Università degli Studi di Padova

Università degli Studi di Padova

Dipartimento di Fisica e Astronomia
Dipartimento di Ingegneria dell'Informazione
Corso di Laurea in Fisica

Tesi di Laurea

Experimental violations of Bell's inequalities with time-bin and polarization entanglement

Violazione delle disuguaglianze di Bell con entanglement in polarizzazione e time-bin

Laureando:
Alexandru Dima
Matricola 1052421

Relatore:
prof. Giuseppe Vallone

Anno Accademico 2014-2015

Darest thou now O soul,
Walk out with me toward the unknown region,
Where neither ground is for the feet nor any path to follow?

Oseresti ora tu, o anima,
Uscire con me verso quella regione sconosciuta,
Dove non c'è né terreno per i piedi né alcun sentiero da seguire?

- W. Whitman, *Darest Thou Now O Soul*, Leaves of Grass, from *Whispers of Heavenly Death*

Contents

Abstract

In this thesis activity the candidate has explored the main aspects of Bell Inequalities theory and eventually executed an experimental Violation test.
This work contains a brief overview of the theoretical aspects around Bell Inequalities: the doubts arisen by Einstein, Podolski and Rosen about Quantum Theory completeness; the hidden variable theories and the bounds on classical correlations; quantum correlations and violation of Bell inequalities; the response given by Bell to the EPR paradox by introducing its famous theorem. In addition, the entanglement phenomenon has been studied, in particular in those dof that are given an important role in the violation of Bell Inequalities. Some documentation work was also done on non linear optics in order to understand fundamental effects that is common practice to exploit in the photon entanglement generation: SHG for the pump photons, SPDC in non-linear crystals. Moreover, the candidate also explored in this thesis some issues more closely related to the experimental aspect:loopholes that could affect the integrity of Bell inequalities proofs; efficiency threshold estimates regarding detection loophole; loophole-free configurations. Eventually, here in this report are also showed, along with the experimental setup brief description and some comments on the apparatus and its setting, the results of an experimental test of the CHSH inequality violation using photons entangled at first in polarization and then in a time-bin entanglement configuration.

Sommario

In questo lavoro di tesi il candidato ha avuto modo di espolare gli aspetti principali della teoria delle Disuguaglianze di Bell e infine di sperimentare direttamente un test di Violazione di tali disuguaglianze.
Questa relazione finale dell'attività di tesi è composta di due parti. La prima si prefigge di essere una breve ma il più possibile esaustiva trattazione dei concetti più importanti dietro alla teoria dei Test di Bell. La lista degli argomenti toccati comprende, ovviamente, il paradosso EPR e la messa in dubbio della completezza della Meccanica Quantistica; di conseguenza, anche la risposta di Bell, contenuta nel suo articolo del '69, alla critica di Einstein, Podolski e Rosen. E quindi necessariamente la trattazione include la teoria delle variabili nascoste e i limiti alle correlazioni classiche; la violazione di tali limiti da parte delle correlazioni quantistiche, ovvero le violazioni delle Disuguaglianze di Bell. In aggiunta, nella tesi è stato approfondito anche l'entanglement, in particolar modo nei gradi di libertà che sono stati sfruttati poi, in fase sperimentale, per la correlazione quantistica delle coppie di fotoni da studiare. A tale scopo il candidato ha approfondito anche alcuni concetti di ottica non lineare utili alla descrizione del processo di generazione dei fotoni entangled: SHG per i fotoni di pompa, SPDC nei cristalli non lineari del secondo ordine.
Infine, gli ultimi concetti teorici qui raccolti riguardano le ipotesi aggiuntive che la maggior parte dei Test di Bell sperimentali richiedono per poter affermare con ragionevole certezza di aver violato le disugaglianze di Bell e aver dimostrato l'incompatibilità delle teorie LHV con le correlazioni tipicamente quantistiche; i cosiddetti *loophole*, falle nei test sperimentali che possono essere sfruttate dalle teorie a variabili nascoste per falsificare i comportamenti quantistici.
Ed infine, si riportano nella seconda parte di questa tesi i risultati dei due test sperimentali di violazione della disuguaglianza CHSH eseguiti in laboratorio dal candidato, il primo su fotoni entangled in polarizzazione e il secondo su coppie entangled in time-bin.

Chapter 1

Entanglement

1.1 Entanglement

Entangled states are a peculiarity of Quantum Physics and can be easily understood and explained once the formalism is known and mastered. Generic states of a quantum system are generally described by an abstract entity, $|\phi\rangle$, named ket. This is actually a vector of a Hilbert space, $\mathcal{H}$, of dimension n. An orthonormal basis can be introduced for $\mathcal{H}$, namely $\{|u_1\rangle, |u_2\rangle, \cdots, |u_i\rangle, \cdots, |u_n\rangle\}$ such that $\langle u_j|u_i\rangle = \delta_{ij}$, $\forall i, j$.
Every ket belonging to $\mathcal{H}$ can be represented in such an orthonormal basis

$$\forall|\phi\rangle \in \mathcal{H}, \;\; |\phi\rangle = \sum_{i=1}^{n} a_i|u_i\rangle$$

Now, given two Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$ of dimensions n respectively m, a higher dimension Hilbert space can be built taking the tensor product of the previous, $\mathcal{H}_1^n \otimes \mathcal{H}_2^m$, of dimension nm. In such a tensor product, an orthonormal basis is given by the tensor products $|u_i \otimes v_s\rangle$, where the $\{|v_s\rangle\}$ kets form an orthonormal basis for $\mathcal{H}_2$, such that $\langle u_i \otimes v_s|u_j \otimes v_t\rangle = \delta_{i,j}\delta_{s,t}$. Now, this means every ket belonging to $\mathcal{H}_1^n \otimes \mathcal{H}_2^m$ can be, in fact, represented as

$$\forall\Phi \in \mathcal{H}_1^n \otimes \mathcal{H}_2^m \qquad \Phi = \sum_{i,s} q_{i,s}|u_i \otimes v_s\rangle$$

The physical meaning of all this is clearer if we postulate that $\mathcal{H}_1^n \otimes \mathcal{H}_2^m$ represents the space of states of two interacting quantum systems, and this seems truly a reasonable assumption. The tensor product space of the quantum states of the system contains kets that can be divided into two categories. The kets which can be factorized as the tensor product of two vectors belonging to $\mathcal{H}_1$ and $\mathcal{H}_2$, which are specific for the case of two independent systems 1 and 2; and those states for which the factorization

$$q_{i,s} = a_i b_s \qquad \exists a, b \in \mathcal{R}$$

is not possible. In this last case Φ represents the state of two strongly interacting systems that act like one, and is called entangled state; the entanglement is an interaction bond between systems which is typical of quantum physics.

1.2 Non-linear optics:

Generally speaking, electromagnetic fields travelling through matter behave quite differently from radiation propagating through simple void. Many peculiar effects arise in these conditions, and two of these are of special interest when it comes to generating entangled photons. In particular, in this sections the *Spontaneous Parametric Downconversion* will be discussed as the main source of entangled photonic systems; also, the SHG phenomenon will be presented, as the two non linear effects often go together well both because of their theoretical affinity and because of their practical usefulness.

