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Judgment aggregation in search for the truth ☆

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ABSTRACT

We analyze the problem of aggregating judgments over multiple issues from the perspective of whether aggregate judgments manage to efficiently use all voters' private information. While new in judgment aggregation theory, this perspective is familiar in a different body of literature about voting between two alternatives where voters' disagreements stem from conflicts of information rather than of interest. Combining the two bodies of literature, we consider a simple judgment aggregation problem and model the private information underlying voters' judgments. Assuming that voters share a preference for true collective judgments, we analyze the resulting strategic incentives and determine which voting rules efficiently use all private information. We find that in certain, but not all cases a quota rule should be used, which decides on each issue according to whether the proportion of 'yes' votes exceeds a particular quota.

1. Introduction

In the by now well-established theory of judgment aggregation, a group needs to form a 'yes' or 'no' judgment on different issues, based on the judgments of the group members on these issues. For instance, the jury in a court trial might need to form a judgment on whether the defendant has broken the contract, and whether the contract is legally valid; the United Nations security council might need to form a judgment on whether country X is threatened by a military coup, and whether the economy of country X is collapsing; and so on. Group judgments matter in practice. They may determine group action: in the court trial example, they may determine whether the defendant is convicted, and in the United Nations example they may determine whether a large-scale international intervention in country X will happen.

So far, nearly the entire judgment aggregation theory follows the classical social-choice theoretic approach of aiming to find out how – and whether – group judgments can reflect the individuals' judgments in a procedurally fair manner, where 'fair' is spelled out in terms of axiomatic conditions on the aggregation rule (such as the anonymity condition or the Pareto-type condition of respecting unanimous judgments). The recent *Symposium on Judgment Aggregation* in Journal

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of Economic Theory (List and Polak, 2010, vol. 145(2)) illustrates well this social-choice theoretic approach, as well as the state of the art of the theory, which we review below. This approach is certainly important in many contexts. The judgment aggregation literature so far, however, has paid only little attention to a different 'epistemic' approach of aiming to *track the truth*, i.e., reach true group judgments. The theory does not model the private information underlying voters' judgments, thereby preventing itself from studying questions of efficient information aggregation. Yet such an epistemic perspective seems particularly natural in the context of aggregating judgments (rather than preferences[1]). In our court trial example, the ultimate goal seems indeed to be to find out independent facts (of whether the defendant has broken the contract and whether the contract is legally valid). So, the jury's voting rule should be optimized with respect to the goal that the resulting group judgments are *true*, not that they are fair to the jurors.

This alters the problem of designing the voting rule. Properties of voting rules standardly assumed in judgment aggregation theory, such as respecting unanimous judgments or anonymity, cannot be taken for granted anymore. If they turn out to be justified, they derive their justification from the truth-tracking goal rather than fairness considerations. To illustrate the contrast, suppose each juror expresses the judgment (opinion) that the contract was broken. A collective 'broken' judgment would then of course count as good from the classical social-choice theoretic perspective of procedural fairness. However, from a truth-tracking perspective, much depends on questions such as whether the jurors' judgments are sufficient evidence for breach of contract, and whether voters have expressed their judgments truthfully.

This paper analyzes judgment aggregation from the truth-tracking and strategic-voting perspective. We model voters' private information, which allows us to ask questions about efficient information aggregation and strategic voting in a Bayesian voting game setting. Though new within judgment aggregation theory, the modeling of private information is well-established in a different body of literature about voting between two alternatives, which started with seminal work by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997) and can be placed in the broader context of work on the Condorcet Jury Theorem (see the review below). In the base-line case, voters share a common interest of finding out the 'correct' alternative, but hold possibly conflicting private information about which of the alternatives might be 'correct'. The voting rule should be designed so as to help finding the 'correct' alternative by making optimal use of all the private information scattered across the voters. So, the goal is *efficient information aggregation*. Such an 'epistemic' binary collective choice problem can in fact be viewed as a special judgment aggregation problem, involving just one issue. Our court trial example involves two issues: firstly, whether the contract was broken, and secondly, whether it is legally valid. If instead only the first issue were on the jury's agenda, the jury would face a single-issue judgment aggregation problem, or equivalently, a binary collective choice problem. The entire machinery and results of the mentioned binary collective choice literature could then be applied in order to design the voting rule.

This paper therefore combines insights from two largely disconnected fields, namely judgment aggregation theory and binary collective choice with common interests. Independently, Ahn and Oliveros (2011) and DeClippel and Eliaz (2011) take a similar approach, but ask different questions, as reviewed below. The two bodies of literature can learn from each other. Indeed, judgment aggregation theory can benefit from methodologies developed in the theory of binary collective choice with common interests, while the latter theory can in turn benefit from an extension beyond single-issue agendas towards more complex agendas with multiple issues. Analyzing this multi-issue case does not reduce to analyzing each issue separately, since preferences establish links between different issues.

It is worth starting simple. This paper therefore assumes that the group faces an agenda with just *two* issues, the simplest kind of multi-issue agenda; but many of our results generalize easily, as discussed in Appendix A. Though simple, agendas with just two issues are important in practice. Our court trial example and United Nations example each involve two issues. To mention further two-issue agendas, a medical commission might need to issue joint judgments on whether a therapy is effective, and whether it is compatible with given ethical standards; members of a political party in charge of elaborating the party programme might seek joint judgments on whether a tax cut is affordable, and whether it is popular; a university hiring committee might seek joint judgments on whether a given candidate is good at research, and whether he or she is good at teaching; and finally, economic advisors to a government during the banking crisis in 2008 might need to issue collective judgments on whether a given bank has short-term liquidity problems, and whether it has long-term liquidity problems.

The issues of an agenda could in principle be mutually interconnected, so that the judgments taken on the issues logically constrain each other; for instance, a 'no' judgment on all issues might be inconsistent. Indeed, interconnections are what render judgment aggregation non-trivial if the usual social-choice theoretic approach of procedural fairness is taken.[2] However, within our truth-tracking approach, designing the voting rule is non-trivial even if the issues are mutually independent. We therefore assume independence between issues (see Bozbay, 2012 for follow-up work on the case of interconnected issues).

Structure of the paper Section 2 introduces our model, in which voters vote on the basis of private information and are guided by 'truth-tracking preferences', i.e., aim for true collective judgments. Section 3 addresses the key question of how

[1] In preference aggregation theory, the core of social choice theory, an epistemic perspective would be less natural since there is no objectively 'true preference' to be found.

[2] In the absence of interconnections one can safely aggregate by taking a separate vote on each issue. This never generates inconsistent collective judgments and meets all standard social-choice theoretic requirements such as anonymity.

to design the voting rule such that it leads to efficient decisions as well as simple-minded, truthful voting behavior in equilibrium. It will turn out that in certain, but not all cases one should use a 'quota rule', which decides on each issue according to whether the number of 'yes' judgments on the issue exceeds a particular quota. The details depend on the exact kind of truth-tracking preferences. For certain preferences, the only voting rule which induces an efficient and truthful Bayesian Nash equilibrium is a quota rule with particular thresholds. For certain other preferences, there is an entire class of such voting rules, including non-monotonic ones. Section 4 analyzes the notion of truthful behavior, by determining the conditions under which a 'sincere' voter directly reveals his information in his vote. In Appendix A, we consider the generalization of our framework and part of results from two to an arbitrary number $m \geqslant 2$ of issues. This links us to the traditional literature on jury theorems that emerges as the special case of $m = 1$ issue. Finally, Appendix B contains all proofs.

Literature review We now selectively review the two bodies of literature to which this paper connects, beginning with judgment aggregation theory. As mentioned, this theory's primary objective has so far been to find out which voting rules can aggregate the judgments of group members over some issues in accordance with certain axiomatic requirements with a classic social-choice theoretic flavor, such as *unanimity preservation* (the counterpart of the *Pareto principle*) and *independence* (the counterpart of Arrow's *independence of irrelevant alternatives*). A series of possibility and impossibility results address this query, by giving answers which depend, firstly, on the axiomatic requirements on the voting rule, and secondly, on the agenda of issues under consideration (e.g., List and Pettit, 2002; Dietrich 2006, 2007, 2010; Nehring and Puppe 2008, 2010; Dietrich and List 2007a, 2008; Dokow and Holzman 2010a, 2010b; Dietrich and Mongin, 2010; see also precursor results by Guilbaud, 1952 and Wilson, 1975; for an introductory overview see List and Polak, 2010). By contrast, a small minority of papers about judgment aggregation take a truth-tracking perspective (e.g., Bovens and Rabinowicz, 2006; List, 2005 and Pivato, 2011). Their innovation is to apply the classical Condorcet Jury Theorem to judgment aggregation. Despite taking a truth-tracking perspective, they have little in common with our work, since private information and strategic incentives are not being considered.[3] List and Pettit (2011) provide the most systematic philosophical analysis of the truth-tracking approach, already discussing strategic incentives and private information and drawing on the second body of literature to which we now turn.

As for this second body of literature, it is concerned with voting rules for binary choice problems in which disagreements are driven (partly or totally) by conflicting information rather than conflicting interests. Specifically, the utilities which voters derive from decisions are affected by the same unknown 'state of the world', about which voters hold private information.[4] Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1997) show that it typically cannot be rational for all voters to vote sincerely, and that the choice of voting rule matters considerably for sincere voting and efficient information aggregation. While the former authors consider the 'purely epistemic' case without conflict of interest, the latter authors introduce some preference heterogeneity (and focus primarily on large electorates). Austen-Smith and Feddersen (2005, 2006) add an extra dimension of pre-voting deliberation. Duggan and Martinelli (2001) extend the approach to continuous rather than binary private information. Feddersen and Pesendorfer (1998), Coughlan (2000) and Gerardi (2000) examine the (in)effectiveness of unanimity rule in 'protecting the innocent' in jury trials. Goertz and Maniquet (2011) analyze efficient information aggregation in large electorates, showing that approval voting outperforms other voting rules in their setting.

Like us (and independently from us), Ahn and Oliveros (2011) and DeClippel and Eliaz (2011) also combine these two aggregation problems by studying elections on multiple binary issues with common preferences and asymmetric information. Each of these papers compares two voting procedures. Ahn and Oliveros (2011) compare resolving each issue by a majority vote among *all* voters with resolving each issue by a majority vote among a subgroup assigned to this issue (where the subgroups for different issues are disjoint and equally large). They show that neither of these procedures is generally more efficient than the other one if the group is large enough. DeClippel and Eliaz (2011) consider a group choice between two possible actions (such as convicting or acquitting the defendant), where the 'optimal' action depends on two or more issues/criteria, and where each voter holds private information on each issue (possibly with correlations across issue). They compare premise-based voting with conclusion-based voting. Under the former, a vote is taken on each issue, and the outcomes determine the group action (conclusion). Under the latter, the group votes directly on which action to take, without forming a group view on the issues. They show that premise-based voting is more efficient than conclusion-based voting, but that the difference vanishes asymptotically as the group size increases. These two works are important advances, into directions different from our work which is not concerned with comparisons of fixed mechanisms but with the design of efficient mechanisms.

[3] Dietrich and List (2007b) analyze strategic voting in judgment aggregation, but in a sense not relevant to us since strategic voting is not modeled as coming from private information and a voter is motivated by the somewhat different goal that the collective judgments match *his own* judgments. Such assumptions are more natural under common knowledge of each other's judgments than under informational asymmetry. See also related work by Nehring and Puppe (2002, 2007).

[4] This contrasts with the scenario of private values of, but common information about, alternatives. Such elections lead to somewhat different Bayesian games. While the case of two alternatives is trivial, a multi-alternative analysis is given by Ahn and Oliveros (2012).

2. The model

2.1. A simple judgment aggregation problem

We consider a group of voters, labeled $i = 1, \ldots, n$, where $n \geqslant 2$. This group needs a collective judgment on whether some proposition p or its negation $\bar{p}$ is true, and whether some other proposition q or its negation $\bar{q}$ is true. In our court trial example, p states that the contract was broken, and q that it is legally valid; in our job candidate example, p states that the candidate is good at research, and q that he or she is good at teaching; and so on for our other examples. The four possible judgment sets are $\{p, q\}$, $\{p, \bar{q}\}$, $\{\bar{p}, q\}$ and $\{\bar{p}, \bar{q}\}$; we abbreviate them by pq, $p\bar{q}$, $\bar{p}q$ and $\bar{p}\bar{q}$, respectively. For instance, $p\bar{q}$ means accepting p but not q. Each voter votes for a judgment set in $\mathcal{J} = \{pq, p\bar{q}, \bar{p}q, \bar{p}\bar{q}\}$. After all voters cast their votes, a collective decision in $\mathcal{J}$ is taken using a voting rule. Formally, a voting rule is a function $f : \mathcal{J}^n \to \mathcal{J}$, mapping each voting profile $\mathbf{v} = (v_1, \ldots, v_n)$ to a decision $d \equiv f(\mathbf{v})$. Among the various voting rules, *quota rules* stand out as particularly natural and common. A quota rule is given by two thresholds $m_p, m_q \in \{0, 1, \ldots, n+1\}$, and for each voting profile it accepts p $[q]$ if and only if at least m_p $[m_q]$ voters accept it in the profile. Quota rules have three salient properties:

- *Anonymity:* For all voting profiles $(v_1, \ldots, v_n) \in \mathcal{J}^n$ and all permutations $(i_1, \ldots, i_n)$ of the voters, $f(v_{i_1}, \ldots, v_{i_n}) = f(v_1, \ldots, v_n)$. Informally, the voters are treated equally.
- *Monotonicity:* For all voting profiles $\mathbf{v}, \mathbf{v}' \in \mathcal{J}^n$, if for each r in $f(\mathbf{v})$ the voters who accept r in $\mathbf{v}$ also accept r in $\mathbf{v}'$, then $f(\mathbf{v}') = f(\mathbf{v})$. Informally, additional support for the collectively accepted propositions never reverses the collective acceptance of these propositions.
- *Independence:* The decision on each proposition $r \in \{p, q\}$ only depends on the votes on r.[5] Informally, the group in effect takes two separate votes, one between p and $\bar{p}$ and one between q and $\bar{q}$.

Remark 1. A voting rule $f : \mathcal{J}^n \to \mathcal{J}$ is a quota rule if and only if it is anonymous, monotonic and independent.

We briefly sketch the proof of the non-trivial direction of implication. As can be shown, if a voting rule $f : \mathcal{J}^n \to \mathcal{J}$ is anonymous and independent, then it is given by two sets $M_p, M_q \subseteq \{0, 1, \ldots, n\}$, in the sense that for each voting profile $\mathbf{v} \in \mathcal{J}^n$ the decision $f(\mathbf{v})$ contains r $(\in \{p, q\})$ if and only if the number of votes in $\mathbf{v}$ containing r belongs to M_r. If f is moreover monotonic, each set M_r can be shown to take the form $\{m_r, m_r+1, \ldots, n\}$ for some threshold $m_r \in \{0, 1, \ldots, n+1\}$. Clearly, f is the quota rule with thresholds m_p and m_q.

