2013-09

Multi-Dimensional Constant Energy Modulation

Heng, Yinghui

Monterey, California. Naval Postgraduate School

NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA

THESIS

MULTI-DIMENSIONAL CONSTANT ENERGY MODULATION

by

Yinghui Heng

September 2013

Thesis Advisor:	Ric A. Romero
Co-Advisor:	Tri T. Ha

THIS PAGE INTENTIONALLY LEFT BLANK

REPORT DOCUMENTATION PAGE			*Form Approved OMB No. 0704-0188*

1. AGENCY USE ONLY *(Leave blank)*	**2. REPORT DATE** September 2013	**3. REPORT TYPE AND DATES COVERED** Master's Thesis

4. TITLE AND SUBTITLE MULTI-DIMENSIONAL CONSTANT ENERGY MODULATION	**5. FUNDING NUMBERS**
6. AUTHOR(S) Yinghui Heng	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000	**8. PERFORMING ORGANIZATION REPORT NUMBER**
9. SPONSORING /MONITORING AGENCY NAME(S) AND ADDRESS(ES) N/A	**10. SPONSORING/MONITORING AGENCY REPORT NUMBER**

11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government. IRB Protocol number ____N/A____.

12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited	**12b. DISTRIBUTION CODE** A

13. ABSTRACT (maximum 200 words)

Multi-dimensional modulation is of interest to communications systems researchers due to its potential for increased spectral efficiency and improved performance in nonlinear channels. We present the multi-dimensional constant energy modulation (CEM) technique, which utilizes *N*-dimensional signal space to improve spectral efficiency while constraining all symbols in a signal constellation to a constant energy. The implementation of the CEM technique is described. Results of the Monte Carlo simulations conducted with linear and nonlinear amplifier effects for *N*=4, *N*=5, *N*=6, *N*=7 and *N*=8 modulation dimensions are presented. Key findings include the need to consider bandwidth, spectral efficiency and power trade-offs when selecting the *N*-dimensional constellation for a required communications system and the need to optimize the constellation to maintain the largest possible minimum Euclidean distance between neighboring vectors while having sufficient vectors to transmit the required symbols. It is observed that the bit error ratio improves with increasing *N* for the same number of bits per symbol. The CEM system also shows better performance when going through a nonlinear amplifier, especially when compared to an equivalent existing modulation technique.

14. SUBJECT TERMS Multi-Dimensional, Constant Energy, Modulation, Probability of Bit Error, Bit Error Ratio (BER), Signal Constellation, Signal Vector	**15. NUMBER OF PAGES** 143
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT Unclassified	**18. SECURITY CLASSIFICATION OF THIS PAGE** Unclassified	**19. SECURITY CLASSIFICATION OF ABSTRACT** Unclassified	**20. LIMITATION OF ABSTRACT** UU

NSN 7540-01-280-5500 Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18

THIS PAGE INTENTIONALLY LEFT BLANK

MULTI-DIMENSIONAL CONSTANT ENERGY MODULATION

Yinghui Heng
Defence Science and Technology Agency, Singapore
B.S., Nanyang Technological University, 2003

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
September 2013

Author: Yinghui Heng

Approved by: Ric A. Romero
Thesis Advisor

Tri T. Ha
Co-Advisor

Clark R. Robertson
Chair, Department of Electrical and Computer Engineering

THIS PAGE INTENTIONALLY LEFT BLANK

ABSTRACT

Multi-dimensional modulation is of interest to communications systems researchers due to its potential for increased spectral efficiency and improved performance in nonlinear channels. We present a multi-dimensional constant energy modulation (CEM) technique, which utilizes N-dimensional signal space to improve spectral efficiency while constraining all symbols in a signal constellation to a constant energy. The implementation of the CEM technique is described. Results of the Monte Carlo simulations conducted with linear and nonlinear amplifier effects for N=4, N=5, N=6, N=7 and N=8 modulation dimensions are presented. Key findings include the need to consider bandwidth, spectral efficiency and power trade-offs when selecting the N-dimensional constellation for a required communications system and the need to optimize the constellation to maintain the largest possible minimum Euclidean distance between neighboring vectors while having sufficient vectors to transmit the required symbols. It is observed that the bit error ratio improves with increasing N for the same number of bits per symbol. The CEM system also shows better performance when going through a nonlinear amplifier, especially when compared to an equivalent existing modulation technique.

THIS PAGE INTENTIONALLY LEFT BLANK

TABLE OF CONTENTS

LIST OF FIGURES

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF TABLES

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF ACRONYMS AND ABBREVIATIONS

AWGN	Additive White Gaussian Noise
BER	Bit Error Ratio
BO	Back-off
CEM	Constant Energy Modulation
FFT	Fast Fourier Transform
IFFT	Inverse Fast Fourier Transform
MDM	Multi-Dimensional Modulation
*M*HPSK	*M*-ary Hyper Phase-Shift Keying
*M*PSK	*M*-ary Phase-Shift Keying
*M*QAM	*M*-ary Quadrature Amplitude Modulation
OFDM	Orthogonal Frequency Division Multiplexing
PAM	Pulse Amplitude Modulation
QAM	Quadrature Amplitude Modulation
SNR	Signal-to-Noise Ratio
TWT	Travelling-Wave Tube

THIS PAGE INTENTIONALLY LEFT BLANK

EXECUTIVE SUMMARY

In modern day digital communication systems, a data set is transmitted in the form of digitized bit streams using coding and modulation. In the transmission process, clipping by a nonlinear power amplifier can significantly degrade the performance of the modulated signal, increasing errors during detection of the received signal. Multi-dimensional modulation is, hence, of interest to communications systems researchers as it can potentially offer increased spectral efficiency when signal constellations are optimized and improved performance in nonlinear channels when equal energy symbols are used. Previous work in [1] proposed *M*-ary hyper phase-shift keying (*M*HPSK) – an equal energy per symbol modulation technique utilizing four dimensions. *M*HPSK was shown to be spectrally efficient, and it outperformed *M*-ary phase-shift keying (*M*PSK) and *M*-ary quadrature amplitude modulation (*M*QAM) with comparable spectral efficiency at low received signal energy levels. Building upon these concepts, this research proposes a multi-dimensional modulation technique – constant energy modulation (CEM) – which utilizes *N*-dimensional signal space to improve spectral efficiency while constraining all symbols in a signal constellation to a constant energy.

The process to design a CEM constellation begins with defining a suitable constraint equation for equal energy symbols. For an *N*-dimensional signal constellation, the normalized constraint equation can be written as

$$\sum_{j=1}^{n} r_j C_j^{\,2} = 1, \tag{1.1}$$

where C_j is the j^{th} non-zero coordinate value, and r_j is the number of times the j^{th} non-zero coordinate value appears in the vector for $j = 1, 2, 3, \ldots, n$ and $n \leq N$. The maximum number of equal energy symbols which can be generated G is given by

$$G = \frac{2^{(n+m)} N!}{r_1! r_2! r_3! \ldots r_n!}. \tag{1.2}$$

Equation (1.2) provides the theoretical maximum number of equal energy symbols which can be generated given the dimension of the modulation *N*, the number of non-zero signal vector coordinates *n*, and the number of times a non-zero vector coordinate is repeated *m*.

The optimum equal energy constellation has to provide the number of symbols within the maximum G while achieving the largest possible minimum Euclidean distance between the vectors in the signal constellation. For a given N-dimensional modulation, assuming that the signal vector coordinate values C_1, C_2, C_3, ..., C_n are such that $|C_1| < |C_2| < |C_3| < ... < |C_n|$, C_1, C_2, C_3, ..., $C_n \in \mathbb{R}$ and $\neq 0$, and $C_1 \neq C_2 \neq C_3 \neq ... \neq C_n$, the set of squared-Euclidean distances which contains the minimum is found to comprise values obtained from the following equations:

$$X = 2|C_1|^2, \tag{1.3}$$

$$Y = (2|C_1|)^2, \tag{1.4}$$

$$\begin{aligned} Z = \{&2\left(|C_2| - |C_1|\right)^2, 2\left(|C_3| - |C_1|\right)^2, 2\left(|C_4| - |C_1|\right)^2, ..., 2\left(|C_{n-1}| - |C_1|\right)^2, \\ &2\left(|C_3| - |C_2|\right)^2, 2\left(|C_4| - |C_2|\right)^2, ..., 2\left(|C_n| - |C_2|\right)^2, \\ &2\left(|C_4| - |C_3|\right)^2, ..., 2\left(|C_n| - |C_3|\right)^2, \\ &..., 2\left(|C_n| - |C_{n-1}|\right)^2\}. \end{aligned} \tag{1.5}$$

Equations (1.3) to (1.5) are used when one of the signal vector coordinates has zero value. Equations (1.4) to (1.5) are used when all the signal vector coordinates have non-zero values. We can summarize two cases as follows:

a. Normalized signal vector coordinates $\hat{s}_i \in \mathbb{R}$ and one zero-value signal vector coordinate

$$d_{\min}^2 = \min\{X, Y, Z\}, \text{ excluding } Z = 2\left(|C_n| - |C_1|\right)^2, \tag{1.6}$$

b. Normalized signal vector coordinates $\hat{s}_i \in \mathbb{R}$ and $\neq 0$

$$d_{\min}^2 = \min\{Y, Z\}, \text{ excluding } Z = 2\left(|C_n| - |C_1|\right)^2. \tag{1.7}$$

By iterating the values of r_j and C_j in Equation (1.1) and calculating the $d_{\min}^2$ for each case using Equations (1.3) to (1.7), we can find the constellation with the largest possible $d_{\min}^2$. This results in the lowest possible bit error ratio (BER), giving us the optimum or near optimum equal energy constellation.

The modulation of CEM via a baseband approach is shown in Figure 1. At the transmitter, data bits are first mapped into symbols depending on the required number of bits per symbol. Each symbol is mapped onto a pre-assigned signal vector. The signal vector coordinates are then mapped onto the in-phase (I) and quadrature (Q) channels of k subcarriers, where k is the modulation dimension N divided by two, rounded up to the nearest whole number. The inverse fast Fourier transform (IFFT) is conducted on all the subcarriers. Complex noise is added when the signal passes through the transmission channel.

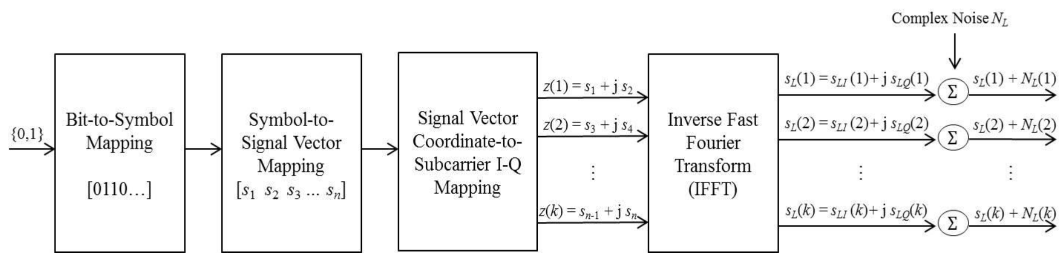

Figure 1. Block diagram of CEM modulator.

The demodulation of CEM is shown in Figure 2. Each received subcarrier signal comprises up to two signal vector coordinates from the transmitted signal vector coupled with noise. The fast Fourier transform (FFT) is conducted on the subcarrier. The resulting values of the signal vector coordinate coupled with noise on the I-channel and Q-channel for each subcarrier are then combined to give a complete signal vector. A minimum Euclidean distance detector is used to locate the signal vector in the pre-determined symbol-to-signal vector list with Euclidean distance closest to that of the received vector, where that signal vector is decided to be the transmitted symbol. Symbol-to-bit mapping is performed to produce the bit sequence at the receiver output.

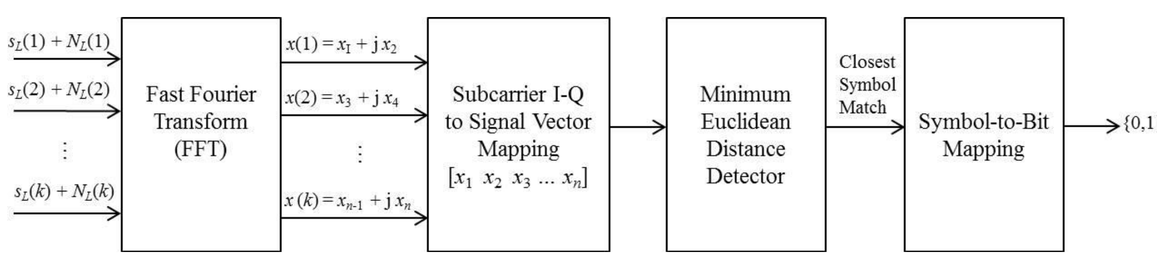

Figure 2. Block diagram of CEM demodulator.

In [1], the author provided three four-dimensional (4D) signal constellations comprising four, five and six bits. We use these constellations, in particular the 4D constellation with six-bit symbols (64-HPSK) to benchmark against the *N*-dimensional constellations developed with CEM in terms of signal power. A summary of the selected constellations with their bandwidth, $d_{\min}^2$, spectral efficiency and power is given in Table 1. We observe from the table that more bandwidth is required due to the increased number of subcarriers for higher modulation dimensions. For the same normalized bit rate R_b, a better spectral efficiency can be obtained by selecting the lowest modulation dimension for which the required bit rate can be achieved.

Table 1. Selected signal sets with bandwidth, spectral efficiency and power. Bandwidth *B* is normalized to half the null-to-null subcarrier bandwidth.

Modulation Dimension	Constraint Equation	Normalized Bit Rate (Number of Bits per Symbol) (R_b)	Normalized Bandwidth (B)	Minimum Squared-Euclidean Distance $\left(d_{\min}^2\right)$	Spectral Efficiency (η)	Power Relative to 64-HPSK [1] in dB
4	$4C_1^2 = 1$ [1]	4	3	1.000	1.333	3.440
4	$3C_1^2 + 0 = 1$ [2]	5	3	0.6667	1.667	1.679
4	$C_1^2 + 3C_2^2 = 1$ [3]	6	3	0.4529	2	0
5	$4C_1^2 + 0 = 1$	6	4	0.5000	1.5	0.4297
5	$C_1^2 + 4C_2^2 = 1$	7	4	0.4069	1.75	-0.4651
5	$C_1^2 + 3C_2^2 + 0 = 1$	8	4	0.2856	2	-2.002
6	$6C_1^2 = 1$	6	4	0.6667	1.5	1.6793
6	$5C_1^2 + 0 = 1$	7	4	0.4000	1.75	-0.5394
6	$C_1^2 + 5C_2^2 = 1$	8	4	0.3693	2	-0.8862
7	$7C_1^2 = 1$	7	5	0.5714	1.4	1.009
8	$8C_1^2 = 1$	8	5	0.5000	1.6	0.4297

[1] From 16-HPSK[1].

[2] From 32-HPSK[1].

[3] From 64-HPSK[1].

An algorithm is developed using MATLAB to conduct the bit-to-symbol and then symbol-to-signal vector mapping. The bit sequence for the required dimension (for example, vector of bits per symbol) is first converted into its Gray code sequence, in which each row differs from the row before it by one bit. A reference vector is selected from the desired signal set of the given dimension and assigned as the first symbol with Gray code bit sequence 0 (first Gray sequence). The remaining vectors are ranked in order of increasing Euclidean distance from the reference vector. The algorithm then assigns the next Gray code sequence to the signal vector, moving down the Gray code sequence and assigning to the vectors in order of increasing Euclidean distance.

We next obtain a theoretical approximation for probability of bit error P_b as a theoretical basis against which to compare the simulated P_b performance. The approximation is obtained using the union bound:

$$P_b \approx \frac{\alpha P_e}{\log_2 M}, \tag{1.8}$$

where M is the number of signal vectors used to generate symbols, α is the average number of nearest neighbors for all symbols in the signal constellation, $1 \le \alpha \le \log_2 M$, and P_e is the probability of symbol error. We then program the CEM modulator and demodulator in MATLAB to conduct Monte Carlo simulations for N=4, N=5, N=6, N=7 and N=8 modulation dimensions and up to normalized R_b = 8 for both linear and nonlinear channels.

For the linear channel, when comparing across different dimensions, we observe that the BER decreases as N increases for the same number of bits per symbol due to the increased minimum Euclidean distance $d_{\min}^2$ between neighboring vectors in the constellation. An example is illustrated in Figure 3 for the case of 64CEM (six bits per symbol). The BER is consistently lower for the higher value of N at each value of E_b/N_o.

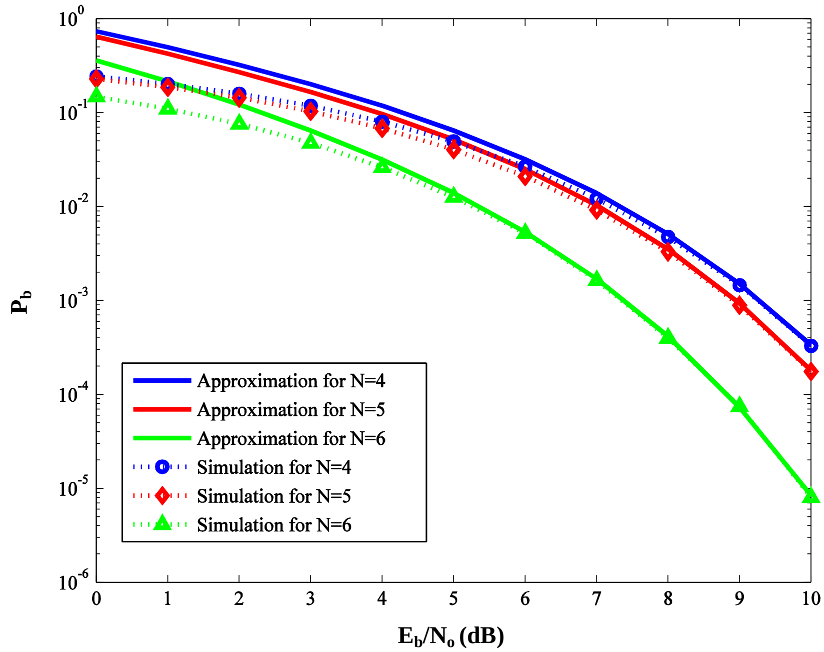

Figure 3. BER comparison with *N*=4, *N*=5 and *N*=6 for 64CEM.

We use the nonlinear model of a travelling-wave tube (TWT) amplifier proposed in [2] to simulate the performance in a channel with nonlinear amplifier effects. From the simulation results shown in Figure 4, we observe that (for *N*=4) when there is no input back-off (0 dB BO), the nonlinear amplifier effects dominate such that improvement in the BER occurs only at very large E_b/N_o. The BER improves significantly with 3 dB input back-off (3 dB BO) and is closer to the performance in a linear channel. For *N*=8 with no back-off, we notice that the BER does not significantly degrade. This is the advantage of using higher order CEM modulation.

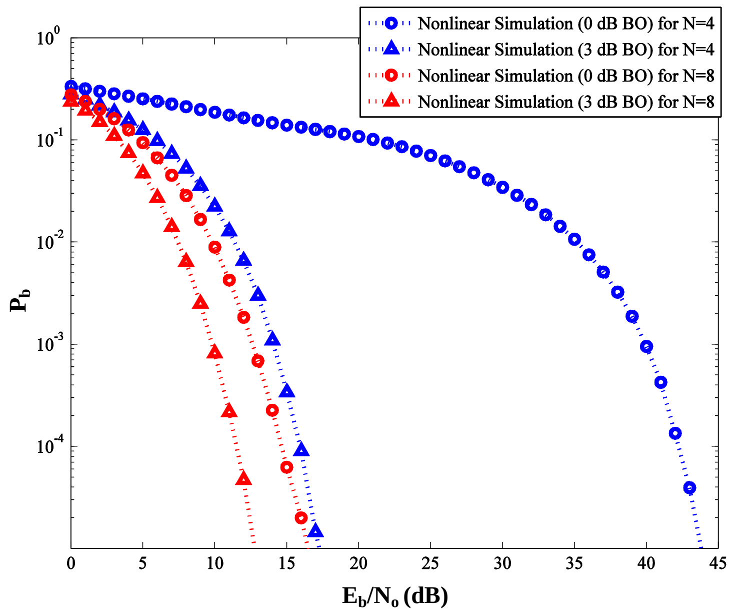

Figure 4. Comparison of BER with nonlinear amplifier effects with *N*=4 and *N*=8.

When a pilot symbol is used for phase correction, the BER improves. This is illustrated using 4D-64CEM in Figure 5.

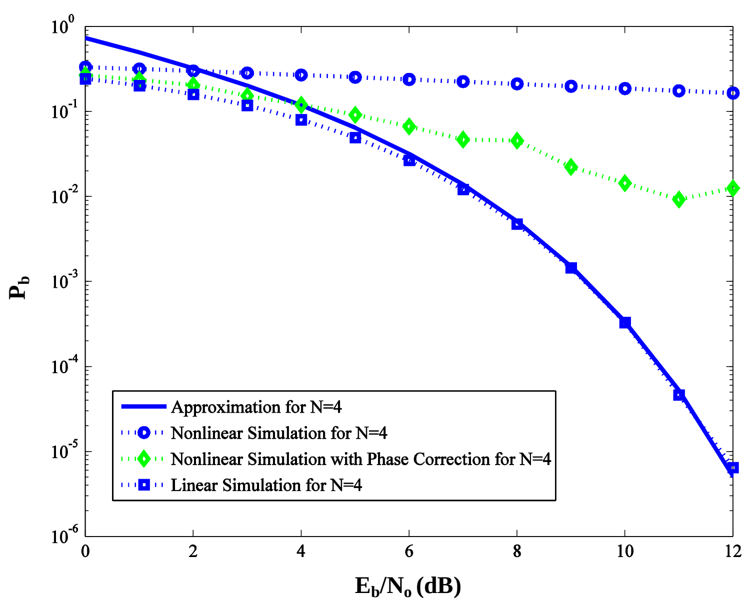

Figure 5. BER with nonlinear amplifier effects at 0 dB input back-off using pilot symbol for phase correction with *N*=4.

Finally, we compare the performance with nonlinear amplifier effects of the 4D-64CEM and 8D-64CEM with an OFDM system using two 8QAM subcarriers. The results as shown in Figure 6 indicate that both the CEM systems exhibit better BER than the OFDM system. When a pilot symbol is used to provide phase correction at 0 dB input back-off, the performance for the CEM systems is further improved, while the OFDM system's performance showed only very slight improvement as shown in Figure 7. The better performance of the CEM system in a nonlinear amplifier could be due to its equal energy symbols, unlike the OFDM-8QAM which goes past saturation for symbols with high energy.

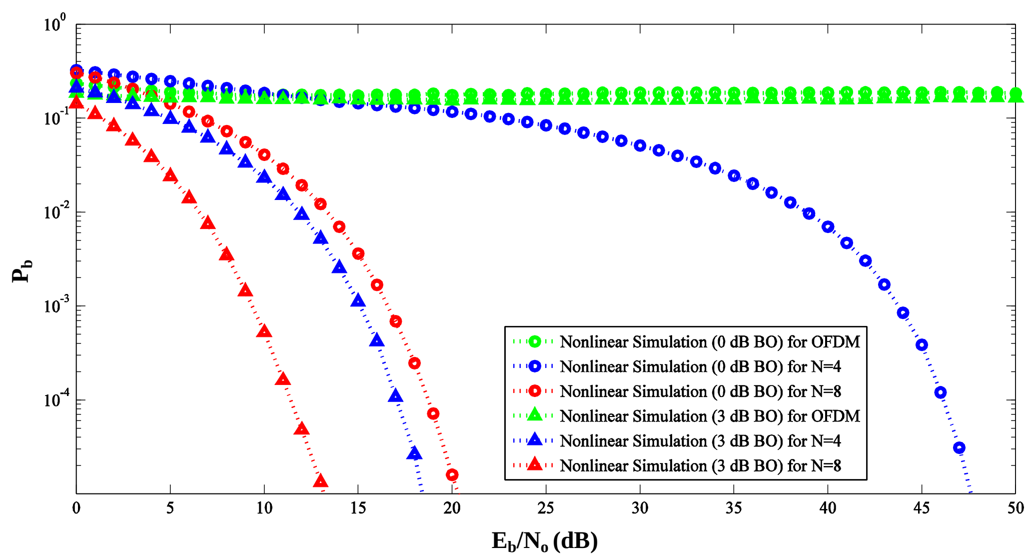

Figure 6. Comparison of BER with nonlinear amplifier effects at 0 dB input back-off with *N*=4, *N*=8 and OFDM-8QAM.

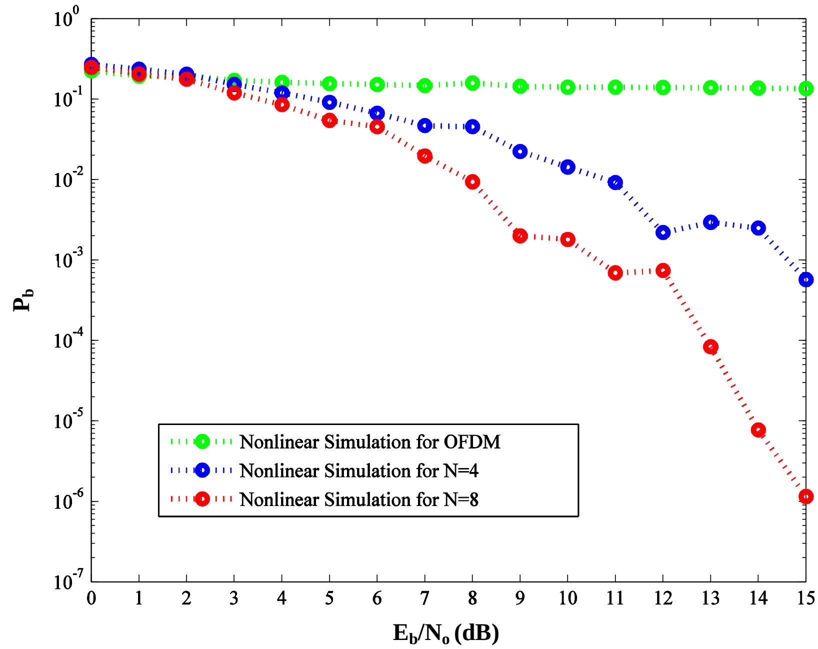

Figure 7. Comparison of BER with nonlinear amplifier effects at 0 dB input back-off using pilot symbol for phase correction with N=4 and OFDM-8QAM.

In summary, the multi-dimensional CEM system has been successfully developed and tested using simulations. The main advantage of the system is the improved BER, in particular in a channel with nonlinear amplifier effects. Recommendations for further work include improving the algorithm to design signal constellations for better BER and extending the research to modulation dimensions beyond N=8 to investigate the added advantages when signal space is further increased and to enhance its performance in nonlinear channels.

References

[1] J. Caldwell, "Hyper phase shift keying modulation," U.S. Patent 8 064 541, November 22, 2011.

[2] A. A. Saleh, "Frequency-independent and frequency-dependent nonlinear models of TWT amplifiers," *IEEE Trans. Commun.*, vol. 29, no. 11, pp. 1715–1720, November 1981.

THIS PAGE INTENTIONALLY LEFT BLANK

ACKNOWLEDGEMENTS

I would like to thank my thesis advisors, Prof Ric Romero and Prof Tri Ha, for their guidance over the course of this thesis and in class. It is not often that one gets the opportunity to develop an idea that is new in the field. Thank you for giving me this opportunity and mentoring me along the way.

To my husband, Wei Ming, thank you for your love and steadfast support all this while. I could not have achieved what I have without you.

And to my friends, Chia Sern, Esther, Lynn, Ahn and Sade, thank you for your help and support during this period. Your friendship has greatly enriched my time here and will leave me with beautiful memories of my stay in Monterey.

THIS PAGE INTENTIONALLY LEFT BLANK

I. INTRODUCTION

A. OVERVIEW

In modern day digital communication systems, a data set is transmitted in the form of digitized bit streams using coding and modulation. A group of bits maps onto a symbol. The symbol then undergoes modulation onto a suitable waveform for transmission. At the receiver end, demodulation is followed by symbol-to-bit mapping to obtain the original transmitted bit stream. During the transmission process, clipping by a nonlinear power amplifier can significantly degrade the performance of the modulated signal, increasing errors during detection of the received signal. An example is the degradation of a quadrature amplitude modulation (QAM) signal due to clipping in the reverse link of fourth generation (4G) cellular systems.

Higher-order modulation schemes, such as *M*-ary quadrature amplitude modulation (*M*QAM) and *M*-ary phase-shift keying (*M*PSK), are spectrally efficient but do not perform well when the received energy is too low [1]. These modulation schemes have two-dimensional signal constellations with symbols that comprise multiple bits. In recent years, there has been increasing interest in multi-dimensional signal constellations which offer the potential to outperform higher-order modulation schemes at low received signal energy levels. Theoretically, optimizing the space in additional dimensions enables an increase in the minimum distance between constellation points for a given transmit power constraint when compared to conventional orthogonal designs. This allows for higher SNR efficiency but at the potential cost of sacrificing bandwidth efficiency [2]. However, increasing the constellation size for a particular dimension improves spectral efficiency. The trade-off among power, bandwidth and spectral efficiency is essentially a compromising procedure between modulation dimension and constellation size.

Previous work based on this concept [1] proposed *M*-ary hyper phase-shift keying (*M*HPSK) – an equal energy per symbol modulation technique utilizing four dimensions. *M*HPSK was shown to be spectrally efficient, and it outperformed *M*PSK and *M*QAM with comparable spectral efficiency at low received signal energy levels.

This research extends *M*HPSK by proposing a multi-dimensional modulation technique—constant energy modulation (CEM)—which constrains all symbols in any multi-dimensional signal constellation to have identical energy. This may offer better power efficiency over a nonlinear channel while transmitting multiple bits per symbol with good spectral efficiency. The development of the multi-dimensional CEM technique is described in this report.

B. SCOPE AND ORGANISATION

An overview of the two-dimensional higher-order (*M*-ary) modulation schemes and orthogonal frequency-division multiplexing (OFDM) used to multiplex modulated signals is given in Chapter II. The intent and research conducted in multi-dimensional modulation techniques are highlighted, in particular, on an attempt to optimize the multi-dimensional signal space and multi-dimensional modulation with energy constraints.

In Chapter III, the approach to develop multi-dimensional constellations is described. The constraint equation for constant energy symbols in a signal constellation is defined and used to derive the closed-loop equation, which calculates the maximum number of symbols for a given *N*-dimensional modulation. Specific signal constellations from the four-dimensional (4D), five-dimensional (5D), six-dimensional (6D), seven-dimensional (7D) and eight-dimensional (8D) signal spaces are selected based on bandwidth and power trade-offs and spectral efficiency. An algorithm for the symbol-to-signal vector mapping is presented and used to generate selected signal constellations. The theoretical approximation for the signal bit error ratio (BER) is given.

In Chapter IV, the generated symbols are passed through simulated linear and nonlinear channels. The BER versus signal-to-noise ratio (SNR) is plotted for the signal constellation operating with different bit rates. The performance of CEM in a linear channel is compared with its performance at 0 dB and 3 dB back-off in a nonlinear channel. Lastly, the performance of CEM is compared with that of the equivalent OFDM system with two *M*QAM subcarriers.

The conclusions and recommendations for future work are presented in Chapter V.

II. MODULATION AND MULTIPLEXING TECHNIQUES

A. M-ARY MODULATION

1. Shannon Channel Capacity

The Shannon channel capacity theorem notes that given an ideal bandlimited channel of bandwidth *B* (Hz), additive white Gaussian noise (AWGN) having a constant power spectral density $N_o/2$ (W/Hz), and a received power *P* (W), the channel capacity *C* (bits/s) is given by $C = B\log_2\left(1 + P/(BN_o)\right)$ [3].

The spectral efficiency of the channel *r* is given as *R/B* where *R* (bits/s) is the data rate. The data rate *R* must be smaller than *C* to achieve reliable transmission. Hence,

$$r = \frac{R}{B} < \frac{C}{B} = \log_2\left(1 + \frac{RE_b}{BN_o}\right). \tag{2.1}$$

The required signal-to-noise ratio per bit E_b/N_o for reliable transmission is constrained by

$$\frac{E_b}{N_o} > \frac{2^r - 1}{r}. \tag{2.2}$$

In dB, (2.2) is given by

$$\frac{E_b}{N_o} > -1.59\text{dB}. \tag{2.3}$$

The Shannon theorem provides a trade-off between E_b/N_o, which determines the power efficiency of a digital modulation, and *r*, which determines the bandwidth efficiency.

2. *M*-ary Modulation Techniques

Digital modulation techniques can be broadly classified into two groups: binary and higher-order *M*-ary modulation. Binary modulation is simpler to implement, but most binary modulation techniques are not spectrally efficient. *M*-ary modulation transmits in symbols, or groups of $\log_2 M$ bits, and can achieve higher spectral efficiency closer to the limit given by the Shannon channel capacity. The ability to offer high spectral efficiency with low power efficiency, or low-to-moderate spectral efficiency with high power efficiency, makes *M*-ary modulation suitable for many real world applications [3]. Two common *M*-ary modulation techniques are *M*PSK and *M*QAM.

a. M-ary Phase-Shift Keying

*M*PSK is a modulation technique which constrains each symbol to have the same energy. Whereas binary phase-shift keying (BPSK) transmits one bit per symbol, *M*PSK uses two basis functions to represent M symbols with $\log_2 M$ bits per symbol. The *M*PSK signal is defined as having a complex envelope formed from setting one of the basis functions as the in-phase (real) component and the other as the quadrature (imaginary) component. An example of the *M*PSK signal constellation with $M = 8$ is shown in Figure 1.

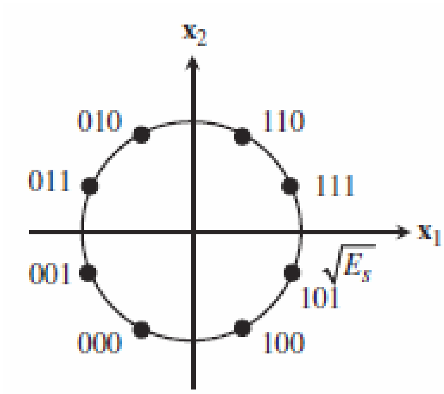

Figure 1. The 8PSK signal constellation. The in-phase and quadrature channels are represented by orthonormal basis functions $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively. From [3].

The null-to-null bandwidth of *M*PSK can be determined from its power spectral density. Using a rectangular pulse, we find the null-to-null bandwidth of the signal to be $2/T_s = 1/T_b$, where T_b is the bit duration and $T_s = T_b \log_2 M$.

