Essays on International Portfolio Allocation and Risk Sharing

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

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Abstract

This thesis contributes to the theoretical literature that analyses the link between international asset trade and international risk sharing. Despite the massive increase in cross-border asset trade since the 1990's, consumption risk sharing across countries remains limited. In standard international business cycle models, efficient risk sharing requires that consumption should be higher in the country where it is cheaper to consume, implying a high positive correlation between relative consumption and real exchange rate, which is strongly rejected in the data. Recent contributions show that it is possible to account for this so-called 'consumption-real exchange rate anomaly' in models with goods and financial market frictions where international asset trade is restricted to a single non-contingent bond.

Chapter 1 analyses whether this class of models can account for the anomaly under a richer asset market structure where agents can trade in domestic and foreign currency bonds. Even such a small departure from the single bond economy implies too much risk sharing compared to the data although the number of assets that can be traded is less than the number of shocks affecting each economy. Introducing demand shocks alongside sector-specific productivity shocks can improve the performance of the model only under specific parameter and monetary policy settings. Chapter 2 extends this analysis to study the implications of international trade in equities, portfolio transaction costs and recursive utility.

Chapter 3 studies the interaction between monetary policy and foreign currency positions in more detail. Different monetary policy regimes can lead to different foreign currency positions by changing the cyclical properties of the nominal exchange rate. These external positions, in turn, affect the cross-border transmission of monetary policy shocks via a valuation channel. The way export prices are set has important implications for optimal foreign currency positions and the valuation channel when prices are sticky and financial markets are incomplete.

Chapter 4 compares the international transmission of uncertainty shocks under alternative asset markets with an emphasis on the behaviour of net foreign assets, exchange rate and currency risk premium and shows that a model with restricted asset trade performs better than a model with complete financial integration in matching certain aspects of the data regarding the dynamics of these variables in response to increased macroeconomic uncertainty.

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To my parents and my sister

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Introduction

International financial markets have become increasingly integrated over the past two decades. External asset and liability positions now exceed 100% of GDP for major industrialised countries. The connections between the external balance sheets of countries have important implications for the cross-border transmission of country-specific business cycle shocks. This thesis contributes to the theoretical literature that analyses the link between international asset trade and international risk sharing.

The first two chapters of this thesis aim to reconcile the observed patterns in international portfolios with the lack of international risk sharing observed in the data. There is a large body of empirical literature which documents that consumption risk sharing across countries remain limited despite the surge in cross-border asset trade. One measure of consumption risk sharing according to standard international business cycle theory is the correlation between cross-country consumption differentials and relative price levels. Efficient risk sharing requires that consumption should be higher in the country where it is cheaper to consume. This implies a high positive correlation between relative consumption and real exchange rate[1], while in the data this correlation is low and often negative. The inability of a large class of international business cycle models -including the models that restrict international asset trade- to account for this robust empirical finding is known as the 'consumption-real exchange rate anomaly'.

Recent contributions show that it is possible to account for this anomaly in models with both goods and financial market frictions, where uninsured supply shocks generate large wealth transfers across countries, which in turn, lead to meaningful deviations from efficient risk sharing. In these models, favourable supply shocks in the domestic tradable goods sector are associated with a rise in relative consumption and a real exchange rate appreciation, which helps generate a negative correlation between these variables. However, these models do not allow for any kind of ex-ante insurance against country-specific shocks as they restrict international asset trade to

[1] Real exchange rate is defined as the price of foreign consumption basket relative to domestic.

a single non-contingent bond. Chapter 1, which is joint work with Gianluca Benigno, shows that allowing a small departure from this set-up and introducing a second internationally traded bond implies too much risk sharing and brings back the anomaly. When relative bond returns are given by the terms of trade, endogenous trade in two bonds spans the uncertainty caused by shocks to tradables and non-tradables in each country and brings the equilibrium very close to the one under complete markets. Also, the bond portfolio that leads to this outcome is implausibly large.

Why does a seemingly small move away from one bond to two bonds bring the model close to complete consumption risk sharing despite the fact that the number of assets that can be traded is less than the number of independent sources of risk? First of all, uninsured tradable sector shocks generate larger deviations from efficient risk sharing compared to uninsured non-tradable sector shocks. Hence, agents would ideally choose a bond portfolio that insulates them from tradable sector shocks. But whether they can do so, depends crucially on how relative bond returns load on the other sources of risk - shocks to the non-tradable sector in this case. Because relative bond returns do not respond strongly to non-tradable sector shocks in most specifications of the model, agents can enjoy a high degree of risk sharing conditional on tradable sector shocks without being exposed to unwanted valuation effects conditional on non-tradable sector shocks.

In this model, monetary policy has important implications for international portfolio allocation and risk sharing because it determines the nominal exchange rate, which in turn determines relative bond returns. When monetary policy in each country is focused on stabilising the domestic consumer price index, relative bond returns are closely related to the real exchange rate, which loads more strongly on non-tradable sector shocks compared to the terms of trade. Hence, trade in bonds implies lower risk sharing when monetary policy stabilises consumer prices rather than the prices of domestically produced tradable goods.

Chapter 1 also explores whether introducing demand shocks can generate enough tension between different hedging motives such that trade in two assets is not sufficient to span all risks that affect relative marginal utilities of consumption. The results show that only under certain parameter and policy settings demand shocks can reduce the degree of risk sharing implied by bonds without comprising the model's ability to match other business cycle facts.

Chapter 2 extends this analysis to study the implications of international trade in equities, portfolio transaction costs and time-non-separable preferences (recursive utility). Trade in equities has similar risk sharing implications as trade in bonds when uncertainty is driven by shocks to the supply of tradable and non-tradable goods. Optimal equity portfolio that achieves full risk sharing in this set-up is foreign biased contradicting the evidence on home equity bias. The similarity between the risk sharing implications of trade in bonds and equities no longer holds in the presence of shocks that redistribute income between capital and labour, which pull optimal portfolio towards home equity and hamper risk sharing conditional on tradable sector shocks.

Portfolio transaction costs generate a wedge between actual and optimal portfolio positions and help the model generate more realistic portfolio positions alongside a low consumption-real exchange rate correlation. In the presence of transaction costs, agents cannot choose the portfolio that minimises the fluctuations in relative marginal utilities of consumption across countries, which lowers the degree of risk sharing. Under recursive utility, due to a preference for intertemporal distribution of risk, equilibrium portfolios are not solely focused on hedging against the fluctuations in relative marginal utilities of consumption across countries as in the case of expected utility. However, this effect is not strong enough to make a significant difference for equilibrium portfolios and consumption-real exchange rate correlations in the baseline model with stationary shocks.

Chapter 3, which is joint work with Bianca De Paoli and Jens Søndergaard, focuses on the interaction between monetary policy and foreign currency positions. Different monetary policy regimes can lead to different foreign currency positions by changing the cyclical properties of the nominal exchange rate. In a flexible price model with only tradable goods, an adverse domestic real shock entails a domestic currency depreciation under a passive money-growth rule, and an appreciation under an inflation-targeting Taylor-rule. Holding everything else constant, this implies that agents are better insured against real shocks by having a long position in foreign currency under the former policy rule, and a short position in the latter.

In this set-up, monetary policy shocks can have real effects even when all prices are fully flexible as long as financial markets are incomplete. When agents can optimally choose a portfolio of domestic and foreign currency bonds, monetary shocks generate endogenous currency movements that trigger international valuation effects. Whether a domestic monetary loosening that depreciates the currency implies an increase or a decrease in net external wealth depends on the country's foreign currency position.

The main result regarding the link between policy regimes and foreign bond positions goes through in a sticky price environment. When prices are sticky and markets are incomplete, the way export prices are set has important implications for optimal foreign currency positions. This is because, conditional on monetary shocks, the covariance between the real exchange rate adjusted relative consumption and the nominal exchange rate depends on whether export prices are set in producer's or buyer's currency. Under producer currency pricing, the strong expenditure-switching effects of a home currency depreciation triggered by a domestic monetary loosening leads to an increase in real exchange rate adjusted relative consumption, making it optimal to have a short position in foreign currency to hedge against monetary shocks irrespective of the policy regime. On the other hand, under local currency pricing, optimal hedge against monetary shocks is a long position, because a domestic monetary expansion that depreciates the home currency also brings a fall in real exchange rate adjusted relative consumption mostly due to the absence of expenditure-switching effects and the high volatility of real exchange rate.

The final chapter investigates the role of financial market integration for the cross-border transmission of country-specific volatility shocks. The extent of financial market integration is important because it affects the extent of precautionary savings in response to higher uncertainty, which in turn affects the net foreign asset accumulation. In a world with complete financial markets, there is no precautionary saving motive because agents can pool all risks. In the other extreme of financial autarky, and assuming away capital accumulation, there is no role for precautionary savings either because agents cannot buy foreign bonds to increase their savings in response to increased uncertainty.

The model used in Chapter 4 is a version of the two-country endowment model with non-tradable goods used in Chapters 1 and 2, modified to allow for recursive preferences and stochastic volatility in endowment processes. In the baseline model, there is international trade in a single bond subject to portfolio adjustment costs. Implications of this model are compared with that of the complete markets model and the model with higher portfolio adjustment costs.

The main result is that an incomplete model with international trade in a single bond performs better than a model with complete financial integration in matching the empirical observations regarding the dynamics of net foreign assets and real exchange rate in response to increased macroeconomic uncertainty. This model can also account for the negative correlation between relative consumption and real exchange rate conditional on both level and volatility shocks. However, it cannot generate meaningful deviations from the uncovered interest rate parity (UIP) condition because in this model real exchange rate appreciates in good states where consumption is higher, which implies a fall in the foreign exchange risk premium following an increase in domestic volatility. Interestingly, increasing the degree of financial market imperfections brings the model close to the complete market model in terms of its implications for the transmission of uncertainty shocks. When there are high portfolio adjustment costs and no means to save other than investing in foreign bonds, agents cannot increase precautionary savings following an increase in income uncertainty, hence consumption and real exchange rate responses to uncertainty shocks remain limited as in the case of complete insurance.

Chapter 1

Portfolio Allocation and International Risk Sharing[1]

1.1 Introduction

The last two decades have witnessed a dramatic increase in international capital flows. Lane and Milesi-Ferretti (2001, 2006) have documented the increase in gross holdings of cross-country bond and equities for various countries. Their analysis show that gross external financial positions now exceed 100% of GDP for major industrialised countries.

Despite this massive wave of financial globalisation, international risk sharing remains low. Efficient risk sharing requires that consumption should be higher in the country where it is cheaper to consume, implying a positive correlation between relative consumption and real exchange rate (RER).[2] However, as first shown by Backus and Smith (1993) and Kollmann (1995), this is strongly rejected in the data. More recently, Obstfeld (2006) measures the degree of risk sharing by looking at averages of consumption growth and real exchange rates for various countries as in the original Backus and Smith (1993) paper. Using this metric, he finds a distinct negative relationship (i.e. faster consumption growth is associated with a real appreciation) in the data for the period going from 1991 to 2006 -the period of financial integration- suggesting a worsening rather than an improvement in international risk sharing.

[1]This chapter draws on a joint work with Gianluca Benigno. It was motivated by Gianluca Benigno's idea to reconcile international portfolio positions with the lack of international risk sharing observed in the data. I have carried out most of the analytical and numerical analyses and written around 90% of the text.

[2]We define real exchange rate as the price of foreign consumption basket in home consumption units, i.e. an increase implies a real depreciation of home currency.

1.1 displays data on international portfolios and international risk sharing (measured by the correlation of relative consumption and real exchange rate) for industrialised countries for 1991 and 2006.

	Financial Globalisation (A+L)/GDP		Net FX exposure as % of GDP		Equity Home Bias		Cor(C-C^{US},Q) Hp-filtered		Cor(C-C^{US}, Q) First-differenced	
	1990	**2004**	**1990**	**2004**	**1990**	**2004**	**1970-1990**	**1991-2006**	**1970-1990**	**1991-2006**
Australia	95.5	218.4	-2.7	17.6	0.88	0.84	-0.26	-0.80	-0.13	-0.62
Austria	130.3	387.9	-7.8	-28.7	0.74	0.44	0.01	-0.65	-0.07	-0.19
Belgium	394.8	802.5	36.9	24.9	0.64	0.57	-0.17	-0.28	-0.12	0.02
Canada	122.1	211.1	-2.2	52.9	0.79	0.72	-0.53	-0.70	-0.16	-0.52
Denmark	195.8	398.7	-27.7	56.4	0.89	0.62	-0.08	-0.59	-0.24	-0.28
Finland	92.1	396.1	-23.5	50.3	0.99	0.61	-0.27	-0.53	-0.06	-0.63
France	128.5	415.1	17.3	37.0	0.86	0.73	-0.13	-0.37	-0.11	-0.27
Germany	118.6	325.6	18.7	19.2	0.85	0.52	-0.32	-0.28	-0.34	0.00
Greece	74.2	194.0	-9.6	10.4		0.95	-0.32	-0.76	-0.13	-0.57
Italy	73.9	222.5	-2.4	9.9	0.84	0.59	-0.12	-0.48	-0.04	-0.32
Japan	111.7	141.9	10.3	58.1	0.97	0.87	0.14	-0.23	0.19	-0.08
Netherlands	260.0	767.4	59.2	87.8	0.65	0.32	-0.45	0.59	-0.41	0.40
New Zealand	133.6	224.8	-27.0	-19.2		0.66	-0.15	-0.92	-0.18	-0.91
Norway	110.1	337.8	3.2	103.8	0.90	0.52	0.19	-0.39	0.01	-0.29
Portugal	85.3	404.0	13.2	2.1	0.89	0.66	-0.60	-0.19	-0.56	0.01
Spain	62.7	285.0	12.3	7.1	0.97	0.85	-0.64	-0.55	-0.45	-0.42
Sweden	147.8	422.8	-11.6	95.1	0.89	0.58	-0.55	-0.43	-0.28	-0.45
Switzerland	378.1	956.6	119.3	317.2	0.66	0.51	0.09	-0.29	0.06	-0.02
UK	349.0	713.3	52.1	99.5	0.77	0.65	-0.56	-0.05	-0.51	0.10
US	80.1	192.2	14.9	46.8	0.90	0.74				
Median	**120.4**	**362.8**	**6.8**	**41.9**	**0.87**	**0.64**	**-0.26**	**-0.43**	**-0.13**	**-0.28**

Table 1.1: International portfolios and relative consumption-real exchange rate correlations for selected industrial countries

Notes: The second column gives the sum of gross assets and liabilities as a share of GDP based on Lane and Milesi-Ferretti (2006) data. The third column contains the net foreign currency exposure as percent of GDP based on Lane and Shambaugh (2010a) data. The fourth column calculates equity home bias as the difference between the share of domestic equity in total equity portfolio and the size of domestic equity market relative to the world using CPIS and GFD data. The last two columns report the correlation between relative consumption and real exchange rate in each country with respect to the U.S. for HP-filtered and first-differenced series. Consumption, exchange rates and prices are from OECD Outlook Database. Consumption is real private consumption index (2000=100) and real exchange rates are constructed using consumer price indices.

While recent contributions (Benigno and Thoenissen, 2008 and Corsetti et al. 2008)[3] have successfully replicated the low degree of international risk sharing in the context of DSGE models, their analysis is based on a simple international financial market structure in which a riskless bond is traded, a structure that is far from reflecting the recent trend in international financial integration.

Our contribution is to examine the extent to which a more plausible asset market structure is compatible with low international risk sharing as current evidence suggests. We find that even in the case where we only allow for international trade in two nominal bonds, the so-called consumption real exchange anomaly is back.

[3]Throughout this chapter we frequently refer to these papers as BT and CDL, respectively.

It is well-known in international risk sharing literature that specifying a model with incomplete financial markets is not sufficient to generate a negative correlation between relative consumption and real exchange rates even when international asset trade is restricted to a non-contingent bond (see Baxter and Crucini, 1995 and Chari et al., 2002). More importantly, Cole and Obstfeld (1991) show that terms of trade movements can provide considerable insurance against supply shocks irrespective of the asset market structure. Therefore, it is important to start from a model which can account for the anomaly when there is trade in a single bond and analyse the implications of introducing a second internationally traded bond to this set-up.

We use a two-country, two-sector model with shocks to tradable and non-tradable sector productivity in each country along the lines of Benigno and Thoenissen (2008) and Corsetti et al. (2008). We first solve the model under the assumption that international asset trade is restricted to a non-contingent bond and review the mechanisms that can account for the anomaly within this framework. These mechanisms rely on the strong wealth effects generated by uninsured country-specific supply shocks. In Benigno and Thoenissen (2008), a favourable supply shock in the domestic tradables sector increases the relative wealth of domestic agents and leads to higher consumption demand in the domestic country, which in turn raises the prices of domestic non-tradable goods relative to foreign, resulting in a real exchange rate appreciation. On the other hand, Corsetti et al. (2008) emphasise the role of low-substitutability between home and foreign goods. They show that the relative increase in domestic wealth following a favourable supply shock leads to a stronger increase in consumption of home goods due to home bias in consumption and increases the relative price of home goods. Since trade elasticity is very low, a rise in the relative price of home goods cannot generate substitution away from home goods to foreign goods, thus the income effect dominates the substitution effect and terms of trade appreciates.

When we allow for international trade in domestic and foreign currency bonds, the above-mentioned wealth effect disappears and the anomaly returns. Why does a seemingly small move away from one-bond to two-bonds bring the model much closer to complete consumption risk sharing despite the fact that markets are incomplete?[4]

First of all, relative consumption risk is affected more by tradable sector shocks than by non-tradable sector shocks. This is because the country that enjoys a rise in non-tradable sector productivity also experiences a fall in the price of non-tradable goods relative to the other country, which in turn reduces the value of the home

[4]Markets are incomplete as there are two bonds and four independent sources of risk - shocks to tradable and non-tradable output in each country. We solve the optimal portfolio using the methodology developed by Devereux and Sutherland (2011).

non-tradable output relative to foreign and offsets the effect of the non-tradable productivity shock on relative consumption.[5] Therefore, agents would want to use bonds mainly to hedge against relative consumption risk coming from tradable sector shocks. But whether they can do so, depends crucially on how relative bond returns are affected by non-tradable sector shocks.

If relative bond returns respond strongly to non-tradable sector shocks, a portfolio that insulates consumers from fluctuations in tradable sector output can make them more vulnerable to fluctuations in non-tradable output due to 'adverse valuation effects'. This in turn would limit the degree of risk sharing that can be provided by bonds. On the other hand, if relative bond returns are weakly related to non-tradable sector shocks, as is the case in most specifications of our model, agents can enjoy a high degree of risk sharing conditional on tradable sector shocks without increasing their exposure to non-tradable shocks, which brings the two bond economy closer to the complete markets economy.

In our model, monetary policy specification has important implications for portfolio allocation and the degree of risk sharing because it determines the nominal exchange rate, and relative bond returns are given by the surprises in the nominal exchange rate. We consider two simple monetary policy rules, domestic tradable price stabilisation and CPI stabilisation, which imply different properties of relative bond returns. Under the former, nominal exchange rate and relative bond returns are determined by the *terms of trade*, whereas under the latter they are given by the *real exchange rate*.

We find that trade in bonds generally leads to higher risk sharing when relative bond returns are determined by the terms of trade as opposed to the real exchange rate. This is because real exchange rate responds more strongly to non-tradable sector shocks, which prevents agents from choosing a portfolio that could insure them fully against the relative consumption risk coming from tradable sector shocks.[6] While the high risk sharing result is robust to different values of trade elasticity when relative bond returns are equal to the terms of trade, this is not the case when relative bond returns are given by the real exchange rate. Our numerical results show that, under CPI stabilisation, the cross-correlation between relative consumption and real

[5] Cole and Obstfeld (1991) show that terms of trade adjustment can offset supply shocks when all goods are tradable, preferences are symmetric and trade elasticity is close to unity. In our model, we are far from the Cole and Obstfeld economy, therefore terms of trade does not ensure high risk sharing against tradable sector shocks.