1.2.1 EM waves propagating in a nonlinear medium

Maxwell's equations in a chargeless matter environment with index of refraction n, are known to be:

$$\nabla \cdot \mathbf{D} = 0 \qquad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{E} = -\frac{\partial B}{\partial t}$$

In addition where it should be remembered that the electric displacement is defined as $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$, where $\mathbf{P}$ is the polarization field; the magnetizing field is $\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$. The wave equation that can be derived, after some calculation, from the previous equation system is

$$\nabla^2 \mathbf{E} - \frac{1}{v_n} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2}$$

This is clearly a non linear second order equation. This is due to the form of the interactions between charges and em fields in matter. This interactions quantify in the relation between polarization and electric fields, that can be linear, $\mathbf{P} = 2\epsilon_0 \chi_e \mathbf{E}$, or depend on a more generic and complicated function, $\mathbf{P} = f(E)$. In the latter case, for weak external electric fields, the phenomena arising can be studied at different level of approximation depending on the Taylor expansion of the polarization field around the $E = 0$ value. For an isotropic medium we can reduce the problem to its components, for simplicity:

$$P = \epsilon_0 (\chi_e^{(1)} E + \chi_e^{(2)} E^2 + \chi_e^{(3)} E^3) + o(E^4)$$

Usually systems in which only linear effects are object of study require a first order approximation. But in order to understand the SHG and SPDC effects, further in these sections second order terms will be taken into consideration.

1.2.2 SHG

We take a second order approximation, thus considering only the second order non-linear term $P = \epsilon_0 \chi_e^{(2)} E^2$.

A generic monochromatic oscillating electric field can be described as:

$$E(t) = [E(\omega) e^{i\omega t} + E^*(\omega) e^{-i\omega t}]$$

Thus, the polarization vector inside our second order dielectric environment, given such an external electric field, would be:

$$P(t) = \epsilon_0 \chi_e^{(2)} [E(\omega)^2 e^{2i\omega t} + E^*(\omega)^2 e^{-2i\omega t} + 2|E(\omega)|^2]$$

Along with the last term, which is constant in time and irrelevant for our purposes, the first two terms depend on 2ω: the final solution of the Maxwell equations in a second order dielectric medium will consist also of second harmonic radiation. That means, a radiation beam entering a SHG crystal will generate exiting radiation with components oscillating also at twice the original frequency.

1.2.3 SPDC - Type-I and Type-II entanglement generation

By the same principle, if an electric oscillating field composed by a sum of two (or more) modes

$$E(t) = \frac{1}{2}[(E(\omega_1)e^{i\omega_1 t} + E^*(\omega_1)e^{-i\omega_1 t}) + (E(\omega_2)e^{i\omega_2 t} + E^*(\omega_2)e^{-i\omega_2 t})]$$

enters the non-linear crystal it can be shown that the exiting radiation can contain 5 different components which are functions of, respectively $\omega'_0 = 0$, $\omega'_1 = 2\omega_1$, $\omega'_2 = 2\omega_2$, $\omega'_3 = \omega_1 + \omega_2$ and $\omega'_4 = \omega_1 - \omega_2$.
The process consisting in the generation of an exiting wave whose frequency is the sum of the two entering waves is called *up-conversion*, while the production of a wave of frequency equal to the difference of the two waves ones is called *down-conversion*.

In a non-linear crystal there is also another phenomenon which can occur: incident radiation on a non-linear crystal gives origin to two different exiting waves. This process is called *spontaneous parametric down-conversion*. If the incident radiation is composed by a single monochromatic wave, when detected the two exiting waves' frequencies and wave vectors appear to be related by the so called *frequency-matching* and *phase-matching conditions* :

$$\omega_p = \omega_i + \omega_s$$

$$\mathbf{k}_p = \mathbf{k}_i + \mathbf{k}_s$$

where p stands for pump, i for idler and s for signal. These two conditions are evidently related to the conservation of energy and momentum in the down-conversion process.

When using uniaxial crystals (as in the experiments conducted in this thesis work) the birefringence properties of the crystal can be used to obtain two types of phase-matching conditions depending on the polarization of the SPDC photons, assuming the pump radiation to be linearly polarized along the optical axis of the crystal: *Type-I conditions*, when the emitted photons share the same polarization state (both ordinarily or extraordinarily polarized); *Type-II conditions*, when the emitted photons are orthogonally polarized respectively on the extraordinary and ordinary directions.

It can be shown that the phase-matching conditions imply the exiting photons paths to belong to the surface of emission cones. In particular, for the Type-I SPDC the idler and signal correlated photons will be found on the surface of a single emission cone centered around the propagation direction (equivalently, the result can be interpreted as two cones tangent one another); while for the Type-II SPDC the two photons exit the crystal on two different cones, centered around the two $\mathbf{k}_i$ and $\mathbf{k}_s$ directions and crossing one another. [7]

1.3 polarization entanglement

About the entanglement, in both types of SPDC the resulting photons are correlated, but for Type-I process they require a different setting or technique in order to be also in a superposition of quantum states. To achieve this a common way is to force the pump radiation to pass through an additional crystal with optical axis perpendicular to the first one. With this setting, the entanglement source becomes the indetermination of the exact position in which SPDC occurs, as it depends on the polarization of the pump beam with respect to the first and second optical axes. If we assume the pump photon's polarization quantum state to be:

$$|pump\rangle = \alpha|H\rangle + \beta|V\rangle$$

Then the bi-photon state given as a result of a Type-I SPDC would be:

$$|\Phi^{\pm}_{\alpha,\beta}\rangle = \beta|H_A\rangle|H_B\rangle \pm \alpha|V_A\rangle|V_B\rangle$$

For a Bell state to be achieved (that is, to achieve maximal entanglement), the pump must be linearly polarized at $\frac{\pi}{4}$ with respect to both optical axes, which translates into $\alpha = \beta$. This setting is often referred to as *Kwiat source*. Another implementation of the Type-I SPDC for obtaining entangled photons is the so called *Roma source* which has the pump beam to pass twice through the crystal means a mirror and a quarter-wave retarder (which changes the polarization in the second passage); the resulting quantum state is equivalent to the previously shown one.

In case of a Type-II process, instead, no additional precaution should be employed to obtain entangled pairs: at the intersections of the two emission cones the indetermination on the actual polarization of the detectable photons is the very source of the entanglement. The state of the pairs that can be at these points detected can be represented as:

$$|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}[|H_A\rangle|V_B\rangle \pm |V_A\rangle|H_B\rangle]$$

That is in fact the SPDC process employed in the experimental test conducted for this thesis project.

1.4 Time-bin entanglement

A completely different degree of freedom with respect to which entanglement can be generated is the time of arrival of the photons. Though necessitating an additional essential component to generate the superposition of states required, it nevertheless involves the use of an SPDC crystal for the actual generation of the correlation. The additional component is a Michelson unbalanced interferometer, where "unbalanced" refers to the fact that the two arms of this device are of different length (precisely, the path difference resulting is constructed to be bigger than the coherence length in order to avoid single photon interference). This setup is meant to create the indetermination of the photons time of arrival by mixing up the two states, for each pump photon, corresponding to a S, short arm propagation state, and L, long arm propagation state:

$$|\phi_0\rangle = \frac{1}{\sqrt{2}}[|S\rangle + e^{i\theta}|L\rangle]$$

In the experimental setup used in this thesis work after the first Michelson interferometer the pump beam is conveyed in the SPDC crystal, where two correlated photons are generated with the same rules described in the previous paragraph. The state of these photons will be described as:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}[|S_A S_B\rangle + e^{i\theta}|L_A L_B\rangle]$$

No Bell states are retrievable other than the correlated ones since the SPDC process requires the entangled photons to be emitted at the same time, preventing anti-correlated time-bin states. Additional Michelson unbalanced interferometers are needed in order to serve as measurement apparata for the quantum state of the photon pairs, as will be discussed in the last chapter of this work.