2.2. A common preference for true collective judgments

Exactly one judgment set in $\mathcal{J}$ is 'correct', i.e., contains propositions which are factually true. It is called the *state (of the world)* and is generically denoted by s. For instance, the state might be $p\bar{q}$, so that p and $\bar{q}$ are true (and $\bar{p}$ and q are false). Voters have identical preferences, captured by a common utility function $u : \mathcal{J} \times \mathcal{J} \to \mathbb{R}$ which maps any decision-state pair (d, s) to its utility $u(d, s)$. Given voters' truth-tracking goal, one would expect $u(d, s)$ to be high if $d = s$, i.e., if the decision is correct. But how exactly should u be specified? We focus on two natural kinds of preferences:

Simple preferences. Here, the utility function is given by

$$u(d, s) = \begin{cases} 1 & \text{if } d = s \text{ (correct decision)} \\ 0 & \text{if } d \neq s \text{ (incorrect decision).} \end{cases} \tag{1}$$

Such preferences are the simplest candidate for truth-tracking preferences. Correct decisions are preferred to incorrect ones, without further sophistication.

Consequentialist preferences. Here, we assume that the decision leads to one of two possible consequences, typically representing group actions. This is captured by a *consequence function* Co which maps the set of possible decisions $\mathcal{J}$ to a two-element set of possible consequences. The consequence function might look as follows in examples given earlier. In our court trial example, the court decision pq leads to conviction, since both premises of guilt are found to be satisfied ($\mathrm{Co}(pq) =$ 'conviction'), while the other decisions all lead to acquittal ($\mathrm{Co}(\bar{p}\bar{q}) = \mathrm{Co}(p\bar{q}) = \mathrm{Co}(\bar{p}q) =$ 'acquittal'). In our job candidate example, the decision pq leads to a hire since the candidate is seen as meeting both criteria ($\mathrm{Co}(pq) =$ 'hire'), while the other decisions all lead to no hire ($\mathrm{Co}(\bar{p}\bar{q}) = \mathrm{Co}(p\bar{q}) = \mathrm{Co}(\bar{p}q) =$ 'no hire'). In our United Nations example, the decisions $p\bar{q}$ and $\bar{p}q$ each lead to a large-scale international intervention in country X ($\mathrm{Co}(p\bar{q}) = \mathrm{Co}(\bar{p}q) =$ 'intervention'), whereas the decisions pq and $\bar{p}\bar{q}$ both lead to no intervention since the United Nations then consider an intervention as being too risky or unnecessary, respectively ($\mathrm{Co}(pq) = \mathrm{Co}(\bar{p}\bar{q}) =$ 'no intervention'). In our bank rescuing example, the decisions $p\bar{q}$ and $\bar{p}q$

[5] Given a voting profile $\mathbf{v}$, the subprofile with respect to r is denoted $\mathbf{v}_r$ $(\in \{r, \bar{r}\}^n)$, and the collective decision with respect to r is denoted $f_r(\mathbf{v})$ $(\in \{r, \bar{r}\})$. Independence means that for all voting profiles $\mathbf{v}, \mathbf{v}' \in \mathcal{J}^n$, if $\mathbf{v}_r = \mathbf{v}'_r$, then $f_r(\mathbf{v}) = f_r(\mathbf{v}')$.

each lead to a governmental rescue plan for the bank ($\mathrm{Co}(p\bar{q}) = \mathrm{Co}(\bar{p}q) =$ 'rescue'), whereas the decisions pq and $\bar{p}\bar{q}$ both lead to no rescue plan since a rescue is seen as infeasible or unnecessary, respectively ($\mathrm{Co}(pq) = \mathrm{Co}(\bar{p}\bar{q}) =$ 'no rescue'). The consequentialist utility function is given by

$$u(d,s) = \begin{cases} 1 & \text{if } \mathrm{Co}(d) = \mathrm{Co}(s) \text{ (correct consequence)} \\ 0 & \text{if } \mathrm{Co}(d) \neq \mathrm{Co}(s) \text{ (incorrect consequence).} \end{cases} \tag{2}$$

Incorrect decisions ($d \neq s$) can have correct consequences ($\mathrm{Co}(d) = \mathrm{Co}(s)$). The hiring committee might view the candidate as good at research and bad at teaching when in fact the opposite is true, so that the resulting consequence ('no hire') is correct for wrong reasons. This gives high utility under consequentialist preferences, but low utility under simple preferences.[6]

In the context of consequentialist preferences, one might wonder whether, given that all that counts is the consequence, it would not suffice to aggregate the voters' judgments on the consequence, ignoring judgments on the underlying issues – leading to a simple group decision problem between two alternatives. This would amount to an informational loss: while a voter's judgment on the consequence (the 'conclusion') follows from his judgments on the two issues (the 'premises'), the latter cannot generally be retrieved from the former. The lost information is valuable information, since, as will turn out, the efficient group action (which is 'found' by an optimal aggregation rule) often makes full use of a voter's judgments on the issues, rather than using only the information of which action follows from these judgments. Put formally, given the votes on the two issues $v_1, \ldots, v_n \in \mathcal{J}^n$, an optimal aggregation rule f leads to the collective action $\mathrm{Co}(f(v_1, \ldots, v_n))$ which depends on the submitted judgments $v_1, \ldots, v_n$ in such a way that *not only* the information of the resulting consequences $\mathrm{Co}(v_1), \ldots, \mathrm{Co}(v_n)$ is used. This suggests that voting directly on the consequence may lead to inefficient group actions, and that it is thus worth voting on the underlying issues.[7]

In real life, many groups vote only on the consequence/action: while the underlying issues/premises typically play a role in the process of group deliberation and discussion prior to voting, the voting itself often only involves a simple yes/no choice on the action. As mentioned, such a conclusion-based procedure can be criticized based on our analysis.

The empirical question of which real-life decision making bodies vote directly on the conclusion and which (as assumed in this paper) vote on underlying issues is beyond the scope of this theoretical paper. In support of the real-life importance of voting on underlying issues, let us merely mention that for some decision-making bodies – such as some criminal courts, constitutional courts, central banks, or political organizations – it is desirable or even legally required to come up with reasons or justifications for the actions which are implemented, for reasons of legitimacy and accountability of public actions. In our terminology, the group needs a collective decision on the issues, not just a collective action/consequence. For instance, many courts must publicly state not just *whether* they convict the defendant, but also *on what grounds* they do so, which involves taking positions on multiple issues. See Kornhauser and Sager (1986) for public accountability of courts, and Pettit (2001) and List and Pettit (2011) for accountability and legitimacy of political organizations.

2.3. Private information and strategies

If voters had not just common preferences, but also common information about what the state might be, then no disagreement could arise. We however allow for informational asymmetry. Each voter has a *type*, representing private information or evidence.[8] A voter's type takes the form of an element of $\mathcal{J}$, generically denoted by t. For instance, a voter of type $t = p\bar{q}$ has evidence for p and for $\bar{q}$. We write $\mathbf{t} = (t_1, \ldots, t_n) \in \mathcal{J}^n$ for a profile of voters' types. Nature draws a state-types combination $(s, \mathbf{t}) \in \mathcal{J}^{n+1}$ according to a probability measure denoted Pr. When a proposition $r \in \{p, \bar{p}, q, \bar{q}\}$ is meant to represent part of voter i's type rather than part of the true state, we often write r_i for r. For instance, $\Pr(p_i|p)$ is the probability that voter i has evidence for p given that p is true. By assumption, the prior probability that r ($\in \{p, \bar{p}, q, \bar{q}\}$) is true is denoted

$$\pi_r = \Pr(r)$$

and belongs to $(0, 1)$, and the probability of getting evidence for r given that r is true is denoted

$$a_r = \Pr(r_i|r),$$

[6] In the judgment aggregation literature, the two possible consequences are often represented by two additional propositions, c and $\bar{c}$, which are referred to as 'conclusion propositions' in contrast to the 'premise propositions' $p, \bar{p}, q, \bar{q}$. In our first two examples, the consequence function is encoded in the biconditional $c \leftrightarrow (p \wedge q)$, whereas in our last two examples it is encoded in the biconditional $c \leftrightarrow ((p \wedge q) \vee (\bar{p} \wedge \bar{q}))$.

[7] A complete argument to the effect that conclusion-based voting is inefficient also requires investigating the strategic incentives under the (different) game form of the conclusion-based procedure. Such an argument is made by DeClippel and Eliaz (2011), albeit in a different framework. Rigorously speaking, our results imply the inefficiency of the conclusion-based procedure only under the hypothesis that, under this procedure, a voter i votes for the consequence $\mathrm{Co}(v_i)$ which follows from his vote v_i under the issue-based procedure. (A violation of this hypothesis would constitute a form of untruthful voting, which could itself be viewed as a shortcoming of the conclusion-based procedure.)

[8] The type could represent information that is not shared with other voters because of a lack of deliberation or limits of deliberation. More generally, a voter i's type could represent uncertainty of other voters about i's beliefs.

belongs to $(1/2, 1)$, and does not depend on the voter i. The parameters $a_p, a_{\bar{p}}, a_q, a_{\bar{q}}$ measure the reliability of private information, as they represent probabilities of receiving 'truth-telling' information. The lower bound of 1/2 reflects the (standard) idea that information is more reliable than a fair coin.

By assumption, voters' types are independent conditional on the state, and in addition the state and the types w.r.t. p are independent of the state and the types w.r.t. q.[9] These independence assumptions allow one to express the joint distribution of the state and the types by just a few parameters, namely $\pi_p, \pi_q, a_p, a_{\bar{p}}, a_q, a_{\bar{q}}$. For instance, the probability that the state is pq and all voters receive the truth-telling evidence pq is

$$\Pr(pq, p_1q_1, p_2q_2, \ldots, p_nq_n) = \Pr(pq)\Pr(p_1q_1, p_2q_2, \ldots, p_nq_n|pq) = \pi_p\pi_q a_p^n a_q^n.$$

Each voter submits a vote in $\mathcal{J}$ based on his type. A *(voting) strategy* is a function $\sigma : \mathcal{J} \to \mathcal{J}$, mapping each type $t \in \mathcal{J}$ to the type's vote $v = \sigma(t)$. We write $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_n)$ for a profile of voters' strategies. Together with a voting rule f and a common utility function u, we now have a well-defined Bayesian game.

For a given type profile $\mathbf{t} \in \mathcal{J}^n$, we call a decision d *efficient* if it has maximal expected utility conditional on the full information $\mathbf{t}$.[10] Some common notions of voting behavior can now be adapted to our framework:

- A strategy σ of a voter is *informative* if $\sigma(t) = t$ for all types t. An informative voter directly reveals his information in his vote.
- A strategy σ of a voter is *sincere* if for every type t, the vote $\sigma(t)$ maximizes the expected utility conditional on the information t. A sincere voter votes for the decision which maximizes the expected utility conditional on his type; so, he acts as if his vote alone determined the decision, neglecting the other voters and their strategies. (Technically, this amounts to optimal behavior in a hypothetical single-player decision problem.)
- A strategy profile $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_n)$ is *efficient* if for every type profile $\mathbf{t} = (t_1, \ldots, t_n)$ the resulting decision $d = f(\sigma_1(t_1), \ldots, \sigma_n(t_n))$ is efficient (i.e., has maximal expected utility conditional on full information $\mathbf{t}$). Hence, all the information spread across the group is used efficiently: the collective decision is no worse than a decision of a (virtual) social planner who has full information.
- A strategy profile $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_n)$ is an *equilibrium* if it is a Nash equilibrium of the corresponding Bayesian game, i.e., if each strategy is a best response to the other strategies. In such a profile, each voter maximizes the expected utility of the collective decision given the strategies of the other voters. (In this maximization exercise, it turns out that a voter must only consider cases in which his vote is pivotal. Under a quota rule with majority thresholds, a voter is for instance pivotal if half of the other voters votes pq and the other half votes $\bar{p}\bar{q}$.)

While informativeness and sincerity are properties of a single strategy (or voter), efficiency refers to an entire profile.

Finally, to avoid distraction by special cases, we make two assumptions. First, we exclude the degenerate case in which some decision in $\mathcal{J}$ is not efficient for any type profile whatsoever. Second, we exclude efficiency ties, i.e., we exclude those special parameter combinations such that some type profile $\mathbf{t}$ leads to different efficient decisions (with different consequences when we assume consequentialist preferences).

3. Which voting rules lead to efficient information aggregation?

3.1. Setting the stage

Our objective is to design the voting rule ('mechanism') in such a way as to yield efficient decisions on the basis of informative votes. In short, the voting rule should render informative voting efficient.[11] We begin by justifying this objective. *Prima facie*, two goals are of interest. The rule should, firstly, lead to efficient outcomes, and, secondly, encourage simple-minded, truthful behavior. By such behavior we mean informative voting.[12] To reach the second goal, informative voting should constitute an equilibrium. If additionally informative voting is efficient, both goals are reached. So, the double goal is that informative voting should be efficient and form an equilibrium. By part (a) of the following theorem, whenever informative voting is efficient, it *a fortiori* defines an equilibrium – which explains our primary objective that informative voting be efficient.

Theorem 1. *Consider an arbitrary common utility function $u : \mathcal{J}^2 \to \mathbb{R}$.*

(a) *For any voting rule, if a strategy profile is efficient, then it is an equilibrium.*
(b) *There is an anonymous voting rule for which informative voting is efficient (hence, an equilibrium).*

[9] Recall that the state consists of a proposition in $\{p, \bar{p}\}$ and another in $\{q, \bar{q}\}$. The first [second] of these propositions is what we call the *state w.r.t. p* [q]. A voter's *type w.r.t. p* [q] is defined similarly.

[10] I.e., d maximizes $E(u(d, S)|\mathbf{t}) = \sum_{s \in \mathcal{J}} u(d, s)\Pr(s|\mathbf{t})$, where '$S$' denotes the random variable generating the state s in $\mathcal{J}$.

[11] By saying "informative voting" without referring to a particular voter, we mean "informative voting by all voters".

[12] One might alternatively mean sincere voting – but in practice there is little difference, since informative and sincere voting coincide under reasonable informational assumptions. As one can show, if informative voting is *not* sincere, then there exists a decision $d \in \mathcal{J}$ such that no voter ever finds himself in an informational position to consider d as best – a rather uninteresting, if not unnatural scenario.

