The London School of Economics and Political Science

Innovation activity, R&D incentives, competition and market value

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Declaration

I declare that the chapters entitled "R&D and market structure in a horizontal differentiation framework" and "Dynamics of R&D investments and the value of the firm" of this thesis are solely my own work.

The chapter entitled "The evaluation of the incentives to firms for innovation: the case of the Fund for Technological Innovation in Italy" is a joint work with dr. Guido De Blasio (Bank of Italy) and prof. Guido Pellegrini (University of Rome "La Sapienza"). My personal share of work for this chapter is 80 per cent, while the remainder is attributed to dr. De Blasio (10 per cent) and prof. Pellegrini (10 per cent).

I certify that the thesis presented by me for examination for the PhD degree is solely my own work other than where I have clearly indicated that it is the work of others, and that the extent of any work carried out jointly by me and any other person is clearly identified in it.

I warrant that this authorization does not, to the best of my belief, infringe the rights of any third party.

Abstract

This thesis examines some characteristics of the interaction between innovation activity of firms, in particular R&D, and economic system. The first main chapter analyses a mechanism of interaction between R&D and market structure, in a horizontally differentiated market where firms invest to increase differentiation among varieties. R&D activity declines over time; prices, output and short-run profits of firms producing the differentiated product move towards the higher steady-state values, production of the non-differentiated good falls. The increasing specialization improves the overall utility of consumers. The comparison with the socially optimal solution shows that firms underinvest in R&D. The second main chapter evaluates the effectiveness of the incentives to development of innovations provided by the Italian Ministry for Economic Development through the Fund for Technological Innovation. We analyse the subsidies to firms supplied by the general and the special sections of this Fund, using a difference-in-differences framework and a regression discontinuity one. We find no hints of effect on investments, dimension, labour productivity, labour costs, financial structure and profitability. For the general section, the effect on assets is positive, suggesting that firms used the subsidy to finance current expenditures. The third main chapter examines the relationship between R&D and market value of firms. We find high heterogeneity in the coefficients of different US manufacturing sectors between 1975 and 1995; sometimes the effects of current R&D on market value are very small or negative. We develop a model with uncertain R&D, where we decompose market value in two components, due to the already concretized assets and to work-in-progress R&D. Risk aversion may cause different evaluations of these components: when investors are risk-averse and managers maximize the long-run firm value, the risk associated with work-in-progress R&D reduces the short-run firm value even if its expected long-run value grows.

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I am responsible for remaining errors. The views expressed in this thesis are those of the author and do not involve the responsibility of the Bank of Italy.

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1 Introduction

The study of the reasons behind economic growth allows to analyse what are the main determinants of wealth and wellbeing of countries. Economic theory found that the main engine behind growth in the long run is the development of new and more productive technologies. The technological choices of firms and more in general of economic agents are endogenous and depend on the characteristics of the economy, first of all the structure of the markets and the institutional framework. Moreover, they influence and modify the economic environment and its structure of incentives for other future innovations. Innovation activity can be defined as the set of all the actions a firm implements in order to introduce a new technological element in the production process. Its output is usually either a product with new or improved characteristics or a more efficient production process for an old product. The main source of innovation is formal research and development (R&D) activity.[1] The knowledge created by innovation slowly spreads among the economic agents.

The aim of this thesis is the study of some aspects of this interdependence between economic system and technological innovation, with a particular

[1] According to OECD (2002), the definitions of the two components of R&D are the following: "Research is experimental or theoretical work undertaken primarily to acquire new knowledge of the underlying foundation of phenomena and observable facts. Development is systematic work, drawing on existing knowledge gained from research and/or practical experience, which is directed to producing new materials, products or devices, to installing new processes, systems and services, or to improving substantially those already produced or installed".

focus on the internal R&D activity of the firm. The remainder of this work is divided in three main chapters, followed by the conclusions.

In the chapter entitled "R&D and market structure in a horizontal differentiation framework" we introduce a mechanism by which firms can influence the shape of market competition introducing innovative characteristics in their products. We analyse how innovation and market structure endogenously interact over time; the relationships between these variables have important policy implications: on one hand, policy measures to stimulate R&D indirectly affect competition and, on the other hand, institutional changes to the market structure influence the incentives to research. In our model, we consider a horizontally differentiated framework where firms invest in R&D to increase differentiation among varieties of the same product. We can think of a product as an instrument allowing us to satisfy some needs. In a differentiated market, each variety has different effectiveness in satisfying each need. A consumer chooses the bundle of varieties giving him the highest level of overall satisfaction. Firms are able to modify the characteristics of their variety through investments in R&D; they may aim towards a more specialized profile, increasing the level of horizontal differentiation. Doing so, they reduce the degree of substitutability with the other varieties and raise their market power. In the limit, they tend to cut the reciprocal influence between varieties and to transform their products in unrelated ones. Moreover, the movement of a variety towards areas of specialization not well fulfilled by other varieties raises the overall satisfaction of consumers. The inclusion of our mechanism in a dynamic framework allows us to determine not only the production and R&D paths, but also the evolution of the market structure over time. Our most important results are that in this environment firms find incentives to invest in R&D to increase their specialization; the quantity of invested resources in research is declining over time, because the returns from further specialization decrease when the firm is more specialized, while prices, output and short-run profits of the firms producing a differentiated product increase. We compare the decentralized outcome and the socially optimal solution and we find that there is a suboptimal investment in R&D, because the socially optimal production is larger than the decentralized one and more output taking advantage of research implies more incentives to invest in R&D; moreover, the firm does not internalize the benefits of reducing substitutability with the other varieties on the profits of the other producers. Afterwards, we examine some empirical evidence using a panel of sectorial data about European firms. The results of the empirical analysis are coherent with the model. We use the ratio between price and marginal cost as market power index. We find that R&D investments and the variation of market power over time are both negatively related to the initial value of market power. Moreover, the number of firms in each sector is independent of all the other variables in the model.

In the chapter entitled "The evaluation of the incentives to firms for innovation: the case of the Fund for Technological Innovation in Italy" we empirically study the effectiveness of one policy instrument created by the

Italian Ministry of Economic Development to stimulate the innovation activity of firms: the subsidies supplied by the Fund for Technological Innovation. An interesting peculiarity is that this Fund, unlike most similar instruments studied in the previous literature, focuses on the development stage of R&D activity. The theoretical models on the topic find that the effects of the incentives to the research and to the development components of R&D are structurally different; but case studies where the policy measures mainly influence only one component are quite unusual in the empirical literature. The Fund is composed of two sections: a general purpose one, where applications from any field of activity and geographical area were accepted and evaluated one-by-one by merit following the chronological order of submission without a set deadline; a special purpose one, periodically issuing calls for applications in specific fields of activity or geographical areas with a set deadline, whose applications are ranked and whose subsidies are assigned to the best projects up to the amount of available resources. For both sections, the policy measures include a concessional loan and a non-refundable grant; the overall amount of the subsidy is the maximum allowed by the European Union regulations. The regular functioning of the general section of the Fund was unexpectedly interrupted after about five months due to shortage of funds; we use this exogenous shock to identify the effect of the policy: we compare the behaviour of the subsidized firms with that of the firms applying to the Fund after the shortage of funds, whose application had been neither assessed nor funded until five years later. The data from the Ministry about the Fund are merged with the 1999-2007 balance sheets of the firms filed at the Centrale dei Bilanci archives. We use two methodologies to evaluate the efficacy of this section of the Fund: a difference-in-differences approach, complemented by a matching procedure to increase the similarity between treated and controls, and a regression discontinuity design approach, using the submission date of the application as the forcing variable. In both cases, we are not able to detect signals of effectiveness of the policy on the investment behaviour of firms in the considered treatment period 2001-2007. The same is true also for sales, capital and employee figures, while there is a positive effect on assets; the additional liquidity from the subsidy seems probably to have been used to finance the current expenditure of firms. Neither the profitability nor the financial structure of the firm seem have been clearly affected by the policy, apart from a reduction in the share of long-term debts over assets, when calculated net of the concessional loan from the subsidy, a result coherent with the hypothesis of lack of effectiveness of the policy. We also evaluate the efficacy of three calls for applications of the special section of the Fund; we merged the application data with the balance sheets from the Cerved archives for the years 2001-2007. We use the regression discontinuity design approach with a normalized ranking of the applying firms as the forcing variable; the results are similar to those from the general section for the treatment period 2003-2007.

In the chapter entitled "Dynamics of R&D investments and the value of the firm" we investigate the relationship between value of firms and their knowledge and conventional capital stocks. We derive some empirical results using the Compustat dataset for the U.S. manufacturing sectors in the period 1975-1995. We see that the version of the relationship used by many previous studies may have effects in terms of efficiency of the estimates; we find a high level of heterogeneity in the coefficients of different sectors, which could undermine the results of the previous analyses usually postulating the same relationship in all sectors after controlling for fixed effects. When separating the current R&D effort from the past one, often the effect of current R&D is much weaker than that one of past R&D and sometimes is negative. To explain this fact, we develop a simple model where there is uncertainty about the results of R&D investments and we explicitly consider the time dimension. These two aspects are relevant because there is a lag between a R&D investment and the achievement of its results on the knowledge asset. The value of a firm takes into account not only the current assets, but also the expected value of the potential ones. Moreover, since research is a risky activity, there can be a different valuation of the already concretized assets and of those still at a work-in-progress stage. Risk-averse investors can penalize the expected returns of the latter in the determination of the market value of the firm. Therefore, when investors are risk-averse and managers maximize the long-run value of the firm, the risk associated with work-in-progress R&D can reduce the short-run firm value even if its expected value grows in the long run.

2 R&D and market structure in a horizontal differentiation framework

2.1 Introduction

The relationship between technological progress and market structure has been a recurrent element of discussion among economists. In particular, many contributions aimed to understand the effects of the different degrees of sectorial market power on the incentives to undertake R&D activity. Less attention has been given to the opposite relationship, how firms can influence, through research, the shape of market competition.

In this chapter we examine a mechanism through which this last relationship can come into effect and how R&D and market structure endogenously interact over time. The relationships between these two variables have important policy implications: on one hand, policy measures to stimulate R&D indirectly affect competition and, on the other hand, institutional changes to the market structure influence the incentives to research.

We consider a horizontally differentiated framework where firms invest in R&D to increase differentiation among varieties of the same product. We can think of a product as an instrument allowing us to satisfy some needs. In a differentiated market, each variety has different effectiveness in satisfying each need. A consumer chooses the bundle of varieties giving him the highest level of overall satisfaction. Firms are able to modify the characteristics of their variety through investments in R&D; they may aim towards a more specialized profile, increasing the level of horizontal differentiation. Doing so, they reduce the degree of substitutability with the other varieties and raise their market power. In the limit, they tend to cut the reciprocal influence between varieties and to transform their products in unrelated ones. An example of this kind of behaviour can be found among food producers: in the market for biscuits some producers specialized their production over time in low fat products (e.g. Misura, in the Italian market) and others in sweet products (e.g. Mulino Bianco).

Moreover, the movement of a variety towards areas of specialization not well fulfilled by other varieties raises the overall satisfaction of consumers. Horizontal differentiation implies a trade-off between level of competition and improvement of consumer welfare, which has been well understood by the antitrust authorities.[2] The introduction of new versions of a product whose characteristics damaged competition with other firms has been justified if the innovative characteristics implied welfare improvements for consumers. This has been one of the main discussions around the Kodak vs. Berkey classic case in the 70s; more recently, when Microsoft has been charged by the U.S. Department of Justice (1998), it defended its choice of selling together Windows and Internet Explorer saying that an integrated platform simplifies the creation of new applications, with advantages for consumers.

[2]See Baker (1997) and Weiss (1974) for some considerations about product differentiation and antitrust activity.

The inclusion of our mechanism in a dynamic framework allows us to determine not only the production and R&D paths, but also the evolution of the market structure over time. Our most important results are that in this environment firms find incentives to invest in R&D to increase their specialization. The quantity of invested resources in research is declining over time, because the returns from further specialization decrease when the firm is more specialized, while prices, output and short-run profits of the firms producing a differentiated product increase.

Moreover, we examine the difference between the previously derived decentralized outcome and the socially optimal solution. We find that the investment in R&D is suboptimal. This is because the socially optimal production is bigger than that one in the decentralized solution and a bigger output taking advantage of research implies the existence of more incentives to invest in R&D. Moreover, the firm does not internalize the benefits of reducing substitutability with the other varieties on the profits of the other producers.

Afterwards, we examine some empirical evidence using a panel of sectorial data about European firms. The results of the empirical analysis are coherent with the model. We use the ratio between price and marginal cost as market power index. We find that R&D investments and the variation of market power over time are both negatively related to the initial value of market power. Moreover, the number of firms in each sector is independent of all the other variables in the model.

The chapter is organized as follows: in the next Section, we review the literature on the relationship between research activity and market structure and we highlight connections and differences between our work and previous ones. In Section 2.3, we formalize the framework and explain the theoretical results. Moreover, we compare the decentralized solution of the model with the social optimum. In Section 2.4 we examine the effects of exogenous shocks and economic policy instruments on the dynamics and the steady-state variables of the model. In Section 2.5, we compare the model predictions about R&D, market power and number of firms with some empirical evidence. In Section 2.6, we conclude and summarize the findings of the chapter and further directions of research.

2.2 Literature review

The first and most influential studies on the relationship between research and market structure are due to Schumpeter (1942) and Arrow (1962).

Schumpeter argues that R&D activity is driven by the attempt to appropriate the monopolistic rents created by innovation. This intuition is interpreted by most of the following literature in the sense that a sector whose structure is a (natural or legal) monopoly is a good ground to nourish research.

On the other hand, Arrow notices that a competitive market provides more incentives to invest in R&D, because research allows a firm to create advantages over the other competitors and therefore to escape the tightness of competition.

These two views define the basic frameworks used by the subsequent theoretical and empirical literature. It is worthy of note that the two points of view are not necessarily in conflict, because what really matters is the differential gain earned by the innovator when moving from the pre-innovative to the post-innovative situation.[3] In fact, on one hand, for a given initial competitive structure an increase in the final profits of the innovator will raise the incentives to innovate; on the other hand, more competition (and therefore a lower rent) in the initial market increases the gain from innovation for a given final degree of market power.

Dasgupta and Stiglitz (1980) provide another pioneering contribution. They are the first to take into account the endogenous nature of the relationship between innovative activity and market structure. They consider the effects of process R&D that allows reducing the marginal cost of a unit of produced good in a Cournot oligopoly. The research expenditure becomes similar to a fixed cost of production and therefore the optimal choices of firms determine the barriers to entry and the number of competitors.

Even though they notice that the market power is better measured by the charged markup than by a concentration index, they use the number of firms as endogenous index of the market structure. Several both theoretical and empirical related works (e.g. Sutton (1998)) do the same. We will see in our model that, using a different framework from that developed by Dasgupta

[3]See for example Aghion, Bloom, Blundell, Griffith and Howitt (2005).

and Stiglitz, concentration and markup may have different behaviours.

Moreover, they use a static framework to simplify the analysis, where number of firms and R&D expenditure are simultaneously determined. In our model, the presence of a dynamic structure allows the perception of the continuous development of the interaction between market structure and incentives to innovate; moreover, we are able to examine the time path of the responses to exogenous changes in the parameters and to policy measures.

The development of the endogenous growth models, in particular the works from Grossman and Helpman (1991a,b,c) and Aghion and Howitt (1992), gives new elements to create theories on the effects of market structure on R&D. In these works, firms perform research either to reduce marginal costs or to increase the quality of their product in a framework of vertical differentiation. In both cases, research improves the efficiency of the production process, increasing the value of the output produced with one monetary unit of input. They emphasize the Schumpeterian view of the relation and therefore imply a negative relationship between competition and research.

However, in the same years several works (e.g. Geroski (1990), Geroski and Pomroy (1990), Blundell, Griffith and Van Reenen (1995, 1999), Nickell (1996)) point out that the empirical evidence seems to be favourable to a positive effect of competition and therefore to Arrow's view. The most recent empirical work by Aghion, Bloom, Blundell, Griffith and Howitt (2005) find an inverted-U relationship, where, when increasing competition, R&D falls in concentrated industries and increases in highly competitive ones. In fact, R&D generates further possibilities of rent extraction and consequently reduces competition. Our empirical analysis shows a positive effect of competition. Moreover, a preliminary analysis of our data showed that the difference between Aghion, Bloom, Blundell, Griffith and Howitt (2005) and the previous literature could be due to the use of the Lerner index as a proxy for the price-marginal cost ratio by the former.

The attempt to reconcile theoretical framework with empirical results follows several lines of research. Peretto (1999) gets results on the same lines as Arrow's argument in an oligopolistic framework where the market structure is endogenously determined following Dasgupta and Stiglitz (1980). An increase in the exogenous level of substitutability among products reduces the equilibrium number of firms and increases the rents from innovation, stimulating R&D.

Aghion and Howitt (1997), Aghion, Harris, Howitt and Vickers (2001), Aghion, Bloom, Blundell, Griffith and Howitt (2005) use a "step-by-step" model of innovation where duopolistic firms run a continuous "innovation race". Sometimes one competitor is able to achieve a monopolistic position and other times the competitors share a symmetric Cournot duopoly. In this kind of model the Schumpeterian effect is balanced by a "competition escaping" increase in R&D when the firms share the market. The relationship between R&D and market structure is cyclical, in the sense that a successful innovation either increases or reduces the distance between firms in the market and every gain of position in the market structure is temporary until the other firm innovates. This is because firms compete to improve production of the same good in a vertical differentiation framework. Therefore, an innovation reduces the effectiveness of the past improvements of the other firms on their own profits.

Nickell (1996), Schmidt (1997), Aghion and Howitt (1997), Aghion, Dewatripont and Rey (1999) try to explain the positive correlation between competition and research using agency considerations: more competition increases the incentives for managers to maintain a tighter discipline in the firm in order to avoid losses, because the margins of profits are lower in a competitive environment. Therefore, managers work to cut the marginal costs as much as possible and invest in R&D to this aim. Moreover, the introduction of an innovation by one firm increases the incentives to innovate of the other firms, because otherwise they lose their market shares.

Aghion and Howitt (1996) make another attempt through separation of research and development activities. An increase in competition raises the speed of adaptability of old production lines to the new standards and through this channel increases development activity and therefore the growth rate of the economy.

Denicol and Zanchettin (2010) study a Cournot vertically differentiated oligopoly with non-drastic innovations and show that the positive relationship between innovation activity and competition can be due to the temporal anticipation of profits from innovation when the market is more competitive.

Other recent related works discuss the correlation between process and product R&D in a simplified static framework similar to ours. Lin and Saggi (2002) compare the incentives to the two kinds of research under Bertrand and Cournot duopolistic structures. Product R&D allows the firm to reduce the level of substitutability of its output with the one of the other firm, while process R&D allows a reduction of the marginal costs. They find a positive correlation between the two kinds of research and show that Bertrand competition gives more incentive to product differentiation. Rosenkranz (2003), working with a similar framework in a monopolistic competition market, shows that cooperation between firms increases product innovation and that the same happens when there is an enlargement of the potential market.

A last work related to our analysis is Bils and Klenow (2001), which empirically examines the expenditure patterns in differentiated and homogeneous products. They find an increase over time of the expenditure in products with increasing differentiation and a fall of the consumption of more static and homogeneous products. Our model explains this behaviour.

The main contribution of this work to the literature regarding the relationship between R&D and market structure is its endogenous development in a dynamic framework of horizontal differentiation, which has not been previously explored. Differently from the previous models based on vertical differentiation, our framework emphasizes that the R&D choices of a firm do not necessarily have negative effects on the strategic environment and on the profits of the other firms producing the same product.

Moreover, while the other models were static, in our case the presence of a time dimension allows us to analyse the transitional dynamics of the firm behaviour, in terms of output, prices and research investments, and the temporal effects of exogenous shocks and policy measures.

In most of the other models, the number of firms was positively correlated with the profit opportunities in the market and was therefore a good approximation of market power. Because in our model outsider and insider firms have different profits opportunities, the number of firms may not always be correlated with the price-marginal cost ratio of the existing firms and therefore the use of concentration measures as a proxy for the market power may be not always appropriate.

2.3 The model

2.3.1 The market framework

We consider an economy where L (normalized to 1) workers/consumers live in continuous time, inelastically supplying their labour.

$N + 1$ goods are produced, using labour as the only production factor. One good is homogeneous and produced under constant returns to scale and perfect competition. The other N goods are differentiated and produced under increasing returns to scale and imperfect competition with strategic interaction among firms. Each firm produces a horizontally different variety of the good. Each variety can be produced in many versions that differ from each other in the degree of substitutability with the other varieties. The set of the currently available versions of a variety depends on the past R&D history of a firm.

The resulting framework is a Cournot oligopolistic market with differentiated products, but the model can be developed with similar results under the hypothesis of monopolistic competition.[4]

Each good aims at satisfying a subset of needs of consumers. Different varieties of the same good have slightly different characteristics; therefore, they are comparatively more or less efficient to satisfy each need.[5] Consumers choose a bundle of varieties to satisfy all their necessities, after a comparison of the overall utility they get from the currently produced versions of the different varieties. We capture this kind of environment saying that consumers have homogeneous preferences and maximize the following intertemporal quasilinear utility function:

$$U(0) = \int_0^\infty \left\{ x_0(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} \left[a_k - \sum_{j=1}^{m_k} \frac{b_{ijk}(t)}{2} x_{jk}(t) \right] x_{ik}(t) \right\} e^{-rt} dt \quad (1)$$

where $x_0(t)$ and $x_{ik}(t)$ are the consumed quantities respectively of the homogeneous good and of the currently produced version of variety i of good k at time t, $m_k \geq 2$ is the number of firms producing a variety of the differen-

[4] The oligopolistic framework seems a better environment because the idea of investing to enhance the idiosyncratic characteristics of the product suggests attention to the other varieties and therefore to the choices of the other firms.

[5] The framework we use here to give an intuition of the meaning of our utility function and of our mechanism of innovation in horizontal differentiation is based on the characteristics utility theory developed by Lancaster (1966a-b; 1975; 1979; 1980) and Gorman (1980).

tiated good k and $r > 0$ is the constant intertemporal discount rate.[6,7,8] The current utility derived from each variety of the differentiated goods depends not only on the consumed quantities of that variety, but also on a weight (the term in square brackets), which negatively depends on consumption of all the different varieties of the good. Therefore, increasing consumption of a variety reduces the marginal utility of additional units not only of the same variety, but also of the others. This is because we assume that there is partial substitutability between varieties: to satisfy its needs, the consumer can substitute

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[6] This utility function is an intertemporal generalization of the quadratic partial equilibrium function used, for example, by Spence (1976), Dixit (1979), Vives (1990), Ottaviano and Thisse (1999). Homogeneity and quadratic quasilinearity of consumer preferences allow us to obtain a linear inverse demand function after aggregating by direct summation the individual demand functions of the consumers. If we weaken one of these two hypotheses, the resulting inverse demand functions would be not linear. In this case, we could not derive explicit solutions of the equations, but the behaviour of the real variables would be qualitatively the same as in our partial equilibrium economy. Therefore, we can consider our simplified model a good approximation of the more general cases with heterogeneous consumers and/or a general equilibrium utility function.

[7] The substitutability parameters b_{ijk} have a time index because firms change the currently produced versions of their variety over time. Newer versions have lower substitutability parameters. A more general formulation of the utility function taking into account all the possible versions of each variety is $U(0) = \int_0^{\infty} \{x_0(t) + + \sum_{k=1}^{N} \sum_{i=1}^{m_k} \int_0^{b_{iik}} ... \int_0^{b_{iik}} \left[a_k - \sum_{j=1}^{m_k} \int_0^{b_{jjk}} ... \int_0^{b_{jjk}} \frac{b_{ijk}}{2} x_{jk}\left(t, \{b_{jlk}\}_{l=1}^{m_k}\right) db_{1jk}...db_{m_k jk}\right] * *x_{ik}\left(t, \{b_{ilk}\}_{l=1}^{m_k}\right) db_{i1k}...db_{im_k k}\} e^{-rt} dt$, where different versions of a variety are indexed by the substitution coefficients b_{ijk} and the own effect of a version of a variety i on its price is b_{iik}. We focus on an equilibrium where only the newest version is produced (that is, $x_{ik}(t, \{b_{ijk}\}_{j=1}^{m_k}) > 0$ only if $b_{ijk} = b_{ijk}(t)\ \forall j$, where the parameter with the time index is the lowest parameter b_{ijk} achievable at time t). Therefore, the utility function can be rewritten as in the main text. See Subsection 2.8.1 in the appendix for a discussion of the choice of the produced versions of a variety and the proof that the production of the newest version is the most common case. When we speak of a variety in the current Section we usually mean the currently produced version of that variety.

[8] The number of firms in each market is here assumed to be constant over time. See Subsection 2.3.3 for a discussion of the endogenization of the number of firms. We assume $m_k \geq 2$ because otherwise the firm has no incentive to invest in R&D to differentiate its own variety.

one variety with another having similar, but not equal, characteristics, which is therefore only partially suitable to satisfy the needs previously satisfied by the other variety. The separation among the behaviours of the homogeneous "static" good and the differentiated ones follows the empirical results of Bils and Klenow (2001).

The utility maximization problem is subject to the budget constraint of the consumer

$$x_0(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} p_{ik}(t)\, x_{ik}(t) \leq w(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} \pi_{ik}(t) \qquad (2)$$

where $w(t)$ is the wage, $\pi_{ik}(t)$ are the redistributed profits of the firm producing variety i of good k at time t and $p_{ik}(t)$ is the price of variety i of good k at time t; good 0 is the numéraire of the economy and its price is normalized to 1.[9]

Lemma 1 *Maximization of the utility function (1) subject to the budget constraint (2), assuming $x_0(t) > 0$ and $x_{ik}(t) > 0$ $\forall i, k$, implies the following linear inverse demand functions:*

$$p_{ik}(t) = a_k - \sum_{j=1}^{m_k} b_{ijk}(t)\, x_{jk}(t) \quad \forall i, k. \qquad (3)$$

Proof. From the first order conditions of the utility maximization problem. See Subsection 2.8.2 in the appendix for further details about the formal

[9] Profits of the firms in the homogeneous good market are null because of the perfect competition assumption.

solution of the maximization problem. ■

The parameters $b_{ijk}(t)$ are a measure of the influence of consumption of variety j on the market of variety i; we assume $b_{ijk} = b_{jik}$ and $b_{iik} = b_{jjk} = b_{0k}$ $\forall i, j$ to complete the symmetry between varieties. If $b_{ijk} = b_{iik}$ $\forall j$, the effect of consuming one more unit of any variety of the same good on the equilibrium price of variety i of good k is the same. Hence, the resulting market structure is a Cournot oligopoly with homogeneous good. If $b_{ijk} < b_{iik}$ $\forall j \neq i$, the equilibrium price of a variety is more sensitive to an increase of the sold quantity of the same variety than to an increase of the sold quantity of another variety and, therefore, the substitutability between varieties is only partial and proportional to the b_{ijk} coefficient.

Let us consider now the production process for the differentiated goods. We use a simple linear production function only requiring labour, equal for varieties of the same good, but that can differ between goods: if the firm producing variety i of good k wants to produce a quantity $x_{ik}(t)$ of its own variety, he needs

$$l_{ik}(t) = d_k + c_k x_{ik}(t) \tag{4}$$

units of labour.

Given the current structure of parameters $b_{ijk}(t)$ of variety i, the price and quantity decisions of firms do not include any intertemporal element; therefore, the firm producing variety i of good k at time t maximizes the current operating profit function:

$$\pi_{ik}^{o}(t) = p_{ik}(t)\, x_{ik}(t) - w(t)\, l_{ik}(t) \tag{5}$$

subject to the inverse demand function (3) and the production function (4). Assuming $x_0(t) > 0$ and $x_{ik}(t) > 0\ \forall i, k$, the first order conditions of maximization of current operating profits imply the following reaction curve:

$$x_{ik}(t) = \frac{a_k - w(t)\, c_k - \sum_{j \neq i} b_{ijk}(t)\, x_{jk}(t)}{2b_{0k}}\ \forall i, k. \tag{6}$$

The only parameters depending on the variety index i are the cross-effect coefficients. Therefore, if the b_{ijk} structure is the same $\forall i$, the optimal choice of $x_{ik}(t)$ is the same for all the firms producing different varieties of the same product.

Proposition 2 *Given a parameters structure such that* $b_{ijk}(t) = b_{ilk}(t)\ \forall j \neq i\ \forall l \neq i$*, maximization of the current operating profit function (5) subject to the inverse demand function (3) and the production function (4), implies a symmetric equilibrium with*

$$x_{ik}(t) = \frac{a_k - w(t)\, c_k}{b_{0k} + \Gamma_k(t)}\ \forall i, k \tag{7}$$

and

$$p_{ik}(t) = \frac{a_k b_{0k} + w(t)\, c_k \Gamma_k(t)}{b_{0k} + \Gamma_k(t)}\ \forall i, k \tag{8}$$

where $\Gamma_k(t) = \sum_{j=1}^{m_k} b_{ijk}(t)$ *is an index of the level of substitutability with the other varieties of the same good.*

Proof. From the first order conditions of the current operating profits maximization problem. See Subsection 2.8.2 in the appendix for further details about the formal solution of the maximization problem. ■

Quantities and prices are negatively related to the index $\Gamma_k(t)$. We require $w(t) < \frac{a_k}{c_k}$ to rule out corner solutions.

A symmetric solution for quantities implies that operating profits of the producer of variety i of good k are the same for all the producers of the same good:

$$\pi_{ik}^{o}(t) = \frac{\left[a_k - w(t)\, c_k\right]^2 b_{0k}}{\left[b_{0k} + \Gamma_k(t)\right]^2} - w(t)\, d_k. \tag{9}$$

Operating profits negatively depend on the price sensitivity with respect to all varieties too. A positive production requires $\pi_{ik}^{o}(t) \geq 0$. We will see in the next Section that in equilibrium $\Gamma_k(t)$ is decreasing over time. Therefore, this condition is always verified if

$$w(t)\, d_k \leq \frac{\left[a_k - w(t)\, c_k\right]^2 b_{0k}}{\left[b_{0k} + \Gamma_k(0)\right]^2}. \tag{10}$$

The overall profit function of the firm producing variety i of good k in period t is

$$\pi_{ik}(t) = p_{ik}(t)\, x_{ik}(t) - w(t)\left[l_{ik}(t) + \sum_{j=1}^{m_k} R_{ijk}(t)\right]$$

where $R_{ijk}(t)$ is the number of workers employed in R&D by the firm producing variety i of good k to improve the level of differentiation with variety j of the same good.[10] We assume perfect substitutability among all the workers, either employed in the production of the different (homogeneous and differentiated) goods or in the R&D activity; the wage is therefore the same for all the workers of the economy.

We close the model deriving the demand of the numéraire good from the budget constraint, which is always binding:

$$x_0(t) = w(t)\left\{1 - \sum_{k=1}^{N}\sum_{i=1}^{m_k}\left[c_k \frac{a_k - w(t)\, c_k}{b_{0k} + \Gamma_k(t)} + d_k + \sum_{j=1}^{m_k} R_{ijk}(t)\right]\right\}. \quad (11)$$

The condition for a positive production of the homogeneous good is

$$\sum_{k=1}^{N}\sum_{i=1}^{m_k}\left\{\frac{c_k\left[a_k - w(t)\, c_k\right]}{b_{0k} + \Gamma_k(t)} + d_k + \sum_{j=1}^{m_k} R_{ijk}(t)\right\} < 1.$$

Let us suppose that we need c_0 units of labour to produce one unit of homogeneous good. If we assume perfect competition in the homogeneous sector and a positive production of the homogeneous good, the zero-profit condition determines the equilibrium wage of the economy $w(t) = \frac{1}{c_0}\ \forall t$.

Coming back to the condition for a positive production of the homogeneous good, we will see at the end of the next Subsection that the amount of labour used in the homogeneous sector is decreasing over time; hence, the

[10] We assume $R_{iik}(t) = 0\ \forall i, k, t$.

condition is always satisfied on the adjustment path if it is satisfied in the asymptotic steady state, where $R_{ijk} = 0$ and $b_{ijk} = 0 \; \forall i \neq j$. Therefore, a necessary and sufficient condition is:

$$\sum_{k=1}^{N} m_k \left[\frac{c_k \left(a_k - \frac{c_k}{c_0} \right)}{2b_{0k}} + d_k \right] < 1.$$

Moreover, the nonnegativity condition of current operating profits (10) requires

$$\frac{d_k}{c_0} \leq \frac{\left(a_k - \frac{c_k}{c_0} \right)^2 b_{0k}}{\left[b_{0k} + \Gamma_k (0) \right]^2}$$

and the positivity constraint of the differentiated goods is

$$\frac{c_k}{a_k} < c_0.$$

We assume in the remaining of the text that all these conditions are satisfied.

2.3.2 The innovation activity

We now model how firms influence the market structure.

The utility of each good for the consumer is determined by the idiosyncratic value of the good in several characteristics. If we associate a numerical value to the consumer evaluation of each characteristic, we can display the position of the good in a characteristics space. Consumers choose their optimal bundle after evaluating the characteristics profiles of the outputs proposed by firms. They consider spatially nearer characteristics profiles more substitutable.

A firm adds to its feasible set of technologies new positions in the characteristics space through investments in R&D. There is a technological trade-off between characteristics: the development of some of them does not allow or even damages the development of others.[11] The optimal choice of the newly added technological positions implies an increase of the level of specialization in some characteristics of the good.

We call a "variety" the set of all the potential positions on the technological frontier of the same good specialized in the same subset of characteristics. For a given variety, a "version" is one of the possible characteristics profiles. Different versions show different degrees of specialization, which translate to different levels of substitutability, with effects on the profits of firms.

We can see in Figure 2.1 an example giving the intuition of the ideas: we show the effects of R&D of two firms in a two-dimensional space of characteristics and the link with the b_{ijk} coefficients.[12] The two axes of the graph are the values of two characteristics z_1 and z_2 of the good.

R&D allows firms to enlarge the set of feasible technologies on the tech-

[11] Lancaster (1966a) shows that the technological frontier of the optimally developed combinations of characteristics must be concave and that the optimal behaviour of firms is staying on the frontier.

[12] We assume that the number of potentially exploited characteristics of a product is not smaller than the potential number of firms in the market. This technical assumption is equivalent to saying that a sensible entrepreneur is always able to find a new specialization to be exploited.

nology frontier, which includes all the technologically possible z_1/z_2 ratios. In our Figure, the level of substitutability between two products (and therefore the value of the b_{ijk} coefficients) is given by the closeness in the z_1/z_2 ratios and by the physical nearness in the Cartesian plane.

Let us suppose that the only available technological position is point A. Both firms must be positioned there and there is perfect substitutability between the produced outputs.

Now the two firms invest in R&D. The farther the produced versions of the varieties are one from the other, the lower is the level of substitutability between them (and the larger are the profits of the two firms). Therefore, the optimal behaviour of the two firms will be adding new positions on the technological frontier towards the opposite axes, for example towards positions B and C.

Without loss of generality, the variety of firm 1 is z_1 intensive and that one of firm 2 is z_2 intensive. Firm 1 (2) learnt how to produce all the versions of its variety between A and B (C), but finds optimal the production of variety B (C) only. The two firms increase the level of specialization of their varieties and move towards the two opposite axes.

Let us go back to the formalization of this situation in our model.

We formally define the dynamics of the lower bound of the achievable substitutability coefficients between the newest versions of two varieties i

and j of the same good with the following equation:

$$\dot{b}_{ijk}(t) = -\gamma b_{ijk}(t)\left[\phi\left(R_{ijk}(t)\right) + \phi\left(R_{jik}(t)\right)\right]. \tag{12}$$

We assume that the dynamics of $b_{ijk}(t)$ depend on the number of R&D workers employed by the two interested firms to reduce substitutability with the other firm. R&D is increasingly difficult to be efficiently organized and consequently there are diminishing returns form R&D when the firm increases the employed quantity of resources. This fact is captured by the function ϕ, which is assumed to be continuous, increasing ($\phi' > 0$) and concave ($\phi'' < 0$), with $\phi(0) = 0$ and $\phi'(0) = \infty$. We assume that the more diversified is the product, the more difficult is finding new useful ways to effectively increase specialization. If we consider the demand functions when the parameters b_{ijk} tend to zero, we find that the varieties tend to become completely unrelated; therefore, the impact of the development of new specialized features on the level of substitutability becomes negligible and a kind of R&D aiming at increasing differentiation with the other varieties becomes useless for the firm. To capture this fact, we assume that a given effort in R&D has the same relative, and not absolute, effect on market power.

The research process is completely deterministic to keep a symmetric simplified outcome, not possible in presence of uncertainty. Moreover, we assume that the firm only produces the most differentiated version of its variety (that is, the version with the lowest values of b_{ijk}).[13]

Last, we assume that, because of patent protection or industrial secrecy, no firm can copy the newly developed version of a variety. Including the ability to imitate some (but not all) characteristics of the new version would weaken the effects of R&D and slow the speed of movement towards the steady state, but would not change the qualitative results.

The R&D choices are an intertemporal decision. Therefore, the firm producing variety i of good k makes its choices maximizing the discounted overall profits:

$$\Pi_{ik}(0) = \int_0^\infty \left\{ p_{ik}(t)\, x_{ik}(t) - w(t) \left[l_{ik}(t) + \sum_{j=1}^{m_k} R_{ijk}(t) \right] \right\} e^{-rt} dt. \quad (13)$$

Assuming a parameters structure such that $b_{ijk}(t) = b_{ilk}(t)\ \forall j \neq i\ \forall l \neq i$, the equilibrium prices and quantities still follow the analysis of the previous Subsection in each period.

We examine now the optimal R&D path.

Proposition 3 *Assuming an initial parameters structure such that* $b_{ijk}(0) = b_{ilk}(0)\ \forall j \neq i\ \forall l \neq i$*, the solution of the optimal control problem where the firm maximizes the discounted sum of its current and future profits (13)*

[13] We show in Subsection 2.8.1 in the appendix that the optimal choice of the firm is the production of the most differentiated version only, if the fixed cost are high enough or if there are three firms or more. Otherwise, the optimal choice could be the production of both the most differentiated and the non-differentiated versions, but not of the intermediate versions. We consider the first case in the main model, but the second case can be easily accommodated.

subject to the inverse demand function (3), to the production function (4) and to the dynamics of the lowest achievable values of the b_{ijk} *coefficients (12) implies the following growth rate of* $R_{ijk}(t)$*:*

$$\frac{\dot{R}_{ijk}(t)}{R_{ijk}(t)} = \frac{1}{\eta_{\phi' R}(R_{ijk}(t))} \left\{ r - \frac{\gamma c_0 \left(a_k - \frac{c_k}{c_0} \right)^2 b_{ijk}(t)\, \phi'(R_{ijk}(t))}{\left[b_{0k} + \Gamma_k(t) \right]^2} \right\} \quad \forall j, t \tag{14}$$

where $\eta_{\phi' R}(R_{ijk}(t))$ *is the absolute value of the elasticity of* $\phi'(R_{ijk}(t))$ *with respect to* $R_{ijk}(t)$ *and* $\phi'(.)$ *is the first derivative of* $\phi(.)$*.*

Proof. From the first order conditions of the optimal control problem. See Subsection 2.8.2 in the appendix for further details about the formal solution of the optimal control problem. ■

We can easily see the dynamics of the model for the case $\phi(R_{ijk}(t)) = \frac{R_{ijk}(t)^{1-\eta}}{1-\eta}$ with $0 < \eta < 1$, which implies a constant absolute value η of the elasticity of the ϕ' function, in the phase diagram in Figure 2.2. The two differential equations (12) and (14) imply that the behaviours of $b_{ijk}(t)$ and of $R_{ijk}(t)$ are the same for all the varieties of good k, given a common initial value $b_{ijk}(0)$ and a common choice of $R_{ijk}(t)$ for some value of t. The locus of points where $\dot{b}_{ijk} = 0$ in the (b_{ijk}, R_{ijk}) space is defined by

$$R_{ijk}\,|_{\dot{b}=0} = 0$$

and

$$b_{ijk}\,|_{\dot{b}=0} = 0$$

while the set of points where $\dot{R}_{ijk} = 0$ is described by

$$R_{ijk}\,|_{\dot{R}=0} = 0$$

and

$$R_{ijk}(t)\,|_{\dot{R}=0} = \left\{ \frac{\gamma c_0 \left(a_k - \frac{c_k}{c_0}\right)^2 b_{ijk}(t)}{r\left[b_{0k} + \Gamma_k(t)\right]^2} \right\}^{\frac{1}{\eta}}.$$

The steady states of the model are identified by the set of points in the phase diagram where both $\dot{b} = 0$ and $\dot{R} = 0$ are satisfied: this set is described by the two conditions $R_{ijk} = 0$ and $0 \leqslant b_{ijk} \leqslant b_{ijk}(0)$. Let us consider the path converging towards $R_{ijk} = 0$ and $b_{ijk} = 0$ as a reference path. A path with a higher initial level of R_{ijk} would be diverging and would imply $R_{ijk}(\infty) = \infty$, which is not feasible. A path with a lower initial level of R_{ijk} would converge towards $R_{ijk}(\infty) = 0$ and $b_{ijk}(\infty) > 0$. If we consider the concentrated profit function (with $p_{ik}(t)$, $x_{ik}(t)$ and $l_{ik}(t)$ already at their optimal value), the returns from one additional infinitesimal unit of R&D tend to infinite when $R_{ijk}(t)$ tends to zero if $b_{ijk}(t) > 0$:

$$\lim_{R_{ijk}(t)\to 0} \frac{\partial \Pi_{ik}(t)}{\partial R_{ijk}(t)} = \lim \left\{ \frac{2\gamma \left(a_k - \frac{c_k}{c_0}\right)^2 b_{0k} b_{ijk}(t)}{r\left[b_{0k} + \Gamma_k(t)\right]^3 R_{ijk}^{\eta}(t)} - w \right\} = \infty.$$

In this case the firm can easily increase its profits with a small R&D investment; therefore, a behaviour converging towards $b_{ijk}(\infty) > 0$ and $R_{ijk}(\infty) = 0$ cannot be optimal. More in general, this limit and the fact that $\dot{R}_{ijk}(t) = 0$ when $R_{ijk}(t) = 0$ imply together that $R_{ijk}(t) = 0$ can never be part of an optimal solution when $b_{ijk}(t) > 0$. The differential equations describing $\dot{b}_{ijk}(t)$ and $\dot{R}_{ijk}(t)$ $\forall j$ are continuous and locally Lipschitz $\forall t$ for $R_{ijk}(t) \neq 0$ and, therefore, a solution to the maximization problem exists and is unique.[14] Hence, the only remaining candidate behaviour, converging towards $b_{ijk}(\infty) = 0$ and $R_{ijk}(\infty) = 0$, is the equilibrium behaviour.[15]

Given the boundary conditions $R_{ijk}(\infty) = 0$ and the initial values $b_{ijk}(0)$, the dynamic behaviour of $R_{ijk}(t)$ is described by the forward-looking equation

$$R_{ijk}(t) = \left\{ \frac{\gamma c_0 \left(a_k - \frac{c_k}{c_0} \right)^2}{r} \int_t^{\infty} \frac{b_{ijk}(s)\, e^{-r(s-t)}}{\left[b_{0k} + \Gamma_k(s) \right]^2} ds \right\}^{\frac{1}{\eta}}$$

while that one of $b_{ijk}(t)$ is described by the backward-looking equation

$$b_{ijk}(t) = b_{ijk}(0)\, e^{-\frac{\gamma}{1-\eta} \int_0^t \left[R_{ijk}^{1-\eta}(s) + R_{jik}^{1-\eta}(s) \right] ds}.$$

Firms gradually reduce the quantity of invested resources in research, because increasing differentiation reduces the pressure of competition, and move

[14] See de la Fuente (2000), p. 433, Theorem 6.2.

[15] Because this model only focuses on a kind of R&D aiming at increasing the degree of differentiation of varieties and does not include other kinds of R&D investments (e.g., to increase productivity), the steady-state situation where $R_{ijk}(\infty) = 0$ should not be interpreted in the sense that firms do not invest in R&D, but that firms exert no effort to increase differentiation among varieties through R&D.

towards the steady-state situation, where the demand functions of different varieties are uncorrelated and there are no incentives to further increase differentiation. The level of research tends to the same steady state with $R_{ijk}(\infty) = 0 \ \forall j$ for all the firms producing different varieties of the same good and, therefore, assuming a common initial value $b_{ijk}(0) \ \forall i, \ \forall j \neq i$, $b_{ijk}(t)$ and $R_{ijk}(t)$ follow the same paths $\forall i \ \forall j \neq i$.

Let us consider now what are the consequences of the implied dynamics on quantities, prices and operating profits of the firms producing the differentiated product. The equilibrium levels of these variables are the static ones for the current b_{ijk} configuration. A review of equations (7), (8) and (9) tells us that they increase during the transitional dynamics and asymptotically tend to the higher steady-state levels. This is because the demand function is less sensitive to the level of output of the other firms when there is more differentiation. Therefore, the residual demand function, which is the space where the firm maximizes its own profits, has a higher intercept. A larger quantity is produced for a given price. Moreover, the firm can better exploit the new residual demand function to charge a higher price for its output.

Instead, the produced quantity of the homogeneous good (11) falls because now the raw utility of one unit of differentiated product is higher (the penalty to the utility for each unit of the other varieties is lower) and, therefore, the differentiated products are preferred.[16] A consequence of this fact is

[16]These dynamics explain the empirical patterns reported by Bils and Klenow (2001) where consumption of the "static" homogeneous good falls and expenditure in the varieties of the differentiated ones increases over time.

that the benefits of the successful research activity are not limited to firms: consumers prefer the bundle of the newly developed varieties, where they obtain a larger quantity of more diversified goods and a smaller one of the homogeneous good.

2.3.3 Endogenization of the number of firms m_k

The previous analysis considered an exogenous number of firms and varieties m_k. We try now to endogenize this variable. The results depend on the market entry conditions of the new firms. In particular, they depend on the initial level of differentiation of the variety produced by the entrant and on the level of the fixed costs.

If we call $b^i_{ijk}(t)$ the value of the substitutability parameter reached by the $m^i_k(t)$ already established firms, we assume that the output of the entrant producing a new variety of good k will be initially characterized by a value of the parameter of substitution with the other older varieties $b^e_{ejk}(t) = \frac{b_{0k} + \frac{1}{m^i_k(t)} \sum_{i=1}^{m^i_k(t)} b^i_{ijk}(t)}{2} < b_{0k}\ \forall j, k$. The new firm indirectly takes advantage of the R&D previously performed by the other firms to differentiate their own varieties and, therefore, the initial value of b_{ijk} of the entrant is weakly lower than b_{0k}. This formulation of $b^e_{ejk}(t)$ is equivalent to assume that the entrant benefits from half the degree of differentiation created by the incumbents for their own products, a situation equivalent to that one in which the incumbent had done its usual research and the entrant had done none in the past periods.

Proposition 4 *When we endogenize the number of firms $m_k(t)$ in the previously described framework, at the initial time $t = 0$ new firms enter the market until the discounted value of the expected profits is null:*

$$\Pi_{ik}(0) = \int_0^{\infty} \left\{ p_{ik}^* (t)\, x_{ik}^* (t) - w(t) \left[l_{ik}^* (t) + \sum_{j=1}^{m_k(t)} R_{ijk}^* (t) \right] \right\} e^{-rt} dt = 0 \tag{15}$$

where the starred variables are the optimal values given by the previous analysis as functions of m_k.

Assuming an initial parameters structure for the incumbents such that $b_{ijk}^i(0) = b_{ilk}^i(0)\ \forall j \neq i\ \forall l \neq i$, no firm has incentives to leave the market in the following periods.

Moreover, assuming also that a firm entering the market in period $\tau > 0$ has an initial parameter structure $b_{ejk}^e(\tau) = \frac{b_{0k} + \frac{1}{m_k^i(t)} \sum_{i=1}^{m_k^i(\tau)} b_{ijk}^i(\tau)}{2}\ \forall j$, two sufficient conditions to ensure no entry after the initial period are that either the incumbents already attained $b_k^i(\tau) \leq \frac{m_k^i - 4}{m_k^i - 2} b_{0k}$ for $m_k^i > 4$ or that $\frac{d_k}{c_0}$ is larger than a threshold.

Proof. Let us consider the possibility that a firm exits. If we examine the path of profits over time we see that

$$\dot{\pi}_{ik}(t) = -\frac{2\left(a_k - \frac{c_k}{c_0}\right)^2 b_{0k} \sum_{j=1}^{m_k} \dot{b}_{ijk}(t)}{\left[b_{0k} + \Gamma_k(t)\right]^3} - \frac{\sum_{j=1}^{m_k} \dot{R}_{ijk}(t)}{c_0} > 0\ \forall i, k \tag{16}$$

because both $\dot{b}_{ijk}(t)$ and $\dot{R}_{ijk}(t)$ are negative. This implies that the discounted value of the expected profits is increasing over time and no firm finds optimal to leave after the initial period.

