A Solution for the Hubble Tension: Late Phantom Energy

Author: Sergi Novell Masot
Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.

Advisor: Eduard Salvador Solé

Abstract: Although cosmology is entering the precision era, there is a quantity for which its measurements are not converging to an exact value, but rather the opposite: the Hubble Constant (H_0). Because its incompatible values stem from observations of the early and late Universe, respectively, in this work we propose a phantom equation of state ($\omega < -1$, ω constant) for dark energy, which could change the Universe evolution in such a way that the two values would be compatible. We will calculate a value of ω that could solve this tension, making the Universe dynamics start to deviate from the ΛCDM standard model at $z \sim 1$. Furthermore, we will study the problems that arise from this exotic dark energy and review alternative theories that also address the Hubble tension. We will solve numerically the Friedman equations to find the Universe scale factor evolution over time, for both phantom and Λ dark energy types.

I. INTRODUCTION

It is a well-known fact that the Universe is not static, but expanding: the general theory of relativity, together with its solutions found by Friedman, set the theoretical grounds upon which the observations were to be able to determine the various parameters that characterise the dynamics of the Universe. And, within all these parameters, there is a quantity that is particularly important: the Hubble constant, H_0, which is namely the expansion rate of the Universe, $H(t)$, at present.

Thanks to the ground-breaking observations by Riess [1] and Perlmutter [2], it was discovered that the expansion of the Universe was accelerating, thus giving birth to our current Λ-Cold Dark Matter (ΛCDM) standard model. In that model, this acceleration is driven by a cosmological constant energy component, Λ, that will cause H_0 to become a constant in the future. This is interesting because, although labeled as "constant", $H(t)$ is a quantity that evolves with time, except precisely if a cosmological constant dark energy is dominant.

In any case, there is a problem: the value of H_0 has been measured using different techniques, and the results so far have not been consistent. The two main methods used are the Cosmic Microwave Background (CMB) and the identification of Type 1a Supernovae (SN1a). With each method being increasingly precise as the experimental errors are being reduced, their discrepancy not only does not disappear, but is instead gaining relevance.

After inflation, the Universe was in a state of hot plasma, that was highly homogenous and with very little deviations from the density and temperature means. By analysing the picture that was left after the decoupling of photons, the CMB, the value of H_0 can be inferred. This is accomplished by taking the power spectrum of the temperature anisotropies in every direction and correlating it to a curve that contains information of several cosmological parameters, including H_0, and relationships between them. The latest value for that quantity, coming from the Planck satellite, is $H_0 = 67.36 \pm 0.54$ [3].

A more direct way of inferring H_0 is measuring the rate at which other galaxies are going away from us, by correlating their distance with their redshift. There have been a handful of relevant ways to do that, but the one that has been studied and modelled the most is the SN1a. It is based on the fact that a Type 1a Supernova always has the same luminosity, given by the Chandrasekhar mass limit of white dwarfs, so by comparing it to its detected brightness, the distance can be calculated. By taking a sufficiently large sample of them, the expansion rate of the Universe is obtained statistically. The problem, or crisis (as is regarded by some), is that the value obtained by SN1a is 74.03 ± 1.42 [4], which differs by more than 4σ with the CMB measurement. This is known as the Hubble tension.

In this work we are going to review some theories that propose specific modifications to the ΛCDM model so that this discrepancy between measurements of H_0 is explained and solved. We will put special emphasis on the Late Phantom Energy solution, which suggests a change in the nature of dark energy, manifested in a higher acceleration of the expansion rate at low redshifts. This, as we will see, results in a higher value of the H_0 inferred from the CMB, thus easing the tension.

II. GOING PHANTOM

The CMB method is based on data from the early Universe ($z \sim 1100$), while SN1a deals with the late Universe ($z \lesssim 1$). Also, while the CMB method calculates the present value of H assuming that the whole Universe evolution has followed ΛCDM hypotheses, SN1a is based on astronomical observations, so the ΛCDM accepted value for H_0 is the one coming from CMB. This is crucial, because one may ask then if this tension may be an indicator that the ΛCDM model needs to be rethought as it may have failed somehow to describe the evolution of the Universe.

As for other ways to obtain H_0 from the early Universe, like the Baryon Acoustic Oscillations method (BAO), they are statistically consistent with the mentioned CMB result, and so are the ones from the late Universe (lensing, quasars. . .) with the SN1a, to some extent. A conservative approach is blaming this discrepancy between H_0 drawn from observations at different redshifts to systematic errors, primarily those concerning the accuracy in measuring distances. This is not the object of this work, but rather pointing at some new physics that could explain this phenomenon, so we will assume that both values are correct by themselves.

A. The Equation of State of Dark Energy

If the present acceleration is higher than what could be expected from fitting to the CMB anisotropies, what could be the physical cause? There have been some solutions proposed so far, and we are going to center on one that involves the so-called phantom energy, at small redshifts. Let us introduce phantom energy first. As we have said before, the evolution of the Universe, after the assumptions of homogeneity and isotropy, can be derived from General Relativity in the Friedman Equations. However, there is a missing piece to be able to fully use them: the relationship between pressure and density. Namely, the equation of state (EOS), usually expressed as $\omega = p/\rho$ (in natural units).

We will not expand a lot here, but will just say that the ΛCDM standard model gives to dark energy the constant value $\omega = -1$. This value means that the density of energy remains constant, thus making the dark energy contribution larger as the Universe expands (as matter density decreases in that scenario). In a Universe with a global EOS of $\omega = -1$, also called De Sitter Universe, the Hubble parameter is constant over time, so we can say that ΛCDM model predicts the Hubble parameter to be approaching a constant value. Phantom energy, in turn, has an EOS of $\omega < -1$, so if the Universe were to have a phantom EOS, when it expanded, the density of this energy would grow, along with the Hubble parameter. This is key, as it could be the cause for the fact that the late Universe derived calculation of H_0 is higher than the one from the early Universe.

To further support this idea with a more involved treatment, it is found in [5] that statistically combining the data from both early and late Universe results in $\omega < -1$ with more than 1σ probability in all cases, thus making the Hubble Constant values compatible. This is considering ω constant, hypothesis that is not so clear should it cross to the phantom domain. In any case, the fact that a phantom EOS can be consistent with the observational data is certainly interesting and is nowadays a subject of ongoing study. This is exactly our purpose in this work, and we will start by working on an analytic phantom solution to the Hubble tension, for the case where ω is constant.

B. Phantom Solution with Constant ω

The Universe is a rather complex system, even at cosmological scales. Matter, radiation, dark energy. . . all have a part in the cosmic picture, and it is clear by just taking a quick look at the Friedman equations that our desired analytic solution cannot be found by taking into account all of them. Thus, the appropriate approximations must be made, together with the assumption of constant ω. We know that, from $z \sim 1$ to our days, the energy in the Universe comes from the contributions of matter and an ever-increasing dark energy, with the radiation and curvature contributions close to none ($\rho_{rad} \approx 0$, $\epsilon \approx 0$). As we are working on the nature of dark energy, and its contribution can be deemed irrelevant before $z \sim 1$, we will greatly simplify our quest by assuming that the Universe is well described by the ΛCDM model until the turning point of $z = 1$. From then on, it is safe to say that the dynamics of the Universe are governed only by matter and dark energy. The key point is that, as we have said, if the Universe expansion were to be due to phantom dark energy, then the Hubble parameter would be higher than what is inferred from ΛCDM. That means that the fact that the SN1a value for H_0 is higher than expected from the CMB could have been caused by the extra acceleration, comparing with ΛCDM model, given by the phantom energy.

Our procedure to quantify this effect will consist in the following: starting at the ΛCDM derived value of the Hubble constant, $H_{0\Lambda} = 67.36$, we will obtain the correspondent value of $H_\Lambda(z = 1)$, assuming $\omega = -1$. At that moment, we impose that H_{ph} and H_Λ coincide, and the same for Ω_{ph} and Ω_Λ. After that, with a constant value of $\omega < -1$, we will find the required value of ω so that $H_{0ph} = 74.03$, which corresponds to the observations from SN1a. All calculations will be based on the Friedman equations, that are:

$$\frac{\ddot{a}}{\dot{a}} = -\frac{4}{3}\pi\left(\sum \rho_i + 3\omega^i\rho_i\right) \tag{1}$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\sum \rho_i - \frac{\epsilon}{a^2} \tag{2}$$

where ρ_i and ω^i are the density contribution and equation of state of each component of the Universe (matter, dark energy...). Regarding densities, it is more convenient to use instead the density parameters: $\Omega_i = \frac{\rho_i}{\rho_c}$, where the term $\rho_c = \frac{3H^2}{8\pi}$ is the critical density of the Universe so that it has zero curvature. We know that in our Universe ($\Omega_m = 0.32$ [3]) the density of dark energy accounts for $\Omega_\Lambda = 1 - 0.32 = 0.68$, and we want to express the Universe evolution in terms of these present density parameters. After solving the Friedman equations for each component ρ_i, we are led to

$$\rho_i = \rho_{i_0} \left(\frac{a_0}{a}\right)^{3(1+\omega_i)} \quad (3)$$

as seen in [6], where the matter EOS is $\omega = 0$. Using this, the following simple calculations give us the energy densities at $z = 1$ (equivalently, $a = (1+z)^{-1} = \frac{1}{2}$):

$$\begin{cases} \rho_{m,\frac{1}{2}} = \rho_{m_0} \cdot 2^3 \\ \rho_{\Lambda,\frac{1}{2}} = \rho_{\Lambda 0} \cdot 1 \end{cases}, \text{ where } \begin{cases} \rho_{m_0} = 0.32 \cdot \frac{3}{8\pi} H_{0\Lambda}^2 \\ \rho_{\Lambda 0} = 0.68 \cdot \frac{3}{8\pi} H_{0\Lambda}^2 \end{cases}$$

It follows that

$$\Omega_{m,\frac{1}{2}} = \frac{\rho_{m,\frac{1}{2}}}{\rho_{c,\frac{1}{2}}} = 0.32 \cdot 2^3 \cdot \frac{H_{0\Lambda}^2}{H_{\frac{1}{2}}^2} \quad (4)$$

$$\Omega_{\Lambda,\frac{1}{2}} = \frac{\rho_{\Lambda,\frac{1}{2}}}{\rho_{c,\frac{1}{2}}} = 0.68 \cdot \frac{H_{0\Lambda}^2}{H_{\frac{1}{2}}^2} \quad (5)$$

Knowing the density parameters at $z = 1$, for $\omega_\Lambda = -1$, we will go forward in time from there, assuming now that the dark energy is phantom. The second Friedman equation can be written as

$$H^2 = H_0^2 \sum \Omega_{i_0} \left(\frac{a_0}{a}\right)^{3(1+\omega_i)} \quad (6)$$

By now setting the initial time as the correspondent to $z = 1$ instead of $z = 0$, we have that:

$$H_{ph}^2 = H_{\frac{1}{2}}^2 \left[\Omega_{m,\frac{1}{2}} \left(\frac{\frac{1}{2}}{a}\right)^3 + \Omega_{ph,\frac{1}{2}} \left(\frac{\frac{1}{2}}{a}\right)^{3(1+\omega_{ph})}\right] \quad (7)$$

By the Equations (4) and (5), for $a = 1$, we have that

$$H_{0ph}^2 = H_{0\Lambda}^2 \left[0.32 + 0.68 \cdot 2^{-3(1+\omega_{ph})}\right] \quad (8)$$

The value of ω that reconciles the Hubble tension will be found for H_{0ph}=74.03 and $H_{0\Lambda}$=67.36:

$$\omega_{ph} = -\frac{ln\frac{\left(\frac{74.03}{67.36}\right)^2 - 0.32}{0.68}}{3ln(2)} - 1 = -1.12 \quad (9)$$

This value falls within the statistical results obtained in [5], and despite the outcome is relevant only insofar as it comes from an approximation, the value for ω_{ph} is interesting, especially for it being close to -1. That means that the theoretical extreme behaviour of phantom energy, that will be reviewed later on this work, is not so radical. If the Universe were to have an EOS given by the obtained result, during a large period of time it would not differ qualitatively much from the ΛCDM model. To see this better, we have obtained the growth of the scale factor, $a(t)$, for both phantom and Λ models, from $z = 1$ (Figure 1). For that, we have used the Euler method to solve numerically the Friedman equations for $\omega_\Lambda = -1$ and $\omega_\Lambda = -1.12$:

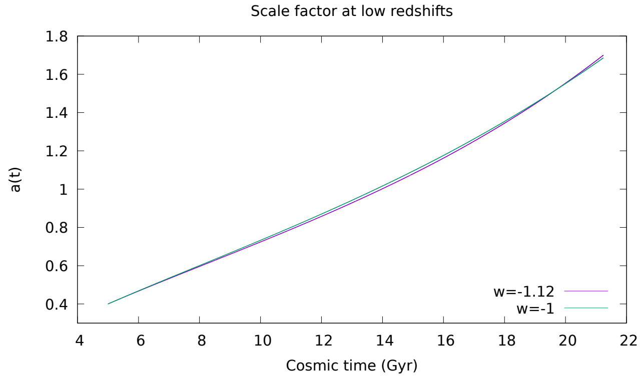

FIG. 1: Evolution of the scale factor for phantom ($\omega_{ph} = -1.12$) and Λ ($\omega_\Lambda = -1$) dark energies, from $a \sim \frac{1}{2}$ (or $z \sim 1$) until $a \sim 1.7$. We can see from the plot that there is very little difference in a(t) between the phantom and ΛCDM models in the past. This is a good thing, because, although sufficient for making the Hubble Constant values come to an agreement, the value of ω_{ph} does not result in an a(t) departing markedly from the standard model, which is well-tuned to the observations.

So far, we have only seen benefits of the Universe possibly having a phantom EOS. However, as we mentioned before, there are some caveats, that must be acknowledged and can become problematic if they are not sorted out.

III. PROBLEMS OF A PHANTOM EOS

One of the reasons that dark energies in the phantom realm have been investigated so little until lately is that they have a number of problems, that are seemingly irreconcilable with the current standard models. These issues stem from the very fundamental nature of this kind of energy and the fact that its origin and characteristics are still relatively unknown.

The first problem one encounters is when asking an elementary question to all of it: where does it come from? The answer is the habitual "we do not know" one gets when addressing dark energy, but is in fact worse than usual. In the case of $\omega = -1$, for example, dark energy has been related to the cosmological constant Λ and to the constant vacuum energy that experimentally is obtained by the Casimir effect. Despite the controversy in this hypothesis, the well-known Cosmological Constant Problem, it is at least a possible real physical interpretation of the mathematical deduction that has been made out of the observations. For a phantom EOS, there is none, still. This issue, together with the next one, may suggest that, were it to exist, the phantom energy would not necessarily obey a constant EOS, but rather fluctuate over time and so it being able to cross the $\omega = -1$ frontier.

Another problem is that a phantom EOS violates the Null Dominant Energy Condition (NDEC), altogether with other stablished energy conditions for stability to be guaranteed (like the Weak and Strong Energy Conditions). As mentioned in [7], by imposing these energy conditions to our isotropic and homogenous Universe, it is derived that $\omega \geq -1$, which collides with our definition of phantom energy. This is not just a problem of formalism and definitions, for it is known that such violations could cause catastrophic vacuum instabilities and other phenomena that lie outside the standard models of physics.

To get a grasp of that, we will peek into the Quantum Field Theory, which is the most common approach to dark energy. This theory states, roughly, that every component of the Universe is an expression of its associated field. In the case of phantom energy, it is usual to subordinate it to a scalar field with a corresponding Lagrangian that has the particular feature that the kinetic term is negative. That means that the Hamiltonian would not be bounded from below, so an instability could push the Hamiltonian of the field to negative infinity. This theoretical singularity of the scalar field is interesting as it is somehow linked to the Big Rip, the predicted dramatic ending of a phantom Universe, that we are going to prove now. Particularly, we will see that, for a constant $\omega < -1$, the scale factor $a(t)$ grows to infinity in finite time.

First, we will see that the density of phantom energy grows with the expansion of the Universe, contrary to the case of mass and radiation. From the equation (3) and the fact that $\omega < -1$, it is clear that when the scale factor grows, the phantom energy density grows too. Hence, for a Universe where the phantom energy becomes dominant, it will continue to dominate. In the case where the energy of the Universe is mainly phantom, as would be our situation, it is safe to assume that the future expansion of the Universe will be driven just by the contribution of phantom energy.

Hence, we can approximate the current Universe energy as its phantom contribution ($\Omega = \Omega_{ph} = 0.68$) when we are to make calculations for the future, greatly simplifying equation (6) to:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \Omega_{ph} a^{-3(1+\omega)} \tag{10}$$

By rearranging the terms, and integrating between t_0 and an arbitrary t:

$$\frac{a^{-\frac{3}{2}(1+\omega)} - a(t_0)^{-\frac{3}{2}(1+\omega)}}{\frac{3}{2}(1+\omega)} = H_0 \Omega_{ph}^{\frac{1}{2}} (t - t_0) \tag{11}$$

which gives us the scale factor evolution,

$$a(t) = \left[\frac{3}{2}(1+\omega) H_0 \Omega_{ph}^{\frac{1}{2}} (t - t_0) + 1\right]^{\frac{2}{3}(1+\omega)} \tag{12}$$

As $(1+\omega) < 0$, if the expression below the power index on the right is equal to zero, then $a(t)$ grows to infinity. That will happen, for the ω we have found previously, when

$$t - t_0 = \frac{2}{3}|\omega + 1|^{-1} H_0^{-1} \Omega_{ph}^{-\frac{1}{2}} = 3.68 \cdot 10^{18} s \tag{13}$$

Thus, in a time of $1.16 \cdot 10^2$ Gyr from now (about ten times the age of the Universe, $t_0 = 13.8$ Gyr) the scale factor would blow up for a constant phantom EOS, leading our Universe to a singularity in finite time.

We can see that the time at which the singularity would occur is so distant in the future that the simplification of constant ω we have made in our first calculus is reasonable, for it applies to just a small window of time in the cosmic evolution ($0 < z < 1$). However, this simplification could become rather erroneous in calculations for larger periods, as also the problems of phantom we have reviewed seem to point to the fact that an EOS like this is prone to change over time.

To mention interesting studies in that direction, we have that the Big Rip is avoided when ω tends asymptotically to -1 in a given pace [8], or when it is allowed that the energy-momentum tensor is not conserved [9], among many other solutions. We find it illustrative to show the Big Rip graphically, by first numerically finding $a(t)$ as before, including the matter density term (Figure 2). Then, in Figure 3 we will compare our diverging phantom scale factor with the evolution of $a(t)$ at the ΛCDM pace.

Separately, there is another difficulty for the success of this phantom theory, that comes from a different source: the observational data. In this work, we have based upon the data of SN1a and CMB. However, it is deduced from the BAO data, which involves measuring the evolution of primordial sound waves over time, that the ΛCDM model cannot really be modified for, precisely, $z < 2$ [10]. That means that maybe our effort to reach an agreement between SN1a and CMB, mediating phantom energy, can be clashing with another valuable observational set, the BAO. Nonetheless, there is still hope for our model. As our result ($\omega_{ph} = -1.12$) is not far from the ΛCDM dark energy EOS ($\omega_\Lambda = -1$) and we saw in Figure 1 that our phantom model resembles the ΛCDM model at low redshifts, maybe we just have a tension with BAO, that can be polished as new and more precise observations are made.

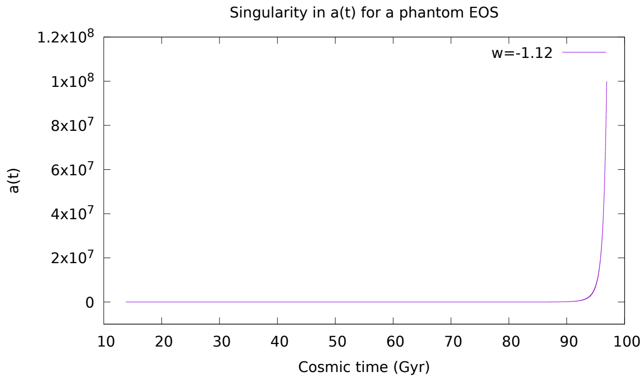

FIG. 2: We can see here that, for $\omega = -1.12$, the moment at which $a(t)$ diverges is of the same order of magnitude as we found in our previous calculus ($\sim$ 100 Gyr), which neglected the matter density term, so we can confirm that the evolution of the Universe in the future will be driven by the dark energy, whatever its nature may be.

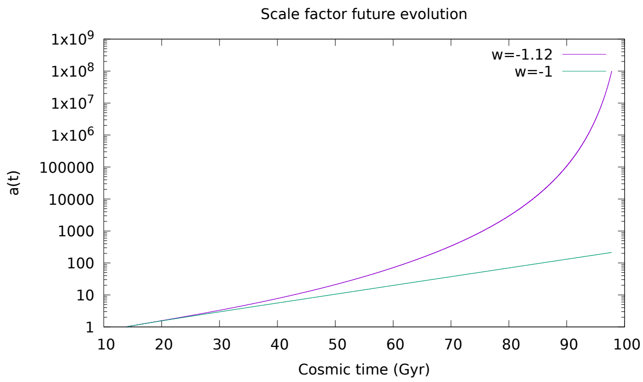

FIG. 3: Comparison, with a logarithmic y axis, of the evolution of scale factor for phantom ($\omega_{ph} = -1.12$) and Λ ($\omega_\Lambda = -1$) dark energy, in the late future. It is interesting to see that the phantom factor scale shows an ever-increasing tendency, even given the fact that the graphic is logarithmic: this is one way to see why the phantom dark energy is said to have a "super-exponential" behaviour.

IV. OTHER POSSIBLE SOLUTIONS TO THE HUBBLE TENSION

Despite it being an interesting and compelling way to solve the Hubble tension, the phantom energy is not the only game in town. One of the newest and less problematic theories is the one of Early Dark Energy (EDE): As stated in [11], it consists in an injection of dark energy in the period before recombination that would raise the value of H_0 inferred from the CMB, thus making it consistent with the SN1a. It is a rather complex theory, but its compatibility with all datasets (including BAO) make it a promising idea, even more than the one we have studied in this work. Also remarkable are the approaches that advocate for a change in the behaviour of neutrinos [12] or, as said before, for the existence of unknown systematic errors in the SN1a measurements.

V. CONCLUSIONS

We have seen that, while the Hubble tension is becoming one of the biggest problems of the present cosmology, a little modification in the EOS of dark energy could iron out this discrepancy in the measurements of H_0. However, it may bring several important issues that still cannot be avoided given the knowledge and data we currently have of the Universe. Perhaps the most promising solution is in the framework of the EDE theory, a novel and ingenious approach to solve the tension that, together with our hypothesis of phantom late energy, should be further examined in the future.

Acknowledgments

I would like to express gratitude to my advisor Eduard Salvador, for introducing me to this fascinating topic, as well as for his guidance and all his valuable suggestions and corrections.
I would also like to thank my family, especially my parents, for their unconditional support.

[1] A. Riess et al, Astron.J. 116:1009-1038 (1998).
[2] S. Perlmutter et al, Astrophys.J. 517:565-586 (1999).
[3] Planck collaboration, [arXiv:1807.06209 [astro-ph.CO]] (2018).
[4] A. Riess et al, Astrophys.J. 876:85 (2019).
[5] E. Di Valentino, A. Melchiorri and J. Silk, Phys.Lett.B. 761:242-246 (2016).
[6] J. Cepa, Cosmología física, p.130, Akal (2007).
[7] S. Carroll, M. Hoffman and M. Trodden, Phys. Rev. D 68, 023509 (2003).
[8] P. Frampton, K. Ludwick, and R. Scherrer, Phys. Rev. D 84, 063003 (2011).
[9] H. Ŝtefanĉić, Phys.Lett.B. 586:5-10 (2004).
[10] L. Verde, T. Treu, and A. Riess, Nat.Astron.3, 891–895 (2019).
[11] V. Poulin et al, Phys. Rev. Lett. 122, 221301 (2019).
[12] C. Kreisch, F. Cyr-Racine, and O. Doré, [arXiv:1902.00534 [astro-ph.CO]] (2019).