Chapter 2

Bell's Theorem: theory

2.1 Intro

What can be sad to be real and what not? When does a theory contain an element of reality? When are we allowed to think that a some level of reality corresponds to an ideal part of a theory? These are difficult questions to answer and even in Physics there have been several different approaches to this issue. However, a significant debate regarding Quantum Physics very principles has been roused by Einstein, Podolski and Rosen starting from the notion of reality and the request of realism addressed to a theory, in particular, to QM. In the famous EPR paper, the authors assumed as a criterion of reality the possibility of successfully predicting a system's physical quantities without in any way influencing it; from this criterion the argument then veers towards the demonstration of Quantum Theory's incompleteness. Even though at first this seems to be a reasonable sufficient condition for realism in theories to be satisfied, from Quantum Theory predictions and results it appears necessary to introduce the simultaneous mensurableness of physical quantities instead of the mere predictability. Under this reinterpretation of the reality condition MQ completeness can be saved. But what would the alternatives to MQ have been and what their predictions? In a nutshell,the existence of a specific bound on some physical quantities involving correlated particles can be claimed by LHV; bound derived only from the local realism axiom. But quantum entanglement shows to be such a surprising phenomenon that has the correlated particles violate the already mentioned classical bound. The consequences are profound, from the theoretical point of view.

2.2 EPR

In 1935 a paper [1] signed by Einstein, Podolski and Rosen called Quantum Theory completeness into question. They claimed that a complete theory should be one which comprehends a theoretical quantity corresponding to each element of reality. In addition to that, the authors single out what they think to be a reasonable criterion of realism according to which a physical quantity is accompanied by a counterpart in the (objective) physical reality if its values can be measured or predicted with *certainty* and *without in anyway disturbing the system.* It is clear now that, taking for granted those assumptions, some questions arise about Quantum Theory. It suffices to think to physical quantities associated to non-commuting operators (say A and B, such that $[A, B] \neq 0$): the simultaneous predictability of both the A and B is hindered by the Heisenberg's Uncertainty Principle.

$$\Delta A \cdot \Delta B \geq \frac{1}{2}|\langle[A, B]\rangle|$$

That means the system which is studied could never be found in a state which is a simultaneous eigenstate of both A and B: when measuring (e.g.) A the wave function collapses in such a way that no consequent certain prediction can be made on the quantity B and vice versa. Hence, the conclusion is that either an alternative theory can be found, which includes additional variables potentially capable of describing both A and B with 100% certainty or A and B cannot have simultaneous reality.

On the other hand, when considering a system of two entangled entities, an interesting paradox can be found. Entangled states are peculiar consequences of the wave function theory of Quantum Physics. Two particles in an entangled state act as they were part of the same entity: if a measurement is operated on one particle the subsequent wave collapse seems to instantaneously affect also the second particle, inducing a predictable change in its state. This change is totally dependent on the measure done on the other particle. As argued by Einstein and his colleagues, two different measurements of non-commuting quantities can be conduced. Depending on the measurement choice on one particle, the effects on the other particle of the wave function collapse can be predicted in terms of certain forecast of the same physical quantity for the entangled partner particle. Take for example a 2-particle (A and B) state described by the state

$$\Psi = \sum_i |u_i\rangle \otimes |v_i\rangle = \sum_j |w_j\rangle \otimes |s_j\rangle$$

Where u,v and w,s are single-particle eigenstates associated with the same physical quantity, respectively G and F, such that:

$$G^A|u_i\rangle = g_i^A|u_i\rangle \qquad G^B|v_i\rangle = g_i^B|v_i\rangle$$

$$F^A|w_j\rangle = g_j^A|w_j\rangle \qquad F^B|s_j\rangle = g_j^B|s_j\rangle$$

And now assume that $[F, G] \neq 0$.

When performing a measurement of G on A the entire system state Ψ collapses. When obtaining g_o^A as a measure of G^A, e.g., the A particle has its wave function collapse on the

eigenket $|u_o\rangle$. At the same time, this means the state of the 2-particle system becomes $\Psi' = |u_o\rangle \otimes |v_o\rangle$. With the same logic, when measuring the F quantity on the A particle we have a similar phenomenon: the Ψ state collapses onto $\Psi'' = |w_k\rangle \otimes |s_k\rangle$ where w_k is the eigenstate of A corresponding to the measurement f_k^A, and s_k is the state on which B collapses after the measure on A.
Thus, after two different measurements on the same part of the 2-particle system there are two different wave functions, s_k and v_o, describing the same reality, that is the second particle. It could be argued that the measurement choice on system A influences somehow the state of particle B, in such a way that the latter changes its state dependently on the measure done on A. However, nothing prohibits us to think of a measure conducted when the two particles are separated by a spacelike interval: a signal communicating to B the operations just done on A would have to travel at several times the speed of light in order to transfer information on measure A before an hypothetically simultaneous measure B could be made. Thus it seems quite reasonable to assume no interaction between A and B after the entanglement has been established. This is evidently inconsistent with Heisenberg's principle consequences on the simultaneous reality of non-commuting physical quantities. Hence, the EPR paradox solution proposed in the famous paper: in order to preserve both locality principle (requiring the two systems cannot instantaneously communicate the measurements just they underwent) and realism principle it must be concluded that there must exist an alternative local theory which include some variables (maybe not even accessible to our cognition) that would give a complete description of the physical reality.

2.3 Local Hidden Variables Theories

The EPR paper, eventually, suggests that Quantum Mechanics could never be the ultimate description of physical reality at the atomic and smaller scales. It suggest instead the existence, at least in principle, of variables knowing which it would become possible to describe the physical phenomena happening at these scales saving both locality and realism (which, in some sense, is a sort of escape hatch for determinism). Nothing, however, it is said about the accessibility of these variables so called *hidden variables*, nothing about the required number of such variables, nor if they have to be discrete or continuos. The local variables theoretical background is unnecessary: the only important aspect are the associated basic assumptions.
The are mainly two possible formulations of a Hidden Variable theory candidate [4], a deterministic LHV model, which is the description used by Bell in his original paper [2], and a more general stochastic model, only later introduced in order to loosen up the assumptions.

2.3.1 Deterministic LHV model

This model is based on the deterministic requirement and on the locality principle. When assuming there is no signaling between the two parts of a correlated system and yet we are able to predict the results of different measurements on one part (as seen in the previous section), it seems legitimate to believe that the behavior of such systems is entirely predetermined by some local variables λ (this is a generic symbol that might also include an entire set of local variables or functions). Let A and B be two correlated particles (that means they interacted at some point in time and then got separated) and Alice and Bob be typical observers that will proceed with x and y choices of measurements on them. Because of the determinism requirement, the hidden factor will be a physical variable which will define a certain outcome of each measurement depending on its particular value. The outcomes, a and b, will in general be functions

$$a = a(x, y, \lambda) \qquad\qquad b = b(x, y, \lambda)$$

of both the settings and λ.
In addition, the locality principle puts a constraint on the dependencies, as the setting choice on one distant particle should not influence the other's outcome. Eventually,

$$a = a(x, \lambda) \qquad\qquad b = b(y, \lambda)$$

thus, if λ is taken to be an aleatory continuous[1] variable with an associated distribution function $\rho(\lambda)$ such that

$$\int \rho(\lambda) d\lambda = 1$$

and, then the expectation value of the outcomes related to the couple of settings (x,y) will be :

$$E(x, y) = \int d\lambda \rho(\lambda) a(x, \lambda) b(y, \lambda)$$

which especially expresses clearly the locality principle preservation.