[6] Real exchange rate consists of the terms of trade and the relative price of non-tradables. Because relative price of non-tradables is directly linked to the relative supply of non-tradables, real exchange rate is affected more strongly by non-tradable sector shocks compared to the terms of trade.

exchange rate can be high or low depending on the value of trade elasticity. But under domestic tradable price stabilisation, the correlation is almost perfect regardless of this parameter.

In light of these results, we enrich the shock structure in our two-sector model and consider demand shocks as well as supply shocks. Our focus is on the implications of this additional source of uncertainty on equilibrium portfolio allocation and, through that, on the international transmission of supply shocks. In other words, we explore whether the presence of demand shocks can generate enough market incompleteness such that the transmission of supply shocks can still be negative as in Benigno and Thoenissen (2008) and Corsetti et al. (2008) even under some endogenous portfolio choice. As demand shocks, we consider shocks to the predictable component of sectoral productivity shocks - 'news shocks' as in Beaudry and Portier (2004), Jaimovich and Rebelo (2008) and Colacito and Croce (2010) among others.[7]

Our numerical results show that only under certain parameter and policy settings, demand shocks can reduce the degree of risk sharing implied by bonds without compromising the model's ability to match other business cycle facts. The intuition for how demand shocks work is as follows. Demand shocks move relative consumption risk in the same direction as supply shocks, but they affect relative bond returns in the opposite direction. Therefore, relative supply and demand shocks require different signs for optimal bond portfolios, which in turn limits the degree of risk sharing ensured by bonds.

For instance, consider the case where demand shocks require a long position in foreign bonds, while supply shocks require the opposite. If demand shocks are sufficiently large, the optimal portfolio will be a long position in foreign currency, which will make home agents worse-off conditional on supply shocks. Given a long position in foreign currency, a negative supply shock that appreciates the domestic currency brings about capital losses, reducing net wealth of agents at a time they need to increase their consumption. This example illustrates the role of adverse valuation effects in accounting for the anomaly in the presence of endogenous portfolio choice.[8]

This chapter is closely related to the literature on country portfolios. Heath-

[7]We want to stress that the demand shocks we consider work in a different way compared to Stockman and Tesar (1995) type 'taste shocks', which are basically shocks to the marginal utility of consumption. Heathcote and Perri (2007) show that these shocks can be used to generate a realistic negative correlation between relative consumption and real exchange rate but their explanation of the anomaly does not rely on market incompleteness.

[8]Ghironi et al.(2010) also focus on the role of valuation channel for international risk sharing. They show that valuation effects can dampen or amplify the response of consumption differential to productivity and government spending shocks in a two-country one-sector DSGE model where there is international trade in equity.

cote and Perri (2007), Kollmann (2006), Collard et al.(2008), Engel and Matsumoto (2009) and Coeurdacier et al.(2010) propose different models that can generate realistic portfolio positions under effectively complete markets. There is also a range of papers that analyse equilibrium portfolios under incomplete markets. Coeurdacier et al.(2007) specify an incomplete market model with supply, demand and redistributive shocks and trade in stocks and bonds to match the basic stylised facts on international portfolios. Hnatkovska (2010) analyses endogenous portfolio choice under incomplete markets in a model with tradable and non-tradable sectors and examines the dynamics of portfolio choice to reconcile the home bias in equity holdings with the high turnover and high volatility of international capital flows. Using different modelling frameworks, Coeurdacier and Gourinchas (2009) and Benigno and Nisticò (2009) also study endogenous bond and equity portfolios under incomplete markets. However, they mainly focus on different hedging motives behind equilibrium portfolio positions, e.g. whether home equity bias is driven by non-diversifiable labour income risk or real exchange rate risk, rather than analysing the implications of portfolio allocation for international risk sharing and consumption-real exchange rate anomaly.

The rest of the chapter proceeds as follows. In sections 1.2 and 1.3, we lay out a two-country two-sector endowment model and solve the model analytically to show how the comovement of relative consumption and real exchange rates is affected by endogenous portfolio choice in the presence of anticipated and unanticipated shocks. Section 1.4 gives the quantitative results of a calibrated production model with capital accumulation. Section 1.5 concludes.

1.2 A two-country two-sector endowment economy

We first develop a basic two-country open economy endowment model. There is a home and a foreign country, each country endowed with a tradable and a non-tradable good. Endowments in each country are stochastic. Households maximise utility over infinite horizon under different asset market configurations: complete markets where agents can trade in a full-set of state-contingent claims, incomplete markets where international asset trade is restricted to a single non-contingent bond and an intermediate case where agents in each country can trade in two nominal bonds denominated in home and foreign currency. The structure of the model is related to the production economies described in BT, CDL and Stockman and Tesar (1995).

1.2.1 Preferences and good markets

Representative agent in home country maximises the expected present discounted value of the utility:

$$U_t = E_t \sum_{s=t}^{\infty} \delta_s \frac{C_s^{1-\rho}}{1-\rho}, \tag{1.1}$$

where C is consumption and δ_s is the discount factor, which is determined as follows:

$$\delta_{s+1} = \delta_s \beta(C_{As}),\ \ \delta_0 = 1, \tag{1.2}$$

where C_A is aggregate home consumption and $0 < \beta(C_A) < 1$. To achieve stationarity under incomplete market specification, we assume $\beta_C(C_A) \leq 0$, which implies that agents discount the future more as aggregate consumption increases, i.e. agents bring consumption forward when aggregate consumption is high. Following Devereux and Sutherland (2011), we assume that the individual takes C_A as given when optimising and specify the discount factor as follows:

$$\beta(C_A) = \omega C_A^{-\eta}, \tag{1.3}$$

with $0 \leq \eta < \rho$ and $0 < \omega \bar{C}_A^{-\eta} < 1$ (for $\eta = 0$ we have the constant discount factor).

C represents a consumption index defined over tradable C_T and non tradable C_N consumption:

$$C_t = \left[\gamma^{\frac{1}{\kappa}} C_{T,t}^{\frac{\kappa-1}{\kappa}} + (1-\gamma)^{\frac{1}{\kappa}} C_{N,t}^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}}, \tag{1.4}$$

where κ is the elasticity of intratemporal substitution between C_N and C_T and γ is the weight that the households assign to tradable consumption. The tradable component of the consumption index is in turn a CES aggregate of home and foreign tradable consumption goods, C_H and C_F:

$$C_{T,t} = \left[\nu^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-\nu)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{1.5}$$

where θ is the elasticity of intratemporal substitution between C_H and C_F and ν is the weight that the households assigns to home tradable consumption. We allow for a home bias in tradable goods by assuming $\nu > \frac{1}{2}$. We adopt a similar preference specification for the foreign country except that variables are denoted with an asterisk. The consumption price index (CPI), which is defined as the minimum expenditure required to purchase one unit of aggregate consumption for the home agent is given by:

$$P_t = \left[\gamma P_{T,t}^{1-\kappa} + (1-\gamma) P_{N,t}^{1-\kappa}\right]^{\frac{1}{1-\kappa}}. \tag{1.6}$$

Meanwhile, the traded goods price index, which is defined as the minimum expenditure required to purchase one unit of a traded good is given by:

$$P_{T,t} = \left[\nu P_{H,t}^{1-\theta} + (1-\nu) P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{1.7}$$

We assume that the law of one price holds, i.e. $P_{H,t}^{*} = P_{H,t}/S_t$,and $P_{F,t} = P_{F,t}^{*} S_t$, where S_t denotes the nominal exchange rate defined as the price of foreign currency in terms of domestic currency. The presence of non-tradable goods and home bias in tradables consumption leads to deviations from purchasing power parity. We define the real exchange rate as $Q = \frac{SP^*}{P}$.

Good market clearing requires $Y_{H,t} = C_{H,t} + C_{H,t}^{*}$, $Y_{F,t}^{*} = C_{F,t}^{*} + C_{F,t}$, $Y_{N,t} = C_{N.t}$ and $Y_{N,t}^{*} = C_{N,t}^{*}$ where C_H and C_F (C_F^* and C_H^*) should satisfy the intratemporal optimisation decisions of home (foreign) households. Endowments of tradable and non-tradable goods follow AR(1) processes of the form:

$$\begin{aligned}
\log Y_{H,t} &= (1-\delta_T)\log \bar{Y}_H + \delta_T \log Y_{H,t-1} + u_{H,t}, && (1.8)\\
\log Y_{F,t} &= (1-\delta_T)\log \bar{Y}_F + \delta_T \log Y_{F,t-1} + u_{F,t}, &&\\
\log Y_{N,t} &= (1-\delta_N)\log \bar{Y}_N + \delta_N \log Y_{N,t-1} + u_{N,t}, && (1.9)\\
\log Y_{N,t}^{*} &= (1-\delta_N)\log \bar{Y}_N^{*} + \delta_N \log Y_{N,t-1}^{*} + u_{N,t}^{*}, &&
\end{aligned}$$

where $0 \leq \delta_T < 1$, $0 \leq \delta_N < 1$, $u_{H,t}, u_{F,t}^{*}, u_{N,t}, u_{N,t}^{*}$ are i.i.d. shocks with $Var(u_H) = Var(u_F) = \sigma_T^2$ and $Var(u_N) = Var(u_N^*) = \sigma_N^2$.

1.2.2 Asset markets

Previous literature establishes the link between international risk sharing and the asset market structure. Backus and Smith (1993) and Kollmann (1995) show that complete markets imply a counterfactual perfect correlation between relative consumption and real exchange rates. Benigno and Thoenissen (2008) and Corsetti et al. (2008) set out the conditions under which it is possible to get a negative correlation between relative consumption and real exchange rates in an incomplete market set-up where only a single non-contingent bond is internationally traded. Here our aim is to see whether their results go through when we allow for endogenous portfolio choice in its simplest form - allowing for trade in two nominal bonds rather than a single non-contingent bond. Hence, we consider three different asset market structures to compare their implications for real exchange rate and relative consumption correlations.

Complete markets

Complete market set-up can be characterised either by assuming that agents in each country can trade in a complete set of state-contingent assets, as in Chari et al.(2002) or Heathcote and Perri (2007) for e.g., or by assuming that there are enough independent assets, bonds and equities, to span all the risks, as in Devereux and Sutherland (2011), Coeurdacier (2009) among others.[9] Here we follow the former approach and do not characterise equilibrium portfolios associated with the complete market equilibrium. We are mainly interested in the risk sharing implications of complete markets, which we will later compare with the implications of incomplete markets.

Assuming that initial wealth levels are equal across countries and discount factors are constant, the following risk sharing condition holds under complete markets:

$$\frac{U_C(C_t^*)}{U_C(C_t)} = \frac{S_t P_t^*}{P_t}, \tag{1.10}$$

which states that marginal utilities of consumption adjusted by the respective CPI's are equalised across countries for each date and state. Backus and Smith (1993) and Kollmann (1995) show that the perfect correlation between relative consumption and real exchange rates implied by equation (1.10) under standard preferences, is strictly rejected in the data. Indeed, in the data relative consumption and real exchange rates are negatively correlated for most of the countries (see Table 1.1).

Incomplete markets: Non-contingent bond economy

In this setting, home and foreign agents hold an international bond, $B_{H,t}$, which pays in units of home currency. The flow budget constraint of the representative home country consumer is given by:

$$B_{H,t} = R_{H,t} B_{H,t-1} + P_{H,t} Y_{H,t} + P_{N,t} Y_{N,t} - P_t C_t, \tag{1.11}$$

where $R_{H,t}$ is the home country nominal interest rate, $P_{H,t}Y_{H,t}$ and $P_{N,t}Y_{N,t}$ are the home currency values of tradable and non-tradable good endowments. In this case,

[9]The spanning condition with trade in equities and bonds requires that there are $n+1$ independent assets for n sources of uncertainty.

there is no portfolio choice problem. International trade in the non-contingent bond only allows for international borrowing and lending and does not provide any other hedging opportunity. This is the standard incomplete markets set-up used in the open economy macro literature.[10]

Maximisation of expected lifetime utility with respect to (1.11) implies the usual bond Euler equation for the home agent:

$$U_C(C_t) = \beta(C_t) E_t U_C(C_{t+1}) R_{H,t+1} \frac{P_t}{P_{t+1}}. \tag{1.12}$$

Foreign agent's optimal choice of home bonds is given by:

$$U_C(C_t^*) = \beta(C_t^*) E_t U_C(C_{t+1}^*) R_{H,t+1} \frac{S_t}{S_{t+1}} \frac{P_t^*}{P_{t+1}^*}, \tag{1.13}$$

$$U_C(C_t^*) = \beta(C_t^*) E_t U_C(C_{t+1}^*) R_{F,t+1}^* \frac{P_t^*}{P_{t+1}^*}, \tag{1.14}$$

where $R_{F,t}^*$ is the nominal interest rate on foreign bond expressed in terms of foreign currency. In the non-contingent bond economy, the risk sharing condition given by equation (1.10) no longer holds. Benigno and Thoenissen (2008) and Corsetti et al.(2008) show that this set-up can account for the consumption-real exchange rate anomaly. We review the main elements of their analysis in section 1.3.2 and show under what conditions this set-up can account for the anomaly.

Incomplete markets: International trade in home and foreign currency bonds

In this set-up we consider a small deviation from the single bond economy and allow for a second bond to be internationally traded. Agents in each country can now trade in bonds denominated in home and foreign currency. Given that the number of independent assets that can be traded internationally is less than the number of shocks, the spanning condition is not satisfied, i.e. markets are incomplete. The flow budget constraint of the home agent in nominal terms is given by:

$$B_{H,t} + S_t B_{F,t} = R_{H,t} B_{H,t-1} + R_{F,t}^* S_t B_{F,t-1} + P_{H,t} Y_{H,t} + P_{N,t} Y_{N,t} - P_t C_t, \tag{1.15}$$

[10] In Benigno and Thoenissen (2008), home agents can trade in both home currency and foreign currency-denominated bonds, while foreign agents can only trade in foreign currency-denominated bonds. Thus international asset trade is restricted to foreign bonds. Stationarity is ensured by assuming international trade of foreign bonds is subject to intermediation costs. This set-up has the same implications as our non-contingent bond economy set-up with international trade in home bonds.

where $B_{H,t-1}$ is the home agent's holdings of internationally traded home bond and $B_{F,t-1}$ is the home agent's holdings of internationally traded foreign bond purchased at the end of period $t-1$ for holding into period t. $R_{H,t}$ and $R^*_{F,t}$ are the risk-free returns on home and foreign bonds.

Letting $\alpha_{H,t} \equiv B_{H,t}$, $\alpha_{F,t} \equiv S_t B_{F,t}$ and defining $NFA_t \equiv \alpha_{H,t} + \alpha_{F,t}$ as the total net claims of home agents on the foreign country at the end of period t (i.e. the net foreign assets of home agents) we can write (1.15) as a net foreign asset accumulation equation[11]:

$$NFA_t = NFA_{t-1}R_{H,t} + \alpha_{F,t-1}R_{x,t} + P_{H,t}Y_{H,t} + P_{N,t}Y_{N,t} - P_tC_t, \tag{1.16}$$

where $R_{x,t} = R_{F,t} - R_{H,t}$ is the excess return on foreign bond relative to home bond expressed in home currency units, with $R_{F,t} = R^*_{F,t}\frac{S_t}{S_{t-1}}$.[12]

Note that once α_F is determined, α_H, α^*_H and α^*_F will also be determined as $\alpha_H = NFA - \alpha_F$ by definition and $\alpha^*_H = -\alpha_H$, $\alpha^*_F = -\alpha_F$ from market clearing conditions. Thus, we only focus on α_F in what follows.

The main difference between the asset accumulation equations (1.16) and (1.11) is the excess return on the portfolio, $\alpha_{F,t-1}R_{x,t}$, which implies state-contingent valuation effects. Therefore, in the set-up with endogenous bond portfolios, agents can smooth consumption not only across time through borrowing and lending in international financial markets, but also across different states of the world to some extent. As we discuss in detail below, the extent of insurance across states provided by trade in bonds depends on the loadings of excess return on different sources of risk.

Consumers' first order conditions imply that as well as the Euler equations given by (1.12) and (1.13), there is also a home Euler equation for foreign bond. These imply the following optimal portfolio choice equations should hold in each country:

$$E_t\left[m_{t+1}R_{x,t+1}\right] = 0, \quad E_t\left[m^*_{t+1}R_{x,t+1}\frac{S_t}{S_{t+1}}\right] = 0, \tag{1.17}$$

where home and foreign stochastic discount factors are given by $m_{t+1} = \beta(C_t)\frac{P_t}{P_{t+1}}\frac{C_{t+1}^{-\rho}}{C_t^{-\rho}}$

[11] Net foreign assets of home agent is defined as net claims of home country on foreign country assets, i.e. $NFA_t = \alpha_{F,t} - \alpha^*_{H,t}$. Since bonds are assumed to be in net zero supply $\alpha_{H,t} = -\alpha^*_{H,t}$. It follows that $NFA_t = \alpha_{H,t} + \alpha_{F,t}$.

[12] A similar budget constraint holds for the foreign agent, where foreign variables are denoted with an asterisk, *. Thus, $\alpha^*_{H,t-1}$ and $\alpha^*_{F,t-1}$ denote the foreign country's holdings of home and foreign bonds, expressed in units of home currency. Bonds are assumed to be in net zero supply in each country. Thus, equilibrium in asset market requires that total bond holdings of home and foreign agents should equal zero, i.e. $\alpha_{H,t} + \alpha^*_{H,t} = 0$ and $\alpha_{F,t} + \alpha^*_{F,t} = 0$.

and $m^*_{t+1} = \beta(C^*_t)\frac{P^*_t}{P^*_{t+1}}\frac{C^{*-\rho}_{t+1}}{C^{*-\rho}_t}$, respectively, and $R_{x,t+1}$ is the excess return on foreign nominal bond, taking home bond as a reference as defined above.

To solve the model in the presence of endogenous portfolio choice under incomplete markets, we use the approximation techniques proposed in Devereux and Sutherland (2011) and Tille and van Wincoop (2010). We approximate our model around the symmetric steady state in which steady-state inflation rates are assumed to be zero.

The second order approximation of the optimal portfolio choice equations in (1.17) together with the property of the model that expected excess returns are zero up to a first order approximation, i.e. $E_t\left[\hat{R}_{x,t+1}\right] = 0 + O(\varepsilon^2)$, gives an orthogonality condition between excess returns and the relative stochastic discount factors denominated in the same currency, which pins down optimal steady-state portfolios:

$$Cov_t\left[(\hat{m}_{t+1} - \hat{m}^*_{t+1} + \Delta\hat{S}_{t+1}), \hat{R}_{x,t+1}\right] = 0 + O(\varepsilon^3). \tag{1.18}$$

As shown by Devereux and Sutherland (2011), to evaluate (1.18) and determine the portfolio shares, it is sufficient to take a first-order approximation of the remaining equilibrium conditions for which the only aspect of portfolio behaviour that matters is the steady-state foreign bond portfolio, $\bar{\alpha}_F$.

1.2.3 Policy rules

We close the model by considering two simple policy rules. Although prices are fully flexible in our model, the way we specify policy rules matters as long as we have a nominal asset. This is because the return differential between home and foreign bonds is given by the rate of (unexpected) nominal exchange depreciation, which is affected by the policy rule in a flexible price setting. Consequently, equilibrium portfolio shares will be affected, which will then feed back into the model (see Devereux and Sutherland, 2008 and Chapter 3).

