The relationship between mathematics teachers' content and pedagogical knowledge and their handling of student contributions: the case of Saudi trainee primary teachers

By

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Abstract

This research aimed to investigate how Saudi mathematics trainee primary teachers respond to their students' contributions and the reasons for their actions. Also, to outline the possible influence of teachers' knowledge, both subject matter knowledge (SMK) and pedagogical content knowledge (PCK), on their handling of students' contributions.

To achieve these goals five participants were observed for eight lessons each and interviewed. The data from three out of the five participants formed three case studies of teachers with differences in terms of their SMK and PCK. Three lesson observations for each case plus the interviews were analysed. The Knowledge Quartet framework was used to analyse the data in order to highlight the knowledge of the trainees and to direct attention towards the incidents where the teachers responded to the students' answers. The interactions in these incidents were also analysed by the (Initiation-Response-Feedback) IRF tool to classify the teachers' responses in relation to the students' contributions.

The findings suggest that the trainees responded to the students' answers usually with one of two types of response. *Confirmation* actions in which the teachers confirmed the correctness or the fault of a given answer and *questioning* actions where the teachers asked further questions usually before making a decision about the answer. They reasoned their ways of accepting correct answers into two types of reasons: *class-focused* and *individual-focused* actions. Conversely, the teachers' justification of their rejection of incorrect answers fell into three groups: *protection* actions, *checking* actions and *other*. Moreover, the teachers' SMK and PCK have influenced their response to the students' answers. Their response approach was mainly shaped by the teachers' beliefs of how mathematics is best learnt whereas PCK influenced the quality of their responses. Also, they depended mainly on their SMK, and to some extent on their PCK, in deciding to accept or reject answers.

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Dedication

Dedicated to my mum, wife, children and siblings.

Chapter 1 : Introduction

1.1 Introduction

Educational school practice is complex, consisting of many elements that interact toward achieving the educational goals. Some believe that the most important component of the teaching process is the teacher, who plays a vital role both in and out of classrooms. Consequently, numerous studies have been conducted by scholars focusing on the character and knowledge of teachers, and this study is not an exception. For instance, Bray (2011), Wilkins (2008) and Hill *et al.* (2005) have investigated the influence of mathematics teachers' knowledge on error handling practices, practice in general and student achievement respectively. Other researchers, such as Cobb and McClain (2006), studied the consequence of teacher performance during mathematical discourse among students and between students and their teachers. However, such linking of teachers' knowledge and mathematical discourse during class interaction still merits deeper analysis.

This chapter offers a brief background of this research in section 1.2 then the research questions and methodology in section 1.3. That is followed by the rational of this study in section 1.4 and the research contribution in section 1.5. Towards the end of this chapter the thesis structure is given in section 1.6.

1.2 Background of this study

Teacher knowledge has been widely investigated by scholars around the world. The seminal work of Shulman (1986; 1987) was the foundation for most of the research that followed. He categorised teacher knowledge and put more emphasis on subject matter knowledge (SMK) and pedagogical content knowledge (PCK) (Shulman, 1987; 1986). For example, Michigan University group's work on mathematical knowledge for teaching (Ball *et al.*, 2001) elaborated these types of knowledge, SMK and PCK, in more detail in their model. Also other work, which built on Shulman's work, was done by Rowland and colleagues (2005) in Cambridge University. They proposed a model called the Knowledge Quartet (KQ) to identify teacher knowledge in classroom teaching. Also, many other researchers have examined the influence of teacher knowledge on teaching practice in general and in certain aspects of teaching, such as dealing with errors (Son, 2013).

For several years it has been the case that a number of trainee mathematics teachers, who enrol at Qassim University in Saudi Arabia, have faced difficulties in teaching students some mathematical topics. The most important difficulty that has been of concern to me for a long time as a mathematics educator is the manner in which trainee teachers deal with students' contributions in class; they vary a great deal in how they respond to their students' actions. This has posed some questions, such as: What is the variation in their responses? Why are the responses so varied, and, particularly, why do they vary from the recommendations made during their training? What skills do they bring to the lessons for handling student contributions and Are these skills sufficient for making the most of these contributions? To answer such questions, I examine in detail the prospective teachers content and pedagogical knowledge, and the ways they deal with their students' contributions, and also the reasons behind their actions in the classroom. Finally, I attempt to link the student teachers' knowledge with their responses to the pupils' contributions.

1.3 Research questions and methodology

This research aims to investigate the ways in which mathematics trainee teachers in Saudi Arabia deal with their students' contributions and the reasons behind them. Also, it examines the trainees' knowledge (SMK and PCK) and how it might influence their responses to the students' contributions. This research answers the following questions:

1. How do trainee mathematics teachers deal with students' contributions in Saudi primary school classes, and why do they respond in these ways?
2. How does the knowledge that trainee mathematics teachers have gained influence the ways in which they handle their students' contributions in the classroom?

My research focuses on male trainee mathematics teachers in Saudi primary schools in grades four, five and six (10-, 11- and 12-year-olds) in the city of Arrass during the second term of the 2013/2014 school year. Multiple case study design was used and qualitative data was collected through interviews and observations and analysed using the Knowledge Quartet framework (Rowland *et al.*, 2005), the Initiation-Response-Feedback (IRF) model (Sinclair and Coulthard, 1992) and thematic analysis (Braun and Clarke, 2006).

1.4 The motivation for this study

Teaching is not a static practice. Rowland and Zazkis (2013) state that contingent moments in mathematics teaching make the teaching unpredictable and cause it to change all the time. These contingent moments could be taken as opportunities for further learning (*ibid*). This point is one of the reasons that I am interested in conducting this project. I want to examine whether the trainees used these opportunities in fruitful ways while they were responding to the students. Also, the variation among the trainees' responses to their students' contributions led me to think about the reasons behind those differences. As the literature shows that teachers' knowledge could be one of the factors that influence teacher responses, I wondered if that was the case with the participants of this research. I then thought of investigating the influence of the Saudi trainees' knowledge on their handling of student contributions.

1.5 Contribution of this thesis

Many studies discussed in the literature review chapter of this research show that investigating teacher knowledge is an ongoing research trend in different countries but is less common in Arab countries. One of the gaps in the literature which is of interest to this study is the absence of studies on teacher knowledge, at least in the Saudi Arabian context generally and in particular in the area of Saudi teacher knowledge and its relationship to teachers' responses to students' contributions. This is one of the potential contributions of this study to the literature as will be discussed later in the last chapter (section 9.4).

Another possible contribution to the field of teacher knowledge is that this research examines the reasons of the teachers' response from their point of view which is not so frequent in the literature as far as I know. Also, it investigates the teacher response to correct answers as well as incorrect answers as the majority of research focused heavily on the errors and misunderstanding more than positive evaluation. This study might extend the work done in this point of research. Furthermore, it may extend the use of the Knowledge Quartet framework as a tool for analysing the data in other non-English context. Moreover, it extends the research's methodology in Saudi context by adding a qualitative research to the very few (if any) qualitative research in that context. Lastly, the finding of this research can help the policy makers in Qassim University to develop their curricula to avoid the weakness points in their graduate knowledge and practice. It aims to enrich the research trend which investigates the relationship between teacher knowledge and practice.

As this research context may be not so well known in the field of mathematics education, I introduce a brief background about the kingdom of Saudi Arabia and its educational system in the next chapter.

1.6 Outline of the thesis

This thesis is organised into nine chapters. The first chapter is the introductory chapter where I give a brief background of the project and a short account of the rational of this research, the research aims, its methodology, its questions and possible contributions. The second chapter is the research context in which I provide background about Saudi Arabia and its educational system. At the end of this chapter I focus on Arrass city as the study site and Arrass Teacher College where the participants were trained. The literature review is discussed in chapter three where I explore the research on teacher knowledge and classroom discourse. That is followed by the methodology chapter where the research approach and design are discussed and data collection and analysis are described. The following three chapters are the analysis and findings of the three case studies where I analyse the data of each and discuss them. The cross-case analysis takes place in chapter eight then summary of the findings, contributions, research limitation, further research and conclusion are explored in chapter nine.

Chapter 2 : The study Background and Context

2.1 Introduction

This study was conducted in the Saudi context which is not so well known in the field of mathematics education research. I introduce a brief background about the Kingdom of Saudi Arabia and a short account of the city of Arrass as the site of this research in the first part of this chapter. Then towards the end of this chapter, I outline the context of the educational system in Saudi Arabia and the teacher college where the participants were trained.

2.2 Brief profile of the country

Saudi Arabia is an Islamic monarchic country located in the southwest corner of Asia. It is considered as the third largest country in the Middle East as it covers around 2,000,000km^2 (772,204 square miles) and its population is a little over 27 million, including 8.4 million foreign residents (Saudi Central Department Of Statistics and Information, 2010). Saudi Arabia has a strong economy due to being the world's largest producer and exporter of oil (The Royal Embassy of Saudi Arabia, 2010). As shown in Figure 2.1, it is bordered by many Arab countries, namely, Jordan and Iraq to the north and northeast, Kuwait, Qatar and the United Arab Emirates to the east, Oman to the southeast, and Yemen to the south; it is also bounded by the Red Sea and the Arabian Gulf to the west and the east, respectively (*ibid*).

Figure 2.1: Kingdom of Saudi Arabia

Saudi Arabia has 13 administrative provinces as follows: Riyadh, Makkah, Madinah, Qassim, Eastern, Asir, Tabouk, Hail, Northern Border, Jizan, Najran, Baha and Al-Jouf, and each one is divided into several governorates (their numbers differing from one region to another) (*ibid*).

The Qassim Region, where the data collection for this study took place, is located to the north of the middle of the Kingdom, and consists of 10 governorates; Arrass is one of the most important cities there. I give more insight into Arras in the following section.

2.3 Arrass city: the study context

In this part I describe the context of Arrass, where the study is conducted, giving a short historical, geographical, demographical and economic background of the region.

Arrass is one of the most important cities in Qassim County. It is located in the middle of the Najd Hills in the heart of the Kingdom, which means that it serves as a crossroads between many Saudi cities; many people who used to live in nearby villages have moved to Arrass to benefit from its services. It is the third biggest city in the Qassim region, as it covers about 20,656km^2 (Saudi Central Department Of Statistics and Information, 2010).

Arrass has a long history. It is located over a group of fresh-water wells where the ancient Arab nomadic tribes used to draw water while they were travelling around looking for grazing pastures for their animals. Over time, it became home to a succession of Arab tribes, who only lived there for short periods before moving on. In 1010, it began to develop and people planted trees (mostly date palms); later they developed farming there. Its people participated in many domestic wars, which made the city famous, but continual unrest made living there difficult. After the Kingdom was founded, peace befell the city, and so it started to grow rapidly; it continues to grow to this day (Alaqel, 2004).

Economically speaking, the majority of the Arrass residents in the past depended on grazing and agriculture. In contrast, 35% of the population now work as teachers; others are self-employed or governmental employees. Arrass remains famous for its trade in dates and for its extensive palm orchards *(ibid*).

The number of residents in greater Arrass has risen from 94,361 in the 2004 census to 133,482 in the 2010 census, including 92,501 people living in the city centre (*ibid*). It has a variety of tribes that form the social fabric of the city. People live both inside and outside the city centre, and as a result the schools are classed as urban or rural. All must provide education in accordance with the guidelines of the Ministry of Education. As mentioned above, Arrass is considered the third city of Qassim in terms of population, and the educational office there is responsible, with regard to primary schools for boys, for 7,833 pupils, 1,063 teachers and 79 schools, including just 21 schools inside the city centre (Alrass Educational Office, 2010). Although most of the schools are modern, with up-to-date equipment and learning facilities, some are still in rented residential buildings, and suffer from a shortage of learning aids. Thus, there is some variability in the schools in Arrass, in terms of size, location and building type.

2.4 A brief description of the Saudi educational system

The Saudi government pays a great deal of attention to its educational system. The Saudi educational system aims to ensure that students are prepared for life and work in the modern world, while meeting the country's religious, social and economic needs (Saudi Educational Policy, 1970). The importance of the education system's aims has led the Saudi government to spend upwards of 150 SR ($40) billion on the education system and human resource developments, which equalled just over 25% of the national budget for 2011 (Saudi Ministry of Finance, 2011). The decision makers in education have decided to complete the development of the educational system by creating new universities and opening 1,500 new schools across the country (*ibid*).

The educational system in Saudi Arabia is divided into two main parts: general and higher education. The Ministry of Education is responsible for the former and the latter is under the supervision of the Ministry of Higher Education.

Both public and private general education contain three compulsory stages, which are primary, intermediate and secondary, and there is an optional kindergarten level. These schools enrol students of ages 6-11, 12-14, 15-17 and 3.5-5, respectively (Saudi Ministry of Education, 2011b). Public education is divided into two main areas: 'Ordinary' and 'Tahfed' schools where the latter focus heavily on teaching the Quran more than other subjects, whereas the former balance the Quran with other studies. In addition, boys and girls are taught separately by male and female teachers respectively in different schools. The school year at all three levels consists of two semesters, which are fifteen weeks long. Classes each last 45 minutes and there are from 28 to 33 classes per week (*ibid*). In addition, the teachers' workload is usually 24 classes per week and there are no teaching assistants. This study focuses on primary schools, and, to some extent, on teacher preparation in higher education institutions.

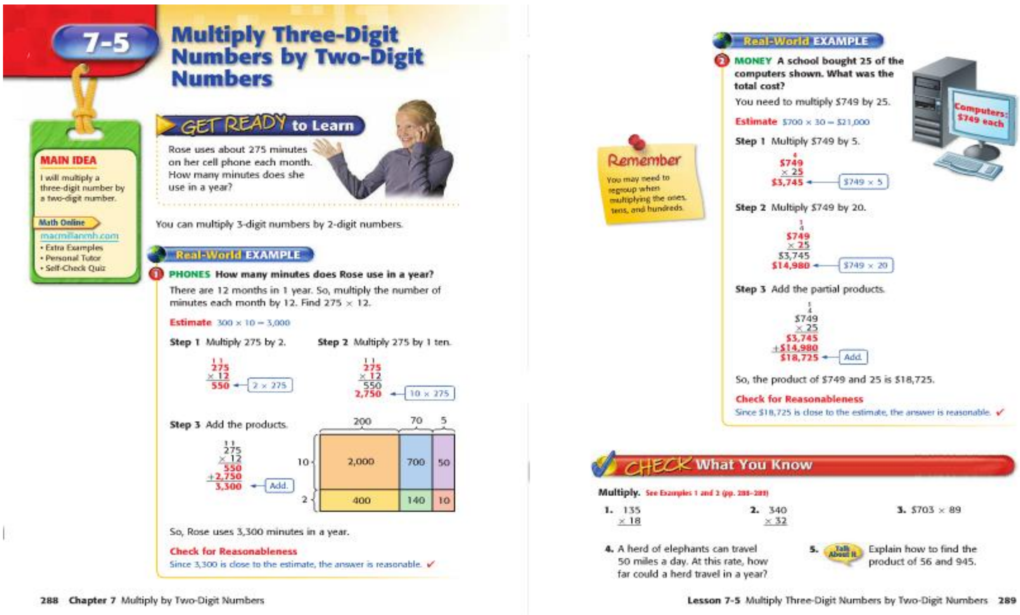
2.5 Saudi primary education

Formal primary education began in the Kingdom in the 1930s as the Directorate of Knowledge was established in 1926. The number of primary schools rose from 226 schools in 1951 to 600 schools in 1960, and then to 1400 schools in 1970. At the beginning of the new millennium, the number of primary schools for boys was 6,148 (Alghamdi and Abdaljwad, 2005). According to the most recent statistics published by the Saudi Ministry of Education, the number of primary schools in 2010 was 6266 for boys and 6086 for girls, and the average number of pupils per class for boys was 19; the ratio of pupils to teachers is 11:1 (Saudi Ministry of Education, 2011a). Teachers in all these schools use a state approved book as shown below.

2.5.1 Saudi primary mathematics textbooks

The primary school curriculum has changed continually, and mathematics is no exception. In order to improve the quality of the education system in the country, a new circular (no. 30324350) dated 07/07/2009 was prepared by the Saudi Ministry of Education, mandating all schools to introduce reformed mathematics and science curricula for years one, four, seven and ten in 2009, and in the following year, for years two, five, eight and eleven. New books for the curricula covered all the general education levels in the 2011 schooling year.

The new primary mathematics curriculum is adapted from the Math Connects series by McGraw-Hill to the Saudi cultural context (see a sample lesson in figure 2.1). It is introduced to the students in five mathematics lessons per week, except in year one which has only three lessons in the general schools, and these numbers decrease by one lesson in the religious schools. It seeks to raise the academic level of lessons and it requires well-trained teachers to be able to use the text book.



7-5 Multiply Three-Digit Numbers by Two-Digit Numbers

MAIN IDEA

I will multiply a three-digit number by a two-digit number.

Math Online
macmillanmh.com
• Extra Examples
• Personal Tutor
• Self-Check Quiz

GET READY to Learn

Rose uses about 275 minutes on her cell phone each month. How many minutes does she use in a year?

You can multiply 3-digit numbers by 2-digit numbers.

Real-World EXAMPLE

1 PHONES How many minutes does Rose use in a year?

There are 12 months in 1 year. So, multiply the number of minutes each month by 12. Find 275 × 12.

Estimate 300 × 10 = 3,000

Step 1 Multiply 275 by 2.

275 × 12 = 550 ← 2 × 275

Step 2 Multiply 275 by 1 ten.

275 × 12: 550, 2,750 ← 10 × 275

Step 3 Add the products.

275 × 12: 550 + 2,750 = 3,300 ← Add.

	200	70	5
10	2,000	700	50
2	400	140	10

So, Rose uses 3,300 minutes in a year.

Check for Reasonableness

Since 3,300 is close to the estimate, the answer is reasonable. ✓

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Real-World EXAMPLE

2 MONEY A school bought 25 of the computers shown. What was the total cost?

You need to multiply $749 by 25.

Estimate $700 × 30 = $21,000

Remember

You may need to regroup when multiplying the ones, tens, and hundreds.

Step 1 Multiply $749 by 5.

$749 × 25: $3,745 ← $749 × 5

Step 2 Multiply $749 by 20.

$749 × 25: $3,745, $14,980 ← $749 × 20

Step 3 Add the partial products.

$749 × 25: $3,745 + $14,980 = $18,725 ← Add.

So, the product of $749 and 25 is $18,725.

Check for Reasonableness

Since $18,725 is close to the estimate, the answer is reasonable. ✓

CHECK What You Know

Multiply. See Examples 1 and 2 (pp. 288–289)

1. 135 × 18
2. 340 × 32
3. $703 × 89
4. A herd of elephants can travel 50 miles a day. At this rate, how far could a herd travel in a year?
5. Talk About It Explain how to find the product of 56 and 945.

Lesson 7-5 Multiply Three-Digit Numbers by Two-Digit Numbers 289

Figure 2.2: lesson 7-5 Multiply three-digit numbers by two-digit numbers

A typical lesson in these textbooks has many sections. It starts with the main idea box which summarises the aim of the lesson which leads in to the first section ‘Get Ready’ where the students are required to do some thinking as a warm up. The ‘Real World Example’ section comes next which gives two worked examples to introduce the idea of the lesson. That is followed by the ‘Check What You Know’ section where the students do some exercises under teacher supervision and then the section ‘Practice and Problem Solving’ where the students solve some exercises and problems from real life either individually or in groups. The lesson ends with the ‘High Thinking Skills’ section where some high level problems challenge the students.

Teachers in Saudi Arabia are requested by the government to use these textbooks as a guide for them to cover all the mathematical topics. They use it in the lessons mostly in a linear order starting from the first section and ending with the last one. Some teachers ask the students to complete a lesson as homework if they could not do it in class. However, although teachers almost always adhere to the books, they omit some parts of it for many reasons which are discussed later in section (5.3.2). Also, some teachers ask the students to solve the questions individually or in groups. In addition, textbooks influence the way in which trainees teach as they consider it the main resource and they follow its instructions of introducing the mathematical concepts and procedures with or without adjustment. Using the textbooks and knowing their content is probably one of the most important roles of the supervision staff in the Saudi universities as some trainees struggle in this matter. This should be taken in account when the trainees enrol in the teacher preparation programmes in the Saudi universities.

2.6 Saudi teacher education

Teacher training and preparation is very important; it is described in detail in this study because I shall investigate teacher knowledge and practice. From 1927, anyone who could read and write was allowed to enrol at the Scientific Institute of Saudi Arabia in Makkah, but in 1954 the Ministry of Education established primary teacher institutes, which accepted anyone who had graduated from primary school. As the numbers of pupils rose, secondary teacher institutes were opened in 1965, to which only intermediate school graduates could apply. As a result of the need to improve the quality of teacher training, thirteen teacher training colleges which awarded a teaching diploma were opened across the country in 1976. These colleges were developed in 1989, extending the years of study to four years, and awarded a bachelor degree in teaching (Alghamdi and Abdaljwad, 2005). Recently, all these institutions have been linked to their nearest universities, and those universities now have the right to restructure the teacher education programmes in the colleges.

2.7 Arrass Teacher College

The college of concern in this study is Arrass Teacher College in the city of Arrass. It was founded in 1979 to prepare teachers for the whole county. Its name was initially The Intermediate Teachers' College, and its graduates were awarded a diploma. However, after twelve years, it was promoted to being allowed to award a bachelor degree, and was renamed Arrass Teacher College. After being linked to Qassim University in 2006, the name was again changed to The Art and Science College, which had ramifications for many of its departments (Qassim University Website, 2011). These departments together build the character of the student teachers by providing them with different subjects.

Recently, the decision makers in the college launched a new department, which is called Primary Education; this consists of staff members from a variety of social science fields, including myself and my colleagues. It is responsible for teaching and training the student teachers before graduating from the college.

Ordinarily, the college tries to balance three kinds of knowledge: general, pedagogical and subject matter knowledge, which are studied by the trainees through the courses provided, and in the final term they practise these three kinds of knowledge in local schools. A member of staff from the college is responsible for checking the progress of the trainees and to see how they are using their newly acquired skills in their teaching by observing their teaching and meeting them several times to discuss feedback.

The two of concern to the study are the departments of Mathematics, and Curricula and Teaching Methods. The former introduces the courses described in Appendix (1) to build the student teachers' mathematical knowledge. The latter, The Curricula and Teaching Methods' department, used to be the pre-eminent department in the college. Its responsibilities are to prepare trainee teachers for work in the country's primary schools and to observe their practice. It usually provides five separate courses as shown in Appendix (2). In this section, I focus heavily on two of these courses, Mathematics Teaching Methodology and Practicum, described in Table 2.1 below:

Course Code	Course Title	Credit Hours	Course Description
431 Method	Mathematics Teaching Methodology	2	The course deals with the effective teaching skills necessary for the mathematics teacher at the primary stage. It also focuses on modern trends in maths teaching (the computer and the Internet), methods and ways peculiar to maths teaching, such as induction, games, small groups, deduction, individual learning, and training students in these by making room for them to practise what they have learned from the course (according to the micro-teaching technique, and the use of modern educational techniques).
449 Method	Practicum	8	This course comes after the trainees have finished both stages of observing and participating while studying both courses: general and specific teaching methodology. The course is aimed at giving trainee teachers some teaching experience before they begin actual teaching, and prepares them psychologically, professionally, administratively and educationally for the teaching profession. This is achieved through providing students with the opportunity to practise real teaching, and making them apply all the knowledge,

			theories and skills under specialist supervision, which guarantees appropriate feedback to help them enhance their academic conduct and encourage them to choose, apply and evaluate what they deem as appropriate teaching methods and educational techniques.

Table 2.1: Two educational courses taught to the trainees

Source: quoted from http://www.tcr.edu.sa/en/Cv/encvcour.ASPx

2.7.1 Mathematics Teaching Methodology

This course aims to prepare the trainees for the Practicum course. They were taught, in this course, various teaching and learning theories such as Cognitive Development Theory by Piaget, Constructivist Theory by Bruner, Theory of Mathematics Learning by Dienes and Behaviourism: Reinforcement by Skinner. These theories were taken from Arabic books such as Almakoshi (2001) and delivered to the trainees through a hand-out given in lectures and workshops. The course contains planning of lessons and exploring, to a lesser extent, the content of primary mathematics textbooks.

2.7.2 Practicum course

As shown in Table 2.1, the practicum course is the final course which the trainees must take before graduating from the college and it is worth eight credits. The aim is to practice what they had learnt under supervision from the college and school. The trainee on this course is supervised by an educational educator from the college who visits him at least twice a month, and a co-operating teacher. The co-operating teacher is a full-time teacher who teaches the subject at the practice school. His role is to give the trainee 8-12 lessons from his 24 lessons weekly teaching load then visit the trainee in the class several times and discuss his teaching. In the first week of term the trainee must register at the Department of Education as a prospective teacher. Then, the list of prospective teachers is checked to make sure that the trainees do not have more than two courses. After this, the trainees are asked to come to the meeting held in the second week of the term. They are met by the head of the department and given some advice about the nature of the training in schools. This is followed by a group meeting in which each supervisor meets his trainee and offers specific advice. After that the trainees are sent to their schools.

When the trainee goes to his school, he meets the head teacher then the co-operating teacher. His teaching load is usually 8-12 classes per week. In the first week, the trainee just attends the co-operating teacher’s classes and discusses the curriculum with him, and they agree on the classes which the trainee will teach. Then the trainee goes to his class alone and starts teaching the textbook and is observed from time to time by his supervisor and the co-operating teacher. Towards the term end he examines the students and takes his report to the college, after which his training is over.

After introducing this short description about the study context, I explore and discuss the literature review on teacher knowledge and classroom discourse in general and in the Saudi context in particular in the next chapter.

Chapter 3 : Review of the Literature

3.1 Introduction

This study focuses heavily on teachers' mathematical content knowledge which includes two kinds of teacher knowledge, subject matter knowledge (SMK) and pedagogical content knowledge (PCK). Also, it investigates the relationship between these two types of knowledge and teaching practice in general, and in particular how teachers deal with pupil contributions in Saudi mathematics classes. Here I define those terms and present the theoretical background of my study as follows. In section 3.2, I give a brief background to the study of teacher knowledge in terms of its history as a field of mathematics education research, its key theories and seminal works in the field that have influenced this study. That is followed in section 3.3 by a short background of classroom discourse which includes teaching exchanges and teachers' moves. I close the chapter with section 3.4 which contains a summary presentation of the conceptual framework and linking seminal works in the field with research in the Saudi context.

3.2 Teacher Knowledge as a field of mathematics education research

Scholars who did research into teacher knowledge almost always describe the forms and the categories of teacher knowledge without mentioning its nature. Barwell (2013, p.597) states that 'knowledge' in Shulman's works can be considered as "something that forms coherent 'bodies', which have a role in teaching". It can also be described, measured, identified and categorized and it is closely connected to reasoning as knowledge forms the bases of reasoning (*ibid*).

In this section I offer a brief overview of the field of teacher knowledge in the following order: historical development of teacher knowledge research, teacher knowledge categories, teacher subject-matter knowledge (SMK), teacher pedagogical content knowledge (PCK), the relationship between SMK and PCK, accounts and investigations of teacher knowledge and then research trends in teacher knowledge.

3.2.1 Historical development of teacher knowledge research

Teacher subject-matter knowledge and pedagogical content knowledge have been the focus of much concern for numerous scholars in recent decades. The growth of this interest began decades ago and has been influenced by various developmental factors. In the following paragraphs I give a brief historical background of the development of research into teacher knowledge.

Teacher knowledge was first examined in California in 1875 in order to license candidates to teach in elementary schools (Shulman, 1986). These tests were heavily focused on the subject-matter that the teacher should know in order to be able to teach but there was no place for testing teaching skills *per se*. According to Shulman (1986, p.5), 'Ninety to ninety-five percent of the test is on the content, the subject matter to be taught'. This reflected the widespread point of view held by the majority of teacher training and preparation courses in that era, which was that the teacher who teaches a certain subject to students must know that subject to a sufficiently high degree in order to be able to introduce it to the pupils. This unbalanced situation between SMK and PCK lasted until the 1980s (*ibid*).

New policies emerged in the USA in the 1980s in order to revise the requirements for licensing teachers. The focus of the new tests was considerably altered, as it then focused on 'The assessment of capacity to teach' (Shulman, 1986, p.5). The test designers were influenced in their shift towards the pedagogical approach by the growing body of research on teaching effectiveness and teacher competency (*ibid*). However, this shift encouraged many other researchers to criticise this move, and Shulman and his colleagues wondered where the subject matter was in these tests. They referred to the lack of focus on subject-matter knowledge in teaching research, calling it 'The missing paradigm'; they emphasized the importance of finding a balance between the content taught and the teaching capacity of the teacher (*ibid*).

In the 1980s and 1990s, there were many movements for increasing government interference in education. These trends, according to Turner-Bisset (1999, p.40), were 'The importance of subject-matter knowledge in teaching, the notion of partnership between schools and HEIs [Higher Education Institutions] and the competency-based movement'. This research development in the education of teachers shifted the focus toward studying teachers' cognitive processes, which in turn led to considering the pedagogical knowledge base upon which the teacher builds his actions in class, and on teacher subject-matter knowledge as an integral part of it (Grossman, 1990).

Research into mathematics teachers' knowledge is developing all the time. The focus of research has moved to 'investigate the nature of the knowledge teachers possess and approaches to support further development of the knowledge they hold or lack' (Chapman, 2013, p.238). The work of Shulman (1986; 1987) can be considered as the theoretical starting point of research into the trends of mathematical knowledge for teaching (Barwell, 2013). A few years after Shulman's (1986; 1987) work, the pioneering research of the Michigan University team (Ball and Bass, 2003; Ball *et al.*, 2001; Ball *et al.*, 2008) emerged and expanded Shulman's ideas of mathematical knowledge for teaching. Many scholars have developed and reformed Shulman's ideas in various directions. For example, some researchers have investigated the influence of teachers' knowledge on practice (Rowland *et al.*, 2000; Fennema and Franke, 1992). Others have examined the relationship between teachers' knowledge and students' achievements such as Hill *et al.* (2005) whereas others searched for the nature of the knowledge needed in teaching such as Ball *et al.* (2008). Some of these studies are fully explored in this chapter (section 3.2.7).

In short, the focus on subject-matter knowledge in the past was followed by shifting the emphasis towards pedagogy in the 1980s, which in turn was criticised by scholars concerned at the absence of subject-matter knowledge. There were gaps in the research on teaching and learning, and to bridge those gaps, the focus of research then shifted from focusing on who the teacher is to what the teacher does (Wilkins, 2008). In these two research approaches, research on teaching characteristics and on teacher knowledge, the latter was actually built upon the former and they are attractive to researchers and policymakers, respectively (Ball *et al.*, 2001).

In the following sections I outline teacher knowledge categorisation proposed by scholars in section 3.2.2. Then in sections 3.2.3 and 3.2.4 I focus on two types of knowledge, SMK and PCK respectively, (which appear in the majority of these categories) by introducing their different definitions given by scholars.

3.2.2 Teacher Knowledge Categories

Teachers' knowledge bases have been of interest to researchers in recent years. However, many scholars have proposed different categories for teacher knowledge (Elbaz, 1983; Leinhardt and Smith, 1985; Shulman, 1986; 1987; Ernest, 1989;

Grossman, 1990; Borko and Putnam, 1996; Turner-Bisset, 1999; Banks *et al.*, 2000; Ball *et al.*, 2008; Hill *et al.*, 2008; Zazkis and Mamolo, 2011; Foster, 2011; Fennema and Franke, 1992; Silberstein and Tamir, 1991; Chick *et al.*, 2006; Tchoshanov, 2010), and some have avoided teacher knowledge categorization (Zazkis and Zazkis, 2011; Watson, 2008).

In this section, I start by giving brief descriptions of the categories proposed by some of the researchers above without explaining all of the components for several reasons. Firstly, it is not relevant to this study to outline all the components of the proposed models as this research focuses on only two kinds of knowledge: subject-matter knowledge (SMK) and pedagogical content knowledge (PCK). Also, I want to avoid misleading the reader by explaining all of them. I intend here to just describe the categories in order to provide a foundation of the following sections where I explain the components of these models but only focussing on SMK and PCK across these models in the following sections 3.2.3 and 3.2.4 respectively. In these sections, I explore how SMK and PCK were defined among these categories to see the similarities and the differences between these definitions and then set this study's definitions.

Various models for teacher knowledge have been proposed by scholars. Elbaz (1983) suggested a model of teacher knowledge that included five categories: 'knowledge of self, knowledge of the milieu of teaching, knowledge of subject matter, knowledge of curriculum development and knowledge of instruction'. In addition, Leinhardt and Smith (1985) advanced two main aspects of teacher knowledge: 'knowledge of lesson structure' and 'subject-matter knowledge'.

A seminal contribution to the field of teacher knowledge is the set of categories that Shulman (1987) introduced. Shulman (1986) focused on the source and the development of teachers' subject matter knowledge (SMK), pedagogical content knowledge (PCK) and curricular knowledge. His initial goal was to probe the significance of SMK in teaching and to identify the knowledge base of teaching. Furthermore, Shulman traced the 'intellectual biography' of four new secondary teachers whose subjects were English, Mathematics, Biology and Social Studies. Interviews and clinical tasks were used to collect the data. From his research, Shulman

(1987) came up with a set of categories of teacher’s knowledge (shown in Figure 3.1) which have influenced research trends in mathematics education.

- General pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter
- Knowledge of learners and their characteristics
- Knowledge of educational contexts, ranging from workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures
- Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds
- Content knowledge
- Curriculum knowledge, with particular grasp of the materials and programs that serve as ‘tools of the trade’ for teachers
- Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding

Figure 3.1: Shulman’s categories of teacher knowledge

Source: (Shulman, 1987, p.9)

Shulman’s work has either been followed by revised or elaborated models of his orignal work. Some researchers expanded his categories by including more components, elaborating others and adding detail by dividing the model’s components into sub-headings. For instance, Ernest (1989) suggested that teacher knowledge can be divided into ‘Teacher's practical knowledge of the teaching of mathematics (both pedagogical and curricular knowledge)’, ‘knowledge of classroom organisation’, and ‘knowledge of the school context’. Grossman (1990) introduced a model of teacher knowledge containing four kinds of knowledge: ‘subject matter knowledge’, ‘general pedagogical knowledge’, ‘pedagogical content knowledge’ and ‘knowledge of content’. Also, Silberstein and Tamir (1991) suggested a model of ‘subject matter knowledge’, ‘general pedogogical knowledge’ and ‘content-specific pedagogical knowledge’. In addition, Borko and Putnam (1996) excluded the ‘knowledge of content’ from the Grossman’s model, proposing three types: ‘subject matter knowledge’, ‘general pedagogical knowledge’ and ‘pedagogical content knowledge’. Turner-Bisset (1999) elaborated on Shulman’s work and added ‘knowledge of self’. A model for English teachers’ professional knowledge was developed by Banks and his colleagues (2000), which contained ‘pedagogical knowledge’, ‘school knowledge’, ‘personal subject construct’ and ‘subject knowledge’. Later, Chick and her colleagues (2006) developed a model including categories that are ‘clearly PCK’, ‘content knowledge in a pedagogical context’, and ‘pedagogical knowledge in a content context’.

However, Shulman’s works were criticised by subsequent researchers. One criticism was that the concept of pedagogical content knowledge lacks ‘adequate definition, and empirical testing’ (Ball *et al.*, 2008, p.389) ,which I believe is a strong statement as Shulman deduced his categories theoretically from some extracts of field observations. Therefore, Ball and colleagues (2008) aimed to develop ‘a practice-based theory’ of teacher content knowledge for teaching by building on Shulman’s (1986) SMK and PCK. They proposed a model for the Mathematical Knowledge for Teaching (see Figure 3.2), which is built on Shulman’s work (Ball *et al.*, 2008; Hill *et al.*, 2008). They divided SMK into common content knoweldge, specialized content knoweldge and horizon content knowledge, while they divded the PCK into knowledge of content and students, knowledge of content and curriculum and knowledge of content and teaching (Ball *et al.*, 2008).

<table>
<tr><th colspan="2">Subject matter knowledge</th><th colspan="2">Pedagogical content knowledge</th></tr>
<tr><td>Common content knowledge (CCK)</td><td rowspan="2">Specialized content knowledge (SCK)</td><td>Knowledge of content and students (KCS)</td><td rowspan="2">Knowledge of content and curriculum</td></tr>
<tr><td>Horizon content knowledge</td><td>Knowledge of content and teaching (KCT)</td></tr>
</table>

Figure 3.2: Domains of Mathematical Knowledge for Teaching

Source: (Ball *et al.*, 2008, p.403)

One critique of Ball’s model is the absence of the mathematical horizon’ explanation (Zazkis and Mamolo, 2011). This explanation came later in (Ball and Bass, 2009). As a result, Zazkis and Mamolo (2011) extended the scope of Horizon content knowledge in Ball’s knowledge model. They considered what the differences between horizon knowledge and the knowledge of curriculum are and the focus of their work was on ‘teachers’ horizon knowledge’. In the same journal, Foster (2011) proposed another kind of knowledge that teachers possess what he refers to as 'Peripheral Mathematical Knowledge'.

Many different models of teachers' knowledge have been developed by scholars. Fennema and Franke (1992) have proposed a model for teacher knowledge which includes 'knowledge of the content, knowledge of pedagogy, knowledge of students' cognition and teachers' beliefs'. Rowland and Turner (2008) have proposed a conceptual framework (The Knowledge Quartet) in their research on mathematics teacher education that consists of four dimensions: foundation which contains substantive subject-matter knowledge and syntactic subject-matter knowledge and transformation which deals with pedagogical content knowledge, connection and contingency. Also, Park and Oliver (2008) proposed a model of PCK that includes 'orientations to science teaching', 'knowledge of students' understanding in science', 'knowledge of science curriculum', 'knowledge of instructional strategies and representations for teaching science' and 'knowledge of assessments of science learning'. Most recently, Tchoshanov (2010) has used a framework of teacher knowledge categories which include 'general pedagogical knowledge', 'pedagogical content knowledge', 'epistemological knowledge', 'other categories of teacher knowledge' and 'content knowledge'. The content knowledge is subdivided into 'cognitive types of teacher content knowledge' which has three subdomains, 'knowledge of facts and procedures', knowledge of concept and connection' and 'knowledge of models and generalization'. A synthesis of models on teacher mathematical knowledge has been proposed by Petrou and Goulding (2011) which consists of intertwined different types of knowledge in the teaching context: 'curriculum knowledge', 'subject matter knowledge' and 'pedagogical content knowledge'.

Although the categorization approach to teacher knowledge has been widely adopted and many researchers believe that teachers' knowledge can be organized in clear categories (Ball *et al.*, 2008; Shulman, 1986), Watson (2008) does not advocate typifying teacher knowledge; she believes that classifying this knowledge is unhelpful for both teacher educators and novices, as this '… can veil the essential mathematical activity in which different kinds of knowledge relate and inform each other' (Zazkis and

Zazkis, 2011, p.249). In addition, Barwell (2013) has critiqued knowledge being categorized as it is interacting all the time and it is hard to distinguish it in its context.

From all these various models of teacher knowledge, I have noted that the majority of researchers have mentioned two key types, which are subject-matter knowledge (SMK) and pedagogical content knowledge (PCK). These two types of knowledge are the focus of this study and are broadly introduced in the following sections.

3.2.3 Teacher Subject-matter Knowledge

Subject-matter knowledge (SMK) or content knowledge, or as some call it, disciplinary knowledge, has been defined and discussed by many scholars since the mid-1980s. For instance, Leinhardt and Smith (1985, p.2) determined that it includes: 'conceptual understanding, the particular algorithmic operations, the connection between different algorithmic procedures, … the number system being drawn upon, understanding of classes of students, errors and curriculum presentation'. Shulman (1986, p.9) has defined SMK as: 'the amount and organization of knowledge *per se* in the mind of the teacher'. Grossman and Richert (1988, p.54) expanded Shulman's discription and included:

> knowledge of the content of one's subject area, including major concepts of the field and the relationships among concepts. In addition to content knowledge, SMK encompasses an understanding of the various ways a discipline can be organized or understood, as well as the knowledge of the ways by which a discipline evaluates and accepts new knowledge.

Similar to the work of Shulman and Grossman, Borko and Putnam (1996) suggested that SMK refers to the relationships between the discipline's facts, concepts and procedures, which may not be unique to teaching. In addition, Fennema and Franke's (1992, p.162) model includes knowledge of mathematics which includes 'teachers' knowledge of the concepts, procedures, and problem solving processes'. It also includes knowing the releationship between concepts and procedures and how to use it in problem solving tasks (*ibid).*

Bromme (1994, p.74) also, defined the SMK of mathematics as, 'what the teacher learns during his or her studies, and it contains, among other things, mathematical propositions, rules, mathematical modes of thinking, and methods'. In turn, Turner-Bisset (1999) divided subject matter into three parts: syntactic knowledge, substantive knowledge and beliefs about the subject. Substantive knowledge refers to the facts and concepts of a discipline and to the framework that orgnanises them, whereas syntactical knowledge is the understanding of the creation of that knowledge. In addition, Krauss *et al* (2008, p.876) has described content knowledge as a 'teacher's understanding of the structures of his or her domain'.

Ball and her colleagues (2008) sub-divided SMK into three domains: 'common content knowledge, specialized content knowledge and horizon content knowledge'. By the first term, they mean 'the mathematical knowledge and skill used in settings other than teaching' (Ball *et al.*, 2008, p.399) such as noticing the incorrect answers of students. In addition, they defined the second domain, specialized content knowledge, as 'the mathematical knowledge and skill unique to teaching'(*ibid*, p:400), for instance, the ability to explain why the student's answer is wrong. The last domain is the understanding of the connection between the subsequent and the previous mathematical topics in the curriculum (Ball *et al.*, 2008). Rowland and Turner (2008, p.92) suggested in their proposed teacher knowledge categorization that the domain of subject matter knowledge can be divided into substantive knowledge and syntactic knowledge in which the former includes knowing 'the key facts, concepts, principles, structures and explanatory frameworks in a discipline' and the latter 'concerns the rules of evidence and warrants of truth within that discipline, the nature of enquiry in the field, and how new knowledge is introduced and accepted in that community'.

In addition, Tchoshanov (2010) stated that teacher content knowledge can be divided cognitively into three kinds of knowledge which are: knowledge of facts and procedures that deal with 'memorization of facts, definitions, formulas, properties, and rules, performing procedures and computations; making observations, conducting measurements, and solving routine problems'. Knowledge of concepts and connections includes 'understanding concepts, making connections, selecting and using multiple representations, transferring knowledge to a new situation, and solving nonroutine problems' and knowledge of models and generalizations which requires 'teachers' knowledge and thinking for generalization of mathematical statements, designing mathematical models, making and testing conjectures, and proving theorems' (*ibid*, p. 148).

The definitions above have some aspects in common: they share the importance of knowing the domain's facts, concepts, principles and procedures and how they fit into an organizational framework. In other words, they specify the knowledge and skills about which the teacher should be fully cognisant, as well as the structures of the domain, in order to be able to teach effectively. In this thesis, I shall define SMK as knowing the content of the subject (mathematics) in terms of the domain's facts, concepts, principles and procedures and the relation between them. These two aspects, understanding the subject as well as how its teaching is organized, are consistent with the three features required to develop a teacher's capacity, as described by Bransford *et al.* (2000): deep understanding of the the subject, full awareness of the ideas in the actual context, and organizing the knowledge into forms that help the student to apply what was learned in new situations. Thus, researchers are seeking out the best ways to help teachers perform their teaching duties by identifing the knowledge they need, understanding how the field is organized, and clarifying how new ideas are added to existing ones and how they can be used in different situations.

However, although scholars share many of the elements of teacher SMK, there are some differences in the components that they have proposed. For instance, some researchers such as Grossman (1990) and Turner-Bisset (1999) sub-divided this knowledge into syntactic and substantive knowledge, whereas others, such as Ball, worked on common and specialized SMK. In fact, Ball and her team's work focuses on the kinds of knowledge needed in the teaching proccess, and they gave specific examples, such as knowing 'why, when you multiply by 10, you 'add a zero'' (Ball *et al.*, 2008, p.401), while some others, who have written about syntactic and substantive knowlegde, have given fewer practical examples, focusing on the structures of the field.

School mathematical knowledge and disciplinary knowledge pertaining to mathematics, i.e. academic mathematical knowledge, are substantively different. The former refers to 'the kind of mathematics needed to teach specific topics at school' (Moreira and David, 2008, p.28), the latter refers to 'what the teacher learns during his or her studies and contains, among other things, mathematical propositions, rules, mathematical modes of thinking and methods' (Bromme, 1994, p.74). It is 'a scientific body of knowledge as produced and organized by professional mathematicians' (Moreira and David, 2008, p.24). These two kinds of knowledge are expected to be consistent, however, Moreira and David (2008) state that they sometimes conflict. For example, academic mathematical knowledge considers creating the real number system as adding 'anything' to the current number system without bearing in mind some pedegogical issues such as how the teachers can deliver it this way to their students, how the teacher can explain adding 'anything' to the current number system and why indeed they want to expand it. Academic mathematical knowledge deals with a high level of abstract forms introduced, to some extent, in schools, which need to be transformed into a more concrete form in order to be accessible to students by using a different type of knowledge, namely, pedagogical content knowledge (PCK).

In short, SMK or disciplinary knowledge, in this study, is the kind of information that pertains to the subject in terms of its facts, concepts, procedures, principles and structures, and the relationships between them. It is the knowledge that teachers need to know with regard to each school subject, not just understanding what and how to do something but also knowing why is it so.

3.2.4 Teacher Pedagogical Content Knowledge

Pedagogical content knowledge (PCK) is arguably the most influential of the three content-related categories, subject matter knowledge, pedagogical content knowledge and curricular knowledge, proposed by Shulman (1986). However, it is very hard to define or characterize PCK (Rowland *et al.*, 2005) and there is no universal agreement about its definition (Van Driel *et al.*, 1998). It has been defined in various ways. For instance Shulman's (1986, p.9) definition of PCK is:

> the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations — in a word, the most useful ways of representing and formulating the subject that make it comprehensible to others … Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the perspectives and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons

In another paper, Shulman (1987, p.8) indicated that PCK is a 'special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding' and it is the 'capacity of a teacher (*not the other content specialist*) to transform the content knowledge he or she possess into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by students' (*ibid*: p. 15, emphasis added). In the same topic, Grossman and Richert (1988) have divided PCK into four components: 'Conceptions of Subject Matter for Teaching', 'Knowledge of Student Understanding', 'Knowledge of Content to be Taught', 'Knowledge of Curriculum Materials'. It includes respectively, 'an awareness of the ways of conceptualizing a subject matter for teaching', 'students' conceptions and misconceptions of particular subject matter', in-depth knowledge of the subject matter and 'knowledge of the materials and resources available for teaching particular subject matter' (*ibid*, p.54).

Later, Grossman in her book *The Making of a Teacher* proposed a model that contains PCK. She divided PCK into similar categories as her previous one with very few changes; it contains: conceptions of purposes for teaching subject matter knowledge (SMK), knowledge of students' understanding, curricular knowledge and knowledge of instructional strategies. These components include knowledge pertaining to the aims of the subject, students' prior knowledge, an awareness of the horizontal and vertical curricula for the subject and deep understanding of representations for teaching particular topics (Grossman, 1990). Borko and Putnam (1996) emphasised that it is a set of SMK used in teaching. Furthermore, Van Driel and others (1998, p.673) have stated that PCK is 'teachers' interpretations and transformations of subject-matter knowledge in the context of facilitating student learning. …[and] encompasses understanding of common learning difficulties and preconceptions of students'. In addition, PCK is considered as a set containing all the following kinds of knowledge sets: substantive knowledge, syntactic knowledge, beliefs about the subject, curriculum knowledge, knowledge of contexts, knowledge of self, knowledge/models of teaching, knowledge of learners - cognitive, knowledge of learners - empirical, knowledge of educational ends and general pedagogical knowledge (Turner-Bisset, 1999).

Niess (2005) considered PCK as a combination of SMK and knowledge for teaching and learning. Similarly, Vail Lowery (2002, p.69) defines it as, 'that domain of teachers' knowledge that combines subject matter knowledge and knowledge of pedagogy'. Moreover, Ball *et al*. (2008) have divided Shulman's pedagogical content knowledge into knowledge of content and students, and knowledge of content and teaching. The former includes a combination of knowledge about mathematics and students, and the interaction between them. For instance, this kind of knowledge contains understanding students' thinking and familiarity with mathematical topics. The latter refers to the interactions between teaching knowledge and mathematical knowledge, such as the ability of selecting appropriate examples to explain some mathematical ideas (*ibid*).

These various definitions all contain the same general idea of combining content with pedagogy. The majority of them can be understood, to some extent, as elaborations on Shulman's work. For example, Grossman and Richert's domain of 'Knowledge of Student Understanding' and Ball's domain of 'Knowledge of Content and Teaching' coincide with Shulman's dimension of PCK regarding students' prior conceptions and misconceptions (Ball *et al.*, 2008). In addition, the other dimension proposed by Shulman, which is the importance of the representations of the content for teaching, can be linked to Grossman and Richert's domain of materials and resources, Grossman's proposal of instructional strategies and the required deep understanding of representations for teaching particular topics and Ball's domain of 'Knowledge of Content and Teaching' (*ibid*).

