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Endogenous growth and property rights over renewable resources

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ABSTRACT

We study how different regimes of access rights to renewable natural resources – namely open access versus full property rights – affect sustainability, growth and welfare in the context of modern endogenous growth theory. Resource exhaustion may occur under both regimes but is more likely to arise under open access. Moreover, under full property rights, positive resource rents increase expenditures on manufacturing goods and temporarily accelerate productivity growth, but also yield a higher resource price at least in the short-to-medium run. We characterize analytically and quantitatively the model's dynamics to assess the welfare implications of differences in property rights enforcement.

1. Introduction

We study how different regimes of access rights to renewable natural resources affect sustainability and welfare in the context of modern endogenous growth theory. There is a long tradition in resource economics of studying access rights in partial equilibrium. The benchmark bioeconomic model – pioneered by Gordon (1954) and Schaefer (1954), and fully characterized by Clark (1973) – typically considers the two polar cases of *open access*, in which the resource is accessible to atomistic harvesters that do not control the evolution of the aggregate resource stock, and *full property rights*, in which the sole owner, or a coordinated group of harvesters, controls the resource stock and therefore adjusts the time profile of harvesting to the dynamics of the resource base. The most popular result, known as the *Tragedy of the Commons* (Hardin, 1968), is that open access may induce resource exhaustion because atomistic harvesters maximize current rents neglecting the effects of current harvesting on future resource scarcity. Related contributions emphasize that when both regimes yield positive resource stocks in the long run, the levels attained under different regimes depend on the specification of harvesting costs and discount rates (Zellner, 1962; Plourde, 1970). Importantly, because this literature focuses on partial-equilibrium models, its results are highly sensitive to the assumption that prices and the interest rate are exogenous (Clark, 2005).

The Gordon–Schaefer–Clark bioeconomic model has been seldom studied in the general equilibrium framework of modern growth theory. Consequently, we still lack a satisfactory treatment of the dependence of growth on access rights. This gap in the existing literature motivates our analysis.

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Notable attempts at integrating resource and growth economics that precede ours are Tahvonen and Kuuluvainen (1991, 1993) and Ayong Le Kama (2001). Both introduce renewable resources and pollution in the neoclassical Solow–Ramsey model and study the interactions between harvesting and negative externalities. Bovenberg and Smulders (1995) analyze the same issues in the context of endogenous growth. Our analysis departs from these contributions in two fundamental respects. First, we abstract from pollution externalities and provide, instead, a detailed comparison of open access and full property rights over a renewable resource that exclusively plays the role of essential production input. Second, we employ a Schumpeterian model of endogenous growth in which different types of innovations coexist.

In our framework productivity growth stems from innovations pursued by incumbent firms as well as by new firms entering the market.[1] Incumbent firms invest in projects aimed at increasing their own total factor productivity (vertical innovation). At the same time, entrants invest in projects that develop new products and set up production and marketing operations to serve the market (horizontal innovation). The rationale for using this approach is three-fold. First, this class of models is receiving strong empirical support in explaining historical patterns of innovation activity and economic growth (Madsen, 2010; Madsen and Timol, 2011). Second, the interaction between the mass of firms and technological change within the firm eliminates scale effects – that is, long-run growth rates do not depend on the size of endowments – a property that is empirically plausible (Laincz and Peretto, 2006; Ha and Howitt, 2007) and is furthermore realistic in the present context where input endowments include a stock of natural resources. Third, the model is analytically tractable, a highly desirable feature in the present context. A major reason for the lack of general-equilibrium analyses of access rights and economic growth is, as Brown (2000) put it, that " Introducing one more differential equation to account for renewable resource dynamics makes it difficult to get general analytical solutions and much of the profession continues to find it tasteless to rely on computer-aided answers". Our model yields a detailed characterization of the dynamics of the resource stock, income levels and productivity growth, both in the transition and in the long run. This allows us to compare equilibrium paths under both regimes and to obtain three sets of results.

The first set of results concerns the effect of property right regimes on resource scarcity and sustainable resource use. Both regimes may yield resource exhaustion or sustained growth in the long run, but the condition for long-run sustainability is always more restrictive under open access: if the intrinsic regeneration rate of the natural resource falls within a specific interval of values, the economy experiences the Tragedy of the Commons under open access but sustained resource extraction – and, hence, sustainable economic growth – under full property rights. When natural regeneration is sufficiently intense to induce sustained growth under both regimes, the resource stock is always higher under full property rights. Importantly, this last result does not imply that the resource price is necessarily lower under full property rights. The reason is that, in our model, the equilibrium value of resource rents is affected by both resource scarcity and income dynamics and – contrary to standard partial equilibrium models – income dynamics are driven by endogenous productivity growth. Specifically, full property rights induce a downward pressure on prices via scarcity effects (i.e., the resource stock tends to be preserved relatively to open access) as well as an upward pressure on prices via rent effects (i.e., resource harvesters with full ownership charge a higher price for given quantity).

The second set of results concerns the impact of resource property rights on market size, innovations and productivity growth. We show that, under full property rights, the market for manufacturing goods is always larger because strictly positive resource rents yield additional income that boost household spending. A larger market size, in turn, attracts entrants so that the economy converges to a steady state with a larger mass of firms. Productivity growth, however, is not faster because the process of entry in the manufacturing sector sterilizes the scale effect: in the long run, firm size and growth rates are the same in the two regimes.

The third set of results concerns the overall effect on consumption and welfare of a regime switch. A shift from full property rights to open access generates negative transitional effects – namely a productivity slowdown and a gradual increase of natural resource scarcity – but also instantaneous level effects having a potentially ambiguous impact on consumption levels: the permanent reduction in expenditure is mitigated by a reduction in the resource price, because open access implies zero net rents from harvesting and thereby lower unit cost for resource inputs. Therefore, switching to open access is welfare reducing only if the utility gain generated by the initial drop in the resource price is more than offset by the static and dynamic losses induced by lower expenditures and transitional growth.

The plan of the paper is as follows. Section 2 describes the model setup. Section 3 derives the general equilibrium relationships that characterize the economy under each regime. Section 4 compares the two regimes in terms of equilibrium outcomes and studies the welfare impact of a regime switch. Section 5 concludes.

2. A model of renewable resources and endogenous growth

The supply side of the economy comprises a final sector producing the consumption good, a manufacturing sector producing differentiated intermediate inputs, and a resource sector that supplies harvest goods to final producers. In the manufacturing sector, incumbents invest in R&D that raise own productivity (i.e., vertical innovations) while outside

[1] The framework, pioneered by Peretto (1998) and Peretto and Connolly (2007), has been recently applied to study the role of resources in fixed supply – like, e.g., land – in a closed economy (Peretto, 2012) and in a two-country, world general equilibrium model with asymmetric trade (Peretto and Valente, 2011). In this paper we extend it to the case of renewable resources.

entrepreneurs develop new varieties of intermediate inputs and start new firms to serve the market (i.e., horizontal innovations). The resource sector may operate under two different regimes – open access or full property rights – that determine different time paths of resource rents and income levels as a result of households choices.

2.1. Final producers

A representative competitive firm produces final output, Y, by means of H units of a "harvest good" drawn from a stock of a renewable natural resource, L_Y units of labor and n differentiated manufacturing goods. The technology is

$$Y(t)=H(t)^{\alpha}L_Y(t)^{\beta}\int_0^{n(t)}X_i(t)^{\gamma}\,di,\qquad \alpha+\beta+\gamma=1, \tag{1}$$

where X_i is the quantity of manufacturing good i and $t\in[0,\infty)$ is the time index. The final producer demands inputs according to the usual conditions equating value marginal productivities to remuneration rates. The demand schedules for labor and resource read

$$L_Y(t)=\beta\frac{P_Y(t)Y(t)}{W(t)}, \tag{2}$$

$$H(t)=\alpha\frac{P_Y(t)Y(t)}{P_H(t)}, \tag{3}$$

where P_Y is the price of final output, W is the wage rate, and P_H is the resource price. The condition for X_i yields

$$X_i(t)=\left[\frac{\gamma P_Y(t)H(t)^{\alpha}L_Y(t)^{\beta}}{P_{Xi}(t)}\right]^{1/(1-\gamma)},\qquad i\in[0,n(t)], \tag{4}$$

where P_{Xi} is the price of good i.

2.2. Manufacturing sector: incumbents

The manufacturing sector consists of single-product firms that supply differentiated goods under monopolistic competition. The typical firm produces with the technology

$$X_i(t)=Z_i(t)^{\theta}\cdot\left(L_{Xi}(t)-\phi\right),\qquad 0<\theta<1,\qquad \phi>0 \tag{5}$$

where Z_i is the firm-specific knowledge, θ is the associated elasticity parameter, L_{Xi} is labor employed in manufacturing production and ϕ is a fixed labor cost. In technology (5), firm's productivity may increase over time by virtue of in-house R&D. Specifically, the firm's knowledge grows according to

$$\dot{Z}_i(t)=\kappa\cdot K(t)L_{Zi}(t),\quad \kappa>0 \tag{6}$$

where κ is an exogenous parameter, L_{Zi} is labor employed in vertical R&D, and K is the stock of public knowledge available to all manufacturing firms. Public knowledge is the average knowledge in the manufacturing industry

$$K(t)=\frac{1}{n(t)}\int_0^{n(t)}Z_j(t)\,dj, \tag{7}$$

which is taken as given at the firm level.[2] The firm maximizes

$$V_i(t)=\int_t^{\infty}\Pi_{Xi}(s)e^{-\int_t^s(r(v)+\delta)dv}\,ds,\quad \delta>0 \tag{8}$$

subject to (6) and (7) and the demand schedule (4), where $\Pi_{Xi}=P_{Xi}X_i-WL_{Xi}-WL_{Zi}$ is the instantaneous profit, r is the interest rate and δ is the exogenous death rate. The solution to this problem, derived in the Appendix, yields a symmetric equilibrium where each firm produces the same output level and captures the same fraction $1/n$ of the market:

$$P_{Xi}(t)X_i(t)=\frac{1}{n(t)}\cdot\gamma P_Y(t)Y(t), \tag{9}$$

where $\gamma P_Y Y$ is the final producer's expenditure on manufacturing goods.

2.3. Manufacturing sector: entrants

Entrepreneurs develop new products and set up new firms to serve the market. This process of horizontal innovations increases the mass of firms over time and the growth rate of n depends on how much labor is employed in start-up operations. For each entrant, denoted i without loss of generality, the labor requirement translates into a sunk cost that is

[2] Peretto and Smulders (2002) provide microeconomic foundations for the knowledge aggregator (7).

proportional to the value of the production good: denoting by L_{N_i} the units of labor employed in start-up activity, the entry cost equals

$$W(t)L_{Ni}(t) = \psi P_{Xi}(t)X_i(t), \quad \psi > 0 \tag{10}$$

where P_iX_i is the value of production of the new good when it enters the market and ψ is a parameter representing technological opportunity. This assumption captures the notion that entry requires more the effort the larger the anticipated volume of production.[3] A free-entry equilibrium requires that the value of the new firm equals the entry cost, that is,

$$V_i(t) = \psi P_{Xi}(t)X_i(t). \tag{11}$$

In symmetric equilibrium the mass of firms grows according to

$$\frac{\dot{n}(t)}{n(t)} = \frac{1}{\psi\gamma} \cdot \frac{W(t)L_N(t)}{P_Y(t)Y(t)} - \delta, \tag{12}$$

where L_N is the total employment in entry (see the Appendix).

2.4. Resource dynamics

The resource stock, S, obeys the regeneration equation

$$\dot{S}(t) = G(S(t)) - H(t), \tag{13}$$

where $G(\cdot)$ is natural regeneration and H is harvesting. Following the benchmark model of renewable resources pioneered by Schaefer (1954), we assume that the regeneration function takes the logistic form

$$G(S(t)) = \eta S(t) \cdot \left(1 - \frac{S(t)}{\overline{S}}\right), \quad \eta > 0, \quad \overline{S} > 0 \tag{14}$$

where η is the intrinsic regeneration (or growth) rate and $\overline{S}$ is the carrying capacity of the habitat, i.e., the maximum level of the resource stock that the natural environment sustains when there is no harvesting.

The harvesting technology is

$$H(t) = BL_H(t) \cdot S(t), \quad B > 0 \tag{15}$$

where B is a productivity parameter also known as the "catchability coefficient" and L_H is the amount of labor employed in harvesting.[4] Employment in harvesting is determined by the choices of the households, who behave like atomistic extractive firms and earn a flow of resource rents given by

$$\Pi_S(t) = P_H(t)H(t) - W(t)L_H(t), \tag{16}$$

where P_H is the price of the harvest good.

2.5. Household behavior and access rights

We consider a representative household endowed with L units of labor that it can either sell in the market for the wage W or use to produce the harvest good that it can then sell in the market for the price P_H. The household has preferences

$$U \equiv \int_0^\infty \log C(t) \cdot e^{-\rho t}\, dt, \quad \rho > 0 \tag{17}$$

where $C(t)$ is the consumption and ρ is the discount rate. Financial wealth, A, consists of ownership claims on firms that yield a rate of return r. The budget constraint reads

$$\dot{A}(t) = r(t)A(t) + W(t)L + \underbrace{[P_H(t)BS(t) - W(t)] \cdot L_H(t)}_{\Pi_S(t) = \text{resource rents}} - P_Y(t)C(t), \tag{18}$$

where the term in square brackets follows from (15) and (16). The household chooses the time paths of consumption, C, and employment in harvesting, L_H, to maximize (17) subject to (18). The choice over L_H depends on the regime of access rights.

[3] See Etro (2004) and, in particular, Peretto and Connolly (2007) for a more detailed discussion of the microfoundations of our assumption. They argue that the entry cost is proportional to the initial variable cost of production because a firm setting up operations incurs the cost of building prototypes of the new products. In symmetric equilibrium (see below) our formulation yields an entry cost equal to $\psi y/n$ (where $y \equiv P_Y Y$). Barro and Sala-i-Martin (2004, Chapter 6) use this assumption to eliminate the scale effect and argue that it is empirically appropriate: the available evidence suggests that rates of innovation are functions of R&D intensity, not of the absolute flow of resources devoted to R&D.

[4] This production function, originally due to Schaefer (1954), exhibits returns to scale of degree two in labor and the resource stock. Following the exhaustive discussion in Brown (2000), we use it for two reasons. First, it is used as is in much of the literature (for both empirical and theoretical reasons) and we follow the practice to make our results directly comparable to the state of the art. Second, modifying it to allow for constant returns does not change our results in any substantial way.

Under *open access*, the household has no control over the total resource stock, the constraint (13) does not appear in the household problem, and the Hamiltonian reads

$$\mathcal{L}^{oa} \equiv \log C(t) + \lambda_a(t)\dot{A}(t), \tag{19}$$

where λ_a is the marginal shadow value of financial wealth. The first-order condition with respect to L_H then yields maximization of current resource rents.

Under *full property rights*, instead, the household has full control over the resource stock and maximizes the present value of the stream of benefits in a forward-looking manner: Eq. (13) is an explicit constraint in the optimization problem and S is an additional state variable. Consequently, the current-value Hamiltonian reads

$$\mathcal{L}^{pr} \equiv \log C(t) + \lambda_a(t)\dot{A}(t) + \lambda_s(t)\dot{S}(t), \tag{20}$$

where λ_s is the marginal shadow value of the resource stock.