Demodulation is usually performed coherently using a bank of L correlators (L-path demodulation) with a minimum Euclidean distance detector or maximum-likelihood detector. Using Gray code bit-to-symbol mapping, we have the BER for coherent *M*PSK:

$$P_b \approx \frac{2}{\log_2 M} Q\left(\sqrt{\frac{2E_b}{N_o}\log_2 M}\sin\frac{\pi}{M}\right), M > 2. \qquad (2.4)$$

b. M-ary Quadrature Amplitude Modulation

MQAM uses a set of $M = 2b$ non-constant amplitude and finite energy signals to represent M distinct symbols with b bits per symbol. Unlike MPSK, which constrains the energy of the symbols, packing them onto a circle of fixed radius in the signal space, MQAM symbols are placed throughout the two-dimensional signal space. Adjacent signal symbols can be designed to have a large Euclidean distance, reducing their vulnerability to errors caused by noise and improving power efficiency. MQAM signal symbols have different amplitudes and phases. The signal space of MQAM can be described using the two-dimensional vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ as shown in Figure 2.

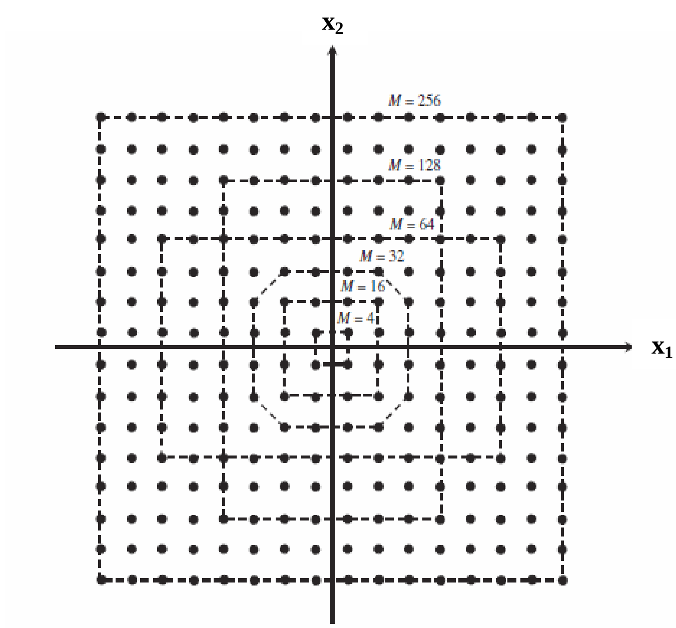

Figure 2. Various signal spaces for rectangular MQAM. After [3].

The power spectral density of the MQAM signaling has the same shape as that of MPSK provided that the same pulse shape is used. The null-to-null bandwidth of the signal is $2/T_s = 1/T_b$, where T_b is the bit duration and $T_s = T_b \log_2 M$. Hence, both modulation techniques offer the same spectral efficiency. The difference is that symbol mapping for *M*QAM generates non-constant symbol amplitudes.

Coherent demodulation is performed similar to *M*PSK, using a bank of *L* correlators (*L*-path demodulation) with a minimum Euclidean distance detector. Using Gray code bit-to-symbol mapping, we find the BER for coherent rectangular *M*QAM to be

$$P_b \approx \frac{4\left(1-\frac{1}{\sqrt{M}}\right)}{\log_2 M} Q\left(\sqrt{\frac{3\log_2 M}{M-1}\left(\frac{E_b}{N_o}\right)}\right). \tag{2.5}$$

B. ORTHOGONAL FREQUENCY-DIVISION MULTIPLEXING

Orthogonal frequency-division multiplexing is a digital multi-carrier modulation method in which a data sequence is encoded onto multiple carrier frequencies [4]. A large number of closely-spaced orthogonal subcarrier signals are used to carry data on several parallel data streams. Each subcarrier is modulated with a conventional modulation scheme, such as *M*PSK and *M*QAM. The OFDM signal is the sum of the modulated subcarriers. Advantages of OFDM include high spectral efficiency and improved robustness in severe channel conditions such as narrowband interference and frequency-selective fading without the need for complex equalization filters.

The implementation of OFDM via the baseband approach is shown in Figure 3 [3]. The incoming data stream of rate $1/T_b$ bits/s is demultiplexed into N parallel streams. Each parallel stream has a rate equal to $1/NT_b$ bit/s and independently modulates one of N orthogonal subcarriers. The modulation technique for each subcarrier is commonly chosen to be the same in practice. The OFDM subcarriers are chosen to be mutually orthogonal with the minimum frequency spacing of $1/T_s$, where T_s is the subcarrier symbol rate. For the k^{th} bit stream, a group of bits are mapped into a subcarrier symbol $Z(k) = I_k + jQ_k$, $0 \le t < T_s$. The sequence of N symbols $\{Z(k)\}_{k=0}^{N-1}$ is fed into the inverse fast Fourier transform (IFFT) to obtain the sequence $\{s_L(n)\}_{n=0}^{N-1}$ which represents the samples of the OFDM complex envelope. The output of the digital-to-analog (D/A) converter is a complex in-phase and quadrature-phase (I-Q) waveform $\hat{s}_L(t) = \sum_{n=0}^{N-1} s_L(n) p\left(t-(nT_s)/N\right)$, where p(t) is the interpolation pulse shape.

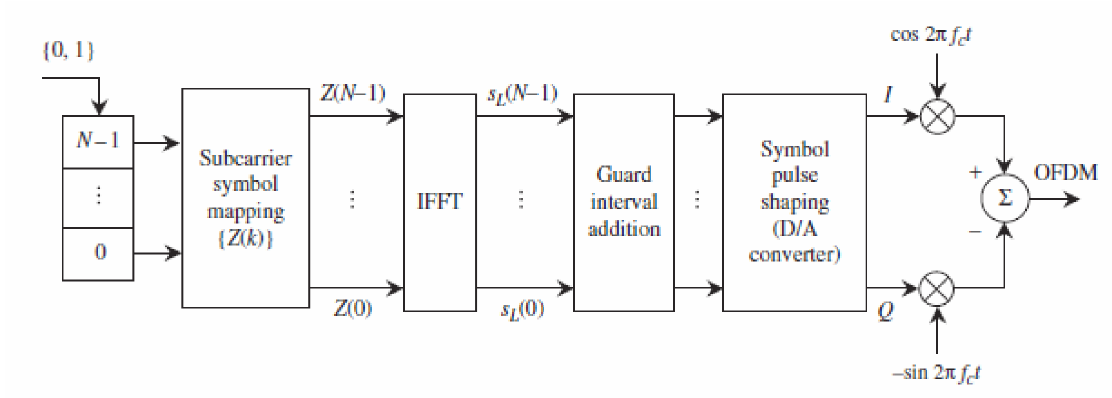

Figure 3. Block diagram of an OFDM modulator. From [3].

Demodulation of the received OFDM signal is shown in Figure 4. The signal is first demodulated into the I-Q baseband waveform containing the subcarrier symbols. The waveform is passed through a filter p(t) which represents the interpolation pulse used by the D/A converter in the transmitter and is then converted into time-domain samples $\{s_L(n)\}_{n=0}^{N-1}$ by the A/D converter at the sampling rate N/T_s. The guard interval is removed, and the fast Fourier transform (FFT) processes the time-domain OFDM samples into the frequency-domain samples $\{Z(k)\}_{k=0}^{N-1}$, which comprise the I-Q values of the subcarrier symbols. Finally, the subcarrier symbols are de-mapped into parallel bit streams.

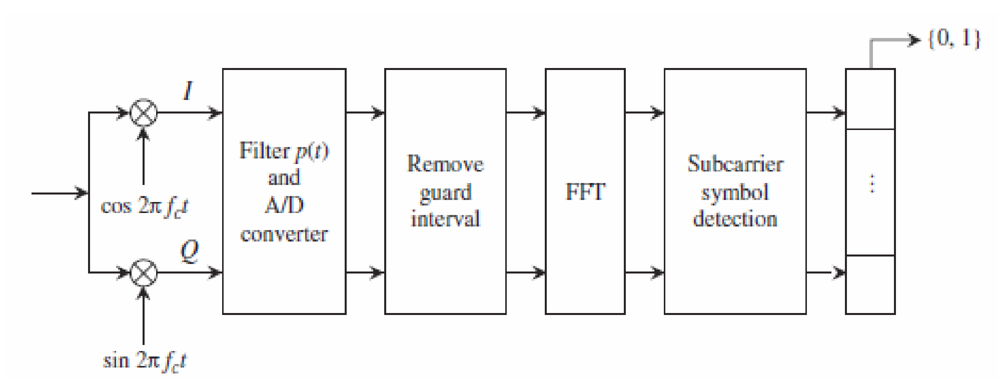

Figure 4. Block diagram of an OFDM demodulator. From [3].

C. MULTI-DIMENSIONAL MODULATION

1. Advantages over Conventional Modulation

Multi-dimensional modulation offers the potential to further increase the number of symbols transmitted, enabling good spectral efficiency while maintaining the power efficiency. It is theorized in [5] that a multi-dimensional signal constellation can achieve performance over the AWGN channel as close as possible to Shannon's bound. The study proposed a new class of modulation schemes called the eight-dimensional constant envelope modulation scheme (8D-CEMS), which utilizes a pulse shape based on a set of pre-determined criteria and an ideal hard limiter to create a transmission signal with a constant envelope. A coding gain of 3.2 dB for a spectral efficiency of 3 bits/s/Hz was obtained for the system with respect to 16QAM systems at BER of 10^{-4}. The study also highlighted the suitability of such signals for transmission over nonlinearly amplified channels due to the constant envelope.

A separate study in [6] presented a generalization of multidimensional modulation to all dimensions that were an integer power of 2. In the study, minimum bandwidth transfer functions were constructed using binary orthogonal Walsh-Hadamard sequences. The sequences were used to enable interference-free transmission at the decision instants as well to as spread incoming parallel data streams and despread them at the receiver. Each of the parallel data streams was transmitted over the same bandwidth as in the case of a two-dimensional modulation system. Results from analysis of the BER and spectral efficiency indicate that the proposed multidimensional modulation systems may not offer improved BER and spectral efficiency over conventional two-dimensional systems for Gaussian channels but may offer advantages when used in nonlinear channels.

2. Optimization of Multi-Dimensional Signal Space

Later works in [2] and [7] proposed a multi-dimensional modulation (MDM) scheme based on the conditions that the distance from the origin to each symbol was equal to $\sqrt{E_s}$ and any two symbols were equidistant. The inter-symbol distance was optimized based on the geometrical concept that two points form a line (1D), three points form a plane (2D) and four points form a space (3D); hence, M symbols need to be positioned in $M-1$ dimensions to obtain the maximum inter-symbol distance. The example of four signals in three dimensions (4-MDM) was used as the test case from which to extrapolate for other values of M and assess the effectiveness of the MDM scheme. Detailed error probability analysis was conducted on the performance of the MDM design over Gaussian and Rayleigh fading channels. The study concluded that the MDM scheme displayed advantages over conventional orthogonal modulations such as frequency-shift keying (FSK) due to the increased minimum Euclidean distance for the constellation points.

3. Multi-Dimensional Modulation with Energy Constraints

Recent research in multi-dimensional modulation focused on maintaining or improving power efficiency while maintaining spectral efficiency. In [8], it was highlighted that the larger the number of dimensions, the worse the spectral efficiency became, although power efficiency was improved; increasing the constellation size while maintaining the number of dimensions led to higher spectral efficiency. The paper presented an optimization procedure which generated optimized constellations by iterating within a sequence of quadratic programming problems under linear inequality constraints. Simulations conducted using this optimization tool showed that the generated constellations outperformed corresponding multi-dimensional pulse amplitude modulation (PAM) constellations by more than one decibel.

In [1], the author built upon previous work on four-dimensional modulation [9] to develop M-ary hyper phase-shift keying. The MHPSK technique used four orthonormal basis functions for binary information modulation. The symbols were assigned to various phases on the surface of a hypersphere in a four-dimensional space; in other words, all symbols were of equal energy. Bit-to-symbol mapping was performed to obtain the 4D signal constellations for four, five and six bits. Monte Carlo simulations conducted using the generated signal constellations validated the improvement in BER over traditional bandwidth efficient modulation schemes such as 8PSK while having a spectral efficiency comparable to single-carrier 16-PSK or 16-QAM. The study concluded that MHPSK is ideal for energy limited scenarios such as satellite communications and for use in nonlinear channels due to its equal energy symbols.

D. CHAPTER SUMMARY

The chapter began with a review of Shannon's channel capacity. An overview of *M*-ary modulation techniques, in particular *M*PSK and *M*QAM, and the OFDM multiplexing technique used to multiplex *M*-ary modulated subcarriers for improved spectral efficiency and robustness in severe channel conditions was given. Key advantages and research interests in multi-dimensional modulation techniques were highlighted. It was noted that multi-dimensional modulation offered increased spectral efficiency when signal constellations were optimized and improved performance in nonlinear channels when equal energy symbols were used.

The next two chapters will build upon these concepts to develop the multi-dimensional constant energy modulation technique.

III. *N*-DIMENSIONAL CONSTANT ENERGY CONSTELLATION

A. CONSTRAINT EQUATION FOR EQUAL ENERGY SYMBOLS

The energy of a signal is the squared length of its signal vector. Given a signal vector $\underline{\mathbf{S}} = [s_1 \;\; s_2 \;\; s_3 \;\; \ldots \; s_N]$, where $s_{1,}$ $s_{2,}$ $s_{3,}$ …, s_N are real, the signal energy is the sum of the square of its vector coordinates. To maintain equal energy for all the symbols within a signal constellation, each symbol must adhere to the constraint equation

$$\sum_{i=1}^{N} s_i^{\,2} = A\,, \tag{3.1}$$

where A is a constant. The normalized constraint equation for constant symbol energy is given by

$$\sum_{i=1}^{N} \hat{s}_i^{\,2} = 1\;. \tag{3.2}$$

To perform a combinatorial analysis [10] to calculate the maximum number of symbols in a given dimension, we propose the following normalized constraint equation:

$$\sum_{j=1}^{n} r_j C_j^{\,2} = 1\,, \tag{3.3}$$

where C_j is the j^{th} non-zero coordinate value, and r_j is the number of times the j^{th} non-zero coordinate value appears in the vector for j = 1, 2, 3, …, n and $n \leq N$.

B. MAXIMUM NUMBER OF SYMBOLS

The maximum number of symbols that can be generated from an N-dimensional modulated signal with N signal vector coordinates can be obtained through permutation of the vector coordinate's polarity (obtained from 2^N) and the dimension in which the coordinate is located (obtained from N!) and is given by $2^N \times N!$.

Taking into consideration the criteria for equal energy symbols given in Equation (3.3), we get the maximum number of equal energy symbols which can be generated G to be

$$G = \frac{2^{(n+m)} N!}{r_1!\, r_2!\, r_3! \ldots r_n!}\,. \tag{3.4}$$

Equation (3.4) provides the theoretical maximum number of equal energy symbols which can be generated given the dimension of the modulation *N*, the number of non-zero signal vector coordinates *n*, and the number of times a non-zero vector coordinate is repeated *m*. In practice, the number of transmit symbols is less than or equal to the maximum, assuming that only whole bits are used to form a symbol.

A summary of the maximum number of generated and transmitted symbols for the *N*-dimensional modulation is shown in Table 1. The *N*-dimensional modulation is able to generate a maximum of $2^N \times N!$ symbols. Hence, the (*N*+1)-dimensional modulation has to be selected when the required number of equal energy symbols is greater than $2^N \times N!$.

Table 1. Maximum number of generated and transmitted symbols for *N*-dimensional constant energy modulation.

N	Maximum Number of Generated Equal Energy Symbols (*G*)	Maximum Number of Transmitted Bits	Maximum Number of Transmitted Equal Energy Symbols (*H*)
1	2	1	2
2	8	3	8
3	48	5	32
4	384	8	256
5	3840	11	2048
6	46080	15	32768
7	645120	19	524288
8	10321920	23	8388608

C. DESIGN OF *N*-DIMENSIONAL CONSTANT ENERGY CONSTELLATION

1. Selection Criteria for *N*-Dimensional Signal Constellation

The selection of a suitable *N*-dimensional signal constellation for a communications system depends on the number of symbols needed. It also depends on the acceptable BER for the system. The worst case BER occurs at the minimum Euclidean distance between any two signal vectors in a signal constellation. The optimum equal energy constellation thus has to provide the number of symbols within the maximum as described in Sections A and B while achieving the largest possible minimum Euclidean distance between the vectors in the signal constellation.

2. Minimum Squared-Euclidean Distance

For a given *N*-dimensional modulation, depending on the signal vector coordinate values C_j and the number of times the coordinate value appears in the vector r_j, there can be many signal constellations which can provide the required number of symbols and bits.

Assume signal vector coordinate values $C_1, C_2, C_3, \ldots, C_n$ such that $|C_1| < |C_2| < |C_3| < \ldots < |C_n|$, $C_1, C_2, C_3, \ldots, C_n \in \mathbb{R}$ and $\neq 0$, and $C_1 \neq C_2 \neq C_3 \neq \ldots \neq C_n$. Using the minimum squared-Euclidean distance $d^2_{\min}$ for ease of calculation, we find the set of squared-Euclidean distances which contains the minimum to comprise values obtained from the following equations:

$$X = 2|C_1|^2, \tag{3.5}$$

$$Y = (2|C_1|)^2, \tag{3.6}$$

$$\begin{aligned} Z = \{ & 2\left(|C_2|-|C_1|\right)^2, 2\left(|C_3|-|C_1|\right)^2, 2\left(|C_4|-|C_1|\right)^2, \ldots, 2\left(|C_{n-1}|-|C_1|\right)^2, \\ & 2\left(|C_3|-|C_2|\right)^2, 2\left(|C_4|-|C_2|\right)^2, \ldots, 2\left(|C_n|-|C_2|\right)^2, \\ & 2\left(|C_4|-|C_3|\right)^2, \ldots, 2\left(|C_n|-|C_3|\right)^2, \\ & \ldots, 2\left(|C_n|-|C_{n-1}|\right)^2 \}. \end{aligned} \tag{3.7}$$

.

Equations (3.5) to (3.7) are used when one of the signal vector coordinates has zero value. Equations (3.6) to (3.7) are used when all the signal vector coordinates have non-zero values. We can summarize two cases as follows:

a. Normalized signal vector coordinates $\hat{s}_i \in \mathbb{R}$ ***and one zero-value signal vector coordinate***

$$d_{\min}^2 = \min\{X, Y, Z\}, \text{ excluding } Z = 2\left(|C_n| - |C_1|\right)^2, \tag{3.8}$$

b. Normalized signal vector coordinates $\hat{s}_i \in \mathbb{R}$ ***and*** $\neq 0$

$$d_{\min}^2 = \min\{Y, Z\}, \text{ excluding } Z = 2\left(|C_n| - |C_1|\right)^2. \tag{3.9}$$

Equations (3.5) to (3.9) reduce the number of options for the minimum squared-Euclidean distance from the Euclidean distance between any two vectors in the signal constellation to a subset of the vector coordinate values. This effectively reduces the number of iterations when conducting the optimization with the normalized constraint equation and, thus, finds the vector set with minimum distance. The software MATLAB is used for simulations.

3. Optimization with Equal Energy Symbols

By iterating the values of r_j and C_j in Equation (3.3) and calculating the $d_{\min}^2$ for each case using Equations (3.5) to (3.9), we can then find the constellation with the largest possible $d_{\min}^2$. This results in the lowest possible BER, giving us the optimum or near optimum equal energy constellation. We call the technique used to obtain the constellation “constant energy modulation".

D. DEVELOPMENT OF *N*-DIMENSIONAL CONSTANT ENERGY CONSTELLATION WITH CEM

1. Implementation of CEM

The CEM technique can be implemented using a concept similar to OFDM. The modulation of CEM via a baseband approach is shown in Figure 5. At the transmitter, data bits are first mapped into symbols depending on the required number of bits per symbol. Each symbol is mapped onto a pre-assigned signal vector. Depending on the modulation dimension N, we map the signal vector coordinates onto the in-phase and quadrature channels of k subcarriers, where k is the modulation dimension N divided by two, rounded up to the nearest whole number. The IFFT is conducted on all the subcarriers. Complex noise is added when the signal passes through the transmission channel.

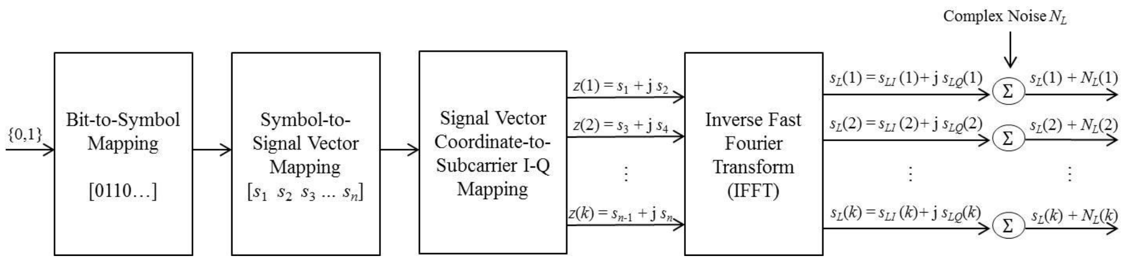

Figure 5. Block diagram of CEM modulator.

The demodulation of CEM is shown in Figure 6. The signal reaching the receiver is the group of the subcarrier signals, each comprising up to two signal vector coordinates from the transmitted signal vector coupled with noise. The FFT is conducted on the subcarrier, which results in the value of a signal vector coordinate coupled with noise on the I-channel and Q-channel for each subcarrier. The coordinates are then combined into a complete signal vector. A minimum-Euclidean distance detector is used to locate the signal vector in the pre-determined symbol-to-signal vector list with Euclidean distance closest to that of the received vector. The symbol corresponding to the selected vector is deemed to be the symbol sent. Lastly, symbol-to-bit mapping is performed to produce the bit sequence at the receiver output.

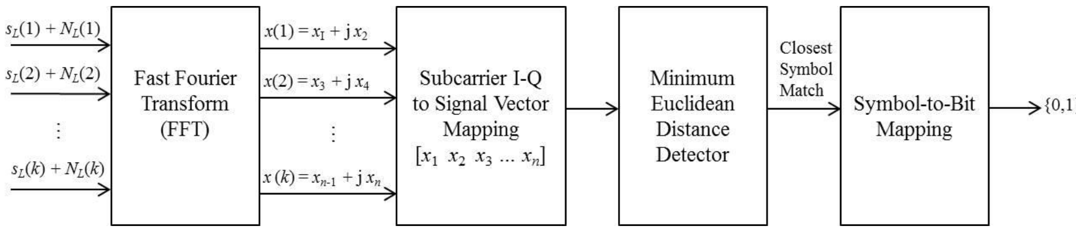

Figure 6. Block diagram of CEM demodulator.

2. Bandwidth, Spectral Efficiency and Power

The CEM technique splits a symbol into its signal vector coordinates for transmission. This could increase the required transmission bandwidth. The bandwidth increment is, however, mitigated by transmitting the vector coordinates using subcarriers which are orthogonal in frequency.

Drawing a reference from the typical OFDM system, the subcarriers in the CEM system are separated in frequency such that the spectrum of each subcarrier has a null at the center frequency of each of the other subcarriers in the system [11]. The spectrum of an OFDM signal with amplitude normalized to one and OFDM subcarriers with center frequencies separated by one (normalized with respect to half the null-to-null subcarrier bandwidth) is illustrated in Figure 7. In the CEM system, up to two signal vector coordinates are mapped onto one subcarrier. Hence, up to two signal vector coordinates occupy one subcarrier bandwidth, and using orthogonal frequencies, up to four signal vector coordinates occupy one and a half times subcarrier bandwidth. A summary of the bandwidth occupancy and spectral efficiency versus bit rate R_b for N=4, N=5, N=6, N=7 and N=8 modulation dimensions and up to R_b=8 (normalized with respect to half the null-to-null subcarrier bandwidth) is given in Table 2. Note that the normalized R_b is exactly the number of bits per symbol.

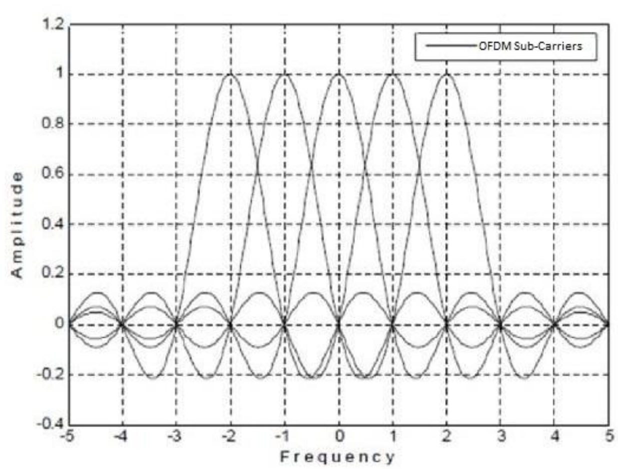

Figure 7. Spectrum of an OFDM signal. After [11].

Table 2. Summary of bandwidth occupancy and spectral efficiency versus bit rate for different modulation dimensions. Bandwidth *B* is normalized to half the null-to-null subcarrier bandwidth.

Modulation Dimension (*N*)	Number of Subcarriers (*SC*)	Normalized Bandwidth (*B*)	Normalized Bit Rate (Number of Bits per Symbol) (R_b)	Spectral Efficiency (η, where $\eta = R_b / B$)
4	2	3	4	1.333
			5	1.667
			6	2
			7	2.333
			8	2.667
5	3	4	4	1
			5	1.25
			6	1.5
			7	1.75
			8	2
6	3	4	4	1
			5	1.25
			6	1.5
			7	1.75
			8	2
7	4	5	4	0.8
			5	1
			6	1.2
			7	1.4
			8	1.6
8	4	5	4	0.8
			5	1
			6	1.2
			7	1.4
			8	1.6

In [1], the author provided three 4D signal constellations comprising four, five and six bits. We use these constellations, in particular the 4D constellation with six-bit symbols (64-HPSK), to benchmark against the *N*-dimensional constellations developed with CEM in terms of signal power. The comparison focuses on CEM constellations with modulation dimensions $4 < N \leq 8$, which have bit rates that are equivalent to or higher than 64-HPSK and minimum squared-Euclidean distance close to or larger than that of 64-HPSK (within 3 dB drop in signal power). It is possible that there could be more than one constellation with the same number of different non-zero vector coordinates for a given modulation dimension. In such cases, the constellation with the larger minimum squared-Euclidean distance is selected for comparison.

A summary of the *N*-dimensional constellations benchmarked against the constellations in [1] in terms of signal power is given in Appendix A. Eleven of these constellations with up to normalized R_b = 8 were selected for further analyses. A summary of the selected constellations with their bandwidth, $d^2_{\min}$, spectral efficiency and power is given in Table 3.

Table 3. Selected signal sets with bandwidth, spectral efficiency and power. Bandwidth *B* is normalized to half the null-to-null subcarrier bandwidth.

Modulation Dimension	Constraint Equation	Normalized Bit Rate (Number of Bits per Symbol) (R_b)	Normalized Bandwidth (*B*)	Minimum Squared-Euclidean Distance $\left(d^2_{\min}\right)$	Spectral Efficiency (η)	Power Relative to 64-HPSK [1] in dB
4	$4C_1^2 = 1$ [4]	4	3	1.000	1.333	3.440
4	$3C_1^2 + 0 = 1$ [5]	5	3	0.6667	1.667	1.679
4	$C_1^2 + 3C_2^2 = 1$ [6]	6	3	0.4529	2	0
5	$4C_1^2 + 0 = 1$	6	4	0.5000	1.5	0.4297
5	$C_1^2 + 4C_2^2 = 1$	7	4	0.4069	1.75	-0.4651
5	$C_1^2 + 3C_2^2 + 0 = 1$	8	4	0.2856	2	-2.002
6	$6C_1^2 = 1$	6	4	0.6667	1.5	1.6793
6	$5C_1^2 + 0 = 1$	7	4	0.4000	1.75	-0.5394
6	$C_1^2 + 5C_2^2 = 1$	8	4	0.3693	2	-0.8862
7	$7C_1^2 = 1$	7	5	0.5714	1.4	1.009
8	$8C_1^2 = 1$	8	5	0.5000	1.6	0.4297

[4] From 16-HPSK [1].

[5] From 32-HPSK [1].

[6] From 64-HPSK [1].

We observe from Table 3 the trade-offs among bandwidth, spectral efficiency and power for the different modulation dimensions. More bandwidth is required due to the increased number of subcarriers for higher modulation dimensions. For the same R_b, a better spectral efficiency can be obtained by selecting the lowest modulation dimension for which the required bit rate can be achieved. As shown in Table 3, for normalized R_b = 6, 4D modulation is the most spectrally efficient as compared to 5D and 6D modulation. In terms of signal power, as the modulation dimension increases, the minimum squared-Euclidean distance $d_{\min}^2$ between the signal vectors in the constellation increases. Hence, for the same R_b, signal power increases with increasing modulation dimension. This is illustrated in Table 3; for normalized R_b = 8, 8D modulation provides the highest power gain compared to 5D and 6D modulation.

3. Algorithm for Designing a Signal Constellation

Bit-to-symbol and then symbol-to-signal vector mapping has to be conducted for the selected symbol vectors in Table 3 in order to conduct further analyses of each constellation's BER. An algorithm is developed using MATLAB to conduct the mapping. The bit sequence for the required dimension (for example, vector of bits per symbol) is first converted into its Gray code sequence, in which each row differs from the row before it by one bit. A reference vector is selected from the desired signal set of the given dimension and assigned as the first symbol with Gray code bit sequence 0 (first Gray sequence). The remaining vectors are ranked in order of increasing Euclidean distance from the reference vector. The algorithm then assigns the next Gray code sequence to the signal vector, moving down the Gray code sequence and assigning to the vectors in order of increasing Euclidean distance. The flowchart of the mapping algorithm is shown in Figure 8. The completed signal constellations with their mapped values are given in Appendix B.

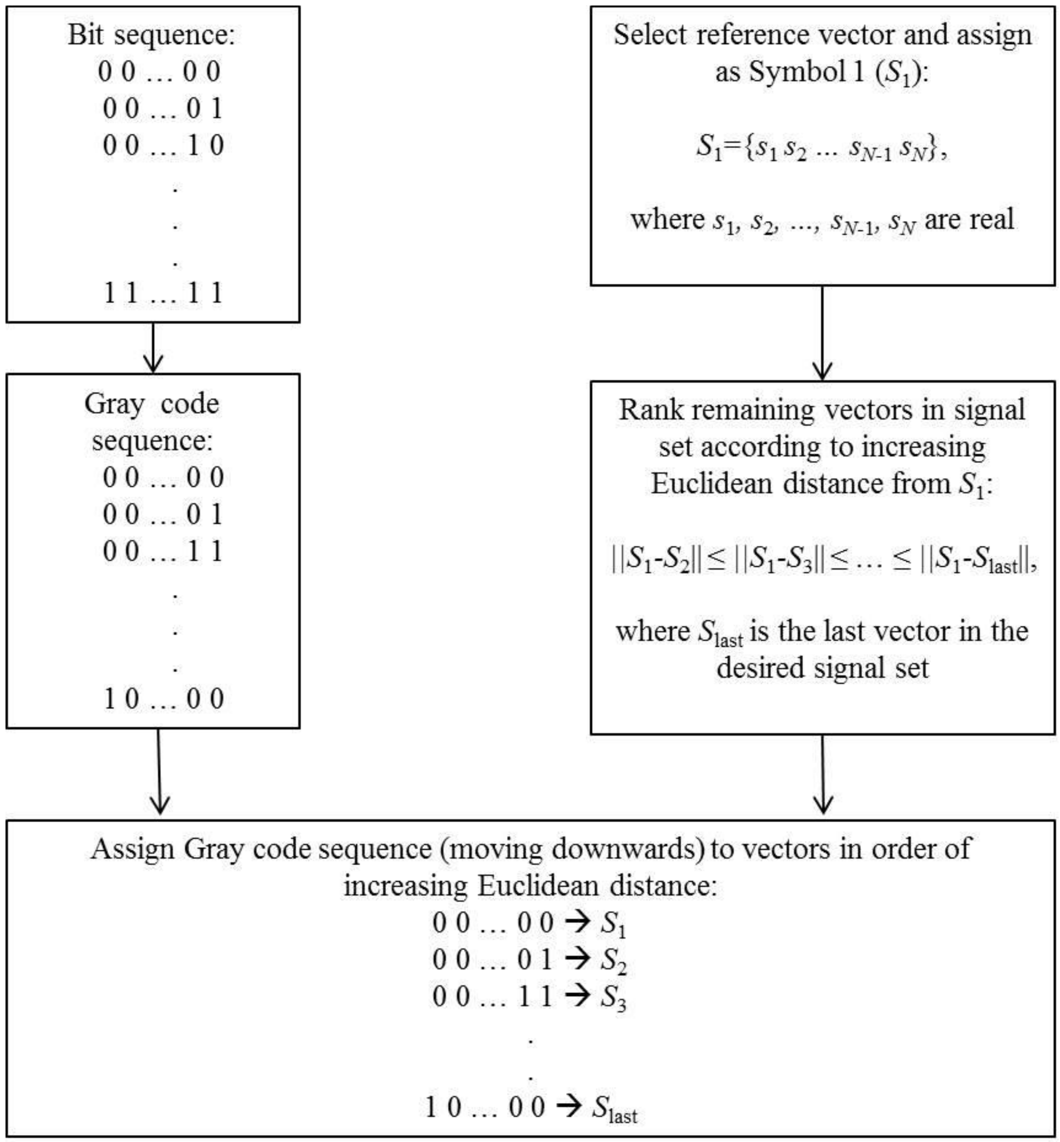

Figure 8. Flowchart of MATLAB algorithm for a signal constellation design.

In cases where there are more signal vectors in a particular constellation than bit streams in the Gray code sequence, only the reference (first) vector up to the vector corresponding to the last bit stream in the Gray code sequence is assigned with bit streams to form symbols. This is performed to maintain the $d_{\min}^2$ value between the two closest vectors for calculations of the BER in the next section.

4. Theoretical Approximation of Bit Error Ratio

Using the union bound, we obtain the total probability of symbol error P_e as

$$P_e \le \frac{1}{M} \sum_{\substack{u=1 \\ u \ne v}}^{M} \sum_{v=1}^{M} Q\left(\frac{du,v}{2\sigma} \right), \tag{3.10}$$

where M is the number of signal vectors used to generate symbols, $d_{u,v}$ is the Euclidean distance of vector s_u with respect to vector s_v and is calculated from

$$d_{u,v} = \left\| s_u - s_v \right\|. \tag{3.11}$$

The variance of Gaussian noise is σ^2 and is equal to $N_o / 2$. The symbol energy E_s is normalized to one. Hence,

$$\frac{d_{u,v}}{2\sigma} = \left\| s_u - s_v \right\| \sqrt{\frac{E_s}{2N_o}} = \left\| s_u - s_v \right\| \sqrt{\left(\frac{E_b}{N_o} \right) \left(\frac{\log_2 M}{2} \right)}. \tag{3.12}$$

Substituting (3.12) into (3.10), we get P_e in terms of E_b / N_o as

$$P_e \le \frac{1}{M} \sum_{\substack{u=1 \\ u \ne v}}^{M} \sum_{v=1}^{M} Q\left(\left\| s_u - s_v \right\| \sqrt{\left(\frac{E_b}{N_o} \right) \left(\frac{\log_2 M}{2} \right)} \right). \tag{3.13}$$

The upper bound for the probability of bit error P_b is given by

$$P_b \le P_e. \tag{3.14}$$

The lower bound for P_b occurs when Gray code is used for bit-to-symbol mapping and is given by

$$P_b = \frac{P_e}{\log_2 M}. \tag{3.15}$$

Hence, a theoretical approximation for P_b can be given as

$$P_b \approx \frac{\alpha P_e}{\log_2 M}, \tag{3.16}$$

where α is the average number of nearest neighbors for all symbols in the signal constellation and $1 \le \alpha \le \log_2 M$.