These two components of PCK proposed by Shulman's work (1986), the knowledge of students' prior conceptions and misconceptions and the importance of the representations of the content for teaching, were agreed by almost all scholars (Van Driel *et al.*, 1998; Park and Oliver, 2008). Teachers' ability to know the students' conceptions and misconceptions helps them to understand their students' actions and to support their teaching (Halim and Meerah, 2002). Teachers can use useful kinds of representations to introduce new topics of the subject that fit their students' prior knowledge and conceptions. The word 'Representations' generally refers to 'external' representation that the teacher uses during teaching. (Barwell, 2013) includes illustrations, examples, models, analogies, explanations, and demonstrations which the teacher uses to ‘transform’ his knowledge into small pieces of information that are easy to be understood by the students. McEwan and Bull (1991) stated that the notion of PCK is embedded in the ‘transmissive’ form of teaching which can be considered as the key words in the majority of the definitions stated above which were ‘transforming’, ‘representing’, ‘formulating’ or ‘psychologizing’ that refers to breaking down the teachers’ subject-matter knowledge gained in the university or experienced into forms that can be acquired by the students. Shulman (1986, p.8) questioned, regarding teachers transforming their knowledge in teaching, ‘How does the successful college student transform his or her expertise in the subject matter into a form that high school students can comprehend?’. The important point here is the necessity of possessing both pedagogy and content by the teacher to teach effectively since he/she must know his/her students’ prior knowledge and some content to choose which presentation he/she will follow to break down the content to make it accessible to the students. This relation between content and pedagogy is explained in the following section.

Others concur with Shulman’s notion of combining the subject with teaching. For instance, Borko and Putnam (1996), Niess (2005) and Vail Lowery (2002) have all emphasised that PCK is a set of SMK used in teaching, which is ultimately the same as Shulman’s (1986, p.8) consideration of PCK as a ‘special amalgam of content and pedagogy’.

In short, this study adapts the definition of PCK stated by Rowland *et al* (2009) as the teachers’ ability to transform their knowledge into forms that students can grasp and understand by breaking down the mathematical ideas into small parts and by using appropriate representations and sources in order to help the students understand the taught topics. That process requires the teacher to be fully rehearsed in the subject, the students’ characteristics and the curriculum as well as how these interact.

3.2.5 Relationship between SMK and PCK

The components of teacher knowledge are hard to distinguish as they frequently interact throughout the teaching process. At this juncture, I examine the relationship between SMK and PCK.

As previously mentioned, Shulman (1987) has explicitly distinguished between SMK and PCK in his classification of a teacher's knowledge base, whereas others, such as McEwan and Bull (1991), Marks (1990), Turner-Bisset and Bennett (1993), Watson and Mason (2007) and Davis and Simmt (2006), disagree with this distinction between the content and the pedagogy. McEwan and Bull (1991, p.318) stress that Shulman's separation of these types of knowledge is unjustifiable, as they believe that ideas and 'all knowledge is, in varying ways, pedagogic' and that there is no epistemological difference between them. This trend is followed by Turner-Bisset and Bennett (1993), as they share the same idea that in the process of actual teaching, all knowledge is presented in a pedagogical way and hence, it is difficult to distinguish between them. Marks (1990) further stresses that PCK as proposed by Shulman, synthesises pedagogy and content and includes content, pedagogy and students. Davis and Simmt (2006) emphasise that the separation of content and pedagogy is unnecessary, and in its place, they propose their branch of knowledge, namely, 'mathematics for teaching'.

PCK as defined above is a broad term. It consists of various types of knowledge, such as content, pedagogy, curriculum and students. Ball and her colleagues (2008) state that Shulman's distinction between PCK and SMK is not clear and it is extremely difficult to distinguish PCK from other forms of teacher knowledge. In addition, Turner-Bisset (1999) suggested that PCK is a combination of several types of knowledge, including SMK. Drawing again on Shulman's work, PCK is a tool for transforming the SMK of a particular discipline into school subject knowledge accessible to students. It includes providing examples, representations and explanations of the elements of SMK to students whereby it can be grasped by those learning, as SMK represents the building blocks of PCK. For instance, in a situation, such as spotting an error in a student's answer, different teachers tend to rely on different kinds of knowledge. As outlined by Ball and others (2008), one can use an aspect of SMK by investigating the source of this error, whereas another can rely on a component of PCK by being already familiar with the student's frequent errors in this particular topic (*ibid*). This demonstrates an understanding of the complexities of teaching, where the teacher's types of knowledge are intertwined and it is impossible to isolate these types of knowledge from each other and from others factors involved in teaching. Moreover, the depth of SMK is one of the most important factors that influence the choice of representations used by the teacher for teaching as an aspect of their PCK (Petrou and Goulding, 2011).

PCK and SMK are complementary in the teaching process. They interact together with other kinds of knowledge and aspects of teaching. The SMK specialist is unlikely to be able to teach effectively without having some pedagogical background, and the pedagogue is unlikely to teach effectively without sufficient knowledge of the subject as Turnuklu and Yesildere's (2007) study shows that having in-depth mathematical knowledge is not sufficient to be able to teach and it is impossible to teach effectively without an acceptable level of subject matter knowledge. Grossman and Richert (1988) state that teachers rely on both general pedagogical knowledge and specific SMK in teaching but also on PCK. Shulman (1986) in his work concerning content knowledge seeks to balance the focus of teacher education on both content and pedagogy. Ball and her colleagues (2008) agreed with Grossman and Richert about the central role of content knowledge and pedagogical content knowledge in terms of the knowledge needed for teaching. In addition, Krauss *et al*.(2008) found that a solid subject matter background is required for successful PCK. Furthermore, Turner-Bisset (1999, p.52) stressed the dangers of focusing only on the teachers' skills and capacities or his/her beliefs, which is the ignorance of 'complex reasoning, thinking and synthesis that underpin the best teaching'.

To summarize, a teacher depends on both content and pedagogy for effective teaching. It is not easy to distinguish between these types of knowledge in practice as the classroom situation also contains other factors. PCK also consists and derives from other types of knowledge, and to be a teacher, it is essential to have both a deep understanding of the subject matter and pedagogic knowledge as well as awareness of the classroom situation.

After giving a brief examination of the terms on knowledge used in this research SMK and PCK in the previous sections, the ways of assessing this knowledge is outlined in the following section.

3.2.6 Accounts and investigations of teacher knowledge

Scholars have described, assessed and explored certain types of teacher knowledge in various ways. Some have used prepared tests while others have described these by using a range of tasks. In addition to these instruments, surveys have been used to describe, assess and explore certain types of knowledge.

Content knowledge can be explored by various methods. In the past, and until today, tests are the main instruments used to explore content knowledge. Shulman (1986) recorded that verbal examinations had taken place as early as 1875 for assessing teacher knowledge and final oral exams for doctorates, used for assessing the candidate's subject matter were introduced in 1958 in the USA. In addition, Krauss *et al*. (2008) introduced the Cognitive Activation in the Classroom Project (COACTIV) instrument to test the subject knowledge of secondary mathematics teachers in Germany. In addition, Kahan, Cooper and Bethea (2003) relied on a test that contained a number of self-assessment tasks, together with some from the literature in order to assess teachers' mathematical knowledge. Other researchers have used a 'paper-and-pencil' content assessment to explore content knowledge, which includes many items requiring direct answers or explanation (Philipp *et al.*, 2007).

This type of knowledge, SMK, can also be explored by using a specialist questionnaire or survey designed by professional teams, such as the 'large-scale survey' designed by Ball *et al.*, at Michigan University aimed at assessing teachers' subject knowledge in the USA, which includes various types of multiple choice questions, focusing on both common and specialist mathematical knowledge (Ball *et al.*, 2005; Hill *et al.*, 2008). Wilkins (2008) used a survey that contained a number of items developed by the researcher and from the literature to explore teacher content knowledge. Similarly, Tchoshanov (2010) used the Teacher Content Knowledge Survey (TCKS) instrument to explore what kind of SMK mathematics teachers in middle grades have. Additionally, Schmidt *et al*. (2011) explored some aspects of teacher knowledge by surveying the pedagogical and content knowledge of mathematics teachers in eight countries. In the pioneering work of members of Michigan State University on the Teacher Education and Development Study in Mathematics (TEDS-M) project questionnaires were used to assess mathematics content knowledge and mathematics pedagogical content knowledge of prospective teachers at the end of their training (Tatto *et al.*, 2008).

Some researchers have applied two instruments to explore teachers' mathematical content, which were a series of performance tasks and a survey (Heritage and Vendlinski, 2006). Also Copur-Gencturk and Lubienski (2013) assessed specialized and common content knowledge for primary mathematics teachers by using two instruments: the Learning Mathematics for Teaching (LMT) and the Diagnostic Teacher Assessments in Mathematics and Science (DTAMS). More recently, Steele (2013) assessed teachers' common content knowledge and specialized content knowledge length, perimeter and area by using specifically designed tasks.

However, scholars such as Bray (2011), Biza *et al* (2007), Empson and Junk (2004) and Nardi *et al*.(2012), have used interviews containing a range of specific tasks (scenarios), in which the teacher was requested to respond to certain situations or to students' answers in order to assess the teacher's SMK. However, this method may also be used to quantify PCK. Others such as Tirosh (2000) have used a questionnaire to assess these two types of knowledge.

Pedagogical content knowledge can be explored through interviews and a range of tasks (Bray, 2011; Empson and Junk, 2004; Biza *et al.*, 2007), also by questionnaires that consist of some aspects of PCK (Chick *et al.*, 2006). The tasks are hypothetical scenarios containing some well-designed teaching situations that require action on the part of the teacher. Although these tasks can quantify the theoretical aspect of teacher knowledge outside the classroom, the practical aspects of PCK can be identified and quantified properly by observing the actual teaching in the classroom (Rowland *et al.*, 2009). This can be followed by an interview where any specific episodes can be discussed with the teacher in order to identify the reasons for his actions (Bray, 2011).

These are the prior works that have shaped the theoretical background of my study. Next, I focus specifically on those studies that have been key to the formation of the conceptual framework of this study by giving a brief description of some research trends in the field of teacher knowledge.

3.2.7 Research on the relationship between teacher knowledge and teaching practice

Teachers' knowledge is of concern for many scholars around the world. It has been researched for various purposes, such as generating categories for teachers' knowledge (e.g. (Shulman, 1987), and focusing on the teacher's level of mathematical knowledge for teaching (MKT) (e.g. (Hill *et al.*, 2008). Other researchers have investigated the influence of teacher knowledge on practice (e.g. (Wilkins, 2008), their characteristics (e.g. (Hill, 2010) and student achievement (e.g. (Campbell *et al.*, 2014). Also, some have investigated the influence of various courses on teacher knowledge (e.g. (Copur-Gencturk and Lubienski, 2013) and some have combined more than one purpose. In addition, these studies were conducted in different countries, with students of different ages and at different times. Moreover, a range of instruments were used to collect the data and various methodologies were followed by scholars, which have all helped in the design of this study. In this section, I focus on the relationship between teacher knowledge and teaching as it is more relevant to my project.

Research on the relationship between teacher knowledge and teaching practice is one of the biggest research trends in teacher knowledge research. It has been carried out by different researchers in different contexts. For example, researchers have investigated this relationship in primary schools with in-service teachers (Chick *et al.*, 2006), with pre-service teachers (Rowland *et al.*, 2000), and in secondary schools with in-service teachers (Baumert *et al.*, 2010) and with pre-service teachers (Kahan *et al.*, 2003). The relationship has also been investigated quantitatively (Wilkins, 2008), qualitatively (Rowland *et al.*, 2005) and using mixed methods (Tchoshanov, 2010). Some researchers have used tasks, interviews, surveys, observations, tests or a combination of two or more tools to collect the data. They were diverse in their focus varying from just focusing on one type of knowledge to different types of knowledge with beliefs and attitude.

Many researchers in this area have investigated the influence of teacher knowledge on different aspects of practice. For example, they have researched the relationship between teacher knowledge and assessing students' thinking (Turnuklu and Yesildere, 2007), handling of students' errors (Bray, 2011), use of inquiry-based instruction (Wilkins, 2008), effective mathematics teaching (Kahan *et al.*, 2003), teaching competence (Rowland *et al.*, 2000), high quality instruction (Baumert *et al.*, 2010), lesson quality (Tchoshanov, 2010), instruction of representing ideas (Steele, 2013), dealing with difficulties of students' standard subtraction algorithm (Chick *et al.*, 2006), awareness of students' misconceptions (Halim and Meerah, 2002) and teaching in general (Rowland *et al.*, 2005). These studies are explained in more detail in the next paragraphs.

However, although these studies varied in their context, aims and methodologies, most of them share the findings that teacher knowledge does influence the practice to a differing extent. For example, Bray (2011) investigated the influence of the beliefs and knowledge of four primary mathematics in-service teachers on their handling of students' errors in the classrooms. She used observations, interviews, including certain tasks, and the Integrating Mathematics and Pedagogy (IMAP) beliefs survey for gathering her data. Analysing the data by using the constant comparative method, she found that both teachers' knowledge and beliefs influence the manner and method of dealing with students' mathematical errors. More specifically, the former facilitates the quality of the response while the latter shapes the class discussions. This finding was shared by Wilkins (2008) who investigated, in a quantitative way, the relationship between 481 primary teachers' content knowledge, beliefs and attitudes, and the effects on teaching practice. He used a model adapted from other researchers to explain this relationship. He assessed the teachers' mathematical content knowledge by using a survey built upon items from the 'Third International Mathematics and Science Study' (TIMSS), the 'Longitudinal Study of American Youth' (LSAY), the 'Second International Mathematics Study' (SIMS) and the author's effort. One of his findings regarding the relationship between knowledge and practice was that teacher knowledge and attitudes directly influence a teacher's beliefs which, in turn, influence their teaching practice. Moreover, teachers' content knowledge was negatively related to the use of inquiry-based instructional practice.

These two studies, from two different angles qualitatively and quantitatively, show that teacher knowledge, beliefs and attitude indeed affect teaching. Also they show that teachers' responses to errors and the use of inquiry-based instruction are linked to knowledge and beliefs and possibly attitudes too. They emphasise the importance of the role of beliefs in the teaching process which fits with the fact that teacher knowledge and beliefs are intertwined. This may highlight the importance of exploring other studies in which researchers have investigated the influence of teacher knowledge alone, if possible, on different aspects of teaching in order to extend the understanding of the phenomena.

Rowland *et al.* (2000) aimed to assess 145 primary mathematics trainee teacher's content knowledge and to link it to their teaching. They used a test (an audit) to assess the knowledge of mathematics trainee teachers at the end of their one year Postgraduate Certificate of Education (PGCE). They stated that trainees with a high score in the audit are more likely to teach numeracy effectively and trainees with a low score showed a lack of teaching competency. This finding shows the possible positive relationship between teacher subject knowledge and teaching and the importance of having solid subject knowledge in order to teach in an effective way.

Similarly, Aubrey (1997b) focused on teacher pedagogical content knowledge of four in-service mathematics teachers of reception year pupils. Her point of interest was to investigate how teachers' pedagogical knowledge appears in the discourse between students and teacher, and how the teacher is made aware of the informal mathematical ideas that children bring with them to class. She used interviews as the research tool and found that a teacher's content knowledge, pedagogical content knowledge and knowledge about the competences of their learners influenced their teaching. The study investigated the influence of teacher knowledge: SMK, PCK and knowledge of students on the ways in which teachers help students learn mathematics. It focused on just teacher knowledge and put emphasis on the discourse in the class. It shares the findings of the previous three studies and all of them were based in the primary context which suggests that the situation could be explored in other contexts by checking the situation in other types of schools.

The positive relationship between teacher knowledge and practice occurs in the middle grades and secondary classes as well. For example, Tchoshanov (2010), in his mixed method study, had three sub-studies and in one of them he examined the relationship between ten middle grade (years 6–8) teachers' cognitive types of content knowledge which relate to knowing facts, procedures, concepts, connections, models and generalizations and lesson quality. He collected data using observations and found that teachers' content knowledge affected the teachers' teaching; more specifically, teachers with conceptual content knowledge teach conceptually. The study differs to others outlined earlier (primary schools) as it was based on teachers of older pupils. It emphasises the importance of the type of content knowledge which effects lesson planning, implementation and content. It highlights the role of teacher knowledge in organising and planning lessons and, more importantly, in helping the teachers to teach the content in appropriate ways.

In the secondary school context, Baumert *et al*. (2010) aimed to investigate the relationship between one hundred secondary teachers' content and pedagogical content knowledge and providing high quality instruction. They used a statistical model to analyse the data. They found that pedagogical and content knowledge can be distinguished theoretically and empirically and that there is a positive relationship between pedagogical content knowledge and teaching practice. In the same type of school, Steele (2013) assessed secondary and primary teachers' common content knowledge (CCK) and specialised content knowledge (SCK) by interviews and written assessment. He found that a strong base of CCK influenced the practice of providing instructions of representations of ideas. In addition, Kahan *et al.* (2003) developed a framework to describe the relationship between teachers' mathematical content knowledge and effective mathematics teaching. The framework consisted of the elements of teaching and the processes of teaching. Their data was collected through observations, interviews and tests, and the sample was a group of 16 prospective secondary mathematics teachers in the US. The findings in this case was that there is a relationship between content knowledge and teaching, but that it does not necessarily follow that having sound mathematical knowledge ensures clear instructions.

These three studies in secondary schools differed in the knowledge they focused on. The quantitative study of Baumert *et al*. (2010) mainly focused on PCK and its relationship to instructions, whereas Steele (2013) investigated the influence of just one component of SMK (common content knowledge) on instructions of representations. In addition, Kahan *et al.* (2003) mainly focused on the effect of SMK on teaching. Considering all three shows that almost every type of teacher knowledge can influence certain aspects of teaching. This highlights the importance of investigating both types of knowledge (SMK and PCK) in my study regarding their influence on certain aspects of teaching.

However, although all the studies outlined above show that teacher knowledge influences practice, some researchers have found that high levels of subject knowledge held by teachers does not guarantee a high standard of teaching. For example, Turnuklu and Yesildere (2007) investigated the relationship between teacher knowledge of pre-service mathematics teachers in Turkey and how they use it to assess their students' thinking. They collected data by using four problem-solving tasks and analysed it quantitatively and qualitatively. Their findings showed that having a solid base of content knowledge is not enough for teaching. The study shows that having a deep understanding of mathematics knowledge is not sufficient to teach it effectively which highlights the importance of the role of PCK in teaching.

In her comparative study of teachers in China and the US, Ma (1999) demonstrates that even the nature of content knowledge held by teachers influences their teaching. She looked for the nature of the knowledge needed for teaching beyond just knowing content for 23 US and 72 Chinese primary mathematics teachers. She used Teacher Education and Learning to Teach project (TELT) interviews. These interviews contained one hypothetical classroom scenarios of teaching topic, responding to a student's mistakes, generating a representation of a certain topic and responding to a novel idea coming from a student. She found that although US teachers had studied advanced mathematics courses and Chinese teachers had just studied an elementary mathematics courses, the Chinese knew the elementary mathematic content much better than the US teachers did.

The Chinese teachers in Ma's study had developed, what she called, Profound Understanding of Fundamental Mathematics (PUFM), while US teachers had a procedurally developed understanding. Having PUFM means that Chinese teachers have more breadth, depth and flexibility of mathematics understanding. This helped the Chinese to build more connections between and within topics, provide more teaching strategies, help the students to make sense of mathematics and to respond to the students more effectively than the US teachers.

Ma's (1999) study sheds light on the importance of investigating the nature of knowledge held by teachers not just the quantity of knowledge. This suggests changes could be made to teacher preparation programmes in order to adjust the balance between the courses of pedagogy and advanced mathematics courses.

The debate of whether having solid content knowledge is enough for competent teaching has led me to consider the importance of investigating the influence of PCK as well. One aspect of PCK is how teachers deal with students' errors, as investigated by Bray (2011). But what has not been dealt with is how teachers deal with students' correct answers. The literature, at least in the Saudi context and to the best of my knowledge, has not addressed the relationship between teacher knowledge and handling of students' contributions, and in particular correct answers. This current study may contribute to the literature regarding this point by investigating the relationship between teacher knowledge and the ways in which they handle their students' contributions.

My study fits with this trend of research on teacher knowledge as I am investigating how trainees respond to their students' contributions and the relationship between their knowledge and practice. However, although my study and other research in this area, are different in context (as my research is in Saudi Arabia), they share the aims, to some extent, to look at how knowledge influences practice.

Other research studies which have influenced mine in terms of the methodology of analysing certain teaching episodes, but not directly connected to teachers' knowledge, deserve to be mentioned as well. For example, Barwell's (2013) research provides a critique of the research inspired by Shulman's categories of teachers' knowledge in terms of a discursive psychological perspective. He argues that in the representational view of knowledge, the researcher has to assume the meaning of the presentations that the teacher has used during teaching and the research does not explain how people make sense of each other's knowledge during this interaction. Also, he argues that the researchers looked carefully at teaching practice to analyse class interactions but without using specific tools. In addition, he makes the point that they emphasized that students and teachers express their knowledge through discussion, which also needed to be analysed (*ibid*).

Barwell analysed two episodes of mathematical knowledge teaching from Hill *et al.'s* (2008) research from the discursive psychological perspective, and showed that the difference between these two perspectives was the representational view of knowledge and knowledge structure. This means that, in discursive psychology, there are no assumptions about the others' knowledge and this type of analysis is based on the interaction itself not on a prior analysis of the tasks. For example, knowledge is socially structured, whereas from the point of view of recent research it is structured in categories in the minds of teachers which the researchers can examine using instruments. This study brings to attention the role of interaction between teachers and students in this study and encourages me to pay more attention to the structure of the interaction in the class to investigate the reasons for teachers' actions while teaching. The discursive perspective, which is outlined in the following sections, showed me how to analyse such kinds of discussions between teachers and students.

3.3 Classroom discourse

Studying how the teachers deal with the students' contribution in order to answer my first question: 'How do trainee mathematics teachers deal with students' contributions in Saudi primary school classes, and why do they respond in these ways?' required me to explore the ways in which they communicate with each other. To do so, I found it useful to explore the field of discourse analysis as the majority of the interactions between the trainees and their students in my data are in question-answer form. Teacher-student interaction is a specialised type of conversation as most of the speech in the classroom has pedagogical purposes (Nassaji and Wells, 2000). This led me to search for the common interaction patterns in the literature to understand how talk in classrooms is constructed. However, although discourse analysis was used originally in language learning classrooms (for example see (Sinclair and Coulthard, 1975)), it was also used in mathematics classrooms (see for example (Drageset, 2014a). In the following sections I focus more on one of the models used in many studies to analyse spoken discourse in classrooms. Then I give a brief background to teacher moves. Finally, I give a short description of how teachers respond to students contributions from the literature.

3.3.1 IRF/E

One of the well-known models for analyzing spoken discourse is proposed by Sinclair and Coulthard (1975) and was revised later by them (Sinclair and Coulthard, 1992). As shown in Figure 3.3, it contains five hierarchical ranks where the lesson is the largest unit and the act is the smallest. They suggest that every lesson is divided into transactions which in turn contain exchanges. Exchanges have many moves that consist of acts (*ibid*).

<table>
<tr><td colspan="16">Lesson</td></tr>
<tr><td colspan="8">Transaction</td><td colspan="8"></td></tr>
<tr><td colspan="4">Exchange</td><td colspan="4"></td><td colspan="4"></td><td colspan="4"></td></tr>
<tr><td colspan="2">Move</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td>Act</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

Figure 3.3: Lesson ranking

In the next paragraphs, I focus on teaching exchange structures and moves which are more relevant to the interaction between teachers and students.

Teacher initiation-student response-teacher feedback (IRF) is a typical exchange in classrooms, which combines with other exchanges to form transactions, (Sinclair and Coulthard, 1992) and it is the most common discourse sequence in classrooms (Waring, 2009). Solomon and Black (2008) state that IRF happens in mathematics classrooms because the nature of interactions there are influenced by the teachers beliefs of mathematics teaching. Dooley (2011) adds that the teacher's type of interaction is likely to be mostly IRF if he believes that mathematics is just facts and procedures that need to be learnt by students. It includes the teacher beginning a discourse with a question, then the student responds and then the teacher gives feedback. If the given feedback is in an evaluative form this exchange fits with Initiation-Response-Evaluation exchange (IRE) proposed by Mehan (1979). For example:

T: How much is five plus two? (teacher initiation move)

S: Seven (student response)

T: Very good. (teacher feedback)

However, although the exchange of IRF has three parts, this does not mean all exchanges must have these three parts (Dailey, 2010). In fact, some exchanges come in the form of 'IR(F)' where the feedback is optional in case the student, for example, responds non-verbally to the teacher request (Sinclair and Coulthard, 1992, p.23). However, though this type of exchange is the most familiar discourse sequence in classrooms, it suffers from some criticisms. For example, Lier (2000) points out that the students' voice is limited in this format as they are just requested to answer the teacher's questions. This kind of criticism encouraged researchers to investigate the benefits that IRF can afford and in doing so much attention was paid to the teachers' moves contained in the initiation and feedback parts of the exchange.

3.3.2 Teacher moves

The IRF exchange contains three different types of moves that form the exchange: opening move, responding move and feedback move, or some call it the follow-up move (Lee, 2007). Each one of these moves can include one or more act which is the smallest part in the Sinclair and Coulthard's (1992) model.

The opening move starts with one of three acts: informative, directive or elicitation. When the teacher uses an informative act in the opening move, which makes an informative exchange, he tends to tell the students something and they are probably not requested to reply (Sinclair and Coulthard, 1992). The teacher uses a directive act (which makes a directing exchange) when he gives a command, and the student responds non-linguistically (*ibid*). The third possible act is elicitation where the teacher asks questions and requests the student to answer (which forms an eliciting exchange) (*ibid*) which is the most common exchange in classrooms (Dailey, 2010).

Much attention has been paid from scholars on the third move (F) in the IRF exchange (Nassaji and Wells, 2000; Lee, 2007). The third turn (move) in the sequence (F) is a contingent action that the teacher does when responding to the student response (Lee, 2007). It could be used in different ways to serve different purposes such as offering evaluation or feedback of the second move from the student, or a follow-up move that generates another three-turn sequence (Nassaji and Wells, 2000). This move is not just an evaluative one but it could display justification, counter-argument, clarification, meta-talk or action to enrich its function (*ibid*). Three types of acts: accept/reject, comment or evaluate can occur in this turn (Sinclair and Coulthard, 1992). The teacher uses an accept or reject act (saying yes or no for example) to confirm the appropriateness or the fault of the reply of the student, and the teacher uses a comment act to comment on the student response by expanding, justifying or providing additional information (*ibid*). Also, the act of evaluation can be used in this slot to judge the quality of the reply (*ibid*).

In addition, Brodie (2008) suggests that teachers can use different type of moves from that proposed by Sinclair and Coulthard (1992) as described in Table 3.1 below.

Insert	The teacher adds something in response to the learner's contribution. She can elaborate on it, correct it, answer a question, suggest something, make a link etc.
Elicit	While following up on a contribution, the teacher tries to get something from the learner. She elicits something else to work on learner's idea. Elicit moves can sometimes narrow the contributions in the same way as funneling.
Press	The teacher pushes or probes the learner for more on their idea, to clarify, justify or explain more clearly. The teacher does this by asking the learner to explain more, by asking why the learner thinks s/he is correct, or by asking a specific question that relates to the learner's idea and pushes for something more.
Maintain	The teacher maintains the contribution in the public realm for further consideration. She can repeat the idea, ask others for comment, or merely indicate that the learner should continue talking.
Confirm	The teacher confirms that s/he has heard the learner correctly. There should be some evidence that the teacher is not sure what s/he has heard from the learner, otherwise it could be press.

Table 3.1: Subcategories of Follow up

Source: (Brodie, 2008, p.212)

Furthermore, Lee (2007) suggests that classroom discourse could include a series of IRF when the learner could not give a correct answer. In this case the teacher can begin a new initiation of a new exchange drawn from the previous one in order to break the big question into small ones, which is similar to the funneling pattern of interaction (Wood, 1994).

Exploring the work of analyzing spoken discourse such as the Sinclair and Coulthard's (1992) model is useful in studying classroom communication. Dailey (2010, p.17) states that 'by breaking up the discourse in layers, the functions of each part become clear and by understanding these functions we can see how they combine to form classroom discourse'. One can probably understand the ways in which teachers respond to their students' contributions in more depth by analysing every move the teacher does on a turn-by-turn basis as the majority of interactions in Saudi classes are in question-answer form. In the following section I explore how teachers handle their students' contributions as presented in the literature.

3.3.3 Research on responding to students contributions in mathematics classroom

Researchers have conducted many studies on classroom interaction and on teacher's responding to students contributions in different subjects as one aspect of this interaction. In this section I focus on mathematics particularly and I explore the research regarding classroom interaction in general and in mathematics classes in particular with more attention paid to how teachers respond to students' answers and comments. I start with research that examine dealing with students' comments (suggestions, answers and questions) then move to responding to students' incorrect answers and finally responding to correct answers.

Teachers response to student comments was investigated by Drageset (2014a). The aim of that study was to examine the ways in which five Norwegian upper primary school mathematics teachers responded to their students' comments (suggestions, answers and questions) in order to build a framework to describe these in detail. The teachers were videotaped for a week then 1800 of their comments were included in the development of the framework categories. He proposed that the teachers responded to students' comments with different types of actions which were categorised into three groups: redirecting, progressing and focusing actions. The directing actions, where the teacher tends to change the students' response, includes putting the comment aside, initiating a new strategy and asking correction questions. In the second group, progressing actions, the teacher tends to move forward beyond the students' comment by providing a demonstration to a solution, simplification by adding information, starting open progress by asking the students to use their own methods and requesting closed progress details in which the teacher guides the class by breaking the big question into many manageable questions. In addition, the third group, focusing actions, in which the teacher intends to stop progressing to focus on details, includes enlighten details in which the teacher asks the students how they reached the answer, justification by asking why, application to similar problems by applying the known roles in a new situation, requesting assessment from other students, a recap in which the teacher repeats and summarizes what has been done and notice where the teacher puts emphasis on important points (Drageset, 2014a).

However, although this study is helpful for my study, it lacks clear distinction of which element of response (suggestions, answers and questions) is associated with which type of teacher action. To extend this work further, Drageset (2014b) has proposed a more explicit account of the nature of students' comments which is outlined in the following paragraph.

Drageset's (2014b) research was built upon his previous work (Drageset, 2014a) where he investigated the influence of a single talk-turn on the mathematical discourse pattern of turn-by-turn form. Besides his proposed three groups of teacher responses in his previous research (redirecting, progressing and focusing actions), he examined the students' responses which fell into five groups: 'explanations, initiatives, partial answers, teacher-led responses, and unexplained answers' (Drageset, 2014b, p.10). Explanations response was when by the teacher requested more detail from the student comment in order to share it with the class. Also, initiatives response is where the student breaks the flow of the discourse by asking or commenting. Partial answers are the answers which are not totally correct or incorrect but somewhere in the middle, including uncompleted ones. In addition, the category of teacher-led responses includes the correct response to basic questions posted by the teacher while leading them towards the answer by breaking the big question into small ones. The last group of responses is unexplained answers in which the student gives a correct or incorrect answer without providing any additional information, or is unable to answer at all.

By combining the teacher and student interventions, Drageset (2014b) found that every turn affected the response pattern. In addition, student explanation was always followed by a teacher focusing action, and teacher progressing actions is tightly connected by the students' teacher-led response. These two repeated patterns are the most observed interaction patterns in his data. Moreover, the most common teacher action was requesting closed progressing details in which the teacher funnels the question, and the least common action was redirecting by asking correction questions. Also, the most common response from the student was teacher-led response and the second most common response was unexplained answers which was always followed by teacher focusing action by asking for justification for example.

This study highlighted the fact that every talk-turn affects the following turn which must be taken in account while analysing interactions between teacher and students when investigating the ways in which they respond to their students' contributions. However, these two studies (Drageset, 2014a; b) investigated the responding patterns to students' comments in general, and in the next paragraphs I am narrowing down my focus to studies that dealt with students' unexpected, incorrect and correct answers.

Research on how teachers respond and deal with students answer is well documented in the literature with more emphasis on incorrect answers. It has been investigated using different approaches of either turn-by-turn or entire episode basis. For example, Son and Crespo (2009) investigated how teachers would respond to students non-traditional mathematical strategies and how they can be influenced by teacher knowledge and beliefs of teaching. They used a hypothetical scenario in which a student (Dan) used a non-traditional solving strategy (unexpected answer) for the division of fractions ($\frac{a}{b} \div \frac{c}{d} = \frac{a \div c}{b \div d}$) to do the division $\frac{2}{9} \div \frac{1}{3}$ and another student (Sally) disagreed with that and used the invert and multiply strategy. The 34 pre-service elementary and secondary teachers' responses and reasoning were examined. The teachers were asked three questions: '(1) Does Dan's strategy works, or not? (2) Is Dan's strategy generalizable to any fraction problem? (3) How and when can Dan's and Sally's strategies be used efficiently?' (*ibid*, p.245). They found that the majority of teachers' responses were teacher-focused in which they tell and explain to their students why the strategy works or not, whereas the rest of them encouraged the students to explain and justify their strategy. Also, the participants stated in their written response that their response was almost always influenced by their beliefs of teaching and to a lesser extent to their knowledge, experience and college programs.

Investigating the ways in which teachers deal with incorrect answers has taken place by many scholars. For example, Santagata (2005; 2004) has examined the actual interaction of teacher-student surrounding errors and she focused on comparing the ways Italian and US teachers deal with errors. Santagata (2004) also examined teachers' stance and attitude towards errors. Sixty videotaped lessons were analysed with 30 lessons in each group. The unit of the quantitative analysis used was a 'mistake management sequence' consisting of the teacher asking, the student answering incorrectly and then the teacher's first response (*ibid,* p.495).

These studies, Santagata (2005; 2004), show that Italian lessons contain more mistakes (11 mistakes per lesson) than US lessons (7.86 mistakes per lesson), so Italian teachers spend 10% of the lesson dealing with mistakes whereas US teachers spend 7%. Also, 97% of mistakes were publicly shared in Italian classes while only 63% were shared in US classes. Errors were corrected in Italian classes by the teachers 31.6% of the time, by the same student who made the mistake 41.2% of the time and by other students 12.4% of the time. They were corrected in US classes by the teacher 25.1% of the time, by the same student 29.8% of the time and by other students 32.4% of the time. Furthermore, the most common response in both classes was the teacher correcting the mistake by himself. The second most common response from Italian teachers was giving the student who made the mistake hints 21.6%, whereas in the US, teachers gave hints 19.2% of the time or redirected the question to other students 19.4% of the time. The third most common response in Italian classes (15.1%) was repeating the question to the student and in US classes (11%) giving hints to other students to help them correct the error. Also, US teachers' beliefs affected their dealing of errors as they avoided the discussion of mistakes publicly, probably as a result of the Behaviorist approach of learning as they focused only on getting the correct answers. Conversely, in the Italian context the teachers tended to share errors in public as a result of cultural specific practice.

Besides these findings, Santagata (2004) stated that Italian teachers criticized the students more than US teachers who usually mitigated their responses. Also, Italian teachers showed their disappointment as they believed that the students are responsible for their performance while US teachers showed positive attitude to the students who made mistakes as they appreciated the students trying.

In the same vein of comparative research, Schleppenbach et al (2007) investigated how primary mathematics teachers treat their students' mistakes in China and the US. They interviewed 24 teachers from China and 12 teachers from the US and recorded 15 first grade lessons and 39 fourth and fifth grade lessons in these countries. They found that the Chinese teachers talked explicitly about errors in the classes more than US teachers did (46 statements versus 18 statements). Also, students felt free to make mistakes and criticize their peers' errors in Chinese lessons. In addition, Chinese teachers tended to ask questions after errors whereas US teachers followed them with statements. Most of the US teachers asked the same student who made the mistake to evaluate the answer, whereas most of the Chinese teachers asked them to correct or explain it. Also, in fraction lessons the US teachers responded by telling the students the answer was wrong and the teachers corrected it, whereas in the Chinese classes the discussion of errors was redirected to other students. Moreover, most of the teachers in both classes see reviewing the incorrect answer with the students, planning their instructions for common errors, and letting the students notice and correct their errors as methods of dealing with errors.

Son and Sinclair (2010) examined how elementary pre-service mathematics teachers interpret and respond to a classroom scenario containing errors on reflective symmetry, and how these approaches are linked to their knowledge. They found that more than half the teachers identified the errors using action-based strategies involving conceptual aspects of symmetry, such as using flipping and folding. The most common pattern of response was the teacher generalizing the properties of reflection in which the teacher showed, talked about or told the students the properties of reflection. The second most common category of response was 'return to the basics' where the teacher used some strategies used in the previous sections (*ibid*, p.38). The last category was 'a Plato-and-the-slave-boy' approach in which the teacher reminded the student about some information about the properties of reflection that the student had forgotten (*ibid*, p.38). In conclusion, teachers identify errors depending on their subject knowledge but their failure to provide good instruction is likely to be related to their PCK.

Chick and Baker (2005) investigated how nine Australian elementary mathematics teachers would respond to four hypothetical tasks containing errors, and how that could be related to their PCK. These tasks were taken from a questionnaire with 17 items used to examine teaching situations and beliefs. The findings of the research were that the teachers used different strategies to respond depending on the nature of the item and topics. They responded by re-explaining some part of the item, using the cognitive conflict strategy, probing student thinking by asking them to explain their work in order to identify the error or to show the teacher their way of thinking and finally by using any other strategy does not fit with the previous categories such as ‘use a simpler example’ (*ibid*, p.251). Most of the responses were re-explaining and cognitive conflict was used in all four items. Furthermore, teachers responded procedurally to subtraction and division items, whereas they responded conceptually to fraction addition and area and perimeter items. In summary, the quality of the teacher response is likely to be linked to their PCK or its application.

However though the previous studies described above dealt with students errors, less frequent research has been done regarding responding to correct answers in which this study contributes. For example, Margutti and Drew (2014) aimed to investigate the ways in which four Italian primary teachers evaluated their 50 students’ answers positively and the relationships between these actions and the discourse they occurred in. They used Conversation analysis to analyse 80 hours of video recording of 50 Italian classes of different subjects. They found that 72.5% of 200 of teachers’ evaluations were positive whereas 10% were negative. The teachers responded in five different formats to various extents. Most of the teachers’ responses were verbal repetitions of the answer and embedded repetition in which the teacher repeated the answer in an extended version. The second most common response was moving to a question-answer sequence without any third turn move. Next was explicit positive assessment where the teachers praised the students and last was using particles like ‘mm’ as a sign of agreement with the correct answers (*ibid*, p.445). Moreover, the teachers chose from these actions to use in the third turn move depending on the pedagogical activity they used, as they usually used repetition strategy as a third turn move with activities containing repetitive question-answer moves that required the same procedure used before to answer correctly.

3.4 Conclusion

In the Saudi context, where this study is conducted, research concerning teachers’ knowledge is completely absent. I have searched for available sources of research on this subject in Saudi Arabia but I have been unable to locate any. Research of this kind in the Saudi educational context requires attention from policymakers and educators, as has been suggested by Almoathem (2009) in his doctoral thesis. His aim was to explore the trends of mathematics education research in post-graduate studies in the country's universities by analysing the content of the 220 Master's and doctoral theses. He focused primarily on the methodology and objectives of each thesis and found that most of the research in mathematics education in the Saudi context was quantitative and descriptive. The research focused on students rather than teachers and the majority used only one tool for data collection, namely the questionnaire. He also identified that the majority of the studies had been conducted at the primary level of education and focused least on the teachers. Moreover, these theses, which focused on teachers, paid more attention to teacher development than lesson preparation.

It is critical to link this study with previous studies in this field. However, there are differences as well as similarities here, as this study will focus on prospective primary level teachers, similar to research conducted by Bray (2011), Rowland *et al*. (2009), Aubrey (1997a), Shulman (1986), Halim and Meerah (2002) and Philipp *et al.* (2007). It also shares the subject (of mathematics) with the studies mentioned above, excluding that of Halim and Meerah (2002), who focused on science. Collecting data through observations and interviews is similar to the studies of Bray (2011), Rowland *et al*.(2009), Aubrey (1997a), Shulman (1986) and Kahan *et al.* (2003). On the other hand, it differs from other studies, as it is conducted in Saudi Arabia and its focus is on students' contributions in the classroom, whereas other scholars such as Bray (2011) focused more on practices related to the handling of student errors.

To summarise, previous studies have pursued different avenues of research. This study follows the trend of investigating the relationship between a teacher's knowledge and his/her teaching practice. To date, and to my knowledge, the focus of this study has not been addressed, this being possible connections between SMK and PCK and teachers' handling of students' contributions in the class whether these be correct or incorrect answers and the rationale for these responses, in the context of Saudi Arabia. This literature review shows some gaps, to which this study may contribute, such as the lack of studies investigating the relationship between teacher knowledge and students' contributions. Also, research on teachers' responses to students' correct answers is not numerous in this field. I have benefited from the literature of teacher knowledge and classrooms discourse in building this study in terms of methodology and data collection instruments and analysis. These are described in the following section of the thesis.

Chapter 4 : The aims and methods of the study

4.1 Introduction

After exploring the literature of teacher knowledge and classroom practice, the focus of this research has been narrowed down and the questions of this research have been shaped. In order to answer the research questions I explore here the methodology I used to achieve the answers. This chapter includes the research aims and questions (section 4.2), the methodology of the research (section 4.3), research setting (section 4.4), research phases (section 4.5), the data gathering methods (section 4.6), the data analysis (section 4.7), research trustworthiness (section 4.8) and the ethical considerations (section 4.9).

4.2 Research Aims and Questions

This study aims to investigate the relationship between Saudi primary prospective teachers' subject matter knowledge (SMK) and pedagogical content knowledge (PCK) and how they use this knowledge to handle their students' mathematical contributions in the lessons. To achieve the study's goals, the following questions are addressed:

1- How do trainee mathematics teachers deal with students' contributions in Saudi primary school classes, and why do they respond in those ways?
2- How does the knowledge that trainee mathematics teachers have gained influence the ways in which they handle their students' contributions in the classroom?

In order to provide clearly delineated answers for the research questions, it was necessary to select a certain methodology and a range of methods. I justify these procedures in the following sections, starting with the research approach or strategy, followed by the design of the study and, finally, the methods of data collection and analysis.

4.3 Choosing research methodology

Research methodology is 'the overall approach to the research process, from the theoretical underpinning to the collection and analysis of the data' (Collis and Hussey, 2003, p.55) which helps the researcher to answer his/her questions (Silverman, 2011). Choosing an appropriate research methodology is a crucial decision that needs to be made by the researcher. In doing so, firstly I need to choose a research approach from the approaches described in the following section 4.3.1 then choose a research design in section 4.3.4.

4.3.1 Research approaches

There are a number of different types of approaches that a researcher can apply to his/her work. Three types of research approaches: quantitative, qualitative, and mixed methods have been discussed in the literature (For details see: Creswell, 2003; Walker, 2010; Roberts-Holmes, 2011; Walliman, 2006; Bryman, 2004; Neuman, 2006; Silverman, 2010; Vaus, 2001). In the next paragraphs I give a brief background to these research approaches.

Quantitative research is, as Creswell (2003, p.18) states:

> one in which the investigator primarily uses postpositivist claims for developing knowledge (i.e., cause and effect thinking, reduction to specific variables and hypotheses and questions, use of measurement and observation, and the test of theories), employs strategies of inquiry such as experiments and surveys, and collects data on predetermined instruments that yield statistical data.

In the quantitative research approach the data is in the form of numbers and it involves a large amount of representative sets of data in order to generalise the findings to the population (Bryman, 2004). The researcher using this approach usually aims to develop a theory or test a hypothesis which is usually associated with the deductive analysis approach from the general to the particular (Blaxter *et al.*, 2008). The prime instruments to collect data in this type of research are survey and experiment (Vaus, 2001). However, although this research approach is widely used in natural science which focuses on positivist analysis and the researcher keeps their distance by being an outsider observer, it has also been used by social science until researchers found it very difficult to measure the subjective human aspects such as emotions and feelings which led to the evolvement of the qualitative approach (Walliman, 2001).

In contrast, qualitative research is defined by Creswell (2003, p.18) as:

> one in which the inquirer often makes knowledge claims based primarily on constructivist perspectives (i.e., the multiple meanings of individual experiences, meanings socially and historically constructed. with an intent of developing a theory or pattern) or advocacy/participatory perspectives (i.e., political, issue-oriented, collaborative. or change oriented) or both.

Therefore, in qualitative research the researcher aims to build a theory by inductive analysis from the particular to the general instead of testing a theory, and focus on individuals' interpretation of their social interaction (Bryman, 2004). The prime research methods used with this type of research is usually the case study and the nature of data is in the form of words (Vaus, 2001). It is also associated with participant observations and in-depth interviews (Walliman, 2001). The researcher here works closely with the subject with the researcher as an insider observer and the researcher focuses on depth more than breadth (Blaxter *et al.*, 2008) and tends to work with small numbers of incidents which reveals rich and subjective data (Collis and Hussey, 2003). This research approach has been criticised for some 'issues of validity and reliability' (Inman, 2011, p.229) which are discussed later in this chapter.

Both quantitative and qualitative research approaches are not better than each other and both have strengths and weaknesses (Dawson, 2009; Denscombe, 2007). It is therefore not an easy task for a researcher to decide which research approach is the most appropriate to his/her research. This decision is influenced by different factors which I outline in the next section.

4.3.2 Factors influencing the choice of research approach

There are many factors that influence the manner of conducting research. These factors include the researcher's personal philosophical background, the purpose of the research, the study sites, the audience and the participants' personalities (Snape and Spencer, 2003). These factors are outlined in brief below.

Researcher's philosophical assumptions

The researcher's own perspective of the reality and the nature of social science research (ontology) and how knowledge is acquired (epistemology) played a vital role in deciding which approach should be taken.

The ontological assumptions held by the researcher influenced the taken approach of research. Ontology is defined as ‘the ideas about the existence of and relationship between people, society and the world in general’ (Eriksson and Kovalainen, 2008, p.18) in which the researcher considers the nature of reality and whether the nature of reality is objective or subjective (Creswell, 2003). The objectivist position of reality believes that human nature is a product of external reality and social entity can be investigated in the same way as the natural scientist examines scientific phenomenon (Bryman, 2004). Conversely, the subjectivists or constructivists perspective, in which my ontological position is in complete agreement, considers the reality of social phenomena as intrinsic to the interaction of the social actors, observing the various social situations and comprehending their meanings as they exist in the social context (*ibid*). In addition, I hold the view that social reality is not definitive, as it is continually altered through social interaction between people (Bryman, 2004; Walliman, 2006). However, although these two ontologies, objectivist and subjectivist positions, are completely distinctive, both shape the epistemological beliefs held by the researchers (Williams and May, 2002) and are associated with positive epistemology and interpretive epistemology respectively (Bryman, 2004) as explained below.

Epistemology is concerned with ‘the very bases of knowledge – its nature and forms, how it can be acquired, and how communicated to other human beings’ (Cohen *et al.*, 2007, p.7). It is shaped by the researcher’s ontological assumptions (Williams and May, 2002) hence one with objective ontology is likely to have positivist epistemology beliefs (Bryman, 2004). In such epistemology, the researcher should be objective and deal with the social world in the same way of dealing with natural science, and believe that they know what they can observe and keep distant from the subject under study (Collis and Hussey, 2003). Positivism values the use of research instruments such as questionnaires to capture reality (Blaxter *et al.*, 2008).

In complete opposition to positivist epistemology, is the interpretivism position. I share the epistemological beliefs with the interpretivism position, which advocates that social science and its components (human) are totally different from natural science objects, and that knowledge is socially created (Bryman, 2004). Therefore, my research focuses on the interaction between people in order to grasp how knowledge is created in the social context of the classroom (Bryman, 2004; Neuman, 2006; Creswell, 2003; Cohen *et al.*, 2007).

Research aims

The purpose of the research, as previously mentioned, is to describe how prospective teachers in Saudi Arabia deal with their students' mathematical contributions in primary mathematics lessons, to investigate the reasons for their actions and, finally, to link these actions with the teachers' knowledge. This research could therefore be considered as a combination of descriptive and explanatory research, its respective purpose being to describe the nature of the social phenomena and to investigate why this is so (Neuman, 2006; Vaus, 2001).

Primarily, these two factors (my philosophical background and the purpose of this research) influenced my choice of approach for conducting this project. Both the epistemological and ontological positions look for the relevant knowledge in a natural context and focus on the interaction between humans to understand meaning. In addition, the study aims to investigate the teachers' knowledge and its relationship to their teaching practice in greater depth; therefore, it was necessary that the study be conducted in the natural context of a classroom. This, in turn, led me to consider a research strategy and design that would be appropriate to the aims of the study.

4.3.3 The research approach of this study

After exploring the different types of research approaches that a researcher can apply to his/her work (quantitative, qualitative, and mixed methods approaches) and after considering the factors that influence the decision of the research approach, I found that an appropriate research approach to my research is the qualitative approach and I justify my choice below.

Firstly, Vaus (2001) states that the research questions, whether descriptive or exploratory, have a great influence on selecting the research approach. In addition, certain words in the research questions can be used as an indication of whether the research is quantitative or qualitative (Dawson, 2009). For example, 'what' questions may fit with any strategy and 'why' and 'how' questions are consistent with certain types of qualitative methods (Yin, 2009). As this study's research questions are 'how' and ‘why’ questions, they normally require the researcher to seek the answers by applying the qualitative strategy.

Secondly, this research requires certain types of data. Neuman (2006) stresses that the nature of the required data affects the choice of research approach. He states that ‘hard’ and ‘soft’ data, which include numbers and words respectively, require a different research approach and data collection techniques. The nature of the data in this study is clearly ‘soft’ data, in the form of words, sentences, impressions and symbols, and therefore, the qualitative approach is the most appropriate one in this case. Similarly, according to Punch (2000), research which uses ‘non-numerical’ data is best served by a qualitative method.