Let us consider what happens if an additional firm tries to enter the market in a period $\tau > 0$, when m_k^i firms are already established. Assuming symmetry, we use $b_k^i(t) \equiv b_{ijk}^i(t)$ and $b_k^e(t) \equiv b_{ejk}^e(t)$ to simplify notation. The formal optimization problems for both the entrant and the incumbent are shown in Subsection 2.8.2 in the appendix. The optimal quantity for an additional firm entering the market in a generic period τ, assuming $b_k^e(\tau) = \frac{b_{0k}+b_k^i(\tau)}{2}$, is

$$x_k^e(\tau) = \frac{2\left(a_k - \frac{c_k}{c_0}\right)\left[\left(4-m_k^i\right)b_{0k} + \left(m_k^i-2\right)b_k^i(\tau)\right]}{\left(16-m_k^i\right)b_{0k}^2 + \left(6m_k^i-8\right)b_{0k}b_k^i(\tau) - m_k^i b_k^i(\tau)^2}$$

which is positive only if $m_k^i < 4$ or $b_k^i(\tau) > \frac{m_k^i-4}{m_k^i-2}b_{0k}$. If the incumbents already differentiated their outputs and achieved a $b_k^i(\tau)$ smaller than this threshold, the entrant is not even able to cover the marginal costs with the revenues and, therefore, does not produce.

Production is also null whether the operative profits of the entrant are negative, which is always verified if

$$\frac{d_k}{c_0} > \begin{cases} \frac{4\left(a_k-\frac{c_k}{c_0}\right)^2}{49b_{0k}} & \text{if } m_k^i = 2 \\ \frac{\left(a_k-\frac{c_k}{c_0}\right)^2}{b_{0k}\left(m_k^i+2\right)^2} & \text{if } m_k^i > 2 \end{cases}.$$

In both cases, the maximum principle conditions imply that no research is undertaken without production and therefore the entrant is inactive. ■

A consequence of the zero-profit condition (15) and of equation (16) is that firms bear negative profits at the beginning and positive ones in the steady state.

2.3.4 Comparison between the social optimum and the decentralized economy solution

Now, let us consider the comparison between the social optimum and the solution of the decentralized economy problem.

We assume that there is a benevolent planner choosing the allocations of the real variables $x_0(t)$, $x_{ik}(t)$, $l_{ik}(t)$, $R_{ijk}(t)$ and $m_k(t)$ to maximize the present value of the utility of consumers. We will see that the socially optimal number of produced varieties $m_k(t)$ is not constant over time. Therefore, to allow a comparison between the two cases, we start by determining the socially optimal amounts of research and production of the differentiated goods for a given m_k and then we discuss the $m_k(t)$ behaviour.

Proposition 5 *The benevolent planner maximizes the utility function (1) subject to the production functions (4) for differentiated products, the production function for the homogeneous product* $l_0(t) = c_0 x_0(t)$*, the full employment condition* $l_0(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} l_{ik}(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} \sum_{j=1}^{m_k} R_{ijk}(t) = 1$ *and the differential equations (12) determining the* $b_{ijk}(t)$ *of all the currently produced versions of the varieties.*

Assuming a parameters structure such that $b_{ijk}(t) = b_{ilk}(t)$ $\forall i$ $\forall j \neq i$ $\forall l \neq i$*, for a given number of produced varieties* m_k *of good* k*, the chosen quantities of the socially optimal solution are given in each period by*

$$x_{ik}^{SO}(t) = \frac{a_k - \frac{c_k}{c_0}}{\Gamma_k(t)} > x_{ik}^{D}(t) \;\; \forall i, k.$$

Proof. From the first order conditions of the benevolent planner's maximization problem. See Subsection 2.8.2 in the appendix for further details about the formal solution of the maximization problem. ■

Here, we can see a first distortion: the socially optimal production is larger than the decentralized output. This is because in the decentralized outcome firms choose quantities to equate marginal cost and marginal revenue, while the socially optimal production equates marginal cost and implicit price.[17] The socially optimal level of production cannot be implemented in the decentralized economy because it would imply a loss for firms due to fixed costs.

Proposition 6 *There is a second distortion in the competitive equilibrium: when taking the decision of investing in R&D the firm does not internalize the benefits of reducing substitutability with the other varieties on the profits of the other producers.*

There are two sides of this fact: on one hand, the firm does not internalize

[17] That is the price that would prevail in a decentralized framework where firms produce the socially optimal quantities.

the positive effect of the R&D activity of the other firms on the substitutability coefficients of the currently chosen version of its variety. On the other hand, it does not internalize the effect of its own research on the level of substitutability of the currently chosen versions of the other varieties. The two sides have opposite effects.[18]

Moreover, the above mentioned distortion in quantities has negative effects on the optimal R&D level because it reduces the production taking advantage of research and therefore its returns.

The overall effect of the externalities is such that the decentralized level of R&D is lower than the socially optimal one.

Proof. Assuming an initial parameters structure such that $b_{ijk}(0) = b_{ilk}(0)$ $\forall i\ \forall j \neq i\ \forall l \neq i$, the solution of the benevolent planner's dynamic optimization problem implies that the R&D path must satisfy:

$$\frac{\dot{R}^{SO}_{ijk}(t)}{R^{SO}_{ijk}(t)} = \frac{1}{\eta_{\phi' R}\left(R^{SO}_{ijk}(t)\right)} \left[r - \frac{\gamma c_0 \left(a_k - \frac{c_k}{c_0}\right)^2 b_{ijk} \phi'\left(R^{SO}_{ijk}(t)\right)}{\Gamma^2_k(t)} \right] \qquad \forall i,k,t\ \forall j \neq i.$$

See Subsection 2.8.2 in the appendix for further details about the formal solution of the dynamic optimization problem.

[18] When we consider the socially optimal level of production, the overall effect of the two sides of the externality is null. Instead, if we consider another level of production (for example, if we implement the decentralized solution quantities or more in general if there are sources of distortions), this is not true anymore. We can show that with a smaller output than the socially optimal one the overall distortion due to this externality is negative.

We can graphically see the comparison between the decentralized and the socially optimal paths in the phase diagram displayed in Figure 2.3. Similarly to what happened in the decentralized case, the steady states of the model are identified by the set of points in the phase diagram with both $R_{ijk} = 0$ and $0 \leqslant b_{ijk} \leqslant b_{ijk}(0)$. Paths with higher initial levels of R_{ijk} than that of the path converging towards $R_{ijk} = 0$ and $b_{ijk} = 0$ would be diverging and would imply $R_{ijk}(\infty) = \infty$, which is not feasible. Paths with lower initial levels of R_{ijk} than that of the path converging towards $R_{ijk} = 0$ and $b_{ijk} = 0$ would converge towards $R_{ijk}(\infty) = 0$ and $b_{ijk}(\infty) > 0$. These paths cannot be optimal, because when considering the concentrated utility function (with $x_0(t)$, $l_0(t)$, $x_{ik}(t)$, $l_{ik}(t)$ $\forall i, k, t$ already at their optimal value) we find that the utility created by one additional infinitesimal unit of R&D tends to infinite when $R_{ijk}(t)$ tends to zero if $b_{ijk}(t) > 0$:

$$\lim_{R_{ijk}(t)\to 0} \frac{\partial U(t)}{\partial R_{ijk}(t)} = \lim \left\{ \frac{2\gamma \left(a_k - \frac{c_k}{c_0}\right)^2 b_{ijk}(t)\, \phi'(R_{ijk}(t))}{r\Gamma_k^2(t)} - \frac{1}{c_0} \right\} = \infty.$$

Once again, this limit also implies that $R_{ijk}(t) = 0$ cannot be part of an optimal solution if $b_{ijk}(t) > 0$; the differential equations describing $\dot{b}_{ijk}(t)$ and $\dot{R}_{ijk}(t)$ $\forall j$ are continuous and locally Lipschitz $\forall t$ for $R_{ijk}(t) \neq 0$ and, therefore, a solution to the maximization problem exists and is unique.[19] Hence, the only remaining candidate behaviour, converging towards $b_{ijk}(\infty) = 0$ and $R_{ijk}(\infty) = 0$, is the optimal choice for the benevolent planner. Let

[19] See de la Fuente (2000), p. 433, Theorem 6.2.

us consider the slopes of the decentralized and the socially optimal paths passing through a given point (b, R) in the phase diagram. We see that:

$$\left.\frac{\partial R_{ijk}^{SO}}{\partial b_{ijk}} - \frac{\partial R_{ijk}^{D}}{\partial b_{ijk}}\right|_{\substack{b_{ijk}=b \\ R_{ijk}=R}} = -\frac{\gamma c_0 \left(a_k - \frac{c_k}{c_0}\right)^2 b_{0k} b \left(b_{0k} + 2\Gamma_k\right) \phi'(R) R}{\eta_{\phi' R} \Gamma_k^2 \left(b_{0k} + \Gamma_k\right)^2 \dot{b}} \tag{17}$$

$$\forall i, k, b, R \; \forall j \neq i$$

which is always positive because in the model $\dot{b}(t) < 0$ for $b > 0$. Therefore, the R&D paths in the socially optimal solution are always steeper than in the decentralized case. It must be true not only for a given point (b, R), but also in a sufficiently small neighbourhood of each point, because the functions of the slopes are continuous. If we consider a value b of b_{ijk} sufficiently near to the steady state, this implies that

$$\begin{aligned} R_{ijk}^{SO}\big|_{b_{ijk}=b} &= \int_0^b \frac{\partial R_{ijk}^{SO}}{\partial b_{ijk}} db_{ijk} \simeq b \frac{\partial R_{ijk}^{SO}}{\partial b_{ijk}}\big|_{b_{ijk}=b} > \\ &> b \frac{\partial R_{ijk}^{D}}{\partial b_{ijk}}\big|_{b_{ijk}=b} \simeq \int_0^b \frac{\partial R_{ijk}^{D}}{\partial b_{ijk}} db_{ijk} = R_{ijk}^{D}\big|_{b_{ijk}=b} \end{aligned}$$

where the approximation error becomes negligible for b small enough. Therefore, in a sufficiently small neighbourhood of the steady state the socially optimal amount of research is always bigger than that one in the decentralized equilibrium. The result $R_{ijk}^{SO} > R_{ijk}^{D}$ can be extended to any value of b_{ijk}: R_{ijk} is continuous in b_{ijk}; therefore, a situation where $R_{ijk}^{D} > R_{ijk}^{SO}$ for some $b_{ijk} > 0$ requires that in the optimal paths $R_{ijk}^{D}\big|_{b_{ijk}=b'} = R_{ijk}^{SO}\big|_{b_{ijk}=b'}$ and

$\frac{\partial R^{D}_{ijk}}{\partial b_{ijk}}\Big|_{b_{ijk}=b'} > \frac{\partial R^{SO}_{ijk}}{\partial b_{ijk}}\Big|_{b_{ijk}=b'}$ for some $b' \epsilon \left(0, b_{ijk}\right)$, which contradicts equation (17). ■

Let us consider now what happens to the socially optimal number of varieties $m_k(t)$ if it is allowed to change over time. In this case, the formal analysis becomes quite complicated, because the optimal number of varieties is not constant and the currently produced versions of different varieties have now different substitution indexes $\Gamma_{ik}(t)$, depending on the period they entered the market. The optimal real variables are now asymmetric and we can have different solutions, where the produced quantities are given by the solutions of the first order conditions with respect to $x_{ik}(t)$:

$$\sum_{j=1}^{m_k(t)} b_{ijk}(t)\, x_{jk}(t) = a_k - \frac{c_k}{c_0}\ \forall i, k, t.$$

The socially optimal R&D decision is symmetric among firms ($R_{ijk}(t) = R_{jik}(t)$) because of the decreasing efficiency of the ϕ function. The path depends on the chosen quantities and on the value of the $b_{ijk}(t)$ coefficients of the currently produced versions:

$$\frac{\dot{R}_{ijk}(t)}{R_{ijk}(t)} = \frac{1}{\eta_{\phi' R}\left(R_{ijk}(t)\right)}\left[r - c_0 \gamma b_{ijk}(t)\, x^{*SO}_{ik}(t)\, x^{*SO}_{jk}(t)\, \phi'\left(R_{ijk}(t)\right)\right] \quad (18)$$

$$\forall i, k, t\ \forall j \neq i$$

where the starred variables are the optimal choices of the benevolent planner for quantities from the previous analysis.

Proposition 7 *Under the hypothesis of constant elasticity of the ϕ' function, the number of varieties in the socially optimal solution implies that in each period the fixed cost of one more variety is approximatively equal to the future gain in terms of differentiation due to R&D:*

$$d_k \tilde{=} \frac{2\eta}{1-\eta} \sum_{j=1}^{m_k(t)} R_{mjk}(t) \;\; \forall k, t \tag{19}$$

where the m index is referred to the marginal variety, which is either the last produced or the last abandoned. Assuming a parameters structure such that $b_{ijk}(t) = b_{ilk}(t)$ $\forall i$ $\forall j \neq i$ $\forall l \neq i$, the number of varieties is increasing over time.

Proof. From the first order conditions of the benevolent planner's dynamic optimization problem; the approximation is due to the fact that $m_k(t)$ is always an integer and therefore its optimal value is usually either slightly smaller or slightly bigger than the solution of the first order conditions. See Subsection 2.8.2 in the appendix for further details about the formal solution of the dynamic optimization problem.

We cannot have a solution where the number of varieties is decreasing: in this case, the solution would be symmetric because, given a symmetric initial situation, the first order conditions are symmetric too. Therefore, all the decisions are always the same for all the varieties. This implies that the R&D and production paths should be positive also for the varieties to be abandoned, which contradicts our assumption of decreasing number of varieties.

A solution where the number of varieties is constant is not possible, because equation (19) implies that the overall R&D level of the marginal firm should be the same $\forall t$, which requires the product $b_{mjk}(t)x_{mk}(t)\,x_{jk}(t)$ in equation (18) to be constant over time and such that $\dot{R}_{mjk}(t) = 0$, a value not compatible with the R&D optimal path. ■

The variation of R&D implied by equation (18) for the marginal variety is concave over time, which implies that the product $b_{mjk}(t)x_{mk}(t)\,x_{jk}(t)$ must decrease. In this case, the economy asymptotically moves towards a situation where the homogeneous good is not produced any longer and all the products are differentiated. The overall number of varieties, in the simplified symmetric case where $a_k = a$, $b_{0k} = b$, $c_k = c$, $d_k = d$ $\forall k$, is given by $m_k = \left\{N\left[d + \frac{c}{b_0}\left(a - \frac{c}{c_0}\right)\right]\right\}^{-1}$ $\forall k$.

In fact, we saw in Subsection 2.3.3 that the endogenous number of firms in the decentralized solution is determined by a zero-profit condition (equation (15)), while the socially optimal one depends on the comparison between the marginal utility of a new variety and the marginal utility of the old one. Because the produced quantities of the old varieties are increasing, their marginal utility is decreasing over time; therefore, the consumer is better off by introducing new varieties. Increasing the number of varieties reduces the marginal utility of an additional one (because it increases the number of b_{ijk}

terms in the demand function). Hence, a situation with increasing quantities and number of varieties is compatible with the first order conditions of the social optimum problem.

When we compare the endogenous number of firms in the decentralized solution (given by equation (15)) and in the social optimum, we see that the former depends on parameters that are not relevant in the steady-state behaviour of the latter, like the intertemporal discount parameter r. Inspection of equation (19) shows that the socially optimal number of varieties always exceeds the number of varieties in the decentralized case when the economy is near the steady state (because average R&D is low, which implies a large socially optimal number of varieties). The comparison in the short run depends on the size of the R&D distortions. If the distortions are big enough and the economy is sufficiently far from the steady state, the socially optimal number of varieties may be smaller than the one in the decentralized case.

2.4 Comparative statics and policy implications

Let us examine now what happens to the model when there are unexpected changes in the exogenous variables. We can interpret these changes either as modifications in the institutional framework or as external shocks to the economic structure of firms.

We assume $\phi\left(R_{ijk}\left(t\right)\right) = \frac{R_{ijk}(t)^{1-\eta}}{1-\eta}$ with $0 < \eta < 1$. Let us consider the vector of the equations determining the equilibrium in the transitional dynamics when m_k is exogenous, that are the choices of firms for quantities and prices, their operating profits, the amount of R&D investments to increase differentiation with one other variety, the wage equation, the equations determining the size of the distortions in the production and in the research activity in the decentralized solution with respect to the social optimum:

$$S\left(t\right)=\begin{cases} x_{0}\left(t\right)=\frac{1}{c_{0}}-\sum\limits_{k=1}^{N}\sum\limits_{i=1}^{m_{k}}\{\frac{c_{k}(a_{k}-\frac{c_{k}}{c_{0}})}{c_{0}[b_{0k}+\sum_{j=1}^{m_{k}}b_{ijk}(t)]}+\frac{d_{k}}{c_{0}}+\sum\limits_{j=1}^{m_{k}}\frac{R_{ijk}(t)}{c_{0}}\} \\ x_{ik}\left(t\right)=\frac{a_{k}-\frac{c_{k}}{c_{0}}}{b_{0k}+\sum_{j=1}^{m_{k}}b_{ijk}(t)} \\ p_{ik}\left(t\right)=\frac{a_{k}b_{0k}+\frac{c_{k}}{c_{0}}\sum_{j=1}^{m_{k}}b_{ijk}(t)}{b_{0k}+\sum_{j=1}^{m_{k}}b_{ijk}(t)} \\ \pi_{ik}^{o}\left(t\right)=\frac{(a_{k}-\frac{c_{k}}{c_{0}})^{2}b_{0k}}{[b_{0k}+\sum_{j=1}^{m_{k}}b_{ijk}(t)]^{2}}-\frac{d_{k}}{c_{0}} \\ R_{ijk}\left(t\right)=\{\int_{t}^{\infty}\frac{\gamma c_{0}(a_{k}-\frac{c_{k}}{c_{0}})^{2}b_{ijk}(s)e^{-r(s-t)}}{[b_{0k}+\sum_{j=1}^{m_{k}}b_{ijk}(s)]^{2}}ds\}^{\frac{1}{\eta}} \\ w\left(t\right)=\frac{1}{c_{0}} \\ x_{ik}^{SO}\left(t\right)-x_{ik}^{D}\left(t\right)=\frac{(a_{k}-\frac{c_{k}}{c_{0}})b_{0k}}{\sum_{j=1}^{m_{k}}b_{ijk}(t)[b_{0k}+\sum_{j=1}^{m_{k}}b_{ijk}(t)]} \\ \frac{\partial R_{ijk}(t)}{\partial b_{ijk}(t)}\mid_{SO}-\frac{\partial R_{ijk}(t)}{\partial b_{ijk}(t)}\mid_{D}=-\frac{\gamma c_{0}(a_{k}-\frac{c_{k}}{c_{0}})^{2}b_{0k}b_{ijk}(t)[b_{0k}+2\sum_{j=1}^{m_{k}}b_{ijk}(t)]R_{ijk}^{1-\eta}(t)}{\eta[\sum_{j=1}^{m_{k}}b_{ijk}(t)]^{2}[b_{0k}+\sum_{j=1}^{m_{k}}b_{ijk}(t)]^{2}\dot{b}_{ijk}(t)}. \end{cases}$$

To determine the effects of a small change in a generic parameter ϑ on the variables in the vector $S\left(t\right)$ in case of exogenous m_k, we differentiate the vector with respect to the parameter:

$$\frac{dS\left(t\right)}{d\vartheta}=\frac{\partial S\left(t\right)}{\partial\vartheta}+\sum_{j=1}^{m_{k}}\frac{\partial S\left(t\right)}{\partial R_{ijk}\left(t\right)}\frac{\partial R_{ijk}\left(t\right)}{\partial\vartheta}+\sum_{j=1}^{m_{k}}\int_{t}^{\infty}\frac{\partial S\left(s\right)}{\partial b_{ijk}\left(s\right)}\frac{\partial b_{ijk}\left(s\right)}{\partial\vartheta}ds.$$

The effects on the steady state variables can be calculated differentiating the vector S^{SS} with the relevant variables in the steady state, where

$$S^{SS}=\begin{cases} x_0^{SS}=\frac{1}{c_0}-\sum_{k=1}^{N} m_k[\frac{c_k(a_k-\frac{c_k}{c_0})}{2c_0 b_{0k}}+\frac{d_k}{c_0}] \\ x_{ik}^{SS}=\frac{a_k-\frac{c_k}{c_0}}{2b_{0k}} \\ p_{ik}^{SS}=\frac{a_k+\frac{c_k}{c_0}}{2} \\ \pi_{ik}^{o,SS}=\frac{(a_k-\frac{c_k}{c_0})^2}{4b_{0k}}-\frac{d_k}{c_0} \\ w^{SS}=\frac{1}{c_0} \\ x_{ik}^{SO,SS}-x_{ik}^{D,SS}=\frac{a_k-\frac{c_k}{c_0}}{2b_{0k}}. \end{cases}$$

When m_k is endogenous, the analysis of the derivatives of $S(t)$ and S^{SS} must be complemented with the effects on the profits of an existing firm producing differentiated good (exit condition) and of a potential entrant in the same market (entry condition):

$$\begin{cases} \Pi_{ik}(t)=\int_t^{\infty}\{\frac{(a_k-\frac{c_k}{c_0})^2 b_{0k}}{[b_{0k}+\sum_{j=1}^{m_k} b_{ijk}(s)]^2}-\frac{d_k+\sum_{j=1}^{m_k} R_{ijk}(s)}{c_0}\}e^{-r(s-t)}ds \\ \Pi_k^e(t)=\int_t^{\infty}\{\frac{(a_k-\frac{c_k}{c_0})^2 b_{0k}[4b_{0k}+(m_k^i-1)b_k^i(s)-(m_k^i+1)b_k^e(s)]^2}{[4b_{0k}^2+2b_{0k}(m_k^i-1)b_k^i(s)-m_k^i b_k^e(s)^2]^2}+ \\ \qquad -\frac{d_k+\sum_{j=1}^{m_k} R_{ejk}^e(s)}{c_0}\}e^{-r(s-t)}ds. \end{cases}$$

The number of firms increases if $\Pi_k^e(t)>0$ and decreases if $\Pi_{ik}(t)<0$ after the change in ϑ. Because the equilibrium profits imply both $\Pi_{ik}(t)>0$ and $\Pi_{ik}^e(t)<0$ if $t>0$, an infinitesimal change in ϑ will never affect m_k and only a bigger variation of ϑ will change it. In the case m_k is affected, we have to sum the direct effect of the variation in ϑ with the indirect one due to the variation in m_k.

We begin now by examining the effects of a change in m_k when this variable is exogenous; this comparative statics not only can be useful to understand the effects of the competition policy, but also may be complementary to explain the effects of changes in other variables when the number of firms is endogenous. A rise in m_k increases the possibilities of substitution between varieties and therefore reduces the market power of firms. This causes a drop in prices, quantities and operating profits of the firms producing differentiated good and in the produced quantity of the homogenous good. The smaller output reduces the investment to increase differentiation towards one single variety, but the effect on the overall investments in R&D (given by $\sum_{j=1}^{m_k} R_{ijk}(t)$) is positive. Another effect is the reduction of the distortions in both quantity and R&D with respect to the socially optimal solution. In fact, a lower price is nearer to the marginal cost and therefore the produced quantity approaches the optimal one. Moreover, this fact reduces one of the R&D distortions. Finally, under the hypothesis that the new firms enter the market with a homogeneous product, increasing the number of firms pushes our economy farther from the steady state, as Γ_k will be larger for all firms. When we consider the steady-state equilibrium, the varieties of the differentiated product are completely unrelated. Therefore, the inclusion of a new variety does not affect prices, quantities and profits in the other differentiated markets, but simply reduces the production of the homogeneous good.

For each other parameter, we now analyze the effect of an infinitesimal change on $S(t)$ and S^{SS} and we examine how a sufficiently big change may affect m_k.

A decrease of the fixed cost d_k, equivalent to a lump sum subsidy to the firms producing differentiated good, does not affect the equilibrium quantities of these firms, but simply increases their profits and increases production of the homogeneous good. A sufficiently strong fall in d_k raises the number of firms through the increase in future profits.

Let us now turn the attention to the γ parameter, which measures the effectiveness of research activity. Changes in this parameter can be associated not only with economic shocks, but also with R&D policy measures. This is because, for example, a subsidy on research, financed through a lump sum tax, increases the efficiency of R&D for the firm, while the tax does not affect the demand functions of the differentiated products, but only that one of the homogeneous one. The most relevant effects of an increase in γ are stronger incentives to invest in R&D. Because a firm does not completely internalize the externalities of the research process of the other firms in the decentralized solution, this increases the distortions and the distance from the socially optimal R&D level. More research speed up the differentiation process and therefore the achievement of the higher profits in the long run and in the steady-state equilibrium. This fact may create incentives to enter the market, and, if the increase in γ is strong enough, it may increase the number of firms producing differentiated good. Because we do not have research in the long run, the steady-state variables are only touched by changes in γ

through the possible increase in the number of varieties and the consequent reduction in the production of the homogeneous good.

Another parameter linked to the efficiency of R&D is the absolute value η of the elasticity of the $\phi'(.)$ function, which measures how quickly the marginal effect of R&D falls. An increase in η does not affect the choices in production, but reduces the chosen level of research. The overall effects on the speed of movement towards the steady state and on the entry choices are uncertain, because, after the shock, R&D is more efficient at low levels and less efficient at high levels. However, we can say that when the chosen level of research is low, greater efficiency more than compensates for the lower level of research. Therefore, the differentiation process is faster and the number of firms m_k may increase, if the change in η is big enough. Once again, the only steady-state effects of changes in η are those eventually due to changes in the number of firms.

Let us consider now the effects of a positive shock on the term $\left(a_k - \frac{c_k}{c_0}\right)$, which can reflect either an increase in the level of demand (through a_k) or a decrease in the marginal labour requirements of production (through c_k). We can see a subsidy on the production activity as a negative shock on the marginal cost. Although we are considering both a demand and a technology shock, the structure of the model is such that the effects on the real variables are symmetric. A positive shock increases production and operating profits. The possibility of exploiting higher profits in the future stimulates R&D activity, quickening the achievement of the steady state, and, if strong enough, may provide incentives for firms to enter the market. In the steady state, we still have the positive effects on production and eventually the increase in the number of firms, which both reduce the production of the homogeneous good.

A comparison of the effects of a production and a R&D subsidy shows that only the former has real consequences on production for a given structure of the substitutability parameters. Inspection of equation (14) shows that this function is linear in γ and quadratic in $\left(a_k - \frac{c_k}{c_0}\right)$. This means that when the intervention is large enough, the production subsidy tends to dominate the R&D one and to increase the slope of the R&D path and the level of R&D more than the R&D subsidy for a given structure of the substitutability parameters.

The last case we consider is the effect of a decrease in the parameter c_0, which is equivalent to an increase in the wage. The effect is the same as that of a joint increase in the marginal cost and drop in the effectiveness of R&D. The overall effect is a reduction in production and profits for the firms producing differentiated good. The drop in production also reduces the incentives to invest in R&D, delaying the achievement of the steady state. Clearly, if the change in the parameter is big enough, the drop in the future profits may reduce m_k. In the steady state, we have a contraction of the production of each differentiated variety and eventually of their number, while there is an increase in the output of the homogeneous product.

2.5 Empirical results

We investigate the empirical evidence on R&D to find out whether it is consistent with our theoretical findings. In particular, we are interested in checking if the R&D and the b paths, described by equations (12) and (14), are coherent with the available data. Then we turn the attention to the relationship between the number of firms and the current level of market power: we check whether they are independent of each other.

We use data from the second and the third Community Innovation Surveys (CIS 2 and CIS 3), coordinated by Eurostat, matched with market power indicators calculated using data from Amadeus.

The CIS are the main innovation monitoring publications of the European Union. The two surveys were carried out in 1996 and in 2001 following the methodological directions of the Oslo manual. They report a lot of information about the innovative activity of European enterprises. We use aggregate data for sector and country in the empirical analysis. The available data allow us to work on an unbalanced panel of 17 sectors, 13 countries and 2 time periods.

Amadeus is a continuously updated database by the Bureau van Dijk Electronic Publishing reporting last ten years balance sheets and other information regarding several million European enterprises. We use the data release updated up to January 2006. Therefore, the available data are about the period 1996-2005.

We work at the sectorial level because the estimates of the market power index, given by the ratio between price and marginal cost, can be done with a sufficient degree of reliability at this level only. In fact, the marginal cost cannot be observed and therefore we have to estimate it by using econometric techniques at the firm level. Alternatively, we could have used an observable approximation, like the ratio between price and average cost. In our framework, this would not be optimal, because firms can have null or even negative profits in a single period, even if they have market power.

We use data for the average firm in each time period, sector and country. We control for time and sector components with the usual panel data fixed effects methodologies and we estimate the trend of the common behaviour of firms.

The first step of the analysis is the estimation of the market power index. We use a variation of the methodology proposed by Klette (1996, 1999) and Hall (1988, 1990), based on the estimate of the price-marginal cost ratio.

Once we have these estimates, we consider a general version of the solution of the differential equation (14). The current R&D level is an undetermined function of the initial market power and the initial level of R&D (which in turn is one-to-one determined by the initial market power on the optimized path and can therefore be neglected, since its effect is absorbed by the initial market power). Therefore, we can estimate the chosen R&D path as a function of the initial market power only.

We use then the average variation in the market power index in periods 1996-1999 and 2000-2003 as a proxy for the variation of the substitution coefficient b in equation (12). After substituting equation (14) for R, we are able to estimate the effect of the initial level of market power on the variation.

We use a semiparametric methodology to estimate the relationships: we add a battery of dummy variables checking sectorial and temporal fixed effects to our nonparametric relationships.

In our model we assume that the number of varieties m_k is constant. This hypothesis is clearly too strong to be verified with real data, because we do not take into account several factors such as the presence of uncertainty in the firm activities and the asymmetries between agents. We test a weaker version: we check whether the number of firms is independent of the current value of the other variables of the model.

In the next Subsection, we estimate the market power indexes. In Subsection 2.5.2, we examine the results of the semiparametric analysis and of the test for independence of the number of firms.

2.5.1 Estimation of the market power indexes

Let us consider the firms of a single sector, country and year.[20] Firm i follows a general production function $Y_i(t) = A_i(t) F_t(X_i(t))$ where the function $F_t(.)$ is common to all firms, but can differ over time, $Y_i(t)$ and $X_i(t)$ are respectively the output produced and the input vector used by firm i at time t.[21]

[20] This Subsection is based on a variation of the methodology proposed by Klette (1996, 1999) and Hall (1988, 1990).

[21] We do not take into account in our empirical analysis the simplifying, but completely unrealistic, hypothesis of the model of a linear production function depending on labour

Using the mean value theorem, we can write the deviation of the logarithm of the output from a benchmark, given by that one of the median firm, as a linear function of the deviation of the logarithm of the inputs from those of the benchmark:

$$y_i(t) - y_0(t) = a_i(t) - a_0(t) + \sum_j \bar{\varepsilon}_i^j(t)\left[x_i^j(t) - x_0^j(t)\right] \tag{20}$$

where the small letters are the logarithms of the capital letters, $\bar{\varepsilon}_i^j(t) = \frac{\bar{X}^j(t)}{F_t(\bar{X}(t))}\frac{\partial F_t(\bar{X}(t))}{\partial X^j(t)}$ is the elasticity of output with respect to input j, $\bar{X}_j(t)$ is the input vector evaluated at an intermediate point between $X_j(t)$ and $X_o(t)$.

Under the hypothesis of imperfect competition in the final output market, the first order conditions of the profit maximization problem imply the following definition of the ratio between price and marginal cost:

$$\frac{P_i(t)}{C'\left(X_i^j(t)\right)} = \frac{P_i(t)}{w^j(t)} A_i(t) \frac{\partial F_t(X_i(t))}{\partial X_i^j(t)} \equiv \theta_i(t)$$

where $C'(X_i^j(t))$ is the marginal cost function.

We see the relationship between the b_{ijk} parameters in the model of Section 2.3 and the $\theta_i(t)$ substituting price and marginal cost in the definition:

only.

$$\theta_i(t) \equiv \frac{P_i(t)}{C'\left(X_i^j(t)\right)} = \frac{a_k b_{0k} + wc_k \Gamma_k(t)}{wc_k\left[b_{0k} + \Gamma_k(t)\right]} \tag{21}$$
$$= \frac{a_k b_{0k} + wc_k \sum_{j=1}^{m_k} b_{ijk}(t)}{wc_k\left[b_{0k} + \sum_{j=1}^{m_k} b_{ijk}(t)\right]} \equiv \Xi_i(\{b_{\underset{-}{ijk}}(t)\}_{j=1}^{m_k}).$$

The price-marginal cost ratio is inversely proportional to the level of the b_{ijk} parameters. Substituting $\theta_i(t)$ in the definition of elasticity $\bar{\varepsilon}_i^j(t)$ we obtain

$$\bar{\varepsilon}_i^j(t) = \theta_i(t)\, \frac{w^j(t)\, \bar{X}^j(t)}{\bar{P}(t)\, \bar{Y}(t)}. \tag{22}$$

The methodology we are using tackles the realistic presence of adjustment costs in the accumulation of capital, that are additional costs, complementary to physical investment, borne when the firm changes its level of physical capital. This element has not been considered in our theoretical model because our production function only included labour for simplicity. Adjustment costs do not allow equation (22) to empirically hold for capital. We solve the problem by using the definition of elasticity of scale in production:

$$\bar{\vartheta}_i(t) = \sum_j \bar{\varepsilon}_i^j(t)$$

which implies for capital K

$$\bar{\varepsilon}_i^K(t) = \vartheta_i(t) - \sum_{j \neq K} \bar{\varepsilon}_i^j(t)\,.$$

Our equation (20) becomes:

$$\tilde{y}_i(t) = \tilde{a}_i(t) + \theta_i(t) \sum_{j \neq K} \frac{w^j(t) \bar{X}^j(t)}{\bar{P}(t) \bar{Y}(t)} \left[\tilde{x}_i^j(t) - \tilde{x}_i^K(t) \right] + \vartheta_i(t) \tilde{x}_i^K(t)$$

where the variables with the tilde are the deviations from the reference point, e.g. $\tilde{y}_i(t) = y_i(t) - y_0(t)$.

Under the basic structure of our model (same production function for all firms, perfect competition in the labour market) and under the additional hypothesis that the difference between the parameters of the individual firm and those of the average one is white noise we can estimate the following equation:

$$\tilde{y}_i(t) = \hat{a}(t) + \hat{\theta}(t) \sum_{j \neq K} \bar{s}_i^j(t) \left[\tilde{x}_i^j(t) - \tilde{x}_i^K(t) \right] + \hat{\vartheta}(t) \tilde{x}_i^K(t) + \omega_i(t)$$

where the coefficients to be estimated have a hat and $\bar{s}_i^j(t)$, the share of the value of the input on the overall value of production at an intermediate point, is approximated by the average of the individual observation and the benchmark. We use in the calculations the vector of the input costs as a measure of the inputs used by the firm. It includes the following elements: material variable costs, cost of employees and depreciation of capital.

The error term $\omega_i(t)$ depends on the value of the regressors, because we include in it the difference between the individual and the average effects. Consequently, we need instruments to correctly estimate the equation. The

$\hat{\theta}(t)$ coefficient is the market power index of the representative firm, while $\hat{\vartheta}(t)$ is the average elasticity of scale in production.

For each year, we use the number of employees and the value of the fixed assets of the firm for the previous years since 1995 as instruments. In some cases, not all the years are available because the variables were not reported. Therefore, we use the available years only.

We use a GMM procedure to estimate the equation; after calling $\omega(t)$ the vector with the values of the error terms and Z the matrix with the instruments, we write our objective equation to be minimized:

$$N\left(\frac{\omega(t)' Z}{N} V(t)^{-1} \frac{Z'\omega(t)}{N}\right)$$

where N is the number of firms in our sample and $V(t)$ is the estimated covariance matrix of $Z'\omega(t)$.

We estimate the coefficients of this equation for each country and sector in the years between 1996 and 2004. Then, we use the estimates of the $\hat{\theta}(t)$ coefficients as market power indexes of the representative firms in the following stages.

2.5.2 Empirical analysis of the relations between R&D, market power and number of firms

After obtaining the estimates of the price-marginal cost ratios, we are now able to work on the relationships between R&D and market power, which are defined by equations (12) and (14) in the model. They require a bit of manipulation before being in the convenient shape for estimation. In particular, we are not able to obtain a closed-form solution to equation (14). However, the system of the two differential equations (12) and (14) has continuous partial derivatives in the open set where $b_{ijk} \in (0, \infty)$ and $R_{ijk} \in (0, \infty)$. The theorems of existence of solutions for differential equations show that, given boundary conditions included in this set, there exists a solution.[22] We can implicitly write the solution for equation (14) as

$$R(t) = f(R(0), b(0), t). \tag{23}$$

The fact that we are excluding the points $b_{ijk} = 0$ and $R_{ijk} = 0$ does not present difficulties. In the non-trivial case $b_{ijk}(0) > 0$, the steady state can be reached only asymptotically. On the other hand, we have shown in Subsection 2.3.2 that $R_{ijk} = 0$ is never an optimal solution of the system in non-trivial cases.[23]

$R(t)$ is the choice variable of the firm. The uniqueness of the optimal path (see Figure 2.2 and the discussion in Subsection 2.3.2) implies that there is a one-to-one relationship between $R(t)$ and $b(t)$. Given the initial parameters, our $R(t)$ can be simply written as a positive function of $b(0)$ and a negative

[22] See for example Boyce and DiPrima (1970), p. 207, Theorem 6.1.

[23] In the case $b_{ijk}(0) = 0$, the solution would collapse to $b_{ijk}(t) = 0$ and $R_{ijk}(t) = 0$ $\forall t$. We rule out this pathological case because outside the aims of our discussion.

one of t. This must be true in particular for $t = 0$ and therefore we can write:

$$R(0) = f(\underset{+}{b(0)}). \tag{24}$$

We can rewrite equation (12) in a discrete time general version, which negatively depends on $R(t)$. Using the general solution to the other differential equation (23) and the one-to-one relationship between $R(0)$ and $b(0)$, we obtain:

$$\frac{\Delta b(t)}{b(t)} = g(R(t)) = g(f(\underset{-}{R(0)}, \underset{-}{b(0)}, t)) = h(\underset{+}{b(0)}, t). \tag{25}$$

We estimate the two functionally undetermined relationships (24) and (25) using a semiparametric procedure.[24] We use the market power index calculated in the previous Subsection as a proxy for the b coefficient. Because there is a negative relation between the market power index and the b coefficients (see equation (21)), the signs of the relationships are inverted.

In equation (24), we use the average (per sector, year and country) R&D levels in 1996 and 2000 as the dependent variable and the average initial market power index in 1996 and 2000 as the regressor.

In equation (25) the dependent variable is the percentage average variation of the market power index between 1996 and 1999 and between 2000 and 2003, while the independent one is the average initial market power index.

We added to both equations dummy variables per sector (17 sectors) and

[24] See as a reference Pagan and Ullah (1999) and Härdle (1990).

time period (2 periods). Hence, the resulting semiparametric structure is:

$$\begin{cases} R = f\left(\hat{\theta}\right) + \alpha' s + \beta t + \varepsilon_1 \\ \frac{\Delta\hat{\theta}}{\hat{\theta}} = h\left(\hat{\theta}\right) + \gamma' s + \delta t + \varepsilon_2 \end{cases} \tag{26}$$

where s is the sectorial dummy variables vector, t is the time period dummy variable and ε_1 and ε_2 are the disturbances. Taking the expectations of these two equations with respect to the market power index $\hat{\theta}$ we eliminate the nonparametric terms of the two regressions:

$$\begin{cases} R - E\left(R|\,\hat{\theta}\right) = \alpha' \left[s - E\left(s|\,\hat{\theta}\right)\right] + \beta\left[t - E\left(t|\,\hat{\theta}\right)\right] + \varepsilon_1 \\ \frac{\Delta\hat{\theta}}{\hat{\theta}} - E\left(\left.\frac{\Delta\hat{\theta}}{\hat{\theta}}\right|\hat{\theta}\right) = \gamma' \left[s - E\left(s|\,\hat{\theta}\right)\right] + \delta\left[t - E\left(t|\,\hat{\theta}\right)\right] + \varepsilon_2. \end{cases} \tag{27}$$

The expectations can be nonparametrically estimated.[25] Afterwards, we obtain the dummy variables coefficients from system of equations (27) by OLS. The nonparametric terms of system of equations (26) are now retrieved using the expectation equations:

$$\begin{cases} \hat{f}\left(\hat{\theta}\right) = \hat{E}\left(R|\,\hat{\theta}\right) - \hat{\alpha}'\hat{E}\left(s|\,\hat{\theta}\right) - \hat{\beta}\hat{E}\left(t|\,\hat{\theta}\right) \\ \hat{h}\left(\hat{\theta}\right) = \hat{E}\left(\left.\frac{\Delta\hat{\theta}}{\hat{\theta}}\right|\hat{\theta}\right) - \hat{\gamma}'\hat{E}\left(s|\,\hat{\theta}\right) - \hat{\delta}\hat{E}\left(t|\,\hat{\theta}\right). \end{cases}$$

We see the estimated nonparametric functions $\hat{f}\,(.)$ and $\hat{h}\,(.)$ in Figures 2.4 and 2.5, where we also report the true data. The predictions shown do

[25] We used a local linear regression estimator with a Gaussian kernel and a bandwidth of 0.2. The results are quite robust to changes in the bandwidth parameters and in the kernel functional form.

not take into account the dummy variables effects, but only the net common trend of the variables.

The relationship between the variation in the market power index of the firm and the initial market power index is negative, as our theory says, and it seems to be quite robust.

Figure 2.5 shows also a negative relationship for R&D as expected. We see that there is a lot of variance in the chosen R&D levels of the firms. This is explained through very strong sectorial differences in the parameters, which are captured by our dummy variables. Moreover, another source of variance can be the share of R&D that is used with other aims, for example to improve production processes.[26]

The relationship shown in this Figure has been broadly discussed in the previous literature on the subject. Our results are on the same lines as most of the works developed in the last years, where a positive relationship between competition and incentives to R&D has been shown.[27] On the other hand, they differ from the most recent work written by Aghion, Bloom, Blundell, Griffith and Howitt (2005), where they find an inverted-U relationship and it can be worthy of note examining why we have this difference. In the cited work, the authors use a panel of U.K. firms in the period 1973-1994 to

[26]It is possible that our results are biased because we do not use instruments to correct measurement error. Anyway, measurement error causes an attenuation bias and therefore the slope of the "true" curve would be even more negative in an estimation using instruments to tackle this problem.

[27]See for example Geroski (1990), Geroski and Pomroy (1990), Blundell, Griffith and Van Reenen (1995, 1999), Nickell (1996).

create a database of observations per year and per sector. Then, they estimate a nonlinear relationship between the citation weighted average number of patents (as a proxy for the research activity) and a function of the average of the Lerner indexes in the sector (as a proxy for the competition level). The essential difference between their and our methodology is that they use the Lerner index, which is the ratio between average cost and sales, to measure competition, while we use the theoretically more accurate ratio between marginal cost and price. If we use their function of the average Lerner index in our calculations, we are able to replicate their result.

The last relationship we take into account regards the number of firms in each sector and we check if it is independent of the other variables of the model. To do this, we use partial correlation coefficients between the number of firms, the level of market power, the variation of the market power index and the average R&D expenditure, net of the dummy variables effects. We add to the usual sectorial and time dummy variables a battery of country variables to account for scale effects due to population and for the different institutional frameworks.

We see in Table 2.1 that the number of firms is uncorrelated with the other variables, whichever dummy variables combination is considered.[28]

[28] Our results about the correlation between concentration and market power are slightly stronger than those presented in the past literature: previous studies (e.g. Salinger, Caves and Peltzman (1990), Bradburd and Owen (1982), Weis (1974)) find very small positive correlations, not usually significant.

2.6 Conclusions

In this chapter, we examined the relationship between R&D and the evolution of market structure over time.

We developed a mechanism of interaction between R&D and market structure based on the idea that firms can invest in research to increase the level of horizontal differentiation between their and the others varieties of a product. Producers try to modify the characteristics of their output to better satisfy needs of consumers that are not fully fulfilled by the other varieties. Doing so, they are able to increase the level of specialization of their product and, therefore, to reduce substitutability with the other varieties. We develop a dynamic framework, which allows us to see how the interaction between market structure and incentives to research changes over time.

Our most important results are that in this environment firms find incentives to invest in R&D to increase their specialization; the quantity of invested resources in research is declining over time, because the returns from further specialization decrease when the firm is more specialized, while prices, output and short-run profits of the firms producing a differentiated product increase.

We compare the decentralized outcome and the socially optimal solution and we find that there is a suboptimal investment in R&D, because the socially optimal production is larger than the decentralized one and more output taking advantage of research implies more incentives to invest in R&D; moreover, the firm does not internalize the benefits of reducing substitutability with the other varieties on the profits of the other producers.

Afterwards, we examine some empirical evidence using a panel of sectorial data about European firms. The results of the empirical analysis are coherent with the model. R&D investments and the variation of the market power index are both negatively related to the initial value of market power. Moreover, the number of firms in each sector is independent of all the other variables in the model.

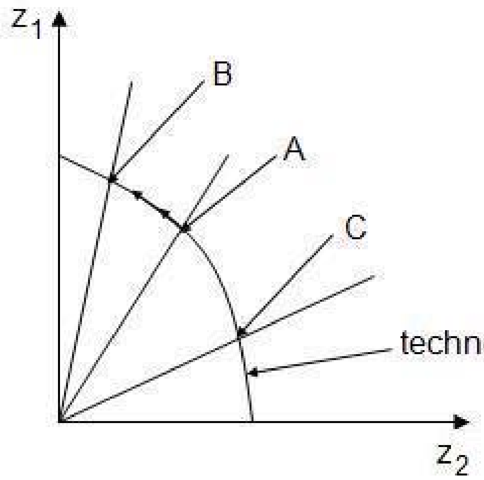
The developed analysis is a good starting point for further extensions: introducing uncertainty would allow greater realism, but afterwards the simplifying hypothesis of symmetry cannot be maintained and therefore the complexity of the model substantially increases. The presence of capital as a production factor could be interesting, because adjustment costs when converting from one variety to another can influence development costs and profits and therefore the incentives to research. The empirical analysis could be extended and deepened, in particular a comparison of the effects of different kinds of R&D (e.g. product and process) may give interesting hints.