The BER of the CEM N-dimensional signal constellations for N=4 with 4-bit, 5-bit and 6-bit symbols are plotted with MATLAB using the theoretical approximation in

Equation (3.14) and shown in Figures 9, 10 and 11, respectively. Two sets of bit-to-symbol assignments are used in the plots: assignment using the algorithm described in Figure 8 and the bit stream assignment via inspection given in [1]. Both assignments achieved comparable BER, although the latter's assignment gave a slightly lower BER. This indicates that the symbol-to-vector mapping algorithm proposed in this chapter can be utilized to generate CEM constellations, and its approximate BER is very close to the approximate BER via inspection in [1]. Note that it is nearly impossible to design a signal constellation for N > 5 using inspection. The BER approximation is used in the next part of the research as a theoretical basis against which to compare the simulated BER.

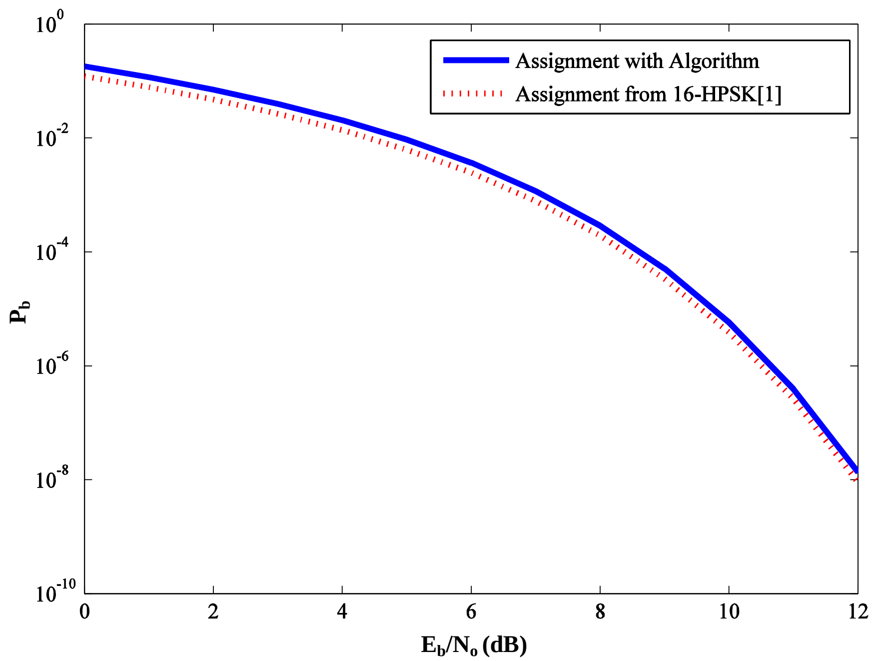

Figure 9. P_b versus E_b / N_o plot for 4D-16CEM.

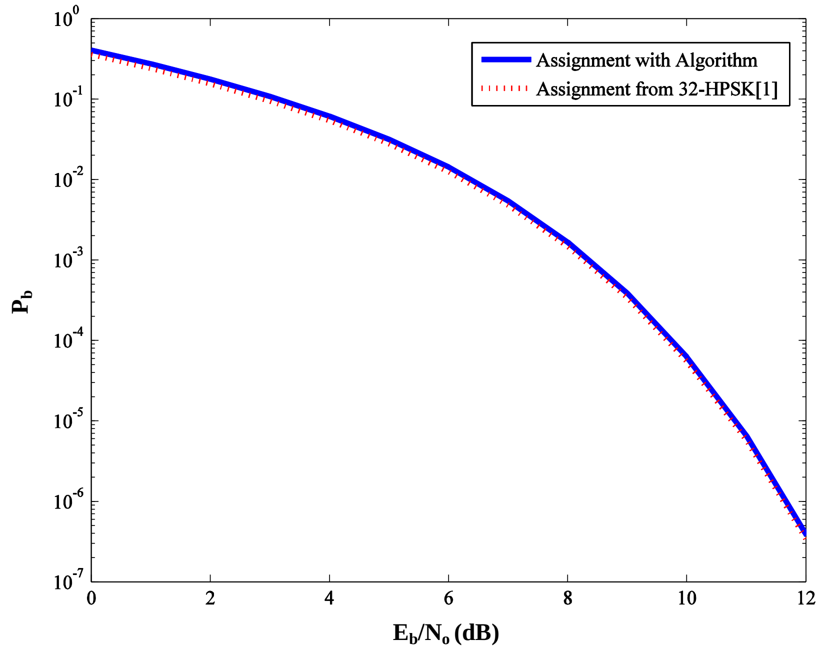

Figure 10. P_b versus E_b / N_o plot for 4D-32CEM.

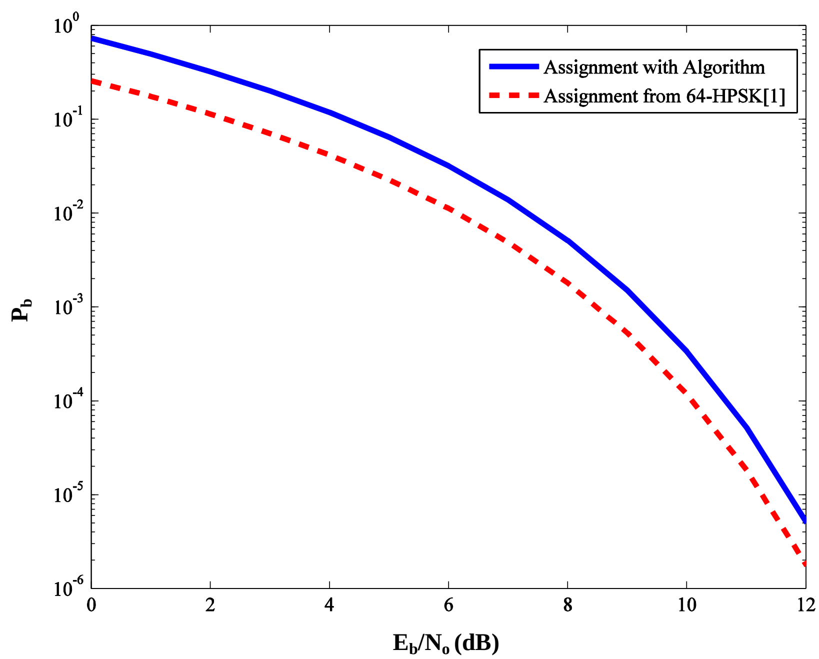

Figure 11. P_b versus E_b / N_o plot for 4D-64CEM.

E. CHAPTER SUMMARY

In this chapter, we described the implementation of the CEM technique, including the selection criteria for the *N*-dimensional signal constellation and the optimization process to obtain signal vector values for the constellation. The bandwidth, spectral efficiency and power trade-offs for N=4, N=5, N=6, N=7 and N=8 modulation dimensions and up to normalized R_b=8 were analyzed. A symbol-to-signal vector mapping algorithm was proposed and used to generate signal constellation mapping values. A theoretical approximation of the CEM's BER was given.

In the next chapter, the constellation data is used to conduct simulations and analyze the performance of CEM in both linear and nonlinear channels.

IV. SIMULATION OF LINEAR CHANNEL AND NONLINEAR AMPLIFIER EFFECTS

A. LINEAR CHANNEL

The CEM modulator and demodulator described in Chapter III were programmed in MATLAB. Monte Carlo simulations were conducted using 1.5 million samples per SNR for N=4, N=5, N=6, N=7 and N=8 modulation dimensions and up to normalized R_b=8.

1. Simulated Results Compared with Theoretical Approximation

The results for N=4 are given in Figures 12 to 14. We observe that for 4D-16CEM (4-bit symbols), 4D-32CEM (5-bits symbols) and 4D-64CEM (6-bit symbols), the simulated results agree closely with the theoretical approximation, in particular for values of E_b/N_o from 6 dB to 10 dB. This indicates that the approximation is most accurate where the probability of bit error is between 10^{-2} and 10^{-5}.

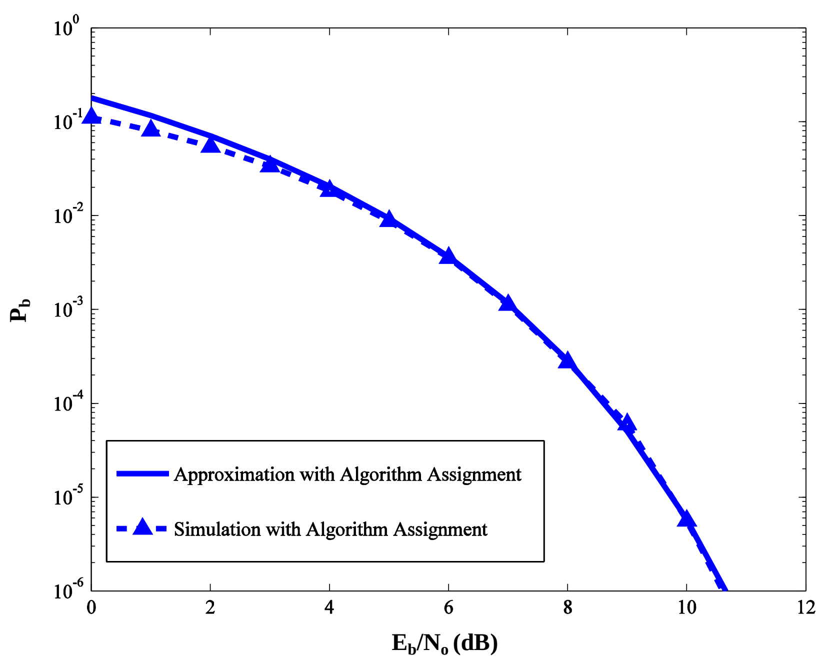

Figure 12. BER for 4D-16CEM using Monte Carlo simulations.

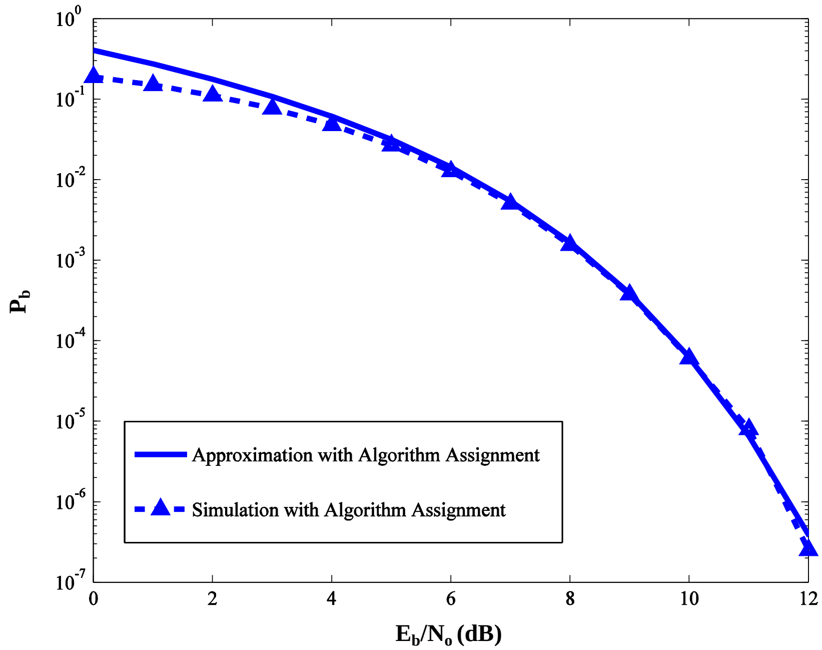

Figure 13. BER for 4D-32CEM using Monte Carlo simulations.

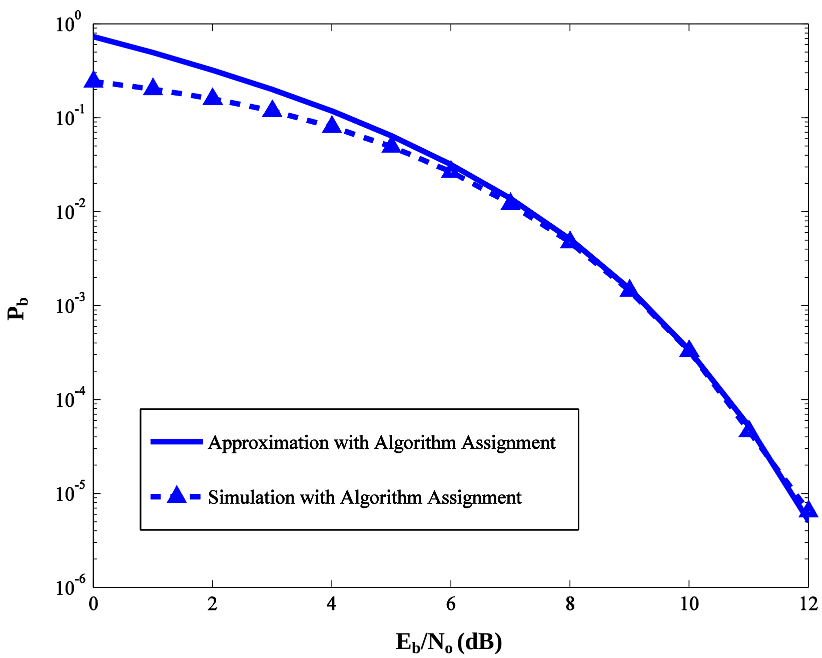

Figure 14. BER for 4D-64CEM using Monte Carlo simulations.

The simulation results for N=5, N=6, N=7 and N=8 modulation dimensions are given in Appendices C, D, E and F, respectively. In general, the simulation results agree with the theoretical approximation as in the case of N=4. We observe that within the same modulation dimension, the simulated results deviate more from the approximation as the bit rate increases. When comparing results for the same bit rate across the different modulation dimensions, we notice that the alignment between the simulation results and theoretical results improve with increasing modulation dimension.

We conclude that the union bound approximation is, in general, a reasonable estimate of BER for CEM systems and is likely to be more accurate for higher modulation dimensions.

2. Comparison across Modulation Dimensions

We next compare the BER of 64CEM (6-bit symbols), 128CEM (7-bit symbols) and 256CEM (8-bit symbols) across different dimensions, as shown in Figures 15, 16 and 17, respectively. We observe that the BER decreases as the modulation dimension N increases for the same number of bits per symbol. As N increases, the increased signal space leads to an increased minimum Euclidean distance $d^2_{\min}$ between neighboring vectors in the constellation, hence, decreasing the BER. This is also in line with the power calculations given in Chapter II, Table 3. When the bit-to-symbol ratio is seven bits per symbol as given in Figure 16, we notice that the BER for N=5 and N=6 are very close for both the approximated and simulated results, with N=5 offering slightly better BER. From Table 3, we see that the two modulations have the same spectral efficiency and similar power, which could explain a closer BER. In this particular instance, the constellation for N=5 has $d^2_{\min}$ = 0.4069, which is slightly larger than that for the N=6 constellation, which has $d^2_{\min}$ = 0.4000 (see Appendix A). This could explain its slightly better BER.

We also notice that at higher bit rates, the larger the modulation dimension, the more the union bound approximation remains applicable for all E_b/N_o. As shown in Figure 17, at the bit rate of eight bits per symbol, the theoretical approximation for N=5 is only applicable for $E_b/N_o > 1.2$ dB and that for N=6 is only applicable for $E_b/N_o > 0.4$ dB. The approximation for N=8, however, is applicable for all E_b/N_o.

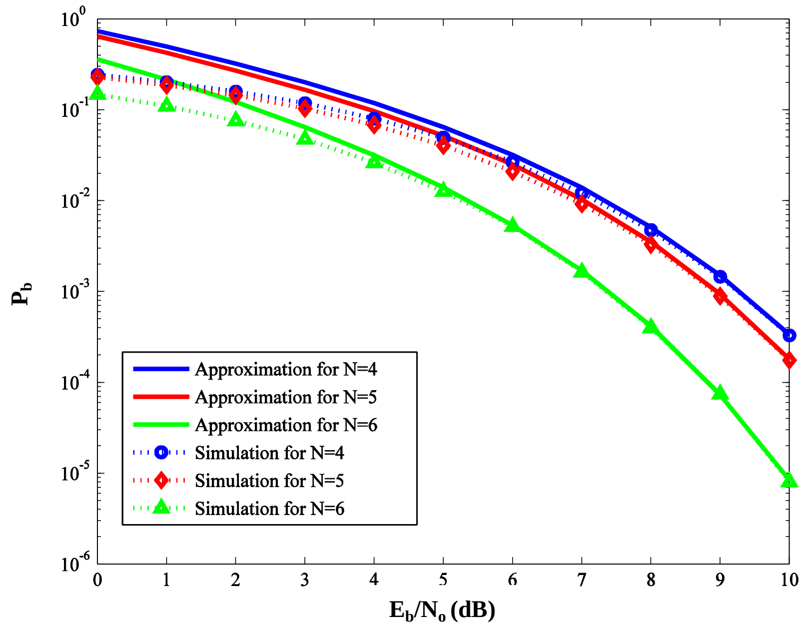

Figure 15. BER comparison across N=4, N=5 and N=6 for 64CEM.

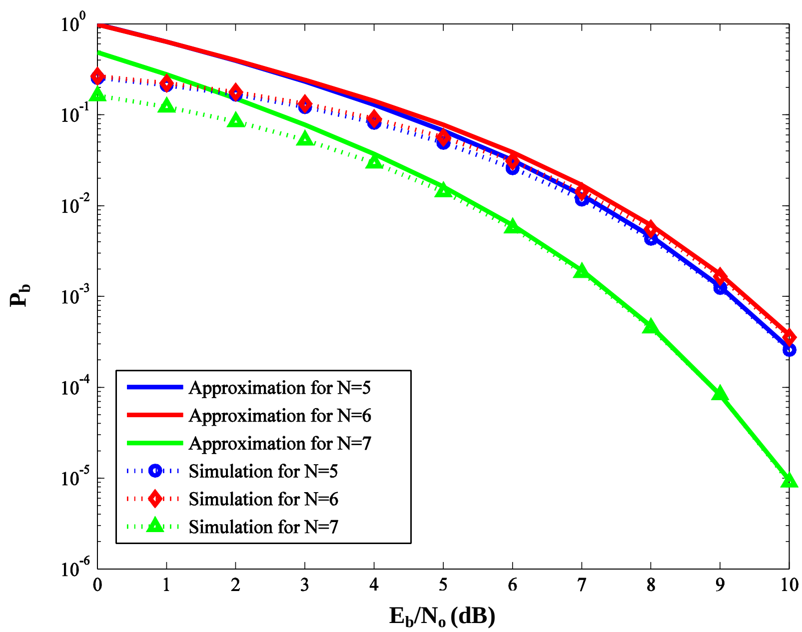

Figure 16. BER comparison across N=5, N=6 and N=7 for 128CEM.

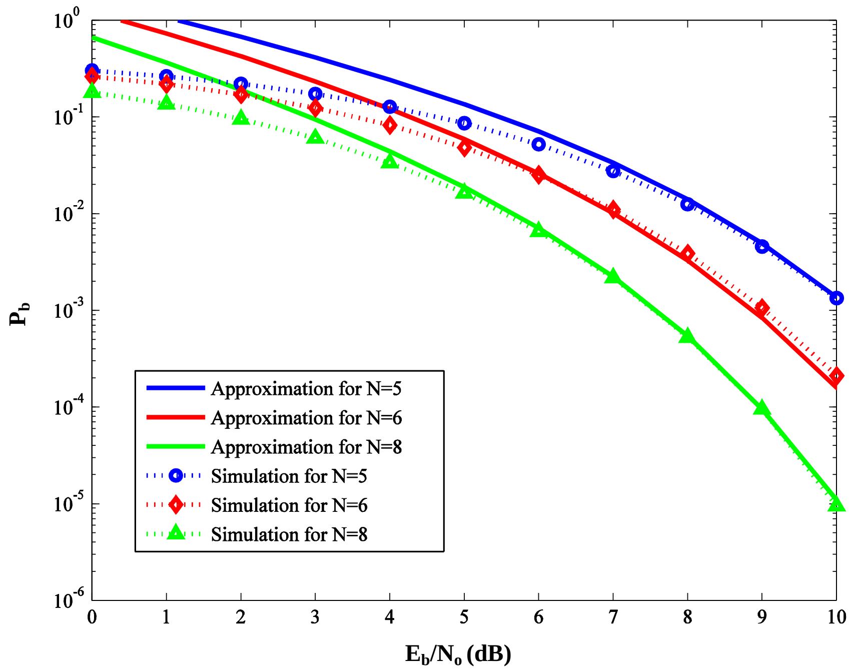

Figure 17. BER comparison across N=5, N=6 and N=8 for 256CEM.

B. NONLINEAR AMPLIFIER EFFECTS

1. Nonlinear Amplifier Model

In the previous section, we analyzed the performance of the CEM system in a linear channel, where the input voltage was linearly amplified to obtain the output voltage as shown in Figure 18. However, real-world applications need to take into account the effects of nonlinear amplifiers. The focus of this section is on analyzing the performance of the CEM system when a nonlinear amplifier is used.

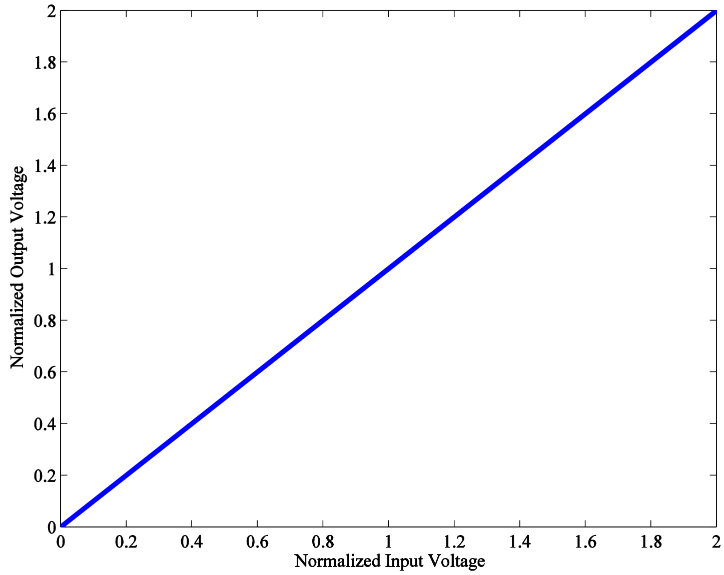

Figure 18. Linear amplifier response.

The nonlinear model of a travelling-wave tube (TWT) amplifier proposed in [12] is used to simulate the effects of a nonlinear channel on the CEM system. The response for the model is given in Figure 19.

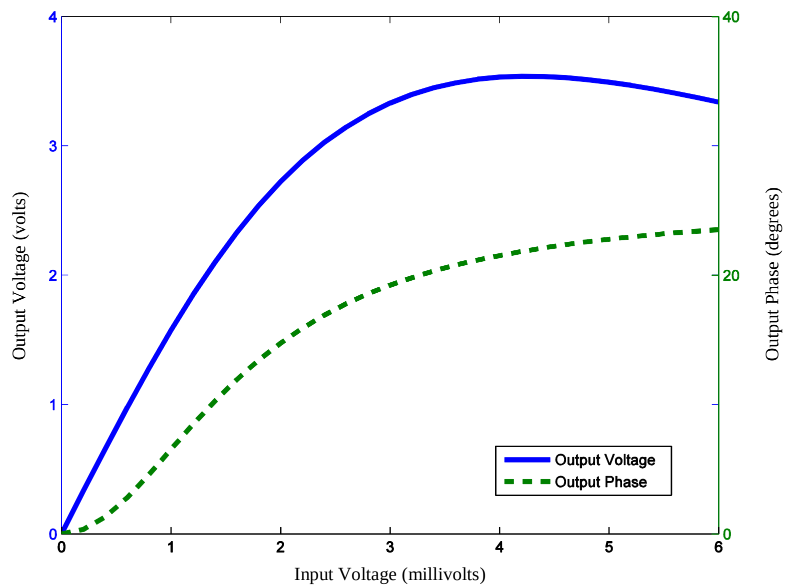

Figure 19. Nonlinear amplifier response. After [12].

The model can be represented by the two-parameter formulas:

$$A(r) = \alpha_a r / (1 + \beta_a r^2) , \quad (4.1)$$

$$\Phi(r) = \alpha_\Phi r^2 / (1 + \beta_\Phi r^2) , \quad (4.2)$$

where r is the input voltage, $A(r)$ is the output voltage, $\Phi(r)$ is the phase offset, and $\alpha_a = 1.6623$, $\beta_a = 0.0552$, $\alpha_\Phi = 0.1533$ and $\beta_\Phi = 0.3456$ are constants obtained from experimental TWT data. We use these equations to simulate the nonlinear amplifier effects on the CEM modulated data after going through the IFFT (see Figure 5).

2. Comparison with Linear Channel

We select the 4D-64CEM and 8D-64CEM systems to compare the BER of the CEM system in a linear channel and in a channel with nonlinear amplifier effects. The selection enables comparison at the same bit rate (six bits per symbol) across the smallest (N=4) and largest (N=8) modulation dimensions studied in this work. For the 8D-64CEM system, the 6-bit symbols are mapped onto the first 64 signal vectors of the constellation given in Table B.11 of Appendix B.

The results for N=4 are given in Figure 20. We observe that when there is no input back-off (0 dB BO), the nonlinear amplifier effects dominate such that improvement in the BER occurs only at very large E_b/N_o. The BER improves significantly with 3 dB input back-off (3 dB BO) and is much closer to the performance in a linear channel.

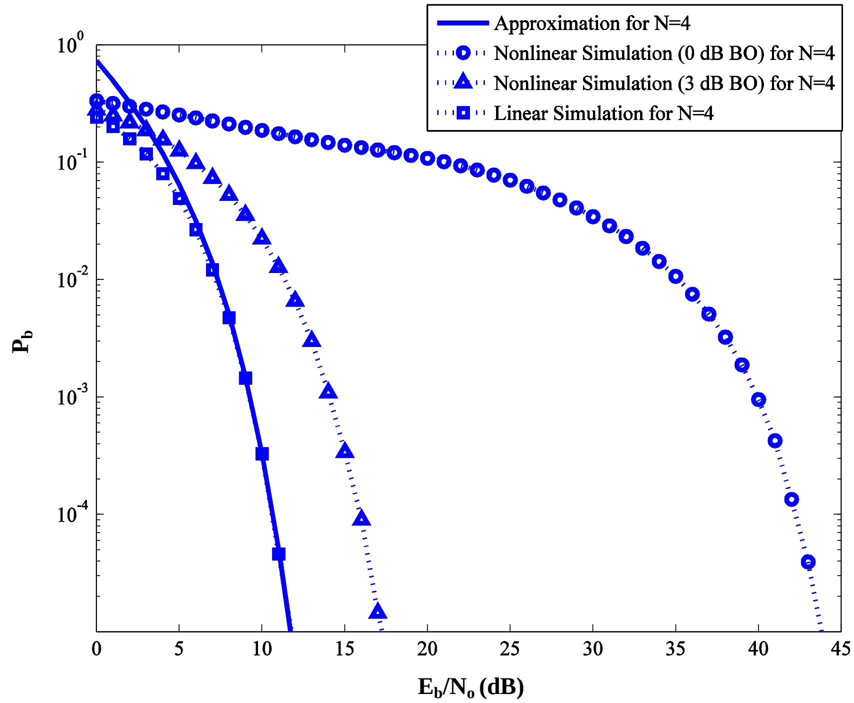

Figure 20. BER in linear channel and with nonlinear amplifier effects for *N*=4.

The results for *N*=8 are given in Figure 21. Likewise for N=4, we observe that the BER improves significantly with 3 dB input back-off (3 dB BO) and is closer to the performance in a linear channel. The comparison of the BER for both *N*=4 and *N*=8 is given in Figure 22. For *N*=8 with no back-off, we notice that the BER does not significantly degrade. This is the advantage of using higher order CEM modulation.

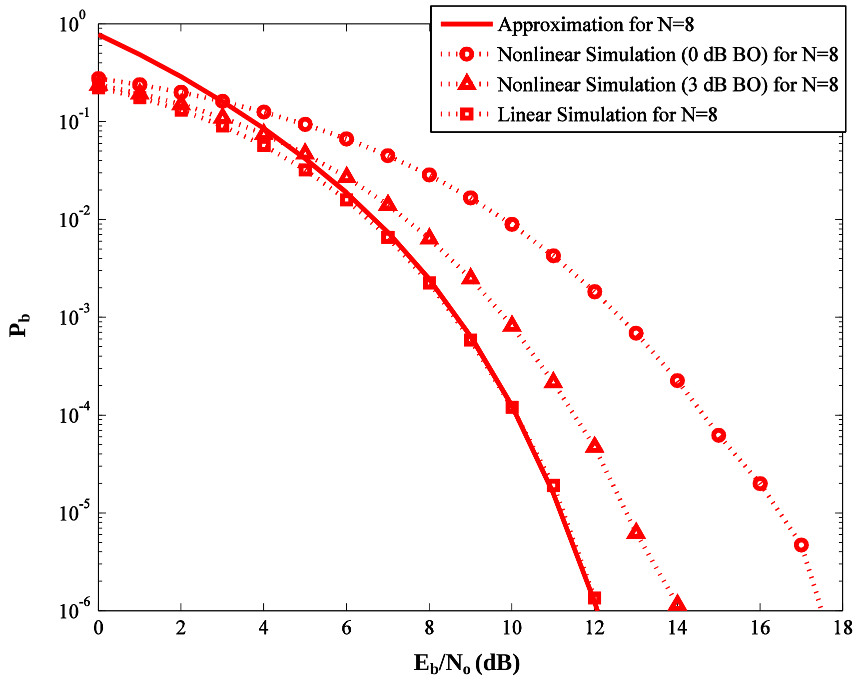

Figure 21. BER in linear channel and with nonlinear amplifier effects for *N*=8.

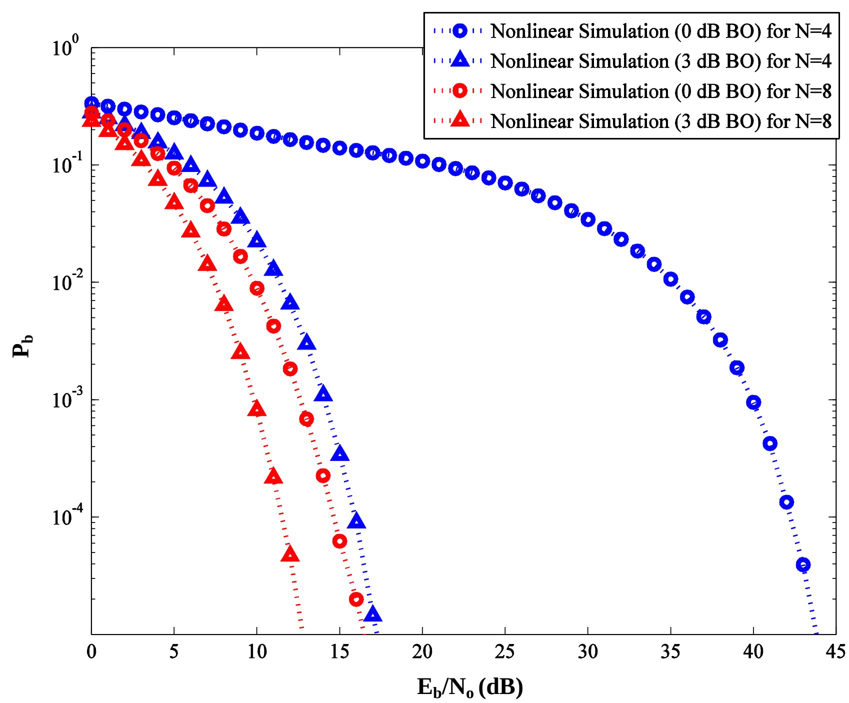

Figure 22. Comparison of BER with nonlinear amplifier effects for *N*=4 and *N*=8.

3. Phase Correction Using Pilot Symbol

In real-world systems, pilot symbols are often used to correct the phase offset and reduce error due to phase change caused by a nonlinear amplifier. We simulate the effect of phase correction by setting the reference (first) vector in the signal constellation as the pilot symbol for the worst case scenario with no input back-off. The results are given in Figures 23 and 24. We see that the inclusion of the pilot symbol improves the BER, bringing the BER somewhat closer to that in a linear channel for both N=4 and N=8.

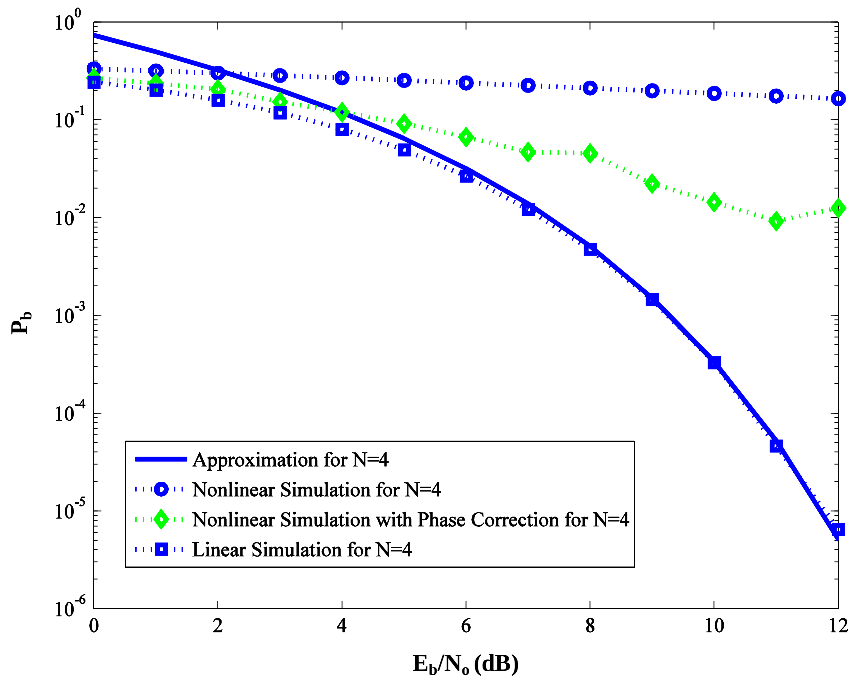

Figure 23. BER with nonlinear amplifier effects at 0 dB input back-off using pilot symbol for phase correction for N=4.