3. General equilibrium

Open access and full property rights yield different harvesting plans, which, in equilibrium, induce different dynamics of consumption and innovation-led growth. Before analyzing in detail the two regimes, we describe the general equilibrium relations that hold in the economy independently of the type of access rights on natural resources. All the expressions discussed below are derived in the Appendix.

3.1. Main features of the equilibrium

The economy allocates labor across five activities: final production, manufacturing production, firm-specific knowledge accumulation (vertical R&D), entry (horizontal R&D), and resource harvesting. Because we have assigned the decision concerning employment in harvesting to the household, we have in fact modeled the household as determining labor supply as $L-L_H$. Consequently, the labor market clearing condition is

$$L - L_H(t) = L_Y(t) + L_X(t) + L_Z(t) + L_N(t), \tag{21}$$

where L_X and L_Z denote, respectively, total employment in production and vertical R&D. Labor mobility yields wage equalization across all activities. We take labor as the numeraire and set $W(t) \equiv 1$. We also denote expenditure on manufacturing goods by $y \equiv P_Y Y$.

The market for the final good clears when output equals consumption, $Y(t) = C(t)$. The household problem yields the Euler equation for consumption growth

$$\frac{\dot{P}_Y(t)}{P_Y(t)} + \frac{\dot{C}(t)}{C(t)} = \frac{\dot{y}(t)}{y(t)} = r(t) - \rho. \tag{22}$$

From (8), the return to financial assets is

$$r(t) = \frac{\Pi_X(t)}{V(t)} + \frac{\dot{V}(t)}{V(t)} - \delta. \tag{23}$$

The free-entry condition yields that financial wealth – the aggregate value of firms – is a constant fraction of the value of final output

$$A(t) = n(t)V(t) = \psi\gamma \cdot y(t). \tag{24}$$

Equilibrium of the financial market requires that all rates of return be equal.

As discussed in detail in Peretto (1998) and Peretto and Connolly (2007), models of this class have well-defined dynamics also when one of the two R&D activities shuts down because it is return-dominated by the other, or even when they both shut down because they fail to generate the household's reservation rate of return on saving. For simplicity, we focus our analysis on the case in which both types of innovation are active and discuss the role of corner solutions in the Appendix. The growth rate of firm-specific knowledge is

$$\frac{\dot{Z}(t)}{Z(t)} = \kappa\theta\gamma \cdot \left(\gamma\frac{y(t)}{n(t)}\right) - (r(t) + \delta). \tag{25}$$

Eq. (25) shows that larger firm size $\gamma y/n$ increases the typical firm's vertical R&D effort, boosting productivity growth. The gross growth rate of the mass of firms is

$$\frac{\dot{n}(t)}{n(t)} + \delta = \frac{1-\gamma-\psi\rho}{\psi} - \frac{n(t)}{\gamma y(t)} \cdot \frac{1}{\psi} \cdot \left(\phi + \frac{1}{\kappa} \cdot \frac{\dot{Z}(t)}{Z(t)}\right). \tag{26}$$

The last term in (26) highlights that firm-specific knowledge accumulation reduces the expansion rate of product variety because it raises the anticipated post-entry expenditure on vertical innovation.

Access rights influence the equilibrium of the economy via their effect on resource use and the income it generates. Eq. (24) implies that expenditure is a constant fraction of labor and resource income

$$y(t) = \frac{1}{1-\rho\psi\gamma} \cdot (L + \Pi_S(t)). \tag{27}$$

Different regimes of access rights over resources yield different dynamics of resource rents and, hence, of consumption expenditure. This mechanism has crucial implications for growth and welfare, as we show in the next section.

3.2. Equilibrium under open access

Under open access, the household chooses employment in harvesting in order to maximize *current* rents, while competition forces the price of the harvest good down to the marginal harvesting cost and thus resource rents to zero. From the Hamiltonian (19), we have

$$P_H^{oa}(t) = \frac{1}{BS^{oa}(t)} \Longrightarrow \Pi_S^{oa}(t) = 0. \tag{28}$$

From (27), this constant time profile of resource income yields an equilibrium with constant expenditure on the final good which, via the saving rule (22), yields that the interest rate equals the discount rate

$$y^{oa}(t) = y^{oa} \equiv \frac{L}{1-\psi\gamma\rho} \quad \text{and} \quad r^{oa}(t) = \rho. \tag{29}$$

This result has important implications for harvesting and innovation.

Proposition 1. (*Natural resource dynamics under Open Access*) *Harvesting is proportional to the existing resource stock*:

$$\frac{H^{oa}(t)}{S^{oa}(t)} = \alpha B y^{oa} = \frac{\alpha BL}{1-\psi\gamma\rho}. \tag{30}$$

The regeneration equation (13) becomes

$$\dot{S}^{oa}(t) = \left(\eta - \frac{\alpha BL}{1-\psi\gamma\rho}\right) \cdot S^{oa} - \frac{\eta}{\overline{S}} \cdot \left(S^{oa}\right)^2, \tag{31}$$

and yields

$$\lim_{t\to\infty} S^{oa}(t) = S_{ss}^{oa} \equiv \begin{cases} \frac{\overline{S}}{\eta} \cdot \left(\eta - \frac{\alpha BL}{1-\psi\gamma\rho}\right) & \text{if } \eta > \overline{\eta}^{oa} \equiv \frac{\alpha BL}{1-\psi\gamma\rho}. \\ 0 & \text{if } \eta \le \overline{\eta}^{oa} \end{cases} \tag{32}$$

There exists a condition on the parameters determining whether the economy experiences natural resource exhaustion or it reaches a steady state with a positive stock. The condition for long-run preservation, $\eta > \overline{\eta}^{oa}$, says that the intrinsic regeneration rate η must be sufficiently high to compensate for the adverse effects of consumers' impatience (a high ρ boosts current consumption and thereby harvesting), resource dependency in production (a high α yields a large demand for the harvesting good), efficiency in harvesting (a high B raises the incentive to hire workers in harvesting), and the size of the population (a high L also raises the incentive to hire workers in harvesting). For $\eta \le \overline{\eta}^{oa}$ the open-access economy experiences the *Tragedy of the Commons*: the rent-maximizing harvesting rule is unsustainable and the resource stock eventually vanishes.

The equilibrium paths of the innovation rates are as follows.

Proposition 2. (*Innovation dynamics under Open Access*) *The rate of accumulation of firm-specific knowledge is*

$$\frac{\dot{Z}^{oa}(t)}{Z^{oa}(t)} = \kappa\theta\gamma \cdot \frac{\gamma y^{oa}}{n^{oa}(t)} - (\rho + \delta). \tag{33}$$

The mass of firms follows a logistic process with constant coefficients:

$$\frac{\dot{n}^{oa}(t)}{n^{oa}(t)} = \nu \cdot \left[1 - \frac{n^{oa}(t)}{\tilde{n}^{oa}}\right], \qquad \nu \equiv \frac{1-\gamma-\theta\gamma-\psi(\rho+\delta)}{\psi} \tag{34}$$

where ν is the intrinsic growth rate, and

$$\tilde{n}^{oa} \equiv \gamma y^{oa} \cdot \frac{1-\gamma-\theta\gamma-\psi(\rho+\delta)}{\phi - (\rho+\delta)\cdot\kappa^{-1}}. \tag{35}$$

is the carrying capacity. In the long run, the mass of firms converges to the carrying-capacity level

$$n_{ss}^{oa} \equiv \lim_{t \to \infty} n^{oa}(t) = \tilde{n}^{oa}.$$

One implication of Proposition 2 is that the engine of growth in the long run is firm-specific knowledge accumulation. Entry generates transitional dynamics in the mass of firms but, due to the fixed operating cost ϕ, it is not self-sustaining. Consequently, in steady state the rate of entry exactly compensates for the death rate of firms δ. The long-run mass of firms n_{ss}^{oa}, on the other hand, determines long-run firm size, $\gamma y^{oa}/n_{ss}^{oa}$, and thus long-run growth. The interpretation is that, at any point in time, the equilibrium of factors market and the consumption/saving decision of the household determine the size of the market for manufacturing goods, γy^{oa}. This, in turn, determines the carrying capacity in the logistic equation characterizing the equilibrium proliferation of firms. Peretto and Connolly (2007) discuss in detail the intuition for why these logistic dynamics arise in a broad class of models and how they relate to the literature. The reason why these models exhibit logistic, instead of exponential, growth of the mass of firms is that they (re)introduce the fixed operating costs of the static theory of product variety that first-generation endogenous growth models set to zero.[5] In this paper's specific application of the Schumpeterian framework, the *finite* amounts of labor and of the natural resource are the force that limits the proliferation of a " specie" – firms/products – through the crowding effect implied by the fixed operating cost.

3.3. Equilibrium under full property rights

Under full property rights, resource owners do not maximize current rents at each point in time but rather the present-discounted value of all rents in a forward-looking fashion. As a consequence, harvesting satisfies the Hotelling rule: the marginal net rent must grow over time at the rate of interest net of the marginal benefits from resource regeneration.[6] We show this result in the Appendix. Here, we focus on the components of the household harvesting plan that identify the key channels through which such plan affects macroeconomic outcomes.

First, resource rents are strictly positive because the household chooses the extraction path so to equalize the profits from harvesting to the marginal shadow value of the resource stock

$$\Pi_S^{pr}(t) = y^{pr}(t) \cdot \lambda_s^{pr}(t) H^{pr}(t). \tag{36}$$

Second, the resource price is

$$P_H^{pr}(t) = \frac{1}{BS^{pr}(t)} + \underbrace{y^{pr}(t) \cdot \lambda_s^{pr}(t)}_{\text{scarcity rent}}. \tag{37}$$

Third, from (27) and (36) we obtain

$$y^{pr}(t) = \frac{L}{1 - \rho\psi\gamma - \lambda_s^{pr}(t) H^{pr}(t)}. \tag{38}$$

According to these expressions, resource rents are not a constant fraction of consumption expenditure because the incentives to harvest depend on the marginal shadow value of the resource stock. As a consequence, full property rights induce an equilibrium path where expenditure and the interest rate are time-varying. The reason is that, differently from the open access regime, the level of harvesting under full property rights is not proportional to the resource stock in each instant: since the harvesting choices of forward-looking resource owners take into account the effect of natural scarcity on future rents, the economy's rate of return continuously adjusts to the dynamics of resource rents generated at each point in time.

We can study the equilibrium path of the economy by constructing a two-by-two system that governs the joint dynamics of the *shadow value of the resource stock*, $m(t) \equiv \lambda_s^{pr}(t) S^{pr}(t)$, and the physical resource stock, $S^{pr}(t)$.

Proposition 3. (*Natural resource dynamics under Full Property Rights*) *Harvesting is a monotonously decreasing function of the shadow value of the resource stock*

$$\frac{H^{pr}}{S^{pr}} = \Lambda(m) \equiv \frac{2\alpha BL}{1 - \rho\psi\gamma + BLm + \sqrt{(1 - \rho\psi\gamma + BLm)^2 - 4\alpha BLm}}. \tag{39}$$

[5] Setting $\phi = 0$ in (34) yields that the mass of firms does not follow a logistic process anymore but grows forever, exactly like in expanding-varieties models *à la* Grossman and Helpman (1991).

[6] The *Hotelling rule* – named after Hotelling (1931) – asserts that an efficient harvesting plan requires that the growth rate of the marginal net rents from harvesting equal the interest rate minus the shadow value of all the positive feedback effects that a marginal increase in the resource stock induces on current rents and on future consumption benefits from resource use. If the resource is non-renewable and harvesting costs are independent of the resource stock, the feedback effects are zero and the Hotelling rule asserts that the growth rate of the marginal net rents from harvesting equal the interest rate.

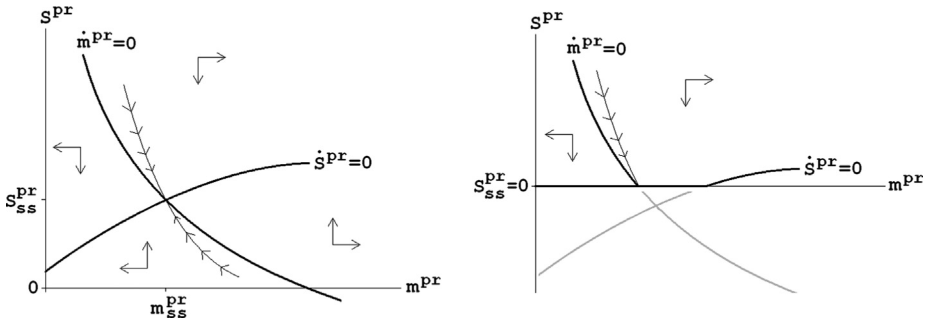

Fig. 1. Dynamics under full property rights according to Proposition 3. Left graph: the case $\eta > \overline{\eta}^{pr}$ implies positive resource stock in the long run. Right graph: the case $\eta < \overline{\eta}^{pr}$ leads to long-run resource exhaustion.

The associated dynamical system consists of the costate equation generated by the Hamiltonian (20) *and the regeneration equation* (13) *evaluated at the harvesting rule* (39)

$$\frac{\dot{m}(t)}{m(t)} = \rho + \frac{\eta}{\overline{S}} S^{pr}(t) - \frac{\alpha}{m(t)}; \tag{40}$$

$$\frac{\dot{S}^{pr}(t)}{S^{pr}(t)} = \eta - \frac{\eta}{\overline{S}} S^{pr}(t) - \Lambda(m(t)). \tag{41}$$

The system is saddle-path stable and converges to

$$\lim_{t\to\infty} m(t) = m_{ss} \equiv \begin{cases} \dfrac{\alpha}{\rho + \left(\eta/\overline{S}\right) S_{ss}^{pr}} & \text{if } \eta > \overline{\eta}^{pr} \equiv \Lambda(m_{ss}) \\ \dfrac{\alpha}{\rho} & \text{if } \eta \le \overline{\eta}^{pr} \end{cases},$$

$$\lim_{t\to\infty} S^{pr}(t) = S_{ss}^{pr} \equiv \begin{cases} \dfrac{\overline{S}}{\eta} \cdot (\eta - \Lambda(m_{ss})) & \text{if } \eta > \overline{\eta}^{pr} \equiv \Lambda(m_{ss}) \\ 0 & \text{if } \eta \le \overline{\eta}^{pr} \end{cases}. \tag{42}$$

Fig. 1 illustrates the dynamics in the two cases: $\eta > \overline{\eta}^{pr}$ yields positive resource stock in the steady state (left diagram) whereas $\eta < \overline{\eta}^{pr}$ leads to resource exhaustion (right diagram). The condition for long-run resource preservation is conceptually analogous to that obtained under open access: if the intrinsic regeneration rate is too low, resource exhaustion occurs. However, the intrinsic regeneration rate triggering exhaustion under full property rights, $\overline{\eta}^{pr} \equiv \Lambda(m_{ss})$, differs from that obtained under open access, $\overline{\eta}^{oa}$. We discuss this point in detail in Section 4.

The convergence results in (42) imply that consumption expenditure and the interest rate are constant in the long run: from (38) and (22), we obtain

$$\lim_{t\to\infty} y^{pr}(t) = y_{ss}^{pr} \equiv \frac{L}{1 - \psi\gamma\rho - m_{ss}\Lambda(m_{ss})} \quad \text{and} \quad \lim_{t\to\infty} r^{pr}(t) = \rho. \tag{43}$$

In light of these results, we can characterize the dynamics of innovation as follows.