2.7 Tables and Figures

Table 2.1: Partial correlation coefficients

	(1)	(2)	(3)	(4)	(5)	(6)
Sectorial dummies?	N	Y	N	N	Y	Y
Year dummies?	N	N	Y	N	Y	Y
Country dummies?	N	N	N	Y	N	Y
between number of firms and market power level						
Correlation coefficient	-0.0796	0.015	-0.1065	-0.129	0.0132	-0.0805
Significance level	0.3976	0.883	0.259	0.194	0.897	0.461
# observations	115					
between number of firms and variation of market power						
Correlation coefficient	0.0138	0.0678	0.0741	-0.0581	0.0896	-0.0156
Significance level	0.8828	0.503	0.432	0.558	0.378	0.886
# observations	116					
between number of firms and average sector R&D						
Correlation coefficient	-0.077	-0.0559	-0.0191	-0.0696	-0.0424	0.0117
Significance level	0.4302	0.599	0.846	0.5	0.692	0.919
# observations	107					

Fig. 2.1: Characteristics space and differentiation



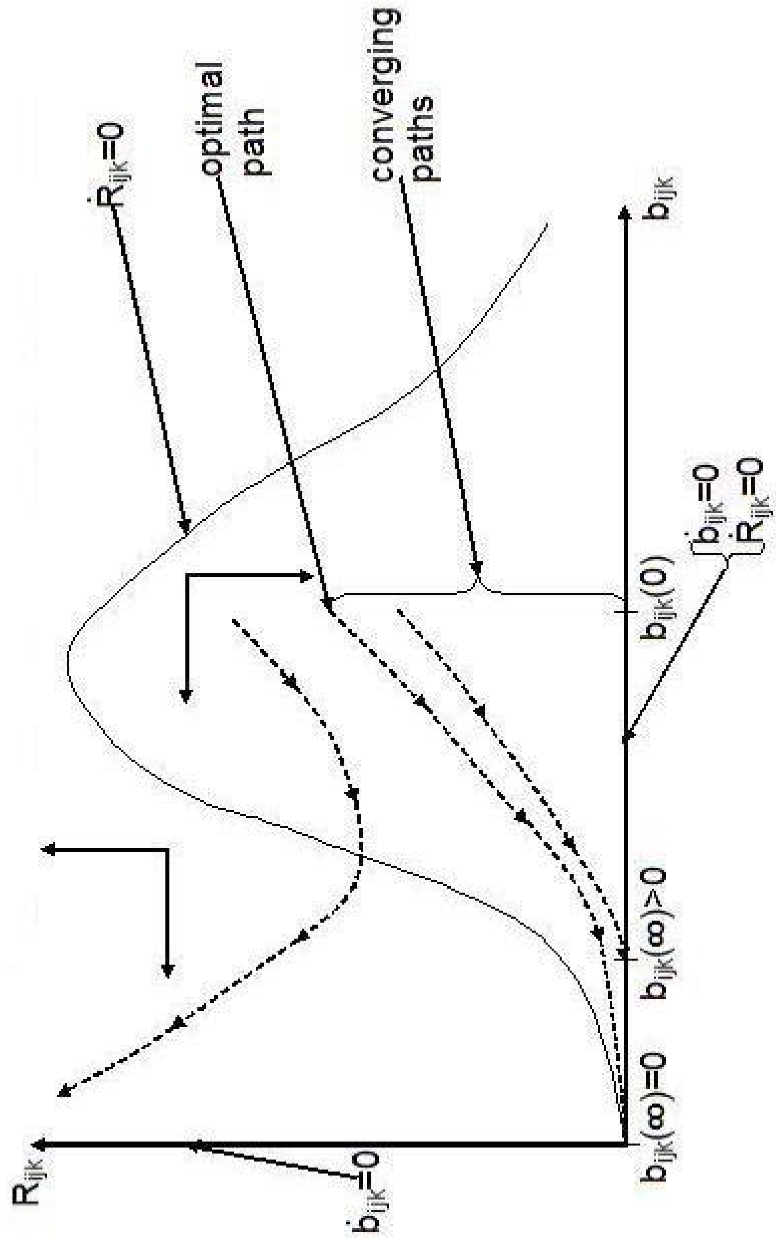


Fig. 2.2: R&D and market power: optimal paths

Fig. 2.3: Comparison between decentralized and socially optimal paths

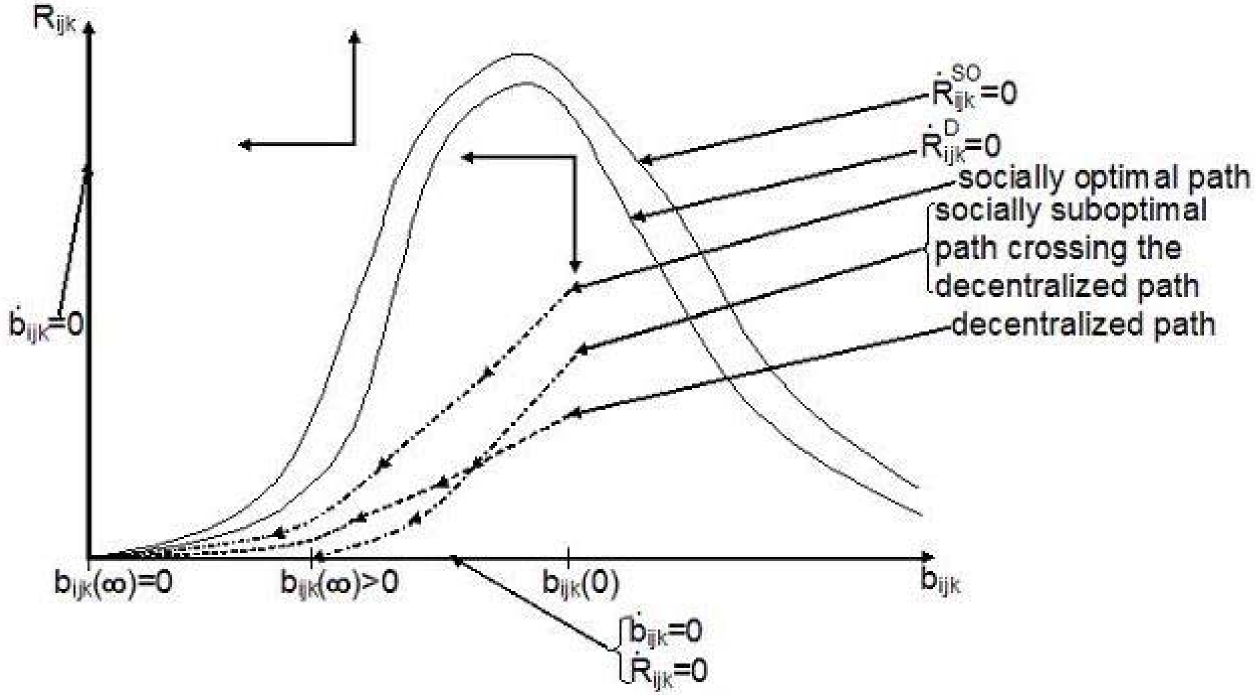
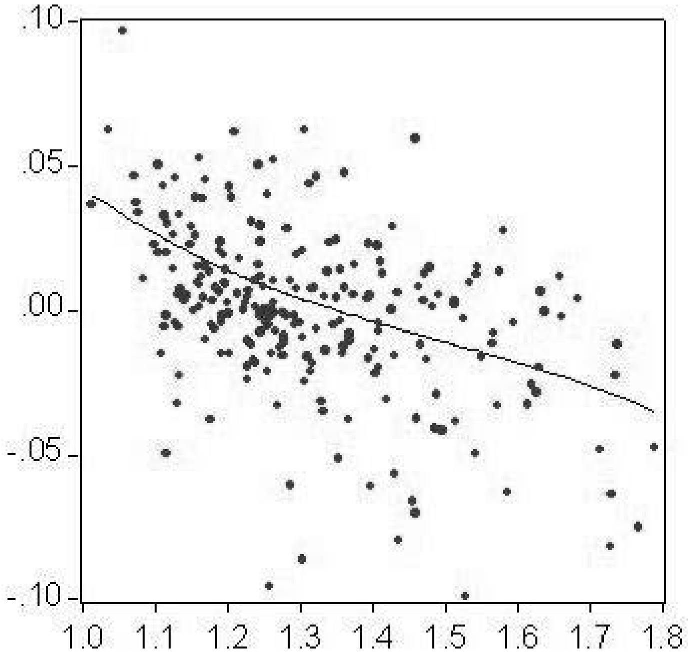
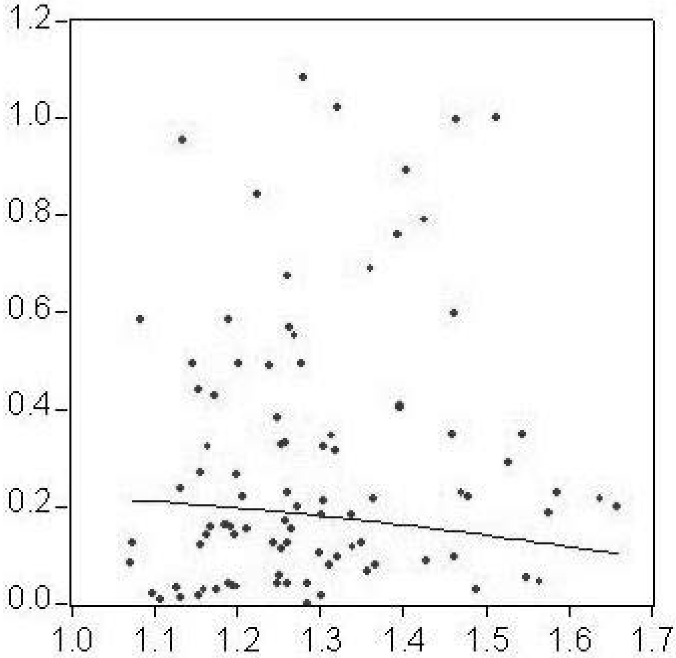


Fig. 2.4: Semiparametric estimation of market power variation on initial market power



The sample includes 115 observations.

Fig. 2.5: Semiparametric estimation of R&D investments on initial market power



The sample includes 107 observations.
R&D investments are in millions of euros.

2.8 Appendix

2.8.1 Endogenous choice of the produced versions

We examine here the conditions under which the optimal behaviour of the firm is the production of the newest version and what happens when these conditions are not satisfied. We find that the case where the only produced version is the one with the lowest b_{ijk} coefficients, examined in the main model, is the right one for most values of the parameters. Moreover, we find that the model can be easily extended to tackle the other case, where the optimal behaviour of a subset of firms is the production of both the most differentiated and the perfectly substitutable versions of their variety.

Let us suppose that we are in the short-run equilibrium described in the main text and one firm (which we suppose is producing variety i of good k) deviates producing both the newest version of its variety and an older version. We can restrict our proof to this case: if the introduction of a second version is not optimal, production of more than two versions will be suboptimal a fortiori. This is because increasing returns to scale imply that differential profits from one additional version are increasing in the produced output of that version and, therefore, decreasing in the number of produced versions.[29] We show that this deviation is only profitable in one case, where our main model can be easily extended. Because the choice of the produced versions

[29] In the case the production of two versions is preferred to the production of the newest version only, we can show using the same methodology of this Subsection that the introduction of a third version is never profitable.

is a pure choice of production and does not require intertemporal elements, we omit the time dimension. Our reasoning can be repeated in each period t.

Given the newest version of a variety, we index all the previously developed versions of the variety using a variable h, which measures the relative distance between the average level of substitutability of the newest and of an older version, calculated at the time of development of the newest version:

$$h = \frac{\frac{1}{m_k-1}\sum_{j\neq i} b_{ijk}^{o,n} - \frac{1}{m_k-1}\sum_{j\neq i} b_{ijk}^{n,n}}{b_{0k} - \frac{1}{m_k-1}\sum_{j\neq i} b_{ijk}^{n,n}} = \frac{b_{ijk}^{o,n} - b_{ijk}^{n,n}}{b_0 - b_{ijk}^{n,n}}$$

where $b_{ijk}^{o,n}$ is the substitutability parameter between the older (with index h) version of variety i and the newest version of variety j, while $b_{ijk}^{n,n}$ is the substitutability parameter between the newest version of variety i and the newest version of variety j; the last equality holds because in equilibrium we have symmetry in the b_{ijk} coefficients. The index h is equal to 1 if we consider the perfectly substitutable version of the good ($b_{ijk}^{o,n} = b_{0k}$), while it tends to 0 as we approach the newest version of the variety.

In the main text, we defined the substitutability level between two varieties, but we did not consider that one between versions of the same variety. We will examine now a reasonable assumption to define substitutability of an old version of a variety with the other varieties and with other versions of the same variety. Let us consider the two extreme cases of $h = 1$ (perfectly substitutable version of the product) and $h = 0$ (a second copy of the newest version of the variety).

In the former case, the substitutability level of the perfectly substitutable version does not benefit at all of the direct past efforts in R&D of firm i, but only of the effort of the other firms to differentiate their variety. In equilibrium, R&D is symmetric for all firms. Therefore, when considering substitutability with another variety, the perfectly substitutable version of variety i benefits of half the current maximum progress on differentiation (that is all the progress attributed to investments on the other varieties). The same is true when we consider substitutability with the newest version of the variety of the same firm. We will call $b_{iik}^{o,n}(h)$ the substitutability parameter between an older (with index h) and the newest version of the same variety i. Therefore, we have that

$$b_{iik}^{o,n}(1) = b_{ijk}^{o,n}(1) = b_{0k} - \frac{b_{0k} - b_{ijk}^{n,n}}{2} = \frac{b_{0k} + b_{ijk}^{n,n}}{2} \ \forall j.$$

If we produce a second copy of the newest version of variety i, it is perfectly substitutable with the other copy of the variety i and has the lowest available level of substitutability with the other varieties. Therefore, we have that

$$b_{iik}^{o,n}(0) = b_{0k} \text{ and } b_{ijk}^{o,n}(0) = b_{ijk}^{n,n}.$$

The level of substitutability between an older version of variety i and the newest version of another variety of the same good linearly depends on h by definition of this parameter. If we suppose that this is also true for the substitutability level between different versions of the same variety, we obtain these two expressions of $b_{iik}^{o,n}(h)$ and $b_{ijk}^{o,n}(h)$:

$$b_{iik}^{o,n}(h) = b_{0k} + h\left(\frac{b_{0k} + b_{ijk}^{n,n}}{2} - b_{0k}\right) = b_{0k} - h\frac{b_{0k} - b_{ijk}^{n,n}}{2} \tag{28}$$

$$b_{ijk}^{o,n}(h) = b_{ijk}^{n,n} + h\left(\frac{b_{0k} + b_{ijk}^{n,n}}{2} - b_{ijk}^{n,n}\right) = b_{ijk}^{n,n} + h\frac{b_{0k} - b_{ijk}^{n,n}}{2}. \tag{29}$$

Firm i now maximizes the sum of the operating profits due to the newest and to the older versions of its variety:

$$\pi_{ik}^{(2)}(h) = p_{ik}^{n}x_{ik}^{n} + p_{ik}^{o}x_{ik}^{o} - w\left(l_{ik}^{n} + l_{ik}^{o}\right)$$

where the indexes n and o discriminate the variables referred respectively to the newest and the older versions of the variety; p_{ik}^{n} and p_{ik}^{o} are the prices implied by the demand function (3), remembering that now we have $m_k + 1$ different versions of the good. The operating profits function of the other firms follows equation (5) as before.

Proposition 8 *When considering the equilibrium described in Section 2.3, let us assume that substitutability among different versions of the same variety is linear in* h*, where* h *has been defined above.*

If $m_k \geqslant 3$*, a deviation from the equilibrium where the firm produces two or more versions of its variety is never profitable.*

If $m_k = 2$ *and* $\frac{d_k}{c_0}$ *is larger than a threshold, a deviation is never profitable too.*

If $m_k = 2$ and $\frac{d_k}{c_0}$ is smaller than a threshold, a deviation can be profitable.

Proof. See Subsection 2.8.2 in the appendix for formal details about the proof and the maximization problem. In the period of deviation, maximization of profits implies the following equilibrium quantities (we call $x^n_{jk}(h)$ the quantities produced by the other firms):

$$\begin{cases} x^n_{ik}(h) = \frac{\chi\left[2(m_k+1)\left(2b_{0k}-b^{n,n}_{ijk}\right)-3h(m_k-1)\left(b_{0k}-b^{n,n}_{ijk}\right)\right]}{2} \\ x^o_{ik}(h) = \chi(2b_{0k} - b^{n,n}_{ijk})(3 - m_k) \\ x^n_{jk}(h)\Big|_{j\neq i} = \chi[4(2b_{0k} - b^{n,n}_{ijk}) - 3h(b_{0k} - b^{n,n}_{ijk})] \end{cases} \tag{30}$$

where $\chi = \frac{a_k - \frac{c_k}{c_0}}{b^{n,n}_{ijk} b_{0k}[8(m_k-2)-3h(m_k-3)] - b^2_{0k}[h(m_k+3)-16] + \left(b^{n,n}_{ijk}\right)^2 [h(4m_k-6)-4(m_k-1)]}$.

We are interested in equilibria where $x^o_{ik}(h) > 0$, otherwise the model collapses to the main text structure. This implies that $x^n_{ik}(h)$ must be greater than zero too, because the residual demand when only the older version of the variety is produced has a lower intercept and the same slope as in the situation where only the newest version is produced. $x^n_{ik}(h) > 0$ implies that the denominator is always positive. Moreover, $\frac{c_k}{a_k} < c_0$ by assumption and $b_{0k} > b^{n,n}_{ijk}$ by construction.

Therefore, the second equation in (30) implies that the older version of variety i of good k is only produced if the number of firms m_k is fewer than three. Otherwise, the optimal production of $x^o_{ik}(h)$ is null.

With $m_k \geqslant 3$, the competition is tight. The negative effects of the introduction of another version on demand are so strong that a positive production of $x^o_{ik}(h)$ yields negative effects on profits, whatever are the fixed costs.

We continue our analysis examining the effects on profits in the case we have two firms in the market of good k[30].

Profits depend on the chosen version h of the variety. There is a trade-off between a high and a low h. If h is high, the version is more substitutable with the other variety, but less substitutable with the newest version of the same variety. The opposite is true when h is low. The choice of the firm depends on the relative weight of these two effects. If we maximize profits with respect to h, we find that its optimal value is always $h = 1$. The effect of substitutability with the newest version of the same variety is always dominating and the deviating firm maximizes its profits producing the perfectly substitutable version of the good together with the newest version of its variety.

Let us consider the comparison between profits when firm i produces the latest version only of the good $(\pi^{(1)}_{ik})$ and when it produces both the latest and an older version. The deviation is the best behaviour if

$$\Delta\pi_{ik}(h) = \pi^{(1)}_{ik} - \pi^{(2)}_{ik}(h) < 0.$$

This inequality implies that producing the newest version only of the variety is the optimal behaviour if $\frac{d_k}{c_0}$ is larger than a threshold defined by the parameters.

With a small $\frac{d_k}{c_0}$, given a situation where all the other firms produce the

[30] We neglect the case $m_k = 1$ because the firm would have no incentives to invest in R&D to differentiate its own product.

newest version of the good, firm i finds optimal a deviation where it produces a positive quantity of both the newest version and the perfectly substitutable version of its variety. ■

We can easily extend our main model to take into account a duopoly where a firm produces both the newest version and the perfectly substitutable version of its variety. In the new situation, one or both firms chooses to produce both the most differentiated and the perfectly substitutable versions of their variety, while the remaining ones produce the differentiated version only. The share of firms producing both versions of their variety is pinned down by the comparison of the profits for the two cases.

While the time path of production of the most differentiated versions of the differentiated good is increasing as in the main model for both types of firms, the firms producing the perfectly substitutable versions of the differentiated good continuously reduce the perfectly substitutable output.

Because the other results about the R&D choices of firms, our main aim, do not qualitatively change, we do not explicitly derive the new version of the model, which is quite straightforward, given the analysis developed in the main text.

2.8.2 Technical details

Lemma 1 The consumer's utility maximization problem in each period t is static and can be written as

$$\max_{V(t)} \left\{ x_0(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} \left[a_k - \sum_{j=1}^{m_k} \tfrac{b_{ijk}(t)}{2} x_{jk}(t) \right] x_{ik}(t) \right\}$$

subject to

$$\begin{cases} x_0(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} p_{ik}(t) x_{ik}(t) \leq w(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} \pi_{ik}(t) \\ x_0(t) > 0, \;\; x_{ik}(t) \geq 0 \; \forall i, k \end{cases}$$

where $V(t) = \{x_0(t), \{\{x_{ik}(t)\}_{i=1}^{m_k}\}_{k=1}^{N}\}$ is the vector with the choice variables of the consumer and we assume $x_0(t) > 0$. The Lagrangian function of this maximization problem is

$$L(t) = x_0(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} [a_k - \sum_{j=1}^{m_k} \frac{b_{ijk}(t)}{2} x_{jk}(t)] x_{ik}(t) + \\ - \lambda(t) [x_0(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} p_{ik}(t) x_{ik}(t) - w(t) - \sum_{k=1}^{N} \sum_{i=1}^{m_k} \pi_{ik}(t)]$$

where $\lambda(t)$ is the Lagrangian multiplier associated with the budget constraint. The optimal values of $x_0(t)$ and $x_{ik}(t)$ $\forall i, k$ are described by the first order conditions:

$$
\begin{cases}
\frac{\partial L(t)}{\partial x_0(t)} = 1 - \lambda(t) = 0 \\
\frac{\partial L(t)}{\partial x_{ik}(t)} = a_k - \sum_{j=1}^{m_k} b_{ijk}(t)\, x_{jk}(t) - \lambda(t)\, p_{ik}(t) \leq 0 \ \forall i, k \\
x_{ik}(t)\, \frac{\partial L(t)}{\partial x_{ik}(t)} = x_{ik}(t)\, [a_k - \sum_{j=1}^{m_k} b_{ijk}(t)\, x_{jk}(t) - \lambda(t)\, p_{ik}(t)] = 0 \ \forall i, k \\
\frac{\partial L(t)}{\partial \lambda(t)} = x_0(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} p_{ik}(t)\, x_{ik}(t) - w(t) - \sum_{k=1}^{N} \sum_{i=1}^{m_k} \pi_{ik}(t) \leq 0 \\
\lambda(t)\, \frac{\partial L(t)}{\partial \lambda(t)} = \lambda(t)\, [x_0(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} p_{ik}(t)\, x_{ik}(t) - w(t) - \sum_{k=1}^{N} \sum_{i=1}^{m_k} \pi_{ik}(t)] = 0 \\
\lambda(t) \geq 0, \ x_0(t) > 0, \ x_{ik}(t) \geq 0 \ \forall i, k.
\end{cases}
$$

The first order conditions are necessary and sufficient for the unique global maximum because the objective function is pseudoconcave and the constraints are quasiconcave.[31]

We assume $x_{ik}(t) > 0 \ \forall i, k$.[32] The system of first order conditions implies:

$$
\begin{cases}
\lambda(t) = 1 \\
p_{ik}(t) = a_k - \sum_{j=1}^{m_k} b_{ijk}(t)\, x_{jk}(t) \ \ \forall i, k \\
x_0(t) = w(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} \pi_{ik}(t) - \sum_{k=1}^{N} \sum_{i=1}^{m_k} [a_k - \sum_{j=1}^{m_k} b_{ijk}(t)\, x_{jk}(t)] x_{ik}(t) \\
x_0(t) > 0, \ x_{ik}(t) > 0 \ \forall i, k.
\end{cases}
$$

[31] See de la Fuente (2000), p. 296, Theorem 1.19.

[32] In the case $x_{ik}(s) = 0$, we find that $R_{ijk}(s) = 0 \ \forall j$ and, therefore, the analysis becomes trivial.

Proposition 2 The choice of quantity and price maximizing current operating profit in each period t for the firm producing variety i of good k is a static problem and can be written as

$$\max_{V_{ik}(t)} p_{ik}(t)\, x_{ik}(t) - w(t)\, l_{ik}(t)$$

subject to

$$\begin{cases} p_{ik}(t) = a_k - \displaystyle\sum_{j=1}^{m_k} b_{ijk}(t)\, x_{jk}(t) \\ l_{ik}(t) = \begin{cases} d_k + c_k x_{ik}(t) & \text{if } x_{ik}(t) > 0 \\ 0 & \text{otherwise} \end{cases} \\ x_{ik}(t) \geq 0, \;\; l_{ik}(t) \geq 0 \end{cases}$$

where $V_{ik}(t) = \{p_{ik}(t), x_{ik}(t), l_{ik}(t)\}$ is the vector with the choice variables of the firm. We assume $x_{ik}(t) > 0$, which implies $l_{ik}(t) > 0$;[33] the Lagrangian function of this maximization problem is

$$L_{ik}(t) = p_{ik}(t)\, x_{ik}(t) - w(t)\, l_{ik}(t) - \lambda_{1ik}(t)\,[p_{ik}(t) - a_k + \\ + \sum_{j=1}^{m_k} b_{ijk}(t)\, x_{jk}(t)] - \lambda_{2ik}(t)\,[l_{ik}(t) - d_k - c_k x_{ik}(t)]$$

where $\lambda_{1ik}(t)$ and $\lambda_{2ik}(t)$ are the Lagrangian multipliers respectively associated with the inverse demand function and the production function. The optimal values of $p_{ik}(t)$, $x_{ik}(t)$ and $l_{ik}(t)$ $\forall i, k$ are determined by the first

[33] In the case $x_{ik}(s) = 0$, we find that $R_{ijk}(s) = 0$ $\forall j$ and, therefore, the analysis becomes trivial.

order conditions and by the condition of nonnegative operative profits:

$$\begin{cases} \frac{\partial L_{ik}(t)}{\partial p_{ik}(t)} = x_{ik}(t) - \lambda_{1ik}(t) = 0 \\ \frac{\partial L_{ik}(t)}{\partial x_{ik}(t)} = p_{ik}(t) - \lambda_{1ik}(t) b_{0k} + \lambda_{2ik}(t) c_k = 0 \\ \frac{\partial L_{ik}(t)}{\partial l_{ik}(t)} = -w(t) - \lambda_{2ik}(t) = 0 \\ \frac{\partial L_{ik}(t)}{\partial \lambda_{1ik}(t)} = p_{ik}(t) - a_k + \sum_{j=1}^{m_k} b_{ijk}(t) x_{jk}(t) = 0 \\ \frac{\partial L_{ik}(t)}{\partial \lambda_{2ik}(t)} = l_{ik}(t) - d_k - c_k x_{ik}(t) = 0 \\ x_{ik}(t) > 0, \; l_{ik}(t) > 0 \\ p_{ik}(t) x_{ik}(t) - w(t) l_{ik}(t) \geq 0. \end{cases}$$

The first order conditions are necessary and sufficient for the unique global maximum because the objective function is pseudoconcave and the constraints are quasiconcave.[34]

We assume a parameters structure such that $b_{ijk}(t) = b_{ilk}(t) \; \forall j \neq i \; \forall l \neq i$. The system of optimality conditions implies

$$\begin{cases} x_{ik}(t) = \lambda_{1ik}(t) = \frac{a_k - w(t) c_k}{b_{0k} + \sum_{j=1}^{m_k} b_{ijk}(t)} \\ p_{ik}(t) = \frac{a_k b_{0k} + w(t) c_k \sum_{j=1}^{m_k} b_{ijk}(t)}{b_{0k} + \sum_{j=1}^{m_k} b_{ijk}(t)} \\ l_{ik}(t) = d_k + \frac{c_k [a_k - w(t) c_k]}{b_{0k} + \sum_{j=1}^{m_k} b_{ijk}(t)} \\ \lambda_{2ik}(t) = -w(t) \\ w(t) < \frac{a_k}{c_k} \\ w(t) d_k \leq \frac{[a_k - w(t) c_k]^2 b_{0k}}{[b_{0k} + \sum_{j=1}^{m_k} b_{ijk}(t)]^2}. \end{cases}$$

[34] See de la Fuente (2000), p. 289, Theorem 1.14.

Proposition 3 The optimal control problem of the firm producing variety i of good k is dynamic and can be written as

$$\max_{V_{ik}} \int_0^{\infty} \left\{ p_{ik}(t)\, x_{ik}(t) - w(t) \left[l_{ik}(t) + \sum_{j=1}^{m_k} R_{ijk}(t) \right] \right\} e^{-rt} dt$$

subject to

$$\begin{cases} p_{ik}(t) = a_k - \sum_{j=1}^{m_k} b_{ijk}(t)\, x_{jk}(t) \ \ \forall t \\ l_{ik}(t) = \begin{cases} d_k + c_k x_{ik}(t) & \text{if } x_{ik}(t) > 0 \\ 0 & \text{otherwise} \end{cases} \quad \forall t \\ \dot{b}_{ijk}(t) = -\gamma b_{ijk}(t) \left[\phi(R_{ijk}(t)) + \phi(R_{jik}(t)) \right] \ \ \forall t\ \forall j \neq i \\ x_{ik}(t) \geq 0,\ l_{ik}(t) \geq 0,\ R_{iik}(t) = 0,\ R_{ijk}(t) \geq 0\ \forall t\ \forall j \neq i \end{cases}$$

where $V_{ik} = \{\{p_{ik}(t), x_{ik}(t), l_{ik}(t), \{R_{ijk}(t)\}_{j=1}^{m_k}\}_{t\in[0,\infty)}\}$ is the vector with the choice variables of the firm. The state variables are $b_{ijk}(t)\ \forall j \neq i$. The concentrated Hamiltonian function of this optimal control problem, where $x_{ik}(t)$, $p_{ik}(t)$ and $l_{ik}(t)$ are at their optimal values (starred variables), is

$$H_{ik}(t) = \{p^*_{ik}(t)\, x^*_{ik}(t) - w(t) [l^*_{ik}(t) + \sum_{j=1}^{m_k} R_{ijk}(t)]\} e^{-rt} + \\ - \sum_{j \neq i} \mu_{ijk}(t)\, \gamma b_{ijk}(t) \left[\phi(R_{ijk}(t)) + \phi(R_{jik}(t)) \right]$$

where $\mu_{ijk}(t)$ is the costate variable associated with $b_{ijk}(t)\ \forall j \neq i$. The behaviour of the candidates for the optimal path of $R_{ijk}(t)\ \forall j \neq i$ is described by the maximum principle conditions and by the nonnegativity condition of the discounted sum of current and future profits:

$$\begin{cases} \frac{\partial H_{ik}(t)}{\partial R_{jk}(t)} = -w(t)\,e^{-rt} - \mu_{ijk}(t)\,\gamma b_{ijk}(t)\,\phi'(R_{ijk}(t)) = 0\ \forall j \neq i \\ \frac{\partial H_{ik}(t)}{\partial b_{ijk}(t)} = -e^{-rt}x^*_{ik}(t)\,x^*_{jk}(t) + \mu_{ijk}(t)\,\frac{\dot{b}_{ijk}(t)}{b_{ijk}(t)} = -\dot{\mu}_{ijk}(t)\ \ \forall j \neq i \\ \frac{\partial H_{ik}(t)}{\partial \mu_{ijk}(t)} = -\gamma b_{ijk}(t)\,[\phi(R_{ijk}(t)) + \phi(R_{jik}(t))] = \dot{b}_{ijk}(t)\ \ \forall j \neq i \\ R_{iik}(t) = 0,\ \ R_{ijk}(t) \geq 0\ \forall j \neq i \\ \lim_{t\to\infty} H_{ik}(t) = 0 \text{ (trasversality condition)} \\ \int_t^\infty \{p^*_{ik}(s)\,x^*_{ik}(s) - w(s)\,[l^*_{ik}(s) + \sum_{j=1}^{m_k} R_{ijk}(s)]\}e^{-r(s-t)}ds \geq 0. \end{cases}$$

We assume an initial parameters structure such that $b_{ijk}(0) = b_{ilk}(0)$ $\forall j \neq i\ \forall l \neq i$; the behaviour of $x_{ik}(t)$, $p_{ik}(t)$ and $l_{ik}(t)$ follows the analysis described in Proposition 2. The system of optimality conditions implies

$$\begin{cases} \mu_{ijk}(t) = -\frac{e^{-rt}w(t)}{\gamma b_{ijk}(t)\phi'\left(R_{ijk}(t)\right)}\ \forall j \neq i \\ \frac{\dot{R}_{ijk}(t)}{R_{ijk}(t)} = -\frac{\phi'\left(R_{ijk}(t)\right)}{\phi''\left(R_{ijk}(t)\right)R_{ijk}(t)}\{r - \frac{\gamma[a_k - w(t)c_k]^2 b_{ijk}(t)\phi'\left(R_{ijk}(t)\right)}{w(t)[b_{0k}+\sum_{j=1}^{m_k} b_{ijk}(t)]^2}\}\ \forall j \neq i \\ \dot{b}_{ijk}(t) = -\gamma b_{ijk}(t)\,[\phi(R_{ijk}(t)) + \phi(R_{jik}(t))]\ \ \forall j \neq i \\ R_{iik}(t) = 0,\ \ R_{ijk}(t) > 0\ \forall j \neq i \\ \int_0^\infty \{\frac{[a_k - w(t)\,c_k]^2\,b_{0k}}{[b_{0k} + \sum_{j=1}^{m_k} b_{ijk}(t)]^2} - w(t)\,[d_k + \sum_{j=1}^{m_k} R_{ijk}(t)]\}e^{-rt}dt \geq 0 \end{cases}$$

where only the initial condition about profits is relevant because they are increasing over time:

$$\dot{\pi}_{ik}(t) = -\frac{[a_k - w(t)\,c_k]^2\,b_{0k}}{[b_{0k} + \Gamma_k(t)]^3}\sum_{j=1}^{m_k}\dot{b}_{ijk}(t) - w(t)\sum_{j=1}^{m_k}\dot{R}_{ijk}(t) > 0.$$

The transversality condition is verified in the solution because

$$\lim_{t\to\infty}\left\{p_{ik}^{*}(t)\,x_{ik}^{*}(t)-w(t)\left[l_{ik}^{*}(t)+\sum_{j=1}^{m_k}R_{ijk}^{*}(t)\right]\right\}<\infty,$$

$\lim\limits_{t\to\infty} e^{-rt}=0$, $\lim_{t\to\infty}\mu_{ijk}^{*}(t)=0$ and

$$\lim_{t\to\infty}-\gamma b_{ijk}(t)\left[\phi(R_{ijk}^{*}(t))+\phi(R_{jik}^{*}(t))\right]=0\ \forall j.$$

Proposition 4 Let us assume that the number of incumbent firms m_k^i is determined according to the zero-profit condition (15); an additional firm enters the market in period $\tau\geq 0$; in a generic period $t\geq\tau$, the substitutability parameters of its variety with the other varieties are $b_{ejk}^{e}(t)\ \forall j$, while $b_{ijk}^{i}(t)$ are the substitutability parameters between two incumbent firms producing varieties i and j of good k. The optimal control problem of an incumbent producing variety i of good k is dynamic and can be written as

$$\max_{V_{ik}^{i}}\int_{0}^{\infty}\left\{p_{ik}^{i}(t)\,x_{ik}^{i}(t)-w(t)\left[l_{ik}^{i}(t)+\sum_{j=1}^{m_k^i}R_{ijk}^{i}(t)+R_{iek}^{i}(t)\right]\right\}e^{-rt}dt$$

subject to

$$\begin{cases}
p_{ik}^{i}(t)=a_k-\sum\limits_{j=1}^{m_k^i}b_{ijk}^{i}(t)\,x_{jk}^{i}(t)-b_{eik}^{e}(t)\,x_{k}^{e}(t)\ \ \forall t\\
l_{ik}^{i}(t)=\begin{cases}d_k+c_k x_{ik}^{i}(t) & \text{if } x_{ik}^{i}(t)>0\\ 0 & \text{otherwise}\end{cases}\quad\forall t\\
\dot{b}_{ijk}^{i}(t)=-\gamma b_{ijk}^{i}(t)\left[\phi(R_{ijk}^{i}(t))+\phi(R_{jik}^{i}(t))\right]\ \forall t\ \forall j\neq i\\
\dot{b}_{eik}^{e}(t)=-\gamma b_{eik}^{e}(t)\left[\phi\left(R_{iek}^{i}(t)\right)+\phi\left(R_{eik}^{e}(t)\right)\right]\ \forall t\\
x_{ik}^{i}(t)\geq 0,\ l_{ik}^{i}(t)\geq 0\ \forall t\\
R_{iek}^{e}(t)\geq 0,\ R_{iik}^{i}(t)=0,\ R_{ijk}^{i}(t)\geq 0\ \forall t\ \forall j\neq i
\end{cases}$$

where $V_{ik}^{i}=\{\{p_{ik}^{i}(t), x_{ik}^{i}(t), l_{ik}^{i}(t), R_{iek}^{e}(t), \{R_{ijk}^{i}(t)\}_{j=1}^{m_k^i}\}_{t\in[0,\infty)}\}$ is the vector with the choice variables of the entrant, $R_{ijk}^{i}(t)$ is the expenditure in R&D of the incumbent producing variety i of good k to increase differentiation with variety j of good k produced by another incumbent and $R_{iek}^{i}(t)$ is the expenditure in R&D of the incumbent to increase differentiation with the variety of the entrant. The state variables are $b_{eik}^{e}(t)$ and $b_{ijk}^{i}(t)$ $\forall j \neq i$.

The optimal control problem of the entrant is dynamic too and can be written as

$$\max_{V_k^e}\int_{\tau}^{\infty}\left\{p_k^e(t)\,x_k^e(t)-w(t)\left[l_k^e(t)+\sum_{j=1}^{m_k^i}R_{ejk}^{e}(t)\right]\right\}e^{-r(t-\tau)}dt$$

subject to

$$\begin{cases} p_k^e(t)=a_k-\sum_{j=1}^{m_k^i}b_{ejk}^{e}(t)\,x_{jk}^{i}(t)-b_{0k}x_k^e(t)\ \ \forall t \\ l_k^e(t)=\begin{cases} d_k+c_kx_k^e(t) & \text{if } x_k^e(t)>0 \\ 0 & \text{otherwise}\end{cases}\quad \forall t \\ \dot{b}_{ejk}^{e}(t)=-\gamma b_{ejk}^{e}(t)\left[\phi(R_{ejk}^{e}(t))+\phi(R_{jek}^{i}(t))\right]\ \ \forall t\ \forall j \\ x_k^e(t)\geq 0,\ \ l_k^e(t)\geq 0,\ \ R_{ejk}^{e}(t)\geq 0\ \forall t\ \forall j \end{cases}$$

where $V_k^e=\{\{p_k^e(t), x_k^e(t), l_k^e(t), \{R_{ejk}^{e}(t)\}_{j=1}^{m_k^i}\}_{t\in[\tau,\infty)}\}$ is the vector with the choice variables of the entrant, $R_{ejk}^{e}(t)$ is the expenditure in R&D of the entrant to increase differentiation with variety j of the same good. The state variables are $b_{ejk}^{e}(t)$ $\forall j$.

We split the analysis in two steps. In the first one we determine the chosen quantities and prices of the incumbents and the entrant for given values of $b^{e}_{ejk}(t)$ and $b^{i}_{ijk}(t)\ \forall i, j, k, t$. In the second one we examine the R&D decisions of entrant and incumbents and the overall dynamics of their profits.

Step 1: the choice of quantity and price in period $t \geq \tau$ is a static decision and does not depend on the dynamics of the lowest achievable values of the b_{ijk} coefficients. We assume $x^{i}_{ik}(t) > 0$ and $x^{e}_{k}(t) > 0\ \forall i, k$, which imply $l^{i}_{ik}(t) > 0$ and $l^{e}_{k}(t) > 0$.[35]

The Lagrangian function of the maximization problem for the incumbent producing variety i of good k is

$$L^{i}_{ik}(t) = p^{i}_{ik}(t)\, x^{i}_{ik}(t) - w(t)\, l^{i}_{ik}(t) - \lambda^{i}_{1ik}(t)\left[p^{i}_{ik}(t) - a_k + \right.$$
$$\left. + \sum_{j=1}^{m^{i}_{k}} b^{i}_{ijk}(t)\, x^{i}_{jk}(t) + b^{e}_{eik} x^{e}_{k}(t)\right] - \lambda^{i}_{2ik}(t)\left[l^{i}_{ik}(t) - d_k - c_k x^{i}_{ik}(t)\right]$$

where $\lambda^{i}_{1ik}(t)$ and $\lambda^{i}_{2ik}(t)$ are the Lagrangian multipliers respectively associated with the inverse demand function and the production function.

The optimal decisions regarding $p^{i}_{ik}(t)$, $x^{i}_{ik}(t)$ and $l^{i}_{ik}(t)$ are functions of the choices of the entrant. They are determined by the first order conditions

[35] In the case $x^{e}_{ik}(s) = 0$, we find that $R^{e}_{ijk}(s) = 0\ \forall j$. If $x^{i}_{ik}(t) = 0$, we find that $x^{e}_{ik}(s) = 0$ and $R^{e}_{ijk}(s) = 0$. In both cases the Proposition is trivially dimostrated.

and by the condition of nonnegative operating profits:

$$\begin{cases} \frac{\partial L_{ik}^{i}(t)}{\partial p_{ik}^{i}(t)} = x_{ik}^{i}(t) - \lambda_{1ik}^{i}(t) = 0 \\ \frac{\partial L_{ik}^{i}(t)}{\partial x_{ik}^{i}(t)} = p_{ik}^{i}(t) - \lambda_{1ik}^{i}(t)\, b_{0k} + \lambda_{2ik}^{i}(t)\, c_k = 0 \\ \frac{\partial L_{ik}^{i}(t)}{\partial l_{ik}^{i}(t)} = -w(t) - \lambda_{2ik}^{i}(t) = 0 \\ \frac{\partial L_{ik}^{i}(t)}{\partial \lambda_{1ik}^{i}(t)} = p_{ik}^{i}(t) - a_k + \sum_{j=1}^{m_k^i} b_{ijk}^{i}(t)\, x_{jk}^{i}(t) + b_{eik}^{e}(t)\, x_k^e(t) = 0 \\ \frac{\partial L_{ik}^{i}(t)}{\partial \lambda_{2ik}^{i}(t)} = l_{ik}^{i}(t) - d_k - c_k x_{ik}^{i}(t) = 0 \\ x_{ik}^{i}(t) > 0,\ l_{ik}^{i}(t) > 0 \\ p_{ik}^{i}(t)\, x_{ik}^{i}(t) - w(t)\, l_{ik}^{i}(t) \geq 0. \end{cases}$$

The first order conditions are necessary and sufficient for the unique global maximum because the objective function is pseudoconcave and the constraints are quasiconcave.[36]

The Lagrangian function of this maximization problem for the entrant is

$$L_k^e(t) = p_k^e(t)\, x_k^e(t) - w(t)\, l_k^e(t) - \lambda_{1k}^e(t)\, [p_k^e(t) - a_k + \\ + \sum_{j=1}^{m_k^i} b_{ejk}^e(t)\, x_{jk}^i(t) + b_{0k} x_k^e(t)] - \lambda_{2k}^e(t)\, [l_k^e(t) - d_k - c_k x_k^e(t)]$$

where $\lambda_{1k}^e(t)$ and $\lambda_{2k}^e(t)$ are the Lagrangian multipliers respectively associated with the inverse demand function and the production function.

The optimal decisions regarding $p_k^e(t)$, $x_k^e(t)$ and $l_k^e(t)$ are functions of the choices of the incumbent firms and are determined by the first order

[36] See de la Fuente (2000), p. 289, Theorem 1.14.

conditions and by the condition of nonnegative operating profits:

$$\begin{cases} \frac{\partial L_k^e(t)}{\partial p_k^e(t)} = x_k^e(t) - \lambda_{1k}^e(t) = 0 \\ \frac{\partial L_k^e(t)}{\partial x_k^e(t)} = p_k^e(t) - \lambda_{1k}^e(t)\, b_{0k} + \lambda_{2k}^e(t)\, c_k = 0 \\ \frac{\partial L_k^e(t)}{\partial l_k^e(t)} = -w(t) - \lambda_{2k}^e(t) = 0 \\ \frac{\partial L_k^e(t)}{\partial \lambda_{1k}^e(t)} = p_k^e(t) - a_k + \sum_{j=1}^{m_k^i} b_{ejk}^e(t)\, x_{jk}^i(t) + b_{0k} x_k^e(t) = 0 \\ \frac{\partial L_k^e(t)}{\partial \lambda_{2k}^e(t)} = l_k^e(t) - d_k - c_k x_k^e(t) = 0 \\ x_k^e(t) > 0,\ l_k^e(t) > 0 \\ p_k^e(t)\, x_k^e(t) - w(t)\, l_k^e(t) \geq 0. \end{cases}$$

The first order conditions are necessary and sufficient for the unique global maximum because the objective function is pseudoconcave and the constraints are quasiconcave.[37]

We assume a parameters structure such that $b_{ijk}^i(t) = b_{ilk}^i(t)\ \forall j \neq i$ $\forall l \neq i$ and $b_{ejk}^e(t) = b_{elk}^e(t)\ \forall j, l$.

