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Conceptions of Functions Among First Degree and Diploma Students

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ABSTRACT

The concept of functions is one of the most important topics in mathematics learning. However many studies have indicated various difficulties students exhibited in learning the concept of functions. Tall & Vinner used the term concept images to describe how students think about concepts. A person's concept image can differ greatly from the formal mathematical definitions of the concept. This study explores an aspect of understanding the concept of functions related to this theme. One hundred and ninety university students from two different universities and with a different academic background participated in the study. The first group of students were diploma students in their second semester of engineering programme. The second group were first year students enroll in science based degree in another higher institutions. Data were gathered from questionnaires which asked students to identify functions in graphic and algebraic forms. They were also requested to give reasons for their choices. A large number of students indicated a strong tendency to observe function only as a formula between x and y. Their justification for rejecting or accepting representations of functions showed a rather rigid and limited understanding of the concept. Many were unable to give clear reasoning explaining their choices according to the formal definition from their previous learning.

Introduction

The concept of functions is one of the important topics in secondary and higher mathematics education (Costello, 1992; NTCM, 1989). Functions can be represented in various ways but the most common ones are tables, algebraic,

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graphs and situations. It should, however be noted that many functions in mathematics, physics or computer sciences are not represented by formula. Hence, the need for students to have the ability to identify various representations other than formulae that they are used to. In Malaysian school curriculum, this concept is introduced formally in Form Four of the secondary level via sets; in terms of domain and range and rule relating each element in the first set with a unique element in the second. In form three, the students are introduced to the concept as a relation between dependent and non-independent variables, specifically as a rule between x and y. The concepts are further developed in the learning of calculus in form four (additional mathematics) throughout form six and at higher level as an introduction to university calculus course. In higher learning, students difficulties in calculus has been associated with weak conceptual understanding of functions (White & Michelmore, 1996). Eisenberg (1992) warns that it is almost impossible to acquire higher concepts in mathematics without understanding the concepts of function.

Literature Review

The idea of function underlies many central concepts in the study of calculus such as limit, derivatives and integral. The importance of this idea has led to a vast amount of research done internationally on students' difficulties in learning the concepts (refer to Lienhart et al. 1992, for a review); however, the difficulties Malaysian students encounter with the concepts has only been reported by Bakar & Tall (1992). It should be pointed out that the definitions students have encountered in form three and form four in Malaysian School Curricullum stresses on two different aspects of function properties. The first definition emphasises the idea of dependency (form three) whilst the definition in form four stresses on arbitrariness and univalence property of the correspondence (Malik, 1982). The latter is considered as a formal definition of function. Hence, it is this gap that causes conflict among students in secondary and higher level courses in Mathematics (Bakar & Tall, 1992).

The way mathematical ideas are presented can be referred to the idea of concept definition and concept images as defined by Tall & Vinner (1981). A mathematical definition of concepts is clearly defined by any mathematical textbooks, whereas concept images include mental pictures, associated properties and processes associated with the mind concerning the concept. Furthermore, concept images may comprises of visual representations, mental pictures or experiences evoke by the concept name. Poor concept image means that the individual is only using a few prototypical examples while applying the concept. Accordingly, a richer concept images mean basing judgement which includes a variety of examples associated with the concepts and its properties. Vinner & Dreyfus (1989) asked college students 'what is a function'. They were further asked to identify if three graphs given were examples of functions. Students responses to definition function were categorized into six patterns: correspondences, dependence relations, rule, formula, operations and representation whereas concept images used to reject particular graphs as functions were summarized into the following categories:

i. One-valuedness: If the correspondence assigns exactly one value to every element in its domain, then it is a function, if not it is not a function.
ii. Discontinuity: If the graph has a gap, the correspondence is discontinuous at one point in its domain.
iii. Split Domain: The domain of the correspondence splits into two sub-domains, in each of which a different rule or correspondence holds.
iv. Exceptional Point: If there is a point of exception for a given correspondence, that is where a point for which the general rule of correspondence does not hold.

Breidenbach & Dubinsky (1992) reported that a typical understanding on the function concept among college students in function is a formula –act of substitution. These students gave satisfactory definitions of the concepts but most of the answers given were either algorithmic or showed a limited view on the concepts.