This theorem, whose part (a) follows from a well-known result by McLennan (1998), is general in that it applies to any kind of (common) preferences. The converse of part (a) does not hold: for instance, under a constant voting rule all strategy profiles are equilibria, typically without being efficient. The message of part (b) is positive but so far vague: it is always possible to make informative voting efficient, but apart from anonymity we do not know anything about the kind of voting rule we can use. And indeed, for some kinds of common preferences, it may not be possible to aggregate in an independent or monotonic way (as counterexamples show). But, once we narrow down to simple or consequentialist preferences, can – or even must – we aggregate in a monotonic resp. independent way? When can – or even must – we use a quota rule? Such questions are answered below.

3.2. Simple preferences

This section addresses the case of simple preferences, given by the common utility function (1). Which rules render informative voting efficient? The answer is 'simple', as we will see. To state our result, we first define two coefficients[13]:

$$k_p := \min\left\{k \in \{0, 1, \ldots, n+1\}\colon \frac{\pi_p}{1-\pi_p} > \left(\frac{1-a_{\bar{p}}}{a_p}\right)^k \left(\frac{a_{\bar{p}}}{1-a_p}\right)^{n-k}\right\}, \tag{3}$$

$$k_q := \min\left\{k \in \{0, 1, \ldots, n+1\}\colon \frac{\pi_q}{1-\pi_q} > \left(\frac{1-a_{\bar{q}}}{a_q}\right)^k \left(\frac{a_{\bar{q}}}{1-a_q}\right)^{n-k}\right\}. \tag{4}$$

These coefficients have an interpretation: as can be proved, for p [q] to be more probably true than false given all information, at least k_p [k_q] individuals need to receive evidence for p [q], i.e., need to have a type containing p [q].

Theorem 2. *Assume simple preferences. Informative voting is efficient if and only if the voting rule is the quota rule with thresholds k_p and k_q.*

This result shows that the quota rule with thresholds k_p and k_q is the *only* rule we may use in view of making informative voting efficient. This result is much more specific than the purely existential claim in part (b) of Theorem 1. This progress was possible by focusing on simple preferences.

3.3. Consequentialist preferences: first type

We now turn to consequentialist preferences. Much depends on the nature of the consequence function. In principle, there exist $2^4 = 16$ potential consequence functions from $\mathcal{J}$ to a binary set of consequences. But, as we shall see shortly, there are only two non-degenerate consequence functions up to isomorphism. We therefore define two types of consequentialist functions. Consequentialist preferences (or the consequence function) are said to be:

- of *type 1* if $\mathrm{Co}(pq) = \mathrm{Co}(\bar{p}\bar{q}) \neq \mathrm{Co}(p\bar{q}) = \mathrm{Co}(\bar{p}q)$;
- of *type 2* if $\mathrm{Co}(pq) \neq \mathrm{Co}(\bar{p}\bar{q}) = \mathrm{Co}(p\bar{q}) = \mathrm{Co}(\bar{p}q)$.

Our first two examples of consequentialist preferences in Section 2.2 are of type 1, while our last two examples are of type 2. But why are *all* non-degenerate consequence functions of one of these two types? Firstly, consequence functions for which each decision in $\mathcal{J}$ has the *same* consequence are of course degenerate and therefore uninteresting. Also consequence functions which depend only on the decision between p and $\bar{p}$, or only on the decision between q and $\bar{q}$, are degenerate, since in this case we are essentially back to a decision problem with a single proposition-negation pair, which has already been studied in the literature.[14] The non-degenerate consequence functions are those which genuinely depend on both propositions. Among all of them, some assign each consequence to exactly two decisions in $\mathcal{J}$, while the others assign one consequence to three decisions and the other consequence to just one decision. As one can show, the former consequence functions are of type 1, while the latter are of type 2 up to isomorphism (i.e., up to exchanging p and $\bar{p}$ and/or exchanging q and $\bar{q}$). Thus, by studying our two types of consequence functions, we will have covered non-degenerate consequentialist preferences exhaustively.

We now address the first type, while the next subsection turns to the second type. One might at first expect there to be little resemblance between the current preferences and simple preferences in terms of the appropriate voting rule. For instance, even when all individuals have type pq, so that there is overwhelming evidence for state pq, the current preferences allow us to efficiently decide for $\bar{p}\bar{q}$, since this decision has the same consequence as pq. Surprisingly, despite

[13] The minimum defining k_p or k_q should be interpreted as $n+1$ if the set whose minimum is being taken is *empty*. In fact, emptiness is impossible under simple preferences. This follows from our non-degeneracy assumption on the model parameters (which also implies that $k_p, k_q \in \{1, \ldots, n\}$). Note that in (3) and (4) the right hand side of the inequality is strictly decreasing in k.

[14] For instance, our UN intervention example would be degenerate if the question of whether to intervene only depended on whether the country is considered as being threatened by a military coup (p or $\bar{p}$). The other pair of propositions (q or $\bar{q}$) could then be eliminated from the voting process.

the differences, consequentialist preferences of type 1 come much closer to simple preferences than to consequentialist preferences of type 2 in terms of the design of the voting rule. The coefficients k_p and k_q, defined earlier for simple preferences, again play a key role.

Theorem 3. *Assume consequentialist preferences of type 1. A voting rule makes informative voting efficient and is monotonic if and only if it is the quota rule with thresholds k_p and k_q.*

So, as for simple preferences, the social planner is led to impose a quota rule with the particular thresholds k_p and k_q. What distinguishes Theorem 3 from Theorem 2 is, for one, its somewhat different (and longer) proof, and secondly, the additional monotonicity requirement. Without this extra condition, a number of other voting rules become possible.[15]

3.4. Consequentialist preferences: second type

We now turn to consequentialist preferences of type 2. The space of aggregation possibilities is somewhat different here. As we shall show, quota rules are *not* always possible, and when they are, the two thresholds must be calculated differently.

For all $k, l \in \mathbb{R}$, we define the coefficient

$$\beta(k,l) = \frac{\pi_p a_p^k (1-a_p)^{n-k}}{\pi_p a_p^k (1-a_p)^{n-k} + \pi_{\bar{p}} a_{\bar{p}}^{n-k} (1-a_{\bar{p}})^k} \times \frac{\pi_q a_q^l (1-a_q)^{n-l}}{\pi_q a_q^l (1-a_q)^{n-l} + \pi_{\bar{q}} a_{\bar{q}}^{n-l} (1-a_{\bar{q}})^l}. \tag{5}$$

One can show that $\beta(k,l)$ has a natural interpretation if $k, l \in \{0, 1, \ldots, n\}$: it is the probability that the state is pq conditional on having k times evidence for (and $n-k$ times evidence against) p and l times evidence for (and $n-l$ times evidence against) q. So, $\beta(k,l) = \Pr(pq|\mathbf{t})$ for some (hence, any) type profile $\mathbf{t} \in \mathcal{J}^n$ containing p exactly k times and q exactly l times; or equivalently,

$$\beta(k,l) = \Pr(p|p_1, \ldots p_k, \bar{p}_{k+1}, \ldots, \bar{p}_n) \times \Pr(q|q_1, \ldots, q_l, \bar{q}_{l+1}, \ldots, \bar{q}_n).$$

As one can prove by drawing on the definition of the consequence function, given a type profile $\mathbf{t}$ containing p exactly k times and q exactly l times, if $\beta(k,l) > 1/2$ then only the decision pq is efficient, while otherwise the three other decisions are all efficient. This implies a first, simple characterization result. Henceforth, the number of votes for a proposition r in a voting profile $\mathbf{v}$ is written $n_r^{\mathbf{v}}$.

Proposition 1. *Assume consequentialist preferences of type 2. A voting rule f makes informative voting efficient if and only if for every voting profile $\mathbf{v} \in \mathcal{J}^n$ the decision $f(\mathbf{v})$ is pq if $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) > 1/2$ and in $\{p\bar{q}, \bar{p}q, \bar{p}\bar{q}\}$ otherwise.*

Which possibilities – if any – are left if we require the rule to be a quota rule? We begin by introducing two coefficients. Given that all voters hold evidence for q, how many voters with evidence for p does it minimally take for the decision pq to become efficient? Similarly, given that all voters hold evidence for p, how many voters with evidence for q does it take for the decision pq to become efficient? The answer to these questions is given by the following numbers, respectively[16]:

$$l_p := \min\{k \in \{0, \ldots, n\}\colon\ \beta(k,n) > 1/2\} \tag{6}$$

$$l_q := \min\{k \in \{0, \ldots, n\}\colon\ \beta(n,k) > 1/2\}. \tag{7}$$

Theorem 4. *Assume consequentialist preferences of type 2. There exists a quota rule making informative voting efficient if and only if $\beta(l_p, l_q) > 1/2$. In this case, that quota rule is unique and has the thresholds l_p and l_q.*

Unlike when preferences are simple or consequentialist of type 1, and unlike in the classic literature for a single pair of propositions $p, \bar{p}$, we have an impossibility:

Corollary 1. *Assume consequentialist preferences of type 2. For some combinations of values of the model parameters ($\pi_p, \pi_q, a_p, a_{\bar{p}}, a_q, a_{\bar{q}}$ and n), no quota rule makes informative voting efficient.*

For instance, if $\pi_p = \pi_q = 0.5$, $a_p = a_q = a_{\bar{p}} = a_{\bar{q}} = 0.7$ and $n = 3$, no quota rule makes informative voting efficient, whereas if instead $\pi_p = \pi_q = 0.6$, the quota rule with thresholds $l_p = l_q = 2$ makes informative voting efficient.

[15] For consequentialist preferences of type 1, one may show that a (possibly non-monotonic) voting rule f makes informative voting efficient if and only if for every voting profile $\mathbf{v} \in \mathcal{J}^n$ the decision $f(\mathbf{v})$ has the same consequence as the decision under the quota rule with thresholds k_p and k_q (i.e., $\mathrm{Co} \circ f = \mathrm{Co} \circ g$, where g is this quota rule). So, once we drop the monotonicity requirement, there is not just one possible voting rule, as for simple preferences, but 2^{4^n} possible rules (since there are 2 allowed decisions for each of the 4^n profiles in $\mathcal{J}^n$).

[16] These two minima are taken over *non-empty* sets of values of k (by the non-degeneracy assumption at the end of Section 2.3).

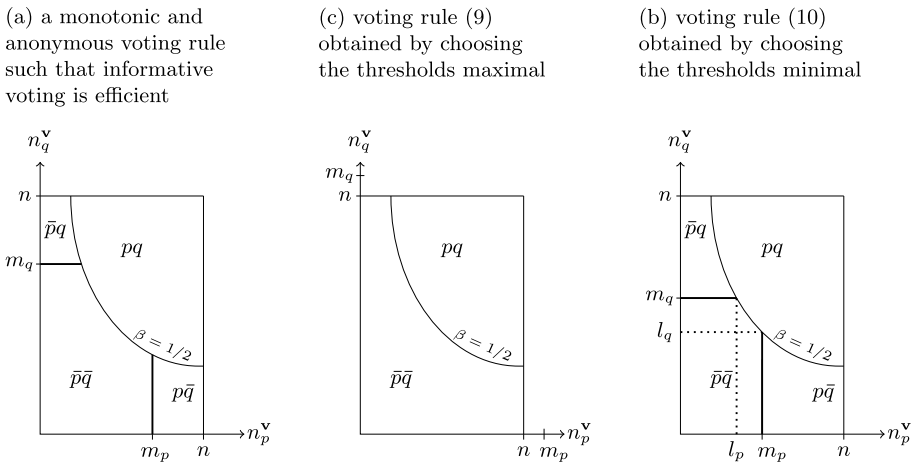

Fig. 1. Illustration of Theorem 5: the decision as a function of the number of votes for p and q.

While by Corollary 1 it may be utopian to aim for a full-fledged quota rule, we now show that one can always achieve two characteristic properties of quota rules, namely anonymity and monotonicity, while often losing the third characteristic property, namely independence. Specifically, we characterize the class of all monotonic and anonymous (but not necessarily independent) aggregation possibilities. As we shall see, this class consists of so-called quota rules 'with exception'. Such rules behave like a quota rule as long as the profile does not fall into an 'exception domain', while generating the 'exception decision' pq on the exception domain. Formally, we define quota rules with exception as follows:

A *quota rule with exception* $f : \mathcal{J}^n \to \mathcal{J}$ is given by thresholds $m_p, m_q \in \{0, \dots, n+1\}$ and an 'exception domain' $\mathcal{E} \subseteq \mathcal{J}^n$, and is defined as follows for all voting profiles $\mathbf{v} \in \mathcal{J}^n$:

- if $\mathbf{v} \notin \mathcal{E}$ then $f(\mathbf{v})$ contains any proposition r in $\{p, q\}$ if and only if $n_r^{\mathbf{v}} \geqslant m_r$ (as for a regular quota rule),
- if $\mathbf{v} \in \mathcal{E}$ then $f(\mathbf{v}) = pq$.

Equivalently, for any r in $\{p, q\}$,

$$f(\mathbf{v}) \text{ contains } r \quad \text{if and only if} \quad \left[n_r^{\mathbf{v}} \geqslant m_r \text{ or } \mathbf{v} \in \mathcal{E}\right].^{17}$$

Standard quota rules arise as special cases with an empty exception domain. In our characterization theorem, the exception domain is $\mathcal{E} = \{\mathbf{v}\colon\ \beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) > 1/2\}$, so that

$$f(\mathbf{v}) \text{ contains } r \Leftrightarrow \left[n_r^{\mathbf{v}} \geqslant m_r \text{ or } \beta\left(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}\right) > 1/2\right], \quad \text{for all } r \in \{p, q\} \text{ and } \mathbf{v} \in \mathcal{J}^n. \tag{8}$$

Theorem 5. *Assume consequentialist preferences of type 2. A voting rule makes informative voting efficient and is monotonic and anonymous if and only if it is the quota rule with exception (8) for some thresholds m_p, m_q such that $\beta(m_p, l_q), \beta(l_p, m_q) > 1/2$.*

Fig. 1 shows three voting rules of the kind given in Theorem 5. They differ in the choice of the thresholds m_p and m_q. Fig. 1(a) shows the generic case. In Fig. 1(b), the thresholds are chosen in a maximal way, i.e., $m_p = m_q = n + 1$. As a result, the decisions $p\bar{q}$ and $\bar{p}q$ are never taken, and the voting rule takes a particularly simple form:

$$f(\mathbf{v}) = \begin{cases} pq & \text{if } \beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) > 1/2 \\ \bar{p}\bar{q} & \text{if } \beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) \leqslant 1/2 \end{cases} \tag{9}$$

for all voting profiles $\mathbf{v} \in \mathcal{J}^n$. In Fig. 1(c), the thresholds are chosen in a minimal way. That is, m_p is the minimal number for which $\beta(m_p, l_q) > 1/2$; in short, m_p is chosen such that $\beta(m_p, l_q) \approx 1/2$, as illustrated by the figure. Similarly, m_q is chosen such that $\beta(l_p, m_q) \approx 1/2$. As a result, the voting rule takes the following form, as the reader may check:

$$f(\mathbf{v}) = \begin{cases} pq & \text{if } \beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) > 1/2 \\ p\bar{q} & \text{if } \beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) \leqslant 1/2 \text{ and } \beta(n_p^{\mathbf{v}}, l_q) > 1/2 \\ \bar{p}q & \text{if } \beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) \leqslant 1/2 \text{ and } \beta(l_p, n_q^{\mathbf{v}}) > 1/2 \\ \bar{p}\bar{q} & \text{otherwise.} \end{cases} \tag{10}$$

[17] The notion of a quota rule with exception could be generalized by allowing the exception decision to differ from pq. The exception decision is pq for us due to the privileged status of pq under consequentialist preferences of type 2.

