Identity and Modelling in Mathematical Literacy: A Case Study in Designing Mathematical Literacy Investigations

by

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Declaration

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Summary

This study was a case study undertaken as qualitative research, from the interpretivist paradigm.

This case study was the case of observing the mathematical identities of an entire cohort of grade 11 Mathematical Literacy learners from a quintile four school. Mathematical Literacy is a uniquely South African subject, offered as an alternative to Mathematics with the aims of improving accessibility to mathematics education and improve mathematical literacy rates in the country.

This study aimed to observe how the learners' mathematical identities may be influenced by their interaction with context-rich Mathematical Literacy material. I focussed on identity in terms of to what extent these learners perceived mathematics to be useful in their present and idealised future lives, and how these views informed the learners' motivations to engage in Mathematical Literacy.

Data collection was done using multiple data sources such as questionnaires, written reflections by the learners, work produced by the learners and a focus group interview. The data was collected over three months, with multiple visits to the school. As a base, an initial Likert Scale questionnaire was administered to all 170 participants to establish their current views about Mathematical Literacy and about themselves as individuals capable of, and willing to learn mathematics. The learners were then invited to participate in two separate mathematical modelling orientation sessions. During these sessions, learners were given the opportunity to discuss and attempt to mathematise problems they were experiencing in their school environment. I used the ideas produced by the learners to formalise two mathematical investigations based on mathematical modelling principles. These mathematical investigations were completed by all the learners as part of their formal school assessment program, within the curriculum requirements and with permission from the school. The learners' work from these investigations were mapped against existing modelling competencies.

Based on their individual reflections to the orientation sessions, their questionnaire responses, and their willingness to participate, a group of 10 learners were selected for a focus group interview. The focus group interviews provided insight into how the learners' experiences with the context-rich investigations, as well as with Mathematical Literacy in general, informed their mathematical identities.

I analysed the focus group data using grounded theory and thematic analysis. From the data it was evident that these learners held an overtly positive mathematical identity that had been established through their keen ability to accept only positive narratives from their immediate environments, and to disregard narratives that threatened their self-held views. The data also indicated that being solely exposed to standardised, contextually shallow materials had hindered the learners' ability to envision the role of mathematics in their lives, thus further misinforming their identities.

In conclusion, I draw on the literature about the global need for mathematical literacy, as well as the nature and intended aims of Mathematical Literacy as a subject to argue a cause for the use of mathematical modelling as a means of instruction to enrich learning experiences and accurately inform the learners' mathematical identities.

Opsomming

Hierdie studie was 'n gevallestudie wat as kwalitatiewe navorsing onderneem is, van die interpretatiewe paradigma.

Hierdie gevallestudie is gebasseer op die waarneming van die wiskundige identiteite van 'n algehele groep leerders in graad 11 Wiskundige Geletterdheid van 'n kwintiel vier skool. Wiskundige Geletterdheid is 'n uniek Suid-Afrikaanse vak, wat as alternatief vir Wiskunde aangebied word, met die doel om beide die toeganklikheid tot wiskunde-onderwys sowel as die wiskundige geletterdheidskoerse in die land te verbeter.

Hierdie studie se doelwit was om te bepaal hoe die leerders se wiskundige identiteite beïnvloed kan word deur hul interaksie met konteksryke Wiskundige Geletterdheid materiaal. My fokus is spesifiek op die bepaling tot watter mate die leerders wiskunde as nuttig beskou in hul huidige en ge-idealiseerde toekomstige lewens, en hoe hierdie sienings die leerders se motiverings om by Wiskundige Geletterdheid betrokke te raak, inlig.

Data-insameling is gedoen deur verskeie databronne soos vraelyste, geskrewe refleksies deur die leerders, werk wat deur die leerders geproduseer is, en 'n fokusgroeponderhoud. Die data is oor drie maande met verskeie besoeke aan die skool ingesamel. As basis is ‘n aanvanklike Likert Skaal-vraelys deur al 170 deelnemers voltooi om hulle oorspronklike sienings oor Wiskundige Geletterdheid vas te stel asook hulle eie siening as individue wat in staat is, en bereid is, om wiskunde te leer. Die leerders is genooi om aan twee afsonderlike wiskundige modellering oriënteringsessies deel te neem. Gedurende hierdie sessies is leerders die geleentheid gegee om probleme wat hulle in hul skoolomgewing ervaar te bespreek en te poog om hierdie probleme wiskundig te verwoord. Ek het die idees wat deur die leerders verwoord is, gebruik om twee wiskundige ondersoeke te formaliseer in ooreenstemming met wiskundige modelleringsbeginsels. Hierdie ondersoeke is daarna deur al die leerders voltooi as deel van hul formele skoolassesseringsprogram, binne die kurrikulumvereistes en met die toestemming van die skool. Die leerders se werk van hierdie ondersoeke is met bestaande modelleringsvaardighede vergelyk.

Gebaseer op hul individuele refleksies van die oriënteringsessies, hul vraelysreaksies, en hul bereidwilligheid om deel te neem, is 'n groep van 10 leerders gekies vir 'n fokusgroeponderhoud. Die fokusgroep onderhoude het insig gegee oor hoe die leerders se ervarings met die konteksryke ondersoeke, asook met Wiskundige Geletterdheid in die algemeen, hulle wiskundige identiteite ingelig het.

Ek het die fokusgroep data ontleed deur gebruik te maak van gegronde teorie en tematiese analise. Uit die data was dit duidelik dat hierdie leerders 'n oordrewe positiewe wiskundige identiteit geopenbaar het wat gevestig is deur hulle ywerige vermoë om slegs positiewe terugvoering uit hul onmiddellike omgewings te aanvaar, en om negatiewe sienings, wat hul selfbeeld bedreig het, te verontagsaam. Die data het ook aangedui dat, om uitsluitlik aan gestandaardiseerde kontekstueel-arm materiaal blootgestel te word, die leerders se vermoë verhinder om die waarde van wiskunde in hul toekoms te sien, wat lei tot verdere misvorming van hul wiskundige identiteit.

Ten slotte, wend ek my tot die literatuur oor die wêreldwye behoefte aan wiskundige geletterdheid, asook die aard en beoogde doelwitte van Wiskundige Geletterdheid as 'n vak, om wiskundige modellering voor te stel as die manier van onderrig om leerervarings te verryk en die leerders se wiskundige identiteit akkuraat in te lig.

Abbreviations, Acronyms and Terms Used

CAPS -	Curriculum and Assessment Policy Statement
DOE -	Department of Education
FET -	Further Educational and Training, the final band of the South African high school system constituted of grades 10 to 12
ID -	Identity, used as, and interchangeably with, the concept of mathematical identity.
ML -	Mathematical Literacy as a South African subject. Distinguished from mathematical literacy as an attribute.
Mathematics -	Refers to the school subject of Mathematics, as defined by the curriculum. Distinguished from the broad field of mathematics.
NCS -	National Curriculum Statement
NSC -	National Senior Certificate
WCED -	Western Cape Education Department

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CHAPTER 1: INTRODUCTION AND PROBLEM STATEMENT

1.1 Introduction

This study is a case study where the relationship between involving learners in the design of context rich Mathematical Literacy material and the learners' mathematical identities, in one high school in South Africa, is explored. In this chapter I will outline the prevalence of the selection of Mathematical Literacy, as a subject, by learners in South Africa. I will also describe the context of the school where this case-study took place. Understanding the context of this study will provide key insight into the description of the rationale for and purpose of this study.

1.2 Orientation and background

1.2.1 The prevalence of Mathematical Literacy as an elective subject in South Africa

Mathematical Literacy (ML) is an FET-band school subject that is offered as an alternative to Mathematics. To choose one or the other, however, is compulsory for Grade 10 to 12. ML has been growing in the number of participants year after year since 2008 (Long, Bansilal, & Debba, 2014). Statistics from the Department of Education show that the number of matric participants had increased by 6362 learners from 2019 to 2020 alone, and were expected to increase further in the coming years (Department of Basic Education, 2020). Furthermore, of all the matriculants who wrote the 2019 NSC examinations, 59% of the learners had chosen ML as a subject over Mathematics. However, despite being a subject that caters to more than half of our matriculants each year, there has been relatively little recent research on ML as a subject (Meyer, 2010), with many articles in my search dating back to 2010 and earlier. In this study, I aimed to, however marginally, address this gap in research and investigate learners' experiences in engaging in ML investigations. This type of research is important to empower both learners and teachers (Meyer, 2010), in order to address the concern that in 2019, 56% of the learners who wrote the ML final examination obtained a mark under 40% (Department of Basic Education, 2020).

1.2.2 The context of the study

This study took place at an underprivileged high school in a township in the Western Cape. For ethical reasons and since this is but one of two high schools in the area, the township cannot be named.

However, the environment in which this school resides, is one that is exposed to gang violence and frequent disruptions. I have had contact with the school for the past four years and I was also a teacher at the school for six months. I have experienced this as a school that does not only face the challenge of lesser resources, but also of circumstantially reduced academic time in relation to teaching the entire cohort of learners. This is not only due to violence but also due to a school culture where, often, learners stay away from school on Fridays, rainy days and most school days after the test cycle has passed.

Majority of the learners in this study performed poorly in mathematics in grade 8 and 9 – poorly referring to barely attaining the pass mark of 40%. The result is that only a handful of learners continued with Mathematics in grades 10 to 12. During this study, I worked with the grade 11 ML learners. The grade 11 group, as a whole, consisted of 209 enrolled learners, of which 179 chose Mathematical Literacy as a subject. My intended sample group was 179 learners, but as mentioned, the culture of attendance was poor and the real sample size declined during the course of the study, losing about 20 learners from the initial to the final questionnaire.

I chose to work with this group of learners because I had been engaging with some of them in a mathematics club in their Grade 8 to 10 years. I knew from the research we had done back then, that these learners were shy and weary of authority. I knew that the fact that I had already developed a relationship of trust with these learners would open the door to more constructive participation on their part, and thus more valid and rich data.

1.3 Rationale and commencement of the study

My interest in this study stemmed from my experience in having taught Mathematical Literacy to grade 11 and grade 12 learners at this very school. I had formed the opinion that Mathematical Literacy is a meaningful, experiential subject with the potential to enrich the lives of learners in practical ways. However, I was also of the opinion that the materials and implementation of the subject (at this school at least), with specific reference to the contexts used, were barriers to creating meaningful learning experiences because they were too far removed from the realities of the learners. As a result, I saw in my learners a belief that the subject was too hard and that they could not do the math, or that the math held no relation to their lives outside of the classroom and was therefore not worth spending more than the necessary time on to pass. In many cases, the learners could not wait to finish school and leave mathematics behind.

Furthermore, I found that research into Mathematical Literacy was primarily focussed on the interactions between the teachers and the learners. Very little attention has been given to how the interaction between the learners and the experience of working with authentic mathematical modelling tasks, influences the learners' perceptions of the subject of ML, as well as their perceptions of themselves as learners capable of doing mathematics.

I approached the school with my idea to conduct my Masters study at their institution. I shared with the school my concerns that, although the school uses materials that are on the required CAPS standard and have for the most part been designed by the WCED, the learners were losing out on meaningful learning opportunities because the contexts used in these materials did not translate to their immediate environment. I felt the potential usefulness of ML was getting lost as learners could not, in my opinion, envision how to use the skills learned in ML in their own lives. I was supported in this notion by the subject head who told me that the most recent Gr 12 ML investigation was centred around the fuselage of an aeroplane. She shared her notions that she did not expect the learners to do well in the investigation as they had never, and probably would never, see the inside of an aeroplane.

I developed the idea of getting the Gr 11 learners directly involved in choosing and developing the contexts of their mathematical investigations. These investigations formed part of their formal assessment program as per curriculum guidelines. The deputy principal and the subject head agreed to the investigation counting as part of the formal assessment for these learners. The only requirement on the part of the school was that this study should be *helpful* to the teachers and not add to their burden. Therefore, when defining the roles of all the educators and myself, the task of setting up the investigation and the memorandum, as well as the marking of every single student's work became my sole responsibility. The teachers agreed only to stand off teaching time for me to run two separate orientation sessions and conduct a pre- and post-study questionnaire with the learners, to lend a hand in the fine tuning and approval of the investigations design and to be present to assist with discipline during the said orientation sessions and questionnaires.

1.4 Problem statement and purpose

The purpose of this study is to describe and analyse the perceptions that learners, who participate in Mathematical Literacy in an under-resourced school, have of themselves as individuals who are (in)capable of doing math and of seeing the applicability of mathematics in their everyday lives, as a result and nature of their participation in ML. The intention is to involve learners in the design of a mathematical investigation task, which is embedded in their immediate contexts, and to study the extent to which learners evaluate this engagement as contributing to a meaningful learning experience, as well as the effect this will have on their mathematical identity.

1.5 Research questions

The design and implementation of this study aimed to answer the following questions:

How does the involvement of learners in the design of context rich modelling tasks for Mathematical Literacy affect their mathematical identities?

a) How does involvement affect their perception of the relevance of mathematics in their lives outside the classroom?
b) How does their involvement affect their alignment to and motives for participating in Mathematical Literacy?

1.6 Research methodology

1.6.1 Research paradigm

This study was undertaken as qualitative research from an interpretivist perspective. Qualitative research is focused on creating an understanding of the processes and contexts that underlie various problems or research topics, and studies people or systems by either interacting with them or merely observing them in their natural environment (Nieuwenhuis, 2014, p.51). Maxwell (2013, p. 168) also brings attention to this, stating that qualitative research design will, by nature, also take the contextual evidence of that which is being studied into account. Merriam (2009, p. 39), enriches this definition by describing qualitative research as research aimed at discovery, collecting insight and "understanding the perspectives of those being studied" in order to make a difference in their lives.

In line with qualitative research design is the interpretivist perspective. This paradigm is focused on understanding human action and is appropriate to the fields of social sciences

(Connole, 1993) such as, in this instance, education. Through an interpretive perspective, one aims to understand the subjective experiences of the individuals partaking in the study with the direct intent of understanding the actions undertaken on their part (Connole, 1993). This approach also acknowledges the existence of many realities (Connole, 1993), implying that participants may yield vastly different experiences and beliefs regarding one topic of study, and that these beliefs could be interpreted in a variety of ways. Finally, in interpretivist design, observation as means of data collection is done, amongst other methods, through means of dialogue aimed at developing an understanding at the level of ordinary language (Connole, 1993).

The purpose of this study was to observe, document and analyse the interaction between learners and Mathematical Literacy material, as well as their subjective experiences, within the natural environment of a school classroom. This was done to establish whether or not engagement in context rich material-design impacted the mathematical identities of the learners. The learners involved were given the opportunity to relay their subjective experiences and personal opinions in an attempt to assist me in forming an understanding of how this specific interaction in the ML classroom potentially exerted influence over learners' beliefs about their mathematical capability and willingness to engage. Therefore, this study can be classified as a qualitative study undertaken from an interpretivist perspective.

1.6.2 Case study research

This study was undertaken as a case study. Case study research can be defined as undertaking a systematic and critical enquiry into a specific situation, in order to generate an understanding that could add to an existing body of knowledge (Nieuwenhuis, 2014; Simons, 2009). It is a study that exists within a bounded system and aims to offer insights into specific dynamics (Nieuwenhuis, 2014). This method is flexible in its data collection and analysis strategies (Timmons & Cairns, 2010), which is beneficial to studies undertaken in education due to multitude of dynamics that influence classroom interactions.

This study was the case of observing the potential changes in learners' mathematical identities in relation to ML, by involving all the Grade 11 learners from an under-resourced school in the modelling of mathematical situations. Because they are learners who historically delivered low marks (and low grade-averages) for the subject of Mathematics, I hypothesised that they would really struggle to apply mathematical concepts to contexts that are too far removed from their immediate environment.

1.6.3 Design based research

This study was designed primarily as a case study. However, due to the nature of my data collection, I have also drawn on elements of design-based research. This is due to the fact that I had two iterations of modelling orientation sessions, whereby the design and experiences of the first, informed the design of the second in order to ensure increased value in the data collected. By drawing on aspects of design-based research, I could address a range of complex education problems (Bakker & Van Eerde, 2015), which I could not foresee when embarking on this study. Therefore, design-based research practice lessened the gap between what I had theorised would be effective orientation session design, and the effects of practically implementing these sessions (Bakker & Van Eerde, 2015).

1.6.4 Literature study

A literature study was completed for this thesis to develop a context rich background against which effective interpretation of the data could take place. The literature study also served the purpose of improving my own understanding of the history, development and intended purpose of ML against an international backdrop.

A potential gap in my literature study pertains to the contextual factors that could implement the identities of the learners. The learners who participated in this study reside on the edge of one of the wealthiest communities in South Africa. They are simultaneously exposed to poverty and affluent lifestyles. The connection between this sort of exposure and the development of identity was not explored. Furthermore, in order to narrow the focus of this study, the effects of classroom culture and teacher input were not considered.

1.6.5 Data collection methods

This study made use of multiple data collection methods.

The first data collection method used was a pre-study and post-study Likert Scale questionnaire. I used this method because questionnaires are the tool that is the most widely used when measuring attitudes (Albaum, 1997; McLeod, 2008; Michalopoulou & Symeonaki, 2017), including those attitudes pertaining to mathematics (Ivanov, Ivanova, & Saltan, 2018; Michalopoulou & Symeonaki, 2017).

Secondly, I made use of the materials produced by the learners during the orientation sessions, including individual, written reflections of their experiences. I also marked and analysed all the work produced during both formal investigation tasks. I had the hard copies of all these materials and responses.

Lastly, I made use of semi-structured focus group interviews. My experiences with these learners had taught me that they would be more forthcoming and willing to talk to the researcher (myself) in the safety of a group. Therefore, I drew on the fact that focus group interviews are ideal in cases where learners tend to be reserved and there is reason to believe that group interaction will be productive in widening the range of responses (Nieuwenhuis, 2014). These interviews were recorded on an audio device and transcribed.

1.6.6 Participant selection

The school at which this case study wat undertaken was selected due to my familiarity with the school and was a case of purposive sampling (Maree & Pietersen, 2014). I was familiar with the ethos and challenges presented in the school and had established relationships with the staff and learners. This created an accessible environment where I could do my study with support from the school, rather than placing a time burden on the school.

The choice to work with the grade 11 learners was also a case of purposive sampling (Maree & Pietersen, 2014). I chose to work with these learners because I had been working with a number of them in a mathematics club, outside of the school context. I had a relationship of trust with them, which I believed would allow them to speak to me more openly and honestly. However, I had never formally taught these learners myself, thus felt I was still far enough removed from their educational situation to make objective observations and interpretations.

1.6.7 Data analysis and interpretation

This study made use of grounded theory to analyse and interpret the data. In this case study, the intent was to produce a theory, that is grounded in the data gathered from the experiences of the learners, that explains this interaction (Miller & Salkind, 2012; Nieuwenhuis, 2014). The analysis of the data was done through a systematic approach, involving the development of codes and the comparison of these codes (Miller & Salkind, 2012; Nieuwenhuis, 2014; Strauss & Corbin, 1990). The comparison of the codes was undertaken as thematic analysis.

The modelling competencies of the learners were also analysed by mapping the data collected against established modelling competencies from the work of Maass (2006). The mapping was done using rubrics I designed to outline the criteria that would be expected for each modelling competency, in relation to the modelling task.

The full methods for data collection, analysis and interpretation are discussed in Chapter 3.

1.7 Structuring of the dissertation

Chapter one of this dissertation provides the background, rationale, and ethical considerations for the study. It also serves to define the research questions and outline the methodology undertaken in this study to answer these questions.

Chapter two is a literature study, compiled of existing literature pertaining to mathematical literacy as an attribute, ML as a subject and mathematical identities. I explored the definition of mathematical literacy and trace the history that lead to the need for and development of ML. I also explored the challenges related to ML and how these aspects connect to the mathematical identities of the learners.

Chapter three is a full discussion of the research design and methodology. In this chapter I outline the justification for the use of qualitative research, case study research and design-based research. I also describe my data analysis and interpretation methods in detail, illustrating how I ensured authenticity and validity of the data, whilst also indicating how I addressed the potential of bias that could arise from my implicit involvement in the research.

Chapter four describes the data after an in-depth analysis was done of all the data sources. The data analysis is described according to the emerging themes, drawing upon multiple relevant data sources for each theme.

In Chapter five I discuss my interpretation of the data analysis by drawing on research from the field and from my literature study. I pose a direct answer to the research questions of this study and develop a theory for the relationship between the use of ML materials, modelling as a means of instruction in ML and the learners' mathematical identities. In this chapter I also draw up the conclusions of the study and discuss the limitations. Finally, I recommend potential directions for further research based on this study.

1.8 Ethical considerations

Before the commencement of this study, I received permission to conduct the research from the Western Cape Department of Education (see permission documentation in Addendum 1) within the time period 4 March to 27 September 2019. Upon the granting of this permission, I received permission from the Ethics Committee of Stellenbosch University (see permission documentation in Addendum 2). After consultation with the school at which this study took place, I also received written consent from the principal to conduct my study (see Addendum 3). I then proceeded to write a letter to the parents and guardians of the Gr 11 learners (see Addendum 4). Only the learners whose parents or guardians returned signed consent forms were approach to partake in this study. In this case, it was all the ML Gr 11 learners. Finally, before conducting the pre-study questionnaire, the learners themselves were briefed on the nature of the study and informed that no part of the data collection – other than the formal assessment task – was compulsory. The learners had the right to refuse to participate or withdraw from participation at any point in the study. The names of the school and learners are also omitted from this dissertation, to protect the anonymity of the participants. Where names have been mentioned, pseudonyms, selected by myself, were used.

1.9 Summary

This chapter provided an overview of the background and rationale of this study. I defined the problem statement, purpose, and research questions. A brief summary of the research methodology was given, and the ethical considerations were described.

Chapter 2 will be a description of the literature study that was undertaken for this study.

CHAPTER 2: LITERATURE STUDY

2.1 Introduction

In this chapter, I review the literature that would create a backdrop which would inform data collection design, and against which the interpretation of the data would take place. As mentioned in Chapter 1, the purpose of this study is to explore how the learners' interaction in the design of context rich ML investigations could influence their mathematical identities. In this chapter I explore the historical view of mathematical literacy as an attribute, the development of ML as a subject in South Africa and the relationship between ML, modelling type tasks and mathematical identity (ID).

2.2 A historical view of mathematical literacy as an attribute

2.2.1 The need for mathematical literacy on a global scale

2.2.1.1 Providing access to the knowledge economy

Mathematical literacy is a concept of US decent and has been defined in various ways by numerous international organisations such as PIAAC and PISA (Jablonka, 2015). One definition as offered by the OECD (2019), is that mathematical literacy is the ability to engage in mathematics in such a way that it enables the practitioner to make well-founded judgements about the role the mathematics plays in their lives and in citizenship. According to the OECD (2019), mathematical literacy is about solving problems and not performing operations, it is about connecting the mathematical content and processes to the situations in which they unfold and thus may not speak to the goals of traditional schooling.

Although there are discrepancies in the definition of mathematical literacy (Julie, 2006), consensus centres around the emphasis that is placed on the development of competencies that transcend school based mathematics and is applicable to a wide array of real-life contexts (Jablonka, 2015). It is a concept that is driven by the economic and political implications of society and therefore, Jablonka (2015) deduces that different mathematical literacy curricula prepare learners in different ways for the 'knowledge economy' – an economy, as defined by Oxford Languages, as one that depends on the quantity, quality and access to information by members of the society. As per this deduction, Jablonka (2015) then argues that regardless of how mathematical literacy is defined, the primary goal of mathematical literacy education, world-wide, is to prepare learners to successfully enter the knowledge economy. In order to achieve this goal, mathematical literacy curricula cannot be focussed on tasks of a purely mathematical nature, but should increase the interaction between practice and theory by foregrounding contexts pertaining to daily life and professional practices (Jablonka, 2015). Mathematical literacy curricula do not need to replace developed mathematics curricula, but can provide many learners with access to mathematics when offered as an alternative to the mainstream (Jablonka, 2015).

2.2.1.2 Addressing social inequalities

In addition to, and perhaps part of, providing access to the knowledge economy, Frankenstein (1990) refers to critical mathematical literacy as a means to address social inequalities. In her study, she defines critical mathematical literacy as understanding numerical data in order to deepen the appreciation of a situation, and as a result question the assumptions about societal structures. Frankenstein (1990) designed a curriculum for statistics based on this premise, that aimed at empowering minorities such as people of colour, women and working- or lower-class employees. Her study was borne from the need to get these minorities to enter the fields of Mathematics and Science (Frankenstein, 1990). In order to accomplish this, she needed to improve the learners' understanding of how mathematics is involved in their practical, daily lives. She designed a curriculum for her statistics module that focussed on real-life data and open-ended problem solving (Frankenstein, 1990). The tasks were designed to encourage her learners to use numerical data to confront race and gender inequalities that were experienced first-hand by these learners (Frankenstein, 1990).

Frankenstein's approach to mathematical literacy education is supported by Verzosa (2015), who defines mathematical literacy as a multidisciplinary approach to mathematics education, that is essential to promote engagement between learners and the issues of values, politics and social justice. Julie (2006) further offers that mathematical literacy provides learners with the proficiency to interact with mathematical constructs as they appear in society. Verzosa (2015) states that, even at middle school level, we can start to develop 'response-able' (Verzosa, 2015, p. 349) members of society by allowing room for the discourse of mathematics education to move beyond the walls of the classroom and address questions pertaining to society, culture and politics (Verzosa, 2015). In engaging with mathematical literacy curricula, learners have the potential to challenge realities that are often taken for granted, and incite action for change (Verzosa, 2015). A curriculum of this nature should, according to Verzosa (2015), foreground real world problems, yet not compromise on mathematical competencies. The contexts used should create a channel that allows learners to understand how mathematics relates to the world around them. In support of this idea, both Verzosa (2015) and Frankenstein (1990) iterate how complex mathematical concepts, such as calculus and statistics, can be used to explore problems pertaining to social and environmental change. Critical mathematical literacy can therefore be seen as “mathematics in and for action” (Julie, 2006, p. 63) as it is focussed on both citizenship and promoting the interest in and understanding of the mathematical models that explain the structures of our societies.

2.2.2 The need for explicit focus on mathematical literacy in South Africa

As a South African, I can personally attest to the fact that our country is rife with social inequalities – inequalities pertaining to wealth, access to medical care and infrastructure, and access to education. Addressing these social injustices is listed as a primary objective of our basic education system (Department of Basic Education, 2011a). However, it was our poor results in international benchmark studies that first sparked the interest in developing a ML curriculum in South Africa.

2.2.2.1 Poor performance in international studies.

South Africa has scored repetitively low on the international assessments for mathematics. We were ranked last of all participating countries according to the Trends in International Math and Science Studies (TIMSS) statistics in 1995, 1998 and 2003 (Bansilal, James, & Naidoo, 2010). In 1998 South Africa was ranked last of 38 participating countries, having scored extremely low in every topic and with averages more than 50 points below our closest competitor (Howie, 1999). Our average score is at a level described as ‘skill not achieved’ (Letaba, 2017). The results showed very little improvement as recently as 2015. The TIMSS tests the mathematical (and language) competencies of learners in grades four and eight in as many as 48 participating countries. South Africa sends learners in grades five and nine to participate in these studies (Letaba, 2017). In 2015 only 1% of our participating learners were displaying advanced skill levels and 83% of grade five’s and 87% of grade nine’s did not achieve any skill level (Letaba, 2017). As a result, South Africa placed 47th out of 48 participating countries (Letaba, 2017). Furthermore, although we do not partake in the OECD assessments (PISA), the organisation ranks South Africa, statistically, as displaying the second lowest levels of mathematical literacy in the world (OECD, 2019). Alongside this ranking, and perhaps as a result of low levels of mathematical literacy, the OECD also ranks

South Africa as having the lowest attainment of tertiary education of all countries associated with the organisation (OECD, 2019).

These results painted a dire picture for mathematics education in South Africa. As a result, due to political will, and not teacher initiative (Buytenhuys & Graven, 2011), it was decided to address these low levels of mathematical literacy explicitly (Bansilal, Mkhwanazi, & Mahlabela, 2012) by creating an intervention programme – Mathematica Literacy (ML) as a school subject (Bansilal et al., 2012). The low levels of mathematical literacy in our population would undoubtably lead to low levels of employment and economic development, thus calling for government action (Bansilal, Webb, & James, 2015). The development of mathematical literacy competencies requires the compulsory study of mathematics (Julie, 2006). At the time in South Africa, Mathematics was an elective subject for learners in grades 10-12, resulting a mere 60% of all learners participating in mathematics beyond the age of 15 (Bansilal et al., 2015). It was evident that in order to improve mathematical literacy levels among our learners, mathematics would have to be a compulsory subject; but that a curriculum would have to be designed that was a more accessible alternative to 'pure mathematics' (Julie, 2006). Therefore, the decision was made to design and implement a curriculum for Mathematical Literacy, as a subject offered to learners in grade 10 to 12. It was intended as a means of offering a differentiated approach to mathematics education, to provide improved access to tertiary studies, and to provide a feasible curriculum to schools that were 'doomed' to low levels of math education due to low socio-economic status (Julie, 2006).

2.2.2.2 Preparing learners for active citizenship

South Africa's low performance in international studies is strongly linked to socio-economic factors (Letaba, 2017). In a Post-Apartheid South Africa, there is great social disparity that needs to be addressed through education. The social injustices experienced by Frankenstein (1990) in the US in the 1990s, of racial, gender and economic bias, are not far removed from those we experience in South Africa today, allowing for the argument that mathematical literacy may be a vehicle to address these issues in our own country. When the Department of Education first assigned a task team to develop the ML curriculum, it was still under the National Curriculum Statement (NCS). In the NCS for Mathematical Literacy (Department of Education, 2003, p. 10), the purpose of the ML curriculum is described as establishing active citizenship in a developing democracy, by developing in learners, a critical stance with relation to mathematical arguments that are presented in the media and on other platforms, as well an understanding of how these numbers can be used to shape policy, thus rendering the learner able to vote effectively. In short, the purpose of this curriculum is to develop a response-able learner and contributing member of society. The curriculum was built on the principles of (not limited to): social transformation, integration and applied competence and human rights, inclusivity and environmental and social justice (Department of Education, 2003).

2.2.3 The envisaged outcomes of Mathematical Literacy

2.2.3.1 Definition of ML

When the subject of ML was first envisioned in South Africa, the definition, purpose and envisaged outcomes for the curriculum were stipulated in the National Curriculum Statement for Mathematical Literacy (Department of Education, 2003). In the document, ML is defined as follows:

> *"Mathematical Literacy is a subject driven by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems."*
> (Department of Education, 2003, p.9)

The NCS curriculum was built on the principles of social transformation, human rights, inclusivity, integration, and applied competence, valuing indigenous knowledge systems and environmental and social justice. Therefore it is deduced that the overarching goal of ML (in coherence with all other subjects) is to develop, among our learners, self-managing people, contributing workers and participating citizens (Department of Education, 2003).

2.2.3.2 The purpose of ML

According to the National Curriculum Statement (NCS) for Mathematical Literacy (Department of Education, 2003), the subject of ML was designed with the purpose of addressing and improving the low rates of literacy and numeracy that was prevalent in the adult population of South Africa, as well as to explicitly address poor performance in international studies. The idea was to increase the levels of engagement in mathematics education, as up to this point, majority of the learners in our schools had not opted to learn mathematics and were described as having 'dropped out' (Department of Education, 2003). The purpose also speaks directly to the overarching goal: (1) creating self-managing persons who are able to successfully manage daily activities such as personal and business finances, map and data reading, and spatial awareness (area and volume); (2) creating a contributing worker who has the numerical and spatial skills needed to deal with work-related problems; and (3) creating a participative citizen who is able to understand and interpret data as it is presented in the media, to understand the effects of numerical data on shaping policies and thus enable themselves to use their democratic vote effectively (Department of Education, 2003, p. 10).

2.2.3.2 The outcomes of ML

The NCS for Mathematical Literacy further states that this purpose and overarching goal will be achieved by allowing learners to engage with, and model relevant situations in order to solve problems that they may encounter in society (Department of Education, 2003). In doing so, learners will be given the opportunity to develop the following outcomes (Department of Basic Education, 2011a, p. 10):

- *use mathematical process skills to identify, pose and solve problems creatively and critically*
- *work collaboratively in teams and groups to enhance mathematical understanding*
- *organise, interpret, and manage authentic activities in substantial mathematical ways that demonstrate responsibility and sensitivity to personal and broader societal concerns*
- *collect, analyse and organise quantitative data to evaluate and critique conclusions*
- *communicate appropriately by using descriptions in words, graphs, symbols, tables and diagrams*
- *use mathematical literacy in a critical and effective manner to ensure that science and technology are applied responsibly to the environment and to the health of others*
- *demonstrate that a knowledge of mathematics assists in understanding the interrelatedness of systems and how they affect each other*
- *be prepared to use a variety of individual and co-operative strategies in learning mathematics*
- *engage responsibly with quantitative arguments relating to local, national and global issues*
- *be sensitive to the aesthetic value of mathematics*
- *explore the importance of mathematical literacy for career opportunities*
- *realise that mathematical literacy contributes to entrepreneurial success.*

The development of these outcomes is aimed at enabling learners to use numbers with understanding to solve real-life problems, use their skills to manage small personal budgets and understand large scale budgets, to model relevant situations graphically and numerically, to analyse situations using spatial reasoning, and to critically engage with data of statistics and chance (Department of Education, 2003). In doing so, this curriculum would provide learners with access to mathematics that would potentially obtain social and economic justice (Bansilal et al., 2015).

2.3 Mathematical Literacy as school curriculum

2.3.1 The specific aims of Mathematical Literacy education in South Africa

ML was conceptualised under the NCS and reviewed again during the curriculum reform that lead to the CAPS curriculum. Although, in these two documents it is clear that the content remained much the same, the specific aims and focus of the curriculum do differ.

2.3.1.1 The National Curriculum Statement

The NCS was a curriculum based on Outcomes Based Education (OBE), which is a leaner-centred, activity based approach to teaching and learning (Department of Education, 2003). In this OBE curriculum for ML, the aims were categorised as learning outcomes, of which there were four: (1) numbers and operations in context; (2) functional relationships; (3) space, shape and measurement; and (4) data handling (Department of Education, 2003).

Each of these learning outcomes are described in terms of the skills learners are expected to develop as they engage with the content and is summarised as follows (Department of Education, 2003, p. 12):

1. Numbers and operation in context: learners must be able to use numbers and relationships to investigate personal, social, and financial contexts.
2. Functional relationships: learners must be able to recognise, interpret, describe, and represent functional relationships in order to solve problems for real and simulated problems.
3. Space, shape, and measurement: learners must be able to measure, estimate and calculate physical quantities, as well as interpret, describe and represent the properties and relationships of 2-dimentional and 3-dimentional shape, in various orientations.

4. Data handling: learners must be able to collect, summarise, display, and analyse data in order to communicate and justify decisions, make predictions, critique findings and draw sound conclusions.

The assessment standards that accompanied this curriculum framework, offered benchmarks in the form of observable traits, against which educators could evaluate the success of the learners, in conjunction with standardised testing (Department of Education, 2008).

Upon analysis of the documents, I found the best way to describe this curriculum was that it was quite a rigid and compartmentalized curriculum, where the content was foregrounded and learner skills were assessed in direct relation to the content. Context seems to be underplayed, especially in the assessment guidelines.

2.3.1.2 The Curriculum and Assessment Policy Statement

The ML curriculum was reviewed not long after its conceptualisation, for the curriculum reform that culminated in the Curriculum and Assessment Policy Statement (CAPS). The CAPS for ML was compiled in 2011 and commissioned in schools in 2012.

The CAPS reframed the definition of ML resulting in something much more elaborate which, in my understanding, embedded the aims of the curriculum. Below follows a summary of the aims of the CAPS curriculum for ML, that I extracted from its definition. In the document, they are referred to as 'key elements' (Department of Basic Education, 2011a, p. 8):

1. Learners should be able to explore real-world contexts and solve authentic problems, using actual resources. When these problems are presented in their real-world messiness, learners may draw on mathematical and non-mathematical skills to solve them.
2. The primary aim of this curriculum is for the skills and knowledge of learners to transcend the familiar contexts and contents to which they are exposed in their personal live. Therefore, they should be exposed to familiar and unfamiliar problems.
3. There should be a focus on the interplay between content, context, and skills, which include estimation, making comparisons, budgeting, analysis, and graphing.
4. Learners must be empowered for the purposes of decision making and communicating. This includes comparing solutions, making justifiable decisions, and communicating ideas using the contextually correct terminology.

5. The learners must be able to make use of integrated content and skills to solve problems – therefore, although the curriculum is divided into topics, learners should draw on the content knowledge and skills from various topics to solve integrated, real life problems.

The CAPS also distinguishes between the teaching of basic skills and applied topics. Basic skill are skills such as calculations with numbers, interpreting answers and calculations, recognizing patterns and relationships, and representing these appropriately. Applied topics are those topics which, more often than not, require the simultaneous use of various basic skills. The topics include finance, measurement, maps, plans and other representations of the world, data handling and probability (Department of Basic Education, 2011a, p. 13).

By comparison, the NCS and CAPS curricula offer the same content, and speak to the same overarching goal. In my review of these two documents, I have come to understand that where the NCS was content focussed, the CAPS is more focussed on the application and use of the content. It can be said that the curriculum has a context-content driven agenda (Bansilal et al., 2015), where the contexts provide the framework within which the content can be used for appropriate interpretation of the scenario (Bansilal et al., 2015). Therefore, there needed to be a shift in the materials used and the methods of implementation of the curricula. Context needed to be foregrounded with the skills deeply embedded within them.

2.3.2 The nature of Mathematical Literacy as a subject today

Mathematical Literacy was formally introduced and implemented in the South African school system in January 2006 (Conradie, 2016; Meyer, 2010). It was, as planned, made compulsory for learners who have not opted to take Mathematics in Grades 10 to 12 (Bansilal et al., 2015; Buytenhuys & Graven, 2011; Long et al., 2014). It is, by definition, a subject that is driven by the application of mathematics in real-life context – as opposed to the mastery of abstract principles (Conradie, 2016) - and aims to develop the ability and confidence of learners to think numerically and spatially, in order to make decisions and solve problems (Beckmann, 2009; Buytenhuys & Graven, 2011; Meaney, 2007; Venkat & Graven, 2008; Vithal & Bishop, 2006). ML is a subject that is unique to South Africa in the sense that it is the only country which offers this subject at secondary school level (Houston & Africa, 2015). A similar approach to mathematics is also taught in other countries such as the USA and Hong Kong, where it is referred to as Quantitative Literacy (QL) (Houston &

Africa, 2015). However, in these countries QL sometimes provides access to advanced mathematics courses, whereas in South Africa, ML was a not a subject that provided access to studying a bachelor's degree until 2018 (Department of Education, 2018) and even then, it does not provide access to many degrees that have a mathematical component to them.