Research Objectives

This study is exploratory in nature, undertaken in a manner similar to Vinner & Dreyfus (1989) and Bakar & Tall (1992), which addresses students' concept images and definitions in a graphical and symbolic form of functions. This study attempts to investigate how university students associate the concept of functions with its representations in symbolic and graphical forms. What concept images do these students have about functions and its graph? Do these images of graphs and function observed among selected group of Malaysian undergraduate students, coherent with the formal definitions that they have learnt? Are there any differences in understanding of function among two groups of students with different academic background.

Research Questions

Since the concept of function is the main prerequisite for calculus course at higher learning, it is therefore important that students should develop the ability to think formally about the concept and have acquired the flexibility to use it in various forms of representations. Specifically, this study investigates the following research questions:

a. What are the common definitions of functions accepted by first year degree students?
b. What are the common definitions of functions accepted by first year diploma students?
c. Which form of representations of function that is the most difficult for both group of students?
d. Are there significant differences in ability in identifying function either in symbolic or graphical form, between two groups of students?
e. Do first year students differ significantly in their conceptual understanding if function as compared to diploma students?

Samples

Samples consist of second semester Diploma Of Engineering students in UiTM Pulau Pinang and university students in their first year Degree science based (non-mathematics major) course from another university. The second group of students have used concept of functions in their first calculus course the previous semester.

Table 1: Students' Distribution

Type of students	Number
First degree students	95
Diploma students	95

Methodology

This study employeded survey methodology adopting both quantitative and qualitative method of analysis. Questions about functions and graphs adapted from Bakar & Tall (1992) and Vinner & Dreyfuss (1989) were distributed during tutorial period. Students were given 30 minutes to answer the questions. The instrument constitutes three part questionnaires. In the first part the students were asked to select the most suitable definition of functions and in the second part the students need to indicate whether the three analytic form of the relations represent a function. The third part has five graphs where the students were asked to indicate whether the graphs represent a function and they were asked to give reasons for their choices.

Result

a. Definition of functions

Question 1 asked students to select the best definition for function. Table 2 demonstrates the distributions of their answers. Majority of students retained the definition of function as a 'formula' or 'rule to do' which are shown in the first three choices. Majority of students were noted to retain school definitions which is formula based (a,b and c) compared to less than 20% of the students recalling the correspondence definition which are mainly used in calculus course in the higher secondary level and in their undergraduate calculus course.

Table 2: Distribution of Answers in Question 1

Question 1	Undergraduate students	Diploma students
a. A function is a formula which produces some value of one variable from a given value of another variable	4	14
b. A function is a table in which for each value of one variable, there correspond a certain value for another variable	22	21
c. A function is an equation for determining the value of the dependent variable, y	43	32
d. A function is a graph that passes the vertical line test	1	2
e. A function is a collection of ordered pairs in which no two ordered pairs have the same inputs which matched with different outputs	10	6

The next part of the questionnaires addresses whether students are able to recognize symbolic representation of functions. Students do not demonstrate any difficulties with the linear form of the equation, however, a large number of students agreed that all of the relation given represent functions (refer to Table 4). This suggests that many students viewed all formula as representing functions. This not surprising as this tendency was earlier shown in the first part of the questionnaires. We also noted that majority of the students also think that a circle represents function. Only 30% of Diploma gave a correct response while the first year degree students perform better. Overall about 30% however, understood that a circle does not represent a function (unless we restrict the domain). Both students perform equally well on the other questions.

b. Concept images

For this part of the questionnaire the students were asked to indicate whether the given graphical forms represent functions. They were also required to give reasons for their answers. These answers are summarized in Table 5. Correct answers mean that they can either state the correspondence form or indicate their use of vertical line test.

Examples of correct reasons given:

for a point in x there is only one y; satisfies (or does not) a vertical line test; linear function

Table 4: Result from Analytic Representations

Question 2	Yes		No	
	Undergraduate	Diploma	Undergraduate	Diploma
a. $y = 3x + 2$	89	91	4	4
b. $x^2 + y^2 = 1$*	62	23	29	71
c. $y = \begin{cases} -x, & x > 1 \\ 0, & -1 \leq, x \leq 1 \\ x, & x < -1 \end{cases}$	73	73	17	19

* Chi Test at 0.05 level there is a significant difference between the groups

Examples of incorrect reasons given are:

there are formula (rules) between x and y; graph is discontinous; it is a circle; it is a parabola; y= f(x) ,the graph is smooth, there is a domain and range, involve variables x and y

We note that for functions in graphical forms the Diploma Students responses almost equally well with first degree students.