This rule is special in that it reduces to the quota rule making informative voting efficient (defined in Theorem 4) whenever such a quota rule exists.

4. When is informative voting sincere?

While the previous section focuses on designing a voting rule, the present section does not depend on the voting rule. We focus on a single voter and answer the question of when informative voting is sincere, that is, when the naive strategy of 'following the evidence' is worthwhile for a sincere voter. For each type of preference, we fully characterize the parameter combinations for which this is so. We begin with simple preferences.

Theorem 6. *Under simple preferences, the informative voting strategy is sincere if and only if* $\frac{a_{\bar{r}}}{1-a_r} \geqslant \frac{\pi_r}{1-\pi_r} \geqslant \frac{1-a_{\bar{r}}}{a_r}$ *for each* $r \in \{p, q\}$.

This result has an intuitive interpretation. We know that necessarily the upper bound $\frac{a_{\bar{r}}}{1-a_r}$ for $\frac{\pi_r}{1-\pi_r}$ exceeds 1 and the lower bound $\frac{1-a_{\bar{r}}}{a_r}$ is below 1, since $a_r, a_{\bar{r}} > 1/2$. For very high or very low values of the prior probabilities π_r, the ratio $\frac{\pi_r}{1-\pi_r}$ is far from 1, so that one of the bounds is violated and informative voting is *not* sincere. This makes sense since if voters have 'strong' prior beliefs, then the evidence collected cannot overrule the prior beliefs: sincere votes cease to be sensitive to evidence, i.e., depart from informative votes. By contrast, for less strong prior beliefs, the inequalities are satisfied, so that informative voting is sincere, i.e., it is worth following the evidence as a sincere voter.

Another useful perspective on the result is obtained by focusing not on the parameters π_r representing prior beliefs, but on the parameters a_r and $a_{\bar{r}}$ representing 'strength of evidence'. The larger a_r and $a_{\bar{r}}$ are (i.e., the 'stronger' private evidence for r and $\bar{r}$ is), the greater the upper bound for $\frac{\pi_r}{1-\pi_r}$ is and the smaller the lower bound is, which makes it easier to meet both inequalities. In summary, sufficiently strong evidence and/or sufficiently weak prior beliefs imply that it is worth voting informatively ('following the evidence') as a sincere voter.

Surprisingly, the characterization remains the same as we move from simple preferences to consequentialist preferences of type 1 (though the proof is quite different):

Theorem 7. *Under consequentialist preferences of type 1, the informative voting strategy is sincere if and only if* $\frac{a_{\bar{r}}}{1-a_r} \geqslant \frac{\pi_r}{1-\pi_r} \geqslant \frac{1-a_{\bar{r}}}{a_r}$ *for each* $r \in \{p, q\}$.

One can interpret this result in a similar way as done for simple preferences.

Finally, we turn to consequentialist preferences of type 2. Here, the characterization is based on the following three coefficients:

$$A := \frac{\pi_p}{1-\pi_p} \times \frac{a_{\bar{q}}}{1-a_q} + \frac{\pi_q}{1-\pi_q} \times \frac{1-a_{\bar{p}}}{a_p} + \frac{1-a_{\bar{p}}}{a_p} \times \frac{a_{\bar{q}}}{1-a_q}$$
$$B := \frac{\pi_p}{1-\pi_p} \times \frac{1-a_{\bar{q}}}{a_q} + \frac{\pi_q}{1-\pi_q} \times \frac{a_{\bar{p}}}{1-a_p} + \frac{a_{\bar{p}}}{1-a_p} \times \frac{1-a_{\bar{q}}}{a_q}$$
$$C := \frac{\pi_p}{1-\pi_p} \times \frac{1-a_{\bar{q}}}{a_q} + \frac{\pi_q}{1-\pi_q} \times \frac{1-a_{\bar{p}}}{a_p} + \frac{1-a_{\bar{p}}}{a_p} \times \frac{1-a_{\bar{q}}}{a_q}.$$

Theorem 8. *Under consequentialist preferences of type 2, the informative voting strategy is sincere if and only if* $A, B \geqslant \frac{\pi_p}{1-\pi_p} \times \frac{\pi_q}{1-\pi_q} \geqslant C$.

Although the characterizing inequalities are more complicated than for the previous two kinds of preference, an interpretation in terms of strength of evidence is again possible. If the voter's evidence is sufficiently strong (i.e., if $a_p, a_{\bar{p}}, a_q, a_{\bar{q}}$ are sufficiently close 1), then C is well below 1 and A and B are well above 1, so that the inequalities are likely to hold; as a result, informative voting is sincere, i.e., it is worth following the evidence as a sincere voter.

Appendix A. Generalization to an arbitrary number of issues

We have so far considered a two-issue agenda. But many of our results generalize to an arbitrary number $m \in \{1, 2, 3, \ldots\}$ of issues. Such a generalization is of obvious interest, firstly because it establishes the link with the traditional literature on jury theorems, which considers $m = 1$ issue, and secondly because judgment aggregation theory rarely limits the size of the agenda.

Let us be precise. Our results for simple preferences generalize smoothly to the m-issue case (where for $m = 1$ issue we recover a key result by Austen-Smith and Banks, 1996). For consequentialist preferences, the picture becomes more complicated, since the number and structural complexity of possible consequence functions grows rapidly with m, so that it does not suffice to consider two types of consequentialist preferences. This is why, although our findings on consequentialist

preferences do partly generalize to the m-issue case, these generalizations cover only a small portion of the large variety of possible consequentialist preferences for $m > 2$ issues. For only $m = 1$ issue, however, there essentially exists only one type of consequentialist preferences, which coincides with simple preferences. We therefore limit our present m-issue analysis to the case of simple preferences, while leaving consequentialist preferences to future research.

We begin by generalizing our model. We consider a fixed number $m \in \{1, 2, 3, \ldots\}$ of issues. For each issue j ($\in \{1, 2, \ldots, m\}$), the group needs to form a collective judgment on whether a proposition p_j or its negation $\bar{p}_j$ is true. Each set $\{p_1^*, \ldots, p_m^*\}$, which for any issue j contains a proposition p_j^* from $\{p_j, \bar{p}_j\}$, is a possible judgment set. Let $\mathcal{J}_m$ be the set of possible judgment sets for our m-issue agenda. (Note that if $m = 2$ then $\mathcal{J}_m$ is our earlier set $\mathcal{J}$, with $p_1 = p$ and $p_2 = q$.) A voting rule $f : \mathcal{J}_m^n \to \mathcal{J}_m$ maps each voting profile $\mathbf{v} = (v_1, \ldots, v_n) \in \mathcal{J}_m^n$ to a collective 'decision' $d \equiv f(\mathbf{v}) \in \mathcal{J}_m$. A *quota rule* is a voting rule defined by m thresholds $h_{p_1}, \ldots, h_{p_m}$ in $\{0, 1, \ldots, n\}$, where, under any given voting profile, a proposition p_j is accepted if and only if at least h_{p_j} voters accept it.

There is an objective fact as to which propositions are true and which false: exactly one (unknown) judgment set in $\mathcal{J}_m$ is 'correct'; it is referred to as the *state* and contains the 'true' propositions. Everyone aims for a correct collective decision. Specifically, each voter holds the same preferences over decision-state pairs $(d, s) \in \mathcal{J}_m^2$, given by a common utility function $u : \mathcal{J}_m^2 \to \mathbb{R}$. We focus on *simple preferences*, defined by the utility function is given by

$$u(d, s) = \begin{cases} 1 & \text{if } d = s \text{ (correct decision)} \\ 0 & \text{if } d \neq s \text{ (incorrect decision).} \end{cases}$$

Each voter i receives a noisy private signal $t_i \in \mathcal{J}_m$, his *type*; intuitively, it contains the propositions for which the voter holds private evidence. A type profile is a vector $\mathbf{t} = (t_1, \ldots, t_n) \in \mathcal{J}_m^n$. Nature draws a state-types combination $(s, \mathbf{t}) \in \mathcal{J}_m \times \mathcal{J}_m^n$ according to a probability measure denoted Pr.

Just as in the two-issue case, one defines the notions of an *efficient* decision given a type profile, a *(voting) strategy*, an *informative* strategy (or strategy profile), a *sincere* strategy (or strategy profile), an *efficient* strategy profile, and an *equilibrium* strategy profile.

When a proposition $r \in \{p_1, \bar{p}_1, \ldots, p_m, \bar{p}_m\}$ is meant to represent part of voter i's type rather than part of the true state, we often write r_i for r. By assumption, the prior probability that $r \in \{p_1, \bar{p}_1, \ldots, p_m, \bar{p}_m\}$ is true is denoted

$$\pi_r = \Pr(r)$$

and belongs to $(0, 1)$, and the probability of getting evidence for $r \in \{p_1, \bar{p}_1, \ldots, p_m, \bar{p}_m\}$ given that r is true is denoted

$$a_r = \Pr(r_i | r),$$

belongs to $(1/2, 1)$, and does not depend on the voter i. Further, voters' types are independent conditional on the state, and in addition the state and the types w.r.t. any proposition are independent of the state and the types w.r.t. any other proposition. To avoid special cases, we exclude that some decision in $\mathcal{J}_m$ is not efficient for any type profile, and that some type profile leads to multiple efficient decisions (i.e., an efficiency tie).

The following two Theorems for simple preferences in the m-issue case generalize Theorems 2 and 6 for the two-issue case as well as classic results by Austen-Smith and Banks (1996) for the one-issue case. We first generalize our earlier coefficients 'k_p' and 'k_q'. For each proposition p_j in $\{p_1, \ldots, p_m\}$, we define the coefficient[18]

$$k_{p_j} := \min\left\{k \in \{0, 1, \ldots, n\}\colon \frac{\pi_{p_j}}{1 - \pi_{p_j}} > \left(\frac{1 - a_{\bar{p}_j}}{a_{p_j}}\right)^k \left(\frac{a_{\bar{p}_j}}{1 - a_{p_j}}\right)^{n-k}\right\}. \tag{11}$$

Interpretationally, for the proposition p_j to be more probably true than false given a type profile, p_j must occur at least k_{p_j} times in this type profile.

Theorem 9. *Under simple preferences, informative voting is efficient if and only if the voting rule is the quota rule with thresholds* $k_{p_1}, \ldots, k_{p_m}$.

Theorem 10. *Under simple preferences, the informative voting strategy is sincere if and only if* $\frac{a_{\bar{p}_j}}{1-a_{p_j}} \geqslant \frac{\pi_{p_j}}{1-\pi_{p_j}} \geqslant \frac{1-a_{\bar{p}_j}}{a_{p_j}}$ *for each* $j \in \{1, \ldots, m\}$.

Finally, Theorem 1 about an arbitrary kind of (common) preferences also generalizes to any number of issues. This theorem continues to hold as stated, simply replacing '$\mathcal{J}$' by '$\mathcal{J}_m$'.

[18] Note that the minimum in (11) is taken over a non-empty set due to our non-degeneracy assumption.

Appendix B. Proofs

We begin by some preliminary derivations (Section B.1). We then prove the results of the main in a new order obtained by clustering the results according to the kind of preferences (Sections B.2–B.5). We finally prove the results of Appendix A (Section B.6).

Conventions. Recall the notation 'f_r' introduced in fn. 5 and the notation 'S' for the random variable generating the state s in $\mathcal{J}$ introduced in fn. 10. Double-negations cancel each other out, i.e., $\bar{\bar{p}}$ stands for p, and $\bar{\bar{q}}$ for q. We refer to the two technical assumptions made at the end of Section 2.3 as 'non-degeneracy' and 'no efficiency ties', respectively.

B.1. Preliminary derivations

The joint probability of a state-types vector $(s, \mathbf{t}) = (s_p s_q, t_{1p} t_{1q}, \ldots, t_{np} t_{nq}) \in \mathcal{J}^{n+1}$ is

$$\begin{aligned}\Pr(s, \mathbf{t}) &= \Pr(s)\Pr(\mathbf{t}|s) = \Pr(s)\prod_i \Pr(t_i|s)\\ &= \Pr(s_p)\Pr(s_q)\prod_i \Pr(t_{ip}|s_p)\Pr(t_{iq}|s_q),\end{aligned}$$

where the last two equations follow from our independence assumptions. A voter's probability of a state $s = p_s q_s \in \mathcal{J}$ given his type $t = p_t q_t \in \mathcal{J}$ is given by $\Pr(s|t) = \Pr(p_s|p_t)\Pr(q_s|q_t)$, which reduces to

$$\Pr(s|t) = \frac{\pi_{p_s} a_{p_s}}{\pi_{p_s} a_{p_s} + \pi_{\overline{p_s}}(1 - a_{\overline{p_s}})} \times \frac{\pi_{q_s} a_{q_s}}{\pi_{q_s} a_{q_s} + \pi_{\overline{q_s}}(1 - a_{\overline{q_s}})} \quad \text{if } p_s = p_t, q_s = q_t \tag{12}$$

$$\Pr(s|t) = \frac{\pi_{p_s} a_{p_s}}{\pi_{p_s} a_{p_s} + \pi_{\overline{p_s}}(1 - a_{\overline{p_s}})} \times \frac{\pi_{q_s}(1 - a_{q_s})}{\pi_{q_s}(1 - a_{q_s}) + \pi_{\overline{q_s}} a_{\overline{q_s}}} \quad \text{if } p_s = p_t, q_s \neq q_t \tag{13}$$

$$\Pr(s|t) = \frac{\pi_{p_s}(1 - a_{p_s})}{\pi_{p_s}(1 - a_{p_s}) + \pi_{\overline{p_s}} a_{\overline{p_s}}} \times \frac{\pi_{q_s} a_{q_s}}{\pi_{q_s} a_{q_s} + \pi_{\overline{q_s}}(1 - a_{\overline{q_s}})} \quad \text{if } p_s \neq p_t, q_s = q_t \tag{14}$$

$$\Pr(s|t) = \frac{\pi_{p_s}(1 - a_{p_s})}{\pi_{p_s}(1 - a_{p_s}) + \pi_{\overline{p_s}} a_{\overline{p_s}}} \times \frac{\pi_{q_s}(1 - a_{q_s})}{\pi_{q_s}(1 - a_{q_s}) + \pi_{\overline{q_s}} a_{\overline{q_s}}} \quad \text{if } p_s \neq p_t, q_s \neq q_t \tag{15}$$