ML is a subject with the potential to transform mathematically weak learners into individuals that are negotiators, participators and sense makers, both in and out of the classroom (Buytenhuys & Graven, 2011). It is a subject aimed at developing logic and problem solving skills rather than focussing on the manipulation of expressions (Meyer, 2010). Thus we can say ML links content knowledge to the relevance of the real world in order to elicit behavioural manifestations in learners such as confidence, critical thinking and problem solving (Bowie & Frith, 2006). Literacy, in any form, can be defined as the use of information and skills to analyse and rationalise problems in a variety of contexts (Ozgen, 2013); so ML should empower learners to use mathematical reasoning, models and content knowledge and skills to solve problems in their everyday lives (Christiansen, 2006; Ozgen, 2013). ML is an attempt to make the abstract discipline of Mathematics more concrete and perceived as 'real', and bring to light the usefulness of Mathematics in the 21st century (Gal, 2009; Vithal, 2006), by learning to view a variety of contexts through a quantitative lens (Geldenhuys, Kruger, & Moss, 2013). These contexts apply, in particular, to those daily contexts of the ordinary South African citizen (Brown & Schäfer, 2006). Including ML in the South African curriculum ensures a future of more numerate citizens, in comparison to 2005 and earlier, where as many as 40% of South African learners were not taking any form of mathematics (Houston & Africa, 2015). It also provides increased opportunity to the development of mathematical skills, as the curriculum itself is not as loaded and as pressured as the Mathematics curriculum (Meyer, 2010).

2.3.3 Mathematical Modelling as a means of instruction

The ML curriculum is designed and intended to be a modelling-based curriculum (Brown & Schäfer, 2006), whereby learners are expected to develop competencies such as reasoning, decision making, problem solving and interpreting mathematical information (Department of Basic Education, 2011a). It was defined as subject where learners make use of life-related applications of mathematics (Department of Education, 2003) and was interpreted to mean that the mathematics be anchored in the real-world, so that mathematics and context may be brought together (Buytenhuys & Graven, 2011). According to Buytenhuys and Graven (2011), in the Teacher's Guide for ML it stated that teachers are challenged to use contexts that reveal underlying mathematics and to then use mathematics to make sense of the contexts. Therefore, it is essential to this study to explore the concept of mathematical modelling.

2.3.3.1 Definition of mathematical modelling

Mathematical modelling is a concept which has been defined and redefined in many ways. However, there is a level of consensus to these definitions. Essentially, mathematical modelling is a process by which we use mathematics to solve real world problems (Hernandez-Martinez & Vos, 2018). It is a vehicle for learning mathematics (Hernandez-Martinez & Vos, 2018) that constitutes any experience in which learners have the opportunity to make decisions about how best to represent real-world scenarios and processes (Gann, Avineri, Graves, Hernandez, & Teague, 2016). It is an educational approach with the goal of making the role of mathematics in society more explicit (Ikeda, 2018).

2.3.3.2 The process of mathematical modelling

Mathematical modelling is a process that has direction, moving from reality into mathematics (Julie & Mbekwa, 2005). In more elaborate terms, it moves from a messy real-world problem, to calling on mathematics to help structure the problem, and then making mathematics of the problem (Cirillo, Pelesko, Felton-Koestler, & Rubel, 2016). However, there is also consensus that mathematical modelling is not a linear, but rather a cyclical process. It does not terminate with the mathematics, but rather the solution is brought back into the real-world context. Blum (2002) and Krawitz and Schukajlow (2018) both describe a three component cycle which I have summarised as follows:

(1) An exploration of a particular context leading to the formation of a mathematical model and question that idealises and provides structure to the context,

(2) the analysis of the created model to answer the question that was formulated, and

(3) the interpretation and validation of the conclusion within the original context.

It can be deduced that if the solution is not valid within the context, the cycle would start again with further exploration of the context and the modification of the model and question until a suitable solution - or solutions - is obtained. One has to start with a real-world problem and end with a real-world solution (Krawitz & Schukajlow, 2018).

Therefore the process of mathematical modelling is a rather demanding process that requires constant back and forth translation between the real world and the mathematics at hand (Krawitz & Schukajlow, 2018).

2.3.3.3 The mathematical modelling task

True mathematical modelling tasks stand in stark contrast with mathematical activities which have not undergone any true translation between reality and mathematics (Krawitz & Schukajlow, 2018). These are mathematical problems that have been, in my own terms, 'dressed up' with a context and presented as a word problem. These problems differ from modelling problems in that the real life context is already cleaned up - there is often a lack of surplus information or not enough missing information (Krawitz & Schukajlow, 2018). These tasks do not evoke rationalisation or logical reasoning (Krawitz & Schukajlow, 2018). They also do not require learners to structure or idealise a context (Krawitz & Schukajlow, 2018) as it is already of an idealised nature. In short, these kinds of problems provide what I refer to as window dressing – a context was placed over an already easy to solve problem, with a pre-determined solution that does not require validation with relation to the real-world context.

If the goal is to engage in authentic modelling, it is essential to bear in mind the goals of the endeavour when designing the problem. From the work of Niss (2008), I deduced the following three purposes of modelling:

(1) To understand a context manifested through representations, explanations and predictions
(2) The use of this information that results in action in terms of decisions being made and problems being solved
(3) The design aspect of the extra-mathematical world whereby artefacts or systems are created

Therefore, it can be said that authentic mathematical modelling only occurs when learners are required to make sense of a situation or context, to represent it mathematically, and to use that information to initiate a real-world decision or solution to a problem, all evident in an artefact of some kind.

2.3.3.4 Assessing mathematical modelling tasks

Mathematical modelling is often thought of as a complex process to initiate and conduct. This complexity holds in terms of measurement of its success, as teachers need to move away from assessing predominantly short numeric answers, to assessing the reasoning behind the answers given (Aydogan Yenmez, Erbas, Cakiroglu, Alacaci, & Cetinkaya, 2017). In fact, sometimes learners may never get to a solution, which is not an indication of failure to successfully engage in mathematical modelling. In this study I will draw on the work of Maass (2006) who formulated the following modelling competencies for the purposes of assessment. These are summarised notions of Maass' (2006) work:

(1) the ability to make assumptions for a problem and simplify situations,

(2) to recognise quantities and variables that influence situations,

(3) to construct relations between variables,

(4) to differentiate between relevant and irrelevant information,

(5) to choose appropriate mathematical notations to represent situations,

(6) to make use of heuristic strategies and mathematical knowledge to solve the problem and

(7) to interpret the results outside of the mathematical context and to generalise the solutions

Mathematical modelling tasks are mapped against these competencies and are assessed according to the degree of evidence available for each competency, based on key identifiers for each competency.

2.4 Challenges to the implementation of the Mathematical Literacy Curriculum

2.4.1 Challenges of stigmatisation

2.4.1.1 Mathematical Literacy is easy mathematics for less capable learners

The initial instruction (according to Brombacher (personal communication, January 2010)) for the design of the ML curriculum was that it should be an “easy mathematics” (Buytenhuys & Graven, 2011), and that is the misleading image still associated with the subject – that it is a watered down version of a Mathematics curriculum, offered to less capable learners (Houston & Africa, 2015) and that it is a less worthy subject than Mathematics (Conradie, 2016). Although Brombacher’s design of the Subject Assessment Guidelines (2008), depicts ML as a different way of doing and learning mathematics, rather than a watered down version of the Mathematics curriculum (Buytenhuys & Graven, 2011), the stigma has stuck. In fact, in many schools in the country, ML is not offered as a positive, open choice for Grade 9 learners who move into the FET band. Rather learners are demoted to ML when they struggle with Mathematics in Grade 10 or 11. On the other hand, in more affluent and competitive schools, learners are openly demoted to ML as early as the end of Grade 9, when teachers have the opinion that learners may struggle with Mathematics in the FET phase. The result is that majority of learners who participate in ML lack enthusiasm and interest in mathematics (Meyer, 2010), which is mistaken for an expression of their intelligence or mathematical capability. They are expected to be learners with low levels of competence (Brown & Schäfer, 2006). In essence, this practice of demoting learners to ML seems beneficial at first glance, but it raises concerns about ML learners’ future prospects, due to the negative image associated with ML in schooling, higher education and in society in general (Venkat & Graven, 2008).

2.4.1.2 Mathematical Literacy is for less capable teachers

It is both learners of and teachers who teach ML who are socially perceived as being less capable than their peers who engage in core Mathematics (Conradie, 2016). They are labelled as being less clever and thus told that they are limiting their own futures by engaging in the subject in the first place (Conradie, 2016). This image exacerbates the problem of poor teaching quality, as there are very few teachers who have and opt to undergo formal ML training (Conradie, 2016). There is also a lack of interest in teaching the subject on the part of mathematics teachers (Bansilal et al., 2012; Conradie, 2016) due to the stigma attached to ML. It is often the misrepresentation that ML is the teaching of basic mathematics in the form of word sums (Bowie & Frith, 2006) that deters teachers from engaging or thinking they can without proper training. Even for those who do understand the intricate nature of ML as a subject, it is often too difficult to break away from the structured curriculum of mathematics (Bowie & Frith, 2006) and teach using a modelling approach. Teachers engaging in ML also undergo public scrutiny, with teachers being told the subject was created for those of them who are too lazy to teach Mathematics and that they are creating kids who feel stupid (Jansen, 2011).

2.4.2 Challenges of contextualisation

2.4.2.1 The importance of authentic contexts

The pedagogy for ML is clearly amenable to modelling and requires that learning must be anchored in the real world. Mathematics and context must not just be brought together, they must be intertwined (Bansilal et al., 2012; Buytenhuys & Graven, 2011). The role of context in ML is described in the CAPS (Department of Basic Education, 2011a), as providing learners with an awareness and understanding of the role of mathematics within our modern society. ML as a subject, where modelling is the main driver (Julie, 2006; Ozgen, 2013), requires contexts that are presented in their authentic, real world messiness, so that learners have, as learning tasks, the formation of mathematical models from the contexts, whereby meaning and implications can be explored (Bansilal et al., 2012). In other words, the contexts serve as messengers to convey information to learners (Meyer, 2010), which is then used to formulate models and solve problems or make decisions. Only when learners are directly involved in the modelling of complex phenomena may they truly grasp how mathematics can be beneficial in their lives (Christiansen, 2006).

2.4.2.2 Problematic textbooks and tasks

Teachers of this subject are encouraged to use contexts which hold relevance and address the needs of the learners (Buytenhuys & Graven, 2011). However, due to the standardised nature of the textbooks (of which there are only 5 available), shallow and outdated contexts are often used as window dressing for abstract mathematical concepts.

The standardised nature of contexts in textbooks also may undermine the accessibility of the subject. This is apparent from one study that compared the ML experiences of learners in a private school, a former model-C school, and a disadvantaged school. Geldenhuys et al. (2013) indicated that learners in a disadvantaged school had a more negative attitude toward ML, and reported lower levels of perceived empowerment, partly due to a lack of applicable (in terms of context) resources and hence significant and enriching learning experiences. A study by Copper and Dunne (1998), in Bansilal & Debba (2012), also concluded that standardised contexts disadvantage the working class because these learners do not find the contexts as easily accessible.

To add insult to injury, the textbooks available, and the supplementary government issued or privately published study guides, are grossly flawed and outdated. There is only one textbook which was issued in 2019, with the other materials dating 2013 or earlier. This automatically creates issues around the realness of contexts pertaining to tax tables, inflation rates, budgets, exchange rates (e.g. R14 to the pound rather than the current over R21 to the pound) and other financial topics. In my experience, some of the more outdated textbooks were so shallow in contextualisation, that gross errors were overlooked – errors such as stating repetitively that there is a 30th of February, or even contexts that were described without critical information rendering learners unable to engage with them. One anecdote comes to mind from a Gr 11 textbook. The learners were required to draw an income and expense sheet for a gardening service to determine the profit or loss of the business. The problem described the various services offered as well as the various expenses to the business. However, it did not describe what the business owner charged for his services offered, thus rendering the whole problem useless as income could not be determined. It is because of flaws such as these that it is said ML textbooks present the content as a watered down version of Mathematics (Meyer, 2010). Using outdated and ill-described contexts leads to pseudo-contextualisation (Bowie & Frith, 2006), which is both demeaning and does not promote true mathematical literacy as it does not allow learners to participate in current discourse (Bowie & Frith, 2006).

When it comes to textbooks and other standardised tasks (such as provincial or national exams), the authenticity of the context is often lost in the writer's interpretation of the scenario (Bansilal & Debba, 2012), leading to the reader trying to answer for the examiner rather than solve the problem by relying on their personal factors and experiences to inform their decision making (Bansilal & Debba, 2012). In other words, an inauthentic context can cause learners to get confused between when to use everyday knowledge and when to use mathematical knowledge. This does not necessarily indicate that they are lacking mathematical knowledge, but rather that they are unclear of what is expected of them due to the nature of the context (Bansilal & Debba, 2012).

2.4.2.3 Creating authentic contexts

In order to develop ML materials that allow for the true mathematization of contexts, we need to use contexts of which all involved have a clear understanding (Bowie & Frith, 2006; Brown & Schäfer, 2006). Learners who live in societies where every day is a constant struggle for survival, are not likely to be interested in information that may only be useful to them in some hypothetical future that they do not see themselves having access to (Prinsloo, 2007). Another anecdote comes to mind here. In the design of this study I consulted with the ML subject head to the relevant high school. She was concerned over the fact that the DoE had just that morning sent them the matric investigation. The theme of the investigation centred around the design of the fuselage of an aeroplane. The learners in this school had never even seen an aeroplane in close proximity. They had no concept of what the fuselage of an aeroplane looked like and there was a real concern that the learners would not be able to engage with the material. When designing materials, the emphasis should be placed on creating an appreciation for mathematics and not the forcing of unfamiliar yet convenient contexts (Meyer, 2010). Learners need to have a thorough understanding of a situation if they stand any chance of using mathematical knowledge to analyse it (Bowie & Frith, 2006).

A big concern with regard to context is that they are decided by curriculum and test designers who are, firstly, not the only stakeholders in education (Julie & Mbekwa, 2005), and secondly, are often far removed from the lives of the learners themselves. For that reason, I argue who better to involve in the selection of those contexts than the learners who reside within them? What better problems to address than those our learners are facing every day? Involving learners in the choices of contexts requires research to understand their choices and to inform how teachers should support learning through modelling.

Thus far I have argued that the aims and definition of ML allows for a meaningful subject, yet that ML is marred by negative perceptions and often seen by learners as a subject for losers. This negative framing of a compulsory subject is bound to influence ML learners' identity negatively.

2.5 Identity in education

Identity, as many constructs in educational research, is a term that is contested in its definition (Marks & O'Mahoney, 2014; Sfard & Prusak, 2005; Wenger, 1998). However, what is agreed upon is that identities have the ability to serve as self-fulfilling prophecies where learning is concerned (Sfard & Prusak, 2005). For the purpose of this study, I will be drawing on the work of Sfard and Prusak (2005) as well as Wenger (1998) in framing a working definition of identity.

2.5.1 Actual and designated Identites

2.5.1.1 Identities are narratives

Sfard and Prusak (2005) define identity as stories or narratives about a person that are reifying, endorsable and significant. Narratives that are reifying use verbs such as 'have' and 'am'; they are endorsable when they are seen as a true reflection of the state of affairs from the perspective of the identity builder; and they are significant if a change in the story can affect the feelings of the identity builder regarding the person who's identity is being narrated (Buytenhuys & Graven, 2011). This indicates that identities are self-authored constructs that are collectively shaped by both story tellers and recipients (Buytenhuys & Graven, 2011) and lead to action being taken in accordance. Sfard and Prusak (2005) draw on the work of Holland, Lachiotte, Skinner and Cain (1998) in describing the productive influence of identity as people telling others who they are but also, and perhaps more importantly, telling themselves who they are and trying to act accordingly.

Our identities are therefore influenced by the stories we tell of ourselves, the stories we are told about ourselves and the stories told about us to others (Sfard & Prusak, 2005). However, it is not so much these stories themselves that shape our identity, but rather our vision and interpretation of them (Sfard & Prusak, 2005) – our evaluation of these stories as having good or bad messages about us that we accept as being true.

2.5.1.2 Actual vs designated identities

Sfard and Prusak (2005) also distinguish between *actual* and *designated* identities. Actual identities are usually present-based narratives about the way one is at a specific given time (Sfard & Prusak, 2005). They are what a person believes about themselves in the here and now, such as "I am not good at mathematics". Designated identities are current, externally imposed narratives based on hypothetical future projections and have the potential to become a part of one's actual identities (Sfard & Prusak, 2005). For example, when a learner is advised to take ML as a subject because they are not perceived by their teacher as being able to cope with Mathematics in higher grades, they may as a result avoid looking into any future career paths that require some form of mathematics. Thus, designated identities give directions to one's actions and deeds, sometimes to a point that is not rational.

2.5.2 Identities as Communities of Practice

Wenger (1998) views identity as a way of being in the world – the extent to which we are (not) involved in a community of practice. This notion is supported by Marks and O'Mahoney (2014) who take the position that identity is concerned with the extent to which a learner can identify or is aligned with a specific group. Wenger (1998) identifies three modes of belonging summarised below:

(1) Engagement, which describes the extent to which learners would voluntarily partake in mathematical events and is influenced by the degree to which their ideas are adopted or marginalised.

(2) Imagination, which is about how mathematics is perceived as useful and meaningful in life outside the classroom, both presently and in the future. It is influenced through experiences and how those experiences are shared with others.

(3) Alignment, which is a response to the imagination face (Anderson, 2007), in which experiences guide the driving force of learners' willingness to engage, i.e. are learners partaking in math because of its perceived value and benefit to their lives or because they are aligning with institutional requirements?

2.6 Mathematical Literacy and mathematical identity

Learners' mathematical identities can be shaped by their experiences of and with ML, as well as the narratives they are exposed to. In this study I wanted to research my hypothesis that, in many cases, ML learners hold negative identities with regards to mathematics and their ability to learn, and therefore revert to the role of the recipient. I hypothesise that they come to accept designated identities without placing much effort into actively forming their own identities. In other words, they believe what is said about them and their abilities as mathematical learners and act accordingly.

2.6.1 Creating misinformed identities

According to Buytenhuys and Graven (2011), the fact that the documents outlining the ML curriculum is so focussed on the development of competence and confidence, is an indication that this curriculum has the aim of developing positive mathematical learner identities. However, the authors also draw the contrast by stating that in reality, it is very often learners who are seen by teachers as mathematically weak (designated identity) and who have low levels of confidence in their mathematical ability, that are more likely to choose ML as a subject. In my experience, public schools, where there are high learner-teacher ratios, limited resources and learners who enter high school with a mathematical 'back-log', the number of learners who fall into this category is becoming increasingly higher.

Furthermore, it is no secret that academic heavy weights such as Jonathan Jansen are not hesitant to voice their disdain for ML in the media. Although I acknowledge that Jansen does not set out to criticize ML explicitly, but rather the faults of our education system, his singling out of ML as a subject contributes to the images of stigmatisation discussed earlier in this chapter. Jansen's public remarks about ML have included:

1. "Mathematical literacy is for dummies" (Smith, 2019)
2. ML is a "lower and strenuous form of math " (Jansen, 2012)
3. "… the consequences of this government's thinking about mathematics are not dissimilar to that of Verwoerd's government: why teach the black child Mathematics? Rather teach them Mathematical Literacy, and condemn them to the kinds of jobs they are fit to occupy." (Jansen, 2012)

Even the curriculum documents designates a shared identity to ML learners when it states that ML is a subject suited to learners who do not perceive mathematics as necessary in future studies or chosen career paths (Buytenhuys & Graven, 2011). This is potentially paving the way to creating an innumerate society, as narratives of success drive learners toward success, where narratives of failure, drive learners toward failure (Sfard & Prusak, 2005). We are creating an unjust categorisation of successful vs unsuccessful learners based on a subject choice that is often influenced by demographical factors such as poverty, race, language and ability (Swanson, 2002). Learners respond to these narratives and the risk is that we are creating a caste system in which learners who opt for ML will always be seen as less able or less likely to succeed.

2.6.2. Is identity influenced by the nature of engagement with Mathematical Literacy?

For my study, I was interested in the extent to which involving ML learners in a modelling process during designing ML investigations, can influence how learners reflect on their own identities. Research conducted by Buytenhuys and Graven (2011) shows that when learners have positive experiences in ML, they are empowered to challenge these designated identities and form their own narratives regarding their competency and the value of ML as a subject. The authors elaborate that ML should empower learners through active participation and engagement in the messiness of real-life scenarios and negotiate a way forward so that learning experiences may developed (modelling). In a study by Venkat and Graven (2008), learners reported that engagement in ML has proven the subject to be more useful than mathematics in addressing their daily and future needs. I hypothesise that when these learning experiences are created - when learners are given the opportunity to be involved in making sense of their own world through modelling relevant contexts - we empower them to make effective changes to those contexts and thus (potentially) positively influence their identities.

2.7 Summary

In this chapter I presented literature that was aimed at establishing an understanding of the attribute of mathematical literacy. I also traced the historical need for, and development and intended outcomes of ML as a South African subject. From the literature it was also clear that ML is amenable to mathematical modelling as a means of instruction, and that the use of mathematical modelling in the classroom may help overcome barriers pertaining to superficial contextualisation and shallow learning materials. The literature also indicated to externally imposed stigmatisation of ML as a subject and how these stigmas may influence the mathematical identities of the learners. I hypothesised, based on the literature, that the use of modelling to create authentic learning experiences could be a means to support the development of more positive learners identities.

CHATER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

In this chapter I define and justify my methodological paradigm for this study, as well as detailing the participant selection. I then describe my data collection methods and the tools designed for these methods. I also illustrate the role of myself as the researcher during data collection and analysis. The methods of data analysis based on grounded theory, thematic analysis and the mapping of modelling competencies is also described and justified.

3.2 Research methodology and paradigm

3.2.1 Qualitative research design

This study operated under qualitative research design, underpinned by the interpretivist paradigm. The purpose of this study was to observe, document and analyse the interaction between learners and ML material, as well as their subjective experiences, within the natural environment of a school classroom. This was done to establish whether or not engagement in context rich material-design, influenced the mathematical identities (ID) of the learners. By making use of qualitative research methodology, I could focus on creating an understanding of the processes and contexts that influenced this study, merely by interacting with and observing the participants' engagement within their natural school environment (Nieuwenhuis, 2014), and taking into account what the contextual evidence is telling me about these learners' mathematical identities (Maxwell, 2013). In accordance with qualitative research practice, in conducting this study I gathered evidence which gave insight into, and an understanding of the learners' perspectives (Merriam, 2009) about ML as a subject and about their views of themselves as (in)capable mathematical learners.

3.2.2 The interpretivist paradigm

In line with qualitative research design is the interpretivist perspective. In this study, I was concerned with how the learners interact with materials and contextual factors within the field of ML (Connole, 1993). I aimed to understand the subjective experiences of the learners partaking in the study, with the specific intent of understanding how their actions or engagement (Connole, 1993) influenced their mathematical ID. In using the interpretivist paradigm I was able to acknowledge the existence of many realities (Connole, 1993) implying that various learners could have vastly different experiences of and with ML, and this could lead to various interpretations of the role and value of ML education in their lives. The learners involved in this study, were given the opportunity to relay their personal opinions and subjective experiences to me in various ways, such as through questionnaires, personal written reflections and focus group interviews. This data allowed me to form an understanding of how their interaction with the ML material and the material design process, influenced the learners' mathematical ID, in terms of their beliefs about their mathematical capability and the relevance of ML in their lives beyond school.

3.2.3 Case study research

This study was undertaken as a case study, in which I did a systematic and critical enquiry into a how learners' involvement in the design of context-rich ML material, influences their mathematical IDs. This undertaking allowed me to generate an understanding that could add to a public body of knowledge (Nieuwenhuis, 2014; Simons, 2009) pertaining to the implementation of the ML curriculum in South Africa.

My case study existed within a bounded system and aimed to offer insights into the dynamics of a specific situation (Nieuwenhuis, 2014). That is: I worked with the Grade 11 ML learners, in an underprivileged school, that exists in a community that is characterised by high poverty levels, low levels of education (and especially tertiary education), and high levels of gang violence that often spills over onto the school grounds. However, this community borders one of South Africa's most affluent communities, meaning the learners regularly bear witness to people with high income levels and luxurious lifestyles, as well as to the thousands of learners who attend the various local tertiary educational institutions. These learners are in a relatively unique situation, where they are exposed to two completely polarised lifestyles – one categorising them as at-risk learners, and one offering a way out of a volatile community. This implies that there are unique dynamics at play that could influence the learners' value position toward ML and thus their mathematical ID.

Case study research allowed me to be flexible in my data collection and analysis strategies (Timmons & Cairns, 2010), which was beneficial due to the wide array of dynamics that influenced the classroom and orientation session interactions. These dynamics included: the resources that were available, the venues offered to me to host modelling orientation sessions and the kind of learning environment they created, the number of learners I was interacting with and who were interacting with each other, the presence or absence of supervising teachers, the willingness of the learners to engage in new methods of learning ML, and the level of content knowledge and mathematical skills of the learners.

This case study also offered insights into ‘how’ and ‘why’ questions (Nieuwenhuis, 2014) such as *how* learners interact with the ML as a subject and ML material, *how* the interaction influences their mathematical ID and *why* they engage in ML as subject at all. Case study research was also a suited method to this study as it allowed me to incorporate evidence what was not necessarily included in my original goals (Timmons & Cairns, 2010), but that arose as a result of the dynamic nature of education, and that proved to be valuable to the findings of this study.

3.2.4 Design-based research

This study was designed primarily as a case study, however, due to the nature of my data collection (discussed below in section 3.4), I have also drawn on elements of design-based research. During my data collection, I made use of two cycles of orientation sessions and investigations (explained in detail in section 3.4.3). The design of these orientation sessions were systematic, yet I had to be flexible in my methodology (Wang, 2017) as I needed to adapt the orientation session according to the situational and dynamic needs of the educational setting. Therefore using multiple orientation sessions as data collection can be seen as iterative practice (Wang, 2017). The analysis of the design and implementation of the first orientation session, informed the design and development of the second orientation session, in order to improve the educational situation and ensure my research would hold contributing value (Calderon, 2010; Wang, 2017). During the first orientation session, I analysed the design of the tasks and means of implementation, with focus on the improvement of the design; whereas the redesigned second orientation session allowed me to focus on the usability and applicability of that data to the core purpose of my study (Calderon, 2010). This interventionist approach, whereby I, as the researcher, deliberately manipulated the design of the orientation sessions and educational setting (Bakker & Van Eerde, 2015), meant that I could simultaneously work toward the overall aim of my study – to determine how learners’ interaction with (the designing of) context rich ML material impacts their mathematical ID - whilst also tending to the specific aims of each stage of the study (Bakker & Van Eerde, 2015). I could create and recreate tasks, as well as alter the educational setting, in order to improve the validity of my data. I could adapt my study to the dynamic environment of the school, as well as the nature of the social interactions taking place during the orientation sessions (Wang, 2017). By drawing on aspects of design-based research, I could address a range of complex education problems (Bakker & Van Eerde, 2015), which I could not foresee when embarking on this study. Therefore, design-based research practice lessened the gap between what I had theorised would be effective orientation session design, and the effects of practically implementing my orientation sessions (Bakker & Van Eerde, 2015). A diagram to illustrate the design and redesign process in included in section 3.4.

3.3 Participant selection

3.3.1 School selection

This case study pertained to learners in an underprivileged school, situated in a volatile community. I had been involved with this school since 2016: first as a part of a team that hosted a mathematics club for learners for research purposes, and later as a substitute teacher for Mathematical Literacy for six months. The choice to work with this school was a case of purposive sampling (Maree & Pietersen, 2014). I was familiar with the ethos of the school, as well as many of the daily challenges posed on teaching and learning. I had well-established relationships with the ML teachers and the management staff, who were very accommodating in creating room for me to conduct my study. However, I was also aware that this school had a context far removed from those in the prescribed ML textbooks that they made use of. I was aware of the low levels of mathematical performance, where less than 30 children per grade (10-12), of about 170, engaged in Mathematics as a subject. Most of the learners chose ML as subject, and still performed poorly, with grade averages hovering around the 40% mark. This school situation provided the opportunity for me to gain valuable insights into the learners' mathematical IDs by creating a wholly unfamiliar opportunity for them to create their own, and engage with, context rich material. I was aware that the educational practices at this school did not make use of mathematical modelling in ML, which meant that it would be a new experience for the learners and potentially influence their mathematical IDs in entirely new ways.

The ML department at the school was invited to work with the me on the study by assisting me in implementing the orientation sessions, designing the investigations, and evaluating the resulting products. The staff declined the opportunity and were content to let me conduct the study on my own, without their input or supervision.

3.3.2 Learner selection

This study was the case of observing, analysing, and describing the potential changes in learners' mathematical identities in relation to ML, by involving the Grade 11 ML learners. As mentioned, these learners delivered low marks and were believed, by the teachers and myself, to really struggle to apply any mathematical concepts to contexts that are too far removed from their immediate environment. Once again, the selection of this particular grade was a use of purposive sampling (Maree & Pietersen, 2014). I deliberately chose this group because most of these learners were familiar to me as I had been engaging with a number of them in a mathematics club since 2016, when they were in grade 8. My experience with these learners were that they are apprehensive to talk about themselves and their academic experiences to anybody that they perceived as exerting authority over them. I knew that, due to pre-existing relationships with many of these learners, there was an established element of trust. This meant that the learners would be more likely to share open, honest, and useful ideas with me, that would provide me with richer and more valid data.

In recruiting the participants, I approached the grade 11 group as a whole (approximately 170 learners) and explained the purpose of this study. I made it clear to the learners that participation in any questionnaires, orientation sessions and interviews would be voluntary and that they could withdraw from the study at any time. However, the investigation that would be designed and administered, would form part of their formal assessment, and therefore they would have to complete it during the allocated class time. A letter of explanation was also sent out to the all the parents and guardians of the learners, and a signed consent form had to be returned before the learners could partake. Both the learners and the parents were assured of the anonymity of the learners and the school (both verbally and in writing) in the write up of this study. No role-call was ever conducted; I merely worked with the learners who had made themselves available at the time.

3.3.3 Focus group selection

All 170 learners were invited to partake in the questionnaires and orientation sessions (discussed in section 3.4). However, a select group of 10 learners were invited to a focus group interview with me personally. Due to the time constraints for data collection set forth by WCED, as well as the many obstacles and delays incurred on the part of the school (such as taking 3 to 4 weeks to complete an investigation instead of the agreed upon 1 week), I did not have much time to select a group of 10 learners and conduct my focus group interviews. Furthermore, by the time the school calendar had allowed me to conduct this focus group interview, it was nearing the end of the school term. Part of the school culture at this institution, was low levels of attendance by learners, after the formal testing cycle had been completed. I knew that there was a time constraint to my participant selection and an added constraint on whether or not the learners I wanted to speak to would even be at school.

Originally, I wanted to use a set of criteria upon which I selected these learners, based on their engagement, questionnaire answers and performance in the investigations, but the latter was not possible as it took too long to obtain their results. Instead, I had to adapt and make use of a combination of convenience sampling and purposive sampling (Maree & Pietersen, 2014).

In convenience sampling one chooses participants based on their availability (Maree & Pietersen, 2014) and to your own convenience (Farrokhi & Mahmoudi-Hamidabad, 2012). Convenience sampling is useful in exploratory research (Ivey, 2011; Maree & Pietersen, 2014), which is what I was busy doing – exploring how the learners' involvement in the design of materials had affected their mathematical identities and views of ML – and is also useful in recruiting participants for focus groups (Stewart, Shamdasani, & Rook, 2007). Many of the learners had voiced their willingness to come and talk to me in a recorded interview, on the condition that there would be food provided. I was aware of which learners these were and had to rely on this convenient availability. However, because I needed to interview learners who exhibited a range of levels of engagement in the study, to obtain the richest data, I also made use (to an extent) of purposive sampling. This helped to avoid the problem of outliers in the group, a risk of using only convenience sampling (Farrokhi & Mahmoudi-Hamidabad, 2012). I chose learners with a specific purpose in mind: those displaying a range of attitudes towards the study (Maree & Pietersen, 2014). In other words, I was looking to engage with learners in a focus group interview to establish their ideas and attitudes toward ML as a subject and about themselves and their mathematical identities. Therefore, I needed to select from the learners who had volunteered, those who had shown both active and reluctant involvement and engagement in the orientation sessions, and those who I knew would be willing to talk to me because of our rapport. These learners were identified from their personal written reflections after each orientation session, as well as the marks they obtained in their investigations. In this selection, I had to rely on both my situational knowledge and my relationship with the participants (Barratt, Ferris, & Lenton, 2015).

3.4. Data collection methods

3.4.1 Overall research design

As part of this study, I aimed to observe the learners with relation to the following questions, aimed at informing my research question as outline in section 1.5:

1. What are the learners' general perception of ML as a subject both before and after the orientation session?
2. How do the learners think other people perceive ML as a subject?
3. Why did the learners choose ML as a subject rather than Mathematics?
4. What is the relation between their mathematical identities (ID) and their ML experiences?

This study was designed in various phases, that constituted different types of engagement from the participants. Each of these phases is described in detail in this section. Below offers a summary of events:

In the first phase, all the learners who had elected to partake in the study, completed a pre-study questionnaire (See Addendum 5). This served the purpose of providing me with insights into their views of ML, as well as their mathematical ID, for comparative purposes with a post-study questionnaire that would be administered at the end of the study.

Thereafter, all the learners who had elected to be a part of the study, took part in the first orientation session. In this first orientation session, we discussed various problems in the school. The learners and I collectively decided on one problem (context) which they would try to solve by making use of mathematical modelling. The learners designed potential questions and problems that could be used in creating an investigation out of this context (See Addendum7).

I then formalised the investigation task by drawing on the ideas of the learners but also adhering to the requirements of the CAPS (Department of Basic Education, 2011a). This assignment (see Addendum 8) was dealt with in term 2. The curriculum outline for term 2, grade 11 is as follows:

GRADE 11: TERM 2		Week Number								
		1	2	3	4	5	6	7	8	9
Topics	Contexts focusing on *Finance* (Interest, Banking, Inflation)									
	Contexts focusing on *Measurement* (Measuring length, measuring weight, measuring volume, measuring temperature)									
	Contexts focusing on *Maps, plans and other representations of the physical world* (Scale and Map work)									
	Revision									
Assessment	Assignment/Investigation Mid-year examinations (2 papers; 1½ hours each; 75 marks each) (covering *Finance*, *Measurement* and *Maps*, integrated with *Numbers* and *Patterns* concepts)									

Figure 1 Curriculum outline for ML Grade 11 Term 2 by topic (Department of Basic Education, 2011a, p. 17)

The investigation designed for this term focussed strongly on the concept of Finance (from term 1 and 2) and drew on the topic of "Numbers in Context" from term 1.

The learners then completed the investigation in class under the supervision of their class teachers, after which, and based on the results, I set to task to evaluate the design of my orientation session and investigation. This was done with the guidance of my supervisor. I redesigned the format of the orientation session to address logistical and curricular problems from the first orientation session (discussed in section 3.4.3). This new format would work with smaller groups and foreground the process of mathematical modelling. Where investigation 1 made use of an authentic context, the structure of the task was still similar to a traditional test-style task – where information was provided, and learners just had to use it in the right way.

This new investigation would require learners to gather information about the context, the problem, and the potential solutions themselves. They would have to discern between useful and irrelevant information and choose effective communication strategies.

The second orientation session was held in term 3. The orientation session was designed as a brainstorming session, whereby learners could plan how to go about collecting, interpreting, and communicating their information in a mathematical context (see Addendum 9). I then took the suggestions and ideas of the learners and formalised a modelling task to serve as their investigation (see Addendum 10), complete with a marking rubric that the learners could use as guidance.

The CAPS (Department of Basic Education, 2011a), offers the following breakdown for term 3:

GRADE 11: TERM 3		Week Number								
		1	2	3	4	5	6	7	8	9
Topics	Contexts focusing on *Measurement* (Perimeter, area and volume)									
	Contexts focusing on *Maps, plans and other representations of the physical world* (Models and Plans)									
	Contexts focusing on *Finance* (Taxation)									
	Contexts focusing on *Probability*									
Assessment	Assignment/Investigation Control test (covering *Measurement, Models and Plans, Finance* and *Probability*, integrated with *Numbers* and *Patterns* concepts)									

Figure 2 Curriculum outline for ML Grade 11 Term 3 by topic (Department of Basic Education, 2011a, p. 17)

The investigation I designed made use of finance topics, but was predominantly based on data handling, which is a topic in the fourth term. This was decided in consultation with, and approved by the ML department at the school.

The learners completed the investigation in class under the supervision of their teachers. This investigation was a group assignment; therefore, it was also supplemented with an individual test (see Addendum 11) that required learners to interpret data from graphs and tables. I also compiled that test myself. I collected and marked the learners' completed group and individual tasks.

At the end of each orientation session, learners wrote a personal reflection on their experiences and opinions based on prompts I provided. I chose 10 learners who had a range of positive and negative responses - who also had a range of marks resulting from their investigation, and who had made themselves available to me - to conduct a focus group interview. In this interview I probed the learners more about their questionnaire responses, as well as their experiences in the orientation session and with ML as a subject (see interview schedule in Addendum 6).

After all the marking was complete, all the learners who were still at school by the end of the third term, completed a post-study questionnaire pertaining to their views of ML as subject and their mathematical ID, and the added component of their views of learning ML by means of modelling orientation sessions.

The diagram below illustrates the process of data collection, as summarised above:

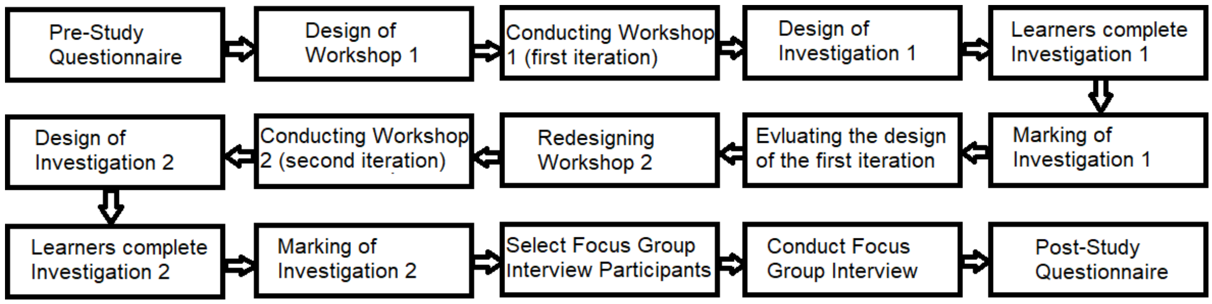


Figure 3 Summary of the process of data collection

As a case study, this study made use of multiple sources of data (Nieuwenhuis, 2014; Simons, 2009) that stemmed from the various phases of data collection. These data sources include:

- Questionnaire data from the questionnaires administered to all the participants, prior to the commencement of the study, as well as after the study was completed. These questionnaires aimed at tracing patterns in identity development as well as the broader image associated with mathematical literacy.
- The learner work from the orientation session and subsequent investigations. This resulted in data in the form of written responses from the learners, as well as learners' marks, which provided insight into their performance.
- A focus group interview with the 10 learners who I selected. The interview was audio recorded and transcribed. The audio and transcribed data are digitally stored as securely as possible.

In the following sections I elaborate on the design of each facet and the process of data collection.