The two systems of optimality conditions imply:

[37] See de la Fuente (2000), p. 289, Theorem 1.14.

$$
\begin{cases}
x^i_{ik}(t) = \lambda^i_{1ik}(t) = \frac{[a_k - w(t)c_k][2b_{0k} - b^e_{eik}(t)]}{2b^2_{0k} + 2b_{0k}\sum_{j=1}^{m^i_k} b^i_{ijk}(t) - b^e_{eik}(t)\sum_{j=1}^{m^i_k} b^e_{ejk}(t)} \\
p^i_{ik}(t) = \frac{a_k b_{0k}[2b_{0k} - b^e_{eik}(t)]}{2b^2_{0k} + 2b_{0k}\sum_{j=1}^{m^i_k} b^i_{ijk}(t) - b^e_{eik}(t)\sum_{j=1}^{m^i_k} b^e_{ejk}(t)} + \\
\quad + \frac{w(t)c_k[2b_{0k}\sum_{j=1}^{m^i_k} b^i_{ijk}(t) - b^e_{eik}(t)\sum_{j=1}^{m^i_k} b^e_{ejk}(t) + b_{0k}b^e_{eik}(t)]}{2b^2_{0k} + 2b_{0k}\sum_{j=1}^{m^i_k} b^i_{ijk}(t) - b^e_{eik}(t)\sum_{j=1}^{m^i_k} b^e_{ejk}(t)} \\
l^i_{ik}(t) = d_k + \frac{c_k[a_k - w(t)c_k][2b_{0k} - b^e_{eik}(t)]}{2b^2_{0k} + 2b_{0k}\sum_{j=1}^{m^i_k} b^i_{ijk}(t) - b^e_{eik}(t)\sum_{j=1}^{m^i_k} b^e_{ejk}(t)} \\
x^e_k(t) = \lambda^e_{1k}(t) = \frac{[a_k - w(t)c_k][b_{0k} + \sum_{j=1}^{m^i_k} b^i_{ijk}(t) - \sum_{j=1}^{m^i_k} b^e_{ejk}(t)]}{2b^2_{0k} + 2b_{0k}\sum_{j=1}^{m^i_k} b^i_{ijk}(t) - b^e_{eik}(t)\sum_{j=1}^{m^i_k} b^e_{ejk}(t)} \\
p^e_k(t) = \frac{a_k b_{0k}[b_{0k} + \sum_{j=1}^{m^i_k} b^i_{ijk}(t) - \sum_{j=1}^{m^i_k} b^e_{ejk}(t)]}{2b^2_{0k} + 2b_{0k}\sum_{j=1}^{m^i_k} b^i_{ijk}(t) - b^e_{eik}(t)\sum_{j=1}^{m^i_k} b^e_{ejk}(t)} + \\
+ \frac{w(t)c_k[b^2_{0k} + b_{0k}\sum_{j=1}^{m^i_k} b^i_{ijk}(t) - b^e_{eik}(t)\sum_{j=1}^{m^i_k} b^e_{ejk}(t) + b_{0k}\sum_{j=1}^{m^i_k} b^e_{ejk}(t)]}{2b^2_{0k} + 2b_{0k}\sum_{j=1}^{m^i_k} b^i_{ijk}(t) - b^e_{eik}(t)\sum_{j=1}^{m^i_k} b^e_{ejk}(t)} \\
l^e_k(t) = d_k + \frac{c_k[a_k - w(t)c_k][b_{0k} + \sum_{j=1}^{m^i_k} b^i_{ijk}(t) - \sum_{j=1}^{m^i_k} b^e_{ejk}(t)]}{2b^2_{0k} + 2b_{0k}\sum_{j=1}^{m^i_k} b^i_{ijk}(t) - b^e_{eik}(t)\sum_{j=1}^{m^i_k} b^e_{ejk}(t)} \\
w(t)\, d_k \le \frac{[a_k - w(t)c_k]^2 b_{0k}[b_{0k} + \sum_{j=1}^{m^i_k} b^i_{ijk}(t) - \sum_{j=1}^{m^i_k} b^e_{ejk}(t)]^2}{[2b^2_{0k} + 2b_{0k}\sum_{j=1}^{m^i_k} b^i_{ijk}(t) - b^e_{eik}(t)\sum_{j=1}^{m^i_k} b^e_{ejk}(t)]^2} \\
\lambda^i_{2ik}(t) = \lambda^e_{2k}(t) = -w(t) \\
\sum_{j=1}^{m^i_k} b^e_{ejk}(t) < \sum_{j=1}^{m^i_k} b^i_{ijk}(t) + b_{0k}.
\end{cases}
$$

Step 2: We assume an initial parameters structure such that $b^i_{ijk}(0) = b^i_{ilk}(0)\ \forall j \neq i\ \forall l \neq i$ and $b^e_{ejk}(\tau) = b^e_{elk}(\tau)\ \forall j, l$. The concentrated Hamiltonian function of the optimal control problem for the incumbent producing variety i of good k, where $l^i_{ik}(t)$, $p^i_{ik}(t)$ and $x^i_{ik}(t)$ are at their optimal values (starred variables), is

$$H_{ik}^{i}(t) = \{p_{ik}^{i,*}(t)\, x_{ik}^{i,*}(t) - w(t)\, [l_{ik}^{i,*}(t) + \sum_{j=1}^{m_k^i} R_{ijk}^{i}(t) +$$

$$+ R_{iek}^{i}(t)]\} e^{-rt} - \mu_{iek}^{i}(t)\, \gamma b_{iek}^{e}(t) \left[\phi\left(R_{iek}^{i}(t)\right) + \phi\left(R_{eik}^{e}(t)\right)\right] +$$

$$- \sum_{j \neq i} \mu_{ijk}^{i}(t)\, \gamma b_{ijk}^{i}(t) \left[\phi(R_{ijk}^{i}(t)) + \phi(R_{jik}^{i}(t))\right]$$

where $\mu_{iek}^{i}(t)$ and $\mu_{ijk}^{i}(t)$ $\forall j \neq i$ are the costate variables associated respectively with $b_{iek}^{i}(t)$ and $b_{ijk}^{i}(t)$ $\forall j \neq i$. The behaviour of the candidates for the optimal path of $R_{iek}^{i}(t)$ and $R_{ijk}^{i}(t)$ $\forall j \neq i$ is described by the following maximum principle conditions and by the nonnegativity condition of the discounted sum of current and future profits:

$$\begin{cases}
\frac{\partial H_{ik}^{i}(t)}{\partial R_{ijk}^{i}(t)} = -w(t)\, e^{-rt} - \mu_{ijk}^{i}(t)\, \gamma b_{ijk}^{i}(t)\, \phi'(R_{ijk}^{i}(t)) = 0 \ \forall j \neq i \\
\frac{\partial H_{ik}^{i}(t)}{\partial R_{iek}^{e}(t)} = -w(t)\, e^{-rt} - \mu_{iek}^{i}(t)\, \gamma b_{iek}^{e}(t)\, \phi'(R_{iek}^{i}(t)) = 0 \\
\frac{\partial H_{ijk}^{inx}(t)}{\partial b_{ijk}^{e}(t)} = -e^{-rt} x_{ik}^{i,*}(t)\, x_{jk}^{i,*}(t) + \mu_{ijk}^{i}(t)\, \frac{\dot{b}_{ijk}^{e}(t)}{b_{ijk}^{e}(t)} = -\dot{\mu}_{ijk}^{i}(t) \ \ \forall j \neq i \\
\frac{\partial H_{ik}^{e}(t)}{\partial b_{iek}^{e}(t)} = -e^{-rt} x_{ik}^{i,*}(t)\, x_{k}^{e,*}(t) + \mu_{iek}^{i}(t)\, \frac{\dot{b}_{iek}^{e}(t)}{b_{iek}^{e}(t)} = -\dot{\mu}_{iek}^{i}(t) \\
\frac{\partial H_{ik}^{i}(t)}{\partial \mu_{ijk}^{i}(t)} = -\gamma b_{ijk}^{i}(t)\, [\phi(R_{ijk}^{i}(t)) + \phi(R_{jik}^{i}(t))] = \dot{b}_{ijk}^{i}(t) \ \ \forall j \neq i \\
\frac{\partial H_{ik}^{i}(t)}{\partial \mu_{iek}^{i}(t)} = -\gamma b_{iek}^{e}(t)\, [\phi(R_{iek}^{i}(t)) + \phi(R_{eik}^{e}(t))] = \dot{b}_{iek}^{e}(t) \\
R_{iik}^{i}(t) = 0,\ R_{iek}^{i}(t) \geq 0,\ R_{ijk}^{i}(t) \geq 0 \ \forall j \neq i \\
\lim_{t \to \infty} H_{ik}^{i}(t) = 0 \text{ (trasversality condition)} \\
\int_{t}^{\infty} \{p_{ik}^{i,*}(s)\, x_{ik}^{i,*}(s) - w(s)\, [l_{ik}^{i,*}(s) + \sum_{j=1}^{m_k} R_{ijk}^{i}(s) + R_{iek}^{i}(s)]\} e^{-r(s-t)} ds \geq 0.
\end{cases}$$

The behaviours of $x^i_{ik}(t)$, $p^i_{ik}(t)$ and $l^i_{ik}(t)$ follow the analysis described in Proposition 2 in each period $t < \tau$ and in Step 1 in this Proposition afterwards. The chosen R&D paths imply that the initial symmetry in the substitutability parameters is carried on; let us call $b^i_k(t) = b^i_{ijk}(t)\ \forall i\ \forall j \neq i$ and $b^e_k(t) = b^e_{ejk}(t)\ \forall j$. The system of optimality conditions implies

$$\begin{cases} \mu^i_{ijk}(t) = -\frac{e^{-rt}w(t)}{\gamma b^i_k(t)\phi'\left(R_{ijk}(t)\right)}\ \forall j \neq i \\ \frac{\dot{R}^i_{ijk}(t)}{R^i_{ijk}(t)} = -\frac{\phi'(R^i_{ijk}(t))}{\phi''(R^i_{ijk}(t))R^i_{ijk}(t)}\{r - \frac{\gamma[a_k - w(t)c_k]^2 b^i_k(t)\phi'(R^i_{ijk}(t))}{w(t)[2b_{0k}+(m^i_k-1)b^i_k(t)]^2}\}\ \forall j \neq i\ \forall t < \tau \\ \frac{\dot{R}^i_{ijk}(t)}{R^i_{ijk}(t)} = -\frac{\phi'(R^i_{ijk}(t))}{\phi''(R^i_{ijk}(t))R^i_{ijk}(t)}\{r - \frac{\gamma[a_k - w(t)c_k]^2 b^i_k(t)[2b_{0k}-b^e_k(t)]^2\phi'(R^i_{ijk}(t))}{w(t)[4b^2_{0k}+2(m^i_k-1)b_{0k}b^i_k(t)-m^i_k b^e_k(t)^2]^2}\} \\ \hfill \forall j \neq i\ \forall t \geq \tau \\ \frac{\dot{R}^i_{iek}(t)}{R^i_{iek}(t)} = -\frac{\phi'(R^i_{iek}(t))}{\phi''(R^i_{iek}(t))R^i_{iek}(t)}\{r+ \\ \qquad - \frac{\gamma[a_k - w(t)c_k]^2 b^e_k(t)[2b_{0k}-b^e_k(t)][2b_{0k}+(m^i_k-1)b^i_k(t)-m^i_k b^e_k(t)]\phi'(R^i_{iek}(t))}{w(t)[4b^2_{0k}+2(m^i_k-1)b_{0k}b^i_k(t)-m^i_k b^e_k(t)^2]^2}\} \\ \hfill \forall j \neq i\ \forall t \geq \tau \\ \dot{b}^i_k(t) = -\gamma b^i_k(t)\left[\phi(R^i_{ijk}(t)) + \phi(R^i_{jik}(t))\right]\ \forall j \neq i \\ \dot{b}^e_k(t) = -\gamma b^e_k(t)\left[\phi(R^i_{iek}(t)) + \phi(R^e_{eik}(t))\right] \\ R^i_{iik}(t) = 0,\ R^i_{ijk}(t) > 0\ \forall j \neq i,\ R^i_{iek}(t) > 0\ \forall t \geq \tau \\ \int_0^\tau \{\frac{[a_k - w(t)c_k]^2 b_{0k}}{[2b_{0k}+(m^i_k-1)b^i_k(t)]^2} - w(t)[d_k + \sum_{j=1}^{m_k} R^i_{ijk}(t) + R^i_{iek}(t)]\}e^{-rt}dt+ \\ +\int_\tau^\infty \{\frac{[a_k - w(t)c_k]^2 b_{0k}[2b_{0k}-b^e_k(t)]^2}{[4b^2_{0k}+2(m^i_k-1)b_{0k}b^i_k(t)-m^i_k b^e_k(t)^2]^2} - w(t)[d_k + \sum_{j=1}^{m_k} R^i_{ijk}(t)+ \\ \hfill +R^i_{iek}(t)]\}e^{-rt}dt \geq 0 \\ \int_\tau^\infty \{\frac{[a_k - w(t)c_k]^2 b_{0k}[2b_{0k}-b^e_k(t)]^2}{[4b^2_{0k}+2(m^i_k-1)b_{0k}b^i_k(t)-m^i_k b^e_k(t)^2]^2} - w(t)[d_k+ \\ \hfill +\sum_{j=1}^{m_k} R^i_{ijk}(t) + R^i_{iek}(t)]\}e^{-r(t-\tau)}dt \geq 0 \end{cases}$$

where only the conditions about profits in periods 0 and τ are relevant because profits are increasing over time ($\dot{\pi}_{jk}(t) > 0$) in subperiods $[0, \tau)$ and $[\tau, \infty)$.

The transversality condition is verified in the solution because
$\lim_{t\to\infty} \left\{p_{ik}^{i,*}(t)\, x_{ik}^{i,*}(t) - w(t)\left[l_{ik}^{i,*}(t) + \sum_{j=1}^{m_k^i} R_{ijk}^{i,*}(t) + R_{iek}^{i,*}(t)\right]\right\} < \infty$,
$\lim_{t\to\infty} e^{-rt} = 0$, $\lim_{t\to\infty} \mu_{ijk}^{i,*}(t) = 0$, $\lim_{t\to\infty} \mu_{iek}^{i,*}(t) = 0$,
$\lim_{t\to\infty} -\gamma b_{iek}^{e}(t)\left[\phi(R_{iek}^{i,*}(t)) + \phi(R_{eik}^{e,*}(t))\right] = 0$ and
$\lim_{t\to\infty} -\gamma b_{ijk}^{i}(t)\left[\phi(R_{ijk}^{i,*}(t)) + \phi(R_{jik}^{i,*}(t))\right] = 0\ \forall j \neq i$.

The concentrated Hamiltonian function of the optimal control problem for the entrant is

$$H_k^e(t) = \{p_k^{e,*}(t)\, x_k^{e,*}(t) - w(t)\,[l_k^{e,*}(t) + \sum_{j=1}^{m_k^i} R_{ejk}^{e}(t)]\}e^{-r(t-\tau)} + \\ - \sum_{j=1}^{m_k^i} \mu_{ejk}^{e}(t)\,\gamma b_{ejk}^{e}(t)\left[\phi(R_{ejk}^{e}(t)) + \phi(R_{jek}^{i}(t))\right]$$

where $\mu_{ejk}^{e}(t)\ \forall j$ is the costate variable associated with $b_{ejk}^{e}(t)\ \forall j$. The behaviour of the candidates for the optimal path of $R_{ejk}^{e}(t)\ \forall j$ is described by the maximum principle conditions and by the nonnegativity condition of the discounted sum of current and future profits:

$$
\begin{cases}
\frac{\partial H_k^e(t)}{\partial R_{ejk}^e(t)} = -w\left(t\right)e^{-r(t-\tau)} - \mu_{ejk}^e\left(t\right)\gamma b_{ejk}^e\left(t\right)\phi'(R_{ejk}^e\left(t\right)) = 0 \ \forall j \\
\frac{\partial H_k^e(t)}{\partial b_{ejk}^e(t)} = -e^{-r(t-\tau)}x_k^{e,*}\left(t\right)x_{ik}^{i,*}\left(t\right) + \mu_{ejk}^e\left(t\right)\frac{\dot{b}_{ejk}^e(t)}{b_{ejk}^e(t)} = -\dot{\mu}_{ejk}^e\left(t\right) \ \forall j \\
\frac{\partial H_k^e(t)}{\partial \mu_{ejk}^e(t)} = -\gamma b_{ejk}^e\left(t\right)[\phi(R_{ejk}^e\left(t\right)) + \phi(R_{jek}^i\left(t\right))] = \dot{b}_{ejk}^e\left(t\right) \ \forall j \\
R_{ejk}^e\left(t\right) \geq 0 \ \forall j \\
\lim_{t\to\infty} H_k^e\left(t\right) = 0 \text{ (trasversality condition)} \\
\int_t^\infty \{[p_k^{e,*}\left(s\right)x_k^{e,*}\left(s\right) - w\left(s\right)[l_k^{e,*}\left(s\right) + \sum_{j=1}^{m_k} R_{ejk}^e\left(s\right)]\}e^{-r(s-t)}ds \geq 0.
\end{cases}
$$

The behaviours of $x_{ik}\left(t\right)$, $p_{ik}\left(t\right)$ and $l_{ik}\left(t\right)$ follow the analysis described in Step 1. The chosen R&D paths imply that the initial symmetry in the substitutability parameters is carried on. The optimality conditions imply

$$
\begin{cases}
\mu_{ejk}^e\left(t\right) = -\frac{w(t)e^{-r(t-\tau)}}{\gamma b_k^e(t)\phi'(R_{ejk}^e(t))} \ \forall j \\
\frac{\dot{R}_{ejk}^e(t)}{R_{ejk}^e(t)} = -\frac{\phi'(R_{ejk}^e(t))}{\phi''(R_{ejk}^e(t))R_{ejk}^e(t)}\{r + \\
\qquad -\frac{\gamma[a_k - w(t)c_k]^2 b_k^e(t)[2b_{0k} + (m_k^i - 1)b_k^i(t) - m_k^i b_k^e(t)]^2\phi'(R_{ejk}^e(t))}{w(t)[4b_{0k}^2 + 2(m_k^i - 1)b_{0k}b_k^i(t) - m_k^i b_k^e(t)^2]^2}\} \ \forall j \\
\dot{b}_{ejk}^e\left(t\right) = -\gamma b_{ejk}^e\left(t\right)[\phi(R_{ejk}^e\left(t\right)) + \phi(R_{jek}^i\left(t\right))] \ \forall j \\
R_{ejk}^e\left(t\right) > 0 \ \forall j \\
\int_\tau^\infty \{\frac{[a_k - w\left(t\right)c_k]^2 b_{0k}[2b_{0k} + (m_k^i - 1)b_k^i\left(t\right) - m_k^i b_k^e\left(t\right)]^2}{[4b_{0k}^2 + 2(m_k^i - 1)b_{0k}b_k^i\left(t\right) - m_k^i b_k^e\left(t\right)^2]^2} - w\left(t\right)[d_k + \\
\qquad + \sum_{j=1}^{m_k} R_{ejk}^e\left(t\right)]\}e^{-r(t-\tau)}dt \geq 0
\end{cases}
$$

where only the condition about profits in period τ is relevant because profits are increasing over time. The transversality condition is verified because

$$\lim_{t\to\infty}\left\{p_k^{e,*}(t)\,x_k^{e,*}(t)-w(t)\left[l_k^{e,*}(t)+\textstyle\sum_{j=1}^{m_k^i}R_{ejk}^{e,*}(t)\right]\right\}<\infty,$$
$$\lim_{t\to\infty}e^{-rt}=0,\ \lim_{t\to\infty}\mu_{ejk}^{e,*}(t)=0 \text{ and}$$
$$\lim_{t\to\infty}-\gamma b_{ejk}^{e}(t)\left[\phi(R_{ejk}^{e,*}(t))+\phi(R_{jek}^{e,*}(t))\right]=0\ \forall j.$$

Propositions 5-6 The utility maximization problem of the benevolent planner is dynamic and can be written as

$$\max_V\int_0^\infty\left\{x_0(t)+\sum_{k=1}^{N}\sum_{i=1}^{m_k}\left[a_k-\sum_{j=1}^{m_k}\frac{b_{ijk}(t)}{2}x_{jk}(t)\right]x_{ik}(t)\right\}e^{-rt}dt$$

subject to

$$\begin{cases} l_{ik}(t)=\begin{cases} d_k+c_kx_{ik}(t) & \text{if } x_{ik}(t)>0\\ 0 & \text{otherwise}\end{cases} \quad \forall t\\ l_0(t)=c_0x_0(t)\ \ \forall t\\ l_0(t)+\displaystyle\sum_{k=1}^{N}\sum_{i=1}^{m_k}\left[l_{ik}(t)+\sum_{j=1}^{m_k}R_{ijk}(t)\right]=1\ \forall t\\ \dot{b}_{ijk}(t)=-\gamma b_{ijk}(t)\left[\phi\left(R_{ijk}(t)\right)+\phi\left(R_{jik}(t)\right)\right]\ \ \forall i,k,t\ \forall j\neq i\\ x_0(t)>0,\ \ l_0(t)\geq 0,\ \ x_{ik}(t)\geq 0,\ \ l_{ik}(t)\geq 0\ \forall i,k,t\\ R_{iik}(t)=0,\ \ R_{ijk}(t)\geq 0\ \forall i,k,t\ \forall j\neq i\end{cases}$$

where $V=\{\{x_0(t),l_0(t),\{\{x_{ik}(t),l_{ik}(t),\{R_{ijk}(t)\}_{j=1}^{m_k}\}_{i=1}^{m_k}\}_{k=1}^{N}\}_{t\in[0,\infty)}\}$ is the vector with the choice variables. We now determine the chosen quantities and labour allocation for given values of $b_{ijk}(t)$. In the second step we examine the R&D choices.

Step 1 (Proposition 5): the choice of the optimal $x_0(t)$, $l_0(t)$, $x_{ik}(t)$ and $l_{ik}(t)$ $\forall i,k$ is static and does not depend on the dynamics of $b_{ijk}(t)$. We assume $x_0(t) > 0$ and $x_{ik}(t) > 0\ \forall i, k$, which imply $l_0(t) > 0$ and $l_{ik}(t) > 0$;[38] the Lagrangian function is

$$L(t) = x_0(t) + \sum_{k=1}^{N}\sum_{i=1}^{m_k}[a_k - \sum_{j=1}^{m_k}\frac{b_{ijk}(t)}{2}x_{jk}(t)]x_{ik}(t) + \\ - \sum_{k=1}^{N}\sum_{i=1}^{m_k}\lambda_{1ik}(t)\left[l_{ik}(t) - d_k - c_k x_{ik}(t)\right] - \lambda_2(t)\left[l_0(t) - c_0 x_0(t)\right] + \\ - \lambda_3(t)\{l_0(t) + \sum_{k=1}^{N}\sum_{i=1}^{m_k}[l_{ik}(t) + \sum_{j=1}^{m_k}R_{ijk}(t)] - 1\}$$

where $\lambda_{1ik}(t)\ \forall i, k$, $\lambda_2(t)$ and $\lambda_3(t)$ are Lagrangian multipliers for the differentiated good and the homogenous good production functions and for the full employment condition. The first order conditions are:

$$\begin{cases} \frac{\partial L(t)}{\partial x_0(t)} = 1 + \lambda_2(t)\, c_0 = 0 \\ \frac{\partial L(t)}{\partial l_0(t)} = -\lambda_2(t) - \lambda_3(t) = 0 \\ \frac{\partial L(t)}{\partial x_{ik}(t)} = a_k - \sum_{j=1}^{m_k} b_{ijk}(t)\, x_{jk}(t) + \lambda_{1ik}(t)\, c_k = 0\ \forall i, k \\ \frac{\partial L(t)}{\partial l_{ik}(t)} = -\lambda_{1ik}(t) - \lambda_3(t) = 0\ \forall i, k \\ \frac{\partial L(t)}{\partial \lambda_{1ik}(t)} = l_{ik}(t) - d_k - c_k x_{ik}(t) = 0\ \forall i, k \\ \frac{\partial L(t)}{\partial \lambda_2(t)} = l_0(t) - c_0 x_0(t) = 0 \\ \frac{\partial L(t)}{\partial \lambda_3(t)} = l_0(t) + \sum_{k=1}^{N}\sum_{i=1}^{m_k}[l_{ik}(t) + \sum_{j=1}^{m_k} R_{ijk}(t)] - 1 = 0 \\ x_0(t) > 0,\ l_0(t) > 0,\ x_{ik}(t) > 0,\ l_{ik}(t) > 0\ \forall i, k. \end{cases}$$

[38]The case $x_{ik}(t) = 0$ is trivial because implies $R_{ijk}(t) = 0\ \forall j$. If $x_0(t) > 0$ and $x_{ik}(t) > 0$ are true in the decentralized solution, they are also verified in the socially optimal solution.

The first order conditions are necessary and sufficient for the unique global maximum because the objective function is pseudoconcave and the constraints are quasiconcave.[39]

We assume a parameters structure such that $b_{ijk}(t) = b_{ilk}(t)\ \forall i,k\ \forall j \neq i\ \forall l \neq i$. The system of first order conditions implies

$$\begin{cases} \lambda_{1ik}(t) = \lambda_2(t) = -\frac{1}{c_0}\ \forall i,k \\ \lambda_3(t) = \frac{1}{c_0} \\ x_{ik}(t) = \frac{a_k - \frac{c_k}{c_0}}{\sum_{j=1}^{m_k} b_{ijk}(t)}\ \forall i,k \\ l_{ik}(t) = d_k + \frac{c_k(a_k - \frac{c_k}{c_0})}{\sum_{j=1}^{m_k} b_{ijk}(t)}\ \forall i,k \\ l_0(t) = 1 - \sum_{k=1}^{N}\sum_{i=1}^{m_k}[\frac{c_k(a_k - \frac{c_k}{c_0})}{\sum_{j=1}^{m_k} b_{ijk}(t)} + d_k + \sum_{j=1}^{m_k} R_{ijk}(t)] \\ x_0(t) = \frac{1}{c_0} - \sum_{k=1}^{N}\sum_{i=1}^{m_k}[\frac{c_k(a_k - \frac{c_k}{c_0})}{c_0\sum_{j=1}^{m_k} b_{ijk}(t)} + \frac{d_k}{c_0} + \frac{\sum_{j=1}^{m_k} R_{ijk}(t)}{c_0}]. \end{cases}$$

Step 2 (Proposition 6): The choice of the optimal value of the $R_{ijk}(t)$ variables is a dynamic decision. The state variables are $b_{ijk}(t)\ \forall i,k\ \forall j \neq i$. The concentrated Hamiltonian function, where $x_{ik}(t)$, $p_{ik}(t)$ and $l_{ik}(t)$ are at their optimal values (starred variables), is

$$\begin{aligned} H(t) = \{x_0^*(t) + \sum_{k=1}^{N}\sum_{i=1}^{m_k}[a_k - \sum_{j=1}^{m_k}\frac{b_{ijk}(t)}{2}x_{jk}^*(t)]x_{ik}^*(t)\}e^{-rt} + \\ -\sum_{k=1}^{N}\sum_{i=1}^{m_k}\sum_{j\neq i}\mu_{ijk}(t)\gamma b_{ijk}(t)[\phi(R_{ijk}(t)) + \phi(R_{jik}(t))] \end{aligned}$$

[39]See de la Fuente (2000), p. 289, Theorem 1.14.

where $\mu_{ijk}(t)$ is the costate variable associated with $b_{ijk}(t)$ $\forall i,k$ $\forall j \neq i$. The behaviour of the candidates for the optimal path of $R_{ijk}(t)$ $\forall i,k$ $\forall j \neq i$ is described by the maximum principle conditions:

$$\begin{cases} \frac{\partial H(t)}{\partial R_{ijk}(t)} = -\frac{e^{-rt}}{c_0} - \gamma \mu_{ijk}(t)\, b_{ijk}(t)\, \phi'(R_{ijk}(t)) = 0 \;\forall i,k\; \forall j \neq i \\ \frac{\partial H(t)}{\partial b_{ijk}(t)} = -e^{-rt} x^*_{ik}(t)\, x^*_{jk}(t) + \mu_{ijk}(t) \frac{\dot{b}_{ijk}(t)}{b_{ijk}(t)} = -\dot{\mu}_{ijk}(t) \;\; \forall i,k\; \forall j \neq i \\ \frac{\partial H(t)}{\partial \mu_{ijk}(t)} = -\gamma b_{ijk}(t)\left[\phi(R_{ijk}(t)) + \phi(R_{jik}(t))\right] = \dot{b}_{ijk}(t) \;\; \forall i,k\; \forall j \neq i \\ R_{iik}(t) = 0,\;\; R_{ijk}(t) \geq 0 \;\forall i,k\; \forall j \neq i \\ \lim\limits_{t\to\infty} H(t) = 0 \text{ (trasversality condition).} \end{cases}$$

We assume an initial parameters structure such that $b_{ijk}(0) = b_{ilk}(0)$ $\forall j \neq i$ $\forall l \neq i$; $x_{ik}(t)$, $p_{ik}(t)$ and $l_{ik}(t)$ follow the analysis described in Proposition 2.

The system of optimality conditions implies

$$\begin{cases} \mu_{ijk}(t) = -\frac{e^{-rt}}{\gamma c_0 b_{ijk}(t) \phi'\left(R_{ijk}(t)\right)} \;\; \forall i,k\; \forall j \neq i \\ \frac{\dot{R}_{ijk}(t)}{R_{ijk}(t)} = -\frac{\phi'\left(R_{ijk}(t)\right)}{\phi''\left(R_{ijk}(t)\right) R_{ijk}(t)} \{ r - \frac{\gamma (a_k - \frac{c_k}{c_0})^2 b_{ijk}(t) \phi'\left(R_{ijk}(t)\right)}{c_0 [\sum_{j=1}^{m_k} b_{ijk}(t)]^2} \} \;\forall i,k\; \forall j \neq i \\ \dot{b}_{ijk}(t) = -\gamma b_{ijk}(t)\left[\phi(R_{ijk}(t)) + \phi(R_{jik}(t))\right] \;\; \forall i,k\; \forall j \neq i \\ R_{iik}(t) = 0,\;\; R_{ijk}(t) \geq 0 \;\forall j \neq i. \end{cases}$$

The transversality condition is verified in the solution because
$\lim\limits_{t\to\infty} \left\{ x^*_0(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} \left[a_k - \sum_{j=1}^{m_k} \frac{b_{ijk}(t)}{2} x^*_{jk}(t) \right] x^*_{ik}(t) \right\} < \infty$,
$\lim\limits_{t\to\infty} e^{-rt} = 0$, $\lim_{t\to\infty} \mu^*_{ijk}(t) = 0$ and
$\lim\limits_{t\to\infty} -\gamma b_{ijk}(t) \left[\phi(R^*_{ijk}(t)) + \phi(R^*_{jik}(t)) \right] = 0 \;\forall i,k\; \forall j \neq i.$

Proposition 7 The intertemporal utility maximization problem of the benevolent planner, assuming $\phi(R) = \frac{R^{1-\eta}}{1-\eta}$, can be written as

$$\max_{V} \int_0^\infty \left\{ x_0(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k(t)} \left[a_k - \sum_{j=1}^{m_k(t)} \frac{b_{ijk}(t)}{2} x_{jk}(t) \right] x_{ik}(t) \right\} e^{-rt} dt$$

subject to

$$\begin{cases} l_{ik}(t) = \begin{cases} d_k + c_k x_{ik}(t) & \text{if } x_{ik}(t) > 0 \\ 0 & \text{otherwise} \end{cases} \quad \forall t \\ l_0(t) = c_0 x_0(t) \;\; \forall t \\ l_0(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k(t)} \left[l_{ik}(t) + \sum_{j=1}^{m_k(t)} R_{ijk}(t) \right] = 1 \; \forall t \\ \dot{b}_{ijk}(t) = -\frac{\gamma b_{ijk}(t)\left[R_{ijk}^{1-\eta}(t) + R_{jik}^{1-\eta}(t)\right]}{1-\eta} \; \forall i,k,t \; \forall j \neq i \\ x_0(t) > 0, \; l_0(t) \geq 0, \; x_{ik}(t) \geq 0, \; l_{ik}(t) \geq 0 \; \forall i,k,t \\ R_{iik}(t) = 0, \; R_{ijk}(t) \geq 0 \; \forall i,k,t \; \forall j \neq i \\ m_k(t) \geq 2 \text{ and integer } \forall k,t \end{cases}$$

where $V = \left\{ \left\{ x_0(t), l_0(t), \left\{ x_{ik}(t), l_{ik}(t), \left\{ R_{ijk}(t) \right\}_{j=1}^{m_k(t)} \right\}_{i=1}^{m_k(t)}, m_k(t) \right\}_{k=1}^{N} \right\}_{t \in [0,\infty)} \right\}$ is the vector with the choice variables of the firm. The first order and maximum principle conditions for $x_0(t)$, $l_0(t)$, $x_{ik}(t)$, $l_{ik}(t)$ and $R_{ijk}(t)$ $\forall i,k$ $\forall j \neq i$, determining the behaviour of the candidates to be optimal choices of benevolent planner, can be obtained following the same procedure as in Propositions 5 and 6; there are now one additional maximum principle condition regarding $m_k(t)$ and the constraint $m_k(t) \geq 2$. The relevant conditions to obtain the result reported in the main text are

$$
\begin{cases}
\frac{\partial L(t)}{\partial x_{ik}(t)} = a_k - \frac{c_k}{c_0} - \sum_{j=1}^{m_k(t)} b_{ijk}(t)\, x_{jk}(t) = 0 \ \forall i,k \\
\frac{\partial H(t)}{\partial R_{ijk}(t)} = -w(t)\, e^{-rt} - \gamma b_{ijk}(t) \left[\mu_{ijk}(t)\, R_{ijk}^{-\eta}(t) + \mu_{jik}(t)\, R_{jik}^{-\eta}(t)\right] = 0 \\
\qquad \forall i,k \ \forall j \neq i \\
\frac{\partial H(t)}{\partial m_k(t)} = \{[a_k - \frac{c_k}{c_0} - \sum_{j=1}^{m_k(t)} b_{mjk}(t)\, x_{jk}(t)] x_{mk}(t) - \frac{d_k}{c_0} + \\
\qquad -\frac{2\sum_{j=1}^{m_k(t)} R_{mjk}(t)}{c_0}\} e^{-rt} + 2\sum_{j=1}^{m_k(t)} \mu_{mjk}(t)\, \dot{b}_{mjk}(t) = 0 \ \forall k
\end{cases}
$$

where the m index in the variables of the third condition is referred to the marginal variety, which is either the last produced or the last abandoned. Also in this case, the transversality condition $\lim_{t\to\infty} H(t) = 0$ is verified in the solution because

$\lim_{t\to\infty} \left\{ x_0^*(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k^*(t)} \left[a_k - \sum_{j=1}^{m_k^*(t)} \frac{b_{ijk}(t)}{2} x_{jk}^*(t) \right] x_{ik}^*(t) \right\} < \infty$,
$\lim_{t\to\infty} e^{-rt} = 0$, $\lim_{t\to\infty} \mu_{ijk}^*(t) = 0$ and
$\lim_{t\to\infty} -\gamma b_{ijk}(t) \left[\phi(R_{ijk}^*(t)) + \phi(R_{jik}^*(t))\right] = 0 \ \forall i,k \ \forall j \neq i.$

Proposition 8 Let us assume that the firm producing variety i of good k deviates from the behaviour described in the main text and decides to produce an older version (with index h) of its variety together with the newest version in period t. We assume, moreover, that substitutability among different versions of the same variety is linear in h, where h has been defined in Subsection 2.8.1.

The deviation does not imply intertemporal elements and, therefore, we consider the static current profits maximization problem. We neglect the time index to simplify notation. The current profits maximization problem of the deviating firm producing variety i of good k can be written as

$$\max_{V_{ik}^{d}(h)} p_{ik}^{n}(h)\, x_{ik}^{n}(h) + p_{ik}^{o}(h)\, x_{ik}^{o}(h) - w\left[l_{ik}^{n}(h) + l_{ik}^{o}(h)\right]$$

subject to

$$\begin{cases} p_{ik}^{n}(h) = a_k - \sum_{j=1}^{m_k} b_{ijk}^{n,n} x_{jk}^{n}(h) - b_{iik}^{o,n}(h)\, x_{ik}^{o}(h) \\ p_{ik}^{o}(h) = a_k - \sum_{j=1}^{m_k} b_{ijk}^{o,n}(h)\, x_{jk}^{n}(h) - b_{0k} x_{ik}^{o}(h) \\ l_{ik}^{n}(h) = \begin{cases} d_k + c_k x_{ik}^{n}(h) & \text{if } x_{ik}^{n}(h) > 0 \\ 0 & \text{otherwise} \end{cases} \\ l_{ik}^{o}(h) = \begin{cases} d_k + c_k x_{ik}^{o}(h) & \text{if } x_{ik}^{o}(h) > 0 \\ 0 & \text{otherwise} \end{cases} \\ x_{ik}^{n}(h) \geq 0,\ x_{ik}^{o}(h) \geq 0,\ l_{ik}^{n}(h) \geq 0,\ l_{ik}^{o}(h) \geq 0 \end{cases}$$

where $V_{ik}^{d}(h) = \left\{\left\{p_{ik}^{r}(h), x_{ik}^{r}(h), l_{ik}^{r}(h)\right\}_{r\in\{o,n\}}\right\}$ is the vector with the choice variables of the deviating firm, the superscripts n and o discriminates between variables of the the old and the new version of the variety, $b_{ijk}^{o,n}(h)$ is the substitutability parameter between the older (with index h) version of variety i and the newest version of variety j of the same good k and $b_{ijk}^{n,n}$ is the substitutability parameter between the newest version of variety i and the newest version of variety j. The current profits maximization problem of a nondeviating firm producing variety j of good k can be written as

$$\max_{V_{jk}^{nd}(h)} p_{jk}^{n}(h)\, x_{jk}^{n}(h) - w l_{jk}^{n}(h)$$

subject to

$$\begin{cases} p_{jk}^{n}(h) = a_k - \sum_{l=1}^{m_k} b_{jlk}^{n,n} x_{lk}^{n}(h) - b_{ijk}^{o,n}(h)\, x_{ik}^{o}(h) \\ l_{jk}^{n}(h) = \begin{cases} d_k + c_k x_{jk}^{n}(h) & \text{if } x_{jk}^{n}(h) > 0 \\ 0 & \text{otherwise} \end{cases} \\ x_{jk}^{n}(h) \geq 0,\; l_{jk}^{n}(h) \geq 0 \end{cases}$$

where $V_{jk}^{nd}(h) = \{p_{jk}^{n}(h), x_{jk}^{n}(h), l_{jk}^{n}(h)\}$ is the vector with the choice variables of the nondeviating firm. We assume $x_{jk}^{n}(h) > 0\ \forall j$, $x_{ik}^{n}(h) > 0$ and $x_{ik}^{o}(h) > 0$, which imply $l_{jk}^{n}(h) > 0$, $l_{ik}^{n}(h) > 0$ and $l_{ik}^{o}(h) > 0$.[40] The Lagrangian function for the deviating firm producing variety i of good k is

$$\begin{aligned} L_{ik}^{d}(h) = {} & p_{ik}^{n}(h)\, x_{ik}^{n}(h) + p_{ik}^{o}(h)\, x_{ik}^{o}(h) - w\left[l_{ik}^{n}(h) + l_{ik}^{o}(h)\right] + \\ & - \lambda_{1ik}^{n}(h)\left[p_{ik}^{n}(h) - a_k + \sum_{j=1}^{m_k} b_{ijk}^{n,n} x_{jk}^{n}(h) + b_{iik}^{o,n}(h)\, x_{ik}^{o}(h)\right] + \\ & - \lambda_{1ik}^{o}(h)\left[p_{ik}^{o}(h) - a_k + \sum_{j=1}^{m_k} b_{ijk}^{o,n}(h)\, x_{jk}^{n}(h) + b_{0k} x_{ik}^{o}(h)\right] + \\ & - \lambda_{2ik}^{n}(h)\left[l_{ik}^{n}(h) - d_k - c_k x_{ik}^{n}(h)\right] - \lambda_{2ik}^{o}(h)\left[l_{ik}^{o}(h) - d_k - c_k x_{ik}^{o}(h)\right] \end{aligned}$$

[40] We are interested in equilibria where $x_{ik}^{o}(h) > 0$, otherwise the model collapses to the main text structure. This implies $x_{ik}^{n}(h) > 0$, because the residual demand when only the older version of the variety is produced has a lower intercept and the same slope as in the situation where only the newest version is produced. Given these two conditions, the first order conditions imply that $x_{jk}^{n}(h) > 0$.

where $\lambda_{1ik}^{n}(h)$, $\lambda_{2ik}^{n}(h)$, $\lambda_{1ik}^{o}(h)$ and $\lambda_{2ik}^{o}(h)$ are the Lagrangian multipliers respectively associated with the inverse demand function and the production function for the new version of the variety and with the inverse demand function and the production function for the old version.

The optimal decisions regarding $p_{ik}^{n}(h)$, $x_{ik}^{n}(h)$, $l_{ik}^{n}(h)$, $p_{ik}^{n}(h)$, $x_{ik}^{n}(h)$ and $l_{ik}^{n}(h)$ are functions of the choices of the nondeviating firms; they are determined by the first order conditions and by the condition that operating profits in case of deviation are bigger than those in absence of deviation:

$$\begin{cases} \frac{\partial L_{ik}^{d}(h)}{\partial p_{ik}^{r}(h)} = x_{ik}^{r}(h) - \lambda_{1ik}^{r}(h) = 0 \; \forall r \in \{n, o\} \\ \frac{\partial L_{ik}^{d}(h)}{\partial x_{ik}^{n}(h)} = p_{ik}^{n}(h) - \lambda_{1ik}^{n}(h)\, b_{0k} - \lambda_{1ik}^{o}(h)\, b_{iik}^{o,n}(h) + \lambda_{2ik}^{n}(h)\, c_{k} = 0 \\ \frac{\partial L_{ik}^{d}(h)}{\partial x_{ik}^{o}(h)} = p_{ik}^{o}(h) - \lambda_{1ik}^{n}(h)\, b_{iik}^{o,n}(h) - \lambda_{1ik}^{o}(h)\, b_{0k} + \lambda_{2ik}^{o}(h)\, c_{k} = 0 \\ \frac{\partial L_{ik}^{d}(h)}{\partial l_{ik}^{r}(h)} = -w - \lambda_{2ik}^{r}(h) = 0 \; \forall r \in \{n, o\} \\ \frac{\partial L_{ik}^{d}(h)}{\partial \lambda_{1ik}^{n}(h)} = p_{ik}^{n}(h) - a_{k} + \sum_{j=1}^{m_{k}} b_{ijk}^{n,n} x_{jk}^{n}(h) + b_{iik}^{o,n}(h)\, x_{ik}^{o}(h) = 0 \\ \frac{\partial L_{ik}^{d}(h)}{\partial \lambda_{1ik}^{o}(h)} = p_{ik}^{o}(h) - a_{k} + \sum_{j=1}^{m_{k}} b_{ijk}^{o,n}(h)\, x_{jk}^{n}(h) + b_{0k} x_{ik}^{o}(h) = 0 \\ \frac{\partial L_{ik}^{d}(h)}{\partial \lambda_{2ik}^{r}(h)} = l_{ik}^{r}(h) - d_{k} - c_{k} x_{ik}^{r}(h) = 0 \; \forall r \in \{n, o\} \\ x_{ik}^{n}(h) > 0, \; l_{ik}^{n}(h) > 0, \; x_{ik}^{n}(h) > 0, \; l_{ik}^{n}(h) > 0 \\ p_{ik}^{n}(h)\, x_{ik}^{n}(h) + p_{ik}^{o}(h)\, x_{ik}^{o}(h) - w\left[l_{ik}^{n}(h) + l_{ik}^{o}(h)\right] \geq \frac{(a_{k} - w c_{k})^{2} b_{0k}}{(b_{0k} + \sum_{j=1}^{m_{k}} b_{ijk}^{n,n})^{2}} - w d_{k}. \end{cases}$$

The first order conditions are necessary and sufficient for the unique global maximum because the objective function is pseudoconcave and the constraints are quasiconcave.[41]

[41] See de la Fuente (2000), p. 289, Theorem 1.14.

The Lagrangian function of this maximization problem for the nondeviating firm producing variety j of good k is

$$L_{jk}^{nd}(h) = p_{jk}^{n}(h)\, x_{jk}^{n}(h) - w l_{jk}^{n}(h) - \lambda_{1jk}^{n}(h) \left[p_{jk}^{n}(h) - a_k + \right.$$
$$\left. + \sum_{l=1}^{m_k} b_{jlk}^{n,n} x_{lk}^{n}(h) + b_{ijk}^{o,n}(h)\, x_{ik}^{o}(h) \right] - \lambda_{2jk}^{n}(h) \left[l_{jk}^{n}(h) - d_k - c_k x_{jk}^{n}(h) \right]$$

where $\lambda_{1jk}^{n}(h)$ and $\lambda_{2jk}^{n}(h)$ are the Lagrangian multipliers respectively associated with the inverse demand function and the production function. The optimal decisions regarding $p_{jk}^{n}(h)$, $x_{jk}^{n}(h)$ and $l_{jk}^{n}(h)$ are functions of the choices of the deviating firm and are determined by the first order conditions and by the condition of nonnegative operating profits:

$$\begin{cases} \frac{\partial L_{jk}^{nd}(h)}{\partial p_{jk}^{n}(h)} = x_{jk}^{n}(h) - \lambda_{1jk}^{n}(h) = 0 \\ \frac{\partial L_{jk}^{nd}(h)}{\partial x_{jk}^{n}(h)} = p_{jk}^{n}(h) - \lambda_{1jk}^{n}(h)\, b_{0k} + \lambda_{2jk}^{n}(h)\, c_k = 0 \\ \frac{\partial L_{jk}^{nd}(h)}{\partial l_{jk}^{nd}(h)} = -w - \lambda_{2jk}^{n}(h) = 0 \\ \frac{\partial L_{jk}^{nd}(h)}{\partial \lambda_{1jk}^{n}(h)} = p_{jk}^{n}(h) - a_k + \sum_{l=1}^{m_k} b_{jlk}^{n,n} x_{lk}^{n}(h) + b_{ijk}^{o,n}(h)\, x_{ik}^{o}(h) = 0 \\ \frac{\partial L_{jk}^{nd}(h)}{\partial \lambda_{2jk}^{n}(h)} = l_{jk}^{n}(h) - d_k - c_k x_{jk}^{n}(h) = 0 \\ x_{jk}^{n}(h) > 0, \;\; l_{jk}^{n}(h) > 0 \\ p_{jk}^{n}(h)\, x_{jk}^{n}(h) - w l_{jk}^{n}(h) \geq 0. \end{cases}$$

The first order conditions are necessary and sufficient for the unique global maximum because the objective function is pseudoconcave and the constraints are quasiconcave.[42]

We assume a parameters structure such that $b_{ijk}^{n,n} = b_{ilk}^{n,n}$ $\forall j \neq i$ $\forall l \neq i$; moreover, we use the definition of $b_{ijk}^{o,n}(h)$ from equation (29) and, given the assumption of linearity in h, the definition of $b_{iik}^{o,n}(h)$ from equation (28). Last, we exploit simmetry between the substitutability coefficients to simplify notation and we call $b_k^n = b_{ijk}^{n,n}$ $\forall j \neq i$. The two systems of conditions imply the following optimal quantities:

$$
\begin{aligned}
x_{ik}^n = \lambda_{1ik}^n &= \frac{(a_k - wc_k)\left[2(m_k+1)\left(2b_{0k}-b_k^n\right) - 3h(m_k-1)\left(b_{0k}-b_k^n\right)\right]}{2b_k^n b_{0k}[8(m_k-2)-3h(m_k-3)] - 2b_{0k}^2[h(m_k+3)-16] + 2\left(b_k^n\right)^2[h(4m_k-6)-4(m_k-1)]} \\
x_{ik}^o = \lambda_{1ik}^o &= \frac{(a_k - wc_k)(3-m_k)\left(2b_{0k}-b_k^n\right)}{b_k^n b_{0k}[8(m_k-2)-3h(m_k-3)] - b_{0k}^2[h(m_k+3)-16] + \left(b_k^n\right)^2[h(4m_k-6)-4(m_k-1)]} \\
x_{jk}^n = \lambda_{1jk}^n &= \frac{(a_k - wc_k)\left[4\left(2b_{0k}-b_k^n\right) - 3h\left(b_{0k}-b_k^n\right)\right]}{b_k^n b_{0k}[8(m_k-2)-3h(m_k-3)] - b_{0k}^2[h(m_k+3)-16] + \left(b_k^n\right)^2[h(4m_k-6)-4(m_k-1)]}.
\end{aligned}
$$

The condition $x_{ik}^n(h) > 0$ implies that the denominators are always positive. The condition $x_{ik}^o(h) > 0$ is verified only if $m_k = 2$; in this case, the overall profits of the deviating firm are

$$
\pi_{ik}^d = \frac{(a_k - wc_k)^2 \left\{9\left[2\left(2b_{0k} - b_k^n\right) - h\left(b_{0k} - b_k^n\right)\right]^2 + 4\left(2b_{0k} - b_k^n\right)^2\right\}}{4\{b_{0k}^2(16-5h) + 3b_k^n b_{0k} h - 2\left(b_k^n\right)^2(2-h)\}^2} - 2wd_k.
$$

Differentiating profits with respect to h yields

$$
\begin{aligned}
\frac{\partial \pi_{ik}^d}{\partial h} = (a_k - wc_k)^2 & (2b_{0k} - b_k^n)(b_{0k} - b_k^n) * \\
& * \frac{4(2b_{0k} - b_k^n)(7b_{0k} + b_k^n) - 9b_{0k}(b_{0k} - b_k^n)h}{[b_{0k}^2(16-5h) - 2\left(b_k^n\right)^2(2-h) + 3b_{0k}b_k^n h]^3} \quad \forall h \in [0,1]
\end{aligned}
$$

[42] See de la Fuente (2000), p. 289, Theorem 1.14.

which is always greater than zero and therefore the optimal choice is $h = 1$; the overall behaviour of the deviating firm in the case $m_k = 2$ is described, therefore, by

$$\begin{cases} x_{ik}^n = \lambda_{1ik}^n = \frac{3(a_k - wc_k)(3b_{0k} - b_k^n)}{22b_{0k}^2 + 6b_k^n b_{0k} - 4(b_k^n)^2} \\ x_{ik}^o = \lambda_{1ik}^o = \frac{(a_k - wc_k)(2b_{0k} - b_k^n)}{11b_{0k}^2 + 3b_k^n b_{0k} - 2(b_k^n)^2} \\ p_{ik}^n = \frac{a_k[11b_{0k}^2 - 2b_{0k}b_k^n - \left(b_k^n\right)^2] + wc_k[11b_{0k}^2 + 8b_k^n b_{0k} - 3\left(b_k^n\right)^2]}{22b_{0k}^2 + 6b_k^n b_{0k} - 4(b_k^n)^2} \\ p_{ik}^o = \frac{a_k[17b_{0k}^2 + 2b_{0k}b_k^n - 3\left(b_k^n\right)^2] + wc_k[27b_{0k}^2 + 10b_k^n b_{0k} - 5\left(b_k^n\right)^2]}{44b_{0k}^2 + 12b_k^n b_{0k} - 8(b_k^n)^2} \\ l_{ik}^n = d_k + \frac{3c_k(a_k - wc_k)(3b_{0k} - b_k^n)}{22b_{0k}^2 + 6b_k^n b_{0k} - 4(b_k^n)^2} \\ l_{ik}^o = d_k + \frac{c_k(a_k - wc_k)(2b_{0k} - b_k^n)}{11b_{0k}^2 + 3b_k^n b_{0k} - 2(b_k^n)^2} \\ x_{jk}^n = \lambda_{1jk}^n = \frac{(a_k - wc_k)(5b_{0k} - b_k^n)}{11b_{0k}^2 + 3b_k^n b_{0k} - 2(b_k^n)^2} \\ p_{jk}^n = \frac{a_k b_{0k}(5b_{0k} - b_k^n) + 2wc_k[3b_{0k}^2 + 2b_k^n b_{0k} - (b_k^n)^2]}{11b_{0k}^2 + 3b_k^n b_{0k} - 2(b_k^n)^2} \\ l_{jk}^n = d_k + \frac{c_k(a_k - wc_k)(5b_{0k} - b_k^n)}{11b_{0k}^2 + 3b_k^n b_{0k} - 2(b_k^n)^2} \\ \lambda_{2ik}^n = \lambda_{2ik}^o = \lambda_{2jk}^n = -w \\ h = 1 \\ wd_k < \frac{9(a_k - wc_k)^2[8b_{0k}^2 - 2(b_k^n)^2 - (b_{0k} - b_k^n)(2b_{0k} + b_k^n)]^2}{4[11b_{0k}^2 + 3b_k^n b_{0k} - 2(b_k^n)^2]^2(2b_{0k} + b_k^n)^2} + \\ + \frac{(a_k - wc_k)^2\{[4b_{0k}^2 - (b_k^n)^2](2b_{0k} + b_k^n) - b_{0k}[11b_{0k}^2 + 3b_k^n b_{0k} - 2(b_k^n)^2]^2\}}{[11b_{0k}^2 + 3b_k^n b_{0k} - 2(b_k^n)^2]^2(2b_{0k} + b_k^n)^2}. \end{cases}$$

3 The evaluation of the incentives to firms for innovation: the case of the Fund for Technological Innovation in Italy.

3.1 Introduction and literature review

The implementation of policy measures to stimulate R&D and the evaluation of their effectiveness is another long-time debated topic in the economic literature. This is because the private management of R&D may be socially inefficient: the external acquisition of knowledge is not always regulated by market mechanisms and agents cannot prevent observation and interaction from other agents, a phenomenon known as spillovers from knowledge in the literature; the social returns from innovation are therefore usually greater than the private ones and the resources allocated by agents to innovate are smaller than the socially optimal amount. The main rationale for public intervention is therefore that subsidies to firms stimulate innovation activities reducing the gap between private and social returns.