The probability of the four states in $\mathcal{J}$ conditional on the *full* information $\mathbf{t} \in \mathcal{J}^n$ is given as follows, where $k := n_p^{\mathbf{t}}$ and $l := n_q^{\mathbf{t}}$:

$$\Pr(pq|\mathbf{t}) = \frac{\pi_p a_p^k (1 - a_p)^{n-k} \pi_q a_q^l (1 - a_q)^{n-l}}{\Pr(\mathbf{t})} \tag{16}$$

$$\Pr(p\bar{q}|\mathbf{t}) = \frac{\pi_p a_p^k (1 - a_p)^{n-k} \pi_{\bar{q}} (1 - a_{\bar{q}})^l a_{\bar{q}}^{n-l}}{\Pr(\mathbf{t})} \tag{17}$$

$$\Pr(\bar{p}q|\mathbf{t}) = \frac{\pi_{\bar{p}} (1 - a_{\bar{p}})^k a_{\bar{p}}^{n-k} \pi_q a_q^l (1 - a_q)^{n-l}}{\Pr(\mathbf{t})} \tag{18}$$

$$\Pr(\bar{p}\bar{q}|\mathbf{t}) = \frac{\pi_{\bar{p}} (1 - a_{\bar{p}})^k a_{\bar{p}}^{n-k} \pi_{\bar{q}} (1 - a_{\bar{q}})^l a_{\bar{q}}^{n-l}}{\Pr(\mathbf{t})}. \tag{19}$$

B.2. General preferences

Proof of Theorem 1. (a) As mentioned, this part follows from McLennan (1998), but for completeness we include a proof. We write T_i $(= T_{ip} T_{iq})$ for the random variable generating voter i's type in $\mathcal{J}$, and $\mathbf{T} = (T_1, \ldots, T_n)$ for the random type profile. Consider any voting rule $f : \mathcal{J}^n \to \mathcal{J}$ and any efficient strategy profile $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_n)$. To show that $\boldsymbol{\sigma}$ is an equilibrium, consider any voter i and type $t_i \in \mathcal{J}$. We have to show that i's vote $\sigma_i(t_i)$ maximizes the expected utility conditional on i's type, i.e., that

$$E\big(u\big(f\big(\sigma_i(t_i), \boldsymbol{\sigma}_{-i}(\mathbf{T}_{-i})\big), S\big)\big|t_i\big) \geqslant E\big(u\big(f\big(v_i, \boldsymbol{\sigma}_{-i}(\mathbf{T}_{-i})\big), S\big)\big|t_i\big) \quad \text{for all } v_i \in \mathcal{J},$$

where $(\sigma_i(t_i), \boldsymbol{\sigma}_{-i}(\mathbf{T}_{-i}))$ and $(v_i, \boldsymbol{\sigma}_{-i}(\mathbf{T}_{-i}))$ of course denote the voting profiles in which i votes v_i resp. $\sigma_i(t_i)$ and each $j \neq i$ votes $\sigma_j(T_j)$. To show this, note that for all $v_i \in \mathcal{J}$,

$$\begin{aligned} E\big(u\big(f\big(v_i, \boldsymbol{\sigma}_{-i}(\mathbf{T}_{-i})\big), S\big)\big|t_i\big) &= \sum_{\mathbf{t}_{-i}\in\mathcal{J}^{n-1}} \Pr(\mathbf{t}_{-i}|t_i) E\big(u\big(f\big(v_i, \boldsymbol{\sigma}_{-i}(\mathbf{t}_{-i})\big), S\big)\big|t_i, \mathbf{t}_{-i}\big) \\ &\leqslant \sum_{\mathbf{t}_{-i}\in\mathcal{J}^{n-1}} \Pr(\mathbf{t}_{-i}|t_i) E\big(u\big(f\big(\sigma_i(t_i), \boldsymbol{\sigma}_{-i}(\mathbf{t}_{-i})\big), S\big)\big|t_i, \mathbf{t}_{-i}\big) \\ &= E\big(u\big(f\big(\sigma_i(t_i), \boldsymbol{\sigma}_{-i}(\mathbf{T}_{-i})\big), S\big)\big|t_i\big), \end{aligned}$$

where the inequality holds because the strategy profile $(\sigma_i, \boldsymbol{\sigma}_{-i}) = \boldsymbol{\sigma}$ is efficient for the type profile $(t_i, \mathbf{t}_{-i}) = \mathbf{t}$.

(b) Since by (16)–(19) the conditional distribution of the state given full information $\mathbf{t} \in \mathcal{J}^n$ depends on $\mathbf{t}$ only via the numbers $n_p^{\mathbf{t}}$ and $n_q^{\mathbf{t}}$, so does the conditional expected utility of each decision, and hence, the set of efficient decisions. For each $(k, l) \in \{0, 1, \ldots, n\}^2$, let $F(k, l) \in \mathcal{J}$ be a decision that is efficient for some (hence, every) $\mathbf{t} \in \mathcal{J}^n$ for which $n_p^{\mathbf{t}} = k$ and $n_q^{\mathbf{t}} = l$. The voting rule f defined by $\mathbf{v} \mapsto f(\mathbf{v}) = F(n_p^{\mathbf{v}}, n_q^{\mathbf{v}})$ is clearly anonymous and renders informative voting efficient. □

B.3. Simple preferences

Although Theorems 2 and 6 about simple preferences are later generalized to the multi-issue case, we give direct proofs of these theorems; this is helpful since the two-issue case is more concrete and elementary than the general m-issue case.

We begin by two lemmas.

Lemma 1. *Assume simple preferences. The expected utility of a decision $d \in \mathcal{J}$ is*

$$E\big(u(d, S)\big) = \Pr(S = d),$$

and the conditional expected utility of d given a type or a type profile is given by the analogous expression with a conditional probability instead of an unconditional one.

Proof. The claim follows immediately from the definition of the utility function. □

The next lemma invokes the coefficients k_p and k_q defined in (3) and (4).

Lemma 2. *Assume simple preferences. For all type profiles $\mathbf{t} \in \mathcal{J}^n$, all $r \in \{p, q\}$, and all decisions $d, d' \in \mathcal{J}$ such that d but not d' contains r, and d and d' share the other proposition,*

$$E\big(u(d, S)\big|\mathbf{t}\big) > E\big(u\big(d', S\big)\big|\mathbf{t}\big) \quad \Leftrightarrow \quad n_r^{\mathbf{t}} \geqslant k_r.$$

Proof. Let $\mathbf{t} \in \mathcal{J}^n$. We first prove the equivalence for $r = p$, $d = pq$ and $d' = \bar{p}q$. By the definition of k_p, the inequality $n_p^{\mathbf{t}} \geqslant k_p$ is equivalent to

$$\frac{\pi_p}{1-\pi_p} > \left(\frac{1-a_{\bar{p}}}{a_p}\right)^{n_p^{\mathbf{t}}} \left(\frac{a_{\bar{p}}}{1-a_p}\right)^{n-n_p^{\mathbf{t}}}, \tag{20}$$

which by (16) and (18) is equivalent to $\Pr(pq|\mathbf{t}) > \Pr(\bar{p}q|\mathbf{t})$, and hence by Lemma 1 to $E(u(pq, S)|\mathbf{t}) > E(u(\bar{p}q, S)|\mathbf{t})$. Next, suppose $r = p$, $d = p\bar{q}$ and $d' = \bar{p}\bar{q}$. Using (17) and (19), the inequality (20) is equivalent to $\Pr(p\bar{q}|\mathbf{t}) > \Pr(\bar{p}\bar{q}|\mathbf{t})$, and hence, to $E(u(p\bar{q}, S)|\mathbf{t}) > E(u(\bar{p}\bar{q}, S)|\mathbf{t})$. The proof for the remaining cases is analogous. □

We are now in a position to prove the two theorems about simple preferences.

Proof of Theorem 2. Consider a rule $f : \mathcal{J}^n \to \mathcal{J}$.

A. First, assume f is the quota rule with thresholds k_p and k_q. Consider a given type profile $\mathbf{t} \in \mathcal{J}^n$. Supposing that voters vote informatively, the resulting voting profile is $\mathbf{v} = \mathbf{t}$. We have to show that the decision $d := f(\mathbf{v})$ is efficient for $\mathbf{t}$, i.e., that (*) $E(u(d, S)|\mathbf{t}) > E(u(d', S)|\mathbf{t})$ for all $d' \in \mathcal{J}\backslash\{d\}$. (We use '>' rather than '⩾' in (*) because of our 'no efficiency ties' assumption.) The property (*) follows from Lemma 2. For instance, if $d = pq$, then by definition of f we have $n_p^{\mathbf{t}} \geqslant k_p$ and $n_q^{\mathbf{t}} \geqslant k_q$, so that Lemma 2 implies the inequality in (*) for $d' = \bar{p}q$ and $d' = p\bar{q}$

For instance, if $d = pq$, then by definition of f we have $n_p^{\mathbf{t}} \geqslant k_p$ and $n_q^{\mathbf{t}} \geqslant k_q$, so that Lemma 2 implies that

$$E\big(u(pq, S)\big|\mathbf{t}\big) > E\big(u(\bar{p}q, S)\big|\mathbf{t}\big), E\big(u(p\bar{q}, S)\big|\mathbf{t}\big) > E\big(u(\bar{p}\bar{q}, S)\big|\mathbf{t}\big),$$

which in turn implies (*); and if $d = \bar{p}\bar{q}$, then $n_p^{\mathbf{t}} < k_p$ and $n_q^{\mathbf{t}} < k_q$, so that Lemma 2 implies that

$$E\big(u(\bar{p}\bar{q}, S)\big|\mathbf{t}\big) > E\big(u(p\bar{q}, S)\big|\mathbf{t}\big), E\big(u(\bar{p}q, S)\big|\mathbf{t}\big) > E\big(u(pq, S)\big|\mathbf{t}\big),$$

which again implies (*).

B. Conversely, suppose informative voting is efficient under f. We consider any $\mathbf{v} \in \mathcal{J}^n$ and $r \in \{p, q\}$, and must show that (**) $f_r(\mathbf{v}) = r \Leftrightarrow n_r^{\mathbf{v}} \geqslant k_r$. Consider the type profile $\mathbf{t} = \mathbf{v}$. Since informative voting is efficient, the decision $d = f(\mathbf{v})$ is efficient for $\mathbf{t}$ $(= \mathbf{v})$, i.e., satisfies condition (*) above. Lemma 2 and (*) together imply (**). For instance, if $f(\mathbf{v}) = pq$, then (**) holds because, firstly, $f_r(\mathbf{v}) = r$, and secondly, $n_r^{\mathbf{v}} \geqslant k_r$ by (*) and Lemma 2. □

Proof of Theorem 6. A. First, assume informative voting is sincere. Equivalently, for any given type $t \in \mathcal{J}$, $E(u(d, S)|t)$ is maximal at $d = t$, i.e., by Lemma 1 (*) $\Pr(d|t)$ is maximal at $d = t$. Applying (*) to type $t = pq$, we have $\Pr(pq|t) \geqslant \Pr(\bar{p}q|t)$, which implies $\frac{\pi_p}{1-\pi_p} \geqslant \frac{1-a_{\bar{p}}}{a_p}$ by (12) and (14). Now applying (*) to type $t = \bar{p}\bar{q}$, we obtain $\Pr(\bar{p}\bar{q}|t) \geqslant \Pr(p\bar{q}|t)$, which by (12) and (14) implies $\frac{a_{\bar{p}}}{1-a_p} \geqslant \frac{\pi_p}{1-\pi_p}$. We have shown both inequalities relating to p. The two inequalities relating to q can be proved analogously.

B. Now suppose $\frac{a_{\bar{r}}}{1-a_r} \geqslant \frac{\pi_r}{1-\pi_r} \geqslant \frac{1-a_{\bar{r}}}{a_r}$ for each $r \in \{p, q\}$. We consider any type $t \in \mathcal{J}$ and have to show that the decision $d = t$ has maximal expected utility given t, or equivalently, that (*) holds.

We show (*) first in the case $t = pq$. Here, the inequality $\frac{\pi_p}{1-\pi_p} \geqslant \frac{1-a_{\bar{p}}}{a_p}$ implies $\Pr(pq|t) \geqslant \Pr(\bar{p}q|t)$ by (12) and (14), and it implies $\Pr(p\bar{q}|t) \geqslant \Pr(\bar{p}\bar{q}|t)$ by (13) and (15). Further, the inequality $\frac{\pi_q}{1-\pi_q} \geqslant \frac{1-a_{\bar{q}}}{a_q}$ implies $\Pr(pq|t) \geqslant \Pr(p\bar{q}|t)$ by (12) and (13). This shows (*) for $t = pq$.

Now we show (*) for the case $t = p\bar{q}$. As $\frac{\pi_p}{1-\pi_p} \geqslant \frac{1-a_{\bar{p}}}{a_p}$, we here have $\Pr(p\bar{q}|t) \geqslant \Pr(\bar{p}\bar{q}|t)$ by (12) and (14), and we have $\Pr(pq|t) \geqslant \Pr(\bar{p}q|t)$ by (13) and (15). As $\frac{a_{\bar{q}}}{1-a_q} \geqslant \frac{\pi_q}{1-\pi_q}$, we also have $\Pr(p\bar{q}|t) \geqslant \Pr(pq|t)$ by (12) and (13). This proves (*) for $t = p\bar{q}$.

By similar arguments, one shows (*) for $t = \bar{p}q$ and for $t = \bar{p}\bar{q}$. □

B.4. Consequentialist preferences: type 1

We begin by two lemmas, which are the counterparts of Lemmas 1 and 2 for the current preferences.

Lemma 3. *Assume consequentialist preferences of type 1. The expected utility of a decision $d \in \mathcal{J}$ is*

$$E\big(u(d, S)\big) = \begin{cases} \Pr(pq) + \Pr(\bar{p}\bar{q}) & \text{if } d \in \{pq, \bar{p}\bar{q}\} \\ \Pr(p\bar{q}) + \Pr(\bar{p}q) & \text{if } d \in \{p\bar{q}, \bar{p}q\}, \end{cases}$$

and the conditional expected utility of d given a type or a type profile is given by the analogous expression with conditional probabilities instead of unconditional ones.