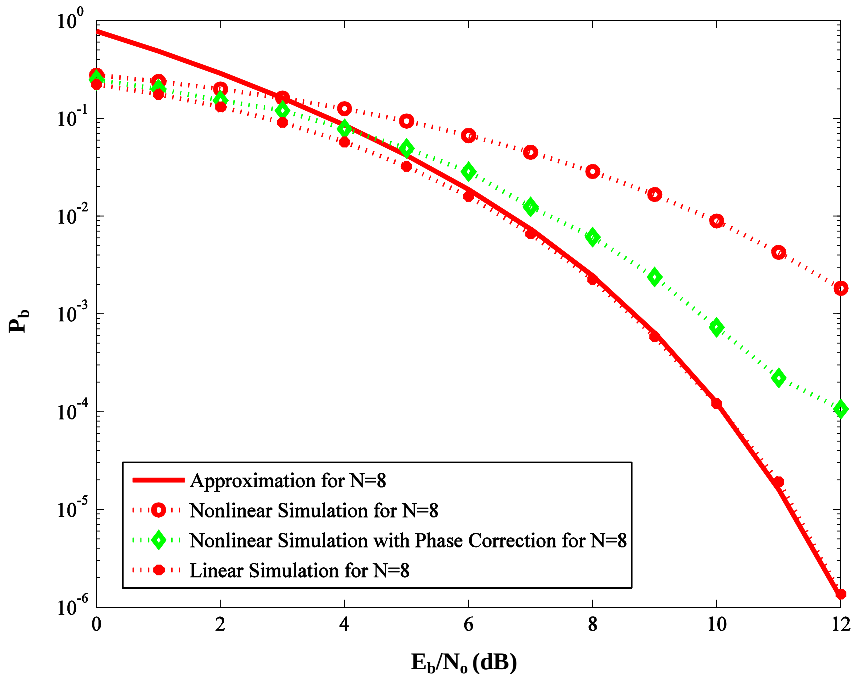

Figure 24. BER with nonlinear amplifier effects at 0 dB input back-off using pilot symbol for phase correction for *N*=8.

4. Comparison with OFDM System Using Two 8QAM Subcarriers

We next compare the performance with nonlinear amplifier effects of the 4D-64CEM and 8D-64CEM to an OFDM system using two 8QAM subcarriers to understand the performance of the CEM versus existing modulation techniques. The results are given in Figures 25 and 26. Both the CEM systems exhibit better BER than the OFDM system at 0 dB and 3 dB input back-offs. When a pilot symbol is used to provide phase correction at 0 dB input back-off, the performance for the CEM systems is further improved, while the OFDM system's performance showed only very slight improvement. The better performance of the CEM system in a nonlinear amplifier could be due to its equal energy symbols. In the case of the OFDM system, symbols transmitted by the 8QAM subcarriers have unequal energy. If more of the transmitted symbols were of higher energy, this would cause the system to operate towards or beyond the compression range of the amplifier, resulting in poor BER.

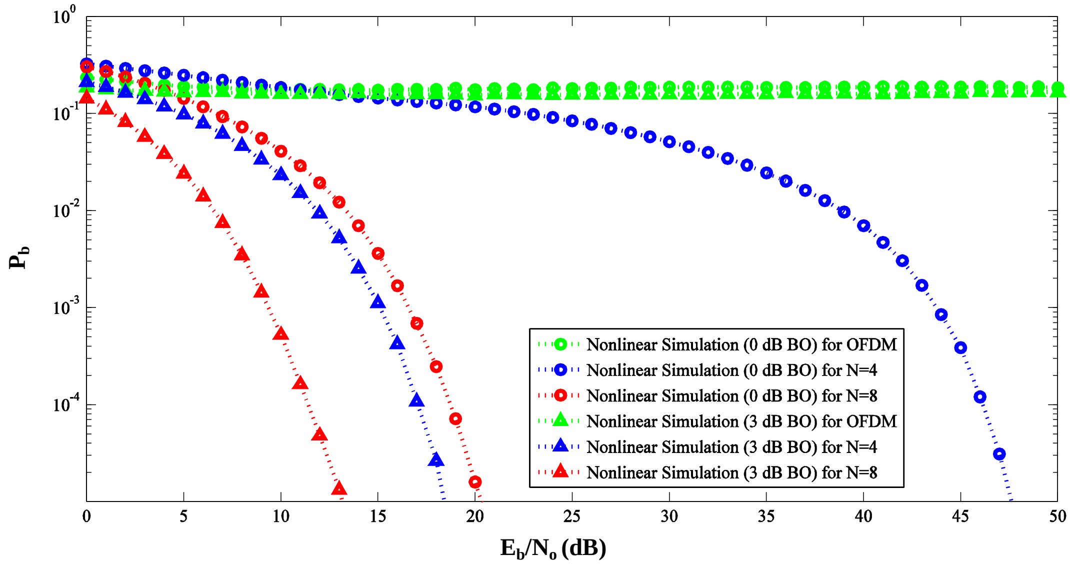

Figure 25. Comparison of BER with nonlinear amplifier effects for *N*=4, *N*=8 and OFDM-8QAM.

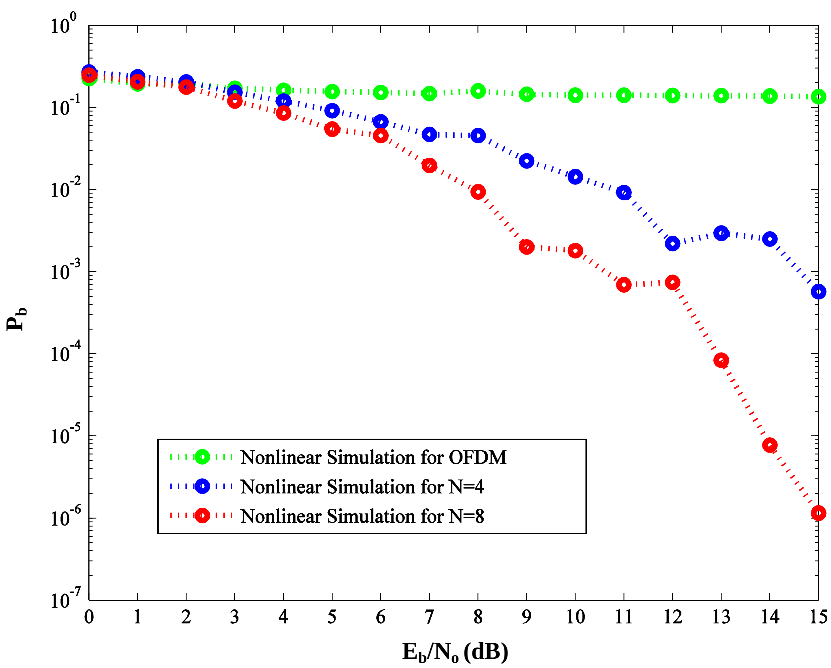

Figure 26. Comparison of BER with nonlinear amplifier effects at 0 dB input back-off using pilot symbol for phase correction for *N*=4, *N*=8 and OFDM-8QAM.

C. CHAPTER SUMMARY

In this chapter, we presented the results of the Monte Carlo simulations for the CEM system operating in a linear channel as well as a channel with a nonlinear amplifier. The simulated results for the linear channel were compared with the theoretical approximation proposed in Chapter III, and a comparison was made of the BER for the N=4, N=5, N=6, N=7 and N=8 modulation dimensions across the same bit rates. The 4D-64CEM and 8D-64CEM systems were used to understand the performance of the CEM technique when nonlinear amplifier effects were considered. The performance of the systems with nonlinear amplifier effects was then compared against that of an OFDM system with two 8QAM subcarriers.

The key findings from this research are summarized and recommendations for future work are offered in the next chapter.

THIS PAGE INTENTIONALLY LEFT BLANK

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

Multi-dimensional modulation is of interest to communications systems researchers due to its potential for increased spectral efficiency and improved performance in nonlinear channels. We presented the multi-dimensional CEM technique, which utilizes the concept of multi-dimensional signal space to improve spectral efficiency while constraining all symbols in a signal constellation to a constant energy. The implementation of the CEM technique was described, and Monte Carlo simulations were conducted with linear and nonlinear amplifier effects for the N=4, N=5, N=6, N=7 and N=8 modulation dimensions.

The results indicate that bandwidth, spectral efficiency and power trade-offs are key considerations when selecting the N-dimensional signal constellation for a required communications system. An optimized signal constellation has sufficient signal vectors to transmit the symbols while maintaining the largest possible minimum Euclidean distance $d_{\min}^2$ between neighboring vectors in the constellation. An efficient bit-to-symbol and symbol-to-signal vector mapping algorithm enables the system to achieve a good BER.

A theoretical approximation of the BER was obtained based on the union bound. Results from the simulations showed that the CEM performed close to the theoretical approximation in a linear channel. When different modulation dimensions N were compared for the same bit rate, it was observed that, as N increased, the BER of the CEM system decreased. This was due to the larger $d_{\min}^2$ between neighboring vectors when signal space was increased. When nonlinear amplifier effects were considered, an increased N likewise resulted in a better BER. In a nonlinear amplifier channel, the use of a pilot symbol for phase correction helped to improve the CEM system's performance. Finally, when compared against an OFDM system with two 8QAM subcarriers, both the 4D-64CEM and 8D-64CEM systems used in the comparison exhibited better performance in a channel with nonlinear amplifier effects. This could be due to the CEM's equal energy constraint on its symbols.

In summary, the multi-dimensional CEM system was successfully developed and tested using simulations. The main advantage of the system was the improved BER, in particular, in a channel with nonlinear amplifier effects.

B. RECOMMENDATIONS

The algorithm to design signal constellations, which was proposed in this research, had referenced Gray coding and assigned symbols based on the minimum Euclidean distance between vectors but could be further optimized. A more efficient mapping algorithm would enable the CEM to achieve a better BER.

The research could also be extended to modulation dimensions beyond N=8 to investigate the added advantages when signal space is further increased and to enhance its performance in nonlinear channels.

APPENDIX A. *N*-DIMENSIONAL CEM CONSTELLATIONS BENCHMARKED AGAINST 64-HPSK FOR SIGNAL POWER

Modulation Dimension (N)	**Constraint Equation ($\sum_{j=1}^{n} r_j C_j^2 = 1$, $j = 1,2,3,\ldots,n$; $n \le N$)**	**Reference Signal Vector (S_0)**	**Minimum Squared-Euclidean Distance $\left(d_{\min}^2\right)$**	**Normalized Bit Rate (R_b)**	**Number of Transmitted Symbols (N_{sym})**	**Power Relative to 64-HPSK, in dB $\left(10\log_2\left(\frac{d_{\min}^2}{0.4529}\right)\right)$**
4	$4C_1^2 = 1$ [7]	$S_0^T=$ *[0.5000 0.5000 0.5000 0.5000]*	1.000	4	16	3.440
4	$3C_1^2 + 0 = 1$ [8]	$S_0^T=$ *[0 0.5774 0.5774 0.5774]*	0.6667	5	32	1.679
4	$C_1^2 + 3C_2^2 = 1$ [9]	$S_0^T=$ *[0.8126 0.3365 0.3365 0.3365]*	0.4529	6	64	0

[7] From 16-HPSK [1].

[8] From 32-HPSK [1].

[9] From 64-HPSK [1].

5	$4C_1^2 + 0 = 1$	S_0^T= [0.5000 0.5000 0.5000 0.5000 0]	0.5000	6	64	0.4297
5	$C_1^2 + 4C_2^2 = 1$	S_0^T= [0.7700 0.3190 0.3190 0.3190 0.3190]	0.4069	7	128	-0.4651
5	$C_1^2 + 3C_2^2 + 0 = 1$	S_0^T= [0.7560 0.3779 0.3779 0.3779 0]	0.2856	8	256	-2.002
6	$6C_1^2 = 1$	S_0^T= [0.4082 0. 4082 0. 4082 0. 4082 0. 4082 0. 4082]	0.6667	6	64	1.6793
6	$5C_1^2 + 0 = 1$	S_0^T= [0.4472 0. 4472 0. 4472 0. 4472 0. 4472 0]	0.4000	7	128	-0.5394

6	$C_1^2 + 5C_2^2 = 1$	S_0^T= *[0.7336 0. 3039 0. 3039 0. 3039 0. 3039 0. 3039]*	0.3693	8	256	-0.8862
6	$C_1^2 + 4C_2^2 + 0 = 1$	S_0^T= *[0.7072 0. 3535 0. 3535 0. 3535 0. 3535 0]*	0.2499	9	512	-2.5824
7	$7C_1^2 = 1$	S_0^T= *[0.3780 0. 3780 0. 3780 0. 3780 0. 3780 0.3780 0.3780]*	0.5714	7	128	1.009
7	$6C_1^2 + 0 = 1$	S_0^T= *[0.4082 0. 4082 0. 4082 0. 4082 0. 4082 0.4082 0]*	0.3333	8	256	-1.3317

7	$C_1^2 + 6C_2^2 = 1$	S_0^T= *[0.7021 0. 2907 0. 2907 0. 2907 0. 2907 0.2907 0.2907]*	0,3380	9	512	-1.2709
7	$2C_1^2 + 5C_2^2 = 1$	S_0^T= *[0.5916 0. 5916 0. 2450 0. 2450 0. 2450 0.2450 0.2450]*	0.2401	11	2048	-2.7561
8	$8C_1^2 = 1$	S_0^T= *[0.3536 0. 3536 0. 3536 0. 3536 0. 3536 0. 3536 0.3536 0.3536]*	0.5000	8	256	0.4297
8	$7C_1^2 + 0 = 1$	S_0^T= *[0.3780 0. 3780 0. 3780 0. 3780 0. 3780 0. 3780 0.3780 0]*	0.2857	10	1024	-2.009

8	$C_1^2 + 7C_2^2 = 1$	S_0^T= [0.6740 0. 2792 0. 2792 0. 2792 0. 2792 0. 2792 0.2792 0.2792]	0.3118	11	2048	-1.6213
8	$2C_1^2 + 6C_2^2 = 1$	S_0^T= [0.6740 0. 2792 0. 2792 0. 2792 0. 2792 0. 2792 0.2792 0.2792]	0.2265	12	4096	-3.009

THIS PAGE INTENTIONALLY LEFT BLANK

APPENDIX B. CONSTELLATIONS WITH BIT-TO-SYMBOL AND SYMBOL-TO-SIGNAL VECTOR MAPPING VALUES

Table B.1. 4D-16CEM constellation mapping values.

Symbol	Bit Stream				Signal Vector			
					s_1	s_2	s_3	s_4
1	0	0	0	0	0.5000	0.5000	0.5000	0.5000
2	0	0	0	1	0.5000	0.5000	0.5000	-0.5000
3	0	0	1	0	0.5000	0.5000	-0.5000	0.5000
4	0	0	1	1	0.5000	0.5000	-0.5000	-0.5000
5	0	1	0	0	0.5000	-0.5000	0.5000	0.5000
6	0	1	1	0	0.5000	-0.5000	0.5000	-0.5000
7	0	1	0	1	0.5000	-0.5000	-0.5000	0.5000
8	0	1	1	1	0.5000	-0.5000	-0.5000	-0.5000
9	1	0	0	0	-0.5000	0.5000	0.5000	0.5000
10	1	1	0	0	-0.5000	0.5000	0.5000	-0.5000
11	1	0	1	0	-0.5000	0.5000	-0.5000	0.5000
12	1	1	0	1	-0.5000	0.5000	-0.5000	-0.5000
13	1	0	0	1	-0.5000	-0.5000	0.5000	0.5000
14	1	1	1	0	-0.5000	-0.5000	0.5000	-0.5000
15	1	0	1	1	-0.5000	-0.5000	-0.5000	0.5000
16	1	1	1	1	-0.5000	-0.5000	-0.5000	-0.5000

Table B.2. 4D-32CEM constellation mapping values.

Symbol	Bit Stream					Signal Vector			
						s_1	s_2	s_3	s_4
1	0	0	0	0	0	0.0000	0.5774	0.5774	0.5774
2	0	0	1	1	0	0.0000	0.5774	0.5774	-0.5774
3	0	0	1	0	1	0.0000	0.5774	-0.5774	0.5774
4	1	1	1	0	0	0.0000	0.5774	-0.5774	-0.5774
5	0	1	1	0	0	0.0000	-0.5774	0.5774	0.5774
6	1	0	1	0	1	0.0000	-0.5774	0.5774	-0.5774
7	1	0	1	1	0	0.0000	-0.5774	-0.5774	0.5774
8	1	1	1	1	1	0.0000	-0.5774	-0.5774	-0.5774
9	0	0	0	0	1	0.5774	0.0000	0.5774	0.5774
10	0	0	1	1	1	0.5774	0.0000	0.5774	-0.5774
11	0	1	1	0	1	0.5774	0.0000	-0.5774	0.5774
12	1	1	1	0	1	0.5774	0.0000	-0.5774	-0.5774
13	0	0	0	1	0	-0.5774	0.0000	0.5774	0.5774
14	0	1	1	1	0	-0.5774	0.0000	0.5774	-0.5774
15	0	1	0	1	1	-0.5774	0.0000	-0.5774	0.5774
16	1	0	1	1	1	-0.5774	0.0000	-0.5774	-0.5774
17	0	0	1	0	0	0.5774	0.5774	0.0000	0.5774
18	1	1	0	0	1	0.5774	0.5774	0.0000	-0.5774
19	0	1	0	1	0	0.5774	-0.5774	0.0000	0.5774
20	1	0	0	1	1	0.5774	-0.5774	0.0000	-0.5774
21	0	1	0	0	0	-0.5774	0.5774	0.0000	0.5774
22	1	1	0	1	0	-0.5774	0.5774	0.0000	-0.5774
23	0	1	0	0	1	-0.5774	-0.5774	0.0000	0.5774
24	0	1	1	1	1	-0.5774	-0.5774	0.0000	-0.5774

25	1	0	0	0	0	0.5774	0.5774	0.5774	0.0000
26	1	1	0	0	0	0.5774	0.5774	-0.5774	0.0000
27	1	0	1	0	0	0.5774	-0.5774	0.5774	0.0000
28	1	1	0	1	1	0.5774	-0.5774	-0.5774	0.0000
29	0	0	0	1	1	-0.5774	0.5774	0.5774	0.0000
30	1	0	0	1	0	-0.5774	0.5774	-0.5774	0.0000
31	1	0	0	0	1	-0.5774	-0.5774	0.5774	0.0000
32	1	1	1	1	0	-0.5774	-0.5774	-0.5774	0.0000

Table B.3. 4D-64CEM constellation mapping values.

Symbol	Bit Stream						Signal Vector			
							s_1	s_2	s_3	s_4
1	0	0	0	0	0	0	0.8126	0.3365	0.3365	0.3365
2	0	0	0	0	1	0	0.8126	0.3365	0.3365	-0.3365
3	0	0	0	0	1	1	0.8126	0.3365	-0.3365	0.3365
4	0	0	1	1	0	1	0.8126	0.3365	-0.3365	-0.3365
5	0	0	0	0	0	1	0.8126	-0.3365	0.3365	0.3365
6	0	0	1	1	0	0	0.8126	-0.3365	0.3365	-0.3365
7	0	0	0	1	0	0	0.8126	-0.3365	-0.3365	0.3365
8	0	1	1	0	0	0	0.8126	-0.3365	-0.3365	-0.3365
9	1	1	1	0	0	0	-0.8126	0.3365	0.3365	0.3365
10	1	0	0	1	0	0	-0.8126	0.3365	0.3365	-0.3365
11	1	0	1	1	0	0	-0.8126	0.3365	-0.3365	0.3365
12	1	0	0	0	0	1	-0.8126	0.3365	-0.3365	-0.3365
13	1	0	1	1	0	1	-0.8126	-0.3365	0.3365	0.3365
14	1	0	0	0	1	1	-0.8126	-0.3365	0.3365	-0.3365
15	1	0	0	0	1	0	-0.8126	-0.3365	-0.3365	0.3365
16	1	0	0	0	0	0	-0.8126	-0.3365	-0.3365	-0.3365
17	0	0	0	1	0	1	0.3365	0.8126	0.3365	0.3365
18	0	0	1	0	0	0	0.3365	0.8126	0.3365	-0.3365
19	0	0	1	0	0	1	0.3365	0.8126	-0.3365	0.3365
20	0	1	1	0	1	0	0.3365	0.8126	-0.3365	-0.3365
21	0	1	0	1	0	1	0.3365	-0.8126	0.3365	0.3365
22	1	1	0	1	1	1	0.3365	-0.8126	0.3365	-0.3365
23	1	1	0	1	1	0	0.3365	-0.8126	-0.3365	0.3365
24	1	1	1	1	1	0	0.3365	-0.8126	-0.3365	-0.3365

25	0	1	0	1	0	0	-0.3365	0.8126	0.3365	0.3365
26	1	1	0	0	1	0	-0.3365	0.8126	0.3365	-0.3365
27	1	1	0	0	1	1	-0.3365	0.8126	-0.3365	0.3365
28	1	1	1	1	1	1	-0.3365	0.8126	-0.3365	-0.3365
29	1	1	1	0	0	1	-0.3365	-0.8126	0.3365	0.3365
30	1	0	1	1	1	1	-0.3365	-0.8126	0.3365	-0.3365
31	1	0	1	1	1	0	-0.3365	-0.8126	-0.3365	0.3365
32	1	0	0	1	1	0	-0.3365	-0.8126	-0.3365	-0.3365
33	0	0	0	1	1	1	0.3365	0.3365	0.8126	0.3365
34	0	0	1	0	1	1	0.3365	0.3365	0.8126	-0.3365
35	0	1	1	1	0	0	0.3365	0.3365	-0.8126	0.3365
36	1	1	0	0	0	1	0.3365	0.3365	-0.8126	-0.3365
37	0	0	1	0	1	0	0.3365	-0.3365	0.8126	0.3365
38	0	1	1	0	1	1	0.3365	-0.3365	0.8126	-0.3365
39	1	1	0	0	0	0	0.3365	-0.3365	-0.8126	0.3365
40	1	1	1	1	0	1	0.3365	-0.3365	-0.8126	-0.3365
41	0	1	1	1	0	1	-0.3365	0.3365	0.8126	0.3365
42	0	1	0	0	0	0	-0.3365	0.3365	0.8126	-0.3365
43	1	1	1	0	1	1	-0.3365	0.3365	-0.8126	0.3365
44	1	0	1	0	1	0	-0.3365	0.3365	-0.8126	-0.3365
45	0	1	0	0	0	1	-0.3365	-0.3365	0.8126	0.3365
46	1	1	1	1	0	0	-0.3365	-0.3365	0.8126	-0.3365
47	1	0	1	0	1	1	-0.3365	-0.3365	-0.8126	0.3365
48	1	0	0	1	1	1	-0.3365	-0.3365	-0.8126	-0.3365
49	0	0	0	1	1	0	0.3365	0.3365	0.3365	0.8126
50	0	1	1	1	1	1	0.3365	0.3365	0.3365	-0.8126
51	0	0	1	1	1	0	0.3365	0.3365	-0.3365	0.8126
52	0	1	0	0	1	1	0.3365	0.3365	-0.3365	-0.8126

53	0	0	1	1	1	1	0.3365	-0.3365	0.3365	0.8126
54	0	1	0	0	1	0	0.3365	-0.3365	0.3365	-0.8126
55	0	1	1	0	0	1	0.3365	-0.3365	-0.3365	0.8126
56	1	1	0	1	0	0	0.3365	-0.3365	-0.3365	-0.8126
57	0	1	1	1	1	0	-0.3365	0.3365	0.3365	0.8126
58	1	1	1	0	1	0	-0.3365	0.3365	0.3365	-0.8126
59	0	1	0	1	1	0	-0.3365	0.3365	-0.3365	0.8126
60	1	0	1	0	0	1	-0.3365	0.3365	-0.3365	-0.8126
61	0	1	0	1	1	1	-0.3365	-0.3365	0.3365	0.8126
62	1	0	1	0	0	0	-0.3365	-0.3365	0.3365	-0.8126
63	1	1	0	1	0	1	-0.3365	-0.3365	-0.3365	0.8126
64	1	0	0	1	0	1	-0.3365	-0.3365	-0.3365	-0.8126

Table B.4. 5D-64CEM constellation mapping values.

Symbol	Bit Stream						Signal Vector				
							s_1	s_2	s_3	s_4	s_5
1	0	0	0	0	0	0	0.5000	0.5000	0.5000	0.5000	0.0000
2	0	0	0	1	0	1	0.5000	0.5000	0.5000	-0.5000	0.0000
3	0	0	1	1	0	0	0.5000	0.5000	-0.5000	0.5000	0.0000
4	1	0	1	0	1	0	0.5000	0.5000	-0.5000	-0.5000	0.0000
5	0	0	1	0	1	0	0.5000	-0.5000	0.5000	0.5000	0.0000
6	1	0	1	1	0	0	0.5000	-0.5000	0.5000	-0.5000	0.0000
7	1	0	0	1	0	1	0.5000	-0.5000	-0.5000	0.5000	0.0000
8	0	0	1	0	0	1	-0.5000	0.5000	0.5000	0.5000	0.0000
9	1	0	0	1	1	0	-0.5000	0.5000	0.5000	-0.5000	0.0000
10	1	0	0	0	1	1	-0.5000	0.5000	-0.5000	0.5000	0.0000
11	0	0	1	1	1	1	-0.5000	-0.5000	0.5000	0.5000	0.0000
12	0	0	0	0	0	1	0.0000	0.5000	0.5000	0.5000	0.5000
13	0	0	0	0	1	0	0.0000	0.5000	0.5000	0.5000	-0.5000
14	0	1	1	0	0	0	0.0000	0.5000	0.5000	-0.5000	0.5000
15	0	1	0	1	0	0	0.0000	0.5000	0.5000	-0.5000	-0.5000
16	0	1	0	0	1	0	0.0000	0.5000	-0.5000	0.5000	0.5000
17	0	1	0	0	0	1	0.0000	0.5000	-0.5000	0.5000	-0.5000
18	0	1	1	0	1	1	0.0000	0.5000	-0.5000	-0.5000	0.5000
19	0	1	1	1	1	0	0.0000	0.5000	-0.5000	-0.5000	-0.5000
20	1	1	0	0	0	0	0.0000	-0.5000	0.5000	0.5000	0.5000
21	1	0	1	0	0	0	0.0000	-0.5000	0.5000	0.5000	-0.5000
22	0	1	1	1	0	1	0.0000	-0.5000	0.5000	-0.5000	0.5000
23	0	1	0	1	1	1	0.0000	-0.5000	0.5000	-0.5000	-0.5000
24	1	1	0	0	1	1	0.0000	-0.5000	-0.5000	0.5000	0.5000

25	1	1	0	1	1	0	0.0000	-0.5000	-0.5000	0.5000	-0.5000
26	0	0	0	1	0	0	0.5000	0.0000	0.5000	0.5000	0.5000
27	1	0	0	1	0	0	-0.5000	0.0000	0.5000	0.5000	0.5000
28	0	0	1	0	0	0	0.5000	0.0000	0.5000	0.5000	-0.5000
29	1	0	0	0	1	0	-0.5000	0.0000	0.5000	0.5000	-0.5000
30	1	0	0	0	0	1	0.5000	0.0000	0.5000	-0.5000	0.5000
31	1	1	0	1	0	1	-0.5000	0.0000	0.5000	-0.5000	0.5000
32	0	0	0	1	1	1	0.5000	0.0000	0.5000	-0.5000	-0.5000
33	1	1	1	1	0	0	-0.5000	0.0000	0.5000	-0.5000	-0.5000
34	0	0	1	1	0	1	0.5000	0.0000	-0.5000	0.5000	0.5000
35	1	1	1	0	1	0	-0.5000	0.0000	-0.5000	0.5000	0.5000
36	0	0	1	1	1	0	0.5000	0.0000	-0.5000	0.5000	-0.5000
37	1	1	1	0	0	1	-0.5000	0.0000	-0.5000	0.5000	-0.5000
38	1	0	1	0	1	1	0.5000	0.0000	-0.5000	-0.5000	0.5000
39	1	0	1	1	1	0	0.5000	0.0000	-0.5000	-0.5000	-0.5000
40	0	1	0	0	0	0	0.5000	0.5000	0.0000	0.5000	0.5000
41	0	0	1	0	1	1	0.5000	-0.5000	0.0000	0.5000	0.5000
42	0	1	1	0	0	1	-0.5000	0.5000	0.0000	0.5000	0.5000
43	1	0	1	1	0	1	-0.5000	-0.5000	0.0000	0.5000	0.5000
44	1	0	0	0	0	0	0.5000	0.5000	0.0000	0.5000	-0.5000
45	0	1	1	0	1	0	0.5000	-0.5000	0.0000	0.5000	-0.5000
46	0	1	1	1	0	0	-0.5000	0.5000	0.0000	0.5000	-0.5000
47	1	0	0	1	1	1	-0.5000	-0.5000	0.0000	0.5000	-0.5000
48	0	1	0	1	0	1	0.5000	0.5000	0.0000	-0.5000	0.5000
49	0	1	1	1	1	1	0.5000	-0.5000	0.0000	-0.5000	0.5000
50	1	1	0	1	1	1	-0.5000	0.5000	0.0000	-0.5000	0.5000
51	0	1	0	1	1	0	0.5000	0.5000	0.0000	-0.5000	-0.5000
52	1	1	1	1	0	1	0.5000	-0.5000	0.0000	-0.5000	-0.5000

53	1	1	1	1	1	0	-0.5000	0.5000	0.0000	-0.5000	-0.5000
54	0	0	0	0	1	1	0.5000	0.5000	0.5000	0.0000	0.5000
55	0	1	0	0	1	1	0.5000	0.5000	-0.5000	0.0000	0.5000
56	1	1	0	0	0	1	0.5000	-0.5000	0.5000	0.0000	0.5000
57	1	1	1	0	1	1	0.5000	-0.5000	-0.5000	0.0000	0.5000
58	1	1	0	0	1	0	-0.5000	0.5000	0.5000	0.0000	0.5000
59	1	0	1	1	1	1	-0.5000	0.5000	-0.5000	0.0000	0.5000
60	1	1	1	1	1	1	-0.5000	-0.5000	0.5000	0.0000	0.5000
61	0	0	0	1	1	0	0.5000	0.5000	0.5000	0.0000	-0.5000
62	1	1	0	1	0	0	0.5000	0.5000	-0.5000	0.0000	-0.5000
63	1	1	1	0	0	0	0.5000	-0.5000	0.5000	0.0000	-0.5000
64	1	0	1	0	0	1	-0.5000	0.5000	0.5000	0.0000	-0.5000

Table B.5. 5D-128CEM constellation mapping values.