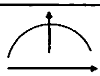
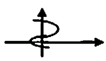
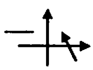
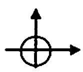
The next part of the questionares asked students to gave justification for their choices of answers in the graphical form of functions. How students justifies their answers to the questionnaires are then analysed and categorized into five categories, as follows: correspondence, vertical line test, discontinuity split domain and others.

Table 5 shows the distribution of all students responses according to the correct answers. Majority of students do not response implying they could not provide justifications for their answers. In Vinner & Dreyfuss studies, their students highest responses was indicated by one valuedness, discontinuity argument being second and split domain third. However, in this study the 'other' category was the most frequent reasons given by students The arguments which are typically used in 'others' categories are mostly an inaccurate attempts.

The examples of these attempts are:

'because there is an x and y, ' there is a dependent and independent variables'

Table 4: Distribution of Answers Given by Students

Question	Yes		No	
	undergraduate	Diploma	undergraduate	Diploma
3a. 	43	44	39	37
3b. 	55	38	30	44
3c. 	47	21	44	43
3d. 	55	36	24	40

*at 0.05 level there is a significant difference between groups

Only 2 persons in this study used split domain argument. It seems to suggest that they have forgotten the idea of correspondence or even the vertical line test while holding on strongly to formula conception We should also note that none of the students includes the aspect of domain or range in their answers as Bakar & Tall (1992) have commented that '*the general concept of domain and range do not seem to stick in the minds of most students*'. Surprisingly, only 4 students mentioned the terms domain or range. Furthermore, we also would like to point out that out of 24 students who chose correspondence definition in Question 1, only five of them used the definition to justify their responses in Question 3.

Table 5: Distribution of Responses According to the Correct Answers

Response Categories	Questions			
	3a	3b	3c	3d
Correspondence	5	6	6	5
Vertical line test	4	5		2
Discontinuity			11	
Split domain			2	
Others	32	10	22	13

The rest of the students displayed conflicting answers. There were those who indicated a circle as a function in graphic form yet rejecting it in algebraic form. Scores for each student are calculated by giving one mark for each correct response. Surprisingly majority of students perform well below 50%. About 35% of Diploma students obtained score below 10 (total 33 marks), while there are about 16% of the first degree students at this level. Only one students from first degree answered all questions correctly.

Conclusion

This study has shown students' difficulties with the formal concepts of functions as demonstrated by their persistence used '*x* and *y* or *f(x)*' instead of referring to the concepts definitions they have learnt and applying it. Many students have difficulties recognizing functions in graphical form yet they find the algebraic form easier. In addition, these university students seem to have developed an understanding of mathematical functions mainly as a rule or procedures in algebraic form or mainly as $y = f(x)$. Students exhibited 'compartmentalization of ideas' that is, the existence of a conflicting scheme in students' cognitive structure (Vinner & Dreyfuss,1989). In general, Diploma students indicated almost similar patterns of difficulties in most of the tasks. All the tasks given are just basic forms of function that students have encountered in their secondary education. This study similarly has shown that knowing the concept definition does not guarantee understanding of mathematical concept as shown by conflicting ideas in students reasoning patterns. The fact that students do not seem to recognize different representations of the same relation further indicates that they do have great difficulties understanding the concept. This was observed in all the students regardless of undergraduate or diploma students. One reason that may contribute towards this difficulty lies in heavy reliance on analytic forms found in the exercises and examples given in textbook and curriculum (Tall & Bakar, 1992; Dubinsky, 1991). It is important that this conflict ought to be restructured if the students are to achieve a better learning as if unattended, it may cause many more learning difficulties (Herscovics, 1989; Tall & Vinner, 1981). As emphasis by Tall (1991)

> 'the curriculum builder and teacher must be aware of the cognitive obstacles that may occur and cause cognitive conflict a when the context is broadened and the learner moves to the next stage'

Varieties of instructional approach should be employed to help students especially UiTM students to make this critical transition from school mathematics to higher mathematics learning easier and more meaningful.

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