Proof. The claim follows easily from the definition of the utility function. □

Lemma 4. *Assume consequentialist preferences of type 1. For each type profile $\mathbf{t} \in \mathcal{J}^n$ and decisions $d \in \{pq, \bar{p}\bar{q}\}$ and $d' \in \{p\bar{q}, \bar{p}q\}$*

$$E\big(u(d, S)\big|\mathbf{t}\big) > E\big(u(d', S)\big|\mathbf{t}\big) \Leftrightarrow \big[n_r^{\mathbf{t}} \geqslant k_r \text{ for both or no } r \in \{p, q\}\big].$$

Proof. Consider any $\mathbf{t} \in \mathcal{J}^n$, $d \in \{pq, \bar{p}\bar{q}\}$ and $d' \in \{p\bar{q}, \bar{p}q\}$. Define $g_r(k) := \pi_r a_r^k (1-a_r)^{n-k}$ and $g_{\bar{r}}(k) := (1-\pi_r)(1-a_{\bar{r}})^k a_{\bar{r}}^{n-k}$ for all $r \in \{p, q\}$ and $k \in \mathbb{R}$. For each $r \in \{p, q\}$, the definition of k_r can now be rewritten as $k_r = \min\{k \in \{0, 1, \ldots, n+1\}: g_r(k) > g_{\bar{r}}(k)\}$. So, (*) for each $k \in \{0, 1, \ldots, n+1\}$, $k \geqslant k_r \Leftrightarrow g_r(k) > g_{\bar{r}}(k)$. (Here, the implication '$\Rightarrow$' uses that $g_r(k)$ $[g_{\bar{r}}(k)]$ is strictly increasing [decreasing] in $k \in \mathbb{R}$.) Now,

$$\begin{aligned}
& E\big(u(d, S)\big|\mathbf{t}\big) > E\big(u(d', S)\big|\mathbf{t}\big) \\
\Leftrightarrow\quad & \Pr(pq|\mathbf{t}) + \Pr(\bar{p}\bar{q}|\mathbf{t}) > \Pr(p\bar{q}|\mathbf{t}) + \Pr(\bar{p}q|\mathbf{t}) \quad \text{by Lemma 3} \\
\Leftrightarrow\quad & g_p\big(n_p^{\mathbf{t}}\big) g_q\big(n_q^{\mathbf{t}}\big) + g_{\bar{p}}\big(n_p^{\mathbf{t}}\big) g_{\bar{q}}\big(n_q^{\mathbf{t}}\big) > g_p\big(n_p^{\mathbf{t}}\big) g_{\bar{q}}\big(n_q^{\mathbf{t}}\big) + g_{\bar{p}}\big(n_p^{\mathbf{t}}\big) g_q\big(n_q^{\mathbf{t}}\big) \quad \text{by (16)–(19)} \\
\Leftrightarrow\quad & \big[g_p\big(n_p^{\mathbf{t}}\big) - g_{\bar{p}}\big(n_p^{\mathbf{t}}\big)\big]\big[g_q\big(n_q^{\mathbf{t}}\big) - g_{\bar{q}}\big(n_q^{\mathbf{t}}\big)\big] > 0 \\
\Leftrightarrow\quad & \big[n_r^{\mathbf{t}} \geqslant k_r \text{ for both or no } r \in \{p, q\}\big] \quad \text{by (*).} \qquad \square
\end{aligned}$$

We can now prove our two theorems about the present preferences.

Proof of Theorem 3. Consider a rule $f: \mathcal{J}^n \to \mathcal{J}$.

A. Assume f is the quota rule with thresholds k_p and k_q. Firstly, f is monotonic. Secondly, to show that informative voting is efficient, consider a given type profile $\mathbf{t} \in \mathcal{J}^n$. Supposing informative voting, the resulting voting profile is then $\mathbf{v} := \mathbf{t}$. We have to show that $d := f(\mathbf{v})$ is efficient for $\mathbf{t}$, i.e., that for each $d' \in \mathcal{J}$ with $\mathrm{Co}(d') \neq \mathrm{Co}(d)$ we have (*)

$E(u(d,S)|\mathbf{t}) \geqslant E(u(d',S)|\mathbf{t})$. Consider any $d' \in \mathcal{J}$ with $\mathrm{Co}(d') \neq \mathrm{Co}(d)$. If $d = pq$, then $n_r^{\mathbf{t}} \geqslant k_r$ for both $r \in \{p,q\}$, implying (*) by Lemma 4. If $d = \bar{p}\bar{q}$, then $n_r^{\mathbf{t}} \geqslant k_r$ for *no* $r \in \{p,q\}$, again implying (*) by Lemma 4. Finally, if d is $\bar{p}q$ or $p\bar{q}$, then $n_r^{\mathbf{t}} \geqslant k_r$ for *exactly one* $r \in \{p,q\}$, so that (*) holds once again by Lemma 4.

B. Conversely, assume f is monotonic and makes informative voting efficient. We consider any $\mathbf{v} \in \mathcal{J}^n$ and must show that (**) $f_r(\mathbf{v}) = r \Leftrightarrow n_r^{\mathbf{v}} \geqslant k_r$ for each $r \in \{p,q\}$. As one can show using our non-degeneracy assumption,

$$k_r \notin \{0, n+1\} \quad \text{for some } r \in \{p,q\}; \tag{21}$$

for instance, if k_r were zero for each $r \in \{p,q\}$, then by Lemma 4 the decisions $\bar{p}q$ and $p\bar{q}$ would be inefficient for each type profile, violating non-degeneracy. We now prove (**) by distinguishing four cases.

Case 1: $n_r^{\mathbf{v}} \geqslant k_r$ for each $r \in \{p,q\}$. We must show that $f(\mathbf{v}) = pq$. Since the decision $f(\mathbf{v})$ is efficient for the type profile $\mathbf{t} = \mathbf{v}$, by Lemma 4, $f(\mathbf{v}) \in \{pq, \bar{p}\bar{q}\}$. Suppose for a contradiction $f(\mathbf{v}) = \bar{p}\bar{q}$. By (21), $k_r \geqslant 1$ for some $r \in \{p,q\}$. Suppose $k_p > 0$ (the case that $k_q > 0$ being analogous). Let $\mathbf{v}'$ be the voting profile obtained from $\mathbf{v}$ by replacing each occurring p by $\bar{p}$. By monotonicity, the decision is $f(\mathbf{v}') = \bar{p}\bar{q}$. By Lemma 4, for the type profile $\mathbf{t}' = \mathbf{v}'$ only $\bar{p}q$ and $p\bar{q}$ are efficient since $n_p^{\mathbf{t}'} = 0 < k_p$ and $n_q^{\mathbf{t}'} = n_q^{\mathbf{v}} \geqslant k_q$. So, the decision $f(\mathbf{v}')$ $(= \bar{p}\bar{q})$ is inefficient, a contradiction since f makes informative voting efficient.

Case 2: $n_p^{\mathbf{v}} \geqslant k_p$ and $n_q^{\mathbf{v}} < k_q$. We must show that $f(\mathbf{v}) = p\bar{q}$. By Lemma 4, $p\bar{q}$ and $\bar{p}q$ are both efficient for the type profile $\mathbf{t} = \mathbf{v}$. So, as informative voting is efficient, $f(\mathbf{v}) \in \{p\bar{q}, \bar{p}q\}$. Suppose for a contradiction $f(\mathbf{v}) = \bar{p}q$. By (21), $k_p > 0$ or $k_q \leqslant n$. First, if $k_p > 0$, define $\mathbf{v}'$ as in Case 1. By monotonicity, the decision is $f(\mathbf{v}') = \bar{p}q$, which is inefficient for the type profile $\mathbf{t}' = \mathbf{v}'$ by Lemma 4 as $n_p^{\mathbf{t}'} = 0 < k_p$ and $n_q^{\mathbf{t}'} = n_q^{\mathbf{v}} < k_q$, a contradiction. Second, if $k_q \leqslant n$, define $\mathbf{v}'$ as the voting profile obtained from $\mathbf{v}$ by replacing each occurring $\bar{q}$ by q. By monotonicity, the decision is $f(\mathbf{v}') = \bar{p}q$, which is again inefficient for the type profile $\mathbf{t}' = \mathbf{v}'$ by Lemma 4 as $n_p^{\mathbf{t}'} = n_p^{\mathbf{v}} \geqslant k_p$ and $n_q^{\mathbf{t}'} = n \geqslant k_q$, a contradiction.

Case 3: $n_p^{\mathbf{v}} < k_p$ and $n_q^{\mathbf{v}} \geqslant k_q$. One can show that $f(\mathbf{v}) = \bar{p}q$ like in Case 2.

Case 4: $n_r^{\mathbf{v}} < k_r$ for each $r \in \{p,q\}$. We must show that $f(\mathbf{v}) = \bar{p}\bar{q}$. By informative voting being efficient and by Lemma 4 applied to $\mathbf{t} = \mathbf{v}$, $f(\mathbf{v}) \in \{pq, \bar{p}\bar{q}\}$. Suppose for a contradiction that $f(\mathbf{v}) = pq$. By (21), $k_r \leqslant n$ for some $r \in \{p,q\}$. We assume that $k_p \leqslant n$ (the proof being analogous if $k_q \leqslant n$). Let the voting profile $\mathbf{v}' \in \mathcal{J}^n$ arise from $\mathbf{v}$ by replacing each occurring $\bar{p}$ by p. By monotonicity, $f(\mathbf{v}') = pq$. This outcome is inefficient for the type profile $\mathbf{t}' = \mathbf{v}'$ by Lemma 4 and $n_p^{\mathbf{t}'} = n \geqslant k_p$ and $n_q^{\mathbf{t}'} = n_q^{\mathbf{v}} < k_q$. □

Proof of Theorem 7. A. First, let informative voting be sincere. Equivalently, for any type $t \in \mathcal{J}$, (*) $E(u(d,S)|t)$ is maximal at $d = t$. Using (*) with $t = pq$, we have $E(u(pq,S)|t) \geqslant E(u(\bar{p}q,S)|t)$, which by Lemma 3 is equivalent to $\Pr(pq|t) + \Pr(\bar{p}\bar{q}|t) \geqslant \Pr(p\bar{q}|t) + \Pr(\bar{p}q|t)$. Using (12)–(15), the latter is equivalent to

$$\frac{\pi_p}{1-\pi_p} \times \frac{\pi_q}{1-\pi_q} + \frac{1-a_{\bar{p}}}{a_p} \times \frac{1-a_{\bar{q}}}{a_q} \geqslant \frac{\pi_p}{1-\pi_p} \times \frac{1-a_{\bar{q}}}{a_q} + \frac{\pi_q}{1-\pi_q} \times \frac{1-a_{\bar{p}}}{a_p},$$

which can be rearranged as

$$\left(\frac{\pi_p}{1-\pi_p} - \frac{1-a_{\bar{p}}}{a_p}\right)\left(\frac{\pi_q}{1-\pi_q} - \frac{1-a_{\bar{q}}}{a_q}\right) \geqslant 0. \tag{22}$$

Analogously, using (*) three more times, with $t = p\bar{q}$, then $t = \bar{p}q$ and finally $t = \bar{p}\bar{q}$, we obtain

$$\left(\frac{\pi_p}{1-\pi_p} - \frac{1-a_{\bar{p}}}{a_p}\right)\left(\frac{1-\pi_q}{\pi_q} - \frac{1-a_q}{a_{\bar{q}}}\right) \geqslant 0 \tag{23}$$

$$\left(\frac{\pi_p}{1-\pi_p} - \frac{a_{\bar{p}}}{1-a_p}\right)\left(\frac{1-\pi_q}{\pi_q} - \frac{a_q}{1-a_{\bar{q}}}\right) \geqslant 0 \tag{24}$$

$$\left(\frac{\pi_p}{1-\pi_p} - \frac{a_{\bar{p}}}{1-a_p}\right)\left(\frac{\pi_q}{1-\pi_q} - \frac{a_{\bar{q}}}{1-a_q}\right) \geqslant 0. \tag{25}$$

Firstly, (i) $\frac{\pi_q}{1-\pi_q} \geqslant \frac{1-a_{\bar{q}}}{a_q}$, since otherwise by (22) we would get $\frac{\pi_p}{1-\pi_p} \leqslant \frac{1-a_{\bar{p}}}{a_p}$ (< 1), whereas by (24) we get $\frac{\pi_p}{1-\pi_p} \geqslant \frac{a_{\bar{p}}}{1-a_p}$ (> 1), a contradiction. Secondly, (ii) $\frac{\pi_p}{1-\pi_p} \geqslant \frac{1-a_{\bar{p}}}{a_p}$, because if (i) holds with a strict inequality, then (ii) follows from (22), whereas if (i) holds with equality, then $\frac{\pi_q}{1-\pi_q} < 1 < \frac{a_{\bar{q}}}{1-a_q}$, which together with (23) implies (ii). We finally show that (iii) $\frac{\pi_p}{1-\pi_p} \leqslant \frac{a_{\bar{p}}}{1-a_p}$ and (iv) $\frac{\pi_q}{1-\pi_q} \leqslant \frac{a_{\bar{q}}}{1-a_q}$. First, suppose (ii) holds with equality. Then $\frac{\pi_p}{1-\pi_p} < 1 < \frac{a_{\bar{p}}}{1-a_p}$, which implies (iii), and with (25) also implies (iv). Second, suppose (ii) holds with a strict inequality. Then with (23) we get (iv). If (iv) holds with a strict inequality, then we get (iii) by (25), while if (iv) holds with equality, then $\frac{1-\pi_q}{\pi_q} = \frac{1-a_q}{a_{\bar{q}}} < 1 < \frac{a_q}{1-a_{\bar{q}}}$, which by (24) implies (iii).

B. Conversely, assume $\frac{a_{\bar{r}}}{1-a_r} \geqslant \frac{\pi_r}{1-\pi_r} \geqslant \frac{1-a_{\bar{r}}}{a_r}$ for each $r \in \{p,q\}$. We have to show that informative voting is sincere, i.e., that (*) holds for each type $t \in \mathcal{J}$. As one can check, the inequalities (22)–(25) all hold. These inequalities imply that (*) holds for each type $t \in \mathcal{J}$. For instance, as shown in part A, (22) reduces to $E(u(pq,S)|t) \geqslant E(u(\bar{p}q,S)|t)$ for $t = pq$. □

B.5. Consequentialist preferences: type 2

We begin by a simple lemma, the counterpart of Lemmas 1 and 3.

Lemma 5. *Assume consequentialist preferences of type 2. The expected utility of a decision $d \in \mathcal{J}$ is*

$$E\big(u(d,S)\big) = \begin{cases} \Pr(pq) & \text{if } d = pq \\ 1 - \Pr(pq) & \text{if } d \neq pq, \end{cases}$$

and the conditional expected utility of d given a type or a type profile is given by the analogous expression with conditional probabilities instead of unconditional ones.

Proof. The claim follows from the specification of the utility function. □

We now prove our results about the current preferences. Some proofs implicitly extend $\beta(k,l)$ to values of $k.l$ not in $\{0,\dots,n\}$, using the expression (5).

Proof of Proposition 1. The claim can easily be shown by elaborating the informal argument given in the text. □

Proof of Theorem 4. A. First, suppose $f : \mathcal{J}^n \to \mathcal{J}$ is a quota rule with thresholds m_p and m_q making informative voting efficient. The following claims must be shown.