Proposition 4. (*Innovation dynamics under Full Property Rights*) *The rate of accumulation of firm-specific knowledge is*

$$\frac{\dot{Z}^{pr}(t)}{Z^{pr}(t)} = \kappa\theta\gamma \cdot \frac{\gamma y^{pr}(t)}{n^{pr}(t)} - \left(r^{pr}(t) + \delta\right).$$

The mass of firms follows the logistic process with time-varying carrying capacity

$$\frac{\dot{n}^{pr}(t)}{n^{pr}(t)} = \nu \cdot \left[1 - \frac{n^{pr}(t)}{\tilde{n}^{pr}(t)}\right], \qquad \nu \equiv \frac{1 - \gamma - \theta\gamma - \psi(\rho + \delta)}{\psi}$$

where ν *is the intrinsic growth rate, and*

$$\tilde{n}^{pr}(t) \equiv \gamma y^{pr}(t) \cdot \frac{1 - \gamma - \theta\gamma - \psi(\rho + \delta)}{\phi - \left(r^{pr}(t) + \delta\right) \cdot \kappa^{-1}}$$

is the carrying capacity. In the long run, since $y^{pr}(t) \to y^{pr}_{ss}$ and $r^{pr}(t) \to \rho$, we have

$$n^{pr}_{ss} \equiv \lim_{t\to\infty} n^{pr}(t) = \lim_{t\to\infty} \tilde{n}^{pr}(t) = \gamma y^{pr}_{ss} \cdot \frac{1-\gamma-\theta\gamma-\psi(\rho+\delta)}{\phi-(\rho+\delta)\cdot\kappa^{-1}}. \tag{44}$$

Proposition 4 can be interpreted along similar lines as Proposition 2: the mass of firms follows a logistic process converging towards a stable carrying-capacity level n^{pr}_{ss}. As we noted above, the finite amounts of labor and of the natural resource limit the proliferation of a " specie" – firms/products – that would otherwise grow exponentially. Unlike the open-access regime, the carrying capacity of firms changes over time due to agents' internalization of the dynamics of the natural resource stock. More generally, human activity – i.e., harvesting – affects the evolution of the resource stock by modifying the habitat in which it grows. The evolution of the resource stock, in turn, affects the " economic habitat" in which firms grow in number and size. Productivity growth in the long run is driven by vertical innovation, whose incentives depend on firm size.

The crucial difference between the open access and the full property rights regimes is that in the former the economy lacks a price signal of scarcity capable of inducing an adaptive response of resource extractors to the changing habitat. This is why, under open access, resource exhaustion – which essentially represents a Tragedy of the Commons – occurs for a larger set of values of the natural regeneration rate under open access than under full property rights, as we show below.

3.4. Resource preservation in the long run

The two regimes imply different conditions for a strictly positive resource stock in the long run. We adopt the standard notion of strong sustainability, according to which a strictly positive resource base must be preserved forever, and interpret such conditions for long-run preservation as conditions for sustainable development. The reason is that in our model the renewable resource is a macro-level essential input that cannot be exhausted without collapsing the entire economy. Our main result is the following.

Proposition 5. (*Resource preservation in the long run*) *The condition for long-run resource preservation is more restrictive under open access*: $\overline{\eta}^{pr} < \overline{\eta}^{oa}$. *Specifically, there are three cases*

(i) *For $\overline{\eta}^{pr} < \overline{\eta}^{oa} < \eta$, both regimes yield resource preservation in the long run, with* $S^{pr}_{ss} > S^{oa}_{ss}$, $y^{pr}_{ss} > y^{oa}$, $n^{pr}_{ss} > n^{oa}_{ss}$.

(ii) *For $\overline{\eta}^{pr} < \eta < \overline{\eta}^{oa}$, the economy preserves the resource under full property rights ($S^{pr}_{ss} > 0$) but experiences the Tragedy of the Commons under open access ($S^{oa}_{ss} = 0$).*

(iii) *For $\eta < \overline{\eta}^{pr} < \overline{\eta}^{oa}$, both regimes yield resource exhaustion in the long run.*

The intuition behind the first statement in Proposition 5 follows immediately from the regeneration equation (13). To preserve the resource in the long run, the intrinsic regeneration rate η must be able to compensate for the depletion due to harvesting. Under open access, cumulative harvesting is more intense because agents do not consider the effects of current exploitation on future scarcity. Also, full property rights generate a higher level of expenditure that induces more intense entry during the transition and, consequently, more firms in the intermediate sector in the long run.[7] It is worth emphasizing that in both cases the threshold for sustainability is increasing in population size. Our general equilibrium model thus captures a crucial channel through which population size puts pressure on the natural environment. For reasons of space we leave the analysis of the implications of this channel to future work. We stress, however, that its presence places strong demands on studies that aim to allow for population growth since, strictly speaking, it rules out simply adding *exogenous* population growth but, rather, requires the explicit modeling of the feedback from the resource stock to population growth.

Fig. 2 illustrates the dynamics of the resource stock $S(t)$ and of its shadow value $m(t)$ in the three scenarios listed in Proposition 5. The grey trajectories along the vertical axis – i.e., a zero shadow value in each instant – represent open access whereas the black trajectories are associated with full property rights. When (i) both regimes exhibit preservation, more intense harvesting under open access yields a lower resource stock in the long run. Alternatively, we may observe (ii) the Tragedy of the Commons under open access, or (iii) asymptotic exhaustion in both regimes. Fig. 2 clarifies that *open access is*

[7] An anonymous referee pointed out that not everything is new in expressions for the sustainability thresholds. The roles of the discount rate and harvesting technology, which also appear in our paper, are well known from the literature, see e.g. the contributions of Clark (1973), Clark and Munro (1975; 1978). What is novel is that we now also have to check the production elasticities of the resource (α) and manufacturing (γ) for final output, as well as the entry parameter (ψ). As the referee points out, these new elements enter the sustainability threshold because of the model's General Equilibrium structure.

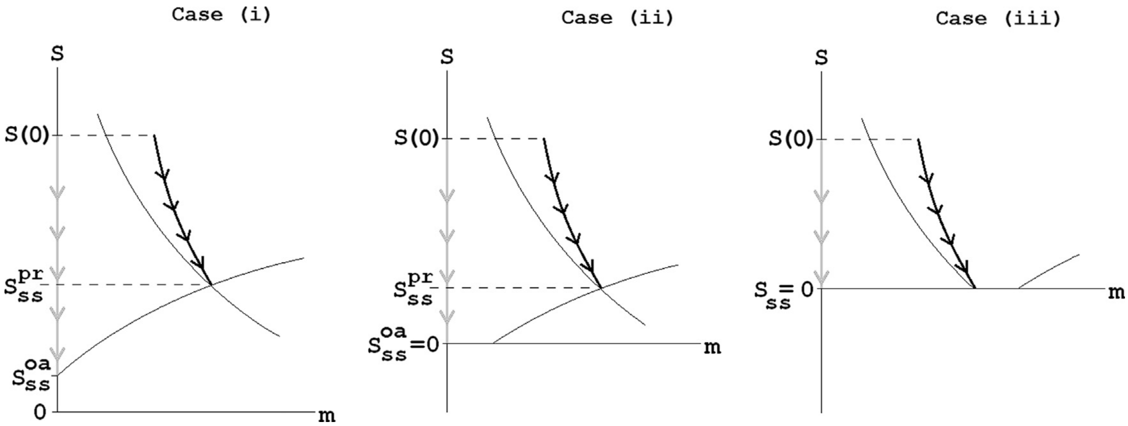

Fig. 2. Regime comparison according to Proposition 5. The black bold trajectory represents full property rights. The grey bold trajectory along the vertical axis represents open access.

a special case of full property rights that obtains for $m(t)=0$ because agents do not internalize the regeneration equation (13) in their intertemporal choices.

3.5. Equilibrium growth rates

To discuss economic growth, we concentrate on the case where long-run preservation obtains in both regimes.[8] In equilibrium, the logarithm of consumption in each instant equals

$$\log C(t)=\log \overline{a}+\underbrace{\log y(t)}_{\text{market size}}-\underbrace{\log P_H(t)^{\alpha}}_{\text{input cost}}+\underbrace{\log\left[(n(t))^{1-\gamma}(Z(t))^{\theta\gamma}\right]}_{\text{TFP}},\tag{45}$$

where we have defined the constant $\overline{a}\equiv\alpha^{\alpha}\beta^{\beta}\gamma^{2\gamma}$. This expression shows that consumption is higher the higher is the value of final output (market-size effect), the lower is the resource price (input-cost effect), and the higher is total factor productivity (TFP) determined by the mass of firms and by the firm-specific knowledge stock. Accordingly, the growth rate of consumption is

$$g(t)\equiv\frac{\dot{C}(t)}{C(t)}=r(t)-\rho-\alpha\frac{\dot{P}_H(t)}{P_H(t)}+(1-\gamma)\frac{\dot{n}(t)}{n(t)}+\gamma\theta\frac{\dot{Z}(t)}{Z(t)}.\tag{46}$$

According to this expression the economy's growth rate has three components. The first is the usual consumption rate of return net of discounting, $r(t)-\rho$. The second is the *resource price drag*, $\alpha\dot{P}_H/P_H$, which represents the negative effect of increased scarcity of the natural input as reflected by the dynamics of the resource price. The third component is TFP growth, a weighted average of the two innovation rates.

In the long run, the resource stock, the harvesting rate, the interest rate and the mass of manufacturing firms are all constant in both regimes. Consequently, the only source of consumption growth in the long run is firm-specific knowledge growth. An important implication of Propositions 2 and 4, then, is that the economy's steady-state growth rate is the same under open access and under full property rights.

Proposition 6. (*Steady-state growth*) *In the long run, firm size is the same in the two regimes, i.e.*

$$\lim_{t\to\infty}\frac{\gamma y^{oa}}{n^{oa}(t)}=\lim_{t\to\infty}\frac{\gamma y^{pr}(t)}{n^{pr}(t)}=\frac{\phi-(\rho+\delta)\cdot\kappa^{-1}}{1-\gamma-\theta\gamma-\psi(\rho+\delta)}\equiv\frac{\gamma y_{ss}}{n_{ss}},\tag{47}$$

implying the same long-run growth rate in the two regimes

$$\lim_{t\to\infty}g(t)=\lim_{t\to\infty}\gamma\theta\frac{\dot{Z}(t)}{Z(t)}=\gamma\theta\cdot\left[\kappa\gamma\theta\cdot\frac{\gamma y_{ss}}{n_{ss}}-(\rho+\delta)\right]>0.\tag{48}$$

Growth rates coincide in the two regimes because, in our framework, long-run growth does not exhibit scale effects – that is, the size of factor endowments does not influence the pace of development in the long run. Importantly, this

[8] There exist corner solutions where vertical and horizontal innovation shut down as the economy becomes smaller due to resource exhaustion. Thus, if long-run preservation fails to hold, growth falls to zero with the level of economic activity. Discussing such dynamics is feasible but much more cumbersome than focusing on the more interesting case that we consider in the text.

invariance result equally applies to the labor endowment and to the *natural resource stock*, conferring further empirical plausibility to this framework.[9] Therefore, regimes characterized by different rates of resource exploitation yield different income levels but equal growth rates in the long run. The reason is that the interaction between horizontal and vertical innovations fragments the intermediate goods market into submarkets whose size does not depend on endowments: although the long-run levels of expenditures and of the mass of firms differ between the two regimes, the size of each firm converges to the same equilibrium level, determined by expression (47), which shows that the fragmentation process makes long-run firm size independent of total market size. Factor endowments, therefore, do not affect the incentives to undertake R&D in the long run because in equilibrium they work through the market size effect, which is sterilized by the entry process in this model. Armed with these results, we can investigate in detail the role of the regime of access rights.

4. Regime comparison

In this section we compare the two regimes in four respects. First, we show that different regimes of property rights induce contrasting effects on the equilibrium value of the resource price (Section 4.1). Second, we distinguish between instantaneous and transitional effects of property rights regimes on consumption (Section 4.2). Third, we characterize analytically the welfare impact of a regimes switch from full property rights to open access (Section 4.3). Fourth, we assess the model predictions quantitatively by performing three numerical simulations that compare the two regimes in three cases: (i) under identical initial conditions; (ii) following a switch from full property rights to open access; (iii) following a switch from open access to full property rights (Section 4.4).

4.1. Resource price: scarcity versus rent effects

The equilibrium value of the resource price is affected by property rights regimes in two ways. First, the resource price at a given instant reflects current *scarcity* – i.e., the current level of the resource stock – and different regimes entail different degrees of resource preservation. Second, under full property rights, the resource price is also affected by income dynamics through the *rent effect* – that is, forward-looking extractors with full ownership make positive profits by charging a higher price than under open access given the same resource stock (cf. Section 3.3). The interplay between scarcity effects and rent effects yields the following result.

Proposition 7. (*Resource price in the two regimes*). *For a given level of the resource stock $S^{oa}(t) = S^{pr}(t) = S(t)$, positive resource rents under full property rights imply a higher resource price than under open access*

$$P_H^{oa}(t) = \frac{1}{BS(t)} < P_H^{pr}(t) = \frac{1}{BS(t)} + \underbrace{y^{pr}(t)\lambda_s^{pr}(t)}_{\text{Rent effect}}.$$

In long-run equilibria with positive preservation, the resource stock is higher under full property rights, $S_{ss}^{pr} > S_{ss}^{oa} > 0$, but the rent effect implies an ambiguous price gap

$$\lim_{t\to\infty} P_H^{oa}(t) = \frac{1}{BS_{ss}^{oa}} \gtreqless \lim_{t\to\infty} P_H^{pr}(t) = \underbrace{\frac{1}{BS_{ss}^{pr}}}_{\text{Scarcity effect}} + \underbrace{\lim_{t\to\infty} y^{pr}(t)\lambda_s^{pr}(t)}_{\text{Rent effect}}.$$

In general, full property rights induce an upward pressure on prices via the rent effect as well as a downward pressure via scarcity effects. If we compare the two regimes at time zero, when the resource stock is given, the resource price is necessarily higher under full property rights because the rent effect is fully operative and is not mitigated by scarcity effects. As the two economies converge to their respective steady states, however, full property rights imply more intense resource preservation (cf. Fig. 2), and the resulting scarcity effect may, but does not necessarily, determine a lower price than under open access. The implications of this tension between scarcity and rent effects for consumption and welfare may be substantial, as we show below.

4.2. Consumption: expenditure-price trade-off and transitional effects

Different harvesting regimes determine different paths of resource price, income and consumption. This mechanism has two main components. The first is *the expenditure-price trade-off* captured by the first two terms in (45). High expenditure levels do not necessarily imply high consumption: if the resource price is also high, the positive impact of market size may be more than offset by the negative impact of input costs. This observation is immediately relevant to our regime comparison. On one hand, full property rights yield higher expenditures relative to open access: from (27), positive resource

[9] In models where production requires the use of natural resources, the absence of scale effects is a particularly desirable property because the available empirical evidence does not support the existence of positive relationships between growth rates and resource stocks in resource-rich countries; e.g., see Bretschger and Valente (2012).

rents imply $y^{pr}(t) > y^{oa}$ in each t. On the other hand, full property rights determine a higher resource price at time zero and, possibly, in the long run (cf. Proposition 7). The expenditure-price trade-off thus suggests that full property rights do not necessarily enhance consumption at each point in time. In particular, open access can yield higher consumption in the short run.