Symbol	Bit Stream							Signal Vector				
								s_1	s_2	s_3	s_4	s_5
1	0	0	0	0	0	0	0	0.7700	0.3190	0.3190	0.3190	0.3190
2	0	0	0	0	0	0	1	0.3190	0.7700	0.3190	0.3190	0.3190
3	0	0	0	0	0	1	0	0.3190	0.3190	0.7700	0.3190	0.3190
4	0	0	0	0	1	0	0	0.3190	0.3190	0.3190	0.7700	0.3190
5	0	0	0	1	0	0	0	0.3190	0.3190	0.3190	0.3190	0.7700
6	0	0	1	0	0	0	0	0.7700	0.3190	0.3190	0.3190	-0.3190
7	0	1	0	0	0	0	0	0.7700	0.3190	0.3190	-0.3190	0.3190
8	1	0	0	0	0	0	0	0.7700	0.3190	-0.3190	0.3190	0.3190
9	0	0	0	0	0	1	1	0.7700	-0.3190	0.3190	0.3190	0.3190
10	0	0	0	0	1	1	0	0.3190	0.7700	0.3190	0.3190	-0.3190
11	0	0	0	0	1	0	1	0.3190	0.7700	0.3190	-0.3190	0.3190
12	0	0	0	1	1	0	0	0.3190	0.7700	-0.3190	0.3190	0.3190
13	0	0	0	1	0	1	0	0.3190	0.3190	0.7700	0.3190	-0.3190
14	0	0	0	1	0	0	1	0.3190	0.3190	0.7700	-0.3190	0.3190
15	0	0	1	1	0	0	0	0.3190	-0.3190	0.7700	0.3190	0.3190
16	0	0	1	0	1	0	0	0.3190	0.3190	0.3190	0.7700	-0.3190
17	0	0	1	0	0	1	0	0.3190	0.3190	-0.3190	0.7700	0.3190
18	0	0	1	0	0	0	1	0.3190	-0.3190	0.3190	0.7700	0.3190
19	0	1	1	0	0	0	0	0.3190	0.3190	0.3190	-0.3190	0.7700
20	0	1	0	1	0	0	0	0.3190	0.3190	-0.3190	0.3190	0.7700
21	0	1	0	0	1	0	0	0.3190	-0.3190	0.3190	0.3190	0.7700
22	0	1	0	0	0	1	0	0.7700	0.3190	0.3190	-0.3190	-0.3190
23	0	1	0	0	0	0	1	0.7700	0.3190	-0.3190	0.3190	-0.3190
24	1	1	0	0	0	0	0	0.7700	0.3190	-0.3190	-0.3190	0.3190

25	1	0	1	0	0	0	0	0.7700	-0.3190	0.3190	0.3190	-0.3190
26	1	0	0	1	0	0	0	0.7700	-0.3190	0.3190	-0.3190	0.3190
27	1	0	0	0	1	0	0	0.7700	-0.3190	-0.3190	0.3190	0.3190
28	1	0	0	0	0	1	0	0.3190	0.7700	0.3190	-0.3190	-0.3190
29	1	0	0	0	0	0	1	0.3190	0.7700	-0.3190	0.3190	-0.3190
30	0	0	0	0	1	1	1	0.3190	0.7700	-0.3190	-0.3190	0.3190
31	0	0	0	1	1	0	1	0.3190	0.3190	0.7700	-0.3190	-0.3190
32	0	0	0	1	1	1	0	0.3190	-0.3190	0.7700	0.3190	-0.3190
33	0	0	0	1	0	1	1	0.3190	-0.3190	0.7700	-0.3190	0.3190
34	0	0	1	1	0	0	1	0.3190	0.3190	-0.3190	0.7700	-0.3190
35	0	0	1	1	0	1	0	0.3190	-0.3190	0.3190	0.7700	-0.3190
36	0	0	1	1	1	0	0	0.3190	-0.3190	-0.3190	0.7700	0.3190
37	0	0	1	0	1	0	1	0.3190	0.3190	-0.3190	-0.3190	0.7700
38	0	0	1	0	1	1	0	0.3190	-0.3190	0.3190	-0.3190	0.7700
39	0	0	1	0	0	1	1	0.3190	-0.3190	-0.3190	0.3190	0.7700
40	0	1	1	0	0	0	1	0.7700	0.3190	-0.3190	-0.3190	-0.3190
41	0	1	1	0	0	1	0	0.7700	-0.3190	0.3190	-0.3190	-0.3190
42	0	1	1	0	1	0	0	0.7700	-0.3190	-0.3190	0.3190	-0.3190
43	0	1	1	1	0	0	0	0.7700	-0.3190	-0.3190	-0.3190	0.3190
44	0	1	0	1	0	0	1	0.3190	-0.7700	0.3190	0.3190	0.3190
45	0	1	0	1	0	1	0	-0.3190	0.7700	0.3190	0.3190	0.3190
46	0	1	0	1	1	0	0	0.3190	0.3190	-0.7700	0.3190	0.3190
47	0	1	0	0	1	0	1	-0.3190	0.3190	0.7700	0.3190	0.3190
48	0	1	0	0	1	1	0	0.3190	0.3190	0.3190	-0.7700	0.3190
49	0	1	0	0	0	1	1	-0.3190	0.3190	0.3190	0.7700	0.3190
50	1	1	0	0	0	0	1	0.3190	0.3190	0.3190	0.3190	-0.7700
51	1	1	0	0	0	1	0	-0.3190	0.3190	0.3190	0.3190	0.7700
52	1	1	0	0	1	0	0	0.3190	0.7700	-0.3190	-0.3190	-0.3190

53	1	1	0	1	0	0	0	0.3190	-0.3190	0.7700	-0.3190	-0.3190
54	1	1	1	0	0	0	0	0.3190	-0.3190	-0.3190	0.7700	-0.3190
55	1	0	1	0	0	0	1	0.3190	-0.3190	-0.3190	-0.3190	0.7700
56	1	0	1	0	0	1	0	0.7700	-0.3190	-0.3190	-0.3190	-0.3190
57	1	0	1	0	1	0	0	0.3190	-0.7700	0.3190	0.3190	-0.3190
58	1	0	1	1	0	0	0	0.3190	-0.7700	0.3190	-0.3190	0.3190
59	1	0	0	1	0	0	1	0.3190	-0.7700	-0.3190	0.3190	0.3190
60	1	0	0	1	0	1	0	-0.3190	0.7700	0.3190	0.3190	-0.3190
61	1	0	0	1	1	0	0	-0.3190	0.7700	0.3190	-0.3190	0.3190
62	1	0	0	0	1	0	1	-0.3190	0.7700	-0.3190	0.3190	0.3190
63	1	0	0	0	1	1	0	0.3190	0.3190	-0.7700	0.3190	-0.3190
64	1	0	0	0	0	1	1	0.3190	0.3190	-0.7700	-0.3190	0.3190
65	0	0	0	1	1	1	1	0.3190	-0.3190	-0.7700	0.3190	0.3190
66	0	0	1	1	0	1	1	-0.3190	0.3190	0.7700	0.3190	-0.3190
67	0	0	1	1	1	1	0	-0.3190	0.3190	0.7700	-0.3190	0.3190
68	0	0	1	1	1	0	1	-0.3190	-0.3190	0.7700	0.3190	0.3190
69	0	0	1	0	1	1	1	0.3190	0.3190	0.3190	-0.7700	-0.3190
70	0	1	1	0	0	1	1	0.3190	0.3190	-0.3190	-0.7700	0.3190
71	0	1	1	0	1	1	0	0.3190	-0.3190	0.3190	-0.7700	0.3190
72	0	1	1	0	1	0	1	-0.3190	0.3190	0.3190	0.7700	-0.3190
73	0	1	1	1	1	0	0	-0.3190	0.3190	-0.3190	0.7700	0.3190
74	0	1	1	1	0	1	0	-0.3190	-0.3190	0.3190	0.7700	0.3190
75	0	1	1	1	0	0	1	0.3190	0.3190	0.3190	-0.3190	-0.7700
76	0	1	0	1	0	1	1	0.3190	0.3190	-0.3190	0.3190	-0.7700
77	0	1	0	1	1	1	0	0.3190	-0.3190	0.3190	0.3190	-0.7700
78	0	1	0	1	1	0	1	-0.3190	0.3190	0.3190	-0.3190	0.7700
79	0	1	0	0	1	1	1	-0.3190	0.3190	-0.3190	0.3190	0.7700
80	1	1	0	0	0	1	1	-0.3190	-0.3190	0.3190	0.3190	0.7700

81	1	1	0	0	1	1	0	0.3190	-0.7700	0.3190	-0.3190	-0.3190
82	1	1	0	0	1	0	1	0.3190	-0.7700	-0.3190	0.3190	-0.3190
83	1	1	0	1	1	0	0	0.3190	-0.7700	-0.3190	-0.3190	0.3190
84	1	1	0	1	0	1	0	-0.3190	0.7700	0.3190	-0.3190	-0.3190
85	1	1	0	1	0	0	1	-0.3190	0.7700	-0.3190	0.3190	-0.3190
86	1	1	1	1	0	0	0	-0.3190	0.7700	-0.3190	-0.3190	0.3190
87	1	1	1	0	1	0	0	0.3190	0.3190	-0.7700	-0.3190	-0.3190
88	1	1	1	0	0	1	0	0.3190	-0.3190	-0.7700	0.3190	-0.3190
89	1	1	1	0	0	0	1	0.3190	-0.3190	-0.7700	-0.3190	0.3190
90	1	0	1	0	0	1	1	-0.3190	0.3190	0.7700	-0.3190	-0.3190
91	1	0	1	0	1	1	0	-0.3190	-0.3190	0.7700	0.3190	-0.3190
92	1	0	1	0	1	0	1	-0.3190	-0.3190	0.7700	-0.3190	0.3190
93	1	0	1	1	1	0	0	0.3190	0.3190	-0.3190	-0.7700	-0.3190
94	1	0	1	1	0	1	0	0.3190	-0.3190	0.3190	-0.7700	-0.3190
95	1	0	1	1	0	0	1	0.3190	-0.3190	-0.3190	-0.7700	0.3190
96	1	0	0	1	0	1	1	-0.3190	0.3190	-0.3190	0.7700	-0.3190
97	1	0	0	1	1	1	0	-0.3190	-0.3190	0.3190	0.7700	-0.3190
98	1	0	0	1	1	0	1	-0.3190	-0.3190	-0.3190	0.7700	0.3190
99	1	0	0	0	1	1	1	0.3190	0.3190	-0.3190	-0.3190	-0.7700
100	0	0	1	1	1	1	1	0.3190	-0.3190	0.3190	-0.3190	-0.7700
101	0	1	1	0	1	1	1	0.3190	-0.3190	-0.3190	0.3190	-0.7700
102	0	1	1	1	1	0	1	-0.3190	0.3190	-0.3190	-0.3190	0.7700
103	0	1	1	1	1	1	0	-0.3190	-0.3190	0.3190	-0.3190	0.7700
104	0	1	1	1	0	1	1	-0.3190	-0.3190	-0.3190	0.3190	0.7700
105	0	1	0	1	1	1	1	-0.7700	0.3190	0.3190	0.3190	0.3190
106	1	1	0	0	1	1	1	-0.3190	-0.7700	0.3190	0.3190	0.3190
107	1	1	0	1	1	0	1	-0.3190	0.3190	-0.7700	0.3190	0.3190
108	1	1	0	1	1	1	0	-0.3190	0.3190	0.3190	-0.7700	0.3190

109	1	1	0	1	0	1	1	-0.3190	0.3190	0.3190	0.3190	-0.7700
110	1	1	1	1	0	0	1	0.3190	-0.7700	-0.3190	-0.3190	-0.3190
111	1	1	1	1	0	1	0	-0.3190	0.7700	-0.3190	-0.3190	-0.3190
112	1	1	1	1	1	0	0	0.3190	-0.3190	-0.7700	-0.3190	-0.3190
113	1	1	1	0	1	0	1	-0.3190	-0.3190	0.7700	-0.3190	-0.3190
114	1	1	1	0	1	1	0	0.3190	-0.3190	-0.3190	-0.7700	-0.3190
115	1	1	1	0	0	1	1	-0.3190	-0.3190	-0.3190	0.7700	-0.3190
116	1	0	1	0	1	1	1	0.3190	-0.3190	-0.3190	-0.3190	-0.7700
117	1	0	1	1	1	0	1	-0.3190	-0.3190	-0.3190	-0.3190	0.7700
118	1	0	1	1	1	1	0	-0.7700	0.3190	0.3190	0.3190	-0.3190
119	1	0	1	1	0	1	1	-0.7700	0.3190	0.3190	-0.3190	0.3190
120	1	0	0	1	1	1	1	-0.7700	0.3190	-0.3190	0.3190	0.3190
121	0	1	1	1	1	1	1	-0.7700	-0.3190	0.3190	0.3190	0.3190
122	1	1	0	1	1	1	1	-0.3190	-0.7700	0.3190	0.3190	-0.3190
123	1	1	1	1	0	1	1	-0.3190	-0.7700	0.3190	-0.3190	0.3190
124	1	1	1	1	1	1	0	-0.3190	-0.7700	-0.3190	0.3190	0.3190
125	1	1	1	1	1	0	1	-0.3190	0.3190	-0.7700	0.3190	-0.3190
126	1	1	1	0	1	1	1	-0.3190	0.3190	-0.7700	-0.3190	0.3190
127	1	0	1	1	1	1	1	-0.3190	-0.3190	-0.7700	0.3190	0.3190
128	1	1	1	1	1	1	1	-0.3190	0.3190	0.3190	-0.7700	-0.3190

Table B.6. 5D-256CEM constellation mapping values.

Symbol	Bit Stream								Signal Vector				
									s_1	s_2	s_3	s_4	s_5
1	0	0	0	0	0	0	0	0	0.7560	0.3779	0.3779	0.3779	0.0000
2	0	0	0	0	0	0	0	1	0.7560	0.0000	0.3779	0.3779	0.3779
3	0	0	0	0	0	0	1	0	0.7560	0.0000	0.3779	0.3779	-0.3779
4	0	0	0	0	0	1	0	0	0.7560	0.3779	0.0000	0.3779	0.3779
5	0	0	0	0	1	0	0	0	0.7560	0.3779	0.0000	0.3779	-0.3779
6	0	0	0	1	0	0	0	0	0.7560	0.3779	0.3779	0.0000	0.3779
7	0	0	1	0	0	0	0	0	0.7560	0.3779	0.3779	0.0000	-0.3779
8	0	1	0	0	0	0	0	0	0.3779	0.7560	0.3779	0.3779	0.0000
9	1	0	0	0	0	0	0	0	0.3779	0.3779	0.7560	0.3779	0.0000
10	0	0	0	0	0	0	1	1	0.3779	0.3779	0.3779	0.7560	0.0000
11	0	0	0	0	0	1	1	0	0.7560	0.3779	0.3779	-0.3779	0.0000
12	0	0	0	0	0	1	0	1	0.7560	0.3779	-0.3779	0.3779	0.0000
13	0	0	0	0	1	1	0	0	0.7560	-0.3779	0.3779	0.3779	0.0000
14	0	0	0	0	1	0	1	0	0.3779	0.0000	0.7560	0.3779	0.3779
15	0	0	0	0	1	0	0	1	0.3779	0.0000	0.7560	0.3779	-0.3779
16	0	0	0	1	1	0	0	0	0.3779	0.0000	0.3779	0.7560	0.3779
17	0	0	0	1	0	1	0	0	0.3779	0.0000	0.3779	0.7560	-0.3779
18	0	0	0	1	0	0	1	0	0.3779	0.3779	0.0000	0.7560	0.3779
19	0	0	0	1	0	0	0	1	0.3779	0.3779	0.0000	0.7560	-0.3779
20	0	0	1	1	0	0	0	0	0.3779	0.7560	0.0000	0.3779	0.3779
21	0	0	1	0	1	0	0	0	0.3779	0.7560	0.0000	0.3779	-0.3779
22	0	0	1	0	0	1	0	0	0.3779	0.7560	0.3779	0.0000	0.3779
23	0	0	1	0	0	0	1	0	0.3779	0.7560	0.3779	0.0000	-0.3779
24	0	0	1	0	0	0	0	1	0.3779	0.3779	0.7560	0.0000	0.3779

25	0	1	1	0	0	0	0	0	0.3779	0.3779	0.7560	0.0000	-0.3779
26	0	1	0	1	0	0	0	0	0.7560	0.0000	0.3779	-0.3779	0.3779
27	0	1	0	0	1	0	0	0	0.7560	0.0000	0.3779	-0.3779	-0.3779
28	0	1	0	0	0	1	0	0	0.7560	0.0000	-0.3779	0.3779	0.3779
29	0	1	0	0	0	0	1	0	0.7560	0.0000	-0.3779	0.3779	-0.3779
30	0	1	0	0	0	0	0	1	0.7560	-0.3779	0.0000	0.3779	0.3779
31	1	1	0	0	0	0	0	0	0.7560	-0.3779	0.0000	0.3779	-0.3779
32	1	0	1	0	0	0	0	0	0.7560	0.3779	0.0000	-0.3779	0.3779
33	1	0	0	1	0	0	0	0	0.7560	0.3779	0.0000	-0.3779	-0.3779
34	1	0	0	0	1	0	0	0	0.7560	0.3779	-0.3779	0.0000	0.3779
35	1	0	0	0	0	1	0	0	0.7560	-0.3779	0.3779	0.0000	0.3779
36	1	0	0	0	0	0	1	0	0.7560	0.3779	-0.3779	0.0000	-0.3779
37	1	0	0	0	0	0	0	1	0.7560	-0.3779	0.3779	0.0000	-0.3779
38	0	0	0	0	0	1	1	1	0.3779	0.7560	0.3779	-0.3779	0.0000
39	0	0	0	0	1	1	0	1	0.3779	0.7560	-0.3779	0.3779	0.0000
40	0	0	0	0	1	1	1	0	0.3779	0.3779	0.7560	-0.3779	0.0000
41	0	0	0	0	1	0	1	1	0.3779	-0.3779	0.7560	0.3779	0.0000
42	0	0	0	1	1	0	0	1	0.3779	0.3779	-0.3779	0.7560	0.0000
43	0	0	0	1	1	0	1	0	0.3779	-0.3779	0.3779	0.7560	0.0000
44	0	0	0	1	1	1	0	0	0.0000	0.7560	0.3779	0.3779	0.3779
45	0	0	0	1	0	1	0	1	0.0000	0.7560	0.3779	0.3779	-0.3779
46	0	0	0	1	0	1	1	0	0.0000	0.3779	0.7560	0.3779	0.3779
47	0	0	0	1	0	0	1	1	0.0000	0.3779	0.7560	0.3779	-0.3779
48	0	0	1	1	0	0	0	1	0.0000	0.3779	0.3779	0.7560	0.3779
49	0	0	1	1	0	0	1	0	0.0000	0.3779	0.3779	0.7560	-0.3779
50	0	0	1	1	0	1	0	0	0.3779	0.0000	0.3779	0.3779	0.7560
51	0	0	1	1	1	0	0	0	0.3779	0.0000	0.3779	0.3779	-0.7560
52	0	0	1	0	1	0	0	1	0.3779	0.3779	0.0000	0.3779	0.7560

53	0	0	1	0	1	0	1	0	0.3779	0.3779	0.0000	0.3779	-0.7560
54	0	0	1	0	1	1	0	0	0.3779	0.3779	0.3779	0.0000	0.7560
55	0	0	1	0	0	1	0	1	0.3779	0.3779	0.3779	0.0000	-0.7560
56	0	0	1	0	0	1	1	0	0.7560	0.3779	-0.3779	-0.3779	0.0000
57	0	0	1	0	0	0	1	1	0.7560	-0.3779	0.3779	-0.3779	0.0000
58	0	1	1	0	0	0	0	1	0.7560	-0.3779	-0.3779	0.3779	0.0000
59	0	1	1	0	0	0	1	0	0.3779	0.0000	0.7560	-0.3779	0.3779
60	0	1	1	0	0	1	0	0	0.3779	0.0000	0.7560	-0.3779	-0.3779
61	0	1	1	0	1	0	0	0	0.3779	0.0000	-0.3779	0.7560	0.3779
62	0	1	1	1	0	0	0	0	0.3779	0.0000	-0.3779	0.7560	-0.3779
63	0	1	0	1	0	0	0	1	0.3779	-0.3779	0.0000	0.7560	0.3779
64	0	1	0	1	0	0	1	0	0.3779	-0.3779	0.0000	0.7560	-0.3779
65	0	1	0	1	0	1	0	0	0.3779	0.7560	0.0000	-0.3779	0.3779
66	0	1	0	1	1	0	0	0	0.3779	0.7560	0.0000	-0.3779	-0.3779
67	0	1	0	0	1	0	0	1	0.3779	0.7560	-0.3779	0.0000	0.3779
68	0	1	0	0	1	0	1	0	0.3779	0.7560	-0.3779	0.0000	-0.3779
69	0	1	0	0	1	1	0	0	0.3779	-0.3779	0.7560	0.0000	0.3779
70	0	1	0	0	0	1	0	1	0.3779	-0.3779	0.7560	0.0000	-0.3779
71	0	1	0	0	0	1	1	0	0.0000	0.3779	0.3779	0.3779	0.7560
72	0	1	0	0	0	0	1	1	0.0000	0.3779	0.3779	0.3779	-0.7560
73	1	1	0	0	0	0	0	1	0.7560	0.0000	-0.3779	-0.3779	0.3779
74	1	1	0	0	0	0	1	0	0.7560	0.0000	-0.3779	-0.3779	-0.3779
75	1	1	0	0	0	1	0	0	0.7560	-0.3779	0.0000	-0.3779	0.3779
76	1	1	0	0	1	0	0	0	0.7560	-0.3779	0.0000	-0.3779	-0.3779
77	1	1	0	1	0	0	0	0	0.7560	-0.3779	-0.3779	0.0000	0.3779
78	1	1	1	0	0	0	0	0	0.7560	-0.3779	-0.3779	0.0000	-0.3779
79	1	0	1	0	0	0	0	1	0.3779	0.7560	-0.3779	-0.3779	0.0000
80	1	0	1	0	0	0	1	0	0.3779	-0.3779	0.7560	-0.3779	0.0000

81	1	0	1	0	0	1	0	0	0.3779	-0.3779	-0.3779	0.7560	0.0000
82	1	0	1	0	1	0	0	0	0.0000	-0.3779	0.7560	0.3779	0.3779
83	1	0	1	1	0	0	0	0	0.0000	-0.3779	0.7560	0.3779	-0.3779
84	1	0	0	1	0	0	0	1	0.0000	0.3779	-0.3779	0.7560	0.3779
85	1	0	0	1	0	0	1	0	0.0000	0.3779	-0.3779	0.7560	-0.3779
86	1	0	0	1	0	1	0	0	0.0000	-0.3779	0.3779	0.7560	0.3779
87	1	0	0	1	1	0	0	0	0.0000	-0.3779	0.3779	0.7560	-0.3779
88	1	0	0	0	1	0	0	1	0.3779	0.0000	0.3779	-0.3779	0.7560
89	1	0	0	0	1	0	1	0	0.3779	0.0000	0.3779	-0.3779	-0.7560
90	1	0	0	0	1	1	0	0	0.3779	0.0000	-0.3779	0.3779	0.7560
91	1	0	0	0	0	1	0	1	0.3779	0.0000	-0.3779	0.3779	-0.7560
92	1	0	0	0	0	1	1	0	0.3779	-0.3779	0.0000	0.3779	0.7560
93	1	0	0	0	0	0	1	1	0.3779	-0.3779	0.0000	0.3779	-0.7560
94	0	0	0	0	1	1	1	1	0.3779	0.3779	0.0000	-0.3779	0.7560
95	0	0	0	1	1	0	1	1	0.3779	0.3779	0.0000	-0.3779	-0.7560
96	0	0	0	1	1	1	1	0	0.3779	0.3779	-0.3779	0.0000	0.7560
97	0	0	0	1	1	1	0	1	0.3779	-0.3779	0.3779	0.0000	0.7560
98	0	0	0	1	0	1	1	1	0.3779	0.3779	-0.3779	0.0000	-0.7560
99	0	0	1	1	0	0	1	1	0.3779	-0.3779	0.3779	0.0000	-0.7560
100	0	0	1	1	0	1	1	0	0.0000	0.7560	0.3779	-0.3779	0.3779
101	0	0	1	1	0	1	0	1	0.0000	0.7560	0.3779	-0.3779	-0.3779
102	0	0	1	1	1	1	0	0	0.0000	0.7560	-0.3779	0.3779	0.3779
103	0	0	1	1	1	0	1	0	0.0000	0.7560	-0.3779	0.3779	-0.3779
104	0	0	1	1	1	0	0	1	0.0000	0.3779	0.7560	-0.3779	0.3779
105	0	0	1	0	1	0	1	1	0.0000	0.3779	0.7560	-0.3779	-0.3779
106	0	0	1	0	1	1	1	0	0.3779	-0.7560	0.3779	0.3779	0.0000
107	0	0	1	0	1	1	0	1	-0.3779	0.7560	0.3779	0.3779	0.0000
108	0	0	1	0	0	1	1	1	0.3779	0.3779	-0.7560	0.3779	0.0000

109	0	1	1	0	0	0	1	1	-0.3779	0.3779	0.7560	0.3779	0.0000
110	0	1	1	0	0	1	1	0	0.3779	0.3779	0.3779	-0.7560	0.0000
111	0	1	1	0	0	1	0	1	-0.3779	0.3779	0.3779	0.7560	0.0000
112	0	1	1	0	1	1	0	0	0.7560	-0.3779	-0.3779	-0.3779	0.0000
113	0	1	1	0	1	0	1	0	0.0000	0.3779	0.3779	-0.3779	0.7560
114	0	1	1	0	1	0	0	1	0.0000	0.3779	0.3779	-0.3779	-0.7560
115	0	1	1	1	1	0	0	0	0.0000	0.3779	-0.3779	0.3779	0.7560
116	0	1	1	1	0	1	0	0	0.0000	0.3779	-0.3779	0.3779	-0.7560
117	0	1	1	1	0	0	1	0	0.0000	-0.3779	0.3779	0.3779	0.7560
118	0	1	1	1	0	0	0	1	0.0000	-0.3779	0.3779	0.3779	-0.7560
119	0	1	0	1	0	0	1	1	-0.3779	0.0000	0.7560	0.3779	0.3779
120	0	1	0	1	0	1	1	0	-0.3779	0.0000	0.7560	0.3779	-0.3779
121	0	1	0	1	0	1	0	1	0.3779	0.0000	-0.7560	0.3779	0.3779
122	0	1	0	1	1	1	0	0	0.3779	0.0000	-0.7560	0.3779	-0.3779
123	0	1	0	1	1	0	1	0	-0.3779	0.0000	0.3779	0.7560	0.3779
124	0	1	0	1	1	0	0	1	-0.3779	0.0000	0.3779	0.7560	-0.3779
125	0	1	0	0	1	0	1	1	0.3779	0.0000	0.3779	-0.7560	0.3779
126	0	1	0	0	1	1	1	0	0.3779	0.0000	0.3779	-0.7560	-0.3779
127	0	1	0	0	1	1	0	1	-0.3779	0.3779	0.0000	0.7560	0.3779
128	0	1	0	0	0	1	1	1	-0.3779	0.3779	0.0000	0.7560	-0.3779
129	1	1	0	0	0	0	1	1	0.3779	0.3779	0.0000	-0.7560	0.3779
130	1	1	0	0	0	1	1	0	0.3779	0.3779	0.0000	-0.7560	-0.3779
131	1	1	0	0	0	1	0	1	0.3779	-0.7560	0.0000	0.3779	0.3779
132	1	1	0	0	1	1	0	0	-0.3779	0.7560	0.0000	0.3779	0.3779
133	1	1	0	0	1	0	1	0	0.3779	-0.7560	0.0000	0.3779	-0.3779
134	1	1	0	0	1	0	0	1	-0.3779	0.7560	0.0000	0.3779	-0.3779
135	1	1	0	1	1	0	0	0	0.3779	-0.7560	0.3779	0.0000	0.3779
136	1	1	0	1	0	1	0	0	-0.3779	0.7560	0.3779	0.0000	0.3779

137	1	1	0	1	0	0	1	0	0.3779	-0.7560	0.3779	0.0000	-0.3779
138	1	1	0	1	0	0	0	1	-0.3779	0.7560	0.3779	0.0000	-0.3779
139	1	1	1	1	0	0	0	0	0.3779	0.3779	-0.7560	0.0000	0.3779
140	1	1	1	0	1	0	0	0	-0.3779	0.3779	0.7560	0.0000	0.3779
141	1	1	1	0	0	1	0	0	0.3779	0.3779	-0.7560	0.0000	-0.3779
142	1	1	1	0	0	0	1	0	-0.3779	0.3779	0.7560	0.0000	-0.3779
143	1	1	1	0	0	0	0	1	0.0000	-0.3779	0.7560	-0.3779	0.3779
144	1	0	1	0	0	0	1	1	0.0000	-0.3779	0.7560	-0.3779	-0.3779
145	1	0	1	0	0	1	1	0	0.0000	-0.3779	-0.3779	0.7560	0.3779
146	1	0	1	0	0	1	0	1	0.0000	-0.3779	-0.3779	0.7560	-0.3779
147	1	0	1	0	1	1	0	0	0.3779	0.0000	-0.3779	-0.3779	0.7560
148	1	0	1	0	1	0	1	0	0.3779	0.0000	-0.3779	-0.3779	-0.7560
149	1	0	1	0	1	0	0	1	0.3779	-0.3779	0.0000	-0.3779	0.7560
150	1	0	1	1	1	0	0	0	0.3779	-0.3779	0.0000	-0.3779	-0.7560
151	1	0	1	1	0	1	0	0	0.0000	0.7560	-0.3779	-0.3779	0.3779
152	1	0	1	1	0	0	1	0	0.0000	0.7560	-0.3779	-0.3779	-0.3779
153	1	0	1	1	0	0	0	1	0.3779	-0.3779	-0.3779	0.0000	0.7560
154	1	0	0	1	0	0	1	1	0.3779	-0.3779	-0.3779	0.0000	-0.7560
155	1	0	0	1	0	1	1	0	0.3779	-0.7560	0.3779	-0.3779	0.0000
156	1	0	0	1	0	1	0	1	0.3779	-0.7560	-0.3779	0.3779	0.0000
157	1	0	0	1	1	1	0	0	-0.3779	0.7560	0.3779	-0.3779	0.0000
158	1	0	0	1	1	0	1	0	-0.3779	0.7560	-0.3779	0.3779	0.0000
159	1	0	0	1	1	0	0	1	0.3779	0.3779	-0.7560	-0.3779	0.0000
160	1	0	0	0	1	0	1	1	-0.3779	0.3779	0.7560	-0.3779	0.0000
161	1	0	0	0	1	1	1	0	-0.3779	-0.3779	0.7560	0.3779	0.0000
162	1	0	0	0	1	1	0	1	-0.3779	0.3779	-0.3779	0.7560	0.0000
163	1	0	0	0	0	1	1	1	-0.3779	-0.3779	0.3779	0.7560	0.0000
164	0	0	0	1	1	1	1	1	0.3779	-0.3779	-0.7560	0.3779	0.0000

165	0	0	1	1	0	1	1	1	0.3779	0.3779	-0.3779	-0.7560	0.0000
166	0	0	1	1	1	1	0	1	0.3779	-0.3779	0.3779	-0.7560	0.0000
167	0	0	1	1	1	1	1	0	0.0000	-0.7560	0.3779	0.3779	0.3779
168	0	0	1	1	1	0	1	1	0.0000	-0.7560	0.3779	0.3779	-0.3779
169	0	0	1	0	1	1	1	1	0.0000	0.3779	-0.7560	0.3779	0.3779
170	0	1	1	0	0	1	1	1	0.0000	0.3779	-0.7560	0.3779	-0.3779
171	0	1	1	0	1	1	0	1	0.0000	0.3779	0.3779	-0.7560	0.3779
172	0	1	1	0	1	1	1	0	0.0000	0.3779	0.3779	-0.7560	-0.3779
173	0	1	1	0	1	0	1	1	-0.3779	0.0000	0.3779	0.3779	0.7560
174	0	1	1	1	1	0	0	1	-0.3779	0.0000	0.3779	0.3779	-0.7560
175	0	1	1	1	1	0	1	0	-0.3779	0.3779	0.0000	0.3779	0.7560
176	0	1	1	1	1	1	0	0	-0.3779	0.3779	0.0000	0.3779	-0.7560
177	0	1	1	1	0	1	0	1	-0.3779	0.3779	0.3779	0.0000	0.7560
178	0	1	1	1	0	1	1	0	-0.3779	0.3779	0.3779	0.0000	-0.7560
179	0	1	1	1	0	0	1	1	0.0000	0.3779	-0.3779	-0.3779	0.7560
180	0	1	0	1	0	1	1	1	0.0000	0.3779	-0.3779	-0.3779	-0.7560
181	0	1	0	1	1	1	0	1	0.0000	-0.3779	0.3779	-0.3779	0.7560
182	0	1	0	1	1	1	1	0	0.0000	-0.3779	0.3779	-0.3779	-0.7560
183	0	1	0	1	1	0	1	1	0.0000	-0.3779	-0.3779	0.3779	0.7560
184	0	1	0	0	1	1	1	1	0.0000	-0.3779	-0.3779	0.3779	-0.7560
185	1	1	0	0	0	1	1	1	-0.3779	0.0000	0.7560	-0.3779	0.3779
186	1	1	0	0	1	1	0	1	-0.3779	0.0000	0.7560	-0.3779	-0.3779
187	1	1	0	0	1	1	1	0	0.3779	0.0000	-0.7560	-0.3779	0.3779
188	1	1	0	0	1	0	1	1	0.3779	0.0000	-0.7560	-0.3779	-0.3779
189	1	1	0	1	1	0	0	1	-0.3779	0.0000	-0.3779	0.7560	0.3779
190	1	1	0	1	1	0	1	0	-0.3779	0.0000	-0.3779	0.7560	-0.3779
191	1	1	0	1	1	1	0	0	0.3779	0.0000	-0.3779	-0.7560	0.3779
192	1	1	0	1	0	1	0	1	0.3779	0.0000	-0.3779	-0.7560	-0.3779

193	1	1	0	1	0	1	1	0	-0.3779	-0.3779	0.0000	0.7560	0.3779
194	1	1	0	1	0	0	1	1	-0.3779	-0.3779	0.0000	0.7560	-0.3779
195	1	1	1	1	0	0	0	1	0.3779	-0.3779	0.0000	-0.7560	0.3779
196	1	1	1	1	0	0	1	0	0.3779	-0.3779	0.0000	-0.7560	-0.3779
197	1	1	1	1	0	1	0	0	0.3779	-0.7560	0.0000	-0.3779	0.3779
198	1	1	1	1	1	0	0	0	-0.3779	0.7560	0.0000	-0.3779	0.3779
199	1	1	1	0	1	0	0	1	0.3779	-0.7560	0.0000	-0.3779	-0.3779
200	1	1	1	0	1	0	1	0	-0.3779	0.7560	0.0000	-0.3779	-0.3779
201	1	1	1	0	1	1	0	0	0.3779	-0.7560	-0.3779	0.0000	0.3779
202	1	1	1	0	0	1	0	1	-0.3779	0.7560	-0.3779	0.0000	0.3779
203	1	1	1	0	0	1	1	0	0.3779	-0.7560	-0.3779	0.0000	-0.3779
204	1	1	1	0	0	0	1	1	-0.3779	0.7560	-0.3779	0.0000	-0.3779
205	1	0	1	0	0	1	1	1	0.3779	-0.3779	-0.7560	0.0000	0.3779
206	1	0	1	0	1	1	0	1	-0.3779	-0.3779	0.7560	0.0000	0.3779
207	1	0	1	0	1	1	1	0	0.3779	-0.3779	-0.7560	0.0000	-0.3779
208	1	0	1	0	1	0	1	1	-0.3779	-0.3779	0.7560	0.0000	-0.3779
209	1	0	1	1	1	0	0	1	-0.7560	0.3779	0.3779	0.3779	0.0000
210	1	0	1	1	1	0	1	0	0.3779	-0.7560	-0.3779	-0.3779	0.0000
211	1	0	1	1	1	1	0	0	-0.3779	0.7560	-0.3779	-0.3779	0.0000
212	1	0	1	1	0	1	0	1	0.3779	-0.3779	-0.7560	-0.3779	0.0000
213	1	0	1	1	0	1	1	0	-0.3779	-0.3779	0.7560	-0.3779	0.0000
214	1	0	1	1	0	0	1	1	0.3779	-0.3779	-0.3779	-0.7560	0.0000
215	1	0	0	1	0	1	1	1	-0.3779	-0.3779	-0.3779	0.7560	0.0000
216	1	0	0	1	1	1	0	1	0.0000	-0.7560	0.3779	-0.3779	0.3779
217	1	0	0	1	1	1	1	0	0.0000	-0.7560	0.3779	-0.3779	-0.3779
218	1	0	0	1	1	0	1	1	0.0000	-0.7560	-0.3779	0.3779	0.3779
219	1	0	0	0	1	1	1	1	0.0000	-0.7560	-0.3779	0.3779	-0.3779
220	0	0	1	1	1	1	1	1	0.0000	0.3779	-0.7560	-0.3779	0.3779

221	0	1	1	0	1	1	1	1	0.0000	0.3779	-0.7560	-0.3779	-0.3779
222	0	1	1	1	1	0	1	1	0.0000	-0.3779	-0.7560	0.3779	0.3779
223	0	1	1	1	1	1	1	0	0.0000	-0.3779	-0.7560	0.3779	-0.3779
224	0	1	1	1	1	1	0	1	0.0000	0.3779	-0.3779	-0.7560	0.3779
225	0	1	1	1	0	1	1	1	0.0000	0.3779	-0.3779	-0.7560	-0.3779
226	0	1	0	1	1	1	1	1	0.0000	-0.3779	0.3779	-0.7560	0.3779
227	1	1	0	0	1	1	1	1	0.0000	-0.3779	0.3779	-0.7560	-0.3779
228	1	1	0	1	1	0	1	1	-0.3779	0.0000	0.3779	-0.3779	0.7560
229	1	1	0	1	1	1	1	0	-0.3779	0.0000	0.3779	-0.3779	-0.7560
230	1	1	0	1	1	1	0	1	-0.3779	0.0000	-0.3779	0.3779	0.7560
231	1	1	0	1	0	1	1	1	-0.3779	0.0000	-0.3779	0.3779	-0.7560
232	1	1	1	1	0	0	1	1	-0.3779	-0.3779	0.0000	0.3779	0.7560
233	1	1	1	1	0	1	1	0	-0.3779	-0.3779	0.0000	0.3779	-0.7560
234	1	1	1	1	0	1	0	1	-0.3779	0.3779	0.0000	-0.3779	0.7560
235	1	1	1	1	1	1	0	0	-0.3779	0.3779	0.0000	-0.3779	-0.7560
236	1	1	1	1	1	0	1	0	-0.3779	0.3779	-0.3779	0.0000	0.7560
237	1	1	1	1	1	0	0	1	-0.3779	-0.3779	0.3779	0.0000	0.7560
238	1	1	1	0	1	0	1	1	-0.3779	0.3779	-0.3779	0.0000	-0.7560
239	1	1	1	0	1	1	1	0	-0.3779	-0.3779	0.3779	0.0000	-0.7560
240	1	1	1	0	1	1	0	1	-0.3779	-0.7560	0.3779	0.3779	0.0000
241	1	1	1	0	0	1	1	1	-0.3779	0.3779	-0.7560	0.3779	0.0000
242	1	0	1	0	1	1	1	1	-0.3779	0.3779	0.3779	-0.7560	0.0000
243	1	0	1	1	1	0	1	1	-0.7560	0.0000	0.3779	0.3779	0.3779
244	1	0	1	1	1	1	1	0	-0.7560	0.0000	0.3779	0.3779	-0.3779
245	1	0	1	1	1	1	0	1	-0.7560	0.3779	0.0000	0.3779	0.3779
246	1	0	1	1	0	1	1	1	-0.7560	0.3779	0.0000	0.3779	-0.3779
247	1	0	0	1	1	1	1	1	-0.7560	0.3779	0.3779	0.0000	0.3779
248	0	1	1	1	1	1	1	1	-0.7560	0.3779	0.3779	0.0000	-0.3779

249	1	1	0	1	1	1	1	1	0.0000	-0.3779	-0.3779	-0.3779	0.7560
250	1	1	1	1	0	1	1	1	0.0000	-0.3779	-0.3779	-0.3779	-0.7560
251	1	1	1	1	1	1	0	1	-0.3779	0.0000	-0.7560	0.3779	0.3779
252	1	1	1	1	1	1	1	0	-0.3779	0.0000	-0.7560	0.3779	-0.3779
253	1	1	1	1	1	0	1	1	-0.3779	0.0000	0.3779	-0.7560	0.3779
254	1	1	1	0	1	1	1	1	-0.3779	0.0000	0.3779	-0.7560	-0.3779
255	1	0	1	1	1	1	1	1	-0.3779	0.3779	0.0000	-0.7560	0.3779
256	1	1	1	1	1	1	1	1	-0.3779	0.3779	0.0000	-0.7560	-0.3779

Table B.7. 6D-64CEM constellation mapping values.