The researcher’s philosophical background affects his/her research approach as well. I tend to look to the interaction between teachers and their students in a natural detailed context, which is consistent with the interpretive position of social science. This fits with the qualitative style, according to Neuman (2006).

Finally, as no such type of research has been conducted previously in the Saudi context, it is essential to examine the phenomena in detail, from first principles, by applying the qualitative approach; as Creswell (2003, p:22) states, ‘if a concept or phenomenon needs to be understood because little research has been done on it, then it merits a qualitative approach’. This uncertainty about the phenomena requires a flexible research approach which fits with the qualitative research approach as Maxwell (2005, p.22) states:

> Qualitative research has an inherent openness and flexibility that allows you to modify your design and focus during the research to understand new discoveries and relationships. This flexibility derives from its particularistic, rather than comparative and generalizing, focus, and from its freedom from the rules of statistical hypothesis testing, which require that the research plan not be significantly altered after data collection has begun

Having made the decision to select a qualitative approach in my research, it is necessary to follow this choice of strategy with a carefully selected research design appropriate to the qualitative strategy.

4.3.4 The design of this study

By using the term ‘design’, I mean the ‘framework for the collection and analysis of data’ (Walliman, 2006, p.42), which links the study questions with the data (Roberts-Holmes, 2011). Scholars have proposed several kinds of research design: cross-sectional or survey, experimental, longitudinal and case study (Walliman, 2006; Vaus, 2001). In addition, Bryman (2004) suggests a fifth design, namely, comparative design.

There are various types of design that are suited to the qualitative approach. Robson (2002) identified the ‘flexible design’, which can be changed during the research process and is associated with the qualitative approach; this includes grounded theory, ethnographic and case study designs (Punch, 2000).

For this study I have selected a case study approach. In order to justify my choice, I focused heavily on Yin’s book (2009) *Case Study Research* in which he elaborates on the case study research design. He states that the case study has a notable strength when, ‘a ‘how’ or ‘why’ question is asked about a contemporary set of events, over which the investigator has little or no control’ (Yin, 2009, p:9). In this research, both how and why questions are used, although other factors affecting the way that teachers deal with their students’ contributions are addressed. Furthermore, it is necessary to find specific details regarding the social situations within the context of mathematics classes in Saudi Arabia. I am therefore addressing the quality of the findings, not seeking generalizations. In the case of this study, the case study is the most appropriate design, according to Walliman (2006).

One of the most frequently used qualitative research methods, which this study used, is the case study (Yazan, 2015). Case is defined by Yin (2014, p:2) as ‘a contemporary phenomenon (the “case”) in its real-world context, especially when the boundaries between phenomenon and context may not be clearly evident’. Stake (1995, p:2) conceptualises case as ‘a specific, a complex, functioning thing,’ more specifically ‘an integrated system’ which ‘has a boundary and working parts’ and ‘in social sciences and human services, [it] is likely to be purposive’. These definitions emphasise the importance of cases being specific and in a real context and they require researchers to pay attention to their boundaries to control the focus of the study.

To design a case study, researchers should consider the five components of a case study: ‘a case study’s questions; its propositions, if any; its unit(s) of analysis; the logic linking the data to the propositions; and the criteria for interpreting the findings’ (Yin, 2014, p:29). After writing the research questions researchers should think about the study propositions and unit of analysis to specify their cases. The research questions help scholars to define their cases (unit of analysis) which could be individuals, groups, organisations, programmes and so on. These units of analysis cannot be studied fully so propositions direct the attention to the things that should be studied in detail.

In this research, to decide what this case study is a case of I thought about the unit of analysis and propositions as, according to Yin (2009), identifying the propositions and the unit of analysis are important components for case studies. As the research focuses on teacher knowledge (proposition) I took this into account and I thought about how the variety of teacher knowledge could affect their responses to the students’ contributions. I believed that the individual teacher was the unit of analysis so I decided to form case studies of trainees with differences in terms of their SMK and PCK.

Once the case and its boundaries have been determined, the type of case study must be considered depending on the overall study purpose. There are several types of case study proposed by scholars. For example, Yin (2014) identifies three types in terms of their outcomes: descriptive, explanatory and exploratory case study. He defines each of them respectively as ‘a case study whose purpose is to described a phenomenon (the “case”) in its real-world context’, ‘a case study whose purpose is to explain how or why some condition came to be’ and ‘a case study whose purpose is to identify the research questions or procedures to be used in a subsequent research study, which might or might not be a case study’ respectively (*ibid*, p:238). Also, he suggests four types of design for case studies. They include a single holistic design, multiple holistic design (holistic designs require one unit of analysis), single embedded design and multiple embedded design (embedded designs require multiple units of analysis) (*ibid*, p:50). Stake (1995) uses three terms to describe case studies: intrinsic, instrumental, and collective. He suggests that if the researcher is interested in a particular case and needs to learn about it, not because he/she wants to know about other cases but because he/she is interested in it, he/she can use the intrinsic case study. Also, if the researcher wants to know about a problem in general an instrumental case study can be used by choosing a case then studying it in order to understand the phenomenon, not the particular case. The collective case study is similar to the instrumental case study but it contains more than one case where, in order to achieve full understanding, each is an instrumental case study.

This research can be considered as a descriptive and explanatory case study. It has some element of description of how trainee teachers deal with their students' contributions in the real life context and it tries to explain the reasons behind the ways of response. Yin(2014) argues that it is a misconception that case studies are appropriate for the exploratory phase only as they can serve two other purposes: descriptive and explanatory. In addition, this research is of a multiple holistic design (Yin, 2014) as it contains more than one trainee teacher with one unit of analysis (a person). It is also in the form of a collective case study design (Stake, 1995) as it includes many instrumental cases to draw a big picture of the phenomenon under investigation. In this multiple case study design, I examine several cases to understand the similarities and differences between the cases then draw a cross-case analysis to compare them. Each case must be selected so that it either, '(a) predicts similar results (a literal replication) or (b) predicts contrasting results but for anticipatable reasons (a theoretical replication)' (Yin, 2014, p:57). In this study, contrasting results had been predicted between teachers with differences in terms of knowledge which may be considered as a theoretical replication between cases.

This project focuses on case studies of three male trainee mathematics teachers (chosen from five participants the project started with) who are practising in three primary schools in Arrass city during the second term of 2013/2014, and each student teacher forms a case study as outlined in the next section.

4.4 Research Setting and participants

This study took place in three different primary schools for boys in the city of Arrass, Saudi Arabia where the five trainee mathematics teachers were practising teaching. I have chosen this city because I live there and I have selected the mathematics trainee teachers because I was a lecturer at Qassim University before I came to the UK. Also it was my responsibility to supervise some of the mathematics trainees in the past and I have experience of working with them. My position as a lecturer in the university did not influence their right to participate or not, as I had not taught them before nor evaluated their teaching, as explained in section 4.9. Moreover, regarding the school choice, I did not control the choice of schools as the trainees were sent to their schools before they participated in the study.

4.4.1 Research participants

In order to choose the study participants many steps needed to be taken. Firstly, I obtained an approval letter from the Dean of my college (Arrass Teacher College) to conduct my research there as an outsider researcher not as one of the staff, in order to have full access to the trainee teachers' classes. Secondly, I intended to meet all the trainee mathematics teachers, as a group, in the first week of the second term of the 2013/2014 school year. I identified 15 trainee mathematics teachers who were to practise teaching in their schools, which had been nominated already. I attended their meeting with the Head of the Department of Education and waited until the Head gave them some advice about participating in the schools, then I remained with them alone. I started by congratulating them on becoming trainees so soon and I provided them with comprehensive details and information about the project, informing them of their rights and responsibilities and asking them about the possibility of taking part in the research. I emphasised the fact that participating, or not participating, in the research would not affect their scores in the training course. I asked them to take their time to consider and to inform me, if interested, by text.

Subsequently, I intended to select about five or six trainee teachers from all those willing to participate, in order to form case studies by using theoretical sampling. Six trainees expressed their interest in participating in the research. Following this, they were asked to read the consent form and the information sheet (appendix 4 and 5) in order to fully understand the purposes and procedures of the study and to decide whether they were still willing to participate. One participant withdrew in the course of the study due to his time commitments so the final number of participants was five trainee teachers. In the next stage, we agreed that I should have a copy of their teaching timetable as soon as possible to enable me to plan my visits to their classes. I asked their supervisors for their permission to let me visit their students in the classrooms and they agreed to do so. These five prospective teachers are fully explored in the next section.

4.4.2 The initial participants of the research

Five trainee mathematics teachers (described in Table 4.1) initially participated in the research while they were practising in three schools (described in Table 4.2). They were named Abdullah, Fallah, Fahad, Onizan and Saad. All of them had been studying for four years at my college (Arrass Teacher College in Qassim University) and they varied with regards to their study load and their training period; hence, some of them had to attend one or two courses at the college while they were practising teaching in the schools.

N	Name	School	Class	Attended courses*	Age
1	Abdullah	Ansar	5/6	0	22
2	Fallah	Tahfed	5/6	1	23
3	Onizan	Tahfed	4/6	0	22
4	Fahad	Hazem	4	1	23
5	Saad	Hazem	6	2	24

Table 4.1 : The initial participants

* attending courses in the college.

N	School	Type of school	Founded	Students per class
1	Hazem	Urban/general	1989	20-25
2	Tahfed	Urban/ religious	1976	22-25
3	Ansar	Rural/general	1980	6-7

Table 4.2 : The study setting

Participant 1

Abdullah was the first trainee who responded to my request to participate in my research. He is a 22-year-old who lives in Arrass city. Abdullah was fully engaged in the school, as he did not have any other commitments at the college. He taught grades five and six in Ansar School for five lessons each per week. He was visited by his academic supervisor twice a month and by his co-operating teacher weekly. They always met and discussed various aspects of Abdullah's teaching. In the first visit to him, I was told that Abdullah was not good with his mathematical content knowledge as he made many mistakes in front of the class, but later towards the end of the term his confidence and knowledge seemed to improve.

Abdullah practised teaching in Ansar School, which is a rural school founded in 1980 with only six and seven students in each class. All mathematics classes take place in the mathematics lab, which has a whiteboard, a projector and tables organised as four groups. He was the only trainee teacher who was practising teaching in this school and was supervised partly by the co-operating mathematics teacher.

Participant 2

The second participant in this research is Onizan who is 22 years old. He was practising teaching in the same school as Fallah, Tahfed School, and taught grades four and six for four lessons each per week. Onizan had completed all the courses required prior to his practice term so he had more time to spend in the school. He was a determined teacher as he showed interest in gaining new information about teaching and always asked me about my comments on the lessons because their supervisor did not come to them regularly as he only visited them three times per term.

Onizan was practising teaching in Tahfed School which was founded earlier than Hazem and Ansar, in 1976. It differs from other schools as it specialises in teaching the Quran, so there are fewer mathematics classes than in the general schools, and the focus is on memorising the Quran. For example, grades four, five and six (which are the focus of my study) in the Tahfed school each have four mathematics lessons per week, whereas in the general schools it is five lessons every week. This puts more time pressure on the trainee to cover the content and it affects the students' mathematical background as well; hence, some topics were skipped by the teachers in compliance with the school's instructions. The school building is new, and includes facilities such as a mathematics lab and resource centre and there were about 22 students per class. Grades four and six had only one class each, whereas grade five had two classes, and these were all taught by the two participants. In addition, as in the Hazem School, there were two co-operating mathematics teachers who partly supervised the two trainee teachers, both in and outside the classes, on a weekly basis.

Participant 3

Fallah, 23 years old, was the third trainee participant in this research. Fallah practised teaching in Tahfed School with Onizan and he taught two classes of grade five for four lessons each and two lessons of grade three per week. He was originally from Riyadh but he lived in Arrass city during the college term time. Fallah failed the Complex Analysis course twice so he had to go to the college to attend the course during his teaching practice. As a result his teaching timetable was overloaded and he was not always free for other commitments. Although his teaching involved some mathematical mistakes and he showed a clear lack of content knowledge when I asked him simple questions, he persisted in asking me about the possibility of taking a Master's degree.

Participant 4

Fahad was the fourth trainee teacher; he is 23 years old and practising teaching at the Hazem School. He taught two classes of grade four for five lessons each per week. He was attending the Complex Analysis course as well as practising in the school. Fahad showed interest in developing his teaching style as he told me that he always looked to the YouTube website for ideas on teaching concepts to bring into the class. Although, Hazem's head teacher gave the trainees the freedom to go out whenever they liked as long as they did not have classes, Fahad remained all day as he was observing and asking others, such as the librarian, about the nature of their jobs. Fahad's supervisor visited him rarely so he said to me he depended heavily on the co-operating teacher's feedback about his teaching.

He was practising in Hazem School which was founded in 1989 and since then it has seen a gradual growth in student numbers. It has two classes in every grade starting from year one until year six. The average number of students was 25 per class. It is a new building with many facilities, such as a resource centre, mathematics lab and a library. There were two co-operating mathematics teachers who worked there and these partly supervised the two trainee teachers Fahad and Saad. All of them met together weekly to discuss various teaching problems faced by the two participants.

Participant 5

The last participant was Saad and he was practising teaching with Fahad at Hazem School. He taught two classes of grade six for five lessons each per week and he had other commitments at the college where he was attending two courses: Complex Analysis and Abstract Algebra, as he had failed them previously. His teaching timetable was very dense at times, but had many gaps which gave him the opportunity to attend college. He lives in a village near Arrass city so he always came to the school just prior to his class and left soon after. This made it inconvenient to arrange the interviews with him. He shared the same supervisor as Fahad so he also depended on his co-operating teacher to improve his teaching.

4.4.3 Research - actual participants: the case studies

After the field work had finished, I came to analyse my data and I looked to all five participants to evaluate the richness of the data. I found that the data was rich so I started to analyse it. I started this analysis by analysing the first lesson of the first teacher and found that the content of the analysis of one lesson was quite long and if I were to analyse all the data in the same depth the thesis would be very long. So I wondered about finding a way of solving this problem to keep the analysis deep and to keep within the allowed time and space. I came to a decision to reduce the number of teachers from five to three due to the limitations of time and space. To do so I had to carefully create criteria to follow in order to choose three participants, to form three case studies, from the five described earlier.

I took the following steps in order to decide which three case studies would be included in my research. Firstly, according to Yin (2009), identifying the propositions and the unit of analysis are important components for case studies. As the research focuses on teacher knowledge (proposition) I took this into account and I wondered how the variety of teacher knowledge could affect their responses to the students' contributions. I believed that the individual teacher was the unit of analysis so I decided to form three case studies of trainees with differences in terms of their SMK and PCK. Consequently, I tried to categorise the five teachers in terms of their SMK and PCK and to select from them the highest and lowest possible.

Categorising the trainees in terms of knowledge was problematic. Some of them knew to teach some topics and did not know to teach others. I tried to collect evidence from their teaching to support or contradict my judgment about their knowledge. I did not intend to quantify their knowledge as my intention was to compare them with each other regarding their knowledge. For example, if the trainee showed no errors in his teaching or went beyond just knowing the subject, I consider his SMK as a solid SMK in the topics he taught as this judgment usually not transferable to other topics. Sometimes I used some specific questions about the subject in the mini task which took place in the interview to check the trainee understands of the taught topics. Evidence from these two sources, observation and interview, gave indication about the depth of the trainees' SMK. PCK was highlighted through the observations and, to some extent, interviews. The trainees were varied in terms of the wealth of their instructions. The one who used more teaching methods and his teaching style differed from the other trainees was categorised as a teacher with solid PCK of the topics he taught.

I found that to some extent Onizan (participant 2) had more depth in his SMK than the others (as the interview and his teaching showed) so I chose him to form a case study of a solid SMK trainee. Also, I found Fahad's (participant 4) teaching was the most different from the others in terms of style and quality so he was chosen to form another case study of a solid PCK trainee. The three other teachers, Abdullah, Fallah and Saad (participants 1, 3 and 5), were closest to each other in having both weak SMK and PCK. I thought of excluding Saad as his timetable was not convenient and he was not in school most of the time. Then I had to choose between Abdullah and Fallah who were similar to each other in terms of knowledge and teaching. Subsequently, I chose Abdullah (participant 1), to form a case study of a weak SMK and PCK trainee for many reasons. Firstly, he had more time to spend in the school as he did not attend a course at the college. Also, regarding the placement, he taught five lessons of each class (general school) instead of four (as in Fallah's religious school) so he had more time to teach and the pressure of time was eliminated to some extent. Finally, to make the setting more diverse I chose to include a rural school (Abdullah's school). To sum up, three trainee teachers Onizan, Fahad and Abdullah were chosen to form three case studies of trainees with differences in terms of SMK and PCK.

4.5 Research phases

In this section I describe the phases of this research which can be divided into three phases: pre-fieldwork, main fieldwork and post-fieldwork as shown below.

4.5.1 Phase 1: Pre fieldwork

During my first year I read many articles on mathematics education to narrow down my thoughts and my study regarding a topic to be researched to obtain a PhD. My reading, and advice from my supervisors, led me to think about teachers' mathematical knowledge and as a result I read intensively in this specific area and developed a proposal for research in a Saudi context. My initial focus was on how teachers' knowledge affects the teachers' practice generally with a preference to emphasise the ways of interaction in classrooms. I wanted to use classroom observation and interview and to analyse the data later after piloting the study. I thought it was necessary to pilot the initial thoughts and instruments while I was in Saudi in May 2011. I went to Saudi and I undertook a pilot study. Then when I returned to the UK in June 2011 I met with my supervisors and discussed in detail the initial findings. The main purpose was to provide my supervisors with the background of the Saudi context and to establish the possibility of undertaking such qualitative research, which was not popular in the Saudi research context. I write about the pilot study in the next section and then I move on to the main fieldwork phase.

4.5.2 Pilot study

The pilot study was conducted in the city of Arrass, Saudi Arabia, between the 1st of May and the 6th of June 2010 in order to test the study's aims and instruments and to gain initial impressions about what actually happens in Saudi classes and also what kind of data I can come up with. In addition, a pilot study helps, as Teijlingen and Hundle (2001, p.2) suggest, in 'developing a research question and research plan [and] Training a researcher in as many elements of the research process as possible'. Thus, I decided to conduct the pilot study at the end of the Saudi school year, even though I was not fully ready in terms of setting the research questions.

In this study, I ensured that I was fully cognisant of ethical issues. I asked the Dean of Arrass Teacher College to provide me with official permission to conduct my project with two trainee teachers, and I obtained another letter from a primary school, allowing me to visit and videotape each teacher in his class.

I selected four teachers to observe and interview; two of them were trainee teachers and the others were in-service teachers. I chose the latter from different learning environments in Saudi schools, which are classified as urban and rural or high and low primary schools. The participants also may represent trainee and in-service teachers, using the reformed and old curriculum. Each class was videotaped, and one successful and one less successful aspect were selected for discussion. In addition, after finishing the data collection I used Sfard's (2001) analytical tool to gain a deeper analysis of one of the negative episodes which provides a model of a typical Saudi class.

This pilot has a number of limitations. One aspect of this study is missing which is the assessment of teachers' knowledge. As this study investigates the relationship between the teachers' knowledge and their handling of the pupils' contributions in the class, it is important to assess beforehand each teacher's level of knowledge, but this has not been done in this study due to the lack of any readily available tasks to explore their level of knowledge. As Biza *et al*. (2007) suggest, such tasks are a good way of exploring the teachers' abilities in a natural, everyday context, but this was not possible because of the time restriction and the lack of specific mathematical resources.

Furthermore, the reforms to the Saudi mathematical curriculum influenced my ability to build my own tasks for measuring teacher knowledge. The Saudi Ministry of Education has introduced new mathematics textbooks in the last two years but I faced difficulties in obtaining any copies as I am abroad, so I was unable to garner a good background to the content of these new textbooks. In addition, if I had been able to access them, I would still not have known what was to be taught in the classes I intended to observe, and so I would have found it difficult to prepare any relevant tasks.

Also, I went there without being able to accurately explore the teachers' pedagogical knowledge by using established tasks and scenarios. Instead, I relied on my experience as teacher educator in estimating their level of knowledge by examining their qualification grades and by seeking their supervisor's impressions. These strategies afforded me an overview of whether the teacher in question has a sufficiently good mathematical background; I did not discuss the teachers' knowledge directly with them in this pilot study.

The variety of the participants in the pilot is the second factor that needs explanation. This study is focused on trainee teachers in Arrass schools, and I was worried about whether or not I would find any trainee teachers. Under time pressure, I asked my colleagues if there were any students who would agree to participate in the project; they made contact with trainees about my intention to visit their classes. They found two trainee teachers who agreed to be observed, and then I arranged to observe two in-service teachers, all of them for just one class each. This is a small number of participants and may not be fully representative for the purposes of the main study but they were nevertheless carefully screened. To give an example of the pilot phase incidents, I summarise one from Hamad's class below then I analyse it in appendix (3).

Hamad is a trainee mathematics teacher in a primary school in Arrass. As I was informed by his supervisor, his grades are the highest among his group. He teaches grade five (11-year-olds).

The observed lesson was entitled Reflections and Graphs, which followed the lesson Translations and Graphs, and it was delivered in the Resource Centre at King Saud Primary School. The majority of the lesson's tasks were exercises on reflections in a horizontal or vertical line.

Generally, the teacher led the class by solving an exercise and then asking the pupils to copy his solution. While the students were copying or solving the task by themselves, Hamad moved around and corrected their work individually. I had expected the pupils to work in groups as the tables were so arranged but they did not.

The lesson went smoothly, and I have noted three types of reaction on Hamad's part towards the pupils' contributions, which were accepting the right answers without asking how they were reached, assisting with the uncompleted answers, and correcting the wrong ones. One positive and one negative incident, i.e. right and wrong responses on the part of the pupils, are discussed in the following paragraphs.

Positive incident

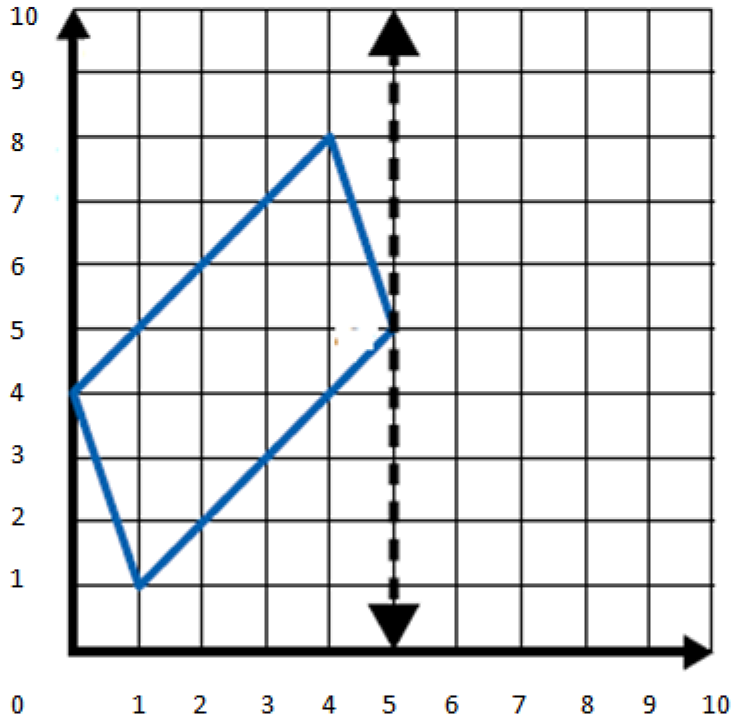
Hamad always dealt with the pupils' correct contributions in the same way. When a pupil was selected to answer a question, Hamad evaluated his answer and if it was correct, he would say, 'excellent'. This frequent behaviour was sometimes followed by an interpretation of the given answer, but it was given by Hamad, not by the pupil. For instance, when the pupil pointed to the coordinates (6, 8) as being the image for the reflected coordinates (4, 8) across the y = 5 axis, Hamad said:

> Excellent, but why is this point [(6, 8)] correct? What did Raied [the pupil] do? He started counting one [from the point (4, 8) to the reflection line], and then he stands in front of the line and counted one [Hamad pointed to the point (6, 8)].

Negative incident

Hamad asked the class to solve the following task:

‘The Diagram has a parallelogram, whose vertices are A (0, 4), B (4, 8), C (5, 5) and D (1, 1). After a reflection in the line, write down the ordered pairs for the new vertices.’



Prior to the pupils’ contributions on the white board, Hamad gave a brief description of the aim of this task and how they could solve it. He randomly asked one pupil to show him where the image of a particular point would be after reflection. After finding the images of points A, B and C, Adel was chosen to give the image of point D (1, 1) on the graph.

Hamad :	[1] [2] [3]	D, D… Look at me Bassam and pay attention… What is the matter with this group…? Why does nobody want to answer? [He points to that group and asks Adel to come to the front of the class to point to the image of point D] Come on Adel, where is D?
Adel	[4]	[Adel goes and points at 6, 1]
Hamad	[5]	No, no.
Pupils	[6]	Teacher, teacher, teacher! [They want to answer]
Hamad	[7]	Come and stand close to D and put your finger on it. Now count

	[8]	until you reach the axis of reflection and tell us how many units you counted.
	[9]	[Adel puts his finger on point D and starts counting one, but before he says two, the teacher says]
Hamad	[10] [11]	No, no, do not count the point of D. [Then Adel repeats counting as the teacher told him] How many? Tell us.
Adel	[12]	[Raises four fingers]
Hamad	[13] [14] [15]	Four, four. Keep in your mind four, keep in your mind four. Now count. [Unfortunately, Adel starts counting from the right-hand edge of the graph, while Hamad was telling another pupil to pay attention]
Hamad	[16] [17]	WHY?? You should stand in front of the axis of reflection and count four from there. [Hamad points to the reflection line on the graph]
	[18]	[Adel puts his finger on the axis of reflection and starts counting 'one', but before he says 'two', Hamad says]
Hamad	[19] [20] [21] [22] [23] [24] [25]	NO, NO, NO! Do not count the axis of reflection. [Adel is already putting his finger on the point 8, 1] [Hamad, to the class] We talked about this in the translation lesson. You always make the same mistake. We said that when you stand on a line or a point, [Hamad stands on a line on the floor and uses it as a number line] do not say 'one' and then move to 'two'. [He takes one step forward] No, no, you stand, then you take one step and only then do you say 'one', then 'two, three, four', that's four steps. DO NOT COUNT THE STARTING POINT. Now recount.
Adel	[26]	[Counts correctly] One, two, three, four. [puts his finger on 9, 1]
Hamad	[27]	Excellent! That's it.

4.5.3 Phase 2: The main fieldwork phase

Through the pilot study I became convinced that it was possible to conduct my research in Saudi schools as I found rich data emerging from the interactions between the student teachers and their students. I believed that focusing only on the trainee teachers and excluding the in-service teachers was more beneficial for my research due to the depth and the diversity of the interactions and also the findings could be useful for my sponsor, Qassim University. As a result I focused heavily on the trainee teachers in the main study and did not modify the data collection methods due to them being compatible with the kinds of data wanted and with the study's questions.

Before obtaining the upgrade to PhD approval from EDU in March 2012, the preparation for the fieldwork started. I communicated with the Dean of Arrass Teacher College to establish how many prospective mathematics teachers would practise teaching in the second school term in Saudi Arabia, which started at the end of February 2012. I was told that there would be about 15-20 trainee mathematics teachers. I could not go at that time for personal circumstances so I decide to go as soon as I could. I went to Saudi Arabia in March 2013 before the registration week of the practicum term. I took with me all the paperwork needed for the fieldwork such as participants' consent forms and information sheets (appendix 4 and 5). I went to the college to collect the access permission from the Dean of the college as I asked him for it prior to arriving in the kingdom. Then I went to the Ministry of Education office in Arrass, which is responsible for schools, and explained to the manager about my research and the need to gain access to some classes in principle. Usually, when my colleagues want to conduct research the college communicates with the school directly but I considered myself an external researcher so I preferred to follow routine procedure to obtain such permission. Two days later, I received the letter of approval from the education office to visit any school under their supervision so I was ready to hand it out to the participants' schools.

When the registration week finished I identified the trainee teachers who were willing to participate in the project as mentioned before. As soon as they were sent to their schools I went to each and gave them the approval letter from the education office and also asked the head teachers for their permission after giving them a short background of my research. They were all happy to help me in my request and I asked them to send me copies of the trainees' timetables when they were ready which they sent two days later. After the trainees had settled into their schools for a week, I asked to meet them at their schools to agree on a proposed plan to visit the classes and to give them consent forms for the parents of their students to allow me to videotape their boys, even though in the Saudi context there is no need for such permission as long as the head teacher agrees to it. I revised the visiting plan with their supervisor to avoid any clash between us.

After agreeing the plan with the trainees and their supervisors, I started to visit the classes and videotape the lessons. During the first visits I asked the trainees about the parents’ consent forms which were still not fully returned. Therefore, I decided to continue visiting the classes and at the end of the fieldwork I checked all the episodes with the student teachers by asking about particular students who were on the tape and if they had their parents’ consent forms; if they did not have consent I distorted their faces in the video.

4.5.4 Phase 3: Post- fieldwork.

After the fieldwork, I went back to the UK with data of about 40 lessons and interviews. I wrote a brief descriptive account of all five teachers to check the richness of the data then I was ready to move to the analysis stage (section 4.7).

4.6 Data collection methods and procedures

Research methods ‘are the tools you use to gather data’ (Dawson, 2009, p.14). As I followed the interpretive paradigm through using a case study design, I searched for methods that are compatible with my research design. Cohen et al (2007, p.261) state that ‘case studies tend to use certain data collection methods, e.g. semi-structured and open interviews, observation,…’. In addition, the nature of the data needed is influenced by my choice of collection method as summarised in Table 4.3. Therefore in this project, two main instruments were used to gather data: observation and semi-structured interview as described below.

N	Data needed to answer research questions	Method(s)
1	How teachers deal with contributions	Observation
2	Why they acted that way	Interview
3	Relationship between knowledge and practice	Observation + interview

Table 4.3: Research methods

4.6.1 Observation

Observation is ‘a method of recording conditions, events and activities through the non-inquisitorial involvement of the researcher’ (Walliman, 2001, p.241). There are two types of observation regarding the involvement of the observer: non-participation and participation observation, in which the observer keeps distant from the subject in the former and takes part in the process or activity in the latter (*ibid*). These, non-participation and participation observations, mirror the terms of ‘direct observation and participant observation’ respectively as proposed by Dawson (2009, p.32). Participant observation can be done overtly with the participant having knowledge of the researcher or covertly without the participant knowing (*ibid*). I used overt participant observation as I accessed the classes with the participants knowing about my research. The teachers were observed and videotaped on eight occasions while they were teaching in order to identify their knowledge and how they dealt with students’ contributions. It is appropriate to identify teachers pedagogical knowledge by observing their actual teaching practice in the classroom (Rowland *et al.*, 2009).

Certain incidents from the observed classes were selected and discussed with the teacher in the interview. By incidents, I mean a situation in which the teacher responded to the students’ contributions whether positively or negatively. The former refers to an incident when the teacher responded to a correct answer or a suggestion from a student, and the latter means the way that the teacher responded to an incomplete or an incorrect answer from a student. These incidents were chosen by me to reveal the interactions between the teacher and their students in order to explore the reasons for their actions.

4.6.2 Interview

The interview is a very powerful instrument used usually if qualitative data is requested (Walliman, 2001). It is a social interaction between at least two persons where one of them is the interviewer who asks questions to obtain some sort of information and the other is the interviewee who answers the questions. It differs from the questionnaire, even if they have the same questions, in the interaction between those people (Silverman, 2010). There are several different types of interviews in terms of the researcher’s attendance: the face-to-face interview in which the interviewer and the interviewee face each other; the telephone interview where the interview is conducted by phone (Walliman, 2006); and e-mail interviews as an alternative to these in which the interviewer and the interviewee exchange multiple e-mails (Meho, 2006).

Dawson (2009) differentiates between three types of interview: structured, unstructured and semi-structured. In structured interviews the interviewer has a fixed list of questions that need to be asked in turn in order to gain specific answers, whereas in unstructured interviews he/she sticks to the general idea and feels free to ask any question regarding the subject under investigation (*ibid*). The third type of interview, which is used in this research, is the semi-structured interview in which the interviewer has a list of questions drawn up prior to the interview schedule; these may be modified as required, based upon the researcher's perceptions of what seems most appropriate to the context (Wisker, 2001).

In this study, there were individual face-to-face semi-structured post-class interviews conducted in the school, after the observed lessons, in order to discuss the selected incidents. In these interviews, a number of mini-tasks were introduced to the participants to explore their SMK about some topics they taught. For example, when I suspected that the teacher omitted an exercise as a result of weak SMK and in the interview he justified it by giving a different reason not contained in my suspect, I asked him to solve one of those exercises and usually he was not able to do so. This gave some indication about his SMK. These mini-tasks came to my attention after reading some research in which the researchers, such as (Bray, 2011; Biza *et al.*, 2007), used hypothetical tasks (scenarios). Teacher responses to these mini-tasks may provide a sufficient description of their knowledge and may be incorporated with other sources (observations and interviews) in order to connect teachers' knowledge with their handling of students' contributions. The interviews, which lasted about 40 minutes each, were conducted in Arabic, and then transcribed and translated into English by myself.

4.6.3 Data collection procedures

I visited all the schools in turn over four days and on the last day of the week I summarised and finalised what I had done during the week. When I went to the school, I first visited the head teacher to inform him about my presence and then I met the trainee teacher whom I intended to visit. I asked him to go to the class before me to give him the chance to ask the students to keep calm. I sat at the back of the class facing the board and started videotaping and taking notes. I also recorded the lessons with a voice recorder in case the voice from the video was not clear. I had a routine that I followed during the observation, see Table 4.4, which helped me to focus my attention.

N	Time	Activities
1	Before the lesson	• I made sure I brought my equipment with me (camera, batteries, voice recorder, textbook, field notes). • I sat at the back to see all the class and focus the camera on the board with some view of the class. • I asked the teacher to put the voice recorder in his pocket to record his speech with individuals and in case the voice was not clear from the camera.
2	During the lesson	• I started the recording by zooming in on the board to see the writing of the teacher. • I wrote notes of what was happening in general such as the topics, how many students there were and how they were organised. • I paid more attention on the interaction between the teacher and the students when he responded to their answers, questions and suggestions. And who the student was. • I zoomed in and out if necessary to record some interaction. • I opened the textbook and followed the lesson and I copied the written work on the board and any examples written there.
3	After the lesson	• I went out before the teacher then waited for him outside. • I spoke with him generally about the lesson and asked him to hand the voice recorder to me. • I interviewed him if we had arranged that in advance or I thanked him and confirmed the date of the next visit. • I went home and wrote my impression about the lesson, with or without seeing the video, and highlighted the possible incidents which I would discuss with him in the interview.

Table 4.4: Observation procedures

When the class had finished I moved to the other trainee at the same school and did the same procedure. After visiting them four or five times I started to interview all the teachers as soon as the observed class had finished. When doing the interview I followed some fixed steps shown in Table 4.5 below.

N	Time	Activities
1	Before the interview	• I made sure I brought my equipment with me (laptop, charger, voice recorder, textbook, field notes). • I was keen to do the interview in a quiet room to avoid noise and possible disruptions. • I turned on the laptop and asked the participant if I could record the interview and I put the voice recorder on the table between us. • I welcomed the interviewee and explained the aims of the interview and how it would go. • I emphasised that his answers would have nothing to do with his marks in the training period.
2	During the interview	• I started with general questions such as: Why did you want to become a mathematics teacher? What was the lesson about? What was the aim? How did it go? Are you happy with it? Do you want to talk about some problem you faced there? • Then I moved to some specific questions related to the content of the topics taught such as: how would you better teach this concept? Which is more important, this concept or that procedure? • Next I told him that I would play a video clip from the lesson discussed earlier and I asked him about some parts of it. Then I played it one more time and then asked him about why he responded in that way? What do you think the student meant here? What was the source of the error in your opinion? • Sometimes we used the book if the teacher needed to see the example which we were discussing and I gave him time to think. • Sometimes I gave the teacher some mathematical question to check his knowledge of the topic if I suspected that he had omitted an exercise, for example because he did not know the answer. • After discussing all the incidents in one lesson we moved to other lessons either in the same interview or if not some other time.

		• At the end I thanked him for his time and asked him if he had any comments or questions.
3	After the interview	• I went home and copied the recording to my laptop and when I had time I started to transcribe it in Arabic.

Table 4.5: Interview procedures

Sometimes they asked me about their teaching, so we discussed my feedback on their classes for their cooperation with me. After visiting them about 8 lessons each and interviewing them between two and four times to discuss certain episodes, the end of the school term approached as well as the fieldwork phase, which was 15/05/2013.

4.7 Data Analysis

In qualitative research, it is better that the researcher analyses his/her data while collecting it (Walliman, 2006) but I could not do that because of time limitation. Below I outline how I analysed the data from the observations then from the interviews.

4.7.1 Observational data analysis

The aim of using observation as a tool in this research is to provide data of the interactions between the teachers and the students, and to highlight the teacher knowledge in order to link these two elements. In the first place, I intended to use the same framework as I used in the pilot study. To probe more deeply into the interactions between teachers and their pupils I wanted to use Sfard's (2001; 2008) analytic approach, which is the process of conceptualizing mathematical thinking as a form of communication. Sfard and her colleagues developed two types of analysis to investigate the ineffectiveness of discursive interactions. These two types are 'Focal' and 'Pre-occupational', and deal with 'Object-level aspects of communication … [and] …the meta-level factors', respectively (Sfard, 2001, p.31). She suggests two main ingredients for focal analysis, namely 'pronounced' and 'attended'. The former refers to the exact words used in a conversation and the latter refers to the non-verbal actions in that conversation, such as gesticulation. She also names the combinations between the interlocutors' statements and actions as 'intended focus'. Moreover, the pre-occupational analysis deals with various other aspects of the interaction. Sfard names it the 'meta-level', which includes the 'interlocutors' concerns about the way the interaction is being managed and… the weighty, and sometimes quite charged, issues of the relationship between the interlocutors' (Sfard, 2001).

However, I felt that Sfard's framework did not fit my data very well. When I tried it on just one incident it produced a very long account of words (see Appendix 3 for an example of my trial of the Sfard framework) which prevented applying it to the whole lesson. This would have lead to just focusing on small parts of the data which would affect the analysis. Therefore, I searched for a framework which pays attention to both teacher knowledge and interactions in classrooms. While I was exploring the literature, I read some materials about Rowland's framework, the Knowledge Quartet (KQ), and found that it fit my data.

4.7.2 The analytic framework

In this research, two tools were used to analyse the data from the observations: the KQ which is described here, and the follow up moves in the IRF frame which was described in section 3.3.2. The KQ was applied to the data to investigate teacher knowledge and the follow up moves in the IRF were used to classify the teachers' responses to the students' contributions into different types of responses such as accept/reject, evaluate and comment actions.

The Knowledge Quartet is a framework proposed by (Rowland *et al.*, 2005) in which they suggest that it can be used to classify how prospective mathematical teachers' knowledge can come to play in classrooms. It is derived from a ground-theory analysis of 24 videotaped mathematics lessons taught by 12 novice teachers in the UK to help teachers and tutors identify and develop teacher's content knowledge. It has four main dimensions: foundation, transformation, connection and contingency which consist in total of 20 codes described in Table 4.6 below.

Dimension	Contributory codes
Foundation: Knowledge and understanding of mathematics *per se* and of mathematics specific pedagogy, beliefs concerning the nature of mathematics, the purposes of mathematics education, and the conditions under which students will best learn mathematics	awareness of purpose (AP); adherence to textbook (ATB); concentration on procedures (COP); identifying errors (IE); overt display of subject knowledge (OSK); theoretical underpinning of pedagogy (TUP); use of mathematical terminology (UT).
Transformation: the presentation of ideas to learners in	choice and use of examples (CE); choice and use of representation (CR);

the form of analogies, illustrations, examples, explanations and demonstrations	use of instructional materials (UIM); teacher demonstration (TD).
Connection: the sequencing of material for instruction, and an awareness of the relative cognitive demands of different topics and tasks	anticipation of complexity (AC); decisions about sequencing (DS); making connections between procedures (MCP); making connections between concepts (MCC); recognition of conceptual appropriateness (RCA).
Contingency: the ability to make cogent, reasoned and well-informed responses to unanticipated and unplanned events	deviation from agenda (DA); responding to students' ideas (RCI); use of opportunities (UO); teacher insight during instruction (TI); responding to the (un)availability of tools and resources (RAT).

Table 4.6: The Knowledge Quartet – dimensions and contributory codes

Source: (Rowland, 2013, p.25)

The foundation dimension, which affects all three dimensions, consists of teacher's mathematical content knowledge, theoretical knowledge of mathematics teaching and learning and teacher beliefs concerning why and how mathematics is learnt. The second dimension, transformation, is the knowledge in action with which the teacher transforms his/her content knowledge into accessible forms which students can learn, by using powerful analogies, explanation and demonstrations. It most similar to the category of PCK proposed by Shulman (Shulman, 1986). The connection dimension is when a teacher makes instructional decisions with awareness of the connections between concepts and procedures, anticipates what the students find hard or easy and recognises the common misconceptions. The last dimension is contingency in which a teacher responds to unplanned situations and unexpected contributions such as questions, suggestions and answers. Also, deviating from the agenda if needed to and coping with the availability or unavailability of materials (Rowland *et al.*, 2009).

The KQ was used in this research because of its capacity to identify teacher mathematical knowledge in practice (Rowland and Turner, 2007). It also has a special dimension, contingency, that deals with teacher-student interactions where the teacher responds to the students' contributions, and therefore it fits well with the aims of this research. In addition, it is an empirical-based framework used mainly with pre-service primary mathematics teachers which is similar to the participants in this research. Furthermore, its use provides a rich analysis albeit in a succinct manner.

However, although the KQ has been used in different contexts in many countries such as UK, Canada and Cyprus, it is not surprising that it might need some adaptation before applying it in this study. When adapting any framework or tests to a new context, one should take into account the differences between the context in which they were derived and the new context in which it will be applied (Petrou, 2009; Delaney *et al.*, 2008). As this framework is derived from mathematics classes in the UK this does not mean it will fit with the Saudi classes without some adjustment. For example, Petrou (2009) argued that the code of ‘adherence to textbook’ does not fit well with teaching practice in Cyprus as she observed that the textbook in Cyprus was used differently from the UK. Therefore, she proposed a new code to be included in the transformation dimension in the framework. This issue also occurs in the Saudi context as the educational system there requires the teachers to follow the textbook page by page so this led to some adjustment in applying this code as all the teachers in this study were adhering to the textbooks. However, although the framework was found to be useful and most of its codes were used in the analysis, I found that the code ‘overt subject knowledge’ did not fit my data. In the English context, at least, the situation is marked by this code when the teacher shows that he knows the topics and the knowledge displayed goes beyond that. In the Saudi context, I found that all the trainees and those in the pilot study, and in fact most of those I have supervised in the past, have weak subject matter knowledge as they simply did not even know the taught topics well. Therefore, just knowing the subject is enough to be coded as overt subject matter in this study as the majority of the trainees did not show that all the time.

In the last dimension, contingency, teachers respond to students’ ideas in one of three possible ways according to Rowland *et al.*(2009, p.150): ‘ignoring what has happened’, ‘acknowledging but sidelining the response from the child’ or ‘responding to the child and incorporating their response into the lesson’. This research elaborates on the last possible response by using the IRF framework to classify the ways in which the trainees respond to their students. Their responses are classified into different types of moves, such as accept/reject, comment and/or evaluate moves and so on (Sinclair and Coulthard, 1992).

To sum up, the KQ was used, with some modification to some codes such as ‘adherence to textbook’ and ‘overt subject knowledge’, to analyse the data from the observations. It is a powerful tool to highlight the teacher knowledge taken part in the action of teaching. Teachers’ interactions with their students marked with ‘responding to children’s ideas’ were classified by applying the IRF tool with more emphasis put on the follow up moves (section 3.3.2).

4.7.3 Interview data analysis

My initial intention was to work within the data-grounded theory paradigm and I was going to use the ‘constant comparative’ technique as proposed by Glaser and Strauss (1967); while coding and comparing the data, it may be possible to identify themes, patterns and thereby generate a theory (Silverman, 2010; 2011; Cohen *et al.*, 2007; Bryman, 2004). Later, I was referred by some students from pharmacy and computer science schools to two other frameworks to analyse qualitative research, which were a framework analysis approach (Furber, 2010; Srivastava and Thomson, 2009; Smith and Firth, 2011) and a thematic approach, which is ‘a method for identifying, analysing, and reporting patterns (themes) within data. It minimally organises and describes your data set in (rich) detail.’ (Braun and Clarke, 2006, p.6). I read about these approaches and I found a similarity between them as they share the strategy of reading the data for familiarity, then coding it followed by searching for themes. All three of them are based on comparisons between codes, data and categories. However, they differ in how they display the data. In the framework approach it is seen as a good idea to use a chart to display the themes, however the other approaches do not display the themes in charts, instead the themes are in written form. Also the framework and thematic approaches can be carried out after data collection, whereas in the constant comparative approach data analysis is best done during data collection.

I chose to use the thematic approach for many reasons. Firstly, for its flexibility as it is compatible with constructionist and essentialist paradigms and it is a theoretical-free tool which provides rich and detailed analysis of data (Braun and Clarke, 2006). Also, it is easy to be learnt and used especially for the early stage qualitative researchers (*ibid*) and, in fact, Guest *et al*. (2011, p.11) stress that it is ‘still the most useful in capturing the complexities of meaning within a textual data set. It is also the most commonly used method of analysis in qualitative research’. Furthermore, Smith and

Firth (2011) state that thematic analysis fits with methods that aim to describe and interpret the participants' view, which is similar to this research as it tries to describe and interpret the participants' view of the ways in which they deal with the students' contributions. Moreover, it shares with other tools the common principles of coding and generating themes as mentioned earlier.

4.7.4 Data analysis procedures

After collecting the data from observations and interviews, the analysis began. There were two types of data, one from the interviews and the other from the observations. Data from the observations focuses on the teachers' actions and their knowledge while the data from the interviews focuses on the reasons given by the teachers for their actions and it feeds to some extent into teachers' knowledge as well. Next, I describe the procedures of data analysis in each case then explain how the cross-case analysis was done.

4.7.4.1. Observation data analysis procedures

When I came back to the UK after doing the fieldwork, I started to explore the entire data with my supervisors to assess its richness. This task required me to transcribe all the important incidents from the observations and all the data from the interviews. I created a word table (see below Table 4.7 for an example) with two columns in which I wrote the lesson's incidents of the first teacher on the left and the related chunks of the interview on the right using number codes. These codes, like (417) mean the seventh question in the first lesson of the fourth teacher, in the example below are used in the interview and the incident to link them together. The number in the incident column means that I asked the trainee about the response before the number and his answer in the interview is the one next to the same number.

Time	Incident	Interview
30:00 – 32:00	Fahad says 'If you go to the shop to buy something, what I am going to buy?' A student says 'Milk or juice'. 'No. These were in the last lesson when we learnt capacity, (417) now we are studying mass. If you buy 24 apples, how many kilo will	(417) Fahad said 'I want to tell the student that there are differences between millilitres and litres, and grams and kilograms, as this might be a common misconception. 'I

	you ask the cashier for?' Some students say '2 kilos' and some say '2.5'. While Fahad is standing still near the board, different answers are reached such as 3 kilos, 3.5 kilos and 4 kilos. (418) Fahad asks 'Why did you say 4 kilos?' Then the student says 'Because 6 plus 6 equals 12 and another 6 plus 6 equals 24' Fahad says 'We said one kilo equals 6 apples so four times 6 equals 24 apples'.	never like to ignore a student, even when I am sure he will answer incorrectly, because if I ignore him many times he will lose confidence and might become shy and not participate in the class'

Table 4.7: A sample of one incident from one lesson

After writing all the lessons' incidents and the related interviews for all the teachers I started to analyse the observation. At first, I analysed the first lesson for the first teacher with Sfard's model as I mentioned earlier in section (4.5.2) and I found that the analysis was very long. Therefore, after changing the framework to the KQ I started with the same lesson and found it was a manageable size.

Next, I recognised that even analysing eight lessons of the three teachers would be very long for the thesis so I decided to include only three lessons from each teacher in the analysis but bearing in mind the danger of missing some important data. I started exploring Abdulla's lessons by looking at all his lessons to find the desired incidents in which he interacted with the students. For example, I made a table to summarise the frequency of these incidents in every lesson as in Table 4.8 below.

Teacher	Observed Lessons	Grade	Incidents per lesson
Abdullah	1	6	4
	2	6	5
	3	5	2
	4	5	3
	5	6	3
	6	6	1
	7	5	5

Table 4.8: A summary of Abdullah lessons

After identifying the incidents in which Abdullah responded to students' contributions, I started to order the lessons decreasingly in terms of the number of incidents in every lesson. I assumed that the higher the number of incidents the more responses occurred which would give more diversity in these responses. I chose the first three lessons with the highest number of incidents to include in the analysis. Before starting the analysis of the first lesson I searched the excluded lessons for any important aspects of responses that needed to be included in the analysis. I had in mind the idea of possibly changing the chosen lessons if I found a lesson with less incidents but richer in content. I found that the response patterns are almost the same among all the lessons. I did the same procedure to Fahad and Onizan's lessons and came back with nine lessons in total.

The first lesson was analysed by writing a short synopsis (about 500-600 words) of the lesson then choosing one incident to be described in detail after the synopsis as an example of interaction. This work was followed by the actual analysis which was done by applying the KQ's codes through MAXQDA software (see http://www.maxqda.com) to the extended version of the lesson, not the short description, in order to give a more detailed picture of the types of knowledge and different responses. The code 'responding to children's idea' was applied to all the sequences of teacher-student-teacher interaction. Then these sequences were categorised into different codes of teachers' responses to answers, suggestions, questions and comments. I re-coded all the lessons of one teacher on this basis.