3.4.2 Questionnaire design

This study made use of a large participant group of about 170 grade 11 learners. All the learners, who had made themselves available, participated in completing both a pre-study and post-study questionnaire. The pre-study questionnaire prompted learners about their motivations to choose ML as a subject, about the personal and external views of themselves as ML learners, and views pertaining to ML as a subject. The post-study questionnaire addressed the same topics, but instead of probing the learners about their choices for ML again, they were probed about their experiences in the orientation sessions. The purpose of these questionnaires was two-fold: (1) to get an overall sense of the learners' view of ML and their mathematical identities and (2) as comparative data to observe any possible changes in their mathematical identities.

3.4.2.1 Designing the items

I set up my questionnaire in 8 different sections – each pertaining to items that relate to a specific aspect of mathematical identity or views regarding ML as a subject. These statements may be analysed separately or summed up with the respective related items (Bertram, 2007). The related sections were split to control the validity of the data, by evaluating to what extent seemingly separate sections corresponded. Each section discussed next is summarised in Table 1 (full questionnaires in Addendum 5) at the end of this section.

Section	Pre-Questionnaire	Post-Questionnaire
1 and 7	**Topic**: Decisions for choosing ML and factors that motivate continued engagement in ML.	**Topic 1:** Learners experiences with the orientation sessions. **Topic 7:** Lost due to erroneous printing by the participating school.
	Alignment aspect of identity: to what extent do the learners believe they made an independent choice vs being guided to make their choice by other stakeholders in their education?	The first topic was changed to probe learners about their experiences with learning by means of mathematical modelling orientation sessions. Section 7 was intended to stay the same as the pre-study questionnaire but was omitted due to printing errors on the part of the school.
2 and 5	**Topic:** How others perceive ML as a subject and the learners who take ML.	Same as the pre-study Questionnaire, for comparative purposes.
	Designated identities: how do the learners believe parents, teachers and peers perceive them in terms of their capability to learn math, considering their choice to engage in ML as a subject?	
3 and 6	**Topic:** How the learners perceive ML as a subject and their own abilities to learn mathematics.	Same as the pre-study Questionnaire, for comparative purposes.
	Actual identities: How do the learners perceive their own mathematical capabilities in relation to their participation in ML? What do they believe of themselves as mathematical learners?	
4 and 8	**Topic:** The relevance of ML in relation to the world outside school and the perceived value of ML as a subject.	Same as the pre-study Questionnaire, for comparative purposes. However, section 8 was omitted from the post-study questionnaire due to printing errors on the part of the school.
	Imagination aspect of identity: to what extent can the learners relate the skills learned in ML to the skills needed for everyday life? What is the nature of the role of ML in the learners' current and future lives?	

Table 1 Summary of each section of the pre-study and post-study questionnaire design

SECTION 1: PERTAINING TO THE DECISION OF TAKING ML AS A SUBJECT AND SECTION 7: PERTAINING TO WHAT MOTIVATES LEARNERS TO ENGAGE IN ML

Sections 1 and 7 of the questionnaires aimed to delve into the alignment aspect of the learners mathematical identities (Anderson, 2007; Wenger, 1998). This was to establish the motivations for participating in ML as a choice subject. Alignment refers to the extent to which learners align their decision to institutional or other requirements (Anderson, 2007). It is how learners interpret the demands of a situation and the structures of power before aligning to a certain decision (Wenger, 1998).

In section 1, I wanted to determine to what extent learners believe they made an independent choice to take ML as a subject vs being guided to do so by the school, parents and guardians, older peers or a combination of the above. In section 7 I delve deeper into finding out *why* learners made the decision for ML as subject and continue to engage. For example: in section 1 the probe reads '*I chose ML because the school said I had to, not because I wanted to'*, whereas in section 7 the probe reads '*the math we do in ML is enjoyable*'.

In the post-study questionnaire, section 1 served as a platform for learners to reflect on their experiences of learning ML through modelling orientation sessions. I probe, for example, '*I would like it if we learned in this way more often'*.

SECTION 2: PERTAINING TO HOW OTHERS PERCEIVE ML AS A SUBJECT AND SECTION 5: PERTAINING TO WHAT OTHERS THINK OF THE LEARNERS THEMSELVES IN RELATION TO THEIR PARTICIPATION IN ML

The stories that are told about these learners, that they are aware of, can be seen as the designated identities (Sfard & Prusak, 2005), which, if believed by the learners, can be internalised and formulate what the learners believe of themselves – their actual identities (Buytenhuys & Graven, 2011; Sfard & Prusak, 2005).

In sections 2 and 5, the items centred around getting a sense of how learners perceive their designated identities, for example, by probing: '*people think that because I take ML I can't go to university*' and '*my teacher sees me as someone who is capable of learning math'*. How do they perceive the views of others, regarding ML as a subject, and them as learners who partake in ML? I was guided by the questions: How do others regard them in terms of their mathematical capabilities and potential for future success?

SECTION 3: PERTAINING TO HOW THE LEARNERS PERCEIVE ML AS A SUBJECT AND SECTION 6: PERTAINING TO HOW THE LEARNERS VIEW THEMSELVES WITH RELATION TO THEIR MATHEMATICAL IDENTITIES

Sections 3 and 6 were focussed on the learners' actual identities (Sfard & Prusak, 2005); the self-authored notion of who they are and what they are capable of. This actual identity is very powerful as it is often the driver for action (Sfard & Prusak, 2005). Actual identities are present based narratives (Sfard & Prusak, 2005), therefore the items are set in the present tense, for example: '*I believe that people who take ML can be successful one day' and 'I can get good marks in ML'*.

The items in sections 3 and 6 focus on learners' views as to the difficulty of ML due to their own capabilities, their perceived capabilities of being a resource or means of support to peers in the field of ML, their satisfaction with their performance in ML and their future projections for their own success as learners who chose ML as a subject.

SECTION 4: PERTAINING TO ML IN RELATION TO THE WORLD AROUND THE LEARNERS AND SECTION 8: PERTAINING TO THE VALUE OF ML AS A SUBJECT

The imagination aspect of identity tells of how learners can envision that which they learn in mathematics (or ML) as being useful in the world around them. In other words how mathematics fits into their broader lives (Anderson, 2007; Wenger, 1998) both now and in the future. This image could drive the idea that learning mathematics (or ML) is either useful or useless (Anderson, 2007).

Sections 4 and 7 focussed on whether or not learners can relate the skills learned in ML to those skills used in everyday life; whether or not ML is important for, or hindering, further study and future success; and the importance of partaking in ML in the present. Items include: '*ML prepares me for life after school' and 'ML is important because I can see how it used in the world around me'*.

It should be noted that both Wenger (1998) and Anderson (2007) view the manifestation of the alignment aspect of identity as a response to the imagination aspect. Therefore, it will be interesting to draw comparisons between these sections, to see if learners are telling a coherent story.

3.4.2.2 Likert Scale design

Both the pre-study and post-study questionnaires were Likert Scale type questionnaires because it is the tool that is the most widely used when measuring attitudes (Albaum, 1997; McLeod, 2008; Michalopoulou & Symeonaki, 2017), including those attitudes pertaining to mathematics (Ivanov et al., 2018; Michalopoulou & Symeonaki, 2017) such as the attitudes of the learners towards ML and their abilities as mathematics learners who are learning ML. The learners were instructed to respond to a statement in terms of the extent to which they agree with the statement (McLeod, 2008). In my design I opted for a four-option response: I totally agree/ I somewhat agree/ I somewhat disagree/I totally disagree. I chose not to make use of the neutral or 'I don't know' option as that would not provide me with data to work with and I did not want to risk 'lazy' learners just checking the neutral column because they did not want to think about what the statements were saying.

One risk with Likert Scale items that I needed to address was to avoid biased responses (Xiao, Liu, & Li, 2017). In setting up the questionnaires, the choice of wording was rather challenging. My experience with these learners had taught me that they generally display underdeveloped vocabularies. For this reason, I kept all the statements simple and direct, using the register of the learners. The statements were also in Afrikaans, the home language of the learners. I alternated between negative and positive statements as is in line with Likert Scaling Theory (Michalopoulou & Symeonaki, 2017) so as to avoid the overt expression of positive or negative attitudes. I also adhered to the notion that biased responses can be limited by using as little survey items as possible (Xiao et al., 2017). I used, at most, only ten items per section and only 61 items in total.

3.4.3 Orientation session and investigation design

3.4.3.1 The first iteration: orientation session 1

The intention with what I have called the 'orientation sessions' was to get the learners themselves involved in the design and development of the context of an ML investigation. The idea was to use the context and problem as the point of departure and then establish the mathematics surrounding the problem. This idea stems from the notion that the ML curriculum is framed as a modelling curriculum and requires learning to be set in the 'real world' (Buytenhuys & Graven, 2011). I argue that ML, as a subject where modelling is the main driver (Julie, 2006; Ozgen, 2013), should make use of contexts that are presented in their authentic, real world messiness, so that learners may learn through the modelling of these contexts. Giving the learners a chance to be directly involved in the modelling of a relevant and complex situation allows them the opportunity to grasp how mathematics can be beneficial in their lives (Christiansen, 2006). My goal here was to create an enriching learning experience for these learners that support the curriculum goals for ML.

The first orientation session was hosted towards the beginning of the second term. Learners were attending school well as there was an active test schedule in place. I was given the last periods of the school day on a Friday to conduct the first orientation session. I was assured that arrangements had been made for the learners in terms of transport so that we could continue working for the full two and a half hours I requested. I would later find out that this was not the case despite several rounds of confirmation.

I conducted the orientation session with approximately 170 learners, completely by myself. The ML teachers and student-teachers rotated shifts, working in pairs to help maintain order and discipline.

The orientation session ran as follows (an instruction sheet is available in Addendum 7):

INTRODUCTION:

I greeted the learners, who were seated in groups of four, and gave them another briefing on the background of the project. I had also done this before the questionnaire which was initiated prior to the study. I spoke about the goals of the project and my research questions. I also gave them a rundown of the day’s events. Each group was given a number card and asked to write the names of each group member on the back of this card and then hand it back to me. (Instruction 1). There were 42 groups. Each group was also given a work pack. The work pack consisted of a page of instructions and then a blank page was given to the group for each instruction numbered 2 to 7. The groups had to write their group numbers on each instruction page so that I could later reassemble and track the various groups. They were instructed not to move ahead and only work on a problem or instruction once told to do so.

ESTABLISHING THE CONTEXT

In instruction 2, learners were given time to talk amongst themselves and think about their school environment. They had to write down any aspects that they felt were problematic or cause for concern in their immediate school environment.

For instruction 3 the groups were asked to collectively choose and elaborate on ONE problem. Guiding questions were given to promote their thinking and detailed description. These answer sheets were collected by me and sorted according to theme. By themes, I am referring to the types of problems that were named e.g. drug abuse, bullying, school fees, school infrastructure. The problem that was mentioned most, was drug abuse on the school grounds. I found this to be a suitable theme which had both an applicable context and a problem to which the solution and execution thereof could be modelled mathematically. I communicated the most frequently mentioned problem to the entire group of learners and asked them to centre their thinking around that specific problemr. The next instruction was for groups to come up with various possible solutions to the problem at hand. I, again, provided guiding questions to help gain detailed explanations of the solutions. These sheets were handed back to me. I scanned through them and picked the idea(s) that were the most frequent and the best described. I shared these selected solutions with the entire group and asked them to, once again, centre their thinking around that solution. The suggested solutions were to hire a security company to patrol the school and use sniffer dogs to search learners, and to employ a (very unethical/illegal) fine system for learners who are caught with drugs on the school property. Both solutions were put on the table because they had strong ties to the financial concepts covered in the ML curriculum and were useable in an ML investigation.

DESIGNING THE INVESTIGATION: FIRST ATTEMPTS AT MODELLING

Instruction 5 required learners to think about the ML curriculum. What mathematical concepts that they had learned would be useful in executing the solution to the problems? Learners could make use of any resources to identify concepts. They had their schoolbooks and textbooks as well as cell phones with them. They were asked to justify their selection of concepts. Instruction 6 asked learners to design model questions that they would expect, pertaining to this context, in a test or investigation. They were told to place themselves in the teachers' shoes and act as if they were setting up these test questions. They had to consider all the aspects that go into question design such as background information needed, tables or graphs to organise information and how questions would follow up on one another. Instruction 7 was the final group task. They were asked to create the memorandum for their question.

PERSONAL REFLECTIONS

The final task of the day was an individual task where each learner was given the opportunity to respond to a series of prompts and reflect on their experiences in the orientation session. This data would later help me identify potential interviewees for the focus group interviews.

The learners' work packs were collected and their ideas were used in the design of their ML investigation that formed part of their formal assessment mark. The design of this investigation is discussed in the next section.

3.4.3.2 The first iteration: investigation 1

After the orientation session, I collected the work the learners had created during the orientation session. I analysed all the learner responses and made use of as many of their ideas that were feasible, in order to create the formal investigation task. The intention was to promote the feeling that the learners had been directly involved in their learning and the creation of this investigation, so as to understand the link between this kind of involvement and potential changes in their reported mathematical ID.

Step one in designing was harvesting the learners' ideas from their produced orientation session materials. I then formalised these ideas according to curriculum requirements. I made a list of each potential question and next to each question I wrote the various groups that had suggested or contributed to the development of that question. I chose the questions that would be included in the questionnaire based on their potential to illicit mathematical reasoning at a standard required of Grade 11 ML learners. I do not claim to be an expert in this regard, therefore (as discussed later) the assignment was also sent to my supervisor and the schoolteachers to carry approval before it was administered. Next to each question in the assignment I indicated which groups (by number) contributed to the design of that question.

I then set out in setting up a coherent investigation that was, by nature, still very theoretical and hypothetical. However, I did research to authenticate the context of the assignment. Thus, the formalisation of this task had two purposes: to align with curriculum demands and to ensure authentic and accurate contextual information.

I started by outlining the context of the problem and indicated, on the investigation, that this was the context selected and discussed by the Grade 11 learners themselves. In the first question they analysed the costs involved in hiring a security company. In order to get authentic prices, I consulted the security staff at our office building. One aspect of this section questioned whether security workers received above or below minimum wage. I ensured to enter the correct information pertaining to minimum wage by consulting the news website Fin 24 (Omarjee, 2019), which indicated the minimum wage as of 1 January 2019. This same section of the investigation addressed the concept of inflation. It should be noted that these are all concepts that either have or should have been taught to the learners by this point in their school career. I got the information about our current inflation rate from a website called Trade Economics (Trade Economics, 2019) which indicated current inflation rates as in May 2019 – which is when this investigation was designed.

In the second question the learners had to analyse the school budget. I used a fictitious school name in the assignment, both to preserve the anonymity of the school but also so that learners were not under the false assumption that they would be analysing their own school's actual budget. I also made sure to add values to the budget that were as accurate as possible without consulting the school's finances, as I did not have access to this. The information included in the budget were as follows:

- School fees per child as indicated by the vice principal for each of the 1154 learners enrolled at the school in 2019.
- The calculated monthly contribution of the Department of Education toward a school for each enrolled learner. I gained information from the Parliamentary Monitoring Group (April 2018) which predicted the value of these contributions for 2019 (Gina, 2018).
- The monthly water and electricity bill as an estimation only. These numbers will drastically vary based on numbers of learners and infrastructure (Hall, 2017).
- Printing as an estimate from the school.
- Maintenance of the grounds as an estimate from the school.
- Internet and telephone accounts as an estimate from the school.
- Tuck shop fees which were my own estimation as the tuck shop is privately stocked and I could not get the information from the shop owner.
- Unplanned expenses, which was also my own estimation.

The third question was slightly controversial, in my opinion. The learners suggested that the school incorporate a fines system, based on a tariff system, that would require learners caught with drugs on the school property to pay money to the school. I was aware that not only is this unethical, it is also illegal. However, because the learners had done the effort to attempt to formulate this tariff system, and because it contributed to mathematical reasoning, I decided to use the question in the investigation. However, in the interest of ethics, the final question of this assignment asked learners to consider the ethical and legal implications of employing such a system in the schools. Learners were asked outright whether such a fines system is right or not and to justify their opinion. This lends itself to the process of modelling whereby learners are to explore the real-world implications of their suggested solutions and would thus provide data on their modelling competencies.

The investigation was sent to the ML department at the school who had no problems and required no alterations. It was printed and administered. Learners completed the investigation in class time, under the supervision of their ML teacher.

3.4.3.3 The second iteration: orientation session 2

CHALLENGES THAT INFORMED THE SECOND ITERATION

The first orientation session presented with a range of challenges that I needed to consider in my design of the second orientation session. I had two goals in mind when I designed the second orientation session. The first was to create a more conducive learning environment, and the second was to work toward the development of an investigation that was more explicitly based on mathematical modelling.

Challenges from the first orientation session that I had to consider included:

- A 1:170 presenter to learner ratio was too much to handle, especially at the end of the day on a Friday when the learners were particularly rowdy.
- The venue was too large, and the learners were too loud for me to be heard when explaining instructions.
- Although I thought the instructions were simple Afrikaans, most learners did not understand the instructions and I had to explain them to almost each group individually again.
- My session was cut short by an hour, as school had come out early and the learners needed to make use of the school-provided transport that left straight after school to get home. This was not communicated in advance.
- A lot of time was lost between instructions, waiting for submissions, and getting learners attention. This frustrated them as much as it did me.

- The learners found the modelling or design of a question difficult. It was not something they had done before and the instruction had to be repeated several times before they were even willing to try.
- Only a handful of groups took the initiative of using their textbooks as guidelines in designing their questions or in identifying applicable mathematical concepts.

My supervisor and I also discussed the nature of the first investigation as being too theoretical and that the learners were not directly and practically involved in the investigating of this context at a deep level. This was my primary concern to address in the second orientation session and investigation.

GOALS OF THE SECOND ORIENTATION SESSION

In the first orientation session learners presented a wide range of potential contexts that were problematic in the school. To save time in the second orientation session, and focus more on the modelling component, I selected the topic for the second investigation from the suggested topics in the first orientation session. I selected a topic that potentially made use of ML concepts other than only the financial concepts, as this orientation session was conducted in the third term, and finances was primarily the focus of the second term. However, most problems that need solving do have a financial component, so the topic did arise again but to a much lesser degree. The goal was to make this investigation more diverse in terms of the mathematics involved and more practical in terms of the actual investigating of the problem.

This second orientation session was aimed at getting the learners to engage with mathematical modelling. However, these learners had never been subjected to experiences where modelling was required, so I did not expect them to have an established skill set. Being submerged in this new mathematical experience would force learners to reflect on their capabilities as ML learners and thus provide richer data about their mathematical ID.

LOGISTICS OF THE SECOND ORIENTATION SESSION

For this orientation session we (myself and the ML teachers) had planned that I work with each class individually, so that the presenter to learner ratio was 1:40 and not 1:170. I was allowed one double period in a week for each class; meaning I had about 70 minutes to conduct this orientation session for each class. It was agreed that the ML class teacher would be present to assist in classroom discipline. This did not happen, and I was left to manage the sessions by myself.

ESTABLISHING THE CONTEXT AND CONTENT

One of the main, and most interesting, complaints or causes of distress for the learners, was the broken toilets in the school. I used this as a starting point in establishing the context, but also tried to bring attention to the water shortage in the Western Cape by providing relevant information. Whether or not they would make use of this information I could not predict. However, that in itself, would say something about their modelling competencies. I planned to incorporate data handling as the main topic on this investigation, as well as including a small financial component.

The orientation session ran as follows:

INTRODUCTION

I greeted the class and presented them with the topic of the day – the broken toilets in the school. The learners worked in groups of 4 again and elaborated on the problem. I provided some guiding questions (see Addendum 9) to encourage them to start thinking mathematically and not emotionally. The learners were instructed that they were to make a presentation about the seriousness of the problem to the principal in the form of the poster. I wanted them to think about what information would need to go onto this poster. I gave some guiding clues and an example from a different scenario.

GROUP WORK

The learners then needed to write a plan on how to go about getting this information, as well as how to best present this information on a poster. The learners did not actually make the poster in the orientation session – rather just develop the ideas on how to best approach this investigation.

The learners then needed to add to their presentation idea, the potential solutions to the problem. Here I was anticipating that they would either say fix the toilets or replace the toilets.

I provided them with information pamphlets about plumbing fees (from real plumbers), the prices of toilets and their installation and information pertaining to the water shortage in Cape Town. I was interested to see if they would consider these external factors when choosing toilets. They needed to discern for themselves what information is important to be presented as part of their solution.

INDIVIDUAL REFLECTION

At the end of the orientation session, each individual learner completed a reflection about their experience in this orientation session, as well as how this orientation session compared to the first one.

After the orientation session, I once again collected the learners' written ideas and analysed them, so that I may include them in the formalisation of the second investigation.

3.4.3.4 The second iteration: investigation 2

As mentioned, two aims in the design of this investigation was, to move away from the topic of finances as this was addressed in detail in the first investigation, and to focus more on the incorporation of modelling competencies.

During the second orientation session, learners were briefed about their chosen topic and instructed to plan for the design of a poster to be presented to the principle to raise factual awareness of the problem. Learners were instructed that they had to start planning *what* data was important to include in this poster, *how* they would go about collecting that data and *how* they should best display the data on the poster (e.g. graphs or tables or written paragraphs).

From their ideas and suggestions, I designed the outline for the actual data collection and creation of the poster, based on a combination of their suggestions on the most efficient data collection methods, and my opinion of the most suited communication strategies. I wanted to avoid a scenario where the learners would resort to using only graphs (or one type of graph) or only tables, I placed guidelines in the investigation that prescribed different methods of data communication at different sections of the poster. Although learners were given freedom in collecting data in any manner they saw fit, they were prescribed to communicate the results by means of paragraphs, tables, bar graphs and pie charts. This is in line with the curriculum requirements for data handling (Department of Basic Education, 2011a).

However, the second investigation would not only encourage the modelling of various communication strategies, but also required the use of a range of calculation skills such as the use of percentages and the calculation of averages. These calculations should have been made in accordance with the information provided and served to provide the data that needed to be displayed in the prescribed methods.

The learners had to collect data about how many people in the school had a problem with the toilets, what the exact nature of the problem was and how many of the toilets were affected. Due to time and logistic constraints, the sample size of people used by the learners was restricted from the whole school, to just the grade 11 group. The grade 11 group is a large group of approximately 200 learners, so the ML learners would have had to carefully plan effective means to collect and interpret that amount of data.

After the learners had collected, interpreted, and communicated data about the problem, they had to start thinking about the solutions. For this section of the investigation, I presented them with the same information pack they had access to during the orientation session. The information pack included the following information that the learners could use to their own discretion:

- A general quote from a plumber pertaining to costs and time involved in repairing and replacing toilets as well as unblocking drains. I got this quote from a family member who owns his own plumbing business in KwaZulu-Natal.
- Information about different toilets including their prices, water consumption, materials and characteristics of those materials, and any other interesting information. I got this information from the websites of Builders Warehouse, Italtile and CTM.
- I got a quote from an American business that offers services in corporate bathroom hygiene management. I cross-referenced this suggested budget with the budget of a South African company employing 1100 workers (a similar number to the number of learners in the school). The amounts corresponded well. This company has asked to remain anonymous in the writing up of this study.

- A template for the development of a cost sheet that they could make use of at their own discretion. The learners were asked to display all the costs involved in the fixing of the toilets on the poster. However, they were not obliged to use this template and their attention was not drawn to it in the outline of the assignment. I did this for two reasons – to see to what extent the learners utilised the information given, and in cases where they did not use to the template, to see how efficiently and clearly, they could communicate such information on their own.
- Examples of graphs that could be used. This was to serve learners as a reminder of the different types of graphs that are covered by the ML curriculum and were available to them to use to their own discretion. The contexts on the example graphs were far removed to that of this investigation and therefore could not simply be copied into the assignment.
- Water levels of the Cape Town dams as of 8 July 2019 (the week of the orientation session and week before the investigation). This information was taken from the official website of the Cape Town Government (City of Cape Town, 2019). I did not draw attention to this information. I aimed to see whether learners would take this kind of information into account when choosing the most effective water replacement toilet.

The creation of the poster was a group assignment. Learners were asked to work in groups of four but many combined or split into their own groups of varying sizes. The nature of group work, informed by my experience with these learners, meant that there was a high risk of some learners doing the majority of the work while others do not contribute effectively to the task. As this task contributed toward the formal assessment marks of the learners, and in the interest of fairness, I also created an individual component to the task. After the poster was finished, every learner wrote a small individual class test of 20 marks (see Addendum 11). This test was centred around the topic of data handling and composed of two questions – the first being the creating of a pie chart based on given data, and the second being the analysis of a bar graph. These questions were typical test style questions as suggested by the Department of Education, and had contexts far removed from the context of our study.

I sent the investigation to the ML department at the school to be evaluated. However, the school did not look at the investigation and just administered it during class time without moderation. Once the investigations were complete, I collected them from the school for marking and analysis.

3.4.4 Focus group interviews

I conducted a focus group with 10 learners who had been identified as described earlier. Focus groups were held, rather than individual interviews, as experience with these learners had shown that they are more forthcoming and willing to talk to the researcher in the safety of their community group (Ivey, 2011; Nieuwenhuis, 2014) . Focus groups have also been described to be an ideal method of data collection in teacher-learner research, when attempting to understand the factors that affect learners' opinions, behaviours and motivations (Xerri, 2018). In this study I attempted to home in on just that – the factors that influenced the learners' beliefs about ML as a subject as well as their capabilities to do and learn mathematics.

In order for a successful focus group to take place I needed to create an environment of open trust and no judgement (Xerri, 2018). I attempted to do this by inviting these learners to a tea-party where we just had an informal chat about their experiences in this study. The idea was to keep the whole process light-hearted from the beginning. Learners were made aware that they would be voice recorded but just so that I can tell their story as truthfully as possible when I report back. They had the option of refusing to answer any questions that made them feel uncomfortable and were also free to leave at any time.

I believed that the group setting would make it more comfortable for the learners to talk – among each other and not just to me – which is an advantage of a focus group as a method, as it removes the 'spotlight' from the individual (Xerri, 2018). Equally important was that I made use of a semi-structured interview sheet, thus avoiding questions that were too rigid. In this way the free flow of conversation was more likely to be promoted (Xerri, 2018). The semi-structured questions allowed learners the freedom to share anything that may come to mind, but provided me with a framework or research agenda (Stewart et al., 2007) to ensure discussions stayed on topic and provided me with coherent data.

3.4.5 Role of the researcher

I was implicitly involved in this research study. I was familiar with many of the learners, although not all the learners, before the study began. I used this familiarity to establish trust and open dialogue with the learners. I was responsible for the design of all data gathering tools including the questionnaires, orientation sessions, investigations, and the interview schedule. I received guidance for these designs from my supervisor.

I designed and administered the pre-study questionnaire to all the learners at once, in a 30-minute time slot provided by the school. I also hosted both orientation sessions by myself. I then had the role of creating the link between the orientation sessions, the resulting investigations, and the formal academic demands of the ML curriculum. I had to formalise the tasks and ensure that they were fair and valid assessment opportunities. Again, I received guidance from my supervisor, as well as the curriculum documents. I conducted the focus groups interviews alone, at the school, in a 1-hour timeslot provided by the school. The schoolteachers administered the post-study questionnaire in their ML lesson, due to time constraints on the part of the school.

3.5. Data analysis methods

The analysis framework for this study is based on grounded theory, where systematic data analysis leads to coding. The interpretation of theses codes was done by means of thematic analysis and the mapping of modelling competencies. The various codes were integrated to create a theory to explain the nature of the interaction between the learners, the context rich ML materials and the learners' mathematical ID.

3.5.1 Grounded theory

3.5.1.1 Justification for grounded theory as a framework

This study aimed to explain the influence which the interaction with the designing of and use of context rich ML materials could have on the mathematical ID of the learners. In this case study, the intent was to produce a theory, that is grounded in the data gathered from the experiences of the learners, that explains this interaction (Miller & Salkind, 2012; Nieuwenhuis, 2014). This study was undertaken from the perspective of exploring the interactions between the learners, the context rich ML materials and the learners' mathematical ID, rather than seeking to (dis)prove any existing theories or hypotheses about ML education (Nieuwenhuis, 2014).

My study made use of a systematic, inductive approach (Nieuwenhuis, 2014), whereby data was collected by multiple visits to the school (Miller & Salkind, 2012) and involved social interaction between myself and the learners (Nieuwenhuis, 2014), such as conducting the orientation sessions and hosting a semi-structured focus group interview.

The analysis of this data was done by means of a very systematic approach, involving the development of codes and the comparison of these codes (Miller & Salkind, 2012; Nieuwenhuis, 2014; Strauss & Corbin, 1990).

3.5.1.2 Systematic data analysis

As per grounded theory, analysing data was a multistep, systematic procedure (Miller & Salkind, 2012; Nieuwenhuis, 2014; Strauss & Corbin, 1990).

I started by analysing the pre-study questionnaire. The questionnaire had been set up with specific themes (or codes) in mind. These core codes (Nieuwenhuis, 2014) included: (1) identity and imagination, (2) identity and alignment, and (3) actual and designated identities. The rest of the data would be analysed with the view of confirming or disconfirming the relevance and influence of these codes in developing a theory about this case study.

The analysis of the rest of the data followed three main steps (Miller & Salkind, 2012):

1. *Open coding*: in open coding I analysed all the various data sources in relation to the core codes and looked for the emergence of sub-themes. Examples of sub-themes that emerged were imagination and the future vs imagination and my society. Within the core code of imagination, learners reflected differently about how they perceive ML in relation to their future lives and how they perceive ML in relation to their current community. Sub-themes emerged for each of the core codes. Two new themes also emerged: learner marks and modelling competencies. During the open coding phase of data analysis, the reciprocal relationship between data analysis and collection became clear (Strauss & Corbin, 1990). All data collected after the initial questionnaire (which was analysed immediately), was aimed at providing information that would speak to the specific established codes, but was not limited to those codes alone (Miller & Salkind, 2012).
2. *Axial Coding*: during axial coding I positioned the main theme of mathematical identities as the central focus. I then explored how each of the codes established in

open coding, related to themes of mathematical ID, as well as to each other. I used thematic analysis as a method for this phase (discussed inn 3.5.2).

3. *Selective coding*: Codes that were established as having weak or irrelevant ties to the main concept of identity were selectively disregarded in this phase. Codes were regarded as irrelevant when they were based on the utterances and actions of individual learners and were not supported by evidence across all or most of the data sources. In selective coding, I described the interrelationship between the various selected codes, to provide a coherent string of analysed data from which I could build a theory or explanation of the actions and interactions that occurred in my case study.

3.5.1.3 Developing a theory

Let it be known that when I refer to a theory, I am referring to a broad explanation (Miller & Salkind, 2012; Nieuwenhuis, 2014) of the actions and interactions within my case study, and my understanding (Nieuwenhuis, 2014) of how they influenced the mathematical ID of the learners.

From my analysis, I interpreted the data in relation to existing literature. However, my theory was not developed directly from theories prevalent in the literature. Rather, the literature informed my theory or explanation, and offered evidence to support my claims from various sources and contexts, far removed from the ones of this study. My theory was developed primarily from the data of the study itself (Miller & Salkind, 2012; Strauss & Corbin, 1990).

Table 2 on the next page summarises my framework for data analysis:

	Event 1	Event 2	Event 3	Event 4	Event 5
	Pre-Study Questionnaire	Open coding	Axial coding	Selective coding	Theory
Explanation	Thematic analysis of the pre-study questionnaire to establish core codes.	Other data sources were analysed in relation to the core codes. New core codes established as well as sub-themes for each code.	The different data sources were compared in terms of the established codes to see how the codes and data sources relate	Irrelevant codes were disregarded (certain sub-themes). The relationship between the relevant codes from various data sources was compiled	Interpretation of the data to develop a theory about the actions and interactions of the case study
Data Source	Pre-study questionnaire	Orientation session 1 and 2 reflections Investigation 1 and 2 Focus group interview Post-study questionnaire	Orientation session 1 and 2 reflections Investigation 1 and 2 Focus group interview Post-study questionnaire	All data sources	Selective coding write-up
Codes	Three main core codes: 1. Identity and Imagination 2. Identity and alignment 3. Actual and designated identities	Two new core codes added. Each sub-theme was awarded a code e.g. Imagination and society was coded as **IS** 1. Identity and imagination 2. Identity and alignment 3. Actual and designated identities 4. Learner Marks 5. Modelling Competencies	All codes and sub-themes	All codes with selected sub-themes	All codes with selected sub-themes

Table 2 Framework for data analysis

3.5.2 Thematic analysis

As mentioned in the previous section, the core codes and sub-themes were analysed according to thematic analysis. Thematical analysis allowed me to deeply analyse the data presented by the learners in this study and produce trustworthy and insightful findings (Nowell, Norris, White, & Moules, 2017). The richness of these findings contributed toward the theoretical nature of this study, but provided me with flexibility in my methods during analysis (Nowell et al., 2017). Each data source was vastly different and needed to be analysed in different ways: some question-by-question and some theme-by-theme. My methods were fluid (Nowell et al., 2017). I constantly compared the emerging themes across the various sources of data. I would, for example, have analysed all of the reflections in orientation session 1 according to the existing themes and sub-themes. In the analysis of orientation session 2, new sub-themes emerged strongly, meaning I had to go back and re-analyse orientation session 1 in accordance with the new codes. This was done to ensure that any claims made in my developed theory would be supported by data throughout all the data sources and from a high number of learners.

3.5.3 Mapping mathematical modelling competencies

The data gathered of the modelling process was mapped against already theorised modelling competencies which include (Maass, 2006): (a) the ability to make assumptions for a problem and simplify situations, (b) to recognise quantities and variables that influence situations, (c) to construct relations between variables, (d) to differentiate between relevant and irrelevant information, (e) to choose appropriate mathematical notations to represent situations, (f) to make use of heuristic strategies and mathematical knowledge to solve the problem and (g) to interpret the results outside of the mathematical context and to generalise the solutions. I analysed the learners' completed tasks according to a set of predetermined criteria for each competency. I designed a rubric for the analysis of the tasks from orientation session/investigation 1, and orientation session/investigation 2. These rubrics are included in Chapter 5. I used these rubrics rigidly to evaluate the extent to which a specific competency was evident or not in the work produced by the learners. The results of this analysis were compared to learner utterances in their personal reflections at the end of each orientation session, as well as to the utterances of the learners who took part in the focus group interviews. The comparison gave me insight into the learners' actual level of competency vs their perceived level of competency as influenced by their mathematical ID.

3.6 Conclusion

In this chapter, I described the research methodology carried out in this case study, and offered a justification for the selected data collection, sampling, and interpretation methods. I described the interpretivist nature of this case study which underpinned the selection of my research methods. The next chapter details the analysis of the various data sources per theme.

CHAPTER 4: DATA ANALYSIS

4.1 Introduction

This chapter details the methods used to analyse each of the various data sources as described in chapter three. These data sources include the pre-study and post-study questionnaires, the work produced by the learners during both orientation sessions and investigations (including the marks obtained), learner utterances in terms of written reflections after each orientation session and a transcribed focus group interview. The description of the data analysis is per theme that emerged. Each theme is discussed in relation to various sources of data that supported its establishment as a significant theme. The themes analysed are: imagination, alignment, designated and actual identities, learner marks and modelling competencies. I also analysed the learners' critiques of the orientation sessions to inform my theory of the feasibility of mathematical modelling as a means of ML instruction in this school.

4.2 Imagination

In this study, I drew on the work of Wenger (1998), to analyse the mathematical identities of the learners. Wenger (1998) views identity as a way of being in the world – the extent to which we are (not) involved in a community of practice. This notion is supported by Marks and O'Mahoney (2014) who take the position that identity is concerned with the extent to which a learner can identify or is aligned with a specific group. Wenger (1998) identifies three modes of belonging:

(1) Engagement - which describes the extent to which learners would voluntarily partake in mathematical events and is influenced by the degree to which their ideas are adopted or marginalised by their teachers and peers.

(2) Imagination - which relates to how mathematics is perceived as useful and meaningful in life outside the classroom, both presently and in the future, and is influenced through experiences and how those experiences are shared with others.

(3) Alignment - which is a response to the imagination aspect (Anderson, 2007), in which experiences guide the driving forces of learners' willingness to engage, i.e. are learners partaking in mathematics because of its perceived value and benefit to their lives or because they are aligning with institutional requirements?

Physical participation in the research was voluntary, although remained semi-obligatory as the investigations that the learners completed formed part of their formal assessment. The nature of their engagement was not entirely based on free-will and is therefore not explored. Instead, I focussed on the imagination and alignment aspects of their identity.

4.2.1 Analysis of the questionnaires

To explore the learners' imagination with relation to ML, I had to probe learners on their beliefs about the value ML holds for their futures and their lives outside of the classroom. In the questionnaires administered both pre- and post-study, sections 4 and 8 pertained to the learners' views of ML in relation to the world, and the value of ML as a subject. The results made it evident that learners strongly perceive ML as being useful outside of the school setting. Items that scored high levels of agreement stated that ML provides knowledge one can use every day and is a subject that prepares you for life after school. Other statements included that it is a subject that is both important and good enough for the future careers that these learners are interested in and that ML can open the doors to various opportunities for studying. Table 3 summarises the response rates of learners who either agreed or strongly agreed with the questionnaire items in the pre- and post-study questionnaires. The cells that were left empty were due to the error in copying on the part of the school, meaning that those items were omitted from the post-study questionnaire.

It is evident from Table 3, on the next page, that the positive views of the learners were maintained from the pre-study to the post-study questionnaire; a case that was true for all sections of this questionnaire. This indicates that the learning experiences offered in this study, did not do much to sway the opinions that this group of learners held.

	Items	Pre-Study Response Rate (%)	Post-Study Response Rate (%)
25	ML teaches me things that I can use outside of school	93	98
26	ML is math that can be used everyday	95	97
27	ML prepares me for life after school	89	89
28	I can see where people make use of the things we learn in ML in my environment	83	84
29	The textbooks and exercises we use help me to see how I can use ML in my own life	91	94
30	The textbooks and exercises don't always make sense to me	55	56
31	The job I want to do one day does not need math; ML is good enough	76	70
58	The math we learn in ML is important for me to make a success of my life	92	
59	My parents and teachers think math and ML is more important for my future than what I think	90	
60	It is important to learn ML because I can see where it is used in the world around me	91	
61	I have to work hard in ML because if I do well, there will be many study opportunities for me	96	

Table 3 Response rate of learners in the pre-study and post-study questionnaires for sections 4 and 8 pertaining to imagination

From the Table 3, it is clear that an overwhelming majority of the nearly 170 learners claim to see the relevance of mathematics in their lives outside of school and of the importance of this subject in their futures. In fact, only 12 learners disagreed with item 25, that ML can be used outside of school, and only 8 learners disagreed that ML teaches them math that they can use every day. What was of interest to me was that, although 92% of these learners deem ML as important for future success, almost all of them felt that their parents or teachers held an even stronger belief of this than they do. Furthermore, a point of contradiction arose as to the relevance of the materials. 91% of the learners felt that the learning materials they use in school help them to see how ML is useful to them in their own lives, but more than half of the learners stated that sometimes the materials don't make sense. This played into my argument that the materials used are often too far removed from the contexts of the learners and hinders it true usefulness. This idea is supported by the slight observed changes in the data of the post-study questionnaire. After the learners' participation in the orientation sessions and investigation, which exposed them to ML embedded within their own real-world contexts, there was a 5% increase in the number of learners who believed ML teaches them things that can be used outside of school. However, there was also a decrease, of 6%, in the number of learners who felt that ML was good enough for the job they want to do when they leave school. I attribute this to an increased awareness of the mathematics requirements for various fields of study and potential future careers as stipulated by tertiary education institutions during information sessions held at the school in term 3.