2.3.2 Stochastic LHV model

This is a natural generalization of the previous model, to which is commonly preferred. This time the hidden factor is conceived to determine only the probabilities of the single results, thus a and b are random functions of λ, $P(a|x, y, b, \lambda)$ and $P(b|x, y, \lambda)$. The basic assumptions in this case is the *local causality* principle which is equivalent to the validity of the following expression:

$$P(a, b|x, y, \lambda) = P(a|x, \lambda) P(b|y, \lambda)$$

in which it is easy to recognize a no-signaling bound [3] under the form of distant *setting* and *outcome independence*. For this latter model, the local expectation values take the form:

$$P(a, b|x, y) = \int d\lambda \rho(\lambda) P(a, b|x, y, \lambda) = \int d\lambda \rho(\lambda) P(a|x, \lambda) P(b|y, \lambda)$$

$$E(x, y) = \sum_{a,b} ab \cdot \int d\lambda \rho(\lambda) P(a|x, \lambda) P(b|y, \lambda)$$

[1] not necessary choice made for simplicity and coherency with Bell's original arguments [2]

The deterministic formulation is clearly a particular case of the stochastic local model. The local causality principle is implicitly satisfied by the deterministic case, as the only additional information is that the probabilities take the extreme values 1 or 0.
In fact, it can be shown that the two models are mathematically equivalent [3] [4].

2.4 Bell's theorem

2.4.1 local bound

In his most famous paper [2], John Bell stated and demonstrated a theorem asserting the incompatibility of local deterministic hidden variable theories with predictions peculiar of quantum mechanics. It is also the very first proposal of an experimental test thought to verify whether LHV theories could ever give a complete description of quantum systems. This remarkable accomplishment takes the form of a inequality which is a bond preserved by LHV models, but not by quantum correlated systems. In fact, the inequality that actually Bell devised is indeed one very specific and difficult to employ in an actual experiment. Here we shall reconstruct Bell's arguments in brief.
Bell actually made two main assumptions

a) the result on one measure does not influence the other, since the two entities are thought to be distant enough

b) the entangled entities are found in a completely anti-correlated state

In particular, the case that Bell studied to derive his inequality was the system of two electrons in a singlet spin state. Because of the entanglement property, when measuring the spin in a chosen arbitrary direction $\vec{v}$ of the first electron, if the result $\vec{\sigma_A} \cdot \vec{v} = +1$ is obtained, then the same measure conducted on the electron's anti-correlated twin will necessarily retrieve $\vec{\sigma_B} \cdot \vec{v} = -1$. Considering the hypothesis a), it must be concluded that there cannot be any kind of light-like signal communicating the result of one measure to the partner electron.
An LHV theory could offer a more complete description of this phenomenon,in terms of predetermined measurement results as functions of a local deterministic variable λ. Bell first described rigorously such a local model and derived its theoretical predictions for the aforementioned case of the electron singlet state. We shall here retread quickly on this steps.
First let's define as $A(\vec{v}, \lambda)$ the measure result of $\vec{\sigma_A} \cdot \vec{v}$, the spin component of the first electron along the $\vec{v}$ direction; and $B(\vec{u}, \lambda)$ the result of measuring $\vec{\sigma_B} \cdot \vec{u}$ on the second electron. Now, the only possible values of A and B are certainly {-1, +1 }. Just as it is done in some paragraphs before, we take the expectation value of the product of the two measurements to be

$$E(\vec{v}, \vec{u}) = \int d\lambda \rho(\lambda) A(\vec{v}, \lambda) B(\vec{u}, \lambda)$$

where, as before, $\int d\lambda \rho(\lambda) = 1$. It is straightforward to notice that P is always bigger than -1. Assuming now that this minimum value can be reached at $\vec{v} = \vec{u}$ only when

$A(\vec{v}, \lambda) = -B(\vec{v}, \lambda)$ (perfect anti-correlation), we can thus redefine the expectation value as

$$E(\vec{v}, \vec{u}) = -\int d\lambda \rho(\lambda) A(\vec{v}, \lambda) A(\vec{u}, \lambda)$$

The next step is then to think of an additional direction along which to measure the spin, $\vec{w}$. It can be deduced the following inequality for the difference of the expectation values

$$E(\vec{v}, \vec{u}) - E(\vec{v}, \vec{w}) = -\int d\lambda \rho(\lambda)[A(\vec{v}, \lambda) A(\vec{u}, \lambda) - A(\vec{v}, \lambda) A(\vec{w}, \lambda)]$$

$$= -\int d\lambda \rho(\lambda) A(\vec{v}, \lambda) A(\vec{u}, \lambda)[1 - A(\vec{u}, \lambda) A(\vec{w}, \lambda)]$$

And hence, immediately can be obtained

$$|E(\vec{v}, \vec{u}) - E(\vec{v}, \vec{w})| \leq \int d\lambda \rho(\lambda)[1 - A(\vec{u}, \lambda) A(\vec{w}, \lambda)]$$

$$= 1 - \int d\lambda \rho(\lambda) A(\vec{u}, \lambda) A(\vec{w}, \lambda) = 1 + E(\vec{u}, \vec{w})$$

Where $|E(\vec{v}, \vec{u}) - E(\vec{v}, \vec{w})| - E(\vec{u}, \vec{w}) \leq 1$ is, finally, the original Bell inequality, to which all classical local realistic models satisfy. Instead, quantum systems in which entangled states are involved do not, as it will be demonstrated in the following section.

2.4.2 quantum predictions and violation

Indeed, for a quantum system like the one described in Bell's seminal work, quantum mechanics predicts

$$E(\vec{v}, \vec{u}) = \langle \Phi^- | (\vec{\sigma_A} \cdot \vec{v})(\vec{\sigma_B} \cdot \vec{u}) | \Phi^- \rangle = -(\vec{v} \cdot \vec{u})$$

to be the expectation value [8]. Bell produces a rigorous and clean mathematical proof that the local realistic bond is incompatible with expectation values coming from quantum physics description. Thus quantum systems can have the Bell inequality violated. However, in the following lines an alternative take will be given in order to support this idea. We shall substitute the classical expectation values with their quantum versions, $< (\vec{\sigma_A} \cdot \vec{v})(\vec{\sigma_B} \cdot \vec{u}) >= -\vec{v} \cdot \vec{u}$, into the Bell inequality.

$$|\vec{v} \cdot \vec{u} - \vec{v} \cdot \vec{w}| \leq 1 - \vec{u} \cdot \vec{w}$$

Now we shall remember that these vectors along which we are measuring the spin component of our electrons are just unit vectors; we can then get to the result

$$|\vec{v}||\vec{u} - \vec{w}| = |\vec{u} - \vec{w}| \leq 1 - cos(\theta)$$

where θ is simply the angle between the $\vec{u}$ and $\vec{w}$ vectors. Again, u and w being unit vectors, their difference' module can be evaluated easily to be

$$|\vec{u} - \vec{w}| = |2sin(\phi/2)| \leq 1 - cos(\theta)$$

by just drawing the vectors on the unitary circle. Eventually, our inequality becomes

$$sin(\phi/2)(sin(\phi/2)-1) \geq 0$$

Clearly, now, this relation is never satisfied (unless for $\phi = \pi$) , this meaning that our quantum system, but more in general every entangled quantum system with that can be brought to the same mathematical description, is expected to violate the original Bell inequality.