Next, I noticed that the interaction in the form of question-answer was the most observed sequence while the other sequences were rare. As a result I thought of focusing the analysis more on dealing with students' *answers* as it was the most frequent in the data. I then subdivided the code 'teacher response to answer' into teacher response to correct and incorrect answer. The latter refers to an incomplete answer or a completely incorrect answer. After identifying the sequence of each response to correct and incorrect answers I investigated the teacher third move (follow up move) which follows the students' answer, and checked the type of these moves (described in section 3.2.2) and the type of exchange according to the Sinclair and Coulthard's (1992) model discussed earlier in the literature review chapter. For example, the teacher's move could be a comment move in which he comments on the answer by expanding it or it could be asking for justification (press move) and so on. By that time, I had, to some extent, some information about teacher knowledge and their ways of responding but I still needed to know their justification of why they responded in that way. This information came from the interview data. Next, I explain how the interview data was analysed.

4.7.4.2. Interview data analysis procedures

The interview data was divided into two types of data: one was generally related to the teachers' background of beliefs of mathematics teaching and the other more relevant to certain parts of the observed lessons. Both of this data provided extra information about teacher knowledge but the latter provided more information about the reasons behind the ways of response. I started to code the data of the first lesson with an open-minded view in which I let the data speak. I paid more attention to parts dealing with teacher knowledge and justification as I coded them according to the aspect of teacher knowledge and response. Then the response code was subdivided into reasons of response to correct and incorrect answers. Next, all the different given reasons for one response were grouped together to form a pattern. For example, in the example above in Table 4.8 the teacher rejected the answer verbally and he reasoned that he was treating a common misconception, so the act of rejecting verbally was associated with treating common misconceptions and other reasons given in different incidents in the lesson. These reasons were grouped together to see if the similarity between them would suggest an overall reason that would join them together. After finding all the responses and their reasons and enriching the teacher knowledge data from the KQ by adding some aspect of teacher knowledge from the interview, I was ready to write the first lesson account.

I started writing the analysis of the first lesson by focusing on the codes of the foundation, transformation, less on connection but heavily on the contingency dimensions. After writing the first lesson I repeated the process for the subsequent lessons. I wrote about teacher knowledge and responses to students' answers in detail in two lessons and in less detail in the third, if it looked like the other two, to avoid repetition. After writing all three lessons analyses I wrote an overview of the teacher knowledge and response patterns among the lessons which was achieved by searching for patterns emerging from the codes of the teacher responses to students' answers. Then I moved to another teacher and checked if he fit with the existing coding or needed re-coding again. After writing all the case reports I moved to the cross-case analysis which is described below.

4.7.4.3 Cross-case analysis procedures

When doing cross-case analysis, researchers look for similarities and differences across the cases in order to fully understand the phenomena under study (Yin, 2009; Stake, 2006). After writing each case report of this research, the three cases were looked at together to see if they had common themes and to highlight the most important findings.

To do this, I re-read all the cases again mainly focusing on the overview in each case. I looked to the common response patterns among the participants and their reasons. Then I grouped the response patterns into two groups of actions and the reasons into two groups of reasons. I investigated the relationships between these two elements and teacher knowledge. Next, I moved to the difference between responses in which I found a variety of responses within the cases and I compared these regarding the various themes. This work was followed by a summary of the findings.

4.8 Research Trustworthiness

Reliability means that when different researchers use the same research methods they are exploring the same problem/issue from various angles (Johnson *et al.*, 2006). In qualitative research, in which the researcher and the researched are human, reliability is an issue, as human behaviour is not static (*ibid*). Regardless of the research approach, the researcher can create reliability in his work by describing his research procedures explicitly (Franklin and Ballan, 2001). In this chapter I tried to increase the reliability of this project by describing the research procedures in detail in terms of the data collection, analysis and choosing the participants. Also, according to Seale (1999), recording the interview can improve the quality of the data so I used a recorder to record the interview and kept my attention on the interviewee's answers to prepare the follow-up questions.

To strengthen the construct validity which refers to 'identifying correct operational measures for the concepts being studied' (Yin, 2009, p.40), I used more than one method to collect the data, as Yin (2009) argues that using multiple sources of evidence increases the construct validity. I observed how Saudi teachers dealt with student responses and I interviewed those teachers to ask them indirectly about their knowledge. I asked informally the participants' co-operating teachers for their views also. Through these procedures, I am triangulating the different data sources (Silverman, 2011) and exploring the issues from various angles.

4.9 Ethical considerations

In this study, I ensured that I was fully aware of all possible ethical issues. I was working as a lecturer in Qassim University before coming to the UK, so I asked the Dean of Arrass Teacher College to provide me with official permission to conduct my project with the trainee teachers as an outsider researcher, and I sought other letters of permission from the primary schools' Head Teachers and students' parents, allowing me to visit and videotape the teacher in their classes. In the Saudi context, these approvals gave me permission to videotape the students as well, even though they were not the focus of the camera lens unless needed to be.

To prevent being judgemental towards the participants I stressed that they were all an important part of the project and we worked together as a team towards achieving the research aims. For example, we agreed that the student teachers would inform me, without any undue pressure, of the dates when I could visit, observe and interview them. In addition, in the Saudi cultural context the trainee is expected to answer and ask questions during the teaching practice as one of their duties in this phase, so they were familiar with being asked for their views.

Privacy was taken into account also. I told all the participating teachers that all information gathered would be kept secure and would only be used for academic purposes. In addition, pseudonyms were used for both teachers and pupils to ensure confidentiality, and some image distortion techniques were applied if requested.

The participants' rights were fully discussed with them; they had the right to decline or withdraw from the project whenever they wanted without prior notice. In fact, one participant withdrew due to time commitment. In the consent form and the information sheet (Appendix 4 and 5), I explained the purposes of the study and the proposed questions; I also told them to contact me if they had any concerns whatsoever.

My position as a lecturer at the university did not influence the participants' right to withdraw from the research, as I had not taught them before nor evaluated their teaching. I was a lecturer in the university before I came to the UK and I tried to emphasise that many times to the participants. I tried to balance the power of my ex-position as a university member and as an insider researcher. I informed the participants that I was no longer a lecturer until I had finished my PhD and I did not have a strong link with their supervisors as they were new staff. That may have given them some freedom to act naturally by eliminating the fear of their supervisors knowing about their teaching from me.

4.10 Conclusion

In this chapter the methodology of this research was described and justified in terms of the research approach, design, data collection methods, data analysis, reliability and validity and ethical considerations. This qualitative study was designed in the form of a multiple case study as the cases shared some commonalities such as all of them were trainee teachers. These case studies are each unique in terms of teacher SMK and PCK. The data consists of forty observed mathematics lessons and interviews. I analysed the data from the observations using the KQ and IRF tools and the interviews by the thematic analysis method. After each case report was written, a cross-case analysis was conducted to investigate the common features and the variety between these cases.

The analysis and the discussion of the research findings are explored in the next four chapters. In order to avoid repetition among the three case reports, as they share some common aspects of practice, I emphasise the codes of adhering to text book and awareness of purpose only in Abdullah's case (Chapter 5) instead of repeating describing the situation in all the cases.

Chapter 5 : The case study of Abdullah

5.1 Introduction

Abdullah (participant 1 in section 4.4.2), a 22-year-old trainee primary teacher practised teaching in a school called Ansar which is a rural school founded in 1980 with a small number of students in every class, as there were between six and ten students per class. All mathematics classes take place in the mathematics lab, which has a whiteboard and a visual presenter. Abdullah lives in Arrass city. Abdullah was fully engaged in the school, as he did not have any other commitments at the college. He taught grade five and six, (11- and 12-year-old) students for five lessons each week. He was visited by his academic supervisor twice a month and observed by his cooperating teacher weekly. Abdullah was observed and interviewed as the table below shows:

<table>
<tr><th>Teacher</th><th>Observed Lessons</th><th>Grade</th><th>Incidents per lesson</th><th>Interviews</th></tr>
<tr><td rowspan="7">Abdullah</td><td>1</td><td>6</td><td>4</td><td rowspan="4">1</td></tr>
<tr><td>2</td><td>6</td><td>5</td></tr>
<tr><td>3</td><td>5</td><td>2</td></tr>
<tr><td>4</td><td>5</td><td>3</td></tr>
<tr><td>5</td><td>6</td><td>3</td><td rowspan="3">2</td></tr>
<tr><td>6</td><td>6</td><td>1</td></tr>
<tr><td>7</td><td>5</td><td>5</td></tr>
</table>

Table 5.1: Observations of Abdullah

Note: lessons 2,7 and 1 are analysed in this thesis

In this chapter, the three lessons that have the highest number of incidents as shown in Table 5.1 are analysed. I start with lesson 2, then 7 and the last lesson is lesson 1. The third lesson (lesson 1) is written in less detail than the first two lessons to avoid repetition. In every lesson there were different types of incidents of which I chose one to show the reader an example of the interactions. The incidents chosen have a variety of teacher responses and the teachers have shown clear evidence of different types of knowledge in these incidents.

The analysis of each lesson starts with a short description of the lesson then a description of one incident, then the KQ is applied to the whole lesson (See Table 4.6 for the used KQ codes). That is followed by applying the IRF framework to the dimension of contingency where the interaction between the teacher and his students are highlighted and the teachers' follow up moves are examined. Towards the end of this chapter, I outline the teacher knowledge and responses to his students' contributions across all analysed lessons.

5.2 The first lesson (lesson 2)

This lesson was entitled 'Problem-Solving Investigation: Looking for a Pattern', which was taught in the Maths lab to Grade 6. Its aim according to the book was to solve problems by looking for a pattern, such as 'Find the three missing numbers: 3, 6, 10, 15, 21, …, …, …'. The lesson was preceded by 'Solving Proportions' lesson and it went as described below:

5.2.1 The lesson description

Abdullah started the lesson standing in front of the class and solved with them two exercises chosen from the book by the students which were: 'solve the proportions $\frac{2}{3} = \frac{n}{9}$ and $\frac{30}{54} = \frac{a}{9}$' . Two minutes later, he asked the students to separate from each other and he distributed a test containing four questions on ratio, rate, ratio tables and proportion, taken from previous lessons. He told them that the test would take just 5 minutes but it took approximately 15 minutes. The questions in their sequence were:

- 'Majed invites 6 men and 15 children into his home. What is the ratio of the children to the men? Explain that'
- 'Find the unit rate for 180 words in 3 minutes'
- 'Ahmed has a discount 7 Riyals every week. What is the total of the discount after four weeks?'

Discount	7			
Weeks	1			4

- 'Solve the proportions $\frac{4}{3} = \frac{f}{45}$ and $\frac{2}{x} = \frac{16}{40}$'.

As soon as they finished solving the test, Abdullah put the test on the board and started to solve the questions, for approximately eight minutes, with the students' help. He asked short questions and they shouted out the answers which he wrote on the board. The last question in the test was solved as shown in the incident below:

Incident 1

This incident is chosen as it shows a variety of responses to students' contributions and evidence of Abdullah's knowledge. It occurred as soon as they solved the previous question, $\frac{4}{3}=\frac{F}{45}$, they moved to the last question of the test which was 'solve the proportion $\frac{2}{x}=\frac{16}{40}$' .

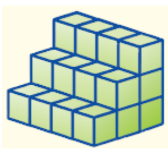
1	Student 1	It is 5.
2	Abdullah	Why is it 5?
3	Student 2	5 times 8
4	Student 1	Because 2 times 8 equals 16 and 5 times 8 equals 40.
5	A	Wait, [He gives the student the wait sign with his fingers], I will tell you, here there is no multiplication or anything else, it is just finding x and that is it.
6	Students	Ok
7	A	So, how much did you say?
8	Student 1	5, because 5 times 8 is 40
9	A	Why 8 times 5 [He goes towards the teacher's book].
10	Student 1	Because 2 times 8 equals 16 and 5 times 8 equals 40.
11	Student 3	We divide
12	A	[Comes back to the board] 2 and 16. We divide by what?
13	Student 4	By 5 ,,, I mean by 8
14	Students	by 8
15	A	8 divided by 2 [He goes to write it on the board]
16	Students	No, 2 times 8 is 16.
17	A	Do I multiply or divide?
18	Some	Multiply
19	Others	Divide
20	Others	Multiply
21	A	Bassam why did you say divide?
22	Bassam	I meant multiply
23	A	Why multiply?
24	Bassam	I forgot
25	A	I multiply by what?
26	Students	By 8
27	A	[He writes$\frac{2\times 8}{x\times 8}=\frac{16}{40}$]. Which number can be multiplied by 8 to get 40?
28	Students	5
29	A	[He writes $\frac{2\times 8}{x\times 8}=\frac{16}{40}=5$].

In the middle of the lesson, Abdullah spent ten minutes doing two activities. For the first he nominated a student to come and solve a proportion which Abdullah chose from the book ($\frac{2}{3}=\frac{\mathrm{n}}{21}$), but the student could not do it so Abdullah asked another to do it. For the second one he asked the students to open their books and complete the exercises from the day before lesson. The exercises were questions 16, 20 and 21 in the book:

- ‘A horse drinks 120 bottles of water every 4 days. How many bottles of water does this horse drink in 28 days?’
- ‘Find x in $\frac{11}{13} = \frac{x}{91}$’
- ‘Find g in $\frac{96}{128} = \frac{12}{g}$’.

He told them not to solve question 22 ($\frac{5}{12} = \frac{x}{6}$) without explaining why. The teacher let the students solve the questions individually and later he asked Thamer to come and write the first two exercises on the board and solve them.

Nine minutes before the end of the lesson, Abdullah started introducing the new lesson, ‘Problem-Solving Investigation: Looking for a Pattern’ which was meant to be the focus of the lesson. On the board he showed the example from the book (Figure 5.1) of a student building a stair by using 4 cubes for the first step, 8 cubes for the second and 12 steps for the third. He wanted to know how many cubes are needed to build 8 steps by using the strategy of looking for a pattern. They together filled in the table.



Steps	1	2	3	…	8
Cubes	4	8	12	…	

Figure 5.1

Three minutes later and towards the end of the lesson, Abdullah moved to questions 4 (Figure 5.2) and 8 which were ‘draw the next two shapes in the pattern:’

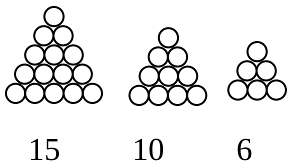


Figure 5.2

and ‘find the missing numbers in : 3 , 6 , 10 , 15 , 21 ,   ,   ,’.

He solved it with the whole class in the same way as was done with solving the test.

5.2.2 Applying the Knowledge Quartet (KQ) and IRF

The following analysis takes into account the class observation and Abdullah's comments in the interview.

Foundation

The purposes of mathematics teaching are varied among individuals and even social groups (Ernest, 2002). Niss (1996) states that there is a strong relationship between the goals of mathematics education and the reasons behind providing it, and that it is sometimes hard to distinguish between them. Each group has their own reasons and aims to teach mathematics in school. For example, Ernest (2002; 2000; 2014) identifies five different social groups with different aims for mathematics education. The groups are industrial trainers, technological pragmatists, old humanists (mathematicians), progressive educators and public educators. Their aims are: to acquire basic mathematical skills and knowledge; acquire a high level of mathematical knowledge and be able to solve practical problems; understand advanced mathematics with increasing confidence and creativity; to enable the learner to be critical; and to produce mathematically knowledgeable citizens in society respectively (*ibid*). The underlying reasons of the first two aims probably focus heavily on the utilitarian aspect of mathematics where the students can use their mathematical knowledge in real life. The reason for the third aim might be the enjoyment and beauty of pure mathematics which mathematicians feel when they work with mathematics. Finally, for the last two aims the reasons could be the importance of personal development and the development of interaction with society respectively. To sum up, mathematics is taught in schools for many reasons and some of them are: it is useful, rewarding, enjoyable, helps to build a personal identity; and, mathematics can be seen as an aspect of culture.

Abdullah started the lesson by revising four topics in order to, as he said in the interview, 'Prepare the students for the actual exam'. His awareness of the revision's purpose (AP) seems to be to some extent utilitarian. He may have introduced the revision in order to pass the exam and thereby move to the next schooling stage. This might justify how he taught this part of the lesson as he did not care whether they enjoyed it or found it fruitful or even understood it. His focus was primarily on getting the correct answer without paying attention to understanding the solution process as he may believe that that was what they needed to know to pass the exam. For example, for the majority of the questions, he asked the students questions such as ‘What can I put here’, ‘What shall we do?’ and ‘Do I multiply or divide?’ which can be considered as signposts to direct the students towards the answer without explaining why they should follow that particular route. He probably has the view that applied mathematics is based on skills more than understanding (Chambers, 2008). Although, this belief may contradict with what he said about the nature of mathematics

> When you face a problem or equation, I find it a challenge to solve it. Even when I give up quickly sometimes I find myself thinking about it so I like this feeling actually. It gives the subject life, not like other subjects which you just need to memorise and then write in the exams.

The test contained similar questions to those which the students had solved in previous lessons. Abdullah justified his choice by saying ‘I gave them something familiar on purpose. I think when they repeat the same exercises they will understand them better and they will be able to solve any exercise like it’. He behaved in the same way when he asked a student to come and solve a proportion on the board, as he said to the student ‘We have solved four exercises with proportions in front of you so you can solve this can’t you?’. He tends to use something familiar, probably a result of his concentration on procedures (COP). He might believe that it is better to learn mathematics this way, in order to help them pass the test, and he wants them to use the same method again and again, even in new situations. This belief may reflect, to a large extent, the ‘transmission’ orientation belief towards mathematics education which focuses heavily on teaching that fits with the view of mathematics as a set of unrelated procedures and routines (Askew *et al.*, 1997). Teaching in this orientation focuses heavily on ‘…clear verbal explanations of routines.’(*ibid*: 33). One aspect of this belief is that the teacher tends to give the students his instructions as steps and the students must do them one by one (*ibid*: 44). Abdullah in fact guided the class through solving the lesson exercises by exchanging questions and answers for fragments of steps. For example, Abdullah said ‘I told them [the students to solve a proportion] to start from the smallest part moving into the biggest part by using multiplication’.

Abdullah showed some hesitancy in his teaching. His subject matter knowledge was not evident all the time (OSK). The lesson had many concepts such as ratio, rate, proportion and ratio tables and their related procedures. It also had the strategy of solving problems by looking for patterns. Abdullah may have introduced the concepts in previous lessons so I did not expect him to outline them in detail. He was not sure sometimes which procedure he should follow. For instance, he asked the students ‘multiply or divide’ ([17] in the incident) several times and when they suggested one or he was faced with an unexpected answer, he sometimes went to check in his book ([9] in the incident). He justified that by saying ‘To be honest, I was not sure about the answer myself so I went [to my book] to check it’. In the beginning I thought this was because he wanted the students to think and decide which way to go, however, later I suspected that he might have some gaps in his knowledge. As a result of this, and for his skipping of question 22 ($\frac{5}{12} = \frac{\mathrm{x}}{6}$), I asked him to solve the same proportion $\frac{5}{12} = \frac{x}{6}$ and he found incorrectly that x = 10, as he said ‘Six times two is twelve so five times two is ten; x is ten’. Then I asked him to think again and he insisted on this answer. I avoided asking him further questions in order to protect him from being judged and to avoid any possible embarrassment.

Transformation

Abdullah did not pay attention to the importance of choice of representation (CR). All the questions except two were solved by using just verbal language without pictorial (iconic) or physical (enactive) representation (Bruner, 1974). He focused on abstract symbols, (symbolic representation) (*ibid*), in order to solve the test and explain its concepts and procedures and then he used two pictures written in the book to introduce the examples of the new lesson.

He demonstrated (DT) several ideas and exercises to the class by asking the students some questions to guide them towards the answer then he wrote the answers on the board. From the transmission orientation belief perspective ‘Interactions between teachers and pupils tend to be question and answer exchanges in order to check whether or not pupils can reproduce the routine or method being introduced to them’ (Askew *et al.*, 1997, p:33). However, he sometimes invited some students individually to participate in the solving process as he said, ‘I asked Azzam to participate as when I saw him he was busy with something else’ and ‘I try my best to encourage Bassam to participate and strengthen his learning’. He tried to convince some students that their answer was incorrect by directing their attention to show that their answer was wrong.

For instance, when a student solved the proportion $\frac{2}{3} = \frac{n}{21,}$ as n=6, Abdullah discussed the solution with the student step by step to convince him that his answer was incorrect. The student justified his answer as, three times two equals six; Abdullah then showed him that three times three equals nine not twenty one. He seemed to deploy the cognitive conflict strategy as he made the student unsure about his answer in order to create a disequilibrium for the student which promoted the student rethink to achieve the accommodation stage (Lerman, 2014). This incident did not fit with the typical image that I had in mind of Abdullah as a teacher who has a weakness of knowledge and someone who teaches in a procedural way. Moreover, he usually wanted to know the source of the students' wrong answers as he said 'I just wanted to know how he [a student] was thinking about the question'.

Connection

Abdullah showed an explicit connection between this lesson and previous and subsequent ones. He used the test to revise the old lessons and prepare the students for the next lesson, which was to be the exam.

However, the connection of the lesson's concepts (MCC) was not always clear (or partly missing). For example, the test contained ratio and proportion and Abdullah did not mention the connection between them, but this may have been explained in the previous lesson. In the interview, Abdullah stressed the importance of connecting the concepts of the greatest common factor and the simplest form but he did not mention that when they simplified the first question $\frac{6}{15}$.

Abdullah was able, to some extent, to anticipate the complexity (AC) of some tasks. For instance, he expected the students to have some difficulties in working with proportions as he said 'They still did not get the idea of working from both sides [of a proportion], left to right or right to left' and he also said 'I think the hardest part in the simplification is finding the greater common factors'.

Contingency

Abdullah responded to the students' ideas (RCI), suggestions and correct and incorrect answers differently. I outline these in the following paragraphs by referring in some parts of the analysis to the incident described above.

Abdullah responded to the students' suggestions by accepting them and sharing them with the class. For example, in this incident, when student 3 [11] suggested division as a way of solving the proportion (student response), Abdullah accepted this suggestion and shifted the student's thinking from multiplication (which student 1 is proposing) to division (accept move followed by press move in which the student is requested to justify his answer (Brodie, 2008)). In other parts of the lesson, he ignored the suggestion. For example, a student suggested that '[they] divide not multiply' (student response) and Abdullah's response was 'Azzam what do you think?' (nomination move followed by elicitation move) Abdullah justified his action by saying 'I asked Azzam to participate as when I saw him he was busy with something else' so wanted to distract him rather than ask Azzam his opinion of the suggestion. Moreover, sometimes he evaluated and commented on the suggestion. For instance, when a student suggested 'Divide the top number by 2 and the bottom one by 3' (student response) for simplifying $\frac{6}{15}$, Abdullah said '… [T]his is not correct … You cannot divide the numerator by 2 and the denominator by 3. It should be divided by the same number' (evaluate move followed by comment).

He dealt with students' correct answers in different ways. For example, he sometimes asked for justification to examine the students' certainty about their answer. For instance, when the student answered 'it is five' [1] (student response), he asked 'why it is five?'[2] (press move). Also he commented on the correct answer ([3] and [4]) by trying to encourage the students to think in another direction [5] '…there is no multiplication or anything else…' (comment move). He was then probably trying to direct the student to solve the question by simplifying the fraction $\frac{16}{40}$ by division. On another occasion, he ignored the correct answer when a student suggested another way to solve the question [10] (reject move). Later in the incident, when Abdullah returned to the board after reading the answer in his book and shifting his strategy from multiplication to division, he was given a correct answer to his question '16 and 2, we divide by what?' and accepted it (accept move) and wanted to write it on the board[15] (revoice move). The same response, asking for justification or accepting it and writing the correct answer, happened again at the end of the incident. In other parts of the lesson, he praised a student (accept move) when he answered correctly by saying 'well done' in order to, as he said, 'encourage him to do more and more'. Moreover, he rejected a correct answer when he asked a student to fill the table (Figure 5.1) and the student said '16 comes after 12' (student response). Abdullah then asked 'Why 16?' (press move) and the student said 'because 4 times 4 is 16' (student response). Abdullah then rejected the answer by saying 'No, it increased from 4 to 8 by four so keep adding fours not multiplying by 4' (reject move followed by comment). He justified his action by saying 'I think that repeated addition differs from multiplication'.

Similarly Abdullah dealt with students' incorrect answers differently. In different parts of the lesson, he repeated the answer to make sure that the student meant this answer and was sure about it. For example, when the student answered by saying 'Subtract' (student response) Abdullah repeated the answer to check his certainty (evaluate move). In addition, he sometimes asked for justification. For example, when a student said 'N is six' in the proportion $\frac{2}{3} = \frac{n}{21}$ (student Response), he asked him 'Why it is 6?' (press move). Moreover, he occasionally responded by making an effort to convince the students about the mistakes in their answers (comment move) and rarely does he say 'your answer is wrong'. Once he asked other student to solve a question after the nominated student could not solve it correctly. In addition, he occasionally responded physically to an incorrect answer by looking at the student who answered as a sign for 'no'. For example, he looked (reject move) at the student who answered 'eight' (student response) when he asked 'how much is 12 plus 4'.

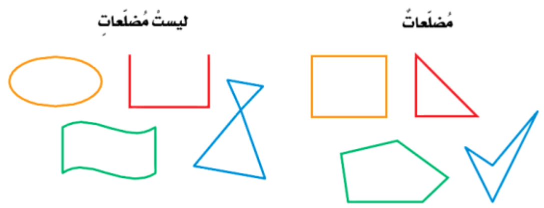
In conclusion, Abdullah showed a weakness in his subject matter knowledge in the taught topic and has a belief in the utilitarian purpose of mathematics. This affected his teaching in the lesson as outlined in the earlier sections. He focused heavily on procedures and did not make a connection between the lesson concept and procedures, such as linking the division with multiplication or connecting multiplication with repeated addition. He seemed to care about the sources of the students' misconceptions. The majority of exchanges in the class were eliciting exchanges ending with a variety of third moves such as accept, reject and press.

5.3 The second lesson (lesson 7)

The lesson was entitled 'Perimeters of Polygons', which was delivered to Grade five in the Maths lab organised as three tables. The book's aim was to find the perimeter of a polygon by adding the lengths of the sides and using the formulae (4X and 2L+2W) to find the perimeter of squares and rectangles. The lesson went as below:

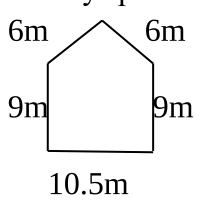
5.3.1 The lesson description:

Abdullah showed the lesson's pages from the textbook on the board using the document camera. He started the lesson from the 'Get Ready' section in the student's book which contains worked examples, and Abdullah explained how they were solved by always asking questions. The students gave answers then he put them on the board. He read out the definition of a polygon with the class, which was written in the student book as, 'A polygon is a plane closed shape with straight sides which doesn't cross over itself'. Below the definition, there were many shapes grouped under two headings: polygons and not polygons as shown below:



After reading the definition, he examined the polygon shapes. He pointed to the triangle and asked 'why is it a polygon? Is it a plane? Is it closed? Does it have straight sides? And does it cross over itself?' Then he repeated the same questions for all the other shapes. During these six minutes there were some questions and answers between Abdullah and his students. For example, a student asked 'why is this shape not a polygon', and another student started to answer him, but then Abdullah asked the second student to stop talking and he answered the question himself.

He then wrote the formula of the perimeter of a polygon on the board as 'it is the sum of its sides'. The class read the formula and Abdullah moved to the first example: 'Find the perimeters of the polygon'. They spent nearly three minutes doing so.



For the next six minutes, Abdullah moved to the next part in the book which was an activity filling in a table of many squares with their sides and perimeters as described in the following incident. They then solved an example from the book of a picture of a square and requested finding the perimeter of a square with sides of 2 units.

Incident 1

This incident is chosen as it shows a variety of responses to students' contributions and evidence of Abdullah's knowledge. After finding the perimeter of a polygon, Abdullah asked the students to do the activity: 'fill in the table below and describe the relationship between the perimeter of the square and its side length'.

Square	1	2	3	4
Side length	1			
Perimeter	4			

Figure 5.3

This dialogue happened during the activity:

1	Abdullah (A)	[Pointing to the square 1] Here the side length is how much?
2	Students	Two
3	A	[silent]
4	Students	One
5	A	Good. What is the perimeter of the first square?
6	Students	Four
7	A	How we find it four?
8	Student1	Because it is the sum of the side lengths
9	A	That would be right for the polygon but this is a square not a polygon. [He writes on the board 'the perimeter of a square =']
10	Student1	I know the answer, it is because one side is one, so the others are one too
11	A	All of these squares are different [pointing to the table]
12	Student 1	I mean the first square has four sides of one, so it is one, one, one and one.
13	A	Do not add the sides because that was correct for the polygon. [He repeats the question] How do we find the perimeter equal to 4 for the first square?
14	Student 3	[Stands and goes to the board and touches the sides of the first square and says] we add one, one, one and one.
15	A	No. [He then reads the formula of the perimeter of a square which is written below the table and writes 'the perimeter of a square = 4 x'] so, what is the perimeter of a square?
16	Students	it is four x

17	A	What does x refer to?
18	Students	[The answers were] one, two, litre, millilitre, rectangle, square, angles, polygon and the length.
19	A	[Listening to the answers and moving from one student to another until he reached the correct answer] It is the length, so the first square has a side of one and its perimeter should be?
20	Students	Four
21	A	Good, we multiply the side of one by four and get four, what about the second one?
22	Students	Two times four equals eight.
23	Student4	Teacher, what is the perimeter?
24	A	Right, you do the same thing to the two squares three and four and you will find that the perimeter will be?
25	Students	Twelve and sixteen.
26	A	Right, memorise the formula to use it elsewhere.

For the next four minutes, they moved to the next part which was about rectangles. They read the definition of the perimeter of a rectangle and Abdullah wrote it on the board as '2W + 2L' under the formula of a square. There was some discussion about the definition of W and L in the formula. Then the class moved to the example of a picture of a rectangle notebook with a width of 18 cm and length of 22cm and they were asked to find the perimeter of the notebook.

Abdullah asked the students to solve the exercise of the 'Make Sure' section individually in the last 25 minutes. He moved around between the three tables and corrected their work until the bell rang.

5.3.2 Applying the Knowledge Quartet (KQ) and IRF

The following analysis takes into account the class observation and Abdullah's comments in the interview.

Foundation

Abdullah's adherence to the textbook (ATB) (students' book) influenced his conduct of the lesson. I considered Abdullah's use of the student book as 'adhering' in the light of Nicol and Crespo's (2006) proposal for some features of using textbooks by teachers. They suggest that the characteristics of teachers who adhere to textbooks are that they are using it as their main source without any modification and they accept it as the authority of what to teach and how. He stuck to the book in every part of the lesson and followed the order of introducing the lesson's concepts, examples and ideas. He tried to solve everything from the lesson's pages and asked the students to complete the exercises for homework. Abdullah’s use of the textbook affects, to some extent, some of his teaching as I illustrate in the other dimensions of the Knowledge quartet (KQ) in the following paragraphs.

Teachers use textbooks differently for many reasons. Their reasons can be categorised into two main kinds: external and internal. By the former, I mean the reasons that relate to the classroom atmosphere and the latter relates to the character of the teacher. Research shows that teachers tend to use textbooks closely as a result of a limitation of time, availability of resources, the influence from colleagues and cooperating teachers (Nicol and Crespo, 2006) and district and school policies (Sosniak and Stodolsky, 1993). The Nicol and Crespo (2006) study found that teachers who did not have time to look for information usually adhered to textbook. Also, they found that when the teacher did not find good alternative sources they stuck to textbook. In addition, they state that the cooperating teacher’s using of textbook has a great influence on the participants as the trainees did the same use of their cooperating teachers by adhering or elaborating or create their teaching materials. Moreover, Sosniak and Stodolsky (1993) found that government and school policies, among others, influence the teacher’s use of the textbook. They found that teachers used selective and variable use of textbook to achieve the educational district aims such as sticking rigidly to mathematics textbook exercises.

The internal reasons of different textbook use, according to research, could be the teacher’s belief about the nature of the subject and how it can be taught (Chavez–Lopez, 2003; Johansson, 2006; Nicol and Crespo, 2006; Sosniak and Stodolsky, 1993). In addition, the teacher’s knowledge, skills and experience shape the use of textbooks (Chavez–Lopez, 2003; Sosniak and Stodolsky, 1993). Moreover, the extent of teacher’s interest in the subject influences the teacher’s use of textbooks (Remillard, 2005).

Abdullah adheres to the textbook as a response to the Saudi educational system which requires all the teachers follow the same book for every stage and every subject. He could modify some parts of the lessons if he wanted to, but he did not do so. As he said in the interview ‘I was told by my supervisor [the cooperating teacher] to try to cover as much as I can from the book so I do not have time to bring any example outside the book’. He emphasised that the limitation of time, the authority of the sources, the education system and the cooperating teacher, played an external role in his adherence to the textbook.

Regarding the internal reasons for adhering to the textbook, it seems that Abdullah's belief and knowledge, and probably lack of experience, influenced his use of the textbook. He believes that mathematics is a challenging subject that required the mastering of procedures and roles and that the teacher should focus on the students' performance. He said:

> I love maths because I feel it challenges me all the time…I am keen to teach the students how to do things rather than confusing them with many details of the concepts… I sometimes prefer to give them the steps to solve the exercise before asking them to solve the exercise by themselves as this avoids them getting stuck.

These beliefs may have influenced his use of the textbook in a way that concentrated on solving as many of the exercises as possible in order to complete the majority of the Saudi mathematical textbooks content. I found that Abdullah always asked the students to use their textbooks and solved the exercises with them one-by-one, putting more emphasis on the procedure (process) rather than learning the concepts.

In this lesson, he seemed to believe that the book is the source of authority and that all of its content must be used. As he said

> I found a written formula for the perimeter of a square in the student book, so I thought … they must apply the formula. I wanted to encourage the students to use the formula instead of adding the side lengths … Because it has its formula, I stuck to it.

Abdullah's knowledge probably affected his use of the textbook as well. According to Nicol and Crespo (2006), teachers' understanding of the content influences their use of textbooks. For example, a teacher who is familiar with the topics does not face difficulties in covering the content. Abdullah did not show some indication of overt subject knowledge (OSK) in different parts of the lessons. For instance, he always kept checking his book to see the answer and also occasionally he skipped some exercises without justification. This is indicative of the subject knowledge difficulties which led him to avoid some difficult exercises. In this lesson, Abdullah seemed not to be fully aware of the way of finding the perimeter of a square as he asked the students to find it just by using the formula. Also, in the incident he thought that a polygon cannot be a square [9]. He made a small change in the order of the textbook to avoid his misconception of the way needed to find the perimeter of a square by adding its side length.

Transformation

Abdullah's demonstration (TD) was influenced by his knowledge, beliefs and adherence to the text book. The concept of a 'polygon' was introduced by reading the definition then showing pictures of many polygons instead of, for example, showing pictures of many polygons and asking about the similarity and differences between and then suggesting the definition (Rowland *et al.*, 2009). He introduced the concept of the perimeter of the square in a way that does not emphasize the importance of understanding it or connecting it to the perimeter definition. His explanation of the concept of the perimeter of the square was done by requiring the students to use the formula which was written in the book without explaining it well. He could have introduced it empirically by starting from the formula of the perimeter of a polygon then using that to find the formula of the perimeter of a square (Steele, 2013). He did not allow the students to use their prior knowledge about finding the perimeter of a polygon in a new situation to generate the new concept or formula. He justified his action by saying 'I found a written formula for the perimeter of a square in the student book, so I thought that to find the perimeter of a square the student cannot add the sides; they must apply the formula'. His focus on procedural knowledge is evidence of his belief of mathematics as a set of procedures and how it ought to be taught, or as a result of his lack of knowledge. He said 'I am keen to teach the students how to do things rather than confusing them with many details of the concept'.

Connection

Abdullah introduced the lesson as it was written in the textbook. He started with the polygon concept and perimeter, and then moved to finding the perimeter of the square and the rectangle without paying attention to the connection between these procedures (MCP). There was no link between these formulae and, as I said earlier, he did not use the procedure of finding the perimeter of the polygon as a base to build up the formula of the square. For instance, in the incident, when the students tried to use what they had just learned about the procedure of finding the perimeter of the polygon in order to find the perimeter of the square, [8, 10, 12 and 14], Abdullah did not allow them to make this link as he probably believed that there was no connection between them.

Contingency

Abdullah responded to the students' ideas differently (RCI). He responded to the students' incorrect answers in the incident by ignoring them and keeping silent ([2] and [3]) then looking for the correct one (reject move followed by nomination); as he said 'They gave me a wrong answer and I was looking for a correct one'. In addition, he kept moving from one student to another until he found the correct answer ([18] and [19]) (reject move followed by nomination). He reasoned that by saying 'I was looking for the correct answer so I kept moving between the students until I found the right person'. Moreover, Abdullah thought some of the correct answers were wrong because he thought they should have stuck to the formulas in the book so he changed the answers himself. For instance, in the section [8] – [15], the students tried to add the side lengths (student response) as they had done with the polygon, which is a correct thing to do, but Abdullah changed their answer by saying 'no' (reject move) and introducing the formula after that (clue move). He justified his action by saying 'I found a written formula…so I thought that…they must apply [it]'.

However, at times he responded to the correct answers by saying words such as 'good' and 'right' ([5], [21], [24] and [26]) (accept move). He said:

> I should correct the wrong answers and praise the correct answers, for example by asking the students to clap their hands for the one who answers correctly to encourage the students to become more active in the lesson.

Also he asked for justification, such as 'How do we find it four?' [7] (press move). He justified that by saying 'I want to make sure that they answered based on an understanding, not guessing'. Moreover, as a result of his adherence to the textbook, he rejected some correct answers [8] – [15] (reject move followed by comment) for his own and changed them to another form, even though they both led to the same answer.

Finally, he was faced with many questions from the students; he answered some of them and ignored others. For instance, in [23] he ignored a fundamental question about the concept of the perimeter which is what the lesson was about (teacher's second turn was missing). In addition, at the beginning of the lesson a student asked the teacher a question but his peer tried to answer so Abdullah asked him to stop and answered the student himself (teacher second turn was directive move followed by comment). He justified that by saying:

> I am keen to stress the importance of the understanding for the students… I prefer to answer the student's question by myself … because I believe that I can deliver the idea to the questioner better than the other student.

This tendency of Abdullah to emphasise the importance of understanding might show some special beliefs about certain aspects of learning.

In conclusion, Abdullah in this lesson tried to cover as much as he could of the lesson content, however he paid less attention to connecting the lesson's concept and procedures. He was supposed to use the perimeter of a polygon as a starting point to work out the formula of the perimeter of the square, but he did not do so because of his knowledge, beliefs and adherence to the student book. This also affected his response to the students' contributions as well. In general, he dealt with students' incorrect answers by rejecting them and with correct answers by accepting them. The main exchanges in the two types of actions were reject move followed by nomination and accept move, followed by comment or press move respectively.

5.4 The third lesson (lesson 1)

The lesson was entitled 'Rate and Ratio', which was taught to Grade 6 in the mathematics lab. It was the first lesson of the chapter of 'Ratio and Proportion', which follows the 'Fraction' chapter. Abdullah's aim was to enable the students to write rates and ratios in fraction forms. The class progressed as described below:

5.4.1 The lesson description

Abdullah started the lesson standing in front of the class on the left of the board and showed the lesson pages on the board by using the visual presenter. The first warming up exercise, which was from the book and took nearly three minutes, was a picture of six blue and two red paper clips and the class was requested to compare the two by using the words 'bigger and smaller', then by a fraction. When he asked for volunteers, the students came up with different answers such as two over six, five over four and two from six. Then he asked a student to read the definition of ratio in the book out loud, ‘A ratio is a comparison between two numbers using division’, and he asked the class to write a ratio that compares the red clips with the blue ones in the simplest form.

For about two minutes, Abdullah read out loud the question of ‘Write the ratio which compares the number of suns with the number of moons [there was a picture of 4 suns and 6 moons] in its simplest form’. He told the students they have two minutes to solve it. A few moments later, Abdullah walked around the class and then solved the exercise with them, whereupon the students answered out loud and Abdullah wrote the answers on the board.

He moved to the third example, which was a table of figures, and asked the students to find out the ratio of 8 pigeons out of a total of 36 birds. They solved it together for three minutes and Abdullah wrote the answer on the board.

During the next six minutes the class read the definition of rate from the book, ‘A ratio that compares two measurements with different units’. Abdullah read an example in the book, which was ($\frac{10\ Saudi\ Riyal}{2\ Jordan\ Dinar} = \frac{5\ Saudi\ Riyal}{1\ Jordan\ Dinar}$) and explained that they must divide the fraction on the left by its denominator to get the unit rate. Then Abdullah wrote the exercises ‘Write the unit rate of 9 Riyals for 3 cupcakes’ (described in the incident below) and ‘Write the rate of a car that covers 232 km in 4 hours’ as practice of the concept of rate. He walked around the tables, asked the students about their answers and then he solved it with the students’ help.

In the last thirty minutes of the lesson Abdullah asked the students to solve the rest of the book exercises individually and he walked around the tables and checked their answers; more than once he corrected the individual students’ answers then solved with them some of the questions on the board. At the end of the lesson he elected a student to come and solve one exercise on the board.

Incident 1

The exercise was ‘Write the unit rate of 9 Riyals for 3 cupcakes’. This dialogue happened during the activity:

1	Abdullah	This exercise is about rate. Can you solve it?
2	Students	We finished
3	A	[Moves toward the board] What is the answer?
4	Students	One third.
5	A	[Moves around the three tables where the students sit and checks their solutions]. Thamer, have you finished?
6	Thamer	Shakes his head.
7	A	[Looking at a group] have you finished?
8	Students	Yes
9	A	Wait for the rest of the class [walks to Thamer’s group (three students) and asks one of them] Why have you written nine over one? [moves to the other one in the same group and asks] What is the denominator?
10	Student 1	Nine
11	A	No
12	Student 1	Wrong?
13	A	Yes, What did we say about the denominator? [Goes to the board and says loud] How much should the denominator be? [looks in the teacher’s book as well] [Fifteen seconds later, a student says]
14	Student 2	It is one
15	A	Ok [goes to the board and starts speaking and writing at the same time], Firstly, you write the fraction … what is it? [Writes the fraction line (-)].
16	Students	Three over nine.
17	A	[Writes $\frac{3}{9}$], OK [waits for the completion of the solution from the students].
18	Students	Divide by three. Three divided by three is one and nine divided by three is three.
19	A	[Moves toward the teacher’s book to check. Following this, he goes to the board and asks] how much must we divide by? [He writes $\frac{3\div}{9\div}$].
20	Students	Three divided by three equals one and nine divided by three equals three.
21	A	[Writes $\frac{1}{3}$]. What have we said about the rate? What is the denominator?
22	Students	It is one
23	A	I told you that it should be 1. But here, why is it not one?
24	Thamer	Divide by nine. Nine divide nine and three …
25	Student 3	Turn it upside down so nine is above and three is under.

26	A	[Deletes the fraction and writes $\frac{9}{3}$, divides it by 3, and writes the answer $\frac{3}{1}$]

5.4.2 Applying the Knowledge Quartet (KQ) and IRF

The following analysis takes into account the class observation and Abdullah's comments in the interview

Foundation

The lesson contained the concepts of ratio and rate which are core topics in primary schools. A ratio is 'A comparison of two things with respect to size and can be represented by a fractional expression, i.e., a/b' (Son, 2013, p.50), whereas rate can be defined as 'A complicated concept comprising many interwoven ideas such as the ratio of two numeric, measurable quantities but in a context where both quantities are changing' (Herbert and Pierce, 2011, p:455). It seems that rate can be considered as a special kind of ratio which compares two measurements with different units. In addition, rate is a hard concept 'To teach and learn' (*ibid*, p.456) and it is probably hard to clearly distinguish it from ratio. Abdullah was not able to fully distinguish between them as he said in the interview 'As far as I know the only difference [between rate and ratio] is that the denominator in the rate must be one, although I am not sure whether this is correct or not'. His statement brought to mind the concept 'Unit rate' which is a rate with a denominator of one, so he might believe that all rates must have a denominator of one. This inability to distinguish between the three concepts (ratio, rate and unit rate) might be due to Abdullah's weak subject knowledge (OSK) regarding this topic and, to some extent, the textbook. The student's book introduced the concepts by stating the definitions and giving some examples without any explicit effort to differentiate between them. This did not help Abdullah to spot the differences between them, apart from the denominator being one in a unit rate.

Abdullah's knowledge of the concept of rate is questionable (OSK). He believes that the rate can only be distinguished from the ratio by identifying that its denominator is always one. It seems that he had some difficulties regarding this concept. For example, he does not notice that every rate has two unit rates not just one. This was obvious when he hesitated over which one to write, $\frac{3}{9}$ or $\frac{9}{3}$ ([17] and [26] in the above incident), when he solved the question of ‘Find the rate of 9 Riyals for 3 cupcakes’. He believed the answer was nine over three as he said to me, ‘Because the question is about the rate so the bigger number should be the one above’ and ‘…I think… three over nine is ratio and nine over three is rate’. During the solving of the question, he kept checking the teacher’s book as he said ‘I want to check the correct answer written in the *helper* [the book]’. In addition, he taught the concept in a procedural way (COP) as he said ‘I find it easier to put the first number as a numerator and the second number as a denominator rather than explain what the concept of rate is’. Another example of these difficulties could be when he omitted solving the exercises in the form of ‘6 Riyals for 12 eggs’ as one solution contained decimals ($\frac{6}{12} = \frac{0.5}{1}$) which may confuse the students in his view. He said ‘I avoid all the exercises that contain decimals as I think that this is hard for their age’. He was probably not aware of the fact that this form and the other which he solved earlier were the same, as he always put the first number as a numerator and the second as a denominator.

Transformation

Abdullah’s choice of representation (CR) was not so useful for the aim of the lesson. He used verbal language with some aid from pictorial representations to express what he knew in order to pass it on to his students. For example, he did not use tables or double number lines or tape diagrams to introduce the concept of ratio and rate. He stuck to the students’ textbook and its pictures while he introduced or solved exercises related to the topic. Even his use of the pictures was just to count the items without any further potential use. Most of the exercises were written in words, e.g. ‘9 Riyals for 3 cupcakes’, without pictures, and he solved them with the students using just symbols. All the class were using improper mathematical language when they talked about certain concepts, such as ‘five over four’ which is an oral representation for the numerator is five and the denominator is four. It seems to me that he considered the use of the document camera as enough of a teaching aid, so he probably assumes that he does not need to use another. In fact, I have encountered this statement from many trainee teachers whom I have supervised on their teaching practice.

Connection

In this lesson account, I could hardly see the connection between the two concepts (MCC) of ratio and rate and its required procedures (MCP). Abdullah only mentioned the definitions and did not spend time connecting them. He introduced them separately and did not mention that rate is a special type of ratio.

Contingency

Abdullah responded to students' correct answers in different ways (RCI). For example, he sometimes asked for confirmation and repeated the answer, as when he got the correct answer 'Two over six' (student response) he said 'How much...two over six' (elicitation move followed by re-voicing move). On other occasions, he probed the students' certainty about their answer by asking for justification. For example, when he asked the students about which operation they would use to simplify a fraction and they said divide, he responded with 'What shall we do, multiply or divide? … Why divide?' (elicitation move followed by press move) as he was testing the confidence of the students about their answer. He said 'I want to make sure that they said divide because of their understanding, not just guessing'.

Furthermore, he rarely accepted the correct answer without comment and wrote it on the board, such as when he asked 'With which number shall we divide the fraction $\frac{232}{4}$?' and the students said 'By four' (student response) ; then he wrote $\frac{58}{1}$ (accept move through written revoicing move). Likewise, in a situation when he was faced with a correct answer followed by a student's suggestion, he ignored the correct answer and accepted the suggestion. For example, when he said 'What shall we do, multiply or divide [to simplify]' some students said 'Divide' (student response) and a student said 'We convert it to multiplication'. He then accepted the suggestion and followed it to solve the task (accept move).

In the incident shown above and with some answers Abdullah was not sure about their correctness and he considered some correct answers as wrong answers due to his misunderstanding of the concept of rate. He ignored a correct answer of 'One third' [4] (student response) and looked around and asked Thamer if he had finished (reject move followed by nomination and elicitation move). He justified his action by saying:

> This is the first lesson on this topic so I think they still have some confusion about the correct ordering of the numbers three and nine. They seemed not to know which should be above and which should be below the fraction line.

In addition, he responded to the correct answer ‘Nine’ (student response) for the question ‘What is the denominator?’ by saying ‘No’ [11] (reject move) as he said ‘He [the student] probably thought I was talking about the numerator …’. This may indicate that Abdullah was looking for the rate nine over three. At other times, He sometimes accepted their answers and commented on them later, such as when he wrote their answer and was trying to convince them that it was not correct in the utterances [16] to [23] (accept move followed by comment). He justified his action by saying:

> In the beginning I was not sure of the solution so I accepted their answer but when I checked it in my book I knew that the answer was wrong. I decided to complete the solution with them to show them that their answer was wrong.

