eq. (A.43) above, this reduces to

$$\delta_f \tilde{K}_f - 2\tilde{\Gamma}\tilde{K}_f - \tilde{K}_f^2 = q. \qquad \text{(A.44)}$$

Note that in order to derive (A.44), we could alternatively sum over the second two of the four equations above.

For the ECB, the optimality condition (A.31) can be decomposed into the four equations

$$\delta_m K_m^{11} - \Gamma^{11} K_m^{11} - \Gamma^{21} K_m^{21} - (K_m^{11} + K_m^{21})^2 - \Gamma^{11} K_m^{11} - \Gamma^{21} K_m^{12} = \tfrac{1}{4}\tau,$$

$$\delta_m K_m^{12} - \Gamma^{11} K_m^{12} - \Gamma^{21} K_m^{22} - (K_m^{12} + K_m^{22})(K_m^{11} + K_m^{21})$$
$$-\Gamma^{12} K_m^{11} - \Gamma^{22} K_m^{12} = \tfrac{1}{4}\tau,$$

$$\delta_m K_m^{21} - \Gamma^{12} K_m^{11} - \Gamma^{22} K_m^{21} - (K_m^{12} + K_m^{22})(K_m^{11} + K_m^{21})$$
$$-\Gamma^{11} K_m^{21} - \Gamma^{21} K_m^{22} = \tfrac{1}{4}\tau,$$
$$\delta_m K_m^{22} - \Gamma^{12} K_m^{12} - \Gamma^{22} K_m^{22} - (K_m^{12} + K_m^{22})^2 - \Gamma^{12} K_m^{21} - \Gamma^{22} K_m^{22} = \tfrac{1}{4}\tau.$$

Given the structure of K_m, we can define a scalar parameter $\tilde{K}_m$ by

$$\tfrac{1}{4}\tilde{K}_m = K_m^{11} = K_m^{12} = K_m^{21} = K_m^{22},$$

so that any of the four equations above can be reduced to

$$\delta_m \tilde{K}_m - 2\tilde{\Gamma}\tilde{K}_m - \tilde{K}_m^2 = \tau. \tag{A.45}$$

From Eq. (A.39), we obtain the four separate equations

$$\Gamma^{11} = r - 2K_f^{11} - (K_m^{11} + K_m^{21}),$$
$$\Gamma^{12} = \quad -2K_f^{12} - (K_m^{12} + K_m^{22}),$$
$$\Gamma^{21} = \quad -2K_f^{21} - (K_m^{11} + K_m^{21}),$$
$$\Gamma^{22} = r - 2K_f^{22} - (K_m^{12} + K_m^{22}).$$

The sum over the first two equations yields

$$(\Gamma^{11} + \Gamma^{12}) = r - 2(K_f^{11} + K_f^{12}) - (K_m^{11} + K_m^{21}) - (K_m^{12} + K_m^{22}),$$

which we can rewrite as

$$\tilde{\Gamma} = r - \tilde{K}_f - \tilde{K}_m. \tag{A.46}$$

The sum over the second two equations above also leads to (A.46).

Next, we decompose (A.42) into two equations, and due to the assumption of symmetry, we can define the scalar parameters $\tilde{k}_f = 2k_{f1} = 2k_{f2}$, $\tilde{c} = c_1 = c_2$, and $\bar{\tilde{d}}_f = \bar{d}_{f1} = \bar{d}_{f2}$, so that either one of the two equations of (A.42) can be written as

$$\delta_f \tilde{k}_f - \tilde{\Gamma}\tilde{k}_f - \tilde{K}_f \tilde{k}_f + q\bar{\tilde{d}}_f = \tilde{K}_f \tilde{c}. \tag{A.47}$$

If we decompose the second optimality condition of the ECB, Eq. (A.32), and define $\tilde{k}_m = 2k_{m1} = 2k_{m2}$ and $\bar{\tilde{d}}_m = \bar{d}_{m1} = \bar{d}_{m2}$, either of the two equations becomes

$$\delta_m \tilde{k}_m - \tilde{\Gamma}\tilde{k}_m - \tilde{K}_m \bar{m}_f + \tau\bar{\tilde{d}}_m = \tilde{K}_m \tilde{c}. \tag{A.48}$$

Finally, the definition $\bar{f} = \bar{f}_1 = \bar{f}_2$ lets us write either equation of (A.40) as

$$\tilde{c} = \bar{f} - \bar{m} - \tilde{k}_f - \tilde{k}_m. \tag{A.49}$$

Thus, the solutions for the matrices K_f, K_m, and Γ, and the solutions for the vectors k_f, k_m, and c can be reduced to the six scalars $\tilde{K}_f$, $\tilde{K}_m$, $\tilde{\Gamma}$, $\tilde{k}_f$, $\tilde{k}_m$, and $\tilde{c}$, which solve the six Eqs. (A.44)–(A.49). These, however, are just the optimality conditions of the feed-back game between an individual country with its own national central bank, as one can verify by setting $\theta = \omega = 1$ in the equations of Section 3.2. □

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