We focus on two cases: in the first one, policy authorities stabilise their own tradable prices ($P_{H,t} = 1$,and $P_{F^*,t} = 1$) and in the second one they stabilise domestic consumer prices ($P_t = 1$,and $P^*_t = 1$).[13] Nominal exchange rate is equal to the terms of trade in the former, while it is given by the real exchange rate in the latter.[14]

[13] Benigno and Thoenissen (2008) close their model by assuming that monetary policy is characterised by CPI targeting whereas Corsetti et al. (2008) take the domestic CPI as numeraire, which are essentially equivalent.

[14] Having a nominal bond with a CPI targeting rule is equivalent to having a real bond (or CPI indexed bond) with any policy rule in terms of equilibrium portfolio and model solution.

1.3 Relative consumption and real exchange rate under alternative asset markets

In this section we first describe the general equilibrium behaviour of relative consumption and real exchange rate in response to sectoral supply shocks under complete markets and illustrate the Backus-Smith-Kollmann condition. Next, we go over the mechanisms put forth by Benigno and Thoenissen (2008) and Corsetti et al.(2008) that can account for the consumption-real exchange rate anomaly when international asset trade is limited to a single non-contingent bond. Then, we analyse how the link between relative consumption and real exchange rate changes when we move from single bond economy to a two bond economy with endogenous portfolio choice.

1.3.1 Complete markets: Backus-Smith-Kollmann condition

Assuming CRRA preferences, log-linearisation of the risk sharing condition in (1.10) gives:

$$\hat{C}_t - \hat{C}_t^* = \frac{\hat{Q}_t}{\rho}, \tag{1.19}$$

which implies that consumption should be higher in the country where it is cheaper to consume.

It is useful to characterise the full general equilibrium solution to relative consumption and real exchange rate under complete markets to compare it with the solution under different configurations of incomplete markets.

$$\hat{C}_t - \hat{C}_t^* = \frac{\gamma\kappa(2\nu-1)}{\Gamma_1}(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + \frac{\Gamma_2}{\Gamma_1}(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*), \tag{1.20}$$

$$\hat{Q}_t = \rho\left(\frac{\gamma\kappa(2\nu-1)}{\Gamma_1}(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + \frac{\Gamma_2}{\Gamma_1}(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*)\right), \tag{1.21}$$

where $\Gamma_1 \equiv 4\theta\nu(1-\nu)(1+\gamma(\kappa\rho-1)) + \kappa(2\nu-1)^2 > 0$ and $\Gamma_2 \equiv (1-\gamma)(\kappa(2\nu-1)^2 + 4\theta\nu(1-\nu)) > 0$ for all possible parameter values. Table 1.2 summarises the definitions of the parameters used in the discussion of analytical results.

The only state variables of the complete market model are the exogenous state variables, i.e. the stochastic endowment processes in each sector and country. Net foreign asset accumulation does not matter for equilibrium dynamics under complete markets. Real exchange rate and relative consumption are perfectly correlated as can be seen from (1.20).

$\Gamma_1 \equiv 4\theta\nu(1-\nu)(1+\gamma(\kappa\rho-1))+\kappa(2\nu-1)^2 > 0.$
$\Gamma_2 \equiv (1-\gamma)(\kappa(2\nu-1)^2+4\theta\nu(1-\nu)) > 0.$
$\Gamma_3 \equiv (2\theta\nu-1)(\gamma\kappa\rho+1-\gamma)-\kappa(2\nu-1) > 0$ for $\theta > \theta_3^*$.
$\Gamma_4 \equiv (\gamma\kappa\rho+1-\gamma)\left(\theta^2+(\theta-1)^2\frac{\sigma_T^2}{\sigma_N^2}\right).$
$\theta_1^* \equiv \frac{1}{2\nu}+\frac{\kappa}{1-\gamma}\frac{2\nu-1}{2\nu}.$
$\theta_2^* \equiv \frac{1+\kappa(2\nu-1)}{2\nu}$, $\theta_2^* < \theta_1^*$ for $1-\gamma<1$.
$\theta_3^* \equiv \frac{1}{2\nu}+\frac{1}{2\nu}\frac{\kappa(2\nu-1)}{\gamma\kappa\rho+1-\gamma}$, $\theta_3^* < \theta_1^*$ for $\nu > \frac{1}{2}$.
$RV_1 \equiv \frac{\Gamma_1(1-\beta\delta_z)^2}{(2\nu-1)\Gamma_3\beta^2}.$

Table 1.2: Some parameter definitions.

1.3.2 Incomplete markets: Non-contingent bond economy

Under incomplete markets, the risk sharing condition no longer holds in levels but in expected future changes in relative consumption and real exchange rate. Combining the home and foreign Euler equations with respect to the international asset gives:

$$E_t(\Delta\hat{C}_{t+1}-\Delta\hat{C}^*_{t+1}) = \frac{1}{\rho}E_t\Delta\hat{Q}_{t+1}. \tag{1.22}$$

Since the risk sharing condition now holds in expected future changes, there will be deviations from the Backus-Smith-Kollmann condition, which can be expressed as $\hat{C}_t-\hat{C}^*_t-\frac{\hat{Q}_t}{\rho}$. Country-specific shocks will create large fluctuations in relative wealth provided that there are significant deviations from this condition.

To simplify the analytical expressions we assume that shocks are permanent, i.e. $\delta_T=\delta_N=1$, so that the general equilibrium solution for relative consumption and real exchange rate dynamics reads:[15]

$$\hat{C}_t-\hat{C}^*_t = \psi_c^{nc}\widehat{NFA}_{t-1}+\frac{\gamma(2\theta\nu-1)}{1+2\nu(\theta-1)}(\hat{Y}_{H,t}-\hat{Y}_{F,t})+(1-\gamma)(\hat{Y}_{N,t}-\hat{Y}^*_{N,t}), \tag{1.23}$$

$$\hat{Q}_t = -\psi_q^{nc}\widehat{NFA}_{t-1}-\left[\frac{(1-\gamma)(2\theta\nu-1)-\kappa(2\nu-1)}{\kappa(1+2\nu(\theta-1))}\right](\hat{Y}_{H,t}-\hat{Y}_{F,t})+\frac{1-\gamma}{\kappa}(\hat{Y}_{N,t}-\hat{Y}^*_{N,t}) \tag{1.24}$$

where $\psi_c^{nc} \equiv \frac{4(1-\beta)\theta\nu}{\beta(1+2\nu(\theta-1))}$ and $\psi_q^{nc} \equiv \frac{(1-\beta)}{\beta}\frac{(\kappa(2\nu-1)^2+4\nu(1-\nu)(1-\gamma)\theta)}{\gamma\kappa(1-\nu)(1+2\nu(\theta-1))}$. $\psi_c^{nc}>0$ and $\psi_q^{nc}>0$ for $\theta > 1-\frac{1}{2\nu}$.

In an incomplete market model, net foreign asset position is an endogenous state variable as reflected by the policy functions in (1.23) and (1.24).[16] Relative consump-

[15]For the analytical derivations, we assume a constant discount factor, i.e. $\eta=0$.

[16]For a sufficiently high elasticity of substitution between home and foreign goods ($\theta > 1-\frac{1}{2\nu}$), higher net foreign assets brought from previous period implies higher consumption at home country

tion and real exchange rate are positively related conditional on non-tradable sector shocks. However, they might move in opposite directions conditional on tradable sector shocks depending on the value of trade elasticity, θ, which in turn can account for the consumption-real exchange rate anomaly as shown by BT and CDL.

To illustrate how the transmission of tradable sector supply shocks changes with trade elasticity, we decompose the real exchange rate into two components- the terms of trade, TOT_t, and the relative price of non-tradables across countries, P_t^N :

$$\hat{Q}_t = \gamma(2\nu - 1)\widehat{TOT}_t + (1-\gamma)\hat{P}_t^N, \tag{1.25}$$

where $\widehat{TOT} = \hat{P}_F^* + \hat{S} - \hat{P}_H$ and $\hat{P}^N = \hat{P}_N^* + \hat{S} - \hat{P}_N$.[17] Equation (1.25) shows clearly that in this model real exchange rates fluctuate due to the presence of home bias in consumption ($\nu > \frac{1}{2}$) and non-traded goods ($\gamma < 1$).

The general equilibrium solution for terms of trade and relative non-tradables price assuming permanent shocks are as follows:

$$\widehat{TOT}_t = -\psi_T^{nc}\widehat{NFA}_{t-1} + \frac{1}{1+2\nu(\theta-1)}(\hat{Y}_{H,t} - \hat{Y}_{F,t}), \tag{1.27}$$

where $\psi_T^{nc} \equiv \frac{(1-\beta)(2\nu-1)}{\beta\gamma(1-\nu)(1+2\nu(\theta-1))}$.[18]

$$\hat{P}_t^N = -\psi_N^{nc}\widehat{NFA}_{t-1} - \left[\frac{(2\theta\nu-1)-\kappa(2\nu-1)}{\kappa(1+2\nu(\theta-1))}\right](\hat{Y}_{H,t} - \hat{Y}_{F,t}) + \frac{1}{\kappa}(\hat{Y}_{N,t} - \hat{Y}^*_{N,t}), \tag{1.28}$$

where $\psi_N^{nc} \equiv \frac{(1-\beta)[\kappa(2\nu-1)^2+4\theta\nu(1-\nu)]}{\beta\gamma\kappa(1-\nu)(1+2\nu(\theta-1))}$.

Using the analytical expressions given in equations (1.23) to (1.28), we can characterise five regions of trade elasticity, each of which implies a different transmission mechanism in response to tradable sector shocks on impact. Figure 1.1 illustrates these regions.

There are two regions of θ for which a positive tradable sector supply shock leads to

($\psi_c^{nc} > 0$) and a more expensive home consumption basket ($\psi_q^{nc} > 0$).

[17] More often, non-tradable prices in each country are expressed relative to the tradable prices, to highlight the Balassa-Samuelson effect:

$$\hat{Q}_t = (2\nu - 1)\widehat{TOT}_t + (1-\gamma)\widehat{RPN}_t \tag{1.26}$$

where terms of trade is defined as above and relative price of non-tradables is defined as $\widehat{RPN}_t \equiv (\hat{P}^*_{N,t} - \hat{P}^*_{T,t}) - (\hat{P}_{N,t} - \hat{P}_{T,t})$.

[18] Note that terms of trade is independent of non-tradable sector shocks because we assume, for ease of exposition, that the persistence of non-tradable endowments, δ_N, is equal to 1. As we show later, terms of trade is independent of non-tradable sector shocks also when $\gamma = 1$ or $\nu = \frac{1}{2}$ or $\kappa\rho = 1$ (utility is separable in tradable and non-tradable consumption).

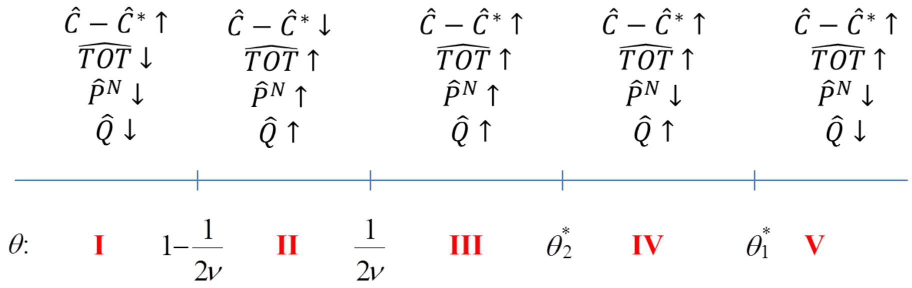


Figure 1.1: Impact responses to a positive tradable endowment shock with respect to trade elasticity, θ, for $\nu > \frac{1}{2}$.

Notes: $\theta_1^* \equiv \frac{1}{2\nu} + \frac{\kappa}{1-\gamma}\frac{2\nu-1}{2\nu}$ and $\theta_2^* = \frac{1+\kappa(2\nu-1)}{2\nu}$.

an increase in relative consumption and a fall in real exchange rate - hence a negative conditional correlation on impact. These regions are region I, where $\theta < 1 - \frac{1}{2\nu}$, and region V, where $\theta > \theta_1^* \equiv \frac{1}{2\nu} + \frac{\kappa}{1-\gamma}\frac{2\nu-1}{2\nu}$. In both of these regions, an unanticipated increase in the tradable endowment of the home country implies a large increase in the relative wealth of home agents, which in turn leads to higher consumption and higher prices in the home country. As we describe in detail below, the main difference between the two regions is that in the former, the increase in relative wealth appreciates both the terms of trade and the relative price of non-tradables, while in the latter it only appreciates the relative non-tradables.[19]

Figure 1.1 shows that there is another region, region II, given by $1 - \frac{1}{2\nu} < \theta < \frac{1}{2\nu}$, where relative consumption and real exchange rate are negatively correlated conditional on tradable endowment shocks. In this region, negative conditional correlation is due to the fact that relative consumption falls in response to a positive tradable sector shock while the real exchange rate depreciates. In what follows we focus our attention on regions I and V, which imply a positive relation between relative consumption and relative income.

Region I: Low trade elasticity

In this region, characterised by $\theta < 1 - \frac{1}{2\nu}$, the mechanism that accounts for the consumption-real exchange rate anomaly is the one emphasised by Corsetti et al.(2008):

[19]CDL shows that there is a sixth region, which gives a transmission mechanism similar to the one described by region I for high values of θ. The main idea is that if endowments are expected to reach a permanently higher level over time, demand exceeds supply in the short-run, increasing relative consumption and appreciating the terms of trade. Because in our set-up shocks bring endowment immediately to its permanent level, we do not get this region. But, we do get it in the production economy version of this two-sector model, which we show in the numerical results section.

Under incomplete markets, home agents become relatively wealthier following a positive home supply shock. Given that consumption is home biased, this positive wealth effect leads to a stronger increase in consumption of home goods, increasing the relative price of home goods. Since price elasticity of tradables is very low, a rise in the relative price of home goods cannot generate substitution away from home goods to foreign goods, thus the income effect dominates the substitution effect and terms of trade appreciates. The strong rise in relative home wealth also appreciates the relative price of non-tradables. In this region, 'negative transmission' of a positive supply shock does not rely on the presence of a non-tradable sector.

To see this more clearly, consider the case where all goods are tradable, such that real exchange rate dynamics are solely driven by the terms of trade. Equation (1.25) shows that when $\gamma = 1$, $\hat{Q}_t = (2\nu - 1)\widehat{TOT}_t$. If trade elasticity is sufficiently low such that $\theta < 1 - \frac{1}{2\nu}$, terms of trade appreciates in response to a positive supply shock at home (see equation (1.27)), which entails an appreciation of the real exchange rate for $\nu > \frac{1}{2}$. On the other hand, the same shock leads to an increase in relative consumption for $\theta < 1 - \frac{1}{2\nu}$, implying a negative correlation between $\hat{C}_t - \hat{C}_t^*$ and $\hat{Q}_t$.

Region V: High trade elasticity

In this region, given by $\theta > \theta_1^*$, the mechanism that generates the conditional negative correlation between relative consumption and real exchange rates is the one emphasised by Benigno and Thoenissen (2008): In the absence of complete markets, a positive supply shock in the home tradable sector implies that home agents become relatively wealthier, which in turn increases the demand for non-tradables in the home country. Given the fixed supply of non-tradables, this increase in demand puts an upward pressure on the price of home non-tradables, more so if the elasticity of substitution between tradables and non-tradables, κ, is low so that the (negative) substitution effect on the demand for non-tradables is weaker than the (positive) income effect. The rise in the relative non-tradable price, in turn, appreciates the real exchange rate (See equations (1.28) and (1.24)). For this mechanism to yield an unconditional negative cross correlation between relative consumption and real exchange rate, it is crucial that tradable sector shocks are sufficiently larger than non-tradable sector shocks.

To build some intuition for why this mechanism is valid for high trade elasticity, consider the other regions where trade elasticity is lower than θ_1^*. For region IV, i.e. $\theta_2^* < \theta < \theta_1^*$, wealth effects of an uninsured positive tradable endowment shock are strong enough to appreciate the relative price of non-tradables. On the other hand, due to home bias in consumption, any increase in the supply of home tradable goods should be absorbed mostly by home agents. When trade elasticity is lower, this implies that the price of home goods should fall by much more to clear the market. Hence, in this region, the depreciation in the terms of trade dominates the appreciation in the relative non-tradables price and the real exchange rate depreciates following the shock, resulting in a positive transmission.

In region III, i.e. for $1 - \frac{1}{2\nu} < \theta < \theta_2^*$, the depreciation of the terms of trade in response to a favourable supply shock is large enough to generate a negative income effect, which in turn would curb the demand for non-tradables and give rise to a depreciation in the relative non-tradable price rather than an appreciation, again leading to a positive transmission where relative consumption and real exchange rate both rise following an increase in tradable goods endowment.

1.3.3 Incomplete markets: International trade in home and foreign currency bonds

In this section, we consider a small departure from the single non-contingent bond economy and look at the risk sharing implications of international trade in nominal bonds denominated in home and foreign currency in the presence of sectoral endowment shocks in each country. Endogenous trade in bonds lets agents hedge ex-ante against the relative consumption risk caused by country-specific shocks. Given that there are two independent assets and four different sources of relative consumption risk (tradable and non-tradable sector shocks in each country), this asset market structure represents an incomplete market set-up. Therefore we would expect the degree of risk sharing provided by trade in nominal bonds to fall somewhere in between the degree of risk sharing provided by trade in a single non-contingent bond and that provided by trade in a complete set of contingent claims. Then the main question is whether the two bond set-up is closer to the single bond set-up so that country-specific supply shocks can still generate changes in relative wealth strong enough to account for the consumption-real exchange rate anomaly, or whether it is closer to the complete market set-up which implies a counterfactual high correlation between relative consumption and real exchange rate.

To answer this question we first solve for the optimal bond portfolio and characterise the policy functions for relative consumption and real exchange rate consistent with this portfolio position. Then we compare relative consumption and real exchange rate responses to supply shocks under this set-up with those under the non-contingent bond and complete market set-ups. We show that whether the risk sharing implications of trade in two nominal bonds is closer to one or the other depends crucially on the properties of relative bond returns, which are in turn determined by the monetary policy specification.

Partial equilibrium analysis of optimal bond portfolio

In order to demonstrate the hedging motives of investors, we first derive a partial equilibrium expression for optimal bond positions as in Benigno and Nisticò (2009) and Coeurdacier and Gourinchas (2009). Specifically, we use the first order approximation to the model equations to evaluate the portfolio orthogonality condition given by (1.18). The partial equilibrium solution for optimal steady-state foreign bond holdings can be written as:

$$\tilde{\alpha}_F = -\frac{1}{2(1-\beta)}\left\{\begin{array}{c} \gamma\frac{Cov_t[\Lambda^T_{Y,t+1},\hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} + (1-\gamma)\frac{Cov_t[\Lambda^N_{Y,t+1},\hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} \\ + \left(\frac{\rho-1}{\rho}\right)\frac{Cov_t[\Lambda_{Q,t+1},\hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} \end{array}\right\}, \tag{1.29}$$

where $\Lambda^T_{Y,t+1} \equiv (E_{t+1}-E_t)\sum_{j=0}^{\infty}\beta^j\Delta\hat{Y}^R_{T,t+1+j}$ and $\Lambda^N_{Y,t+1} \equiv (E_{t+1}-E_t)\sum_{j=0}^{\infty}\beta^j\Delta\hat{Y}^R_{N,t+1+j}$ denote relative (non-financial) income risk in both sectors and $\Lambda_{Q,t+1} \equiv E_{t+1}\sum_{j=0}^{\infty}\beta^j\Delta\hat{Q}_{t+1+j} - E_t\sum_{j=0}^{\infty}\beta^j\Delta\hat{Q}_{t+1+j}$ denotes real exchange rate risk. We define $\hat{Y}^R_T$ and $\hat{Y}^R_N$, as $\hat{Y}^R_{T,t} \equiv \hat{Y}_H - \hat{Y}^*_F - \widehat{TOT}_t$ (relative tradable income adjusted for relative tradable prices) and $\hat{Y}^R_{N,t} \equiv \hat{Y}_{N,t} - \hat{Y}^*_{N,t} - \hat{P}^N_t$ (relative non-tradable income adjusted for relative non-tradable prices).