On the other hand, Abdullah dealt with students’ incorrect answers in different ways. He sometimes asked for justification. For example, when a student said the ratio of six blue and two red paper clips was ‘Five over four’ (student response) he asked them ‘Why do you say five?’ (press move). He said ‘I wanted to see how he found this answer, whether it is built on something wrong or just a random answer’. In addition, he occasionally responded physically to an incorrect answer by shaking his head as a sign for ‘No’ and correcting the answer himself, as when he got an answer ‘By one’ (student response) he then responded by shaking his head and saying ‘No, by itself’ (reject move followed by comment). He corrected the answer, ‘To minimise timewasting, I gave them the answer directly’ as he said. Also he sometimes ignored the incorrect answer and did something else. For example, he ignored the students who said ‘One third’ [4] in the incident and moved around the class to check their answers.

Additionally, he occasionally responded by saying just the word ‘No’ (reject move) as he responded that way to a student’s answer ‘Nine’ [10] in the incident. Furthermore, when he was faced with two answers, one of which was correct and the other was not, he ignored the incorrect answer and responded to the correct one. For instance, when he wanted to simplify the fraction $\frac{2}{6}$ he asked ‘We divide the two numbers (2 and 6) by what?’ and he got the answers ‘We can divide them by two’ and ‘three’. Then he responded with ‘Why do we divide by two?’ (press move) due to ‘Time pressure’ as he said. In addition, he sometimes ignored answers, both correct and incorrect ones, and did another action, such as when he said ‘We want the greatest common factor for two and six’ and the responses were ‘Two’ and ‘Three’. He then ignored the answers and broke the question down into two parts, two and six separately and asked for clarification about their response (elicit moves making funnelling patterns of interaction). When they insisted on their answer he started to act in agreement with them when he said ‘So what about six?’ in order to convince them about the incorrectness of their proposal (press move). He then got the same type nature of an incorrect answer and corrected them explicitly by saying ‘These are the multiples, not the factors’ (reject move followed by comment).

Some suggestions were proposed by some students and Abdullah dealt with them in different ways. For example, he accepted a student’s suggestion, even though it seemed not to be what Abdullah had expected, when he said ‘What shall we do, multiply or divide [to simplify $\frac{2}{6}$]?’. A student said ‘We convert it to multiplication’ then he accepted the suggestion and responded by saying ‘Multiply by what?’ (accept move followed by press move). Later he realised that the suggestion was not what he wanted to hear so he responded again by saying ‘No, we divide the two numbers (2 and 6) by what the definition says?’ (reject move followed by press move). He said:

> In the beginning yes [I thought the suggestion was correct] but when the students said division? I recognised that it was wrong. I think that I have not thought carefully about it.

Furthermore, in the incident where the students suggested that $\frac{3}{9}$ can be divided by three [18], Abdullah did not respond because it seems that he was not sure about the solution as he then went to check in his book. Later, when he found that the suggestion was different from the one in his book, he responded by writing the suggestion on the board (revoicing move) and discussed it with them and guided them through the question to convince them that the solution was not correct. This effort from him resulted in other suggestions from two students who said ‘Divide by nine. Nine divide nine and three…’ [24] and ‘Turn it upside down so nine is above and three is under’ [25] respectively.

They tried to move from $\frac{3}{9}$ to a fraction where the denominator was one but they were not encouraged by the teacher to do so as he seemed to have accepted the second suggestion and deleted the old suggestion, $\frac{3}{9}$, and had written $\frac{3}{1}$ on the board.

In conclusion, Abdullah faced some difficulties regarding the lesson's concepts: rate, ratio and unit rate. He was not fully aware of the differences between them and he omitted some of the lesson's exercises as a result, probably due to some gaps in his subject matter knowledge. He taught the lesson in a procedural way which may indicate some limitation in his pedagogical content knowledge as well. The correct answers were always accepted and commented on, or Abdullah asked for justification if he was not sure about the correctness of them, otherwise they were rejected. Furthermore, incorrect answers were rejected then commented on or checked for justification. Moreover, Abdullah helped the students to achieve the correct answer by offering them the opportunity to correct themselves and by breaking the big questions into small ones. The majority of the exchanges in the class were elicitation exchanges with accept or reject moves, followed by press or comment moves.

5.5 An overview of Abdullah's knowledge

The analysis using the Knowledge Quartet framework shows that Abdullah was a teacher who showed limitations in his subject and pedagogical knowledge, and had some difficulty in teaching mathematics regarding the topics he taught. He showed explicit reliance on textbooks and a belief in the utilitarian purpose of teaching mathematics. He often confused many concepts himself such as the concepts of ratio and rate in the last lesson. He was not sure about the correctness of the majority of the answers suggested in the classes so he kept checking the teacher's book to see the correct answers. He did that almost in all the observed lessons which may give rise to some questions about his subject matter knowledge. In addition, he responded incorrectly to many questions posed by the students and many examples of that were outlined in the lessons' accounts above. Other evidence of having gaps in his subject matter knowledge was that he omitted some parts of the lessons that he was not sure about as he was not confident about his ability to deliver them to the students. All of this evidence may consider Abdullah as a teacher with weak subject matter knowledge of the taught topics.

On many occasions Abdullah emphasised the importance of teaching in a procedural way rather than concentrating on concepts. He rarely connected concepts and procedures with each other. The majority of lessons were introduced in the same way and no teaching aids were employed. However, he also wanted to investigate the students' difficulties and tried to help them. Also he applied some learning strategies (e.g. cognitive conflict) to convince the students about the incorrectness of their answers in order to support their learning.

5.6 An overview of Abdullah Responses to students' contributions

Most of exchanges in Abdullah's class were eliciting exchanges. He opened the interaction by an elicitation move then a student respond. That followed by an evaluation or comment move (Sinclair and Coulthard, 1992).

Abdullah responded to correct and incorrect answers differently. Sometimes he accepted correct answers, other times he asked for justification and occasionally he even rejected the students' correct answers. Conversely, he usually rejected the incorrect answers but sometimes he accepted them for many reasons. In the following sections I outline the response patterns of Abdullah towards the students' correct and incorrect answers and the reasons for his actions then linking these to his knowledge.

5.6.1 Abdullah's response pattern to correct answers: 1) Accepting the answers

Abdullah's most observed action towards correct answers was accepting them in different ways (accept move). He always accepted them and wrote them on the board (W) with or without further actions (A). The actions usually came in different forms such as comment, asking for justification or praise in order to serve many purposes as questions to move to the next step in the solving process. For instance, when a student answered 'Divide' (student response) to Abdullah's question 'how to solve the proportion $\frac{30}{54} = \frac{A}{9}$?' in lesson 3, Abdullah said 'Ok' then wrote $\frac{30\div}{54\div}$ and asked 'Divide by what?' (accept move followed by revoicing then elicitation move). In addition, the action (A) could come as a clue, for example, when Abdullah said 'Read the definition, it says using…' (clue move) to help the student start his answer. Furthermore, Abdullah sometimes asked for justification as a further action after accepting the answer. For instance, when a student said 'By two' (student response) as an answer for Abdullah's question ‘What do we divide [$\frac{4}{6}$] by [to simplify it]?’, Abdullah responded by saying ‘Why by two?’ (press move) in order to give the student the opportunity to justify his answer.

This type of response pattern happened during the solving of the exercises (practice questions) more than in the introduction part of the lessons. It seemed that towards the end of lessons time pressure was a concern as Abdullah accepted (or even ignored) the answers without any discussion. In addition, Abdullah seemed more likely to accept some answers if they came from ‘good’ students rather than the others. Furthermore, when the majority of the class agreed on an answer it was likely to be accepted by Abdullah even if it was not what he expected.

He asked the students to justify their correct answers on many occasions in order to, as he said, ‘Check their sureness about the answers and whether they answered depending on understanding or guessing’. It seemed that there were other factors that encouraged him to do so. For example, when the answer did not fit Abdullah’s expectation he asked for justification. This was clear when Abdullah sometimes, for example, expected the students to solve a question ($\frac{x}{25} = \frac{2}{5}$) by multiplication but the students chose to solve it by division. While the student was solving the question of ($\frac{x}{25} = \frac{2}{5}$) by division, Abdullah probably asked for justification as a result of his beliefs that using multiplication is easier than division or because of his preference to adhere to specific procedures.

Abdullah’s subject knowledge played a role in this matter, as he asked for justification when he was in doubt about the correctness of their answers. He sometimes asked the students to see how they found the solution while he was checking his book to see the answers. Moreover, he usually asked for justification when he reached the main point in the solving process on which the exercise focused to check the understanding of that point. For example, when the students answered that the perimeter of the first square was 4 (in the incident of lesson 3) he asked ‘why?’ as that was the main focus of the lesson. What is more, on some occasions he justified asking for justification as it was supported by his beliefs in the benefit of peer learning, so he asked the ‘good’ ones to justify their answers in order to reintroduce the knowledge to the others. Moreover, sometimes when Abdullah subject knowledge came into play, for example when he was confused about a step in solving a question, he accepted the students' answers without evaluation, but then sometimes later he found the answer incorrect and declared it as incorrect, even though he declared it correct originally.

5.6.2 Abdullah's response pattern to correct answers: 2) Rejecting correct answers

Another response pattern to a correct answer was rejecting or ignoring it (reject move). For instance, a student answered '8 times 2 equals 16' (student response) to Abdullah's question 'What comes next [number 8 in order to find its multiples]?', and Abdullah said 'No, that's wrong. You add 8, then you get 16' (reject move followed by comment).

Abdullah tended to ignore answers when he was faced with two or more answers where one of them was incorrect. In these cases he focused on the incorrect one as he mentioned that he wanted to know the source of the errors. He tended to reject correct answers towards the end of the lesson because of time pressure. This also happened when the answer was not the focus of the exercise.

Sometimes he rejected some of correct answers as he treated them as incorrect ones because he was not aware that they were correct. This could be, to some extent, related to limitations in his subject matter knowledge. It also could be related to his pedagogical content knowledge in terms of his tendency to control the student while solving the exercise or to avoid using certain teaching methods that are difficult to put into practice. He probably did not want the students to steer away from his preferred methods of solving or calculating. For example, his tendency to use a certain method to find the multiples of 8 by applying the repeated addition instead of using multiplication as described earlier.

5.6.3 Abdullah's response pattern to incorrect answers: 1) Rejecting incorrect answers

Abdullah usually rejected incorrect answers (reject move) physically or verbally by shaking his head, staring, saying the answer again or saying 'No'(S), usually followed by further actions. These actions could be one or more of: asking for justification (J), asking other students (O), commenting (C) or doing it himself (D). For instance, Abdullah wanted to find out the factors of 16 so he asked the students 'What are the factors of 16?' and two students said '32' (student response). He then said 'Why do you say 32? (J), you have to differentiate between multiples and factors (C). Khalid can you answer? (O)' (press move followed by comment and nomination move). This exchange was a (JCO) type as Abdullah asked for justification then commented and elected a student to answer.

Abdullah responded to incorrect answers by repeating them (S) as he said in the interview he wanted to confirm what he had heard and to share it with the class (confirm move). He also dealt with them by asking for justification (J) in order to make sure about the source of the mistake, to deal with it, and also to check whether the answer was just a guess or not (press move). This action was most likely to occur when the class was solving exercises and with weaker students. Also, Abdullah ignored some incorrect answers due to the limitation of time, especially if they happened at the beginning or end of the lesson. Sometimes, in different parts of the lessons, he would correct the answer himself (D) when he felt that time did not allow him to give details (maintain move). I noticed that he explained and corrected the wrong answers in detail when they occurred in the main part of the lesson, in other words, when they happened during the focus of the lesson he felt he had to explain more. For example, when the students answered incorrectly during the introduction of the concept of the perimeter of a square, he rejected their answers and commented (C) on them because the focus of the lesson was finding the perimeter of polygons.

Some aspect of Abdullah's pedagogical content knowledge and subject matter knowledge, such as a lack of awareness of connections between concepts (e.g. polygon and square) may have influenced his dealing of the polygon parameter situation described above. In addition, when the incorrect answers came from certain students with either stronger or weaker mathematical knowledge, he spent more time with them in explaining, and corrected them in order to support their learning. Furthermore, Abdullah's subject matter knowledge probably did not help him respond to the incorrect answers in a productive way, probably because he did not know how to respond to them. When he received a wrong answer regarding a new concept or idea, he ignored it and looked around for other answers, as he expected the students to make mistakes on these occasions. He also asked for justification which was one aspect of his PCK.

5.6.4 Abdullah's response pattern to incorrect answers: 2) Accepting incorrect answers

Conversely, Abdullah accepted (accept move) some incorrect answers in order to investigate the source of the confusion. He usually applied the cognitive conflict strategy (Powell, 2006) to show the students that they were solving incorrectly. For example, he did so in the second lesson when the student tried to solve the proportion $\frac{2}{3} = \frac{n}{21}$ by saying 'N is 6 because 2 times 3 is 6' (student response) then Abdullah responded by saying 'Ok, 3 times 3?' (accept move) then he commented by saying 'If you multiply by 3 over 3 … the dominator will be 9 not 21' (insert move as it inserts something to the students' contribution (Brodie, 2008)). He used the cognitive conflict strategy with some specific students who were struggling with mathematics in order to support their learning. In the example above Abdullah knew the answer before the student started solving because he chose it from the book where it was already solved and then wrote it on the board for the student. Another example happened in the incident of the third lesson when Abdullah checked the book [13] and then was faced with the answer of 3 over 9 which was incorrect from his point of view. He knew the answer but acted in agreement with the student until he showed them that their answer did not fit with the unit rate concept [21] and [23]. The last example happened in the third lesson when Abdullah asked the student to find out the factors of 2 and 6. He said that 'The factors of 2 are 1 and 2' and wrote them on the board but the students did not agree and said '4, 6 and 8'. He accepted their answers and wrote them on the board and said 'What about 6?'. They responded '12, 18 and 24'. He then wrote these and said 'These are multiples not factors'. In this example he was clearly sure about his answer as he wrote it in the first place but when the students insisted on their answers he preferred to convince them about the incorrectness about their answers by creating cognitive conflict. By doing so he showed some confidence in his teaching which was missing on other occasions. This highlighted the fact that he did this action in situations where he was clearly depending on his secure subject knowledge which in turn led him to use his PCK to teach in a different way.

In conclusion, the findings of applying the KQ on Abdullah's classes showed evidence of limitations in his SMK and less so of his PCK. Abdullah responded to correct answers by accepting them almost all the time and there were some times when he did not. Most of his moves regarding correct answers were accept moves with press, revoice and comment moves. Conversely, he always rejected incorrect answers unless the error occurred in some part of the lessons which he knew well. In these cases he applied the cognitive conflict strategy to convince the students about their errors. Most of his moves regarding incorrect answers were reject moves with press, insert and comment moves. His SMK, PCK and belief of peer learning as well as time pressure influenced the ways in which he responded to correct and incorrect answers.

Chapter 6 : The case study of Onizan

6.1 Introduction

Onizan (participant 2 in section 4.4.2) was one of the trainee teachers who participated in this research. He is 22 years old who was practising teaching in Tahfed School and taught grades four and six (10 and 12 year olds) for four lessons each per week. Onizan had completed all the courses required prior to his practice term so he had more time to spend in the school. He was a determined teacher as he showed interest in gaining new information about teaching and always asked me for comments on his lessons because his supervisor did not come to him regularly and only visited hem three times per term. Onizan was observed and interviewed as shown in Table 6.1.

Teacher	Observed Lessons	Grade	Incidents per lesson	Interviews
Onizan	1	6	4	1
	2	4	5	
	3	6	3	2
	4	6	3	
	5	4	2	3
	6	6	3	
	7	4	4	4
	8	4	3	

Table 6.1: Observations of Onizan

Note: Lessons 1,2 and 7 are analysed in this thesis

In this chapter, the three lessons that have the highest number of incidents as shown in Table 6.1 are analysed. I start with lesson 2 then 7 and the last lesson is lesson 1. The third lesson (lesson 1) is written in less detail than the first two lessons to avoid repetition. In every lesson there were different types of incidents of which I chose one to show the reader an example of the interactions. The incidents chosen have a variety of teacher responses and the teachers have shown clear evidence of different types of knowledge in these incidents.

6.2 The first lesson (lesson 2)

This lesson, entitled ‘Time’, was introduced to Grade 4. The aim of the lesson was to calculate the elapsed time between two points of time. For example, ‘Khalid travels to his grandmother's houses. If he leaves at 3:20 pm and arrives at 5:40 pm, how long does he take to arrive at his grandmother's house?' The lesson is described below:

6.2.1 The lesson description

The class of twenty students sat in five columns of four rows each. The first exercise in the textbook on this lesson contained a table consisting of three activities: writing the alphabet, writing the names of ten countries and jumping twenty times. For each, the start and end time, and elapsed time needed to be calculated. However, Onizan omitted this exercise and did something else of his own design. During the first six minutes, Onizan asked the students 'What is the purpose of time in our life?' He got different answers such as 'To know the time' and 'To read a clock'. He then asked them how many minutes are in an hour and how many seconds in a minute. After that he turned the projector on and asked the class to do two activities as a warm-up. Firstly, he selected a student to write the numbers from one to thirty on the board while one of his peers counted to see how many seconds it took the first student to write them. It took him one minute and ten seconds to write on the board and Onizan asked 'How many seconds is that? ... This is our lesson today; how can we write it?'. He wrote '1:10 M' on the board as a student suggested. For the second activity, he asked a student to write the names of four peers on the board, while another counted how long it took. It took twenty three seconds and he wrote '23 S' on the board then moved to the textbook.

Onizan explained to the class the following two examples from the book (the first one is a worked example):

- 'It takes Abdulaziz an hour and 30 minutes to travel to his farm. If he leaves his house at 4:00 pm, what time will he arrive at his farm?'.

$$4{:}00 \xrightarrow[1\ hour]{} 5{:}00 \xrightarrow[30\ minute]{} 5{:}30$$

Figure 6.1

- 'The clock on the side shows the time when the team started training and if they finished training at 5:30 pm, how long did the training last for?'

They took nearly eight minutes to explore the first example, as it turned out that some students could not read the clock, and seven minutes to do the other one. In the first example, Onizan asked the students individually to read the analogue clocks and some students could not read the clock (5:30) correctly so Onizan spent some time explaining how to read the analogue clock. Then he asked them about the amount of time between each of the three clocks (e.g. how many hours between 4 and 5 and between 5 and 5:30) and he wrote their answers on the board and asked them to copy them in their books. I describe in detail how he explained the second example in incident 1 below.

Incident 1:

‘The clock shows the time when the team started training and if they finished training at 5:30 pm, how long did the training last for?’ 

The following dialogue happened while solving the example:

1	Onizan	So, looking at the board, when did the training start?
2	Student 1	Three and quarter
3	Onizan	Excellent [writes 3:15 on the board]. And when did the training finish?
4	Students	Five and thirty minutes; five and a half
5	Student 2	It is thirty and five minutes
6	Onizan	Five and a half or a half and five? Who said it is thirty and five? [no answer] It is five and a half [writes on the board two points on a line: one for the beginning and one for the end]. How long does the training last for? How can we find that? [students raise their hands]
7	Student 3	3 and a quarter
8	Onizan	[asks others]
9	Student 4	3
10	Onizan	No, we follow the same method as the first example [he writes an arrow from 3:15]
11	Student 5	From 3 to 5 it is two hours and …[Onizan interrupted him]
12	Onizan	We want to solve it in the same way we did earlier, [asks another one to participate]
13	Student 6	Draw a line from 3:15 to 4:15.
14	Onizan	Right, we need an hour [write 3:15 $\xrightarrow{\text{1 h}}$ 4:15]. Who can complete it?
15	Student 7	Draw a line to 5:15
16	Onizan	[Completes the solution and writes 4:15 $\xrightarrow{\text{1 h}}$ 5:15] And then?
17	Student 8	Draw a line until 6:15

18	Onizan	What do we write? [asks another one]
19	Student 9	Until 5:30
20	Onizan	[Writes 5:15 $\xrightarrow{15\text{ m}}$ 5:30] How long is the training from the beginning until the end? [students rise hands] I want someone who has not participated?
21	Student 10	It is two hours and a quarter.
22	Onizan	Excellent, [Writes 2 hours and 15 minutes on the board].
23	Student 11	Teacher, why did you write 15 minutes?
24	Onizan	A quarter means 15 [minutes].
25	Student 12	Why is it 2 hours and 15 minutes? It is 3 hours and 15 minutes as you count 3,4 and 5.
26	Onizan	Your method is wrong and if you want to find the time you should start from here [pointing to the solution on the board] and finish here (at 5:30). From 3 to 4 is an hour and from 4 to 5 is an hour and from 15 to 30 is a quarter.
27	Student13	If we want to write it [2 hours and 15 minutes] as numbers how can we do that?
28	Onizan	Your peer asks how we can write it by numbers. Who can do that? [asks a student to write it on the board]
29	Student14	[writes on the board 1 2 3 4 5]
30	Onizan	No, [deletes the writing and asks another one]
31	Student 15	[comes and writes 2:15M]
32	Onizan	Excellent. This is how it can be written.

For the next ten minutes, and towards the end of the lesson, the students were asked to solve the exercise of 'for how long does each activity last?'

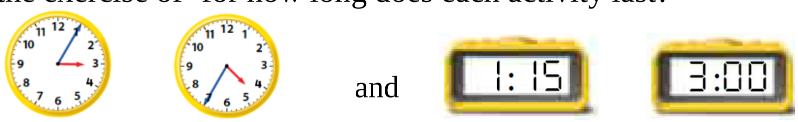


Figure 6.2

Onizan solved it with them in the same way he did for the examples in the introductory part of the lesson by jumping in hours and minutes from the start to the end.

6.2.2 Applying the Knowledge Quartet (KQ) and IRF

The following analysis takes into account the class observation and Onizan's comments in the interview.

Foundation

Research shows that solving elapsed time problems is a hard task for some students (Dixon, 2008) and for some teachers to teach (Kamii and Russell, 2012). This may be because the students do not have the ability to coordinate hierarchical units of time (hours and minutes) as they treat them separately (*ibid*). It requires them to know (at least) two of the three components of the task: starting point, elapsed time and end point. Also, they have to know the sequence (order) of events (Monroe *et al.*, 2002) and to be able to count forwards and backwards in different units, such as hours and minutes. The known start and end time type of problem can be solved by regarding the task as a comparison (difference between) subtraction between two points of time (Anghileri, 2014). Then, using an iconic representation such as the empty number line (Klein *et al.*, 1998; Van de Walle, 2006) to apply the counting up strategy from the smaller amount of time (Dixon, 2008). The students calculated the steps to find elapsed time. For the second type of problem where one of the points is missing, Dixon (2008) suggests applying the counting on/back strategy to move a certain time from the start/end point in order to reach the other point which is then the answer. Onizan seemed to follow these recommendations to solve the exercises, which the book also suggested.

Telling the time is problematic for some students (Friederwitzer and Berman, 1999). The ability of telling the time does not necessarily mean understanding the concept of time (Dickson *et al.*, 1984) which requires grasping that the events occur in a temporal order and that the duration of the intervals between them needs to be appreciated (Piaget, 1969). Some of the problems of telling the time are that the students cannot read the clock because of the complexity of using the analogue clock and having to differentiate between the hands, and remember that each number has two different meanings (Friederwitzer and Berman, 1999). In addition, it requires the students to understand the continuous nature of time (Dickson *et al.*, 1984). Some of Onizan's students had difficulty in this matter at the beginning of the lesson and Onizan tried to explain how to read analogue clocks. He said in the interview:

> I consider reading the time correctly is vital for the lesson so I spent a long time correcting the errors related to this point at the beginning of the lesson and towards the end.

The lesson contained the concepts of time, elapsed time, hour and minute. Also it had the procedures of telling the time, finding elapsed time and converting between hours and minutes. Onizan had secure subject knowledge of the lesson concepts and procedures (OSK) as he dealt with them correctly in the lesson apart from tiny mistakes such as writing two hours and fifteen minutes as 2:15 M. This mistake may have occurred as a result of him being unaware of the relation between decimals and time since it should be written as 2.25 H. He did not concentrate heavily on the concepts of time, minutes and seconds as he said in the interview ‘I expected them to know the time as some of them were wearing watches’. However, the first exercise which he omitted would have been a good starting point to give the students the sense of what a minute is. Instead, he started the lesson by counting the time spent on writing from one to thirty (seconds) and did not mention the start and end time in this activity.

Onizan started the lesson by asking ‘What is the point of time in our life?’ He seemed to believe that mathematics is useful in real life (AP) which fits with the utilitarian view of mathematics (Ernest, 2014). He also added in the interview that ‘I saw most of them were wearing watches and I am sure they will benefit from the lesson in their life’. This highlights the view of using mathematics outside the class to help the students in the real world, such as being able to read a clock or solving a real problem about elapsed time.

Transmission-orientated teachers towards mathematics teaching concentrate on procedures while teaching (Askew *et al.*, 1997). The teaching process contains exchanges of questions and answers between the teacher and the class to make sure they have mastered the methods they have been taught. They believe that the students’ own strategies for solving problems are less important than their instructions and the students’ misunderstandings need to be treated by more practice (*ibid*). Onizan seemed to believe that mathematics is better learned by engaging the class with questions and answers and by sticking to the teacher’s instructions (COP). He was clearly concentrating on finding the elapsed time correctly more than the related concept. His instructions for doing so were a combination of questions and sequenced steps that needed to be followed. He sometimes undervalued the students’ strategies for solving a problem which fits with this orientation. For instance, when the student said ‘From 3 to 5 it is two hours’ [11] in the incident, Onizan interrupted him and said ‘We want to solve it in the same way we did earlier’. This indicates that Onizan preferred to stick to his method rather than encourage the students to develop their own methods. He treated the students’ errors by asking others in the class to correct them and this could be related to his beliefs of the importance of peer learning.

Transformation

Onizan demonstrated (TD) the lesson's concepts and procedures differently. He missed the main point of the lesson in the introductory part as he did not mention the start and end time between which the elapsed time is calculated. He tried to introduce the time spent on doing something rather than the time between two points of time, then moved to telling them how to tell the time. He was probably aware of the importance of knowing how to read analogue clocks as he said in the interview 'I consider reading the time correctly is vital for the lesson…'. Later in the examples, he introduced the clocks as they were displayed in the book with some extension questions to direct the class through the answers. Moreover, he did not suggest the idea of counting back as he could have done with the student who said 'Draw a line to six fifteen' [17]. He could have counted back from 6:15 until 5:30 but he did not do so.

He valued the idea of using the textbook's examples (CE) to introduce new concepts and procedures (Rowland *et al.*, 2009) as a model that he should adhere to and train the students to follow. He said in the interview 'I prefer to give and solve the lessons' examples to show the students how it works and I want them to follow the same way to avoid making mistakes'. This view may shed light on his actions on the student's suggestion of jumping two hours instead of one in utterance [11] in the incident.

Onizan's choice of representation (CR) fits with Dixon's (2008) recommendations of introducing elapsed time. He used an effective representation to display the concept of elapsed time as he used an empty time line to explain how to calculate the elapsed time, and before that he used the pictures of the three clocks (iconic representation) (Rowland *et al.*, 2009; Bruner, 1974) to show the students how to read the clocks.

Connection

Onizan did not make connections between the required procedures of the lesson (MCP). For example, the link between calculating the elapsed time and subtraction was missing. However, the book stated explicitly that 'to find the elapsed time for each activity you have to find the difference between the start time and the end time'. Onizan did not mention this link between the two. He did not mention the concept of subtraction at all during the lesson; instead he asked how long something took to do. He might not be aware of this connection. Moreover, he did not connect reading the clock with multiplication. For instance, instead of pointing out that all numbers on the clock have two meanings, one for the hour and the other for the minutes (e.g. 2 is two o'clock for the short hand and 10 minutes for the long hand), he counted up by 5s as he said 'between every two numbers there are five minutes'.

Contingency

Correct answers were dealt with by accepting, ignoring or rejecting them (RCI). Onizan accepted some of them without comment. For example, when he asked 'if the long hand is on 3 then 6' (elicitation move) and he received the correct answers (student response), he did not comment on them (no third turn). The absence of the comment in the third turn can be considered as a positive evaluation (Waring, 2008). It seemed that he did not comment on the correct answer if it came between incorrect ones that had been commented on and if it was not to do with the main lesson idea. In addition, he sometimes accepted correct answers and praised the students by saying 'Excellent', 'Good' and 'Well done' (evaluation move). For instance, he praised the students who answered correctly in ([2] and [21]) as they answered the important points in the solving process (the focus of the exercise) which was finding the start point and the elapsed time respectively. Also he praised a student when they gave an answer which the majority of the class did not know. For example, he praised the student who wrote '2:15 M' in the incident ([31]) which was correct from Onizan's point of view. Onizan said in the interview 'I always say good words such as 'Excellent' and 'Well done' to reward the students for their correct answers. The students love these words and it encourages them to do their best'. Moreover, Onizan accepted some correct answers and wrote them on the board if they were in the process of solving an exercise (third turn is accepted followed by the elicitation move to start another exchange). For instance, he wrote the students' answers on the board in the incident to build up the steps of the solving process as in the utterances [14], [16], [20] and [22].

Onizan ignored and rejected some correct answers. He ignored some of them when he thought they were incorrect but were actually correct. For example, in the incident when the student said 'Until 6:15' [17] (student response to elicitation opening move 'And then?') Onizan ignored him (third turn is missing then another exchange started by elicitation move [18]) as he said in the interview 'As this an example of one of the main ideas of the lesson I thought the answer was incorrect so I chose to ignore it'. Moreover, he rejected some correct answers when the students had not followed his instructions. For instance, when the student said, [11] in the incident, 'From 3 to 5 it is two hours' (student response to the head elicitation initiation [6]) Onizan rejected this answer and commented on it (the third turn is providing a clue to help the students' reply [12]). That was because it did not fit with his instructions of moving in hourly steps not two hour steps. He acted so in order to, as he said in the interview, 'I want to bring the student to my method of solving the exercise as I am keen to avoid the possible errors or confusion caused by different ways of solving the problems'.

However, Onizan dealt with the students' incorrect answers in different ways: he rejected them by asking other students, commented on them, corrected them, repeated them, or used a combination of two or more of the strategies. Onizan always asked other students to give their answers when an incorrect one occurred. For example, when he received the incorrect answer in the utterance [7] in the incident (student response to elicitation opening) he asked other students (third turn of implicit negative evaluation followed by another elicitation move [8]). He justified his action by saying:

> I believe that some students accept the information from their peers more than they do from the teacher. This is built on personal experience as a teacher and as a student. Sometimes there is some competition or jealousy among the students so it also encourages them to work harder to correct each other so at the end of the day no one is better than anyone else.

He sometimes commented on the answers after they were corrected by others, especially when he thought that the point was very important. For example, when he asked 'if the long hand is on number two, How many minutes is that?' (elicitation opening) a student said twenty (student response). Onizan received the correct answer later and then commented (comment move or insert move) by saying 'between every two numbers there are five minutes', as this an important fact about telling the time. In addition, he rejected wrong answers by saying 'no' and commented on them or corrected them himself. For instance, Onizan said 'every small line between the numbers is a minute so the third line after number 3, what it could be?'(opening turn with a clue giving more information, then an elicitation act) A student said '20' (student response) and Onizan said ‘no 20 means that the hand is on 4. When it is on 3 it means 15 so there are 16, 17, 18 and 19 between these two numbers…’ (third turn: rejection and comment act).

Onizan sometimes corrected wrong answers himself when the answer was considered an error in an important part of the lesson. For example, in the incident when the student said ‘it is 3 h and 15 m as you count 3,4 and 5’ [25] (student response), Onizan said ‘your method is wrong and if you want to find the time you should start from here … and finish here …’ [26] (third turn: evaluation or insert move). He justified that in the interview by saying ‘I felt that was a big problem and it was not just a random answer so I wanted to treat it carefully’. Moreover, Onizan sometimes repeated the answer as a way of questioning the answer which can be understood as ‘change the answer’ by the students. This exchange contains an elicitation act and a reply followed by the teacher’s evaluation act through the repetition of the reply with a rise in intonation. For instance, in the first exercise after the two examples, Onizan asked when the activity ended and nominated Ali to answer (elicitation move). Ali said ‘the time is seven and fifteen minutes’ (it was 4:35 on the analogue clock) (student response). Onizan said ‘No’ (evaluation with no initiation to the next exchange) then Ali changed the answer to four and half (student response). Onizan said ‘four and a half?’ (maintain move or evaluation by repetition with no initiation to the next exchange) so Ali said ‘four’ (student response). Onizan said ‘four?’ (maintain move with no initiation to the next exchange) and then Ali said four and seven minutes (student response). He did that with some students as he said in the interview:

> Ali is a determined boy who likes to participate in the class even though he is not very good at mathematics. He always becomes annoyed when I put the X mark on his homework as he asks me to tell him where the mistake is instead of putting the X mark. I spent long time with him to give him gentle support even though there was only a few minutes of the lesson left.

In conclusion, Onizan showed some indication of having secure subject knowledge as he very rarely made mistakes. He used a powerful representation to show the students how to calculate the elapsed time by using the empty line. However, his demonstration of the lesson was mostly done in a procedural way, since he tended to force the students to follow his instructions regardless of their own ideas. Onizan's most used type of exchange during the interactions with the class was eliciting exchange. Most of his follow up moves were accept/reject and comment moves (insert, elicit and maintain). Most of the students' correct answers were accepted and shared and some of them were rejected if they did not fit with Onizan's expectations. He mostly dealt with students incorrect answers by rejecting them and asking others for their answers.

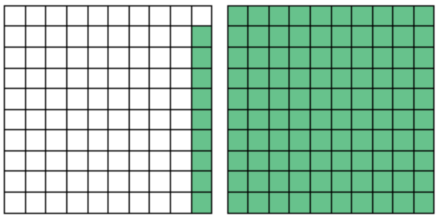
6.3 The second lesson (lesson 7)

The lesson of 'Mixed Numbers and Decimals' was introduced to Grade 4 in the mathematics lab. Its aim was to enable the students to convert a mixed number to a decimal, such as 'Write 1 $\frac{2}{10}$ as a decimal'. This lesson seemed to be the second on this particular topic and was a continuation of what the class did the day before.

6.3.1 The lesson description:

Onizan started the lesson by revising the previous lesson (the first part of the topic) for about ten minutes. He wrote $1\frac{1}{2}$ on the board and said 'This is a mixed number' and then he wrote 5.40 and said 'This is a decimal number'. With the class, he then converted three mixed numbers of his choosing to decimals ($15\frac{7}{10}$, $16\frac{14}{100}$ and $7\frac{5}{100}$). In particular, Onizan wrote the mixed number $15\frac{7}{10}$ on the board and asked the students how to convert it to a decimal. One student volunteered and said the decimals number digit by digit. Onizan then said 'We write the same number down here' then he wrote 15 under the mixed number and said: 'Now we put a point instead of the fraction line'. Next, he drew an arrow from the fraction line to the point and wrote 7. He then asked the students if the answer was correct or not. Later, they converted two decimals to mixed numbers (7.03 and 15.1), however, the conversion from decimals to mixed numbers was not included in the book. For instance, Onizan elected a student who came and wrote $7\frac{3}{100}$ on the board. Onizan then explained it by drawing an arrow from 7 in the decimal to 7 in the mixed number and another arrow from the point to the fraction line. He said: '7 becomes 7 and the point becomes a fraction line. Here we write the denominator 100 because 03 is two digits'.

The class then moved to the book and Onizan showed them the first example on the board through the projector, which took four minutes. It was a picture of a lizard and the text on the side was ‘the length of the animal in the picture is $1\frac{9}{100}$ metre. Write $1\frac{9}{100}$ as a decimal. You can use the area model or the place value table below’.

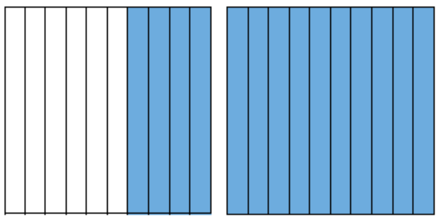


Hundreds	Tens	Units	. Tenths	Hundredths
		1	0	9

Figure 6.3

Onizan asked the class how many coloured parts there are in the square on the right (of the model). Then, he said that it makes a whole number of one and the decimal is nine out of 100 then he wrote 1.09 on the board.

For the next seven minutes, Onizan and the class solved the first two exercises in the book, which were ‘Write the following area models as a mixed number and a decimal’:

1) 

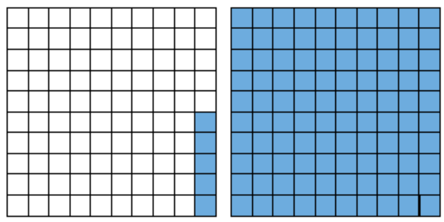
2) 

Figure 6.4

Onizan elected a student to start with the model on the left. The student wrote the model as 1.4 and then Onizan asked another student to convert 1.4 to a mixed number. The student stated the number as ‘one then fraction line then 4 above and ten under’ without referring to the model. Then they correctly did the same for the model on the right. Next Onizan asked the class to solve the third exercise, which was ‘Write twelve and three tenths as a mixed number and a decimal’. This took four minutes: I describe in detail how they solved it incident 1.

Incident 1:

The exercise was ‘Write twelve and three tenths as a mixed number and a decimal’. The following dialogue occurred during this exercise:

1	Onizan	How can we write twelve and three tenths?
2	A student	Twenty above …
3	Onizan	No. [asks another student]
4	Students	It is twelve…
5	Onizan	[writes 12.], How can we write three tenths as a decimal? Adel
6	Adel	[no answer]
7	Onizan	Who has not answered yet? Ali?
8	Ali	Twelve is above and thirty is under.
9	Onizan	No. Why did you say that ?
10	Ali	[no answer]
11	Onizan	I will give a present to whoever answers this question correctly and I will not help you. [students raise hands and he is looking at them] Khalid?
12	Khalid	Three and then five
13	Onizan	Majed?
14	Majed	Twelve and point then thirteen
15	Onizan	[writes 12.13], No, you are close to it
16	A student	Twelve and two above and thirteen under
17	Onizan	[writes 12 $\frac{2}{13}$ and deleted it], You have one more chance. [students raise hands]. Thamer?
18	Thamer	Twelve and thirty one
19	Onizan	[asks another one]
20	A student	Twelve and point then a hundred and thirty
21	Onizan	We agreed on 12 so focus on the rest. Ibrahim?
22	Ibrahim	Point thirteen
23	Onizan	No
24	A student	Point twelve
25	Onizan	[shakes his head], Saleh?
26	Saleh	Twelve then point and one
27	Onizan	No, [writes 12.3] it is 3 out of ten. Who can write the other form?
28	Saad	Twelve then a line. 3 above and ten under.

Then, they solved the exercise ‘Write twelve and three hundredths as a mixed number and a decimal’ during the next two minutes by electing a student who said ‘put a line then 3 above and 100 under then put 12 at the side’. For the next eighteen minutes, towards the end of the lesson, Onizan asked the students to solve some exercises individually from the activity book.

6.3.2 Applying the Knowledge Quartet (KQ) and IRF

The following analysis takes into account the class observation and Onizan's comments in the interview.

Foundation

Rational numbers, including fractions and decimals, are particularly hard to teach and learn (Clarke *et al.*, 2006; Bonotto, 2006; Sweeney and Quinn, 2000; Nunes *et al.*, 2009; Domoney, 2002). This might be due to the variety of ways of representing and interpreting rational numbers (Clarke *et al.*, 2006; Dickson *et al.*, 1984; Vamvakoussi and Vosniadou, 2010). Also, it could be because of the natural number knowledge interference, where students apply their knowledge of natural numbers inappropriately to rational numbers, and because of the problems in rational number notation (Iuculano and Butterworth, 2011; Ni and Zhou, 2005). In addition, in this lesson in particular, difficulties could have arisen due to the way that these concepts, mixed numbers and decimals were introduced to the students without linking them together (Sweeney and Quinn, 2000). While learning fractions and decimals, students find it easier to deal with fractions in everyday contexts rather than in an abstract way (Dickson *et al.*, 1984). Also they encounter some difficulties in transition from diagrams to words and symbols and in understanding the mixed numbers (*ibid*). Moreover, students have difficulty in grasping the idea of place value in the base-ten number system (Martinie, 2014).

Some scholars suggest that to increase the understanding of decimals and fractions, teachers should use multiple representations (e.g. a number line, an area model/grid, money or a place value) (Martinie, 2014) and teach both at same time (Glasgow *et al.*, 2000) or even teach them earlier to avoid 'whole number bias' (forming decimals and fractions in terms of their whole number parts) (Ni and Zhou, 2005). In addition, playing trading games which uncover relations between two numbers could help the students' teacher to understand fractions (Amato, 2006). Students also need to connect quantities and representations of fractions to use them meaningfully and they should be able to connect the two different notations (e.g. 0.25 and $\frac{1}{4}$) that represent the same quantity of a rational number (Nunes *et al.*, 2009). In addition, it is advised that providing students with some experience of rational numbers in class helps them develop the concept, as rational numbers are rarely found out of the school setting (Skypek, 1984; Bonotto, 2006). Moreover, students, in order to understand fractions, need to know equivalence and order in the same way as whole numbers, and prior informal knowledge about division can be used to understand fraction quantity (Nunes *et al.*, 2009).

In this lesson, students struggled to deal with decimals correctly. One possible reason for this is how they were presented in the book. Decimals can be defined as 'Rational numbers of the form $a/10^n$ where a and n are integers' (Skypek, 1984, P.11) or simply as a special kind of fraction with the base-ten number system in the denominators (Martinie, 2014). The student book defined them as 'A number which a point and a place value are used to show part of the whole' without any explanation of the concepts place value, point, part and whole which Onizan presumably explained in the first part of the topic (the previous lesson). In addition, research shows that another reason is probably that teaching in a procedural way leads to a weaker understanding of the concepts (*ibid*). Onizan concentrated on procedures (COP) while he introduced the lesson. For example, he converted between mixed numbers and decimals by using arrows to show the relation between the decimal point and the fraction line in a procedural way. It seemed that his focus was on the product not the process as he was just looking for correct answers by asking many students. This fits with the transmission orientation towards mathematics belief which believes that mathematics is best learned by obtaining procedures with clear verbal instructions containing question and answer interactions between the teacher and the class (Askew *et al.*, 1997). Within this orientation, the students become heavily dependent on teacher instructions (*ibid*). This was clear in the incident when Onizan stopped his help after saying 'I will give a present…' The students had to depend on their own reasoning, so many strange answers came to the surface, which may have been an indication of the misconceptions they had regarding decimals or were just guesses in order to obtain the extrinsic reward. Furthermore, Onizan said in the interview 'because I found that the lesson on fractions was skipped by their previous teacher [of the previous year], I faced difficulties in introducing the lesson', which may emphasise his belief in the importance of building new knowledge on old knowledge.

The lesson contained many concepts and procedures. It had the concepts of mixed numbers, decimals, models and a place value table which interact together to form the lesson. In addition, the main procedure was to convert between mixed numbers and decimals. Onizan showed secure subject knowledge in teaching these components (OSK). He did not make any mistakes in the lesson and he responded correctly to all questions regarding these concepts. However, although he understood the place value table (see Figure 6.3) well as he explained it to me in the interview, he did not mention it to the class at all even though it was written in the book. He justified his action by saying:

> In this school, there are fewer mathematics lessons (4 lessons) than other normal schools (5 lessons) [as this is a religion school] so we suffer from that, and I was asked to skip some of the book. I chose to skip the value place table because it was more complicated for the students than the models

Moreover, he was able to identify the students' errors (IE) of decimals as he responded to almost all these errors and he wanted to know the sources of these errors (e.g. in the utterance [9]). This may indicate that a reasonable common content knowledge was held by Onizan (Ball *et al.*, 2008).

However, the lesson in the book was designed to introduce the conversion from mixed numbers to decimals only, and Onizan added to that the movement from decimals to mixed numbers. He seemed to believe that prior knowledge students have can be used to build new knowledge (Martinie, 2014). Before this lesson, the students were taught the conversion between fractions and decimals, and improper fractions and mixed numbers and vice versa. He said in the interview 'They knew how to write mixed numbers as decimals and fractions and vice versa so I thought it would be a good idea to extend the lesson to include writing decimals as mixed numbers'. This effort to extend the book's aims might also show some confidence of secure subject matter knowledge (OSK) as he was able to rely on it to voluntarily introduce the new procedure to the class.

Transformation

Onizan demonstrated (TD) the conversion between mixed numbers and decimals almost in the same way. He tended to introduce it to the class as a set of mechanical steps that were needed to be done in order to arrive at a final answer. For example, to convert 15.1 to a mixed number he said to the class ‘You write 15 up here and the point becomes a line. Then 1 goes above the line and we put ten under because 1 is one figure’. While he was speaking he drew these arrows on the board.

$$15\ \tfrac{1}{10}$$

$$15\ .\ 1$$

In this example, no effort was made towards understanding the reasons for doing this. The students were just told to do so, to arrive at the answer 15 $\frac{1}{10}$ and it was not clear whether they understood it or not. I asked Onizan in the interview to explain the procedure more and he said ‘the whole number is the same (15) and the decimal part (point 1) is one tenth which means one part of a set of ten. We then write it as fraction of one part of ten whole’. I then asked him why he did not explain it in this way to the class and he said ‘I think most of them will not understand it well so I preferred to show them a different way so they could solve it correctly’. Onizan knew the procedure very well as he expressed secure subject knowledge regarding it. He might have used the method he did because of a limitation in his pedagogical content knowledge, as he felt that he was not able to deliver it in an acceptable way or because of his belief of the importance of getting the final answer correct. His focus on getting correct answers influenced his teaching as he applied the procedural way of doing the conversion in order to help the students to achieve the correct answers regardless of their understanding of the meaning of the conversion.

Connection

Onizan did not make connections between mixed numbers and decimals (MCC). He taught them separately and did conversions between them only in procedural ways, which made it difficult for the students to see the connection between them (Scaptura et al., 2007). Some textbooks link fractions with decimals only by conversion, (Sweeney and Quinn, 2000) as the Saudi textbook did. Onizan’s weak pedagogical knowledge may have influenced his ability to link them explicitly together as he mentioned that it is hard to explain to the students the meaning of doing the conversion between these two concepts. He might have anticipated the complexity of this conversion for the students so he chose to do it in this way.

Contingency

Onizan responded to the students' ideas (RCI) in a variety of ways. Most of his interactions with the class were in eliciting exchanges. There were few correct answers in the lesson as the students made many mistakes regarding decimals because of the difficulty of this topic.

Onizan dealt with the students' correct answers by accepting and writing them on the board. At the beginning of the lesson, he dealt with them by doing this and using them as opportunities to check the students' capacity to do the conversion. That may have been because the lesson started with revision, so Onizan found it a good idea to evaluate the students' ability to do the conversion. For example, when a student converted $16\frac{14}{100}$ to 16.14 (student response to Onizan's elicitation move), Onizan asked 'is that right?' in order to see if there was anyone who did not know how to do it (third move is missing but it could be understood as an implicitly evaluative/maintain move which occurred before initiating another eliciting exchange). He then found some students who said it was wrong (student response) and then he commented on their answers (comment move). In this example, Onizan was the primary knower when he asked 'is that right?' as he asked the student who converted the mixed number in first place to wait and not change his answer when he saw his peers disagreed with him.

Later, in the main part of the lesson, he accepted correct answers by praising the students and writing them on the board with or without comments. He kept receiving them then moving on to the next steps. For instance, when a student wrote model (1) in Figure 6.4 as 1.4 (student response to elicitation move), Onizan accepted the answer and moved to the next point by asking him to convert it to a mixed number (accept move followed by another eliciting exchange). Onizan's tendency to share whatever the students said and write it on the board may have been a result of his secure subject knowledge and the high level of confidence he has.

Students' incorrect answers were dealt with by asking for justification, rejecting them or writing then deleting them. Onizan asked some students to justify their incorrect answers in order to, as he said in the interview,

> know what the students were thinking so I asked them about their answers. I like discussing errors with some of them which may be common but some are sensitive to share them with the class. I was keen to help the students correct their errors when I knew the reasons behind their errors.

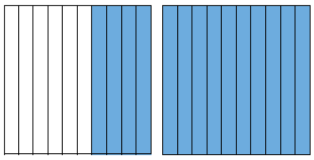
For example, in the incident [8], Onizan asked the student who said 'Twelve is above and thirty is under' [9] why he said that (the former comment being a student response to Onizan's elicitation initiation and the latter a press move while the primary knower shifted to the student). He wanted to know the source of his error which he believed, as he mentioned in the interview, some other students may have but unfortunately the student did not answer. He did this at the beginning of the lesson and at the beginning of the incident. It seems that he tended to ask for justification when he was faced with many incorrect answers (e.g. [2], [4], [6] and [8]) regarding the same point (the position of 12), as he considered it a common mistake that needed to be dealt with. Moreover, he wrote some incorrect answers on the board and then deleted them. For instance, in the utterance [17], the student answered incorrectly (student response to an elicitation opening) and Onizan wrote the answer and deleted it (this action might be considered as a re-voicing move (Forman and Larreamendy-Joerns, 1998) when Onizan repeated the student's answer in a written form to make it explicit to the class). This could be related to his secure subject knowledge as he was ready to share the common errors with the students as he wanted to unpack them.

Onizan usually rejected incorrect answers physically or verbally and asked other students to correct them. Sometimes he said no or shook his head if he was faced with incorrect answers (reject move) and sometimes he corrected them himself or by asking other students. For example, the majority of Onizan's responses in the incident were asking other students (e.g. [2] and [3]) (rejection move followed by nomination and elicitation opening move of another exchange). He justified his action by saying in the interview:

> If a student makes a mistake in part two of the lesson, which is just a solving exercise, I usually ask others, as some students accept a correction better if it comes from other students. Then I tend to spend more time on the discussion with the students about their answers

It seems that he believed that some students accept corrections from other students, therefore he used this strategy in his teaching.