I wanted to probe the learners more on their responses to the questionnaire about how they use ML knowledge in their everyday lives and what role they envision for ML education in their futures. Thus, I included questions pertaining to imagination in the focus group interviews.

4.2.2 Analysis of the focus group interviews

During the focus group sessions held after the orientation sessions and investigations were completed, I probed the participating learners in various ways about how they see the importance of ML with regards to their future, and also how they make use of ML in their everyday lives. At first the learners seemed clear on their view of the level of importance ML plays in their futures, but as the interview progressed, their answers became increasingly contradicting.

Initially, all 10 participants agreed that ML plays a crucial role in their futures once they leave school, particularly at university. Furthermore, they all agreed that that they had to achieve good marks in ML to gain access to university. One student, Larry, stated his favourite part of ML is that it is easy because "*then you get better marks*" (FG 1 Larry), which was important "*so that we can be accepted when we apply to different universities*" (FG 1 Larry). He understood that being accepted with ML was dependent on the direction of study and his marks for the subject. He expressed concern that ML could become more difficult which left him less confident that he would be accepted. What was interesting to me, was that acceptance to University seemed to be the end point of ML in the lives of these learners. When asked if they could see how they would be using ML in their futures outside of school, Rebecca stated, "*I don't' know. I am not going that way. It's only for working out time and so on*" (FG 2 Rebecca). Larry also did not see the relevance of ML once he starts studying even though he aimed to be studying sport administration. He stated that "*math will not be a problem for me"* (FG 1 Larry) when he studies, implying he will not have to do it. When I probed him about budgeting as an aspect of sport administration, he laughingly stated that he simply does not want to work with budgets. In his mind they would all be done with math once they leave school. At this point it seemed that the intrinsic value of ML was in actual fact much lower than what was depicted in the questionnaire responses.

Robert objected to Larry's statements, stating that he believed they would always use math as it was part of their everyday lives such as "*time management and when you go to the shops"* (FG 3 Robert). I probed the learners on other aspects in their lives where they make use of ML. Their responses mentioned determining the distance between home and school and determining how long it would take you to move between the two so that you can be punctual; for mixing ingredients (such as in baking) and for domestic chores, specifically determining how much time you have to do a chore and determining the amount of water and washing powder needed for the laundry as well as when to add fabric softeners to the washing load. However, they all felt that they do not think of it as doing math when they are participating in these tasks. This indicates that the learners were displaying low levels of reasoning pertaining to the practicality of mathematics in their everyday lives. They could make superficial connections between mathematics and daily tasks, but in essence, mathematics remains an abstract concept to them.

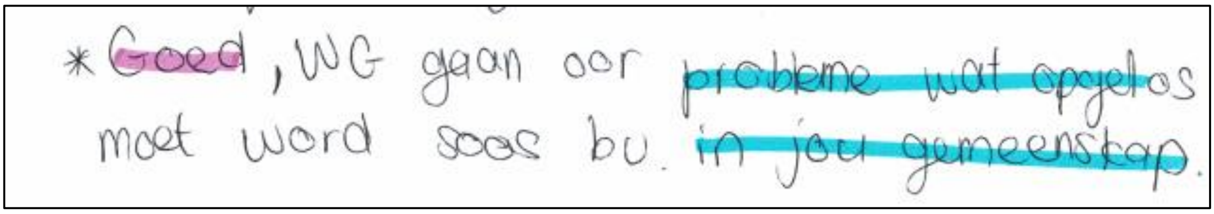
4.2.3 Analysis of learners' individual reflections in the orientation sessions

In both orientation sessions, learners were required to complete individual reflections during which they responded to specific prompts. Neither orientation session 1 nor orientation session 2 prompted learners specifically about the relevance and importance of ML outside of school and in their futures. However, learners still brought up several responses that pertained to this imagination aspect. The prompts for the reflections are attached in Addendum 7 and Addendum 9.

In orientation session 1, 28 learners stated responses pertaining to the use of ML in their societies and 11 learners had responses that linked ML to their futures. In orientation session 2 those numbers increased to 49 and 12 responses respectively. Table 4 summarises some of the phrases that learners used in these responses. These phrases are not direct quotes but are compiled from key words identified in learner responses. The key words are bold:

	Phrase	Number of Responses in WS 1	Number of Responses in WS 2
ML in Society	ML helps with **problems in society/in your area/any problems/real life**	20	9
	Make use of/address/improve school based/real/our problems	0	36
	Gives **clarity/ teaches us** about school context	0	2
	Has our **best interests** in mind	0	2
	It is **impossible to separate** ML from our lives	1	0
	ML gives me **insight into/knowledge of** the world **around me**	7	0
ML and the Future	ML **prepares** me for my **future/what I want to do** in the future	9	0
	I can **study ML** beyond school	1	0
	ML **opens doors**	1	1
	Worth it for the future	0	1
	Prepares/Helps you or others for a better future	0	7
	Helps achieve me **goals**	0	2
	Will help in **other subjects**	0	1

Table 4 Learners responses pertaining to imagination in orientation sessions 1 and 2



The translation reads: "(the orientation session made me feel) good. ML is about problems that must be solved like e.g. in your community".

Figure 4 A learner's response pertaining to imagination.

Table 4 indicates that when learners reflect on their ML practice, many of them are aware of the role this subject plays in their lives outside of the classroom. When looking at the first two responses, it is evident that the orientation sessions may have made more learners aware of how ML can be used to address problems that exist within these learners' immediate contexts. However, it is also clear that the sentiments of the learners who partook in the focus group, reside in the majority of the group as well – that ML is not really considered as a major feature in the futures of the learners. Table 4 indicates that collectively, between orientation session 1 and 2 fewer than 13% of the learners spontaneously connected ML practice to their futures.

At this point, the data indicates that learners do not autonomously consider the role of ML in their lives and in their future, but need to be prompted to do so.

4.3 Alignment

Anderson (2007) defines alignment as those factors that drive learners to engage and is often a response to the imagination aspect of their identity. The data above showed that learners did not freely and consciously associate ML practices with their futures, nor the improvement of or participation in their immediate environments. Therefore, it was necessary to probe learners further to understand what underlying motives they have for participating in ML, other than finishing school. I was interested in why these learners selected ML rather than Mathematics and hypothesised that they may have been influenced by the school or their caregivers in making this decision.

4.3.1 Analysis of the questionnaires

To explore the learners' alignment to ML, I had to probe learners on why they chose this subject, rather than Mathematics and what it is that makes them believe it was the right choice. In the pre-study questionnaire, sections 1 and 7 probed learners on their decision to take ML as a subject, and their motivation behind engagement in the subject. Unfortunately, as mentioned, due to a school printing error, section 7 of the post-questionnaire was not administered and, as a result, there is no comparative data for the alignment aspect. However, the pre-study did provide insights into the learners' motivations behind their subject choice.

Based on the learner responses, it would appear that the decision to engage in ML is one these learners feel that they have taken autonomously. 77% of the learners stated that they were not obligated to take ML by the school, due to their math marks for grade 9 or any other reason, and a staggering 93% stated that their decisions were not taken under the obligation of their parents. Only 2 out of 168 learners indicated that they strongly feel their parents made the decision for them.

One factor that is an apparent driving force behind the decisions of the learners, is their math marks and the potential to obtain better marks. 63% of the learners acknowledge that their marks were too low to take Mathematics in Grade 10, and 78% of the learners chose ML because they thought it would be easier than Mathematics, implying a better chance at higher marks. This implication is supported by a 73% response rate in learners who agree that they chose ML because past (older) learners said that it would be easy to get good marks.

Interestingly to me, the learners do acknowledge the value of taking a mathematics-based subject. Only 38% of the learners agreed with the statement that they chose ML because “you don’t really need math”, with 40% of the learners strongly disagreeing with that statement. Supplementary to that, a mere 23% of learners stated that they don’t really want to partake in Mathematics or ML and this was contradicted by 62% of the learners who strongly disagreed.

As a result of participating in ML since grade 10, 92% of the learners’ state that it is an enjoyable subject, with only 3 learners strongly disagreeing. Only 20% of the learners felt that they continued to engage simply because it is a compulsory part of their schooling, with 78% of the learners stating that their chosen future career will make use of ML. A small number of learners, 13%, state that they would rather be partaking in Mathematics as a subject.

The responses of the learners are summarised in Table 5:

	Item	**Pre-Study Response Rate (%)**
1	I chose ML because the school said I had to, not because I wanted to	77% disagree
2	I chose ML because my parents said I had to, not because I wanted to	93% disagree
3	I chose ML because my marks were too bad for Math	63% agree
4	I chose ML because I thought it was easier than Math	78% agree
6	I chose ML because I don’t think you really need Math	38% agree
9	I chose ML because learners that already take it say it is easy to get good marks	73% agree
52	The math we do in ML is enjoyable	92% agree
53	I take ML purely because the school makes it compulsory	20% agree
55	I actually don’t want to take Math or ML at all	23% agree
57	The job I want to do one day will make use of the math we learn in ML	78% agree

Table 5 Response rate of learners in the pre-study questionnaires for sections 1 and 7 pertaining to alignment

4.3.2 Analysis of the focus group interviews

To gain more insight into the driving forces behind the learners’ engagement, namely their alignment, I probed the 10 participating learners about the appeal of ML to them. Three clear driving forces were identified: marks, recognition, and group work.

First, all the learners agreed that marks are the most important part of learning math, because marks are the gateway to universities and colleges. Their opinion was that marks are important now to set them up for the future they have imagined for themselves. However, as seen in the imagination aspect, once they reach that future, mathematics itself would no longer be important other than menial, daily, routine tasks. The learners affirmed that the best part of ML is that it is easy because '*then you get good marks*' (FG1 Larry). Their concept of ease in relation to ML was associated with the fact that ML, in their opinion, does not have as many calculations as Mathematics. They prioritised marks to the extent that their favourite ML topics were related to the ease with which they can obtain higher marks. Robert stated his favourite part of ML was "*area and volume…because they give the formulas* [in the tests]" (FG3 Robert). He said that made it more '*interesting than problem solving…because I understand better*' (FG3 Robert), although in this case I would argue that he had used the word interesting in the place of appealing. Rebecca also stated that her favourite part of ML was '*about the centimetres*' (FG2 Rebecca), to which she affirmed she meant conversions between units of measurement. She could not give a reason why this is her favourite part of ML. I attribute it to the familiarity of the concept. She would most likely be able to convert units with confidence, as she has been doing it since primary school and it is a section of work that relies on basic knowledge about the rules of multiplying and dividing by 10. She does not need to understand how the units of measurement is being used in the context when simply required to convert the units. Furthermore, the learners were in agreement that their textbooks are good because, as Larry stated, '*the answers are in the back… then you work out and then you can see your working out is right*' (FG1 Larry). I found this statement interesting because, despite follow up statements that their books are old and full of mistakes that confuse them, they were still considered good books because they provided access to the answers, and to these learners, answers correlate strongly with marks. When probed about what constitutes satisfactory marks, Larry bragged that "*since grade 10, I have never failed math again. I get a 4 and then a 5 and then a 4 again and then a 5 again and so on*"[1](FG1 Larry). All the learners gave nods of agreement that these were impressive marks - marks which translate to percentages in the 50s and 60s. These marks are significantly higher than the class averages of the investigations which are discussed later in the chapter, which affirms the source of Larry's pride. Previous encounters with these learners, outside of the study has also taught me that these learners believe 40% is a good mark because it is 10% above the current official pass mark. However, for these learners

[1] The South African system allocates scores as follows: 7 – 80 to 100%; 6 – 70 to 70%; 5 – 60 to 69%; 4 – 50 to 59%; 3 – 40 to 49%; 2 – 30 to 39% and 1 – less than 30%

who have college and university in mind, it seems they have set their standard (aligned) to the minimum required marks of 50% – 60%.

Second, the role of recognition and power as a driving force was clear. What came to light from the analysis of the work produced by the learners in orientation session 1 to orientation session 2, was the extent to which the learners aligned with and engaged with the investigation. It became apparent that the learners were personally invested in the second orientation session and engaged deeply in the experience. Despite being given information sheets and price lists needed to complete the tasks, Rebecca and the other girls in the group informed me that they had walked a distance of 3,6km (one way) to go and investigate the prices of toilets, taps and locking mechanisms for the bathrooms at school. Rebecca described this as being '*more exciting*' (FG2 Rebecca). Another reason that may have prompted increased alignment was that after orientation session 1, where the learners named and explored the drug problem in the school and suggested cameras as a solution, the school implemented cameras and, according to the learners, dealt with the drug problems more effectively. Rebecca said this made them feel very good. Her feelings were that '*they listened [to us], they saw [us]'* (FG2 Rebecca). This gave the learners a sense of unity with Larry stating that '*we stood together. We all stood together over one thing'* (FG1 Larry).

This highlighted a third driving force behind engagement, namely groupwork. Claire stated that she thoroughly enjoyed orientation session 2 because "*then people could talk*" (FG4 Claire). The learners also stated that with trickier ML topics, such as probability (according to Robert), it is much more enjoyable when learners can work on the problems together.

The above-mentioned sentiments were echoed by the group as a whole in the orientation session reflections.

4.3.3 Analysis of learners' individual reflections of orientation session 1 and orientation session 2

The individual responses of the learners to the prompts for reflection at the end of both orientation sessions, alluded to the learners' motivations for engaging in the orientation sessions, which offer insights into why learners would align with ML in general.

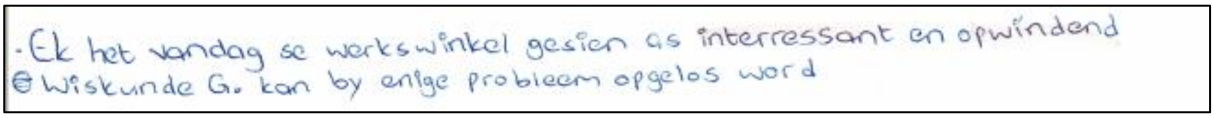
There were 167 responses in orientation session 1 and 68 responses in orientation session 2 pertaining to engagement due to enjoyment. It should be noted that the prompts in orientation session 2 made use of the word ‘enjoyable’, so I did not acknowledge a response of ‘yes’. Responses were only tallied if they made use of the word or synonyms of the word ‘enjoyable’ of their own accord.

Furthermore, there were 57 and 34 responses respectively, pertaining to the orientation session as being a worthwhile learning experience, and the number of responses about having their (the learners’) opinions and ideas heard increased from 33 to 35. Groupwork was also highlighted as a valuable motivator for engagement by 31 responses in orientation session 1 and 23 responses in orientation session 2.

Table 6 summarises some of the phrases or key words learners used in these responses. These phrases are not direct quotes but are compiled from key words identified in learner responses and were translated from Afrikaans to English.

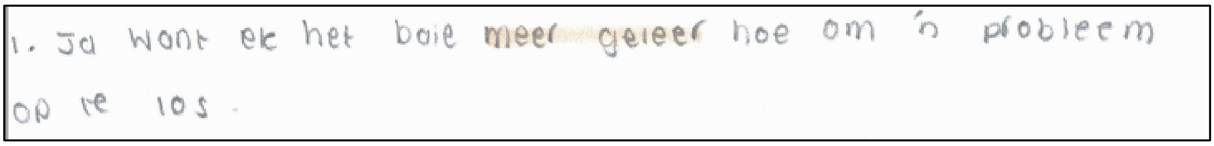
	Phrase/Key Words	**Number of Responses in WS 1**	**Number of Responses in WS 2**
Enjoyment	Interesting	54	13
	Inspiring/encouraging	10	0
	Relevant	0	4
	Enjoyable (lekker)	75	46
	Exciting/look forward to	25	2
	Comfortable (to ask questions/discuss)	0	3
Worthwhile Learning	Learned/experienced a lot/something/new things	57	29
	Think outside the box/differently	0	5
Being Heard	Talk about my problems/what bothers me	0	12
	Raise/Hear my/others’ opinions/ideas	33	19
	Our input/ideas matter/are shared	0	4
Group Work	Good Experience/Enjoyable	31	23

Table 6 Learners responses pertaining to alignment in orientation sessions 1 and 2


·Ek het vandag se werkswinkel gesien as interressant en opwindend
© Wiskunde G. kan by enige probleem opgelos word

The translation reads: "I saw today's orientation session as interesting and exciting. ML can be used in solving any problem".[2]

Figure 5 A learner's response pertaining to alignment


1. Ja want ek het baie meer geleer hoe om 'n probleem op te los.

The translation reads: "Yes (I enjoyed the orientation session), because I learned more about how to solve a problem".

Figure 6 A second learner's response pertaining to alignment.

From Table 6 it is evident that there is a strong social aspect to alignment. Although there are fewer responses (and fewer participants) in orientation session 2 to each of these attributes, these were still aspects that the learners brought up without being prompted to do so. Therefore, it is evident that, not only do learners enjoy working in groups because they are surrounded by their friends, they enjoy talking about relevant problems, sharing their opinions surrounding these problems and their ideas on how to become a part of the solution. It is clear that incorporating this social element has led almost one third of the learners to describe these orientation sessions as worthwhile learning experiences. Over and above the responses mentioned, 5 learners indicated that this experience made them feel cared for and 10 learners explicitly asked for this approach to be repeated.

4.4 Actual and designated identities

Identity can be described as narratives about a learner that have the ability to serve as self-fulfilling prophecies in terms of the learning of that individual (Sfard & Prusak, 2005). These narratives have to be reifying, endorsable and significant (Sfard & Prusak, 2005) in order to contribute to the development of a learners identity. In other words, the individual about whom these stories are told, must view them as being a true reflection of themselves and

[2] Interest and excitement were coded for only once, under the theme of enjoyment

their situation. These narratives are collectively composed between the individual at the centre of the narratives, and those in their environment who tell stories about them (Buytenhuys & Graven, 2011). Sfard and Prusack (2005) refer to these as actual and designated identities: where actual identities refer to those stories a person tells and believes of themselves, while designated identities are externally imposed narratives with future projections (Sfard & Prusak, 2005).

In this section I focussed on the learners' perception of their narratives. I probed about what stories they believe are being told about them, as well as the narratives they have of themselves. The intention was to explore the correlation between the two and how they could have influenced the Imagination and Alignment of the learners.

4.4.1 Analysis of the questionnaire

To explore the learners' designated and actual identities, I had to probe learners on their perceptions of the narratives being told about them, by themselves and others. In the questionnaires administered, both pre- and post-study, sections 2 and 5 pertained to the learners' views of how others perceive ML and think of them as learners who partake in ML (designated identities), while sections 3 and 6 pertained to the learners' personal narratives (actual identities).

4.4.1.1 Designated identities

According to learners' responses to the questionnaires, the stories being told about them by their peers, parents and teachers are, overall, positive narratives of support, equality, and belief in the learners' ability to succeed.

Initially, almost all the learners (98%) agreed that their parents believed they can get good marks in ML, with only two of about 170 learners strongly disagreeing in the pre-questionnaire and zero strongly disagreeing in the post-questionnaire. However, 49% of respondents acknowledged that their parents do not believe they work hard enough to obtain those marks, with that number increasing to 54% by the post-questionnaire, administered just before the June exams. A mere 13% of the learners in the pre-study questionnaire and 22% in the post-study questionnaire felt their parents actually wanted them to take Mathematics instead of ML. I again attribute this increase to learners becoming more aware of university requirements due to the information sessions presented at the school.

Similarly, in both questionnaires, 92% of the learners agree that their teachers believe they can achieve good marks in ML and 87% of the learners state that the teachers actually motivate them to try and achieve their goal marks. In the pre-study questionnaire, 71% of the learners feel that their classmates see them as someone who is 'good at' ML (with this number increasing to 80% post-study) and in both cases about 55% feel their peers trust them enough to ask them for help in ML. It was also clear from both questionnaires that more than 85% of the learners have the view that others believe that people who take ML can achieve success in life, and in both cases, more than 75% of the respondents hold the view that others see ML as 'real' mathematics.

However, there were also some items that pointed out potentially negative connotations to ML and these learners. In the pre-questionnaire, 46% of the respondents felt that other people believe ML is for people who cannot do mathematics and by the post-study, this number had increased to 54%. It was also clear from both questionnaires that between 79% and 84% of the learners believe people think of ML as being easy. Initially only 51% agreed that people think learners who take ML cannot go to university, but by June this number had increased to two thirds of the group, which supports my hypothesis about learners becoming aware of university criteria from information sessions. Furthermore, the number of learners who agreed that people think ML is for dumb[3] people increased from 33% to 40% in the pre-study and post-study questionnaires.

The above data is summarised in Table 7 on the next page.

[3] In this thesis the word dumb is not used to describe someone with a lack of power of speech, but rather as a less derogatory synonym for stupid to translate Afrikaans expressions of that tone.

	Items	Pre-Study Response Rate (%)	Post-Study Response Rate (%)
11	People think ML is for people who cannot do math	46	54
12	People think ML is easy	84	79
13	People think that if I take ML, I won't be able to go to university	51	66
14	People believe that learners who take ML can achieve success one day	86	88
15	People believe that ML is still a form of real math	77	75
16	People believe ML is for dumb people	33	40
32	My parents/caretakers believe I can get good marks for ML	98	97
33	My parents/caretaker think I don't work hard enough at ML	49	54
34	My parents/caretaker actually want me to take normal math	13	22
35	My teacher believes I can get good marks in ML	92	92
36	My teacher motivates me to work harder in ML	87	87
38	My classmates see me as someone who is good at ML	71	80
39	My classmates regularly ask me for help in ML	55	56

Table 7 Response rate of learners in the pre-study and post-study questionnaires for sections 2 and 5 pertaining to designated identities

From the data, it is apparent that in the minds of the learners, people can hold a potentially less positive view of ML as a subject, but that it is not directly linked to the learner. For most of these learners, the narratives told about them as individuals capable of learning and doing mathematics, is overwhelmingly positive. However, it does seem that the criteria and standards set forth by individuals and institutions (such as universities) outside of the learners' immediate environment, may be viewed by the learners as being more negative and critical of ML. They are seemingly exposed to two narratives: an inner circle that tells them they can succeed, and farther removed influence that frowns upon ML and paints it as a barrier to further education.

4.4.1.2 Actual identities

The stories learners tell of themselves have the same characteristic tones of positivity as the designated identities. It was clear from the pre-study and post-study questionnaires that, at face value, these learners believe in themselves and their ability to succeed. They believe that they can do and learn math and that they are achieving satisfactory marks for whatever purpose they intend.

Table 8 summarises the responses of the learners in the pre-study and post-study questionnaire. The cells left empty are, again, due to erroneous printing meaning those items were not administered in the post-study questionnaire:

	Items	Pre-Study Response Rate (%)	Post-Study Response Rate (%)
17	I think ML is for people who cannot do math	14	15
18	I think ML is easy	74	60
20	I think if I take ML, I won't be able to go to university	23	23
21	I believe that people who take ML can achieve success one day	97	97
22	I think that ML is for dumb people	2	4
41	I can get good marks in ML	96	97
42	I understand the math we learn in ML	88	94
43	I can master the math we learn in ML even if it is difficult	89	91
44	I believe I can offer ML help to my peers	80	80
47	I determine how good I am at ML by looking at my marks	91	91
48	I am satisfied with my ML marks	66	72
49	I chose ML because I don't understand normal math[4]	41	
50	I feel dumb because I take ML and not normal math	14	

Table 8 Response rate of learners in the pre-study and post-study questionnaires for sections 3 and 6 pertaining to actual identities

In analysing Table 8, it is clear that learners do believe they are capable to learn and do mathematics. A mere 14% to 15% of learners felt that ML is for people who cannot do mathematics, but perhaps more telling was that in both questionnaires between 50% and 60% of the respondents *strongly* disagreed with that statement. In support of this, only 2% and 4% respectively, state that ML is for dumb people and in the pre-study questionnaire only 14% of respondents said they feel dumb for not taking Mathematics. However, in this questionnaire, 41% of the learners did agree that they took ML because they did not understand Mathematics in the lower grades. Furthermore, in both questionnaires only 23% of the learners believed that their choice to participate in ML is hindering their chances of going to university. In fact, in both questionnaires 97% of the respondents believe that, although they take ML, they will succeed in life. This indicates that for a number of learners, success is not directly related to tertiary education.

[4] In this case 'normal math' refers to the subject Mathematics as opposed to Mathematical Literacy. It is a colloquial reference to the subject.

What has been established is that, in the minds of the learners, success is related to their marks (91% agree on this in both questionnaires). In both questionnaires, more than 95% of the learners believe they can get good marks in ML and the number of learners who believe they can understand ML increased from 88% to 94% from pre-study to post-study questionnaire. Interestingly to me, despite the fact that the number of learners who view ML as easy decreased between the two questionnaires from 74% to only 60%, the number of learners who feel they can master ML even when it is difficult remained relatively constant at 89% and 91% respectively. The number of learners who felt they are capable of offering help to other also remained constant at 80%. To me this indicates a contradiction in what the learners are telling me – that many of them are finding the mathematics they are learning increasingly more difficult, but that they are still a good source of support to others and are capable of understanding ML well. Nonetheless, despite the reports of experiencing increased difficulty in ML, as well as my experience of the learners' marks in the investigations, the number of learners who reported that they are satisfied with their marks increased from 66% to 72%.

Upon the above analysis, I did not feel that the learners had a clear sense of their mathematical identities. Although I acknowledge an identity as a dynamic and ever-changing construct, the questionnaires posed some contradicting notions, which indicates to me that the learners do not have a clear and concise picture of what they believe of themselves in relation to ML. It would also appear that their immediate environment and the support received from their inner circle, allows them to form positive and self-empowering narratives. However, when they step into a wider society, the standards and norms of different environments may cause these learners to question their beliefs of themselves and their mathematical ability.

4.4.2 Analysis of the focus group interviews

To gain better insight into the learners' mathematical identities, I probed the 10 participating learners to give more details on how they think their peers, teachers and parents perceive them, as well as how they perceive themselves. Again, I was met with optimism and positivity intermingled with downplayed negative comments.

4.4.2.1 Designated identities

Having worked in this school, I understand that the learners attach a high value to a given instruction. An instruction given by a teacher or authoritative figure is to be carried out to the letter. In the minds of the learners, wrongdoing or mistakes are almost always achieved if instructions are not followed explicitly. This sentiment was echoed by Larry, who, when I mentioned that their poster turned out much different to that which I had asked, responded in pure panic and cried out, '*so is it actually wrong?'* (FG1 Larry). The concept of open-ended questions was not familiar to these learners at all, the idea that there is not one definitive right or wrong answer is incomprehensible. In the formal school curriculum that these learners were exposed to, ML tasks were typically designed to have a correct answer, which could be obtained by the learners if they followed the methods they were taught in class. I believe this curriculum design may speak to the perceptions of the learners. It tells of curriculum designers and implementers who are wary of providing learners with opportunities to self-regulate or explore academic content outside of a rigid, scaffolded structure. It speaks to a power dynamic that does not support learners to develop agency. This idea was supported by the learners when Larry stated that his ML teacher offered very little assistance during investigation 2 but that '*we dare not ask him questions…maybe just to spell certain words…we did not want to ask him questions*' (FG1 Larry). This does not support the illustration of a good and motivating support base from the teachers that was indicated in the questionnaires, where most learners stated that their teachers were encouraging and motivating. This contradiction was further mirrored in the responses from the learners. When explicitly asked if the learners feel they have a good support base in their families and teachers, the response was a very assertive '*yeeeeeeees'* (FG3 Robert) from Robert and nodded agreement by his peers. However, Robert also elaborated on his teacher's behaviour in class aimed at humiliating the learners for not getting answers correct. Robert explained that his favourite part of ML is that the teacher places kids in the '*spotlight'* (FG3 Robert) to give an answer and responds to them by calling them *'a horrible person'* (FG3 Robert) in front of the whole class. To the boys in the focus group this was acknowledged as a joke that lets them bond with their teacher. In my view, this is another example of poor support and lack of a nurturing culture. The only evidence that suggests what I would consider as support, is that the learners all felt that their support base did not compare them to learners who take Mathematics and accepted their choice for ML.

When probed about what their peers, who take 'normal' Mathematics, think about them as ML learners and whether or not they ever felt compared, they all responded that they did not feel compared by anyone. Larry further elaborated that, *'some of them brag they are smarter than us, but that is not true because Mathematics and ML both have calculations that you need to do'* (FG1 Larry). Claire also discounted the notion that Mathematics learners are smarter than ML learners because *'my marks are better than the person who does normal math'* (FG4 Claire). This indicates that the learners have an acute ability to discount negative narratives about them, based on the marks, regardless of the subject content, and by making use of humour as an excuse for unsupportive behaviour. This provides some clue as to why they are able to maintain such positive actual identities despite evidence that should suggest the contrary, and might in different contexts.

4.4.2.2 Actual identities

During the focus group interviews, it was clear to me that the learners do not easily waver from the narrative of belief in themselves and a positive outlook on their capabilities.

Larry confidently stated during the interview that he wanted to share the honest sentiment that he repetitively failed Mathematics in Gr 8 and 9, but that he now understands the mathematics and has not failed since Gr 10. This affirms his belief in himself as someone who is capable of understanding mathematics. In fact, later on in the interview he confidently states that for him, *'math is not a problem'* (FG1 Larry). However, although the preoccupation with marks has been well established, Robert was supported by the others to say that ML does not need to increase in difficulty because as it stands, they get *'better marks because it is easy'* (FG3 Robert). This was indicative to me that there is, albeit subconscious, a fear that increased cognitive demand may not be met. If the work is harder, they expect to achieve lower marks, which may challenge, what now appears to be, a very fragile sense of identity.

According to the interview, the learners also felt re-energized to have had a role in the way the school addressed the drug problem. As mentioned before, the learners stated that they felt good about it and as if they had been seen and heard. The pride that the learners were feeling in that moment was clearly visible in their body language – smiling faces, sitting up straight, and puffing out their chests. Rebecca then went on to say that she believed they stood a chance in initiating change in the toilet problem addressed in investigation 2 as well. What I take away from this is that when ML activities lead to tangible results, marks are no longer the overwhelming focus on which they base their identities. Instead they are supplemented by successes that can be measured with visible results and an impact on their environment.

4.4.3 Analysis of learners' individual reflections of orientation session 1 and orientation session 2

A flaw in my research methodology was to not prompt learners more about their actual identities – that which they think and tell of themselves – at the end of each of the orientation sessions. However, a small number of learners did mention aspects pertaining to their actual or designated identities of their own volition. Their responses are summarised in Table 9. The phrases used are not direct quotes but have been developed from key words in the learners' responses and translated from Afrikaans:

	Phrase/Key Words	**Number of Responses in WS 1**	**Number of Responses in WS 2**
Actual Identities: Positive	I feel good/positive/comfortable/better/proud	56	2
	Self-confident	9	1
	I can do math/I know what I am doing	10	0
	I can achieve	6	0
	I am smart (have brains)	8	0
	I can do my own orientation session	0	1
	Enjoy math	0	1
Actual Identities: I understand	I can understand (better)	15	14
	I know more	2	0
Actual Identities: Negative	I am a failure	1	0
	I feel stupid	3	0
Designated Identities: Negativity	People say we are stupid	2	0
	People think we cannot achieve	2	0

Table 9 Learners responses pertaining to designated and actual identities in orientation sessions 1 and 2

The translation reads: "It (the mathematics in the orientation session) made me feel good (about myself)"

Figure 7 A learner's response pertaining to actual identity

Table 9 suggests that not many learners spontaneously reflect on their ability and willingness to learn and do well in mathematics. However, of those who do, the response is overwhelmingly positive. This is in trend with the rest of the data, which suggest these learners have an unconditional optimism when it comes to their self-image and perceptions of their capabilities. Despite evidence to the contrary, in the products produced in the investigations, some of these learners seem to honestly believe that they (can) understand mathematics. There are only single individuals (eight responses from orientation session 1) who openly speak against the trend and admit they feel less capable or that they feel others see them in that way.

4.5 Learner marks

At this point it has been clearly established that the learners have a confident yet fragile sense of identity, one that is almost unnaturally positive and hopeful when viewed within their educational context, and that is strongly influenced by their marks. This is especially true when it is considered that the learners identify their main criteria against which to consider their success, as being their ML marks. For this study, I did not have access to all the marks of the learners in terms of their previous test, exam, and term marks. However, the investigations that I designed, and which the learners completed after orientation sessions 1 and 2, formed part of their formal assessment program and contributed to 10% of both their Term 2 and Term 3 marks. These marks are indicative of learners who are barely reaching the minimum pass mark and who do not display competent levels of understanding.

4.5.1 Analysis of investigation 1

Investigation 1 was closely related to a typical school investigation task that the learners were used to. The information was given, and learners were required to respond to various contextual questions and to do calculations of varying levels of difficulty. The investigation (Addendum 8) consisted of 3 questions, composed of 15 sub-questions that served as scaffolds to systematically increase the cognitive demand. In other words, the questions were broken down into manageable chunks, starting with easy and simple calculations, progressing to more complex calculations and culminating in the interpretation of their calculations in relation to society. The learners completed these investigations individually. There were 173 submissions in total. The results were extremely poor and are summarised in Table 10 and Table 11.

Average Percentage for Investigation 1	
Grade Overall	39
Classes taught by Teacher A (11A and E)	53
Classes taught by Teacher B (11 B, C and D)	31
11A	50
11B	35
11C	32
11D	27
11E	55

Table 10 Summary of average percentages obtained in investigation 1

Analysis of Rating Code Level Achieved in Investigation 1[5]						
Rating Code	**11A**	**11B**	**11C**	**11D**	**11E**	**Total**
1 (0-29%)	0	13	17	24	0	**54**
2 (30-39%)	5	6	13	7	1	**32**
3 (40-49%)	5	3	9	6	7	**30**
4 (50-59%)	5	5	2	2	20	**34**
5 (60-69%)	11	1	1	0	8	**21**
6 (70-79%)	1	0	0	0	1	**2**
7 (80-100%)	0	0	0	0	0	**0**
	27	**28**	**42**	**39**	**37**	**173**

Table 11 Summary of rating code levels achieved in investigation 1

The pass percentage for learners in Grade 11 in ML is currently 30%. I verified this with the ML department at the school. The average for the 173 assignments submitted to me to be marked, was 39%. I am aware of 8 assignments that were not submitted and received a zero mark, which would further lower the grade average, but that was not considered in the analysis of these marks so as to avoid skewing the data. The grade average mark is described as 'elementary achievement' according to the official rating scale (Department of Basic Education, 2011a), and I argue that it should not be considered sufficient to pass. What is more, 50% of these learners achieved a level 1 or 2, a mark that is a barely a pass or not at all. Only 13% of the learners achieved a mark of at least 60%.

Evidently, there is a big discrepancy, of 22%, between the averages of the classes taught by Teacher A and Teacher B. The cause of this discrepancy is not a difference in the quality of teaching, as one would assume. Teacher A had clearly made the memorandum of the assignment accessible to the learners. There was clear evidence that at least 15 learners in 11A and 28 learners in 11B had copied answers directly from the memorandum. I had

[5] Classes 11 A and 11 B had fewer submission because in these classes, 14 and 13 learners respectively take Mathematics instead of ML.

concrete proof, such as quoting answers that start with the words, '*for this question learners should…'* for certain questions on the papers of certain learners. I gave a zero mark for these questions on the advice of the subject head of the school. However, despite those zero marks, these two classes still performed much better than the other classes, indicating to me that the use of the memorandum by the learners could have been much more widespread.

I further analysed each investigation to see the frequency with which the learners obtained less than half the marks allocated to each individual sub-question. That data is summarised in Table 12:

Question	Number of Learners	Percentage of Learners
1.1	30	17
1.2	26	15
1.3.1	66	38
1.3.2	60	35
1.3.3	62	36
1.3.4	114	66
2.1	69	40
2.2	58	34
2.3	108	62
2.4	161	93
2.5	162	94
3.1.1	172	99
3.1.2	167	97
3.1.3	172	99
3.1.4	171	99

Table 12 The number of learners who obtained less than 50% for each question in investigation 1

In my opinion, one of the main goals of education is to enable learners to transfer their knowledge to various contexts and make sense of the meaning of the calculations within that context. Table 11 indicates that almost all the learners failed to answer questions 2.4 through 3.1.4 with at least 50% accuracy. I found it interesting that three of these questions (2.4, 2.5 and 3.1.4) were interpretation questions, where learners had to simply give their opinion on a matter and support their opinion with their calculations. These questions asked why it was important for the school to create a budget, and how the proposed budget could be improved. Many learners did not even try to answer these questions, so either they did not have an opinion, or they could not form an opinion from the mathematics. Question 3.4 was an ethically grey question, asking whether it is ethical for the school to make money by fining learners who are caught with drugs on the school property. This question was suggested by the learners and I was conflicted about using it in the investigation. In the end, I decided that I would use it, because it would indicate whether learners can create the links between mathematics and context. In this case most learners answered that it was a good thing, because to them the mathematics said they would make money, and regardless of the context, that is a good idea. The legal implications of such a system was deemed irrelevant. All of this indicates to me that these learners will easily say that they use mathematics in their daily lives, but they are not truly able to make the link between mathematical calculations and societal ideas or problems. In reality, despite what the learners are telling me, and themselves, there may be stark separation between ML and the life outside the classroom.

1.4.2 Analysis of investigation 2

The intention behind the design of investigation 2 (Addendum 10), was to encourage the learners to engage more deeply with the context of the problem, and so foster the transfer of knowledge. They were given some information that may have been useful to them in creating the poster, but they were also required to gather a lot of the information themselves. In the orientation session prior to the investigation, the learners were given the opportunity to plan how to gather, use and best represent their information, to help propose a solution to the problem at hand. I was there to guide the learners through the planning process and used their ideas to stipulate guidelines in the actual investigation. However, the posters the learners created as their final product left a lot to be desired. It did not really come as a surprise, as both the learners and the teachers mentioned on more than one occasion that they did not have experience in these types of modelling tasks and that they were unsure of how to proceed. My role was to ensure them that uncertainty and making sense of one’s ideas was a normal part of this process.

The posters were completed as groups, which I marked, but the marks were so poor that a second, individual task (Addendum 11) was given as part of this investigation. The second task was of a more familiar test-like structure that required the learners to interpret information from graphs. The results of this section of the assignment was much better by comparison but still did not speak to the ‘satisfactory’ marks the learners were portraying. I found this particularly concerning because the ‘test’ was of a low standard, in my opinion.

The questions were centered around reading the number of responses for specific categories in a bar graph, and constructing a simple pie chart – outcomes already described at primary school level (Department of Basic Education, 2011b). The questions were simple and straight forward and pertained to work that the learners had been exposed to since grade 7 (Department of Basic Education, 2011b).

The results of both tasks, the poster and the test, are summarised in Table 13 to Table 16.