The second source of consumption gaps between the two regimes is given by *differences in transitional growth rates*. Expression (46) captures the relevant components. On one hand, equilibrium interest rates differ during the transition because full property rights yield positive and time-varying profits from harvesting (cf. Section 3.3). On the other hand, productivity growth rates differ between regimes during the transition: entry in manufacturing proceeds at different speeds because, starting from a given initial condition $n(0)$, the mass of firms must reach different long-run levels, n_{ss}^{pr} or n_{ss}^{oa}, in the two regimes.

All these mechanisms jointly determine the overall impact of property rights regimes on consumption and thereby on present-value welfare. In particular, the expenditure-price trade-off suggests that open access is not necessary welfare-reducing. Although open access is by definition a regime that fails to maximize present-value resource rents, it is not possible to conclude that full property rights are always Pareto-superior because access rights interact with other market failures – namely monopolistic competition in manufacturing and non-decreasing returns to R&D – and the favorable impact of open access on resource prices can be substantial. The next subsection sheds further light on this issue by characterizing analytically the welfare effects of a regime shift.

4.3. Switching from property rights to open access: analytical results

Suppose that the economy is initially in the steady-state equilibrium of the full property rights regime with positive resource stock (i.e., $\overline{\eta}^{pr} < \eta$). At time $t=0$, the economy suddenly shifts to open access – as a result of, e.g., failure in enforcing property rights. The overall impact of the regime switch on welfare depends on the combination of the instantaneous and transitional effects discussed below.

Instantaneous level effects: At time zero, the regime switch induces two instantaneous adjustments: expenditure jumps down, from y_{ss}^{pr} to y_{ss}^{oa}, and the resource price jumps down, from $(P_H^{pr})_{ss} = (1+By_{ss}^{pr}m_{ss}^{pr})/BS_{ss}^{pr}$ to $P_H^{oa} = 1/BS_{ss}^{pr}$. From expression (45), the ratio between consumption levels (immediately) before and (immediately) after the switch is

$$\frac{C^{pr}(0^-)}{C^{oa}(0^+)} = \frac{y_{ss}^{pr}}{y_{ss}^{oa}} \cdot \left(\frac{P_H^{oa}(0^+)}{P_H^{pr}(0^-)}\right)^{\alpha} = \frac{y_{ss}^{pr}}{y_{ss}^{oa}} \cdot \left(\frac{1}{1+By_{ss}^{pr}m_{ss}^{pr}}\right)^{\alpha}$$

This ratio may be above or below unity in view of the expenditure-price trade-off. Hence, the overall level effect is generally ambiguous: at the time of the regime switch, we may observe either an instantaneous drop or an instantaneous increase in consumption.

Transitional growth effects: After time 0, there are two types of transitional effects respectively induced by productivity growth and resource scarcity. First, there is a transitional slowdown in productivity growth via both horizontal and vertical innovations: the switch to open access reduces the mass of firms over time (n must move from the initial state n_{ss}^{pr} to the new steady state $n_{ss}^{oa} < n_{ss}^{pr}$) and also reduces the growth rate of firm-specific knowledge as a result of reduced expenditure. As a consequence, the transitional growth rate of TFP after the switch is smaller than the rate enjoyed before – in fact, it may even be negative because the mass of firms is shrinking. The second transitional effect results from increased scarcity: the resource stock moves from the initial state S_{ss}^{pr} to $S_{ss}^{oa} < S_{ss}^{pr}$, and this decline increases the resource price after the initial instantaneous drop.

Overall effect on welfare: After the switch, the consumption path generated by open access may be above or below the baseline path – i.e., the path, characterized by permanent full property rights, that the economy would have followed without the regime switch. The reason is the ambiguous impact of the instantaneous level effects: while the transitional growth effects (i.e., productivity slowdown and increased scarcity) tend to reduce consumption after the regime switch, the initial drop in the resource price may be strong enough to raise consumption above the baseline level at time zero. Fig. 3 describes the possible outcomes according to three scenarios. If the initial jump in consumption is downward, the entire time profile of consumption for $t > 0$ is strictly below the baseline path – in which case, the switch to open access yields a welfare loss. If the initial consumption jump is upward, the impact on welfare is positive if consumption remains forever above the baseline path, and is generally ambiguous if consumption falls short of the baseline path at some finite time.

Since the model yields a closed-form solution for the equilibrium path after the regimes switch, we can assess the scope of possible ambiguities in welfare effects analytically:

Proposition 8. *The welfare change experienced by an economy that switches to the open access regime is*

$$\rho(U^{oa} - U_{ss}^{pr}) = \underbrace{\alpha B m_{ss}^{pr} y_{ss}^{pr}}_{\text{Initial price drop}} - \underbrace{\left(1 - \frac{y^{oa}}{y_{ss}^{pr}}\right)\left(1 + \frac{\varphi}{\rho+\nu}\right)}_{\text{Expenditure fall amplified by productivity slowdown}} - \underbrace{\frac{\alpha}{\rho+\omega}\left(1 - \frac{S_{ss}^{oa}}{S_{ss}^{pr}}\right)}_{\text{Increased scarcity}} \tag{49}$$

where we have defined $\varphi \equiv (\kappa/\nu)(\theta\gamma)^2 x_{ss} + (1-\gamma)$ *with* $x_{ss} \equiv \lim_{t\to\infty} \gamma y(t)/n(t)$, *and* $\omega \equiv \eta - BL_{Hss}^{oa}$ *with* $L_{Hss}^{oa} \equiv \lim_{t\to\infty} L_H^{oa}(t)$.

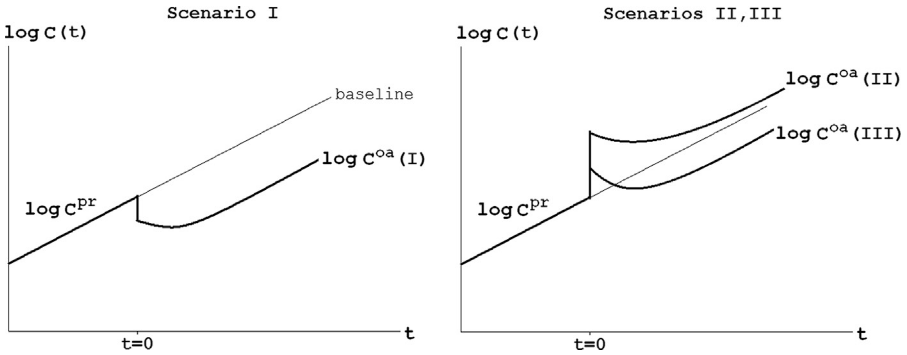

Fig. 3. Effects of a regime switch from full property rights to open access at time $t=0$. Scenario I: welfare loss (consumption is always below the baseline level). Scenario II: welfare gain (consumption is always above the baseline level). Scenario III: ambiguous welfare effect.

Proposition 8 formally establishes that the switch to open access yields a welfare loss unless the positive effect of the initial drop in the resource price is large enough to compensate for the negative effects induced by (i) the instantaneous fall in expenditure due to the destruction of the flow of resource rents; (ii) the transitional slowdown of TFP growth induced by reduced expenditure; (iii) the gradual increase in resource scarcity. This result suggests a more general conclusion that abstracts from the experiment of regime switching: full property rights improve welfare relative to open access if the utility cost induced by positive resource rents is more than offset by the static gains generated by higher expenditure and the dynamic gains induced by faster (transitional) productivity growth.

It should be clear that our results concerning the welfare effects of property-right regimes depend crucially on the endogenous nature of both the resource price and the productivity growth rate. This property differentiates our analysis from the traditional resource economics literature, which typically employs partial equilibrium models.

4.4. Numerical simulations: parallel paths and regime switching

This subsection presents a quantitative assessment of our model with particular focus on three questions. First, what is the dominant source of welfare differentials between the two regimes if we assume identical initial conditions? Second, assuming a failure of property rights enforcement, is the switch to open access welfare-reducing under plausible parameter values? Third, what are the dynamics of endogenous variables assuming an opposite regime switch, from open access to full property rights?

The first question highlights the main difference between our theory and partial equilibrium models. Since both resource harvesting and aggregate productivity are endogenous in our analysis, different regimes of property rights yield different welfare levels via the three channels emphasized in expression (45): the size of the market, the cost of resource inputs and TFP. We compute for each regime the equilibrium path from $t=0$ to $t\to\infty$ starting from the same initial conditions $(S(0), n(0), Z(0))$ and assuming identical parameters. These *parallel paths* show differences in economics outcomes arising only from differences in property rights regimes. Quantitatively, the key channel of such differences is TFP growth rather than to the resource price drag (cf. expression (46) above).

To tackle the second and third questions we depart from the method of "parallel paths" and run two further simulations, respectively labelled "switch to open access" and " switch to full property rights". In both cases, the switch to the new regime occurs starting from the long-run equilibrium of the old regime. The simulation "switch to open access" complements our theoretical results in Section 4.3 and shows the extent to which the collapse of full property rights implies a welfare loss. The simulation "switch to full property rights" clarifies what are the dynamics of endogenous variables in the opposite case where the economy abandons open access.

All simulations use the following set of parameter values. The input elasticities in production of the natural resource and of manufacturing goods are, respectively, $\alpha=0.15$ and $\gamma=0.35$, reflecting conventional empirical estimates for resource-rich countries. Total labor is $L=1$ and the utility discount rate is $\rho=4\%$. In the resource sector we set $\eta=3\%$, $B=0.15$ and $\overline{S}=50$. The technological parameters determining innovation rates are set so as to generate a long-run growth rate of consumption of 2% per year: $\delta=0.1$, $\kappa=0.32$, $\theta=0.5$, $\psi=2.5$, $\phi=1$. The initial conditions vary according to the exercise, as described below.

4.4.1. Simulation 1 – parallel paths

This simulation calculates for each regime the equilibrium path from $t=0$ to $t\to\infty$ starting from the same initial condition: $S(0)=23.65$, $n(0)=0.0725$, $Z(0)=1$. Table 1 reports the associated steady-state values of the relevant endogenous variables, along with calculated present-value welfare.

Fig. 4 shows the equilibrium paths. The quantitative exercise illustrates nicely the forces highlighted in the qualitative analysis. Initially the open access regime features more harvesting and a lower resource price. However, over time the resource becomes more expensive than in the property rights regime. Given the common initial condition, the resource

Table 1
Long-run values in the simulation " Parallel Paths".

Regime	y_{ss}	S_{ss}	H_{ss}	$P_{H(\infty)}$	n_{ss}	g_{ss}	U
Full Property Rights	1.090	22.02	0.37	0.44	0.085	0.02	113.1
Open Access	1.036	11.14	0.26	0.60	0.081	0.02	107.6

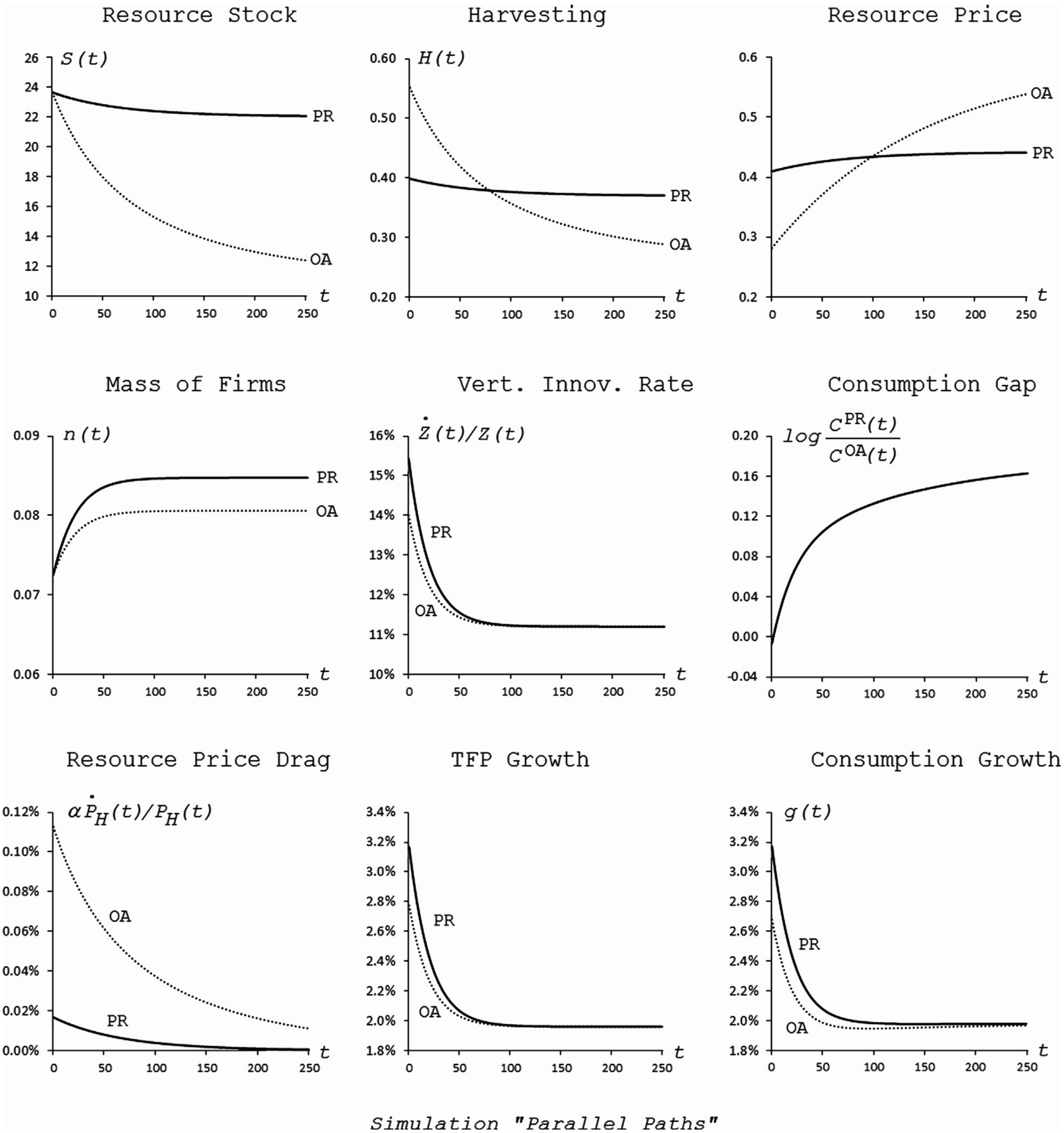

Fig. 4. Simulation results – Parallel Paths. The equilibrium paths of the relevant variables under full property rights and open access are depicted with bold lines and dotted lines, respectively.

stock is always higher under property rights. Consequently, the size of the market for manufacturing goods is larger and the economy grows faster. Such difference in TFP growth explains the bulk of the quantitative difference in the consumption paths because the difference in resource price drag is very small. It is worth re-iterating the key point of this exercise:

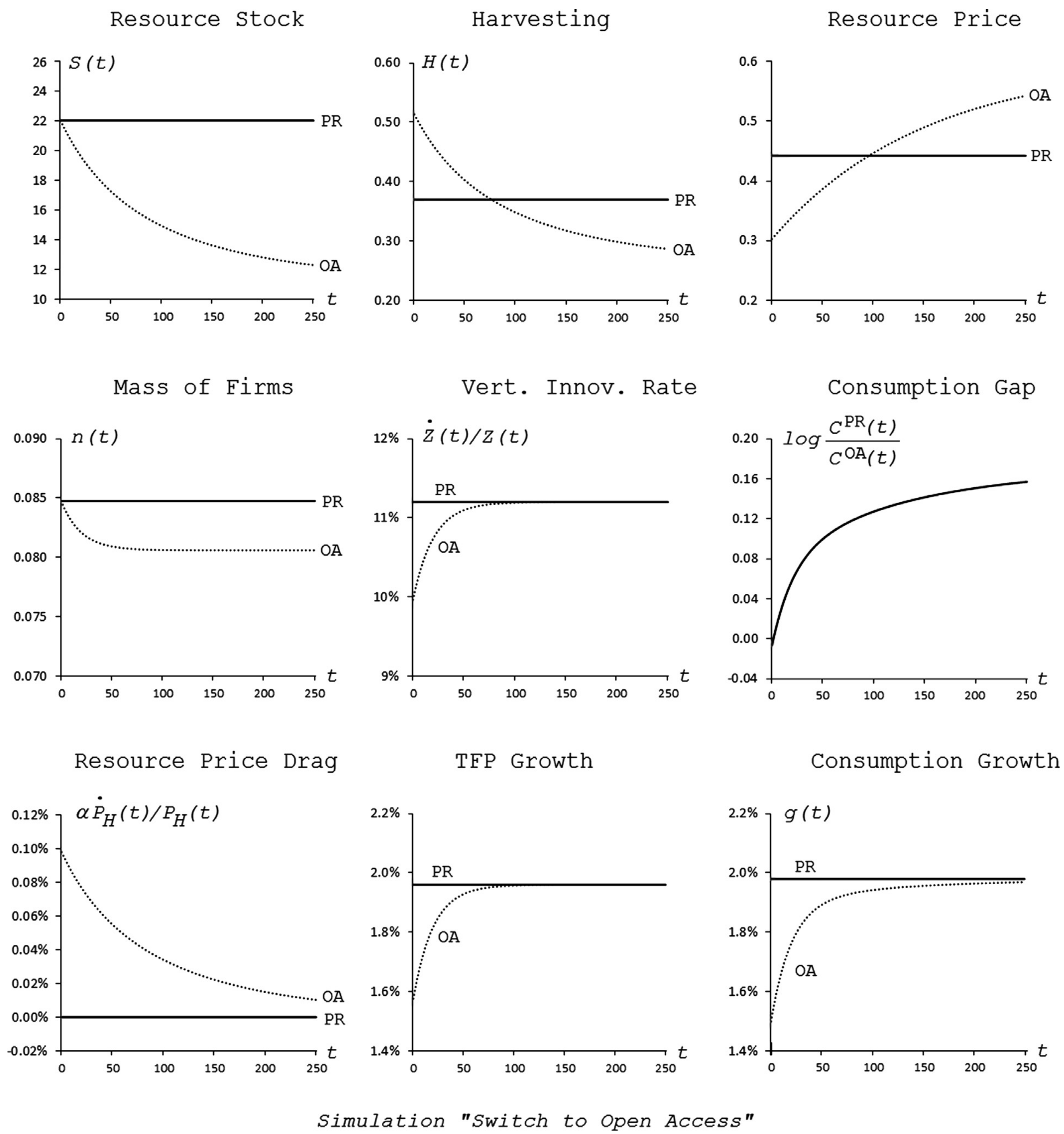

Fig. 5. Simulation results – Switch to Open Access. The equilibrium paths of the relevant variables under full property rights (baseline path) and open access (actual path) are depicted with bold lines and dotted lines, respectively.

because we hold everything but the property rights regime the same in constructing these two paths, the analysis quantifies differences due *solely* to access rights. The next two exercises, in contrast, quantify the effects of moving from one regime to the other and thus do not hold everything but the access rights constant since in each case the dynamics features a different initial condition.

4.4.2. Simulation 2 – switch to open access

In this simulation the economy is initially in the long-run equilibrium with full property rights and, at time $t=0$, experiences a permanent switch to the open access regime. From the values reported in Table 1, we thus have the initial conditions $S^{oa}(0)=22.02$ and $n^{oa}(0)=0.085$, keeping $Z^{oa}(0)=1$ without loss of generality. Fig. 5 depicts the equilibrium path

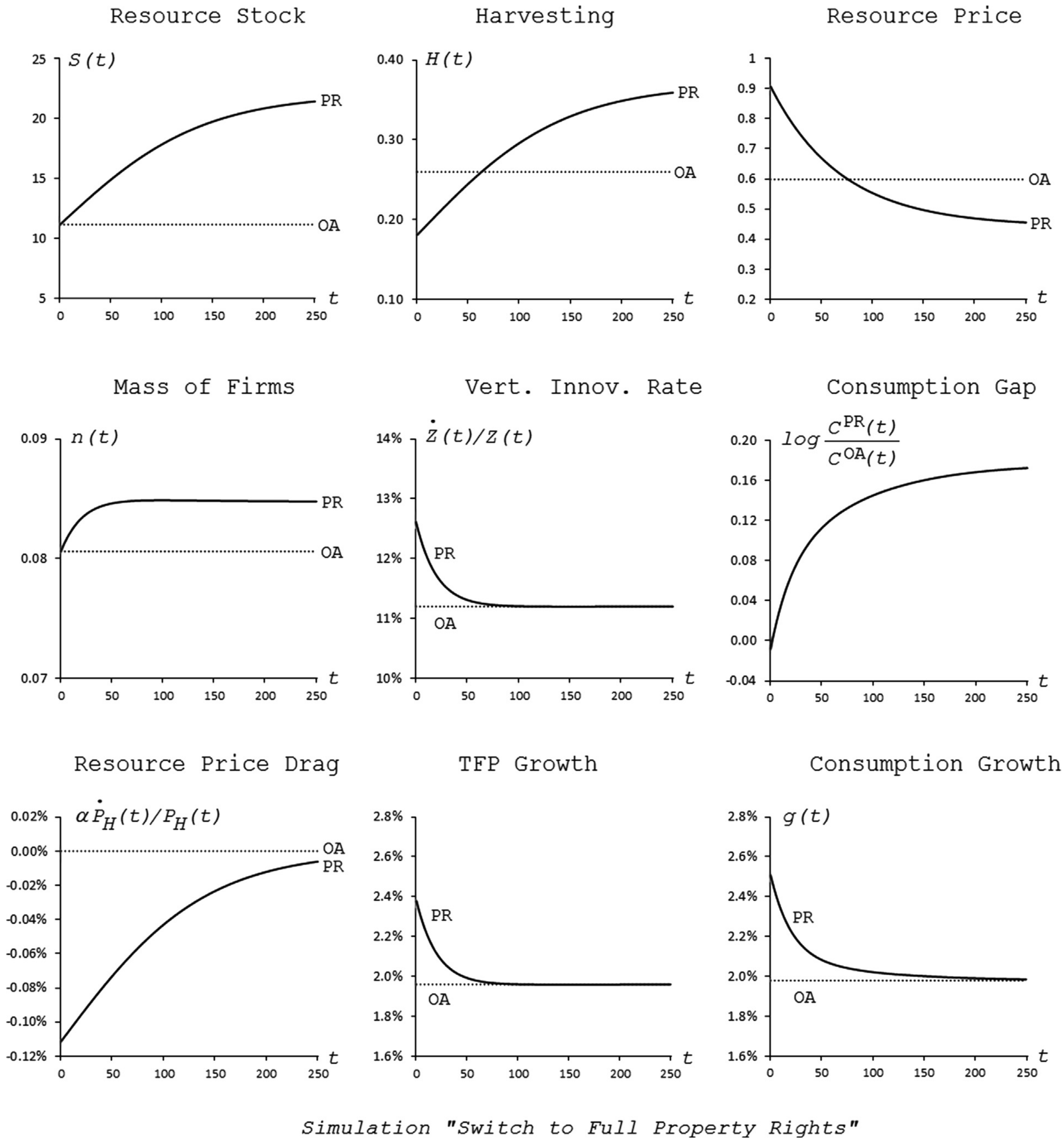

Fig. 6. Simulation results – Switch to Full Property Rights. The equilibrium paths of the relevant variables under full property rights (actual path) and open access (baseline path) are depicted with bold lines and dotted lines, respectively.

with open access, which is followed from $t=0$ onwards, and the baseline path with full property rights, i.e., the path that the economy would have followed if the regime switch had not occurred.

The consumption gap is negative between $t=0$ and $t=2$ because immediately after the switch there is more harvesting and more production. The effect is quantitatively small, however (it not very evident in Fig. 5 because of the scale but it exists). This result illustrates numerically what we have called Scenario III in our theoretical analysis of Section 4.3 (see Fig. 3), namely that upon switching to open access consumption jumps instantaneously up but, due to the associated productivity slowdown, it then goes below the baseline level. Recall that in this scenario the sign of the welfare change is generally ambiguous. In this numeric simulation we find that the switch to open access is welfare-reducing since

$$U^{pr}_{(\text{baseline})} = 104.7 > 99.7 = U^{oa}.$$

Several other simulations based on alternative parameter values (not shown here) suggest that switching to open access is welfare reducing as long as crucial parameters like α and γ are kept within ranges that are empirically plausible for modern industrialized economies, i.e., $\alpha < 0.2$ and $\gamma > 0.2$. However, for higher values of α and/or lower values of γ – which may be plausible for regions or (less industrialized) countries where production is heavily dependent on resources – it seems possible that the switch to open access becomes welfare improving.

4.4.3. Simulation 3 – switch to full property rights

The last simulation assumes that the economy is initially in the long-run equilibrium with open access. The switch at time $t=0$ to the regime with full property rights thus features the initial conditions $S^{pr}(0)=11.14$ and $n^{pr}(0)=0.081$, with $Z^{pr}(0)=1$. Fig. 6 depicts the equilibrium path followed from $t=0$ onwards, and the baseline path with open access as a reference benchmark. Although on impact harvesting falls, during the transition both harvesting and the resource stock are increasing, eventually converging to higher values. Because of these dynamics, the resource price initially rises and then gradually falls to a lower level. This evolution of the resource sector drives a gradual expansion of the market for manufacturing goods, which, in turn, drives a temporary acceleration of TFP and consumption. The temporary acceleration of consumption growth drives the rise in welfare.

5. Conclusion

This paper analyzed the impact of different regimes of access rights to renewable natural resources on sustainability conditions, innovation rates and welfare levels in a Schumpeterian model of endogenous growth. The crucial difference between open access and full property rights is that, in the former, the economy lacks a price signal of scarcity capable of inducing an adaptive response of resource extractors to the changing habitat. Consequently, the critical condition for long-run sustainability is always more restrictive under open access: the economy might experience the Tragedy of the Commons under open access and sustained economic growth under full property rights.

Full property rights yield positive rents from harvesting and therefore higher expenditure relative to open access: the bigger market size induces faster productivity growth during the transition via both horizontal and vertical innovations. However, positive rents also imply that the resource price is lower under open access given the same resource stock. Consequently, a failure in property-rights enforcement that induces a regime switch to open access generates negative transitional effects via slower productivity growth but also ambiguous level effects on consumption because reduced resource prices mitigate the impact of lower expenditures. The closed-form solution delivered by the model shows that switching to open access is welfare reducing if the utility gain generated by the initial drop in the resource price is more than offset by the static and dynamic losses induced by reduced expenditure.

The crucial role played by endogenous prices and endogenous productivity growth in our conclusions confirms that a proper understanding of the relationship between long-term sustainability and property-right regimes requires a full general equilibrium analysis. In particular, the vertical structure of production that characterizes our model implies that prospects for sustainability hinge on the link between price formation in upstream extraction/harvesting and the incentives to innovate faced by downstream industries: this topic deserves further research at both the theoretical and the empirical levels.

Acknowledgments

Disclaimer: The views expressed in this paper are those of the authors and do not necessarily represent those of the IMF or IMF policy.

Appendix A

Manufacturing sector (incumbents): maximization problem: Using the demand schedule (4) and the technology (5), the incumbent firm's profit equals

$$\Pi_{Xi}=\left[\frac{\gamma P_Y H^{\alpha} L_Y^{\beta}}{P_{Xi}}\right]^{1/(1-\gamma)}\left[P_{Xi}-WZ_i^{-\theta}\right]-WL_{Zi}-W\phi. \tag{A.1}$$

The firm maximizes (8) subject to (A.1) and (6)-(7), using P_{Xi} and L_{Zi} as control variables, firm-specific knowledge Z_i as the state variable, taking public knowledge K as given. The current-value Hamiltonian is

$$\mathcal{L}_i^x \equiv \Pi_{Xi}=\left[\frac{\gamma P_Y H^{\alpha} L_Y^{\beta}}{P_{Xi}}\right]^{1/(1-\gamma)}\left[P_{Xi}-WZ_i^{-\theta}\right]-WL_{Zi}-W\phi+\lambda_i^x\cdot\kappa K L_{Zi}, \tag{A.2}$$

where λ_i^x is the dynamic multiplier associated to (6). Since the Hamiltonian is linear in L_{Zi}, we have a bang-bang solution. The necessary conditions for maximization read

$$1=\frac{1}{1-\gamma}\left[\frac{P_{Xi}-WZ_i^{-\theta}}{P_{Xi}}\right], \tag{A.3}$$

$$\lambda_i^x\cdot\kappa K-W\le 0 \qquad (<0 \text{ if } L_{Zi}=0, =0 \text{ if } L_{Zi}>0), \tag{A.4}$$

$$(r+\delta)\cdot\lambda_i^x-\dot{\lambda}_i^x=\theta\cdot X_i W Z_i^{-\theta-1}. \tag{A.5}$$

Condition (A.3) follows from $\partial\mathcal{L}^x/\partial P_{Xi}=0$ and yields the standard mark-up rule

$$P_{Xi}=\frac{1}{\gamma}\cdot WZ_i^{-\theta}. \tag{A.6}$$

Condition (A.4) is the Kuhn–Tucker condition for R&D investment: in an interior solution, the marginal cost of employing labor in vertical R&D activity (W) equals the marginal benefit of accumulating knowledge ($\lambda_i^x\kappa K$). Condition (A.5) is the costate equation for knowledge: with strict equality in (A.4), substitution of both $\lambda_i^x=W/(\kappa K)$ and (A.6) in (A.5) yields

$$r+\delta=\gamma\theta\cdot\frac{X_iP_{Xi}}{W}\cdot\kappa\frac{K}{Z_i}+\frac{\dot{W}}{W}-\frac{\dot{K}}{K}. \tag{A.7}$$

Manufacturing sector (incumbents): symmetry: The symmetry of the equilibrium is established in detail in Peretto (1998: Proposition 1) and Peretto and Connolly (2007). Applying the same proof to the present model, the mark-up rule (A.6) is invariant across varieties and implies the same price P_{Xi}, the same quantity X_i, and the same employment in production L_{Xi} for each $i\in[0,n]$. Therefore, we can combine (A.3) and (A.6) to write each firm's market share as in expression (9) in the main text. Concerning the knowledge stock, from (7) and (6), the equilibrium growth rate under symmetry is

$$\dot{K}/K=\dot{Z}_i/Z_i=\dot{Z}/Z=\kappa\cdot L_{Zi}, \tag{A.8}$$

where we can substitute $L_Z=nL_{Zi}$ to obtain

$$\frac{\dot{Z}}{Z}=\kappa\cdot\frac{L_Z}{n}. \tag{A.9}$$

Manufacturing sector (entry): derivation of (12): Given a constant death rate of firms δ, the mass of entrants in each instant equals the gross variation in the mass of firms $\dot{n}+\delta n$. This implies that total labor employed in entry activities equals $L_N=L_{Ni}(\dot{n}+\delta n)$, and Eq. (10) may be written as

$$WL_N=(\dot{n}+\delta n)\psi P_{Xi}X_i. \tag{A.10}$$

Rearranging terms, we have

$$\frac{\dot{n}}{n}=\frac{WL_N}{\psi nP_{Xi}X_i}-\delta, \tag{A.11}$$

where we can substitute (9) to obtain (12).