Onizan rarely dealt with students incorrect answers by guiding them through questions to the desired point. When he focused on a particular point and tried to bring the students' attention to it, he faced some unexpected answers, which he responded to by paraphrasing his questions or by giving some hints. This fits with what Wood (1994) called the 'Funnel pattern of interaction' where the teacher guides the students toward the desire answer by a series of questions. For example, when they tried to write model (1), see below, as a mixed number and a decimal, Onizan asked 'How many coloured parts are there on the right?' and one student said ten.



Model (1) from Figure 6.4

Onizan said 'All ten of them are coloured so how many is that?' The student said 100 and Onizan said 'If we say this (the right side) is fully coloured what does that mean?' He got no answer. Then, Onizan asked the others to answer and one student said 'It is a rectangle'. Onizan said 'One fully coloured rectangle equals what?' and a student said 'It is a whole number of one'. In this extract, Onizan tried to help the student to notice that one fully coloured rectangle is a whole number of one but he was faced with some answers that he considered incorrect. He reacted to them by guiding the students with questions towards the point. It seems that in the beginning his question was not clear so he tried to change it which might have been because of his lack of experience.

In conclusion, Onizan showed secure subject content knowledge of the taught topic on which he relied while teaching. He extended the lesson's content by adding the conversion from decimals to mixed numbers confidently and the lesson was taught without error. His reliance on procedures was highly noticeable and he avoided presenting certain parts of the lesson such as the place value table, possibly because of limitations of time and pedagogical content knowledge. He wanted to investigate the students' source of errors in order to deal with them. On the whole, he used eliciting exchanges in most of his interactions with the students and he dealt with students' correct answers by accepting them (mostly by accept moves but also to a lesser extent maintain moves). He dealt with the incorrect ones by rejecting them, asking other students or asking for justification (reject and press moves).

6.4 Onizan third lesson (lesson 1)

The lesson of 'Percentages and Fractions' was introduced to Grade 6. Its aim was to enable students to write a percentage as a fraction or a mixed number and vice versa. For example, 'Write 80% as a fraction in the simplest form' and 'Write $\frac{2}{5}$ as a percentage'. The lesson was preceded by 'Rate and Ratio' and 'Proportion' lessons.

6.4.1 The lesson description:

Onizan started the introductory part of the lesson, which lasted for four minutes, by asking 'What is percentage?' Then he selected three students to answer in turn. He received the answers 'It is a ratio', no answer and 'A number out of 100'. Onizan confirmed the last answer by repeating it and then he wrote on the board 50% and 30% and said 'You may see this percentage if there is sale on in the market. Today we will learn how we can rewrite it as a fraction in the simplest form'. A student suggested that it can be written as $\frac{50}{100}$ and Onizan asked him to simplify it. The class, with Onizan, divided the numerator and the denominator by 50 so Onizan wrote $\frac{1}{2}$ on the board.

For the next two minutes, Onizan moved to the first example in the book which was 'Write $\frac{9}{20}$ as a percentage'. It was worked out in the book by writing the proportion $\frac{9}{20} = \frac{x}{100}$ then finding that $\frac{9\times5}{20\times5} = \frac{45}{100}$ = 45%. Onizan explained the process to the class.

Onizan spent the next five minutes explaining the second example 'Write the percentage of the model' (see Figure 6.5 below).

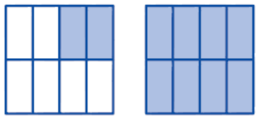


Figure 6.5

Before starting the example, Onizan said 'I will give you an example' then he wrote $\frac{125}{100}$ on the board and asked the students to simplify it. The final outcome of the discussion with the class was that it was $1\frac{25}{100} = 1\frac{25 \div 25}{100 \div 25} = 1\frac{1}{4}$. He then returned to the model and followed the book's instructions with the class. They simplified $1\frac{2}{8}$ and wrote it as $1\frac{1}{4}$ then they converted it to an improper fraction $\frac{5}{4}$. Next, they wrote the proportion $\frac{5}{4} = \frac{x}{100}$ and found that the percentage was 125%.

Onizan moved to the first exercise 'Write each percentage as a fraction or a mixed number in the simplest form: A) 15% B) 80% C) 180%', and spent nine minutes solving it with the class. At the end, he asked the students to copy the solution in their books. For example, they solved the exercise of 180% as $\frac{180}{100} = 1\frac{80}{100}$ then Onizan said 'we solved 80 in B) so he wrote $1\frac{80 \div 20}{100 \div 20} = 1\frac{4}{5}$.

The second exercise was 'Middle schools make up 30% of the total of Saudi's schools. What fraction is this of Saudi's schools?' Onizan elected a student to solve it on the board and it took two minutes.

Next, Onizan moved to the exercise 'Write each of the fractions and mixed numbers as a percentage: A) $\frac{1}{4}$ B) $\frac{2}{5}$ C) $2\frac{1}{4}$ ', and spent nine minutes solving it. I describe in detail in incident 1 how they solved (c). He then asked the students to copy the answers and to open their activity book and to solve the first exercise which was 'Write 60% as a fraction'. He then went around the class and corrected their answers individually for the last 15 minutes of the lesson.

Incident 1:

The exercise was 'Write $2\frac{1}{4}$ as a percentage' and the following dialogue occurred.

1	Onizan	So who can solve it and get a present?
2	Students	[Raise hands]

3	Onizan	Ali?
4	Ali	[Comes to the board and spends a few seconds thinking] We multiply 4 by 2 then add one?
5	Onizan	I do not know, just solve it by yourself. I cannot help you as this a competition question.
6	Ali	[Writes $\frac{9}{4}$ and thinks].
7	Onizan	[While Ali is thinking he deletes the answer] Go to your seat.
8	Khalid	[comes to the board]
9	Onizan	We want to solve it in the same way we did earlier to $\frac{125}{100}$.[The second example]
10	Khalid	[Writes $\frac{9}{4}$] Is this right?
11	Onizan	No, [deletes it] who will come next? Saad
12	Saad	[Comes to the board]
13	Onizan	Saad can you solve it as we did for that example? [pointing to the other part of the board where he wrote $1\frac{25}{100} = 1\frac{25 \div 25}{100 \div 25} = 1\frac{1}{4}$]
14	Saad	[Writes $2\frac{1}{4} = 2\frac{1\times 25}{4\times 25} = \frac{225}{100}$]
15	Onizan	Excellent, [Writes = 225% and deletes the solution] so we solve it by writing 2 on the left, then how do we write one over four as a pecentage?? We have just done it here [pointing to A)]. We multiplied the fraction by 25 so we found 25 over 100. [Writes 225%]. Copy it to your books.
16	A student	If there is another number instead of 2 [In $2\frac{1}{4}$]?
17	Onizan	Any number comes on the left, for example 20 or 16 should be moved to the left of the percentage. [Writes $5\frac{1}{4}$ = 525%]

6.4.2 Applying the Knowledge Quartet (KQ) and IRF

The following analysis takes into account the class observation and Onizan's comments in the interview

Foundation

The lesson had many concepts and procedures. It had the concepts of percentage, fraction, ratio and proportion which are hard to understand (Iuculano and Butterworth, 2011). It also had procedures of converting a percentage to a fraction and vice versa. Onizan did not encounter any problems while introducing these components which may indicate that he showed overt subject knowledge (OSK) regarding these concepts and procedures. However, I was in doubt about this when Onizan rejected the two student answers in the incident ([6] and [10]). I asked him about their answers and he responded by saying ‘yes their answers were absolutely correct but I preferred to encourage them to solve it as we did …’.

The transition from the definition of the percentage to the conversion to a fraction was done with a high emphasis on procedures (COP) as I outlined in the previous lessons. With the class, Onizan solved the exercises almost all in the same way by relying heavily on a process to do the conversion. For example, in the incident, two students tried the same method ([6] and [10]) which was not Onizan’s preference so he told them to try the one they had just used to solve an earlier example. He justified his action as he said in the interview ‘I was keen to prevent them from making mistakes when they only relied on their own methods’.

Transformation & Connection

Onizan stuck to the book’s examples and representations when he demonstrated the conversion between a percentage and a fraction (TD). He introduced the conversion in a procedural way where he concentrated on obtaining the correct answers with less attention paid to the understanding. This teaching approach may lead to the absence of making connections between percentages and fractions (MCC), which in turn may hinder a child’s easy movement between different representations of them, numerically or visually (Suh *et al.*, 2008).

Contingency

Onizan accepted the students’ correct answers in different ways (RCI). He sometimes repeated them in order to confirm and share them with the class. For example, he repeated the student answer (student response of elicitation initiation) of the definition of the percentage (accept move) in order to, as he said in the interview, confirm the correctness of the definition and make it explicit to all the children. In addition, he praised some students who answered correctly in order to encourage them to participate more in the class. For instance, when the student gave the correct answer (student response to an elicitation opening) to the question in the incident, Onizan said to him ‘excellent’ (accept move) [15] may because he deserved this appreciation as he was the only one who could solve the problem among the three participants. Moreover, on some occasions, he accepted correct answers and wrote them on the board and then moved on to the next step in the process. For example, when Onizan wanted to write 80% as a fraction, a student said ‘Write 80 above 100’ (student response to an elicitation opening). Onizan accepted this (accept move) then wrote 80/100 and asked ‘How can we simplify this?’ (another elicitation exchange). He was probably concerned with saving class time as he mentioned that point many times during the interview. Lastly, although he usually accepted correct answers, he rejected some that did not fit with his expectations or his solving methods. For example, as he justified in the interview, he rejected the two answers in the incident, [7] and [11] (reject move), because the students used another method that he did not want them to follow. He was keen for them to avoid any possible mistakes when they use their own solving methods so he preferred to tie them to his methods.

Onizan dealt with incorrect answers by rejecting them in three different ways. Firstly, at the beginning of the lesson he directly asked other students for their answers when he was given incorrect ones. For example, when he asked ‘What is the percentage?’ (elicitation opening) he received an incorrect answer (student response) then he asked another student and so on until he found the correct one (implicit evaluation move followed by nomination and a new eliciting exchange). He justified that by saying in the interview, ‘I believe that some students prepare themselves for the lesson in advance. In this case, I was looking for those students to give them the chance to express themselves…’. Onizan’s second response was to correct answers himself. For instance, when the class was simplifying $\frac{25}{100}$, Onizan asked ‘Can we simplify the fraction ($\frac{25}{100}$)?’ (elicitation move). A student said ‘Yes, we can divide it by 4’ (student response) then Onizan said ‘We divide by 25’ and wrote $\frac{25\div 25}{100\div 25} = \frac{1}{4}$ (reject move followed by insert move since Onizan added new information to the student’s contribution). He did that as he said:

> I was disappointed when I found that the students were very weak in their times table… I gave them the answer as this was just an example not a real exercise and I was concerned with saving class time.

It seems that Onizan gave the answer if he believed that the students would not be able to provide it. In the original quote Onizan mentioned that the students would not be able to divide the denominator by the numerator or less so he preferred to give them the correct answer.

Lastly, he rejected incorrect answers by saying ‘no’ then commenting on the answers. For example, when they were writing 80% as a fraction, Onizan asked ‘How can we simplify $\frac{80}{100}$?’ (elicitation initiation). A student said ‘We divide 80 by 40’ (student response). Onizan said ‘No, we want a number that divides both 80 and 100’ (reject move followed by a clue) and the student said ‘50?’ (student response). Onizan then said ‘No’ (reject move) and asked another student to answer (nomination followed by elicitation opening). Onizan justified his action as he thought that dividing the numerator by a number regardless of the denominator was a common mistake held by many students that needed to be dealt with. He then asked other students as he believes that some students accept corrections better if they come from other students.

In conclusion, Onizan showed secure subject knowledge while teaching this lesson since there were no problems in introducing the topic. He taught in a procedural ways as he did in the other lessons and most exchanges between him and his students were eliciting exchanges. He dealt with correct answers by accepting them and sharing them with the class (almost always accept moves followed by maintain moves). His protectiveness of the students making mistakes led him to reject some correct answers as he wanted the students to follow his methods. Incorrect answers were dealt with by rejecting them and redirecting the question to other students (reject moves followed by insert and press moves).

6.5 An overview of Onizan’s knowledge

The analysis of Onizan’s teaching with reference to the KQ showed many indications of Onizan’s secure subject knowledge of the concepts and procedures he taught. He was aware of the lessons’ components and did not make mistakes when introducing them. He also confidently and correctly answered all student questions and he showed, to some extent, the ability to anticipate the sources of some errors. For example, when a student said ‘we divide by 3’ to simplify $\frac{15}{100}$, Onizan demonstrated the reasoning behind this, as he thought that the student was focusing on the numerator only which Onizan believes is a common problem for his students. On some occasions, Onizan extended the content of certain lessons, such as the lesson of ‘Mixed Numbers and Decimals’ by including the conversion from decimals to mixed numbers which was not part of the lesson. This may indicate that Onizan has secure content knowledge of these topics.

Conversely, some evidence of a limitation in Onizan’ pedagogical content knowledge was identified. He demonstrated almost all the lessons in a procedural way where he put more emphasis on doing certain routines and methods without explaining why they work. For example, when he explained to the class how to convert a decimal to a fraction, he used verbal instructions supported with arrows to show how to do it, with no explanation of why it was done that way. This could be because he felt that he was not able to deliver it in an acceptable way as he said ‘I think most of them will not understand it well’. In addition, he wanted to see correct answers which on some occasions required him to reject some of the students’ own strategies in order to prevent them making mistakes. Moreover, he sometimes used inadequate activities to introduce some parts of the lessons and he missed the main point of the activity. For example, he started the lesson of ‘Elapsed Time’ by asking a student to write the numbers from one to thirty while another one counted. Then he calculated the elapsed time. He did not mention the start and end time between which the elapsed time is calculated, which was the main point of the lesson. Therefore, the activity of counting became the main focus rather than the time that had elapsed between the two points of time.

6.6 An overview of Onizan’s response to students’ contributions

Most of the interactions between Onizan and his students were eliciting exchanges where Onizan initiates an elicitation move, a question usually, which is followed by a student’s reply, then a follow up move by Onizan (Sinclair and Coulthard, 1992). I am particularly interested in the third turn in this exchange as it is a contingent action dependent on the second turn in the exchange (Lee, 2007). In this section I focus on this third move in Onizan’s dealing with students correct and incorrect answers.

6.6.1 Onizan's response pattern to correct answers: 1) Accepting correct answers

Onizan responded to correct answers by accepting, ignoring or rejecting them. Among all the lessons, the majority of his responses were to accept them in one common way. He usually gave accept moves through one or more of the following linear four steps: praising the student (P), writing the answer on the board (W), acting (e.g. asking and commenting) (A) and moving to next point (M). For example, when Onizan asked the students to write 15% as a fraction (elicitation initiation), he received the answer of '15 over 100' (student response). He responded by saying 'Excellent' (P) (accept move) and then wrote $\frac{15}{100}$ on the board (W) (written re-voicing) and asked 'How can we simplify this?' (M) (starting another eliciting exchange). His response in this example was PWM as he praised, then wrote the answer and next moved to the simplification of the fraction. If he had asked the students, for example, 'why did you say that?' as he did in another example, after writing the answer (press move), that would fit with the acting step (A) and then the response would have been PWAM. The pattern of this response shows that Onizan's third turn usually has a combination of many acts varying from accept, re-voice and press, followed by initiating a new eliciting exchange.

The reasons his responses were varied were explained in the interviews. For example, Onizan justified his praise (P) (accept) as a way of encouraging the students to participate more in the class and it was provided to certain pupils for that reason and for the main points of the lesson. In addition, he wrote (W) (re-voice) the answers on the board to share them with the class and to confirm it. Furthermore, he acted (A) by asking a question (press/maintain) such as 'is that right' to use it to check the students' understanding of certain concepts and their ability of doing and justifying certain procedures.

Onizan's subject matter knowledge and pedagogical knowledge came to play in this type of response. He clearly identified the answers as correct ones depending on his subject knowledge and then he accepted them. He praised (P) the students probably as a result of his belief that the students learn better if they are encouraged by the teacher, which is part of his pedagogical knowledge. Writing and sharing the answers (W) may have been done because he has secure subject matter knowledge so he has the confidence to do so or as an aspect of his pedagogical knowledge. Inviting the class to participate in the lesson could have been seen as Onizan's preferred way to introduce the lesson, which may also be considered an aspect of his pedagogical knowledge. Onizan's action (A) could be related to his pedagogical knowledge more than his subject knowledge as he used some techniques in order to check understanding or as a way of sharing the answers with the class.

6.6.2 Onizan's response pattern to correct answers: 2) Rejecting correct answers

Onizan rejected correct answers (reject move) less frequently than accepting them. He rejected some answers as they were not what he expected to hear. On more than one occasion, Onizan rejected correct answers because the students had used different methods to his. For instance, he rejected the students' answers of writing $2\frac{1}{4}$ as $\frac{9}{4}$ (reject move) in order to write it as a percentage. He wanted them to solve it by using his method of writing the mixed number as $2\frac{1\times 25}{4\times 25}$. This pattern shows that Onizan's third move when the student uses different methods, is always rejection. He justified his response as he was keen to protect the students from making mistakes if they used their own methods.

This response could be related to both Onizan's subject knowledge and his pedagogical knowledge as well as time pressure. To check Onizan's subject knowledge regarding this point, I asked him about the correctness of these solutions and he said 'yes they were correct'. He explained to me how the students would continue their answers if he let them do so. This may be considered evidence of Onizan's knowledge of the subject matter and I started to think about his pedagogical knowledge regarding this situation. It seemed to me that his demonstration was built on doing blind procedures which may have shaped his teaching. The possible limitation that he has in his pedagogical content knowledge may influence his dealing with this situation as he was not brave or confident enough to let the students try their strategies. Instead, his over protectiveness affected his teaching by forcing the students to copy his method of solving the problem.

6.6.3 Onizan's response pattern to incorrect answers: 1) Rejecting incorrect answers

Onizan's most observed response to incorrect answers was to reject them. He gave reject moves by using at least one step out of three sequential steps. Firstly, he started by refusing (R) (reject move) the answer through different ways such as saying no, shaking his head, deleting the answer or repeating it. This was followed by asking (A) other students to participate (elicitation move) by answering or correcting their peers. The last step was him commenting on or correcting (C) the answers (comment or insert move). For example, when Onizan said no to a student who answered incorrectly and then asked another, he followed the RA response type. Also, if he corrected the incorrect answer by himself after asking many students, he followed the AC type of response. I found that he responded in different formats such as A, AC, RC, RA and so on and he justified this in many ways. The common moves Onizan made, when an incorrect answer occurred, were rejection followed by nomination and elicitation. These were followed by either comments or correcting.

Onizan always asked other students when he was faced with incorrect answers, which could have been done for many reasons. On many occasions, Onizan stated that he believes that some students accepted corrections by their peers more than the teacher so this influenced him to do so. Also, he did that in the beginning of some lessons to give those who had prepared the lessons in advance the chance to express themselves and as an encouragement to them. Moreover, in the second part of the lessons, which was just solving exercises and had less time pressure, he spent more time with the students checking their answers.

Also, he commented on some incorrect answers when he thought that the point was very important or the error could be considered as a common mistake that needed to be dealt with. In addition, he corrected them himself when the answer was considered an error in an important part of the lesson or because of the limitation of time and his doubt in the ability of the students to answer it correctly.

Onizan's subject knowledge and pedagogical knowledge may have influenced his dealing with incorrect answers. Firstly, his ability to identify the incorrect answers relied heavily on his subject knowledge. Secondly, he reacted in different forms (R) to refute these answers, such as writing and deleting them, which can be considered a confident action and demonstrating good subject knowledge that he was able to share with the class. Also, repeating the answers with a question sound in his voice as a sign that ‘the answer is wrong’ may form an aspect of his pedagogical knowledge as he was using this strategy to break the routine in his teaching. Thirdly, asking other students (A) seems to be related to Onizan’s beliefs of the importance of peer learning and the importance of praising the children for their preparation of the lessons. Fourthly, Onizan's comments on them (C) can be related to both subject and pedagogical knowledge. He used the former to judge the answer and identify the gaps as he has secure content knowledge. Also, an aspect of his pedagogical content knowledge was that he commented on some answers because he was keen to alert the students to some common errors and wanted to tie them to his solving methods. Finally, he corrected (C) some incorrect answers by himself because he has a secureknowledge that he could rely on to do so. Also, time pressure came to play in this point as he wanted to save lesson time.

6.6.4 Onizan’s response pattern to incorrect answers: 2) Asking for justification

Onizan less frequently asked for justification of some incorrect answers (press move). This was clear when Onizan was faced with many wrong answers regarding the same point which he then treated as a common mistake that needed to be dealt with.

This action showed that Onizan has a good subject knowledge that enables him to investigate the sources of the errors. However, it could be related more to the belief of the importance of knowing the source of the misunderstanding before treating the error. He believed that some students do not like to share their mistakes with the class. In spite of this, Onizan took mistakes as opportunities to explore the sources of the mistakes and then to treat them accordingly.

In conclusion, the analysis of Onizan’s lesson using the KQ shows that Onizan has secure content knowledge but insecure pedagogical content knowledge. The most common interaction pattern was eliciting exchange with a different type of third turn moves. He dealt with students’ correct answers by accepting them and sharing them with the class if they fit with his instructions. His third turn was always accepting then eliciting a new exchange. Conversely, he rejected incorrect answers and asked other students to answer, and his third turn was rejection followed by nomination and eliciting move. His SMK, PCK, beliefs and time limitation affected his response patterns to correct and incorrect answers.

Chapter 7 : The case study of Fahad

7.1 Introduction

Fahad (participant 4 in section 4.4.2) was one of the trainee teachers who participated in this research. He is 23 years old and practising teaching at the Hazem School. He taught two classes of grade four (9-10-year-olds) for five lessons each per week. He was attending the Complex analysis course as well as practising in the school. Fahad showed interest in developing his teaching style as he told me that he always looked to the YouTube website for ideas about teaching certain concepts to bring into the class. Although, the Head Teacher at Hazem's School gave the trainees the freedom to go out whenever they liked as long as they did not have classes, Fahad always remained at school all day as he was observing and asking others, such as the librarian, about the nature of their jobs. Fahad's supervisor visited him rarely so he told me that he depended heavily on the co-operating teacher feedback about his teaching. Fahad was observed and interviewed as the Table 7.1 below shows:

Teacher	Observed Lessons	Grade	Incidents per lesson	Interviews
Fahad	1	4A	3	1
	2	4B	2	
	3	**4B**	**5**	2
	4	**4A**	**5**	
	5	4B	3	3
	6	4B	3	
	7	**4B**	**4**	4
	8	4A	2	

Table 7.1: Observations of Fahad

Note: lessons 3,4 and 7 are analysed in this thesis

In this chapter, the three lessons that have the highest number of incidents as shown in Table 7.1 are analysed. I start with lesson 3 then 4 and the last lesson is lesson 7. The third lesson is written in less detail than the first two lessons to avoid repetition. In every lesson there were different types of incidents of which I chose one to show the reader an example of the interactions. The incidents chosen have a variety of teacher responses and the teachers show clear evidence of different types of knowledge in these incidents.

7.2 The first lesson (lesson 3)

The lesson of ‘Compare and Order Fractions’ was taught to Grade 4 in their classroom which was made up of 5 rows and 4 columns of 20 students. The lesson’s aim was to compare two fractions by using one of three ways (models (fraction bar), a number line or equivalent fractions), and to order three fractions increasingly or decreasingly. It was preceded by the lessons of ‘Parts of a Whole’, ‘Parts of a Set’, ‘Locate Fractions on a Number Line’ and ‘Equivalent Fractions’.

7.2.1 The Lesson Description

For the first two minutes, Fahad showed the lesson page on the board and asked a student to read the first statement on the page: ‘To compare fractions you can use models or a number line or equivalent fractions’. He asked ‘What is a model?’ and got different answers such as ‘Fractions’ and ‘Rectangles’. Then he wrote $\frac{5}{8}$ and $\frac{3}{8}$ and asked how to compare them by using models. A student suggested that they can multiply 5 by 8 and 3 by 8 (cross multiplication) and Fahad asked him to wait for a moment and then he showed the first example (see Figure 7.1), from the book, on the board:

‘Which is bigger: the height of $\frac{5}{8}$ of red ribbon or $\frac{3}{8}$ of yellow ribbon?’.

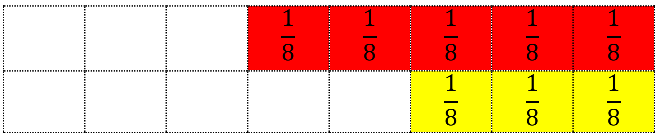


Figure 7.1

Fahad spent nearly three minutes explaining the example to the class by mentioning the comparison signs ($<, >$ *and* $=$) and how they apply to integer numbers (e.g. 10>5 and 5=5). Then he showed them how to draw fraction bars and how to use them to compare two fractions by counting the parts then comparing them visually.

Fahad then moved to the second example ‘Which is bigger: $\frac{1}{2}$ or $\frac{1}{4}$? Use the number line shown below’. He spent three minutes on this section and he started by saying ‘A half is bigger than a quarter is not it? Can you prove that?’ A student suggested that they see the denominator and the numerator and Fahad went back to the board and pointed to the written fractions $\frac{5}{8} \ldots \frac{3}{8}$. Fahad said ‘To compare them, a student suggested to multiply 5 by 8 and 3 by 8. Is that right?’ The students did not agree with that as they suggested different strategies like multiplying 8 by 8, multiplying the denominator by the denominator, multiplying the numerator by the numerator and multiplying the first fraction by the denominator of the second one. Fahad stressed that they use multiplication when the denominators are different and wrote $\frac{5}{8} > \frac{3}{8}$. He then showed a worked example of ordering three fractions $\frac{2}{3}, \frac{1}{2}$ and $\frac{7}{12}$ with the number line and equivalent fractions. He spent a minute on this example and he asked the class to just look at it. Then he mentioned that in the equivalent fractions method they use multiplication.

After that he moved to the exercises section where he spent two minutes solving the first two shown in Figure 7.2: 'Compare by using ($<, >$ *and* $=$)

1)

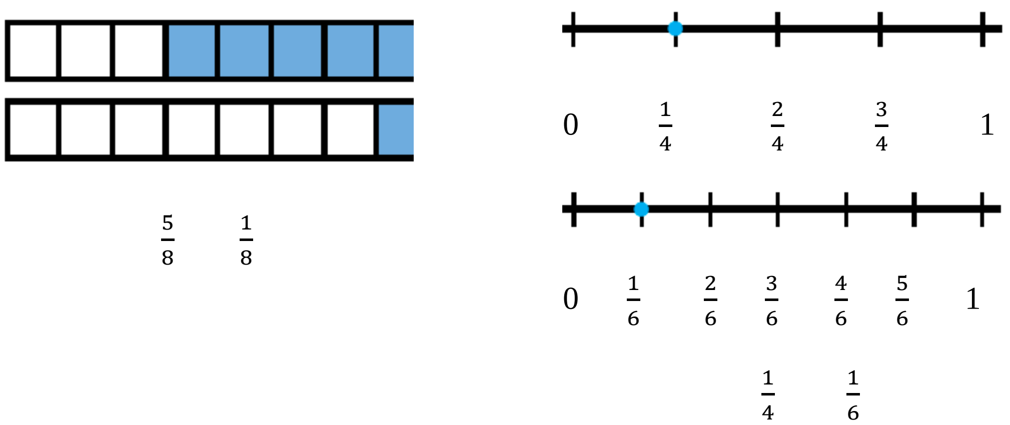


Figure 7.2

He solved the first one by counting the parts and comparing 5 to 1. The second one was solved by multiplying 4 by 1 and 6 by 1, then by comparing 6 to 4 (cross multiplication).

Next, Fahad solved the exercise 'Compare $\frac{3}{4}$ _ $\frac{3}{6}$', within the next two minutes by asking the students for their thoughts, and he suggested using cross multiplication then comparing 18 to 12. Later, he asked if there was anyone who did not understand and five students raised their hands so he repeated the last exercise. During that time Fahad was asked many times about the number line model and he responded by different answers. For example he responded by saying 'it could be solved by using the cross multiplication method', 'wait I will explain the number line later' and 'it is used in situations when dealing with three fractions'. After explaining the last exercise, Fahad wrote two examples of his choice, $\frac{3}{4}$ $\frac{1}{2}$ and $\frac{3}{5}$ $\frac{1}{5}$,and asked the students to come and solve them on the board. The first one, which is similar to the last worked exercise, was solved by cross multiplication and the other was solved, with Fahad's help, by direct comparing, as the student was suggesting using cross multiplication as well. This activity lasted for three minutes.

For the next six minutes, Fahad solved the next exercise with the class: 'Order $\frac{1}{16}$, $\frac{7}{8}$, and $\frac{3}{4}$ from the least to the greatest' by unifying the denominators to be 16 as he mentioned that 16 exists in the 8 and 4 times tables. He wrote them as $\frac{1}{16} \times -$, $\frac{7}{8} \times -$ and $\frac{3}{4} \times -$ then $\frac{1}{16} \times \frac{1}{1}$, $\frac{7}{8} \times \frac{2}{2}$ and $\frac{3}{4} \times \frac{4}{4}$. Finally he wrote $\frac{1}{16} < \frac{3}{4} < \frac{7}{8}$ and then moved to the next group of fractions, $\frac{3}{8}$, $\frac{2}{6}$, and $\frac{4}{8}$, which took ten minutes and which is described in the incident below.

Incident 1:

The exercise was 'Order $\frac{3}{8}$, $\frac{2}{6}$, and $\frac{4}{8}$ from the least to the greatest' and the following dialogue happened while solving the exercise:

1	Fahad	How we can solve this one?
2	Student 1	These denominators do not exist in each other's times tables.
3	Fahad	Right, in this case you should multiply the second fraction by 8 to have the same denominator 48, then order them. [Writes $\frac{3}{8} \times \frac{6}{6}$, $\frac{2}{6} \times \frac{8}{8}$ and $\frac{4}{8} \times \frac{6}{6}$] Who can complete it?
4	Student 2	21 over 48.
5	Fahad	No, who can help?
6	Student 3	It is 18 over 48, 16 over 48 and 24 over 48.
7	Fahad	Well done. Now can you put them in order starting with the smallest? Adel
8	Adel	[Comes to the board and writes $\frac{2}{6} < \frac{3}{8} < \frac{4}{8}$].
9	Fahad	Excellent Adel, can you all clap Adel please. So we should use a different method here. I can tell you a short method that helps solving these kinds of ordering questions. These fractions' denominators do not exist in each other's times tables right?
10	Students	Yes
11	Fahad	So, remember I will tell you this method but do not overuse it as it might not work all the time ok? Firstly keep your eyes on the numerators and then here they are 2,3 and 4 right?

12	Students	Right
13	Fahad	Now, you just focus on these numbers and just simply order them. So which one is the smallest?
14	Student 2	2
15	Fahad	Right. Which comes next?
16	Student 3	3 then 4
17	Fahad	Good. Now the correct order will be $\frac{2}{6}$ then $\frac{3}{8}$ then $\frac{4}{8}$. The same answer. Easy, is not it?
18	Students	Yes
19	Fahad	If we do it to this exercise [Pointing to $\frac{1}{16}$, $\frac{7}{8}$, $\frac{3}{4}$ on the board], you find it works as well. But it will not work if there are similar numerators like the next exercise $\frac{3}{6}$, $\frac{2}{3}$, and $\frac{3}{3}$.

He spent the next three minutes ordering $\frac{3}{6}$, $\frac{2}{3}$, and $\frac{3}{3}$ by asking a student to solve it on the board and the student solved it correctly by multiplying the second and third fractions by 2 as per Fahad's instruction of the previous example. The rest of the lesson was spent solving the rest of the exercises.

7.2.2 Applying the Knowledge Quartet (KQ) and IRF

The following analysis takes into account the class observation and Fahad's comments in the interview.

Foundation

Many students have difficulties in comparing and ordering fractions (Liu *et al.*, 2013). Some students make more mistakes when comparing fractions with the same numerators (e.g. $\frac{1}{32} > \frac{1}{18}$ because 32>18) (Smith *et al.*, 2005) than comparing fractions with common denominators (e.g. $\frac{5}{8} > \frac{1}{8}$ as 5> 1) (Meert *et al.*, 2009). Stafylidou and Vosniadou (2004) found that students have two contradictory beliefs when comparing fractions: one is that when the numerator or denominator increases, the fraction value also increases; and the other is that when the numerator or denominator decreases, the fraction value increases. In addition, some students compare two fractions by looking at the value of their components not the real numerical value (Liu *et al.*, 2013) and even some adults do that (Bonato *et al.*, 2007). Conversely, Meert and colleagues (2010) found that children access the magnitude of the whole fraction when comparing fractions. These difficulties could be related to the inference of natural numbers on learning fractions (Ni and Zhou, 2005), the treatment of the fractions components as independent numbers (Liu *et al.*, 2013) or because teachers' understanding of fractions affects students understanding (Van Steenbrugge *et al.*, 2014).

Fractions can be compared and ordered using different methods. Students can use equivalent fractions, a number line, models, referring to the distance of a fixed point, or an informal equivalence fractions based strategy to compare two or more fractions (Nunes *et al.*, 2009; Keijzer and Terwel, 2001; Dickson *et al.*, 1984). Although a number line was absent in this lesson, it is worth saying a few words about it. A number line is harder than the area model and sub-set of discrete objects as it does not represent the fraction as a part of the whole or as a set of objects format, but shows it as an abstract number on a line (Dickson *et al.*, 1984). In addition, finding the unit of the number line, the subdivision of the unit whether it is equal to the denominator or not and locating a point in different patterns of cues (partitions) whether consistent, incomplete or irrelevant are all problems associated with the number line method (Keijzer and Terwel, 2001). Using a number line to locate a fraction fits with the measurement situation type in primary schools (Dickson *et al.*, 1984) where the students need to represent a part of a whole unit like ¼ bar (Nunes *et al.*, 2009). This type of using fractions as a measure provides less help in understanding equivalence and ordering fractions than division situations where the students do a division when the dividend is less than the divisor (*ibid*).

Fahad showed some limitation in his subject knowledge (OSK) regarding the lesson's components: comparison and ordering fractions concepts and procedures. Firstly, he omitted some parts of the lesson where the book suggested using the number line. Even though some students asked him directly about it, he did not give a direct answer. Fahad justified his action in the interview by saying:

> using the number line is hard to introduce to the class because it needs to be prepared in advance to make sure the partitions are placed in right places. I need to seek a ready labelled number line because you know I cannot do it myself. I do not know how to partition a number line into twelve parts to show the fraction five over twelve.

However, the book provided a number line model as in the example of comparing a half with a quarter, and Fahad did not use it and instead he used the cross multiplication method to compare those fractions. This might indicate that he did not know how to use it or how to make it, or both.

Secondly, it seemed that Fahad was not fully aware of using the cross multiplication method while comparing fractions. When the student suggested using this method to compare $\frac{5}{8}$ and $\frac{3}{8}$ at the beginning of the lesson, Fahad asked him to wait. I thought that was because he did not want to disturb the lesson flow, then in the interview I asked him ‘Does the suggestion work here?’. He replied ‘I do not think it works as the fractions here are common denominators’ so I asked him to try that. He tried and found it worked, ($5 \times 8 > 3 \times 8$), and said ‘I thought he suggested something wrong but now I realise that I was wrong’. This might indicate some confusion held by Fahad in terms of the name and use of cross multiplication. He called it cross multiplication which may be considered as an incorrect name because cross multiplication indeed is used to solve proportional problems (Ellis, 2013). What he had used was, in a sense, the equivalent fractions method as he, with or without awareness, multiplied $\frac{5\times8}{8\times8}$ and $\frac{3\times8}{8\times8}$ then compared 40 with 24. It seemed that Fahad did not know the term cross multiplication well and he was not aware about the relation between his method and the equivalent fractions methods.

Lastly, at the end of the lesson, Fahad came up with a comparing method that he claimed works sometimes and he highlighted its limitations. He told the class to compare just the numerators regardless of the denominators and, unfortunately, most of the book examples supported this misconception. He clearly applied the ordering role of the natural numbers on the fractions which usually leads to more errors and misconceptions (Ni and Zhou, 2005). He accessed the component of the fractions and did not take into account the importance of the value of the fractions in ordering fractions. He said in the interview that he came up with this method from some students in the other class. He then tested some examples and found it worked, then he generalised it. He also could not justify why it worked for these examples when I asked him about that. I asked him to apply it to fractions $\frac{2}{4}, \frac{4}{6}$ and $\frac{3}{8}$ (counter example) and he found it did not work even though the numerators were all different. Proposing this inadequate method, as well as omitting the number line and using cross multiplication, shows some evidence of weakness in Fahad's subject matter knowledge regarding the taught topic.

Fahad relied heavily on procedures in his teaching (COP). He turned the lesson into a list of steps that were needed to be done one after another (and this is explained in detail in the next section). He dwelt mostly on one method (cross multiplication) and did not give the students the chance to develop their own. This fits with the transmission-oriented beliefs of mathematics as a set of procedures that are needed to be done correctly and mathematics is better learned using clear verbal instructions (Askew *et al.*, 1997). Moreover, the interaction between Fahad and the students in the form of questions and answers is, in a sense, one aspect of transmission-oriented belief (*ibid*).

Transformation

Fahad demonstrated (TD) comparison of fractions in a procedural way. He used some specific words such as 'keep your eye on …' that indicate a procedural voice in his instructions. It seemed that the students should start by checking the denominators then if they are the same, comparing the numerators without explaining why. Later in the lesson he adopted the cross multiplication method to compare fractions. He was focusing on finding the correct answers even if they came without understanding. In addition, ordering fractions was introduced in steps starting by writing them in separate rows then writing the multiplication signs then the fraction lines and so on. And, again, he did not introduce the reasons for following these steps. Instead, he invented a method to help the student achieve the answer quicker than using the equivalent fraction method. However, though he showed some care about the students learning as he asked about whether they understood or not, his re-explanations were a repetition of the original demonstrations. He probably taught that way as a result of his weak subject matter knowledge which did not help him to cover all the content of the lesson as he applied some strategies and omitted others that he was not fully sure of.

Connection

The lesson components were not connected during the lesson. Fahad did not make explicit connections between fractions with common numerators and denominators with their value (MCC) . For example, when they compared two fractions with common numerators, Fahad did not highlight to the students the idea that when the denominator increases the fraction value conversely decreases. Also he did not make it clear that when the numerator increases, when comparing fractions with common denominators, the fraction value increases. He might not be aware of this relation or not believe in its importance.

Fahad showed some signs of being able to anticipate the lesson complexity (AC). He came up with the new strategy of ordering three fractions as he was expecting some problems with using multiplications to order fractions, as he said in the interview:

> I hesitated to introduce this suggested idea as it does not always work. Some students whose parents teach at home gave me this idea. I tested it and found that it worked in some exercises. I was looking for a way the students could use to compare three fractions. They are weak when it comes to using times tables, so I thought to use this idea to help them.

Contingency

Fahad dealt with students' correct answers by accepting or rejecting them (RCI). He accepted some correct answers by praising the students then commenting on them and conducting further eliciting exchanges. For example, Fahad asked 'When do we use multiplication? [to compare fractions]' (elicitation opening) and a student said 'When the denominators are different' (student response). Fahad responded 'Well done (accept move), here the denominators are 8 (comment), so which one is greater 5 or 3?' (elicitation move to start a new eliciting exchange). He justified his action by saying

> I like to praise the students. I think rewarding the students by using good words such as good, well done or excellent is a good way to build confidence and encourage them to participate in the class.

In addition, he accepted correct answers by writing them on the board with or without further action. For example, Fahad asked while comparing $\frac{3}{5}$ and $\frac{2}{5}$ 'Which is bigger 2 or 3?'(elicitation initiation) and the student said '3' (student response). Fahad then wrote $\frac{3}{5} > \frac{2}{5}$ (accept move) and moved to the next exercise.

Some correct answers were rejected by Fahad. He said 'No' or 'Stop' for some correct answers and commented on them. For instance, Fahad asked 'How to compare $\frac{3}{8}$ and $\frac{5}{8}$?' (elicitation opening) and got many correct answers such as 'We multiply the first fraction by the denominator of the second one …[Fahad interrupted him]' (student response). Fahad said 'No, look, multiplication is another method that we do not need here … Khaled?' (reject move followed by comment and nomination). In this situation, Fahad justified his action by saying:

> I think Majed [the student] preferred to cross multiply rather than anything else, so he suggested it as a solving plan. I had not expected his answer as I was looking for a student who can spot that the denominators were different and I wanted him to work with us on the model methods

This showed that Fahad responded in that way probably due to his tendency to adhere to the textbook order by starting with the models before jumping to further strategies. Also, his response could be justified as a result of the influence of his subject matter knowledge as he was not sure about the correctness of using cross multiplication to compare two fractions that have the same denominator, as discussed in the foundation dimension above.

Similarly, incorrect answers were rejected in different ways. He did so verbally (no and stop) or physically (shake of the head) then commenting, asking other students or correcting them himself. For example, in the incident, [4] and [5], he said 'No, who can help?' (reject move followed by nomination) as a response to the student who said '21 over 48' (the student response). Fahad justified his action by saying 'Sometimes I prefer to look for another answer from the students to avoid saying wrong to the student who answers incorrectly, to protect him from being disappointed'. In addition, Fahad commented on a student's incorrect answer regarding, from Fahad's point of view, comparing fractions with common denominators: $\frac{3}{5} \ldots \frac{2}{5}$ in which the students answered 'five times three equal fifteen and …' (student response) by saying 'Stop, the first step is to keep your eye on the denominators. If they are the same, compare the numerators …' (reject move followed by a clue). He justified that by saying:

> this student is a naughty boy, so I spent more time with him to explain how to solve such a comparison. I do not like to say go to your seat to any student, as this may hurt his feelings. The class always has the idea of using cross multiplication in any fraction comparison, regardless of the denominator, which needs serious treatment

He commented on incorrect answers when it seemed like a major problem for the students.

In conclusion, Fahad showed some limitation in his subject matter and, to less extent, pedagogical knowledge. He taught the lesson in a procedural way and no connection between the lesson components was made. Students' correct answers were dealt with by accepting them with praise (accept move) or rejection (reject move) when they did not meet his expectation or were thought to be wrong by Fahad. Similarly, he rejected incorrect answers verbally or physically (reject move) and tried to comment on them when they related to serious misconceptions of the students (comment move).

7.3 The second lesson (lesson 4)

This lesson was entitled 'Mixed Numbers' and was taught to Grade 4. Its aims were for students to learn how to write area models as mixed numbers and improper fractions, locate points on the number line and convert between mixed numbers and improper fractions. For example, 'Write $1\frac{1}{3}$ as an improper fraction' and 'Write $\frac{50}{6}$ as a mixed number'. It followed the lesson of 'Compare and Order Fractions'.

7.3.1 The Lesson Description:

Fahad spent the first five minutes of the lesson explaining the different kinds of fractions by asking the students to read the first statement in the book: 'A mixed number is one which contains a whole number and a fraction'. He then asked the students to explain the meaning of this statement and wrote $1\frac{1}{2}$ as an example. The second statement was 'An improper fraction is a fraction in which the numerator is more than, or equal to, the denominator'. He wrote examples such as $\frac{4}{2}$ and $\frac{4}{4}$ and asked about the meaning of them. Next, he wrote $\frac{2}{3}$ and called it 'An integer fraction' where the numerator is less than the denominator. Fahad then summarised those statements before moving to an example of his own creation which was a drawing of a pizza (circle) cut into four with 3 parts shaded. He then asked how to write it as a fraction and he received different answers such as 'It is 3 above 1' and 'It is 3 above 4'. Fahad then explained to the class how to write the fraction correctly by labelling the parts as numbers then writing the shaded parts as the numerator and the total of all parts as the denominator.

For the next three minutes, Fahad introduced the first worked example in the book which was: ‘Sarah made two circle cakes and cut them into five. If she eats three pieces write the remaining cake as a mixed number and an improper fraction’.

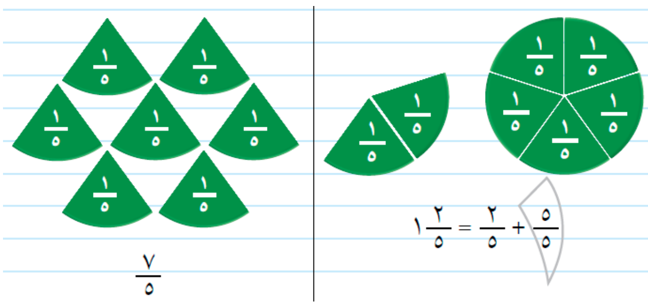


Figure 7.3

The book worked out the mixed number by considering the whole cake as one and the two pieces as a fraction of 2 over 5 ($\frac{5}{5}+\frac{2}{5}=1\frac{2}{5}$). Also the book counted one fifth seven times to find out the improper fraction $\frac{7}{5}$.

Fahad moved to the next part of the lesson which was the conversion between mixed numbers and improper fractions which took four minutes. He explained two worked examples: ‘Convert $1\frac{3}{8}$ to an improper fraction’ and ‘$\frac{11}{8}$ to a mixed number’. However, though the book worked the former as $1\frac{3}{8}$ = 1+ $\frac{3}{8}=\frac{8}{8}+\frac{3}{8}=\frac{11}{8}$, Fahad used the multiplication method which was multiplying 8 by 1 then adding 3 with some arrows to find out $\frac{11}{8}$. He introduced the second example using division as it was in the book.

The next ten minutes were spent doing the worked example: ‘Write point A as a mixed number and an improper fraction’

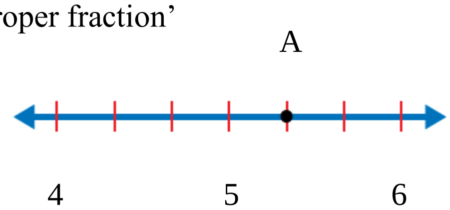


Figure 7.4

In the book this example was worked by stating that every interval is a third so A is $5\frac{1}{3}$ and it could be written as $5\frac{1}{3}=\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{3}{3}+\frac{1}{3}=\frac{3+3+3+3+3+1}{3}=\frac{16}{3}$. Fahad explained it by telling the students that the number on the left is the whole number of the mixed number. Then he asked them to count the small marks between 5 and 6 (including 6) then write the answer (number 3) as the denominator of the mixed number as shown below (Figure 7.5). He wrote $5\frac{1}{3}$ and converted it to an improper fraction by multiplying 5 by 3 then adding 1 so the answer was $\frac{16}{3}$.

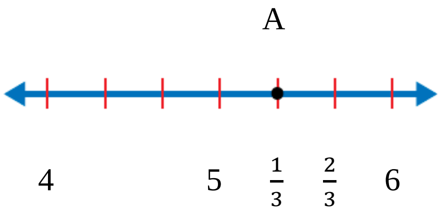


Figure 7.5

He repeated that explanation as many students did not understand how to write it as a mixed number. The students asked many questions such as 'If the number 6 is not in this place, what will the denominator be?' Fahad responded by saying 'If 6 is not here we continue the counting as the denominator is still the same and does not change'. He then wrote:

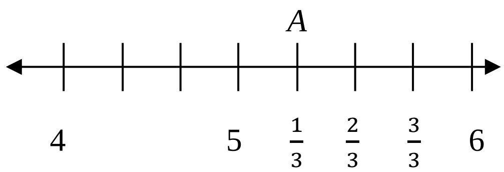


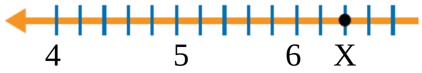
Figure 7.6

And the student corrected him by mentioning that the denominator should be 4 then Fahad said 'It is ok everyone makes mistakes' then amended the fractions. Another question was asked by a student as described in the following incident.

Incident 1:

Fahad was asked by a student what would happen if there was no number on the left or the right in the previous example in Figure 7.4 and Fahad responded as the following dialogue shows:

1	Fyras	If there was no number on the left or the right how can we do it?
2	Fahad	You must have the two numbers otherwise you cannot solve the question.
3	Fyras	But question (9) does not have a number on the right hand point.

4	Fahad	[Looks to question 9 in the book]  Figure 7.7 Mmm, the most important number is the one on the left. Let us move to the first exercise… no let us move to question 9.
5	Students	We will skip the first 8 exercises?
6	Fahad	Yes, we will return to Fyras’ question here. See the figure on the board. How can we do it?
7	Student 1	The whole number is 6 and we count the marks.
8	Fahad	Well done. We count from the small marks of 6 as one, two, three, four and five [he counts with the class out loud]. If there is no number on the right just count until the arrowhead. Who can do it?
9	Hassan	Me
10	Fahad	Come on, Hassan
11	Hassan	[Comes to the board] The number is 6 [Writes 6 then a fraction line]
12	Fahad	Good. What is the denominator?
13	Hassan	We count one, two, three, four and five. It is five [Write 5 under the line]
14	Fahad	What is the numerator?
15	Hassan	It is 3.
16	Students	No. [Give different answers] 3, 4 and 2
17	Fahad	Shush, he knows. Look at the board. Where is x?
18	Hassan	It is 2 [Writes $6\frac{2}{5}$]. Now we multiply 6 by 5 and add 2 [Write $\frac{32}{5}$].
19	Fahad	Well done. Clap for Hassan please.

Fahad spent the rest of the lesson solving two exercises with the same idea of locating a point on the number line.

7.3.2 Applying the Knowledge Quartet (KQ) and IRF

The following analysis takes into account the class observation and Fahad’s comments in the interview.

Foundation

Students find it hard to understand fractions greater than one unit especially when displayed through area models (Dickson *et al.*, 1984). When students see a diagram with two identical divided shapes where one is fully shaded and some parts of the other are shaded, they can hardly find the represent fraction in the diagram (*ibid*). For example, if the shapes were divided into five parts and one of them was fully shaded and the other one had two shaded parts, some students may write $\frac{7}{10}$ instead of $1\frac{2}{5}$ or $\frac{7}{5}$ (*ibid*).