Class	Average Percentage
11A	16
11B	42
11C	43
11D	33
11E	27
Overall	31
31 group submissions, 1 group poster 'lost' (zero mark), 167 learners	

Table 13 Summary of class averages for investigation 2 poster

Class	Average Percentage
11A	60
11B	35
11C	40
11D	35
11E	45
Overall	40
163 submitted tests, 4 unsubmitted	

Table 14 Summary of class averages for investigation 2 test

Analysis of Rating Code Level Achieved in Investigation 2 Poster						
Rating Code	**11A**	**11B**	**11C**	**11D**	**11E**	**Total**
1 (0-29%)	31	8	15	13	29	**96**
2 (30-39%)	0	4	4	11	10	**29**
3 (40-49%)	0	4	17	0	0	**21**
4 (50-59%)	0	8	4	0	0	**12**
5 (60-69%)	0	4	0	5	0	**9**
6 (70-79%)	0	0	0	0	0	**0**
7 (80-100%)	0	0	0	0	0	**0**
	31	**32**	**40**	**29**	**39**	**167**

Table 15 Summary of rating code levels achieved in investigation 2 poster

Analysis of Rating Code Level Achieved in Investigation 2 Test						
Rating Code	**11A**	**11B**	**11C**	**11D**	**11E**	**Total**
1 (0-29%)	0	10	7	14	4	**35**
2 (30-39%)	1	5	10	7	6	**29**
3 (40-49%)	1	5	10	5	8	**29**
4 (50-59%)	14	5	7	6	10	**42**
5 (60-69%)	12	1	2	1	4	**20**
6 (70-79%)	1	1	0	2	2	**6**
7 (80-100%)	1	1	0	0	0	**2**
	30	**28**	**36**	**35**	**34**	**163**

Table 16 Summary of rating code levels achieved in investigation 2 test

From Table 15 and Table 16, one can see that there is a discrepancy in the number of submissions for the posters and the tests. The 'missing' submissions were not included in the data analysis as I did not want to skew the data with zero marks. I was not informed whether these learners had just been absent or had left school indefinitely. The teachers at the school suggested that they would manage this discrepancy for the purposes of formal processing of the learners' marks.

An aspect of interest to me, was that in the poster, the classes (11A and 11E) which had done significantly better in investigation 1, did worse than the other classes with the poster aspect of investigation 2. The 'high' (relative to this case study) performance of classes A and E in investigation 1, is attributed to their access to the memorandum, as discussed earlier. The task where the learners had to design a poster was of a much higher cognitive demand, with much less scaffolding and structure. Not only did they have to engage in new ways of thinking and approaching mathematical problems, they also had to evaluate their own progress against a rubric. This was something they had never done before in ML. The classes who did not have access to the memorandums in investigation 1, outperformed (albeit slightly) the classes A and E in the poster task. However, classes A and E got the highest scores in the individual test component again. This indicates to me that the learners in classes A and E are being taught in a manner that foregrounds the structure and content of a traditional school-based assessment, more so than the other classes. These learners are not exposed to learning opportunities that allow them to openly explore contexts mathematically but are trained to answer specific mathematical questions in a specific way.

4.6 Modelling competencies

The orientation sessions and investigations used in this study, were designed to explore the learners' modelling competencies. The first orientation session and investigation straddled the line between modelling and traditional test type tasks, whereas the second orientation session and investigation aimed at being a more explicit modelling type task that would yield more valuable data about the learners' modelling competencies.

The data gathered of these two modelling processes were mapped against the following theorised modelling competencies (Maass, 2006): (1) the ability to make assumptions for a problem and simplify situations, (2) to recognise quantities and variables that influence situations, (3) to construct relations between variables, (4) to differentiate between relevant and irrelevant information, (5) to choose appropriate mathematical notations to represent situations, (6) to make use of heuristic strategies and mathematical knowledge to solve the problem and (7) to interpret the results outside of the mathematical context and to generalise the solutions. The broad framework I used was as in Table 17 (MM stands for mathematical modelling and the number refers to which competency was observed as numbered above):

	MM1: the ability to make assumptions for a problem and simplify situations	MM2: to recognise quantities and variables that influence situations	MM3: to construct relations between variables	MM4: to differentiate between relevant and irrelevant information,	MM5: to choose appropriate mathematical notations to represent situations	MM6: to make use of heuristic strategies and mathematical knowledge to solve the problem	MM7: to interpret the results outside of the mathematical context and to generalise the solutions
A	No Evidence						
B	Attempts made but largely unsuccessful						
C	Attempts made with some degree of success						
D	The competency was displayed well and clearly.						

Table 17 The broad framework for analysing modelling competencies

These two assignments were the first time in their high school careers that the learners had ever been exposed to modelling type tasks. They had never before experienced materials outside of the prescribed textbook and the tests and investigations that were recommended by the Department of Education. Therefore, when analysing the data, I took a very liberal view. I established key markers I was looking to observe for each modelling competency and then determined the relative level of success. I had to revert to a norm-based assessment, where I compared the work of the different groups to establish who had achieved a level of success, and who had not by comparison. I also took contextual factors into consideration such as the colloquial language use of the learners, which often needs to be interpreted in relation to their backgrounds.

4.6.1 Analysis of orientation session 1

During orientation session 1, learners carried out various instructions (Addendum 7) under the main task of helping me design their investigation for the term. They could make use of their textbooks as guides when identifying topics and setting up questions that could be used, but the creation of context was dependent purely on their own experiences. I analysed the orientation session responses of each group, to each instruction, in relation to the modelling competencies. Table 18 to Table 24 indicate the markers and criteria used to establish the learners' relative level of success in displaying each modelling competency. The numbers refer to the number of groups that attained each level:

MM1 – Ability to make assumptions for a problem Can the learners describe the problem clearly to give the reader a clear but summarised scope of what is being dealt with? Does the description leave room for mathematising the problem?		
Level Obtained	**Number of Groups**	**Description**
A (No evidence)	22	Learners either did not describe their problem situation at all, or the description was of such a nature that I could not determine the nature and extent of their problem.
B (Attempts made but largely unsuccessful)	17	Learners were able to describe their problem situation with some success. However, the description was very socially driven and did not leave much room for making math of the problem.
C (Attempts made to some degree of success)	1	Learners described their problem with relative clarity, allowing the flow of ideas around how to make math of the problem. The description was relatively objective, and solution orientated.
D (The competency was displayed well and clearly)	0	The problem is described clearly in terms of nature and scope. Numerical data is included that summarised the problem well but also allowed for mathematization of the problem.

Table 18 Summary of levels obtained in relation to MM1 in orientation session 1

MM2 – Recognising quantities and variables that influence the situation Could the learners identify significant role players involved in solving the problem? Could the learners identify resources needed to solve the problem?		
Level Obtained	**Number of Groups**	**Description**
A (No evidence)	8	Learners either did not complete the instruction, or did not correctly identify any role players or resources needed to solve the problem.
B (Attempts made but largely unsuccessful)	23	Learners attempted to identify the role players and resources needed for the solution but named very few and did not describe their influence on the situation well. Critical resources were missed.
C (Attempts made to some degree of success)	9	Learners were able to identify most of the resources and role players, and were able to describe some form of a relationship between these variables (more focused on social dynamics).
D (The competency was displayed well and clearly)	0	All the relevant role players and resources are identified. The relationship between these variables and the implementation are made clear in mathematical terms e.g. use of proportions.

Table 19 Summary of levels obtained in relation to MM2 in orientation session 1

MM3- Construct relations between variables Could the learners quantify the resources needed? Could they map out a realistic timeline for the implementation of the proposed solution?		
Level Obtained	**Number of Groups**	**Description**
A (No evidence)	22	Learners either did not submit any work to analyse or they were completely unable to quantify or numerically describe the relationship between the variables and the implementation of the solution.
B (Attempts made but largely unsuccessful)	7	Learners attempted to quantify the relationship between the variables (role players and resources) and the implementation of the solution but the description was either incorrect (nonsensical) or incomplete.
C (Attempts made to some degree of success)	3	Learners were able to show evidence of correctly quantifying and mathematically anticipating the influence of variables on the solution of the problem at hand. In many cases the numbers used to describe these relationships were not realistic.
D (The competency was displayed well and clearly)	0	The relationships established in MM2 are quantified in a manner that is sensical. Realistic numbers are used to describe these relationships. This description is indicative of which mathematical concepts the problem lends itself to.

Table 20 Summary of levels obtained in relation to MM3 in orientation session 1

MM4- Differentiate between relevant and irrelevant information Could the learners identify and describe how concepts from their ML curriculum are related to the specific problem at hand? Is the description of the solution and the identifying of variables pertinent to the problem at hand?		
Level Obtained	**Number of Groups**	**Description**
A (No evidence)	23	Learners identified ML topics that are in no way related to the problem or solution they described, or their choice in topics was not supported by an explanation as to how it pertains to the problem and solution. Some learners did not complete this section.
B (Attempts made but largely unsuccessful)	7	Learners identified and justified one or two concepts to satisfaction, but also mentioned several concepts and ideas which were off topic or not realistic within the context of the problem.
C (Attempts made to some degree of success)	8	Most of the topics chosen and their justification was on topic and relevant to the context. One or two ideas were ambiguous in terms of their relation to the situation at hand and in some cases not all aspects of the problem and solution were considered.
D (The competency was displayed well and clearly)	2	Learners clearly described the topics they would use in relation to the problem and the link between the topic and the context was explicit, correct and sensical. There was no ambiguity or unrelated topics or ideas mentioned. Learners have considered most aspects regarding the solution to problem.

Table 21 Summary of levels obtained in relation to MM4 in orientation session 1

MM5 – Choose appropriate mathematical notations to represent the situations Could the learners design questions for an investigation into their solution that is relevant to their ML curriculum and aimed at implementing the solution to their problem? They had access to their ML textbooks to serve as a model.		
Level Obtained	**Number of Groups**	**Description**
A (No evidence)	22	Learners either did not design any questions at all or designed questions that were not mathematically driven, i.e. social questions that do not require mathematical reasoning or calculations to be answered.
B (Attempts made but largely unsuccessful)	16	Attempts were made to design mathematical questions, but they were not sensical or did not pertain to the context of the problem or solution at hand.
C (Attempts made to some degree of success)	2	There is clear evidence of the development of mathematics-based questions that hold relevance to the problem and solution at hand. Questions may be flawed in their wording or structure but can be adapted to be used in a formal investigation.
D (The competency was displayed well and clearly)	0 .	Good mathematical questions were designed that help to solve the problem at hand. The questions are worded and structured well. The values and variables used are realistic. The questions can be used (almost) as is in a formal investigation

Table 22 Summary of levels obtained in relation to MM5 in orientation session 1

MM6- Make use of heuristic strategies & mathematical knowledge to solve the problem Could the learners set to work mathematically solving their own designed questions (creating a memorandum)? This section will be supplemented with data of learners' individual performance in the actual conducting of the investigation.		
Level Obtained	**Number of Groups**	**Description**
A (No evidence)	28	Learners either did not attempt to answer their own designed questions or had not designed any to answer. Some answers reflected no mathematics and were socially orientated.
B (Attempts made but largely unsuccessful)	9	Learners attempted to use mathematics to provide answers to their questions but were largely incorrect in their calculations and application of mathematical knowledge. Their answers as well as their methods, were incorrect or did not make sense and did not hold relevance to the context of the problem.
C (Attempts made to some degree of success)	3	Learners were able to use mathematics to answer their own designed questions with some success. Their answers may have been incorrect but there was evidence of mathematical thinking in their methods. Their 'answers' held relevance to the context of the problem.
D (The competency was displayed well and clearly)	0	Learners were able to demonstrate mathematical thinking and skills by correctly answering their own questions. Their methods are clear and concise, and their work can be used (almost) as is to create an informative and guiding memorandum.

Table 23 Summary of levels obtained in relation to MM6 in orientation session 1

MM7 – To interpret results outside the mathematical context and generalize the solution For this section I did not make use of the orientation session data as no instructions allowed the learners to display this competency. I made use of three questions (2.4; 2.5; 3.1.4) in the individual investigation which pertained to the ethical and legal implications of their proposed solutions as well as the implications of their calculations in social contexts. Could the learners look beyond the math to the social implications of their suggested solutions. The numbers here indicate individual responses and not group responses.		
Level Obtained	**Number of Learners**	**Description**
A (No evidence)	148	Learners did not answer any of the three questions to satisfaction (more than 50% correct). Display no evidence of skills pertaining to relating math beyond the immediate context.
B (Attempts made but largely unsuccessful)	20	Learners show limited ability to relate their mathematics to the context of the real world, i.e. transfer their knowledge. Learners managed to answer one of the three question correctly.
C (Attempts made to some degree of success)	5	Learners demonstrated relative success in relating their mathematical answers to a societal context. Learners managed to answer 2 out of three questions correctly.
D (The competency was displayed well and clearly)	0	Learners successfully related their mathematical answers to the societal context in all three questions.

Table 24 Summary of levels obtained in relation to MM7 in investigation 1

As expected, none of the learners displayed any of the modelling competencies well and only a few individuals managed to display some relative level of competency at all. Most of these learners could not even describe the problem effectively which set them up to fail in the other areas as well. What was clear here again, was that almost all the learners were unable to interpret the mathematics within the context of the problem, highlighting the gap between ML and their immediate environment.

4.6.2 Analysis of investigation 2

In the second investigation, I wanted to give learners a chance to observe modelling behaviour before having to enact it themselves for the second time. During the orientation session, learners were given the opportunity to plan for the investigation by making a rough draft of the poster they were required to design and elaborating on their ideas on how they were going to go about collecting, interpreting and displaying data. I also used an example scenario to illustrate to them how the process could look. I guided them through the orientation session by asking them questions to direct their thinking and clearing up any misconceptions that were identified.

The teachers were, by their own choice, not present in this orientation session, so when the time came for the learners to complete the investigation in class, they were left largely to their own devices. The modelling required of the learners in the task was of a more demanding level than in the previous task and the marks were, perhaps as a result, worse.

Table 25 to Table 31 indicate the markers and criteria used to establish the learners' relative level of success in displaying each modelling competency in investigation 2. The numbers refer to the number of groups that attained each level.

MM1 – Ability to make assumptions for a problem Could the learners identify and describe the scope of the problem clearly?		
Level Obtained	**Number of Groups**	**Description**
A (No evidence)	**4**	Scenario not described at all. If there is little description it is vague and ambiguous.
B (Attempts made but largely unsuccessful)	**20**	Some elements of the problem are mentioned but not described in enough detail to get the full scope of the problem.
C (Attempts made to some degree of success)	**7**	Some of the elements pertaining to the problem are described. The reader gets a sense of the problem but is left with many questions.
D (The competency was displayed well and clearly)	**0**	The problem is well described. The elements are quantifiable. A clear picture of the problem has been created.

Table 25 Summary of levels obtained in relation to MM1 in investigation 2

MM2 – Recognising quantities and variables that influence the situation Could learners identify role players and variables in the situation and gather data pertaining to these role players and variables effectively?		
Level Obtained	**Number of Groups**	**Description**
A (No evidence)	23	No evidence of information having been gathered. Role players/variables not identified.
B (Attempts made but largely unsuccessful)	5	Methods for gathering data does not make sense. The data does not make sense in the context. Confusion as to the variables and role players involved but attempts have been made.
C (Attempts made to some degree of success)	0	Data has been gathered about various role players and variables, but the data is incomplete.
D (The competency was displayed well and clearly)	3	Data gathered about all necessary and relevant role players and variables in a sensical way. Data is clear and complete.

Table 26 Summary of levels obtained in relation to MM2 in investigation 2

MM3- Construct relations between variables Could the learners visually display the quantified relationship between the problem and the variables/role players through the use of tables and graphs?		
Level Obtained	**Number of Groups**	**Description**
A (No evidence)	3	No tables or graphs have been created to display the data. The data is not described at all.
B (Attempts made but largely unsuccessful)	16	Attempts made at creating graphs and tables. Information used does not hold relevance to the context of the problem.
C (Attempts made to some degree of success)	9	Graphs and tables have been created but are incomplete or partially incorrect leading to a relatively unclear picture of the relations between the variables.
D (The competency was displayed well and clearly)	3	Relations between variables are clearly portrayed in graphs and tables. May have minor errors on graphs or tables but these do not compromise the information represented.

Table 27 Summary of levels obtained in relation to MM3 in investigation 2

MM4- Differentiate between relevant and irrelevant information Could the learners use relevant information as a justification of the choice for the solution of the problem? Learners had access to various sources of (ir)relevant information.		
Level Obtained	**Number of Groups**	**Description**
A (No evidence)	5	Too much missing or irrelevant information on the poster. No suggested solution to the problem (no toilet suggested).
B (Attempts made but largely unsuccessful)	5	Some irrelevant information on the poster. Toilet was selected without relevant information to offer sufficient justification.
C (Attempts made to some degree of success)	10	Minor bits of irrelevant information but not problematic to the overall picture of the problem and solution. Toilet chosen with some evidence of relevant information used as justification for the choice made.
D (The competency was displayed well and clearly)	11	Clear and relevant information on the poster. Choice of toiler is clear with ample relevant information to justify the selection made.

Table 28 Summary of levels obtained in relation to MM4 in investigation 2

MM5 – Choose appropriate mathematical notations to represent the situations Could the learners display all data and solutions to problems (cost sheet) in a mathematically appropriate and clear way?		
Level Obtained	**Number of Groups**	**Description**
A (No evidence)	14	No logical layout to the poster. No relevant tables and graphs to represent the data. No relevant costing sheet as part of the solution to the problem.
B (Attempts made but largely unsuccessful)	15	Overall poster layout not logical, no flow to the pattern of thinking. Tried to make use of graphs, tables, and costing sheets but the information does not make sense or is not completely relevant to the problem.
C (Attempts made to some degree of success)	2	Graphs, tables, and costing sheets are on the poster. Data is relatively clear but there is an ample amount of error in the representation (not calculations). Overall layout of poster can be followed, displaying some logic to the layout but with room for improvement.
D (The competency was displayed well and clearly)	0	Graphs, tables, and costing sheets are clear and make sense. The overall layout of the poster is easy to follow. The way the information is represented makes is useable.

Table 29 Summary of levels obtained in relation to MM5 in investigation 2

MM6- Make use of heuristic strategies & mathematical knowledge to solve the problem . Could the learners make efficient and correct calculations as to the cost of the solution to the problem? Could they make sensible recommendations based on the numbers?		
Level Obtained	**Number of Groups**	**Description**
A (No evidence)	23	No costing sheet shown. No calculations to mathematically justify the recommendation made.
B (Attempts made but largely unsuccessful)	6	Incomplete costing sheet or highly flawed calculations, resulting in the information not being usable to sufficiently justify the recommendations made. However, attempts were made.
C (Attempts made to some degree of success)	1	Complete but partially flawed costing sheet in terms of calculations. Enough viable information to justify the recommendations made.
D (The competency was displayed well and clearly)	1	Minor to no flaws in the costing sheet. The costing sheet is complete. The information is accurately used to justify the recommendations made.

Table 30 Summary of levels obtained in relation to MM6 in investigation 2

MM7 – To interpret results outside the mathematical context and generalize the solution Could the learners interpret everything about the problem and solutions to form a sensible conclusion that is described well?		
Level Obtained	**Number of Groups**	**Description**
A (No evidence)	17	No mathematical calculations or data are used in the conclusion. The problem has no (suggested) solution.
B (Attempts made but largely unsuccessful)	10	Minor pieces of mathematical information are used in the conclusion. There is no (suggested) sufficient solution the problem. Most elements of the problem remain unaddressed.
C (Attempts made to some degree of success)	3 .	Evidence of mathematical information used to a relatively large extent in the conclusion. More elements of the problem are addressed than unaddressed. The problem has a partial (suggested) solution.
D (The competency was displayed well and clearly)	1	All the elements of the problem have been addressed and are supported by mathematical evidence. Information is well integrated, and the problem has a satisfactory (suggested) solution.

Table 31 Summary of levels obtained in relation to MM7 in investigation 2

During investigation 2, there was a minor improvement in the competency levels displayed by the learners, yet the levels displayed by the learners remain, in my opinion, far from adequate. It is clear that most of the learners still struggle with even the first competency, which is to describe the scope of a problem in such a way that there is enough information to make mathematics of the situation. The learners continuously left out statistical and numerical information leading to superficial descriptions that leave the reader with no real sense of the nature of the problem. This happened despite explicit guidelines given during the orientation session and in the investigation instructions. In the second instance, there were only 3 groups whose posters showed evidence that they carried out the instruction to gather data from, and about, the various role players and variables in this problem. This lack of data meant that learners could not construct and communicate the relations between these variables and role players. In most of the posters the learners did make some kind of a table or graph to display the tendency of data, but the information was seemingly random, pertaining to prices of toilets and not the variables within the problem.

During the focus group interviews, Larry stated that his teacher provided no guidance during the making of the posters whereas Rebecca stated that their teacher had told them which graph to draw. This explains why most groups had the same graph and table in their poster, which neither adhered to the requirements of the task, nor communicated relevant information. However, some learners did communicate the data effectively, adhering to the norms associated with tabulating and graphing data (layout).

In the fourth competency, many learners showed that they could successfully justify a choice if it can be backed with numerical evidence. These learners could choose and communicate their choice of a toilet that met their needs both financially and structurally (in terms of material and design). However, most learners did not consider any factors outside of those two elements, of which information was provided, such as water use during the drought and installation fees. In fact, the learners disregarded almost all the information provided as part of the assignment and based their justification solely on the limited information they gathered themselves. This meant that the learners did not sufficiently make use of the information at their disposal to carry out MM5 and MM6. Their calculations were flawed and incomplete, if there were any, meaning the financial implications of addressing this problem was not properly analysed. Furthermore, disregarding the instruction sheet and rubrics meant that 94% of the posters submitted did not communicate the information well. There was no logical flow to the poster, there was information missing and often replaced with information that was hard to interpret in terms of the problem and solution. In the end, MM7 also fell short as learners delivered some kind of conclusion that did not summarize the problem and solution, and did not address the contextual elements of the problem. Yet both learners and teachers felt that these investigations were adequate to be submitted for formal assessment.

Figure 8 is an example of one of the posters submitted. The introduction to the problem is in the bottom right corner (counter-intuitive in a country that reads from left to right). The table does not have a heading or give an indication as to what the data is about. There is a pie chart that has been done well. The pink paper indicates the data collected for the pie chart. There is no cost analysis done for the solving of the problem, no suggested replacement for the toilets or maintenance plan, and no overall conclusion.

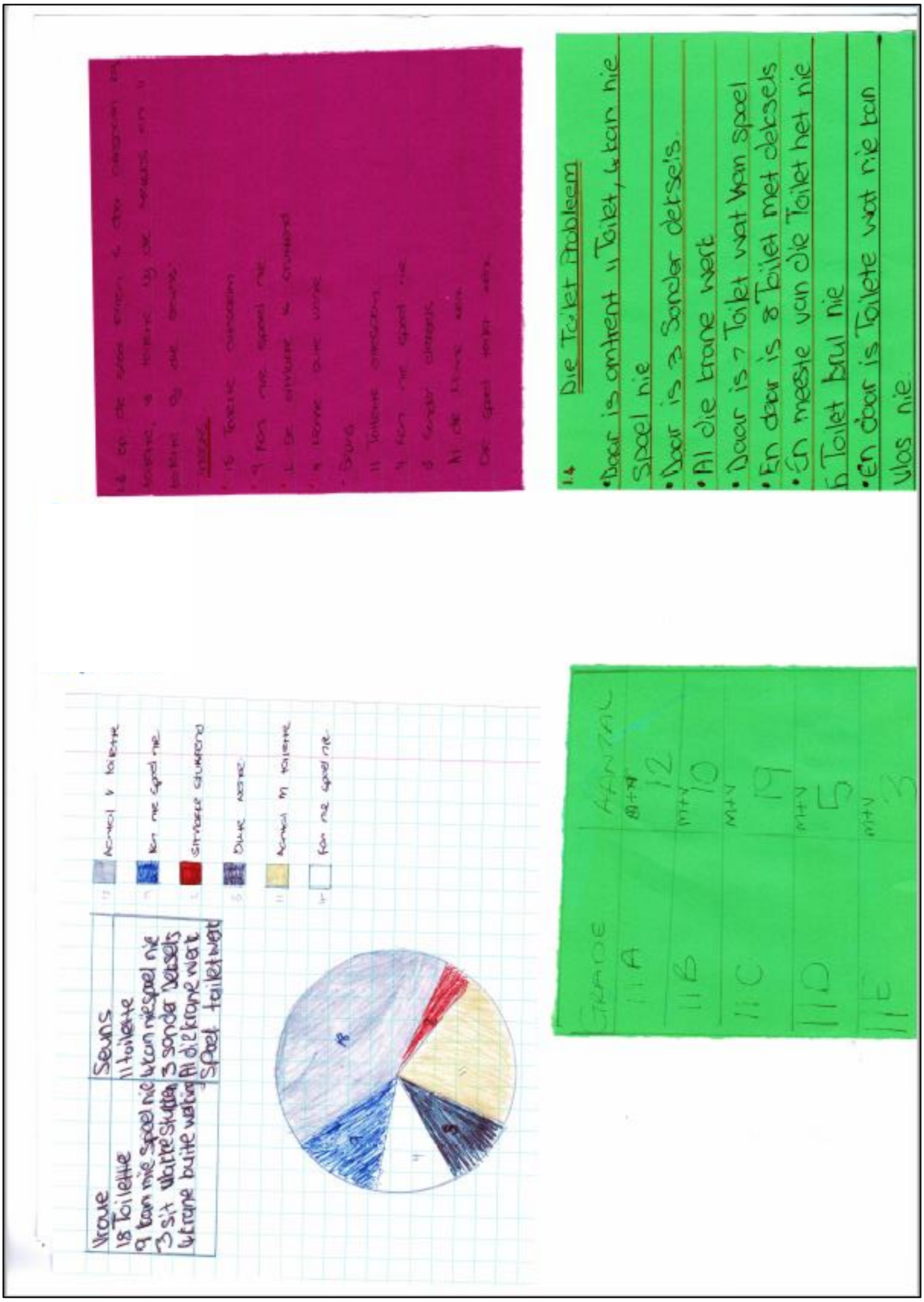


Figure 8 A poster submitted in investigation 2 by one of the groups.

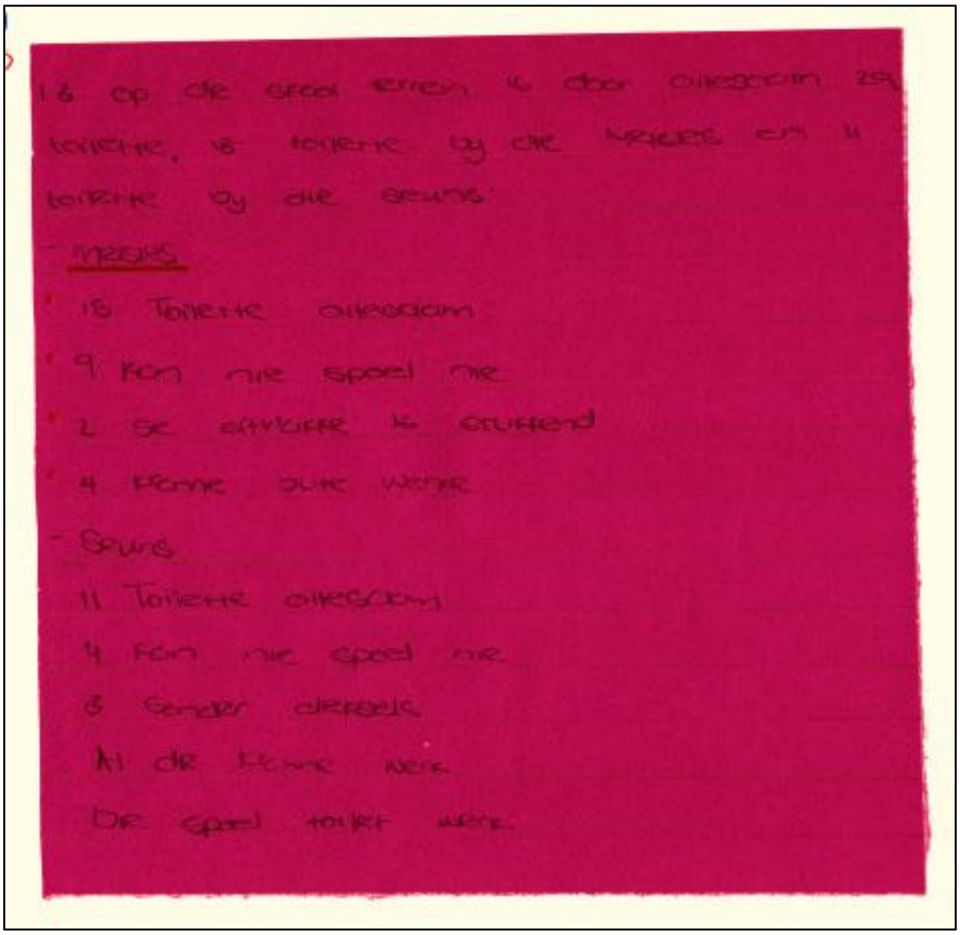


Figure 9 An enhanced image of the data collected for the pie chart

4.7 Critique of the orientation sessions

As a final aspect of this case study, I wanted to explore how learners had perceived and experienced the orientation sessions themselves, in comparison to the traditional, contextually shallow investigations they were used to. I prompted the learners about the orientation sessions in the post-study questionnaire, in the focus group interviews and at the end of the second orientation session (individual reflection). As expected, there were negative comments about the orientation sessions, both of which had many circumstantial hiccups. What I found of interest was that the learners complained much more about the learning environment than the orientation sessions and investigations themselves. Complaints ranged from pertaining to wasted time and not partaking in what they considered 'real mathematics' to the venue being too loud or too full.

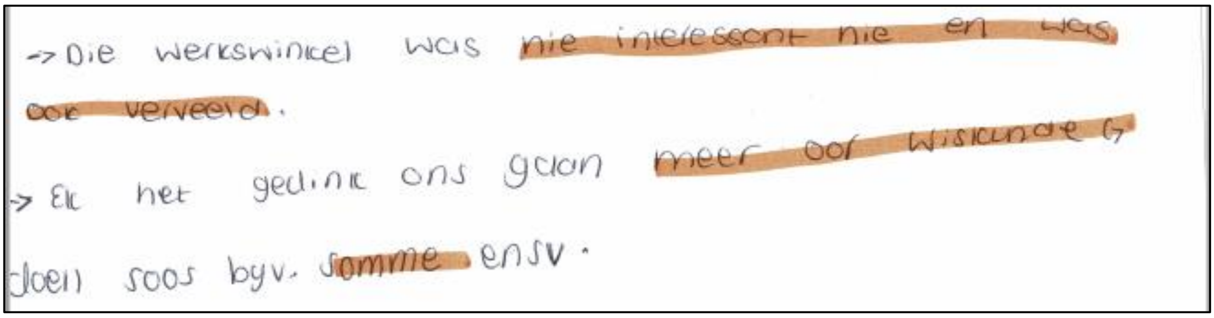
4.7.1 Analysis of the post-study Questionnaire

Initially, the post-study questionnaire indicated an overwhelmingly positive response to the orientation sessions. 92% of the learners agreed with the statement that the orientation sessions were an enjoyable way to learn, with 52% indicating that they strongly agree. Similarly, 92% of the learners agreed that the orientation session showed them how mathematics can be used in their environment, with only one learner strongly disagreeing. Only 5 learners were strongly opposed to making use of this method of learning more often, and 85% were open to the possibility. Furthermore, 85% of the learners agreed that the orientation sessions had improved their understanding of mathematics. 70% of the learners indicated that their marks after the orientation sessions were an improvement on their grade 10 marks.

However, 61% of the learners felt this method of learning was more difficult than the methods they are exposed to in class, 30% of the learners felt that the orientation sessions weren't really about mathematics at all and 18% of the learners stated that they would not want to waste time with another orientation session. Two of the learners refused to complete the post-study questionnaire stating that it asked "*stupid questions that do not matter*". These learners did not want to partake in an interview to explain their position either. However, the learners' individual responses as reflection in the orientation sessions, as well as the responses in the focus group did provide insights into what elements were being critiqued and why some learners consider the orientation session a waste of time.

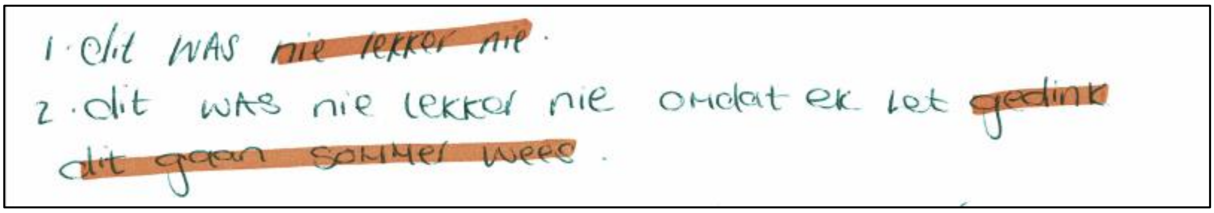
4.7.2 Analysis of the individual reflections of orientation session 1

In orientation session 1 learners were prompted about how they experienced the orientation session and whether or not they would like to participate in such a session again. Of all the learners who participated in the study, most were positive, and seemed to enjoy the orientation session. However, 29 learners did offer critique. Their critique was centred around the orientation session being" *boring*" or "*uninteresting*" or "*not being about real mathematics*" because there was "*not enough sum"* or *"no sums"*. Individual learners stated that it was *"unclear what was expected*" of them and that the orientation session was ineffective, that they "*did not like their group"*, that there was "*too much thinking"* involved and that it was pointless because "*teachers don't care about our opinions"*.


-> Die werkswinkel was nie interessant nie en was ook verveeld.
-> Ek het gedink ons gaan meer oor Wiskunde G doen soos byv. somme ensv.

A direct translation reads: "The orientation session was not interesting and was also boring. I thought we would learn more about doing ML e.g. sums etc".

Figure 10 A learner's critique of the orientation session.


1. dit was nie lekker nie.
2. dit was nie lekker nie omdat ek het gedink dit gaan sommer weer.

A direct translation reads: "It was not fun (nice) because I thought it would be more sums".

Figure 11 A second learner's critique of the orientation session.

The data is summarised in Table 32 and indicates the key words used to categorise learners' responses to the orientation session:

Key Words in Learner Responses	Number of Responses
Boring/not interesting	12
Unclear	2
Not good	1
Not Math (no sums)	10
Groupwork uncomfortable	1
Too much thinking	1
Our opinions don't matter to teachers	1
Not effective	1

Table 32 Summary of the learners' critique of orientation session 1

4.7.3 Analysis of the individual reflections of orientation session 2

At the end of orientation session 2, learners were prompted about whether or not they enjoyed the second orientation session, as well as which orientation session was more enjoyable and why.

4.7.3.1 Preference for orientation session 1 or orientation session 2

It was apparent from the data that the second orientation session was preferred by more of the learners. In fact, 31% of the learners preferred orientation session 1, 58% of the learners preferred orientation session 2, 2% of the learners did not enjoy either and 9% enjoyed both equally. Not all the learners offered up reasons for their preference, but of those that did it was clear that there was not one overwhelming factor that made one orientation session better than the other. Those who preferred orientation session 1 cited many reasons such as "*speaking about more than one problem*", actually having improved the school environment (referring to the installation of cameras), having "*better discussions"* and enjoying working with members from other classes. Reasons for preferring orientation session 2 also varied and included having *"more information"* at their disposal, having a "*better group*" to work in, that it was "*easier to understand*" (mostly due to familiarity) and it being more productive or focussed with less disruption.

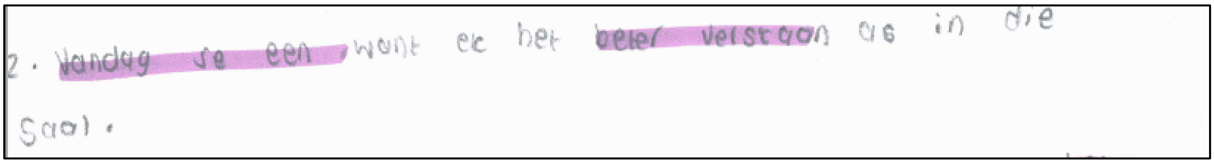
The learners' responses are summarised in Table 33 and Table 34. The phrases used as indicators are paraphrased and based on key words extracted from learner responses:

Key Words in Learner Responses	Number of Responses
Spoke about more than one problem	3
Improved the school environment	9
Questions made more sense	2
Learners could make and solve their own questions	1
Better discussions	3
Better venue	1
Like working with kids from the other class	5
Easier	1

Table 33 Reasons why learners preferred orientation session 1

Key Words in Learner Responses	Number of Responses
More enjoyable or interesting	7
More solutions/information given	6
Better group	10
Better questions (thought provoking and relevant)	7
Easier to understand	12
More productive/focussed/more stuff done/less disruption	5

Table 34 Reasons why learners preferred orientation session 2



The translation reads: "Today's one was better because I understood it better than in the hall". The hall refers to the venue of the first orientation session.

Figure 12 A learner's response as to why they preferred they second orientation session

From the responses in Table 33 and Table 34, I deduce that what made learners prefer one orientation session to the other had more to do with the learning environment, including the participants, and the visible results, than the actual task.

4.7.3.2 Explicit critique of orientation session 2

The importance of the learning environment was echoed again in the negative comments offered by the learners after orientation session 2. The critiques were in a similar vein to orientation session 1 and have a lot to do with the learning environment, although some critique does also pertain to the task itself. Critique for the second orientation session was delivered in phrases such as the orientation session was *"not cool"* or *"not fun"*, that it was *"not ML because there were no sums"*, that the *"venue was too small"* and the discipline was poor and that it was" *boring"* or a "*waste of time".*

The learners' responses are summarised in Table 35. The phrases used as indicators are paraphrased and based on key words extracted from learner responses:

Key Words in Learner Responses	Number of Responses
Not fun/cool	9
Not related to ML/not sums	4
Venue too small	2
No discipline/co-operation/too loud	6
Not nice questions	1
Boring	5
Waste of time	1
Not explained well	2

Table 35 Summary of learners' critique of orientation session 2

4.7.4 Analysis of the Focus Group Interview

The sentiments shared by the learners during the focus group interviews, supported those offered by the learners at the end of each orientation session, and strengthened my theory that the learning experiences of these learners are more severely influenced by the learning environment than the actual learning task.

When asked about the extent to which they enjoyed the first orientation session, Larry immediately complained that it was too loud, and he could not really focus and comprehend the work. Claire stated that she enjoyed the first orientation session, but the second orientation session was too limited in terms of time which did not allow her to focus on the work effectively. This sentiment was met with agreement from Larry. Robert stated that the second orientation session was more '*comfortable´* (FG3 Robert), because he was more familiar with the experience. Candice furthered the discussion on the learning environment by saying that the second orientation session was more enjoyable because '*then we could talk to people (better)'* (FG5 Candice), by comparison the first orientation session which was '*too messy'* (FG5 Candice).