General equilibrium: derivation of (22): In both regimes of access rights – see the Hamiltonians (19) and (20) – the household problem yields the necessary conditions

$$1/C=\lambda_a P_Y, \tag{A.12}$$

$$\dot{\lambda}_a=\lambda_a(\rho-r), \tag{A.13}$$

from which we obtain the standard Keynes–Ramsey rule (22).

General equilibrium: derivation of (23): Time-differentiating (8) yields (23).

General equilibrium: derivation of (24): Combining (9) with (11), we obtain (24).

General equilibrium: derivation of (25): Substituting (A.8) in (A.7) yields

$$\frac{\dot{Z}}{Z}=\frac{\dot{W}}{W}+\kappa\theta\gamma^2\cdot\frac{P_YY}{Wn}-(r+\delta). \tag{A.14}$$

Setting $W=1$ in (A.14) yields Eq. (25) in the text.

General equilibrium: derivation of (26): Time-differentiating the free entry condition (24), we obtain

$$\frac{\dot{V}_i}{V_i}=\frac{\dot{P}_Y}{P_Y}+\frac{\dot{Y}}{Y}-\frac{\dot{n}}{n}. \tag{A.15}$$

Substituting (A.15) in (23) to eliminate $\dot{V}_i/V_i$ yields

$$r+\delta+\frac{\dot{n}}{n}=\frac{\dot{P}_Y}{P_Y}+\frac{\dot{Y}}{Y}+\frac{\Pi_{Xi}}{V_i} \tag{A.16}$$

where, because $C=Y$, we can use the Keynes–Ramsey rule (22) to obtain

$$\frac{\dot{n}}{n}=\frac{\Pi_{Xi}}{V_i}-\rho-\delta. \tag{A.17}$$

Substituting (A.6) and (A.9) in the definition of profits Π_{Xi}, we have

$$\Pi_{Xi}=\gamma(1-\gamma)\cdot\frac{P_Y Y}{n}-W\phi-W\frac{1}{\kappa}\frac{\dot{Z}}{Z}. \tag{A.18}$$

Substituting (A.18) in (A.17), and using (24) to eliminate V_i, we have

$$\frac{\dot{n}}{n}=\frac{1-\gamma}{\psi}-W\frac{n}{\psi\gamma P_Y Y}\cdot\left[\phi+\frac{1}{\kappa}\frac{\dot{Z}}{Z}\right]-\rho-\delta,$$

which reduces to (26) for $W=1$.

General equilibrium: derivation of (27): Substituting $A=\psi\gamma\cdot P_Y Y$ from (24), as well as $Y=C$, in the wealth constraint (18), we obtain

$$\frac{\dot{P}_Y}{P_Y}+\frac{\dot{Y}}{Y}=r+\frac{L-y}{\psi\gamma\cdot y}+\frac{\Pi_S}{\psi\gamma\cdot y}. \tag{A.19}$$

The Keynes–Ramsey rule (22) then yields

$$y(1-\psi\gamma\rho)=L+\Pi_S, \tag{A.20}$$

which yields (27) in the text.

Equilibrium under open access: derivation of (28) and (29): Under open access, normalizing $W\equiv 1$ and recalling expression (18), the Hamiltonian (19) reads

$$\mathcal{L}^{oa}\equiv\log C^{oa}+\lambda_a^{oa}\cdot\left[r^{oa}A^{oa}+L-P_Y^{oa}C^{oa}+\left(P_H^{oa}BS^{oa}-1\right)\cdot L_H^{oa}\right], \tag{A.21}$$

where C^{oa} and L_H^{oa} are control variables and A^{pr} is the only state variable. The necessary conditions for maximization are

$$1/C^{oa}=\lambda_a^{oa}P_Y^{oa}; \tag{A.22}$$

$$P_H^{oa}BS^{oa}=1; \tag{A.23}$$

$$\dot{\lambda}_a^{oa}=\lambda_a^{oa}\cdot\left(\rho-r^{oa}\right). \tag{A.24}$$

From (A.23), we have $P_H^{oa}BS^{oa}=1\Longrightarrow\Pi_S^{oa}=0$, which is expression (28) in the text. Substituting (28) in (27), we have

$$P_Y^{oa}Y^{oa}=L(1-\psi\gamma\rho)^{-1}, \tag{A.25}$$

which, substituted into the Keynes–Ramsey rule (22), yields $r^{oa}=\rho$.

Equilibrium under open access: proof of Proposition 1: Combining (28) with (3), we obtain (30). Substituting (30) into (13), we obtain the differential equation (31) which converges to the unique steady state

$$\lim_{t\to\infty}S^{oa}(t)=\left(\overline{S}/\eta\right)\cdot\left(\eta-\frac{\alpha BL}{1-\psi\gamma\rho}\right).$$

Imposing the non-negativity restriction on physical quantities $S^{oa}\geq 0$ determines the critical threshold reported in (31).

Equilibrium under open access: proof of Proposition 2: As noted in the main text, Proposition 2 assumes that innovation activities are operative in each instant: that is, the economy uses positive amounts of labor in vertical R&D and entry activities ($L_Z>0$ and $L_X>0$) implying positive rates of public knowledge growth ($\dot{Z}(t)>0$) and of gross entry $((\dot{n}(t)/n(t))+\delta>0)$. A detailed analysis of the implied restrictions on parameters is reported at the end of this Appendix. Expression (33) is obtained by substituting $y=y^{oa}$ and $r^{oa}=\rho$ from (29) into Eq. (25). Substituting (33) into (26) yields (34), which is dynamically stable around the unique positive steady state $n_{ss}^{oa}\equiv\lim_{t\to\infty}n^{oa}(t)=\tilde{n}^{oa}$.

Equilibrium under Full Property Rights: the Hotelling rule: Under full property rights, normalizing $W\equiv 1$ and recalling expression (18), the Hamiltonian (20) reads

$$\mathcal{L}^{pr}\equiv\log C^{pr}+\lambda_a^{pr}\cdot\left[r^{pr}A^{pr}+L-P_Y^{pr}C^{pr}+\Pi_S^{pr}\right]+\lambda_s^{pr}\cdot\dot{S}^{pr},$$

that is

$$\mathcal{L}^{pr}\equiv\log C^{pr}+\lambda_a^{pr}\cdot\left[r^{pr}A^{pr}+L-P_Y^{pr}C^{pr}+\Pi_S^{pr}\left(S^{pr},L_H^{pr}\right)\right]$$
$$+\lambda_s^{pr}\cdot\left[G\left(S^{pr}\right)-H^{pr}\left(S^{pr},L_H^{pr}\right)\right], \tag{A.26}$$

where C^{pr} and L_H^{pr} are the control variables, A^{pr} and S^{pr} are the state variables, and the functions

$$\Pi_S\left(S^{pr},L_H^{pr}\right)\equiv\left(P_H^{pr}BS^{pr}-1\right)\cdot L_H^{pr}, \tag{A.27}$$

$$G(S^{pr}) \equiv \eta S^{pr} \cdot \left[1-\left(S^{pr}/\bar{S}\right)\right], \tag{A.28}$$

$$H(S^{pr}, L_H^{pr}) \equiv BL_H^{pr} S^{pr}, \tag{A.29}$$

directly follow from definitions (16), (14) and (15). The necessary conditions for maximization are

$$1/C^{pr} = \lambda_a^{pr} P_Y^{pr}; \tag{A.30}$$

$$\frac{\partial \Pi_S(S^{pr}, L_H^{pr})}{\partial L_H^{pr}} = \frac{\lambda_s^{pr}}{\lambda_a^{pr}} \cdot \frac{\partial H(S^{pr}, L_H^{pr})}{\partial L_H^{pr}}; \tag{A.31}$$

$$\frac{\dot{\lambda}_a^{pr}}{\lambda_a^{pr}} = \rho - r^{pr}; \tag{A.32}$$

$$\frac{\dot{\lambda}_s^{pr}}{\lambda_s^{pr}} = \rho - \frac{\partial G(S^{pr})}{\partial S^{pr}} + \frac{\partial H(S^{pr}, L_H^{pr})}{\partial S^{pr}} - \frac{\lambda_a^{pr}}{\lambda_s^{pr}} \frac{\partial \Pi_S(S^{pr}, L_H^{pr})}{\partial S^{pr}}; \tag{A.33}$$

along with the transversality conditions

$$\lim_{t\to\infty} \lambda_a^{pr}(t) A^{pr}(t) e^{-\rho t} = 0, \tag{A.34}$$

$$\lim_{t\to\infty} \lambda_s^{pr}(t) S^{pr}(t) e^{-\rho t} = 0. \tag{A.35}$$

Henceforth, we denote the *marginal net rent* from employing an additional unit of labor in harvesting as

$$\Pi'_S \equiv \frac{\partial \Pi_S(S^{pr}, L_H^{pr})}{\partial L_H^{pr}} = (P_H^{pr} BS^{pr} - 1) = \frac{\Pi_S(S^{pr}, L_H^{pr})}{L_H^{pr}}. \tag{A.36}$$

Time-differentiating (A.31), we obtain

$$\frac{\dot{\lambda}_s^{pr}}{\lambda_s^{pr}} - \frac{\dot{\lambda}_a^{pr}}{\lambda_a^{pr}} = \frac{\dot{\Pi}'_S}{\Pi'_S} - \frac{\dot{S}^{pr}}{S^{pr}},$$

where we can substitute (A.32) and (A.33) to obtain

$$\frac{\dot{\Pi}'_S}{\Pi'_S} = r^{pr} - \left\{ \frac{\lambda_a^{pr}}{\lambda_s^{pr}} \cdot \frac{\partial \Pi_S(S^{pr}, L_H^{pr})}{\partial S^{pr}} + \left[\frac{\partial G(S^{pr})}{\partial S^{pr}} - \frac{\partial H(S^{pr}, L_H^{pr})}{\partial S^{pr}} \right] - \frac{\dot{S}^{pr}}{S^{pr}} \right\}. \tag{A.37}$$

Eq. (A.37) is a generalized *Hotelling rule*: an efficient harvesting plan requires that the growth rate of the marginal net rents from resource harvesting equal the interest rate minus the term in curly brackets – which represents the shadow value of all the positive feedback effects that a marginal increase in the resource stock induces on current rents and on future consumption benefits from resource use. If the resource were non-renewable ($\eta = 0$ implies that $\partial G/\partial S = 0$) and harvesting costs were independent of the resource stock ($\partial H/\partial S = 0$), the term in curly brackets would be zero: in that case, Eq. (A.37) would collapse to the basic Hotelling's (1931) rule $\dot{\Pi}'_S/\Pi'_S = \dot{P}_H/P_H = r^{pr}$.

Equilibrium under Full Property Rights: derivation of (36) *and* (37): From (A.27) and (A.29), the first-order condition (A.31) can be re-written as

$$\lambda_a^{pr} \cdot (P_H^{pr} BS^{pr} - 1) = \lambda_s^{pr} \cdot BS^{pr},$$

where we can substitute $\lambda_a^{pr} = 1/(P_Y^{pr} Y^{pr})$ from (A.30), and multiply both sides by L_H^{pr}, to obtain

$$(P_H^{pr} BS^{pr} - 1) \cdot L_H^{pr} = \lambda_s^{pr} \cdot BS^{pr} L_H^{pr} \cdot y^{pr}, \tag{A.38}$$

which yields expression (37) in the main text. The left-hand side of (A.38) equals current net rents from harvesting, Π_S^{pr}. Therefore, substituting $H^{pr} = BS^{pr} L_H^{pr}$ from (15) into (A.38), we obtain Eq. (36) in the text.

Equilibrium under Full Property Rights: derivation of (38): From (27), we can rewrite the relation between expenditure and resource rents as

$$\Pi_S^{pr}(t) = y^{pr}(t)(1 - \rho\psi\gamma) - L. \tag{A.39}$$

Substituting Π_S^{pr} in (A.39) by means of (36), we obtain

$$y^{pr}(t) = \frac{L}{1 - \rho\psi\gamma - \lambda_s^{pr}(t) H^{pr}(t)}, \tag{A.40}$$

that is Eq. (38) in the text. For future reference, notice that – using the resource demand schedule (3) – resource rents can also be written as

$$\Pi_S^j(t) = P_H^j(t) H^j(t) - L_H^j(t) = \alpha y^j(t) - L_H^j(t). \tag{A.41}$$

Combining (A.39) with (A.41) under full property rights, it follows that

$$L - L_H^{pr}(t) = (1 - \rho\psi\gamma - \alpha) \cdot y^{pr}(t). \tag{A.42}$$

In any equilibrium with positive final output, we must have $L > L_H^{pr}(t)$ and, consequently, the parameter restriction

$$1 - \rho\psi\gamma - \alpha > 0. \tag{A.43}$$

Equilibrium under Full Property Rights: proof of Proposition 3: The proof hinges on three steps: (i) the derivation of the dynamic system (40) and (41); (ii) the proof of saddle-point stability; (iii) the proof of results (42).

(i) *Dynamic system*: First, we derive Eq. (40). From (A.28) and (A.29), we have

$$\frac{\partial G(S^{pr})}{\partial S^{pr}} - \frac{\partial H(S^{pr}, L_H^{pr})}{\partial S^{pr}} = \eta - 2\left(\eta/\overline{S}\right) \cdot (S^{pr}) - BL_H^{pr}. \tag{A.44}$$

From (A.30) and (A.27), we respectively have

$$\lambda_a^{pr} = 1/y^{pr} \quad \text{and} \quad \frac{\partial \Pi_S(S^{pr}, L_H^{pr})}{\partial S^{pr}} = P_H^{pr} BL_H^{pr}. \tag{A.45}$$

Substituting (A.44) and (A.45) into (A.33), as well as $H^{pr}/S^{pr} = BL_H^{pr}$ from (15), we have

$$\frac{\dot{\lambda}_s^{pr}}{\lambda_s^{pr}} = \rho - \eta + 2\left(\eta/\overline{S}\right) \cdot (S^{pr}) + \frac{H^{pr}}{S^{pr}} - \frac{1}{\lambda_s^{pr}} \cdot \frac{P_H^{pr} H^{pr}}{y^{pr} S^{pr}}. \tag{A.46}$$

Substituting P_H^{pr} by means of (3), we obtain

$$\frac{\dot{\lambda}_s^{pr}}{\lambda_s^{pr}} = \rho - \eta + 2\left(\eta/\overline{S}\right) S^{pr} + \frac{H^{pr}}{S^{pr}} - \frac{1}{\lambda_s^{pr}} \cdot \frac{\alpha}{S^{pr}}. \tag{A.47}$$

From (13) and (15), the growth rate of the resource stock is

$$\frac{\dot{S}^{pr}}{S^{pr}} = \eta - \left(\eta/\overline{S}\right) S^{pr} - \frac{H^{pr}}{S^{pr}}. \tag{A.48}$$

Eqs. (A.47) and (A.48) imply

$$\frac{\dot{\lambda}_s^{pr}}{\lambda_s^{pr}} + \frac{\dot{S}^{pr}}{S^{pr}} = \rho + \left(\eta/\overline{S}\right) S^{pr} - \frac{\alpha}{\lambda_s^{pr} S^{pr}}. \tag{A.49}$$