Measurement is one of the situations where fractions are used in primary schools (Nunes *et al.*, 2009). In this kind of situation, the student needs to 'represent a quantity by means of a number' (*ibid*: p.3), like representing a fraction as a number on the number line (Van Steenbrugge *et al.*, 2014; Hecht *et al.*, 2003). Understanding the concept of a fraction as a point on the number line model is harder than in area and sub-set models (Dickson *et al.*, 1984). That is because, in the number line, the fraction is thought of as an abstract number not as part of a whole object or part of a set of objects (*ibid*). In order to locate a fraction on the number line meaningfully, the students are required to gain two intertwined notions: the fraction as a number (quantity) and as a distance from a given point (Charalambous and Pitta-Pantazi, 2007). However, although locating a fraction on the number line may look easy, many students find it difficult to do so (Hannula, 2003; Charalambous and Pitta-Pantazi, 2007). Many students make mistakes in locating a point on the number line as they count the partition marks instead of the intervals of the number line (Baturo, 2004) or fail to identify the unit of the given number line (Keijzer and Terwel, 2001). Moreover, it requires the ability to order and compare the given fractions (Smith, 2002).

The lesson contained mixed number and improper fraction concepts and the procedures of converting between these two concepts, and locating a point on the number line. However, Fahad spent more time introducing mixed numbers and locating a point on the number line. He showed some limitation in his subject matter knowledge (OSK) regarding the concept of mixed numbers which is illustrated in the following paragraph.

Fahad's knowledge of the mixed number concept is questionable. However, although he showed the ability to deal with the definition and gave some examples of it, he was not fully aware of the meaning of a mixed number. For example, he omitted the way in which the book converted $1\frac{3}{8}$ to $1+\frac{3}{8}=\frac{8}{8}+\frac{3}{8}=\frac{11}{8}$ because he did not know that $1\frac{3}{8}=1+\frac{3}{8}$ as he confirmed that in the interview. It seemed that he did not know that the fractional part in a mixed number is another value that must be added to the whole number to form the mixed number. In addition, in the interview, he could not justify his method of multiplying 8 by 1 then adding 3 to convert the mixed number to an improper fraction as he was not aware of the fact that $\frac{(1\times8)+3}{8} = 1 + \frac{3}{8} = 1\frac{3}{8}$. This again showed that Fahad did not grasp the idea of a mixed number as a fraction added to a whole number.

Locating a point on the number line is another difficulty Fahad and his class struggled with. In this matter, Fahad faced some problems in identifying the fractional part of the given point on the number line. For example, Fahad asked the students to count the small marks in Figure 7.4 instead of the intervals on the number line, which may be considered as a mistake (Baturo, 2004). In addition, identifying the unit of the intervals was problematic for the class. Fahad was asked how to find the unit if the point on the right/left has moved or disappeared as in Figure 7.5, and he could not provide a correct answer. He responded incorrectly by repeating the same denominator as Figure 7.6 then he was corrected by the students or stating the impossibility of doing the answers without knowing the two points. He justified the former action by saying ‘I did not understand what the student meant by his question’. Also, he justified the latter by explaining that:

> This is the first time I have seen the exercise in the book. When I saw it I had to admit that what I had said was wrong, and to do that clearly I moved to solve the question that I had just seen.

This justification shows that Fahad did not have the ability to determine the proper unit of an interval on a number line. To support this assumption, in the interview I asked Fahad to tell me what point x is in the figure below:

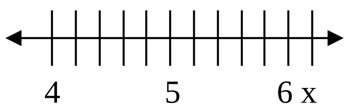


Figure 7.8

He answered ‘It is six and a half’ as he counted up to the arrowhead. I also asked him about the point of showing the numbers 4 and 5 and the small marks between them in the question, but he did not know why they were there.

Transformation

Although the lesson had three parts, the definitions of mixed numbers and improper fractions, the conversion between them, and locating a point on the number line, Fahad spent most of the lesson on the latter. He introduced (TD) it in a procedural way by following steps that needed to be done to locate the point correctly. For example, he asked the students to count the small marks instead of counting the intervals between the two numbers. Some of the students were confused about whether they should count the starting number or not. However, although Fahad showed some concern about the students' understanding of the lesson on occasion, he was not able to explain the procedures more when the students asked him to do so. He just repeated what he had already said using the same examples.

Connection

Fahad did not anticipate the complexity (AC) of locating a point on the number line. He found many students did not understand what he just had introduced to them which required him to explain it again. In addition, he was not aware of the possibility of solving certain kinds of questions where there is just one number on the left as he said 'This is the first time I have seen this kind of question' when he saw an exercise with just one number on the left. The lack of ability to anticipate the complexity of locating a point on the number line may have lead, as well as other factors, to spending more time on illustrating this part of the lesson.

Contingency

There were few occasions in the lesson where the students provided correct answers as the majority of the time was spent explaining how to locate a point on the number line which was hard for the class. Fahad dealt with correct answers by accepting them then praising or asking for justification (RCI). For example, he praised by saying 'Excellent' (accept move) to the student who answered correctly (student response) for his question (elicitation opening). He justified his action of praising the students at the beginning of the lesson as it might encourage the students to pay more attention to the lesson. In addition, in the incident [12] and [19], Fahad praised a student who had struggled to answer correctly because he considered the student to be very good at maths, but lacking in confidence. Fahad wanted to encourage him to participate more and be more active in class as he said in the interview. Moreover, Fahad asked some students about the reasoning behind their correct answers, then commented on that answer. For example, in the drawing of the pizza (circle) cut in four with 3 shaded parts, Fahad asked the students how to write it as a fraction (elicitation opening) and he got the answer of ‘It is 3 above 4’ (student response). Then he asked ‘Why do we write it as $\frac{3}{4}$?’ (press move) and said ‘This is because there are 3 coloured parts and the number of parts in the whole is 4. Do you understand?’ (comment move followed by an elicitation move). He did that, as he said in the interview, ‘I would like to stress the importance of writing the coloured parts above the total. I think that a common mistake needed to be dealt with, as the students always mixed it up’.

Conversely, incorrect answers were dealt with by rejecting them then asking other students, or by supporting the answerer until he achieved the correct answer. For example, Fahad rejected the answer which considered a mixed number as just a fraction (reject move) at the beginning of the lesson and asked others to answer (nomination move). Fahad mentioned in the interview that he did not want to blame the students for not knowing something they had just learned, so he preferred to ask others students, especially in the first part of the lesson. In addition, when a student made a mistake which Fahad considered a common problem, he asked other students then commented on the situation to correct the incorrect answers as he did in the pizza example mentioned above. Moreover, some students who Fahad had expected to obtain the correct answer were supported when they made some errors. For instance, in the incident [15] the student gave an incorrect answer (student response) then his peers tried to correct him in [16]. Fahad in [17] asked them to keep calm while he supported the student and helped them to focus his thinking by providing an extra question (directive move followed by clue move). Fahad justified his action by stating that the student was a good student who lacked confidence and needed encouragement to do better.

Fahad responded to the students’ questions and this lead him to deviate from his agenda (DA). Two important questions described above were when Fahad responded by giving improper answers, probably as a result of his lack of subject matter knowledge. These questions were asked publicly and Fahad chose to respond in the same way. For example, when Fahad was asked what to do if there were no points on the number line, he answered incorrectly then he missed out the first 8 questions in order to start with the question he was asked about. That was clear in the incident when the students asked him ‘We will skip the first 8 exercises?’ [5] and he agreed with them. He commented on the situation in the interview by saying:

> When I saw it [Q9 where there was no point on the right] I had to admit that what I had said was wrong, and to clarify the answer I moved to solve the question that I had just seen.

He might have wanted to cover the asked point well and therefore he preferred not to postpone it until after solving the first 8 questions. However, although these questions showed some limitations in Fahad’s subject matter knowledge, he was confident when he was answering and keen to correct his answers.

Fahad used the opportunity (UO) of him being corrected by the students to increase the awareness of errors and make them a part of a public discussion in his class. He said to the class: ‘It is ok, everyone makes mistakes’ then he justified that in the interview by saying:

> I was happy when he corrected me as I do not want to be correct all the time, from the students’ point of view. I wanted them to know that even the teacher can make mistakes as this give them some confidence to share errors. Also, some of them may be happy because they can correct the teacher. I want my students to generate new ideas all the time

This justification shows that Fahad’s class is open to discussing errors publicly and he wanted them to generate ideas without thinking of errors and these may be considered a sign of confidence and a solid PCK.

In conclusion, in this lesson Fahad showed some limitation in his subject matter knowledge regarding the concept of mixed numbers, and he was not fluent in locating a point on the number line. He kept teaching in a procedural way, hence he taught the students to follow fixed steps in order to reach the final product. He showed some interest in treating some common mistakes that came to the surface and also showed some confidence by sharing questions asked by the students. He dealt with the students’ correct answers by accepting them and praising the students. He sometimes asked the students to justify their correct answers when it related to some problematic part of the lesson (accept and press move). Incorrect answers were always rejected (reject move) and some support was given to students to help them correct their answers.

7.4 The third lesson (lesson 7)

The lesson of 'Problem-Solving Investigation: Choosing a Strategy' was introduced to Grade 4. Its aim was to check the students' understanding of solving some written problems by choosing an appropriate strategy from several strategies, such as using a drawing, tables and rational interpretation.

7.4.1 The Lesson Description:

At the beginning of the lesson, Fahad asked the students to close their books and he asked them about the previous few lessons. They revised and summarised what they had learned about locating a point on a number line, mixed numbers and improper fractions and how to convert them. It took them nearly five minutes to do this and then Fahad drew two circles divided into 6 parts. One of them was fully shaded and the other had 4 shaded parts and he asked them to write the model as a mixed number ($1\frac{4}{6}$) and an improper number ($\frac{10}{6}$). They said the answers out loud and he wrote them on the board.

Next, in the following two minutes, Fahad divided the class into two teams A and B (12 and 13 students) to start a competition. Then he asked them to listen to his instructions (e.g. do not answer without a permission) and told them that the aim of this lesson was to encourage them to think effectively.

Fahad asked team A to answer the first question, which took 10 minutes, 'Lila can make a dish of food in 20 minutes. If she wants to make 8 dishes on the condition that she finishes at 8:00 pm, when should she start?'. He asked the students for their attention and tried to break the question down into small parts. While doing that, a student (Ibrahim) was disturbing Fahad by giving the answer 'She starts at five twenty'. Fahad asked him to keep quiet until he had finished explaining the question. Ibrahim suggested the answer of going backwards 20 minutes 8 times from 8 pm and Fahad's preference was to calculate the total time (160 minutes = 2h and 40 minutes) then go forward from 5:20. There was disagreement among the team about whether the final answer was 5:40 or 5:20 until they settled on 5.40. Fahad agreed that the answer was 5:40 as he thought the total time was 2 hours and twenty minutes. Then, with Ibrahim insisting on his answer of 5.20, Fahad was convinced to change his answer to 5:20. He then returned to the other student who answered 5:40 and tried to prove the incorrectness of the solution by counting forward 2h and 40 minutes from 5:40 until he reached 8:20 not 8pm.With the class, Fahad spent the next six minutes solving the question described in the incident below.

Incident 1:

The question was ‘Saleh and three of his friends rent a boat for three hours. If the cost of renting the boat is 80 Riyals per hour, how much will each of them pay?. It was solved as the dialogue shows:

1	Fahad	Can anyone from team B read the second question?
2	Student 1	[Read the first sentence and was stopped by Fahad]
3	Fahad	So, if you are Saleh and you have three friends, how many persons are you?
4	Students	Four
5	Fahad	If they pay 80 riyals how much must each one pay for one hour?
6	Mohammed	Ten
7	Students	No, wrong
8	Fahad	It is ok, he is right, there is nothing wrong, [moves toward Mohammed’s table] Why did you say ten? Answer Mohammed, you are four people and you paid 80 Riyals. If you said you paid ten what is the total?
9	Mohammed	Four
10	Fahad	Forty you mean?. You paid forty so is it correct or not?
11	Mohammed	No
12	Fahad	So how much does each one pay? [Points at another student]
13	Student 2	Twenty
14	Fahad	You are right, ok Mohammed each one pays twenty Riyals so all pay eighty riyals right? [Returns to the board] They used the boat for three hours, We want to find the rent of the boat for three hours. How much?
15	Student 3	250
16	Fahad	Why did you say 250, I want to know where you got 250 from?
17	Student 3	Because double eighty plus eighty… [Interrupted by Fahad]
18	Fahad	Not double, we add it. [Writes 80+]
19	Student 3	Eighty plus eighty plus eighty equals two hundred and fifty
20	Fahad	[Writes 80+80+80] excellent, you are right. Count with me eighty plus eighty equals what?
21	Student 3	160
22	Fahad	And another eighty?
23	Student 3	240
24	Fahad	You have added ten more, so how much does everyone pay?
25	Student 4	It is 220 not 240
26	Fahad	[Calculates mentally] 160 plus 80 [Writes 220]
27	Students	[Giving different answers] it is 240 , it is 220
28	Fahad	Calm down, 80 plus 80 is how much?

29	Students	160
30	Fahad	And 80?
31	Student 5	170, 180, 190, 200, 210, 220, 230, 240, it is 240.
32	Fahad	[Asking the class to calm down angrily] so 240. How much does every one pay?
33	Yasser	80
34	Fahad	Yasser said 80, is that right? [Writes 80+80+80+80 and shouts to some boys to be quiet].
35	Student 6	60
36	Fahad	Excellent, Yasser's solution [Deletes 80+80+80+80 and writes 60+60+60+60] equals 120+120, so it is 240 which is right. Well done Yasser.

He then challenged the class to solve the following question within five minutes: 'Nada bought cloths of different sizes: large (cost: 20 R), medium (cost: 15R) and small (cost: 13R). If she paid 65 riyals, how many cloths of each size did she buy?' The rest of the lesson was spent solving other questions from the text book.

7.4.2 Applying the Knowledge Quartet (KQ) and IRF

The following analysis takes into account the class observation and Fahad's comments in the interview.

Foundation

In this lesson, Fahad was not able to identify some of the students' errors (IE). However, although they were tiny mistakes related to simple mathematical operations like addition, they made a big difference to the solving process. For instance in the first exercise, Fahad was confused and could not spot the error of converting the total time of 160 minutes into two hours and twenty minutes. Also, in the incident [26], he could not identify the error of calculating the rent of three hours (80×3) as 220 until it was noticed by some students. This inability to identify errors could be related to stress Fahad was experiencing, or his insecure subject matter knowledge. He mentioned in the interview that Ibrahim's distractions made him confused over the correctness of the students' answers which led him to accept any answer. He said: 'I got confused and lost control when I was faced with two different answers from two students who appeared confident. This confusion led me to accept some answers without evaluating them'. Also, the student contribution in the incident [25] made him unsure about the answer of 240. It is clear that Fahad was not confident about his answers as he changed them immediately when a student suggested another answer. This might show some limitation in his subject matter knowledge (OSK) as if he had secure foundation knowledge of a simple operation such as 160 + 80 is 240, he would not have been confused by 220. He also counted twenty times eight using his fingers to find out the cooking time which was 20×8=160 which may give an indication of the difficulty Fahad faced with mental calculation.

The solving problem lesson in the textbook is designed as linear steps starting with understanding the question request, understanding the problem, planning, solving and checking the answer. Fahad was not clear about this process as he started to ask questions without showing a clear plan to solve the problem. He showed a lack of knowledge (OSK) regarding the solving and checking phases in the first exercise. When he calculated the total cooking time (2 hours and 40 minutes) Ibrahim suggested the answer of starting at 5:20. Fahad then tended to use the checking phase before the solving one. Ibrahim provided the solving step as he suggested that they subtract 2h and 40m from 8:00 to reach 5:20 then Fahad asked him to explain it more. It seemed that Fahad tried to solve the question by using trial and error as he tried starting at 5:20 and later he tried 5:40 as well.

Transformation & Connection

The lesson required flexibility in solving the question and that was partly missing in Fahad's teaching as the lesson went the same way for all the solved questions (TD). However, Fahad mentioned that the purpose of the lesson was 'to encourage them to think effectively' and he introduced the lesson using some short questions to help the students get started. The main role of the students was to answer Fahad's questions while he was guiding them towards the solution, instead of thinking on their own as the lesson's aim stated. Even though he divided the class into two teams, they did not work as groups; instead they worked individually responding to Fahad's instructions. In addition, he showed some flexibility in changing the direction of the solution in order to extend the benefit gained from the exercises, especially the second one in the incident. He could have finished the second exercise from the utterance [13] by multiplying 20 by 3 to get 60 but he said in the interview:

> In the beginning I was going to multiply 20 by 3 to get 60 …then I thought if I take the other way which was finding the total rent 80 times 3 then divide by 4 which I think contained more mathematical skills.

This shows that Fahad tried his best to support the students' learning by combining many operations together in order to challenge the students to calculate the answers correctly. Moreover, he used an improper example in the introductory part of the lesson as not all the exercises contained mixed numbers or improper fractions and the connection between the previous lessons and this one was missing (MCC).

Contingency

Fahad accepted correct answers in different ways (RCI). He accepted some of them with hesitancy and some were accepted straightaway. When he was not sure about the correctness of the answer or felt nervous he did not accept it straightaway as he would think about it or ask for an explanation before accepting it. For example, when Ibrahim said that 160 minutes equals 2 hours and forty minutes (student response) Fahad did not accept it straightaway as he calculated it out loud by subtracting 60 then 60 then 40 (comment move). Later, he changed it to 2 hours and twenty minutes which may show some doubt about the answer. Also, he asked Ibrahim to further explain his solving strategy (press move) when Ibrahim suggested a correct answer that Fahad was probably not expecting. Ibrahim explained 'If you subtract two hours from eight o'clock it will be six and then if you subtract forty minutes you will get five twenty'. Fahad then accepted this answer. His hesitancy in accepting this answer straightaway might have been related to the nervousness that Fahad felt when Ibrahim kept disturbing him at the beginning of the lesson, or the lack of ability to calculate mentally.

Correct answers were sometimes accepted straightaway by praising, asking for justification or repeating also, Fahad sometimes rejected them. Fahad liked to say words such as 'right', 'excellent' when the students answered correctly in order to encourage them to participate more in the lesson. For instance, he praised Yasser in the incident [36] when he provided a correct answer (accept move). In addition, Fahad mentioned in the interview that he asked for justification to check the student's way of producing the answer. For example, when Ibrahim said 'Five twenty' (student response), Fahad responded 'right, why five twenty?' (accept move followed by press move). He wanted to make Ibrahim's thinking public to the class. Also, he repeated the student's answers as a confirmation of their correctness. When Ibrahim explained what he had done as 'from eight to six is two hours' (student response), Fahad repeated what Ibrahim said (accept move/Re-voicing move). Moreover, Fahad rejected a correct answer, whether he knew it was correct or not, when it appeared at the beginning of the solving process in order to avoid ending the solution early. For example, when Ibrahim gave the answer of ‘five twenty’ (student response) directly after reading the question, Fahad asked him to keep quiet until he had finished explaining the question (directive move). It seemed that Fahad believed in the importance of using the exercises as opportunities to diagnose the students’ understanding of the subject. Fahad justified that in the interview:

> I did not take the correct answer and finish the exercise as we were still at the beginning of the lesson. I wanted to see the other students’ contributions to check if there was any action to be taken.

Incorrect answers were dealt with by rejection, acceptance, supporting the answerer and asking for justification. Fahad rejected incorrect answers verbally (e.g. wait) and physically (shaking head). For example, he said ‘Wait, we do not want random answers…’ (reject move followed by comment move) for the student who answered ‘Five forty’ (student response). It seemed that Fahad was sure that the answer was wrong as he described it as a random answer which naturally required a rejection. In addition, he accepted some incorrect answers with or without praising the answerer. For instance, when there was a debate between the two students who answered ‘Five twenty’ and ‘Five forty’ (student response), Fahad was confused and accepted the incorrect answer and ignored the correct one (accept move). He mentioned that he was affected by the confidence of the students who gave him the two answers. That showed some gaps in Fahad’s subject matter knowledge and a lack of confidence which did not help him to avoid being confused by the confidence of others.

Fahad showed some sympathy for some students who gave incorrect answers. Fahad supported some students by asking the class to pay attention to the answerer and saying some encouraging words such as ‘He can do it’. In addition, Fahad tried to apply the cognitive conflict strategy (Powell, 2006) to convince students, with either a good or bad mathematical background, who made errors. For example, in utterances [5]-[11] of the incident, Fahad was faced with an incorrect answer ‘Ten’ (student response) from Mohammed. Fahad then replied by supporting Mohammed and encouraging him to continue participating in the exchange. He said ‘It is ok, he is right, (evaluate move) there is nothing wrong’ (comment move). Then he asked for justification by saying ‘Why did you say ten?’ (press move). When he did not receive an answer from

Mohammed, Fahad changed his strategy and shifted to apply the cognitive conflict strategy by asking Mohammed some questions to guide him to discover his error. Fahad tried to make Mohammed aware of his mistake by showing him that if each pays 10 Riyals the total payment will be forty Riyals not eighty. The same situation occurred with student 3 in utterances [15]-[24] where Fahad accepted the answer then asked for justification and applied the cognitive conflict to convince the student that he was wrong. He justified that by saying:

> I do not like to say to the student that he is wrong because he will sit down and keep silent. Instead of that I ask him to answer even if the answer is wrong, then I try to correct him by getting him to self-correct. By that I mean that he discovers that his answer is incorrect by himself. With this boy I tried to convince him that ten Riyals was not enough so that he would understand that his answer was wrong

Fahad in this lesson took the opportunity (UO) of Mohammed's errors to increase the chance of discussing flawed solutions in front of the class which may have showed some element of confidence and secure PCK.

In conclusion, in this lesson Fahad was nervous due to some students' interruptions which may have affected his ability to identify errors. Also, he was not flexible enough to accept the students' solving strategy as he wanted to be the only one to give the solving instructions. These two elements may show some gaps in his subject matter knowledge and his teaching method. He dealt with most of the students' correct answers by accepting them either immediately or after some thinking. Also, incorrect ones were dealt with by rejecting them or accepting a few of them when he was not sure whether they were correct or not. On some occasions, he showed some willingness to help the students when they faced some problems while solving questions. He did that by guiding them with questions and applying the cognitive conflict strategy to convince the students about their mistakes.

7.5 An overview of Fahad's knowledge

The results of applying the KQ on Fahad's lessons showed much evidence in the weakness of his subject matter knowledge regarding the concepts and the procedures he taught. Some of this evidence was his insistence to teach procedurally which makes him a transform-oriented teacher who controls the class. Another indication of his weak subject knowledge was that he omitted essential parts of the lessons, like comparing fractions using the number line, which the students needed in the following lessons. In addition, making errors in his answers to the students' questions or in his knowledge about some concepts and procedures were other indicators of weak subject matter knowledge. For example, his wrong answers to the students' questions about locating a point on the number line and his unawareness of the notion of a mixed number being a fraction added to a whole number, for example, $5\frac{3}{7} = 5 + \frac{3}{7}$. Furthermore, he was easily confused by students who answered confidently as Fahad did not depend on his knowledge on these occasions and was not able to spot their errors. Moreover, inventing a strange strategy to order three fractions showed some gaps in his knowledge about the nature of the comparison procedure. All this evidence showed that Fahad can be considered as a teacher with limitations in his subject matter knowledge of the taught topics.

Fahad taught the lessons in a procedural way as he structured the lessons as question-answer forms. This meant that he asked the students to take steps without knowing why they should do it that way. However he also showed some signs of caring about the students' learning as he kept repeating the original explanation when a student asked him to explain more. Fahad was concerned with helping the students achieve correct answers in a quicker way as he came up with some self-invented strategies that he thought might help in this matter. However, he showed less flexibility compared to the textbook as he adhered to his own method and rarely give the students chance to use their own strategies. On the other hand, he was supportive to some students who faced difficulties in solving exercises by giving them more attention and time. In addition, he tended to create the culture of discussing the students' errors publicly in his class by raising the awareness of the possibility of making mistakes and asking the students to correct their peers' incorrect answers. Also, he was determined and looked to different resources to develop his teaching such as the YouTube website. Moreover, he kept asking the students if they understood the taught lesson and repeated instructions again and again. In the following I highlight the response patterns of Fahad to the students' contributions (correct and incorrect answers).

7.6 An overview of Fahad's response to students' contributions

The most common exchange type between Fahad and his students was elicitation exchanges (Sinclair and Coulthard, 1992). He asked the students a question and they provided an answer then he evaluated their response. In the next section, I focus on this third turn when Fahad dealt with the students' correct and incorrect answers.

7.6.1 Fahad's response pattern to correct answers: 1) Accepting correct answers

Fahad nearly always dealt with the students' correct answers by accepting them (accept move). The majority of them were accepted immediately when he was sure about their correctness and some were questioned before being accepted. Occasionally, Fahad asked the students to explain their answers before he accepted or rejected it, especially if the answer came through a route that Fahad was not familiar with. For example, in the third lesson (lesson 7), a student suggested subtracting two hours and forty minutes from eight o'clock and Fahad asked him to explain that procedure in more detail. When a correct answer was accepted by Fahad, he usually followed a linear process of dealing with it starting by praising (P), acting (A) (e.g. asking for justification, repeating and writing the answer), commenting (C) then moving to the next point (M) (e.g. starting an eliciting exchange). However, occasionally he missed out some parts of this linear process depending on the situation. For instance, when a student answered 'forty' (student response) to Fahad's question 'how many minutes are left [from 120] until 160?' (opening move), Fahad responded by saying 'Good boy (P), it is then two hours and forty minutes (C) so when should she start? (M)' (accept move followed by comment then elicitation move). This exchange was in the form of PCM and the pattern of this response shows that Fahad's third turn usually has a combination of many acts varying from acceptance, re-voice and press, followed by initiating a new eliciting exchange.

Fahad revealed his reasons for each part of the response process (PACM) in the interview. Firstly, he tended to believe that praising (P) the students will encourage them to participate in the class discussion and will raise their confidence. The acting (A) part involved asking for justification and repeating the answer which Fahad usually did to check the extent of the confidence of the answerer and how he reached the answer.

He repeated some correct answers to confirm their correctness and to allow the students to hear them again. In addition, he commented (C) on correct answers for many reasons such as giving the students clues to help them answer correctly. Also, he commented to summarise the solving process and to emphasise the way of doing tasks correctly to avoid common mistakes. Lastly, he usually moved (M) to the next point by initiating a further eliciting exchange in order to complete the solution or check the students' understanding.

This response pattern could be related to Fahad's mathematical content knowledge (SMK and PCK). The response process has many intertwined actions which make it more complicated for the observer to try to link these actions with Fahad's knowledge. Firstly, Fahad asked for explanation when his subject matter could not help him cope with the suggested solving plan. This clearly could be related, to some extent, to the lack of his subject matter knowledge. Fahad decided whether to accept an answer or not depending on his subject matter knowledge. Then, when an answer was regarded as a correct one he moved to apply the PACM form which built on Fahad's subject matter knowledge and pedagogical content knowledge. Fahad's set of beliefs may come to play in the praising part of the response pattern, as he believed that a student learns better, becomes more confident and participates more in class when he is praised. This beliefs and subsequent action may form part of Fahad's general pedagogical knowledge. The acting (A) part could be related more to Fahad's pedagogical content knowledge than the subject matter knowledge as he wanted to check the students' understanding and would like to share their thinking with the other. Also, his action of repeating some answers to confirm their correctness may be considered part of his pedagogical knowledge as well as depending on his subject matter knowledge. Fahad's moving (M) could be related to his pedagogical knowledge as he used different kinds of questions to check understanding or move forward in solving the exercises.

7.6.2 Fahad's response pattern to correct answers: 2) Rejecting correct answers

Fahad rarely rejected correct answers (reject move) in the taught lessons but when he did he did so verbally (saying no) or physically (shake head) then commenting on that. For instance, one occasion was when a student provided a different solving method and the other occasion was when a correct answer was suggested in the early stage of solving a problem. When a student answered correctly (student response), Fahad thought it was wrong then responded by saying no (reject move) then a comment (comment move) followed by further questions (elicitation move) as described in the lessons 1 and 3.

The reasons behind this action were that Fahad was not expecting the answer and therefore thought it was wrong and this can be related to the lack of his subject matter knowledge. The second reason was that Fahad did not want to take the answer and finish the exercise as he believed that the exercises are a good opportunity to check the students' understanding and misconceptions. The latter reason could be related more to his beliefs about teaching which may be considered as an aspect of his pedagogical knowledge.

7.6.3 Fahad's response pattern to incorrect answers: 1) Rejecting incorrect answers

Most of Fahad's responses to incorrect answers were to reject them (reject move). In doing so, it seemed that he followed some intertwined steps. First he refused the answer (R) whether verbally or physically Next, he commented on it (C) in order to correct the answer, giving a clue or providing clarification. Then that was sometimes followed by asking other students (A) to correct their peers' answers or provide their own answers. This pattern did not occur each time but it was the most common scenario when Fahad was faced with an incorrect answer. At other times he missed out one of the RCA steps depending on the situation. For instance, a student answered incorrectly by saying '21 over 48' (student response) and Fahad acted by saying 'No (R), who can help? (A)' (reject move followed by nomination). This interaction was in the form of RA and he sometimes responded by A, RC or RCA form. In fact, most of the exchanges between Fahad and his students in this matter were reject moves followed by one or more of comment and/or nomination move.

Fahad justified his different responses to student incorrect answers in the interview. He justified commenting (C) on the incorrect answers by stating that he used the errors as good opportunities to treat some common student mistakes. He also wanted to help the students get correct answers by giving them some hints. In addition, he stated that he sometimes corrected the incorrect answers himself as a result of time pressure. Fahad asked (A) other students for different reasons such as his tendency to not disappoint any student for their errors. Also, he found it a good chance to check the students' understanding while they were correcting each other.

Fahad's knowledge may be related to this response pattern to some extent. Identifying that the given answer was incorrect then rejecting it (R) clearly depended on the subject matter knowledge held by Fahad. The comments (C) to the answers could be related to both subject matter knowledge and pedagogical knowledge. When Fahad commented by correcting the answer himself, he depended on his subject matter knowledge and he might have done that as a result of time limitation when he did not want to spend more time on simple mistakes. In addition, providing the students with some clues could be connected to his subject matter knowledge as he clearly has the information which he passes to them. Also, this action might relate to his pedagogical knowledge as he probably wanted the students to achieve the answers by themselves. Furthermore, when he commented by giving further explanations, again he depended on his subject matter knowledge and probably his belief in the importance of students understanding the concepts being taught and to tie them to his solving methods depending on his pedagogical content knowledge. Also his reliance on his ability to reintroduce parts of the lessons considered an aspect of his pedagogical knowledge. Moreover, asking (A) other students might be related more to the belief in the importance of protecting the students' emotions, encouraging peer learning in some situations and to the tendency of checking understanding in others. These factors seemed to be related to his pedagogical knowledge more than his subject matter knowledge.

7.6.4 Fahad's response pattern to incorrect answers: 2) Supporting some answerers

Fahad gave support (comment move) when an incorrect answer occurred, either from students with weak or strong mathematics knowledge. In this case he acted differently by saying words such as 'good' then asking further questions and trying to convince those students about the incorrectness of their answers. The questions he asked served two major purposes: asking for justification in order to know the source of the errors or helping the students to focus their thinking during solving by applying the tunnelling strategy (Wood, 1994). Another type of support given was convincing the students about their answers which was done by Fahad by applying the cognitive conflict strategy (Lerman, 2014) to emphasise the conflict between the student's answer and the problem context. The majority of Fahad's moves in this matter were press moves followed by elicitation moves, as he did not explicitly reject the answers as he asked further questions.

Many different justifications were given by Fahad regarding this response pattern. In general, Fahad avoided disappointing the students about their errors so he tried to support the students who failed to reach a correct answer. For example, when the mistake came from a student who Fahad thought was weak in mathematics, Fahad acted that way as he wanted to encourage him to participate more and to give him the chance to express himself and build confidence. Similarly, if it came from a student who was strong in mathematics, Fahad acted that way as he believed that the student needed to be supported in order to be able to answer correctly as the student had the information to answer correctly but might lose it temporarily. Furthermore, Fahad tried to convince the students about their errors because he believed in the importance of self-correction. Fahad believes that, when the student is encouraged to see that his answer is incorrect, the student needs to be guided through questions to spot his error and this leads to effective learning.

It seems that Fahad's beliefs and knowledge, to some extent, influenced this response pattern. The idea of supporting some students may relate more to Fahad's pedagogical content knowledge and beliefs more than his subject matter knowledge. His tendency to avoid disappointing the students about their errors in order to encourage them to take part in the lessons could be related to his pedagogical knowledge and beliefs. Also his tendency to find the source of errors, in order to treat them, and his help to let the students answer correctly might be considered as aspects of his pedagogical content knowledge. In addition, his way of convincing the students about their errors could be related to secure pedagogical content knowledge that Fahad used to support the students' learning. It seems that he believed in the importance of learning by discovering the incorrect part in the solving process. Moreover, Fahad's subject matter knowledge came to play when errors occurred in certain parts of the lessons which he was fully confident about in terms of the content. If the errors happened in some parts of the lessons, where Fahad did not have secure knowledge, he probably would not act as he had in this response pattern.

In conclusion, Fahad showed some limitations in his subject matter knowledge but less so in his pedagogical content knowledge which both affected his response patterns to his students' answers. He dealt with students' answers differently depending on his knowledge and beliefs. He accepted and rejected some correct answers and rejected most of the incorrect ones. Also, he supported some students when difficulties surfaced during the lessons. The most common type of exchange between Fahad and his class was elicitation exchange with accept and press moves for the response of correct answers and reject and comment moves for the response of incorrect answers.

Chapter 8 : Cross-case Analysis & Discussion

8.1 Introduction

When doing cross-case analysis, researchers look to similarities and differences across the cases in order to fully understand the phenomena under study (Yin, 2009; Stake, 2006). After writing each case report of this research, the three cases needed to be looked at together to see if they have common themes and to highlight the most important findings.

This research revealed findings from the cross-case analysis which are introduced and discussed in two sections. Firstly, I describe and discuss the findings of the *common* responses (similarity) of the trainees to students' correct and incorrect answers (section 8.2). Then most of the discussion is focused on three *different* dimensions (themes) to which the trainee teachers responded differently (difference) (section 8.3). Through these discussions, this research answers the following questions:

1. How do trainee mathematics teachers deal with students' contributions in Saudi primary school classes, and why do they respond in these ways?
2. How does the knowledge that trainee mathematics teachers have gained influence the ways in which they handle their students' contributions in the classroom?

8.2 Teachers' common responses to students' correct and incorrect answers

The research participants responded to their students' correct and incorrect answers in a variety of ways. The response patterns and the teacher's justifications of their actions are included in Table 8.1 to build the bigger picture of how and why they dealt with the answers. This provides easier access to the data.

Table 8.1 has two main sections (cells) that contain teachers' responses and students' answers. These cells are: teachers' acceptance of correct answers and incorrect answers regarded as correct (ACI), and teachers' rejection of incorrect answers and correct answers regarded as incorrect (RIC). I start my discussion with the ACI section then the RIC section.

	Correct answers/ *Incorrect* answers (regarded as correct answers)	
	Actions	Reasons*
(ACI) Teachers *accept* by doing one or more of	-Writing it on the board	- Sharing it with the class
	-Praising	- Increased participation/confidence
	- Asking for justification	- Checking sureness/confidence - Interest in the process
	- Commenting on it (clue, summary)	- Avoid common mistakes
	- Repeating it	- Confirm correctness
	- Progressing to next point	
(RIC) Teachers *reject* by doing one or more of	*Incorrect* answers/ *Correct* answers (regarded as incorrect answers)	
	Actions	Reasons*
	- Rejecting verbally and/or physically	- Changing routine
	- Correcting it	- Time limitation
	- Asking for justification	- Knowing the source of errors - Checking certainty
	- Asking other students	- Avoid disappointment - Check understanding - Peer learning
	- Repeating it	- Checking sureness
	- Commenting on it	- Treat common mistakes

Table 8.1: A summary of the trainees' *common* response patterns to students' correct and incorrect answers, and their justification

* As identified in the interviews based on explicit statements.

8.2.1 Teachers' acceptance of correct answers/incorrect answers (ACI)

Accepting correct answers is a natural action most teachers do when they are faced with a correct answer. In this study, as shown by the KQ analysis in the contingency dimension, most of the teachers accepted correct answers (and incorrect answers regarded as correct) in a variety of ways. The teachers' third turns in the IRF (Initiation-Response-Feedback) (Sinclair and Coulthard, 1992) interactions with students were almost always accept moves with or without further moves, such as revoice, press and/or comment moves. These ways can be categorised into two types of actions: *confirming actions* (praising, writing and repeating it, progressing and commenting) and *questioning actions* (asking and/or giving a clue). The former happens after the answer is regarded as correct in the teacher's mind and is therefore regarded as a post-acceptance action. For example, when the teacher decides that the answer is correct he praises, writes it on the board, repeats it and progresses to the next point. The interaction between the teacher and his students in these actions is in the form of teacher-student-teacher (T-S-T) which is typically one round of IRF. The latter, questioning actions, usually occur before the answer is accepted as the teachers ask for justification or give the students clues before accepting the answer. This, therefore is regarded as a pre-acceptance action. This interaction between the teacher and his students is in the form of T-S-T-S-T-…, which forms two or more rounds of IRF either with the same student or with different students.

The teachers in the study gave different reasons for their actions. These reasons fall in two different groups depending on the target. The first group of responses was *class-focused*, as the teacher aimed his response on the class as a group rather than the individual level. For example, the teachers wrote the correct answer on the board to share with class so here they emphasised the importance of the class in their justification. This group contained the commenting action where the teacher gave a summary to share with class in order to avoid common mistakes. The second group of responses was *individual-focused*, as the teacher focused more on the individuals by checking the sureness of their answers and their ways of solving problems. Also, it included the teacher's praising of the student to increase their confidence and participation.

Teachers' mathematical content knowledge and beliefs, as proposed by the KQ analysis, probably influenced the way that they accepted correct answers. However, knowledge and beliefs are intertwined in teaching practice and most of the teachers' responses to correct answers in this research could be related to their pedagogical knowledge. Focusing on the interactions at the turn-by-turn level shows that writing the answer on the board to share it with the class and repeating it to confirm its correctness are aspects of general pedagogical knowledge. These actions probably do not belong to the content of mathematics as teachers of other subjects would probably do them. Also, it could be related to the trainees' SMK as they depend on their knowledge to decide the correctness of the given answers. In addition, commenting on the answers by giving clues and a summary are related to pedagogical knowledge and, to some extent, to SMK. The teachers gave clues and summaries depending on their SMK in order to avoid common mistakes, and this has also been influenced by their PCK. Furthermore, teacher's beliefs about how mathematics is better learnt, as an aspect of the foundation dimension of the KQ, come to play in the praising action as they believe that praising the students increases their confidence and participation, which ultimately leads to better learning from the teacher's point of view. Moreover, asking for justification can be linked to SMK and PCK as the teacher checks the correctness depending on his SMK, and showing his interest in the student's thinking is one aspect of his PCK. In short, SMK plays a role in deciding the correctness of the answer and the pedagogical knowledge forms the substance of the instructions, while the beliefs of how mathematics is best learnt shapes the response approach. Also confirmation actions could be linked strongly to pedagogical knowledge and to some extent to beliefs, while questioning actions are mainly related to PCK.

As indicated in the literature review chapter (section 3.2.2), I observed that research focused on negative evaluation (teachers' handling of errors/misconceptions) outnumbered the research focused on positive evaluation (teachers' dealings of correct answers). Margutti and Drew (2014) emphasise that research on teachers' positive evaluations and how they are constructed is unexplored. That is probably because mathematics education research tends to focus on problems and difficulties of teaching and learning, rather than what is working well, in order to understand them in more depth and treat them accordingly. That might explain the lack of studies on teachers' responses to correct answers and justifications for their actions, which I intend to discuss in this part. This study might contribute to this area of research as outlined in the research contribution section (section 9.4).

Evidence from the contingency dimension of the KQ analysis of the data suggests that the teachers in this study responded positively with confirming actions (praising, repeating, summarising and/or progressing) and questioning actions (asking for justification and giving clue). These finding (about the confirmation actions) echo, to some extent, the findings of Margutti and Drew's (2014) research, as they found that teachers' positive responses to correct answers can be grouped into five groups. They evaluated 145 instances and categorised the teachers' responses into 'explicit positive assessments' (praise), 'verbatim full repetitions of students' answers' (repeating the answer), 'embedded repetitions' (writing the answer), 'formulaic receipt particles' (e.g.

'mm') and/or 'direct transition to the next Q–A sequence' (progressing) (*ibid*; p.439). In addition, questioning actions are found in the categories proposed by Drageset (2014b; a) who categorised the interactions between teachers and students on a turn-by-turn basis. He found that teachers respond to students' answers by doing one or more action that falls into three main categories: redirecting, progressing and focusing actions. The former mainly deal with the incorrect answers (which are discussed later) and the latter is where the teacher provides a summary for what has been done in the interaction up to that point. Also the progressing actions are where the teachers simplify the lesson element by adding information (clues).

Dragest's (2014a; b) research did not investigate the reasons behind teachers' responses to students' answers. In fact, he categorised the response patterns and looked at how the repeating patterns are structured. Whereas, Margutti and Drew (2014) found that the teachers chose which strategy to respond with depending on the pedagogical context they built. The findings of other research and this research shows that these trainee teachers do almost the same actions as teachers in other cultural contexts, which might give an indication of limited influence of cultural context that could be explored in further research.

To sum up, in this part of the response (accepting correct and incorrect answers) ACI where teachers respond to correct answers by providing accept moves with or without other moves, their responses can be categorised into two groups: confirming actions and questioning actions. Teachers justified their actions in two different ways depending on their focus: class focus or individual focus. In addition, evidence from the KQ analysis suggested that mainly the teachers' pedagogical knowledge and beliefs, and to some extent SMK and PCK, played roles in the teachers' responses. The literature shows similar response techniques with no explicit emphasis put on the participants' justifications by the researchers.

8.2.2 Teachers' rejection of incorrect answers/correct answers (RIC)

Teachers in general reject incorrect answers in many different ways. The teachers' third turns in the IRF interactions with students were almost always reject moves with or without further moves such as revoice, press and/or comment moves. In this research they rejected incorrect answers, and correct answers regarded as incorrect, by doing one or more of the following actions: explicit rejection (verbal or physical), correcting the answer, commenting on it, asking for justification, asking other students and/or repeating the answer with the sound of '?'. I believe that the first three responses can be grouped as confirming actions, as the teacher confirmed the fault in the answer and did not request a response (T-S-T type). Whereas the last three actions can be grouped as questioning actions where the teacher questioned the answer and wanted a response from either the same student or another (T-S-T-S-T-…).

The teachers in this study gave many reasons for their responses. These reasons fall into three groups: protection actions (such as changing routine, knowing the source of errors, avoiding disappointment and treating common mistakes). For example, teachers changed the routine as they wanted to protect the students from boredom. In addition, they wanted to learn the source of errors and treat common mistakes in order to protect the students from making mistakes in the future. Also, the teachers wanted to avoid the students feeling disappointed. The other group of reasons was checking actions (such as checking sureness and understanding) where the teachers tended to ask questions in order to check these two elements. Finally, further reasons were time pressure and the belief of peer learning but these do not fit with the first two groups so are considered together in a separate group 'other'.

The participants' pedagogical content knowledge, general pedagogical knowledge and beliefs all played a role in shaping their interactions with the students when they rejected the students' incorrect answers, and correct answers they regarded as incorrect. As identifying errors is an element from the propositional knowledge highlighted in the foundation dimension in the KQ, it is clear that the teachers depended on their SMK to decide whether to reject the answer or not. Next when the answer was regarded as incorrect they responded by rejecting verbally or physically in order to change the class technique, which is evidence of general pedagogical knowledge. In addition, their pedagogical knowledge encouraged them to ask other students for their answers to avoid the disappointment of the students and to check their sureness by asking for justification or repeating the answer. All of these responses can be related to generic classroom practices. Also, one aspect of the foundational knowledge in the KQ is teacher beliefs of how mathematics is best learnt so teachers' beliefs of peer learning may have affected some teachers' responses as they asked other students to answer in order for them to tell their peers how to solve the problems correctly. Moreover, probably the type of knowledge that could be mostly linked to teachers' responses to incorrect answer is PCK (Hill *et al.*, 2005). The teachers' PCK, which they gained in the Mathematics Teaching Methodology course in Table 2.1, has influenced their practice regarding the investigating of the source of errors, its treatment and the students' understanding. These responses are evident of some aspect of PCK where teachers show an interest in student thinking and in investigating the sources of misconceptions and treating them (Chick *et al.*, 2006; Graeber, 1999). In short, the analysis of the data with regard to the two dimensions of foundation and transformation from the KQ showed that the teachers' mathematical content knowledge (SMK and PCK) and beliefs about how mathematics is learnt influenced the way they dealt with their students' answers.

However, teachers' identifying and responding to students' errors is a major task teachers need to do (NCTM, 2000), research shows that some teachers avoid treating them (Son and Sinclair, 2010). Researchers have paid great attention to the treatment of errors and suggest that it should be considered as the starting point of learning (Schleppenbach *et al.*, 2007; Son, 2013). Many researchers have investigated how teachers respond to students' errors and misconceptions. For example, Chick and Baker (2005) in their study investigated how teachers would respond to a hypothetical task containing a misconception. They found that the strategies teachers would respond with were re-explaining the error, applying cognitive conflict, probing students' thinking. Also, Santagata's (2005) research compared Italian and US teacher's beliefs and practice of error handling and found that US teachers typically repeated the procedures until the students recognise the error. Also, it showed that teachers responded firstly by correcting it themselves or asking other students, then commenting by giving hints and asking for justification. Other studies showed that the Chinese teachers, who participated in the study by Schleppenbach *et al* (2007), shared their students' errors with them without shame whereas the US teachers were less open to errors and usually posed the question to other students (*ibid*). Moreover, Drageset (2014b) found that teachers sometimes respond to student answers by redirecting their approach through asking correction questions, asking the students to justify their answers and/or asking other students.

The research findings in this study mainly share the same responses of other research as described above. All the participants in this research showed, to some extent, an interest in students understanding and thinking so they asked for justification as other teachers did. Also they commented on some answers to treat common mistakes and they asked other students to avoid embarrassment as the US teachers did. In addition, evidence from this study and Santagata's (2005) study about the belief held by some teachers of the importance of peer learning could be considered an aspect of the socio-constructivist view of learning (Son and Crespo, 2009). In some parts of the interactions, the teachers in this research share the US teachers' actions, in Schleppenbach *et al*'s (2007) study, of hiding the students' errors and quickly redirecting the questions to other students. This could have been a potential influence from the behaviourist perspective of Skinner (1974) which is a part of the trainees background education (as they learned about learning theories in the Mathematics Teaching Methodology course in Table 2.1). From the behaviourist perspective of learning, teachers are encouraged not to discuss errors with students and to just reinforce correct answers (*ibid*). Moreover, any differences between my findings and the others may be due to the fact that instructional practices in different cultures lead to different responses to mistakes (Santagata, 2005). Santagata (2005, p.492) states that 'Teachers are often observed falling back on culturally based practices, particularly, when they try to adopt innovative teaching techniques'. The practice in the Saudi culture which the teachers use, may differ from other practices in different cultures. This might shed light of possible differences between this study and others.

To sum up, in this research teachers responded to students' incorrect answers with two different kinds of actions. Firstly, confirming actions where the teacher confirms the incorrectness of the answer by rejecting it verbally or physically, correcting it and/or commenting on it. The other kind of action is questioning actions where the teachers pose some questions to explore the students' thinking and understanding, asking other students or/and repeating the answers. These actions were done by the teachers for many reasons such as protecting students from boredom, errors or embarrassment. Other reasons were checking the students' understanding and sureness. Evidence from the KQ analysis shows that teachers' mathematical content knowledge, both SMK and PCK, played a role in these responses as well as time pressure, beliefs and pedagogical knowledge. Moreover, the findings of this research mirror other research results as the teachers in other contexts do almost the same as those participating in this research but possibly for different reasons.

In the preceding sections I elaborated on the similarity between the three cases and now I am going to discuss the themes of differences among the cases.

8.3 Themes of differences among teachers' responses to students' answers

Here, I focus on the variety between the cases regarding the teachers' responses to students' answers. Firstly, the different reasons for the teachers accepting or rejecting the students' answers (teachers' rationales for judging the students' answers in certain ways) are investigated in the first dimension (section 8.3.1). Next, teacher's routines when errors occurred are discussed in the second dimension (section 8.3.2). In the third dimension (section 8.3.3) I discuss the ways the teachers supported the students' learning in the light of how they handled their students' responses.

8.3.1 Teachers' reasons (rationales) behind accepting or rejecting answers

Although, as shown in the KQ analysis of the cases, teachers in this study share the fact that their SMK played a role in their decision of whether to accept or reject a given answer, they were influenced by other factors as discussed below.