In relation to the task, Rebecca stated that second orientation session was more exciting because it culminated in a task that let them go and gather information from local businesses, which was enjoyable to her and her friends. Larry expressed that the task was helpful to them in their ML because it helped them to work though information and '*turn it into a math problem'* (FG1 Larry). He believed this was a good thing because it "*allows your brain to grow because you actually have to understand what you are reading and how to use the information that has been given to you'* (FG1 Larry). This was met by agreement from all the learners.

When prompted on whether it would be feasible to use this orientation session strategy in their school more often, all the learners said yes. However, Larry brought it to my attention that many of the learners could not understand how these orientation sessions were related to ML when there were no sums involved. To these learners, mathematics is synonymous with making calculations. He stated that "*they don't like it*"(FG1 Larry). By 'it' he was referring to the change in method of instruction and the deviation from the norm. Rebecca then suggested that the teachers use this method "*maybe just like twice a month"* (FG2 Rebecca) rather than too regularly (in her frame of reference) and was convinced that these orientation sessions could work for the learners.

From the above responses, it became clear to me that the learners in this group like the comfort zone of routine and predictability. They preferred a learning environment and activity that mimics their traditional school structure because it provides a greater level of certainty about what is expected of them, and offers up instructions that are accepted by the learners as giving a definitive direction. Although many learners may be intrigued by the tasks themselves, this approach to learning and the learning environment it creates, establishes unease among the learners and was thus not something they felt ready to buy into at this stage. Simultaneously, however, there was no direct rejection of this method of instruction. Therefore, I hypothesize that using modelling sessions as a means of ML instruction is feasible – at least within the context of this case study.

4.8 Conclusion

In this chapter I set to simultaneously present and analyse the data collected in this study. The chapter set out to analyse the data per theme and presented evidence from each data source to support the emergence of that theme as significant. From the data we have seen that the learners have a very superficially developed imagination aspect of their identity. Therefore, the alignment aspect, which reacts to imagination, is also questionable. These learners have an ever-optimistic mathematical identity that is supported by the narratives within their immediate circle, but that is overtly challenged by the narratives of the broader society. Their self-constructed identities are also brought under question when it is considered that the learners draw a direct relationship between success and their ML marks. From the work produced in this study, it is evident that the learners produce low marks that should not necessarily support such overly optimistic identities. In the next chapter, I will interpret these findings in relation to the existing theory to answer my research questions explicitly.

In this chapter I also analysed the learners' mathematical modelling competencies according to the work they produced in the investigations. It was clear that the learners displayed low levels of competencies that were attributed to their inexperience with modelling tasks. The data indicated that even the second round of mathematical modelling, in investigation 2, already showed some improvement in the learners' competencies, but was by no means at a high level yet. This improvement, in coherence with the learners' overall positive feedback about their experiences in the modelling orientations, lead to the conclusion that modelling orientation sessions can be a feasible method of instruction for ML. In the next chapter I will interpret these findings in relation to existing literature about modelling and ML to support my hypothesis.

CHAPTER 5: INTERPRETATION, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

In this chapter I will interpret the analysed data about mathematical identities of the learners, and the learners' experiences of working with authentic contexts in modelling tasks.

This chapter is structured to address, firstly, the theme of identity, followed secondly by the interpretation of the experience with authentic contexts. I will then establish the link between the two themes through the development of a grounded theory and thus answer my research questions. The interpretation culminates in the drawing up of my conclusions of this case study.

I also discuss the limitations of the study, as well as make recommendations for future studies.

5.2 Identity

5.2.1 Actual and designated identities

As defined in chapter two and four, Sfard and Prusak see identity as stories or narratives about a person that are reifying, endorsable and significant (Sfard & Prusak, 2005). They are present-based narratives (Sfard & Prusak, 2005) that have the potential to lead to action on the part of the identity builder (Buytenhuys & Graven, 2011). In other words, through the development of mathematical ID, ML has the potential to transform weak learners into negotiators, participators and sense-makers (Buytenhuys & Graven, 2011). This occurs as learners try to enact that which they are told about themselves and believe about themselves inherently.

5.2.1.1 Actual identities

From the data collected in this study, the learners reported an ever-present mood of optimism surrounding the narratives they perceive is being told about them and by them. Throughout the study the learners held fast to their opinion that, not only can they do mathematics and obtain satisfactory marks, but they also have enough skills to assist their peers with confidence. Even though most learners acknowledge that they did not understand mathematics in the lower grades, at that present moment they felt confident in their ability to succeed at school mathematics and beyond. They strongly felt that ML was not for 'dumb' people and that in fact, they were smarter than the learners who take Mathematics as a subject as their marks were often higher.

Marks were a big factor for these learners in their personal narratives, which form their actual identities (Sfard & Prusak, 2005). Majority of the learners did not waver from the opinion that they are satisfied with their marks, even though the marks I was given access to rarely exceeded the 40%-50% range. Most of these learners felt pride because even though their marks were low, they had improved since grade 8 and 9, where they struggled to pass. However, the learners also made it clear that their confidence was strongly linked to the perceived ease of ML. They viewed ML as a subject in which they could easily obtain a pass mark, which, to them, constitutes a good mark. In their opinion, ML was at the ideal level of cognitive demand. They actually seemed to hold some apprehension to the idea of ML becoming more demanding, because they were not confident that they would then be able to maintain their marks and be accepted to university. This sentiment made it clear to me that the actual identities of the learners, based on their personal narratives are actually quite fragile. They outwardly portrayed a lot of confidence and optimism within their own environment, but when faced with the potential demands of the broader society they wish to join in the future, they were visibly more cautious about their sentiments of themselves and their abilities as mathematics learners.

5.2.1.2 Designated identities

The actual identities of learners are also influenced by their designated identities – by those stories told about them that they perceive as accurate and true (Buytenhuys & Graven, 2011). The data indicated to me that these learners were either entirely oblivious to any negativity associated with ML, or they had collectively decided to ignore it.

These learners were from a socio-economically low area and in an under-resourced school. It is no secret that schools such as these are considered to be doomed to low levels of performance in mathematics (Julie, 2006). In fact, it is expected that in a school such as this, in the area in which it resides, the majority of the learners would opt to take ML as a subject, because ML is often seen as an alternative for learners with poor mathematical competence (Brown & Schäfer, 2006). In this case study, this was the reality. The majority of the learners opted for ML, despite the widespread and misleading stigma that ML is an easy and less worthy form of mathematics (Buytenhuys & Graven, 2011; Conradie, 2016; Houston & Africa, 2015). This stigma is broadcasted all over the media with utterances from public figures such as Jonathan Jansen claiming ML is designed to make learners feel stupid, and to hinder their opportunities at future success (Smith, 2019).

Yet somehow, these learners had managed to reject this unjust categorization (Swanson, 2002) of ML and the learners who partake in it. As mentioned, it may be that the learners did not follow the media and were not privy to public opinion about their choice subject. More likely, though, is that within their community, ML is considered the normal choice. It is not a demotion as it is in many wealthier schools, it is not a sign of failing at mathematics or of being less capable than the learners who do take Mathematics. Instead ML is a smart choice that increases their chances of gaining access to tertiary education institutions, because they are more likely to pass, or to pass better, than if they had chosen to take Mathematics.

The learners were also supported in their choice by the members of their immediate environment, such as their parents, teachers, and peers. Within this community, ML is not viewed as an easy, worthless subject, but as one that paves a more accessible road to success.

However, I did note two points of contradiction to this ever-optimistic narrative.

First, although, according to the learners, the teachers did not compare them to Mathematics learners, and believed in the learners' ability to succeed in life beyond school, teaching practices in the school may not have reflected this level of support. Despite that fact that ML was intended as a curriculum based on modelling (Brown & Schäfer, 2006), the learners had never engaged in any modelling type tasks. One possible reason for this, mentioned by the learners, was that these modelling type tasks are time-consuming. This study proved that there is plenty of time within the ML curriculum to conduct modelling type tasks. What may have been occurring here, was that the real problem was the messiness and the struggle of navigating the high cognitive demands of modelling tasks. It may also be that, in reality, limits had been placed on what is believed about the learners' capabilities. This formed part of a designated identity that was not being accepted by the learners within their immediate environment.

In the second instance, there was a slight decrease in the number of learners who reported that they believed learners who take ML can go to university, and a slight increase in learners who reported that ML is for dumb people. I attributed this to the learners who had been exposed to university information sessions, where many of them were made aware of the requirements of Mathematics as a subject for certain directions that they may have been interested in. In the eyes of a 17-year-old, this may have seemed like the end of the road and may have made them more open to accepting the stigma that ML is for “stupid people”.

Again, I saw reflected, the idea that the learners were confident in and willing to accept those positive narratives told about them within their immediate context. As soon they cast their eyes and ears to the narratives and expectations of the broader society, they were less secure in what they were willing to accept about themselves and their mathematical abilities.

5.2.2 Imagination

As defined in previous chapters, the imagination aspect of identity is about how mathematics is perceived as useful and meaningful by the learners in their lives outside the classroom, both presently and in the future (Wenger, 1998).

5.2.2.1 Imagination and future projections

At face value, the data presented another ideal situation: the learners reported that they experienced ML as a subject that equipped them with knowledge that they could use every day, that it prepared them for life after school, that it would open doors for studying opportunities and that the knowledge and skills obtained in ML would be useful in their planned career paths. The learners did not waver on these opinions throughout the study. From these optimistic views, it would seem that the international goal of ML, providing learners with access to the knowledge economy (Jablonka, 2015), would be realised.

However, upon further prompting of the learners in the focus group interviews, it became abundantly clear that this was not the case. The learners were only able to offer shallow and generic examples of how ML is useful in their daily lives – such as time management, money management when shopping, and baking. Furthermore, in their investigations it was also clear that they were largely unable to use mathematical knowledge and calculations to justify choices in the real-world contexts of the problems they were working on. The learners also only believed that ML, due to its ease in comparison to Mathematics, was useful as it provided a gateway to get into tertiary education institutions – it offered a means to an end. The learners held the firm belief that once they leave school, they could leave anything to do with mathematics or ML behind, regardless of being pointed out areas in their chosen career paths where they would have to make use of mathematical knowledge. The learners did not understand the true relevance and role of ML (and the mathematics it entails) in their lives and in broader society.

2.1.2.2 Using contexts to influence imagination

In order to enable learners to access the knowledge economy, teaching practices need to increase the amount of interaction between practice and theory, by foregrounding the real-world contexts in which mathematics is used (Jablonka, 2015; Verzosa, 2015). Contexts are supposed to be used as vehicles to help learners understand the world around them (Verzosa, 2015) and address social issues in their countries (Frankenstein, 1990). The inauthentic contexts that these learners had been exposed to had failed to do just that. If learners are not exposed to ML in relation to authentic contexts, they cannot possibly understand the role of mathematics in potential career opportunities (Department of Education, 2003), as was one of the primary aims of this curriculum. Furthermore, if learners are not exposed to authentic contexts, we cannot expect them to move beyond low levels of mathematical literacy, and place them at risk of low levels of employment and economic development (Bansilal et al., 2015). If they do not understand the role of mathematics in their futures, and are not prepared to embrace mathematics in their futures, there is a risk of maintaining our status as the country with lowest rates of access to tertiary education (OECD, 2019), and exacerbated joblessness.

The imagination aspect of identity sets the tone for how the learners will respond to mathematics (and ML) – whether they will be driven to align their goals with that of societal demands and to what extent they would be willing to engage in ML practice (Anderson, 2007). If the learners are merely exposed to textbooks which, by their own words, are often confusing and lacking information, textbooks which makes use of outdated and shallow contexts, the learners cannot be expected to understand the role of ML in their lives outside of school. The learners in this study had a very minimal understanding of the relevance of ML in both their present and future lives, which placed a restriction on the development of their mathematical identities through misinforming their drivers for alignment. These learners reported that they engaged in ML because it would pave the way for success in their future lives. In reality, engagement in ML was only deemed necessary, by the learners, to finish school and gain entry to tertiary education. They could not envision the long-term role, beyond the context of education, of mathematics in their lives. They did not recognise mathematics as a vehicle that could be used to make sense of the world and inform their daily decisions.

5.2.3 Alignment

As defined by literature, alignment is a response to the imagination face (Anderson, 2007), in which the learners' perceptions about the relevance of ML guide the driving force of learners' willingness to engage (Wenger, 1998), i.e. the imagination aspect informs why learners are either willing to engage in ML education or not .

After analysing the imagination component of the learners' identities, and realising how superficial and limited their imagination aspect of identity was, I wondered what would be the drivers behind the learners' high levels of engagement in the ML orientation sessions I presented to them as a part of this study.

ML is a compulsory subject for any learner in South Africa who does not take Mathematics (Bansilal et al., 2015; Buytenhuys & Graven, 2011; Julie, 2006; Long et al., 2014), and often a subject that many learners are guided into choosing by the school, due to their academic performance in grade 9. In this study, an overwhelming majority of the learners believed that they had made an autonomous choice to take ML as a subject – that they were not guided into this choice under the influence of the school nor parents. In fact, only 23% of the learners stated that they would rather not partake in any form of mathematics education, thus only engaged in order to complete school and obtain a matric certificate. This indicated to me that most of the learners did have some sense of value attached to mathematics education – but what was that value?

5.2.3.1 Alignment for good marks

One goal of the ML curriculum that has been undeniably achieved, is that it has offered a more accessible form of mathematics education to learners in South Africa (Julie, 2006). Over three quarters of the learners in this study acknowledged that they chose ML because it was the easier subject that offered the potential for higher marks than those they had achieved in grade 9 Mathematics. They also acknowledged that they were influenced by older peers who held the opinion that it was easy to get good marks in ML. This potential for higher marks guided the learners to align their actions with ML and engage in ML – in fact to them it made ML enjoyable. They even prioritised their favourite topics as those where formulae are provided and their potential for successfully completing the calculations is the highest, because the cognitive demand is the lowest.

The focus group interviews also repetitively pointed to marks as the most important factor in their mathematics education, purely because good marks meant they could access tertiary education (and thus good careers). To these learners, good marks were anything above 40% (which is already 10% above pass) and which met the minimum requirements for their intended course of study. Once they left school, in their minds, ML would become irrelevant and there would no longer be a need to align and engage with any form of mathematics education. Once again it was clear that the value the learners attached to mathematics education was in it serving as a means to an end – a way to transcend their economic status by accessing tertiary education and thus a wider pool of career opportunities. The mindset of the learner also further highlighted the fragility of their mathematical identities. If they expected ML to be easy and for them to pass with certainty, their identities would be seriously challenged if the cognitive demand of the subject were to increase. I wondered if their narratives about themselves and their peers would hold should some learners in this group fail. I hypothesise that it would not.

5.3.3.2 Alignment for social interaction

It was evident from the data that the learners in this study placed a high value, and thus aligned well, with group work. Skills pertaining to working in a collaborative setting are one of the intended outcomes of the ML curriculum (Department of Education, 2003). These learners also drew attention to the enjoyment aspect of working together in groups during the orientation sessions and investigations. This by no means indicated that the quality of the group work was of a high standard. In fact, the work produced by the learners in investigation 2 indicated that there was much to be desired in terms of effective collaboration. However, the learners felt that the social nature of these learning tasks increased their willingness to engage (alignment) because it created an enjoyable learning experience. Furthermore, solving problems that lead to visible changes in their environment, to the benefit of their society also provided these learners with a sense of pride. In this case I am referring to the installation of cameras on the school grounds as suggested by the grade 11 ML learners to address the drug problem identified during orientation session 1. They enjoyed the entire process, from talking about their societal problems, to trying to find a solution to them, to witnessing a resulting physical change. In the minds of the learners this constituted a worthwhile learning experience and something they would willingly engage in again. What the learners were experiencing here was ML in and for change (Julie, 2006).

What I deduced from the learners' responses pertaining to alignment, was that their reasons for alignment are as superficial as their imagination. Their main drivers for alignment were groupwork – perhaps more so for the social aspect than the benefits of learning in a group setting – and the potential for good marks. In both cases, alignment was likely to end abruptly once the learners finished school.

5.2.4 Contradicting marks

As mentioned, the learners placed the highest value on their ML marks. Their marks were, for them, the sole indicator of their level of success and their mathematical ability. When discussing their alignment, the learners highlighted their appreciation for the ease at which they could obtain good marks and the satisfaction with the marks that they had obtained up to that point in their schooling. However, the marks obtained in the investigations were so low, that they stand in direct contradiction to what the learners are indicating about their mathematical identities. This is even though the learners had seen their marks for investigation 1 before the post-study questionnaire and focus group interviews. They knew that they had obtained low marks yet did not waver from their narrative that their marks were satisfactory.

In the first investigation, which closely resembled a typical school-based task, the grade average achieved was 39% - barely a pass and a level described as elementary achievement (Department of Basic Education, 2011a). The largest portion of the learners received less than 30% and only half of the learners achieved the self-proclaimed satisfactory mark of 40% and above. As the cognitive demand of the task increased, more learners were starting to fail at answering the questions, with over 95% of the learners being unable to answer any questions that pertained to using mathematics to justify decisions being taken in the real-world context, such as how to improve a budget. For these learners, the mathematics stopped at the basic calculations; they could not bridge the gap between context and content. In this task, even the learners who did well only did well because they had access to the memorandum, indicating they were unable to navigate their task without assistance.

In the second investigation, the results were even worse. Despite a planning and modelling session, the learners only obtained a 31% average for their modelling task. I had to supplement this task with a traditional, school-based type test, which also only obtained an average of 40%. More than half of the learners completely failed the task, and those who no longer had access to the memorandum did marginally worse than the other learners by comparison.

Neither investigation showed evidence of the satisfactory marks the learners were speaking of – the marks that would provide them with access to tertiary education. Perhaps more concerning is the fact that these low marks illustrate the inability of the learners to make judgements about the role of ML in their lives (OECD, 2019). The low marks obtained in these investigations indicate the need for these learners to be exposed to learning activities that move away from traditional learning styles, which are exclusively focused on performing operations and not enough on problem solving (OECD, 2019). At the time of this study, the learners did not have mathematical literacy skills that could transcend school mathematics (Jablonka, 2015), or even effectively solve problems within their own immediate contexts – not because they were unable but because they had not been exposed to learning situations to develop mathematical literacy skills. The learners' current inability to understand numerical data in order to appreciate and question the problem and structures in their immediate environment indicates a lack of critical mathematical literacy (Frankenstein, 1990). This implies that the goals of addressing the low rates of mathematical literacy in South Africans is not being achieved (Department of Education, 2003). This is clear from data pertaining to formal standardized exams (Department of Basic Education, 2020), as well as the site-based assessments in this study.

Once again, the same predicament was illustrated: when learners are not presented the opportunity to use ML content in combination with authentic contexts, they cannot bridge the gap between the two – they cannot be expected to understand how the mathematics can be used to inform and justify the decisions taken in solving real-world problems. It is no surprise that here, again, the learners were seemingly selectively deciding which marks to consider to feed their ever-optimistic ID, focusing on the good and disregarding or turning a blind eye to the bad.

5.3 The experience of authentic contexts

5.3.1 The modelling tasks

As previously defined, ML is a subject that is driven by life-related applications (Department of Education, 2003) where learners should not only explore real-world contexts, but engage critically with and solve authentic problems (Bakker & Van Eerde, 2015; Beckmann, 2009;

Buytenhuys & Graven, 2011; Department of Basic Education, 2011a; Gann et al., 2016; Venkat & Graven, 2008; Vithal & Bishop, 2006). In this study, I invited the learners to actively participate in two orientation sessions where they were able to decide on problems from their own contexts, pertaining to their immediate school environment, for which they would go on to (attempt to) mathematically model solutions. By using a modelling approach, the learners were presented the opportunity to engage with authentic contexts and explore the role of ML within those contexts. The contexts that were provided by the learners themselves were used as framework within which different ML content was explored (Bansilal et al., 2015; Conradie, 2016). The idea was to develop the content in such a way that the focus was on logic and problem solving rather than on traditional school-style manipulation of expressions (Meyer, 2010).

The first task straddled the gap between traditional school-based tasks and authentic modelling tasks. An authentic context was explored, but with a lot of scaffolding intended to orientate the thinking of the learners toward solving the problem at hand – being the problem they had identified as drugs on the school premises. The task required learners to make use of knowledge from the sections of numbers and calculations, and predominantly, finances.

The second task was aimed at widening the range of topics that would need to be drawn on in order to solve a real life problem (Department of Basic Education, 2011a) – being the learner-identified problems of broken and unusable toilets – by drawing on numbers and calculations, finances and data handling. The second task also set out to explicitly require the learners to use reasoning, models and content in order to solve the problem (Christiansen, 2006; Ozgen, 2013). Merely doing calculations would not have sufficiently contributed to this task. The learners were required to make a poster whereby they had to illustrate their calculations and collected data, describe how this mathematical information impacted their context and the problem at hand, and find suitable ways to communicate their ideas via tables, graphs and written paragraphs.

5.3.2 Modelling competencies

The learners' performance in these tasks were mapped against pre-determined modelling competencies (Maass, 2006), as illustrated again in Table 17. Each of the two orientation sessions/investigations had their own set of criteria which indicated at what level the learners had displayed the specific competency.

Going into these investigations, I was aware that the learners had never been exposed to modelling type tasks before. This was literally a first-time-experience for them. For this reason, I did not have high expectations for the level of competencies that the learners would display. That said, I also had no intention of lowering the demand of the tasks. Instead, I structured the two tasks as discussed above – the first to straddle the line between the familiar and the unknown, and the second to immerse them in authentic modelling as much as possible. As mentioned in the analysis, I also took a liberal and norm-based approached to evaluating the work of the learners. Their work was compared to the criteria but also to each other, to judge what could be considered ‘relative success’ (as in levels B and C).

5.3.2.1 The first iteration

In the first orientation session and investigation, more than half of the learners were not able to adequately describe the problem within the frame of the context (MM1). Their explanations and descriptions were shallow, incomplete and did not open the door to numerically explore the situation. This set them up to struggle with the subsequent modelling competencies. The number of learners who were unable to meet levels B-D (indicating moderate success) increased with each competency evaluated: from 55%-58% (MM1-MM5) and jumping to a staggering 93% and 86% in MM6 and MM7. This indicates that being unable to effectively describe a problem and identify the relationships and influence of various role-players, renders a problem largely unsolvable as the learners cannot effectively work with incomplete, non-sensical numbers and calculations. Furthermore, without a thorough understanding of the context and the problems in relation to the mathematics, more than 90% of the learners were unable to describe how the mathematics (the numbers) influenced that context and thus inform possible solutions to the problems.

5.3.2.2 The second iteration

The second investigation was more cognitively demanding than the first as it required more intense modelling on the part of the learners with much less scaffolding. However, despite this, the learners did show improvement in the level of competencies displayed.

In the second investigation over 60% of the learners were able to describe the context (MM1) and the problem to some degree of success, creating a workable problem in which some role players, variables and their relations were established (MM2). The learners attempted to communicate these relations more clearly using self-engineered graphs and tables, and for the most part, direct this communication at relevant information. The big improvement here, in my opinion, was the learners' ability to consult various sources of information and decide what is not relevant to this context. That said, there was still a severe lack of useful information used, which hindered the success of the modelling tasks. This was again because of (although improved) too shallow descriptions of the problems, leading to partial modelling, resulting in insufficient solutions to the problem. The latter part was evident again in the work of the learners, where 94% of the learners could not obtain relative success at MM6 due to incomplete, absent, or non-sensical work with numbers. In the end, only one group was able to deliver a well-formed, clear, and concise solution to the toilet problem in their school.

5.3.2.3 Comparing the first and second iteration

There was a minor improvement in the modelling competencies displayed by the learners from the first to the second iteration, indicating that mathematical modelling is not something entirely out of reach for these learners. However, the results were still not satisfactory or indicative of any real levels of successful modelling. It can be deduced that a lack of exposure to modelling, and a resulting lack of modelling competencies means that the purpose of modelling was not achieved. Niss (2008) lists the following three purposes of modelling:

(1) To understand a context manifested through representations, explanations, and predictions.
(2) The use of this information that results in action in terms of decisions being made and problems being solved.
(3) The design aspect of the extra-mathematical world whereby artefacts or systems are created.

The purpose can be achieved when the learners engage wholly, and regularly in the modelling process, which Blum (2002) and Krawitz and Schukajlow (2018) both describe as a three-component cycle whereby:

(1) There is an exploration of a context leading to the formation of a mathematical model and question that idealises and provides structure to the context.

(2) There is an analysis of the mathematical model aimed at answering the question that was posed.

(3) There is an interpretation and validation of the conclusion within the original context. It can be deduced that if the solution is not valid within the context, the cycle would start again with further exploration of the context and the modification of the model and question, until a suitable solution - or solutions - is obtained. One has to start with a real-world problem and end with an applicable real-world solution (Krawitz & Schukajlow, 2018).

In both iterations, the learners were barely able to work through the first step of the modelling process. They were not able to explore the context in such a way that an effective model could be created, but their models did improve from one iteration to the next. Because these models were largely incomplete or non-sensical, the learners could not analyse the model, and virtually none of the learners could interpret their results and conclusions in relation to the problem at hand.

In terms of the purpose of mathematical modelling, it was clear that the learners did not have a true and deep understanding of the contexts they themselves resided in, or they had an extreme challenge in communicating their understanding. This meant that the learners were not able to use the mathematical information to justify their decisions in relation to societal and ethical norms, and thus their skills could not be expected to transcend these immediate tasks to be applied to other contexts and models.

5.3.3 Modelling orientation sessions as a means of instruction

If learners are going to successfully model problems within their immediate contexts, we need to expose them to this teaching and learning approach from younger grades, offering them enough time to explore these contexts and providing them with the guidance needed. This process could start as early as senior phase (middle school), grades 7 to 9 (Verzosa, 2015).

5.3.3.1 Feedback from the learners

In this study, the learners indicated that they enjoyed the modelling orientation sessions as a means of instruction, and that, at the very least, the majority of them were open to making use of this method more often. Many of the learners commented that the orientation sessions had opened their eyes to how mathematics can be used in their environment and 85% of the learners felt that the modelling process had helped improved their understanding of ML. It was acknowledged that the modelling orientation sessions were more difficult than traditional teaching methods. One third of the learners felt it was too far removed from ML as there were too little calculations. I argue that, due to their struggles with defining and describing the problems and role-players, these learners were not aware of all the calculations required to solve the problems.

One fifth of the learners felt the orientation sessions were a waste of time. The reasons cited were boredom and not being sure what is expected of them. In this case I argue that the learners were presented with many struggles, trying to deal with an open-ended problem. It was the first time they were exposed to solving problems that did not have explicit right or wrong answers. The learners had no familiar way of gauging if they were meeting expectations. For these learners, the uncertainty and unpleasantness of that kind of cognitive demand would easily be described as 'boredom' – a deflection for not understanding the purpose of the task nor the demands of the task, therefore feeling forced to engage when they would otherwise have moved on from the task.

It was also reported, by more than half of the learners, that the second orientation session was more enjoyable than the first. The learners felt that the second orientation session provided them with clearer expectations than the first – despite being a less scaffolded, more open-ended task. This signifies that the learners were showing progress in their understanding of, or at least willingness to engage in, the demands of the modelling process. However, a few learners still complained that the investigation was still too far removed from traditional ML practices.

The learners in this study presented no real opposition to using modelling orientation sessions as a means of instruction. In fact, their main critique offered to these orientation sessions centred around the classroom atmosphere – venues that were too large or too small, groups that were nice to work with or not, and interaction that was too loud and chaotic. Although I acknowledge that creating an ideal learning environment for these types of tasks presents with an array of challenges (not focussed on in this study), it should not be used as an excuse to avoid mathematical modelling in the ML classroom.

5.3.3.2 A cause for the use of modelling orientation sessions as a means of instruction

ML was intended as a modelling curriculum (Brown & Schäfer, 2006), that is supposed to be offering a differentiated approach to mathematics education (Julie, 2006; Verzosa, 2015). The curriculum has the goal of focussing on authentic problems that require a combination of mathematical and non-mathematical skills in order to solve them (Department of Basic Education, 2011a). If we are to provide our learners with rich and worthwhile learning experiences, we need to create a shift in the materials that are used – to move away from solely using standardised outdated textbook tasks, to including tasks where the immediate contexts of the learners are foregrounded. We cannot rely only on generic window-dressed contexts that do not constitute true modelling tasks (Krawitz & Schukajlow, 2018). Instead, we need to make time for exploring contexts that apply directly to the ordinary South African, in their daily lives (Brown & Schäfer, 2006).

To rewrite textbooks yearly to ensure relevant and current information and numbers are used, and to centrally design investigations that cater to the current contextual needs of the diverse array of schools that can be found in one district, is not realistically feasible. I argue that we should be creating orientation sessions and investigations at ground level – addressing problems in the immediate environment of the learners and which they deem to be important.

Although it is difficult to break away from structured mathematics curricula (Bowie & Frith, 2006), and although modelling is a very demanding process with back and forth interaction between the learners, teachers, modelling material and contexts (Krawitz & Schukajlow, 2018), this study has shown that it can yield some wonderful real world results – even when the modelling itself is messy. In this study, the first iteration culminated in an improvement of the school environment with the implementation of cameras which, according to the learners, had helped manage the drug problem they identified. The learners, as a result, felt confident in their ability to bring about change through ML and proud in the action they had taken as a group. This echoes the sentiments of Frankenstein (1990) that if we want to empower our learners, we need to use open-ended problems, with real-life data embedded within the first-hand experiences of the learners. Modelling tasks can also address the stigma attached to ML teachers, of being less capable (Conradie, 2016), as the mathematics education provided by modelling orientation sessions far outranks the cognitive demands of teaching basic word sums (Bowie & Frith, 2006).

Moreover, in not using authentic contexts and not exploring the potential for change offered as through modelling orientation sessions, we are ensuring a situation where, for many learners, the purpose of the ML curriculum is not being achieved. The traditional teaching methods as experienced in this school was not creating learners who were able to interpret data, solve work-related problems, display spatial awareness or manage finances (Department of Education, 2003).

We risk creating learners who will never truly understand the role of mathematics in their lives (imagination), who cannot confidently establish why they should be undergoing mathematics education other than to finish school (alignment). Thus, we cannot expect high levels of deep and persistent engagement in their mathematics education. In essence we place limitations on the development of the learners' mathematical identities through rote teaching practices, which create discrepancies between mathematical content and the contexts of the learners.

5.4 Identity and mathematical modelling in ML

5.4.1 An unanswered question

This study set out to answer the following research questions:

How does the involvement of learners in the design of context rich modelling tasks for Mathematical Literacy affect their mathematical identities?

c) How does involvement affect their perception of the relevance of mathematics in their lives outside the classroom?
d) How does their involvement affect their alignment to and motives for participating in Mathematical Literacy?

From the data analysis and interpretation, it was apparent that there was no significant change in what the learners believed about themselves and their ability and willingness to learn and do mathematics, as a result of their participation in the study. At face value it seems that the answer to my research question was that involving learners in designing context rich modelling tasks does not affect or influence their mathematical identities.

However, I argue that these questions cannot be answered as the learners do not have a true sense of their mathematical identities. Their stringent clinging to an ever-optimistic view of their abilities and willingness to engage, despite evidence that should be pointing to the contrary, indicated to me that these learners have an acute ability to disregard any evidence or narratives that may challenge their views of themselves. They cling almost solely to the positive messages they encounter in their immediate environment and seemingly avoid ideas from a broader society that threatens this.

Although the orientation sessions yielded low marks (which was identified as the main factor considered to inform the learners' view of their abilities) and created cognitive distress and uncertainty among the learners, they did not waver from their optimistic reported identities. I argue this to mean that they disregarded these experiences as they did not fit in with learning experiences they have come to trust as the norm. These learners very easily disregard designated identities that do not adhere to their perceived personal identities (Sfard & Prusak, 2005), to the point that they cannot even be influenced, especially by something as fleeting as two orientation sessions of a day each.

Furthermore, this norm of rote teaching practices, which train learners to engage with rigidly structured tasks also has not, in my opinion, fairly presented learners with the opportunity to explore and come to terms with their true limitations and potentials. Their perceived ability of their mathematics skills is limited to the extent to which they can complete similarly structured tasks with success. To them it is about how mathematical calculations are done, rather than why or how it can be used in their lives outside of the classroom.

The learners could not imagine a use for ML in their work or university lives once they left school. I consider again, the ideas of Wenger (1998) and Anderson (2007), who define mathematical identity as being composed of three faces or communities of practice: imagination, alignment and engagement.

These learners were clearly at a disadvantage, where rote practices and inauthentic, outdated contexts had limited their imagination. They did not have a sense of how ML is useful in their lives outside of generic events such as time-management. I argue that if they do not have a well-formed imagination, there cannot be a well-formed sense of alignment or resulting engagement. Therefore, I deduced that these learners do not have an established mathematical identity, which consequently cannot be affected or influenced by a once-off learning experience. If we truly want to understand how to influence the identities of the learners, and to empower the learners, they need to be exposed to relevant, meaningful learning opportunities, such as modelling orientation sessions, on a regular basis.

2.3.1 A grounded theory

From the literature about mathematical literacy, identity and modelling, in coherence with the findings from this study, I have theorised that in order to influence the mathematical identities of our learners, we actually need to start with mathematical modelling. We need to directly include the learners in the modelling process. It is only when our learners are explicitly involved in modelling problems and events in their own environments that they will truly be able to understand how mathematics can be useful in their lives (Christiansen, 2006). Furthermore, it is only once learners can imagine the role of mathematics in their lives, that they can start to develop an accurate mathematical identity – one that reflects and incorporates the realities our learners experience. By involving the learners we gain the benefit of minimizing the loss of authenticity of a context as a risk of interpretation from the perspective of an external writer or creator of the tasks. (Bansilal et al., 2012). Developing tasks surrounding true authentic contexts, and thus allowing for true mathematization, requires the input of all those involved in the process (Bowie & Frith, 2006; Brown & Schäfer, 2006). We have the responsibility of including our learners in modelling ML investigations if we want them to make mathematical sense of their immediate contexts. In order for learners to successfully mathematically analyse and interpret contexts, they need to have a thorough understanding of them (Bowie & Frith, 2006) and what better way to position them for success than to use the contexts in which they reside.

ML is supposed to be multi-disciplinary in its approach (Verzosa, 2015), indicating that through the use of orientation sessions (as a supplement to traditional teaching practices), learners will be able to explore and address questions pertaining to society and culture (Verzosa, 2015). However, it is essential that these orientation sessions foreground messy, complex, real-world problems that are used as frameworks to explore and address societal issues (Bansilal & Debba, 2012; Frankenstein, 1990; Verzosa, 2015). Through taking this approach to teaching ML, we allow learners to solve their own experienced problems through mathematics (Hernandez-Martinez & Vos, 2018). From the study, it was clear that learners enjoyed, and felt empowered through being part of the solution to or addressing of the drug problem in their school. They also enjoyed sourcing information to attempt to solve the toilet problem as well – being practically involved. Using real, applicable contexts in creating investigations that address real, applicable problems, we can develop an appreciation for mathematics in our learners (Meyer, 2010) which can inform the alignment and engagement aspects of their mathematical identities.

This study has also shown that, even in an under-resourced school, orientation sessions are feasible both timewise and logistically. The orientation sessions, through real-life applications and visible results (such as the cameras), also have the potential of dethroning marks as the key informer of the learners' identities and gives a voice to the learners' learning experiences within their contexts. We have also seen that the learners are ready and willing to partake in these orientation sessions.

Through involving these learners in open-ended problems offered by modelling type tasks, we also remove the externally imposed notion of what the learners are capable of in mathematics. The learners will be presented with the opportunity to explore their own capabilities and willingness to learn – thus more accurately informing their developing identities. Furthermore, if learners are involved in solving or addressing actual, real problems, they are also presented with the opportunity to become contributing members of society, as per curricula goals (Department of Basic Education, 2011a). As contributing members of society, they may be exposed to both internal and external narratives of a positive, yet substantial nature, which may further influence their identities.

Finally, by incorporating modelling tasks with high cognitive demands, we can shift the thinking that often expects teachers to have low expectations of ML learners because they are widely considered to be learners who are weak in mathematics. Research shows that publicly held definitions about a situation or a group of people (such as our learners) can become an integral part of how we treat and interact with these learners, thus affecting the development of the learners (Straehler-Pohl, Fernández, Gellert, & Figueiras, 2014). In other words, if we hold preconceived limited beliefs about our learners' mathematical abilities and potential, we may hold lower expectations for them in terms of the ML curriculum. The expectations of teachers can directly impact the framing of educational outcomes and learning experiences of the learners (Mazenod et al., 2019). When teachers teach with low expectations, what manifests in learners is lower educational attainment and decreased levels of self-confidence (Mazenod et al., 2019). Whether it is done with purpose or not, teachers who hold low expectations of their learners, often provide a significantly reduced curriculum with limited pedagogical strategies that actually decrease the opportunities for autonomous learning to take place and subsequently hinder the development of identities (Mazenod et al., 2019). I believe this is what I was witnessing in the learners in this case study – a preconceived notion of their abilities that defined the learning opportunities they were regularly exposed to and which, sadly, had created a cycle of restricted opportunities (Mazenod et al., 2019). The learners presented with poor marks in mathematics in grade 8 and 9. The level of what is deemed a satisfactory for grades 10 to 12 was adjusted downward to 40%, a mark that suited the externally held expectations of what these learners can achieve. The learning activities these learners were being exposed to were aimed at coaching them to achieve this level of satisfaction. Little time and effort were placed into actively developing mathematical literacy skills. Without mathematical literacy skills, these learners will remain to be viewed as mathematically weak learners who are not expected to do well, and therefore amount to just that.

If we as teachers really want to create autonomous and mathematically literate learners, who possess a skillset that can transcend the school environment, we need to make use of modelling tasks with high cognitive demands. These tasks will expect more of our learners, whilst also giving the space they need to reflect on their own mathematical skills and journeys. In this way, I argue, an accurate yet dynamic mathematical identity can be developed by the learners.