Symbol	**Bit Stream**						**Signal Vector**					
							s_1	s_2	s_3	s_4	s_5	s_6
1	0	0	0	0	0	0	0.4082	0.4082	0.4082	0.4082	0.4082	0.4082
2	0	0	0	1	0	1	0.4082	0.4082	0.4082	0.4082	0.4082	-0.4082
3	0	0	1	1	0	0	0.4082	0.4082	0.4082	0.4082	-0.4082	0.4082
4	1	0	1	0	1	0	0.4082	0.4082	0.4082	-0.4082	0.4082	0.4082
5	0	0	1	0	1	0	0.4082	0.4082	-0.4082	0.4082	0.4082	0.4082
6	1	0	1	1	0	0	0.4082	-0.4082	0.4082	0.4082	0.4082	0.4082
7	1	0	0	1	0	1	-0.4082	0.4082	0.4082	0.4082	0.4082	0.4082
8	0	0	1	0	0	1	0.4082	0.4082	0.4082	0.4082	-0.4082	-0.4082
9	1	0	0	1	1	0	0.4082	0.4082	0.4082	-0.4082	0.4082	-0.4082
10	1	0	0	0	1	1	0.4082	0.4082	0.4082	-0.4082	-0.4082	0.4082
11	0	0	1	1	1	1	0.4082	0.4082	-0.4082	0.4082	0.4082	-0.4082
12	0	0	0	0	0	1	0.4082	0.4082	-0.4082	0.4082	-0.4082	0.4082
13	0	0	0	0	1	0	0.4082	0.4082	-0.4082	-0.4082	0.4082	0.4082
14	0	1	1	0	0	0	0.4082	-0.4082	0.4082	0.4082	0.4082	-0.4082
15	0	1	0	1	0	0	0.4082	-0.4082	0.4082	0.4082	-0.4082	0.4082
16	0	1	0	0	1	0	0.4082	-0.4082	0.4082	-0.4082	0.4082	0.4082
17	0	1	0	0	0	1	0.4082	-0.4082	-0.4082	0.4082	0.4082	0.4082
18	0	1	1	0	1	1	-0.4082	0.4082	0.4082	0.4082	0.4082	-0.4082
19	0	1	1	1	1	0	-0.4082	0.4082	0.4082	0.4082	-0.4082	0.4082
20	1	1	0	0	0	0	-0.4082	0.4082	0.4082	-0.4082	0.4082	0.4082
21	1	0	1	0	0	0	-0.4082	0.4082	-0.4082	0.4082	0.4082	0.4082
22	0	1	1	1	0	1	-0.4082	-0.4082	0.4082	0.4082	0.4082	0.4082
23	0	1	0	1	1	1	0.4082	0.4082	0.4082	-0.4082	-0.4082	-0.4082
24	1	1	0	0	1	1	0.4082	0.4082	-0.4082	0.4082	-0.4082	-0.4082

25	1	1	0	1	1	0	0.4082	0.4082	-0.4082	-0.4082	0.4082	-0.4082
26	0	0	0	1	0	0	0.4082	0.4082	-0.4082	-0.4082	-0.4082	0.4082
27	1	0	0	1	0	0	0.4082	-0.4082	0.4082	0.4082	-0.4082	-0.4082
28	0	0	1	0	0	0	0.4082	-0.4082	0.4082	-0.4082	0.4082	-0.4082
29	1	0	0	0	1	0	0.4082	-0.4082	0.4082	-0.4082	-0.4082	0.4082
30	1	0	0	0	0	1	0.4082	-0.4082	-0.4082	0.4082	0.4082	-0.4082
31	1	1	0	1	0	1	0.4082	-0.4082	-0.4082	0.4082	-0.4082	0.4082
32	0	0	0	1	1	1	0.4082	-0.4082	-0.4082	-0.4082	0.4082	0.4082
33	1	1	1	1	0	0	-0.4082	0.4082	0.4082	0.4082	-0.4082	-0.4082
34	0	0	1	1	0	1	-0.4082	0.4082	0.4082	-0.4082	0.4082	-0.4082
35	1	1	1	0	1	0	-0.4082	0.4082	0.4082	-0.4082	-0.4082	0.4082
36	0	0	1	1	1	0	-0.4082	0.4082	-0.4082	0.4082	0.4082	-0.4082
37	1	1	1	0	0	1	-0.4082	0.4082	-0.4082	0.4082	-0.4082	0.4082
38	1	0	1	0	1	1	-0.4082	0.4082	-0.4082	-0.4082	0.4082	0.4082
39	1	0	1	1	1	0	-0.4082	-0.4082	0.4082	0.4082	0.4082	-0.4082
40	0	1	0	0	0	0	-0.4082	-0.4082	0.4082	0.4082	-0.4082	0.4082
41	0	0	1	0	1	1	-0.4082	-0.4082	0.4082	-0.4082	0.4082	0.4082
42	0	1	1	0	0	1	-0.4082	-0.4082	-0.4082	0.4082	0.4082	0.4082
43	1	0	1	1	0	1	0.4082	0.4082	-0.4082	-0.4082	-0.4082	-0.4082
44	1	0	0	0	0	0	0.4082	-0.4082	0.4082	-0.4082	-0.4082	-0.4082
45	0	1	1	0	1	0	0.4082	-0.4082	-0.4082	0.4082	-0.4082	-0.4082
46	0	1	1	1	0	0	0.4082	-0.4082	-0.4082	-0.4082	0.4082	-0.4082
47	1	0	0	1	1	1	0.4082	-0.4082	-0.4082	-0.4082	-0.4082	0.4082
48	0	1	0	1	0	1	-0.4082	0.4082	0.4082	-0.4082	-0.4082	-0.4082
49	0	1	1	1	1	1	-0.4082	0.4082	-0.4082	0.4082	-0.4082	-0.4082
50	1	1	0	1	1	1	-0.4082	0.4082	-0.4082	-0.4082	0.4082	-0.4082
51	0	1	0	1	1	0	-0.4082	0.4082	-0.4082	-0.4082	-0.4082	0.4082
52	1	1	1	1	0	1	-0.4082	-0.4082	0.4082	0.4082	-0.4082	-0.4082

53	1	1	1	1	1	0	-0.4082	-0.4082	0.4082	-0.4082	0.4082	-0.4082
54	0	0	0	0	1	1	-0.4082	-0.4082	0.4082	-0.4082	-0.4082	0.4082
55	0	1	0	0	1	1	-0.4082	-0.4082	-0.4082	0.4082	0.4082	-0.4082
56	1	1	0	0	0	1	-0.4082	-0.4082	-0.4082	0.4082	-0.4082	0.4082
57	1	1	1	0	1	1	-0.4082	-0.4082	-0.4082	-0.4082	0.4082	0.4082
58	1	1	0	0	1	0	0.4082	-0.4082	-0.4082	-0.4082	-0.4082	-0.4082
59	1	0	1	1	1	1	-0.4082	0.4082	-0.4082	-0.4082	-0.4082	-0.4082
60	1	1	1	1	1	1	-0.4082	-0.4082	0.4082	-0.4082	-0.4082	-0.4082
61	0	0	0	1	1	0	-0.4082	-0.4082	-0.4082	0.4082	-0.4082	-0.4082
62	1	1	0	1	0	0	-0.4082	-0.4082	-0.4082	-0.4082	0.4082	-0.4082
63	1	1	1	0	0	0	-0.4082	-0.4082	-0.4082	-0.4082	-0.4082	0.4082
64	1	0	1	0	0	1	-0.4082	-0.4082	-0.4082	-0.4082	-0.4082	-0.4082

Table B.8. 6D-128CEM constellation mapping values.

Symbol	Bit Stream							Signal Vector					
								s_1	s_2	s_3	s_4	s_5	s_6
1	0	0	0	0	0	0	0	0.4472	0.4472	0.4472	0.4472	0.4472	0.0000
2	0	0	0	0	0	0	1	0.0000	0.4472	0.4472	0.4472	0.4472	0.4472
3	0	0	0	0	0	1	0	0.0000	0.4472	0.4472	0.4472	0.4472	-0.4472
4	0	0	0	0	1	0	0	0.4472	0.0000	0.4472	0.4472	0.4472	0.4472
5	0	0	0	1	0	0	0	0.4472	0.0000	0.4472	0.4472	0.4472	-0.4472
6	0	0	1	0	0	0	0	0.4472	0.4472	0.0000	0.4472	0.4472	0.4472
7	0	1	0	0	0	0	0	0.4472	0.4472	0.0000	0.4472	0.4472	-0.4472
8	1	0	0	0	0	0	0	0.4472	0.4472	0.4472	0.0000	0.4472	0.4472
9	0	0	0	0	0	1	1	0.4472	0.4472	0.4472	0.0000	0.4472	-0.4472
10	0	0	0	0	1	1	0	0.4472	0.4472	0.4472	0.4472	0.0000	0.4472
11	0	0	0	0	1	0	1	0.4472	0.4472	0.4472	0.4472	0.0000	-0.4472
12	0	0	0	1	1	0	0	0.4472	0.4472	0.4472	0.4472	-0.4472	0.0000
13	0	0	0	1	0	1	0	0.4472	0.4472	0.4472	-0.4472	0.4472	0.0000
14	0	0	0	1	0	0	1	0.4472	0.4472	-0.4472	0.4472	0.4472	0.0000
15	0	0	1	1	0	0	0	0.4472	-0.4472	0.4472	0.4472	0.4472	0.0000
16	0	0	1	0	1	0	0	-0.4472	0.4472	0.4472	0.4472	0.4472	0.0000
17	0	0	1	0	0	1	0	0.0000	0.4472	0.4472	0.4472	-0.4472	0.4472
18	0	0	1	0	0	0	1	0.0000	0.4472	0.4472	0.4472	-0.4472	-0.4472
19	0	1	1	0	0	0	0	0.0000	0.4472	0.4472	-0.4472	0.4472	0.4472
20	0	1	0	1	0	0	0	0.0000	0.4472	0.4472	-0.4472	0.4472	-0.4472
21	0	1	0	0	1	0	0	0.0000	0.4472	-0.4472	0.4472	0.4472	0.4472
22	0	1	0	0	0	1	0	0.0000	0.4472	-0.4472	0.4472	0.4472	-0.4472
23	0	1	0	0	0	0	1	0.0000	-0.4472	0.4472	0.4472	0.4472	0.4472
24	1	1	0	0	0	0	0	0.0000	-0.4472	0.4472	0.4472	0.4472	-0.4472

25	1	0	1	0	0	0	0	-0.4472	0.0000	0.4472	0.4472	0.4472	0.4472
26	1	0	0	1	0	0	0	-0.4472	0.0000	0.4472	0.4472	0.4472	-0.4472
27	1	0	0	0	1	0	0	0.4472	0.0000	0.4472	0.4472	-0.4472	0.4472
28	1	0	0	0	0	1	0	0.4472	0.0000	0.4472	0.4472	-0.4472	-0.4472
29	1	0	0	0	0	0	1	0.4472	0.0000	0.4472	-0.4472	0.4472	0.4472
30	0	0	0	0	1	1	1	0.4472	0.0000	0.4472	-0.4472	0.4472	-0.4472
31	0	0	0	1	1	0	1	0.4472	0.0000	-0.4472	0.4472	0.4472	0.4472
32	0	0	0	1	1	1	0	0.4472	0.0000	-0.4472	0.4472	0.4472	-0.4472
33	0	0	0	1	0	1	1	0.4472	-0.4472	0.0000	0.4472	0.4472	0.4472
34	0	0	1	1	0	0	1	-0.4472	0.4472	0.0000	0.4472	0.4472	0.4472
35	0	0	1	1	0	1	0	0.4472	-0.4472	0.0000	0.4472	0.4472	-0.4472
36	0	0	1	1	1	0	0	-0.4472	0.4472	0.0000	0.4472	0.4472	-0.4472
37	0	0	1	0	1	0	1	0.4472	0.4472	0.0000	0.4472	-0.4472	0.4472
38	0	0	1	0	1	1	0	0.4472	0.4472	0.0000	0.4472	-0.4472	-0.4472
39	0	0	1	0	0	1	1	0.4472	0.4472	0.0000	-0.4472	0.4472	0.4472
40	0	1	1	0	0	0	1	0.4472	0.4472	0.0000	-0.4472	0.4472	-0.4472
41	0	1	1	0	0	1	0	0.4472	0.4472	-0.4472	0.0000	0.4472	0.4472
42	0	1	1	0	1	0	0	0.4472	-0.4472	0.4472	0.0000	0.4472	0.4472
43	0	1	1	1	0	0	0	-0.4472	0.4472	0.4472	0.0000	0.4472	0.4472
44	0	1	0	1	0	0	1	0.4472	0.4472	-0.4472	0.0000	0.4472	-0.4472
45	0	1	0	1	0	1	0	0.4472	-0.4472	0.4472	0.0000	0.4472	-0.4472
46	0	1	0	1	1	0	0	-0.4472	0.4472	0.4472	0.0000	0.4472	-0.4472
47	0	1	0	0	1	0	1	0.4472	0.4472	0.4472	0.0000	-0.4472	0.4472
48	0	1	0	0	1	1	0	0.4472	0.4472	0.4472	0.0000	-0.4472	-0.4472
49	0	1	0	0	0	1	1	0.4472	0.4472	0.4472	-0.4472	0.0000	0.4472
50	1	1	0	0	0	0	1	0.4472	0.4472	-0.4472	0.4472	0.0000	0.4472
51	1	1	0	0	0	1	0	0.4472	-0.4472	0.4472	0.4472	0.0000	0.4472
52	1	1	0	0	1	0	0	-0.4472	0.4472	0.4472	0.4472	0.0000	0.4472

53	1	1	0	1	0	0	0	0.4472	0.4472	0.4472	-0.4472	0.0000	-0.4472
54	1	1	1	0	0	0	0	0.4472	0.4472	-0.4472	0.4472	0.0000	-0.4472
55	1	0	1	0	0	0	1	0.4472	-0.4472	0.4472	0.4472	0.0000	-0.4472
56	1	0	1	0	0	1	0	-0.4472	0.4472	0.4472	0.4472	0.0000	-0.4472
57	1	0	1	0	1	0	0	0.4472	0.4472	0.4472	-0.4472	-0.4472	0.0000
58	1	0	1	1	0	0	0	0.4472	0.4472	-0.4472	0.4472	-0.4472	0.0000
59	1	0	0	1	0	0	1	0.4472	0.4472	-0.4472	-0.4472	0.4472	0.0000
60	1	0	0	1	0	1	0	0.4472	-0.4472	0.4472	0.4472	-0.4472	0.0000
61	1	0	0	1	1	0	0	0.4472	-0.4472	0.4472	-0.4472	0.4472	0.0000
62	1	0	0	0	1	0	1	0.4472	-0.4472	-0.4472	0.4472	0.4472	0.0000
63	1	0	0	0	1	1	0	-0.4472	0.4472	0.4472	0.4472	-0.4472	0.0000
64	1	0	0	0	0	1	1	-0.4472	0.4472	0.4472	-0.4472	0.4472	0.0000
65	0	0	0	1	1	1	1	-0.4472	0.4472	-0.4472	0.4472	0.4472	0.0000
66	0	0	1	1	0	1	1	-0.4472	-0.4472	0.4472	0.4472	0.4472	0.0000
67	0	0	1	1	1	1	0	0.0000	0.4472	0.4472	-0.4472	-0.4472	0.4472
68	0	0	1	1	1	0	1	0.0000	0.4472	0.4472	-0.4472	-0.4472	-0.4472
69	0	0	1	0	1	1	1	0.0000	0.4472	-0.4472	0.4472	-0.4472	0.4472
70	0	1	1	0	0	1	1	0.0000	0.4472	-0.4472	0.4472	-0.4472	-0.4472
71	0	1	1	0	1	1	0	0.0000	0.4472	-0.4472	-0.4472	0.4472	0.4472
72	0	1	1	0	1	0	1	0.0000	0.4472	-0.4472	-0.4472	0.4472	-0.4472
73	0	1	1	1	1	0	0	0.0000	-0.4472	0.4472	0.4472	-0.4472	0.4472
74	0	1	1	1	0	1	0	0.0000	-0.4472	0.4472	0.4472	-0.4472	-0.4472
75	0	1	1	1	0	0	1	0.0000	-0.4472	0.4472	-0.4472	0.4472	0.4472
76	0	1	0	1	0	1	1	0.0000	-0.4472	0.4472	-0.4472	0.4472	-0.4472
77	0	1	0	1	1	1	0	0.0000	-0.4472	-0.4472	0.4472	0.4472	0.4472
78	0	1	0	1	1	0	1	0.0000	-0.4472	-0.4472	0.4472	0.4472	-0.4472
79	0	1	0	0	1	1	1	-0.4472	0.0000	0.4472	0.4472	-0.4472	0.4472
80	1	1	0	0	0	1	1	-0.4472	0.0000	0.4472	0.4472	-0.4472	-0.4472

81	1	1	0	0	1	1	0	-0.4472	0.0000	0.4472	-0.4472	0.4472	0.4472
82	1	1	0	0	1	0	1	-0.4472	0.0000	0.4472	-0.4472	0.4472	-0.4472
83	1	1	0	1	1	0	0	0.4472	0.0000	0.4472	-0.4472	-0.4472	0.4472
84	1	1	0	1	0	1	0	0.4472	0.0000	0.4472	-0.4472	-0.4472	-0.4472
85	1	1	0	1	0	0	1	-0.4472	0.0000	-0.4472	0.4472	0.4472	0.4472
86	1	1	1	1	0	0	0	-0.4472	0.0000	-0.4472	0.4472	0.4472	-0.4472
87	1	1	1	0	1	0	0	0.4472	0.0000	-0.4472	0.4472	-0.4472	0.4472
88	1	1	1	0	0	1	0	0.4472	0.0000	-0.4472	0.4472	-0.4472	-0.4472
89	1	1	1	0	0	0	1	0.4472	0.0000	-0.4472	-0.4472	0.4472	0.4472
90	1	0	1	0	0	1	1	0.4472	0.0000	-0.4472	-0.4472	0.4472	-0.4472
91	1	0	1	0	1	1	0	-0.4472	-0.4472	0.0000	0.4472	0.4472	0.4472
92	1	0	1	0	1	0	1	-0.4472	-0.4472	0.0000	0.4472	0.4472	-0.4472
93	1	0	1	1	1	0	0	0.4472	-0.4472	0.0000	0.4472	-0.4472	0.4472
94	1	0	1	1	0	1	0	-0.4472	0.4472	0.0000	0.4472	-0.4472	0.4472
95	1	0	1	1	0	0	1	0.4472	-0.4472	0.0000	0.4472	-0.4472	-0.4472
96	1	0	0	1	0	1	1	-0.4472	0.4472	0.0000	0.4472	-0.4472	-0.4472
97	1	0	0	1	1	1	0	0.4472	-0.4472	0.0000	-0.4472	0.4472	0.4472
98	1	0	0	1	1	0	1	-0.4472	0.4472	0.0000	-0.4472	0.4472	0.4472
99	1	0	0	0	1	1	1	0.4472	-0.4472	0.0000	-0.4472	0.4472	-0.4472
100	0	0	1	1	1	1	1	-0.4472	0.4472	0.0000	-0.4472	0.4472	-0.4472
101	0	1	1	0	1	1	1	0.4472	0.4472	0.0000	-0.4472	-0.4472	0.4472
102	0	1	1	1	1	0	1	0.4472	0.4472	0.0000	-0.4472	-0.4472	-0.4472
103	0	1	1	1	1	1	0	0.4472	-0.4472	-0.4472	0.0000	0.4472	0.4472
104	0	1	1	1	0	1	1	-0.4472	0.4472	-0.4472	0.0000	0.4472	0.4472
105	0	1	0	1	1	1	1	-0.4472	-0.4472	0.4472	0.0000	0.4472	0.4472
106	1	1	0	0	1	1	1	0.4472	-0.4472	-0.4472	0.0000	0.4472	-0.4472
107	1	1	0	1	1	0	1	-0.4472	0.4472	-0.4472	0.0000	0.4472	-0.4472
108	1	1	0	1	1	1	0	-0.4472	-0.4472	0.4472	0.0000	0.4472	-0.4472

109	1	1	0	1	0	1	1	0.4472	0.4472	-0.4472	0.0000	-0.4472	0.4472
110	1	1	1	1	0	0	1	0.4472	-0.4472	0.4472	0.0000	-0.4472	0.4472
111	1	1	1	1	0	1	0	-0.4472	0.4472	0.4472	0.0000	-0.4472	0.4472
112	1	1	1	1	1	0	0	0.4472	0.4472	-0.4472	0.0000	-0.4472	-0.4472
113	1	1	1	0	1	0	1	0.4472	-0.4472	0.4472	0.0000	-0.4472	-0.4472
114	1	1	1	0	1	1	0	-0.4472	0.4472	0.4472	0.0000	-0.4472	-0.4472
115	1	1	1	0	0	1	1	0.4472	0.4472	-0.4472	-0.4472	0.0000	0.4472
116	1	0	1	0	1	1	1	0.4472	-0.4472	0.4472	-0.4472	0.0000	0.4472
117	1	0	1	1	1	0	1	0.4472	-0.4472	-0.4472	0.4472	0.0000	0.4472
118	1	0	1	1	1	1	0	-0.4472	0.4472	0.4472	-0.4472	0.0000	0.4472
119	1	0	1	1	0	1	1	-0.4472	0.4472	-0.4472	0.4472	0.0000	0.4472
120	1	0	0	1	1	1	1	-0.4472	-0.4472	0.4472	0.4472	0.0000	0.4472
121	0	1	1	1	1	1	1	0.4472	0.4472	-0.4472	-0.4472	0.0000	-0.4472
122	1	1	0	1	1	1	1	0.4472	-0.4472	0.4472	-0.4472	0.0000	-0.4472
123	1	1	1	1	0	1	1	0.4472	-0.4472	-0.4472	0.4472	0.0000	-0.4472
124	1	1	1	1	1	1	0	-0.4472	0.4472	0.4472	-0.4472	0.0000	-0.4472
125	1	1	1	1	1	0	1	-0.4472	0.4472	-0.4472	0.4472	0.0000	-0.4472
126	1	1	1	0	1	1	1	-0.4472	-0.4472	0.4472	0.4472	0.0000	-0.4472
127	1	0	1	1	1	1	1	0.4472	0.4472	-0.4472	-0.4472	-0.4472	0.0000
128	1	1	1	1	1	1	1	0.4472	-0.4472	0.4472	-0.4472	-0.4472	0.0000

Table B.9. 6D-256CEM constellation mapping values.

Symbol	Bit Stream								Signal Vector					
									s_1	s_2	s_3	s_4	s_5	s_6
1	0	0	0	0	0	0	0	0	0.7336	0.3039	0.3039	0.3039	0.3039	0.3039
2	0	0	0	0	0	0	0	1	0.3039	0.7336	0.3039	0.3039	0.3039	0.3039
3	0	0	0	0	0	0	1	0	0.7336	0.3039	0.3039	0.3039	0.3039	-0.3039
4	0	0	0	0	0	1	0	0	0.7336	0.3039	0.3039	0.3039	-0.3039	0.3039
5	0	0	0	0	1	0	0	0	0.7336	0.3039	0.3039	-0.3039	0.3039	0.3039
6	0	0	0	1	0	0	0	0	0.7336	0.3039	-0.3039	0.3039	0.3039	0.3039
7	0	0	1	0	0	0	0	0	0.7336	-0.3039	0.3039	0.3039	0.3039	0.3039
8	0	1	0	0	0	0	0	0	0.3039	0.3039	0.7736	0.3039	0.3039	0.3039
9	1	0	0	0	0	0	0	0	0.3039	0.3039	0.3039	0.7736	0.3039	0.3039
10	0	0	0	0	0	0	1	1	0.3039	0.3039	0.3039	0.3039	0.7736	0.3039
11	0	0	0	0	0	1	1	0	0.3039	0.3039	0.3039	0.3039	0.3039	0.7736
12	0	0	0	0	0	1	0	1	0.3039	0.7336	0.3039	0.3039	0.3039	-0.3039
13	0	0	0	0	1	1	0	0	0.3039	0.7336	0.3039	0.3039	-0.3039	0.3039
14	0	0	0	0	1	0	1	0	0.3039	0.7336	0.3039	-0.3039	0.3039	0.3039
15	0	0	0	0	1	0	0	1	0.3039	0.7336	-0.3039	0.3039	0.3039	0.3039
16	0	0	0	1	1	0	0	0	0.7336	0.3039	0.3039	0.3039	-0.3039	-0.3039
17	0	0	0	1	0	1	0	0	0.7336	0.3039	0.3039	-0.3039	0.3039	-0.3039
18	0	0	0	1	0	0	1	0	0.7336	0.3039	0.3039	-0.3039	-0.3039	0.3039
19	0	0	0	1	0	0	0	1	0.7336	0.3039	-0.3039	0.3039	0.3039	-0.3039
20	0	0	1	1	0	0	0	0	0.7336	0.3039	-0.3039	0.3039	-0.3039	0.3039
21	0	0	1	0	1	0	0	0	0.7336	0.3039	-0.3039	-0.3039	0.3039	0.3039
22	0	0	1	0	0	1	0	0	0.7336	-0.3039	0.3039	0.3039	0.3039	-0.3039
23	0	0	1	0	0	0	1	0	0.7336	-0.3039	0.3039	0.3039	-0.3039	0.3039
24	0	0	1	0	0	0	0	1	0.7336	-0.3039	0.3039	-0.3039	0.3039	0.3039

25	0	1	1	0	0	0	0	0	0.7336	-0.3039	-0.3039	0.3039	0.3039	0.3039
26	0	1	0	1	0	0	0	0	0.3039	0.3039	0.7736	0.3039	0.3039	-0.3039
27	0	1	0	0	1	0	0	0	0.3039	0.3039	0.7736	0.3039	-0.3039	0.3039
28	0	1	0	0	0	1	0	0	0.3039	0.3039	0.7736	-0.3039	0.3039	0.3039
29	0	1	0	0	0	0	1	0	0.3039	-0.3039	0.7736	0.3039	0.3039	0.3039
30	0	1	0	0	0	0	0	1	0.3039	0.3039	0.3039	0.7736	0.3039	-0.3039
31	1	1	0	0	0	0	0	0	0.3039	0.3039	0.3039	0.7736	-0.3039	0.3039
32	1	0	1	0	0	0	0	0	0.3039	0.3039	-0.3039	0.7736	0.3039	0.3039
33	1	0	0	1	0	0	0	0	0.3039	-0.3039	0.3039	0.7736	0.3039	0.3039
34	1	0	0	0	1	0	0	0	0.3039	0.3039	0.3039	0.3039	0.7736	-0.3039
35	1	0	0	0	0	1	0	0	0.3039	0.3039	0.3039	-0.3039	0.7736	0.3039
36	1	0	0	0	0	0	1	0	0.3039	0.3039	-0.3039	0.3039	0.7736	0.3039
37	1	0	0	0	0	0	0	1	0.3039	-0.3039	0.3039	0.3039	0.7736	0.3039
38	0	0	0	0	0	1	1	1	0.3039	0.3039	0.3039	0.3039	-0.3039	0.7736
39	0	0	0	0	1	1	0	1	0.3039	0.3039	0.3039	-0.3039	0.3039	0.7736
40	0	0	0	0	1	1	1	0	0.3039	0.3039	-0.3039	0.3039	0.3039	0.7736
41	0	0	0	0	1	0	1	1	0.3039	-0.3039	0.3039	0.3039	0.3039	0.7736
42	0	0	0	1	1	0	0	1	0.3039	0.7336	0.3039	0.3039	-0.3039	-0.3039
43	0	0	0	1	1	0	1	0	0.3039	0.7336	0.3039	-0.3039	0.3039	-0.3039
44	0	0	0	1	1	1	0	0	0.3039	0.7336	0.3039	-0.3039	-0.3039	0.3039
45	0	0	0	1	0	1	0	1	0.3039	0.7336	-0.3039	0.3039	0.3039	-0.3039
46	0	0	0	1	0	1	1	0	0.3039	0.7336	-0.3039	0.3039	-0.3039	0.3039
47	0	0	0	1	0	0	1	1	0.3039	0.7336	-0.3039	-0.3039	0.3039	0.3039
48	0	0	1	1	0	0	0	1	0.7336	0.3039	0.3039	-0.3039	-0.3039	-0.3039
49	0	0	1	1	0	0	1	0	0.7336	0.3039	-0.3039	0.3039	-0.3039	-0.3039
50	0	0	1	1	0	1	0	0	0.7336	0.3039	-0.3039	-0.3039	0.3039	-0.3039
51	0	0	1	1	1	0	0	0	0.7336	0.3039	-0.3039	-0.3039	-0.3039	0.3039
52	0	0	1	0	1	0	0	1	0.7336	-0.3039	0.3039	0.3039	-0.3039	-0.3039

53	0	0	1	0	1	0	1	0	0.7336	-0.3039	0.3039	-0.3039	0.3039	-0.3039
54	0	0	1	0	1	1	0	0	0.7336	-0.3039	0.3039	-0.3039	-0.3039	0.3039
55	0	0	1	0	0	1	0	1	0.7336	-0.3039	-0.3039	0.3039	0.3039	-0.3039
56	0	0	1	0	0	1	1	0	0.7336	-0.3039	-0.3039	0.3039	-0.3039	0.3039
57	0	0	1	0	0	0	1	1	0.7336	-0.3039	-0.3039	-0.3039	0.3039	0.3039
58	0	1	1	0	0	0	0	1	0.3039	-0.3039	-0.3039	0.7736	0.3039	0.3039
59	0	1	1	0	0	0	1	0	0.3039	0.3039	-0.3039	-0.3039	0.7736	0.3039
60	0	1	1	0	0	1	0	0	0.3039	-0.3039	0.3039	-0.3039	0.7736	0.3039
61	0	1	1	0	1	0	0	0	0.3039	-0.3039	-0.3039	0.3039	0.7736	0.3039
62	0	1	1	1	0	0	0	0	0.3039	0.3039	0.3039	-0.3039	-0.3039	0.7736
63	0	1	0	1	0	0	0	1	0.3039	0.3039	-0.3039	0.3039	-0.3039	0.7736
64	0	1	0	1	0	0	1	0	0.3039	0.3039	-0.3039	-0.3039	0.3039	0.7736
65	0	1	0	1	0	1	0	0	0.3039	-0.3039	0.3039	0.3039	-0.3039	0.7736
66	0	1	0	1	1	0	0	0	0.3039	-0.3039	0.3039	-0.3039	0.3039	0.7736
67	0	1	0	0	1	0	0	1	0.3039	-0.3039	-0.3039	0.3039	0.3039	0.7736
68	0	1	0	0	1	0	1	0	0.3039	0.3039	0.7736	0.3039	-0.3039	-0.3039
69	0	1	0	0	1	1	0	0	0.3039	0.3039	0.7736	-0.3039	0.3039	-0.3039
70	0	1	0	0	0	1	0	1	0.3039	0.3039	0.7736	-0.3039	-0.3039	0.3039
71	0	1	0	0	0	1	1	0	0.3039	-0.3039	0.7736	0.3039	0.3039	-0.3039
72	0	1	0	0	0	0	1	1	0.3039	-0.3039	0.7736	0.3039	-0.3039	0.3039
73	1	1	0	0	0	0	0	1	0.3039	-0.3039	0.7736	-0.3039	0.3039	0.3039
74	1	1	0	0	0	0	1	0	0.3039	0.3039	0.3039	0.7736	-0.3039	-0.3039
75	1	1	0	0	0	1	0	0	0.3039	0.3039	-0.3039	0.7736	0.3039	-0.3039
76	1	1	0	0	1	0	0	0	0.3039	0.3039	-0.3039	0.7736	-0.3039	0.3039
77	1	1	0	1	0	0	0	0	0.3039	-0.3039	0.3039	0.7736	0.3039	-0.3039
78	1	1	1	0	0	0	0	0	0.3039	-0.3039	0.3039	0.7736	-0.3039	0.3039
79	1	0	1	0	0	0	0	1	0.3039	0.3039	0.3039	-0.3039	0.7736	-0.3039
80	1	0	1	0	0	0	1	0	0.3039	0.3039	-0.3039	0.3039	0.7736	-0.3039

81	1	0	1	0	0	1	0	0	0.3039	-0.3039	0.3039	0.3039	0.7736	-0.3039
82	1	0	1	0	1	0	0	0	0.3039	-0.7336	0.3039	0.3039	0.3039	0.3039
83	1	0	1	1	0	0	0	0	-0.3039	0.7336	0.3039	0.3039	0.3039	0.3039
84	1	0	0	1	0	0	0	1	-0.3039	0.3039	0.7736	0.3039	0.3039	0.3039
85	1	0	0	1	0	0	1	0	-0.3039	0.3039	0.3039	0.7736	0.3039	0.3039
86	1	0	0	1	0	1	0	0	-0.3039	0.3039	0.3039	0.3039	0.7736	0.3039
87	1	0	0	1	1	0	0	0	-0.3039	0.3039	0.3039	0.3039	0.3039	0.7736
88	1	0	0	0	1	0	0	1	0.3039	0.3039	-0.7736	0.3039	0.3039	0.3039
89	1	0	0	0	1	0	1	0	0.3039	0.3039	0.3039	-0.7736	0.3039	0.3039
90	1	0	0	0	1	1	0	0	0.3039	0.3039	0.3039	0.3039	-0.7736	0.3039
91	1	0	0	0	0	1	0	1	0.3039	0.3039	0.3039	0.3039	0.3039	-0.7736
92	1	0	0	0	0	1	1	0	0.3039	0.7336	0.3039	-0.3039	-0.3039	-0.3039
93	1	0	0	0	0	0	1	1	0.3039	0.7336	-0.3039	0.3039	-0.3039	-0.3039
94	0	0	0	0	1	1	1	1	0.3039	0.7336	-0.3039	-0.3039	0.3039	-0.3039
95	0	0	0	1	1	0	1	1	0.3039	0.7336	-0.3039	-0.3039	-0.3039	0.3039
96	0	0	0	1	1	1	1	0	0.7336	0.3039	-0.3039	-0.3039	-0.3039	-0.3039
97	0	0	0	1	1	1	0	1	0.7336	-0.3039	0.3039	-0.3039	-0.3039	-0.3039
98	0	0	0	1	0	1	1	1	0.7336	-0.3039	-0.3039	0.3039	-0.3039	-0.3039
99	0	0	1	1	0	0	1	1	0.7336	-0.3039	-0.3039	-0.3039	0.3039	-0.3039
100	0	0	1	1	0	1	1	0	0.7336	-0.3039	-0.3039	-0.3039	-0.3039	0.3039
101	0	0	1	1	0	1	0	1	0.3039	-0.3039	-0.3039	0.7736	0.3039	-0.3039
102	0	0	1	1	1	1	0	0	0.3039	-0.3039	-0.3039	0.7736	-0.3039	0.3039
103	0	0	1	1	1	0	1	0	0.3039	0.3039	-0.3039	-0.3039	0.7736	-0.3039
104	0	0	1	1	1	0	0	1	0.3039	-0.3039	0.3039	-0.3039	0.7736	-0.3039
105	0	0	1	0	1	0	1	1	0.3039	-0.3039	-0.3039	0.3039	0.7736	-0.3039
106	0	0	1	0	1	1	1	0	0.3039	-0.3039	-0.3039	-0.3039	0.7736	0.3039
107	0	0	1	0	1	1	0	1	0.3039	0.3039	-0.3039	-0.3039	-0.3039	0.7736
108	0	0	1	0	0	1	1	1	0.3039	-0.3039	0.3039	-0.3039	-0.3039	0.7736

109	0	1	1	0	0	0	1	1	0.3039	-0.3039	-0.3039	0.3039	-0.3039	0.7736
110	0	1	1	0	0	1	1	0	0.3039	-0.3039	-0.3039	-0.3039	0.3039	0.7736
111	0	1	1	0	0	1	0	1	0.3039	0.3039	0.7736	-0.3039	-0.3039	-0.3039
112	0	1	1	0	1	1	0	0	0.3039	-0.3039	0.7736	0.3039	-0.3039	-0.3039
113	0	1	1	0	1	0	1	0	0.3039	-0.3039	0.7736	-0.3039	0.3039	-0.3039
114	0	1	1	0	1	0	0	1	0.3039	-0.3039	0.7736	-0.3039	-0.3039	0.3039
115	0	1	1	1	1	0	0	0	0.3039	0.3039	-0.3039	0.7736	-0.3039	-0.3039
116	0	1	1	1	0	1	0	0	0.3039	-0.3039	0.3039	0.7736	-0.3039	-0.3039
117	0	1	1	1	0	0	1	0	0.3039	-0.7336	0.3039	0.3039	0.3039	-0.3039
118	0	1	1	1	0	0	0	1	0.3039	-0.7336	0.3039	0.3039	-0.3039	0.3039
119	0	1	0	1	0	0	1	1	0.3039	-0.7336	0.3039	-0.3039	0.3039	0.3039
120	0	1	0	1	0	1	1	0	0.3039	-0.7336	-0.3039	0.3039	0.3039	0.3039
121	0	1	0	1	0	1	0	1	-0.3039	0.7336	0.3039	0.3039	0.3039	-0.3039
122	0	1	0	1	1	1	0	0	-0.3039	0.7336	0.3039	0.3039	-0.3039	0.3039
123	0	1	0	1	1	0	1	0	-0.3039	0.7336	0.3039	-0.3039	0.3039	0.3039
124	0	1	0	1	1	0	0	1	-0.3039	0.7336	-0.3039	0.3039	0.3039	0.3039
125	0	1	0	0	1	0	1	1	-0.3039	0.3039	0.7736	0.3039	0.3039	-0.3039
126	0	1	0	0	1	1	1	0	-0.3039	0.3039	0.7736	0.3039	-0.3039	0.3039
127	0	1	0	0	1	1	0	1	-0.3039	0.3039	0.7736	-0.3039	0.3039	0.3039
128	0	1	0	0	0	1	1	1	-0.3039	-0.3039	0.7736	0.3039	0.3039	0.3039
129	1	1	0	0	0	0	1	1	-0.3039	0.3039	0.3039	0.7736	0.3039	-0.3039
130	1	1	0	0	0	1	1	0	-0.3039	0.3039	0.3039	0.7736	-0.3039	0.3039
131	1	1	0	0	0	1	0	1	-0.3039	0.3039	-0.3039	0.7736	0.3039	0.3039
132	1	1	0	0	1	1	0	0	-0.3039	-0.3039	0.3039	0.7736	0.3039	0.3039
133	1	1	0	0	1	0	1	0	-0.3039	0.3039	0.3039	0.3039	0.7736	-0.3039
134	1	1	0	0	1	0	0	1	-0.3039	0.3039	0.3039	-0.3039	0.7736	0.3039
135	1	1	0	1	1	0	0	0	-0.3039	0.3039	-0.3039	0.3039	0.7736	0.3039
136	1	1	0	1	0	1	0	0	-0.3039	-0.3039	0.3039	0.3039	0.7736	0.3039

137	1	1	0	1	0	0	1	0	-0.3039	0.3039	0.3039	0.3039	-0.3039	0.7736
138	1	1	0	1	0	0	0	1	-0.3039	0.3039	0.3039	-0.3039	0.3039	0.7736
139	1	1	1	1	0	0	0	0	-0.3039	0.3039	-0.3039	0.3039	0.3039	0.7736
140	1	1	1	0	1	0	0	0	-0.3039	-0.3039	0.3039	0.3039	0.3039	0.7736
141	1	1	1	0	0	1	0	0	0.3039	0.3039	-0.7736	0.3039	0.3039	-0.3039
142	1	1	1	0	0	0	1	0	0.3039	0.3039	-0.7736	0.3039	-0.3039	0.3039
143	1	1	1	0	0	0	0	1	0.3039	0.3039	-0.7736	-0.3039	0.3039	0.3039
144	1	0	1	0	0	0	1	1	0.3039	-0.3039	-0.7736	0.3039	0.3039	0.3039
145	1	0	1	0	0	1	1	0	0.3039	0.3039	0.3039	-0.7736	0.3039	-0.3039
146	1	0	1	0	0	1	0	1	0.3039	0.3039	0.3039	-0.7736	-0.3039	0.3039
147	1	0	1	0	1	1	0	0	0.3039	0.3039	-0.3039	-0.7736	0.3039	0.3039
148	1	0	1	0	1	0	1	0	0.3039	-0.3039	0.3039	-0.7736	0.3039	0.3039
149	1	0	1	0	1	0	0	1	0.3039	0.3039	0.3039	0.3039	-0.7736	-0.3039
150	1	0	1	1	1	0	0	0	0.3039	0.3039	0.3039	-0.3039	-0.7736	0.3039
151	1	0	1	1	0	1	0	0	0.3039	0.3039	-0.3039	0.3039	-0.7736	0.3039
152	1	0	1	1	0	0	1	0	0.3039	-0.3039	0.3039	0.3039	-0.7736	0.3039
153	1	0	1	1	0	0	0	1	0.3039	0.3039	0.3039	0.3039	-0.3039	-0.7736
154	1	0	0	1	0	0	1	1	0.3039	0.3039	0.3039	-0.3039	0.3039	-0.7736
155	1	0	0	1	0	1	1	0	0.3039	0.3039	-0.3039	0.3039	0.3039	-0.7736
156	1	0	0	1	0	1	0	1	0.3039	-0.3039	0.3039	0.3039	0.3039	-0.7736
157	1	0	0	1	1	1	0	0	0.3039	0.7336	-0.3039	-0.3039	-0.3039	-0.3039
158	1	0	0	1	1	0	1	0	0.7336	-0.3039	-0.3039	-0.3039	-0.3039	-0.3039
159	1	0	0	1	1	0	0	1	0.3039	-0.3039	-0.3039	0.7736	-0.3039	-0.3039
160	1	0	0	0	1	0	1	1	0.3039	-0.3039	-0.3039	-0.3039	0.7736	-0.3039
161	1	0	0	0	1	1	1	0	0.3039	-0.3039	-0.3039	-0.3039	-0.3039	0.7736
162	1	0	0	0	1	1	0	1	0.3039	-0.3039	0.7736	-0.3039	-0.3039	-0.3039
163	1	0	0	0	0	1	1	1	0.3039	-0.7336	0.3039	0.3039	-0.3039	-0.3039
164	0	0	0	1	1	1	1	1	0.3039	-0.7336	0.3039	-0.3039	0.3039	-0.3039

165	0	0	1	1	0	1	1	1	0.3039	-0.7336	0.3039	-0.3039	-0.3039	0.3039
166	0	0	1	1	1	1	0	1	0.3039	-0.7336	-0.3039	0.3039	0.3039	-0.3039
167	0	0	1	1	1	1	1	0	0.3039	-0.7336	-0.3039	0.3039	-0.3039	0.3039
168	0	0	1	1	1	0	1	1	0.3039	-0.7336	-0.3039	-0.3039	0.3039	0.3039
169	0	0	1	0	1	1	1	1	-0.3039	0.7336	0.3039	0.3039	-0.3039	-0.3039
170	0	1	1	0	0	1	1	1	-0.3039	0.7336	0.3039	-0.3039	0.3039	-0.3039
171	0	1	1	0	1	1	0	1	-0.3039	0.7336	0.3039	-0.3039	-0.3039	0.3039
172	0	1	1	0	1	1	1	0	-0.3039	0.7336	-0.3039	0.3039	0.3039	-0.3039
173	0	1	1	0	1	0	1	1	-0.3039	0.7336	-0.3039	0.3039	-0.3039	0.3039
174	0	1	1	1	1	0	0	1	-0.3039	0.7336	-0.3039	-0.3039	0.3039	0.3039
175	0	1	1	1	1	0	1	0	-0.3039	0.3039	0.7736	0.3039	-0.3039	-0.3039
176	0	1	1	1	1	1	0	0	-0.3039	0.3039	0.7736	-0.3039	0.3039	-0.3039
177	0	1	1	1	0	1	0	1	-0.3039	0.3039	0.7736	-0.3039	-0.3039	0.3039
178	0	1	1	1	0	1	1	0	-0.3039	-0.3039	0.7736	0.3039	0.3039	-0.3039
179	0	1	1	1	0	0	1	1	-0.3039	-0.3039	0.7736	0.3039	-0.3039	0.3039
180	0	1	0	1	0	1	1	1	-0.3039	-0.3039	0.7736	-0.3039	0.3039	0.3039
181	0	1	0	1	1	1	0	1	-0.3039	0.3039	0.3039	0.7736	-0.3039	-0.3039
182	0	1	0	1	1	1	1	0	-0.3039	0.3039	-0.3039	0.7736	0.3039	-0.3039
183	0	1	0	1	1	0	1	1	-0.3039	0.3039	-0.3039	0.7736	-0.3039	0.3039
184	0	1	0	0	1	1	1	1	-0.3039	-0.3039	0.3039	0.7736	0.3039	-0.3039
185	1	1	0	0	0	1	1	1	-0.3039	-0.3039	0.3039	0.7736	-0.3039	0.3039
186	1	1	0	0	1	1	0	1	-0.3039	-0.3039	-0.3039	0.7736	0.3039	0.3039
187	1	1	0	0	1	1	1	0	-0.3039	0.3039	0.3039	-0.3039	0.7736	-0.3039
188	1	1	0	0	1	0	1	1	-0.3039	0.3039	-0.3039	0.3039	0.7736	-0.3039
189	1	1	0	1	1	0	0	1	-0.3039	0.3039	-0.3039	-0.3039	0.7736	0.3039
190	1	1	0	1	1	0	1	0	-0.3039	-0.3039	0.3039	0.3039	0.7736	-0.3039
191	1	1	0	1	1	1	0	0	-0.3039	-0.3039	0.3039	-0.3039	0.7736	0.3039
192	1	1	0	1	0	1	0	1	-0.3039	-0.3039	-0.3039	0.3039	0.7736	0.3039

193	1	1	0	1	0	1	1	0	-0.3039	0.3039	0.3039	-0.3039	-0.3039	0.7736
194	1	1	0	1	0	0	1	1	-0.3039	0.3039	-0.3039	0.3039	-0.3039	0.7736
195	1	1	1	1	0	0	0	1	-0.3039	0.3039	-0.3039	-0.3039	0.3039	0.7736
196	1	1	1	1	0	0	1	0	-0.3039	-0.3039	0.3039	0.3039	-0.3039	0.7736
197	1	1	1	1	0	1	0	0	-0.3039	-0.3039	0.3039	-0.3039	0.3039	0.7736
198	1	1	1	1	1	0	0	0	-0.3039	-0.3039	-0.3039	0.3039	0.3039	0.7736
199	1	1	1	0	1	0	0	1	0.3039	-0.3039	-0.3039	-0.7736	0.3039	0.3039
200	1	1	1	0	1	0	1	0	0.3039	0.3039	-0.3039	-0.3039	-0.7736	0.3039
201	1	1	1	0	1	1	0	0	0.3039	-0.3039	0.3039	-0.3039	-0.7736	0.3039
202	1	1	1	0	0	1	0	1	0.3039	-0.3039	-0.3039	0.3039	-0.7736	0.3039
203	1	1	1	0	0	1	1	0	0.3039	0.3039	0.3039	-0.3039	-0.3039	-0.7736
204	1	1	1	0	0	0	1	1	0.3039	0.3039	-0.3039	0.3039	-0.3039	-0.7736
205	1	0	1	0	0	1	1	1	0.3039	0.3039	-0.3039	-0.3039	0.3039	-0.7736
206	1	0	1	0	1	1	0	1	0.3039	-0.3039	0.3039	0.3039	-0.3039	-0.7736
207	1	0	1	0	1	1	1	0	0.3039	-0.3039	0.3039	-0.3039	0.3039	-0.7736
208	1	0	1	0	1	0	1	1	0.3039	-0.3039	-0.3039	0.3039	0.3039	-0.7736
209	1	0	1	1	1	0	0	1	0.3039	0.3039	-0.7736	0.3039	-0.3039	-0.3039
210	1	0	1	1	1	0	1	0	0.3039	0.3039	-0.7736	-0.3039	0.3039	-0.3039
211	1	0	1	1	1	1	0	0	0.3039	0.3039	-0.7736	-0.3039	-0.3039	0.3039
212	1	0	1	1	0	1	0	1	0.3039	-0.3039	-0.7736	0.3039	0.3039	-0.3039
213	1	0	1	1	0	1	1	0	0.3039	-0.3039	-0.7736	0.3039	-0.3039	0.3039
214	1	0	1	1	0	0	1	1	0.3039	-0.3039	-0.7736	-0.3039	0.3039	0.3039
215	1	0	0	1	0	1	1	1	0.3039	0.3039	0.3039	-0.7736	-0.3039	-0.3039
216	1	0	0	1	1	1	0	1	0.3039	0.3039	-0.3039	-0.7736	0.3039	-0.3039
217	1	0	0	1	1	1	1	0	0.3039	0.3039	-0.3039	-0.7736	-0.3039	0.3039
218	1	0	0	1	1	0	1	1	0.3039	-0.3039	0.3039	-0.7736	0.3039	-0.3039
219	1	0	0	0	1	1	1	1	0.3039	-0.3039	0.3039	-0.7736	-0.3039	0.3039
220	0	0	1	1	1	1	1	1	0.3039	0.3039	0.3039	-0.3039	-0.7736	-0.3039

221	0	1	1	0	1	1	1	1	0.3039	0.3039	-0.3039	0.3039	-0.7736	-0.3039
222	0	1	1	1	1	0	1	1	0.3039	-0.3039	0.3039	0.3039	-0.7736	-0.3039
223	0	1	1	1	1	1	1	0	-0.7336	0.3039	0.3039	0.3039	0.3039	0.3039
224	0	1	1	1	1	1	0	1	-0.3039	-0.7336	0.3039	0.3039	0.3039	0.3039
225	0	1	1	1	0	1	1	1	-0.3039	0.3039	-0.7736	0.3039	0.3039	0.3039
226	0	1	0	1	1	1	1	1	-0.3039	0.3039	0.3039	-0.7736	0.3039	0.3039
227	1	1	0	0	1	1	1	1	-0.3039	0.3039	0.3039	0.3039	-0.7736	0.3039
228	1	1	0	1	1	0	1	1	-0.3039	0.3039	0.3039	0.3039	0.3039	-0.7736
229	1	1	0	1	1	1	1	0	0.3039	-0.7336	0.3039	-0.3039	-0.3039	-0.3039
230	1	1	0	1	1	1	0	1	0.3039	-0.7336	-0.3039	0.3039	-0.3039	-0.3039
231	1	1	0	1	0	1	1	1	0.3039	-0.7336	-0.3039	-0.3039	0.3039	-0.3039
232	1	1	1	1	0	0	1	1	0.3039	-0.7336	-0.3039	-0.3039	-0.3039	0.3039
233	1	1	1	1	0	1	1	0	-0.3039	0.7336	0.3039	-0.3039	-0.3039	-0.3039
234	1	1	1	1	0	1	0	1	-0.3039	0.7336	-0.3039	0.3039	-0.3039	-0.3039
235	1	1	1	1	1	1	0	0	-0.3039	0.7336	-0.3039	-0.3039	0.3039	-0.3039
236	1	1	1	1	1	0	1	0	-0.3039	0.7336	-0.3039	-0.3039	-0.3039	0.3039
237	1	1	1	1	1	0	0	1	-0.3039	0.3039	0.7736	-0.3039	-0.3039	-0.3039
238	1	1	1	0	1	0	1	1	-0.3039	-0.3039	0.7736	0.3039	-0.3039	-0.3039
239	1	1	1	0	1	1	1	0	-0.3039	-0.3039	0.7736	-0.3039	0.3039	-0.3039
240	1	1	1	0	1	1	0	1	-0.3039	-0.3039	0.7736	-0.3039	-0.3039	0.3039
241	1	1	1	0	0	1	1	1	-0.3039	0.3039	-0.3039	0.7736	-0.3039	-0.3039
242	1	0	1	0	1	1	1	1	-0.3039	-0.3039	0.3039	0.7736	-0.3039	-0.3039
243	1	0	1	1	1	0	1	1	-0.3039	-0.3039	-0.3039	0.7736	0.3039	-0.3039
244	1	0	1	1	1	1	1	0	-0.3039	-0.3039	-0.3039	0.7736	-0.3039	0.3039
245	1	0	1	1	1	1	0	1	-0.3039	0.3039	-0.3039	-0.3039	0.7736	-0.3039
246	1	0	1	1	0	1	1	1	-0.3039	-0.3039	0.3039	-0.3039	0.7736	-0.3039
247	1	0	0	1	1	1	1	1	-0.3039	-0.3039	-0.3039	0.3039	0.7736	-0.3039
248	0	1	1	1	1	1	1	1	-0.3039	-0.3039	-0.3039	-0.3039	0.7736	0.3039

249	1	1	0	1	1	1	1	1	-0.3039	0.3039	-0.3039	-0.3039	-0.3039	0.7736
250	1	1	1	1	0	1	1	1	-0.3039	-0.3039	0.3039	-0.3039	-0.3039	0.7736
251	1	1	1	1	1	1	0	1	-0.3039	-0.3039	-0.3039	0.3039	-0.3039	0.7736
252	1	1	1	1	1	1	1	0	-0.3039	-0.3039	-0.3039	-0.3039	0.3039	0.7736
253	1	1	1	1	1	0	1	1	0.3039	-0.3039	-0.3039	-0.7736	0.3039	-0.3039
254	1	1	1	0	1	1	1	1	0.3039	-0.3039	-0.3039	-0.7736	-0.3039	0.3039
255	1	0	1	1	1	1	1	1	0.3039	0.3039	-0.3039	-0.3039	-0.7736	-0.3039
256	1	1	1	1	1	1	1	1	0.3039	-0.3039	0.3039	-0.3039	-0.7736	-0.3039

Table B.10.   7D-128CEM constellation mapping values.

Symbol	Bit Stream							Signal Vector						
								s_1	s_2	s_3	s_4	s_5	s_6	s_7
1	0	0	0	0	0	0	0	0.3780	0.3780	0.3780	0.3780	0.3780	0.3780	0.3780
2	0	0	0	0	0	0	1	0.3780	0.3780	0.3780	0.3780	0.3780	0.3780	-0.3780
3	0	0	0	0	0	1	0	0.3780	0.3780	0.3780	0.3780	0.3780	-0.3780	0.3780
4	0	0	0	0	1	0	0	0.3780	0.3780	0.3780	0.3780	-0.3780	0.3780	0.3780
5	0	0	0	1	0	0	0	0.3780	0.3780	0.3780	-0.3780	0.3780	0.3780	0.3780
6	0	0	1	0	0	0	0	0.3780	0.3780	-0.3780	0.3780	0.3780	0.3780	0.3780
7	0	1	0	0	0	0	0	0.3780	-0.3780	0.3780	0.3780	0.3780	0.3780	0.3780
8	1	0	0	0	0	0	0	-0.3780	0.3780	0.3780	0.3780	0.3780	0.3780	0.3780
9	0	0	0	0	0	1	1	0.3780	0.3780	0.3780	0.3780	0.3780	-0.3780	-0.3780
10	0	0	0	0	1	1	0	0.3780	0.3780	0.3780	0.3780	-0.3780	0.3780	-0.3780
11	0	0	0	0	1	0	1	0.3780	0.3780	0.3780	0.3780	-0.3780	-0.3780	0.3780
12	0	0	0	1	1	0	0	0.3780	0.3780	0.3780	-0.3780	0.3780	0.3780	-0.3780
13	0	0	0	1	0	1	0	0.3780	0.3780	0.3780	-0.3780	0.3780	-0.3780	0.3780
14	0	0	0	1	0	0	1	0.3780	0.3780	0.3780	-0.3780	-0.3780	0.3780	0.3780
15	0	0	1	1	0	0	0	0.3780	0.3780	-0.3780	0.3780	0.3780	0.3780	-0.3780
16	0	0	1	0	1	0	0	0.3780	0.3780	-0.3780	0.3780	0.3780	-0.3780	0.3780
17	0	0	1	0	0	1	0	0.3780	0.3780	-0.3780	0.3780	-0.3780	0.3780	0.3780
18	0	0	1	0	0	0	1	0.3780	0.3780	-0.3780	-0.3780	0.3780	0.3780	0.3780
19	0	1	1	0	0	0	0	0.3780	-0.3780	0.3780	0.3780	0.3780	0.3780	-0.3780
20	0	1	0	1	0	0	0	0.3780	-0.3780	0.3780	0.3780	0.3780	-0.3780	0.3780
21	0	1	0	0	1	0	0	0.3780	-0.3780	0.3780	0.3780	-0.3780	0.3780	0.3780
22	0	1	0	0	0	1	0	0.3780	-0.3780	0.3780	-0.3780	0.3780	0.3780	0.3780
23	0	1	0	0	0	0	1	0.3780	-0.3780	-0.3780	0.3780	0.3780	0.3780	0.3780
24	1	1	0	0	0	0	0	-0.3780	0.3780	0.3780	0.3780	0.3780	0.3780	-0.3780

25	1	0	1	0	0	0	0	-0.3780	0.3780	0.3780	0.3780	0.3780	-0.3780	0.3780
26	1	0	0	1	0	0	0	-0.3780	0.3780	0.3780	0.3780	-0.3780	0.3780	0.3780
27	1	0	0	0	1	0	0	-0.3780	0.3780	0.3780	-0.3780	0.3780	0.3780	0.3780
28	1	0	0	0	0	1	0	-0.3780	0.3780	-0.3780	0.3780	0.3780	0.3780	0.3780
29	1	0	0	0	0	0	1	-0.3780	-0.3780	0.3780	0.3780	0.3780	0.3780	0.3780
30	0	0	0	0	1	1	1	0.3780	0.3780	0.3780	0.3780	-0.3780	-0.3780	-0.3780
31	0	0	0	1	1	0	1	0.3780	0.3780	0.3780	-0.3780	0.3780	-0.3780	-0.3780
32	0	0	0	1	1	1	0	0.3780	0.3780	0.3780	-0.3780	-0.3780	0.3780	-0.3780
33	0	0	0	1	0	1	1	0.3780	0.3780	0.3780	-0.3780	-0.3780	-0.3780	0.3780
34	0	0	1	1	0	0	1	0.3780	0.3780	-0.3780	0.3780	0.3780	-0.3780	-0.3780
35	0	0	1	1	0	1	0	0.3780	0.3780	-0.3780	0.3780	-0.3780	0.3780	-0.3780
36	0	0	1	1	1	0	0	0.3780	0.3780	-0.3780	0.3780	-0.3780	-0.3780	0.3780
37	0	0	1	0	1	0	1	0.3780	0.3780	-0.3780	-0.3780	0.3780	0.3780	-0.3780
38	0	0	1	0	1	1	0	0.3780	0.3780	-0.3780	-0.3780	0.3780	-0.3780	0.3780
39	0	0	1	0	0	1	1	0.3780	0.3780	-0.3780	-0.3780	-0.3780	0.3780	0.3780
40	0	1	1	0	0	0	1	0.3780	-0.3780	0.3780	0.3780	0.3780	-0.3780	-0.3780
41	0	1	1	0	0	1	0	0.3780	-0.3780	0.3780	0.3780	-0.3780	0.3780	-0.3780
42	0	1	1	0	1	0	0	0.3780	-0.3780	0.3780	0.3780	-0.3780	-0.3780	0.3780
43	0	1	1	1	0	0	0	0.3780	-0.3780	0.3780	-0.3780	0.3780	0.3780	-0.3780
44	0	1	0	1	0	0	1	0.3780	-0.3780	0.3780	-0.3780	0.3780	-0.3780	0.3780
45	0	1	0	1	0	1	0	0.3780	-0.3780	0.3780	-0.3780	-0.3780	0.3780	0.3780
46	0	1	0	1	1	0	0	0.3780	-0.3780	-0.3780	0.3780	0.3780	0.3780	-0.3780
47	0	1	0	0	1	0	1	0.3780	-0.3780	-0.3780	0.3780	0.3780	-0.3780	0.3780
48	0	1	0	0	1	1	0	0.3780	-0.3780	-0.3780	0.3780	-0.3780	0.3780	0.3780
49	0	1	0	0	0	1	1	0.3780	-0.3780	-0.3780	-0.3780	0.3780	0.3780	0.3780
50	1	1	0	0	0	0	1	-0.3780	0.3780	0.3780	0.3780	0.3780	-0.3780	-0.3780
51	1	1	0	0	0	1	0	-0.3780	0.3780	0.3780	0.3780	-0.3780	0.3780	-0.3780
52	1	1	0	0	1	0	0	-0.3780	0.3780	0.3780	0.3780	-0.3780	-0.3780	0.3780

53	1	1	0	1	0	0	0	-0.3780	0.3780	0.3780	-0.3780	0.3780	0.3780	-0.3780
54	1	1	1	0	0	0	0	-0.3780	0.3780	0.3780	-0.3780	0.3780	-0.3780	0.3780
55	1	0	1	0	0	0	1	-0.3780	0.3780	0.3780	-0.3780	-0.3780	0.3780	0.3780
56	1	0	1	0	0	1	0	-0.3780	0.3780	-0.3780	0.3780	0.3780	0.3780	-0.3780
57	1	0	1	0	1	0	0	-0.3780	0.3780	-0.3780	0.3780	0.3780	-0.3780	0.3780
58	1	0	1	1	0	0	0	-0.3780	0.3780	-0.3780	0.3780	-0.3780	0.3780	0.3780
59	1	0	0	1	0	0	1	-0.3780	0.3780	-0.3780	-0.3780	0.3780	0.3780	0.3780
60	1	0	0	1	0	1	0	-0.3780	-0.3780	0.3780	0.3780	0.3780	0.3780	-0.3780
61	1	0	0	1	1	0	0	-0.3780	-0.3780	0.3780	0.3780	0.3780	-0.3780	0.3780
62	1	0	0	0	1	0	1	-0.3780	-0.3780	0.3780	0.3780	-0.3780	0.3780	0.3780
63	1	0	0	0	1	1	0	-0.3780	-0.3780	0.3780	-0.3780	0.3780	0.3780	0.3780
64	1	0	0	0	0	1	1	-0.3780	-0.3780	-0.3780	0.3780	0.3780	0.3780	0.3780
65	0	0	0	1	1	1	1	0.3780	0.3780	0.3780	-0.3780	-0.3780	-0.3780	-0.3780
66	0	0	1	1	0	1	1	0.3780	0.3780	-0.3780	0.3780	-0.3780	-0.3780	-0.3780
67	0	0	1	1	1	1	0	0.3780	0.3780	-0.3780	-0.3780	0.3780	-0.3780	-0.3780
68	0	0	1	1	1	0	1	0.3780	0.3780	-0.3780	-0.3780	-0.3780	0.3780	-0.3780
69	0	0	1	0	1	1	1	0.3780	0.3780	-0.3780	-0.3780	-0.3780	-0.3780	0.3780
70	0	1	1	0	0	1	1	0.3780	-0.3780	0.3780	0.3780	-0.3780	-0.3780	-0.3780
71	0	1	1	0	1	1	0	0.3780	-0.3780	0.3780	-0.3780	0.3780	-0.3780	-0.3780
72	0	1	1	0	1	0	1	0.3780	-0.3780	0.3780	-0.3780	-0.3780	0.3780	-0.3780
73	0	1	1	1	1	0	0	0.3780	-0.3780	0.3780	-0.3780	-0.3780	-0.3780	0.3780
74	0	1	1	1	0	1	0	0.3780	-0.3780	-0.3780	0.3780	0.3780	-0.3780	-0.3780
75	0	1	1	1	0	0	1	0.3780	-0.3780	-0.3780	0.3780	-0.3780	0.3780	-0.3780
76	0	1	0	1	0	1	1	0.3780	-0.3780	-0.3780	0.3780	-0.3780	-0.3780	0.3780
77	0	1	0	1	1	1	0	0.3780	-0.3780	-0.3780	-0.3780	0.3780	0.3780	-0.3780
78	0	1	0	1	1	0	1	0.3780	-0.3780	-0.3780	-0.3780	0.3780	-0.3780	0.3780
79	0	1	0	0	1	1	1	0.3780	-0.3780	-0.3780	-0.3780	-0.3780	0.3780	0.3780
80	1	1	0	0	0	1	1	-0.3780	0.3780	0.3780	0.3780	-0.3780	-0.3780	-0.3780

81	1	1	0	0	1	1	0	-0.3780	0.3780	0.3780	-0.3780	0.3780	-0.3780	-0.3780
82	1	1	0	0	1	0	1	-0.3780	0.3780	0.3780	-0.3780	-0.3780	0.3780	-0.3780
83	1	1	0	1	1	0	0	-0.3780	0.3780	0.3780	-0.3780	-0.3780	-0.3780	0.3780
84	1	1	0	1	0	1	0	-0.3780	0.3780	-0.3780	0.3780	0.3780	-0.3780	-0.3780
85	1	1	0	1	0	0	1	-0.3780	0.3780	-0.3780	0.3780	-0.3780	0.3780	-0.3780
86	1	1	1	1	0	0	0	-0.3780	0.3780	-0.3780	0.3780	-0.3780	-0.3780	0.3780
87	1	1	1	0	1	0	0	-0.3780	0.3780	-0.3780	-0.3780	0.3780	0.3780	-0.3780
88	1	1	1	0	0	1	0	-0.3780	0.3780	-0.3780	-0.3780	0.3780	-0.3780	0.3780
89	1	1	1	0	0	0	1	-0.3780	0.3780	-0.3780	-0.3780	-0.3780	0.3780	0.3780
90	1	0	1	0	0	1	1	-0.3780	-0.3780	0.3780	0.3780	0.3780	-0.3780	-0.3780
91	1	0	1	0	1	1	0	-0.3780	-0.3780	0.3780	0.3780	-0.3780	0.3780	-0.3780
92	1	0	1	0	1	0	1	-0.3780	-0.3780	0.3780	0.3780	-0.3780	-0.3780	0.3780
93	1	0	1	1	1	0	0	-0.3780	-0.3780	0.3780	-0.3780	0.3780	0.3780	-0.3780
94	1	0	1	1	0	1	0	-0.3780	-0.3780	0.3780	-0.3780	0.3780	-0.3780	0.3780
95	1	0	1	1	0	0	1	-0.3780	-0.3780	0.3780	-0.3780	-0.3780	0.3780	0.3780
96	1	0	0	1	0	1	1	-0.3780	-0.3780	-0.3780	0.3780	0.3780	0.3780	-0.3780
97	1	0	0	1	1	1	0	-0.3780	-0.3780	-0.3780	0.3780	0.3780	-0.3780	0.3780
98	1	0	0	1	1	0	1	-0.3780	-0.3780	-0.3780	0.3780	-0.3780	0.3780	0.3780
99	1	0	0	0	1	1	1	-0.3780	-0.3780	-0.3780	-0.3780	0.3780	0.3780	0.3780
100	0	0	1	1	1	1	1	0.3780	0.3780	-0.3780	-0.3780	-0.3780	-0.3780	-0.3780
101	0	1	1	0	1	1	1	0.3780	-0.3780	0.3780	-0.3780	-0.3780	-0.3780	-0.3780
102	0	1	1	1	1	0	1	0.3780	-0.3780	-0.3780	0.3780	-0.3780	-0.3780	-0.3780
103	0	1	1	1	1	1	0	0.3780	-0.3780	-0.3780	-0.3780	0.3780	-0.3780	-0.3780
104	0	1	1	1	0	1	1	0.3780	-0.3780	-0.3780	-0.3780	-0.3780	0.3780	-0.3780
105	0	1	0	1	1	1	1	0.3780	-0.3780	-0.3780	-0.3780	-0.3780	-0.3780	0.3780
106	1	1	0	0	1	1	1	-0.3780	0.3780	0.3780	-0.3780	-0.3780	-0.3780	-0.3780
107	1	1	0	1	1	0	1	-0.3780	0.3780	-0.3780	0.3780	-0.3780	-0.3780	-0.3780
108	1	1	0	1	1	1	0	-0.3780	0.3780	-0.3780	-0.3780	0.3780	-0.3780	-0.3780

109	1	1	0	1	0	1	1	-0.3780	0.3780	-0.3780	-0.3780	-0.3780	0.3780	-0.3780
110	1	1	1	1	0	0	1	-0.3780	0.3780	-0.3780	-0.3780	-0.3780	-0.3780	0.3780
111	1	1	1	1	0	1	0	-0.3780	-0.3780	0.3780	0.3780	-0.3780	-0.3780	-0.3780
112	1	1	1	1	1	0	0	-0.3780	-0.3780	0.3780	-0.3780	0.3780	-0.3780	-0.3780
113	1	1	1	0	1	0	1	-0.3780	-0.3780	0.3780	-0.3780	-0.3780	0.3780	-0.3780
114	1	1	1	0	1	1	0	-0.3780	-0.3780	0.3780	-0.3780	-0.3780	-0.3780	0.3780
115	1	1	1	0	0	1	1	-0.3780	-0.3780	-0.3780	0.3780	0.3780	-0.3780	-0.3780
116	1	0	1	0	1	1	1	-0.3780	-0.3780	-0.3780	0.3780	-0.3780	0.3780	-0.3780
117	1	0	1	1	1	0	1	-0.3780	-0.3780	-0.3780	0.3780	-0.3780	-0.3780	0.3780
118	1	0	1	1	1	1	0	-0.3780	-0.3780	-0.3780	-0.3780	0.3780	0.3780	-0.3780
119	1	0	1	1	0	1	1	-0.3780	-0.3780	-0.3780	-0.3780	0.3780	-0.3780	0.3780
120	1	0	0	1	1	1	1	-0.3780	-0.3780	-0.3780	-0.3780	-0.3780	0.3780	0.3780
121	0	1	1	1	1	1	1	0.3780	-0.3780	-0.3780	-0.3780	-0.3780	-0.3780	-0.3780
122	1	1	0	1	1	1	1	-0.3780	0.3780	-0.3780	-0.3780	-0.3780	-0.3780	-0.3780
123	1	1	1	1	0	1	1	-0.3780	-0.3780	0.3780	-0.3780	-0.3780	-0.3780	-0.3780
124	1	1	1	1	1	1	0	-0.3780	-0.3780	-0.3780	0.3780	-0.3780	-0.3780	-0.3780
125	1	1	1	1	1	0	1	-0.3780	-0.3780	-0.3780	-0.3780	0.3780	-0.3780	-0.3780
126	1	1	1	0	1	1	1	-0.3780	-0.3780	-0.3780	-0.3780	-0.3780	0.3780	-0.3780
127	1	0	1	1	1	1	1	-0.3780	-0.3780	-0.3780	-0.3780	-0.3780	-0.3780	0.3780
128	1	1	1	1	1	1	1	-0.3780	-0.3780	-0.3780	-0.3780	-0.3780	-0.3780	-0.3780

Table B.11. 8D-256CEM constellation mapping values.

Symbol	Bit Stream								Signal Vector							
									s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
1	0	0	0	0	0	0	0	0	0.3536	0.3536	0.3536	0.3536	0.3536	0.3536	0.3536	0.3536
2	0	0	0	0	0	0	0	1	0.3536	0.3536	0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536
3	0	0	0	0	0	0	1	0	0.3536	0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536
4	0	0	0	0	0	1	0	0	0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536
5	0	0	0	0	1	0	0	0	0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536
6	0	0	0	1	0	0	0	0	0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536
7	0	0	1	0	0	0	0	0	0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536	0.3536
8	0	1	0	0	0	0	0	0	0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536	0.3536	0.3536
9	1	0	0	0	0	0	0	0	-0.3536	0.3536	0.3536	0.3536	0.3536	0.3536	0.3536	0.3536
10	0	0	0	0	0	0	1	1	0.3536	0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536
11	0	0	0	0	0	1	1	0	0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536
12	0	0	0	0	0	1	0	1	0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536
13	0	0	0	0	1	1	0	0	0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536
14	0	0	0	0	1	0	1	0	0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536
15	0	0	0	0	1	0	0	1	0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536
16	0	0	0	1	1	0	0	0	0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536
17	0	0	0	1	0	1	0	0	0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536
18	0	0	0	1	0	0	1	0	0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536
19	0	0	0	1	0	0	0	1	0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536
20	0	0	1	1	0	0	0	0	0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536
21	0	0	1	0	1	0	0	0	0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536
22	0	0	1	0	0	1	0	0	0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536
23	0	0	1	0	0	0	1	0	0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536
24	0	0	1	0	0	0	0	1	0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536

25	0	1	1	0	0	0	0	0	0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536
26	0	1	0	1	0	0	0	0	0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536
27	0	1	0	0	1	0	0	0	0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536
28	0	1	0	0	0	1	0	0	0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536
29	0	1	0	0	0	0	1	0	0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536
30	0	1	0	0	0	0	0	1	0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536	0.3536
31	1	1	0	0	0	0	0	0	-0.3536	0.3536	0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536
32	1	0	1	0	0	0	0	0	-0.3536	0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536
33	1	0	0	1	0	0	0	0	-0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536
34	1	0	0	0	1	0	0	0	-0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536
35	1	0	0	0	0	1	0	0	-0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536
36	1	0	0	0	0	0	1	0	-0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536	0.3536
37	1	0	0	0	0	0	0	1	-0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536	0.3536	0.3536
38	0	0	0	0	0	1	1	1	0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536
39	0	0	0	0	1	1	0	1	0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536
40	0	0	0	0	1	1	1	0	0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536
41	0	0	0	0	1	0	1	1	0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536
42	0	0	0	1	1	0	0	1	0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536
43	0	0	0	1	1	0	1	0	0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536
44	0	0	0	1	1	1	0	0	0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536
45	0	0	0	1	0	1	0	1	0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536
46	0	0	0	1	0	1	1	0	0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536
47	0	0	0	1	0	0	1	1	0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536
48	0	0	1	1	0	0	0	1	0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536
49	0	0	1	1	0	0	1	0	0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536
50	0	0	1	1	0	1	0	0	0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536
51	0	0	1	1	1	0	0	0	0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536
52	0	0	1	0	1	0	0	1	0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536

53	0	0	1	0	1	0	1	0	0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536
54	0	0	1	0	1	1	0	0	0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536
55	0	0	1	0	0	1	0	1	0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536
56	0	0	1	0	0	1	1	0	0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536
57	0	0	1	0	0	0	1	1	0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536
58	0	1	1	0	0	0	0	1	0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536
59	0	1	1	0	0	0	1	0	0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536
60	0	1	1	0	0	1	0	0	0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536
61	0	1	1	0	1	0	0	0	0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536
62	0	1	1	1	0	0	0	0	0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536
63	0	1	0	1	0	0	0	1	0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536
64	0	1	0	1	0	0	1	0	0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536
65	0	1	0	1	0	1	0	0	0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536
66	0	1	0	1	1	0	0	0	0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536
67	0	1	0	0	1	0	0	1	0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536
68	0	1	0	0	1	0	1	0	0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536
69	0	1	0	0	1	1	0	0	0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536
70	0	1	0	0	0	1	0	1	0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536
71	0	1	0	0	0	1	1	0	0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536
72	0	1	0	0	0	0	1	1	0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536
73	1	1	0	0	0	0	0	1	-0.3536	0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536
74	1	1	0	0	0	0	1	0	-0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536
75	1	1	0	0	0	1	0	0	-0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536
76	1	1	0	0	1	0	0	0	-0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536
77	1	1	0	1	0	0	0	0	-0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536
78	1	1	1	0	0	0	0	0	-0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536
79	1	0	1	0	0	0	0	1	-0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536
80	1	0	1	0	0	0	1	0	-0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536

81	1	0	1	0	0	1	0	0	-0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536
82	1	0	1	0	1	0	0	0	-0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536
83	1	0	1	1	0	0	0	0	-0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536
84	1	0	0	1	0	0	0	1	-0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536
85	1	0	0	1	0	0	1	0	-0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536
86	1	0	0	1	0	1	0	0	-0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536
87	1	0	0	1	1	0	0	0	-0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536
88	1	0	0	0	1	0	0	1	-0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536
89	1	0	0	0	1	0	1	0	-0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536
90	1	0	0	0	1	1	0	0	-0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536
91	1	0	0	0	0	1	0	1	-0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536
92	1	0	0	0	0	1	1	0	-0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536
93	1	0	0	0	0	0	1	1	-0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536	0.3536
94	0	0	0	0	1	1	1	1	0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536
95	0	0	0	1	1	0	1	1	0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536
96	0	0	0	1	1	1	1	0	0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536
97	0	0	0	1	1	1	0	1	0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536
98	0	0	0	1	0	1	1	1	0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536
99	0	0	1	1	0	0	1	1	0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536
100	0	0	1	1	0	1	1	0	0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536
101	0	0	1	1	0	1	0	1	0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536
102	0	0	1	1	1	1	0	0	0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536
103	0	0	1	1	1	0	1	0	0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536
104	0	0	1	1	1	0	0	1	0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536
105	0	0	1	0	1	0	1	1	0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536
106	0	0	1	0	1	1	1	0	0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536
107	0	0	1	0	1	1	0	1	0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536
108	0	0	1	0	0	1	1	1	0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536

109	0	1	1	0	0	0	1	1	0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536
110	0	1	1	0	0	1	1	0	0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536
111	0	1	1	0	0	1	0	1	0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536
112	0	1	1	0	1	1	0	0	0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536
113	0	1	1	0	1	0	1	0	0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536
114	0	1	1	0	1	0	0	1	0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536
115	0	1	1	1	1	0	0	0	0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536
116	0	1	1	1	0	1	0	0	0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536
117	0	1	1	1	0	0	1	0	0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536
118	0	1	1	1	0	0	0	1	0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536
119	0	1	0	1	0	0	1	1	0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536
120	0	1	0	1	0	1	1	0	0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536
121	0	1	0	1	0	1	0	1	0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536
122	0	1	0	1	1	1	0	0	0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536
123	0	1	0	1	1	0	1	0	0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536
124	0	1	0	1	1	0	0	1	0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536
125	0	1	0	0	1	0	1	1	0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536
126	0	1	0	0	1	1	1	0	0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536
127	0	1	0	0	1	1	0	1	0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536
128	0	1	0	0	0	1	1	1	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536
129	1	1	0	0	0	0	1	1	-0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536
130	1	1	0	0	0	1	1	0	-0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536
131	1	1	0	0	0	1	0	1	-0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536
132	1	1	0	0	1	1	0	0	-0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536
133	1	1	0	0	1	0	1	0	-0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536
134	1	1	0	0	1	0	0	1	-0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536
135	1	1	0	1	1	0	0	0	-0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536
136	1	1	0	1	0	1	0	0	-0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536

137	1	1	0	1	0	0	1	0	-0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536
138	1	1	0	1	0	0	0	1	-0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536
139	1	1	1	1	0	0	0	0	-0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536
140	1	1	1	0	1	0	0	0	-0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536
141	1	1	1	0	0	1	0	0	-0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536
142	1	1	1	0	0	0	1	0	-0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536
143	1	1	1	0	0	0	0	1	-0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536
144	1	0	1	0	0	0	1	1	-0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536
145	1	0	1	0	0	1	1	0	-0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536
146	1	0	1	0	0	1	0	1	-0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536
147	1	0	1	0	1	1	0	0	-0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536
148	1	0	1	0	1	0	1	0	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536
149	1	0	1	0	1	0	0	1	-0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536
150	1	0	1	1	1	0	0	0	-0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536
151	1	0	1	1	0	1	0	0	-0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536
152	1	0	1	1	0	0	1	0	-0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536
153	1	0	1	1	0	0	0	1	-0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536
154	1	0	0	1	0	0	1	1	-0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536
155	1	0	0	1	0	1	1	0	-0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536
156	1	0	0	1	0	1	0	1	-0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536
157	1	0	0	1	1	1	0	0	-0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536
158	1	0	0	1	1	0	1	0	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536
159	1	0	0	1	1	0	0	1	-0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536	-0.3536
160	1	0	0	0	1	0	1	1	-0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536	0.3536
161	1	0	0	0	1	1	1	0	-0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536	0.3536
162	1	0	0	0	1	1	0	1	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536	0.3536
163	1	0	0	0	0	1	1	1	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536	0.3536
164	0	0	0	1	1	1	1	1	0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536

165	0	0	1	1	0	1	1	1	0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536
166	0	0	1	1	1	1	0	1	0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536
167	0	0	1	1	1	1	1	0	0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536
168	0	0	1	1	1	0	1	1	0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536
169	0	0	1	0	1	1	1	1	0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536
170	0	1	1	0	0	1	1	1	0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536
171	0	1	1	0	1	1	0	1	0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536
172	0	1	1	0	1	1	1	0	0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536
173	0	1	1	0	1	0	1	1	0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536
174	0	1	1	1	1	0	0	1	0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536
175	0	1	1	1	1	0	1	0	0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536
176	0	1	1	1	1	1	0	0	0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536
177	0	1	1	1	0	1	0	1	0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536
178	0	1	1	1	0	1	1	0	0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536
179	0	1	1	1	0	0	1	1	0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536
180	0	1	0	1	0	1	1	1	0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536
181	0	1	0	1	1	1	0	1	0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536
182	0	1	0	1	1	1	1	0	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536
183	0	1	0	1	1	0	1	1	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536
184	0	1	0	0	1	1	1	1	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536
185	1	1	0	0	0	1	1	1	-0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536
186	1	1	0	0	1	1	0	1	-0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536
187	1	1	0	0	1	1	1	0	-0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536
188	1	1	0	0	1	0	1	1	-0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536
189	1	1	0	1	1	0	0	1	-0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536
190	1	1	0	1	1	0	1	0	-0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536
191	1	1	0	1	1	1	0	0	-0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536
192	1	1	0	1	0	1	0	1	-0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536

193	1	1	0	1	0	1	1	0	-0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536
194	1	1	0	1	0	0	1	1	-0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536
195	1	1	1	1	0	0	0	1	-0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536
196	1	1	1	1	0	0	1	0	-0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536
197	1	1	1	1	0	1	0	0	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536
198	1	1	1	1	1	0	0	0	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536
199	1	1	1	0	1	0	0	1	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536
200	1	1	1	0	1	0	1	0	-0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536
201	1	1	1	0	1	1	0	0	-0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536
202	1	1	1	0	0	1	0	1	-0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536
203	1	1	1	0	0	1	1	0	-0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536
204	1	1	1	0	0	0	1	1	-0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536
205	1	0	1	0	0	1	1	1	-0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536
206	1	0	1	0	1	1	0	1	-0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536
207	1	0	1	0	1	1	1	0	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536
208	1	0	1	0	1	0	1	1	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536
209	1	0	1	1	1	0	0	1	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536
210	1	0	1	1	1	0	1	0	-0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536	-0.3536
211	1	0	1	1	1	1	0	0	-0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536	-0.3536
212	1	0	1	1	0	1	0	1	-0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536	0.3536
213	1	0	1	1	0	1	1	0	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536	-0.3536
214	1	0	1	1	0	0	1	1	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536	0.3536
215	1	0	0	1	0	1	1	1	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536	0.3536
216	1	0	0	1	1	1	0	1	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536	-0.3536
217	1	0	0	1	1	1	1	0	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536	0.3536
218	1	0	0	1	1	0	1	1	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536	0.3536
219	1	0	0	0	1	1	1	1	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536	0.3536
220	0	0	1	1	1	1	1	1	0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536

221	0	1	1	0	1	1	1	1	0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536
222	0	1	1	1	1	0	1	1	0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536
223	0	1	1	1	1	1	1	0	0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536
224	0	1	1	1	1	1	0	1	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536
225	0	1	1	1	0	1	1	1	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536
226	0	1	0	1	1	1	1	1	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536
227	1	1	0	0	1	1	1	1	-0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536
228	1	1	0	1	1	0	1	1	-0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536
229	1	1	0	1	1	1	1	0	-0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536
230	1	1	0	1	1	1	0	1	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536
231	1	1	0	1	0	1	1	1	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536
232	1	1	1	1	0	0	1	1	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536
233	1	1	1	1	0	1	1	0	-0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536
234	1	1	1	1	0	1	0	1	-0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536
235	1	1	1	1	1	1	0	0	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536
236	1	1	1	1	1	0	1	0	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536
237	1	1	1	1	1	0	0	1	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536
238	1	1	1	0	1	0	1	1	-0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536	-0.3536
239	1	1	1	0	1	1	1	0	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536	-0.3536
240	1	1	1	0	1	1	0	1	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536	-0.3536
241	1	1	1	0	0	1	1	1	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	0.3536
242	1	0	1	0	1	1	1	1	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536	-0.3536
243	1	0	1	1	1	0	1	1	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536	-0.3536
244	1	0	1	1	1	1	1	0	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	0.3536
245	1	0	1	1	1	1	0	1	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536	-0.3536
246	1	0	1	1	0	1	1	1	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	0.3536
247	1	0	0	1	1	1	1	1	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	0.3536
248	0	1	1	1	1	1	1	1	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536

249	1	1	0	1	1	1	1	1	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536
250	1	1	1	1	0	1	1	1	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536
251	1	1	1	1	1	1	0	1	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536	-0.3536
252	1	1	1	1	1	1	1	0	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536	-0.3536
253	1	1	1	1	1	0	1	1	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536	-0.3536
254	1	1	1	0	1	1	1	1	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536	-0.3536
255	1	0	1	1	1	1	1	1	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	0.3536
256	1	1	1	1	1	1	1	1	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536	-0.3536

THIS PAGE INTENTIONALLY LEFT BLANK

APPENDIX C. MONTE CARLO SIMULATION RESULTS FOR *N*=5

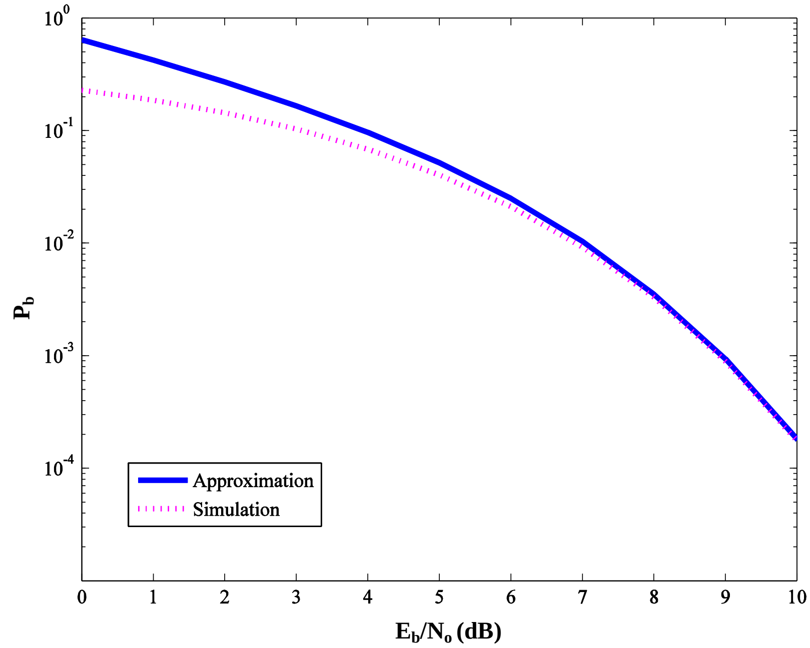

Figure C.1. BER for 5D-64CEM using Monte Carlo simulations.

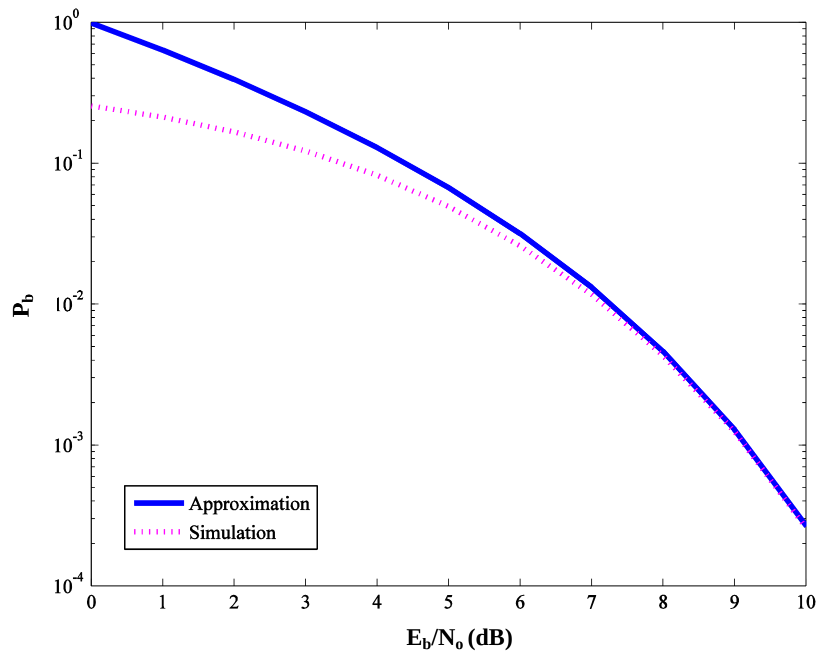

Figure C.2. BER for 5D-128CEM using Monte Carlo simulations.

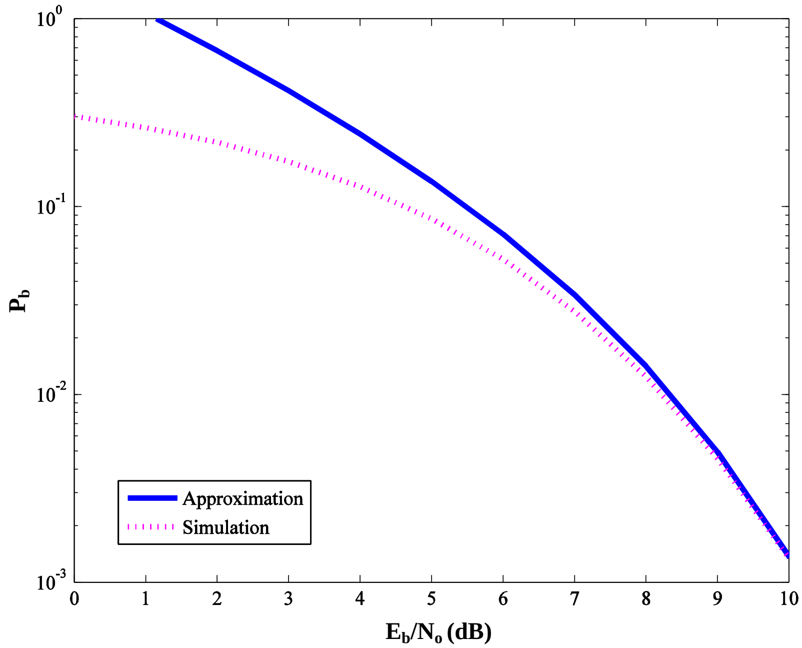

Figure C.3. BER for 5D-256CEM using Monte Carlo simulations.

APPENDIX D. MONTE CARLO SIMULATION RESULTS FOR *N*=6

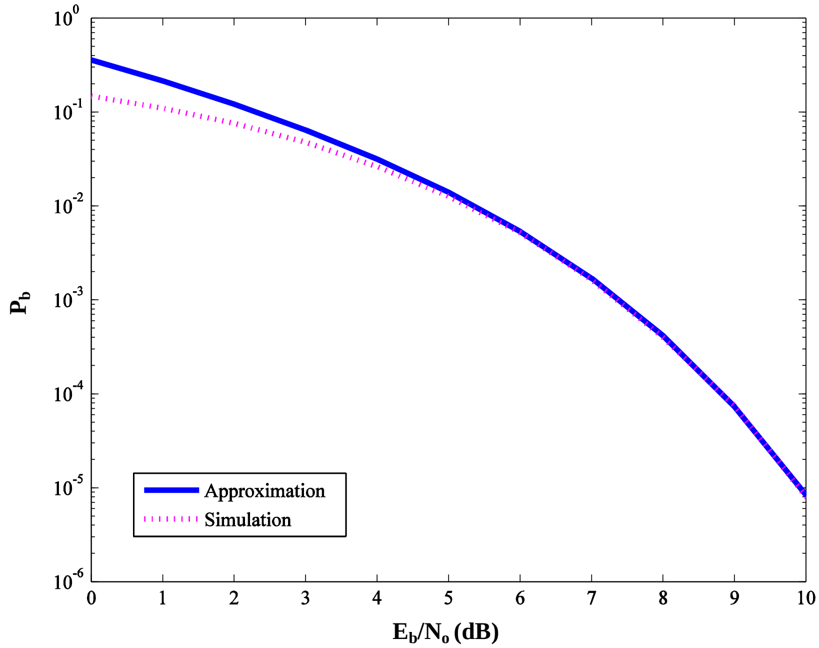

Figure D.1. BER for 6D-64CEM using Monte Carlo simulations.

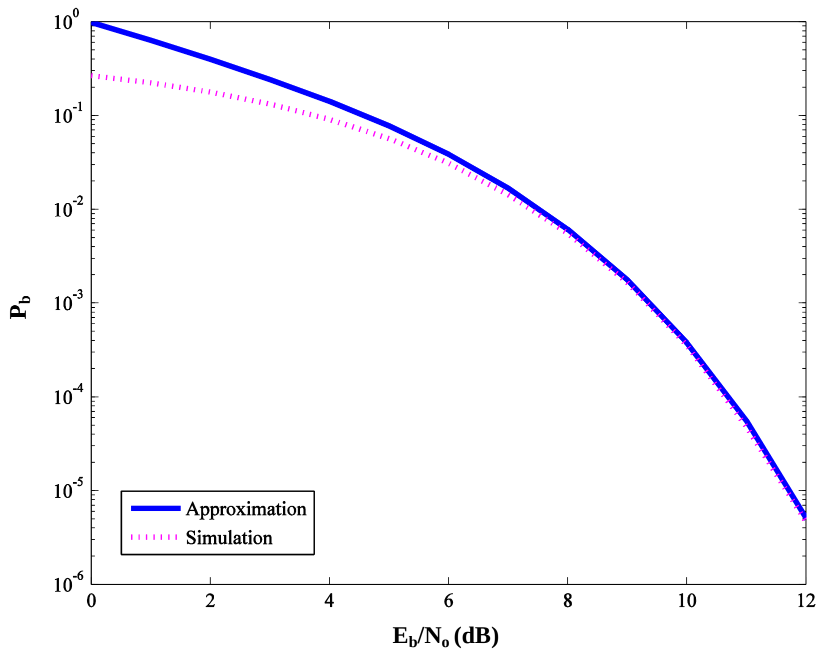

Figure D.2. BER for 6D-128CEM using Monte Carlo simulations.

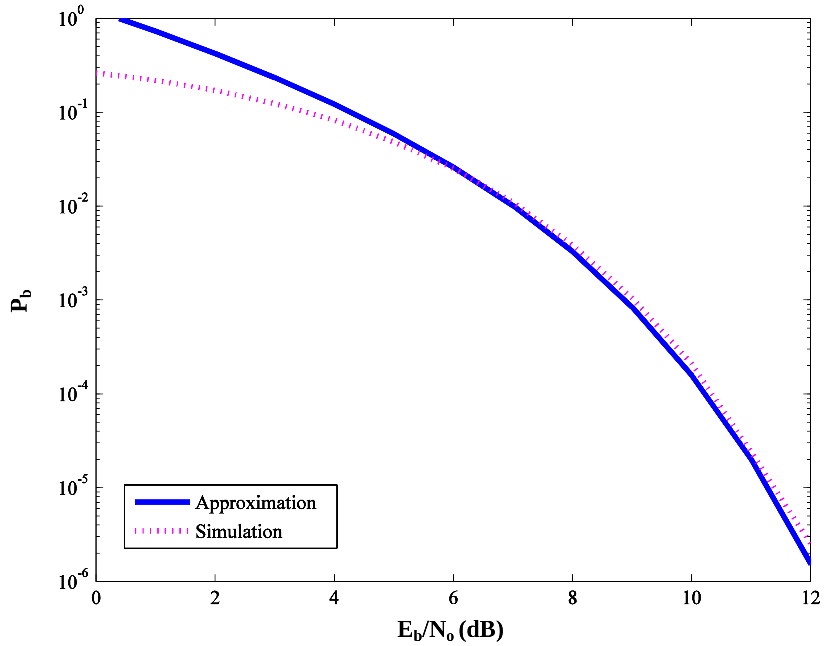

Figure D.3. BER for 6D-256CEM using Monte Carlo simulations.

APPENDIX E. MONTE CARLO SIMULATION RESULTS FOR *N*=7

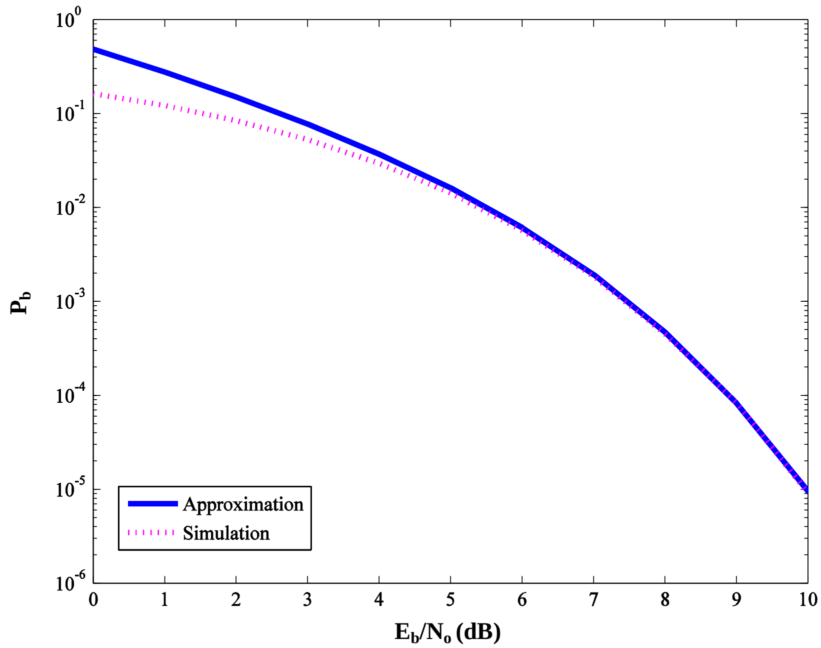

Figure E.1. BER for 7D-128CEM using Monte Carlo simulations.

THIS PAGE INTENTIONALLY LEFT BLANK

APPENDIX F. MONTE CARLO SIMULATION RESULTS FOR *N*=8

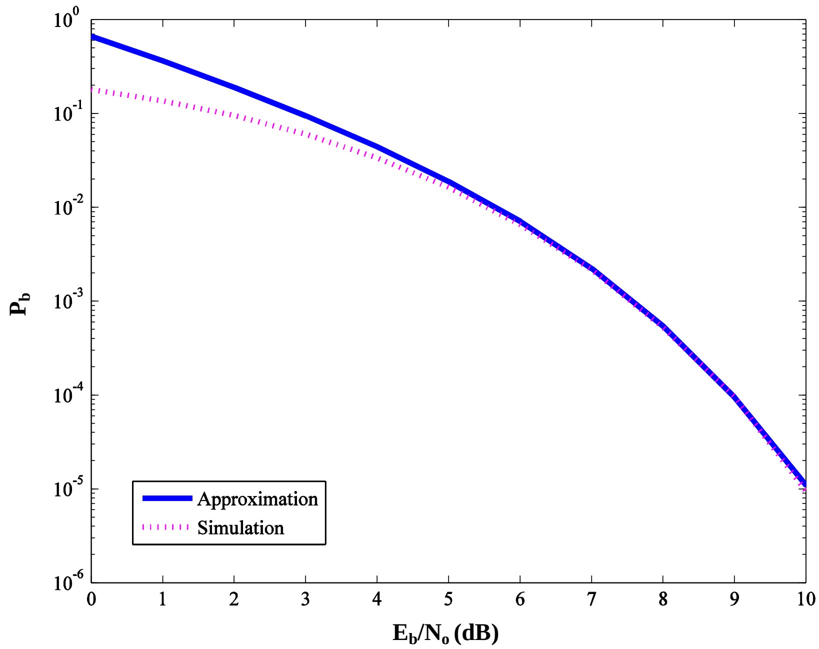

Figure F.1. BER for 8D-256CEM using Monte Carlo simulations.

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF REFERENCES

[1] J. Caldwell, "Hyper phase shift keying modulation," U.S. Patent 8 064 541, November 22, 2011.

[2] T. C. Wong and H. M. Kwon, "Multi-dimensional modulation for multi-path fading channels: Performance analysis," in *Proc. IEE*E *Military Communications Conference*, pp. 1743–1746, San Jose, CA, 2010.

[3] T. T. Ha, *Theory and Design of Digital Communicaton Systems*. New York: Cambridge University Press, 2011.

[4] Orthogonal frequency-division multiplexing. From Wikipedia, 29 July 2013. [Online]. Available: http://en.wikipedia.org/wiki/Orthogonal_frequency-division_multiplexing. [Accessed 5 August 2013].

[5] S. L. Drakul and E. Biglieri, "A class of constant envelope multi-dimensional constant envelope schemes for non-Linearly amplified channels," in *Proc. IEEE Singapore International Conference on Communication Systems*, vol. 1, pp. 114–118, Singapore, 1994.

[6] H. Sari, “A generalization of multidimensional modulation,” in *Proc. IEEE International Conference Communications*, vol. 2, pp. 683–687, Seattle, WA, 1995.

[7] T. C. Wong et al., "Hybrid multi-dimensional modulation for Gaussian and fading channels," in *Proc. IEEE Vehicular Technology Conference Fall*, pp. 1–5, Ottawa, ON, 2010.

[8] M. Beko and R. Dinis, “Designing good multi-dimensional constellations,” *IEEE Wireless Communications Letters*, vol. 1, no. 3, pp. 221–224, 3 April 2012.

[9] G. R. Welti and J. S. Lee, “Digital transmission with coherent four-dimensional modulation,” *IEEE Trans. Inf. Theory*, pp. 497–502, July 1974.

[10] R. C. Bose and B. Manvel, *Introduction to Combinatorial Theory*. New York: Wiley, 1984

[11] S. Gilson et al., A Report on the Project Implementation of Channel Coding in OFDM System, May 2013. [Online]. Available: http://share.pdfonline.com/94f58321cdc94c76ad0df1b322e39414/black_final.pdf. [Accessed 9 August 2013].

[12] A. A. Saleh, "Frequency-independent and frequency-dependent nonlinear models of TWT amplifiers," *IEEE Trans. Commun.*, vol. 29, no. 11, pp. 1715–1720, November 1981.

INITIAL DISTRIBUTION LIST

1. Defense Technical Information Center
   Ft. Belvoir, Virginia

2. Dudley Knox Library
   Naval Postgraduate School
   Monterey, California