Claim 1. *$m_p = l_p$ and $m_q = l_q$.*

Consider a type profile $\mathbf{t} \in \mathcal{J}^n$ for which $n_p^{\mathbf{t}} = n$ and $n_q^{\mathbf{t}} = l_q$. Assuming informative voting, the resulting voting profile is $\mathbf{v} = \mathbf{t}$. By definition of l_q, $\beta(n, l_q) > 1/2$. So $f(\mathbf{v}) = pq$ by Proposition 1. Thus, $l_q \geqslant m_q$ by definition of f. One analogously shows that $l_p \geqslant m_p$. To show the converse inequalities, consider a voting profile $\mathbf{v} \in \mathcal{J}^n$ for which $n_p^{\mathbf{v}} = m_p$ and $n_q^{\mathbf{v}} = n$ ($\geqslant m_q$). The resulting decision is $f(\mathbf{v}) = pq$ by definition of f. So, by Proposition 1, $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) = \beta(m_p, n) > 1/2$. Hence, $m_p \geqslant l_p$ by definition of l_p. Analogously, one shows that $m_q \geqslant l_q$.

Claim 2. *$\beta(l_p, l_q) > 1/2$.*

For any voting profile $\mathbf{v} \in \mathcal{J}^n$ for which $n_p^{\mathbf{v}} = l_p$ ($= m_p$) and $n_q^{\mathbf{v}} = l_q$ ($= m_q$), we have $f(\mathbf{v}) = pq$ by definition of f, so that by Proposition 1 $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) > 1/2$, i.e., $\beta(l_p, l_q) > 1/2$.

B. Conversely, assume $\beta(l_p, l_q) > 1/2$. We show that the quota rule f with thresholds l_p and l_q makes informative voting efficient. We first prove that for all $k, l \in \{0,\dots,n\}$,

$$\beta(k,l) > 1/2 \quad \Leftrightarrow \quad [k \geqslant l_p \text{ and } l \geqslant l_q]. \tag{26}$$

Let $k, l \in \{0,\dots,n\}$. If $k \geqslant l_p$ and $l \geqslant l_q$, then $\beta(k,l) \geqslant \beta(l_p, l_q) > 1/2$, where the first inequality holds because β is increasing in each argument. If $k < l_p$, then $\beta(k,l) \leqslant \beta(k,n) \leqslant 1/2$, where the last inequality holds by definition of l_p ($> k$). Analogously, if $l \leqslant l_q$, then $\beta(k,l) \leqslant 1/2$.

Now consider any type profile $\mathbf{t} \in \mathcal{J}^n$. Assuming informative voting, the resulting voting profile is $\mathbf{v} = \mathbf{t}$. We have to show that the decision $f(\mathbf{v})$ is efficient for $\mathbf{t}$ ($= \mathbf{v}$). First, if $n_p^{\mathbf{t}} \geqslant l_p$ and $n_q^{\mathbf{t}} \geqslant l_q$, the decision is $f(\mathbf{v}) = pq$, which is efficient by Proposition 1 since $\beta(n_p^{\mathbf{t}}, n_q^{\mathbf{t}}) > 1/2$ by (26). Second, if $n_p^{\mathbf{t}} < l_p$ or $n_q^{\mathbf{t}} < l_q$, the resulting decision $f(\mathbf{v})$ is in $\{\bar{p}q, p\bar{q}, \bar{p}\bar{q}\}$, which is efficient by Proposition 1 since $\beta(n_p^{\mathbf{t}}, n_q^{\mathbf{t}}) \leqslant 1/2$ by (26). □

Proof of Theorem 5. Consider a rule $f : \mathcal{J}^n \to \mathcal{J}$. We repeatedly draw on the fact that (*) $\beta(k,l)$ is strictly increasing in each argument.

A. First, assume f is defined by (8) for thresholds m_p and m_q satisfying $\beta(m_p, l_q)$, $\beta(l_p, m_q) > 1/2$. Clearly, f is anonymous. To show that informative voting is efficient, it suffices by Proposition 1 to prove that for all $\mathbf{v} \in \mathcal{J}^n$,

$$f(\mathbf{v}) = pq \quad \Leftrightarrow \quad \beta\big(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}\big) > 1/2. \tag{27}$$

If $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) > 1/2$, then clearly $f(\mathbf{v}) = pq$ by (8). Conversely, assume $f(\mathbf{v}) = pq$. Then, by definition of f, either $n_r^{\mathbf{v}} \geqslant m_r$ for each $r \in \{p, q\}$, or $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) > 1/2$. In the second case, we are done. Now assume the first case. Since $\beta(m_p, l_q) > 1/2$, we have $\beta(m_p, n) > 1/2$ by (*), whence $m_p \geqslant l_p$ by definition of l_p. Using (*) and that $n_p^{\mathbf{v}} \geqslant m_p \geqslant l_p$ and $n_q^{\mathbf{v}} \geqslant m_q$, we have $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) \geqslant \beta(l_p, m_q)$. Moreover, $\beta(l_p, m_q) > 1/2$ by definition of m_q. So, $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) > 1/2$, which completes the proof of (27).

It remains to show monotonicity of f. Take two voting profiles $\mathbf{v}, \mathbf{v}' \in \mathcal{J}^n$ such that for all $r \in f(\mathbf{v})$, the voters who vote for r in $\mathbf{v}$ also vote for r in $\mathbf{v}'$.

Case 1: $f(\mathbf{v}) = pq$. Then, $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) > 1/2$ by (27). Also, $n_p^{\mathbf{v}'} \geqslant n_p^{\mathbf{v}}$ and $n_q^{\mathbf{v}'} \geqslant n_q^{\mathbf{v}}$, so that $\beta(n_p^{\mathbf{v}'}, n_q^{\mathbf{v}'}) \geqslant \beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}})$ by (*). It follows that $\beta(n_p^{\mathbf{v}'}, n_q^{\mathbf{v}'}) > 1/2$, so that $f(\mathbf{v}') = pq$ by (27).

Case 2: $f(\mathbf{v}) = p\bar{q}$. We have to show that $f(\mathbf{v}') = p\bar{q}$, i.e., that

$$n_p^{\mathbf{v}'} \geqslant m_p, \qquad n_q^{\mathbf{v}'} < m_q, \quad \text{and} \quad \beta\big(n_p^{\mathbf{v}'}, n_q^{\mathbf{v}'}\big) \leqslant 1/2. \tag{28}$$

Since $f(\mathbf{v}) = p\bar{q}$, the definition of f implies $n_p^{\mathbf{v}} \geqslant m_p$ and $n_q^{\mathbf{v}} < m_q$, and the definition of $\mathbf{v}'$ implies $n_p^{\mathbf{v}'} \geqslant n_p^{\mathbf{v}}$ and $n_q^{\mathbf{v}'} \leqslant n_q^{\mathbf{v}}$; hence, the first two inequalities in (28) hold. As $\beta(m_p, l_q) > 1/2$ and $n_p^{\mathbf{v}} \geqslant m_p$, we have $\beta(n_p^{\mathbf{v}}, l_q) > 1/2$ by (*). Also, since $f(\mathbf{v}) = p\bar{q}$, we have $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) \leqslant 1/2$ by (27). Hence, $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) < \beta(n_p^{\mathbf{v}}, l_q)$. So, $n_q^{\mathbf{v}} < l_q$ by (*), whence $n_q^{\mathbf{v}'} < l_q$ as $n_q^{\mathbf{v}'} \leqslant n_q^{\mathbf{v}}$. Thus, by definition of l_q, $\beta(n, n_q^{\mathbf{v}'}) \leqslant 1/2$, so that $\beta(n_p^{\mathbf{v}'}, n_q^{\mathbf{v}'}) \leqslant 1/2$ by (*), proving (28).

Case 3: $f(\mathbf{v}) = \bar{p}q$. One can show that $f(\mathbf{v}') = \bar{p}q$ analogously to Case 2.

Case 4: $f(\mathbf{v}) = \bar{p}\bar{q}$. Then, $n_p^{\mathbf{v}} < m_p$, $n_q^{\mathbf{v}} < m_q$, and $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) \leqslant 1/2$. We have to show that $f(\mathbf{v}') = \bar{p}\bar{q}$, i.e., that these three inequalities still hold if $\mathbf{v}$ is replaced by $\mathbf{v}'$. This follows from the fact that $n_p^{\mathbf{v}'} \leqslant n_p^{\mathbf{v}}$ and $n_q^{\mathbf{v}'} \leqslant n_q^{\mathbf{v}}$ (by definition of $\mathbf{v}'$) and from (*).

B. Conversely, let f be monotonic and anonymous, and make informative voting efficient. For each $r \in \{p, q\}$, define

$$m_r := \min\big\{n_r^{\mathbf{v}}\colon\ \mathbf{v} \in \mathcal{J}^n \text{ such that } f_r(\mathbf{v}) = r \text{ and } \beta\big(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}\big) \leqslant 1/2\big\},$$

where this minimum is interpreted as $n+1$ if it is taken over an empty set. We prove that f has the required form with respect to the so-defined thresholds m_p and m_q. The proof proceeds in several steps and is completed by Claims 5 and 6 below.

Claim 1. *For all $\mathbf{v} \in \mathcal{J}^n$, if $n_p^{\mathbf{v}} \geqslant l_p$, $n_q^{\mathbf{v}} \geqslant l_q$ and $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) \leqslant 1/2$, then $f(\mathbf{v}) = \bar{p}\bar{q}$.*

Let $\mathbf{v} \in \mathcal{J}^n$ satisfy the antecedent condition. First assume $f(\mathbf{v}) = p\bar{q}$ for a contradiction. Let $\mathbf{v}'$ be the voting profile obtained from $\mathbf{v}$ by replacing each $\bar{p}$ by p. By monotonicity, $f(\mathbf{v}') = p\bar{q}$. However, Proposition 1 implies that $f(\mathbf{v}') = pq$, since $\beta(n_p^{\mathbf{v}'}, n_q^{\mathbf{v}'}) = \beta(n, n_q^{\mathbf{v}}) \geqslant \beta(n, l_q) > 1/2$ (where the first inequality holds by $n_q^{\mathbf{v}} \geqslant l_q$, and the second by definition of l_q). This contradiction proves that $f(\mathbf{v}) \neq p\bar{q}$. One similarly proves that $f(\mathbf{v}) \neq \bar{p}q$. So, as $f(\mathbf{v}) \in \{p\bar{q}, \bar{p}q, \bar{p}\bar{q}\}$ by Proposition 1, we have $f(\mathbf{v}) = \bar{p}\bar{q}$, proving the claim.

Claim 2. *For all $\mathbf{v} \in \mathcal{J}^n$, if $n_p^{\mathbf{v}} \leqslant l_p$, $n_q^{\mathbf{v}} \leqslant l_q$ and $\beta(l_p, l_q) \leqslant 1/2$, then $f(\mathbf{v}) = \bar{p}\bar{q}$.*

Consider any $\mathbf{v} \in \mathcal{J}^n$ satisfying the antecedent condition. Let $\mathbf{w} \in \mathcal{J}^n$ arise from $\mathbf{v}$ by replacing $l_p - n_p^{\mathbf{v}}$ occurrences of $\bar{p}$ by p, $l_q - n_q^{\mathbf{v}}$ occurrences of $\bar{q}$ by q. Note that $n_p^{\mathbf{w}} = l_p$ and $n_q^{\mathbf{w}} = l_q$, whence by Claim 1 $f(\mathbf{w}) = \bar{p}\bar{q}$. By monotonicity, it follows that $f(\mathbf{v}) = \bar{p}\bar{q}$.

Claim 3. *$m_p \geqslant l_p$ and $m_q \geqslant l_q$.*

Suppose for a contradiction $m_p < l_p$. By definition of m_p, there is a $\mathbf{v} \in \mathcal{J}^n$ such that $m_p = n_p^{\mathbf{v}}$, $f_p(\mathbf{v}) = p$ and $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) \leqslant 1/2$. As by Proposition 1, $f(\mathbf{v}) \in \{\bar{p}q, p\bar{q}, \bar{p}\bar{q}\}$, it follows that $f(\mathbf{v}) = p\bar{q}$. We consider two cases.

Case 1: $n_q^{\mathbf{v}} \geqslant l_q$. Let $\mathbf{v}' \in \mathcal{J}^n$ be the voting profile arising from $\mathbf{v}$ by replacing each $\bar{p}$ by p. By monotonicity, the resulting decision is $f(\mathbf{v}') = p\bar{q}$. But $f(\mathbf{v}') = pq$ by Proposition 1 as $\beta(n_p^{\mathbf{v}'}, n_q^{\mathbf{v}'}) = \beta(n, n_q^{\mathbf{v}}) \geqslant \beta(n, l_q) > 1/2$ (where the first inequality holds by $n_q^{\mathbf{v}} > l_q$ and the second by definition of l_q).

Case 2: $n_q^{\mathbf{v}} < l_q$. Then by Claim 2 $f(\mathbf{v}) = \bar{p}\bar{q}$, a contradiction since $f(\mathbf{v}) = p\bar{q}$.

We have shown one inequality of Claim 3; the other one has an analogous proof.

Claim 4. *For all $\mathbf{v} \in \mathcal{J}^n$ with $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) \leqslant 1/2$, if $n_p^{\mathbf{v}} \geqslant m_p$ then $f(\mathbf{v}) = p\bar{q}$, and if $n_q^{\mathbf{v}} \geqslant m_q$ then $f(\mathbf{v}) = \bar{p}q$.*

Consider any $\mathbf{v} \in \mathcal{J}^n$ with $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) \leqslant 1/2$. Suppose for a contradiction that $n_p^{\mathbf{v}} \geqslant m_p$ but $f(\mathbf{v}) \neq p\bar{q}$. Then, as by Proposition 1 $f(\mathbf{v}) \in \{\bar{p}q, p\bar{q}, \bar{p}\bar{q}\}$, either $f(\mathbf{v}) = \bar{p}q$ or $f(\mathbf{v}) = \bar{p}\bar{q}$.

Case 1: $f(\mathbf{v}) = \bar{p}q$. Let $\mathbf{v}' \in \mathcal{J}^n$ be the voting profile arising from $\mathbf{v}$ by replacing each $\bar{q}$ by q. By monotonicity, the resulting decision is $f(\mathbf{v}') = \bar{p}q$, whereas by Proposition 1 $f(\mathbf{v}') = pq$ because $\beta(n_p^{\mathbf{v}'}, n_q^{\mathbf{v}'}) = \beta(n_p^{\mathbf{v}}, n) \geqslant \beta(l_p, n) > 1/2$, where the first inequality holds because $n_p^{\mathbf{v}} \geqslant l_p$ (by Claim 3) and the second inequality holds by definition of l_p.