Diagram 1 below illustrates my theorised relationship between mathematical modelling in ML, and learner identities:

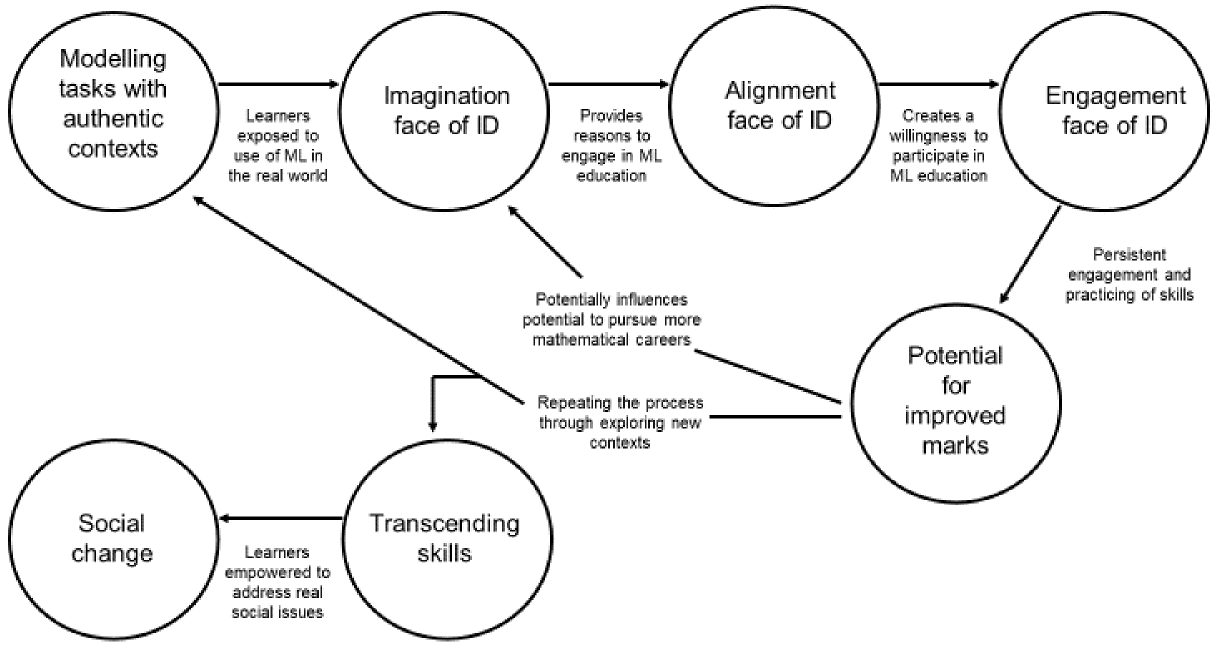


Figure 13 A proposed relationship between identity and mathematical modelling in ML

If learners are exposed to opportunities to solve real-world problems by mathematically modelling authentic contexts in cognitively demanding tasks, it will help them to understand the role of mathematics in their lives. This will influence their mathematical identities by developing a realistic well-formed imagination. Their imagination can then accurately inform their alignment which can create a willingness to engage. If learners deeply and persistently engage in ML tasks, they will, through use and practice, improve on their ML knowledge and develop mathematical skills. This can potentially lead to improved marks. If the learners' marks improve, it may open more doors for tertiary education or potential careers the learners could consider. This in turn, exerts more influence on their imagination by informing the role of ML in their future lives. Regardless of marks, through exploring new problems and contexts in different modelling opportunities, learners can also further inform their imaginations by seeing mathematics in action in different ways. Through constantly being exposed to different contexts, the skills and knowledge of the learners can transcend the classroom. Learners may then be empowered to address and explore social issues through a mathematical lens, even once they have left school.

5.5 Conclusions

This study was undertaken as a case study at an under-resourced school, where most of the learners engaged in ML as a subject rather than Mathematics. The study explored how the learners' engagement in the design of context-rich ML material, through mathematical modelling of their own experienced contexts, influenced their mathematical identities. The conclusions of this study can be summarised in three main points:

1) The effect of low expectations on mathematical identity.
2) The influence of task design on mathematical identity.
3) The feasibility of modelling as a means of instruction to enhance opportunities for the development of mathematical literacy.

Each of these points is summarised in the following sections.

5.5.1 The effect of low expectations on mathematical identity

Studies have shown that when teachers perceive learners as having lower academic attainment, they often make pedagogical decisions that expose learners to a more restricted curriculum in terms of the opportunities afforded them for autonomous learning (Mazenod et al., 2019). This is often done in order to prioritise outcomes with more immediate or tangible results, such as foregrounding performance in standardised tests or simply maintaining classroom discipline (Mazenod et al., 2019). As participants in ML, the learners in this study are under a stigma that expects them to have low levels of mathematical attainment (Brown & Schäfer, 2006), which is furthered by the link between poor performance and the low socio-economic status of these learners and their community (Letaba, 2017). It was evident from the study that the learners performed better in traditional, standardised, test-type tasks than in modelling tasks. These learners had also never before been exposed to modelling type tasks that allowed them to explore mathematics in relation to their world. By allowing low-expectations to inform pedagogic practice, and lower the cognitive demand of the learning experience, a cycle of restricted opportunity is created (Mazenod et al., 2019), which is associated with low levels of employment (Bansilal et al., 2015). In this case study, the accepted mark of what is good was anything above a 40% pass mark. Learners were driven toward this goal by the foregrounding of calculations and mathematical procedures, which the learners found much easier than engaging with authentic contexts. However, through exposure to university information sessions, the learners became aware that the bar that had been set within their community did not necessarily coincide with those of the broader community, as 40% would not grant them access to many of their choices for tertiary study. As a result, the learners found the ever-optimistic identities and narratives they had of themselves being challenged. Through low expectations and the accompanying pedagogical choices, barriers are created in the developing of accurate learner identities (Mazenod et al., 2019). If learners are to develop a true sense of their identity, we need to embrace ML as a subject that is intended to make mathematics more accessible (Julie, 2006), rather than a subject intended to make mathematics easier.

5.5.2 The influence of task design on mathematical identity

A key aspect in ensuring accessible, yet appropriately challenging mathematical education in ML, is task design. For learners to develop an accurate mathematical identity, that can be supported within their immediate context as well as in the broader society, learners need to understand the (potential) role of mathematics in their lives outside of the classroom. They need to understand the value of mathematics in order to align themselves with the subject and direct their engagement with mathematical tasks. Christiansen (2006) summarises the link between task design and mathematical identity well when he states that the only way learners can really understand the importance of mathematics in their lives, is if they are actively involved in the modelling (or task design) process. If we want to develop mathematical literacy in our learners, and in doing so provide access to the knowledge economy (Jablonka, 2015), we need to move away from materials that hinder mathematical literacy through the use of shallow context and outdated materials. We need to foreground the real-world contexts of the average South African (Brown & Schäfer, 2006) and use them as channels through which our learners can understand and question issues in their society

(Frankenstein, 1990; Verzosa, 2015). The learners in this case study showed that the existing stigma surrounding ML did not create the expected lack of enthusiasm or interest in mathematics (Meyer, 2010), and I argue that we should use this to our advantage and involve the learners directly in the design of ML investigations. In doing so, we can offer learners the intended differentiated approach to ML (Julie, 2006; Verzosa, 2015). If learners are involved in modelling authentic contexts, embedded in their immediate environment, not only are they, in my perception, empowered to participate as active citizen, who form a part of the solution; they are also provided the opportunities to use mathematics to make judgements about their lives (OECD, 2019), and use numerical data to appreciate and question various situations (Frankenstein, 1990). Through involving learners directly in designing up-to-date, authentic ML tasks, we are promoting true mathematical literacy (Bowie & Frith, 2006) in our learners and also positively influencing accurate mathematical identities.

5.5.3 The feasibility and necessity of modelling as a means of instruction to enhance opportunities for the development of mathematical literacy

This study indicated that, within the context of this school, and the CAPS curriculum, using modelling and orientation sessions as the termly investigation task, is a feasible approach to ML education in terms of time, resources and the willingness of the learners to participate. The learners in this study reported high levels of enjoyment in these tasks as well as high levels of engagement. The learners also reported feeling positively empowered in having been a part of the solution to problems they had faced in their school environment, and that they had developed a greater understanding of the role of mathematics in their lives. ML, and the South African curriculum in general, was designed to address social injustices (Department of Basic Education, 2011a), which can be done actively by learners in modelling tasks pertaining to issues in their immediate environment. Modelling tasks are also a good way to strengthen the link between the abstract mathematics and its more concrete, useful applications (Gal, 2009; Vithal & Bishop, 2006). These tasks can promote the confidence and critical thinking of our learners (Bowie & Frith, 2006), as well as realising the goal of making mathematics more accessible (Julie, 2006), whilst painting ML as a subject for change (Julie, 2006). In order to truly empower our learners, to make them the agents of change, we need to immerse them in open-ended problems that make use of real-life data (Frankenstein, 1990), and we need to start as early as grade 7, 8 and 9 (Verzosa, 2015). By the time our learners are in grade 11, and are preparing themselves for future career choices, they need to have a well-formed idea of the role of mathematics in society, of the career opportunities presented to mathematically literate persons (Department of Education, 2003), and the extent of their own abilities and willingness to pursue mathematical education – their identities. The use of modelling and orientation sessions as a means of instruction, promotes ML as a modelling- or context-driven subject (Brown & Schäfer, 2006; Conradie, 2016; Julie, 2006), whilst also developing mathematical literacy in our learners. For our learners to develop accurate identities, and understand their roles in society, we need to move away from the idea that a mathematically literate person is someone who can memorise formulae and do mental calculations well. Instead, we should be focussing on creating learners who can use technology, such as calculators, in accordance with mathematics, not only to do calculations, but also to relate those numbers to the world around them in terms of their meaning and impact on the contexts they describe.

5.6 Limitations of the study

This case study was presented with a number of challenges and flaws that need to be improved upon if the intent is to duplicate the study.

The first limitation of this study was the chaos experienced in the two orientation sessions that limited the effectiveness of these sessions. In the first session, the venue was too big and the student to teacher/researcher ratio, was too big to manage effectively. The teachers who were there to provide assistance in managing discipline, intervened in the orientation session according to their own understanding and this caused confusion where their interpretation of the instructions was different to mine as the researcher. The instructions themselves, although aimed at being simple and understandable, were not clear enough to the learners who, as a result, were hesitant to engage. The session was also cut short when the school day ended earlier than what had been discussed in our arrangement of this event. Similar problems, pertaining to poor communication and arrangements, were experienced in the second orientation session. I was given one small classroom to accommodate three classes (about 95 learners), with no teacher assistance. In this case, the venue was much too small and loud as groups discussed their ideas and worked through the instructions. It was also a challenge to move around in the venue and monitor the progress of the learners due to a lack of space.

If this study were to be replicated, I would implement a more stringent planning process with the teachers at the school to ensure that the logistics of the orientation sessions are planned down to the last detail, and that all the teachers have the same understanding of the course of events. I would also give the teachers a more active role in the planning and running of the orientation sessions to ensure consistency in the communication of expectations and clarifications of instructions. Lastly, I would also set out time to preview the venues to ensure a complete understanding of the interactions expected to take place and also to ensure the correct technology and equipment is arranged to maximise the efficiency of the session.

The second limitation to this study was the incorrect copying of the post-study questionnaire. The school did not have or could not make time for me to conduct the post-study questionnaire myself. They offered to print the questionnaire for me and administer it during class time. It was only once I started analysing these questionnaires that I noticed a number of questions had not been printed. The result was that I did not have direct comparisons for all the sections of the pre-study and post-study questionnaires. Although this did not hinder my data analysis, it could have provided more data to support my interpretation.

This is a small limitation that can be overcome in future studies by printing the questionnaire myself and delivering it to the school. In this way I can ensure that the entire document is printed correctly.

A third limitation to this study, was the limitation imposed on my participant selection for the focus group interviews. Many learners were absent in the time I needed to conduct these interviews, and as a result I had to select my participants from a smaller group. I was unable to secure interviews with learners who had indicated negative responses to the orientation sessions and questionnaires, as they were either not at school in that week or unwilling to partake in the interview. This was the case even though I suggested multiple dates for the interview to take place. Interviewing these learners could have further enriched my data on the shortcomings of the orientation sessions, which would have been useful to inform future studies.

This is a challenging limitation to overcome, as learner presence at school is not something that is under the direct control of the researcher or even the teachers. However, in employing a more comprehensive planning session for data collection, better dates for interviews could have been identified, such as time periods where the teachers knew most of the learners would be attending school.

The final limitation I can describe for this study was the actual design of the orientation sessions for the modelling tasks. Feedback from the learners suggested that the instruction and delivery of the orientation sessions was somewhat confusing. It should be noted that these sessions were not only a first for the learners, but also for me as a presenter. I have learned through these sessions that I need to develop clearer outcomes and formulate my instructions in such a way that they reflect the language used by the learners. Involving teachers in the design of these orientation sessions would be greatly helpful in understanding the learners' potential reactions to the way in which instructions are phrased, the examples that are used to illustrate the expectations and the navigating of down-time between instructions.

5.7 Recommendations for further study

This study was undertaken as case study within a very specific environment. My first suggestion is that this study be replicated – with the above limitations addressed – in a different quintile 4[6] school in a similar community, to establish if the findings of this study are applicable to other, similar environments.

Secondly, I would recommend that this study also be replicated in a quintile 1 school to explore the similarities and differences of the findings across socio-economic boundaries. In completing this study in differing environments, we can establish a more complete picture about ML education in South Africa as a whole, as well as exploring how we can bridge the gap in educational approaches across quintiles.

I also recommend that this study expand to include the point of views, identities, and willingness of the teachers to use modelling sessions and authentic contexts in their classrooms. This study could inform how teachers' identities influence their teaching practices, thus the learning experiences of the learners, as well as provide indication as to

[6] In SA schools are described in a quintile system where a quintile 1 school can be described as a more affluent school where government funding is supplemented by high parent-paid school fees, whilst a quintile 5 school is wholly dependent on government funding.

how teachers should be supported in successfully implementing modelling tasks in their classrooms.

Finally, I would also recommend research be done on the development of a framework for the design of modelling tasks, specifically within the South African context. If we want to include modelling as a means of instruction and ensure effective modelling in the wide array of classroom situations in our schools, we need to provide guidance to assist teachers in navigating the contextual challenges that may arise. This framework should be adaptable yet adhere to both modelling principles and the principles of CAPS. It should empower teachers to design and assess modelling tasks regardless of the resources and contextual factors of their schools.

5.8 Closing Remarks

This study was a case study undertaken from the interpretivist perspective. I set out to explore to what extent engagement in the design of context rich ML materials, could influence the identities of a group of grade 11 ML learners in a quintile 4 school. I conducted my research through creating learning experiences for the learners in the form of modelling orientations sessions. These sessions were not interventions, but rather intended as sessions to create valuable learning experiences and to orientate the learners toward the formal assessment investigations we collectively designed. The learners defined the contexts for the investigations, and I formalised the investigation tasks to ensure the mathematics would adhere to curriculum and school demands.

Through engagement in and reflection of these orientation sessions on the part of the learners, and through questionnaire and interview data, it became apparent that these learners did not have accurately formed mathematical identities. These learners had not had exposure to mathematical tasks that would adequately inform them of the role that mathematics plays in their lives. Thus, the learners could not direct their alignment to mathematics beyond the school setting.

This study made it clear to me that rote teaching practices hinder the development of mathematical identities and true mathematical literacy. Hence, I developed an argument for the inclusion of mathematical modelling tasks in the subject of Mathematical Literacy to empower our learners and to develop in them, the envisioned contributing citizens.

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Addenda

Addendum 1: WCED approval letter

Western Cape Government

Education

Directorate: Research

Audrey.wyngaard@westerncape.gov.za

tel: +27 021 467 9272

Fax: 0865902282

Private Bag x9114, Cape Town, 8000

wced.wcape.gov.za

REFERENCE: 20181128–9227

ENQUIRIES: Dr A T Wyngaard

Ms Jeanne-Mari Du Plessis
68 Lovell Road
Die Boord
Stellenbosch
7613

Dear Ms Jeanne-Mari Du Plessis

RESEARCH PROPOSAL: IDENTITY AND MODELLING IN MATHEMATICAL LITERACY: A CASE STUDY IN DESIGNING MATHEMATICAL LITERACY INVESTIGATIONS

Your application to conduct the above-mentioned research in schools in the Western Cape has been approved subject to the following conditions:

1. Principals, educators and learners are under no obligation to assist you in your investigation.
2. Principals, educators, learners and schools should not be identifiable in any way from the results of the investigation.
3. You make all the arrangements concerning your investigation.
4. Educators' programmes are not to be interrupted.
5. The Study is to be conducted from **04 March 2019 till 27 September 2019**
6. No research can be conducted during the fourth term as schools are preparing and finalizing syllabi for examinations (October to December).
7. Should you wish to extend the period of your survey, please contact Dr A.T Wyngaard at the contact numbers above quoting the reference number?
8. A photocopy of this letter is submitted to the principal where the intended research is to be conducted.

9. Your research will be limited to the list of schools as forwarded to the Western Cape Education Department.
10. A brief summary of the content, findings and recommendations is provided to the Director: Research Services.
11. The Department receives a copy of the completed report/dissertation/thesis addressed to:

The Director: Research Services
Western Cape Education Department
Private Bag X9114
CAPE TOWN
8000

We wish you success in your research.

Kind regards.
Signed: Dr Audrey T Wyngaard
Directorate: Research
DATE: 28 November 2018

Addendum 2: Ethics committee approval letter

UNIVERSITEIT
STELLENBOSCH
UNIVERSITY

NOTICE OF APPROVAL

REC Humanities New Application Form

28 March 2019

Project number: 8824

Project Title: Identity and Modelling in Mathematical Literacy: A Case Study in Designing Mathematical Literacy Investigations

Dear Miss Jeanne-Mari Du Plessis

Your REC Humanities New Application Form submitted on **12 March 2019** was reviewed and approved by the REC: Humanities.

Please note the following for your approved submission:

Ethics approval period:

Protocol approval date (Humanities)	Protocol expiration date (Humanities)
28 March 2019	27 March 2021

Please take note of the General Investigator Responsibilities attached to this letter. You may commence with your research after complying fully with these guidelines.

If the researcher deviates in any way from the proposal approved by the REC: Humanities, the researcher must notify the REC of these changes.

Please use your SU project number **(8824)** on any documents or correspondence with the REC concerning your project.

Please note that the REC has the prerogative and authority to ask further questions, seek additional information, require further modifications, or monitor the conduct of your research and the consent process.

FOR CONTINUATION OF PROJECTS AFTER REC APPROVAL PERIOD

Please note that a progress report should be submitted to the Research Ethics Committee: Humanities before the approval period has expired if a continuation of ethics approval is required. The Committee will then consider the continuation of the project for a further year (if necessary)

Included Documents:

Document Type	File Name	Date	Version
Research Protocol/Proposal	Masters Research Proposal_J du Plessis _20606273 final	07/11/2018	
Proof of permission	Jeanne-Marie_[2]	07/11/2018	
Default	J du Plessis WCED supervisor letter	07/11/2018	
Informed Consent Form	parental consent Jeanne-Mari du Plessis (1)	27/11/2018	
Data collection tool	masters questionnaire and interview roster (1)	27/11/2018	
Data collection tool	masters questionnaire and interview roster (1)	27/11/2018	
Data collection tool	observation roster (1)	27/11/2018	
Default	WCED application Jeanne-Mari du Plessis (1)	27/11/2018	
Proof of permission	Research approval WCED	14/01/2019	
Default	Addressing Comments of the REC on Ethics Proposal	16/01/2019	
Parental consent form	Parental Consent Form	17/01/2019	

If you have any questions or need further help, please contact the REC office at cgraham@sun.ac.za.

Sincerely,

Clarissa Graham

REC Coordinator: Research Ethics Committee: Human Research (Humanities)

National Health Research Ethics Committee (NHREC) registration number: REC-050411-032.
The Research Ethics Committee: Humanities complies with the SA National Health Act No.61 2003 as it pertains to health research. In addition, this committee abides by the ethical norms and principles for research established by the Declaration of Helsinki (2013) and the Department of Health Guidelines for Ethical Research: Principles Structures and Processes (2nd Ed.) 2015. Annually a number of projects may be selected randomly for an external audit.

Investigator Responsibilities

Protection of Human Research Participants

Some of the general responsibilities investigators have when conducting research involving human participants are listed below:

1.Conducting the Research. You are responsible for making sure that the research is conducted according to the REC approved research protocol. You are also responsible for the actions of all your co-investigators and research staff involved with this research. You must also ensure that the research is conducted within the standards of your field of research.

2.Participant Enrollment. You may not recruit or enroll participants prior to the REC approval date or after the expiration date of REC approval. All recruitment materials for any form of media must be approved by the REC prior to their use.

3.Informed Consent. You are responsible for obtaining and documenting effective informed consent using **only** the REC-approved consent documents/process, and for ensuring that no human participants are involved in research prior to obtaining their informed consent. Please give all participants copies of the signed informed consent documents. Keep the originals in your secured research files for at least five (5) years.

4.Continuing Review. The REC must review and approve all REC-approved research proposals at intervals appropriate to the degree of risk but not less than once per year. There is **no grace period.** Prior to the date on which the REC approval of the research expires, **it is your responsibility to submit the progress report in a timely fashion to ensure a lapse in REC approval does not occur.** If REC approval of your research lapses, you must stop new participant enrollment, and contact the REC office immediately.

5.Amendments and Changes. If you wish to amend or change any aspect of your research (such as research design, interventions or procedures, participant population, informed consent document, instruments, surveys or recruiting material), you must submit the amendment to the REC for review using the current Amendment Form. You **may not initiate** any amendments or changes to your research without first obtaining written REC review and approval. The **only exception** is when it is necessary to eliminate apparent immediate hazards to participants and the REC should be immediately informed of this necessity.

6.Adverse or Unanticipated Events. Any serious adverse events, participant complaints, and all unanticipated problems that involve risks to participants or others, as well as any research related injuries, occurring at this institution or at other performance sites must be reported to Malene Fouche within **five (5) days** of discovery of the incident. You must also report any instances of serious or continuing problems, or non-compliance with the RECs requirements for protecting human research participants. The only exception to this policy is that the death of a research participant must be reported in accordance with the Stellenbosch University Research Ethics Committee Standard Operating Procedures. All reportable events should be submitted to the REC using the Serious Adverse Event Report Form.

7.Research Record Keeping. You must keep the following research related records, at a minimum, in a secure location for a minimum of five years: the REC approved research proposal and all amendments; all informed consent documents; recruiting materials; continuing review reports; adverse or unanticipated events; and all correspondence from the REC

8.Provision of Counselling or emergency support. When a dedicated counsellor or psychologist provides support to a participant without prior REC review and approval, to the extent permitted by law, such activities will not be recognised as research nor the data used in support of research. Such cases should be indicated in the progress report or final report.

9.Final reports. When you have completed (no further participant enrollment, interactions or interventions) or stopped work on your research, you must submit a Final Report to the REC.

10.On-Site Evaluations, Inspections, or Audits. If you are notified that your research will be reviewed or audited by the sponsor or any other external agency or any internal group, you must inform the REC immediately of the impending audit/evaluation.

Addendum 3: School consent letter

To Whom It May Concern

Hereby I, [redacted] principal of [redacted] confirm that Jeanne-Mari du Plessis (student number 20606273, University of Stellenbosch) has my consent to conduct het Masters Study at this school starting in 2019.

Jeanne-Mari has informed us of the scope of the study and we offer our full permission and support for her to conduct data collection on our school premises, through interaction with our teachers and grade 11 students.

Jeanne-Mari will also be offered support in terms of a school liaison through which any necessary arrangements can be made, and who will serve a channel of communication between Jeanne-Mari and myself, the principal.

Signed

Printed name: [redacted]

Date:
30/10/2018

Addendum 4: Parental consent form

English translation to follow Afrikaans.

Liewe Ouers/Voogde

Ek, Jeanne-Mari du Plessis, beplan om n studie by [redacted] te onderneem wat die betrokkenheid van u kind as 'n Graad 11, Wiskunde Geletterdheid leerder sal vereis.

Wiskundige geletterdheid is 'n vak met baie potensiaal wat u kinders praktiese alledaagse wiskundige vaardighede wat vereis word in die moderne samelewing, kan leer. Ongelukkig wys navorsing dat min tot dusver gedoen is met betrekking tot die ontwikkeling van konteks-spesifieke materiaal – materiaal wat die wiskunde ooglopend van toepassing in hulle onmiddelike omgewing maak. Aangesien ek WG by hierdie skool aangebied het, kan ek sien hoe kontekste wat ver verwyder is van hierdie kinders se lewens, hindernisse tot betekenisvolle leer veroorsaak.

Gedurende kwartale 2 en 3 hierdie jaar, wil ek graag twee werksessies aanbied waar ek met u kinders werk om WG materiaal te ontwikkel wat spesifiek focus op die konteks van [redacted] Hierdie material wat ons ontwikkel sal dien as die "ondersoek" wat kwartaaliks deur die WKOD vereis word van Gr 11 leerders. Dit sal dus deel vorm van hulle formele skool ervaring en sal ook tot hulle punte bydrae.

Gedurende hierdie proses sal ek ook opnames en onderhoude met u kinders voer ten einde die volgende vrae te beantwoord:

1. Hoe beïnvloed hulle betrokkenheid in die ontwikkeling van hierdie materiaal hulle oortuigings van hulself as wiskundig vaardige individue?
2. Hoe beïnvloed hulle betrokkenheid in die ontwikkeling van hierdie materiaal hulle seining rakende die nut van wiskunde buite skool verband?
3. Hoe beïnvloed hulle betrokkenheid hulle seining oor hoekom dit nodig is om wiskunde te kan doen?

My doel is om te sien of hierdie tipe betrokkenheid u kinders sal inspireer om meer geredelik deel te neem in wiskundeige geletterdheid of nie, ten einde kritiese wiskundige vaardighede te ontwikkel.

U ondersteuning in hierdie verband word baie waardeer.

Hiermee gee ek, ____, ouer/voog van ______________________________ toestemming vir hierdie leerder om deel te neem aan die studie gelei deur Jeanne-Mari du Plessis (20606273) as deel van haar meestersgraad. Ek bevestig dat die doel, doelwitte en riglyne van hierdie studie duidelik aan my gestel is en dat ek verstaan wat hierdie studie behels. Ek bevestig ook dat ek bewus gemaak is van die leerder se reg tot anonimiteit, asook ons gemeenskaplike reg om ter enige tyd aan die studie te onttrek.

Geteken:__________________________ Datum:__________________________

Dear Parents/Guardians

I, Jeanne-Mari du Plessis, wish to undertake a study at [redacted] that will require the participation of your child as a Grade 11, Mathematical Literacy student.

Mathematical Literacy is a subject with a lot of potential to teach your children practical mathematical skills, needed for everyday life in modern society. However, research indicates that not much as has been done in the way of developing context specific materials – materials that will make the math evident to your children in their immediate situation. Having taught ML at this school, I can see how using contexts far removed from the lives of these children, creates a barrier to meaningful learning.

In terms 2 and 3 of this year, I would like to host two workshops where I work with your children in developing ML materials that are focussed on the context of [redacted] specifically. This material we develop will serve as the "Investigation" that the WCED requires Gr 11 students to do each term. It will thus form part of their formal school experience and also contribute to their marks.

During this process I will also be conducting surveys and interviews with your children in order to answer the following questions:

1. How does their involvement in the design influence their beliefs of themselves as individuals capable of doing mathematics?
2. How does their involvement in design influence their view of the usefulness of mathematics outside of school?
3. How does their involvement influence their views on why one should be able to do mathematics?

My goal is to see whether this sort of engagement would or would not inspire your children to engage more readily in mathematical literacy, in order to develop critical mathematical skills.

Your support in the matter is greatly appreciated.

Hereby, I ____, parent/guardian of _________________________________ give consent for this learner to participate in the study conducted by Jeanne-Mari du Plessis (20606273) as part of her masters degree. I acknowledge that the purpose, aims and outline of this study has been made clear to me and that I understand what this study will entail. I also acknowledge that I have been made aware of the learners rights to anonymity and our collective right to withdraw from the study at any given time.

Signed:___________________________ Date:___________________________

Addendum 5: Questionnaires

English translations available upon request.

Vooraf Vraelys oor Wiskundige Geletterdheid

Lees die volgende stellings en besluit tot watter mate jy daarmee saamstem. Maak 'n kruisie in die toepaslike blokkie. Let op dat WG staan vir Wiskundige Geletterdheid.

NAAM: ____________________

		Ek stem saam	Ek stem soortvan saam	Ek stem nie heeltemaal saam nie	Ek stem glad nie saam nie
As ek dink oor my besluit om Wiskundige Geletterdheid as vak te neem in Gr 10					
1	Ek het WG gekies want die skool het gesê ek moet en nie omdat ek wou nie				
2	Ek het WG gekies want my ouers het gesê ek moet en nie omdat ek wou nie				
3	Ek het WG gekies want my punte was te swak vir wiskunde				
4	Ek het WG gekies want ek het gedink dit is makliker as wiskunde				
5	Ek het WG gekies want ek het gedink ek sal van die vak hou				
6	Ek het WG gekies want ek dink nie regtig 'n mens het wiskunde nodig nie				
7	Ek het WG geskies want ek dink ek kan die wiskunde van WG meer gebruik as 'skoon wiskunde'				
8	Ek het WG gekies want leerders wat dit reeds neem sê hulle hou daarvan				
9	Ek het WG gekies want leerders wat dit reeds neem sê dit is maklik om goeie punte te kry				
10	Ek het WG gekies want ek hou nie van Wiskunde nie				
As ek dink oor hoe ander mense Wiskundige Geletterdheid as 'n vak sien					

11	Mense dink WG is vir mense wat nie kan wiskunde doen nie				
12	Mense dink WG is baie maklik				
13	Mense dink as ek WG neem gaan ek nie Universiteit toe kan gaan nie				
14	Mense glo dat kinders wat WG neem eendag suksesvol kan wees				
15	Mense glo dat WG nogsteeds 'n vorm van regte wiskunde is				
16	Mense dink WG is vir dom mense				
As ek dink oor hoe ek self Wiskundige Geletterdheid as 'n vak sien					
17	Ek dink WG is vir mense wat nie kan wiskunde doen nie				
18	Ek dink WG is baie maklik				
19	Ek dink WG is behulpsaam vir my toekoms				
20	Ek dink as ek WG neem gaan ek nie Universiteit toe kan gaan nie				
21	Ek glo dat mense wat WG neem eendag suksesvol kan wees				
22	Ek dink WG is vir dom mense				
23	Ek glo dat WG nogsteeds 'n vorm van regte wiskunde is.				
24	WG is meer opwindend as wiskunde want dit maak gebruik van stories wat ek kan verstaan				
As ek dink oor dit wat ek in Wiskundige Geletterdheid leer, en die wêreld om my					
25	WG leer my dinge wat ek kan gebruik buite die skool				
26	WG is wiskunde wat elke dag gebruik kan word				
27	WG berei my voor vir die lewe na skool				
28	Ek kan sien waar mense dit wat ons in WG leer gebruik in my omgewing				

29	Die handboeke en oefeninge wat ons gebruik help my om te sien hoe ek WG in my eie lewe kan gebruik				
30	Die hanboek en oefeninge gebruik stories wat nie altyd vir my sin maak nie.				
31	Die werk wat ek eendag wil doen het nie Wiskunde nodig nie, WG is goed genoeg.				
	As ek dink oor dit wat mense van my sê				
32	My ouers/voogde glo ek goeie punte vir WG kan kry				
33	My ouers/voogde dink ek werk nie hard genoeg aan my WG nie				
34	My ouers/voogde wil eintlik he ek moet 'skoon wiskunde' doen				
35	My onderwyser glo ek kan goeie punte kry in WG				
36	My onderwyser motiveer my om harder te werk in WG				
37	My onderwyser sien my as iemand wat die wiskunde wat ons in WG leer goed kan doen				
38	My klasmaats sien my as iemand wat goed is in WG				
39	My klasmaats vra my gereeld vir hulp met WG				
	As ek dink oor dit wat ek van myself sê				
40	Al neem ek WG as vak, is ek nog steeds iemand wat wiskunde kan doen				
41	Ek kan goeie punte in WG kry				
42	Ek verstaan die wiskunde wat ons in WG leer				
43	Al is van die wiskunde wat ons in WG leer moeilik, kan ek dit baas raak				
44	Ek glo ek kan my klasmaats hulp aanbied in WG				
45	Ek het genoeg selfvertroue om my antwoorde in die WG klas voor almal te bespreek				
46	As die wiskunde probleme in WG moeilik raak dan druk ek deur totdat ek die regte antwoord self gekry het				

47	Ek bepaal hoe goed ek is in WG deur na my punte te kyk				
48	Ek is tevrede met my WG punt				
49	Ek neem WG want ek verstaan nie wiskunde nie				
50	Ek voel dom omdat ek WG neem en nie skoon wiskunde nie				
51	Ek raak hopeloos as ek swak doen in WG				
As ek dink oor wat my motiveer om Wiskundige Geletterdheid te doen					
52	Die wiskunde wat ons in WG doen is vir my lekker				
53	Ek neem WG slegs omdat die skool my verplig				
54	Ek sou eintlik eerder skoon wiskunde wou neem				
55	Ek wil eintlik glad nie wiskunde of WG neem nie				
56	My ouers/voogde dink wiskunde is belangriker as wat ek dink				
57	Die werk wat ek eendag wil doen gaan gebruik maak van die wiskunde wat ons in WG leer				
As ek dink oor die belangrikheid van Wiskundige Geletterdheid					
58	Die wiskunde wat ons in WG leer is belangrik om 'n sukses van my lewe te maak				
59	My ouers/voogde en onderwysers dink wiskunde en WG is belangriker vir my toekoms as wat ek dink				
60	Om WG te leer is belangrik want ek sien hoe WG in die wêreld om my gebruik word				
61	Ek moet hard werk in WG want as ek goed doen in WG sal daar vir my baie studiegeleenthede wees				

Eindvraelys oor Wiskundige Geletterdheid

Lees die volgende stellings en besluit tot watter mate jy daarmee saamstem. Maak 'n kruisie in die toepaslike blokkie. Let op dat WG staan vir Wiskundige Geletterdheid.

NAAM: ____

		Ek stem saam	Ek stem soortvan saam	Ek stem nie heeltemaal saam nie	Ek stem glad nie saam nie
As ek dink oor die werksinkels wat vir my aangebied was in Wiskunde Geletterdheid					
1	Die werkswinkel in ondersoeke was vir my 'n lekker manier om te leer				
2	Die werkswinkel en ondersoeke het my laat sien hoe ek wiskunde in my omgewing kan gebruik				
3	Ek sal daarvan hou as ons meer gereeld op so 'n manier leer				
4	My punte vir die ondersoek was beter as wat ek in Gr 10 gekry het				
5	Die manier van leer het my verstaan van wiskunde verbeter				
6	Dit was vir my interessant om ons eie wiskunde ondersoek te ontwikkel				
7	Na die ondersoek voel ek nou meer in staat om wiskunde in my lewe buite die klaskamer te gebruik				
8	Hierdie manier van wiskunde leer is moeiliker as hoe ons dit in die klas doen				
9	Ek dink nie die werkswinkel was oor wiskunde nie				
10	Ek wil nie weer my tyd mors met so 'n werkswinkel nie				
As ek dink oor hoe ander mense Wiskundige Geletterdheid as 'n vak sien					
11	Mense dink WG is vir mense wat nie kan wiskunde doen nie				
12	Mense dink WG is baie maklik				
13	Mense dink as ek WG neem gaan ek nie Universiteit toe kan gaan nie				

14	Mense glo dat kinders wat WG neem eendag suksesvol kan wees				
15	Mense glo dat WG nogsteeds 'n vorm van regte wiskunde is				
16	Mense dink WG is vir dom mense				
	As ek dink oor hoe ek self Wiskundige Geletterdheid as 'n vak sien				
17	Ek dink WG is vir mense wat nie kan wiskunde doen nie				
18	Ek dink WG is baie maklik				
19	Ek dink WG is behulpsaam vir my toekoms				
20	Ek dink as ek WG neem gaan ek nie Universiteit toe kan gaan nie				
21	Ek glo dat mense wat WG neem eendag suksesvol kan wees				
22	Ek dink WG is vir dom mense				
23	Ek dink dat WG nogsteeds 'n vorm van regte wiskunde is.				
24	WG is meer opwindend as wiskunde want dit maak gebruik van stories wat ek kan verstaan				
	As ek dink oor dit wat ek in Wiskundige Geletterdheid leer, en die wêreld om my				
25	WG leer my dinge wat ek kan gebruik buite die skool				
26	WG is wiskunde wat elke dag gebruik kan word				
27	WG berei my voor vir die lewe na skool				
28	Ek kan sien waar mense dit wat ons in WG leer gebruik in my omgewing				
29	Die handboeke en oefeninge wat ons gebruik help my om te sien hoe ek WG in my eie lewe kan gebruik				
30	Die hanboek en oefeninge gebruik stories wat nie altyd vir my sin maak nie.				
31	Die werk wat ek eendag wil doen het nie Wiskunde nodig nie, WG is goed genoeg.				

As ek dink oor dit wat mense van my sê					
32	My ouers/voogde glo ek goeie punte vir WG kan kry				
33	My ouers/voogde dink ek werk nie hard genoeg aan my WG nie				
34	My ouers/voogde wil eintlik he ek moet 'skoon wiskunde' doen				
35	My onderwyser glo ek kan goeie punte kry in WG				
36	My onderwyser motiveer my om harder te werk in WG				
37	My onderwyser sien my as iemand wat die wiskunde wat ons in WG leer goed kan doen				
38	My klasmaats sien my as iemand wat goed is in WG				
39	My klasmaats vra my gereeld vir hulp met WG				
As ek dink oor dit wat ek van myself sê					
40	Al neem ek WG as vak, is ek nog steeds iemand wat wiskunde kan doen				
41	Ek kan goeie punte in WG kry				
42	Ek verstaan die wiskunde wat ons in WG leer				
43	Al is van die wiskunde wat ons in WG leer moeilik, kan ek dit baas raak				
44	Ek glo ek kan my klasmaats hulp aanbied in WG				
45	Ek het genoeg selfvertroue om my antwoorde in die WG klas voor almal te bespreek				
46	As die wiskunde probleme in WG moeilik raak dan druk ek deur totdat ek die regte antwoord self gekry het				
47	Ek bepaal hoe goed ek is in WG deur na my punte te kyk				
48	Ek is tevrede met my WG punt				
49	Ek neem WG want ek verstaan nie wiskunde nie				
50	Ek voel dom omdat ek WG neem en nie skoon wiskunde nie				

51	Ek raak hopeloos as ek swak doen in WG				
As ek dink oor wat my motiveer om Wiskundige Geletterdheid te doen					
52	Die wiskunde wat ons in WG doen is vir my lekker				
53	Ek neem WG slegs omdat die skool my verplig				
54	Ek sou eintlik eerder skoon wiskunde wou neem				
55	Ek wil eintlik glad nie wiskunde of WG neem nie				
56	My ouers/voogde dink wiskunde is belangriker as wat ek dink				
57	Die werk wat ek eendag wil doen gaan gebruik maak van die wiskunde wat ons in WG leer				
As ek dink oor die belangrikheid van Wiskundige Geletterdheid					
58	Die wiskunde wat ons in WG leer is belangrik om ‘n sukses van my lewe te maak				
59	My ouers/voogde en onderwysers dink wiskunde en WG is belangriker vir my toekoms as wat ek dink				
60	Om WG te leer is belangrik want ek sien hoe WG in die wêreld om my gebruik word				
61	Ek moet hard werk in WG want as ek goed doen in WG sal daar vir my baie studiegeleenthede wees				

Addendum 6: Interview schedule

English translation available upon request.