Eq. (A.49) can be transformed into a differential equation governing the *shadow value of the resource stock*, $m \equiv \lambda_s^{pr} S^{pr}$, which depends on the resource stock

$$\frac{\dot{m}}{m} = \rho + \left(\eta/\overline{S}\right) S^{pr}(t) - \frac{\alpha}{m}, \tag{A.50}$$

which is Eq. (40) in the text. We now derive (41). From (A.38), we have

$$P_H^{pr} BS^{pr} = 1 + \lambda_s^{pr} S^{pr} \cdot By^{pr}. \tag{A.51}$$

Substituting $P_H^{pr} = \alpha y^{pr}/H^{pr}$ from (3) into (A.51), and using $m \equiv \lambda_s^{pr} S^{pr}$, we have

$$\frac{S^{pr}}{H^{pr}} = \frac{1}{\alpha B y^{pr}} + \frac{m}{\alpha}. \tag{A.52}$$

Using (A.40) to substitute y^{pr} in (A.52), we obtain

$$\frac{S^{pr}}{H^{pr}} = \frac{1 - \rho\psi\gamma - m \cdot \dfrac{H^{pr}}{S^{pr}}}{\alpha BL} + \frac{m}{\alpha},$$

which generates the second-order static equation

$$\alpha BL \left(\frac{S^{pr}}{H^{pr}}\right)^2 - (1 - \rho\psi\gamma + BLm) \cdot \frac{S^{pr}}{H^{pr}} + m = 0. \tag{A.53}$$

Eq. (A.53) determines, at each point in time, the equilibrium stock-flow ratio S^{pr}/H^{pr} for given m. The roots of (A.53) are

$$\frac{S^{pr}}{H^{pr}} = \frac{1 - \rho\psi\gamma + BLm \pm \sqrt{(1 - \rho\psi\gamma + BLm)^2 - 4\alpha BLm}}{2\alpha BL}. \tag{A.54}$$

Notice that, in order to ensure a real value for S^{pr}/H^{pr}, the term under the square root is constrained to be strictly positive

$$(1 - \rho\psi\gamma + BL \cdot m)^2 - 4\alpha BL \cdot m = (1 - \rho\psi\gamma)^2 + 2(1 - \rho\psi\gamma - 4\alpha)BLm + (BLm)^2 > 0. \tag{A.55}$$

In order to isolate the admissible root in (A.54), notice that $H^{pr} = BL_H^{pr} S^{pr}$ from (15) and $L_H^{pr} < L$ from the requirement of strictly positive labor (see (A.43) above) imply that $H^{pr} < BLS^{pr}$ must hold in each instant in an equilibrium with positive harvesting and positive final production. Imposing this inequality in (A.54), we have

$$\frac{BLS^{pr}}{H^{pr}} = \frac{1-\rho\psi\gamma + BLm \pm \sqrt{(1-\rho\psi\gamma+BLm)^2 - 4\alpha BLm}}{2\alpha} > 1. \tag{A.56}$$

The above inequality can only be satisfied by the solution exhibiting the plus sign in front of the square root.[10] Inverting the stock-flow ratio in (A.56), we can thus write

$$\frac{H^{pr}}{S^{pr}} = \Lambda(m) \tag{A.57}$$

in each instant t in which there is an equilibrium with positive production, where

$$\Lambda(m) \equiv \frac{2\alpha BL}{1-\rho\psi\gamma + BL\cdot m + \sqrt{(1-\rho\psi\gamma + BL\cdot m)^2 - 4\alpha BL\cdot m}}. \tag{A.58}$$

Notice that, given the restriction (A.55), (A.58) implies

$$\Lambda'(m) \equiv \frac{\partial \Lambda(m)}{\partial m} < 0. \tag{A.59}$$

These results allow us to complete the autonomous two-by-two system: Eq. (40) is (A.50) above; substituting result (A.57) into (A.48), we obtain (41).

(ii) *Saddle-point stability*: The steady-state loci of system (40) and (41) are given by

$$\dot{m}(t) = 0 \to S^{pr}(t) = \frac{\overline{S}}{\eta}\cdot\left(\frac{\alpha}{m(t)} - \rho\right); \tag{A.60}$$

$$\dot{S}^{pr}(t) = 0 \to S^{pr}(t) = \frac{\overline{S}}{\eta}\cdot\left[\eta - \Lambda(m(t))\right]. \tag{A.61}$$

The steady state (m_{ss}, S_{ss}^{pr}) is therefore characterized by

$$S_{ss}^{pr} = \frac{\overline{S}}{\eta}\cdot\left(\frac{\alpha}{m_{ss}} - \rho\right); \tag{A.62}$$

$$\Lambda(m_{ss}) = \eta - \left(\eta/\overline{S}\right) S_{ss}^{pr}. \tag{A.63}$$

Therefore, there exists a steady state with positive resource stock if and only if parameters are such that

$$m_{ss} < \frac{\alpha}{\rho} \quad \text{and} \quad \Lambda(m_{ss}) < \eta. \tag{A.64}$$

Linearizing system (40) and (41) around the steady-state (m_{ss}, S_{ss}^{pr}), we have

$$\begin{pmatrix} \dot{m}/m \\ \dot{S}^{pr}/S^{pr} \end{pmatrix} \simeq \begin{pmatrix} \varsigma_1 \equiv (\alpha/m_{ss}^2) & \varsigma_2 \equiv \left(\eta/\overline{S}\right) \\ \varsigma_3 \equiv -\Lambda'(m_{ss}) & \varsigma_4 \equiv -\left(\eta/\overline{S}\right) \end{pmatrix} \begin{pmatrix} m - m_{ss} \\ S^{pr} - S_{ss}^{pr} \end{pmatrix},$$

where (recalling result (A.59) above), the coefficients have definite signs: $\varsigma_1 > 0, \varsigma_2 > 0, \varsigma_3 > 0, \varsigma_4 < 0$. These signs imply $(\varsigma_4\varsigma_1 - \varsigma_2\varsigma_3) < 0$. As a consequence, the characteristic roots of the linearized system, given by the eigenvalues

$$\frac{(\varsigma_1+\varsigma_4) \pm \sqrt{(\varsigma_1+\varsigma_4)^2 - 4(\varsigma_4\varsigma_1 - \varsigma_2\varsigma_3)}}{2},$$

are necessarily real and of opposite sign. The steady state (m_{ss}, S_{ss}^{pr}) thus displays saddle-point stability: given the initial state $S^{pr}(0) = S_0$, there is a unique trajectory determined by the jump variable $m(0)$ driving the system towards (m_{ss}, S_{ss}^{pr}). Ruling out explosive paths by standard arguments,[11] the saddle-path determines a unique equilibrium path which converges to a positive stationary level of the resource stock $S_{ss}^{pr} > 0$ provided that the restrictions (A.64) are satisfied.

(iii) *Steady states*: Results (42) follow from the condition for positive steady-state resource stock implied by (41). If the parameters are such that $\eta > \Lambda(m_{ss})$, restrictions (A.64) are satisfied and saddle-point stability implies that $(m(t), S^{pr}(t))$ converge to the steady state (m_{ss}, S_{ss}^{pr}) with $S_{ss}^{pr} > 0$ determined by (A.62) and (A.63): see Fig. 1, left graph. If parameters

[10] The proof of this statement is by contradiction: picking the solution with the minus sign, inequality (A.56) would imply $4\alpha\{\alpha - (1-\rho\psi\gamma)\} > 0$, which is not possible because we would violate the parameter restriction (A.43).

[11] Explosive paths would violate either the transversality condition $\lim_{t\to\infty} m(t)e^{-\rho t} = 0$ appearing in (A.35) or the intertemporal resource constraint (13).

imply $\eta \leq \Lambda(m_{ss})$, instead, the steady state (A.62) and (A.63) is not feasible in view of restrictions (A.64) and the dynamics generated by the loci (A.60) and (A.61) imply that $(m(t), S^{pr}(t))$ converge to a steady state with $\lim_{t\to\infty} S^{pr}(t) = 0$ and $\lim_{t\to\infty} m(t) = \alpha/\rho$, as shown in Fig. 1, right graph.

Equilibrium under Full Property Rights: proof of Proposition 4: As noted in the main text, Proposition 4 assumes that innovation activities are operative in each instant (i.e., $\dot{Z}(t) > 0$ and $(\dot{n}(t)/n(t)) + \delta > 0$: see the further details reported at the end of this Appendix). The equilibrium growth rate of Z^{pr} follows directly from (25), and can be substituted into (26) to obtain

$$\frac{\dot{n}^{pr}(t)}{n^{pr}(t)} = \frac{1-\gamma-\theta\gamma-\psi(\rho+\delta)}{\psi} - \frac{1}{\psi}\cdot\frac{n^{pr}(t)}{\gamma y^{pr}(t)}\cdot\left[\phi - \frac{1}{\kappa}\cdot\left(r^{pr}(t)+\delta\right)\right]. \tag{A.65}$$

Defining $\nu \equiv (1-\gamma-\theta\gamma-\psi(\rho+\delta))/\psi$ and

$$\tilde{n}^{pr}(t) \equiv \gamma y^{pr}(t)\cdot\frac{1-\gamma-\theta\gamma-\psi(\rho+\delta)}{\phi - \frac{1}{\kappa}\cdot\left(r^{pr}(t)+\delta\right)},$$

expression (A.65) reduces to

$$\frac{\dot{n}^{pr}(t)}{n^{pr}(t)} = \nu\cdot\left[1 - \frac{n^{pr}(t)}{\tilde{n}^{pr}(t)}\right]. \tag{A.66}$$

Having established that $\lim_{t\to\infty} y^{pr}(t) = y^{pr}_{ss}$ and $\lim_{t\to\infty} r^{pr}(t) = \rho$ in (43), the carrying capacity $\tilde{n}^{pr}(t)$ is asymptotically constant

$$\lim_{t\to\infty}\tilde{n}^{pr}(t) = \gamma y^{pr}_{ss}\cdot\frac{1-\gamma-\theta\gamma-\psi(\rho+\delta)}{\phi-(\rho+\delta)\cdot\kappa^{-1}},$$

implying that Eq. (A.66) is dynamically stable around the steady state $\lim_{t\to\infty} n^{pr}(t) = \lim_{t\to\infty}\tilde{n}^{pr}(t)$.

Long-Run Equilibria: proof of Proposition 5. The proof hinges on two steps: (a) proving that $\overline{\eta}^{oa} > \overline{\eta}^{pr}$, and (b) comparing the three subcases (i)–(iii).

(a) *Proof that* $\overline{\eta}^{oa} > \overline{\eta}^{pr}$. From (32) and (42), the difference between the critical levels $\overline{\eta}^{oa} - \overline{\eta}^{pr} = \alpha BL/(1-\psi\gamma\rho) - \Lambda(m_{ss})$. Substituting the definition of $\Lambda(m_{ss})$ from the third expression in (42), we have

$$\overline{\eta}^{oa} - \overline{\eta}^{pr} \equiv \frac{\alpha BL}{1-\psi\gamma\rho}\cdot\left[1 - \frac{2(1-\psi\gamma\rho)}{1-\psi\gamma\rho + BLm_{ss} + \sqrt{(1-\rho\psi\gamma + BLm_{ss})^2 - 4\alpha BLm_{ss}}}\right].$$

The term in square brackets is strictly positive if and only if $1-\rho\psi\gamma > \alpha$, a condition that surely holds given the restriction (A.43).

(b) *Proof of subcases* (i)-(iii): Considering subcase (i), suppose that $\overline{\eta}^{pr} < \overline{\eta}^{oa} < \eta$. Then, both regimes yield positive stock in the long run with the following property. From (32) and (42),

$$S^{oa}_{ss} - S^{pr}_{ss} = \frac{\overline{S}}{\eta}\cdot\left[\Lambda(m_{ss}) - \frac{\alpha BL}{1-\psi\gamma\rho}\right] = \frac{\overline{S}}{\eta}\cdot\left(\overline{\eta}^{pr} - \overline{\eta}^{oa}\right) < 0,$$

because the last term is strictly negative ($\overline{\eta}^{oa} > \overline{\eta}^{pr}$). Hence, $S^{pr}_{ss} > S^{oa}_{ss}$. Under full property rights, positive harvesting in the long run implies a positive asymptotic shadow value of the resource stock: $m_{ss} > 0$ and $\Lambda(m_{ss}) > 0$. Consequently, (29) and (43) yield $y^{pr}_{ss} > y^{oa}$. Concerning the mass of firms, from (35) and (44), we have $n^{oa}_{ss}/n^{pr}_{ss} = y^{oa}/y^{pr}_{ss}$, which implies that $n^{pr}_{ss} > n^{oa}_{ss}$. Considering subcase (ii), suppose that $\overline{\eta}^{pr} < \eta < \overline{\eta}^{oa}$. Then, we have $S^{pr}_{ss} > 0$ from (42) and $S^{oa}_{ss} = 0$ from (32); since the production function (1) implies that the resource is essential, resource exhaustion under open access yields zero production/consumption under open access. Considering subcase (iii), suppose that $\eta < \overline{\eta}^{pr} < \overline{\eta}^{oa}$. Then, we have $S^{pr}_{ss} = 0$ from (42) and $S^{oa}_{ss} = 0$ from (32) that imply zero production/consumption under both regimes.

Equilibrium growth rates: derivation of (45). Symmetry in the manufacturing sector implies that final output (1) and the market share of intermediates (9) can be re-written as $Y = H^{\alpha} L_Y^{\beta} n X^{\gamma}$ and $X_i = \gamma(P_Y Y/nP_{Xi})$, respectively. Combining these two expressions to eliminate X, and using the profit-maximizing conditions of the final sector (2) and (3) to eliminate H and L_Y, we have

$$Y = H^{\alpha} L_Y^{\beta} n^{1-\gamma}\left(\gamma\frac{P_Y Y}{P_{Xi}}\right)^{\gamma} = \left(\frac{\alpha P_Y Y}{P_H}\right)^{\alpha}\left(\frac{\beta P_Y Y}{W}\right)^{\beta}\cdot n^{1-\gamma}\left(\frac{\gamma P_Y Y}{P_X}\right)^{\gamma}.$$

Observing that Y drops out, and solving the above expression for P_Y, we obtain

$$P_Y = \frac{1}{\alpha^{\alpha}\beta^{\beta}\gamma^{\gamma}}\cdot P_H^{\alpha}\cdot W^{\beta}\cdot n^{-1+\gamma}\cdot P_X^{\gamma}.$$

Observing that $C = Y = y/P_Y$, the above expression yields

$$\log C = \log \alpha^{\alpha}\beta^{\beta}\gamma^{\gamma} + \log y - \log P_H^{\alpha} + \log n^{1-\gamma} - \log\left(W^{\beta}P_X^{\gamma}\right).$$

Using the pricing rule (A.6) to eliminate P_X, the above expression becomes

$$\log C = \log \alpha^{\alpha}\beta^{\beta}\gamma^{\gamma} + \log y - \log P_H^{\alpha} + \log n^{1-\gamma} - \log\left[W^{\beta}\cdot\left(\frac{1}{\gamma}\cdot WZ_i^{-\theta}\right)^{\gamma}\right],$$

where, setting $W=1$ and rearranging terms, we have

$$\log C = \log \alpha^{\alpha}\beta^{\beta}\gamma^{2\gamma} + \log y - \log P_H^{\alpha} + \log\left(n^{1-\gamma}Z^{\theta\gamma}\right).$$

Defining $\overline{a} \equiv \alpha^{\alpha}\beta^{\beta}\gamma^{2\gamma}$, the above expression yields (45) in the text.