Case 2: $f(\mathbf{v}) = \bar{p}\bar{q}$. By definition of m_p there is a $\mathbf{w} \in \mathcal{J}^n$ such that $n_p^{\mathbf{w}} = m_p$, $f_p(\mathbf{w}) = p$ and $\beta(n_p^{\mathbf{w}}, n_q^{\mathbf{w}}) \leqslant 1/2$. As by Proposition 1 $f(\mathbf{w}) \in \{p\bar{q}, \bar{p}q, \bar{p}\bar{q}\}$, it follows that $f(\mathbf{w}) = p\bar{q}$. Let $\mathbf{v}'$ [$\mathbf{w}'$] be the voting profile arising from $\mathbf{v}$ [$\mathbf{w}$] by replacing each q by $\bar{q}$. By monotonicity, $f(\mathbf{v}') = \bar{p}\bar{q}$ and $f(\mathbf{w}') = p\bar{q}$. Now let $\mathbf{w}''$ be a voting profile arising from $\mathbf{w}'$ by replacing $n_p^{\mathbf{v}'} - n_p^{\mathbf{w}'}$ $(= n_p^{\mathbf{v}} - m_p \geqslant 0)$ occurrences of $\bar{p}$ by p. By monotonicity, $f(\mathbf{w}'') = p\bar{q}$. So, $f(\mathbf{w}'') \neq f(\mathbf{v}')$, a contradiction by anonymity since $\mathbf{w}''$ is a permutation of $\mathbf{v}'$.

This shows the first implication in Claim 4. The second one can be shown similarly.

Claim 5. $\beta(m_p, l_q), \beta(l_p, m_q) > 1/2$.

We only show that $\beta(m_p, l_q) > 1/2$; the other inequality is analogous. Suppose for a contradiction that $\beta(m_p, l_q) \leqslant 1/2$. So, since $\beta(n+1, l_q) > \beta(n, l_q) > 1/2$ (by definition of l_q), we have $m_p \neq n+1$. Hence, there is a $\mathbf{v} \in \mathcal{J}^n$ such that $n_p^{\mathbf{v}} = m_p$ and $n_q^{\mathbf{v}} = l_q$. By Claim 4, $f(\mathbf{v}) = p\bar{q}$. Let $\mathbf{v}'$ be the voting profile arising from $\mathbf{v}$ by replacing each $\bar{p}$ by p. By monotonicity, $f(\mathbf{v}') = p\bar{q}$, a contradiction since by Proposition 1 $f(\mathbf{v}') = pq$ since $\beta(n_p^{\mathbf{v}'}, n_q^{\mathbf{v}'}) = \beta(n, l_q) > 1/2$.

Claim 6. *f is given by (8).*

Consider any $\mathbf{v} \in \mathcal{J}^n$ and $r \in \{p, q\}$. We show the equivalence (8) by distinguishing different cases. If $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) > 1/2$, then $f(\mathbf{v}) = pq$ by Proposition 1, implying (8). If $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) \leqslant 1/2$ and $n_r^{\mathbf{v}} \geqslant m_r$, then (8) holds by Claim 4. Finally, if $\beta(n_p^{\mathbf{v}}, n_q^{\mathbf{v}}) \leqslant 1/2$ and $n_r^{\mathbf{v}} < m_r$, then $f_r(\mathbf{v}) \neq r$ by definition of m_r, whence (8) again holds. □

Proof of Theorem 8. A. First, suppose informative voting is sincere. Equivalently, for any given type $t \in \mathcal{J}$, the decision $d = t$ has maximal conditional expected utility, i.e., (*) $E(u(d, S)|t)$ is maximal at $d = t$. Applying (*) with $t = pq$, we have $E(u(pq, S)|t) \geqslant E(u(\bar{p}\bar{q}, S)|t)$, which by Lemma 5 reduces to $\Pr(pq|t) \geqslant 1 - \Pr(pq|t)$, i.e., to $\Pr(pq|t) \geqslant 1/2$. Using (12), one derives that $\frac{\pi_p}{1-\pi_p} \times \frac{\pi_q}{1-\pi_q} \geqslant C$. Now applying (*) with $t = p\bar{q}$, we have $E(u(p\bar{q}, S)|t) \geqslant E(u(pq, S)|t)$, which by Lemma 5 reduces to $1 - \Pr(pq|t) \geqslant \Pr(pq|t)$, so that $\Pr(pq|t) \leqslant 1/2$. Using (13), one obtains $\frac{\pi_p}{1-\pi_p} \times \frac{\pi_q}{1-\pi_q} \leqslant A$. Finally, applying (*) with $t = \bar{p}q$, we have $E(u(\bar{p}q, S)|t) \geqslant E(u(pq, S)|t)$, which by Lemma 5 reduces to $1 - \Pr(pq|t) \geqslant \Pr(pq|t)$, whence $\Pr(pq|t) \leqslant 1/2$. Using (14), one derives $\frac{\pi_p}{1-\pi_p} \times \frac{\pi_q}{1-\pi_q} \leqslant B$. This proves all inequalities.

B. Conversely, suppose $A, B \geqslant \frac{\pi_p}{1-\pi_p} \times \frac{\pi_q}{1-\pi_q} \geqslant C$. For each given type $t \in \mathcal{J}$, one has to show (*). As the reader can verify using Lemma 5 and (12)–(15), if $t = pq$ then (*) follows from $\frac{\pi_p}{1-\pi_p} \times \frac{\pi_q}{1-\pi_q} \geqslant C$; if $t = p\bar{q}$ then (*) follows from $A \geqslant \frac{\pi_p}{1-\pi_p} \times \frac{\pi_q}{1-\pi_q}$; if $t = \bar{p}q$ then (*) follows from $B \geqslant \frac{\pi_p}{1-\pi_p} \times \frac{\pi_q}{1-\pi_q}$; and if $t = \bar{p}\bar{q}$ then (*) can be derived from $A \geqslant \frac{\pi_p}{1-\pi_p} \times \frac{\pi_q}{1-\pi_q}$ or from $B \geqslant \frac{\pi_p}{1-\pi_p} \times \frac{\pi_q}{1-\pi_q}$. □

B.6. An arbitrary number of issues

In this subsection, we assume the generalized model of Appendix A, with m ($\in \{1, 2, 3, \ldots, \}$) issues. To prove Theorem 9, we begin with a lemma. As in the two-issue case, we write 'S' for the random variable generating the state, and $n_r^{\mathbf{t}}$ for the number of types in type profile $\mathbf{t}$ ($\in \mathcal{J}_m^n$) which contain proposition r ($\in \{p_1, \bar{p}_1, \ldots, p_m, \bar{p}_m\}$). This latter notation can also be used with a voting profile $\mathbf{v}$ instead of a type profile $\mathbf{t}$.

Lemma 6. *Under simple preferences, for all type profiles $\mathbf{t} \in \mathcal{J}_m^n$ and decisions $d, d' \in \mathcal{J}_m$, $E(u(d, S)|\mathbf{t}) > E(u(d', S)|\mathbf{t})$ if and only if*

$$\prod_{r \in d} \pi_r a_r^{n_r^{\mathbf{t}}} (1 - a_r)^{n - n_r^{\mathbf{t}}} > \prod_{r \in d'} \pi_r a_r^{n_r^{\mathbf{t}}} (1 - a_r)^{n - n_r^{\mathbf{t}}}. \tag{29}$$

Proof. Assume simple preferences and consider any $\mathbf{t} \in \mathcal{J}_m^n$. One easily shows that for all $d \in \mathcal{J}_m$

$$E\big(u(d, S)|\mathbf{t}\big) = \Pr(S = d|\mathbf{t}) = \prod_{r \in d} \frac{\pi_r a_r^{n_r^{\mathbf{t}}} (1 - a_r)^{n - n_r^{\mathbf{t}}}}{\Pr(\mathbf{t})}.$$

This implies the claimed equivalence. □

Proof of Theorem 9. Consider a voting rule $f : \mathcal{J}_m^n \to \mathcal{J}_m$.

A. First, assume that f is the quota rule with thresholds $k_{p_1}, \ldots, k_{p_m}$. Take any type profile $\mathbf{t}$ and suppose informative voting. Then the resulting voting profile is $\mathbf{v} = \mathbf{t}$. We show that $d := f(\mathbf{v})$ is the only efficient decision for $\mathbf{t}$, i.e., that (*) $E(u(d, S)|\mathbf{t}) > E(u(d', S)|\mathbf{t})$ for all $d' \in \mathcal{J}_m \setminus \{d\}$. For all issues j, if $p_j \in d$, then $n_{p_j}^{\mathbf{v}} = n_{p_j}^{\mathbf{t}} \geqslant k_{p_j}$, so that by (11)

$$\pi_{p_j} a_{p_j}^{n_{p_j}^{\mathbf{t}}} (1 - a_{p_j})^{n - n_{p_j}^{\mathbf{t}}} > \pi_{\bar{p}_j} a_{\bar{p}_j}^{n - n_{p_j}^{\mathbf{t}}} (1 - a_{\bar{p}_j})^{n_{p_j}^{\mathbf{t}}},$$

or in other words (since $n - n_{p_j}^{\mathbf{t}} = n_{\bar{p}_j}^{\mathbf{t}}$)

$$\pi_{p_j} a_{p_j}^{n_{p_j}^{\mathbf{t}}} (1 - a_{p_j})^{n - n_{p_j}^{\mathbf{t}}} > \pi_{\bar{p}_j} a_{\bar{p}_j}^{n_{\bar{p}_j}^{\mathbf{t}}} (1 - a_{\bar{p}_j})^{n - n_{\bar{p}_j}^{\mathbf{t}}};$$

while if $\bar{p}_j \in d$, then $n_{\bar{p}_j}^{\mathbf{v}} = n_{\bar{p}_j}^{\mathbf{t}} < k_{p_j}$, so that by (11)

$$\pi_{p_j} a_{p_j}^{n^{\mathbf{t}}_{p_j}} (1-a_{p_j})^{n-n^{\mathbf{t}}_{p_j}} < \pi_{\bar{p}_j} a_{\bar{p}_j}^{n-n^{\mathbf{t}}_{p_j}} (1-a_{\bar{p}_j})^{n^{\mathbf{t}}_{p_j}},$$

i.e.,

$$\pi_{\bar{p}_j} a_{\bar{p}_j}^{n^{\mathbf{t}}_{\bar{p}_j}} (1-a_{\bar{p}_j})^{n-n^{\mathbf{t}}_{\bar{p}_j}} > \pi_{p_j} a_{p_j}^{n^{\mathbf{t}}_{p_j}} (1-a_{p_j})^{n-n^{\mathbf{t}}_{p_j}}.$$

In summary, for all $r \in d$ (whether of the form p_j or $\bar{p}_j$), we have

$$\pi_r a_r^{n^{\mathbf{t}}_r} (1-a_r)^{n-n^{\mathbf{t}}_r} > \pi_{\bar{r}} a_{\bar{r}}^{n^{\mathbf{t}}_{\bar{r}}} (1-a_{\bar{r}})^{n-n^{\mathbf{t}}_{\bar{r}}}.$$

It follows that for all $d' \in \mathcal{J}_m \backslash \{d\}$ we have

$$\prod_{r \in d} \pi_r a_r^{n^{\mathbf{t}}_r} (1-a_r)^{n-n^{\mathbf{t}}_r} > \prod_{r \in d'} \pi_r a_r^{n^{\mathbf{t}}_r} (1-a_r)^{n-n^{\mathbf{t}}_r},$$

i.e., $E(u(d,S)|\mathbf{t}) > E(u(d',S)|\mathbf{t})$ by Lemma 6. This proves (*).

B. Now suppose informative voting is efficient under f. Consider any voting profile $\mathbf{v} \in \mathcal{J}_m^n$ and issue $j \in \{1,\ldots,m\}$; we must show that (**) $p_j \in f(\mathbf{v})$ if and only if $n^{\mathbf{v}}_{p_j} \geqslant k_{p_j}$. Since informative voting is efficient, $d := f(\mathbf{v})$ is efficient for the type profile $\mathbf{t} := \mathbf{v}$. In fact, d is the only efficient decision for $\mathbf{t}$ by our 'no ties' assumption. So, any other decision $d' \in \mathcal{J}_m \backslash \{d\}$ yields a lower expected utility given $\mathbf{t}$ than d, i.e., $E(u(d,S)|\mathbf{t}) > E(u(d',S)|\mathbf{t})$, which by Lemma 6 implies that

$$\prod_{r \in d} \pi_r a_r^{n^{\mathbf{t}}_r} (1-a_r)^{n-n^{\mathbf{t}}_r} > \prod_{r \in d'} \pi_r a_r^{n^{\mathbf{t}}_r} (1-a_r)^{n-n^{\mathbf{t}}_r}.$$

If we apply this inequality to the case that d' differs from d only on the jth issue (i.e., $d \triangle d' = \{p_j, \bar{p}_j\}$), then after cancellation the inequality simplifies to

$$\pi_{p_j} a_{p_j}^{n^{\mathbf{t}}_{p_j}} (1-a_{p_j})^{n-n^{\mathbf{t}}_{p_j}} > (<) \pi_{\bar{p}_j} a_{\bar{p}_j}^{n^{\mathbf{t}}_{\bar{p}_j}} (1-a_{\bar{p}_j})^{n-n^{\mathbf{t}}_{\bar{p}_j}} \quad \text{if } p_j \in (\notin) d.$$

After replacing $n^{\mathbf{t}}_{\bar{p}_j}$ by $n - n^{\mathbf{t}}_{p_j}$ on the right hand side, it follows that

$$\pi_{p_j} a_{p_j}^{n^{\mathbf{t}}_{p_j}} (1-a_{p_j})^{n-n^{\mathbf{t}}_{p_j}} > \pi_{\bar{p}_j} a_{\bar{p}_j}^{n-n^{\mathbf{t}}_{p_j}} (1-a_{\bar{p}_j})^{n^{\mathbf{t}}_{p_j}} \Leftrightarrow p_j \in d.$$

This entails (**), since by (11) the left hand side of this equivalence holds if and only if $n^{\mathbf{t}}_{p_j} \geqslant k_{p_j}$, where $\mathbf{t} = \mathbf{v}$. □

Proof of Theorem 10. A voter's probability of a state $s \in \mathcal{J}_m$ given his type $t \in \mathcal{J}_m$ is given by the following equation:

$$\Pr(s|t) = \prod_{r \in t \cap s} \frac{\pi_r a_r}{\pi_r a_r + \pi_{\bar{r}}(1-a_{\bar{r}})} \times \prod_{r \in t \backslash s} \frac{\pi_r (1-a_r)}{\pi_r (1-a_r) + \pi_{\bar{r}} a_{\bar{r}}} \tag{30}$$

Based on this expression, it is straightforward to generalize our earlier proof for the two-issue case (Theorem 6) to the current m-issue case. □

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