1. Het jy die werkswinkel wat Juf JM aangebied het geniet?
    a. Wat daarvan het jy geniet?
    b. Wat daarvan het jy nie geniet nie?
    c. Watter van die twee werkswinkels het jy meer geniet?
        i. Hoekom?
2. Wat is jou opinie oor die tipe manier van WG onderrig?
    a. Dink julle dit is behulpsaam?
        i. Hoe?
        ii. Hoekom of hoekom nie?
    b. Dink julle dit is iets wat gereeld gedoen kan word?
        i. Hoekom of hoekom nie?
    c. Het dit jou anders laat dink oor WG?
        i. Hoe?
3. Wat was julle ervaring met die fisiese doen van die ondersoeke na die ervaring?
    a. Was dit makliker of moeiliker as die ondersoeke in Graad 10?
        i. Wat het dit makliker of moeiliker gemaak?
        ii. Wat sou dit nog makliker gemaak het?
        iii. Dink julle, julle was genoeg uitgedaag in graad 10?
            1. Was julle tyd gegee om self probleme op te los (voorbeeld)?
            2. Was julle gereeld gesê hoe om die wiskunde te doen?
                a. Hou julle daarvan of nie? Verduidelik.
    b. Was die ondersoeke in die jaar meer interessant en relevant to julle lewens?
4. Hoe het die benadering tot die leer van WG julle laat dink oor jul eie vermoeë om wiskunde te doen?
    a. Dink julle julle kan wiskunde goed doen?
        i. Hoekom sê jy so?
    b. Geniet julle dit om wiskunde te doen?
        i. Hoekom sê jy so?
    c. Sien jy hoe jy wiskunde buite skool kan gebruik?
        i. Kan jy vir my voorbeelde gee?
    d. Dink julle wiskunde is belangrik?
        i. Hoekom of hoekom nie?
        ii. Net nou of ook in die toekoms?
            1. Hoekom sê jy so?
5. Wat dink julle van WG as ‘n vak?
    a. Is dit bruikbaar?
    b. Watter tipe leerders kies WG bo Wiskunde?
        i. Wat het jou laat besluit om WG te kies bo Wisk?
        ii. Hoekom neem jy deel in WG?
        iii. As jy kon, sou jy die vak los? Hoekom of hoekom nie?
6. Wat dink die onderwysers van WG as ‘n vak en die leerders wat dit neem?
    a. Sê hulle die vak is belangrik?
        i. Kan jy voorbeelde gee van wat hulle sê?
    b. Voel jy dat hulle in julle as WG leerders glo?

            i. Kan jy voorbeelde gee van wat hulle sê, vir jou om so te antwoord
    c. Maak hulle moeite om klas te gee op 'n manier wat relevant is en behulpsaam is vir julle?
            i. Kan jy voorbeelde gee van hoekom jy so sê?
7. Wat dink jou vriende en familie van WG as 'n vak en die leerders wat dit neem?
    a. Dink hulle die vak is belangrik?
            i. Kan jy 'n voorbeeld gee van wat hulle sê?
    b. Voel jy hulle glo in jou as WG leerder om goed te kan doen in WG?
            i. Kan jy voorbeelde gee van wat hulle sê om jou so te laat dink?
    c. Glo hulle WG leerders is net so slim en bevoeg soos leerder wat normale Wisk doen?
            i. Gee 'n voorbeeld van hoekom jy so sê?
8. Wat is jou gunstelling deel van WG?
    a. Hoekom? Wat daarvan is vir jou lekker?
9. Wat hou jy die minste van WG?
    a. Hoekom?
10. Het julle enige ander kommentaar wat julle wil deel?

Addendum 7: Orientation session 1 instruction sheet

English translation available upon request.

WISKUNDE GELETTERDHEID ORIENTASIE-SESSIE: 26 APRIL 2019

Werk saam in 'n groep om die volgende instruksies uit te voer. Wag vir Juf JM om vir julle te sê wanneer om aan 'n spesefieke opdrag te werk.

Julle het vir elke opdrag 'n A4 blaai waarop julle kan skryf.

Instruksie 1

Skryf die name van jul groeplede agter op die nommer wat aan jul groep toegeken is. Sit die kaartjie in die boks wat Juf JM ombring. Skryf ook julle GROEPNOMMER op elke blaai wat julle het.

Instruksie 2

Dink aan julle skoolomgewing. Dink aan die klaskamers, die snoepie, die sosiale aktiwiteite, die sport en so aan. Dink aan die dinge wat lekker is. Dink aan die dinge wat minder lekker is.

Kan julle as 'n groepie dalk dink aan 'n paar probleme of uitdagings wat julle graag sou wil hê die skool moet aanspreek? Gebruik die spasie om die probleme waaraan julle kan dink neer te skryf.

Instruksie 3

Kies nou EEN van die idees in Instruksie 2 om op te fokus. Beskryf die probleem in meer detail.

Wat is dit wat julle pla?

Wat is al gedoen aan die probleem?

Hoekom is dit vir julle belangrik?

Gebruik die spasie hier onder om jul idees neer te skryf.

Gee die blaai in by Juf JM as julle klaar is.

Instruksie 4

Ons het nou saam een probleem gekies om op te fokus. In jul groepe, dink aan 'n moontlike oplossing vir die probleem. Beskryf die oplossing hier onder in detail.

Wat kan gedoen word?

Wie moet dit doen?

Wat het ons nodig om die oplossing uit te voer?

Hoe lank het mens nodig om die oplossing uit te voer?

Gee die blaai in by Juf JM as julle klaar is.

Instruksie 5

Ons het nou as 'n groep gestem vir een oplossing om aan te werk. Nou moet ons bietjie nadink oor hoe dit wat ons in WG leer ons kan help om die oplossing uit te voer.

Dink oor die onderwerpe wat al in WG hanteer is in Gr 10 en 11:

- Berekeninge met getalle
- Getalpatrone
- Meet me tyd, afstand en gewig
- Finansiële dokumente en tariewe
- Oppervlaktes en Volumes
- Kaartwerk en skale
- Waarskynlikheid
- Finansies wat behels oor bankstate en BTW
- Data hantering (gemiddelde, modus, mediaan)
- Inkomste- en uitagawesstate
- Wins en verlies
- Begrotings
- Gelykbreek punte
- Inflasie

Watter van hierdie onderwerpe kan ons van gebruik maak om die oplossing uit te voer?

Skryf elke onderwerp wat julle dink van toepassing neer en skryf 'n rede hoekom julle so sê.

Instruksie 6

Met die onderwerp in gedagte, watter tipe vrae dink julle moet mens vir jouself vra om die oplossing uit te voer? Skryf al julle idees neer.

Instruksie 7

Gaan te werk om te wys hoe mens van die vrae in Instruksie 6 sou beantwoord?

Instruksie 8: Refleksie tyd (individueel)

- Hoe het jy vandag se werkswinkel ervaar en hoekom?
- Hoe het vandag se werkswinkel jou laat voel oor WG as 'n vak? Hoekom?
- Hoe het vandag se werkswinkel jou laat voel oor jouself? Hoekom?
- Sou jy weer wil deelneem aan so werkswinkel? Hoekom of hoekom nie?

Addendum 8: Investigation 1 and memorandum

Wiskundige Geletterdheid Ondersoek: Kwartaal 2

Graad 11

Tema: Dwelmmisbruik op die skoolgronde

Totaal: 65 Punte

DIE PROBLEEM

Dit het onlangs onder die leerders van Hoërskool K se aandag gekom dat daar vreeslik baie dwelmisbruik op die skoolgronde plaasvind. Die leerders het dit onder die personeel se aandag gebring en die skool het besluit om op te tree daarteen deur, onder andere, spesiale sekuriteit aan te stel. Die Wiskundige Geletterdheid Gr 11 leerders bied die skool ondersteuning aan deur die wiskunde agter die oplossing van die dwelmprobleem te doen.

Hierdie probleem en konteks is deur die Gr 11 groep voorgestel.

VRAAG 1: DIE SEKURITEITSMAATSKAPPY

Die Hoof van Hoërskool K nader 'n Sekuriteitsmaatskappy met 'n goeie reputasie. Die maatskappy se naam is *BEWAAR!*. Die hoof vra *BEWAAR!* om vir hom 'n kwotasie op te stel. Die Hoof wil hê daar moet wagte wees wat elke gang sowel as die res van die gronde patrolleer. Hy will ook kameras in elke gang opsit en een in die kantoor, en snuffelhonde een keer per week inkry. Hy wil weet wat die sekuriteit die skool per maand sal kos. Voordat *BEWAAR!* die kwotasie kan opstel het hulle sekere inligting nodig. Dit is die werk van die Gr 11 leerders om die inligting te bereken.

1.1 As die skool wil gebruik maak van 2 wagte om elk van die skool se 4 gange te patrolleer, sowel as 2 wagte wat die res van die gronde patrolleer, hoeveel wagte het die skool nodig? (1)

Die idee vir die vraag is van groep 8 en 31

1.2 Die sekuritetswagte word per dag betaal. Hoeveel werksdae is daar gemiddeld in 'n maand as Januarie 23 dae, Februarie 20 dae, Maart 21 dae en April 22 dae gehad het? (3)

1.3.1 *BEWAAR!* het die volgende kwotasie deurgestuur. Help om die kwotasie te voltooi deur die onbekende waardes te bereken: (12)

Die idee vir die vraag is van groep 31

KOWTASIE VIR SEKURITEIT VIR HOëRSKOOL K		**NO: 13428**
Item	**Koste**	**Totale Bedrag**
Instaleer en monitor van kameras	R850 per kamera per maand	**A**
Sekurtietswagte	R300 per dag per persoon x die gemiddelde aantal werksdae per maand	**B**
Gebruik van snuffelhonde	R200 per hond. Stel voor 2 honde een keer per week vir gemiddeld van 3 weke per maand	**C**
Administrasiefooie	Per maand	**R150**
Totaal	**Per maand**	**D**
Totaal	**Per jaar**	**E**
Hierde kwotasie is geldig vir 30 dae na uitreiking.		Datum: 30/04/2019

Die idee vir die vraag is van groep 8 en 32

1.3.2 Teen watter datum moet die skool besluit of hulle die kwotasie aanvaar of nie? (1)

1.3.3 Minimumloon in Suid-Afrika is R20 per uur. Dit beteken 'n maatskappy mag nie 'n werker minder as R20 per uur betaal nie. As die sekuriteit van 07h00 to 15h00 per dag by die skool moet wees, bereken of hulle meer of minder as die minimum loon verdien. Toon alle berekeninge.

(4)

1.3.4 Indien die inflasie vir 2019 4,5% is, bereken die totale prys per jaar van sekuriteit vir die volgende skooljaar. (3)

Die idee vir die vraag is van groep 15, 19 en 34.

VRAAG 2: DIE SKOOL SE BEGROTING

Voordat die skool kan besluit om *BEWAAR!* se kwotasie te aanvaar, moet die Hoof na die skool se begroting gaan kyk. Die skool kry inkomste vanaf die regering maar ook van skoolfooie. Die regering gee vir die skool ongeveer R1330 per kind per maand. Die maandelikse skoolfooie is R220 per maand per kind. Daar is 1154 leerders in die skool. Die skool begroot vir die volgende maandelikse uitgawes:

Salarisse:	R 1 053 300
Water en Elektrisiteit:	R89 000
Drukwerk:	R100 700
Onderhoud van gronde:	R206 000
Internet en telefoon:	R122 000
Snoepie:	R78 000
Onvoorsiende uitgawes:	R75 000

2.1 Bereken die skool se totale inkomste. (3)

2.2 Trek 'n inkomste en uitgawestaat vir die skool op. Toon die totale inkomste en uitgawes aan. (10)

Die idee vir die vraag is van groep 2, 6, 15, 17, 19, 20, 23, 26, 36 en 43

2.3 Het die skool genoeg oor om die maadelikse fooie aan *BEWAAR!* te betaal? Staaf jou antwoord deur bewerkings. (3)

Die idee vir die vraag is van groep 9

2.4 Hoekom is dit belangrik vir die skool om 'n maandelikse begroting op te stel? (2)

Die idee vir die vraag is van groep 1

2.5 Hoe dink jy kan die skool te werk gaan om die begroting aan te pas sodat hulle *BEWAAR!* se sekuriteitsfooi kan betaal? Gee twee moontlike maniere. (2)

Die idee vir die vraag is van groep 6

VRAAG 3: BOETESISTEEM

Om fondse in te samel vir die sekuriteit, stel die Gr 11 leerders aan die Hoof voor dat persone wat gevang word met dwelms, beboet moet word. Hulle voel dat persone volgens 'n tariewe stelsel beboet moet word en stel die volgende voor:

Die idee vir die vraag is van groep 3

Gewig van dwelms	**Koste per gram**
0 – 1,2 g	R15
1,3 – 2 g	R19
2,1 – 2,5 g	R22
2,6 – 3 g	R27
3,1 – 4 g	R35
Meer as 4g	R50

3.1 Veronderstel die skool het *BEWAAR!* se kwotasie aanvaar. Die sekuritietswagte het in die eerste maand 6 persone betrap met dwelms. Die dwelms het die volgende geweeg:

Persoon 1: 1,5 g

Persoon 2: 1,8 g

Persoon 3: 2,2g

Persoon 4: 0,8 g

Persoon 5: 3,2 g

Persoon 6: 0,3 g

3.1.1 Bereken die totale inkomste wat die boetesisteem ingebring het. (12)

3.1.2 Maak die skool dan aan die einde van die spesefieke maand 'n wins of verlies ten opsigte van hul begroting? Toon alle bewerkings. (3)

Die idee vir die vrae is van groep 36

Die idee vir die vraag is van groep 13 en 23

3.1.3. Bereken die gewig van die dwelms per maand waarvoor leerders beboet sal moet word om elke maand gelyk te breek. (4)

3.1.4 Dink jy dit is eties korrek en verantwoordelik om so boetestelsel te implementeer? Gee 'n rede vir jou antwoord. (2)

J du Plessis 20606273 M Education

Ondersoek 1 : Memorandum [redacted]

Totaal : 65 Punte

Vraag 1

1.1 10 wagte

1.2 (23 + 20 + 21 + 22) ÷ 4 = 21,5

∴ Gemiddeld van ongeveer 22 werksdae per maand

1.3.1 A = R850 x 5

= R4250

B = R300 x 10 x 22

= R66 000

C = R200 x 2 x 3

= R1200

D = 4250 + 66000 + 1200 + 150

= R 71 600

E = R71 600 x 12

= R 859 200

3.2 30 Mei 2019

3.3 Dagfooi : R300

1.3.4 R859 200 × $\frac{4,5}{100}$ = R38 664

∴ Bedrag vir 2020 R859 200 + R38 664
= R897 864

Vraag 2

2.1 Inkomste is vanaf skoolfooie en regering
∴ R1330 × 1154 = R1534 820 (regering)
R220 × 1154 = R253 880 (skoolfooie)
Totaal: R1534 820 + R253 880
= R1788 700

2.2

Inkomste	R	Uitgawes	R
Regering	1534 820	Salarisse	1053 300
Skoolfooie	253 880	Water & Elektrisiteit	89 000
		Drukwerk	100 700
		Onderhoud	208 000
		Internet en telefoon	122 000
		Snoepie	78 000
		Onvoorsiende uitgawes	75 000
Totaal	1788 700	Totaal	1724 000

2.3 Skool benodig: R71 600
Skool het beskikbaar: R1788 700 − R1724 000
= R64 700

Nee, die skool het nie genoeg geld nie.

1.3.4. R859 200 × $\frac{4,5}{100}$ = R 38 664

∴ Bedrag vir 2020 = R 859 200 + R38 664
= R897 864

Vraag 2

2.1 Inkomste is vanaf skoolfooie en regering
∴ R1330 × 1164 = R1534 820 (regering)
R220 × 1164 = R 253 880 (Skoolfooie)
Total: R1534 820 + R253 880
= R1788 700

2.2

Inkomste	R	Uitgawes	R
Regering	1534 820	Salarisse	1 053 300
Skoolfooie	253 880	Water & Elektrisiteit	89 000
		Drukwerk	100 700
		Onderhoud	206 000
		Internet en telefoon	122 000
		Snoepie	78 000
		Onvoorsiende uitgawes	75 000
Totaal	1788 700	Totaal	1724 000

2.3 Skool benodig: R71 600
Skool het beskikbaar: R1788 700 − R1724 000
= R64 700

Nee, die skool het nie genoeg geld nie.

3.1.3 Boetes moet nog R6728,10 in bring om gelyk te breek

~~∴ R6728,10 1,2 g × R15 =~~

R6728,10 − (1,2 × R15) − (0,8 × 19) − (0,5 × 22) − (0,5 × 27)
 − (1 × 35)
 = R6635,40

∴ R6635,40 ÷ R50 = 132,71 g

∴ Totale gewig = 132,71 + 1,2 + 0.8 + 0,5 + 0,5 + 1
 = 136,71g

3.1.4 Hier moet leerders aanspreek dat dit onwettig is om geld te maak uit onwettige aktiwiteite op die skoolgronde. Dit is nie eties nie en dwelmmisbruikers moet die wettige nagevolge dra.

Addendum 9: Orientation session 2 instruction sheet and information booklet

English translations available upon request.

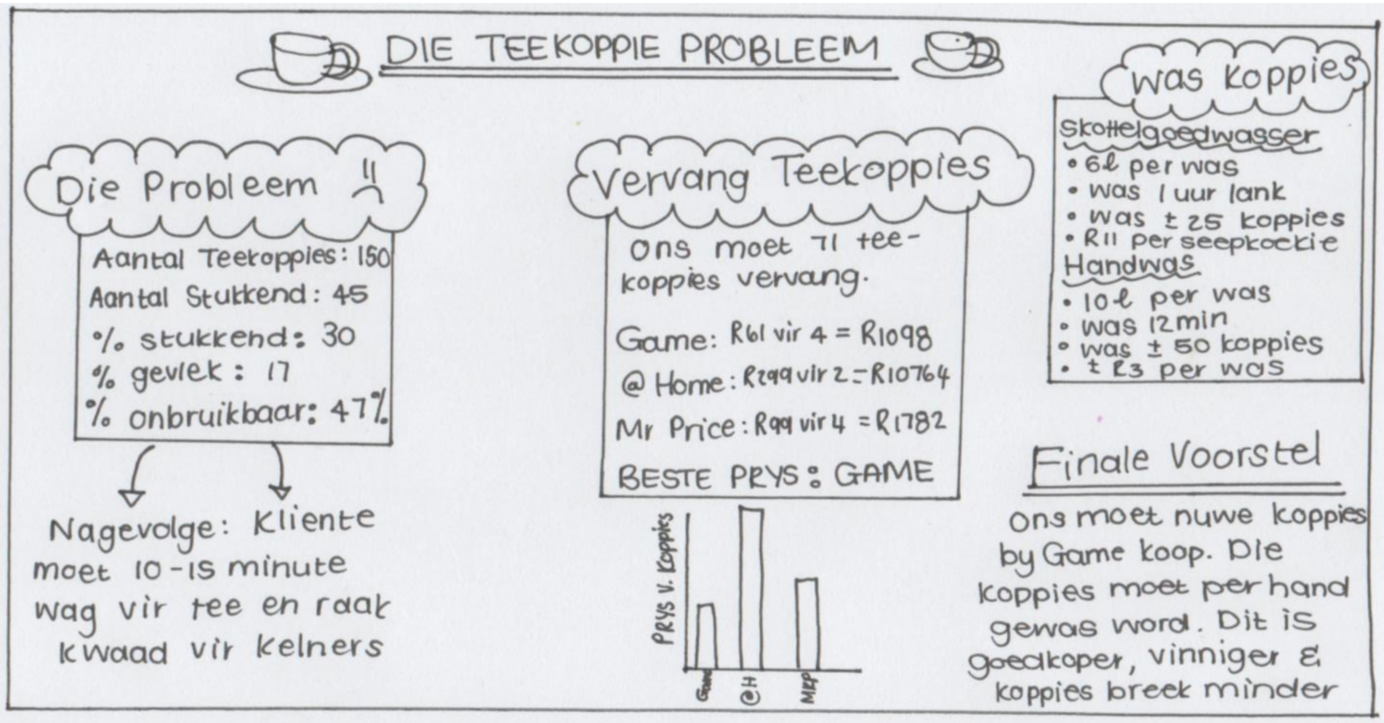
Orientasie-Sessie 2: Die Toilet Probleem

Julle as Graad 11 groep het die toilette in die skool aangewys as een van die grootste probleme in die skool.

Vandag gaan ons saam werk aan 'n plan om die probleem aan te spreek.

Kom ons kyk na die volgende storie van Cynthia en Ben:

Cynthia en Ben is kelners by 'n klein koffiewinkel. Hulle het agtergekom dat die koffiewinkel se teekoppies gereeld opraak omdat baie van hulle chip in die skottelgoedwasser en daar nie genoeg is wat nog bruikbaar is nie. Die kliente raak soms vies omdat hulle moet wag vir teekoppies. Cynthia en Ben is innoverend en in plaas daarvan om net te kla oor die probleem wil hulle ook deel wees van die oplossing. Hulle het besluit om die probleem te analiseer, om op te kom met 'n voorstel oor hoe om die probleem aan te spreek en dit aan die eienaar oor te dra. Hulle het hul bevindings en idees in 'n plakkaat soos hieronder aangeteken en voorgestel:



Dink nou bietjie na oor die toiletprobleem by die skool. Wat as ons self met 'n analise en oplossing vir die probleem opkom en dit aan die skoolhoof deurgee deur middel van 'n plakkaat? Jy sal die volgende in ag moet neem:

1. Beskryf en analiseer die proleem met die toilette
2. Dink aan moontlike oplossings and ondersoek hulle
3. Maak 'n voorstel in detail oor die beste oplossing.

STAP 1: BESKRYF EN ANALISEER DIE PROBLEEM

Neem tyd om die toiletprobleem in detail te beskryf. Dink aan al die inligting wat die hoof behoort te hê.

1.1. Hoeveel mense in die skool voel daar is 'n probleem met die toilette? Hoe sal jy inligting daaroor insamel?
1.2. Hoeveel toilette is daar? Hoe weet jy?
1.3. Hoeveel toilette is heeltemaal onbruikbaar? Wat laat jou so sê?
1.4. Wat maak die toilette onbruikbaar?
1.5. Is dit onbruikbaar deur almal of net onder sekere omstandighede? Watter omstandighede?
1.6. Is dit heeldag onbruikbaar of eers van 'n sekere tyd af? Wanneer?
1.7. Enige ander inligting wat julle dink belangrik is

STAP 2: DINK AAN MOONTLIKE OPLOSSINGS

Dink oor wat aan die toilette gedoen moet word sodat hulle permanent bruikbaar sal wees. Beskryf die oplossing in detail aan die hoof. Wees spesifiek. Julle mag gebruik maak van die inligtingstuk om julle idees aan te vul. Onthou om spesifiek te wees in terme van die aantal items, koste en tyd. Dit sal goed wees om met 'n paar idees op te kom en hulle te vergelyk voordat julle die beste een kies. Neem die volgende ook in ag:

2.1. Is dit nodig om al die toilette te vervang? Is dit nodig om die hele toilet te vervang?

2.2. Hoeveel toilette moet heeltemaal vervang word en hoeveel moet net herstel word?

2.3. Wat sal dit kos om toilette te vervang?

2.4. Hoe sal jy die beste toilet kies om die huidige toilette mee te vervang?

2.5. Hoe gaan die toilette skoongehou word?

2.6. Hoe gaan die situasie gemonitor word tydens die skooldag?

STAP 3: MAAK 'N VOORSTEL VIR DIE BESTE OPLOSSING

Volgende week gaan julle al die nodige inligting insamel en die probleem op 'n plakkaat voorstel.

Orientasie-Sessie 2: INLIGTINGSUK

Die volgende is 'n kwotasie van 'n loodgieter oor die kostes rondom toiletherstel.

DU PLESSIS PLUMBING SERVICES	
Item	**Prys**
Toiletdrein uitwas en ontblok (neem 1 uur)	1 uur se arbeid per badkamer
Toilette wat nie spoel nie herstel (neem 1 uur)	1 uur se arbeid + R80 per toilet vir materiale
Nuwe toilette installeer (neem 2 ure)	2 ure se arbeid + R300 per toilet vir materiale. Jy moet ook jou eie toilet koop.
Per uur fooi vir arbeid (werkers)	R500 vir die eerste uur R450 per uur daarna

Die volgende is Pryse van toilette (vanaf Builders Warehouse, CTM en Italtile):

Toilet	**Prys per toilet**	**Waterverbruik**	**Materiaal**	**Waarborg**	**Ekstra**
Betta Iqwa	R1795	3 liter per spoel	Keramiek (baie hard)	10 jaar	Is ekstra higiënies
Betta Low Level	R495	6 liter per spoel	Keramiek (baie hard)	10 jaar	
Builders Close Couple	R 745	6 liter per spoel	Porselein (breek maklik)	10 jaar	
Coral Dual Top Flush	R849,90	3 liter per spoel	Plastiek	10 jaar	

Oor toilette skoonhou

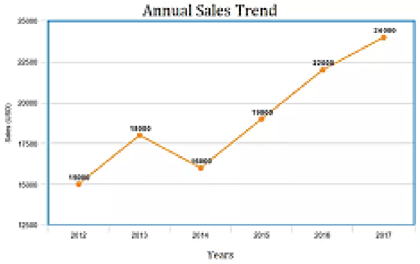
Groot maatskappye glo dat 'n toilet elke twee ure skoongemaak moet word en dat iemand aangestel moet word vir die rol. Die gemiddelde kostes verbonde is R2600 per maand.

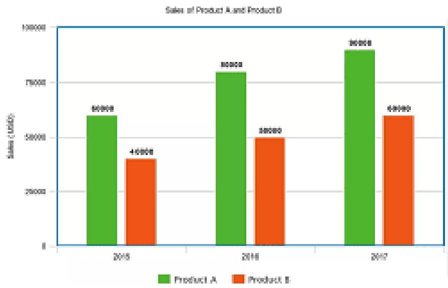
Inligting vanaf: https://www.cleanermatch.com/commercial/cost-of-commercial-restroom-sanitation.html

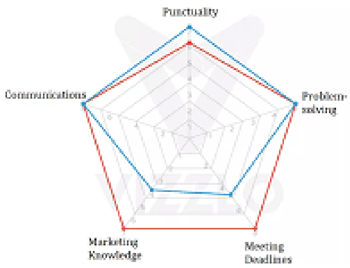
Templaat vir kostestaat

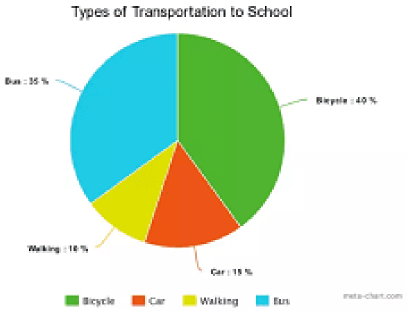
Item	**Prys per eenheid**	**Aantal items**	**Totale Prys**
Kitkat	R8,90	30	R267
Totale Prys			**Som**

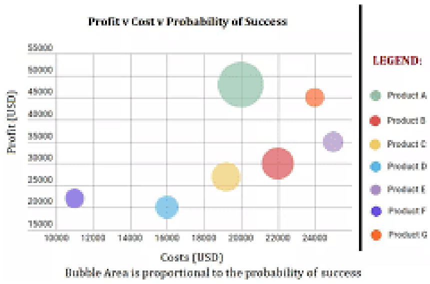
Soms is dit beter om inligting in grafieke voor te stel want dit lees maklik. Onthou julle watter soort grafieke tot jul beskiking is?











Addendum 10: Investigation 2 and rubric

English translations available upon request.

Wiskunde Geletterdheid Ondersoek   Graad 11   Totaal: 45

Die Toilet Probleem: Groepsopdrag

Julle as Graad 11 groep het die toilette in die skool aangewys as een van die grootste probleme in die skool.

In hierdie opdrag gaan julle ‘n plakkaat maak waarop julle die probleem beskryf en ook ‘n voorstel gaan maak vir ‘n moontlike oplossing. **HIERDIE IS ‘N GROEPSOPDRAG.**

Om die probleem te beskryf gaan julle die situasie eers moet analiseer. Tydens die werkswinkel het julle beplan hoe julle dit gaan doen. Die onderstaande instruksies is afkomstig van jul eie idees.

Volg die stappe om die plakkaat te voltooi. Maak ook gebruik van die rubriek op die laaste bladsy – dit is waarvolgens julle plakkaat gemerk gaan word.

Stap 1: Analiseer die probleem

1.1 Skryf ‘n inleidende paragraaf wat die probleem beskryf. Beskryf die toestand van die badkamers. Maak seker dat alle nodige inligting in die paragraaf staan – dink aan al die ekstra inligting wat julle genoem het tydens die werkswinkel.

1.2 Kies jul eie metode en gaan samel data in oor hoeveel van die Graad 11 leerders ‘n probleem het met die toilette. Julle kan enige manier gebruik, maar maak seker elke Graad 11 se opninie word in ag geneem.

- Teken die data aan in ‘n tabel met ‘n bypassende staafgrafiek.

1.3 Gaan vind uit hoeveel toilette daar in die skool is wat beskikbaar is vir die leerders. Julle moet die seuns- en dogterstoilette in ag neem.

- Teken aan hoeveel toilette daar is
- Teken aan hoeveel van die toilette onbruikbaar is en beskryf wat hulle onbruikbaar maak (bv heeltemaal stukkend, kan nie spoel nie, geblok, ens.)
- Teken die data in ‘n tabel aan met ‘n bypassende sirkelgrafiek

1.4 As daar enige ander nodige inligting oor die probleem is, maak seker dit verskyn ook op die plakkaat.

Stap 2: Stel 'n moontlike oplossing voor

2.1 Gebruik die inligting in die inligtingstuk en stel voor (indien nodig) watter toilet die beste sal wees om die stukkende toilette mee te vervang.

- Beskryf wat julle daardie spesefieke toilet laat kies het
- Gee redes vir jou antwoorde
- Teken die inligting in 'n paragraaf aan

2.2 Gebruik die inligting in die inligtingstuk en bereken al die kostes om die toilette reg te maak (wenk: die loodgieter moet dit reg maak).

- Maak gebruik van die kostestaat templaat en teken die inligting in die kostestaat aan
- Wys hoe julle die berekeninge gedoen het

2.3 Maak 'n voorstel oor hoe om die badkamers in die toekoms skoon en bruikbaar te hou

- Wie is verantwoordelik daarvoor?
- Hoe gereeld moet die badkamers gediens word?
- Wat moet gedoen word om seker te maak dit bly skoon en bruikbaar?
- Wat is die kostes hierby betrokke?

Stap 3: Die gevolgtrekking

Skryf 'n gevolgtrekking waarin julle al julle idees oor die probleem en oplossing kort en kragtig opsom.

Rubriek – Een punt word gegee vir elke vereiste

	1	2	3	4	5	6	7	8
Plakkaat Opskrif • Duidelik leesbaar • Kreatief • Beskryf die probleem goed								
Uitleg van die plakkaat • Netjies • Logies (vloei goed) • Aantreklik (mooi om na te kyk) • Maklik om te lees								
Inleiding • Goeie beskrywing van die probleem • Dit gee duidelike en relevante inligting oor die toestand van die badkamers • Goeie taalgebruik (nie emosioneel)								
Metodes om inligting te verkry oor hoeveel Gr 11 leerders 'n probleem het met toilette • Logies (maak sin) • Haalbaar (maklik om uit te voer) • Goed uitgevoer								
Tabel en grafiek 1 • Die inligting pas by mekaar • Tabel het gepaste opskrifte (2 punte) • Inligting op tabel is voldoende • Grafiek het gepaste opskrifte (2 punte) • Maak gebruik van kleur om te onderskei tussen die inligting • Grafiek is die regte formaat								
Tabel en grafiek 2 • Die inligting pas by mekaar • Tabel het gepaste opskrifte (2 punte) • Inligting op tabel is voldoende • Grafiek het gepaste opskrifte (2 punte) • Maak gebruik van kleur om te onderskei tussen die inligting • Grafiek is die regte formaat								
Die toilet wat gekies is • Die keuse is duidelik gemaak • Die inligting en redes wat gebruik word om die keuse te staaf is voldoende (2 punte)								
Kostestaat • Templaat korrek gebruik • Die korrekte inligting is gebruik • Bewerkings is korrek (2 punte) • Tabel is maklik om te lees								
Voorstel vir badkameronderhoud • Voorstel maak sin • Beskryf wie verantwoordelik is • Beskryf wat moet gedoen word en hoe gereeld • Beskryf die betrokke kostes								
Gevolgtrekking • Spreek elke aspek in die inleiding aan • Duidelike opsomming van alle idees en voorstelle • Opsomming van alle betrokke kostes • Goeie taalgebruik (nie emosioneel)								
TOTAAL: 45								

Addendum 11: Investigation 2 test and memorandum
English translations available upon request.

Wiskunde Geletterdheid Individuele Klastoets Graad 11 Totaal: 20

Die volgende klastoets is **INDIVIDUELE WERK.** Die punt van hierdie toets word saam met jou plakkaat punt getel om die finale punt vir Ondersoek 2 te bereken.

Vraag 1 (14 punte)

Die Du Preez-gesin se oudste seun word in Desember 18. Hy wil dadelik sy lisensie kry en begin kar bestuur. Sy ma is op haar senuwees daaroor. Sy gaan doen navorsing oor die hoeveelheid 18-jarige bestuurders wat al in motorongelukke was. Sy vind die onderstaande tabel. Dit is die resultate van 1680 18-jarige bestuurders wat al in motorongelukke was.

	Groot ongeluk (besering)	Klein ongeluk (geen besering).
Boys	12%	48%
Girls	4%	36%

4.1 Hoeveel dogters was al in 'n ongeluk gewees? (2)

4.2 Hoeveel seuns was al beseer in 'n ongeluk? (2)

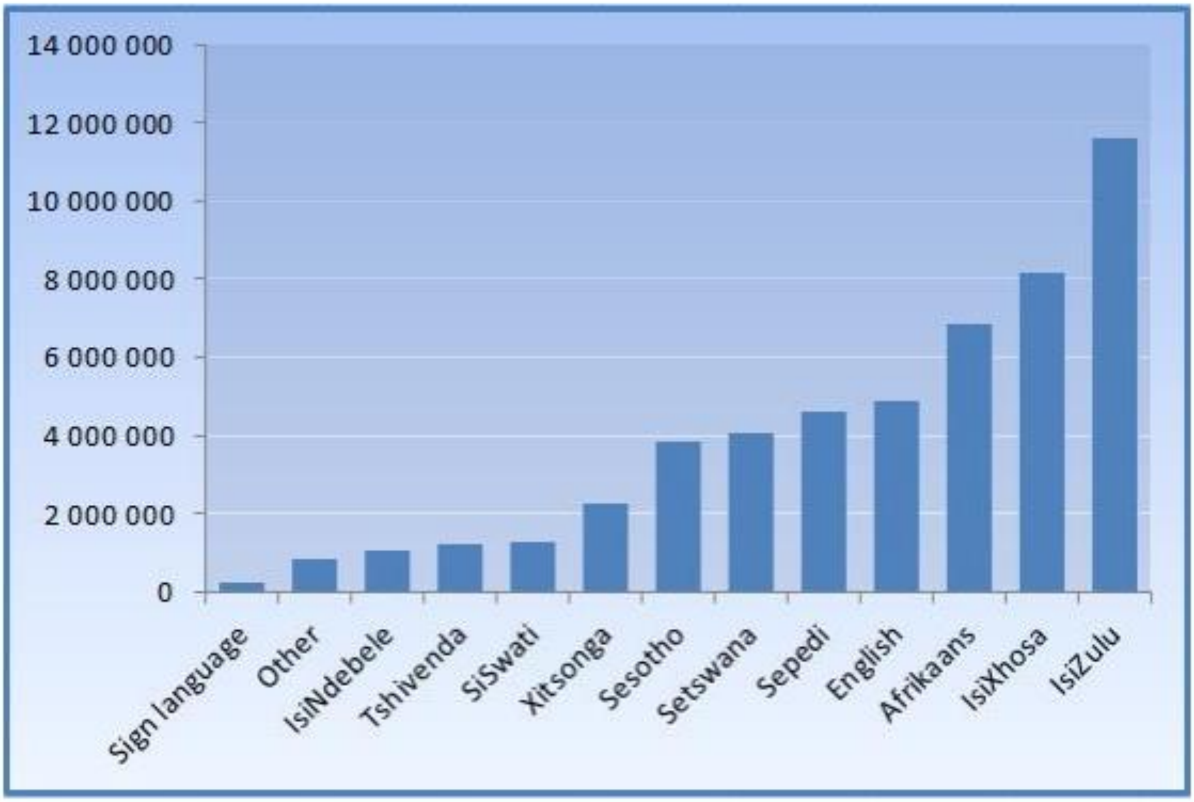
4.3 Bereken die persentasieverhouding van **seuns tot meisies** wat al in 'n ongeluk was in die eenvoudigste vorm. (2)

4.3 Stel die data in 'n sirkeldiagram voor. (6)

4.4 Is die data diskreet of kontinu? Motiveer jou antwoord. (2)

Vraag 2 (6 punte)

Die onderstaande tabel dui aan die aantal mense wat een van die 11 tale van Suid-Afrika , of gebaretaal, as huistaal praat.



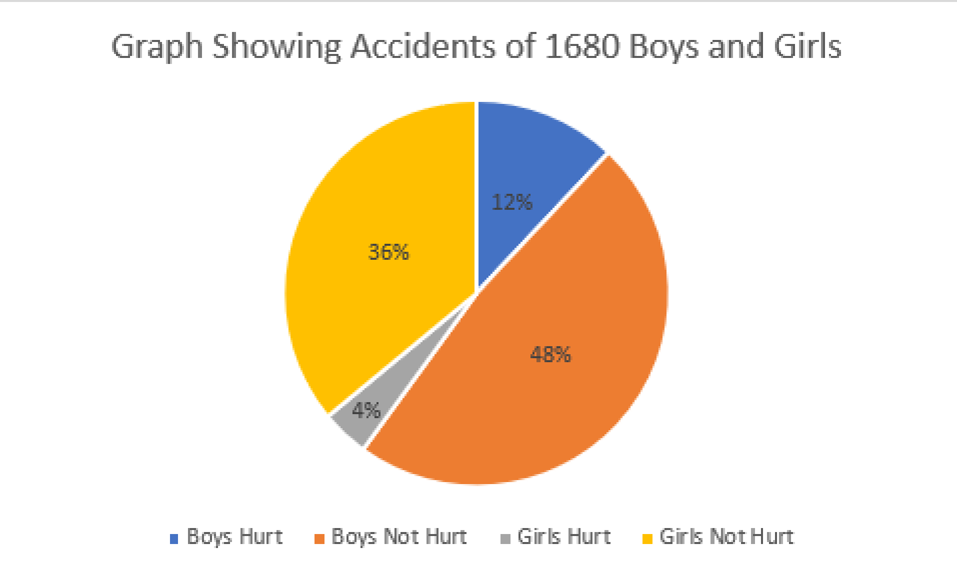
Grafiek verkry vanaf: http://www.mathsatsharp.co.za/wp-content/uploads/2015/01/Worksheet_5_Data_Handling_2_Grd_12_Math_Literacy.pdf

2.1 Gee 'n geskikte opskrif vir die grafiek (1)

2.2 Watter taal word die minste gepraat onder Suid-Afrikaners? Hoekom is dit so? (2)

2.3 Hoeveel mense praat nie Zulu of Xhose nie (min of meer)? (2)

2.4 Wat beteken die staaf wat "Ander" (Other) as taal aandui? (1)



Memorandum for the pie chart.

Addendum 13: Originality report

20606273_MastersThesis

ORIGINALITY REPORT

9%	8%	5%	4%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS