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Game Theory of the Past Decade, the Contribution of the Netherlands

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1 Introduction

Game theory is a mathematical theory. Its aim is to model decision situations where two or more parties have strategic interaction, and to develop and to analyse solution concepts. A party can be a person, a firm, a political party, a nation, etc.. Many of the game theoretical models stem from situations of economic interaction. Both at the micro-economic level and at the macro-economic level abundant examples can be found where strategic behavior is essential. Game theory delivers the tools, comprising ideas about what is called a solution and about how the decision situation should be strategically played by the parties.
It is generally acknowledged that game theory started with the fundamental book 'Theory of Games and Economic Behavior' (1944) by John von Neumann and Oskar Morgenstern. Their main motivation comes from the following phrase (p. 31).

> We wish to find the mathematically complete principles which define rational behavior for the participants in a social economy and to derive from them the general characteristics of that behavior.

Already from the beginning of the development of game theory it was clear that two main branches should be discerned, namely noncooperative games and cooperative games. Obviously in noncooperative games the players have not the possibility or no incentive to cooperate with each other in order to derive joint profit. Noncooperative game theoretical models presume that the strategical moves of the players are taken simultaneously and independently. However each players outcome depends on the collection of strategical moves of all the players.
In cooperative games the assumption is made that any subset of players can form a coalition which has a certain strength or worth. The main question in cooperative game theory concerns a fair division of the worth of the grand coalition, taking into account the strength of all the subcoalitions.
Not long after the appearing of the book of Von Neumann and Morgenstern, the main lines of development of game theory were set by four recent Nobel prize winners, namely John Nash (1994), Reinhard Selten (1994), John Harsanyi (1994) and William Vickrey (1996) of

which the last one died from a heart attaque a few days after the announcement. Nash (1950) introduced the concept of equilibrium point as a solution method for noncooperative games and still nowadays Nash-equilibria are recognized as the most appropriate solution idea for such games. Selten (1975) extended the equilibrium concept to multi-move decision situations. He argued that it makes sense not only to anticipate on rational behavior of the opponents but on irrational behavior as well. He introduced the concepts of perfect equilibria and subgame perfectness which can be considered as refinement concepts with respect to Nash-equilibria. Harsanyi (1967), motivated by problems of arms control, proposed models for decision situations with incomplete information. Before that the rules of the games included that all players know all the data of the game and games with incomplete information were considered to be unsolvable. Harsanyi introduced for games with incomplete information the concept of Bayesian-Nash equilibrium, a method based on statistical techniques. The present information-economy in which crucial incomplete information or knowledge is continuously updated can be seen as being formed by the work of Harsanyi. Finally Vickrey approached game theory from the economic side. He was very much interested in incentives related to asymmetric information, like public economies with as central question how far should a government influence or control economic markets. Though less mathematically rigorous, Vickry introduced many game theoretical concepts in a qualitative sense, some of them even before they were "discovered" by the mathematicians.
Besides the roles of these four Nobel prize winners one should not underestimate the historical contributions of other researchers of which Lloyd Shapley and Robert Aumann should be especially mentioned.

2 Developments in the Netherlands

Game theory in the Netherlands started with Stef Tijs. In 1975 he finished his Ph.D. Thesis "Semi-infinite and infinite matrix games and bimatrix games" at the Catholic University at Nijmegen. Already in 1987 about 15 researchers in game theory were affiliated to Dutch universities and 12 of them straightforwardly branched from Tijs. The state of the art of game theory in the Netherlands in 1987 can be found in a book dedicated to the 50-th anniversary of Tijs in which contributions are presented by his students (Peters and Vrieze(1987)).
After 1987 a turbulent development in game theory took place. The number of workers in the field largely increased, not only in the Netherlands but all over the world. Partly this is due to a revolution in economic theory where more and more game theory is accepted as delivering the toolbox for explanations of economic behavior. Of course, the above mentioned Nobel prize to game theorists increased the status of the field. Presently in the Netherlands more than 30 researchers, among which 6 at a professor level, in game theory are active and moreover one can argue whether certain researchers in economics are in fact game theoretists. In the past 10 years more than 20 Ph.D. Theses in game theory appeared in the Netherlands. In the next section most of them will be mentioned. Another 8 till 10 dissertations will be finished in the near future. Numerous papers appeared in respected international journals.
At the organisational side we can mention the editorial activities for international journals by Eric van Damme, Hans Peters, Stef Tijs and Koos Vrieze. Further, Stef Tijs is the editor of a Kluwer book series on Theory and Decisions in which 15 books appeared during the past decade, either directly concentrated on fields in game theory or related via economic decision situations. Recently 3 international conferences were organized (Tilburg, 1991 (Van Damme), Maastricht, 1996 (Peters), Twente 1996 (Driessen, Hoede, Faigle)) and next year a fourth one is planned (Maastricht, 1998 (Vrieze)). Also since 1982 a monthly game theory

seminar is organized by Tijs.

3 Developments in Noncooperative Games

A noncooperative two-person game can be characterized by a four-tuple $< A, B, r_1, r_2 >$, where A and B are the nonempty strategy sets of player 1 and player 2 respectively and where $r_k : A \times B \rightarrow IR$ is the payoff function of player $k, k = 1,2$. Generally, the players are allowed to randomize their strategies, that is they may select a strategy according to a probability measure on their available set of strategies. When $|A|$ is finite, such a probability measure is just a probability vector in a finite dimensional Euclidian space and when $|A|$ is infinite the probability measure is assumed to be defined on a given probability space. Let x denote a randomized strategy for player 1 and let y denote a randomized strategy for player 2. $r_k(x, y)$ will denote the expectation of $r_k(a, b)$ with respect to x and y.
The generally accepted solution method for non-cooperative games is the Nash-equilibrium. A Nash-equilibrium is defined as a pair of strategies (x, y), such that none of the players has a unilateral incentive for a deviation:

$$\begin{aligned} r_1(x,y) &\geq r_1(\tilde{x},y), \forall \tilde{x} \\ r_2(x,y) &\geq r_2(x,\tilde{y}), \forall \tilde{y} \end{aligned}$$

In the early eighties the structure of the set of Nash equilibria for bimatrix games (the case where both (A) and (B) are finite) were extensively studied by Jansen (1981, 1987). Van Damme studied games in extensive form and bimatrix games and he succeeded in connecting most of the refinement concepts of Nash-equilibria (Van Damme, 1987). In the thesis of Borm (1990) some remaining gaps in the properties of the refinement concepts were filled and furthermore he derived specific results in case $|A| = 2$ and $|B| = n \in IN$. In the thesis of Jurg (1993) a thorough study is done for perfect and proper equilibria and refinements thereoff. Recently the long lasting question whether any bimatrix game possesses a quasi-strict equilibrium (already suggested by Harsanyi in 1973) was solved by Norde (1994).
Kohlberg and Mertens (1986) introduced a stability concept which leads to a selection of a set of equilibria which satisfy certain properties. Vermeulen, in his thesis (1996), studied extensively this stability concept and several variations of the original definition. He succeeded in defining the essential properties of the set-valued solution concepts for non-cooperative games.
It appears as if the attemps to refine the set of equilibria to one single point have temporarily reached an endpoint by the introduction of the stable set. Peleg and Tijs (1996) introduced the consistency principle for games in strategic form. Norde, Potters, Reijnierse and Vermeulen (1996) were able to prove that there do not exist consistent equilibrium selections and consistent refinements. It is by no means overdone to say that in the past 15 years Dutch researchers have had great influence on the developments of the general theory of equilibria of noncooperatieve-games and many of the characterizing properties were elucidated in our own surroundings.
In a thesis, which opens new horizons, Hurkens (1995) relaxed the rules of a noncooperative game situation. He allowed the players to make strategical moves like commitments and communications. He considered sets of solutions instead of one point solutions. Among other results he showed that so-called persistent sets or "closed under rational behaviour" sets are appropriate for games in an evolutionary context. Further he derived unexpected results where he allowed players to burn part of their fortune, which appeared to be equivalent of buying communication lines at a high prize.

4 Developments in Stochastic Games

Stochastic games are dynamic games that evolve along a discrete time path. At any decision moment the players have to play a game identical with a static noncooperative game with the extension that the strategy choices determine a probability distribution according to which the next state (again a noncooperatieve game) is chosen, etc.. Usually the infinite horizon model is studied and the evaluation criteria for an infinite stream of payoffs normally concentrate on discounted rewards or limiting average rewards defined respectively as

$$v_k^{\beta}(x,y) = \sum_{n=0}^{\infty} \beta^n r_k^n(x,y) \quad , \quad \beta \in (0,1)$$

$$a_k(x,y) = \liminf_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N} r_k^n(x,y)$$

One should be aware of the fact that here x and y, rather than one shot strategies, denote rules that prescribe the players how to play the game along the infinite time path.
The first Dutch thesis came from Vrieze (1983) in which a complete study of the zero-sum case ($r_1 + r_2 = 0$) was elaborated. For non-zero sum stochastic games, especially for the limiting average criterion, the existence of Nash-equilibria is not yet known,and no doubt presently this is one of the most challenging open problems in game theory. In the thesis of Thuijsman (1989) a new idea based on threat stretegies is introduced. That idea is later on exploited and until now all existence results for Nash-equilibria for subclasses of stochastic games can be founded on the concepts of threat strategies. The fundamental result in this ares can be found in a paper of Vrieze and Thuijsman (1989). Stochastic games can be seen as extension of Markov Decision Problems to the competitive case. This viewpoint has recently been worked out in the first book on stochastic games by Filar and Vrieze (1997).
A promising new branche in the area of stochastic games is introduced in Joosten (1996), in which he studies stochastic games where the game parameters change over time. For instance a game with vanishing action is defined as a game where a player will forget an action in case he does not make use of it for a sufficiently long time. In Joosten, this one and related stochastic games with changing parameters are solved.
Passchier (1996) explored the application of stochastic games to light traffic control in which he knew to exploit optimally the similarities between stochastic games and Markov Decision Problems.

5 Developments in Cooperative Games

The theory of cooperative games is based on the idea of a characteristic function. Let N be the set of players, then any subset $S \subset N$ is called a coalition and a characteristic function is a function v:

$$v : 2^N \to I\!R,$$

that is, to every coalition S, a real number is associated reflecting the strength of that coalition. The main goal of the theory of cooperative games is to define and motivate solution concepts in which $v(N)$, the worth of the grand coalition, is divided among the players in a relatively fair way. Interesting work in this area is done by Driessen published in his book (1988) in which he connects several of the existing solution concepts and studies the properties of a new one, the τ-value, introduced by Tijs. In the thesis of Derks (1991) it

was shown how certain classes of games can be seen to be polyhedral cones, which enabled him to give easy proofs and more understanding of the different properties for many classes of cooperative games.
In the thesis of Otten (1995) several characterizations of solution concepts are given. Some of them apply to NTU-games, where the "utility" is no longer transferable between the players.
Finally, we want to mention the work of Curiel. In cooperation with Potters and Tijs she introduced the idea of combinatorial games. In combinatorial optimization several classical problems like travelling salesman, minimum cost spanning tree, assignment problems, etc., are studied in a one-person sense, that is, there is only one optimizer. In practice often more parties at least jointly influence the interaction and the outcome. When the worth of a coalition has to be computed, this coalition can be considered to be one unified player that has to optimize a certain type of combinatorial optimization problem. In the thesis of Curiel (1988) this idea is applied for the first time in a systematic way. In the next section we will see that several studies followed this approach. See also Curiel (1997).
Nowadays it is widely recognized that cooperative game theory has a broad range of application. Especially with respect to schemes of costs allocation cooperative game theory offers convincing tools as was already shown by Littlechild (?). More recently, applications to allocation of reduction measures in order to abate the environmental pollution were worked out (Van der Ploeg and De Zeeuw (1994), Tol (1996)). Further, in a paper of Van den Nouweland, they report about profit allocation in games constructed from cooperation of phoning in planes.
In the classical cooperative game model it is assumed that there is unrestricted communication between the players. Evidently, in practice this is not always the case and recently this phenomenon is tried to express in game theoretical terms in the thesis of Van den Nouweland (1993). She modelled communication restriction by a communication graph. As solution concepts she proposed the Myerson value and the position value. In general these two values are hard to compute, however Van den Nouweland developed elegant algoritm for several subclasses of communication games.
In the algorithmic sphere the work of Reijnierse (1995) should be mentioned. He has developed methods to compute the nucleolus. The emphasis of his work lies in TU-games and besides the algorithmic results he defined properties that make solution concepts unique and he showed all kind of relations between the different solutions.

6 Developments in Combinatorial Games

In a short period of time four dissertations appeared around this subject. In the preceding section we already motivated this area. In Kuipers (1994) numerous classical combinatorial optimization problems are studied in a game setting. For instance the travelling salesman games (especially routing games), the minimum cost spanning tree game (represented as an information graph game) and assignment games. The main results concern existence and properties of the anti-core and anti-nucleolus. "Anti" because the characteristic function has the meaning of costs and obviously players prefer the lesser costs. In the work of Kuipers the properties of the underlying combinatorial structure were fully exploited in computational schemes or other aspects of the solution concepts. Aarts (1994) mainly concentrated on minimum cost spanning tree games. For several special cases he derived new and useful results. Also Veltkamp (1995) concentrated on minimum cost spanning tree games. However in his thesis the emphasis lies on the axiomatic characterization of different solution concepts for different instances of the games. Obviously, a good characterization shows whether a concept is applicable in the light of the combinatorial properties of the decision problem. Hamers (1995) explored sequencing games in which several agents want to use one machine that can only serve one agent at a time. The obvious question concerns the sequence of the agents that determine the order in which they are served, taking into account all kind of cost structures. Hamers compared several solution methods and derived many new properties. In 1993 a special issue of ZOR (now MMOR) was edited by Borm and Tijs, dealing with games, graphs and O.R..

7 Developments in Bargaining Games

Bargaining theory can be seen as part of cooperative game theory, where the utility is not transferable. When N players are involved an N-dimensional set is defined an element of which has to be appointed, where component k is associated the player $k, k = 1, 2, \ldots, N$. In the thesis of Peters (1986) the relations between several solution concepts for bargaining games were studied. In his book, Peters (1992) worked out the axiomatic characterizations for the different solution concepts, where the axioms refer to intuitive appealing properties. Especially he studied non-symmetrical extensions of the classical solutions like Nash solutions and Kalai-Smorodinski solutions.
Houba (1994) studied a few extensions of the classical bargaining model. For a three player situation the rules of the game allow just two players to cooperate. He derived many axiomatic characterizations for solutions of such games. Further he studied bargaining games motivated by policy making decision situations. The main feature of this type of games concerns the fact that the players can influence the outcome in case of a disagreement leading to additional strategical aspects.

8 Developments in Social Choice Theory

Social Choice is about making decision in a societal surrounding. The preference relations among the alternatives of the players determine the choice behavior. In Wakker (1986, 1989) the main assumption concerns the fact that the set of alternatives can be structured as a cartesian product. Any component refers to a specific aspect of the alternative. Wakker obtains many mathematical properties of this type of decision problems. In Storcken (1989) the ordering of the alternatives reflected in the preference relations, stand central. His goal is to derive possibility theorems for the different models related to different assumptions on the orderings. In fact, his work can be considered as the first cohesive approach for the study of possibility theorems. In Van der Stel (1993) voting schemes are studied in which all players call one alternative and next a voting rule determines the alternative chosen. He derives many new results with respect to the concept of strategy-proof choices, which are choices that are non-manipulative. Finally in Otten (1995) effectivity functions for social choices are studied. Effectivity functions define sets of alternatives that are reachable for coalitions of players in the sense that they are able to veto alternatives outside this set. Many interesting results about effectivity functions can be found in Otten.
Monsuur (1994) studied choice problems, where alternatives are ordered by pairwise comparison with respect to importance, preference, dominance, quality, etc.. The idea is to come to a priority list of the alternatives based on these pairwise comparisons. Further, Monsuur studied circularity measures for tournaments.

9 Conclusions

From the above it might be clear that the contribution of the Dutch researchers to the developments of game theory is overwhelming. Nearly all the fields of game theory are covered and many new and appealing ideas and results come from our country. Together with Israel and the United States, The Netherlands is one of the leading countries in game theory. Obviously, the researchers do the job and they need to possess the mathematical level to apply their creativity. It is in this respect that the activities of the National School of Operations Research (LNMB) tremendously contributed to the fundamental knowledge of our young bright newcomers. In the same spirit, we need to mention our counterpart in economics, the National School of Quantitative Economics (NAKE), that performs a similar excellent job as the LNMB. We are quite sure that without the continuous endeavours of these two Schools to maintain the outstanding level of our young researchers, the Netherlands would never have reached their present position.

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