Another reason to justify public policy is that financial constraints in the borrowing market can lead to suboptimal investments by firms.[43] R&D investments are risky and subject to asymmetric information between firms and lenders; a higher interest rate than that equating demand and supply of credit can help lenders to discriminate between good and bad projects,

[43] A literature review of the topic is in Hall (2002).

at the social cost of a suboptimal overall credit amount. Public intervention through a concessional loan can loosen the financial constraints.

But the successful implementation of public policies to stimulate innovation is not easy for many reasons, firstly asymmetric information. For instance, project assessment is more difficult for the policymaker than for the firm proposing it: problems of adverse selection might arise. Moreover, once the subsidy has been granted, firms have less incentives to exert effort and there can be moral hazard. Public intervention should therefore ideally aim at stimulating only the projects not undertaken otherwise (additionality), possibly with high social returns; but these requirements are hard to observe and in particular the additionality can be verified only by ex post econometric techniques: the evaluation of the efficacy of the policy is therefore necessary to understand if the design is good enough to achieve its proposed goal of stimulating innovation and economic activity.

In the empirical literature, the general evidence about the efficacy of R&D subsidies is controversial. David, Hall and Toole (2000) critically discuss the findings of forty years of international studies and conclude that there is no definitive evidence on whether public and private R&D expenditures are substitutes or complements. In the analysis of the Small Business Innovation Research program in the U.S., Wallsten (2000) finds that public grants displace firm expenditures dollar for dollar. Lach (2002), on a panel of Israeli firms, finds that subsidies have been effective for small firms, while the policy had a negative effect on large firms. Gonzalez, Jaumandreu and Pazò (2005) in analysing Spanish data find that only a small subset of firms would not have undertaken R&D activity in the absence of the subsidy, while there is no evidence of crowding out among the innovation active firms. Gorg and Strobl (2007), in analysing an Irish sample of firms, conclude that public subsidies replace private R&D expenditure when the award is substantial. Czarnitzki, Ebersberger and Fier (2007) find a positive effect of cooperation on the effectiveness of subsidies in a panel of firms from Germany and Finland.

In contrast with such a wide range of international empirical literature, very few studies examine the efficacy of Italian R&D policies, even if the number of interventions and the amount of invested resources have been relevant in the last decades. Merito, Giannangeli and Bonaccorsi (2008) evaluate the efficacy of the subsidies awarded in 2000 by the Special Fund for Applied Research of the Ministry of University and Research, introduced with the aim of supporting the research component of industrial R&D; they find that four years after the award of the subsidy, the policy had had little effect on number of employees, sales, productivity, labour costs and patent applications. Carboni (2008) examines the efficacy of the main national R&D incentives on a sample from the Capitalia/MCC survey of manufacturing firms in 2001-2003 and does not find crowding out effects.

This study evaluates the effectiveness of the subsidies supplied by the Fund for Technological Innovation (FTI), a financial tool created by the Italian Ministry of Economic Development (MED) to stimulate the innovation activity of firms; this study fills a hole in the literature, given the scarce attention for the Italian policy measures and for several, worthy of note, characteristics of the policy and of the evaluation exercise. An interesting peculiarity is that this Fund, unlike most similar instruments studied in the previous literature, focuses on the development stage of R&D activity. The theoretical models on the topic (e.g. Aghion and Howitt (1996)) find that the effects of the incentives to the research and to the development components of R&D are structurally different; but case studies where the policy measures mainly influence only one of the two components are quite unusual in the empirical literature.

The FTI is composed of two sections: a general purpose one, where applications from any field of activity and geographical area were accepted and evaluated one-by-one by merit following the chronological order of submission without a set deadline; a special purpose one, periodically issuing calls for applications in specific fields of activity or geographical areas with a set deadline, whose applications are ranked and whose subsidies are assigned to the best projects up to the amount of resources available. For both sections, the policy measures include a concessional loan and a non-refundable grant; the overall amount of the subsidy is the maximum allowed by the European Union regulations.

Another interesting feature of the study is in the identification procedure used for the general section: the regular functioning of this section of the Fund was unexpectedly interrupted after about five months due to shortage of funds; we use this exogenous shock to identify the effect of the policy: we compare the behaviour of the subsidized firms with that of the firms applying to the Fund after the shortage of funds, whose application had been neither assessed nor funded until five years later.

The data from the MED about the FTI are merged with the 1999-2007 balance sheets of the firms filed at the Centrale dei Bilanci archives. We use two methodologies to evaluate the efficacy of this section of the Fund: a difference-in-differences approach, complemented by a matching procedure to increase the similarity between treated and controls, and a regression discontinuity design approach, using the submission date of the application as the forcing variable. In both cases, we are not able to detect signals of effectiveness of the policy on the investment behaviour of firms in the considered treatment period 2001-2007. The same is true also for sales, capital and employee figures, while there is a positive effect on assets; the additional liquidity from the subsidy seems probably to have been used to finance the current expenditure of firms. Neither the profitability nor the financial structure of the firm seem have been clearly affected by the policy, apart from a reduction in the share of long-term debts over assets, when calculated net of the concessional loan from the subsidy, a result coherent with the hypothesis of lack of effectiveness of the policy.

We also evaluate the efficacy of three calls for applications of the special section of the FTI; we merged the application data with the balance sheets from the Cerved archives for the years 2001-2007. We use the regression discontinuity design approach with a normalized ranking of the applying firms as the forcing variable; the results are very similar to those from the general section for the treatment period 2003-2007.

Several robustness checks are applied to the general section (alternative methodologies – difference-in-differences with matching and caliper, unbalanced panel – are shown in the main text; an alternative balance sheet dataset – Cerved – is shown in the appendix) and the results are very similar; moreover, we evaluate the effectiveness of the policy for some subsamples of firms whose characteristics could improve the effect of the policy (small and medium firms, those with high average debt cost, those with a high ratio of subsidy over net investments) and the hints of effectiveness, even if slightly stronger than for the full sample, are not consistent in these cases either.

The remainder of the chapter is organized as follows: Section 3.2 describes the characteristics of the FTI policy intervention; in Section 3.3 we present the datasets for both the general and the special purpose sections and discuss the main empirical analysis, empirically examining the effectiveness of the FTI. For the general purpose section of the Fund, in Section 3.4 we check the robustness of the results under variations in the used methodology, while Section 3.5 examines whether the policy affected investments in some relevant subsamples of firms. Section 3.6 summarizes the results and conclusions are reached.

3.2 Description of the policy intervention

The Italian Ministry of Economic Development created the Fund for Technological Innovations with Law n. 46/1982; after almost twenty years of activity, the Fund was reorganized in 2001 introducing a new selection mechanism and a more accurate definition of its aims. The purpose of the FTI is the stimulation of innovations through subsidies to firms for R&D projects with a prevailing share of development costs. This definition distinguishes the field of activity of the FTI from that of the Fund for Support to Research of the Ministry of University and Research, oriented to stimulate the research component of R&D.

The FTI is organized in a general purpose section and in a special purpose one. The former is not restricted to specific fields of economic activity or geographical areas and its applications are evaluated one-by-one by merit following the chronological order of presentation without a predetermined deadline. The latter periodically issues calls for applications in specific fields of activity or geographical areas by a predetermined deadline; firms' applications are ranked and the subsidies are assigned to the best projects up to the available amount of resources.

In both cases, the applications are preliminarily evaluated by the banks charged with the procedure to verify the necessary requirements and the economic, financial and technical profiles of the applicants and of the projects. The report of this assessment is sent to the Ministry. If the research costs prevail on the development costs, the application is sent to the Ministry of University and Research instead of to the Ministry of Economic Development.[44] The MED asks the opinion of a Technical Committee, which considers the application and the preliminary reports and expresses a synthetic judgment on the merit of the project. The MED bases its decision on granting the subsidy on the Committee's opinion. Should the application be rejected, the reasons are explained.

The stated costs of the project for which subsidies are granted must be explicitly imputed either to the research or to the development components and might include expenditures for labour, machinery, consulting, general and consumption costs, feasibility studies and research centre organization.

The overall amount of subsidy is the same for the two sections and is equal to the upper bound allowed by the regulations of the European Union. The basic share is 25 per cent of costs for the development component and 50 per cent for the research component. As the FTI requires that the development costs must be at least half of the overall ones in the projects, the basic share of subsidized costs is between 25 and 37.5 per cent. The subsidy can be augmented up to an overall additional 25 per cent of costs in the following cases: areas with problems of economic development (defined by the Objectives 1 and 2 of the European Union regional policies for the period 2000-2006), projects included in the objectives of the R&D framework programs of the European Union, projects in cooperation with other firms

[44] The applications transferred to the Ministry of University and Research are not included in our sample.

or public research organizations, small and medium firms.[45]

The MED awards the selected firms a concessional loan at an interest rate of one fifth of the market rate, covering 60 per cent of the costs of the project; the remaining contribution is a grant. The financial plan of the loan has a maximum duration of ten years, plus a grace period during the execution of the project.

The financial plan of the subsidy may include up to five installments. Payment is made 60 days after presentation of the fiscal documentation of the R&D expenditures by the firm performing the project. The small and medium firms could ask advance payment of the initial installments.

Projects for the general section of the Fund could begin between 12 months before and 6 months after submission of the application. The subsidized R&D expenditure should have been completed between 18 and 48 months after the date of submission; the MED could allow an extension of 12 months in case of unexpected technical difficulties.

Regarding the special section, the calls for applications required firms to submit a preliminary project with the application. After the rankings of the projects were published, firms had to submit a final version within two months; projects should have been concluded between 18 and 36 months after the submission of the final project, plus eventually the 12-month extension.

[45] The MED used the following criteria to define the small and medium firms: in the last fiscal year the number of employees must be less than 250, the firm must be independent and either the overall annual sales revenue must be less than 40 millions of euros or the overall assets must be less than 27 millions of euros.

The timeline of the FTI policy measures is shown in Figure 3.1. After the reorganization of the FTI in 2001, the general section of the Fund regularly worked from 27/10/2001 to 17/03/2002. 879 applications submitted in this period were positively evaluated and received the subsidy. The Fund lent more than 1100 millions of euros as concessional loans and paid more than 500 millions as grants. The subsidized projects were performed between 01/01/2001 and 01/04/2006.

The number of submitted applications and the amount of financial resources required by the Fund exceeded the expectations of the Ministry; because of shortage of funds, the MED communicated on 07/05/2002 that the assessment of the applications received after 17/03/2002 had been suspended; applications were allowed to be submitted until 13/01/2003, when became clear that the shortage of funds would not have been solved shortly. The necessary financial resources to fund this second group of applicants were recovered four years later and the assessment of these projects began only after 11/12/2007.[46] Theoretically these projects should have been performed between 18/03/2001 and 13/01/2008.

The special section of the FTI was not used in the period of regular operation of the general section. Many calls for applications have since been issued. We will restrict our attention to the three first calls for application,

[46]The Ministry either funded the already completed project or allowed some modifications in the timing and the content of the projects to update it. Anyway, the assessment of the projects of the control group began after the end of the timeline considered in this work and therefore did not impact the behaviour of these firms.

issued in 2003-2004. The length of the available data period for the calls issued after 2004 is too short for the analysis. In the analysed calls, the projects were performed between the second half of 2004 and the beginning of 2009.

The first call, issued in 2003, was directed to projects developed by small and medium firms in Lombardy on the subjects included in the fifth and sixth R&D framework programs of the European Union (e.g. biotechnologies, information and communication technologies, robotics, space research, food safety). The final ranking of the projects, published in June 2004, was determined positively considering the cooperation with other firms, research centres and universities, the location in areas with problems of economic development and the industrial development of patented innovations. 455 applications were submitted and 39 grants were assigned. The overall amount of subsidy paid to the selected firms was around 50 millions of euros.

The second call, issued in 2003, was directed to projects developed by small and medium firms to apply information and communication technologies (ICT) to the production and organizational processes. The final ranking of the projects, published in August 2004, was determined considering novelty, feasibility and impact of the project and the cooperation with other firms, research centres and universities. 530 applications were submitted and 112 grants were assigned. The overall amount of subsidy paid to the selected firms was around 140 millions of euros.

The third call, issued in 2004, was directed to medium-high and high technology projects developed by small and medium firms in areas with problems of economic development in Northern and Central Italy (Lombardy excluded). The final ranking of the projects, published in December 2004, was determined positively considering the novelty of the product innovations developed, the industrial development of patented innovations and the cooperation with other firms, research centres and universities. 387 applications were submitted and 40 grants were assigned. The overall amount of subsidy paid to the selected firms was around 50 millions of euros.

3.3 Econometric analysis of the effectiveness of the FTI

3.3.1 The dataset

Information about submitted applications and their outcomes has been provided by the MED. About the general part of the FTI, data regarded all the applications submitted by the 778 subsidized firms; moreover, we have information about the 1083 firms whose projects were submitted after 17/3/2003, whose assessment had been suspended.[47]

The MED did not release data on the firms whose applications were assessed and rejected as the information was considered confidential. According to the Ministry officers, the number of rejections was extremely low. Virtually all the projects passed the initial report of the banks on the economic

[47] Firms were allowed to submit multiple projects; 879 projects were subsidized and 1242 were submitted, but not assessed.

and financial profile of the applicants; rejections were mainly due to technical deficiencies in the proposed project in the judgment of the Technical Committee. This fact probably pushed firms to apply and may explain why the number of applications was higher than expected. The dataset provided by the MED included information about the firm (name, fiscal code, address), the cost structure of the project (timeline, research and development components), the awarded funding (loan and grant, timeline of the financial plan).

Regarding the special section of the FTI, we know the basic data of the firm, the overall cost of the project, the amount of the awarded subsidy and the complete final rankings, with 191 grants assigned and 1151 rejected projects in the three calls.

We recovered the balance sheets of the firms of interest matching our data with the Cerved and Centrale dei Bilanci archives. The Cerved dataset includes the balance sheets of almost all the Italian companies; the Centrale dei Bilanci dataset has more detailed information about a subset including the largest firms. Cerved final dataset does not include variables depending on employees, as there were too many missing observations in this variable, and gross investments; moreover data from this archive are slightly less reliable because balance sheets are not controlled for coherence. The Centrale dei Bilanci dataset has a bias towards larger firms. This is not a problem as the firms that invest in research are generally large ones.

We matched data from the FTI general section with the Centrale dei

Bilanci dataset; we have been able to find around 75 per cent of the applying firms in one or more years. We also replicated both the matching and the analysis with the Cerved dataset, where have been able to find the 95 per cent of the firms. The results are very similar. The Tables of the main estimates with Cerved are given, without comments, in Tables $a3.1$ and $a3.2$ in the appendix. The data from the FTI special section were matched with the Cerved dataset, because the smaller number of observations required a better coverage of the sample, even if the price is a smaller number of available variables.

We restricted our attention to manufacturing firms to ensure greater homogeneity (around 80 per cent of the matched sample). We removed the outliers in the general section dataset excluding the observations whose percentage variation of investments, sales and employees was in the first and the last percentile of the distributions of the variables separately for subsidized and unsubsidized firms. In the special section dataset we excluded the first and the last percentile of the levels of the same variables. The procedure we used for the FTI special section is less effective in the elimination of the outliers, but allows a more homogeneous sample to be constructed, improving comparability in the absence of matching. Moreover, for both datasets we do not include the observations with missing data in the variables of interest in any considered year and the firms experiencing structural changes (such as mergers and acquisitions).

For the FTI general section, the final dataset is a balanced panel of 387 firms (171 treated) observed in the period 1999-2007. Year 2000 is the pretreatment year, in which no firm had already started the submitted projects; they were started between 2001 and 2002 and concluded between 2003 and 2006. In 2007 all subsidized projects were concluded. Data for the year 1999 allowed the use of some predated variables to scale investments for dimension. The descriptive statistics in the pretreatment year for subsidized, unsubsidized and all firms are shown in Table 3.1.

For the FTI special section, the final dataset with all the three calls for applications includes 345 firms observed in the period 2001-2007. 129 were from the Lombardy call (7 treated), 108 were from the ICT one (15 treated) and 108 from the Central and Northern Italy one (16 treated). In this case the pretreatment year is 2002, the projects were started between the second half of 2004 and the first half of 2005 and they were concluded between the beginning of 2006 and the beginning of 2009. At the end of the considered period, therefore, some projects had not been concluded. The descriptive statistics are shown in Table 3.2.

3.3.2 FTI general section

The evaluation of a policy aims to assess if the firm receiving the subsidy ("treated") behaved differently because of the public intervention. This situation is clearly counterfactual: we cannot simultaneously observe the behaviour of a firm under the hypotheses of receiving a subsidy and not receiving it. We econometrically solve the problem comparing the behaviour of the treated firms with that one of others (the "control" group) with similar characteristics, but which did not receive the subsidy. All the standard methodologies used in the evaluation of the incentives are essentially mean comparisons.

Two main approaches are available. The difference-in-differences methodology compare the variation of the behaviour of treated and controls between a pre-policy period, $t = 0$, and a post-policy period, $t = \tau$:

$$\beta = E\left[y^t_\tau - y^t_0 \middle| s = 1 \right] - E\left[y^c_\tau - y^c_0 \middle| s = 0 \right]$$

where y is the outcome variable used to measure the behaviour of firms, the superscripts t and c are respectively for treated and controls, the subscripts show the time period and s is equal to 1 if the firm received the subsidy. After differencing to eliminate firm effects, we can therefore estimate the policy effect by comparison between treated and controls. The difference-in-differences approach can also be implemented estimating over the pooled sample of treated and controls in the different periods a panel fixed effects regression function of the type

$$y_{it} = \alpha_i + \gamma_t + \beta s_{it} + \varepsilon_{it}$$

where the index i is the firm index, α_i and γ_t are the firm and time fixed effects and the dummy variable s_{it} is equal to 1 if firm i is treated and year t is in the treatment period.

Several hypotheses are assumed: one is that the introduction of the policy does not modify the market prices and the number of firms in the market; if this is not the case, the effects on the global prices affect the behaviour of the control group and the empirical results are biased: for example, an observed positive difference between treated and control firms could be due to the fact that control firms invest less than in the absence of the policy. Moreover, because the changes induced by the policy in the number of firms in the market are not captured by a mean comparison, we are not able to conclude anything about the aggregate expenditure in R&D.

The most critical assumption is that the two groups – treated and controls – have to be similar enough to allow the use of the control group as a proxy for the behaviour of the treated group in the absence of the policy intervention. This is not necessarily verified, because there could be self-selection within the two groups and therefore the observed results could be due to structural differences between treated and controls and not to the direct effect of the policy (selection bias). To improve comparability between the two groups we use a nearest-neighbour matching technique: for each treated firm we choose a vector of characteristics X and we select (with replacement) the firm of the control group whose characteristics minimize a distance objective function from the characteristics of the treated. The objective function that we use is the Mahalanobis distance, which is a sum of the squares and the cross-products of the differences between characteristics, weighted using the inverse of the covariance matrix of the distribution of X. This type of matching procedure weights the effect of the policy on the outcome according to the distribution of the treated; we estimate therefore the average treatment effect on the treated (ATT).

If the matching was exact and the vector X was the same for each selected couple of treated and control, we would assume that, for a given value of the vector of characteristics $X = x$, treated and controls only differ for fixed (firm and time) effects and that in particular the counterfactual behaviour of the firm in the absence of the policy is the same for both groups:

$$E\left[y_\tau^t - y_0^t \middle| s = 0, X = x \right] - E\left[y_\tau^c - y_0^c \middle| s = 0, X = x \right] = 0 \ \forall x. \tag{31}$$

In the literature two slightly stronger conditions are assumed to estimate the ATT (strong ignorability conditions; Heckman, Ichimura and Todd, 1998):

$$\text{assignment to treated or controls} \perp y_\tau - y_0 \middle| s = 0, \ X = x \ \forall x \tag{32}$$

and

$$\exists c > 0 : c < P\left(s = 1 \middle| X = x \right) < 1 - c \ \forall x \tag{33}$$

where the first one (conditional independence assumption between assignment and outcome in absence of treatment) is the counterfactual assumption, while the second one (common support between treated and controls) is necessary to empirically allow comparability between treated and controls for a given x.

Given the continuous nature of the variables we use in the characteristics vector, matching cannot be exact; in this case the estimator is biased and must be corrected by including a term depending on the difference between the matching variables of the matched observations, as shown by Abadie and Imbens (2010). There is another issue: the loss of degrees of freedom due to the replacement in the matching does not allow a correct estimation of the variance of the estimator with the standard methods either in the regression or in the mean difference formulations of the estimator. Abadie and Imbens (2008; 2010) show that bootstrap estimation may be not consistent and propose an alternative variance estimator under regularity conditions. We will use this estimator.

In our analysis the control group is the pool of firms whose applications were submitted after 17/03/2002 and whose projects were not assessed. These applications are a particularly good control group, since the suspension of the assessment was an exogenous event for firms.

We use as characteristics in the matching vector X a dimensional variable (log of employees for Centrale dei Bilanci and log of sales for Cerved), investments ratios (normalized by employees for Centrale dei Bilanci and by sales for Cerved) and any other reported variable with a significant difference in the not matched distributions. The characteristics vector is chosen in the pretreatment year. The matching has been stratified per technological sector (according to the classification from O.E.C.D. (2007)). The effects of the matching procedure on the differences between treated and controls are shown in Table 3.3.[48] The subsidized firms in the prematching sample invest significantly more than the control firms when normalizing for employees and assets, they have a higher share of long-term debts and a higher average cost of capital and their profitability is smaller in terms of ROE. They are larger in size, but this is never significant. There are still similar differences after the matching, except that now the unsubsidized firms are larger than the subsidized ones, but no mean difference is significant.

There are two main relevant remaining differences between the pools of treated and control firms. The first one is in the timing of the application (subsidized firms applied earlier); we can expect the resulting bias to be positive for the subsidized firms, because, if a firm is faster in preparing the application, it is probably better organized and more effective when making investments. The second is that the distribution of treated and control firms is not symmetric because the control group includes firms whose project may have been rejected after the assessment. The matching procedure should allow us to select the most similar controls to the treated, minimizing the distortions between the two distributions. Any possible remaining bias from this source should be positive too, because we can suppose that the potentially rejected firms included in the control group are the worse ones. Because our final result is that the subsidized firms did not invest more than the unsub-

[48] The corresponding results for the matching with the Cerved dataset are shown in the appendix in Table $a3.1$.

sidized ones, we can be sufficiently confident that, in our case, the resulting bias from both these sources is not relevant.

The alternative methodology is the regression discontinuity design, which can be implemented in this case using the date of application as the forcing variable; we compare the value of the outcome variable in a neighbourhood of the cutoff between treated and controls in the forcing variable. We assume that in a sufficiently small neighbourhood of the cutoff the assignation of the subsidy may be approximated by a random variable, not dependent on the characteristics of the firms; because of randomness, the characteristics of the treated and of the control groups are similar and their average behaviour in the absence of the subsidy will be similar too. In formal terms, the counterfactual hypothesis is that:

$$\lim_{R_i^t \to \bar{R}} E\left(y_\tau^t \middle| s = 0, R_i^t \right) = \lim_{R_i^c \to \bar{R}} E\left(y_\tau^c \middle| s = 0, R_i^c \right) \tag{34}$$

where $\bar{R}$ is the value of the forcing variable at the cutoff (in this case the variable is the date of submission of the application) and R_i^t and R_i^c are the value of the forcing variable for the firm i respectively of the treated and the control groups. After ordering the observations according to the value of the forcing variable, we estimate its relationship with the outcome separately for treated and controls and we compare the two estimates at the cutoff.

This approach gives us a local estimate of the effect of the policy; it has better internal validity than the difference-in-differences approach, which is a weighted average of the effects for different characteristics vectors, but it relies on a smaller number of observations and therefore its properties are less well approximated by those implied by the asymptotic theory. It tackles the problem of the timing of the applications by construction (while exclusion from the sample of the rejected firms is still an issue), but it has another drawback in this specific evaluation exercise: the frequency distribution of the application dates shows a discontinuity at the cutoff; this fact, due to the way the MED administrated the classification of the dates, could in any case cast some doubts on the hypothesis of absence of ordering of the firms in a neighbourhood of the cutoff (Figure 3.2).

Even if both available methodologies are not perfect for this evaluation exercise, the difference-in-differences approach is probably more reliable ex post, because the expected bias from the possible sources of distortion does not invalidate our results; we show in the following Subsections the results using this methodology. The results with the regression discontinuity design are presented in the appendix; the conclusions are very similar and this is another comfortable signal that the final result is robust enough to the highlighted problems.

Difference-in-differences estimates We monitored several aspects of firms' activity: investments, sales, assets, number of employees, capital, financial structure, average productivity, average labour cost and profitability. Investments have been considered either in raw levels or scaled by a dimensional variable (sales, assets, number of employees, capital) taken from the balance sheet of two years before the start of the policy measure (pre-pretreatment year; 1999) to ensure exogeneity. We considered gross and net overall investments and net intangible investments.[49] Net investments are the sum of the variation of the (either overall or intangible) capital in the year; they therefore include direct investment and disinvestment activity, net of depreciation in the year. This is a good proxy to determine the additional amount of accumulated capital in the year, after substituting depreciated and sold physical capital. Gross investments are the sum of the variation of the gross overall capital, without considering depreciation; they are therefore an index of firms' investments activity for both accumulating additional capital and substituting old capital and are not influenced by any discretionary policy of management of depreciation. The main stimulation effect of the subsidies should be captured by the investments variables for two main reasons: Italian accounting rules require that the costs of projects with a prevailing share of development costs must be capitalized and included in investments;[50] the effect of the work-in-progress project will be therefore captured by them. Moreover, R&D projects subsidized by the FTI are already in an applied step of their life; if the project is successful, the developed innovation will be immediately available for inclusion in the production process.

[49] We only have the full details of the investment activity components for the sum of tangible and intangible investments and therefore we have not been able to construct the gross intangible investments variables.

[50] See for example Pisoni, Bava, Busso and Devalle (2009) as a reference of the Italian accounting rules.

The behaviour of investments therefore captures changes in the economic activity of the firm not only for the development of the project, but also for the immediate application of its results.

Another group of variables considered to check the policy effects of stimulation of the economic activity are those regarding the dimension of the firm; when completed, a successful project should become an additional asset for the firm, producing value and increasing sales, and may stimulate the employment of additional workers. We use here the logarithms of employees, assets, sales and capital as dimensional indexes, to determine the effects of the policy in relative, and not absolute, terms, but the results both for the matching and the program evaluation are qualitatively the same when we use the raw value of the variables. Capital is determined as the sum of the intangible and the tangible fixed assets as reported in the balance sheet.

The value added per employee is the average productivity of labour and should be able to capture possible positive effects of the R&D project on the efficiency of the firm; the cost of labour per employee is a proxy for the average value of the human capital of the firm and should show whether the project modified the quality of the human capital used within the firm.

The financial variables are those regarding the debt structure of the firm; this is because we should eventually detect signals of substitution of private with public funding in these variables, in particular the long-term debt. They have been calculated net of the policy: neither debts nor assets in this case include the current value for each period of the borrowed amount of the loan associated with the subsidy.

Another possible effect of the subsidy is in terms of reputation: lenders could perceive as a positive signal of reliability the fact that the firm received a public subsidy; this possible effect should be shown by the average cost of debt. We removed from the financial charges the component due to the payments associated with the concessional loan of the subsidy, to isolate the effect of the policy on the cost of the unsubsidized debts.

We examine moreover two profitability indexes: the return on equity (ROE), calculated as net income before taxes over equity, monitors variations in the capability of the firm of producing income for the shareholders; the return on assets (ROA), calculated as gross operating surplus over assets, is an index of the general profitability of the firm's assets.

We can see the difference-in-differences estimates and standard errors for each year between 2001 and 2007 with respect to the pretreatment year (2000) in Tables 3.4a and 3.4b.[51]

The 95 per cent confidence intervals of the estimated coefficients, which show the differences in the variation between treated and controls, do not allow to exclude a null effect for both gross and net investments, in raw levels and for any dimensional normalization. We also considered the differences in the net intangible investments to understand if there is a capitalization of knowledge for future investments, but also in this case we are not able

[51] The corresponding results using the Cerved dataset are shown in the appendix in Table a3.2.

to exclude a null effect. When we consider the dimensional variables, we find a significant, increasing over time, positive difference in the amount of the overall assets of the firm, while it does not happen for the number of employees and the amount of sales and fixed capital. Subsidized firms therefore probably used the subsidy to increase the share of current assets. Moreover, we are not able to exclude a null effect for labour productivity and firms' average labour cost. The same is true for the overall share of debts, while the coefficient is negative and becomes significant towards the end of the period when we consider the long-term debts. These last two results can be signals that the firm substituted its already existing long-term debts with the concessional loan associated with the subsidy and increased the amount of short-term debts. We find, moreover, no coherent differences for the average cost of capital, even if there are some hints of reduction at the end of the period. Lastly, the confidence intervals do not allow to exclude a null effect for the profitability indexes too.

Discussion of the results There can be several reasons explaining why the detected signals of efficacy of the policy are very weak.

The first one is clearly that the effectiveness of the policy has been very limited and that the Fund's attempt to stimulate R&D investments substantially failed; in this case public funding would have completely substituted the already existing private one and no additional investment would have been performed. The presence of a substitution effect between private financial resources and public ones is coherent with the negative coefficient for the share of long-term debts.

A second possibility is that the dimensions we are considering do not capture the effects of the policy. Given the current Italian laws, expenditures for R&D projects with a prevailing share of development costs have to be included in investments and therefore we should be sufficiently confident that this set of variables captures the additional expenditures due to the subsidized project while it is in progress. Some doubts can be cast on the capability of the monitored variables to capture the consequences after the conclusion of the project: there could be a time lag before its results affect the performance and the characteristics of the firm, in terms of sales or even of introduction of the innovation in the production structure of the firm; the subsidized projects should aim at the industrial development of applied projects and therefore their use within the firm should be carried out shortly, but given the limited number of available years after the completion of the projects we cannot completely rule out this case. Anyway, if there are no hints of stimulation in the behaviour of the firm when the project is still in progress, the monitoring of the behaviour of the firm after the conclusion of the project becomes less relevant since the subsidy probably did not triggered any additionality effect.

Another possibility is that the precision of the estimates is too low to allow a reliable estimate of the effects. This could be the case in particular for the FTI special section, where the number of observations is low, while for the general section this risk should be less relevant. Another source of low precision in the estimates could come from the fact that the relevance of the subsidized project for the firm is relatively small and therefore its impact on the behaviour is hidden by the overall noise in the variation of the variables. We checked this possibility in Section 3.5, where we restricted the sample to the firms with the highest subsidy over overall net investments ratio; the policy seems to be ineffective once again, suggesting that this should not be the case.

Other reasons to explain the results could come from the required hypotheses for the unbiasedness of the estimates, in particular from the assumption that the control group is not affected by the policy. The subsidy may indirectly affect the activity of the control group because the unsubsidized firms indirectly benefit from the policy by means of spillovers from the R&D activity of the subsidized firms. Spillovers require some already acquired knowledge to be "spilled over"; moreover a reasonable amount of time is usually required for knowledge to spread. For these two reasons, this possibility should not be particularly relevant during the progress of the project, while it could be after its conclusion.

3.3.3 FTI special section

To analyse the effectiveness of the special section of the FTI we used a regression discontinuity design approach. The reason for the different approach is that in this case the natural control group is made up of the rejected firms and this group of firms can intrinsically differ from the treated ones because the ranking is a signal that they are worse. The regression discontinuity design approach allows us to tackle this problem, because we locally compare the firms immediately before and after the cutoff between subsidized and unsubsidized firms, which are the most similar between the two groups.

As the rankings of the three calls for applications are not comparable, we create a new variable $\hat{R}_{ij}$, which we call normalized ranking, giving the relative distance of each firm from the last ranked subsidized firm:

$$\hat{R}_{ij} = 1 - \frac{R_{ij}}{\bar{R}_j}$$

where $\frac{R_{ij}}{\bar{R}_j}$ is the ratio between the position of the firm in the raw ranking and the ranking of the last subsidized firm. Therefore, all the subsidized firms have a value of normalized ranking between 1 and 0; the last subsidized firm has a value $\tilde{R}_j = 0 \; \forall j$ for all the call for applications. For the subsidized firms of a specific call for application, the normalized ranking gives the share of subsidized firms with a worse relative position. The normalized ranking is negative for the rejected firms; the number of rejected firms included between 0 and -1 is the same as the number of subsidized firms. Given that the overlapping in the timing of the calls for applications is strong and we have few observations for each call, we estimated the effects for the three calls together, including fixed effects to take into account the presence of heterogeneity between calls. We order the firms according to the normalized ranking (which is our forcing variable in the terminology of the regression discontinuity design) and we estimate the relationship between this variable and the outcome separately for treated and controls.[52]

We cannot check the counterfactual hypothesis (34), but we can reasonably reject it in the case treated and controls around the cutoff are different on average in the pretreatment period. We can see the results of this comparison in Table 3.5. In the first two columns there are the comparisons for the full sample, while in the third and the fourth ones the sample of the controls has been restricted to those with a normalized ranking of at least -1, which implies that we considered approximately the same number of treated and controls. Treated firms invest slightly more than controls in the pretreatment year, but the difference is not significant. Dimension, profitability and long-term debts are very similar. One possible reason for this strong similarity is that the main criteria of the rankings are novelty of the project and cooperation with other firms and public research centers, which are not directly related to the economic and financial structure of firms.

In Figure 3.3 are presented the semiparametric estimates and the 95 per cent confidence intervals of the relationship between normalized ranking and the outcome variables for the net overall investments variables, scaled by the three dimensional variables.[53] Table 3.6 shows the value of the jump between

[52] The ICT call for applications awarded subsidies to some low ranking firms because the call reserved some funds to specific projects. We excluded these firms from the analysis.

[53] The graphs for the other variables are not shown for sake of brevity, but they are available.

the estimates for treated and controls at the cutoff and the bootstrapped standard errors of the effect of the policy. In the estimation we used a triangular kernel, which has good boundary properties, and added dummies for call for applications, geographical area and technological sector. The optimal bandwidth has been separately calculated for treated and controls using the rule of thumb from Silverman (1986). The standard errors of the jump have been calculated using a bootstrap procedure with 200 replications stratified by technological level.

The outcome variable in the case of the investments variables is the cumulated sum of (overall or intangible) net investments between 2003 and 2007, either in raw levels or scaled by a dimensional variable taken from the balance sheet of two years before the start of the policy measure (pre-pretreatment year; 2001); in the other cases, we used either the log of the mean (for sales, assets and capital) or the ratio of the means (for ROA and ROE) in the period 2003-2007. The financial variables have not been included in the analysis because we do not know the amount of the awarded loan and the time structure of the subsidy and therefore we cannot remove the amount associated with the subsidy from the variables as we did for the general section.

The confidence interval of the difference in the estimates for treated and controls does not allow to exclude a null effect and the coefficients are often negative for net investments, dimension and profitability. In the case of net intangible investments, the difference becomes significant, but negative.

In Table 3.7 we report the alternative estimates obtained using a parametric specification of the relationship between outcome and forcing variable. In this case too we included dummy variables for call for applications, geographical area and technological sector. In each column we used a polynomial of a different degree; in the first column it is a zero-degree polynomial, which is equivalent to a mean comparison, in the other columns the relationship is respectively linear and quadratic. The reported number is the value of the coefficient of the treatment dummy variable, which is the difference in the estimates at the cutoff.

Similarly to the semiparametric methodology, the confidence intervals of the coefficients do not usually allow to exclude a null effect and, in the few cases where the coefficients are significantly different from zero, they are negative. We can conclude, therefore, that, whatever is the methodology we use, we do not detect signals that the policy measures from the special section of the FTI have been able to induce differences in the behaviour of treated and controls.

The possible reasons and their discussion for the results may be the same as for the general section; there is only one more caveat: given the smaller number of observations the results for the special section are slightly less robust.

3.4 FTI general section: robustness checks

The results we presented in the last Section are quite robust to several checks. In this Section we examine two modifications in the difference-in-differences approach for the FTI general section: in the first one we introduce a caliper in the matching procedure and we just consider the most similar matched couples, while in the second one we expand our sample including all firms observed in at least both the pre-pretreatment and the pretreatment years (1999 and 2000) and one year in the treatment period (2001-2007).

3.4.1 Matching and caliper with radius at the median of the distance

In the first additional check, we use a variation of the matching methodology (matching with caliper). After matching the observations, we consider the distribution of the distance between treated and control for each matched couple and we include in our sample only the couples with a lesser distance than the median of the distribution. On one hand, the selected couples of the sample will be the most similar and therefore the difference in their behaviour is a better proxy for the efficiency of the policy; on the other hand, we halve the sample and therefore its properties are less well approximated by those implied by the asymptotic theory.

The balancing properties of the sample matched with this methodology are reported in the last two columns of Table 3.3. No mean difference is significant and in most cases the p-value is higher than before.

The results of the difference-in-differences estimates are reported in Table 3.8. For sake of brevity we just show the estimates for the overall investments, but the conclusions are quite similar for the other variables too. We do not find any substantial difference with the effects reported in Table 3.4a: all the confidence intervals do not allow to exclude a null effect and sometimes the coefficients are negative.

3.4.2 Unbalanced panel

In the balanced panel we only included in the sample the firms with a complete time series in all the considered years. We now relax this assumption and include also firms with breaks in the time series or that left the panel for any reason before the end of the considered period. We include, therefore, all the firms with at least two observations in the pre-pretreatment and in the pretreatment years (1999 and 2000) and one observation during the treatment period (2001-2007).The effects of this change of sample are several. Now we can use more observations and therefore the properties of the sample are better approximated by those implied by the asymptotic theory. Moreover, the sample now includes also the firms which ended their activity before the end of the period; it is therefore more representative of the overall impact of the policy. The most important negative consequence is that now the panel changes its composition every year and the results are less stable. The matching procedure has been repeated in each year and the pairing can change over time.

The results for investments are reported in Table 3.9.[54] We find some positive significant effects at the beginning of the period for gross investments, while for net investments the confidence intervals do not allow to exclude a null effect for the whole time period.

3.5 FTI general section: evaluation of the effectiveness in subsamples

We now examine whether the subsidies from the FTI general section have been successful in stimulating the investments for some subsamples, where we could expect a stronger effect of the policy. For each of them, we calculated the effect of the policy on net and gross investments ratios using the difference-in-differences approach with matching. The mean differences in the pretreatment year before and after the matching for the three considered subsamples are reported in Tables $a3.4$, $a3.5$ and $a3.6$ in the appendix.

A first group we consider are small and medium firms; the rules of the policy are somehow more favourable for them: the paid net overall subsidy is at least 10 per cent higher; moreover, the criteria of selection are looser: a direct contribution to R&D is not required and they can simply coordinate an external R&D effort; the benchmarks for the evaluation of the novelty of the project, of its financial sustainability and of its economic impact are slightly relaxed; big firms are also required to formally show additionality

[54] The balancing properties for the overall subsample of firms entering the estimation in at least one year are shown in the appendix in Table $a3.3$.

with respect to the customary R&D activity. This more favourable treatment for small and medium firms is justified by the fundamental role that they have in the Italian economy, by the scale economies existing in the organization of the R&D activity and the appropriation of its results and by the more severe constraints that small and medium firms usually experience in access to the credit markets. The results for this subsample are reported in Table 3.10.

A second subsample includes the firms with a higher average debt cost than the median of the distribution of this variable. These firms are more likely to experience borrowing constraints, because the higher average debt cost implies that lenders have perceived them as riskier. Moreover, even in the absence of strict borrowing constraints, the cost of additional financial resources in the market is probably higher for these firms; because the amount of subsidy received through the concessional rate of the loan is calculated on the market rate and not on the opportunity cost of capital of the firm, the amount of subsidy actually received by these firms is higher than the figure officially stated by the Ministry. Both these facts imply that the effectiveness of the policy for this group of firms can be stronger than for the other firms. The results are reported in Table 3.11.

In the third subsample there are the firms whose subsidy over net investments ratio is higher than the median of the distribution of this variable. This ratio should capture how relevant the subsidy is with respect to the investment activity of the firm. A high ratio implies that a large share of the overall investments plans is subsidized and therefore we can expect that the impact on the overall behaviour of the firm is more likely to be significant. The results in this case are reported in Table 3.12.

The results are very similar for the three subsamples. We find positive coefficients and sometimes the confidence intervals allow us to exclude a null impact, showing that the signals of effectiveness of the policy are slightly stronger than for the full sample. But these positive results are quite isolated: we do not find any positive consistently significant temporal trend and even in each year a significantly positive coefficient is not confirmed by the other considered variables. We cannot therefore conclude with certainty that the policy had an overall positive effect even for these subgroups of firms.

3.6 Conclusions

In this work we have analysed the effectiveness of the Italian Fund for Technological Innovation development subsidies.

We separately examined the subsidies supplied by the general and the special purpose sections of this Fund. For the former, we used both a difference-in-differences approach and a regression discontinuity design one. We have not been able to detect consistent signals of effectiveness on gross, net and intangible investments. When we checked the effect on the other variables, the confidence intervals do not allow to exclude null effects on employment, sales, capital, labour productivity, labour costs, financial structure and profitability. We found a positive effect on the overall value of assets; this result, given that we cannot rule out a null effect on investments and capital, suggests that the subsidy has been mainly used by firms to finance current assets. The results are similar under several additional robustness checks.

When we consider three subsamples of firms where the policy could have been more effective (small and medium firms, those with high average debt costs, those with a high subsidy over net investment ratio) we only find very weak hints of effectiveness. The results are the same also for the special purpose section of the FTI: in this case we used a regression discontinuity design approach and we have not been able to detect signals of effects either on investments or on any other considered variable.

3.7 Tables and Figures

Table 3.1: FTI general section, descriptive statistics of firms in pretreatment year

	Treated		Controls		Overall				
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Min	Median	Max
Gross Investments	3325	6465	7808	71809	5814	53722	-13235	1223	1044440
Gross Investments/Employees	16.39	19.17	14.92	22.26	15.57	20.96	-43.59	9.60	190.75
Gross Investments/Sales	0.08	0.10	0.08	0.10	0.08	0.10	-0.23	0.05	0.88
Gross Investments/Assets	0.09	0.10	0.07	0.08	0.08	0.09	-0.24	0.06	0.65
Gross Investments/Capital	0.47	0.81	0.41	0.51	0.44	0.66	-0.97	0.28	7.14
Net Investments	2902	36702	5047	46154	4092	42219	-188283	341	667446
Net Investments/Employees	12.54	31.52	8.36	17.64	10.21	24.84	-47.95	3.32	275.57
Net Investments/Sales	0.06	0.13	0.04	0.10	0.05	0.11	-0.15	0.02	0.86
Net Investments/Assets	0.06	0.12	0.04	0.09	0.05	0.11	-0.16	0.02	0.68
Net Investments/Capital	0.37	0.81	0.29	0.71	0.32	0.76	-0.80	0.11	7.23
Net Int. Investments	1701	34109	573	3681	1074	22914	-185246	7	400906
Net Int. Investments/Employees	1.96	12.72	1.12	6.00	1.49	9.58	-41.29	0.07	141.76
Net Int. Investments/Sales	0.01	0.07	0.00	0.02	0.01	0.05	-0.10	0.00	0.80
Net Int. Investments/Assets	0.01	0.05	0.01	0.03	0.01	0.04	-0.07	0.00	0.62
Net Int. Investments/Int. Capital	1.37	5.68	1.19	5.19	1.27	5.41	-1.00	0.05	61.75
Employees	268	498	273	717	271	630	14	117	8620
Sales	66750	162540	61899	161298	64043	161840	3444	22546	1997563
Assets	72205	242120	62763	199275	66935	219254	2072	22150	2847149
Capital	24407	136837	21623	130103	22853	133106	65	5282	1864455
Value Added/Employees	64.31	33.17	60.31	27.34	62.08	30.12	12.31	54.52	309.17
Labour Cost/Employees	35.02	7.09	34.18	8.80	34.55	8.10	18.28	33.58	114.87
Debts/Assets	0.64	0.17	0.64	0.15	0.64	0.16	0.07	0.67	0.95
Long-Term Debts/Assets	0.12	0.10	0.11	0.09	0.11	0.09	0.00	0.10	0.50
Financial Charges/Debts	0.03	0.02	0.03	0.02	0.03	0.02	0.00	0.03	0.22
ROA	0.07	0.07	0.08	0.06	0.07	0.07	-0.16	0.06	0.36
ROE	0.13	1.07	0.28	0.59	0.21	0.84	-13.04	0.19	6.71

Balanced panel. The number of firms is 387 (171 treated). The statistics regards the pretreatment year (2000). The denominator of the investments variables is the value of the variable in 1999.

Table 3.2: FTI special section, descriptive statistics of firms in pretreatment year

	Treated		Controls		Overall				
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Min	Median	Max
Net Investments	327	858	212	863	225	863	-1771	18	6857
Net Investments/Sales	0.04	0.12	0.03	0.09	0.03	0.10	-0.14	0.00	0.67
Net Investments/Assets	0.04	0.11	0.03	0.09	0.03	0.09	-0.18	0.00	0.53
Net Investments/Capital	0.23	0.69	0.20	0.59	0.20	0.60	-0.67	0.02	4.03
Net Int. Investments	97	396	28	293	35	306	-1527	-3	2926
Net Int. Investments/Sales	0.01	0.04	0.01	0.03	0.01	0.03	-0.05	0.00	0.33
Net Int. Investments/Assets	0.01	0.03	0.01	0.03	0.01	0.03	-0.05	0.00	0.23
Net Int. Investments/Int. Capital	0.87	3.35	0.67	2.44	0.69	2.55	-0.91	0.00	22.71
Sales	9525	7487	10956	9228	10798	9055	18	8796	56095
Assets	10677	11063	10489	9222	10510	9425	178	7751	55649
Capital	2616	3994	2583	3210	2587	3299	17	1374	23910
Debts/Assets	0.63	0.22	0.70	0.16	0.69	0.17	0.14	0.73	1.01
Long-Term Debts/Assets	0.12	0.12	0.11	0.11	0.11	0.11	0.00	0.09	0.52
Financial Charges/Debts	0.03	0.02	0.03	0.02	0.03	0.02	0.00	0.03	0.09
ROA	0.24	0.26	0.19	0.81	0.20	0.77	-11.07	0.19	2.40
ROE	0.07	0.06	0.06	0.08	0.06	0.07	-0.52	0.06	0.37

Balanced panel. The number of firms is 345 (38 treated). The statistics regards the pretreatment year (2002). The denominator of the investments variables is the value of the variable in 2001.

Table 3.3: Mean comparison before and after the matching, balanced panel, FTI general section

	Not matched		Matched		Matched with caliper	
	Mean difference	p-value	Mean difference	p-value	Mean difference	p-value
Gross Investments	4483	0.42	-609	0.29	190	0.69
Gross Investments/Employees	1.4616	0.50	2.5246	0.18	-0.8265	0.69
Gross Investments/Sales	0.0039	0.71	0.0076	0.39	-0.0006	0.96
Gross Investments/Assets	0.0129	0.17	0.0119	0.19	-0.0032	0.75
Gross Investments/Capital	0.0606	0.37	0.0717	0.32	-0.0444	0.29
Net Investments	2145	0.62	-504	0.84	-51	0.85
Net Investments/Employees	4.1795	0.10	3.2127	0.24	0.0311	0.99
Net Investments/Sales	0.0127	0.27	0.0101	0.38	0.0030	0.71
Net Investments/Assets	0.0180	0.10	0.0121	0.30	0.0040	0.59
Net Investments/Capital	0.0812	0.30	0.0485	0.57	0.0218	0.66
Net Int. Investments	1128	0.63	-989	0.71	-70	0.46
Net Int. Investments/Employees	0.8410	0.39	0.5353	0.64	0.2295	0.71
Net Int. Investments/Sales	0.0049	0.32	0.0039	0.49	0.0035	0.11
Net Int. Investments/Assets	0.0040	0.35	0.0024	0.63	0.0033	0.11
Net Int. Investments/Int. Capital	0.1805	0.75	0.3923	0.48	0.5639	0.12
log(Employees)	-0.0198	0.85	-0.1183	0.28	-0.1285	0.29
log(Sales)	0.0275	0.81	-0.0330	0.77	-0.1384	0.27
log(Assets)	0.0532	0.66	-0.0159	0.90	-0.0727	0.58
log(Capital)	0.1002	0.50	-0.0650	0.65	-0.0749	0.68
Value Added/Employees	4.0090	0.19	4.8356	0.11	0.3663	0.86
Labour Cost/Employees	0.8399	0.31	0.8888	0.22	-0.0103	0.99
Debts/Assets	0.0032	0.85	0.0050	0.77	0.0077	0.74
Long-Term Debts/Assets	0.0169	0.08	0.0090	0.37	0.0024	0.83
Financial Charges/Debts	0.0046	0.04	0.0020	0.40	-0.0006	0.74
ROA	-0.0020	0.77	-0.0042	0.54	-0.0076	0.31
ROE	-0.1498	0.08	-0.1096	0.19	-0.0058	0.85

The statistics regards the pretreatment year (2000).
The denominator of the investments variables is the value of the variable in 1999.
In the last two columns, the caliper has radius equal to the median of the distribution of the distance between matched firms.
The prematching number of firms is 387 (171 treated).