The teachers in this study accepted some incorrect answers in differing frequencies due to a variety of reasons. Apparently, Abdullah was the teacher who most frequently accepted incorrect answers and Onizan did so the least. It seems that Abdullah did so due to his mathematical content knowledge (SMK and PCK). Evidence from his case report showed that he usually could not identify errors and this formed some aspect of his knowledge according to the foundation dimension in the KQ analysis. In other words, his SMK was largely weak in the topics he taught. This lead Abdullah to accept incorrect answers as he thought they were correct until he checked the teachers' book or was notified by some students about the error. Also, he accepted incorrect answers intentionally, when they occurred in an area where he has solid SMK, in order to investigate the source of the misconception and to convince the students about the incorrectness of their answers. He did that by asking for justification and applying the cognitive conflict strategy (as learnt in the Mathematics Teaching Methodology course in Table 2.1), or Disequilibrium in Piaget's theory which is 'a state of being uncomfortable when one has to adjust his or her thinking (schema) to resolve conflict and become more comfortable' (Powell, 2006, p.27). That could be considered an aspect of PCK. These reasons shed light on the role of teachers' mathematical content knowledge (SMK and PCK) in their decision to accept incorrect answers.

Conversely, Fahad's acceptance of incorrect answers was due to a lack of confidence he displayed in some observed lessons. His SMK could have played some part in this matter as it did not protect him from being confused when he was faced with many different answers. Similarly, Onizan accepted incorrect answers rarely when he thought they were correct, which could be linked to his SMK. In short, these findings show that teachers depend on their SMK, PCK and experience while they evaluate the given answers.

Some correct answers were rejected intentionally or unintentionally by the participants in this research and some justifications were given. It seems that the teachers' ability to identify errors, which is an aspect of subject knowledge in the foundation dimension in the KQ, took a main role in this matter as evidence showed that all of the teachers thought some of these answers were incorrect, so they rejected them. Also, teachers' belief was another factor that influenced some of their decisions. For example, Fahad's beliefs in the importance of using exercises to diagnose the students' understanding as well as extending their learning led him to reject the correct answers given at the beginning of the solving of the exercises. In addition, PCK could be related to the decisions made in Abdullah and Onizan's cases more than Fahad's case, as there was little explicit evidence regarding this matter in Fahad's lessons. Abdullah and Onizan rejected correct answers when the students used other solving strategies as they wanted to control the class and tried to avoid using some strategies for various reasons. For example, Abdullah tended to avoid the strategies that are hard to explain to the students, whereas Onizan prevented the students from choosing other strategies in order to protect them from making mistakes. In contrast, Fahad rejected some strategies as he preferred to use ones that extend the students' skills and use of mathematical operations. To sum up, teachers' SMK, beliefs and PCK influenced the teachers' response to students' answers in terms of accepting or rejecting them. These factors interact with each other before the teachers in this study decided to reject incorrect answers.

Research shows that there are many factors that influence teacher’s practice (Wilkins, 2008; Blömeke and Delaney, 2012; Santagata, 2005; Son and Crespo, 2009; Chick and Baker, 2005; Bray, 2011). For example, Wilkins (2008) suggests that teachers’ SMK, attitude toward mathematics and their beliefs about effective instruction are related to teachers’ practice. Also, he proposes that beliefs partially act as an intermediary between knowledge, attitude and practice which supports the finding of Blömeke and Delaney (2012) that beliefs connect knowledge with practice. In addition, some studies mentioned above focused in particular on handling students’ errors and solving strategies. For example, Son and Crespo (2009) find that belief, knowledge, experience and college programmes affected teachers’ responses towards non-traditional strategies. Also, research shows that cultural beliefs and practice (Santagata, 2005), the nature of the items (Chick and Baker, 2005) and knowledge and beliefs (Bray, 2011) influence teachers’ error handling. Most of these findings share the effect of knowledge and belief on teachers’ practice.

The research findings from the KQ analysis here show that teachers’ mathematical knowledge, identified in the foundation dimension mainly, SMK, PCK and beliefs, affect their handling of students’ answers. This fits with the body of literature described above, as the findings confirm the relationship between teachers’ knowledge and beliefs with their practice. However, this study differed in the context and the focus as it looked to students’ correct and incorrect answers, whereas the majority of the research mentioned above just focused on incorrect answers. This could be one potential contribution of this study to the research on teacher knowledge as outlined in section 9.4.

8.3.2 Teachers’ routines towards students’ errors

Teachers’ response patterns to students’ errors varied across the cases. All the trainees in this research had their strategies to protect the students from errors (that is to prevent their occurrence) and to treat the errors which I discuss below.

The trainees used many strategies to avoid errors in their classes. For example, Abdullah and Onizan rejected the answers that students gave when they used their own strategies to solve them. One of the reasons that the teachers wanted the students to follow the given strategies (which were almost always the textbook methods) was to protect them from making mistakes. Conversely, Fahad was less strict about the students following his methods as he gave them sometime to use their own strategies and, in fact, he did bring other strategies to his class. He also responded to errors by highlighting the fact that everyone makes mistakes and this probably made his class more open to share the flawed solutions. To sum up, preventing the students from using their own solving strategy was a common technique among the trainees in order to protect the students from making mistakes, however the teachers differed in the extent to which they prepared their classes to discuss the errors publicly.

Treating students' errors in this research varied as to the extent and approach. All the teachers, to some extent, asked other students when they were faced with errors. For example, Onizan was the trainee who redirected questions to other students the most when an error occurred. He did that due to his beliefs about peer learning and as a result of his own experience as a student. On the contrary, teacher's emotional feelings came to play in this matter in Fahad's case as he tried to avoid hurting the feelings of the student who answered incorrectly.

The strategy of the teachers correcting the answers themselves was used to different extents. Fahad and Abdullah corrected the errors by themselves less than Onizan. All of them felt the effect of time pressure but more so in Onizan' case as he just had four lessons per week whereas the others had five. Also, Onizan tended to correct the answers when he felt that the answer was a big mistake whereas Fahad usually commented on them and gave hints to help the student correct themselves. In the interview Onizan said that his class was not prepared to share and discuss errors, whereas Fahad helped his students discuss their errors by telling them that everyone make mistakes.

All the teachers asked the students to justify their answers in order to investigate the source of these errors. However, I noticed only a few incidences where the students justified their answers. Furthermore, Fahad asked the students to provide more information, not just why they did it that way but how they did it. Moreover, the teachers provided some kind of support for the students when they were faced with errors which is discussed later in the third dimension. In short, Fahad's class was most open to discussing errors as he encouraged the students to explain how and why they answered in the way they did, and a lack of time affected all the teachers' responses.

The routines of protecting and treating errors discussed earlier could be related to the trainees' mathematical content knowledge and beliefs. The tendency to protect students from errors could be related mainly to the theoretical underpinning of pedagogy which is a part of teachers' PCK as Chick et al (2006) state that addressing students' misconceptions is an element of the teachers' PCK. Also it could be related, to some extent, to their beliefs about the students' incapacity of solving problems on their own or their beliefs of the importance of mathematics students not making mistakes. In addition, from Onizan' point of view, evidence from the interview shows that asking other students is linked to his beliefs in peer learning, and from Fahad's point of view, to pedagogical knowledge as it prevents embarrassment which could lead to more participation from the class. Also, the teachers correcting answers and asking for justification can be linked to PCK, as this is a strategy to treat common mistakes and for correcting answers, especially under the pressure of time.

It seems that there were different types of class discussions where teachers responded to errors in this research. Abdullah and Onizan's class probably fit with the conventional textbook culture proposed by Wood and others (2006) where interaction in the class was typically of IRF. Those two teachers and, to a lesser extent, Fahad guided their students by establishing a series of short questions. However, Fahad was more open and gave his students some hints to help them correct the answers, and this strategy might fit with the conventional problem-solving culture (*ibid*). In addition, Onizan and Abdullah always redirected questions to other students as a strategy which could have hindered the opportunity of using errors to establish further learning. It seems that there is an inverse relation between redirecting questions and sharing the flawed answers in their classes. Their strategy of asking other students to focus on achieving the correct answer by redirecting the question to avoid the mistake might fit the behaviourist view of learning (as taught in the Mathematics Teaching Methodology course in Table 2.1) (Skinner, 1974) as outlined earlier (section 8.2.2). Also, Abdullah and Onizan less frequently share the errors with their class compared to Fahad's class, by avoiding the discussion or even preventing them from happening. This mainly contradicts the literature that suggests that teachers were encouraged to discuss mistakes and use them as a start to further learning (Son and Sinclair, 2010). Fahad's approach seems to fit, to some extent, with the constructivist approach of learning which considers it normal for errors to exist in class discussions, as he prepared the class to accept errors publicly and was prepared to be corrected himself by the students (Santagata, 2005).

To sum up, the teachers tended to protect their students from making errors by forcing them to follow the given strategies and by treating errors in different ways. The teachers, Onizan and Abdullah, who tried to prevent their students from making errors did not see them as opportunities to create further learning, as they saw them as traps. The trainees treated errors by redirecting the questions to other students (maintain move), correcting the errors by themselves (insert move) and asking the students to justify their answers (press move) in order to investigate the source of these errors. This variation in the trainees' practice created two different types of class discussions: conventional textbook culture (by guiding students by establishing a series of short questions) and the conventional problem-solving culture (by giving hints to help students correct the answers) as proposed by Wood *et al.* (2006). The teachers' responses toward errors and the interaction with the students were built on their PCK, beliefs, class discussion culture and their theoretical approach of learning.

8.3.3 Supporting students' learning

The third dimension, in which the trainees' responses to students' answers vary, is in the extent of supporting students' learning. The participants of this research differed in the extent of inviting the students to participate in the class activities and in providing support to them when they faced difficulties. I discuss both below.

In this research, one strategy of inviting the students to participate in the class was by praising them when they gave a correct answer. Fahad and Onizan praised their students more than Abdullah did. They did that to increase the students' participation and confidence when they contributed to the lessons. Abdullah was the teacher who praised less as he just praised particular students who helped him by answering correctly, when the majority of the class did not know the answer. He praised them to 'encourage [them] to do more and more' as he said in the interview. The teachers in this study varied in terms of who they praised, as Fahad and Onizan praised any students who answered correctly whereas Abdullah just praised the students with good mathematical background. 'I do not believe in the importance of praising all the students as this may hinder the lesson flow' said Abdullah in the interview. This shows that teachers' beliefs about the importance of praising, which represents a classroom management strategy (Gable *et al.*, 2009), may influence some of the teachers' response to correct answers as to whether to praise all the students or just some of them.

Praising is a way of reinforcing the desired behaviour (of answering correctly). It fits with 'operant conditioning', from a behaviourist prospective, which means that reinforcing the response increases the probability of repeating that response (see for example the first chapter of Skinner (1938)). It also informs the students of the teacher's approval when their behaviour fits with the teachers' expectation (Burnett, 2001). The teachers in this research seemed to use, what they learnt in the Mathematics Teaching Methodology course (Table 2.1), to some extent, the idea of using praise as a positive motivator to encourage the students to answer correctly, which ultimately increases their participation and confidence. The goal of praising was to boost the accuracy of the students' contributions rather than their effort (Daly *et al.*, 2007) as Onizan and Abdullah only praised the correct answerers, whereas Fahad sometimes praised the students who tried to answer as well. To sum up, teachers' beliefs and knowledge influenced the teachers' decision to praise students' answers.

All the teachers in this research tried to help the students by applying strategies learnt in the Mathematics Teaching Methodology course (Table 2.1), to help them reach the desired answers or by convincing them about their errors. Occasionally, Fahad and Abdullah applied the cognitive conflict strategy (called 'disequilibrium' in Piaget's theory (Powell, 2006, p.27)) when incorrect answers were given, while Onizan used funnelling questions (Wood, 1994) to guide the students to reach the correct answers. For example, Fahad sometimes provided support to the students who could not answer correctly by spending more time with them and encouraging them to revise their answers. Also he motivated the students by telling them that they can do it and convinced them about their errors using the cognitive conflict strategy. Similarly, Abdullah used the cognitive conflict strategy to show the students' errors as he asked them to compare his initial answer with the one found after he had asked them a series of questions. Onizan usually used a different strategy as he broke down the big question into many small questions in order to guide the class to the desired answers.

Research shows that cognitive conflict is a well-known strategy to deal with misconceptions (Chick and Baker, 2005; Ernest, 1996) which involve conceptual change (Druyan, 1997), however, it does not always lead to conceptual change as sometimes the conflict is not recognised by some students (Dekkers and Thijs, 1998). Teachers in this research did not help the students recognise the conflict as they stopped interacting with the students after showing them their error without further action, which probably meant that the change in their concepts was unlikely. Also when they applied this strategy with some students who are weak at mathematics, it might have harmed their knowledge construction as they do not have the ability to resolve the conflict (*ibid*). Moreover, by combining this strategy with others, such as asking other students, the students who have conflict might be able to resolve the conflict when they see how the questions were answered by their teachers or peers.

Some researchers examine the use of the cognitive conflict strategy in classrooms. For example, Chick and Baker (2005) investigated the way teachers would respond to hypothetical students' misconceptions in the topics of subtraction, division, fraction addition and area/perimeter. They found that teachers would use the cognitive conflict strategy more than using probing their thinking strategy and less than using the re-explain strategy, depending on the nature of the items and the topics (*ibid*). Evidence from this research suggests that teachers' knowledge, to some extent, contributes to the decision of applying this strategy. For example, Abdullah applied the cognitive conflict strategy just when the error occurred in an area where he had secure knowledge. In addition, it seems that teachers' PCK plays a role in this matter as they tried to help the student deal with errors, which could be considered a strategy stemming from teachers' PCK (Ernest, 1996). In short, evidence from applying the KQ on analysing the data in this research shows that both teachers' content knowledge and pedagogical content knowledge could be linked to the way that the teachers support their students' learning when they are faced with some difficulties.

In conclusion, in this chapter I investigated and discussed the findings from the three cases by focusing on *common* and *different* response patterns among the participants. The findings of this research show that the teachers accepted and rejected the students' answers in different ways. They gave accept/reject moves combined with other moves such as press, maintain and insert, and these responses have been influenced by many factors, such as SMK, PCK, beliefs and experience. I also investigated the reasons behind these choices. Theses findings are summarised in order to answer the research questions in the next chapter.

Chapter 9 : Conclusion and Implications

9.1 Introduction

This research aims to investigate the ways in which pre-service mathematics teachers handle their students' mathematical contributions (correct and incorrect answers) in Saudi primary classes, and the reasons behind these ways. It also aims to explore the interaction between mathematical content knowledge (both teachers' subject matter knowledge (SMK) and pedagogical content knowledge (PCK)), beliefs and the ways of dealing with their students' mathematical contributions. Therefore to achieve these aims, the following questions were addressed:

1- How do trainee mathematics teachers deal with students' contributions in Saudi primary school classes, and why do they respond in those ways?

2- How does the knowledge that trainee mathematics teachers have gained influence the ways in which they handle their students' contributions in the classroom?

In order to answer these questions, three case studies were constructed and data was gathered from observations and interviews. The Knowledge Quartet (KQ) framework was used to analyse the observation data in order to explore the trainees' knowledge. Also, patterns of responding to students' answers were identified by the (Initiation-Response-Feedback) IRF framework. Also data from interviews was used to explore the reasons behind teachers' responses to the students and was nested to the transcripts of the observations in order to have easy access to them.

This chapter contains six sections which are structured as follows. I start the first two sections (section 9.2, 9.3) by introducing a summary of the study's findings in order to answer the research questions. That is followed by the study's contribution to research and the literature (section 9.4). Next, the implications and recommendations are discussed in the third section (section 9.5). That is followed by an identification of the limitations, difficulties of the research and suggestions of further research (section 9.6).

9.2 Summary of the ways and reasons behind teachers' responses to students' answers

To answer the first question of this research: 'How do trainee mathematics teachers deal with students' contributions in Saudi primary school classes, and why do they respond in those ways?' the KQ framework was used to analyse data from three case studies. That was followed by a cross-case analysis which provided answers to the research questions. In the following paragraphs I will answer the first part of the first question by summarising the ways teachers responded to the students' answers, and then the second part of the question by summarising the reasons behind their actions.

Evidence from the KQ analysis, particularly the contingency dimension, shows that all the teachers dealt with their students' answers mostly in two types of actions (see Table 8.1). The first type was *confirmation* actions where the teacher confirms the correctness or the fault of the given answer. These actions happened after the decision of accepting or rejecting the answers had been made so they might be considered as post-decision actions forming one round of teacher-student-teacher interaction. For example, in this study, the teachers confirmed the correctness of the answer (accept move) by writing it in the board (maintain move), praising (accept move), commenting on (comment move) it by giving a summary (maintain move), repeating it and/or progressing to the next point. Also, they confirmed the fault of an answer (reject move) by providing explicit rejection verbally or physically, correcting it (insert move), repeating it and/or commenting on it (maintain move). The other form of response was *questioning* actions where the teachers asked further questions usually before making decision about the answer. This type of response contained actions where the teachers asked for justification (press move), gave clues (elicit move) and/or asked other students about their thoughts (maintain move) about the given answers. These actions required more than one round of teacher-student-teacher interaction. Moreover, at the teachers' individual level, they applied different strategies, learnt in the Mathematics Teaching Methodology course (Table 2.1), when the students had difficulties. For example, Fahad and Abdullah applied the cognitive conflict strategy whereas Onizan usually used tunnelling questions when their students faced a problem solving the exercises.

Many reasons for the ways, in which teachers responded to their students answers (summarised above and in Table 8.1), were given by the participants. Their ways of accepting the students' correct answers fall into two types of reasons. The first one is *class-focused*, as the teacher aimed his response to the class as a group rather than the individual level. For example, the teachers wrote the correct answer on the board to share with class so here they emphasised the importance of the class in their justification. This group contained the commenting action where the teacher gave a summary to share with the class in order to avoid common mistakes. The second group of responses is *individual-focused*, as the teacher focused more on the individuals by checking the sureness of their answers and their ways of solving problems by asking them for justification. Also, it included the teacher's praising of the student to increase their confidence and participation.

The reasons for the ways in which the teachers rejected incorrect answers fall into three groups. The first one is *protection* reasons, such as changing routine, knowing the source of errors, avoiding disappointment and treating common mistakes. For example, the teachers rejected incorrect answers verbally or physically to protect the students from boredom. Also, they asked other students to avoid embarrassment and commented on the errors to protect the students from making mistakes in the future. The second one is *checking* reasons, such as checking sureness and understanding, where the teachers tended to ask questions in order to check these two elements. The final reasons group is '*other*' which contained time pressure which lead the teacher to correct the errors by himself, and peer learning where the teacher asked others to let the answerer learn from his peers.

9.3 Summary of the influence of teachers' knowledge on the handling of students' contributions.

To answer the second question of this research: 'How does the knowledge that trainee mathematics teachers have gained influence the ways in which they handle their students' contributions in the classroom?' three dimensions of different themes where the teachers vary in their response were identified and discussed. These dimensions, as well as the common response patterns discussed earlier, highlight the possible influence of teachers' knowledge on their practice as outlined in the following paragraphs.

Evidence from the foundation dimension in the KQ analysis shows that the teachers' mathematical content knowledge (SMK, PCK and beliefs) influence the ways in which the teachers responded to the students' answers. For example, the teachers depended mainly on their SMK, and to some extent on PCK, in deciding to accept or reject a given answer as discussed in the first dimension of the teachers rational to accept or reject the answers. Teachers' SMK affected their decision to accept or reject answers. Those with weak SMK usually accepted incorrect answers as they thought they were correct and rejected correct answers as they thought they were incorrect.

After the answers were rejected or accepted, teachers acted in different ways depending on different kinds of knowledge. When teachers accepted the answers, their knowledge and beliefs affected their ways of responding. Confirmation actions of correct answers could be linked strongly to pedagogical knowledge and to some extent to beliefs, while questioning actions are mainly related to PCK. For example, teachers depended on their general pedagogical knowledge when they wrote the answer on the board in order to share it with the class and repeat it. Also, their PCK and, to some extent SMK, came to play while they were commenting on the answers by giving clues and a summary, and when they asked for justification. In addition, teacher's beliefs about how mathematics is better learnt influenced the trainees' decision to praise the students, as they believe that praising the students increases their confidence and participation. Moreover, the teachers' PCK influenced their practice regarding the investigating of the source of errors, their treatment and the students' understanding.

Teachers' beliefs about the ways in which mathematics is best learnt, as shown by the foundation dimension, played a main role in deciding which approach the teachers took to respond. For example, teachers who believe that the students learn better when errors are corrected by peers usually responded by asking other students to correct the errors.

PCK influences the quality of the ways teachers' responded, as it provides the basis which teachers depend on to achieve the intended goals effectively. For example, teachers with weak PCK cannot respond to students' errors with rich instructions as they cannot go beyond identifying the errors and just probably asked for justification without further actions. Whereas teachers who have a stronger PCK provided more support to the students when they faced difficulties and increased their class's willingness to share errors.

9.4 Research contribution

This project has potential contribution in the Saudi research context and the field of mathematics education. To the best of my knowledge, this research is the first research in the Saudi context that investigates the area of teachers' knowledge and how it might relate to their practice. Also, it contributes to extending the use of qualitative research in the Saudi context as the majority of research there has been done quantitatively (Almoathem, 2009). Furthermore, it might add a new block to the literature in the Saudi context specifically and in the field of education in general by providing more opportunity to conduct comparative research among international cultures.

Another contribution is that the KQ was adopted and used in this research, which could be considered as the first use of the KQ in the Saudi context and in Arabic language speaking countries. This adds new insight into the use of this framework internationally besides its use in the UK and other countries such as Cyprus. Some of The KQ's codes were modified such as 'Overt subject matter' and 'adherence to textbook' to fit with the Saudi context. As outlined in the methodology chapter (section 4.7.2), these codes were applied to the data with a slight adjustment due to the differences between the UK context (where the codes were created) and the Saudi context. Also this research focuses mainly on the foundation and contingency dimensions with less reference to the transformation and connection dimensions in the KQ. This may extend the research on contingent situations in teaching which is an important part of mathematics teaching (Rowland and Zazkis, 2013). Moreover, discourse analysis was combined with the KQ to enrich the level of analysis of the teachers' response in the turn-by-turn basis of speech between the teachers and students. This type of analysis which built in turn-by-turn basis is less common in the literature as most of the research focuses on incidents as a whole.

Furthermore, this project adds to the literature of how teachers deal with students' answers. The majority of research previously has focused on how teachers deal with errors, whereas research of positive evaluation is less explored. This study expands the research of teachers' responses to students' answers by investigating how these Saudi teachers dealt with both correct and incorrect answers. Also, it investigates the reasons behind the ways in which teachers dealt with the students' answers from their perspective, which may extend the relatively limited research literature on these reasons. Lastly, it might encourage research on teachers' taxonomy of responding to students' contributions by offering a new refinement of the current taxonomies.

9.5 Research implications and recommendations

Research findings were discussed and the research questions were answered in the previous sections. These findings have several implications for the policy maker in Qassim University, the co-operating teachers and supervisors of the trainees. This study shows that the teachers' knowledge including beliefs influences their practice, so presumably much more attention should be paid to this knowledge in order to improve practice. One main finding is that the mathematical content knowledge of mathematics trainees in Qassim University is generally not promising. One possible recommendation to the policy makers in the university is that they should reform their curricula to improve their graduate's mathematical content knowledge. Also, they should introduce some background about the New-Reform mathematics textbooks as the trainees protested that they do not have sufficient knowledge about the content of the primary curricula before they went into schools. In addition, changes could be made to teacher preparation programmes in order to adjust the balance between the courses of pedagogy and advanced mathematics courses. This could help the trainees to develop Profound Understanding of Fundamental Mathematics (PUFM) which help them to teach more effectively (Ma, 1999).

The research shows that the teachers almost always tended to teach procedurally rather than conceptually. This puts more pressure on the co-operating teachers and supervisors to help the trainees change their method of teaching as one of their duties is to advise the trainees to not do so. This project also suggests that the co-operating teachers and supervisors should put more effort in encouraging the trainees to build a productive learning environment in their class by sharing the students' errors as most of the trainees did not take this chance to create further learning. Also, they should encourage the trainees to increase the opportunity for group work, as the majority of the classes were organised in separate rows which prevents the students working together. In addition, the trainees' use of teaching aids was not promising, so the co-operating teachers and supervisors should encourage them to use more representations.

Furthermore, evidence from the foundation dimension shows that the trainees sometimes not to identify errors in the students' answers which needs more consideration from the head teachers, supervisors, co-operating teachers and the curricula developer in Qassim University to strength the trainees' knowledge.

9.6 Research limitations, difficulties and suggestions for future research

One possible limitation of this research is that its scope was limited to investigate the ways in which teachers respond to their students' answers and the reasons for that. Due to the nature of the data, which was almost always in the form of question-answer, space and time restrictions I could not extend the focus to include other types of contributions, such as student suggestions or questions. Also, as other qualitative studies, this study's findings cannot be generalised to other settings which may limit the potential implications to the study set.

I faced many difficulties during and after gathering the data. Time constraints limited my work during fieldwork. I planned to visit five trainees for about eight lessons each during a maximum period of three months given by my sponsor in order to complete the fieldwork. I found it difficult to move from class to class then review the content of the lessons to prepare questions for the interviews, which meant postponing the analysis until the end of the fieldwork. After collecting the data, I found that the KQ was a good choice to analyse the data with instead of using the proposed framework of Sfard's (2001; 2008) analytic approach which was used in the pilot study. However, although spending time searching for a different approach to analyse data increased my knowledge about the methods of research, spending time reading about the KQ before starting the analysis made the schedule run late.

After analysing the data with the KQ I found it is worth suggesting some further research that can be done to extend this research further. This research is new in the Saudi context and its results may help in future research in Saudi Arabia and in mathematics education as well. The teacher education programmes in Saudi Arabia can be improved by outlining the levels of the trainee teachers' knowledge which may enhance further research of why their knowledge is at that level and also how to improve it. The content of the preparation courses at Qassim University could be reformed as a result of the study's finding. Research on how to prepare the trainees for

Practicum is worth doing in order to enrich the content of some courses at the university to increase their ability to cope with the new stage in their study. In addition, the same study could be done for female trainee teachers as well. They take almost the same courses which may provide an opportunity to compare male and female trainees regarding the objectives of the research. Furthermore, research about other aspects of teachers' characteristics that may influence their response to student contributions, or extending the research to include the teachers' dealings of other types of students' contributions, such as their questions, is advised in order to fully understand the phenomena.

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Appendix (1) Mathematical courses taught to the trainees

Course Code	Course Title	Credit Hours	Course Description
140 Math	Introduction to mathematics	2	Linear equations and their applications-linear inequalities - absolute value equations and inequalities - complex numbers- quadratic equations and their applications - odd and even functions - function algebra - exponent and logarithmic functions - cones- matrices
150 Math	Calculus	3	Concept of limit - calculation of limits - continuity and its consequences - limit at infinity and infinity limits – derivatives - correlated averages, mean value theorem
200 Math	Math Lab	2	Introduction to child's early discoveries - primary school math and methods of teaching it – counting - whole numbers – arithmetic – measurement
206 Math-m	Basics of Mathematics	3	Ordering - principle of well-ordering - principle of mathematical reasoning or logic, proving methods – inference - sets and operations - Descartes' multiplication of sets, binary relations - set partitioning - equivalence classifications - limited sets – tables
211 Math-m	Calculus	3	Riemann's concept of calculus - exponent and logarithmic function – curves - partial and complete calculus – geometric applications
230 Math-m	Topics in Euclidean geometry	3	Euclid's five postulates - history of geometry - segments and rays – angles - angle measurement - geometric inequalities - Pythagoras' theory - the circle and its area - lines and planes - transformations - areas of some solids
242 Math-m	Statistics and probability (1)	2	Definition of basic statistics and terminology - Distributions iterative - Graphical representation of data - Measures of central tendency - Measures of Dispersion - Sprains and kurtosis - Correlation (Pearson coefficient, Spearman coefficient of rank) - Simple linear regression - Method of least squares - Accidents and the sample space - Probability axioms - Conditional probability - Biz theory - Autonomy

251 Math-m	Mathematical programming	2	linear programming: the ultimate practical problem-solving model reduction (design algorithms, prove limits, classify problems - the ultimate theoretical problem-solving model) combinatorial search (coping with intractability)
207 Math-m	Theory of numbers	3	Divisibility - prime numbers - fundamental theorem of arithmetic -Fermat's numbers - Fermat's method of analysis - linear Diophantine equations - basic qualities of identities - linear identities - Euler's Theorem - Chinese remainder theorem - Wilson's theorem - numerical functions - perfect numbers - Fermat's conjecture
235 Math-m	Analytical Geometry	3	vector space - definition of the plane distance - problems the dot product, to get the angle of two vectors - the cross product, to get a perpendicular vector of two known vectors (and also their spatial volume) intersection problems, angle between two planes - algebraic properties of vectors, numerical multiplication of vectors
244 Math-m	Statistics and probability 2	3	Random variable and probability function - expectation and variance of the random variable - some probability distributions - Bernoulli distribution - Poisson distribution - random variables -related variables - sample distribution-central limit theorem - homework estimation and testing - trust periods – kinds of moments
273 Math-m	Discrete Math	3	Logic: propositional logic, logical equivalence, predicates & quantifiers, and logical reasoning. Sets: basics, set operations Functions: one-to-one, onto, inverse, composition, graphs. Integers: greatest common divisor, Euclidean algorithm. Sequences and summations mathematical reasoning: proof strategies, mathematical induction, recursive definitions, Structural Induction. Relations: properties, combining relations, closures, equivalence, partial ordering. Counting: basic rules, pigeon hall principle, permutations and combinations, binomial coefficients and Pascal triangle. Probability: discrete probability. Expected values and variance. Graph Theory - Circle Graph - Bar Graph - Line Graph -Histogram - Picture Graph

302 Math-m	Linear Algebra	3	Matrices and operations on them - matrix rank – determinants - inverse of a matrix - linear equations - vector space – linear dependence and independence - radix and dimension - coordinates and change of radix – verticality - Cauchy-Schwarz inequality, gram algorithm - linear transformations - distinctive vector values
312 Math-m	Advanced Calculus	4	Successions – sequences- convergence – N-root test - RAP-test- functions in coefficients - partial derivation - complete calculus - binary calculus – tri-calculus - coordinates - maximum and minimum values of functions in two variables - multiple calculus - physical applications - tri-calculus in cylindrical and circular coordinates – Stoke's theorem - linear calculus
363 Math-m	Ordinary differential equations	3	Basic concept - formulation of differential equations - classification of and solution to first rate differential equations - applications of first rate differential equations - high ranking differential equations with constant coefficients - theory of solution existence and singleness - liner independence - parameter changes - unspecified coefficient methods - rank reduction
305 Math-m	Abstract Algebra	4	Groups - partial group - circular group - p-local structure of groups; Sylow Theorems and some applications - Lagrange Theorem - properties of homomorphism – basic theorem of homomorphism – automorphisms - inner automorphism - Kelly theory- circular set classification - outer direct multiplication – inner direct multiplication
313 Math-m	Introduction to real analysis	3	Field axioms and ordered sets - the set of real numbers and completeness axiom - countable and uncountable sets - sequences and convergence - monotonic sequences - Cauchy sequences – Cauchy criterion and Bolzano-Weierstrass theorem - subsequences - limits of functions - main properties of limits - monotonic functions and their limits - continuous functions & their properties on interval - uniform continuity - compact sets and continuity - derivative of functions – Riemann's calculus

353 Math-m	Linear programming	2	Mathematical model (linear program) as a solution to some problems - graphic way of solving problems – duality - precise analysis
383 Math-m	Problems and Solutions	2	Unconventional solutions – problem-solving strategies - selected algebraic problems - selected problems in analysis - selected problems in finite mathematics - selected problems in statics and probability
415 Math-m	Complex Analysis	3	Field of complex numbers – Cartesian & polar representation of complex numbers - DeMoivre law – roots of unity – topological properties of complex number and Riemann sphere – complex functions – limit & continuity of complex functions – analytic functions – Cauchy-Riemann equations – harmonic functions – exponential function – trigonometric and hyperbolic functions – logarithmic function – complex integration – Cauchy-Goursat theorem – Cauchy integral formula – Liouville theorem – fundamental theorem of algebra – complex sequences & series - representation of analytic functions by series – Taylor series – Laurent series - zeros and singularities for analytic functions – residue calculus – evaluation some real integrals using residue calculus.
454 Math-m	Numerical Analysis	3	Error special functions numerical linear algebra basic concepts -solving systems of linear equations - Eigenvalue algorithms - other concepts and algorithms5 interpolation - polynomial interpolation -Spline interpolation - trigonometric interpolation - other interpolates - approximation theory -Miscellaneous6 finding roots of nonlinear equations - optimization - basic concepts - linear programming - nonlinear programming - uncertainty and randomness - theoretical aspects - applications - numerical quadrature (integration) - numerical ordinary differential equations - numerical partial differential equations - finite difference methods - finite element methods techniques for improving these methods Monte Carlo method applications
462 Math-m	Topics in applied	3	The content is not final and subject to change from term to term

	mathematics		
R7-m	Optional Math course	3	According to course selection

Source: http://tcr.edu.sa/defaulteng.ASPx

Appendix (2): Educational courses taught to the trainees

Course Code	Course Title	Credit Hours	Course Description
331 Method	Primary Education Curricula	2	The course aims at providing students with the modern concept of the approach, principles and standards on which the primary education curricula and their constitutive elements are based. It also aims at supplying students with information on the forms, schemas, development, rationale, measures, steps and obstacles of the curricula.
332 Method	General Education Methodology	2	The course aims at providing students with basic principles related to teachers such as qualities, responsibilities and duties. It also seeks to familiarize students with the nature of teaching and its basic stages, general guidelines on lesson planning in terms of procedural aims, teaching procedures and evaluation methods. As well, the course deals with a number of general teaching methods that can be used in teaching any subject, such as the method of debate and discussion, deductive methods, cooperative learning and guided inference along with making room for students to train themselves in these methods through micro teaching.
333 method	School Activities	2	The course aims at providing students with information on the concept of school activities, and the stages of their development, the types of activities and mechanisms, together with the role of the activity leader in planning and carrying out school activities; it also provides students with performing skills to plan and carry out different school activities. It also deals with the problems of school activities and ways and suggestions of solving them.
431 Method	Mathematics Teaching	2	The course deals with the effective teaching skills necessary for the mathematics teacher at the primary stage. It also focuses on modern trends in maths

	Methodology		teaching (the computer and the Internet), methods and ways peculiar to maths teaching, such as induction, games, small groups, deduction, individual learning, and training students in these by making room for them to practise what they have learned from the course (according to the miniature teaching technique, and the use of modern educational techniques).
449 Method	Practicum	8	This course comes after the trainees have finished both stages of observing and participating while studying both courses: general and specific teaching methodology. The course is aimed at giving trainee teachers some teaching experience before they begin actual teaching, and prepares them psychologically, professionally, administratively and educationally for the teaching profession. This is achieved through providing students with the opportunity to practise real teaching, and making them apply all the knowledge, theories and skills under specialist supervision, which guarantees appropriate feedback to help them enhance their academic conduct and encourage them to choose, apply and evaluate what they deem as appropriate teaching methods and educational techniques.

Source: quoted from The Teacher College website (http://www.tcr.edu.sa/en/Cv/encvcour.ASPx).

Appendix (3): Discussion of the analysis of one incident from the pilot study

This lesson is a good example of a Saudi teacher struggling to teach his pupils a new concept. It shows a combination of eliciting correct answers and the benefit of sharing a pupil's error as an opportunity to discuss a common mistake with the pupils in order to prevent reoccurrence. In these excerpts, Hamad was inviting the pupils to participate in the class by solving a particular mathematical problem (on the whiteboard). He was looking, at the beginning, just for the correct answers, by saying, for instance, 'Who can find the image of the point…'. Then, his intention shifted to evaluating each pupil's solution, and when Adel showed some misunderstanding of the task, Hamad exploited the situation in order to address a common mistake.

In most of the positive situations (when the pupils gave correct answers), Hamad said, 'Excellent'; this is a common form of verbal praise, which Brophy (1981) defined as a word 'to commend the worth of or to express approval or admiration'. It seems that Hamad relied heavily on praise as a reinforcement technique, and hence he said the word 'excellent' more than five times in the lesson. This process is the Q-A-R technique, which means that a question from Hamad is followed by an answer from a pupil, which is then reinforced by Hamad. As mentioned above, Hamad asked the pupil about the image, the pupil answered and then Hamad praised him by saying 'excellent'. Moreover, Hamad himself sometimes gave a brief interpretation of how the pupil had arrived at the correct answer. This procedure may be thought of as sharing the correct answer with the class but Hamad did not give it much attention, as he tended to say it quickly and without much emphasis. Whatever the task was, Hamad always behaved in the same way, and this lack of variety could, to some extent, be less than stimulating for the pupils. He could have possibly employed other methods to encourage the pupils to give correct solutions, such as giving the pupil a small gift or praising him in some nonverbal manner.

As for how Hamad dealt with his pupils' errors, Adel had clearly not understand the concept of reflection, and thus, he was not able to find the image of point (1, 1).

Hamad intentionally used Adel's contribution as an opportunity for deepening the understanding of the concept among the pupils but, unfortunately, in an inappropriate and unproductive way. He tended to guide the children towards the solution through unexplained procedures; in this case, he pointed to the image of a given point, which did not make sense, at least, to Adel, who failed to count the units correctly even after being told not to count the starting point. Hamad instead could have focused on why he had taken these steps. When Hamad was asked about why Adel was making mistakes, he answered:

> 'Adel was absent last week […] He missed the last lesson [translation] so he did not know how to count on a graph. […] He may be embarrassed. He does not like to stand in front of his peers.'

It is clear that in this statement Hamad believes that new knowledge is built upon previously acquired knowledge. In addition, he seems to be a sensitive teacher because he considered that Adel's emotions influenced his performance.

There are several possible comments to be made about Hamad's teaching style. The first relates to his knowledge base. Bray (2011) indicates that teachers with a weaker mathematical background often focus on procedures to obtain correct answers. Hamad, as mentioned above, has strong mathematical subject-matter knowledge according to his CV, so he may have been facing obstacles in his efforts to make the mathematical concepts accessible to his pupils due to a lack of pedagogical content knowledge (PCK). Rowland *et al*. (2009) stress the importance of the role that PCK plays in supporting teachers to break down concepts into easily digestible ideas. This kind of knowledge cannot be assessed through a teacher qualification, and the trainee teacher may be expected to rate poorly on it (*ibid*), so Hamad's PCK should be checked in order to make sure that it is at an acceptable level.

Another explanation may be time pressure. Hamad taught with the reformed textbook, which was only introduced this year. He feels that this new textbook is too long to be taught in one term, as Hamad, when he was asked about the textbook in general, said:

> ‘Oooh, it is too long. You know, when I go home, I take my brother’s textbook who is teaching in Grade Five and see where they stopped today. To be honest, they covered more than us so I should move faster.’

I understand from this answer that Hamad tended to teach according to instrumental understanding in order to catch up with other schools, so he was concentrating on quantity not quality. He was also influenced by his assistant teacher (whose responsibility is to support Hamad in his classes) and his supervisor because they kept pressing Hamad to move forward in the textbook, which, as he told me, forces him to spend less than adequate time on many important aspects.

The final teaching approach needing to be addressed is limited exploration of the sources of pupils’ errors. Hamad did not ask Adel either why he initially pointed to 6, 1, which resulted from counting four units from the right-hand edge of the graph, or why he pointed to 8, 1 on the second occasion. Hamad also did not investigate the difficulties Adel had in finding an image in the previous lesson; he said, ‘We talked about this in the translation lesson. You always make the same mistake’, even though all the pupils should have mastered it before starting this lesson. Moreover, Hamad sometimes explained a pupil’s answer himself, rather than asking the pupil how he had arrived at the correct answer.

In conclusion, limitations in Hamad’s pedagogical content knowledge have affected his teaching effectiveness. Being asked to cover all parts of the syllabus in a short period of time is also a challenge for Hamad, who struggled to perform his duties and satisfy his supervisor.

Further analysis

Adel’s situation needs further analysis if his problems are to be understood more deeply. As noted above, Adel struggled with how to identify the image of point D, and hence he failed to perform this task correctly more than three times. To probe deeper to the root of his misconceptions, I shall revisit Adel’s episode and apply Sfard’s approach, which is the process of conceptualizing mathematical thinking as a form of communication, which has greatly inspired me.

Sfard and her colleagues developed two types of analysis to investigate the ineffectiveness of discursive interactions. These two kinds are Focal and Preoccupational, and they deal with 'Object-level aspects of communication … [and] …the meta-level factors', respectively (Sfard, 2001).

Focal analysis:

Sfard (2001) suggests two main ingredients for this kind of analysis, which are pronounced and attended. The former refers to the exact words used in a conversation, and the latter refers to the nonverbal actions in that conversation, such as gesticulation. She also names the combinations between the interlocutors' statements and actions as 'intended focus'.

To apply this type of analysis to the Hamad and Adel episode, I have outlined their actions in the following table:

Person	Utterances	Pronounced	Attended	Intended
Hamad	[3], [5], [27]	Where is D? No, no, that's it.	Using a graph	The image
Adel	[4], [26]		Pointing (6, 1), puts his finger on 9, 1.	
Hamad	[8], [16], [17]	Until you reach the axis of reflection. Why? From there.		Movement direction
Adel	[14]		Counting from the right-hand edge of the graph	
Hamad	[7], [8], [11], [13]	Now count, how many units? How many? Four.	Procedural interpretation on the board.	The distance between

Adel	[12]		Raises four fingers.	the image and the reflection line.
Hamad	[10], [13], [19], [23], [25]	Do not count the point of D. Now count. Do not count the axis. Move one step. Do not count the starting point.	Using the floor as an instrument to show his movement and direction	Counting the starting point.
Adel	[9], [18], [26]		His finger is on point D, and he starts counting one, two, three. Four.	

Table 0.1: Adel episode; analysis of Hamad's and Adel's actions

Hamad's intended focus developed in an appropriate manner. His focus evolved from the whole picture, which involved finding the image, to its details. Finding the image of a given point in this lesson happened in a sequential and logical order, which started with calculating the distance between the point and the reflection line, looking to the direction of the movement, and then applying this combination of the length and the direction in order to locate the image.

The main aim of the task was to find the reflective image for point D. To do so, Hamad's initial intended focus was on locating the image. Hamad began the task by asking 'Where is D?', and he responded to Adel's gesticulation by saying 'No, no', before finally saying in the last part of the task (when Adel had found the correct answer) 'That's it'. This diversity of utterances occurred in different stages of the task; the teacher used a question structure to initiate the task and then he used the word 'No as an evaluation for the answer. In addition, when the solution had been reached, the teacher praised the pupil by saying 'Excellent'. Thus, Hamad showed an ability to control his pronouncements in order to deal with different situations, for example, when he needed to encourage the pupils to answer by saying ‘What is the matter with this group, why does nobody want to answer?’, or when he evaluated an answer by saying ‘No, no’, or even when he wanted to show respect and praise the pupil who answered correctly by using the word ‘Excellent’.

Hamad faced serious problems in dealing with Adel’s lack of ability in identifying an image. Adel’s initial answer was 1, 6, as in [4], which could be because Adel had counted four from the right-hand edge of the graph; he re-counted in the same way but in the opposite direction in [14], reinforcing his incorrect method. These two actions may serve to better understand the source of Adel’s difficulties in following Hamad’s guidance. Adel has a problem regarding the correct direction for counting, and Hamad had given him a sequence of blind steps, which confused him for the rest of the task. Hamad seemed to be unaware of how Adel had arrived at his first answer, and did not ask him why he had given that incorrect answer; this could be seen as a lack of teaching knowledge. His responses to Adel’s contributions in this matter were to give Adel a direct verbal order, such as ‘Now count until you reach the axis of reflection’, and asking ‘Why?’ when there was a mistake in the answer in order to prepare the pupil for further actions; he finally gave a direct visible sign by indicating the correct direction with his hand.

The figure above shows that Hamad spent much time calculating the distance between point D and the reflection line, and how to count from the starting point. These two actions overlap all the time. Adel counted the starting point three times, in [9], [14] and [18], but he knew in [12] that the distance was four. Adel was initially asked by Hamad to calculate the distance, and then when Adel counted the starting point, Hamad responded by saying ‘No, no. Do not count the point of D’, as in [10]; after that, Adel knew that the distance was four units. Adel found it difficult to repeat Hamad’s steps on the right-hand side of the reflection line on the graph, and hence he counted the starting point again. Hamad again did not ask about the source of this error, and again responded by stressing his previous statement ‘No, no, no. Do not count the axis of reflection’. Hamad shared this mistake with the rest of the class when he said ‘We learnt about that in the translation lesson. You always make the same mistake. We said that when you stand on a line or point [Hamad stood on a line on the floor, using it as a number line], do not say ‘one’ and move to two [he moves one step forward]. No, No, when you stand, move one step and then say ‘one, two, three, four‘. Four steps. ‘Do not count the starting point’.

The table above assists in identifying Hamad’s responses to Adel’s contributions. Most of Hamad’s responses were verbal; he also used the floor as an instrument in order to illustrate the principle of reflection. He kept giving Adel direct commands, such as count, do not count, and move; the majority of those orders occurred with respect to counting the starting point. In addition, Hamad used a mixture of methods to help Adel achieve the correct answer; he employed the graph on the board, verbal instructions and explanations, gesticulations and the floor of the classroom, all to respond to Adel’s misunderstanding of how to count from the starting point. Hamad’s pedagogical content knowledge helps him to be able, to some extent, to guide Adel to the end of the task. It helps him to zoom in from the whole picture to the parts; in other words, he relied on his experience to shift his focus from finding the image of point D to the part within that process that was causing difficulty, which was calculating the distance and counting the units correctly.

Preoccupational analysis:

This kind of analysis deals with various other aspects of the interaction. Sfard names it the meta-level, which includes the ‘Interlocutors’ concerns about the way the interaction is being managed and… the weighty, and sometimes quite charged, issues of the relationship between interlocutors’ (Sfard, 2001).

The interaction between Hamad and his student is a typical example of the relationship between a teacher and a student in Saudi Arabia. Hamad adopted a leading role all the time (as is expected of a teacher in Saudi Arabia), and Adel did not speak. The majority of this trainee teacher’s statements were proactive, which means that he invited the pupil to react; for instance, in [3], [7], [13], [23], [8], [11] and [16], Hamad asked Adel to perform some action, such as to count, to move and not to count, and he wanted Adel to respond. On the other hand, Hamad’s reactive utterances, i.e. reactions to the pupil’s contributions, were only four, in [5], [10], [19] and [25]. Hamad gave Adel direct orders or cautions as responses to the pupil’s actions. Adel often used only nonverbal actions in this discourse, such as pointing to the coordinates 6, 1, and counting with a finger. This interaction shows that Adel depended heavily on Hamad's instructions, and thus lacked confidence in taking part effectively, which is not good for him as he is merely a listener. Hamad's tendency, here, could be linked to his lack of experience or even knowledge of how to balance the weights of the participants.

In conclusion, Hamad's performance as outlined through the focal analysis; most of his pronouncements and attendant actions complemented each other smoothly throughout the task. He zoomed in from locating the image of point D to the particular stages of the process. Hamad responded to all of Adel's contributions in a variety of ways, which were designed to encourage, evaluate and praise. These responses helped Adel to achieve the correct answer in a procedural manner. The interaction between Adel and his teacher was not clear until a preoccupational analysis had been conducted. This helped to identify that the interaction between them was not particularly efficacious. In fact, Hamad maintained control all the time without Adel having any part to play and who spoke very infrequently. I would suggest that if Adel was working with another teacher, he would have been more active (depending on the balance of effort expected of him).

Appendix (4): Participant information sheet

I am conducting a study entitled, *The relationship between mathematics teachers' content and pedagogical knowledge and their handling of student contributions in their lessons in Saudi primary schools*. This is a requirement toward the completion of my PhD degree in Education at the University of East Anglia, Norwich, England.

In my research I shall observe you, as the participant, while you are teaching in your class. This observation will be videotaped and some episodes will be chosen to focus on in a face-to-face individual interview after the class has finished. The interview will last about 40 minutes and will comprise of some questions about the lesson, and you will be asked to perform some tasks as well. You may decide when and where we conduct the interview. It will be recorded unless you disagree. Your participation in responding to this interview would be greatly appreciated.

All the data collected from you will be kept strictly confidential and will be used for educational purposes only by me and my supervisors. All names will be anonymised and no specific references involving your name or the name of your school will be made in my report. Also, you are free to withdraw from participating in this study at any time. If you do withdraw, no information you have given to me will be used.

I would like to thank you in advance for your participation in this study also I and you will work as a team to obtain the project's aims. Finally, should you have any enquiries about the conduct of the research and what is involved in participating in this study, please contact me by e-mail and in the unlikely events please complains to the Head of school Dr. Nalini Boodhoo via N.Boodhoo@uea.ac.uk.

Sincerely,

Bader Aldalan

E- mail: B.Aldalan@uea.ac.uk

Supervisor's contact details:

Dr Elena Nardi

University of East Anglia

School of Education

Norwich

United Kingdom

E-mail: E.Nardi@uea.ac.uk

Appendix (5): Participant consent form

Participation in Research Project

PARTICIPANT CONSENT FORM

I have read the information sheet setting out the research project, *The relationship between mathematics teachers' content and pedagogical knowledge and their handling of student contributions in their lessons in Saudi primary schools.,* undertaken by *Bader Aldalan*, who is doing a PhD in Education at the University of East Anglia, Norwich, and I agree to take part.

I understand that my participation will involve being *observed in the class and interviewed and I will be asked to do some tasks.*

I understand that all the views and information I provide will remain anonymous and will be treated confidentially. I will not be identified in any report, dissertation or other material produced as a result of this project.

I understand that any information I provide will be stored securely.

I also understand that I am free to withdraw from participating in the research at any time and that my views and information will not then be used.

I agree / I do not agree [please delete as appropriate] to take part in the research project, *The relationship between mathematics teachers' content and pedagogical knowledge and their handling of student contributions in their lessons in Saudi primary schools.*

Name (please print):

Signature: Date:

Thank you