Equilibrium growth rates: derivation of (46): Time-differentiating (45) and substituting $\dot{y}/y$ by means of the Keynes–Ramsey rule (22), we obtain (46).

Equilibrium growth rates: proof of Proposition 6: Propositions 2 and 4 imply the result of identical firm size in the long run (47). Substituting (47) in (25), we obtain identical asymptotic rates of vertical innovation, $\lim_{t\to\infty}\dot{Z}(t)/Z(t) = \kappa\gamma\theta(\gamma y_{ss}/n_{ss}) - (\rho+\delta)$. Letting $t\to\infty$ in expression (46), we obtain $\lim_{t\to\infty}g(t) = \lim_{t\to\infty}\gamma\theta(\dot{Z}(t)/Z(t))$ and therefore (48).

Resource price: proof of Proposition 7: From (28) and (37), the resource prices under the two regimes read

$$P_H^{oa} = \frac{1}{BS^{oa}} \quad \text{and} \quad P_H^{pr} = \frac{1+Bmy^{pr}}{BS^{pr}}.$$

For a given level of the resource stock $S(t) = \tilde{S}$, the above expressions imply $P_H^{oa}|_{S=\tilde{S}} < P_H^{pr}|_{S=\tilde{S}}$ because $Bm(t)y^{pr}(t) > 0$. In the long-run equilibria of the two regimes, resource prices equal

$$\lim_{t\to\infty} P_H^{oa}(t) = \frac{1}{BS_{ss}^{oa}} \quad \text{and} \quad \lim_{t\to\infty} P_H^{pr}(t) = \frac{1+Bm_{ss}y_{ss}^{pr}}{BS_{ss}^{pr}}.$$

Consequently, the sign of the gap $\lim_{t\to\infty}P_H^{oa}(t) - \lim_{t\to\infty}P_H^{pr}(t)$ is determined by the inequality $1/BS_{ss}^{oa} \gtrless (1+Bm_{ss}y_{ss}^{pr})/BS_{ss}^{pr}$, that is,

$$\frac{S_{ss}^{pr}}{S_{ss}^{oa}} \gtrless 1 + Bm_{ss}y_{ss}^{pr}. \tag{A.67}$$

Substituting S_{ss}^{oa} and S_{ss}^{pr} by (32) and (42), and eliminating y_{ss}^{pr} by (43), the above expression reduces to

$$\frac{\eta - \Lambda(m_{ss})}{\eta - \dfrac{\alpha BL}{1-\psi\gamma\rho}} \gtrless 1 + \frac{Bm_{ss}L}{1-\psi\gamma\rho - m_{ss}\Lambda(m_{ss})}. \tag{A.68}$$

The sign is generally ambiguous because, defining $\Upsilon \equiv 1-\psi\gamma\rho$ and $\Delta \equiv \Lambda(m_{ss})$, we can rewrite (A.68) as

$$\underbrace{(\Upsilon - m_{ss}\Delta)}_{\text{positive}}\underbrace{(\alpha BL - \Delta\Upsilon)}_{\text{positive}} - \underbrace{(m_{ss}BL)}_{\text{positive}}\underbrace{(\eta\Upsilon - \alpha BL)}_{\text{positive}} \gtrless 0.$$

Regime switch: proof Proposition 8: Rewrite (45) as

$$\log C(t) = \log \overline{a} + \log y(t) - \alpha \log P_H(t) + \log T(t), \tag{A.69}$$

where we have defined total factor productivity as

$$\text{TFP} = T(t) \equiv (n(t))^{1-\gamma}(Z(t))^{\theta\gamma}. \tag{A.70}$$

Starting from (A.69) and (A.70), the derivation of expression (49) involves three intermediate steps: deriving explicit expressions for (i) TFP, (ii) the resource price, and (iii) present-value utility, under the regime of open access.

(i) *Total factor productivity*: For future reference, we denote the rate of vertical innovation by $\hat{Z}(t) \equiv \dot{Z}(t)/Z(t)$, its asymptotic value by $\hat{Z}_{ss} \equiv \lim_{t\to\infty}\hat{Z}(t)$, and the long-run growth rate of the economy by $g_{ss} \equiv \lim_{t\to\infty}g(t) = \theta\gamma\hat{Z}(t)$. Under open access, the TFP term can be re-expressed as follows. By definition

$$\log T^{oa}(t) = \theta\gamma \log Z_0 + \theta\gamma\int_0^t \hat{Z}^{oa}(s)ds + (1-\gamma)\log n_0 + (1-\gamma)\log\left(\frac{n^{oa}(t)}{n_0}\right),$$

where we can add and subtract $\hat{Z}_{ss}$ from $\hat{Z}(t)$, obtaining

$$\log T(t) = \log\left(Z_0^{\theta\gamma}n_0^{1-\gamma}\right) + g_{ss}\cdot t + \theta\gamma\int_0^t \left[\hat{Z}(s) - \hat{Z}_{ss}\right]ds + (1-\gamma)\log\left(\frac{n(t)}{n_0}\right). \tag{A.71}$$

Denoting $x^j(t) \equiv \gamma y^j(t)/n^j(t)$, and recalling that $y^{oa}(t) = y^{oa}$ is constant over time, we have $\dot{n}^{oa}/n^{oa} = -\dot{x}^{oa}/x^{oa}$. Therefore, the differential equation for n^{oa} in (34) yields $\dot{x}^{oa} = \nu(x^{oa}_{ss} - x^{oa})$, the solution of which is

$$x^{oa}(t) = x^{oa}_0 e^{-\nu t} + x^{oa}_{ss}(1 - e^{-\nu t}). \tag{A.72}$$

Result (A.72) implies that

$$\begin{aligned}\theta\gamma \int_0^t \left(\hat{Z}(s) - \hat{Z}^*\right) ds &= \kappa(\theta\gamma)^2 \int_0^t (x(t) - x_{ss})\, ds \\ &= \frac{\kappa(\theta\gamma)^2 x_{ss}}{\nu} \left(\frac{x_0}{x_{ss}} - 1\right)(1 - e^{-\nu t}).\end{aligned} \tag{A.73}$$

Also, from the solution (34), we have

$$\frac{n(t)}{n_0} = \frac{1 + \left(\frac{n_{ss}}{n_0} - 1\right)}{1 + \left(\frac{n_{ss}}{n_0} - 1\right) e^{-\nu t}},$$

where we can take logarithms and approximate the resulting terms to obtain

$$\log\left(\frac{n(s)}{n_0}\right) = \left(\frac{n_{ss}}{n_0} - 1\right)(1 - e^{-\nu t}). \tag{A.74}$$

Observing that $(n_{ss}/n_0) - 1 = (x_0/x_{ss}) - 1$, results (A.73) and (A.74) yield

$$\log T(t) = \log\left(Z_0^{\theta\gamma} n_0^{1-\gamma}\right) + g_{ss} \cdot t + \varphi\left(\frac{n_{ss}}{n_0} - 1\right)(1 - e^{-\nu t}), \tag{A.75}$$

where we have defined $\varphi \equiv (\kappa(\theta\gamma)^2 x_{ss}/\nu) + (1-\gamma)$.

(ii) *Resource price*: Since open access implies a constant harvesting rate, the resource stock follows the logistic process

$$\frac{\dot{S}^{oa}(t)}{S^{oa}(t)} = \omega \cdot \left(1 - \frac{S^{oa}(t)}{S^{oa}_{ss}}\right), \tag{A.76}$$

where, denoting by $L_{Hss} = \lim_{t \to \infty} L_H(t)$, we have defined the constants $\omega \equiv \eta - BL_{Hss}$ and $S^{oa}_{ss} \equiv \overline{S} \cdot (\eta - BL_{Hss})/\eta$. The solution of (A.76) is

$$\frac{S(t)}{S_0} = \frac{1 + \left(\frac{S_{ss}}{S_0} - 1\right)}{1 + \left(\frac{S_{ss}}{S_0} - 1\right) e^{-\omega t}},$$

where we can take logarithms and approximate the resulting terms to obtain

$$\log \frac{S(t)}{S_0} = \left(\frac{S_{ss}}{S_0} - 1\right)(1 - e^{-\omega t}).$$

Since (28) implies $-\log(P^{oa}_H(t)/P^{oa}_H(0)) = \log(S(t)/S_0)$, we have

$$-\log \frac{P^{oa}_H(t)}{P^{oa}_H(0)} = \left(\frac{S_{ss}}{S_0} - 1\right)(1 - e^{-\omega t}). \tag{A.77}$$

(iii) *Present-value utility*: Under open access, expression (A.69) reads

$$\log C^{oa}(t) = \log \overline{a} + \log\left(\frac{y^{oa}}{(P^{oa}_H(0))^\alpha}\right) - \alpha \log\left(\frac{P^{oa}_H(t)}{P^{oa}_H(0)}\right) + \log T^{oa}(t). \tag{A.78}$$

Without loss of generality, let us normalize $\log \overline{a} + \log\left(Z_0^{\theta\gamma} n_0^{1-\gamma}\right) \equiv 0$.[12] Substituting (A.75) and (A.77) in (A.78), we obtain

$$\begin{aligned}\log C^{oa}(t) &= \log y^{oa} - \alpha \log P^{oa}_H(0) + g_{ss} \cdot t \\ &\quad + \alpha\left(\frac{S^{oa}_{ss}}{S_0} - 1\right)(1 - e^{-\omega t}) + \varphi\left(\frac{n^{oa}_{ss}}{n_0} - 1\right)(1 - e^{-\nu t}).\end{aligned} \tag{A.79}$$

Substituting (A.79) in the welfare functional (17), and integrating, we obtain

$$U^{oa} = \frac{1}{\rho}\left[\log y^{oa} - \alpha \log P^{oa}_H(0) + \frac{g_{ss}}{\rho} + \frac{\alpha}{\rho + \omega}\left(\frac{S^{oa}_{ss}}{S_0} - 1\right) + \frac{\varphi}{\rho + \nu}\left(\frac{n^{oa}_{ss}}{n_0} - 1\right)\right], \tag{A.80}$$

which is the level of welfare associated to the transition dynamics under open access given generic initial conditions S_0, Z_0, n_0.

[12] This normalization only simplifies the notation and does not affect the results.

On the basis of these results, we consider the initial conditions at time zero as determined by the steady state of the full property rights regime. In particular, given the general expression (A.69), the *baseline consumption path* that the economy would follow if full property rights were maintained after time zero is given by

$$\log C^{pr}(t) = \log \overline{a} + \log y_{ss}^{pr} - \alpha \log P_{H,ss}^{pr} + \log T^{pr}(t), \tag{A.81}$$

where TFP grows at the asymptotic rate g_{ss} in each instant after time zero. Consequently, the *baseline present-value welfare* generated by the baseline consumption path (A.81) equals

$$U_{ss}^{pr} = \frac{1}{\rho}\left[\log y_{ss}^{pr} - \alpha \log (P_H^{pr})_{ss} + \frac{g_{ss}}{\rho}\right]. \tag{A.82}$$

Taking the difference between (A.80) and (A.82), we obtain expression (49).

Further details: operativeness of innovation activities: Both Propositions 2 and 4 assume that innovation activities are operative in each instant – that is, $\dot{Z}(t)/Z(t) > 0$ and $(\dot{n}(t)/n(t)) + \delta > 0$. The parameter restrictions that guarantee this outcome can be derived as follows. For simplicity, consider the open access regime. From (29), substitute $y^{oa} = y^{oa}$ and $r^{oa} = \rho$ into Eq. (25): the non-negativity constraint on firm-specific R&D implies

$$\frac{\dot{Z}^{oa}(t)}{Z^{oa}(t)} = \begin{cases} \kappa\theta\gamma \cdot \dfrac{\gamma y^{oa}}{n^{oa}(t)} - (\rho+\delta) & \text{if } n^{oa}(t) < \overline{n}^{oa} \equiv \dfrac{\kappa\theta\gamma^2}{\rho+\delta} y^{oa} \\ 0 & \text{if } n^{oa}(t) \geq \overline{n}^{oa} \end{cases}. \tag{A.83}$$

Substituting this result in (26), we obtain

$$\frac{\dot{n}^{oa}(t)}{n^{oa}(t)} + \delta = \begin{cases} \dfrac{1-\gamma-\psi\rho-\theta\gamma}{\psi} - \dfrac{\phi-(\rho+\delta)\cdot\kappa^{-1}}{\psi\gamma L(1-\psi\gamma\rho)^{-1}} \cdot n^{oa}(t) & \text{if } n^{oa}(t) < \overline{n}^{oa} \\ \dfrac{1-\gamma-\psi\rho}{\psi} - \dfrac{\phi}{\psi\gamma L(1-\psi\gamma\rho)^{-1}} \cdot n^{oa}(t) & \text{if } n^{oa}(t) \geq \overline{n}^{oa} \end{cases}. \tag{A.84}$$

Imposing the non-negativity constraint on employment in entry, L_N, expression (A.84) implies two threshold levels on the mass of firms. First, when $\dot{Z}^{oa}(t) > 0$, there exists

$$n_{T1}^{oa} \equiv \frac{\gamma L(1-\gamma-\psi\rho-\theta\gamma)}{(1-\psi\gamma\rho)\left[\phi-(\rho+\delta)\kappa^{-1}\right]},$$

such that

$$n^{oa}(t) < \overline{n}^{oa} \quad \text{and} \quad \frac{\dot{n}^{oa}(t)}{n^{oa}(t)} + \delta = \begin{cases} \dfrac{1-\gamma-\psi\rho-\theta\gamma}{\psi} - \frac{\phi-(\rho+\delta)\cdot\kappa^{-1}}{\psi\gamma L(1-\psi\gamma\rho)^{-1}} \cdot n^{oa}(t) & \text{if } n^{oa}(t) < n_{T1}^{oa} \\ 0 & \text{if } n^{oa}(t) \geq n_{T1}^{oa} \end{cases}. \tag{A.85}$$

Second, when $\dot{Z}^{oa}(t) = 0$, there exists $n_{T2}^{oa} \equiv (\gamma(1-\gamma-\psi\rho)L)/(\phi(1-\psi\gamma\rho))$, such that

$$n^{oa}(t) \geq \overline{n}^{oa} \quad \text{and} \quad \frac{\dot{n}^{oa}(t)}{n^{oa}(t)} + \delta = \begin{cases} \dfrac{1-\gamma-\psi\rho}{\psi} - \dfrac{\phi}{\psi\gamma L(1-\psi\gamma\rho)^{-1}} \cdot n^{oa}(t) & \text{if } n^{oa}(t) < n_{T2}^{oa} \\ 0 & \text{if } n^{oa}(t) \geq n_{T2}^{oa} \end{cases}. \tag{A.86}$$

It follows from (A.85) and (A.86) that a sufficient condition for positive gross entry is

$$n^{oa}(t) < \min\{n_{T1}^{oa}, n_{T2}^{oa}\}. \tag{A.87}$$

Provided that (A.87) holds, the mass of firms obeys the logistic process described in (A.84). Proposition 2 thus assumes implicitly that condition (A.87) is satisfied. This assumption is without loss of generality: because the logistic process (34) always converges to a finite steady state, satisfying (A.87) is equivalent to imposing restrictions on the exogenous parameters appearing in (A.84) and on the initial condition $n^{oa}(0)$. The alternative cases in which parameters yield no horizontal R&D are a less interesting special case because the model collapses to an economy with vertical R&D only.

Appendix B. Supplementary data

Supplementary data associated with this paper can be found in the online version at http://dx.doi.org/10.1016/j.euroecorev.2015.02.003.

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