Table 3.4a: Difference-in-differences estimates with matching, balanced panel, FTI general section: investments

	2001	2002	2003	2004	2005	2006	2007
Gross Investments	1046.03	691.27	665.97	685.35	88.94	580.42	616.67
	(757.84)	(1057.54)	(754.20)	(474.38)	(495.25)	(628.64)	(941.87)
Gross Investments /Employees	4.3659	-0.2767	1.1576	3.4516	3.9790	2.1318	0.2648
	(3.2483)	(3.8393)	(3.0022)	(2.1233)	(4.1033)	(3.0934)	(3.2531)
Gross Investments /Sales	0.0132	-0.0101	-0.0033	0.0042	0.0136	0.0103	-0.0098
	(0.0144)	(0.0183)	(0.0130)	(0.0105)	(0.0172)	(0.0164)	(0.0162)
Gross Investments /Assets	0.0131	-0.0150	-0.0033	0.0052	0.0119	0.0082	-0.0079
	(0.0157)	(0.0187)	(0.0137)	(0.0107)	(0.0175)	(0.0164)	(0.0164)
Gross Investments /Capital	0.0723	-0.0871	-0.0274	0.0500	0.1218	0.0969	0.0512
	(0.0963)	(0.1107)	(0.0826)	(0.0773)	(0.1193)	(0.1062)	(0.1139)
Net Investments	-343.74	-1409.37	-687.70	2389.63	-2373.55	-937.38	-1055.63
	(3144.30)	(3155.44)	(3159.17)	(5336.14)	(3843.51)	(3185.29)	(2960.60)
Net Investments /Employees	3.3020	-1.8684	2.9115	3.9110	-0.0489	1.7782	-0.2091
	(3.6304)	(3.5095)	(3.5567)	(2.9463)	(4.2621)	(3.9276)	(2.9858)
Net Investments /Sales	0.0127	-0.0082	0.0081	0.0069	-0.0031	0.0174	-0.0012
	(0.0136)	(0.0143)	(0.0127)	(0.0097)	(0.0186)	(0.0201)	(0.0140)
Net Investments /Assets	0.0150	-0.0109	0.0138	0.0118	0.0006	0.0138	0.0032
	(0.0149)	(0.0143)	(0.0125)	(0.0096)	(0.0177)	(0.0190)	(0.0120)
Net Investments /Capital	0.0467	-0.0676	0.0211	0.0518	-0.0553	0.0436	0.1003
	(0.0985)	(0.1053)	(0.1033)	(0.0895)	(0.1305)	(0.1062)	(0.1149)
Net Int. Investments	-1825.76	-1975.15	-1804.91	1880.63	-3082.32	-1278.95	-1337.36
	(3108.25)	(3096.64)	(3006.91)	(5429.59)	(3867.55)	(3078.92)	(3050.19)
Net Int. Investments /Employees	-0.1703	-0.3420	-1.7361	0.9798	-0.2606	1.2420	0.2102
	(1.4269)	(1.4845)	(1.5087)	(1.7618)	(2.3748)	(1.8080)	(1.6295)
Net Int. Investments /Sales	-0.0042	-0.0007	-0.0088	-0.0006	-0.0042	0.0045	-0.0015
	(0.0072)	(0.0077)	(0.0073)	(0.0076)	(0.0142)	(0.0094)	(0.0078)
Net Int. Investments /Assets	0.0001	0.0014	-0.0046	0.0022	-0.0053	0.0058	0.0024
	(0.0067)	(0.0071)	(0.0071)	(0.0069)	(0.0129)	(0.0092)	(0.0081)
Net Int. Investments /Int. Capital	-0.5048	4.4711	2.2578	0.7210	3.8817	3.5612*	3.0953
	(2.2795)	(3.7467)	(4.0780)	(3.3083)	(2.8835)	(1.9540)	(2.7031)

Baseline year: 2000. The dataset includes 288 not repeated firms (171 treated; 3728 overall observations) after the matching.
The denominator of the investments variables is the value of the variable in 1999.
Standard errors are reported between brackets.
Estimates and standard errors use the procedure from Abadie and Imbens (2002; 2009).

Table 3.4b: Difference-in-differences estimates with matching, balanced panel, FTI general section: other variables

	2001	2002	2003	2004	2005	2006	2007
log(Employees)	0.0146	0.0215	0.0375*	0.0256	0.0373	0.0347	0.0449
	(0.0139)	(0.0179)	(0.0227)	(0.0298)	(0.0328)	(0.0380)	(0.0426)
log(Sales)	0.0042	0.0332	0.0515	0.0404	0.0528	0.0395	0.0722
	(0.0194)	(0.0268)	(0.0402)	(0.0376)	(0.0449)	(0.0481)	(0.0540)
log(Assets)	0.0432**	0.0524*	0.0565	0.0773**	0.0746*	0.1158*	0.1083*
	(0.0217)	(0.0285)	(0.0373)	(0.0385)	(0.0453)	(0.0613)	(0.0657)
log(Capital)	0.0450	0.0564	0.0784	0.1229	0.1255	0.1798	0.1760
	(0.0586)	(0.0719)	(0.0824)	(0.0883)	(0.0964)	(0.1271)	(0.1354)
Value Added	-2.1639	0.7784	0.6799	1.1402	0.9008	3.9598	5.4717
/Employees	(1.9877)	(1.8982)	(2.2494)	(2.4097)	(2.6890)	(3.0796)	(3.9916)
Labour Cost	-0.0972	-0.1681	-0.7766*	-0.1699	-0.2114	0.2861	0.4252
/Employees	(0.3523)	(0.3892)	(0.4498)	(0.4237)	(0.5817)	(0.6834)	(0.6715)
Debts/Assets	0.0134	-0.0006	-0.0119	-0.0074	-0.0030	-0.0006	-0.0159
	(0.0082)	(0.0109)	(0.0137)	(0.0144)	(0.0163)	(0.0174)	(0.0164)
Long-Term Debts	0.0060	-0.0168*	-0.0157	-0.0174	-0.0095	-0.0191*	-0.0248*
/Assets	(0.0065)	(0.0087)	(0.0108)	(0.0111)	(0.0104)	(0.0113)	(0.0134)
Financial Charges	0.0009	0.0065	0.0038*	0.0002	0.0016	-0.0030**	-0.0019
/Debts	(0.0021)	(0.0053)	(0.0021)	(0.0019)	(0.0028)	(0.0013)	(0.0019)
ROE	-0.0249	0.1408	-0.0887	0.0956	-0.2800	0.0840	0.1686
	(0.0857)	(0.1704)	(0.1080)	(0.1086)	(0.3765)	(0.2998)	(0.1327)
ROA	-0.0041	0.0047	0.0071	-0.0060	0.0012	0.0029	0.0073
	(0.0074)	(0.0080)	(0.0093)	(0.0086)	(0.0087)	(0.0096)	(0.0088)

Baseline year: 2000. The dataset includes 288 not repeated firms (171 treated; 3728 overall observations) after the matching.
All the variables regarding the financial structure of firm have been calculated excluding the amount of the loan associated to the subsidy.
Standard errors are reported between brackets.
Estimates and standard errors use the procedure from Abadie and Imbens (2002; 2009).

Table 3.5: Regression discontinuity design, balancing properties, FTI special section

	Full sample		Normalized ranking ≥-1	
	Mean difference	p-value	Mean difference	p-value
Net Investments	-114	0.44	-276	0.14
Net Investments/Sales	0.0091	0.58	0.0276	0.26
Net Investments/Assets	0.0132	0.42	0.0265	0.29
Net Investments/Capital	0.0292	0.78	-0.0203	0.92
Net Intangible Investments	-69	0.19	-44	0.55
Net Intangible Investments/Sales	0.0024	0.68	0.0083	0.21
Net Intangible Investments/Assets	0.0008	0.86	0.0064	0.22
Net Intangible Investments/Intangible Capital	0.1934	0.66	0.4487	0.52
log(Sales)	-0.0610	0.74	0.0242	0.92
log(Assets)	-0.0394	0.83	0.0268	0.92
log(Capital)	-0.0776	0.75	0.2017	0.57
ROA	0.0018	0.89	-0.0078	0.60
ROE	0.0511	0.70	0.0377	0.64

The statistics regards the balanced panel in the pretreatment year (2002).
The denominator of the investments variables is the value of the variable in 2001.
The number of firms is 345 (38 treated) in the full sample, 72 (38 treated) in the sample restricted to the firms with normalized ranking ≥-1.

Table 3.6: Regression discontinuity design, kernel estimates of the discontinuity at the cutoff, balanced panel, FTI special section

	Semiparametric estimates	Bandwidth for treated	Bandwidth for controls
Net Investments	158.65 (696.67)	0.2737	1.1896
Net Investments /Sales	-0.0439 (0.1666)	0.2485	1.4062
Net Investments /Assets	0.0140 (0.1425)	0.1901	1.2952
Net Investments /Capital	-0.4238 (0.9788)	0.1703	1.2236
Net Int. Investments	-268.83** (130.14)	0.2407	1.5743
Net Int. Investments /Sales	-0.0506*** (0.0138)	0.3311	1.8273
Net Int. Investments /Assets	-0.0444** (0.0215)	0.3474	1.5426
Net Int. Investments /Int. Capital	-2.3881 (5.3753)	0.2087	1.1832
log(Sales)	-0.0422 (0.2760)	0.2243	1.3416
log(Assets)	-0.3018 (0.1999)	0.2203	1.2303
log(Capital)	-0.1986 (0.4189)	0.3249	1.4516
ROA	0.0174 (0.0192)	0.1723	1.3742
ROE	0.2107 (0.1709)	0.2173	2.2113

The dataset includes 345 firms (38 treated).
Bootstrapped standard errors (200 replications) are reported between brackets.
The bandwidths for both treated and controls have been determined using the rule of thumb procedure from Silverman (1986).
The denominator of the investments variables is the value of the variable in 2001.
The difference is positive when the estimate for treated is greater than the estimate for controls.
Dummies for geographic area, technological sector and call for applications are included.

Table 3.7: Regression discontinuity design, polynomial estimates of the discontinuity at the cutoff, balanced panel, FTI special section

degree dummies	0 yes	1 yes	2 yes	3 yes
Net Investments	-1031.55 (650.39)	-1072.16 (675.40)	928.39 (1286.30)	-885.38 (1111.59)
Net Investments/Sales	-0.0964* (0.0528)	-0.0589 (0.1040)	0.0353 (0.1849)	-0.1718 (0.1729)
Net Investments/Assets	-0.0523 (0.0525)	-0.0482 (0.1103)	0.0574 (0.1699)	-0.1137 (0.1818)
Net Investments/Capital	-0.1110 (0.3027)	-0.2622 (0.5868)	-0.0484 (0.8590)	-1.5708* (0.9445)
Net Int. Investments	-32.22 (73.93)	-310.84* (180.00)	-149.26 (212.40)	-476.11* (280.10)
Net Int. Investments/Sales	-0.0136 (0.0135)	-0.0491 (0.0341)	-0.0312 (0.0500)	-0.0855 (0.0564)
Net Int. Investments/Assets	0.0053 (0.0194)	-0.0516 (0.0350)	-0.0457 (0.0603)	-0.0450 (0.0583)
Net Int. Investments/Int. Capital	-1.1465 (1.3000)	-2.0917 (1.7574)	-5.0893* (3.0416)	-6.0287* (3.2185)
log(Sales)	-0.0420 (0.1852)	-0.3726 (0.3074)	0.2299 (0.2498)	0.2367 (0.6904)
log(Assets)	-0.0759 (0.1970)	-0.5168* (0.2672)	-0.1473 (0.2331)	-0.1702 (0.7780)
log(Capital)	-0.1441 (0.2682)	-0.6311 (0.5192)	-0.2287 (0.3774)	0.1879 (0.7665)
ROA	0.0093 (0.0127)	0.0000 (0.0246)	0.0443 (0.0292)	-0.0066 (0.0264)
ROE	0.0353 (0.2919)	0.3097 (0.4957)	0.3406 (0.7876)	0.0087 (1.0832)

The dataset includes 345 firms (38 treated).
Standard errors are reported between brackets.
The denominator of the investments variables is the value of the variable in 2001.
The coefficient is positive when the estimate of the effect for treated is greater than that one for controls.
Dummies for geographic area, technological sector and call for applications are included.

Table 3.8: Difference-in-differences estimates with matching and caliper with radius at the median of the distance, balanced panel, FTI general section

	2001	2002	2003	2004	2005	2006	2007
Gross Investments	9.86	151.68	-13.30	-31.81	-128.23	-308.94	-723.99*
	(322.40)	(685.01)	(682.01)	(274.01)	(450.49)	(498.05)	(439.18)
Gross Investments	0.8350	5.0960	-0.0522	2.5624	2.2316	-0.4536	-2.1268
/Employees	(2.2822)	(3.6864)	(2.7272)	(1.8708)	(2.5987)	(3.1004)	(2.9864)
Gross Investments	0.0069	0.0168	-0.0099	0.0067	0.0090	-0.0014	-0.0186
/Sales	(0.0141)	(0.0201)	(0.0136)	(0.0116)	(0.0176)	(0.0193)	(0.0193)
Gross Investments	0.0070	0.0086	-0.0084	0.0108	0.0107	0.0001	-0.0117
/Assets	(0.0143)	(0.0185)	(0.0134)	(0.0111)	(0.0175)	(0.0186)	(0.0171)
Gross Investments	0.0414	0.0673	0.0083	0.1243**	0.1571	0.0129	0.0407
/Capital	(0.0677)	(0.0796)	(0.0699)	(0.0568)	(0.1286)	(0.1031)	(0.1033)
Net Investments	-279.75	-267.91	767.13	-632.91	-91.74	-1074.32	-443.11
	(321.72)	(575.28)	(881.50)	(549.37)	(590.76)	(762.53)	(505.01)
Net Investments	-2.0933	3.0098	2.8391	2.3284	-1.3060	-1.5665	-0.9018
/Employees	(2.1752)	(2.3426)	(3.4928)	(2.3870)	(3.0098)	(3.6600)	(2.6461)
Net Investments	-0.0111	0.0102	-0.0006	0.0025	-0.0041	-0.0096	-0.0095
/Sales	(0.0107)	(0.0143)	(0.0153)	(0.0113)	(0.0161)	(0.0215)	(0.0180)
Net Investments	-0.0135	0.0017	0.0034	0.0054	-0.0052	-0.0086	-0.0024
/Assets	(0.0120)	(0.0152)	(0.0140)	(0.0109)	(0.0155)	(0.0210)	(0.0141)
Net Investments	-0.0841	-0.0072	0.0129	0.0636	-0.0319	-0.0602	0.0133
/Capital	(0.0769)	(0.0783)	(0.0851)	(0.0700)	(0.0898)	(0.0972)	(0.0945)

Baseline year: 2000. The dataset includes 148 not repeated firms (85 treated; 1184 overall observations) after the matching with caliper.
The caliper has radius equal to the median of the distribution of the distance between matched firms.
The denominator of the investments variables is the value of the variable in 1999.
Standard errors are reported between brackets.
Estimates and standard errors use the procedure from Abadie and Imbens (2002; 2009).

Table 3.9: Difference-in-differences estimates with matching, unbalanced panel, FTI general section

	2001	2002	2003	2004	2005	2006	2007
Gross Investments	1310.10*	1016.40	1079.04*	585.30	497.96	357.55	911.80
	(675.40)	(929.87)	(654.25)	(408.73)	(439.47)	(615.64)	(907.40)
Gross Investments	7.3246**	4.6500	11.5164***	1.3938	4.9133	1.6543	2.9468
/Employees	(3.1336)	(4.1622)	(3.9904)	(2.3051)	(4.1554)	(2.9514)	(3.3528)
Gross Investments	0.0235*	0.0129	0.0216	0.0077	0.0305*	0.0152	0.0100
/Sales	(0.0133)	(0.0181)	(0.0132)	(0.0105)	(0.0175)	(0.0161)	(0.0202)
Gross Investments	0.0235*	0.0093	0.0212	0.0045	0.0253	0.0085	0.0003
/Assets	(0.0138)	(0.0198)	(0.0135)	(0.0108)	(0.0180)	(0.0156)	(0.0164)
Gross Investments	0.0805	0.0306	0.1146	0.0304	0.1388	0.0880	-0.0109
/Capital	(0.0837)	(0.1302)	(0.1046)	(0.1007)	(0.1273)	(0.1174)	(0.1297)
Net Investments	-1219.85	-1874.44	-830.88	1490.02	-2032.07	-1516.52	-1106.20
	(2812.97)	(2844.36)	(2820.26)	(4707.67)	(3074.19)	(2959.40)	(2947.29)
Net Investments	-7.6662**	-5.8341*	5.9266*	0.0154	-1.2839	1.1744	2.7780
/Employees	(3.8556)	(3.3839)	(3.3651)	(2.5216)	(4.2642)	(3.6081)	(3.2751)
Net Investments	0.0030	-0.0098	0.0199	0.0027	0.0039	0.0117	0.0177
/Sales	(0.0116)	(0.0146)	(0.0143)	(0.0102)	(0.0206)	(0.0164)	(0.0186)
Net Investments	0.0027	-0.0144	0.0250*	0.0038	0.0066	0.0056	0.0116
/Assets	(0.0110)	(0.0144)	(0.0150)	(0.0090)	(0.0203)	(0.0144)	(0.0131)
Net Investments	-0.0286	-0.0965	0.0905	0.0498	-0.0995	0.0931	0.0686
/Capital	(0.0763)	(0.1233)	(0.1248)	(0.0864)	(0.1556)	(0.1133)	(0.1061)

Baseline year: 2000. The dataset includes 353 not repeated firms (208 treated, 2675 overall observations) after the matching.
The denominator of the investments variables is the value of the variable in 1999.
Estimates and standard errors use the procedure from Abadie and Imbens (2002;2009).
Standard errors are reported between brackets.

Table 3.10: Difference-in-differences estimates with matching, small and medium firms, balanced panel, FTI general section

	2001	2002	2003	2004	2005	2006	2007
Gross Investments	433.57*	142.96	461.63	266.89	259.25	450.08*	323.99
	(226.45)	(391.43)	(294.96)	(208.40)	(235.92)	(263.77)	(374.77)
Gross Investments	3.5145	-0.4656	6.6159	6.7261*	7.8970	7.3826*	7.7937*
/Employees	(2.8472)	(4.5193)	(4.6879)	(3.5215)	(5.3088)	(3.9007)	(4.5869)
Gross Investments	0.0246	-0.0121	0.0341	0.0177	0.0351	0.0346**	0.0306
/Sales	(0.0158)	(0.0218)	(0.0208)	(0.0133)	(0.0225)	(0.0161)	(0.0256)
Gross Investments	0.0198	-0.0236	0.0261	0.0129	0.0245	0.0238	0.0225
/Assets	(0.0163)	(0.0223)	(0.0220)	(0.0141)	(0.0234)	(0.0166)	(0.0237)
Gross Investments	0.0925	-0.1149	0.1507	0.0913	0.0327	0.1487	0.0657
/Capital	(0.0999)	(0.1405)	(0.1849)	(0.1232)	(0.1789)	(0.1670)	(0.1846)
Net Investments	214.74	205.41	587.85*	177.09	322.27	485.88*	139.31
	(254.15)	(319.23)	(317.11)	(335.05)	(379.89)	(293.49)	(346.64)
Net Investments	-3.2571	-2.7345	9.6079	3.9142	3.8602	4.3019	5.1908
/Employees	(3.7002)	(4.9071)	(6.3213)	(3.1987)	(6.0470)	(3.6558)	(4.2466)
Net Investments	-0.0107	-0.0141	0.0420**	0.0124	0.0194	0.0248	0.0270
/Sales	(0.0119)	(0.0224)	(0.0211)	(0.0132)	(0.0257)	(0.0161)	(0.0238)
Net Investments	-0.0095	-0.0201	0.0409**	0.0130	0.0177	0.0162	0.0278
/Assets	(0.0134)	(0.0218)	(0.0197)	(0.0145)	(0.0270)	(0.0149)	(0.0204)
Net Investments	0.0707	-0.0377	0.2974*	0.1502	0.1667	0.1039	0.2207
/Capital	(0.1251)	(0.1343)	(0.1681)	(0.1678)	(0.2103)	(0.1522)	(0.1605)

Baseline year: 2000. The dataset includes 158 not repeated firms (95 treated, 1264 overall observations) after the matching.
The denominator of the investments variables is the value of the variable in 1999.
Estimates and standard errors are calculated using the procedure from Abadie and Imbens (2002;2009).
Standard errors are reported between brackets.

Table 3.11: Difference-in-differences estimates with matching, high debt cost firms, balanced panel, FTI general section

	2001	2002	2003	2004	2005	2006	2007
Gross Investments	1805.42	3254.59	3626.92	2239.53*	684.65	2522.37	2353.60
	(1522.37)	(2629.65)	(2443.82)	(1273.90)	(983.31)	(1815.53)	(2078.23)
Gross Investments /Employees	8.8079**	8.2347	12.0818	11.9546***	6.1154	10.2856*	6.7157
	(4.4285)	(8.0292)	(7.9746)	(4.1752)	(7.8791)	(5.6676)	(4.7773)
Gross Investments /Sales	0.0263	0.0250	0.0318	0.0263	0.0157	0.0319	0.0002
	(0.0224)	(0.0409)	(0.0307)	(0.0182)	(0.0276)	(0.0324)	(0.0265)
Gross Investments /Assets	0.0265	0.0248	0.0399	0.0292	0.0082	0.0354	0.0061
	(0.0228)	(0.0417)	(0.0354)	(0.0187)	(0.0284)	(0.0356)	(0.0265)
Gross Investments /Capital	0.1336	0.2237	0.2807	0.2621*	0.1050	0.2897	0.1628
	(0.1369)	(0.2292)	(0.2623)	(0.1527)	(0.2275)	(0.2100)	(0.2177)
Net Investments	5105.79	721.93	1407.38	6808.60	1517.33	1720.01	-118.63
	(3451.43)	(1972.27)	(1772.04)	(9860.94)	(1908.42)	(2463.32)	(1849.83)
Net Investments /Employees	8.2061	-3.4750	7.1064	0.1785	5.3210	4.5766	0.7331
	(5.6388)	(5.7366)	(6.7269)	(4.8059)	(6.7593)	(6.3279)	(4.4475)
Net Investments /Sales	0.0328	-0.0171	0.0135	-0.0052	0.0246	0.0315	-0.0116
	(0.0206)	(0.0213)	(0.0240)	(0.0166)	(0.0207)	(0.0411)	(0.0235)
Net Investments /Assets	0.0294*	-0.0177	0.0257	-0.0035	0.0259	0.0315	-0.0080
	(0.0177)	(0.0219)	(0.0266)	(0.0152)	(0.0213)	(0.0435)	(0.0198)
Net Investments /Capital	0.1376	0.0294	0.2486	0.1038	0.1915	0.2197	0.1516
	(0.1249)	(0.1348)	(0.1997)	(0.1310)	(0.1810)	(0.2082)	(0.1649)

Baseline year: 2000. The dataset includes 127 not repeated firms (81 treated, 1016 overall observations) after the matching.
The denominator of the investments variables is the value of the variable in 1999.
Estimates and standard errors are calculated using the procedure from Abadie and Imbens (2002;2009).
Standard errors are reported between brackets.

Table 3.12: Difference-in-differences estimates with matching, high subsidy over net investments intensity firms, balanced panel, FTI general section

	2001	2002	2003	2004	2005	2006	2007
Gross Investments	209.34	400.12	-213.27	378.23	432.51	508.29	924.88
	(323.85)	(598.75)	(354.69)	(308.55)	(363.77)	(347.40)	(622.90)
Gross Investments /Employees	4.7645	3.3987	1.4133	7.9762	8.4624	5.5387	11.3745
	(3.8355)	(5.5169)	(4.3448)	(5.1817)	(5.2687)	(4.8174)	(8.6387)
Gross Investments /Sales	0.0531*	0.0508	0.0447	0.0597	0.0669*	0.0720	0.1102
	(0.0300)	(0.0351)	(0.0356)	(0.0383)	(0.0388)	(0.0455)	(0.0798)
Gross Investments /Assets	0.0316	0.0246	0.0126	0.0374	0.0389	0.0380	0.0503
	(0.0219)	(0.0264)	(0.0213)	(0.0264)	(0.0293)	(0.0323)	(0.0459)
Gross Investments /Capital	0.0119	0.1122	0.0358	0.1956*	0.0109	-0.0108	-0.0384
	(0.0723)	(0.1166)	(0.0850)	(0.1075)	(0.1883)	(0.1716)	(0.2112)
Net Investments	1318.99*	1394.09	-1010.36	1879.86*	1917.89	1657.84**	1903.99**
	(683.26)	(971.73)	(638.19)	(1017.39)	(2137.66)	(687.00)	(897.91)
Net Investments /Employees	6.8785	6.4900	-4.1344	13.0662	6.7631	11.2897	14.3249
	(8.0678)	(10.4896)	(5.8500)	(10.2057)	(11.7711)	(9.5809)	(10.8170)
Net Investments /Sales	0.0478	0.0749	0.0306	0.0838	0.0522	0.1034	0.1255
	(0.0340)	(0.0584)	(0.0357)	(0.0566)	(0.0702)	(0.0632)	(0.0805)
Net Investments /Assets	0.0270	0.0358	-0.0017	0.0555	0.0176	0.0559	0.0548
	(0.0243)	(0.0394)	(0.0201)	(0.0386)	(0.0498)	(0.0397)	(0.0352)
Net Investments /Capital	0.0067	0.1476	-0.0158	0.3138**	-0.1824	0.0947	0.0557
	(0.0707)	(0.1324)	(0.1126)	(0.1445)	(0.2159)	(0.1162)	(0.1323)

Baseline year: 2000. The dataset includes 146 not repeated firms (78 treated; 1168 overall observations) after the matching.
The denominator of the investments variables is the value of the variable in 1999.
Estimates and standard errors are calculated using the procedure from Abadie and Imbens (2002;2009).
Standard errors are reported between brackets.

Fig. 3.1: Timeline of the FTI policy measures

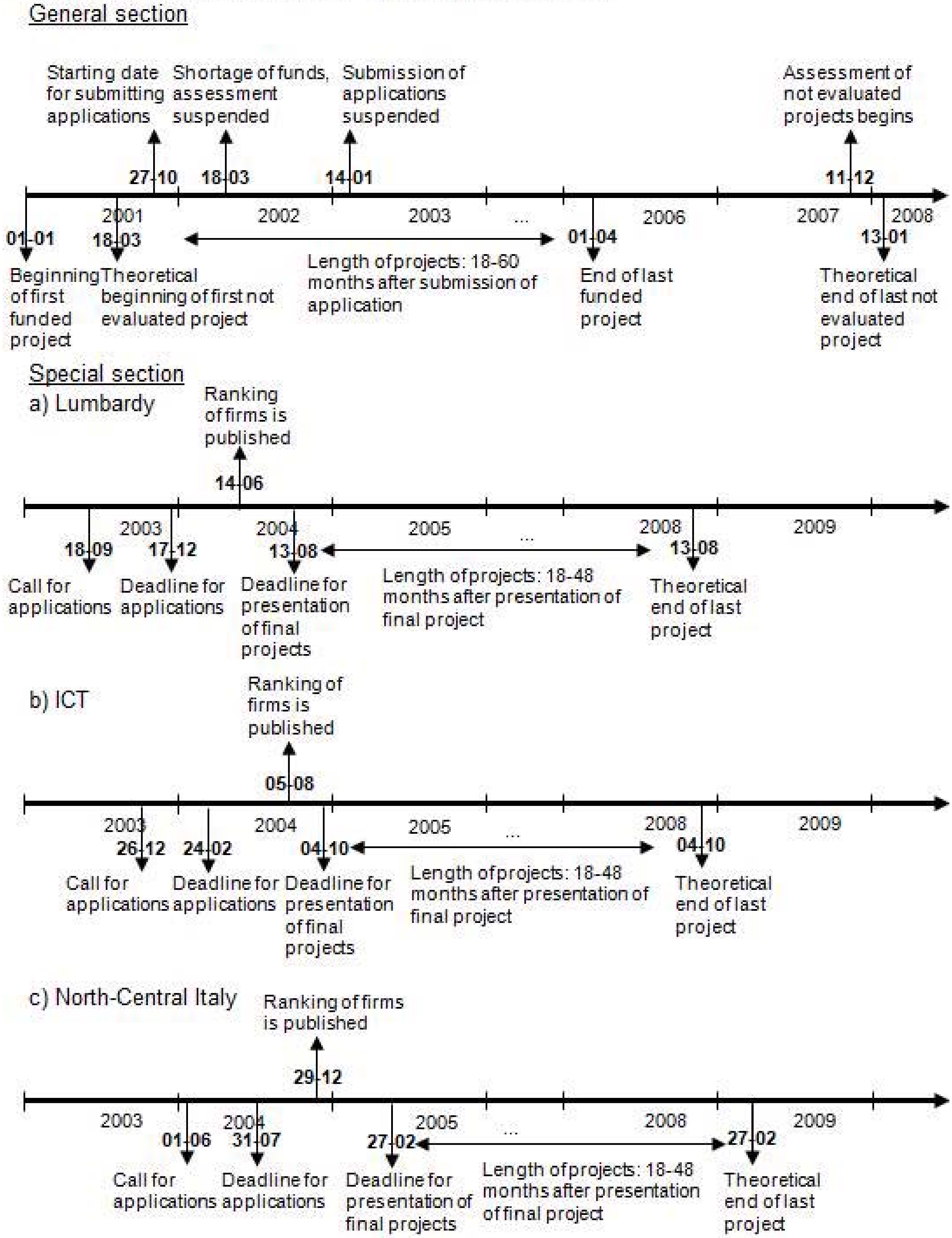


Fig. 3.2: Distribution of the dates of submission of the applications

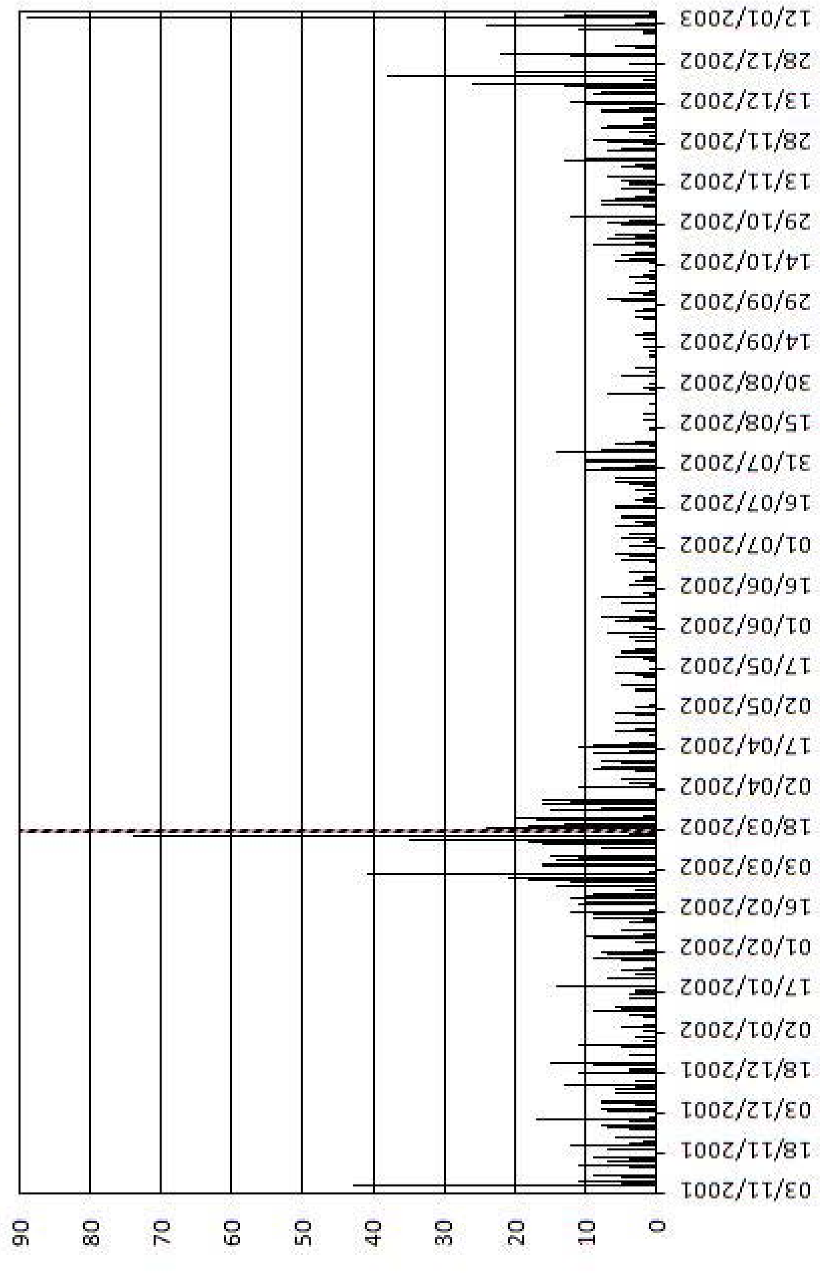
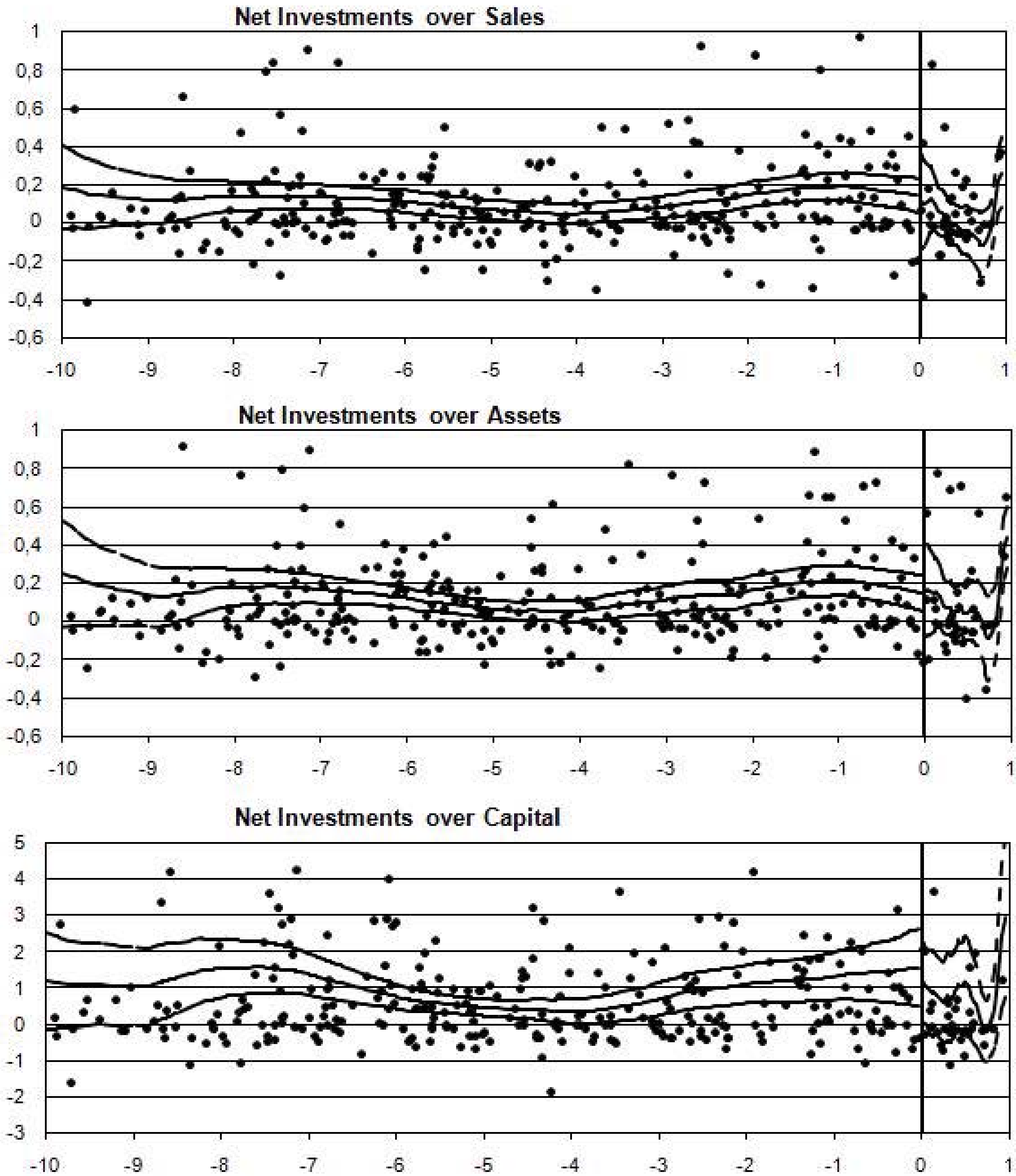


Fig. 3.3: Regression discontinuity design, semiparametric estimation, FTI special section: net overall investments



3.8 Appendix

3.8.1 Additional Tables

Table a3.1: Mean comparison before and after the matching, balanced panel, FTI general section, Cerved dataset

	Not matched		Matched	
	Mean difference	p-value	Mean difference	p-value
Net Investments	907.44	0.59	1303.58	0.49
Net Investments/Sales	-0.0016	0.92	0.0075	0.44
Net Investments/Assets	0.0026	0.88	0.0121	0.23
Net Investments/Capital	-0.0128	0.90	0.0153	0.87
Net Int. Investments	751.91	0.62	812.18	0.64
Net Int. Investments/Sales	-0.0060	0.55	0.0017	0.67
Net Int. Investments/Assets	-0.0086	0.47	0.0002	0.96
Net Int. Investments/Int. Capital	0.1882	0.77	0.3703	0.60
log(Sales)	0.1667	0.12	-0.0091	0.94
log(Assets)	0.1941	0.08	0.0346	0.77
log(Capital)	0.1956	0.14	0.0552	0.69
Debts/Assets	0.0251	0.05	0.0066	0.62
Long-Term Debts/Assets	0.0126	0.12	0.0061	0.49
Financial Charges/Debts	0.0063	0.02	0.0032	0.32
ROA	0.0029	0.57	0.0010	0.85
ROE	0.5133	0.29	0.5063	0.36

The statistics regards the pretreatment year (2000).
The denominator of the investments variables is the value of the variable in 1999.
The prematching number of firms is 653 (280 treated).

Table a3.2: Difference-in-differences estimates with matching, balanced panel, FTI general section, Cerved dataset

	2001	2002	2003	2004	2005	2006	2007
Net Investments	-1916.09	-1109.72	-1775.49	564.02	-2117.78	-2753.34	-1990.33
	(2116.51)	(2196.66)	(2159.96)	(3661.83)	(2283.54)	(2277.86)	(2188.27)
Net Investments	0.0068	0.0072	0.0248*	0.0222*	0.0358	-0.0073	-0.0119
/Sales	(0.0119)	(0.0133)	(0.0132)	(0.0130)	(0.0265)	(0.0135)	(0.0134)
Net Investments	0.0018	0.0046	0.0342**	0.0177*	0.0124	-0.0106	-0.0123
/Assets	(0.0114)	(0.0120)	(0.0142)	(0.0103)	(0.0160)	(0.0125)	(0.0126)
Net Investments	0.0036	0.1146	0.4860**	0.1770	0.2014	0.0315	0.0283
/Capital	(0.1104)	(0.1098)	(0.2383)	(0.1338)	(0.1546)	(0.1181)	(0.1153)
Net Int. Investments	-1430.67	-1502.83	-1558.46	845.57	-1408.76	-1651.70	-1410.16
	(2085.39)	(2085.76)	(2102.87)	(3669.46)	(2181.91)	(2138.79)	(2124.14)
Net Int. Investments	0.0065	0.0076	0.0046	0.0054	0.0048	-0.0048	-0.0076
/Sales	(0.0064)	(0.0085)	(0.0072)	(0.0090)	(0.0106)	(0.0071)	(0.0072)
Net Int. Investments	0.0090	0.0103	0.0114	0.0089	0.0040	-0.0006	-0.0035
/Assets	(0.0063)	(0.0076)	(0.0088)	(0.0079)	(0.0088)	(0.0071)	(0.0067)
Net Int. Investments	0.3123	0.4005	1.2385	4.9594	2.4521	-0.3565	-1.4947
/Int. Capital	(1.2059)	(1.4968)	(1.1715)	(3.7844)	(2.8635)	(1.5300)	(1.7483)
log(Sales)	0.0016	-0.0053	-0.0073	-0.0248	0.0178	0.0066	-0.0033
	(0.0203)	(0.0262)	(0.0283)	(0.0307)	(0.0359)	(0.0397)	(0.0437)
log(Assets)	-0.0114	0.0014	0.0210	0.0348	0.0583*	0.0765**	0.0502
	(0.0174)	(0.0242)	(0.0276)	(0.0300)	(0.0335)	(0.0369)	(0.0401)
log(Capital)	-0.0209	-0.0070	0.0358	0.1007*	0.1315**	0.1446**	0.1105
	(0.0317)	(0.0450)	(0.0546)	(0.0569)	(0.0641)	(0.0687)	(0.0716)
Debts/Assets	0.0045	-0.0010	-0.0199*	-0.0185*	-0.0074	0.0006	-0.0130
	(0.0056)	(0.0084)	(0.0107)	(0.0112)	(0.0119)	(0.0119)	(0.0125)
Long-Term Debts	0.0038	0.0026	-0.0178*	-0.0118	-0.0022	-0.0105	-0.0090
/Assets	(0.0066)	(0.0074)	(0.0098)	(0.0100)	(0.0091)	(0.0099)	(0.0100)
Financial Charges	0.0025	0.0000	0.0006	-0.0009	-0.0034	-0.0053***	-0.0047***
/Debts	(0.0035)	(0.0036)	(0.0026)	(0.0015)	(0.0021)	(0.0014)	(0.0015)
ROE	-0.5968	-0.5187	-0.6304	-0.5892	-0.8021	-0.5598	-0.8224
	(0.6852)	(0.6825)	(0.6942)	(0.6868)	(0.7262)	(0.6968)	(0.8486)
ROA	-0.0056	-0.0080	0.0033	-0.0067	0.0005	-0.0005	0.0009
	(0.0053)	(0.0068)	(0.0068)	(0.0081)	(0.0076)	(0.0072)	(0.0079)

Baseline year: 2000. The dataset includes 461 not repeated firms (280 treated, 3688 overall observations) after the matching.
The denominator of the investments variables is the value of the variable in 1999.
Estimates and standard errors use the procedure from Abadie and Imbens (2002;2009).
Standard errors are reported between brackets.

Table a3.3: Mean comparison before and after the matching, unbalanced panel, FTI general section

	Not matched		Matched	
	Mean difference	p-value	Mean difference	p-value
Gross Investments	-3603.25	0.41	509.00	0.31
Gross Investments/Employees	-0.9337	0.70	-1.8028	0.49
Gross Investments/Sales	-0.0042	0.65	-0.0092	0.34
Gross Investments/Assets	0.0033	0.70	-0.0003	0.98
Gross Investments/Capital	-0.0287	0.66	-0.0360	0.60
Net Investments	-1614.29	0.64	1142.56	0.63
Net Investments/Employees	3.1424	0.14	2.6895	0.26
Net Investments/Sales	0.0073	0.46	0.0061	0.57
Net Investments/Assets	0.0108	0.25	0.0082	0.41
Net Investments/Capital	0.0270	0.69	0.0321	0.64
Net Int. Investments	978.64	0.60	1025.02	0.64
Net Int. Investments/Employees	0.4453	0.58	0.3616	0.69
Net Int. Investments/Sales	0.0032	0.41	0.0028	0.53
Net Int. Investments/Assets	0.0028	0.42	0.0028	0.49
Net Int. Investments/Int. Capital	0.1238	0.78	0.3365	0.45
log(Employees)	-0.0106	0.91	-0.0071	0.94
log(Sales)	0.0305	0.76	0.0710	0.49
log(Assets)	0.0552	0.60	0.0254	0.81
log(Capital)	0.0950	0.46	0.0597	0.65
Value Added/Employees	3.3252	0.23	3.1698	0.27
Labour Cost/Employees	1.1272	0.13	1.1009	0.12
Debts/Assets	-0.0045	0.77	0.0127	0.44
Long-Term Debts/Assets	0.0122	0.16	0.0062	0.53
Financial Charges/Debts	0.0017	0.57	0.0027	0.35
ROA	-0.0024	0.71	-0.0070	0.29
ROE	-0.1493	0.03	-0.0866	0.21

The statistics regards the pretreatment year (2000).
The denominator of the investments variables is the value of the variable in 1999.
The prematching number of firms is 497 (208 treated).

Table a3.4: Mean comparison before and after the matching, small and medium firms, balanced panel, FTI general section

	Not matched		Matched	
	Mean difference	p-value	Mean difference	p-value
Gross Investments	87.069	0.60	77.828	0.66
Gross Investments/Employees	3.7049	0.20	2.0110	0.52
Gross Investments/Sales	0.0144	0.22	0.0134	0.28
Gross Investments/Assets	0.0237	0.06	0.0211	0.10
Gross Investments/Capital	0.0384	0.69	0.0460	0.63
Net Investments	150.29	0.43	155.699	0.42
Net Investments/Employees	5.5123	0.12	4.5648	0.24
Net Investments/Sales	0.0171	0.21	0.0190	0.17
Net Investments/Assets	0.0221	0.12	0.0242	0.09
Net Investments/Capital	0.0048	0.97	0.0526	0.56
Net Int. Investments	11.041	0.83	-44.667	0.51
Net Int. Investments/Employees	0.4037	0.53	-0.4903	0.55
Net Int. Investments/Sales	0.0024	0.34	-0.0010	0.70
Net Int. Investments/Assets	0.0028	0.27	-0.0009	0.75
Net Int. Investments/Int. Capital	0.4131	0.22	0.1218	0.76
log(Employees)	-0.1026	0.24	-0.0342	0.71
log(Sales)	-0.0308	0.70	0.0069	0.94
log(Assets)	0.0189	0.84	0.1181	0.23
log(Capital)	0.1070	0.47	0.2079	0.17
Value Added/Employees	2.8360	0.52	5.2280	0.26
Labour Cost/Employees	0.8681	0.48	1.5346	0.25
Debts/Assets	0.0014	0.95	-0.0089	0.68
Long-Term Debts/Assets	0.0069	0.57	0.0116	0.33
Financial Charges/Debts	0.0058	0.10	0.0021	0.58
ROA	-0.0024	0.80	-0.0014	0.88
ROE	-0.2332	0.10	-0.1729	0.24

The statistics regards the pretreatment year (2000).
The denominator of the investments variables is the value of the variable in 1999.
The prematching number of firms is 214 (95 treated).

Table a3.5: Mean comparison before and after the matching, high debt cost firms, balanced panel, FTI general section

	Not matched		Matched	
	Mean difference	p-value	Mean difference	p-value
Gross Investments	137.00	0.91	838.61	0.41
Gross Investments/Employees	1.6619	0.48	-0.9722	0.69
Gross Investments/Sales	0.0069	0.58	0.0091	0.44
Gross Investments/Assets	0.0117	0.34	0.0114	0.34
Gross Investments/Capital	0.0470	0.65	0.0107	0.92
Net Investments	-3126.92	0.20	-2161.59	0.40
Net Investments/Employees	4.9988	0.21	2.6416	0.55
Net Investments/Sales	0.0190	0.21	0.0165	0.30
Net Investments/Assets	0.0208	0.17	0.0185	0.24
Net Investments/Capital	0.0354	0.78	0.0401	0.72
Net Int. Investments	-2061.00	0.35	-1869.56	0.43
Net Int. Investments/Employees	0.5880	0.56	-0.5631	0.63
Net Int. Investments/Sales	0.0018	0.64	-0.0005	0.91
Net Int. Investments/Assets	0.0033	0.33	0.0009	0.80
Net Int. Investments/Int. Capital	-0.2615	0.64	0.0975	0.75
log(Employees)	-0.2509	0.13	-0.0316	0.86
log(Sales)	-0.1002	0.57	-0.0355	0.84
log(Assets)	-0.0794	0.67	-0.0153	0.94
log(Capital)	-0.1390	0.53	0.0267	0.91
Value Added/Employees	7.8352	0.02	4.0094	0.24
Labour Cost/Employees	1.7326	0.11	0.7739	0.51
Debts/Assets	-0.0016	0.94	-0.0273	0.25
Long-Term Debts/Assets	0.0209	0.16	0.0091	0.58
Financial Charges/Debts	0.0063	0.09	0.0020	0.64
ROA	0.0004	0.96	-0.0046	0.55
ROE	-0.2587	0.11	-0.2232	0.19

The statistics regards the pretreatment year (2000).
The denominator of the investments variables is the value of the variable in 1999.
The prematching number of firms is 180 (81 treated).