Equation (1.29) shows that the foreign bond portfolio, $\tilde{\alpha}_F$, depends on the loadings of relative bond returns on relative income and real exchange rate risk. It is optimal to go long in foreign bond (and short in home bond) if foreign bonds pay more when relative income is lower at home or when home consumption basket is more expensive. That is, $Cov_t(\Lambda^T_{Y,t+1},\hat{r}_{x,t+1}) < 0$, $Cov_t(\Lambda^N_{Y,t+1},\hat{r}_{x,t+1}) < 0$ and $Cov_t(\Lambda_{Q,t+1},\hat{r}_{x,t+1}) < 0$ for $\rho > 1$ imply a long position in foreign currency bonds, i.e. $\tilde{\alpha}_F > 0$.[20]

Using the property of the model that expected returns are zero up to a first order approximation, i.e. $E_t\hat{r}_{x,t+1} = 0 + O(\varepsilon^2)$, we can write relative bond returns, $\hat{r}_{x,t+1}$,

[20]Note that terms of trade and relative non-tradable price affect relative consumption risk through two channels; first by affecting the value of non-financial income in each country and second by affecting the price of the consumption basket.

as the surprises in the nominal exchange rate:

$$\hat{r}_{x,t+1} = \hat{S}_{t+1} - E_t\hat{S}_{t+1} + O(\varepsilon^2). \tag{1.30}$$

Therefore, loading factors and equilibrium portfolios depend crucially on the behaviour of the nominal exchange rate, which in turn is determined by policy specification.

Portfolio allocation and risk sharing under domestic tradable price stabilisation

Assuming monetary policy in each country stabilises respective domestic tradable prices, excess return on foreign bonds is given by the terms of trade:

$$\hat{P}_{H,t} = \hat{P}^*_{F,t} = 0 \Rightarrow \hat{S}_t = \widehat{TOT}_t \Rightarrow \hat{r}_{x,t} = \widehat{TOT}_t - E_{t-1}\widehat{TOT}_t. \tag{1.31}$$

In this case, due to the monetary policy rule, nominal bonds act like bonds indexed to domestic tradable price index.

To get the analytical solution for the bond portfolio, we characterise closed form expressions for the two components of the portfolio orthogonality condition, real exchange rate adjusted relative consumption and relative bond returns, in terms of the structural shocks and the excess return on portfolio $\tilde{\alpha}_F\hat{r}_{x,t}$. Assuming $\delta_T = 1$, $\delta_N = \delta < 1$ we get the following:[21]

$$\begin{aligned}\hat{C}_t - \hat{C}^*_t - \frac{\hat{Q}_t}{\rho} \quad = \quad & \psi_{rcq}\widehat{NFA}_{t-1} + \frac{\Gamma_3}{\kappa\rho(1+2\nu(\theta-1))}(\hat{Y}_{H,t} - \hat{Y}_{F,t}) \\ & + \frac{(1-\beta)(1-\gamma)(\kappa\rho-1)}{(1-\beta\delta)\kappa\rho}(\hat{Y}_{N,t} - \hat{Y}^*_{N,t}) \\ & + \frac{(1-\beta)\Gamma_1}{\kappa\rho\gamma(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t},\end{aligned} \tag{1.32}$$

where $\psi_{rcq} \equiv \frac{(1-\beta)\Gamma_1}{\beta\kappa\rho\gamma(1-\nu)(1+2\nu(\theta-1))}$ and $\Gamma_3 \equiv (2\theta\nu - 1)(\gamma\kappa\rho + 1 - \gamma) - \kappa(2\nu - 1)$. $\Gamma_3 > 0$

[21] We first consider the case with $\delta_N = \delta < 1$, instead of setting $\delta_N = 1$ as we do in the analysis of the non-contingent bond economy. We do this to understand how relative bond returns (terms of trade) responds to non-tradable shocks. Because when $\delta_N = 1$, terms of trade is independent of non-tradable sector shocks.

for $\theta > \theta_3^* \equiv \frac{1}{2\nu} + \frac{1}{2\nu}\frac{\kappa(2\nu-1)}{\gamma\kappa\rho+1-\gamma}$. Note that $1-\frac{1}{2\nu} < \theta_3^* < \theta_1^*$ (see Figure 1.1).

$$\begin{aligned}\hat{r}_{x,t} &= \widehat{TOT}_t - E_{t-1}\widehat{TOT}_t = \frac{1}{1+2\nu(\theta-1)}(u_{H,t}-u_{F,t}) \\ &+\frac{\beta(1-\gamma)(1-\delta)(2\nu-1)(\kappa\rho-1)}{(1-\beta\delta)\Gamma_1}(u_{N,t}-u^*_{N,t}) \\ &-\frac{(1-\beta)(2\nu-1)}{\gamma(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t}.\end{aligned} \tag{1.33}$$

Consider first real exchange rate adjusted relative consumption and excess returns under the zero-portfolio solution ($\tilde{\alpha}_F = 0$) to build intuition for the optimal bond position. The zero-portfolio solution corresponds to the solution that would arise when agents can only trade in a single non-contingent bond. First note that for $\nu > \frac{1}{2}$, hedging against non-tradable endowment shocks requires a short position in foreign bonds irrespective of the substitutability between tradables and non-tradables or any other parameter. On the other hand, optimal hedge against tradable endowment shocks depends crucially on the value of trade elasticity in line with the arguments following Figure 1.1 . For values of θ in region I, a positive tradable endowment shock leads to an increase in $\hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho}$ and fall in $\hat{r}_{x,t}$, pulling the equilibrium portfolio towards a long position in foreign bonds. For values of θ that lie in region V, both $\hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho}$ and $\hat{r}_{x,t}$ increase following a positive tradable endowment shock, which makes it optimal to go short in foreign bonds.[22]

In what follows, to simplify algebra and facilitate the discussion of different cases, we focus on the case where both tradable and non-tradable endowment shocks have unit root, $\delta_T = \delta_N = \delta = 1$ as we do in the analysis in section 1.3.2. Solving equations (1.32), (1.33) and the portfolio orthogonality condition given in (1.18) under this assumption implies the following optimal bond portfolio:

$$\tilde{\alpha}_F = -\tilde{\alpha}_H = -\frac{\gamma(1-\nu)\Gamma_3}{(1-\beta)(\gamma\kappa\rho+(1-\gamma))}, \tag{1.34}$$

where $\Gamma_3 \gtrless 0$ for $\theta \gtrless \theta_3^*$. Therefore, the sign of the optimal bond portfolio depends on the value of trade elasticity. For θ belonging to region I, optimal portfolio is long in foreign currency whereas for θ in region V, it is the opposite.[23] Although there are four shocks affecting each country and only two assets that can be internationally

[22]Note that for $\theta = \theta_3^*$, $\Gamma_3 = 0$ and there is perfect risk sharing conditional on tradable endowment shocks even under zero-portfolio. When $\nu = \frac{1}{2}$, $\Gamma_3 = 0$ for $\theta = 1$. This is the knife-edge case described by Cole and Obstfeld: If $\nu = \frac{1}{2}$ and $\theta = 1$, terms of trade ensures complete risk sharing conditional on tradable sector shocks irrespective of the assets that are traded.

[23]This follows from the fact that for $\nu > \frac{1}{2}$, $1 - \frac{1}{2v} < \theta_3^* < \theta_1^*$.

traded, optimal bond portfolio does not depend on the relative variance of different shocks. This is because under the assumption that $\delta = 1$, terms of trade is independent of non-tradable endowment shocks as shown in equation (1.33). Hence, agents can choose a portfolio to insure themselves perfectly against tradable sector shocks, without being subject to unwanted valuation effects conditional on non-tradable endowment shocks.[24] For more general parameter values, terms of trade loads on relative non-tradable income shocks, hence equilibrium portfolio becomes a complicated object that depends on the relative variance of tradable versus non-tradable income shocks. However, as we discuss below, even in this case, portfolios will be biased more towards hedging against tradable income shocks as terms of trade loads weakly on non-tradable income shocks even when tradable and non-tradable goods are complements in consumption.

Optimal portfolio allocation has important implications for the relative consumption and real exchange rate dynamics in response to tradable endowment shocks. The solution for relative consumption and real exchange rate in this case becomes:

$$\hat{C}_t - \hat{C}_t^* = \psi_c^{nc} NFA_{t-1} + \frac{\gamma\kappa(2\nu - 1)}{\Gamma_1}(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + (1-\gamma)(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*), \quad (1.35)$$

$$\hat{Q}_t = -\psi_q^{nc} NFA_{t-1} + \frac{\rho}{\Gamma_1}\gamma\kappa(2\nu - 1)(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + \frac{(1-\gamma)}{\kappa}(\hat{Y}_{N,t} - \hat{Y}_{N,t}^*), \quad (1.36)$$

where ψ_c^{nc} and ψ_q^{nc} are as defined in section 1.3.2. Comparison of equations (1.35) and (1.36), with equations (1.23) and (1.24), which give the solution in the case of a single bond, shows clearly that $\hat{C}_t - \hat{C}_t^*$ and $\hat{Q}_t$ are no longer negatively correlated conditional on tradable endowment shocks. Indeed, the response of $\hat{C}_t - \hat{C}_t^*$ and $\hat{Q}_t$ to tradable endowment shocks in this two bonds set-up is exactly the same as that under the complete market set-up given by equations (1.20). On the other hand, due to the fact that terms of trade is independent of non-tradable endowment shocks, agents cannot use bonds to hedge against these shocks and hence relative consumption and real exchange rate response to non-tradable sector shocks is the same as that under the single bond set-up.

Hence, when excess returns are given by the terms of trade, trade in two nominal bonds ensures perfect risk sharing across countries conditional on tradable endowment shocks for all possible values of θ. Thus, when central bank stabilises the domestic

[24]The appendix shows the decomposition of the equilibrium portfolio given in (1.34) in terms of the loadings of excess returns on relative non-financial income risk by sector and real exchange rate risk in line with (1.29).

tradable price index, a slight departure from a single bond economy to a two bonds economy kills the wealth effects associated with tradable income shocks.

How do the risk sharing implications of bonds change when terms of trade loads on non-tradable endowment shocks, that is when $\delta < 1$? A closer inspection of equation (1.33) suggests that even under a general parameter setting, terms of trade loads more strongly on tradable sector shocks compared to non-tradable shocks. This is intuitive as the terms of trade is directly linked to relative supply of tradables whereas it is only indirectly affected by changes in the relative supply of non-tradables through the complementarity/substitutability between tradables and non-tradables. Thus, bonds would be mainly used to hedge against the risks they can span more effectively, implying high insurance in response to tradable income shocks, which implies high insurance overall.[25]

Portfolio allocation and risk sharing under consumer price stabilisation

When monetary policy in each country stabilises the respective consumer price index, excess return on foreign bonds is given by the real exchange rate:

$$\hat{P}_t = \hat{P}_t^* = 0 \Rightarrow \hat{S}_t = \hat{Q}_t \Rightarrow \hat{r}_{x,t} = \hat{Q}_t - E_{t-1}\hat{Q}_t. \tag{1.37}$$

In this case, nominal bonds act like CPI-indexed bonds because of the monetary policy specification. For $\delta_T = 1$ and $\delta_N = 1$, excess return on foreign bonds is given by:

$$\begin{aligned}\hat{r}_{x,t} &= \hat{Q}_t - E_{t-1}\hat{Q}_t = -\left[\frac{(1-\gamma)(2\theta\nu - 1) - \kappa(2\nu - 1)}{\kappa(1 + 2\nu(\theta - 1))}\right](u_{H,t} - u_{F,t}) \\ &\quad + \frac{1-\gamma}{\kappa}(u_{N,t} - u^*_{N,t}) - \frac{(1-\beta)[4\theta\nu(1-\nu)(1-\gamma) + \kappa(2\nu-1)^2]}{\gamma\kappa(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t}.\end{aligned} \tag{1.38}$$

The other component of the portfolio orthogonality condition, real exchange rate adjusted relative consumption, is still given by equation (1.32), where $\tilde{\alpha}_F\hat{r}_{x,t}$ is suitably adapted to the new policy specification and $\delta = 1$ is imposed to make it compatible with (1.38).

To build intuition for the optimal bond position, we consider the zero-portfolio solution once again. As we established during our discussion of the non-contingent bond economy, real exchange rate appreciates in response to a positive supply shock

[25]Numerical results for the endowment economy with stationary shocks ($\delta < 1$) show that when excess returns are given by the terms of trade, the cross-correlation between relative consumption and real exchange rate is robustly high (i.e. 0.999) regardless of the calibration of parameters.

in home tradables sector for θ taking values in regions I and V. For these values of θ, real exchange rate adjusted relative consumption also increases in response to the same shock. Therefore, hedging against the consumption risk coming from tradable sector shocks require a long position in foreign currency for values of θ in region I and and region V (Figure 1.1).

The optimal hedge against non-tradable income shocks depends on whether tradable and non-tradable goods are substitutes or complements in consumption. Under the former specification, i.e. $\kappa\rho < 1$, relative consumption adjusted by the real exchange rate falls in response to a positive non-tradable income shock (see equation (1.32)), while the opposite is true for $\kappa\rho > 1$. When the two goods are complements, demand for tradables also increase following a positive non-tradable supply shock. Given that the supply of tradable goods is fixed, this leads to an excess demand for tradables, which appreciates the terms of trade and leads to a fall in real exchange rate adjusted consumption differential. On the other hand, under the zero portfolio solution, real exchange rate depreciates in response to an increase in relative home non-tradable income irrespective of any parameter specification (see equation (1.38)). Therefore, hedging against the consumption risk coming from non-tradable sector shocks requires a long position in foreign currency when $\kappa\rho < 1$, and a short position when $\kappa\rho > 1$.

Since $\hat{r}_{x,t}$ is a complicated expression even for permanent shocks, we impose the additional restriction that preferences for tradable goods are symmetric ($\nu = \frac{1}{2}$) to be able to display analytical results for optimal portfolio allocation and show its implications for risk sharing. Note that for $\nu = \frac{1}{2}$, real exchange rate movements are driven only by movements in the relative price of non-tradables, i.e. $\hat{Q}_t = (1-\gamma)\hat{P}_t^N$.

Evaluating the portfolio orthogonality condition using (1.32) and (1.38) under the parameter restrictions $\delta_T = \delta_N = 1$ and $\nu = \frac{1}{2}$, we get the following optimal foreign bond position:[26]

$$\tilde{\alpha}_F = \frac{\gamma\left[(\theta-1)^2(\gamma\kappa\rho+(1-\gamma))\frac{\sigma_T^2}{\sigma_N^2} - \theta^2(1-\gamma)(\kappa\rho-1)\right]}{2(1-\beta)(1-\gamma)\theta^2\rho}. \tag{1.39}$$

[26]To compare this foreign currency position with the one obtained under domestic tradable price stabilisation, impose $\nu = \frac{1}{2}$ in equation (1.34):

$$\tilde{\alpha}_F = -\frac{\gamma(\theta-1)}{2(1-\beta)}$$

The optimal foreign bond position under domestic tradable price stabilisation is thus negative for $\theta > 1$.

For the reasons discussed above, assuming complementarity between tradables and non-tradables, i.e. $\kappa\rho < 1$, is sufficient to have a long position in foreign bonds. If tradable sector shocks are sufficiently large compared to non-tradable sector shocks, optimal portfolio will still be a long position in foreign currency also for $\kappa\rho > 1$.[27]

Given the optimal portfolio allocation in (1.39), relative consumption and real exchange rate dynamics are as follows:

$$\begin{aligned} \hat{C}_t - \hat{C}_t^* &= \psi_{rc}\widehat{NFA}_{t-1} \\ &+\frac{1}{\Gamma_4}\left[\begin{array}{c} \gamma\kappa\rho\theta(\theta-1)(\hat{Y}_{H,t}-\hat{Y}_{F,t}) \\ +\left(\theta^2(1-\gamma)+(\theta-1)^2(\gamma\kappa\rho+1-\gamma)\frac{\sigma_T^2}{\sigma_N^2}\right)(\hat{Y}_{N,t}-\hat{Y}_{N,t}^*) \end{array}\right] \end{aligned} \tag{1.40}$$

$$\begin{aligned} \hat{Q}_t &= -\psi_q\widehat{NFA}_{t-1} \\ &+\frac{\rho(1-\gamma)}{\Gamma_4}\left[-\theta(\theta-1)(\hat{Y}_{H,t}-\hat{Y}_{F,t})+\theta^2(\hat{Y}_{N,t}-\hat{Y}_{N,t}^*)\right] \end{aligned} \tag{1.41}$$

where $\Gamma_4 \equiv (\gamma\kappa\rho + 1 - \gamma)\left(\theta^2 + (\theta-1)^2\frac{\sigma_T^2}{\sigma_N^2}\right) > 0$ for all possible parameter values. Equations (1.40) and (1.41) show that relative consumption and real exchange rate are negatively correlated conditional on tradable endowment shocks for all possible values of θ, given our parameter restrictions $\delta_T = \delta_N = 1$ and $\nu = \frac{1}{2}$. This is because when relative bond returns are given by the real exchange rate, bonds are almost equally good in hedging against the relative consumption risks coming from tradable and non-tradable sector shocks. Therefore, optimal bond portfolio in this case is torn between hedging against tradable and non-tradable shocks, which in turn implies that the consumer cannot insure fully against any of these shocks. This gives rise to international wealth transfers that imply lower risk sharing compared to the case where relative bond returns are equal to the terms of trade.[28]

Even though the parameter restrictions we impose here might seem somewhat

[27] See the appendix for a discussion of the loading factors that show the breakdown of the optimal portfolio according to different hedging motives.

[28] We should acknowledge that the parameter restrictions we impose here, particularly the restriction that $\nu = \frac{1}{2}$, make it easier to get the negative comovement between real exchange rate and relative consumption conditional on tradable income shocks. This is because when $\nu = \frac{1}{2}$, real exchange rates move only due to relative non-tradable prices, which reflect the income effect more strongly. When $\nu > \frac{1}{2}$ and $\theta > 1 - \frac{1}{2\nu}$ such that terms of trade depreciates in response to tradable endowment shocks, it will be more difficult to get the real exchange rate to appreciate following the appreciation in relative non-tradable prices as there will be an offsetting effect coming from terms of trade. Nevertheless, numerical results show that this set-up can still generate a negative correlation between relative consumption and real exchange rate conditional on tradable sector shocks for $\nu > \frac{1}{2}$ and θ in region V.

limited, analytical results help us compare the equilibrium outcomes in the single bond economy with that in the two bonds economy and facilitates the understanding of the hedging properties of bonds under the two simple policy rules we consider. These results highlight the parameters that are important for optimal portfolios and the transmission of shocks and guide us in the calibration of the model in the numerical analysis.

To summarise, moving away from trade in a single non-contingent bond to trade in two bonds makes a huge difference for international risk sharing and transmission of supply shocks. When monetary policy rules are such that relative bond returns are associated with the terms of trade, sectoral supply shocks do not create a meaningful tension on equilibrium portfolios and hence agents can ensure high risk sharing by taking the correct portfolio position. On the other hand, when relative bond returns are given by the real exchange rate, trade in bonds ensures less risk sharing because the real exchange rate loads equally well on both tradable and non-tradable income risks, which implies that having a portfolio to hedge against one source of shock would imply unwanted valuation effects conditional on the other. Whether this set-up can generate reasonable portfolio positions alongside a negative relative consumption-real exchange rate correlation is a quantitative question which we explore later in section 1.4.

1.3.4 Including demand shocks

In the previous section, we showed that even a small move away from the non-contingent bond set-up leads to very high risk sharing in response to supply shocks, especially when agents can have claims to terms of trade (can trade in nominal bonds when domestic tradable price index is stabilised in each country). The insight from the analysis on the nominal bonds under CPI- targeting is that we can limit the risk sharing implied by endogenous asset trade if excess returns load equally well on all sources of risks and different risks imply different portfolio positions. In this case, equilibrium portfolios will depend on the relative size of shocks and valuation effects will have the potential to impede risk sharing depending on the type of shock that hits the economy.

In this section, we introduce shocks to the anticipated component of tradable endowments- 'news shocks', which act as demand shocks in our two-sector endowment model and show how these shocks can change the risk sharing properties of nominal bonds conditional on supply shocks.[29] We present analytical results only for the case of tradable price targeting since this is the setting under which trade in two bonds brings the equilibrium close to that under complete markets. The intuition we build for this case can be used to understand the case of CPI stabilisation. We discuss the role of demand shocks in detail in the numerical results section.

We assume that tradable endowment process now has a predictable component in each country. $u_{H,t}$ and $u_{F,t}$ are unanticipated home and foreign tradable endowment shocks at time t, $z_{H,t}$ and $z_{F,t}$ are information that arrive at time t about the $t+1$ values of home and foreign tradable endowments. When there is positive news today (an increase in $u_{zh,t}$), agents anticipate home tradable endowment to be higher in the next period. The formulation we use is similar to Colacito and Croce (2010):[30]

$$\log Y_{i,t} = \delta_T \log Y_{i,t-1} + \log z_{i,t-1} + u_{i,t}, \tag{1.42}$$

$$\log z_{i,t} = \delta_z \log z_{i,t-1} + u_{Zi,t} \text{ for } i = H, F, \tag{1.43}$$

where $0 \leq \delta_T < 1$, $0 \leq \delta_z < 1$, $u_{H,t}, u_{F,t}, u_{ZH,t}, u_{ZF,t}$ are i.i.d. shocks with $Var(u_H) = Var(u_F) = \sigma_T^2$ and $Var(u_{ZH}) = Var(u_{ZF}) = \sigma_Z^2$. The stochastic processes for non-tradable endowments are still given by equations (1.9).[31]

To understand how the presence of news shocks affects optimal portfolios, consider the general equilibrium expressions for the two components of the portfolio orthogonality condition given by (1.18), where we again assume that $\delta_N = \delta_T = 1$ for ease of exposition:

[29] We also derive analytical results for i-pod shocks as in Coeurdacier et al. (2007), which can be found in the appendix.

[30] Colacito and Croce (2010) consider endowment processes which grow at a constant rate and follow an integrated process of order 1 in each country. Schmitt-Grohe and Uribe (2008) introduce a more general shock structure in which each structural shock has an unanticipated and anticipated component which can be known up to three quarters in advance. They specify a fully-fledged closed economy RBC model with stationary and non-stationary neutral productivity shocks, non-stationary investment productivity shocks and government spending shocks. Their estimates show that the most important news are the shocks to the stationary component of productivity anticipated 3 quarters in advance. Since in our model a period corresponds to one year, specifying one-period ahead anticipation shocks is roughly consistent with this finding.

[31] We initially introduce "news" only to the tradable sector, because trade in nominal bonds under domestic tradable price stabilisation ensures too much risk sharing conditional on tradable endowment shocks. In the numerical part, we consider news to both sectors.

$$
\begin{aligned}
\hat{C}_t - \hat{C}_t^* - \tfrac{\hat{Q}_t}{\rho} &= \psi_{rcq}\widehat{NFA}_{t-1} \\
&+\tfrac{\Gamma_3}{\kappa\rho(1+2\nu(\theta-1))}\left[(\hat{Y}_{H,t}-\hat{Y}_{F,t})+\tfrac{\beta}{1-\beta\delta_z}(\hat{z}_{H,t}-\hat{z}_{F,t})\right] \\
&+\tfrac{(1-\gamma)(\kappa\rho-1)}{\kappa\rho}(\hat{Y}_{N,t}-\hat{Y}^*_{N,t}) \\
&+\tfrac{(1-\beta)\Gamma_1}{\gamma\kappa\rho(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t},
\end{aligned} \tag{1.44}
$$

$$
\begin{aligned}
\hat{r}_{x,t} &= \widehat{TOT}_t - E_{t-1}\widehat{TOT}_t \\
&= \tfrac{1}{1+2\nu(\theta-1)}\left((u_{H,t}-u_{F,t})-\tfrac{\beta(2\nu-1)\Gamma_3}{(1-\beta\delta_z)\Gamma_1}(u_{ZH,t}-u_{ZF,t})\right) \\
&-\tfrac{(1-\beta)(2\nu-1)}{\gamma(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t},
\end{aligned} \tag{1.45}
$$

where Γ_1 and Γ_3 are as defined before.[32] Note that the coefficients on unanticipated tradable and non-tradable shocks and the excess return on the portfolio ($\tilde{\alpha}\hat{r}_{x,t}$) are identical to the ones given in equations (1.32) and (1.33). Shocks to the anticipated component of tradable endowments affect real exchange rate adjusted relative consumption in the same way as unanticipated shocks, only discounted by $\frac{\beta}{1-\beta\delta_z}$. In other words, for $\theta > \theta_3^*$ such that $\Gamma_3 > 0$, or for $\theta < 1 - \frac{1}{2\nu}$, $\hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho}$ rises in response to an increase in both the anticipated and unanticipated components of tradable endowments.[33]

On the other hand, as shown by equation (1.45), terms of trade responds differently to anticipated and unanticipated shocks. For $\nu > \frac{1}{2}$ and $\theta > \theta_3^*$, a positive shock to the predictable component of tradables endowment, which increases relative consumption gap in favour of home agents, appreciates the terms of trade. This is because after receiving the positive news about future endowment, home agents increase their demand for tradables in the current period. Given that the supply of tradables is still fixed when agents receive the news, this leads to an excess demand for tradables in the current period, which in turn appreciates the terms of trade as consumption is home biased. Since news about future supply conditions increase current demand and appreciate the terms of trade, news shock act as a demand shock.[34]

Due to the fact that real exchange rate adjusted consumption differential and excess returns are positively correlated conditional on unanticipated shocks but negatively correlated conditional on anticipated shocks, relative variance of the two shocks

[32]See Table 1.2 for a summary of the definitions of the convoluted parameters.

[33]The extent to which anticipated shocks affect $\hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho}$ is determined crucially by δ_z. As δ_z increases, $\frac{\beta}{1-\beta\delta_z}$ increases, amplifying the response of relative consumption to anticipated shocks.

[34]Note that when $\theta < 1 - \frac{1}{2\nu}$, both anticipated and unanticipated endowment shocks work as demand shocks, because terms of trade appreciate following an unanticipated increase in tradable endowment in this region of θ.

will determine the sign of optimal portfolio as displayed below:

$$\tilde{\alpha}_F = -\frac{\gamma(1-\nu)\Gamma_3}{(1-\beta)(\gamma\kappa\rho+1-\gamma)}\left(1-\frac{\Gamma_3}{\Gamma_1}\frac{\beta^2(2\nu-1)\frac{\sigma_Z^2}{\sigma_T^2}}{(1-\beta\delta_z)^2}\right). \tag{1.46}$$

As shown in (1.46), the optimal bond portfolio in the presence of news shocks is the optimal bond portfolio given in (1.34) plus an expression that depends on the relative variance of anticipated shocks with respect to unanticipated shocks to tradables endowment. Therefore, for $\nu > \frac{1}{2}$, $\theta > \theta_3^*$ and a sufficiently high σ_Z^2/σ_T^2, i.e. $\sigma_Z^2/\sigma_T^2 > \frac{\Gamma_1(1-\beta\delta_z)^2}{(2\nu-1)\Gamma_3\beta^2} \equiv RV_1^*$, it is optimal to have a long position in foreign bonds rather than a short position which would be optimal to hedge againts unanticipated endowment shocks. This would then imply adverse valuation effects in the face of unanticipated shocks to tradable endowments and potentially impede risk sharing. In this case, endogenous trade in nominal bonds will not be enough to hedge perfectly against any of these two shocks. Thus there will be deviations from the perfect risk sharing condition, which might potentially give rise to a negative correlation between relative consumption and real exchange rate conditional on unanticipated supply shocks in the tradables sector.

The general equilibrium expressions for $\hat{C}_t - \hat{C}_t^*$ and $\hat{Q}_t$ are very complicated especially after plugging in the optimal portfolio. Thus to show the risk sharing implications of nominal bonds in the presence of news shocks, we report the solution for the relative consumption and real exchange rate as the zero-portfolio (or non-contingent bond) solution plus the response to the excess return on the portfolio, $\tilde{\alpha}_F\hat{r}_{x,t}$, which is characterised by equations (1.45) and (1.46) in equilibrium.

$$\begin{aligned}\hat{C}_t - \hat{C}_t^* &= \psi_c^{nc}\widehat{NFA}_{t-1} + \frac{\gamma(2\theta\nu-1)}{1+2\nu(\theta-1)}(\hat{Y}_{H,t}-\hat{Y}_{F,t}) \\ &+\frac{\beta}{1-\beta\delta_z}\frac{\gamma 4\theta\nu(1-\nu)}{1+2\nu(\theta-1)}\frac{\Gamma_3}{\Gamma_1}(\hat{z}_{H,t}-\hat{z}_{F,t}) \\ &+(1-\gamma)(\hat{Y}_{N,t}-\hat{Y}_{N,t}^*) + \frac{4\theta\nu(1-\beta)}{(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t}\end{aligned} \tag{1.47}$$

$$
\begin{aligned}
\hat{Q}_t \;=\; & -\psi_q^{nc}\widehat{NFA}_{t-1} - \left[\frac{(1-\gamma)(2\theta\nu-1)-\kappa(2\nu-1)}{\kappa(1+2\nu(\theta-1))}\right](\hat{Y}_{H,t}-\hat{Y}_{F,t}) \qquad (1.48)\\
& -\frac{\beta}{1-\beta\delta_z}\frac{\left[4\theta\nu(1-\nu)(1-\gamma)+\kappa(2\nu-1)^2\right]}{\kappa(1+2\nu(\theta-1))}\frac{\Gamma_3}{\Gamma_1}(\hat{z}_{H,t}-\hat{z}_{F,t})\\
& +\frac{1-\gamma}{\kappa}(\hat{Y}_{N,t}-\hat{Y}^*_{N,t}) - \frac{(1-\beta)[4\theta\nu(1-\nu)(1-\gamma)+\kappa(2\nu-1)^2]}{\gamma\kappa(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t}
\end{aligned}
$$

Table 1.3 gives the signs of the responses of relative consumption and real exchange rate to anticipated and unanticipated endowment shocks under certain parameter restrictions to illustrate how the introduction of demand shocks might affect the comovement of these variables through an adverse valuation channel. We construct Table 1.3 under the assumptions that $\delta_T = \delta_N = 1$, $\nu > \frac{1}{2}$, $\theta > \theta_1^*$ and $\sigma_Z^2/\sigma_T^2 > RV_1^*$ so that $\tilde{\alpha}_F > 0$ as suggested by the news shocks.[35]

	$\hat{C}_t - \hat{C}_t^*$	$\hat{Q}_t$	$\tilde{\alpha}_F\hat{r}_{x,t}$
$\hat{Y}_{H,t} - \hat{Y}_{F,t}$	$+$	$-$	$+$
$\hat{z}_{H,t} - \hat{z}_{F,t}$	$+$	$-$	$-$
$\hat{Y}_{N,t} - \hat{Y}^*_{N,t}$	$+$	$+$	0
$\tilde{\alpha}_F\hat{r}_{x,t}$	$+$	$-$	

Table 1.3: Impact responses of relative consumption and real exchange rate to relative supply and demand (news) shocks with trade in two bonds

Notes: This table gives the sign of the impact responses of relative consumption, $\hat{C}_t - \hat{C}_t^*$, and real exchange rate, $\hat{Q}_t$, to the relative shocks described in the first column assuming that $\nu > \frac{1}{2}$, $\theta > \theta_1^*$ and $\sigma_Z^2/\sigma_T^2 > RV_1^*$ so that $\tilde{\alpha}_F > 0$.

Under the zero-portfolio/non-contingent bond economy solution ($\tilde{\alpha}_F = 0$), real exchange rate and relative consumption are negatively correlated in response to both supply and demand shocks in the tradable sector. As shown in Table 1.3 , $\hat{Q}_t$ is negatively related to $\tilde{\alpha}_F\hat{r}_{x,t}$, which means that the real exchange rate appreciates with an increase in the excess return on portfolio.

Therefore, for a given short position in foreign bonds- as in the case of only unanticipated shocks- an increase in home tradable endowment that depreciates the terms of trade ($\hat{r}_{x,t} \uparrow$), leads to a negative valuation effect ($\tilde{\alpha}_F\hat{r}_{x,t} \downarrow$), which in turn offsets any positive wealth effect that would arise under the non-contingent bond economy in response to this shock and hence improve international risk sharing.

However, if news shocks are sufficiently large, optimal bond portfolio switches sign, i.e. $\tilde{\alpha}_F > 0$, and a positive tradable endowment shock that depreciates the terms of trade, implies a positive wealth transfer to the home agent, ($\tilde{\alpha}_F\hat{r}_{x,t} \uparrow$), which in turn

[35] We remind the reader that $\theta > \theta_1^*$ implies $\theta > \theta_3^*$ because $\theta_1^* > \theta_3^*$ for $\nu > \frac{1}{2}$.

appreciates the real exchange rate even more than it would under the non-contingent bond economy and impede risk sharing. Therefore, for sufficiently large news shocks, real exchange rate and relative consumption are negatively correlated conditional on tradable sector supply shocks as well as demand shocks.

As we explore numerically in the next section, even if news shocks are not large enough to overturn the sign of the optimal portfolio, they can still limit risk sharing conditional on unanticipated endowment shocks by changing the size of the optimal portfolio.

1.4 Numerical analysis in a calibrated two-country, two-sector RBC model

In this section, we calibrate a two-country, two-sector production economy model with capital accumulation along the lines of Benigno and Thoenissen (2008) and Corsetti et al. (2008) and look at the quantitative implications of introducing a second internationally traded asset for optimal portfolios and relative consumption-real exchange rate correlation alongside standard business cycle moments.

We first describe the model briefly, then proceed to the calibration and the discussion of numerical results under various asset market set-ups when there are only unanticipated sectoral productivity shocks. Numerical results confirm the intuition provided by the analytical results regarding the endowment economy that trade in two international bonds brings the equilibrium closer to complete market equilibrium hence implies too much risk sharing compared to the Backus-Smith-Kollmann evidence. Finally we consider implications of introducing news shocks alongside unanticipated shocks.

1.4.1 The model

The model we use for quantitative analysis follows closely Benigno and Thoenissen (2008). Each country specialises in the production of a tradable and a non-tradable intermediate good. Final goods are obtained by combining domestic and foreign tradable inputs with domestic non-tradable inputs. All trade between the two countries is in intermediate goods and final goods are only used for domestic consumption. Capital and labour are immobile across countries.

Producers

Final good producers combine home and foreign intermediate goods, C_T and C_N, according to the CES function given by equation (1.4) to yield the final home consumption good $Y \equiv C$. Tradable intermediate inputs, C_T, are obtained by combining home and foreign intermediates according to (1.5). The intratemporal elasticity of substitution between tradable and non-tradable inputs is given by κ, while θ governs the substitutability between home and foreign tradable inputs. There is home bias in the demand for tradable inputs, i.e. $\nu > \frac{1}{2}$. Price indices corresponding to final output and the output of tradable goods are given by equations (1.6) and (1.7).

Intermediate goods firm in each sector choose labour, capital and investment to maximise the expected discounted value of profits:

$$\max_{K_{i,t+1},L_{i,t},X_{i,t}} E_0 \sum_{t=0}^{\infty} \delta_t \frac{U_C(C_t,(1-L_t))}{U_C(C_0,(1-L_0))} \frac{P_0}{P_t} [P_{i,t}Y_{i,t} - P_t w_t L_{i,t} - P_{H,t}X_{i,t}], \tag{1.49}$$

subject to the production function in each sector,

$$Y_{i,t} = F(A_{i,t}, K_{i,t-1}, L_{i,t}) = A_{i,t} L_{i,t}^{\alpha_i} K_{i,t-1}^{1-\alpha_i}, \tag{1.50}$$

where the subscript i, for $i = H, N$,marks variables associated with tradable and non-tradable sectors. Y_i denotes the output in sector i, w_t is the real wage, $X_{i,t}$ denotes investment by intermediate firms producing sector i. A_i denotes sector-specific total factor productivity, L_i and K_i are labour and capital input used in sector i. It is assumed that investment is in units of the domestic tradable good, hence investment price in both sectors is given by P_H. Aggregate capital accumulation equation is:

$$K_t = (1-\delta)K_{t-1} + X_t. \tag{1.51}$$

Aggregate capital and investment are given simply by $K_t = K_{H,t} + K_{N,t}$ and $X_t = X_{H,t} + X_{N,t}$. Intermediate firms' labour demand functions imply the following wage equation

$$\alpha \frac{P_{H,t}}{P_t} A_t \left(\frac{K_{H,t-1}}{L_{H,t}}\right)^{1-\alpha} = w_t = \alpha_N \frac{P_{N,t}}{P_t} A_{N,t} \left(\frac{K_{N,t-1}}{L_{N,t}}\right)^{1-\alpha_N},$$

while optimal investment is determined simply by:

$$P_{H,t} = E_t m_{t+1} \left\{P_{i,t+1} MPK_{i,t+1} + P_{H,t+1}(1-\delta)\right\}, \ i = H, N.,$$

where m_t is the stochastic discount factor of domestic agents defined as $m_t = \frac{\beta(C_t,1-l_t)U_C(C_{t+1},1-L_{t+1})}{U_C(C_t,1-L_t)} \frac{P_t}{P_{t+1}}$.

Consumers

Consumers behave similarly to what is described in the endowment economy. Representative agent in home economy maximises the expected present discounted value of utility,

$$U_t = E_t \sum_{s=t}^{\infty} \delta_s U(C_s, (1-L_s)), \tag{1.52}$$

where utility now depends on leisure, $1-L$, as well as consumption, C. We modify the endogenous discount factor δ_s accordingly:

$$\delta_{s+1} = \delta_s \beta(C_{As}, 1-L_{As}),\ \delta_0 = 1, \tag{1.53}$$

where C_A is aggregate home consumption and L_A is aggregate leisure and $0 < \beta(C_A, 1-L_A) < 1$. To achieve stationarity under incomplete market specification, we assume $\beta_C(C_A, 1-L_A) \leq 0$ and $\beta_{1-L}(C_A, 1-L_A)$.

As before, we solve the model under alternative asset market structures. Consumer's first order conditions and net foreign asset accumulation equations under each market structure is as described in section 1.2.2, where marginal utility functions are adjusted accordingly, i.e. $U_C(C)$ is replaced by $U_C(C, 1-L)$ and net foreign asset accumulation equations are modified to account the fact that agents also spend their income on investment, $P_{H,t}X_t$. In addition to optimal consumption and portfolio decisions characterised by the first order conditions given in subsection 1.2.2, there is an optimal labour supply decision given by:

$$w_t = \frac{u_{1-L}\left(C_t, (1-L_t)\right)}{u_C\left(C_t, (1-L_t)\right)}.$$

Similar equations hold for the foreign country.

Market clearing

Market clearing for intermediate goods requires:

$$\begin{aligned} Y_{H,t} &= F(A_{H,t}, L_{H,t}, K_{H,t-1}) = C_{H,t} + C^*_{H,t} + X_t, \\ Y_{F,t} &= F(A_{F,t}, L_{F,t}, K_{F,t-1}) = C^*_{F,t} + C_{F,t} + X^*_t, \\ Y_{N,t} &= F(A_{N,t}, L_{N,t}, K_{N,t-1}) = C_{N,t}, \\ Y^*_{N,t} &= F(A^*_{N,t}, L^*_{N,t}, K^*_{N,t-1}) = C^*_{N,t}, \end{aligned}$$

while for final goods we have $Y_t = C_t$ and $Y^*_t = C^*_t$.

Factor market clearing implies,

$$\begin{aligned} L_H + L_N = L, \quad & L_F + L^*_N = L^*, \\ K_H + K_N = K, \quad & K_F + K^*_N = K^*, \end{aligned}$$

while asset market clearing is as described before for the endowment economy. We close the model by two different policy rules as before.

1.4.2 Calibration

We calibrate the model along the lines of BT and CDL assuming symmetry across countries. Our baseline calibration is given by Table 1.4 . Most of the parameter values are the same as the ones used by BT. We are considering three different trade elasticity values, i.e. $\theta = 0.25$, 2.5, 8, to discuss how the introduction of a second internationally traded asset affects each of the transmission mechanisms that can account for the anomaly when there is only one internationally traded bond.

Following BT and CDL and most of the international RBC literature, we assume that preferences are non-separable in consumption and leisure. We use specification used by Backus et al.(1992) and CDL:[36]

$$U(C, 1-l) = \frac{[C^{\omega}(1-l)^{1-\omega}]^{1-\rho} - 1}{1-\rho}, \quad 0 < \omega < 1, \ \rho > 0. \tag{1.54}$$

We calibrate the consumption share in utility, ω, such that at the steady-state, agents devote one-third of time to work. Risk aversion parameter is equal to 2. As

[36] BT calibrates the utility function as in Stockman and Tesar (1995) who use the following form:

$$U(C, 1-l) = \frac{C^{1-\rho}(1-l)^{\eta}}{1-\rho}$$

Parameter	Description	Baseline values
$\bar{\beta}$	Steady-state discount factor	0.96
ρ	Coefficient of constant relative risk aversion	2
ω	Consumption share in utility	0.34
θ	Elasticity of substitution between domestic and foreign goods	0.5, 2.5, 8
κ	Elasticity of substitution between tradables and non-tradables	0.5, 2.5, 8
ν	Preference for domestic goods in the production of tradables	0.72
γ	Preference for tradables in consumption	0.55
$\alpha = \alpha_N$	Labour share in production	0.67
δ	Depreciation rate of capital	0.10
Λ	Productivity shocks (persistence and spill-overs)	$\begin{bmatrix} 0.88 & 0 & 0.22 & 0 \\ 0 & 0.88 & 0 & 0.22 \\ 0 & 0 & 0.30 & 0 \\ 0 & 0 & 0 & 0.30 \end{bmatrix}$
$V(u)$	Variance-covariance matrix of productivity shocks in percent	$\begin{bmatrix} 0.0376 & 0.0159 & 0.0072 & 0.0044 \\ 0.0159 & 0.0376 & 0.0044 & 0.0072 \\ 0.0072 & 0.0044 & 0.0051 & 0.0021 \\ 0.0044 & 0.0072 & 0.0021 & 0.0051 \end{bmatrix}$

Table 1.4: Baseline calibration

in CDL, we specify the endogenous discount factor in line with the period utility function.

$$\beta(C, 1-l) = \frac{1}{1 + \psi[C^{\omega}(1-l)^{1-\omega}]}$$

where we set the Uzawa convergence parameter, ψ, such that the steady state discount factor, $\overline{\beta}$, is equal to 1/1.04, consistent with a steady-state real interest rate of 4% per year.

We set the parameters pertaining to the consumption basket in the following way. The share of tradable goods in final consumption, γ, is 0.55, while the share of home goods in tradable consumption, ν, is 0.72. The calibration of this parameter is the same across both BT and CDL.

We assume an elasticity of substitution between traded and non-traded goods, κ, of 0.44, as suggested by Stockman and Tesar (1995) and adopted by BT.[37] For

[37] CDL use a higher value of $\kappa = 0.74$ following Mendoza (1991). Ostry and Reinhart (1992) estimate this parameter to be higher in the range of 0.66-1.44. We provide a sensitivity analysis with respect to κ below.

$\rho = 2$, this implies that utility is non-separable between traded and non-traded goods. Given that $\kappa\rho < 1$, our benchmark calibration implies traded and non-traded goods are complements.

The share of labour input in the production of tradable and non-tradable intermediates are set equal to each other at $\alpha_H = \alpha_N = 0.67$ and the rate of depreciation of capital is set to 10% per annum.

In calibrating the processes for tradable and non-tradable sector productivity shocks, we mainly rely on BT, who estimate these processes for the US relative to EU15 and Japan using annual data between 1979-2002. We calibrate the persistence of tradable sector productivity shocks slightly higher to 0.88 (BT calibration sets it to 0.84) while keeping the rest of the calibration as in their paper.[38] The persistence of non-tradable productivity shocks are set to 0.30 and tradable sector shocks are bigger than non-tradable sector shocks, with a variance-covariance matrix given in 1.1.

After solving the model in terms of the state variables, we use the autoregressive processes for the shocks to generate simulated time series of length T (T=600) for the variables of interest. We repeat this procedure J (J =200) times and then compute the average of the moments from logged and HP-filtered series excluding the first 100 periods of simulation.

1.4.3 Results with unanticipated productivity shocks

We first look at the performance of the model in a single bond set-up when there are only unanticipated sectoral productivity shocks in line with BT and CDL. As shown by our analytical results for the endowment economy version of this model, the comovement between relative consumption, real exchange rate and its components depends crucially on the value of the trade elasticity, θ. Figure 1.2 shows the impact responses of real exchange rate, terms of trade, relative price of non-tradables and relative consumption conditional on a 1% increase in tradable sector productivity in the non-contingent bond economy for different values of the trade elasticity parameter,

[38]The utility function used by BT following Stockman and Tesar (1995), implies a slightly higher volatility of relative consumption compared to the utility function we use here. This in turn yields somewhat lower consumption-real exchange rate correlations for a given shock calibration. To make-up for this difference between the two preference specifications, we slightly increase the persistence of tradable sector shocks to make the wealth effects of these shocks more important and to emphasise their mechanism. (See Baxter and Crucini (1995) and Baxter (1995) on how higher shock persistence makes market incompleteness more important in international RBC models).

θ.[39] There are six different regions of θ (divided by vertical lines and colored in white and grey to ease identification), which imply different signs of comovement between relative consumption and relative prices on impact. The upper panel shows four of these six different regions that lie to the left of $\theta = 1$ and the lower panel shows the last two regions that cover values of θ greater than 1.

Regions of trade elasticity that we focus on for our calibration are regions I, V and VI, which all imply an increase in relative consumption and an appreciation in the real exchange rate following an increase in tradable sector productivity- implying a negative conditional correlation on impact. CDL emphasises regions I and VI, while BT analysis is valid for region V where θ takes values between 0.93 and 4.6 when parameters other than θ are calibrated according to Table 1.4 .

In section 1.3.2, we explain the different transmission mechanisms that occur when θ itakes values in regions I and V. The intuition is similar for production economies hence we do not repeat it here. But it is worth to say a few words about the transmission mechanism that occurs in region VI. As CDL explain, for very high degrees of substitutability between home and foreign goods, a sufficiently persistent shock can increase the relative wealth of domestic agents such that in the short-run the increase in the demand for home goods exceeds the increase in the output, which peaks later due to the dynamics of capital. Hence, terms of trade appreciates on impact, while relative consumption increases. However, terms of trade appreciation in this region is quite limited compared to that in region I. [40]

Next, we briefly discuss how the different transmission mechanisms highlighted in Figure 1.2 reflect into Backus-Smith correlations and other second moments. Table 1.5 reports various business cycle statistics for three different values of θ belonging to regions I, V and VI under alternative asset markets. Results for the non-contingent bond economy are given in the first column of each θ panel in Table 1.5 .

[39]Impact responses to a non-tradable sector productivity shock do not yield a negative transmission between relative consumption and real exchange rate except for a very limited range of low θ parameters (for θ between 0.31 and 0.36 a positive NT shock appreciates the real exchange rate by appreciating the terms of trade while increases relative consumption at home). Figures are available from authors on request.

[40]Note that the two mechanisms that are highlighted in CDL would still be present in a one-sector model with only tradable goods as they rely on the role of the terms of trade in generating a negative correlation between relative consumption and real exchange rate.

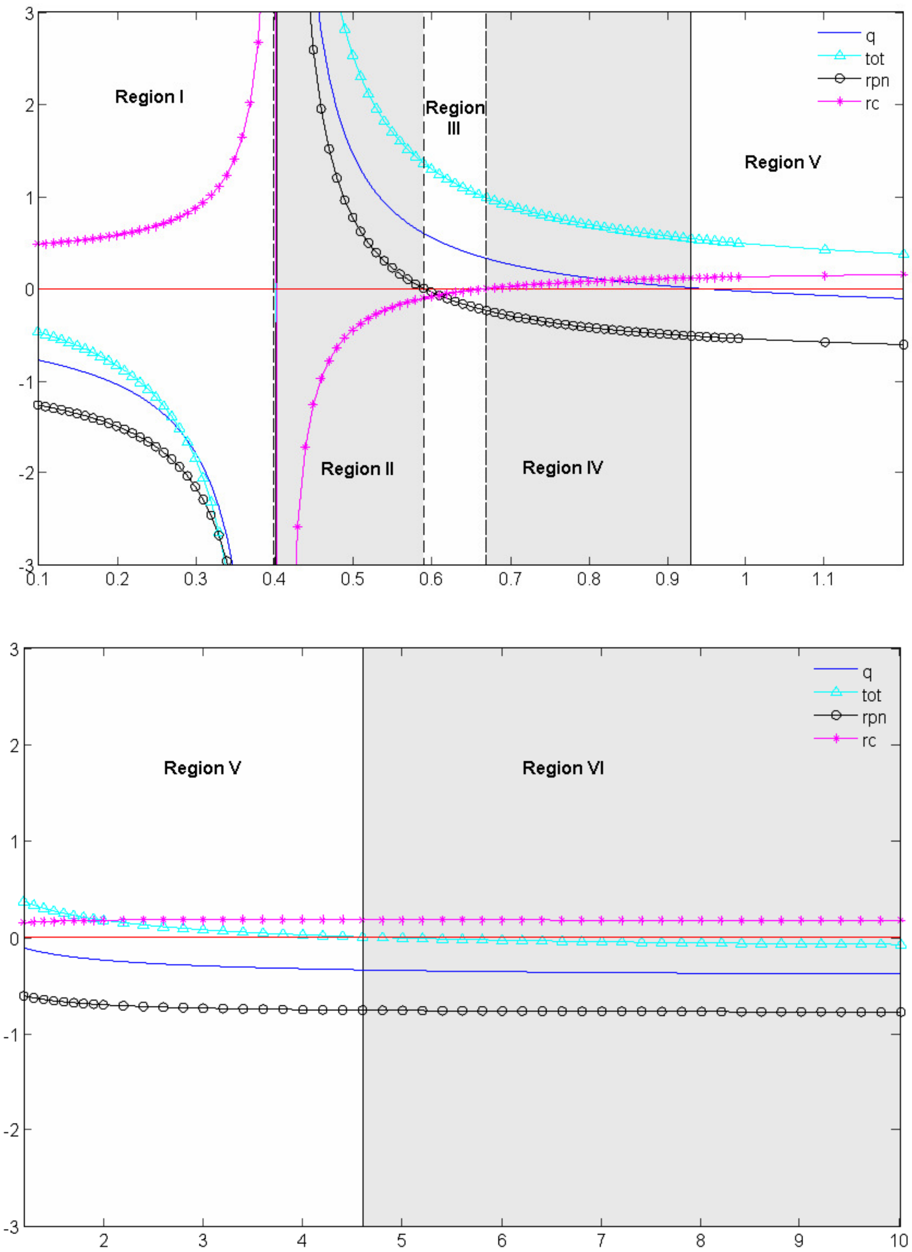


Figure 1.2: Impact responses of relative consumption and relative prices to a 1% increase in tradable sector productivity for different values of trade elasticity, θ.

Notes: x-axis shows the values of trade elasticity, θ. y-axis shows the impact responses of real exchange rate (q), terms of trade (tot) and relative price of non-tradables (rpn) in percentage deviations from steady-state.

	data	Θ=2.5				Low Θ, Θ=0.25				High Θ, Θ=8			
		nc	2bonds (rx=Q)	2bonds (rx=TOT)	complete market	nc	2bonds (rx=Q)	2bonds (rx=TOT)	complete market	nc	2bonds (rx=Q)	2bonds (rx=TOT)	complete market
Std dev of GDP	1.57	1.80	1.82	1.85	1.85	1.73	1.69	1.69	1.69	1.87	1.94	1.98	1.98
Std dev rel. to GDP													
Real exchange rate (RER)	6.16	0.37	0.30	0.29	0.30	1.39	0.38	0.43	0.41	0.44	0.36	0.37	0.36
Terms of trade (TOT)	2.12	0.31	0.37	0.42	0.42	1.31	1.10	1.08	1.08	0.22	0.24	0.24	0.24
Rel. price of non-traded	1.46	0.92	0.82	0.80	0.81	1.84	0.64	0.72	0.70	0.95	0.83	0.87	0.85
Consumption	0.76	0.38	0.37	0.36	0.35	0.57	0.41	0.40	0.40	0.37	0.35	0.33	0.33
Investment	4.33	3.22	3.21	3.19	3.19	3.14	3.17	3.17	3.17	3.60	3.63	3.63	3.64
Hours worked	0.31	0.41	0.42	0.44	0.44	0.37	0.36	0.36	0.36	0.43	0.48	0.50	0.50
AR(1) coefficients													
Real exchange rate	0.67	0.44	0.54	0.39	0.36	0.43	0.68	0.43	0.45	0.47	0.56	0.45	0.47
GDP	0.50	0.50	0.50	0.51	0.51	0.49	0.49	0.49	0.49	0.53	0.53	0.54	0.54
Consumption	0.66	0.47	0.46	0.47	0.47	0.50	0.47	0.48	0.48	0.47	0.46	0.47	0.47
Cross corr btw H and F													
GDP	0.35	0.38	0.35	0.31	0.31	0.50	0.56	0.57	0.57	0.28	0.19	0.14	0.14
Consumption	0.06	0.69	0.72	0.83	0.84	-0.18	0.61	0.70	0.70	0.69	0.74	0.86	0.85
Investment	0.07	0.29	0.26	0.24	0.24	0.47	0.50	0.50	0.50	-0.05	-0.13	-0.16	-0.17
Cross corr btw													
RER and Relative consumption	-0.45	**-0.07**	**0.19**	**0.74**	**0.76**	**-0.90**	**0.60**	**0.97**	**0.98**	**-0.28**	**0.27**	**0.70**	**0.71**
RER and Terms of trade	0.32	-0.16	-0.20	-0.10	-0.09	0.98	0.80	0.75	0.76	0.18	0.03	-0.03	-0.04
TOT and Relative consumption	-0.74*	0.65	0.41	0.41	0.44	-0.94	0.53	0.86	0.84	0.19	0.15	0.25	0.23
Cross corr btw GDP and													
Real net exports	-0.26	0.09	0.27	0.43	0.44	-0.40	-0.44	-0.44	-0.44	-0.05	0.11	0.17	0.17
Real exchange rate	-0.09	-0.37	-0.35	-0.22	-0.21	-0.49	0.27	0.29	0.30	-0.48	-0.50	-0.44	-0.45
Consumption	0.78+	0.85	0.79	0.74	0.74	0.86	0.85	0.86	0.86	0.84	0.70	0.65	0.65
Investment	0.93+	0.97	0.97	0.97	0.97	0.98	0.98	0.98	0.98	0.92	0.91	0.91	0.91
Hours worked	0.86+	0.95	0.95	0.95	0.95	0.93	0.91	0.91	0.91	0.96	0.94	0.94	0.94
Foreign bond position/GDP	0.47		6.64	-6.47			6.22	1.69			10.37	132.52	

Notes : Data column contains statistics calculated by Benigno and Thoenissen (2008). Home country refers to the United States, and foreign is a weighted average of Japan and EU-15 and the period for calculation spans 1970-2002. Both the data and simulated moments are of annual frequency, logged and HP-filtered. Data statistics marked by * are taken from Corsetti et al (2008) while those marked by + are taken from Raffo (2010). Data for foreign bond position as a share of GDP is net foreign currency exposure as share of GDP for the US taken from Lane and Shambaugh (2010).

Table 1.5: Business cycle statistics with unanticipated shocks to sectoral TFP for different values of trade elasticity.

Region V: Benchmark calibration

Naturally, the business cycle statistics that we obtain under the calibration with $\theta = 2.5$ are similar to those reported by BT.[41] The model is able to generate a negative cross-correlation between relative consumption and real exchange rate that is around -0.07. Comparing this with a correlation of 0.76 which arises under complete markets (fourth column of first panel in Table 1.5) shows that market incompleteness really matters in this set-up.[42] The mechanism that generates the negative correlation between relative consumption and real exchange rate for this calibration also implies a negative correlation between the real exchange rate and terms of trade. This is because real exchange rate and terms of trade move in opposite directions in response to a tradable productivity shock for values of θ inside region V as depicted in Figure 1.2 and tradable sector shocks are dominant in driving the business cycle according to our calibration.

An apparent drawback is the low volatility and persistence of the real exchange rate. Because the law of one price holds for traded goods, only sources of volatility in real exchange rate are the fluctuations in terms of trade and relative price of non-tradables. Due to a relatively high value of trade elasticity, terms of trade volatility is limited. Although large wealth effects that are present under incomplete markets make relative non-tradables prices more volatile compared to complete markets, this effect does not raise real exchange rate volatility much.

The model for this calibration cannot account for the quantity puzzle, which refers to the failure of a general class of international RBC models in generating higher cross-country correlations between GDPs compared to consumption levels. Comparing the first and fourth columns of the first panel of Table 1.5 shows that market incompleteness goes in the right way as it reduces the cross-country consumption correlations with respect to complete markets, but it is not sufficient to account for the puzzle.[43] Also, net exports, which are countercyclical in the data, are weakly procyclical for $\theta = 2.5$ in the non-contingent bond set-up. Large wealth effects following a tradable sector productivity shock increase the demand for imported goods at home but the

[41] Although the model and calibration we use here are in the same spirit as BT, they are not equivalent. For example, we specify a different utility function, we use endogenous discount factor to make the model stationary and we set investment adjustment costs to zero since the volatility of investment relative to GDP is already around 3 without any adjustment costs in the non-contingent bond economy.

[42] The fact that the consumption-real exchange rate correlation is below unity under complete markets is due to the non-separability of consumption and leisure in the utility function.

[43] CDL show that modelling distribution sector can account for the quantity puzzle whether risk sharing is complete or not. It also increases the volatility of terms of trade and real exchange rate.

complementarity between tradable and non-tradables limits this demand to some extent as non-tradables supply is fixed. This in turn, makes it harder for the model to generate countercyclical net exports.[44]

Region I: Low trade elasticity

The calibration with $\theta = 0.25$ yields a large negative correlation (-0.90) between relative consumption and real exchange rate in the non-contingent bond economy (see first column of panel 2 in Table 1.5) in line with the transmission mechanism highlighted in the first region depicted in Figure 1.2. The correlation between the terms of trade and real exchange rate shoots up to 0.98, which is quite high compared to 0.32 implied by the data.

The non-contingent bond economy with low trade elasticity performs better than that with $\theta = 2.5$ in terms of real exchange rate volatility, though volatility still remains quite below its empirical counterpart. With low trade elasticity, the cross correlation between home and foreign consumption is lower than that of home and foreign GDP, but it is negative, which is not supported by the data. Also with low θ, net exports become strongly countercyclical mainly due to large terms of trade appreciation that makes imports more expensive during good times.

Region VI: High trade elasticity

The terms of trade appreciation for θ belonging to region VI is much more limited compared to the terms of trade appreciation for θ belonging to region I (See Figure 1.2). This leads to a Backus-Smith-Kollmann correlation of around -0.28, which is more in line with the data than -0.90 implied by $\theta = 0.25$. Also, the fact that the terms of trade depreciates over the long-run for high θ implies a more realistic real exchange rate-terms of trade correlation (0.18) compared to the other two trade elasticity parameters. However, high trade elasticity makes the quantity puzzle much worse, resulting in a much higher correlation between home and foreign consumptions than home and foreign GDPs. It also leads to a counterfactual negative correlation between home and foreign investment.

[44] Indeed, changing the value of κ to 0.83 reduces the correlation of net exports and GDP to 0.02, while the cross-correlation between relative consumption and real exchange rate becomes -0.09.

Implications of a second internationally traded bond for international risk sharing and business cycles

As we discussed before, portfolio choice affects international risk sharing and transmission of shocks through the valuation effect that enters net foreign asset accumulation. Using the goods market clearing conditions and approximating up to first order, change in the net foreign asset position can be written as:

$$\underbrace{\widehat{NFA}_t - \frac{1}{\beta}\widehat{NFA}_{t-1}}_{\Delta NFA_t} = \underbrace{\frac{\bar{C}_H^*}{\bar{Y}}(\hat{P}_{H,t} - \hat{P}_t + \hat{C}_{H,t}^*) - \left(\frac{\bar{C}_F}{\bar{Y}}(\hat{P}_{F,t} - \hat{P}_t + \hat{C}_{F,t})\right)}_{CA_t} + \underbrace{\tilde{\alpha}_F(\hat{r}_{x,t})}_{VAL_t} + O(\varepsilon^2). \tag{1.55}$$

For the level of approximation we use here, valuation effect is given by the excess return on the steady-state foreign bond portfolio. We know that the steady-state portfolio is determined by the orthogonality condition given by (1.18). For the utility function specified in (1.54), this condition can be written as:

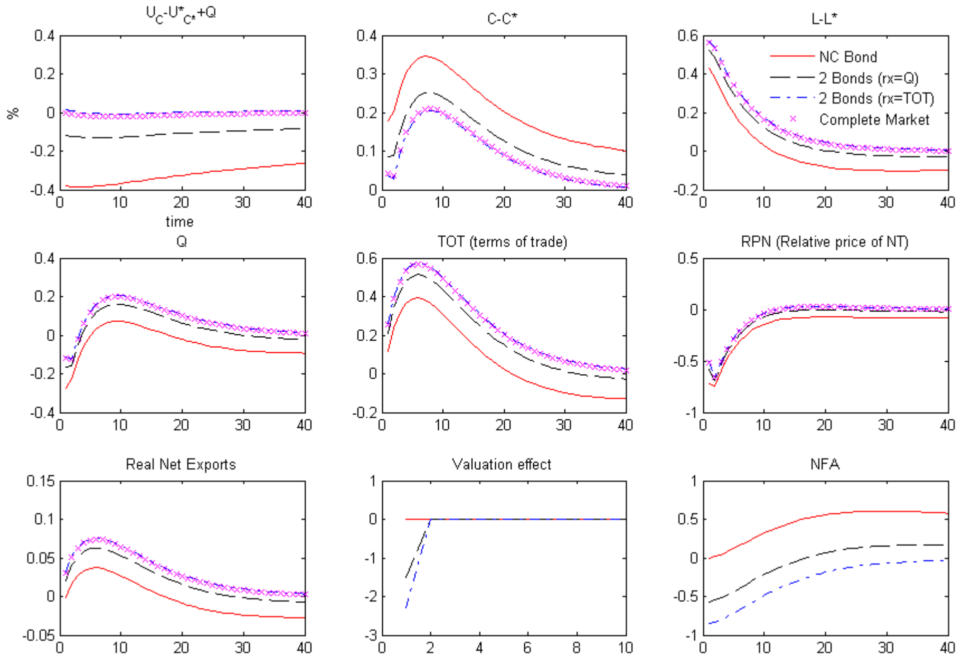
$$Cov_t\left[\left(a_1(\hat{C}_{t+1} - \hat{C}_{t+1}^*) + a_2(\hat{l}_{t+1} - \hat{l}_{t+1}^*) - \hat{Q}_{t+1}\right)\hat{r}_{x,t+1}\right] = 0 + O(\varepsilon^3), \tag{1.56}$$

where $a_1 \equiv 1 - \omega(1-\rho)$ and $a_2 \equiv (1-\omega)(1-\rho)\frac{\bar{l}}{1-\bar{l}}$ and $a_1 > 0$, $a_2 < 0$ for $\rho > 1$. Thus, hedging against fluctuations in relative marginal utilities of consumption means hedging against fluctuations in relative consumption and relative labour supplies adjusted by the real exchange rate. It is optimal to have a long position in foreign bonds, if the excess on foreign bond, $\hat{r}_{x,t}$, is higher when consumption is lower in the home country and/or when total hours worked is higher in the home country. Excess returns are determined according to policy rules as described in equations (1.31) and (1.37).

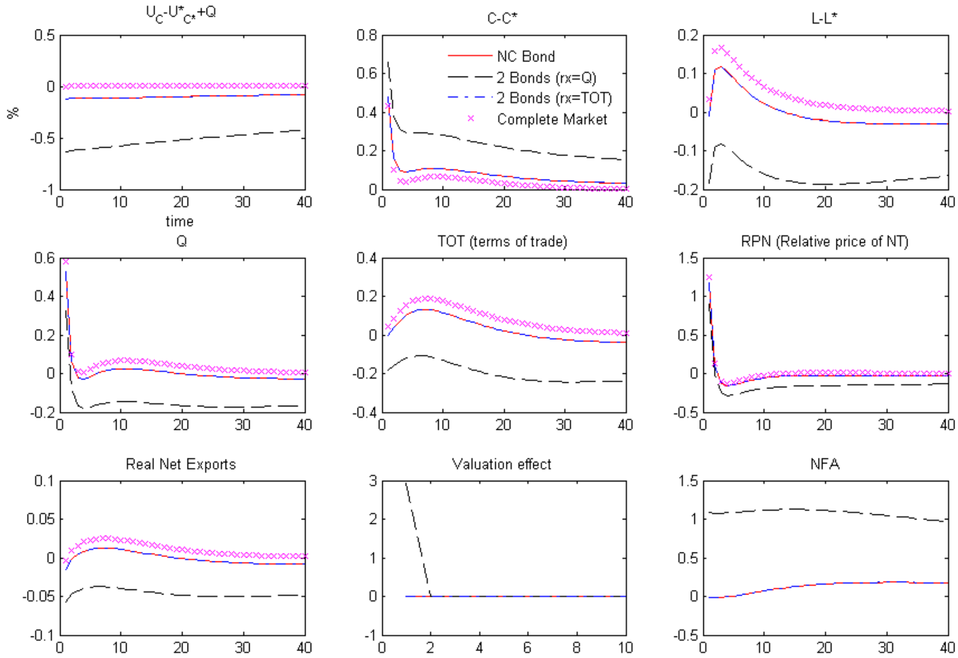
Region V: Benchmark calibration

First, consider the baseline calibration with $\theta = 2.5$. To understand the equilibrium portfolio position, it is useful to analyse the components of equation (1.56), namely the response of relative marginal utilities of consumption adjusted by the real exchange rate and excess return under the zero-portfolio solution (non-contingent bond economy). Figure 1.3 plots the impulse responses of these variables for $\theta = 2.5$ under four different asset market structures.

For now, let us just focus on the straight line that depicts the non-contingent bond economy solution (NC economy) to understand the equilibrium portfolio. Following a positive tradable sector shock in the home country, relative consumption and hours



Tradable sector productivity shock



Non-tradable sector productivity shock

Figure 1.3: Impulse responses to sector-specific productivity shocks with $\theta = 2.5$.

Notes: x-axis shows periods measures in years. y-axis shows the percentage deviations from steady-state.

worked increase. Home agents work more compared to foreign agents because wages are higher in the home country following the increase in productivity. While the increase in relative consumption implies a fall in relative marginal utility, the increase in relative labour effort implies a rise, limiting the overall fall in relative marginal utility on impact.[45] Given the dynamics of the real exchange rate and the terms of trade under $\theta = 2.5$, which we explain in detail above, hedging against tradable sector shocks require a long position in foreign bonds under CPI stabilisation, but a short position when domestic tradable prices are stabilised.

What is the optimal hedge against non-tradable sector shocks? The lower panel of Figure 1.3 shows that shocks to non-tradable sector productivity do not generate large deviations from the efficient risk sharing condition, i.e. the response of relative marginal utilities of consumption to a non-tradable sector under the non-contingent bond economy is close to that under the complete markets. Therefore, optimal hedge against these shocks is a near-zero portfolio. This creates a tension in the determination of equilibrium portfolio. As our calibration gives a larger weight to tradable sector shocks, equilibrium portfolios would be biased towards hedging against tradable sector shocks. But depending on the strength of the response of excess returns to a non-tradable sector shock, a portfolio that is a good hedge against tradable sector shocks can be a bad hedge against non-tradable sector shocks, which in turn would limit the size of the portfolio and impede risk sharing conditional on both shocks.

Table 1.5 reports the optimal foreign currency bond position as a share of GDP along with other business cycle statistics for the two bonds economy under the two policy rules we consider (columns 2 and 3 in the first panel of Table 1.5). Under CPI stabilisation, the model implies a large long position in foreign bonds (around 6.6 times GDP) and a positive but low consumption-real exchange rate correlation around 0.19. We can see from the second column of the first panel of Table 1.5 that the partial insurance provided by this trading opportunity limits the volatility of relative non-tradables price and the volatility of the real exchange rate compared to the non-contingent bond set-up. Nevertheless, Balassa-Samuelson effect still operates to some extent as we can see from the negative correlation between real exchange rate and terms of trade and the negative correlation between relative non-tradables price and relative consumption.

Figure 1.3 shows that the impulse responses to a tradable sector shock in this case (labelled by 2 bonds (rx=Q)) lies in between the impulse responses of the NC

[45] Hence in this case, non-separability of consumption and leisure limits the size of the total risk to be hedged.

economy and complete markets, highlighting the partial insurance against tradable shocks. But, interestingly, impulse responses in the lower panel of Figure 1.3 show that, having access to two international bonds makes the fluctuations in relative marginal utilities of consumption even larger than they are under the non-contingent bond economy conditional on non-tradable sector shocks. Hence, the lower panel of Figure 1.3 illustrates very nicely how valuation effects can actually go in the wrong way when market incompleteness matters.

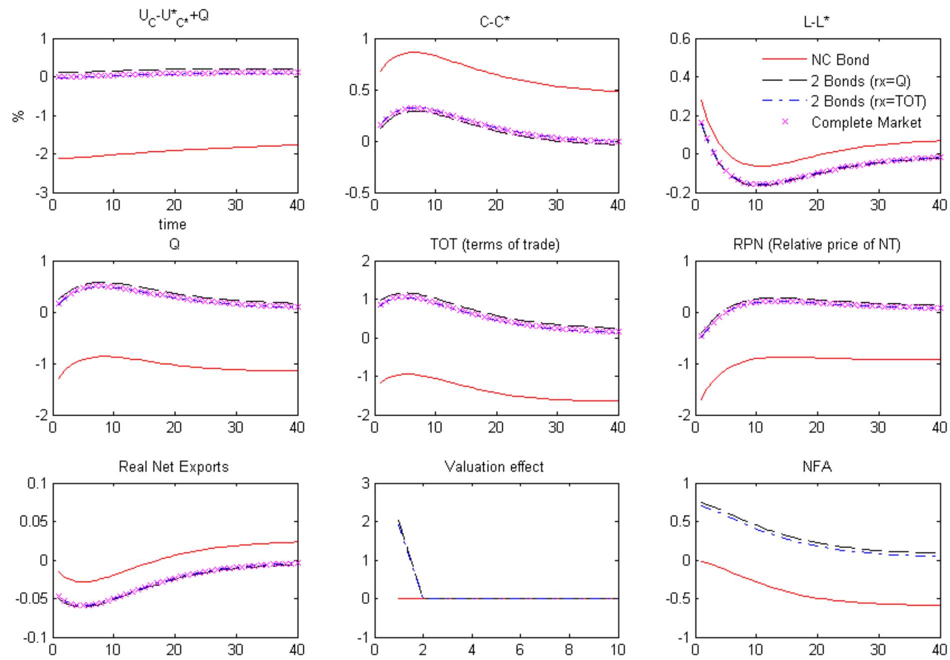
Under domestic tradable price stabilisation, the model implies an equally large short position in foreign bonds, but a high consumption-real exchange rate correlation (0.74) which is very close to the correlation implied by complete markets (0.76). Indeed, comparing columns 3 and 4 of the first panel in Table 1.5 shows that allowing agents to have claims to the terms of trade almost completes the markets despite the fact that relative marginal utilities of consumption are subject to two different sources of risk (relative T and NT productivity shocks). Also, Figure 1.3 shows how the impulse responses obtained under this set-up (labelled by 2 bonds (rx=TOT)), sit on top of the complete market impulses for both shocks. Hence we confirm the intuition provided by the analytical results within the context of a more general production economy. This result is interesting as it shows that risk sharing can be higher when bonds cannot load on all sources of uncertainty in the economy.

Region I: Low trade elasticity

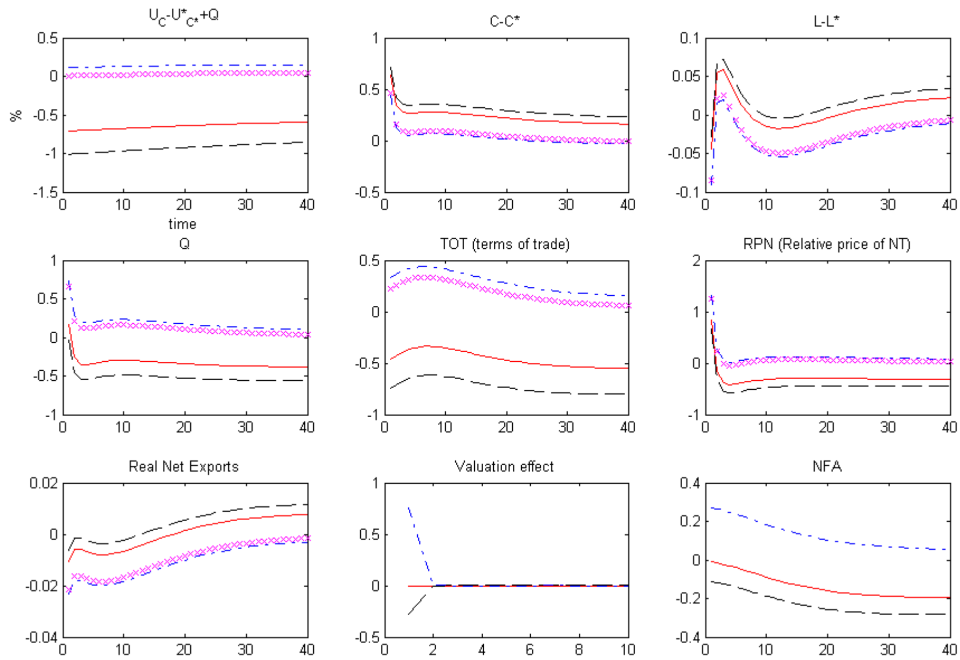
The result that trade in bonds under tradable price stabilisation almost completes markets also holds here (compare the third and fourth columns of the second panel in Table 1.5). What is more, trade in bonds implies a high positive correlation between relative consumption and real exchange rate also under the CPI stabilisation.

Figure 1.4 shows impulse responses to tradable and non-tradable productivity shocks for $\theta = 0.25$. Again, focus on the plots for the non-contingent bond economy to understand the portfolio implications of the model. For this calibration, home terms of trade appreciates on impact following both sectoral shocks, which in turn strengthens the increase in the relative wealth of home agents compared to the calibration with $\theta = 2.5$. This means that the marginal utility gap (the deviation from efficient risk sharing) is bigger under low θ for both shocks, i.e. there is more risk to be shared through the bond portfolio for low θ.

In fact, for low θ, hedging against non-tradable shocks also requires a non-zero portfolio. This is because when tradable and non-tradable goods are complements, an increase in non-tradable goods consumption goes hand in hand with an increase



Tradable sector productivity shock



Non-tradable sector productivity shock

Figure 1.4: Impulse responses to sector-specific productivity shocks with $\theta = 0.25$.

Notes: x-axis shows periods measures in years. y-axis shows the percentage deviations from steady-state.

in the demand for tradables. Given that tradable consumption is home biased and trade elasticity is low, this increased demand for tradables given an initially fixed supply leads to a home terms of trade appreciation. This, in turn, leads to higher wealth in the domestic country and widens the gap between the marginal utilities of consumption across the two countries conditional on non-tradable shocks.

When excess return on bonds is given by the real exchange rate, hedging against tradable sector shocks implies a long position in foreign bonds whereas hedging against non-tradable sector shocks implies a short position. This is because real exchange rate appreciates in response to a positive tradable sector shock that lowers relative marginal utility of consumption at home but depreciates in response to a positive non-tradable sector that affects relative marginal utility in a similar way (see Figure 1.4). The resulting portfolio is a long position in foreign bonds around 6 times the GDP, which is comparable to that obtained under $\theta = 2.5$.

On the other hand, when nominal bonds give claims to the terms of trade, it is optimal to have a long position in foreign bonds to hedge against both sources of shocks because for each shock, home terms of trade appreciates (foreign bonds pay less) precisely when marginal utility is lower in the home country. Thus, optimal portfolio switches sign compared to the case of $\theta = 2.5$ and shrinks in size to 1.7 as a share of GDP (Since terms of trade volatility is higher with low θ, a smaller portfolio can achieve higher risk sharing). Despite the smaller portfolio position, consumption-real exchange rate correlation goes up to 0.97, which is close to the value under complete markets (see the second panel of Table 1.5).

It is interesting to note that for low elasticity values, impulse responses to tradable sector shocks with trade in two bonds under *both* policy rules are almost identical to those under complete markets. The main difference in the risk sharing implications of bonds across the two policy rules is with regards to non-tradable sector shocks: Tilting the bond portfolio towards tradable sector shocks, implies larger unwanted valuation effects in response to non-tradable sector shocks under CPI stabilisation (Figure 1.4). But this is not enough to generate a low consumption-real exchange correlation. These results suggest that it is actually harder to account for the consumption-real exchange rate anomaly in the presence of endogenous portfolio choice when θ is low.

Region VI: High trade elasticity

As in the case of trade elasticities belonging to regions I and V, trade in home and foreign bonds under domestic tradable price stabilisation brings the model very close to the complete market outcome also in region VI (see the last panel of Table 1.5).