2.5 Other more general inequalities

2.5.1 CHSH, 1969

Although representing a sure milestone in profound debate on the very foundations of quantum mechanics, on the experimental side the original Bell inequality lacks a bit of practical convenience for many reasons. In 1969 a generalization of Bell's theorem and inequality was given in a notorious paper by Clauser, Horne, Shimony and Holt [9]. The main difference with the derivation made by Bell is not to consider perfectly correlated pairs. In a nutshell

$$|E(\vec{v},\vec{u}) - E(\vec{v},\vec{w})| \leq \int_V d\lambda\rho(\lambda)|A(\vec{v},\lambda)B(\vec{u},\lambda) - A(\vec{v},\lambda)B(\vec{w},\lambda)|$$

$$= \int_V d\lambda\rho(\lambda)|A(\vec{v},\lambda)B(\vec{u},\lambda)|[1 - B(\vec{u},\lambda)B(\vec{w},\lambda)]$$

$$= 1 - \int_V d\lambda\rho(\lambda)B(\vec{u},\lambda)B(\vec{w},\lambda)$$

where V is the entire domain to which λ belongs.
Now we shall assume that for some $\vec{u'}$ the expectation value is $E(\vec{u},\vec{u'}) = 1-\epsilon$, with $0 \leq \epsilon \leq 1$. This assumption is "experiment-friendly" since it replaces the claim for a perfect (anti-)correlation of the Bell derivation: ϵ values close but not precisely equal to zero is the best we can actually do [9]. Let's now split the λ-domain in two components and let them be $V_\pm = \{\lambda : A(\vec{u},\lambda) = \pm B(\vec{u},\lambda)\}$.
It's trivial to notice that, because of the normalization condition and of the correlation assumption we just made, the following $\int_{V_-} d\lambda\rho(\lambda) = \frac{\epsilon}{2}$ will stand true. Now, after few steps of simple calculations, we obtain

$$\int_V d\lambda\rho(\lambda)B(\vec{u},\lambda)B(\vec{w},\lambda) \geq E(\vec{u'},\vec{w}) - \epsilon$$

And by substituting in the previous results, eventually

$$|E(\vec{v},\vec{u}) - E(\vec{v},\vec{w})| \leq 2 - E(\vec{u'},\vec{w}) - E(\vec{u},\vec{u'})$$

which is the CHSH inequality, more commonly found in the form (with generalized parameters)

$$S = E(a,b) - E(a,b') + E(a',b) + E(a',b') \leq 2$$

The Quantum Mechanical predictions instead give expectation values of S bigger than the LHV bond. For example, in time-bin entangled systems analogous to the one employed in this thesis experimental phase, the entangled photons state in described by the formulas in the paragraph 2.4 . The measurements correspond to projections onto the quantum states

$$|\Phi_A\rangle = \frac{1}{\sqrt{2}}[|S\rangle_A + e^{i(\theta+\alpha)}|L\rangle_A]$$

$$|\Phi_B\rangle = \frac{1}{\sqrt{2}}[|S\rangle_B + e^{i(\theta+\beta)}|L\rangle_B]$$

And thus the correspondent projector operator would take the form

$$P_{\tau_A} = \frac{1}{2}[|S\rangle_A + e^{i(\theta+\alpha)}|L\rangle_A\langle S|_A + e^{-i(\theta+\alpha)} + \langle L|_A]$$

Hence, the coincidence probabilities are easily calculated as

$$C_{tau}(\alpha,\beta) = \langle\Phi|P_{\tau_A} \otimes P_{\tau_B}|\Phi\rangle = \frac{1}{2}(1 + cos(\alpha - \beta))$$

It's immediate to see that substituting the $\{(0,\frac{\pi}{4}),(\frac{\pi}{2},\frac{\pi}{4}),(0,\frac{3\pi}{4}),(\frac{\pi}{2},\frac{3\pi}{4})\}$ parameter values and evaluating the $E(\alpha,\beta)$ values, finally it can be seen that the quantum model predicts an expectation value for S of

$$S_Q = 2\sqrt{2}$$

The calculation of the theoretical expectation value of the S-value with entanglement in polarization DOF follows quite a similar path as for spin entangled systems (for example see [8]), only changing the polarisation vector states and polarisation projector operators. The final S_Q value is still predicted to be $2\sqrt{2}$.

2.5.2 CH74, 1974

Some years later, Clauser and Holt published another noticeable paper in which they raised the first doubts about additional assumptions required for the incompatibility theorems, such as Bell's theorem, to work [10]. In particular Clauser and Horne addressed directly the fair-sampling assumption, which it will be seen in the next chapter to be responsible for the detection loophole.

The so called CH74 inequality is fair-sampling-free, that means no additional fair-sampling assumption needs to be made to plug the leak of the detection loophole, as also no-click events are implicitly considered [2] in the correlations inequality

$$-1 \leq p_{12}(a,b) - p_{12}(a',b) + p_{12}(a,b') + p_{12}(a',b') - p_1(a') - p_2(b) \leq 0$$

[2] The derivation of this inequality starts by considering also non detected photons- see original article for details [10]

Here $p_{12}(a,b)$ stands for the coincidence probability of a pair event detection respectively in the a-set detector 1 and in the b-set detector 2. Instead, $p_1(a)$ represents the single-count probability in detector 1 set with the a parameter. In addition, a N-independent form of the previous inequality can be given [11]

$$T = \frac{p_{12}(a,b) - p_{12}(a',b) + p_{12}(a,b') + p_{12}(a',b')}{p_1(a') + p_2(b)} \leq 1$$

where N is the number of particles emitted by the source. This is sometimes referred to as as *CH-inequality for ratio inequality* which can reveal even more useful in actual experimental use since N is quantity really difficult to measure without interfering with the system [10].
However, the CH74 inequality still needs an additional (though weaker) assumption of *no-enhancement*, which for maximally entangled photons based experiments asserts that event detecting counting rate without a polariser in place are at least equal to the same counting rates in presence of any direction set polariser [10].
Moreover, a CH-inequality based violation test requires for a minimum efficiency threshold (this argument will be better explored in the next chapter), that is of $\eta_{ch74} \geq 82.3\%$ [11], for maximally entangled states.
There are several works, though, that demonstrate this minimum required efficiency is lower for non maximally entangled states: one noteworthy is [13], in which the authors demonstrate that the actual minimum efficiency threshold for CH-based Bell tests is reached for almost product states.

Chapter 3

Loopholes

3.1 Introduction to loopholes problem

There have been several experimental tests that already proved Bell's violations in many different configurations and on many different systems. However none of these gave a conclusive demonstration of local realism inconsistency with quantum behavior. In fact, the entire history of Bell tests experiments since nowadays is affected by some important leaks exploiting which Nature could in principle be able to trick us and fake local realism violation. That is, since now nobody can claim for sure quantum correlations special infraction of locality and realism principles are 100 % proved. Of course, there are no surprises to be really expected from an ideal experiment which would close simultaneously all of the loopholes that could affect an ultimate test. However, such an achievement would be important not only for scientific accuracy of the theory, but also in order to neutralize possible flaws in quantum communication protocols deriving from loopholes that could be used by eavesdroppers to undermine the security of the protocol. In the following chapter the main types of loopholes will be presented and eventually some of the solutions proposed for a loophole-free configuration will be briefly discussed.

3.2 Locality loophole

This loophole consists basically on the violation of the *outcome* (in short, $P(A|\lambda, a, b, B) = P(A|\lambda, a, b)$), or *setting independence* ($P(A|\lambda, a, b) = P(A|\lambda, a)$) principles. There could be the possibility of a signal transmitting information from one measurement location to the other, thus counterfeiting the quantum violation. The same could happen in principle between the setting choice point in spacetime and the distant measurement point: a signal transmitting information at a speed equal or smaller than the light one could forge the violation even in this case. The best way to deal with this loophole is to address to special relativity laws. By simply spatially separating each measurement on one part of the system from the other and from the distant measurement setting choice, the job is done: no classical physical signal can - because of the causal structure of spacetime, according to General Relativity- in principle transmit informations in spatially separated regions. A way to exploit such a spatial separation in an real experiment is to randomize the setting choices and, meanwhile, place the two random switching devices at such a distance that light would take longer to travel it than the switch of setting to happen. [5] However, the true randomness of the switching algorithms can be questioned as, in fact, in most cases they are periodic and thus only *almost random.* Is is clear now that the issue is moving on the random source employed and in principle one could never stop asking whether the source is truly random or is included in a wider predetermination that could fake the quantum correlations.

3.2.1 Superdeterminism

Quantum random numbers generators could be suggested as solutions for the random switch of setting, but someone could argue that even those quantum processes could be predetermined at a hidden level. Using human pure arbitrary choice or even the signals captured from two galaxies located very far one from another in the universe could be suggested either, but even there the ghost of a superdeterministic theory could in principle still be a threat to quantum theory [3]. However, such a possibility would be indeed very conspiratorial and counterproductive to support it, as it would imply very little chances of correctly interpreting nature's laws.

3.3 Detection loophole

Real detecting devices never reach the ideal 100% efficiency, due to their limited performance. Moreover, natural losses can occur during the transmission of the entangled particles from the entanglement source to the measurement devices. For this reason, in every Bell there is in fact an additional outcome, the "no-click" event (symbolized by $\perp$), and there are two possible ways of processing this outcome: either by counting it along with the "+" and "-" standard outcomes in the calculations or by discarding each no-click event and considering only the "valid" + and - . These two different approaches are necessary to prevent the possibility of a LHV theory that could fake the violation of a Bell inequality : this possibility represents the detection loophole.

3.3.1 fair-sampling and post-selection

Conducing a conclusive Bell test based only on the "proper" + or - outcomes without an ideal detection efficiency necessarily requires the so called *fair-sampling* assumption: the revealed set of events is a fair sample, that is a well representative subset of the total events a 100%-efficient experimental setting would have counted. However, there have been proposed some LHV simple models which are able to counterfeit the quantum correlations violation of the inequalities just by exploiting the low efficiencies of the detectors [6].
In fact, it can be shown that an actual efficiency threshold exists, below which Bell's inequalities violation can be faked by the local models mentioned above. In particular, it has been demonstrated that the minimum efficiency required, in a CHSH inequality test

in which no-click events are discarded, in order to close the detection loophole is $\eta = 2/3$ [3].

3.3.2 no-click as an additional valid outcome

Even in the more elegant solution of considering $\perp$ events as valid outcomes, a minimum efficiency threshold can be evaluated. Following the argument presented by Brunner [3], the minimum efficiency required in a CHSH test with two maximally entangled particles. If we assume that our observers, Alice and Bob, have imperfect detectors with η efficiency, the no-click event probability will be $1 - \eta$. And let them decide to pick the $+1$ result each time a no-click event occurs. In ideal conditions, the quantum correlated system would give a result for the S quantity of $S = 2\sqrt{2}$. But in our imperfect conditions, this would happen only with probability η^2, that is the probability of the two detectors to click. On the other hand, if both detectors don't click, with probability $(1 - \eta)^2$, the outcomes will be completely uncorrelated, leading thus to the classical result of $S = 2$. If our observers still hope to violate the local bound, they should achieve:

$$\eta^2 2\sqrt{2} + (1 - \eta)^2 2 > 2$$

which translates into

$$\eta > \eta* = \frac{2}{1 + \sqrt{2}} \approx 83\%$$

This result stands for maximally entangled systems with a defined Hilbert space of dimension 2. However it can be shown that the violation threshold can be lowered when considering partially entangled states, e.g. $|\Psi_\theta\rangle = \cos\theta|00\rangle + \sin\theta|11\rangle$, for which in the limit $\theta \to 0$ the relation $\eta* = 2/3$ is valid. Moreover, it can also be demonstrated that this threshold can be lowered arbitrarily close to zero by using Bell inequalities for systems in higher dimensional Hilbert spaces [3].

3.4 Freedom-of-choice loophole

There is another important assumption which is commonly required in a local model derivation: the *freedom-of-choice* (or measurement independence) assumption. Apart from asking for the measurement settings and single measures to be independent, it is essential to ask for the measurement setting choices to be independent from the hidden variable(s) λ. That means, the validity of the following formula is crucial in the discussion of each Bell inequality:

$$P(\lambda|x, y) = P(\lambda)$$

the meaning of which is more clear after invoking the Bayes theorem ($P(x, y|\lambda)P(\lambda) = P(\lambda|x, y)P(x, y)$):

$$P(x, y|\lambda) = P(x, y)$$

In a nutshell, what is here required is that the observers should be truly free to choose the measurement to be conducted on the A and B particles without being, in some way, deceived by the hidden factor. To avoid statistical dependence between the settings and the hidden factor, one simple solution is to space-like separate the generation of the entangled states from the measurement setting choice.

3.5 Coincidence-time loophole

The following loophole is based again on a failure of a "minor" assumption implicitly made in some kind of experimental Bell's inequalities violation tests. Specifically, the assumption here addressed is the *fair-coincidence*, which is similar somehow to the fair-sampling assumption: it is generally assumed that the successfully revealed pairs are well representative of the entire set of pairs that would have been detected in ideal conditions of efficiency. This loophole addresses especially those experimental settings in which almost coincidental arrival times are used to identify the pairs of particles to be measured. It opens, generally, when the detection time depends not only on the hidden factor but also on the shifts in the local measurement setting: a if changes in setting have repercussions on the detection time, then naturally the number of correctly identified pairs changes, possibly invalidating the test. A satisfactory way to deal with this loophole is to implement *locally predefined time-slots or a window-sum method* [4].

3.6 Memory loophole

Also essential for every experimental test of Bell's inequalities is the no-memory assumption: it consists in the requiring successive measurements to be independent and identically distributed. That means the memory loophole gets opened every time the n-th measurement on A (or B) is not space-like separated from the *(n-1)*-th one made either on A or B, thus letting available the possibility of a statistical dependency of the n-th result from either the previous results and settings, just as if the experimental setting remembered the past runs of the test. The consequences are not sensible in terms of adjustments of the local bound or of efficiency thresholds, but yet this loophole implies the necessity of a wider significance interval of sigma value in order to assess the experimental results are distant enough from the local realistic bound [4]. Thus, the only solution to close this loophole is to achieve a wide enough violation of the mentioned bound such that the LHV theory hypothesis can be safely rejected.

3.7 Proposed loophole-free tests

Two are the main loopholes that need to be closed and seem they can be only one at a time, as well as two are the main roads that have been trodden until now in the attempt of achieving an ultimate loophole-free test, that means the closing of all the loopholes simultaneously. One involving photonic entangled quantum states, because of the photons natural bent to be used to close locality loophole. The other is represented by the use of atomic entangled systems, because of the high detection efficiency reached by detectors which make the atoms good candidates for the detection loophole closing. However both approaches reveal some experimental issues and challenges which still need to be successfully managed.

3.7.1 Photons

The use of photons as entangled systems brings as main advantage the relatively easy implementation of a space-like separation between the A and B parties. However modern technologies only recently allowed superconducting detectors to reach a 95% of detection efficiency, but still the use of these apparata has been employed only in limited environments where the distance between setting components needed to close the locality loophole have never been reached; a required 300 m, approximately, distance is required since the detection process when using this type of detectors is relatively slow, of the order of 10^{-6} seconds [3].

3.7.2 Atoms

Atoms entanglement, achieved with the entanglement swapping technique, instead reveals to be an optimal solution to close the detection loophole as almost 100% efficiency can be reached in detecting atoms. By absorption of two entangled photons, two distant atoms can become as well entangled thus opening a promising way of closing simultaneously the locality and detection loopholes. Until now there has not been reported any decisive result, though.

3.8 Loophole-free candidate experiment

Recently, an impressive work has been reported which seems, until now, the best loophole-free candidate experiment [12]. In this electron spin-based experiment all locality, detection and freedom-of-choice loopholes were meant to be closed, while violating a CHSH inequality, $S \leq 2$, without any further assumption, by obtaining $S = 2.42 \pm 0.20$, which stands for a probability that LHV to produce this results of $p = 0.039$ [12].

Chapter 4

Experimental results

In the experimental part of this thesis, the aim of the candidates work was to execute two violation tests with photons entangled in two different degrees of freedom: time-bin DOF and polarization. In order to successfully make the incompatibility between local realism and quantum entangled systems stand out, it was necessary to assume some of the previously mentioned additional assumptions: fair-sampling, freedom-of-choice and no-memory mainly.
In both experiments a pulsed laser source in the near ultraviolet region of the spectrum is employed as a photon pump. In the SPDC crystal the ultraviolet photons are converted in entangled pairs of red-wavelength photons. After the second order crystal, the quantum state measures are operated on the entangled pairs by Michelson interferometers and polarization analysers respectively. Eventually, the photons are collected by detectors (photodiodes) which count the incoming entangled pairs for each polarization or phase parameter choice.

4.1 Time-bin entangled photons

4.1.1 Measurement description

The measurement consists, from the quantum theory point of view, in a projection on the states given in paragraph 3.5.1, following the procedure there explained. From the physical point of view it consists in the use of an additional Michelson unbalanced interferometer positioned between the SPDC crystal and the detectors. This is the instrument responsible for the state projection. The measurement parameters, in other words, the $\alpha(\beta)$ phase, are controlled by modifying the length of one arm of the measuring interferometer.
The registered photon counts are of three types: SS, LL and SL. The interesting pairs are of course the SL type, which correspond to the entangled pairs, in which the actual path followed by the photons is undetermined.

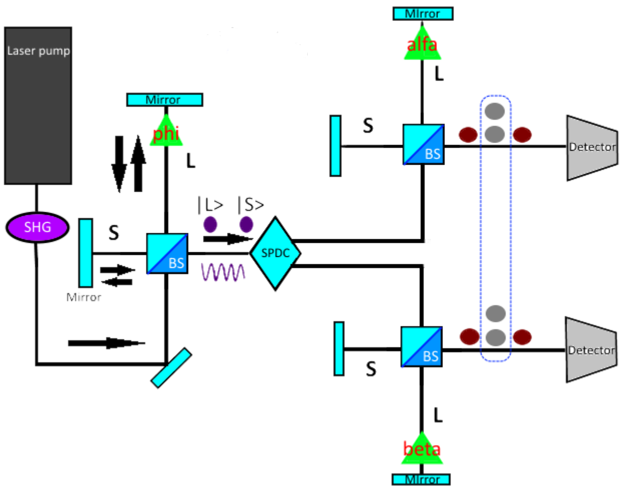

Figure 4.1: Experimental setup scheme. The green triangles represent the phase shifters. The violet and red curve lines are a pictorial representation of the SPDC frequency-matching. The violet dots before the SPDC crystal are the two components of the pump photon quantum state. Analogously, after the crystal the red dots represent the SS and LL pairs. The grey dots are the undetermined couples, the ones that need to be counted. In this setup the BS crystals are half-reflecting mirrors.

4.1.2 Results analysis

The measurement obtained in this phase of the experiment returned the following coincidence rates

	$(0, \frac{\pi}{4})$	$(\frac{\pi}{2}, \frac{\pi}{4})$	$(0, \frac{3\pi}{4})$	$(\frac{\pi}{2}, \frac{3\pi}{4})$
$(+,+)$	408	387	375	92
$(-,+)$	75	76	67	344
$(+,-)$	69	78	56	358
$(-,-)$	381	402	387	111

These are the coincidence rates for the 4 x 4 configurations of the channel A and B interferometers, the interferometer phases being $\alpha \in \{0, \frac{\pi}{2}\}$ and $\beta \in \{\frac{\pi}{4}, \frac{3\pi}{4}\}$. As just said before, the experimental setting employs Michelson interferometers and thus, for convenience reasons, the apparatus has been built with two outputs (instead of the standard 4, as in other classic works). Due to this choice in the original preparation of the experimental apparatus, the violation measure requires to reacquire all the measurements after rotating the previously listed phases by a π quantity for the sake of calculating the S quantity.
The expectation values for each choice of (α, β) is given by

$$E(\alpha, \beta) = \frac{C(+,+) + C(-,-) - C(-,+) - C(+,-)}{C(+,+) + C(-,-) + C(-,+) + C(+,-)}$$

where $+$ stands for the proper phase value while - for its orthogonal value.

$E(0, \frac{\pi}{4})$	$E(\frac{\pi}{2}, \frac{\pi}{4})$	$E(0, \frac{3\pi}{4})$	$E(\frac{\pi}{2}, \frac{3\pi}{4})$
0.69 ± 0.02	0.67 ± 0.02	0.72 ± 0.02	-0.55 ± 0.03

The last step is the calculation of the S$(0, \frac{\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4})$ value. S is constructed following the formula:

$$S(0, \frac{\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}) = E(0, \frac{\pi}{4}) - E(\frac{\pi}{2}, \frac{\pi}{4}) + E(0, \frac{3\pi}{4}) + E(\frac{\pi}{2}, \frac{3\pi}{4})$$

with which the following results have been achieved:

$S_{measured}$	σ_S	S_{LHV}	LHV compatibility
2.64	0.05	2	13

4.2 Polarization entangled photons

4.2.1 Description of the measures

The measurements are conducted by means of two polarisation analyzers composed, in fact, by a polarisation beam splitter, PBS, and a half-wave plate, $\frac{\lambda}{2}$, set between the

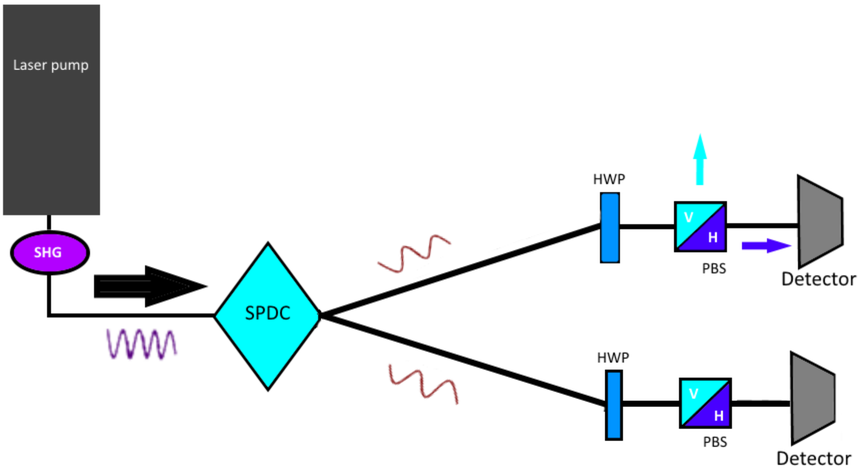

Figure 4.2: The scheme of the polarization experimental setup is represented. The main components are the pulsed laser pump and the SHG crystal upraising the laser pulses to 400nm. The SPDC crystal generating the entangled pairs, the half-wave plates and the beam splitters, followed by the detectors.

SPDC entanglement source and the detectors. The PBS is a crystal which separates the differently polarized components of an entering beam, thus representing actually a polarisation state projector. The half-wave plates, instead, are used to rotate the polarization state of linearly polarized radiation. Or, equivalently, rotates the axis along which the PBS executes the measure. The polarization parameters of these analyzers were manually set to the values $\{(0,\frac{\pi}{8}),(\frac{\pi}{4},\frac{\pi}{8}),(0,\frac{3\pi}{8}),(\frac{\pi}{4},\frac{3\pi}{8})\}$, where the factor $\frac{1}{2}$ from the parameters values set used in the previous test is due to the fact that in these experiments half-wave plates have been used: a 2π phase shift is obtained after manually rotating the polariser by π radians.

4.2.2 Results analysis

The coincidence rates measured are collected in the following table:

$C(\alpha,\beta)$	$\alpha=0$	$\alpha=\frac{\pi}{4}$	$\alpha=\frac{\pi}{2}$	$\alpha=\frac{3\pi}{4}$
$\beta=\frac{\pi}{8}$	717	558	3304	3388
$\beta=\frac{3\pi}{8}$	3414	3321	663	606
$\beta=\frac{5\pi}{8}$	3415	659	522	3383
$\beta=\frac{7\pi}{8}$	575	3331	3289	625

Using these experimental coincidence rates, the expectation values have been calculated, following the formula:

$$E(\alpha,\beta) = \frac{C(0,\frac{\pi}{8}) + C(0',\frac{\pi}{8}') - C(0',\frac{\pi}{8}) - C(0,\frac{\pi}{8}')}{C(0,\frac{\pi}{8}) + C(0',\frac{\pi}{8}') + C(0',\frac{\pi}{8}) + C(0,\frac{\pi}{8}')}$$

Where the ' symbol stands for the orthogonal of the indexed polarization angle. The result are:

$E(0,\frac{\pi}{8})$	$E(\frac{\pi}{4},\frac{\pi}{8})$	$E(0,\frac{3\pi}{8})$	$E(\frac{\pi}{4},\frac{3\pi}{8})$
-0.659 ± 0.008	0.704 ± 0.008	0.719 ± 0.008	0.679 ± 0.008

S is constructed following the formula:

$$S(0,\frac{\pi}{4},\frac{\pi}{8},\frac{3\pi}{8}) = E(0,\frac{\pi}{8}) - E(\frac{\pi}{4},\frac{\pi}{8}) + E(0,\frac{3\pi}{8}) + E(\frac{\pi}{4},\frac{3\pi}{8})$$

Eventually obtaining:

$S_{measured}$	σ_S	S_{LHV}	LHV compatibility
2.76	0.02	2	47

Chapter 5

Conclusion

Both the conducted experiments gave satisfactory results that confirm theoretical predictions. In both experiments the Bell inequalities were violated with significant σ-distance from the local bound. For the test in polarization the precision achieved is considerably finer than the one reached in the time-bin measurement, since the quantum correlations deviate from the predicted local bound by a 46 σ-distance; this was mainly due to the fewer photon losses in the polarization setup (e.g. less half-reflecting mirrors were involved) and thus a higher number of photons were available for the detectors to collect. Nevertheless, the experimental setup was necessarily unable to close the loopholes that affect this kind of experiments, and hence the experiments both require additional assumptions to be able to claim a successful violation.

With regard to the *locality loophole*, neither of the two experiments conducted had it successfully closed as the measurement settings were predetermined and fixed; in fact, the two parameter switch events were never thought to be spatially separated.

As for the *detection loophole*, neither this loophole has been closed. Unfortunately, due to not optimal setting of the apparata (mainly not ideal laser pump efficiency) the required efficiency has not been reached, exposing the experimental test to LHV theories exploiting the unfair sampling.

Further on, even the freedom-of-choice assumption is in these tests necessary to correctly certify the violation: no spacelike interval has been set between the source of entangled photons and the measure-parameter settings. Thus it is impossible to be 100 % sure that the choice of the measurement settings is independent of the hidden variable(s) λ.

The polarization entanglement and the time-bin test are, with respect to the fair-coincidence assumption, immune to the time-coincidence loophole thanks to the particular laser pump used in these experiments: the pulsed laser grants the possibility to know exactly at which time we expect the photons to arrive. This furnishes a clock following which true entangled pairs can be distinguished from unfair coincidences.

Eventualy, also the memory loophole is on the list of the loopholes affecting the conducted experiments: each single measure was obviously not spacelike separated from the previous ones, allowing for their influence in the subsequent measures.

In conclusion, this work has again confirmed the quantum theory's predictions and the Bell inequalities violation by entangled systems. Indeed, it was not possible to avoid the necessity of additional assumptions. The closing of all those loopholes and thus the achievement of the ultimate proof of quantum nonlocality incompatibility with local realistic hidden variable models requires more complicated setups and non-trivial solutions, like in [12].

Ringraziamenti

Un ringraziamento va prima di tutto al Prof. Giuseppe Vallone che mi ha seguito durante tutto il periodo della tesi, sopportando tutti i miei dubbie le mie domande e trovando sempre la pazienza per rispondere anche a quelle più strampalate e sconclusionate.
Desidero ringraziare anche il prof. Paolo Villoresi, per avere per primo risposto alle mie curiosità riguardanti la *Quantum Information* e per avermi dato la possibilità di intraprendere questo percorso di tesi indirizzandomi verso il prof. Vallone, e Marco Tomasin, per la pazienza dimostratami nei pomeriggi di laboratorio.
Devo necessariamente ringraziare i miei amici, Paolo, Marcello, Matteo, Stefano, Eugenio e Giorgio e tutti gli altri, con i quali ho condiviso dei fantastici momenti di evasione e dai quali ho sempre avuto ottimi consigli.
Ringrazio mia madre, mio padre e i miei nonni, la mia famiglia, che hanno fatto così tanto per permettermi di essere qui.
Infine, ringrazio la persona che in questi tre anni non è mai mancata, tanto nei momenti più bui quanto nelle ore più felici.
Grazie Adele.

Bibliography

[1] A. Einstein, B. Podolski, N. Rosen, *Can Quantum-Mechanical description of physical reality be considered complete?*, Phys. Rev. 47 (777), 1935.

[2] J.S. Bell, *On the Einstein-Podolski-Rosen paradox*, Physics 1 3, 195-200, Nov. 1964.

[3] N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, S. Wehner, *Bell nonlocality*, Rev. Mod. Phys. 86, 419 – Published 18 April 2014; Erratum Rev. Mod. Phys. 86, 839 (2014)

[4] Johannes Kofler, Marissa Giustina,*Requirements for a loophole-free Bell test using imperfect setting generators* arXiv:1411.4787 [quant-ph], submitted 18 Nov 2014

[5] A. Aspect,P. Grangier and G. Roger, *Experimental realization of Einstein-Podolski-Rosen-Bohm Gedankenexperiment: a new violation of Bell's inequalities*, 1982, Phys. Rev. Lett. 49, 91.

[6] E. Santos, *Critical analysis of the experimental tests of local hidden-variable theories*, 1992, Phys. Rev. A 46, 3646.

[7] Giuseppe Vallone, Paolo Mataloni, *Advances in Atomic, Molecular and Optical Physics*, Vol 60, pg.291-314.

[8] Michel Le Bellac, *Quantum Physics*, Cambridge University Press, 2006.

[9] John F. Clauser, Michael A. Horne, Abner Shimony, Richard A. Holt ,*Proposed experiment to test local hidden-variable theories*, Phys. Rev. Lett. 23 (15) , 1969.

[10] John F. Clauser, Michael A. Horne, *Experimental consequences of objective local theories*, Phys. Rev. D 10, 526 , 1974

[11] Andrei Khrennikov, Sven Ramelow, Rupert Ursin, Bernhard Wittmann, Johannes Kofler, Irina Basieva, *On the equivalence of Clauser-Horne and Bernhard inequality based test*, arXiv:1403.2811 [quant-ph]

[12] B. Hensen et al. , *Experimental loophole-free violation of a Bell inequality using entangled electron spins separated by 1.3 km*, arXiv:1508.05949 [quant-ph], 24 August 2015.

[13] Giuseppe Vallone, Gustavo Lima, Esteban S. Gomez, Gustavo Canas, Jan-Ake Larsson, Paolo Mataloni, Adan Cabello, *Bell scenarios in which nonlocality and entanglement are inversely related*, Phys. Rev. A 89, 012102, 2014.