Table a3.6: Mean comparison before and after the matching, high subsidy over net investments intensity firms, balanced panel, FTI general section

	Not matched		Matched	
	Mean difference	p-value	Mean difference	p-value
Gross Investments	-6115.64	0.46	-242.74	0.59
Gross Investments/Employees	0.7335	0.80	0.0407	0.99
Gross Investments/Sales	-0.0019	0.88	-0.0163	0.35
Gross Investments/Assets	0.0132	0.27	0.0062	0.69
Gross Investments/Capital	0.0221	0.73	0.0581	0.34
Net Investments	-3940.72	0.46	-516.96	0.31
Net Investments/Employees	4.3715	0.15	1.7660	0.69
Net Investments/Sales	0.0081	0.55	-0.0089	0.64
Net Investments/Assets	0.0154	0.23	0.0089	0.57
Net Investments/Capital	0.0221	0.80	0.0817	0.24
Net Int. Investments	-478.58	0.26	-54.01	0.63
Net Int. Investments/Employees	-0.2028	0.79	-0.2440	0.77
Net Int. Investments/Sales	-0.0006	0.85	-0.0002	0.95
Net Int. Investments/Assets	-0.0001	0.97	0.0018	0.58
Net Int. Investments/Int. Capital	0.5599	0.47	1.0612	0.23
log(Employees)	-0.3684	0.00	-0.1063	0.41
log(Sales)	-0.3499	0.01	-0.0900	0.49
log(Assets)	-0.3503	0.02	-0.1220	0.39
log(Capital)	-0.3821	0.04	-0.1312	0.51
Value Added/Employees	4.4575	0.23	3.4489	0.41
Labour Cost/Employees	0.5878	0.59	0.7433	0.51
Debts/Assets	-0.0015	0.94	0.0224	0.44
Long-Term Debts/Assets	0.0106	0.38	0.0141	0.37
Financial Charges/Debts	0.0044	0.11	0.0029	0.47
ROA	0.0086	0.33	0.0011	0.92
ROE	-0.2127	0.09	-0.1655	0.35

The statistics regards the pretreatment year (2000).
The denominator of the investments variables is the value of the variable in 1999.
The prematching number of firms is 294 (78 treated).

3.8.2 FTI general section: regression discontinuity design

We implement here the regression continuity design to the unmatched sample of the FTI general section using the date of submission of the applications as the forcing variable. We want to understand if the results are the same when we compare firms applying immediately before and after the date of suspension of the assessment of the applications.[55]

We focus our analysis in this appendix on investments, which are our most important outcomes; the results for all the mean values of the other considered variables (dimension, financial structure, etc.) in the treatment period are not reported for sake of brevity, but they are very similar to those reported in the main text and are available on request. The outcome variables we use here are the cumulated sum of gross and net investments in the treatment period (2001-2007), divided by a dimensional variable calculated in the pre-pretreatment year (1999).

The estimates of the effect of the policy and the bootstrapped standard errors for investments when we use either a nonparametric or a semiparametric methodology are shown in Table $a3.8$. We use a triangular kernel and, in the semiparametric case, we add geographical area and technological

[55] To ensure that the hypotheses of the regression discontinuity design were satisfied we used here a different procedure to polish data: instead of removing the first and the last percentile of the variations, we eliminated the first and the last percentile of the levels of the outcome variables in the pretreatment year and in the average of the treatment years. This procedure is less effective in the elimination of the outliers, but allows the construction of a more homogeneous sample, improving comparability in absence of matching. The balancing properties of the final sample are shown in Table $a3.7$, both for the full sample and for the restricted local sample of firms applying between 90 days before and 90 days after the cutoff date.

sector dummies. The bandwidth is separately determined for treated and controls using the rule of thumb procedure suggested by Silverman (1986). The standard errors of the jump are calculated using a bootstrap procedure with 200 replications stratified per technological level. Once again, almost all the estimates are not significant, with the exception of the semiparametric estimates for the cumulated net investments over employees.

In Table $a3.9$ are reported the results for investments when we use a parametric specification of the relationship between outcome and forcing variable, in a polynomial form. In the first three columns we respectively use polynomials of degree zero, one and two. While there seems to be some significant differences in the raw mean comparison in the first column, their significance is not confirmed when we consider the linear or the quadratic specification.

In the forth column we include geographical area and technological sector dummies in the linear specification; under our hypotheses there should be no correlation between treatment assignment and the dummy variables in a neighbourhood of the cutoff and we expect therefore the results to be very similar to the second column. The results are confirmed.

In the last column we consider a linear local polynomial regression restricting the sample to the firms applying between 90 days before and after the day of suspension of the assessment of the applications. This subsample includes around one half of the applications. The estimated differences are slightly bigger in this case and the coefficients are weakly significant only when the denominator is the number of employees.

3.8.3 Tables for the regression discontinuity design

Table a3.7: Regression discontinuity design, balancing properties, FTI general section

	Full sample		90 days around the cutoff	
	Mean difference	p-value	Mean difference	p-value
Gross Investments	-1.00	1.00	32.26	0.95
Gross Investments/Employees	1.5596	0.34	1.9230	0.49
Gross Investments/Sales	0.0014	0.84	-0.0007	0.95
Gross Investments/Assets	0.0088	0.25	0.0096	0.42
Gross Investments/Capital	0.0351	0.53	0.0255	0.65
Net Investments	135.30	0.75	-34.03	0.96
Net Investments/Employees	2.1753	0.22	3.0193	0.28
Net Investments/Sales	0.0061	0.43	0.0057	0.60
Net Investments/Assets	0.0111	0.18	0.0108	0.35
Net Investments/Capital	0.0451	0.50	0.0835	0.26
Net Int. Investments	-115.76	0.25	-3.47	0.97
Net Int. Investments/Employees	-0.0113	0.98	0.1753	0.81
Net Int. Investments/Sales	0.0005	0.75	0.0008	0.73
Net Int. Investments/Assets	0.0009	0.60	0.0012	0.64
Net Int. Investments/Int. Capital	0.7673	0.38	-0.7357	0.51
log(Employees)	-0.0684	0.46	-0.0640	0.66
log(Sales)	-0.0091	0.93	0.0268	0.86
log(Assets)	-0.0029	0.98	0.0177	0.91
log(Capital)	0.0038	0.98	0.1117	0.58
Value Added/Employees	4.4404	0.08	2.3173	0.55
Labour Cost/Employees	1.1582	0.09	0.9181	0.39
Debts/Assets	0.0026	0.88	0.0187	0.44
Long-Term Debts/Assets	0.0153	0.10	0.0136	0.36
Financial Charges/Debts	0.0041	0.09	0.0039	0.18
ROA	0.0023	0.72	-0.0056	0.55
ROE	-0.0860	0.28	-0.0800	0.62

The statistics regards the balanced panel in the pretreatment year (2000). The denominator of the investments variables is the value of the variable in 1999.
The number of firms is 421 (182 treated) in the full sample, 202 (75 treated) in the sample restricted to 90 days around the cutoff.

Table a3.8: Regression discontinuity design, kernel estimates of the discontinuity at the cutoff, balanced panel, FTI general section

	Nonparametric estimates	Semiparametric estimates	Bandwidth for treated	Bandwidth for controls
Cumulated Gross Investments	-3404.27 (5825.86)	-5260.42 (6113.16)	19.4437	40.8260
Cumulated Gross Investments/Employees	18.3466 (19.6993)	23.9116 (20.9197)	12.7867	35.7277
Cumulated Gross Investments/Sales	0.1094 (0.0839)	0.1331 (0.0854)	15.1285	31.8514
Cumulated Gross Investments/Assets	0.0971 (0.0917)	0.1161 (0.0961)	17.0482	45.4119
Cumulated Gross Investments/Capital	1.1463 (1.0625)	1.2089 (1.1149)	15.8922	29.6688
Cumulated Net Investments	-506.51 (2089.21)	122.67 (2411.99)	19.0224	29.2701
Cumulated Net Investments/Employees	18.7384 (12.5106)	24.0207* (14.4165)	12.2811	23.5627
Cumulated Net Investments/Sales	0.0793 (0.0596)	0.1024 (0.0623)	18.3330	23.6368
Cumulated Net Investments/Assets	0.0530 (0.0691)	0.0812 (0.0737)	16.5183	26.6993
Cumulated Net Investments/Capital	0.8672 (0.7803)	1.0286 (0.7994)	14.2335	26.4042

The dataset includes 421 firms (182 treated).
The bandwidths for both treated and controls have been determined using the rule of thumb procedure from Silverman (1986).
The denominator of the investments variables is the value of the variable in 1999.
Bootstrapped standard errors (200 replications) are reported between brackets.

Table a3.9: Regression discontinuity design, polynomial estimates of the discontinuity at the cutoff, balanced panel, FTI general section

sample	full	full	full	full	±90 days
degree	0	1	2	1	1
dummies	no	no	no	yes	no
Gross Investments	3029.85	-229.99	-3242.22	-1546.69	-2292.19
	(2260.58)	(3467.52)	(4650.28)	(3498.82)	(5061.93)
Gross Investments /Employees	27.2238***	24.4996	29.3591	23.0479	40.3892*
	(9.6494)	(17.5858)	(19.8105)	(16.5727)	(21.6237)
Gross Investments /Sales	0.0684*	0.0425	0.1092	0.0263	0.1113
	(0.0382)	(0.0716)	(0.0714)	(0.0673)	(0.0805)
Gross Investments /Assets	0.0775*	0.0124	0.0938	0.0070	0.1171
	(0.0401)	(0.0836)	(0.0797)	(0.0827)	(0.0975)
Gross Investments /Capital	0.9154**	0.5599	0.7311	0.7837	1.4627
	(0.4241)	(0.7256)	(0.9053)	(0.8101)	(0.9291)
Net Investments	351.86	768.29	-341.74	1087.59	7.17
	(1068.01)	(1629.16)	(2002.00)	(1746.78)	(1981.42)
Net Investments /Employees	11.9560*	9.8160	9.9552	13.3543	27.1878*
	(6.3652)	(11.5510)	(12.6864)	(12.3870)	(14.6108)
Net Investments /Sales	0.0398*	0.0327	0.0450	0.0382	0.0898
	(0.0237)	(0.0453)	(0.0496)	(0.0495)	(0.0580)
Net Investments /Assets	0.0400	0.0086	0.0197	0.0205	0.0750
	(0.0244)	(0.0533)	(0.0552)	(0.0575)	(0.0711)
Net Investments /Capital	0.5581*	0.4954	0.2528	0.7494	1.0886
	(0.3337)	(0.5128)	(0.6135)	(0.5563)	(0.6967)

The number of firms is 421 (182 treated) in the full sample, 202 (75 treated) in the sample restricted to 90 days around the cutoff.
The denominator of the investments variables is the value of the variable in 1999.
Standard errors are reported between brackets.
In the fourth column, dummies for geographic area and technological sector are included.

4 Dynamics of R&D investments and the value of the firm

4.1 Introduction

The economic literature has investigated the relationship between the value of a firm and its knowledge capital stock since the pioneering work of Griliches (1981). It defined the basic static framework of all the following research on this subject. Many advances have been achieved in the field since then. Almost all of them have been based on empirical studies, while the theoretical framework is still only partially developed.

In this work, we examine some sectorial empirical evidence regarding the relationship and we try to explain some puzzling results using a simple model of firm choices where we introduce uncertainty in the results of R&D activity, an element not fully taken into account in the previous studies.

In more detail, in the first half of our work, we derive some empirical results about the relationship between the value of a firm and the knowledge and physical capital assets, sector by sector, in the U.S. manufacturing industries for the period 1975-1995. We see that the theoretical assumptions of the relationship used by many previous studies may have effects in terms of efficiency of the estimates. We find a high level of heterogeneity in the coefficients of different sectors, which could cast some doubts on the validity of the results of the previous analyses usually postulating the same relationship in all sectors after controlling for fixed effects. When separating the current research effort from the past one, often the effect of current R&D is much weaker than that one of past R&D and sometimes is negative.

The usual theoretical framework is not able to explain these results and therefore we develop, in the second half of the chapter, a simple model where there is uncertainty about the results of R&D investments. The fundamental improvements of our model upon the previous theoretical framework are the introduction of a risk of failure in R&D activity and the explicit consideration of the time dimension. They are relevant because there is a lag between a R&D investment and the achievement of its results on the knowledge asset. In the meantime, the value of a firm will take into account not only the current assets, but also the expected value of the potential ones. Moreover, since research is a risky activity, there can be a different valuation of the already concretized assets and of those still at a work-in-progress stage. Risk-averse investors can penalize the expected returns of the latter in the determination of the market value of the firm. Therefore, when investors are risk averse and managers maximize the long-run value of the firm, the risk associated with work-in-progress R&D can reduce the short-run firm value even if its expected value grows in the long run.

The rest of the chapter is organized as follows: Section 4.2 examines the past literature on the subject, highlighting and discussing the most relevant results; Section 4.3 estimates and interprets our empirical results on the relationship between firm value and its determinants; Section 4.4 develops the dynamic model and discusses its implications for firm value; Section 4.5 concludes and highlights the main results and further directions of research.

4.2 Literature review

The relationship between firm value and knowledge assets has been examined in many empirical works over the last thirty years. In almost all cases, the estimations consider U.S. firms during the 1970s and the 1980s.

The first attempt at this work is undertaken by Griliches (1981), who delimits the framework of most of the following works. The starting point of his analysis is the following static definition:

$$V = q\left(\gamma A + K\right)$$

where V is the value of the firm, K and A are, respectively, the conventional and the knowledge stock of assets and q is the current market valuation of each unit of asset, reflecting market conditions and the firm's position in the market.

After some manipulations and approximations, we derive the estimated equation:

$$\ln\left(Q\right) = \gamma \frac{A}{K} + d + u$$

where $Q = \frac{V}{K}$ is the usual Tobin's Q, d is a dummy vector capturing the effects due to the temporal and the cross-sectional dimensions of fixed effects and u is the error term.

The number of patents and a weighted sum of current and past R&D expenditure approximate the knowledge stock. The coefficients of all these variables are positive and significant. Including a lagged Q term results in the past R&D coefficients becoming much less significant and sometimes negative. This fact has been interpreted by Griliches as a sign that changes in the market valuation are due to the unanticipated component of current R&D. Other regressions decomposing the R&D effect in the predicted and surprise components seem to confirm such an interpretation.

Griliches' analysis leaves many unanswered questions and encourages several extensions. One of the most interesting works in subsequent years is Pakes (1985), which tries further to develop the relationships between firm value, patents, anticipated and unanticipated R&D. Pakes' first step is the construction of a simple theoretical framework based upon the investment theory of Lucas and Prescott (1971).

The research program of a firm may be modeled as a sequence of random variables, which are the R&D investments in each future period. The management of the firm modifies the program in each period to maximize the expected discounted value of profits derived from the program subject to the previous R&D expenditures and any other currently available information.

Under the hypothesis of rational expectations, the impact on program value caused by the update in the information set in each period is assumed to be white noise. The same is true for the difference between the valuation of the management and that of the market. The observed one period rate of return on the equity market will therefore be equal to the sum of the market interest rate and these last two white noises.

A consequence of profit maximization is that the chosen level of R&D is a function of the unexpected gains and losses in the value of the program. To complete the model, the patent stock generating function is assumed to depend on the effective value of the research program and on the propensity to patent (assumed once again to be white noise) in each previous period. The three equations of the model are

$$\begin{cases} q_t = \epsilon_t + \eta_{1,t} \\ r_t = c\,(L)\,\epsilon_t \\ p_t = d\,(L)\,\epsilon_t + b\,(L)\,\eta_{3,t} \end{cases}$$

where q_t is the portion of the rate of return in excess of the market rate, r_t is the logarithm of the expenditure in R&D, p_t is the logarithm of the patent stock, ϵ_t, $\eta_{1,t}$ and $\eta_{3,t}$ are independent white noises associated respectively with the innovation in the information set of the management, with the difference of valuation between the market and the management and with the propensity to patent in period t, L is the lag operator, the functions in L are polinomials of the kind $c\,(L) = \sum_{\tau=0}^{\infty} c_\tau L^\tau$. Pakes assumes $b_0 = 1$.

After some manipulations, the three equations can be one-by-one recursively estimated in the following constrained autoregressive formulation:

$$
\begin{cases}
q_t = f_1(L)\, q_{t-1} + f_2(L)\, r_{t-1} + f_3(L)\, p_{t-1} + \eta_{1,t} \\
r_t = c_0 \frac{\sigma_\epsilon^2}{\sigma_q^2} q_t + f_4(L)\, r_{t-1} + \eta_{2,t} \\
p_t = f_5(L)\, r_t + f_6(L)\, p_{t-1} + \eta_{3,t}
\end{cases}
$$

where $\eta_{2,t}$ can be expressed as a function of $\frac{\sigma_\epsilon^2}{\sigma_q^2}$, ϵ_t and $\eta_{1,t}$ and is uncorrelated with both q_t and the past values of all the variables; the polynomials $f_i(L)$ $\forall i$ can be expressed as functions of $b(L)$, $c(L)$ and $d(L)$. The results from this estimation are compared with a similar non-recursive VAR model and with an unrestricted model where all the current and past values of the variables were included in all the equations. An initial result is that, while the exclusion of z_t from the r_t and q_t equations is confirmed, the same is not true for u_t in the r_t equation. However, in general, the restricted model seems to be coherent with the estimation results. We have a confirmation of the link between positive changes in the firm value and those in R&D and in the patent policies; the latter relationship is noisier, because changes in the propensity to patent do not affect the r_t and q_t equations.

A later work by Griliches, Hall and Pakes (1991) reinterprets and further develops these results. The basic theory tested in the work is that, in the framework of Pakes (1985), R&D activity is subject to the effects of market shocks (linked to demand and therefore associated with the u_t term of Pakes (1985)) and technological opportunities (linked to supply and therefore associated with the e_t term). If the patent stock generating process is not influenced by demand factors, we can use it to discriminate between demand

and supply shocks. Except for the pharmaceutical industry, the attempt fails because of the noisiness in the patents data due to the changes in the propensity to patent.

Hall, Jaffe and Trajtenberg (2005) use a citation weighted patents stock as a proxy for the knowledge capital to explore the relationship between firm value and knowledge stock using the framework from Griliches (1981). The results are quite coherent with previous ones. R&D explains the changes in firm value better than the patents stock, but the weighted patents stock is a much better index than the simple one and it does not lose much significance when both R&D and weighted patents stock are included.

An interpretational issue arises if the weighted patents stock and a function of past R&D expenditures either are or are not two equivalent proxies of the knowledge stock. A good reason why the two can measure different aspects is that R&D expenditure is an input of the knowledge creation process and therefore is an index of the initial effort, while the patent stock is the output and hence is nearer to a measure of the success of the process.

Apart from the works examined in these few pages, there are many other studies worth noting. Just to cite some of them, Hall (1993) finds hints of structural changes in the valuation of intangible R&D assets after 1985. Johnson and Pazderka (1993) examine how the effect of R&D on market value changes in the presence of market power in the Canadian market. Blundell, Griffith and Van Reenen (1999) find that firms with a larger market share experiment more innovations and their market value benefits more from it.

4.3 Some empirical results

We begin our investigation with an empirical microeconometric analysis of the relationship between firm value and knowledge capital. We use observations at the firm level from the Compustat database in the period 1975-1995.

The Compustat dataset allows us to know the balance sheet data and several other indexes of the U.S. firms since the 1950. We extract data about a subset of firms that consistently did R&D activity in the considered period.

We deflate all the nominal variables using the U.S. GDP deflator, we clean the datasets removing the duplicated observations and the firms experiencing substantial mergers and acquisitions, erasing the first and last percentile of all the used variables (except R&D, where we just removed the last percentile).

The final version of our dataset contains about 20000 observations regarding 1600 firms. A subset of observations had crucial missing variables and therefore the available observations for estimation were about 14000.

We construct now the variables. The knowledge capital of the firm is approximated using the cumulated overall R&D stock, constructed with the perpetual inventory method. In each period, we add the current R&D expenditure (proxy for the contribution to the knowledge capital in the current period) to the discounted stock of past R&D, a proxy for the current value of past ideas; we assume that past ideas become obsolete over time and lose

15 per cent of their current value after each year:[56]

$$
\begin{aligned}
R\&Dcum_t &= (1-\delta)\, R\&Dcum_{t-1} + R\&D_t \\
&= 0.85 R\&Dcum_{t-1} + R\&D_t \\
&= \sum_{i=0}^{\infty} (1-\delta)^i\, R\&D_{t-i}.
\end{aligned}
$$

The use of the same depreciation rate for all the economic sectors is clearly a strong assumption and we will see in our analysis that it can be the reason behind some results. Unfortunately, estimations of sector-by-sector depreciation rates are not currently available and therefore we have to rely on this simplifying assumption.

There is one more source of bias in the construction of the cumulated R&D stock. We restricted the subset of observations to the firms with no missing data on R&D in the period 1975-1995. In the construction of the cumulated R&D stock, we also used the data for the period 1950-1975 to improve the accuracy of our index. Unfortunately, the available data about this period are not complete and there are firms beginning their activity before 1950. Therefore, sometimes the stock slightly underestimates the effective amount of accumulated knowledge capital.

We construct two other variables. The value of the firm is the sum of the total current value of the outstanding shares (end of the fiscal year value of one share times the number of shares corrected to take into account the

[56] This value for the depreciation rate is common in the literature. See for example Cockburn and Griliches (1988), Hall (1993), Hall, Jaffe and Trajtenberg (2000).

preferred ones), of the total dividend paid to the shareholders and of the long and short-term debt. The conventional capital stock is the sum of the net book value of plants and equipment, of the inventory, of investments in unconsolidated subsidiaries: it includes all the assets of the firm except that of knowledge.

Following the previous empirical literature, we work on a linear version of the relationship between the market value of the firm (V_t), the value of the physical assets (K_t) and that one of the knowledge capital, approximated by the discounted sum of past and current R&D ($R\&Dcum_t$)

$$V_t = a_0 + a_1 K_t + a_2 R\&Dcum_t + \eta. \tag{35}$$

This equation can also be seen as a linear approximation of the equation (43) of the model derived in the next Section.

In equation (35) we neglect labour and other variable inputs because we suppose they are always adjusted to their optimal value and, therefore, they are a function of the other variables. This means that our coefficients in the empirical analysis will include the indirect effect of the capital and knowledge stocks through the adjustment of the employed variable inputs.

In a second stage of the analysis, we weaken the assumption that past and current R&D have the same effect. Therefore, we split the cumulated R&D stock in two terms: current R&D and cumulated past R&D ($R\&Dpast_t$), which is the overall cumulated R&D stock, calculated using the methodology we explained before, net of the current R&D contribution. We can write the definition of the cumulated past R&D stock as

$$
\begin{aligned}
R\&Dpast_t &= (1-\delta)\left(R\&Dpast_{t-1} + R\&D_{t-1}\right) \\
&= \sum_{i=1}^{\infty} (1-\delta)^i \, R\&D_{t-i}.
\end{aligned}
$$

The estimated equation in this case will be

$$
V_t = a_0 + a_1 K_t + a_2 R\&Dpast_t + a_3 R\&D_t + \eta. \tag{36}
$$

Given the variables and the equations, we begin our analysis examining the descriptive statistics. In Table 4.1, we see the overall descriptive statistics. In general, we notice an extremely high level of dispersion, given by the fact that for all the variables the standard deviation is at least three times the average value. The distributions are extremely skewed towards the left, with the mean after the 75th percentile.

The extremely high dispersion suggests that a sector-by-sector analysis can allow us to take into account one of the most likely sources of heterogeneity and that therefore we can improve the quality of our analysis.

In Table 4.2, we see the disaggregation used in the analysis, based on the ISIC code. In general, we use a two-digit codification, but in some cases we have enough observations to consider a finer disaggregation to highlight important sectors, which show a different behaviour from the rest of the two digit classification parent sector.

Table 4.3 reports the descriptive statistics sector by sector. The ratio between the standard deviation and the mean is now generally between 1.5 and 2 for all the variables, a sign that the between sectors component of the heterogeneity is quite relevant. The analysis of the structure of percentiles shows us that the differences between sectors are not only in the magnitude of the means, but also in the shape of the distributions.

Almost all the sectors where internal heterogeneity is still particularly relevant are those producing high technology output (computers, electrical equipment, electronic components, precision instruments...). A possible reason is that the technological innovation rate is particularly high in these sectors and there is less standardization of the production system of firms. These sectors will show a different behaviour from the others in the rest of the analysis, with an extremely significant positive impact of current R&D.

We examine now the estimates of the relationship between the value of the firm and the accumulated assets in an econometric framework. We use a panel model allowing sector specific fixed effects, but with common slopes among sectors for the R&D regressors. The structure of the error term is therefore $\eta = \sum d_i + \varepsilon$, where d_i are sectorial dummies and ε is the orthogonal error term. We see the results in Table 4.4.

In the second half of the Table, we use our linear specification of the relationship (equations (35) and (36) of the previous discussion). Regression (3) of the Table uses as a proxy for the knowledge capital the overall cumulated

R&D stock, while we split this regressor in current R&D and past cumulated R&D in regression (4). All the variables have positive and strongly significant coefficients; the R^2 is quite high.

It is clear comparing their coefficients and from simple equality tests that current and past R&D have strongly different effects on the market value; therefore, the use of the overall cumulated R&D stock can be a wrong approximation. Moreover, we see that the effect of current R&D is stronger than that of cumulated past R&D at this level of aggregation.

The traditional specification pioneered by Griliches (1981) and then used in most of the following studies is estimated in the first half of Table 4.4 using the final form $\ln(Q) = \ln(\frac{V}{K}) = a_0 + a_1\gamma\frac{A}{K} + \eta$. The lower R^2 with respect to our linear specification is partially due to the effect of the dimension, which is removed in Griliches' version. However, in this case the estimation results improve much more when decomposing the R&D stock, suggesting that the restrictions induced by the approximation used to develop Griliches' relationship should be carefully considered.

In Table 4.5, we estimate completely separate regressions for each sector using our specification. We immediately see that the previous fixed effects model hardly explains the differences between sectors. Not only the constant terms, but also the other coefficients are strongly different among sectors, implying that the effects of our independent variables are quite heterogeneous. The relevant increase of the R^2 observed in many sectors confirms this fact. Moreover, the restriction that the coefficients are equal in all sectors is rejected when tested with equality tests.

We can find several reasons to explain this result.

First, we are using a linear approximation. If the returns from the accumulated assets are not linear in the quantity of the asset, sectors that, on average, are more knowledge intensive or invest more in conventional capital will show different coefficients from the others.

A second reason could be that sectors are extremely heterogeneous. In this case, the previous studies based on the aggregate observations would be just partially reliable.

Another result worthy of note regards the R&D coefficients. In most sectors we have a very small effect of current R&D, while the effect of past R&D is more relevant. Sometimes the current R&D coefficient is strongly negative, a result in contrast with the hypothesis that the firm is maximizing its profits: indeed, if the effect on the value of a firm is negative the optimal choice would be not to invest in research.

The most relevant exceptions to this behaviour are the pharmaceutical sector (ISIC 2423) and the highly technological ones (ISIC 3000-3300), where the technological cycle is faster than in the others. Strongly significant positive coefficients of current R&D show the high value of innovations in these sectors. The rate of depreciation is likely to be much larger than in the other sectors and this fact explains why the contribution of past R&D to the market value is usually near zero.[57]

[57] The large number of observations in the high tech sectors may be the reason why

We try to explain why current R&D has small or negative effect on the market value in many sectors with a model that examines the choices of the firm and its market value in presence of uncertainty about the results of R&D activity.

4.4 Firm value, knowledge capital and R&D in a dynamic model

As we saw in Section 4.2, almost all the works concerning the relationship between knowledge stock and firm value are based on the static definitional model proposed by Griliches (1981).

In this Section we try to analyse the relationship in a slightly different framework: a simple dynamic theoretical scheme based on the maximizing choices of a firm. It allows us to analyse the consequences of an R&D decision during the different stages of the process. The most important innovations of this framework with respect to the static model of Griliches (1981) are the time dimension and the presence of uncertainty in research activity; a consequence is that we must take into account not only the accumulated knowledge stock, but also the work-in-progress R&D effort in determining the value of the firm. We see that this fact can be particularly relevant to explain our previous empirical results if the market is risk averse and therefore penalizes the uncertainty of a R&D investment before its conclusion.

---

in the fixed effects model with common slopes the dominant behaviour is a significantly positive effect of current R&D.

Let us consider a profit maximizing firm producing one good; it sells its product in a perfectly competitive market, where the market price of the product is normalized to 1.

A continuously differentiable quasiconcave function describes production; we suppose it shows increasing returns to scale for small quantities of product and decreasing returns for large quantities:

$$Y_t = A_t F(K_t, L_t) \tag{37}$$

where A_t is the accumulated knowledge asset, K_t is the net physical capital and L_t is the labour, whose supply is infinitely elastic given the wage w_t.[58]

We can formally state our assumption on the returns to scale of the production function saying that $\exists \bar{Q} \in R_+$ satisfying the following property:

– $\forall K, L$ such that $F(K, L) = Q < \bar{Q} \Rightarrow F(kK, kL) < kQ \; \forall k \in (0, 1)$

– $\forall K, L$ such that $F(K, L) = Q > \bar{Q} \Rightarrow F(kK, kL) < kQ \; \forall k \in (1, +\infty)$

Capital is accumulated through investment, without any adjustment costs, but with a constant depreciation rate δ:

$$K_t = \delta K_{t-1} + i_t. \tag{38}$$

Therefore, physical investment has an immediate effect on the capital stock. We use this timing to simplify the analysis of the capital accumulation,

[58] This hypothesis about the production function is coherent with the perfect competition assumption; moreover, it allows us to be sure that the solution of the maximization problem implies a finite and strictly positive output at firm level.

which is not our main aim in this work.

The firm can invest in R&D to increase its knowledge assets. The formalization of the R&D process is the focal point of our analysis. We assume that in each period the firm receives opportunities of investment in R&D, which could yield an increase of the knowledge asset in the following period. Given a research expense of $R\&D_t$ at time t, at the beginning of period $t+1$ (before taking decisions regarding capital investments, R&D, etc.) the firm observes the results of the investment. With probability π, it is successful and yields an increase of the knowledge asset of $\Delta A_{t+1} = \gamma R\&D_t^{\rho}$ with $0 < \rho < 1$. With probability $1 - \pi$, it fails and the knowledge asset remains the same value as in period t.

Therefore, we consider two elements in the creation of knowledge asset that make it different from the physical asset accumulation. The first one is uncertainty: there is a probability that the investment will be unfruitful. The second is the presence of diminishing returns to R&D.

The last constraint states the equality between sources and uses of funds:

$$Y_t = w_t L_t + i_t + d_t + R\&D_t. \tag{39}$$

The returns from sales in each period t are employed to pay wages (w_t) and dividends (d_t)and to finance investments in physical capital (i_t) and R&D ($R\&D_t$).

The value of the firm on the financial market, under the hypotheses that investors are risk neutral and that managers maximize the expected value of profits, is given by the following Bellman equation:

$$V_t = \max_{c_t} \left[d_t + \beta E_t \left(V_{t+1} \right) \right] \tag{40}$$

where $c_t = \{d_t, L_t, i_t, R\&D_t\}$ is the vector with the choice variables of the firm and β is the discount factor.

Proposition 9 *Maximization of the value function (40) subject to the production function (37), the capital accumulation equation (38), the resources constraint (39) and the accumulation process of* A_t *implies the following optimality conditions, which determine the optimal* K_t*,* L_t *and* $R\&D_t$*:*

$$\begin{cases} A_t F_K \left(K_t, L_t \right) = 1 - \beta\delta \\ A_t F_L \left(K_t, L_t \right) = w_t \\ R\&D_t = \left[\gamma\rho \sum\limits_{j=1}^{\infty} \beta^j \sum\limits_{i=1}^{j} \binom{j-1}{i-1} \pi^i \left(1 - \pi \right)^{j-i} F\left(K_{t+j}^{*,i}, L_{t+j}^{*,i} \right) \right]^{\frac{1}{1-\rho}} \end{cases} \tag{41}$$

where $K_{t+j}^{*,i}$ *and* $L_{t+j}^{*,i}$ *are the optimal values of* K *and* L *in period* $t + j$ *in case the R&D activity has been successful* i *times between* t *and* $t + j$*; these values are implicitly determined by the first order conditions of the following periods.*

Proof. From the first order conditions of the intertemporal profit maximization problem.[59] ■

[59] The second order conditions are satisfied at the critical points given the hypothesis of

If we assume w_t to be constant (a reasonable hypothesis in a short-run analysis) and complementarity between K and L, we see the following dynamics. The optimal quantities of physical capital and labour used in the process are constant for a given A_t. If the R&D activity is successful, A_t increases and therefore the optimal amounts of used capital and labour increase too. $R\&D_t$ is an increasing function of the optimal values of K and L in case of success and of the parameters β, π, γ and ρ. This means that the chosen level of R&D is larger when the firm gives more value to the future, when there is a larger probability of successful research and when the loss of efficiency due to the diminishing returns to R&D is smaller. Moreover, because the optimal values of K and L increase when research is successful, the R&D level increases after each success.[60]

Let us now analyse the dynamics of the dividend and of the value function following the investment in R&D. Once again, wages are supposed to be constant.

To be able to observe the effects of the investment in each period, we consider, in this first step of the analysis, a firm that did not make R&D investments in the past. In period t, the firm can exploit an unexpected research opportunity, whose results will be observed in the following period

quasiconcavity of the production function.

[60] Larger firms have more incentives to invest in R&D, because the output taking advantage of a successful research is larger.

$t+1$. In the periods up to $t-1$ the dividend of the firm will be:

$$d_1 = A_1 F\left(K_1, L_1\right) - wL_1 - \left(1-\delta\right) K_1 \tag{42}$$

where K_1 and L_1 are the optimal values of K and L given A_1 and the other parameters.

The value of the firm up to $t-1$ will be

$$V_1 = d_1 + \beta d_1 + \beta^2 d_1 + \ldots = \frac{d_1}{1-\beta}.$$

We can interpret the value V_1 as the static value of the firm at time $t-1$, which is the value produced by the already accumulated physical and knowledge assets.

At time t there is a research opportunity where $R\&D > 0$ is optimal in our framework. Therefore, the firm decides to invest a positive quantity of R&D. The dividend in this period is

$$d_2 = d_1 - R\&D.$$

In the case in which R&D is unfruitful, the firm comes back to the previous dividend d_1 in all the periods after t. Hence, the value is again V_1 from period $t+1$.

If the research investment is successful, the dividend in period $t+1$ is

$$d_3 = (A_1 + \gamma R\&D^{\rho})\, F(K_2, L_2) - wL_2 - K_2 + \delta K_1$$

where K_2 and L_2 are the new optimal values of K and L given the increased value of A. The value of the dividend in $t+2$ and in the following periods is

$$d_4 = (A_1 + \gamma R\&D^{\rho})\, F(K_2, L_2) - wL_2 - (1-\delta)\, K_2.$$

In the case of success, the value of the firm is from period $t+2$

$$V_4 = d_4 + \beta d_4 + \beta^2 d_4 + \ldots = \frac{d_4}{1-\beta}$$

while in period $t+1$ it is

$$\begin{aligned} V_3 &= d_3 + \beta d_4 + \beta^2 d_4 + \ldots \\ &= d_4 - \delta (K_2 - K_1) + \beta d_4 + \beta^2 d_4 + \ldots \\ &= V_4 - \delta (K_2 - K_1). \end{aligned}$$

In period t, because of the uncertainty of the R&D investment, the value function is

$$
\begin{aligned}
V_2 &= d_2 + \beta\left[\pi V_3 + (1-\pi) V_1\right] \qquad (43) \\
&= (1-\beta\pi) V_1 + \beta\pi V_3 - R\&D \\
&= V_1 + \beta\pi (V_3 - V_1) - R\&D.
\end{aligned}
$$

The value of the firm in this period is equal to the value due to the already accumulated physical and knowledge assets (which is V_1) plus the expected discounted profits from the investment, net of the R&D expenditures.

Proposition 10 *Given the dynamics of the optimal production implied by the first order conditions previously examined, if the research investment decision is optimal, then* $V_3 > V_2 \geq V_1$.

Moreover, if we assume $F_{KL}(K, L) > 0 \ \forall K, L$, *that is an increase in the use of one production factor increases the marginal productivity of the other factor (complementarity), then* $V_4 > V_3$.

Proof. In each stage the optimal choices of the previous stages are still available under the constraints of the problem, but they are not chosen. Therefore, the new strategies yield a weakly higher value function.

The strong inequality $V_3 > V_2$ derives from the fact that V_2 is a weighted average of V_3 and V_1, net of the amount of the R&D expenditures.

Moreover, the hypothesis $F_{KL} > 0$ and the first order conditions (41) imply

$$\begin{aligned}(A_t + \gamma R\&D_t^{\rho}) F_K(K_{t+1}, L_{t+1}) &= A_t F_K(K_t, L_t) \\ \Rightarrow F_K(K_{t+1}, L_{t+1}) &< F_K(K_t, L_t) \\ \Rightarrow K_{t+1} &> K_t.\end{aligned}$$

This fact and $V_3 = V_4 - \delta(K_2 - K_1)$ imply $V_4 > V_3$. ■

We see the resulting dynamics of the firm value in both cases of success and failure in Figure 4.1.

The value of the firm weakly increases at time t after the investment in R&D, even if there has not been any growth in knowledge capital and the future increase is not sure, because of the uncertainty of the investment. This is because the value function incorporates the expected profit value of the investment and therefore, in our simplified case, it is an average of the profits on the success and failure paths, weighted according to the probability of the two events, net of the research costs.

In period $t+1$, the firm observes the results of the investment. In the case of success, the market value increases again to include all the profits due to the investment in R&D (after one period of physical capital adjustment). In the case of failure, it decreases to the value before the R&D investment.

Proposition 11 *Let us consider the full dynamics assuming that there are R&D opportunities in each period and the firm optimizes its R&D invest-*

ments. If we assume that R&D has been successful in the last period, we can write the firm value after s successes in R&D activity:

$$V(s) = \sum_{r=0}^{\infty} \frac{(\beta\pi)^r \left\{d_{[s+r]} - [1-\beta(1-\pi)]\,\delta\left(K_{[s+r]} - K_{[s+r-1]}\right)\right\}}{[1-\beta(1-\pi)]^{r+1}} \tag{44}$$

where $d_{[s+r]}$ and $K_{[r+s]}$ are the optimal dividend and capital after $r+s$ successes in research activity.[61,62]

Proof. Let us consider the definition of the value function at time t after s successes:

$$\begin{aligned} V_t(s) &= \sum_{r=0}^{\infty} \beta^r E_t\left(d_{t+r} \mid s\right) \\ &= \sum_{r=0}^{\infty} \beta^r \sum_{g=0}^{r} \{P\left[d_{[s+g]}^{t+r}\right] d_{[s+g]} + \\ &\quad -P\left[d_{[s+g]}^{t+r}, d_{[s+g-1]}^{t+r-1}\right] \delta\left(K_{[s+g]} - K_{[s+g-1]}\right)\} \end{aligned}$$

where $P\left[d_{[s+g]}^{t+r}\right]$ is the probability of receiving the dividend yielded after $s+g$ successes at time $t+r$ and $P\left[d_{[s+g]}^{t+r}, d_{[s+g-1]}^{t+r-1}\right]$ is the probability of achieving the $(s+g)$-th success at time $t+r$. We substitute the value of the probabilities and simplify the power series. ■

Each time R&D is successful, the value function in the following periods

[61] The formula can be trivially adapted to the case of failure in R&D activity in the last period.

[62] Under the hypothesis of free exit from the market, the market value must be positive. In our analysis we assume $V(s) < +\infty$. We can rule out the case $V(s) = +\infty$ in a more general analysis, where we consider the full behaviour of the market; we do not extend the analysis in this direction because it is beyond the scope of this chapter.

is shifted upwards (Figure 4.2). The firm will be evaluated in each period at a weakly higher value than that one due to its current assets (this is implied by the inequality $V_2 \geq V_1$ of Proposition 10).

Let us consider now the consequences of weakening the hypothesis that investors are risk neutral. If they are risk averse, their valuation equation could be now, for example:

$$\begin{aligned} V_t &= d_t + \beta E_t\left(V_{t+1}\right) - \nu Var_t\left(V_{t+1}\right) \\ &= \sum_i \beta^i E_t\left(d_{t+i}\right) - \nu Var_t\left(\Sigma_i \beta^i d_{t+i}\right). \end{aligned} \tag{45}$$

We added to the risk neutral valuation a negative correction proportional to the variance of dividends in the following periods. Because in our model the only stochastic element is the outcome of R&D activity, the main practical consequence of introducing risk aversion is a penalization of the market valuation of R&D investments when the research outcome is still uncertain.

There are two possible behaviours of managers, depending on their objectives.

On one hand, if managers maximize the short-term valuation of the firm on the market (which is V_t, calculated by equation (45)), they aim to maximize the same objective equation of investors, that is, they will include the risk correction into equation (40). The dynamics of the model are almost the same as before; the only difference is that the optimal quantity of R&D invested in each period is reduced and therefore the productivity of the firm and the value of the firm grow less in the long run.

On the other hand, if managers maximize the long-run valuation of the firm the conclusions change.

Proposition 12 *If managers maximize the long-run market valuation of the firm (or equivalently the expected value of profits), they will exactly follow the same behaviour analysed in the previous pages. However, the market now evaluates the firm according to equation (45) and the short-run market valuation no longer reflects the valuation of the managers. The chosen R&D level is higher than the level maximizing the valuation of investors and the market valuation is lower than the expected value of the profits because there is uncertainty about the investment results. If the market is risk averse enough, in the short run investors can evaluate the firm less than its static value without considering the work-in-progress investments in R&D. Anyway, in the long run the firm grows more than in the previous case and after a certain number of periods the market valuation is larger.*

The considerations we made in this Section can be extended and deepened in several directions. A first possibility is the use of other models of the mechanism of knowledge stock creation. A first partial exploration seems to show that the basic dynamic behaviour of the model presented here is robust to many specification changes. It is qualitatively similar to that one we would have if R&D had a probability of success for more than one period or if we were using many degrees of effectiveness (not just success or failure) in the R&D process. We have the same results if we do not consider research to be a one-shot process, but we condition the quantity of created knowledge capital in a case of success to a function of all past R&D expenditures.

Other interesting extensions go beyond the aims of this work, although they are worthy of note as hints for further explorations, since they can shed light on the long-run behaviour of firms.

We considered the behaviour of a single firm in a perfectly competitive market under the hypothesis of constant wages. There may be interesting results if we consider the market as a whole. Under the hypothesis of perfect competition and no spillovers between firms, there is heterogeneity in the firm structure, because of the different outcomes of investments. The long-run market equilibrium depends on the conditions of the entrant firms. If they enter the market at the initial level of efficiency, after a while the technological growth will create entry barriers because the new firms will not be able to achieve positive profits. In the long run, firms already in the market that have grown too little will leave the market and therefore there will be concentration. A competitive market outcome could be re-established if we allow spillovers or if the new firms enter the market at an increasing efficiency level.

4.5 Conclusions

In this chapter, we examined some econometric results about the relationship between market value and assets of a firm; we tried to explain some puzzling evidence using an innovative simple model where we introduce uncertainty about the R&D process and a risk-averse stock market.

We see in the empirical analysis that the usual positive correlation between the firm value and the physical and knowledge assets is confirmed at an aggregate level in a panel model with fixed effects. We notice a high level of heterogeneity and try to explain part of it through the differences between sectors. Equality tests and separate regressions in each industry seem to confirm that this source of heterogeneity is important because the coefficients of the regressors are significantly different and the R^2 of the separated OLS regressions in most industries are consistently higher than the R^2 of the single regression with common slopes and fixed effects. In the separate regressions, we split the contributions of current and past R&D and report several very small or even negative coefficients for current R&D.

We try to explain this last fact in a theoretical framework where we modify the usual basic static model including dynamic elements and where R&D is an activity with uncertain results. This alternative approach to the problem allows us to examine the consequences of R&D investments in their various stages, highlighting the fact that research decisions have effects on the market value of the firm even before the conclusion of the R&D program.

Therefore, the market valuation of the firm not only depends on the already accumulated assets, but also on the expected return of the work-in-progress research.

Moreover, the uncertain nature of R&D investments can have other consequences. In the presence of risk aversion, the market penalizes the expected value of an investment before knowing if it has been successful. If managers aim to maximize the long-run value of the firm, this fact induces a difference between the valuation of the management and that of the market: the latter can negatively perceive long-run positive investments because of the associated risk.

4.6 Tables and Figures

Table 4.1: Overall descriptive statistics

	Mean	St. Dev.	Min	0.05	0.25	Median	0.75	0.95	Max	# obs
Total Value of Firm	527.86	1514.21	1.97	7.78	29.12	93.21	370.45	2388.81	25710	19335
Conventional Assets	275.91	1001.35	0.04	2.48	12.88	41.72	175.92	1184.79	26861	20359
Overall Cumulated R&D	69.12	295.26	0.00	0.38	2.30	8.85	34.72	282.56	9533	14854
Current R&D	16.27	69.94	0.00	0.11	0.62	2.08	7.89	66.24	2209	14854
Past Cumulated R&D	52.85	230.40	0.00	0.00	1.28	6.12	25.69	215.40	7324	14854

All the data are expressed in millions of dollars.

Table 4.2: ISIC sectors

ISIC Code	Sector
1500	Food
1700	Textile
2100	Paper
2200	Printing/Publishing
2300	Petroleum
2400	Chemicals
2423	Pharmaceuticals
2500	Rubber/Plastic
2600	Mineral Products
2700	Metal Works
2800	Metal Products
2900	Machinery
2910	General Purpose Machinery
3000	Computers
3100	Electrical Equipment
3200	Electronic Components
3300	Precision Instruments
3400	Car/Truck
3500	Aerospace
3600	Other Manufacturing

Table 4.3a: Descriptive statistics by sector

	ISIC	Mean	St. Dev.	Min	0.05	0.25	Median	0.75	0.95	Max	# obs
Total Value of Firm	1500	1267.1	2077.3	6.5	23.3	92.4	391.3	1511.1	5554.3	17736.1	910
	1700	185.2	253.0	2.8	7.5	29.8	83.1	226.1	759.8	1615.2	1016
	2100	897.5	1184.0	7.9	11.8	110.3	387.9	1204.2	3480.1	6093.7	742
	2200	551.6	860.4	8.0	13.2	50.8	130.2	517.9	2492.3	4090.7	473
	2300	3627.1	6068.1	23.8	61.6	240.7	603.7	4132.3	18524.3	23550.9	297
	2400	579.4	870.4	3.1	8.9	57.5	221.0	754.8	2257.1	7150.1	1372
	2423	1645.4	3134.2	3.2	10.6	43.3	221.8	1711.1	7777.9	19185.5	838
	2500	345.5	631.8	4.8	9.3	18.8	49.5	349.7	1532.2	4700.2	731
	2600	413.7	503.1	8.0	14.8	65.7	252.4	522.8	1572.1	2591.6	396
	2700	456.0	702.2	10.8	18.9	56.0	148.4	411.0	2103.0	4307.5	779
	2800	173.2	259.4	4.3	8.6	25.5	74.2	181.0	693.1	2045.2	1170
	2900	302.1	593.4	4.0	6.9	26.9	71.1	179.9	2017.9	6133.9	783
	2910	268.1	543.1	2.6	8.0	27.0	70.4	179.7	927.5	7649.0	1494
	3000	360.8	823.4	2.0	5.4	22.9	76.6	286.4	1820.4	6400.6	1089
	3100	350.6	805.3	2.5	7.6	24.1	74.0	370.2	1306.1	12625.2	1246
	3200	205.8	536.5	2.2	4.9	15.6	44.6	133.3	972.4	5133.5	1672
	3300	186.7	435.6	2.0	5.5	18.0	46.4	121.1	992.5	3568.5	2109
	3400	1004.0	1917.0	3.0	12.5	45.4	150.6	587.2	3228.5	25709.6	853
	3500	871.8	1414.7	2.0	4.8	35.2	235.2	1072.4	4193.8	7989.6	395
	3600	316.9	677.9	4.6	8.4	26.0	72.5	265.7	1646.0	6276.7	970
Conventional Assets	1500	517.6	732.3	4.5	12.0	54.7	204.0	724.7	1815.2	5803.8	964
	1700	102.6	140.0	1.4	4.5	19.8	45.7	136.3	364.2	935.8	1049
	2100	624.9	847.5	5.1	10.3	63.7	243.8	941.8	2667.4	4453.9	758
	2200	209.2	331.1	1.4	6.5	22.1	54.0	180.1	987.1	1386.8	483
	2300	2716.6	4520.7	12.4	24.0	203.2	454.0	2919.5	13772.9	17472.8	313
	2400	287.5	443.6	0.7	3.6	24.4	83.2	342.5	1254.1	4755.6	1442
	2423	350.7	574.2	0.1	0.8	4.3	49.5	479.2	1650.0	3259.8	871
	2500	188.6	409.1	1.6	4.3	10.7	23.0	169.8	967.7	3013.6	774
	2600	259.3	300.2	2.1	8.2	39.7	167.4	367.7	760.3	1621.2	416
	2700	325.0	501.8	4.0	12.2	36.2	103.5	282.3	1651.6	2200.1	828
	2800	88.8	127.7	1.4	3.6	15.7	41.4	96.3	404.4	804.7	1234
	2900	147.9	304.0	1.2	2.7	12.0	37.4	88.7	718.6	4331.6	845
	2910	142.8	335.0	1.1	3.2	13.4	32.7	84.3	586.6	5770.2	1590
	3000	115.2	308.4	0.0	1.0	5.7	18.8	70.5	521.3	2402.4	1151
	3100	263.6	1050.0	0.9	2.9	11.5	37.6	197.0	749.4	13708.8	1369
	3200	67.8	179.2	0.5	1.8	6.2	15.4	43.5	311.8	1539.5	1737
	3300	72.2	188.1	0.2	1.3	5.1	16.0	44.6	418.9	1779.8	2194
	3400	793.3	1701.1	0.9	5.7	20.3	64.2	310.1	3249.5	26861.2	910
	3500	525.0	876.1	0.7	1.7	18.2	140.7	618.7	2448.0	4251.4	414
	3600	169.3	317.6	1.6	4.6	15.1	36.1	123.2	756.4	3162.4	1017

Table 4.3b: Descriptive statistics by sector

	ISIC	Mean	St. Dev.	Min	0.05	0.25	Median	0.75	0.95	Max	# obs
Overall Cumulated R&D	1500	46.64	70.39	0.00	0.41	2.81	15.37	63.20	172.33	593.00	476
	1700	7.28	13.13	0.01	0.27	1.30	3.02	7.22	25.99	90.61	357
	2100	32.61	51.00	0.06	0.48	2.11	11.73	35.98	126.96	318.77	330
	2200	4.80	6.73	0.15	0.28	0.77	1.39	5.23	21.63	21.91	86
	2300	189.55	219.29	0.00	0.42	13.43	112.28	320.31	628.77	904.62	122
	2400	61.37	106.68	0.00	0.71	4.25	16.60	67.19	303.27	621.07	1223
	2423	219.08	403.25	0.07	0.73	6.14	33.48	219.99	1118.92	2464.66	828
	2500	42.17	107.81	0.00	0.22	1.43	5.09	29.34	181.83	909.25	472
	2600	29.95	41.84	0.00	0.15	1.21	8.04	45.71	140.90	160.06	201
	2700	33.68	54.72	0.04	0.28	2.11	12.35	36.55	145.91	313.77	446
	2800	12.50	23.21	0.00	0.23	1.27	5.12	12.69	51.79	191.38	794
	2900	36.22	107.02	0.01	0.42	2.07	6.95	23.62	136.19	767.46	705
	2910	29.41	124.93	0.00	0.28	1.67	6.20	17.96	81.99	1740.78	1275
	3000	66.72	154.48	0.00	0.50	3.90	14.06	54.37	316.35	1620.07	1138
	3100	77.68	303.53	0.04	0.45	2.50	8.56	44.31	260.08	3727.66	1079
	3200	34.29	104.61	0.00	0.38	1.92	7.68	27.14	115.96	1842.13	1581
	3300	44.52	127.41	0.00	0.33	2.39	7.80	26.97	208.78	1814.85	2103
	3400	295.03	1047.89	0.03	0.50	2.98	14.13	57.85	1209.66	9533.08	696
	3500	243.79	520.94	0.11	0.97	5.86	39.59	189.45	1323.71	3027.93	300
	3600	17.36	32.00	0.00	0.32	1.65	4.62	15.56	91.06	185.57	642
Current R&D	1500	11.95	19.67	0.00	0.15	0.77	5.06	16.57	43.57	217.92	476
	1700	1.79	2.90	0.00	0.10	0.42	0.86	1.65	6.00	24.28	357
	2100	7.96	11.41	0.06	0.08	0.77	2.90	8.70	28.34	65.48	330
	2200	1.41	2.51	0.01	0.12	0.20	0.60	2.38	4.26	21.42	86
	2300	46.23	63.26	0.00	0.13	3.73	26.07	81.27	125.59	360.85	122
	2400	13.97	22.16	0.00	0.17	1.27	4.38	17.73	67.37	157.29	1223
	2423	55.34	98.46	0.05	0.20	1.37	7.20	65.51	262.40	771.96	828
	2500	8.68	16.67	0.00	0.93	0.41	1.40	5.68	43.24	157.26	472
	2600	6.55	8.36	0.00	0.05	0.32	2.68	9.15	26.10	33.75	201
	2700	7.02	11.04	0.00	0.07	0.59	2.33	7.74	28.71	58.15	446
	2800	2.83	5.38	0.00	0.07	0.35	1.02	2.65	12.48	35.24	794
	2900	8.16	17.27	0.00	0.11	0.52	1.71	5.14	39.50	159.63	705
	2910	6.12	15.30	0.00	0.08	0.49	1.54	4.30	17.82	292.79	1275
	3000	17.71	42.64	0.00	0.14	1.03	3.49	12.28	81.67	437.76	1138
	3100	18.39	68.24	0.00	0.11	0.67	2.42	10.12	53.92	690.64	1079
	3200	8.24	24.93	0.00	0.10	0.49	1.51	5.66	32.71	447.08	1581
	3300	8.09	20.78	0.00	0.11	0.63	1.66	4.81	49.35	200.98	2103
	3400	72.02	153.31	0.00	0.15	0.74	3.29	15.21	388.88	2208.82	696
	3500	55.81	109.52	0.00	0.20	1.21	8.71	39.73	325.42	569.81	300
	3600	4.23	7.60	0.00	0.08	0.41	1.17	3.92	21.11	50.57	642

Table 4.3c: Descriptive statistics by sector

	ISIC	Mean	St. Dev.	Min	0.05	0.25	Median	0.75	0.95	Max	# obs
Cumulated Past R&D	1500	34.69	53.50	0.00	0.00	1.30	9.74	47.68	139.43	375.07	476
	1700	5.49	10.87	0.00	0.00	0.72	2.01	5.13	20.36	77.02	357
	2100	24.65	40.58	0.00	0.00	1.11	7.66	26.80	103.53	253.29	330
	2200	3.40	5.38	0.00	0.00	0.47	0.87	2.70	18.38	18.62	86
	2300	143.32	175.67	0.00	0.00	7.04	58.62	230.82	506.96	624.67	122
	2400	47.40	86.96	0.00	0.00	2.23	11.21	49.40	236.38	551.98	1223
	2423	163.74	312.73	0.00	0.00	3.86	24.66	154.32	875.95	1868.37	828
	2500	33.50	87.97	0.00	0.00	0.83	3.33	23.00	154.56	770.06	472
	2600	23.41	34.87	0.00	0.00	0.60	5.79	31.71	116.09	128.65	201
	2700	26.66	45.15	0.00	0.00	0.99	9.36	28.01	122.97	261.56	446
	2800	9.67	18.50	0.00	0.00	0.63	3.34	10.31	38.09	156.31	794
	2900	28.05	86.79	0.00	0.00	1.27	5.16	17.89	98.52	643.76	705
	2910	23.29	105.12	0.00	0.00	1.03	4.43	13.60	63.68	1470.51	1275
	3000	49.01	115.67	0.00	0.00	2.04	9.36	40.55	245.05	1182.31	1138
	3100	59.29	240.07	0.00	0.00	1.27	5.76	32.68	213.42	3154.10	1079
	3200	26.05	81.40	0.00	0.00	1.15	5.23	20.40	91.29	1395.05	1581
	3300	36.43	113.43	0.00	0.00	1.31	5.79	19.61	165.00	1813.29	2103
	3400	223.01	809.39	0.00	0.00	1.91	9.45	40.93	946.82	7324.26	696
	3500	187.98	420.61	0.00	0.00	3.59	26.94	144.07	1084.77	2591.69	300
	3600	13.13	25.14	0.00	0.00	1.09	3.19	11.31	65.15	142.55	642

Table 4.4: Fixed effects regressions of firm value on conventional and knowledge capital

$\log(V/K) = a_0 + a_1\gamma A/K + \eta$

	(1)		(2)	
	Coefficients	St. Error	Coefficients	St. Error
Overall Cumulated R&D/Conventional Assets	0.012	0.003		
Current R&D/Conventional Assets			0.747	0.077
Cumulated Past R&D/Conventional Assets			0.007	0.001
R^2	0.039		0.252	

Equality tests:		p-value
Current R&D vs Cumulated Past R&D	F(1,14079)=93.26	0.00

$V = a_0 + a_1{*}K + a_2{*}A + \eta$

	(3)		(4)	
	Coefficients	St. Error	Coefficients	St. Error
Conventional Assets	1.300	0.039	1.238	0.041
Overall Cumulated R&D	1.769	0.259		
Current R&D			6.713	2.001
Cumulated Past R&D			0.618	0.473
R^2	0.814		0.820	

Fixed effects by sector are included.
The sample includes 14101 observations.

Equality tests:		p-value
Current R&D vs Cumulated Past R&D	F(1,14079)=6.50	0.01

Table 4.5a: OLS regressions of firm value on conventional and knowledge capital by sector

ISIC	Regressors	Coefficients	St. Error	R^2	# obs
1500	Conventional Assets	2.535	0.204	0.819	438
	Current R&D	-25.674	11.043		
	Cumulated Past R&D	7.936	3.669		
	Constant	-150.614	82.780		
1700	Conventional Assets	1.144	0.056	0.843	346
	Current R&D	0.229	3.974		
	Cumulated Past R&D	2.749	0.931		
	Constant	20.529	3.077		
2100	Conventional Assets	1.176	0.054	0.930	329
	Current R&D	1.175	5.060		
	Cumulated Past R&D	5.667	1.622		
	Constant	59.642	16.459		
2200	Conventional Assets	3.973	0.377	0.904	86
	Current R&D	-79.021	15.073		
	Cumulated Past R&D	8.223	2.248		
	Constant	6.693	9.094		
2300	Conventional Assets	1.139	0.055	0.955	108
	Current R&D	1.988	4.869		
	Cumulated Past R&D	7.463	1.441		
	Constant	482.847	150.836		
2400	Conventional Assets	1.624	0.098	0.869	1163
	Current R&D	-2.733	2.920		
	Cumulated Past R&D	2.694	0.518		
	Constant	56.306	8.850		
2423	Conventional Assets	0.464	0.255	0.915	801
	Current R&D	30.025	2.360		
	Cumulated Past R&D	-0.341	0.319		
	Constant	-86.776	27.624		

Table 4.5b: OLS regressions of firm value on conventional and knowledge capital by sector

ISIC	Regressors	Coefficients	St. Error	R^2	# obs
2500	Conventional Assets	1.451	0.119	0.789	441
	Current R&D	-11.931	3.555		
	Cumulated Past R&D	2.472	0.471		
	Constant	107.149	14.645		
2600	Conventional Assets	1.154	0.108	0.880	196
	Current R&D	-6.063	5.441		
	Cumulated Past R&D	8.294	1.145		
	Constant	31.598	15.731		
2700	Conventional Assets	1.176	0.451	0.909	415
	Current R&D	-4.251	3.556		
	Cumulated Past R&D	4.010	0.815		
	Constant	21.139	7.605		
2800	Conventional Assets	1.363	0.077	0.890	758
	Current R&D	7.992	3.523		
	Cumulated Past R&D	2.799	0.884		
	Constant	7.886	3.210		
2900	Conventional Assets	2.840	0.258	0.896	679
	Current R&D	-6.168	4.376		
	Cumulated Past R&D	1.378	0.811		
	Constant	-22.596	7.808		
2910	Conventional Assets	1.425	0.202	0.917	1206
	Current R&D	4.225	6.724		
	Cumulated Past R&D	-0.077	1.180		
	Constant	39.114	10.571		
3000	Conventional Assets	1.525	0.175	0.808	1070
	Current R&D	10.276	2.007		
	Cumulated Past R&D	-1.382	0.584		
	Constant	54.350	11.941		

Table 4.5c: OLS regressions of firm value on conventional and knowledge capital by sector

ISIC	Regressors	Coefficients	St. Error	R^2	# obs
3100	Conventional Assets	1.918	0.099	0.934	995
	Current R&D	2.955	1.421		
	Cumulated Past R&D	0.600	0.361		
	Constant	-21.551	8.480		
3200	Conventional Assets	2.529	0.312	0.784	1507
	Current R&D	2.670	2.354		
	Cumulated Past R&D	-0.759	0.452		
	Constant	26.090	6.189		
3300	Conventional Assets	1.683	0.117	0.869	2027
	Current R&D	3.870	1.079		
	Cumulated Past R&D	0.159	0.092		
	Constant	24.997	3.609		
3400	Conventional Assets	2.512	0.249	0.931	645
	Current R&D	-19.406	3.186		
	Cumulated Past R&D	2.344	0.345		
	Constant	-7.430	40.101		
3500	Conventional Assets	1.112	0.071	0.940	284
	Current R&D	-0.945	1.168		
	Cumulated Past R&D	1.310	0.256		
	Constant	149.878	21.193		
3600	Conventional Assets	1.297	0.088	0.935	607
	Current R&D	22.863	5.819		
	Cumulated Past R&D	-0.209	2.172		
	Constant	14.077	7.443		

Equality tests among sectors:		p-value
Conventional Assets	F(19,14021)=14.64	0.00
Current R&D	F(19,14021)=12.79	0.00
Cumulated Past R&D	F(19,14021)=15.29	0.00

Fig. 4.1: Effects of one R&D attempt on firm value

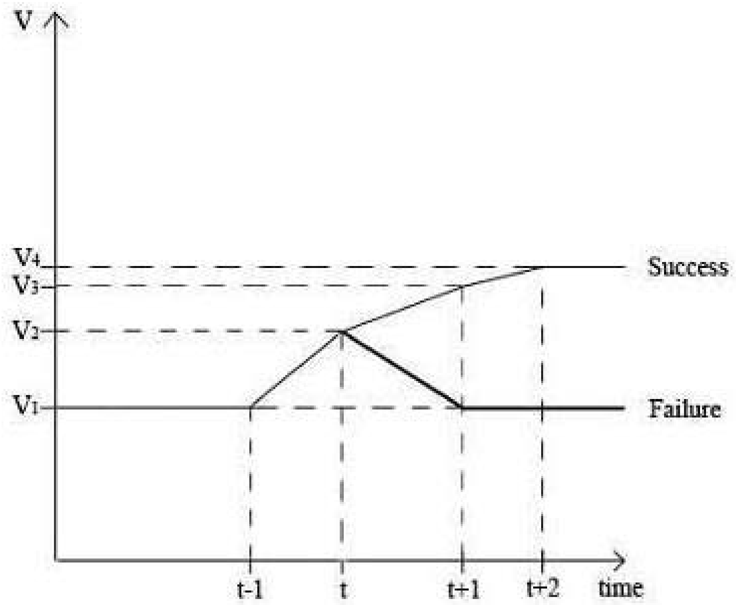
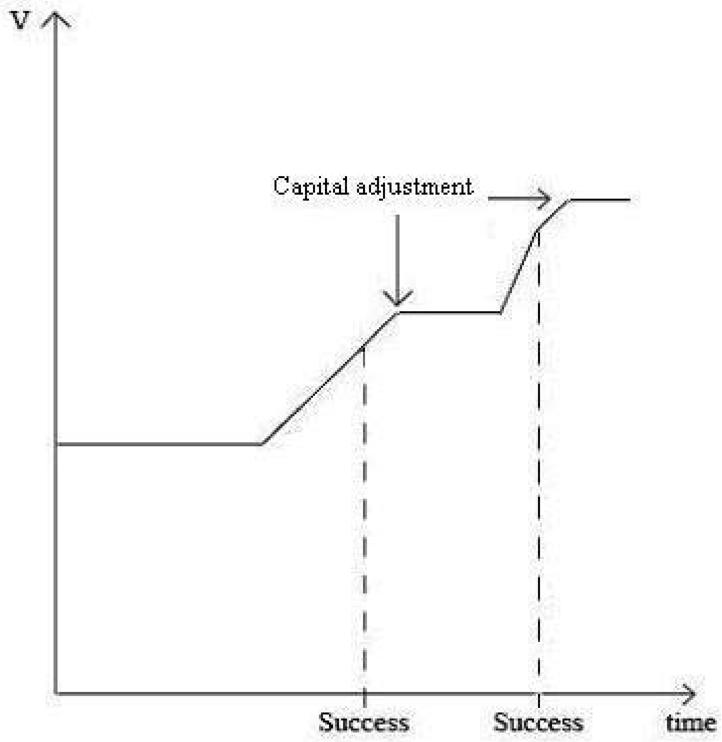


Fig. 4.2: Effects of continuous R&D attempts on firm value



5 Conclusions

This thesis examined some characteristics of the interaction between innovation activity of firms, in particular research and development investments, and economic system. We analysed some aspects of the relationships of R&D activity with market structure, public incentives to development and market value of firms.

In the chapter "R&D and market structure in a horizontal differentiation framework" we studied how firms can influence market competition introducing innovative characteristics in their products. We analysed how innovation and market structure endogenously interact over time. We considered a horizontally differentiated framework where firms can invest in R&D to increase differentiation among varieties of the same product. We thought of a product as an instrument allowing us to satisfy some needs. In a differentiated market, each variety had different effectiveness in satisfying each need. A consumer chose the bundle of varieties giving him the highest level of overall satisfaction. Firms were able to modify the characteristics of their variety through investments in R&D; they aimed towards a more specialized profile, increasing the level of horizontal differentiation and raising their market power. The movement of a variety towards areas of specialization not well fulfilled by other varieties raised the overall satisfaction of consumers. The inclusion of our mechanism in a dynamic framework allowed us to determine not only the path of production and R&D, but also the evolution of the market structure over time. Our most important results were that in this environment firms found incentives to invest in R&D to increase their specialization; the quantity of invested resources in research was declining over time, because the returns from further specialization decreased when the firm is more specialized, while prices, output and short-run profits of the firms increased. We compared the decentralized outcome and the socially optimal solution and we found a suboptimal investment in R&D, because the socially optimal production was larger than the decentralized one and more output taking advantage of research implied more incentives to invest in R&D; moreover, the firm did not internalize the benefits of reducing substitutability with the other varieties on the profits of the other producers. We examined the empirical evidence on the subject using a panel of sectorial data about European firms and the results of the empirical analysis were coherent with the model.

In the chapter "The evaluation of the incentives to firms for innovation: the case of the Fund for Technological Innovation in Italy" we empirically studied the effectiveness of the Fund for Technological Innovation, a policy instrument created by the Italian Ministry of Economic Development to stimulate private innovations by firms. This Fund focused on the development stage of R&D activity and was composed of two sections: a general purpose one, where applications from any field of activity and geographical area were accepted and evaluated one-by-one by merit following the chronological order of submission without a set deadline; a special purpose one, periodically issuing calls for applications in specific fields of activity or geographical areas with a set deadline, whose applications were ranked and whose subsidies were assigned to the best projects up to the amount of available resources. For both sections, the policy measures included a concessional loan and a non-refundable grant. The regular functioning of the general section of the Fund was unexpectedly interrupted after about five months due to shortage of funds; we used this exogenous shock to identify the effect of the policy: we compared the behaviour of the subsidized firms with that of the firms applying to the Fund after the shortage of funds, whose application had been neither assessed nor funded until five years later. The data from the Ministry about the Fund were merged with the 1999-2007 balance sheets of the firms filed at the Centrale dei Bilanci archives. We used two methodologies to evaluate the efficacy of this section of the Fund: a difference-in-differences approach, complemented by a matching procedure to increase the similarity between treated and controls, and a regression discontinuity design approach, using the submission date of the application as the forcing variable. In both cases, we were not able to detect signals of effectiveness of the policy on the investment behaviour of firms in the considered treatment period 2001-2007. The same was true also for sales, capital and employee figures, while there was a positive effect on assets; the additional liquidity from the subsidy seemed probably to have been used to finance the current expenditure of firms. Neither the profitability nor the financial structure of the firm seem had been clearly affected by the policy, apart from a reduction in the share of long-term debts over assets, when calculated net of the concessional loan from the subsidy, a result coherent with the hypothesis of lack of effectiveness of the policy. We also evaluated the efficacy of three calls for applications of the special section of the Fund; we merged the application data with the balance sheets from the Cerved archives for the years 2001-2007. We used the regression discontinuity design approach with a normalized ranking of the applying firms as the forcing variable; the results were very similar to those from the general section for the treatment period 2003-2007.

In the chapter "Dynamics of R&D investments and the value of the firm" we investigated the relationship between value of firms and their knowledge and conventional capital stocks. Analysing this relationship in a panel of observations from Compustat regarding the U.S. manufacturing sectors for the period 1975-1995, we saw that the theoretical assumptions of the relationship used by many previous studies may have effects in terms of efficiency of the estimates. We found a high level of heterogeneity in the coefficients of different sectors, which could undermine the results of the previous analyses usually postulating the same relationship in all sectors after controlling for fixed effects. When separating the current research effort from the past one, often the effect of current R&D was much weaker than that one of past R&D and sometimes was negative. To explain this fact, we developed a simple model where there was uncertainty about the results of R&D investments and we explicitly considered the time dimension. These two aspects were relevant because of the lag between a R&D investment and the achievement of its results on the knowledge asset. The value of a firm therefore took into account not only the current assets, but also the expected value of the potential ones. Moreover, since research is a risky activity, there could be a different valuation of the already concretized assets and of those still at a work-in-progress stage. Risk-averse investors could penalize the expected returns of the latter in the determination of the market value of the firm. Therefore, when investors are risk averse and managers maximize the long-run value of the firm, the risk associated with work-in-progress R&D could reduce the short-run firm value even if its expected value grows in the long run.

References

[1] Abadie, A., & Imbens, G. W. (2002), *Simple and Bias Corrected Matching Estimators for Average Treatment Effects*, N.B.E.R. Technical Working Paper N. 283

[2] Abadie, A., & Imbens, G. W. (2006), "Large Sample Properties of Matching Estimators for Average Treatment Effects", *Econometrica*, 74(1), pp. 235-267

[3] Abadie, A., & Imbens, G. W. (2008), "On the Failure of the Bootstrap for Matching Estimators", *Econometrica*, 76(6), pp. 1537-1557

[4] Abadie, A., & Imbens, G. W. (2010), "Bias Corrected Matching Estimators for Average Treatment Effects", *Journal of Business and Economic Statistics*, forthcoming

[5] Aghion, P., Bloom, N., Blundell, R., Griffith, R., & Howitt, P. (2005), "Competition and Innovation: an Inverted-U Relationship", *The Quarterly Journal of Economics*, 120(2), pp. 701-728

[6] Aghion, P., Dewatripont, M., & Rey, P. (1999), "Competition, Financial Discipline and Growth", *The Review of Economic Studies*, 66(4), pp. 825-852

[7] Aghion, P., & Griffith, R. (2005), *Competition and Growth: Reconciling Theory and Evidence (Zeuthen Lectures)*, The M.I.T. Press

[8] Aghion, P., Harris, C., Howitt, P., & Vickers, J. (2001), "Competition,

Imitation and Growth with Step-by-Step Innovation", *The Review of Economic Studies*, 68(3), pp. 467-492

[9] Aghion, P., & Howitt, P. (1992), "A Model of Growth through Creative Destruction", *Econometrica*, 60(2), pp. 323-351

[10] Aghion, P., & Howitt, P. (1996), "Research and Development in the Growth Process", *Journal of Economic Growth*, 1(1), pp. 49-73

[11] Aghion, P., & Howitt, P. (1998a), *Endogenous Growth Theory*, The M.I.T. Press

[12] Aghion, P., & Howitt, P. (1998b), "Market Structure and the Growth Process", *Review of Economic Dynamics*, 1(1), pp. 276-305

[13] Aghion, P., & Schankerman, M. (2004), "On the Welfare Effects and Political Economy of Competition-Enhancing Policies", *The Economic Journal*, 114(498), pp. 800-824

[14] Albareto, G., Bronzini, R., de Blasio, G., & Rassu, R. (2008), "Sussidi e Investimenti: un Test sui Vincoli Finanziari delle Imprese", *Politica Economica*, 24(1), pp. 107-134

[15] Arrow, K. J. (1962), "Economic Welfare and Allocation of Resources for Invention", in Nelson, R. (ed.), *The Rate and Direction of Inventive Activity: Economic and Social Factors*, Princeton University Press

[16] Atkinson, A. B., & Stiglitz, J. E. (1969), "A New View of Technological Change", *The Economic Journal*, 79(315), pp. 573-578

[17] Baker, J. B. (1997), "Product Differentiation through Space and Time: Some Antitrust Policy Issues", *Antitrust Bulletin*, 42(1), pp. 177-196

[18] Banerjee, A. V., & Duflo, E. (2009), "The Experimental Approach to Development Economics", *Annual Review of Economics*, 1, pp. 151-178

[19] Barrell, R., Mason, G., & O'Mahony, M. (2000), *Productivity, Innovation and Economic Performance*, Cambridge University Press

[20] Barro, R. J., & Sala-i-Martin, X. (2004), *Economic Growth*, The M.I.T. Press

[21] Bertrand, M., Duflo, E., Mullainathan, S. (2004), "How Much Should We Trust Difference-in-Differences Estimates?", *The Quarterly Journal of Economics*, 119(1), pp. 249-275

[22] Bils, M., & Klenow, P. J. (2001), "The Acceleration in Variety Growth", *The American Economic Review*, 91(2), pp. 274-280

[23] Bloom, N., Bond, S., & Van Reenen, J. (2001), *The Dynamics of Investment under Uncertainty*, I.F.S. Working Paper N. W01/05

[24] Bloom, N., Schankerman, M., & Van Reenen, J. (2007), *Identifying Technological Spillovers and Product Market Rivalry*, N.B.E.R. Working Paper N. W13060

[25] Bloom, N., & Van Reenen, J. (2002), "Patents, Real Options and Firm Performance", *The Economic Journal*, 112(478), pp. C97-C116

[26] Blundell, R., & Costa Dias, M. (2000), "Evaluation Methods for Non-Experimental Data", *Fiscal Studies*, 21(4), pp. 427-468

[27] Blundell, R., Griffith, R., & Van Reenen, J. (1995), "Dynamic Count Data Models of Technological Innovation", *The Economic Journal*, 105(429), pp. 333-344

[28] Blundell, R., Griffith, R., & Van Reenen, J. (1999), "Market Share, Market Value and Innovation in a Panel of British Manufacturing Firms", *The Review of Economic Studies*, 66(3), pp. 529-554

[29] Boone, J. (2000), "Competitive Pressure: the Effects on Investments in Product and Process Innovation", *The RAND Journal of Economics*, 31(3), pp. 549-569

[30] Boone, J. (2001), "Intensity of Competition and the Incentive to Innovate", *International Journal of Industrial Organization*, 19(5), pp. 705-726

[31] Booth, G. G., Junttila, J., Kallunki, J.-P., Rahiala, M., Sahlström, P. (2006), "How Does the Financial Environment Affect the Stock Market Valuation of R&D Spending?", *Journal of Financial Intermediation*, 15(2), pp. 197-214

[32] Boyce, W. E., & DiPrima, R. C. (1970), *Introduction to Differential Equations*, John Wiley & Sons Editors

[33] Bradburd, R. M., & Over, A. M. (1982), "Organizational Costs, 'Sticky' Equilibria and Critical Levels of Concentration", *The Review of*

Economics and Statistics, 64(1), pp. 50-58

[34] Branstetter, L. G., & Sakakibara, M. (2002), "When Do Research Consortia Work Well and Why? Evidence from Japanese Panel Data", *The American Economic Review*, 92(1), pp. 143-159

[35] Bresnahan, T. F., Stern, S., & Trajtenberg, M. (1997), "Market Segmentation and the Sources of Rents from Innovation: Personal Computers in the Late 1980s", *The RAND Journal of Economics*, 28(0), pp. S17-S44

[36] Bronzini, R., & de Blasio, G. (2006), "Evaluating the Impact of Investments Incentives: the Case of Italy's Law 488/1992", *Journal of Urban Economics*, 60(2), pp. 327-349

[37] Brown, M. A., Curlee, T. R., & Elliott, S. R. (1995), "Evaluating Technology Innovation Programs: the Use of Comparison Groups to Identify Impacts", *Research Policy*, 24(5), pp. 669-684

[38] Bucci, A. (2003), "Horizontal Innovation, Market Power and Growth", *International Economic Journal*, 17(1), pp. 57-82

[39] Budd, C., Harris, C., & Vickers, J. (1993), "A Model of the Evolution of Duopoly: Does the Asymmetry between Firms Tend to Increase or Decrease?", *The Review of Economic Studies*, 60(3), pp. 543-573

[40] Caballero, R. J., & Jaffe, A. B. (1993), "How High Are the Giants' Shoulders: an Empirical Assessment of Knowledge Spillovers and Creative Destruction in a Model of Economic Growth", in Blan-

chard, O. J., & Fischer, S. (eds.), *N.B.E.R. Macroeconomics Annual 1993 Vol. 8*, The M.I.T. Press

[41] Cannari, L., D'Aurizio, L., de Blasio, G. (2007), "The Effectiveness of Investment Subsidies: Evidence from Survey Data", *Rivista Italiana degli Economisti*, 12(3), pp. 3-19

[42] Carboni, O. A. (2008), *The Effect of R&D Subsidies on Private R&D: Evidence from Italian Manufacturing Data*, C.R.E.No.S. Working Paper N. 2008/15

[43] Chan, L. K. C., Lakonishock, J., & Sougiannis, T. (2001), "The Stock Market Valuation of Research and Development Expenditures", *The Journal of Finance*, 56(6), pp. 2431-2456

[44] Chaney, P. K., Devinney, T. M., & Winer, R. S. (1991), "The Impact of New Product Introductions on the Market Value of Firms", *The Journal of Business*, 64(4), pp. 573-610

[45] Chiang, A. C. (1992), *Elements of Dynamic Optimization*, McGraw Hill

[46] Cockburn, I., & Griliches, Z. (1988), "Industry Effects and Appropriability Measures in the Stock Market's Valuation of R&D and Patents", *The American Economic Review*, 78(2), pp. 419-423

[47] Connolly, R. A., & Hirschey, M. (1984), "R&D, Market Structure and Profits: a Value-Based Approach", *The Review of Economics and Statistics*, 66(4), pp. 682-686

[48] Connolly, R. A., & Hirschey, M. (1988), "Market Value and Patents: a Bayesian Approach", *Economics Letters*, 27(1), pp. 83-87

[49] Connolly, R. A., Hirsch, B. T., & Hirschey, M. (1986), "Union Rent Seeking, Intangible Capital and Market Value of the Firm", *The Review of Economics and Statistics*, 68(4), pp. 567-577

[50] Czarnitki, D., Ebersberger, B., & Fier, A. (2007), "The Relationship between R&D Collaboration, Subsidies and R&D Performance: Empirical Evidence from Finland and Germany", *Journal of Applied Econometrics*, 22(7), pp. 1347-1366

[51] Dasgupta, P., & Stiglitz, J. E. (1980), "Industrial Structure and the Nature of Innovative Activity", *The Economic Journal*, 90(358), pp. 266-293

[52] David, P. A., & Hall, B. H. (2000), "Heart of Darkness: Modeling Public-Private Funding Interactions inside the R&D Black Box", *Research Policy*, 29(9), pp. 1165-1183

[53] David, P. A., Hall, B. H., & Toole, A. A. (2000), "Is Public R&D a Complement or Substitute for Private R&D? A Review of the Econometric Evidence", *Research Policy*, 29(4-5), pp. 497-529

[54] de Blasio, G., & Lotti, F. (eds.; 2008), *La Valutazione degli Aiuti alle Imprese*, Il Mulino Editore

[55] Dehejia, R. H., & Wahba, S. (2002), "Score-Matching Methods for Non-Experimental Causal Studies", *The Review of Economics and*

Statistics, 84(1), pp. 151-161

[56] de la Fuente, A. (2000), *Mathematical Methods and Models for Economists*, Cambridge University Press

[57] Denicol, V., & Zanchettin, P. (2010), "Competition, Market Selection and Growth", *The Economic Journal*, 120(545), pp. 761-785

[58] Dixit, A. K. (1979), "A Model of Duopoly Suggesting a Theory of Entry Barriers", *The Bell Journal of Economics*, 10(1), pp. 20-32

[59] Duflo, E., Glennerster, R., Kremer, M. (2007), "Using Randomization in Development Economics Research: a Toolkit", in Schultz, T., & Strauss, J. (eds.), *Handbook of Development Economics Vol. 4*, Elsevier

[60] Eurostat (2004), "Innovation in Europe: Results for the E.U., Iceland and Norway - Data 1998-2001", *Panorama of the European Union Collection*, Office for Official Publications of the European Communities

[61] Eurostat (2004), "Statistics on Science and Technology in Europe - Data 1991-2002", *Panorama of the European Union Collection*, Office for Official Publications of the European Communities

[62] Eurostat (2004), "Statistics on Innovation in Europe - Data 1996-1997", *Panorama of the European Union Collection*, Office for Official Publications of the European Communities

[63] Eurostat & O.E.C.D. (2005), "Oslo Manual", *Methods and Nomenclatures Collection*, Office for Official Publications of the European Communities

[64] Fan, J., & Gijbels, J. (1996), *Local Polynomial Modelling and Its Applications*, Chapman & Hall Editors

[65] Frolich, M. (2004), "Finite-Sample Properties of Propensity-Score Matching and Weighting Estimators", *The Review of Economics and Statistics*, 86(1), pp. 77-90

[66] Geroski, P. A. (1990), "Innovation, Technological Opportunity and Market Structure", *Oxford Economic Papers*, 42(3), pp. 586-602

[67] Geroski, P. A., Machin, S., & Van Reenen, J. (1993), "The Profitability of Innovating Firms", *The RAND Journal of Economics*, 24(2), pp. 198-211

[68] Geroski, P.A., & Pomroy, R. (1990), "Innovation and the Evolution of Market Structure", *The Journal of Industrial Economics*, 38(3), pp. 299-314

[69] Gonzalez, X., Jaumandreu, J., & Pazò, C. (2005), "Barriers to Innovation and Subsidy Effectiveness", *The RAND Journal of Economics*, 36(4), pp. 930-950

[70] Gonzalez, X., & Pazò, C. (2008), "Do Public Subsidies Stimulate Private R&D Spending?", *Research Policy*, 37(3), pp. 371-389

[71] Gorg, H., & Strobl, E. (2007), "The Effect of R&D Subsidies on Private R&D", *Economica*, 74(2), pp. 215-234

[72] Gorman, W. M. (1980), "A Possible Procedure for Analysing Quality Differentials in the Egg Market", *The Review of Economic Studies*, 47(5), pp. 843-856

[73] Griliches, Z. (1981), "Market Value, R&D and Patents", *Economics Letters*, 7(2), pp. 183-187

[74] Griliches, Z. (1984), *R&D, Patents and Productivity*, The University of Chicago Press

[75] Griliches, Z. (1990), "Patents Statistics as Economic Indicators: a Survey", *Journal of Economic Literature*, 28(4), pp. 1661-1707

[76] Griliches, Z. , Hall, B. H., & Pakes, A. (1991), "R&D, Patents and Market Value Revisited: Is There a Second (Technological Opportunity) Factor?", *Economics of Innovation and New Technology*, 1(3), pp. 183-202

[77] Greenwood, J., & Jovanovic, B. (1999), "The Information Technology Revolution and the Stock Market", *The American Economic Review (Papers and Proceedings)*, 89(2), pp. 116-122

[78] Grossman, G. M., & Helpman, E. (1991a), "Quality Ladders in the Theory of Growth", *The Review of Economic Studies*, 58(1), pp. 43-61

[79] Grossman, G. M., & Helpman, E. (1991b), *Innovation and Growth in the Global Economy*, The M.I.T. Press

[80] Grossman, G. M., & Helpman, E. (1991c), "Quality Ladders and Product Cycles", *The Quarterly Journal of Economics*, 106(2), pp. 557-586

[81] Hall, B. H. (1993), "The Stock Market Valuation of R&D Investment during the 1980s", *The American Economic Review*, 83(2), pp. 259-264

[82] Hall, B. H. (2002), "The Financing of Research and Development", *Oxford Review of Economic Policy*, 18(1), pp. 35-51

[83] Hall, B. H., Jaffe, A. B., & Trajtenberg, M. (2005), "Market Value and Patent Citations", *The RAND Journal of Economics*, 36(1), pp. 16-38

[84] Hall, B. H., & Oriani, R. (2006), "Does the Market Value R&D Investment by European Firms? Evidence from a Panel of Manufacturing Firms in France, Germany, and Italy", *International Journal of Industrial Organization*, 24(5), pp. 971-993

[85] Hall, B. H., & Van Reenen, J. (2000), "How Effective Are Fiscal Incentives for R&D? A Review of the Evidence", *Research Policy*, 29(4-5), pp. 449-469

[86] Hall, R. E. (1988), "The Relation between Price and Marginal Cost in U.S. Industry", *The Journal of Political Economy*, 96(5), pp.

921-947

[87] Hall, R. E. (2001), "The Stock Market and Capital Accumulation", *The American Economic Review*, 91(5), pp. 1185-1202

[88] Hahn, J., Todd, P., & Van Der Klaauw, W. (2001), "Identification and Estimation of Treatment Effects with a Regression-Discontinuity Design", *Econometrica*, 69(1), pp. 201-209

[89] Härdle, W. (1990), *Applied Nonparametric Regression*, Cambridge University Press

[90] Heckman, J., Ichimura, H., Smith, J. A., & Todd, P. (1998), "Characterizing Selection Bias Using Experimental Data", *Econometrica*, 66(5), pp. 1017-1098

[91] Heckman, J., Ichimura, H., & Todd, P. (1998), "Matching as an Econometric Evaluation Estimator", *The Review of Economic Studies*, 65(2), pp. 261-294

[92] Ho, Y. K., Keh, H. T., Ong, J. M. (2005), "The Effects of R&D and Advertising on Firm Value: an Examination of Manufacturing and Non Manufacturing Firms", *I.E.E.E. Transactions on Engineering Management*, 52(1), pp. 3-14

[93] Hobijn, B., & Jovanovic, B. (2001), "The Information Technology Revolution and the Stock Market: Evidence", *The American Economic Review*, 91(5), pp. 1203-1220

[94] Hussinger, K. (2008), "R&D and Subsidies at the Firm Level: an Application of Parametric and Semiparametric Two-Step Selection Models", *Journal of Applied Econometrics*, 23(6), pp. 729-747

[95] Isaac, R. M., Reynolds, S. S. (1988), "Appropriability and Market Structure in a Stochastic Invention Model", *The Quarterly Journal of Economics*, 103(4), pp. 647-671

[96] Jaffe, A. B. (1986), "Technological Opportunity and Spillovers of R&D: Evidence from Firms' Patents, Profits and Market Value", *The American Economic Review*, 76(5), pp. 984-1001

[97] Jaffe, A. B. (2002), "Building Program Evaluation into the Design of Public Research-Support Programmes", *Oxford Review of Economic Policy*, 18(1), pp. 22-34

[98] Johnson, L. D., & Pazderka, B. (1993), "Firm Value and Investment in R&D", *Managerial & Decision Economics*, 14(1), pp. 15-24

[99] Klepper, S., & Graddy, E. (1990), "The Evolution of New Industries and the Determinants of Market Structure", *The RAND Journal of Economics*, 21(1), pp. 27-44

[100] Klette, T. J. (1996), "R&D, Scope Economies and Plant Performance", *The RAND Journal of Economics*, 27(3), pp. 502-522

[101] Klette, T. J. (1999), "Market Power, Scale Economies and Productivity: Estimates from a Panel of Establishment Data", *The Journal of Industrial Economics*, 47(4), pp. 451-476

[102] Klette, T. J., & Griliches, Z. (1996), "The Inconsistency of Common Scale Estimators When Output Prices Are Unobserved and Endogenous", *Journal of Applied Econometrics*, 11(4), pp. 343-361

[103] Klette, T. J., & Griliches, Z. (2000), "Empirical Patterns of Firm Growth and R&D Investment: a Quality Ladder Model Interpretation", *The Economic Journal*, 110(463), pp. 363-387

[104] Klette, T. J., Moen, J., & Griliches, Z. (2000), "Do Subsidies to Commercial R&D Reduce Market Failures? Microeconometric Evaluation Studies", *Research Policy*, 29(4-5), pp. 471-495

[105] Lach, S. (2002), "Do R&D Subsidies Stimulate or Displace Private R&D? Evidence from Israel", *The Journal of Industrial Economics*, 50(4), pp. 369-390

[106] Lach, S., & Schankerman, M. (1989), "Dynamics of R&D and Investment in the Scientific Sector", *The Journal of Political Economy*, 97(4), pp. 880-904

[107] Laitner, J., & Stolyarov, D. (2003), "Technological Change and the Stock Market", *The American Economic Review*, 93(4), pp. 1240-1267

[108] Lalive, R. (2008), "How Do Extended Benefits Affect Unemployment Duration? A Regression Discontinuity Approach", *Journal of Econometrics*, 142(2), pp. 785-806

[109] Lancaster, K. J. (1966a), "A New Approach to Consumer Theory",

The Journal of Political Economy, 74(2), pp. 132-157

[110] Lancaster, K. J. (1966b), "Change and Innovation in the Technology of Consumption", *The American Economic Review*, 56(1-2), pp. 14-23

[111] Lancaster, K. J. (1975), "Socially Optimal Product Differentiation", *The American Economic Review*, 65(4), pp. 567-585

[112] Lancaster, K. J. (1979), *Variety, Equity and Efficiency*, Columbia University Press

[113] Lancaster, K. J. (1980), "Competition and Product Variety", *The Journal of Business*, 53(3/2), pp. S79-S103

[114] Leahy, D., & Neary, J. P. (1997), "Public Policy towards R&D in Oligopolistic Industries", *The American Economic Review*, 87(4), pp. 642-662

[115] Lee, D. S., & Lemieux, T. (2010), "Regression Discontinuity Designs in Economics", *Journal of Economic Literature*, 48(2), pp. 281-355

[116] Lin, P., & Saggi, K. (2002), "Product Differentiation, Process R&D and the Nature of Market Competition", *European Economic Review*, 46(1), pp. 201-211

[117] Lucas, R. E., & Prescott, E. C. (1971), "Investment under Uncertainty", *Econometrica*, 39(5), pp. 659-681

[118] Merito, M., Giannangeli, S., & Bonaccorsi, A. (2008), "L'Impatto degli Incentivi Pubblici per la R&S sull'Attività delle PMI", in de Blasio,

G., & Lotti, F. (eds.), *La Valutazione degli Aiuti alle Imprese*, Il Mulino Editore

[119] Miguel, E., & Kremer, M. (2004), "Worms: Identifying Impacts on Education and Health in the Presence of Treatment Externalities", *Econometrica*, 72(1), pp. 159-217

[120] Mookherjee, D., & Ray, D. (1991), "On the Competitive Pressure Created by the Diffusion of Innovations", *Journal of Economic Theory*, 54(1), pp. 124-147

[121] Nickell, S. J. (1996), "Competition and Corporate Performance", *The Journal of Political Economy*, 104(4), pp. 724-746

[122] O.E.C.D. (2002), *Frascati Manual: Proposed Standard Practice for Surveys on Research and Experimental Development*, O.E.C.D.

[123] O.E.C.D. (2007), *Science, Technology and Industry Scoreboard*, O.E.C.D.

[124] Ottaviano, G. I. P., & Thisse, J.-F. (1999), *Monopolistic Competition, Multiproduct Firms and Optimum Product Diversity*, C.E.P.R. Discussion Paper N. 2151

[125] Pagan, A., & Ullah, A. (1999), *Nonparametric Econometrics*, Cambridge University Press

[126] Pakes, A. (1985), "On Patents, R&D and the Stock Market Rate of Return", *The Journal of Political Economy*, 93(2), pp. 390-409

[127] Peretto, P. F. (1999), "Cost Reduction, Entry and the Interdependence of Market Structure and Economic Growth", *Journal of Monetary Economics*, 43(1), pp. 173-195

[128] Pisoni, P., Bava, F., Busso, D., & Devalle, A. (2009), *Bilancio 2008*, M.A.P. Editore

[129] Rogers, M. (2002), "The Influence of Diversification and Market Structure on the R&D Intensity of Large Australian Firms", *The Australian Economic Review*, 35(2), pp. 155-172

[130] Romer, P. M. (1990), "Endogenous Technological Change", *The Journal of Political Economy*, 98(5), pp. S71-S102

[131] Rosenkranz, S. (2003), "Simultaneous Choice of Process and Product Innovation When Consumers Have a Preference for Product Variety", *Journal of Economic Behavior & Organization*, 50(2), pp. 183-201

[132] Salinger, M., Caves, R. E., & Peltzman, S. (1990), "The Concentration-Margins Relationship Reconsidered", *Brookings Papers on Economic Activity, Microeconomics*, pp. 287-335

[133] Schmidt, K. (1997), "Managerial Incentives and Product Market Competition", *The Review of Economic Studies*, 64(2), pp. 191-213

[134] Schumpeter, J. (1942), *Capitalism, Socialism and Democracy*, Harper Editor

[135] Shane, S. (2001), "Technological Opportunities and New Firm Creation", *Management Science*, 47(2), pp. 205-220

[136] Shortridge, R. T. (2004), "Market Valuation of Successful versus Non-successful R&D Efforts in the Pharmaceutical Industry", *Journal of Business Finance and Accounting*, 31(9-10), pp. 1301-1325

[137] Silverman, B. (1986), *Density Estimation for Statistics and Data Analysis*, Chapman & Hall Editors

[138] Singh, N., & Vives, X. (1984), "Price and Quantity Competition in a Differentiated Duopoly", *The RAND Journal of Economics*, 15(4), pp. 546-554

[139] Smith, J. A., & Todd, P. (2001), "Does Matching Overcome LaLonde's Critique of Nonexperimental Estimators?", *Journal of Econometrics*, 125(1-2), pp. 305-353

[140] Spence, A. M. (1976), "Product Selection, Fixed Costs and Monopolistic Competition", *The Review of Economic Studies*, 43(2), pp. 217-236

[141] Sutton, J. (1998), *Technology and Market Structure*, The M.I.T. Press

[142] Thistlewaite, D., & Campbell, D. (1960), "Regression-Discontinuity Analysis: an Alternative to the Ex-Post Facto Experiment", *Journal of Educational Psychology*, 51, pp. 309-317

[143] Tirole, J. (1988), *The Theory of Industrial Organization*, The M.I.T. Press

[144] Toivanen, O., Stoneman, P., & Bosworth, D. (2002), "Innovation and the Market Value of the U.K. Firms, 1989-1995", *Oxford Bulletin of Economics and Statistics*, 64(1), pp. 39-61

[145] Vives, X. (1990), "Trade Association Disclosure Rules, Incentives to Share Information and Welfare", *The RAND Journal of Economics*, 21(3), pp. 409-430

[146] Vives, X. (2008), "Innovation and Competitive Pressure", *The Journal of Industrial Economics*, 56(3), pp. 419-469

[147] Wallsten, S. J. (2000), "The Effects of Government-Industry R&D Programs on Private R&D: The Case of the Small Business Innovation Research Program", *The RAND Journal of Economics*, 31(1), pp. 82-100

[148] Weiss, L. W. (1974), "The Concentration-Profits Relationship and Antitrust", in Goldschimd, H., Mann, H., & Weston, J. (eds.), *Industrial Concentration: the New Learning*, Little Brown & Co. Editors

[149] Weiss, P. (2003), "Adoption of Product and Process Innovations in Differentiated Markets: the Impact of Competition", *Review of Industrial Organization*, 23(3-4), pp. 301-314

[150] Wolff, G. B., & Reinthaler, V. (2008), "The Effectiveness of Subsidies Revisited: Accounting for Wage and Employment Effects in Business R&D", *Research Policy*, 37(8), pp. 1403-1412