Trade in bonds under CPI stabilisation leads to a consumption-real exchange rate correlation of around 0.27 which is lower than what is implied by trade in bonds under tradable PPI stabilisation but still higher than the empirical counterpart. Not surprisingly, the implied portfolio positions are extreme and are far from matching the data just as the Backus-Smith-Kollmann correlations are. The fact that the terms of trade volatility falls dramatically with high trade elasticity means that agents should hold a much larger foreign currency position to ensure a given degree of risk sharing.[46]

Sensitivity analysis

Our finding that trade in nominal bonds ensures too much risk sharing is robust to different calibrations of key parameters. We already discuss the role of trade elasticity, θ, for optimal portfolios and degree of risk sharing with reference to Figures 1.2, 1.3, 1.4 and Table 1.5 . In Figure 1.5, we plot consumption-real exchange rate correlation alongside optimal foreign bond positions for different values of intratemporal elasticity of substitution between tradables and non-tradables, κ, under alternative asset market and policy combinations. As mentioned in section 1.4.2, values of κ generally used in the literature varies between 0.44 and 1.44. In this range, the non-contingent bond set-up yields a negative consumption-real exchange rate correlation. For high values of κ, i.e. for κ larger than 3, relative price of non-tradable goods adjusts less in response to supply shocks hence the correlation turns positive even in the absence of any portfolio choice. The foreign bond portfolio as a share of GDP is quite sensitive to κ when excess return is given by the real exchange rate. For high values of κ, real exchange rate depreciates with respect to a positive tradable sector shock, while relative consumption increases. Hence it becomes optimal to have a short position in foreign bonds rather than a long position. On the other hand, κ has a limited impact on the dynamics of the terms of trade and hence on optimal portfolio under domestic tradable price stabilisation. Under this policy rule, trade in bonds yields a consumption-real exchange rate cross-correlation that is very close to the complete market outcome regardless of the value of κ.

Figure 1.6 analyses the effects of varying the share of non-traded goods in the consumption of final goods, γ. For very low values of γ, consumption-real exchange

[46]There is a special case where θ is set such that the terms of trade response to a tradable sector productivity shock is almost zero which means that relative bond returns cannot load on the relative consumption risk created by relative tradable sector shocks. For our calibration this occurs for values of θ between 4 and 5 as can be seen from 1.2. In particular, for $\theta = 4.6$, relative consumption-real exchange rate correlation is around -0.20 both in the non-contingent bond economy and the two bonds economy with tradable PPI stabilisation, whereas the implied foreign bond position as a share of GDP is -73.

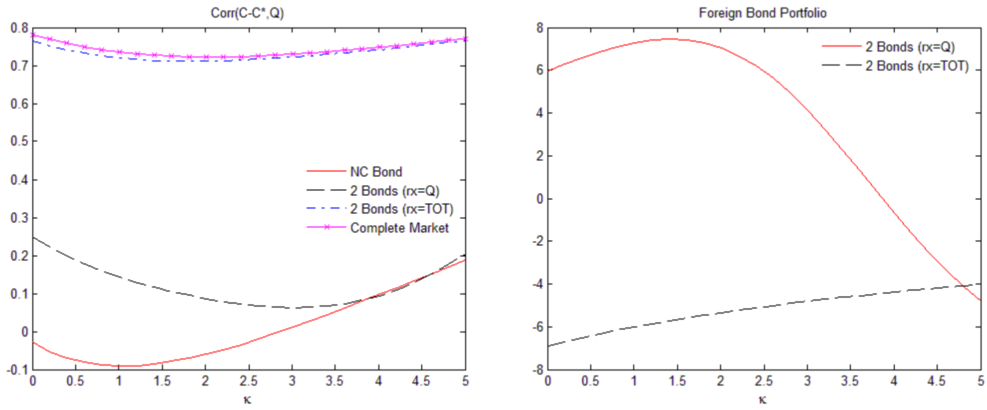


Figure 1.5: Sensitivity with respect to the elasticity of substitution between tradables and non-tradables, κ, in the baseline model with unanticipated productivity shocks in each sector.

rate correlation is very high because most of the final goods are non-tradable and relative price of non-tradable goods moves in a way to offset the changes in the relative supply of non-tradables as we mention above. As γ increases, tradable sector shocks become more important hence we get the mechanism that generates the negative correlation between relative consumption and real exchange rate. As γ becomes very high, the Balassa-Samuelson effect diminishes and correlation picks up again. This U-shaped pattern is valid for all asset market structures. For any value of γ, trade in bonds complete the markets when excess returns are given by the terms of trade while correlations implied by trade in bonds under CPI stabilisation are closer to those that arise with trade in a single non-contingent bond. Equilibrium portfolios increase in absolute value as the share of tradable goods increases. When γ is close to 1, real exchange rate is determined mainly by the terms of trade hence the optimal bond portfolio under CPI stabilisation also becomes negative.

In Figure 1.7, we present sensitivity analysis with respect to different values of home bias in consumption, ν. The cross-correlation rises after a certain value of consumption home bias. Optimal foreign currency portfolio approaches to zero as ν approaches to 1, i..e complete home bias. Figure 1.8 repeats this exercise for the relative variance of non-tradable sector shocks with respect to tradable sector shocks. As we increase the relative size of non-tradable shocks, cross-correlation increases under all asset market structures. Optimal foreign currency position falls under CPI stabilisation but it is not affected under domestic tradable PPI stabilisation because terms of trade does not respond significantly to non-tradable sector shocks.

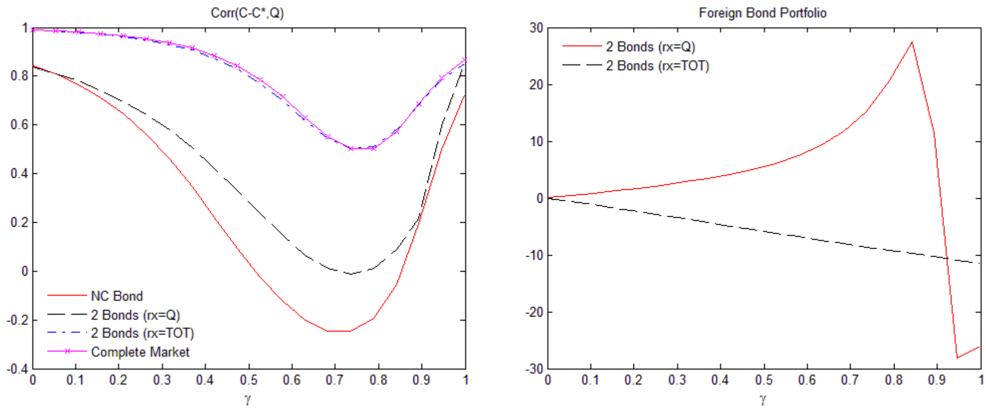


Figure 1.6: Sensitivity with respect to the the share of non-tradables in final consumption, γ, in the baseline model with unanticipated productivity shocks in each sector.

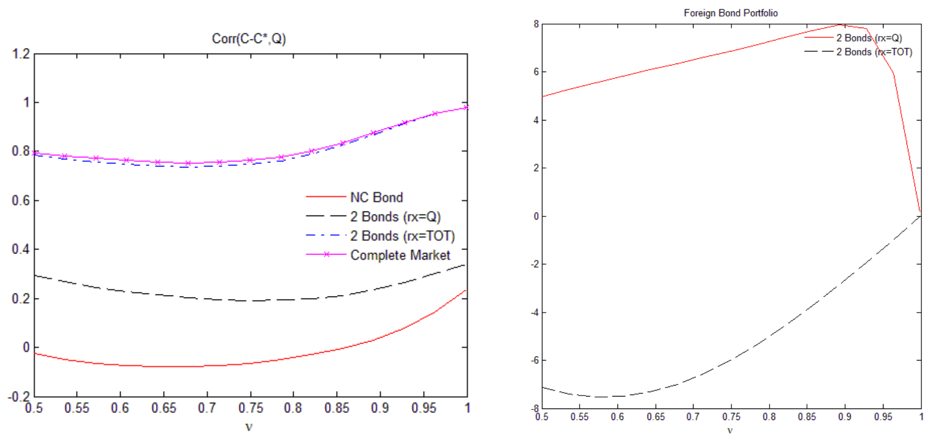


Figure 1.7: Sensitivity with respect to the degree of home bias in consumption, ν, in the baseline model with unanticipated productivity shocks in each sector.

1.4.4 Results with anticipated productivity shocks

Next, we analyse the consequences of introducing news shocks alongside unanticipated productivity shocks in tradable and non-tradable sectors. As we discussed before in the analytical section, news about future productivity work as a typical demand shock, increasing consumption and prices at the same time. Therefore, relative consumption and real exchange rate would generally be negatively correlated conditional on news shocks, which would potentially help in accounting for the anomaly

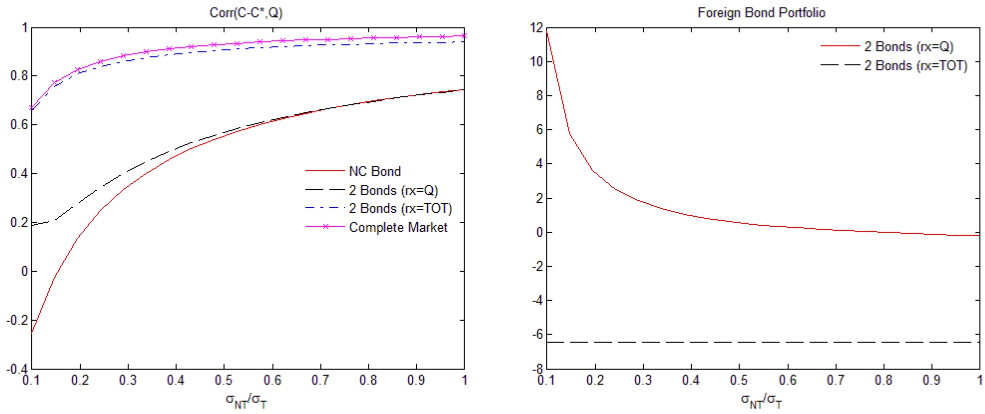


Figure 1.8: Sensitivity with respect to the relative variance of non-tradable shocks, σ^2_{NT}/σ^2_T, in the baseline model with unanticipated productivity shocks in each sector.

in the presence of some endogenous portfolio choice.[47] We are mainly interested in the effect of news shocks on optimal portfolios and risk sharing. Provided that anticipated and unanticipated shocks pull the equilibrium portfolio towards different directions, we can generate a meaningful market incompleteness to account for the anomaly.

We specify the exogenous processes for sectoral productivity shocks that incorporate news as follows:

$$\begin{aligned}
\log A_{H,t} &= (1-\delta_T)\log \bar{A}_H + \delta_T \log A_{H,t-1} + \log z_{H,t-1} + u_{H,t}, \\
\log A_{F,t} &= (1-\delta_T)\log \bar{A}_F + \delta_T \log A_{F,t-1} + \log z_{F,t-1} + u_{F,t}, \\
\log A_{N,t} &= (1-\delta_N)\log \bar{A}_N + \delta_N \log A_{N,t-1} + \log z_{N,t-1} + u_{N,t}, \\
\log A_{N^*,t} &= (1-\delta_N)\log \bar{A}_{N^*} + \delta_N \log A_{N^*,t-1} + \log z_{N^*,t-1} + u_{N^*,t}, \\
\log z_{i,t} &= \delta_{z_i} \log z_{i,t-1} + u_{Zi,t}, \quad \text{for } i = H, F, N, N^*.
\end{aligned}$$

where $0 \le \delta_T < 1$, $0 \le \delta_N < 1$, $0 \le \delta_z < 1$. We first consider a calibration where news shocks are persistent and small which is along the lines of Colacito and Croce (2010). Table 1.6 reports the business cycle statistics obtained from a model which is calibrated according to Table 1.4 for different values of trade elasticities, where persistence of news to tradable and non-tradable sector productivity are set equal to the persistence of unanticipated productivity shocks in these sectors, i.e. $\delta_{z_H} = \delta_{z_F} = \delta_T = 0.88$, $\delta_{z_N} = \delta_{z_{N^*}} = \delta_N = 0.30$ and the relative variance of news to unanticipated

[47] Opazo (2006) looks at the role of expectation shocks in accounting for the Backus-Smith puzzle in a single bond economy with only tradable goods.

shocks in each sector is 0.01, i.e. $\sigma^2_{z_H}/\sigma^2_T = \sigma^2_{z_F}/\sigma^{*2}_T = \sigma^2_{z_N}/\sigma^2_N = \sigma^2_{z_{N^*}}/\sigma^{*2}_N = 0.01$.

Comparing Table 1.6 with Table 1.5 for $\theta = 2.5$, shows that small and persistent news shocks make the consumption-real exchange rate correlation more negative, -0.16, under the non-contingent bond economy without worsening the model's performance to fit other business cycle statistics. In fact, introduction of news shocks makes the model more compatible with the data as it turns the correlation between the real exchange rate and terms of trade from negative to positive and reduces the correlation between terms of trade and relative consumption. Because news shocks are small in our calibration, they do not reduce the comovement of consumption, investment and hours worked with GDP in a significant way.

In line with our intuition and the analytical results presented before, introducing news shocks does not change the risk sharing properties of bonds under CPI stabilisation whereas it makes a big difference under domestic tradable price stabilisation. This is because under the latter, excess return is given by the terms of trade, which covaries negatively with relative consumption risk conditional on anticipated shocks, but positively conditional on unanticipated shocks. This tension makes the short position in foreign currency smaller and implies a negative consumption-real exchange rate correlation of -0.08. Hence, in the presence of small and persistent news shocks, trade in bonds that give claims to terms of trade can no longer replicate the complete market outcome.

As the second and third panels of Table 1.6 shows, news shocks are more effective for $\theta = 2.5$ (or in general for θ belonging to region V), because under $\theta = 0.25$ and $\theta = 8$, unanticipated shocks to tradable sector productivity affect the terms of trade in a similar way to news shocks, i.e. they also work as demand shocks, hence news shocks cannot reduce consumption-real exchange rate correlation to low levels with endogenous trade in bonds.

For larger news shocks, optimal foreign currency position switches sign under tradable price stabilisation, i.e. it becomes optimal to have a long position in foreign currency rather than a short position, and consumption-real exchange rate correlation becomes more negative but this comes at the cost of creating too much volatility in GDP. Sensitivity analysis with respect to the variance and persistence of news shocks are available from authors upon request.

	data	Θ=2.5				Low Θ, Θ=0.25				High Θ, Θ=8			
		nc	2bonds (rx=Q)	2bonds (rx=TOT)	complete market	nc	2bonds (rx=Q)	2bonds (rx=TOT)	complete market	nc	2bonds (rx=Q)	2bonds (rx=TOT)	complete market
Std dev of GDP	1.57	1.83	1.86	1.84	1.88	1.74	1.71	1.72	1.71	1.91	2.00	2.00	2.03
Std dev rel. to GDP													
Real exchange rate (RER)	6.16	0.40	0.30	0.38	0.31	1.73	0.40	0.48	0.44	0.45	0.36	0.39	0.37
Terms of trade (TOT)	2.12	0.35	0.41	0.38	0.43	1.93	1.11	0.96	1.11	0.25	0.27	0.26	0.26
Rel. price of non-traded	1.46	0.94	0.82	0.91	0.83	2.06	0.68	0.88	0.72	0.96	0.82	0.90	0.86
Consumption	0.76	0.38	0.37	0.38	0.35	0.62	0.41	0.42	0.40	0.37	0.34	0.33	0.33
Investment	4.33	3.23	3.22	3.23	3.21	3.19	3.23	3.22	3.23	3.58	3.62	3.63	3.63
Hours worked	0.31	0.42	0.44	0.43	0.45	0.38	0.38	0.38	0.38	0.44	0.50	0.49	0.51
AR(1) coefficients													
Real exchange rate	0.67	0.47	0.60	0.46	0.39	0.44	0.63	0.39	0.44	0.48	0.58	0.48	0.49
GDP	0.50	0.51	0.51	0.51	0.52	0.49	0.49	0.49	0.49	0.54	0.55	0.55	0.55
Consumption	0.66	0.47	0.46	0.47	0.47	0.49	0.47	0.48	0.48	0.47	0.46	0.47	0.47
Cross corr btw H and F													
GDP	0.35	0.36	0.31	0.34	0.28	0.49	0.54	0.53	0.54	0.24	0.13	0.13	0.10
Consumption	0.06	0.65	0.69	0.67	0.83	-0.31	0.59	0.57	0.68	0.62	0.70	0.82	0.84
Investment	0.07	0.27	0.24	0.26	0.22	0.44	0.46	0.45	0.46	-0.05	-0.15	-0.16	-0.18
Cross corr btw													
RER and Relative consumption	-0.45	**-0.16**	**0.20**	**-0.08**	**0.76**	**-0.92**	**0.58**	**0.34**	**0.97**	**-0.36**	**0.28**	**0.44**	**0.71**
RER and Terms of trade	0.32	0.01	-0.09	0.05	-0.07	0.97	0.78	0.63	0.76	0.26	0.09	0.01	-0.01
TOT and Relative consumption	-0.74*	0.42	0.28	0.31	0.45	-0.91	0.54	0.65	0.84	-0.04	0.04	0.38	0.30
Cross corr btw GDP and													
Real net exports	-0.26	0.13	0.32	0.23	0.45	-0.38	-0.42	-0.41	-0.42	0.04	0.18	0.18	0.22
Real exchange rate	-0.09	-0.33	-0.31	-0.28	-0.21	-0.36	0.20	-0.02	0.26	-0.43	-0.47	-0.46	-0.45
Consumption	0.78+	0.81	0.74	0.78	0.72	0.76	0.83	0.86	0.83	0.79	0.63	0.68	0.62
Investment	0.93+	0.97	0.97	0.97	0.97	0.97	0.98	0.98	0.98	0.90	0.90	0.90	0.90
Hours worked	0.86+	0.94	0.94	0.94	0.95	0.92	0.89	0.89	0.89	0.95	0.94	0.94	0.94
Foreign bond position/GDP	0.47		10.22	-2.95			6.74	2.20			13.74	69.32	

Notes : Data column contains statistics calculated by Benigno and Thoenissen (2008). Home country refers to the United States, and foreign is a weighted average of Japan and EU-15 and the period for calculation spans 1970-2002. Both the data and simulated moments are of annual frequency, logged and HP-filtered. Data statistics marked by * are taken from Corsetti et al (2008) while those marked by + are taken from Raffo (2010). Data for foreign bond position as a share of GDP is net foreign currency exposure as share of GDP for the US taken from Lane and Shambaugh (2010).

Table 1.6: Business cycle statistics with anticipated and unanticipated shocks to sectoral TFP for different values of trade elasticity.

1.5 Conclusion

In this chapter, we review and compare different mechanisms that rely on good market frictions and market incompleteness to account for the consumption-real exchange rate anomaly. We show that the performance of these models worsen considerably when we move away from a single bond economy and allow for ex-ante risk sharing in the form of home and foreign currency bonds. Irrespective of the value of trade elasticity, relative consumption-real exchange rate correlations increase dramatically to the values implied by complete markets when agents can trade in bonds which give claims to the terms of trade. Although trade in bonds leads to less risk sharing when relative bond returns are given by the real exchange rate, correlations implied by this asset-market and policy combination are much higher than that in the data. A common characteristic of optimal portfolios among different policies and trade elasticity values is that they are implausibly large. Therefore, two-sector models with sectoral productivity shocks fail in both generating realistic portfolio positions and a low degree of risk sharing when we allow for portfolio choice between two assets.

We explore the role of news shocks in generating meaningful market incompleteness in the presence of endogenous portfolio choice and show that only under certain trade elasticity and policy combinations anticipated and unanticipated shocks can create a significant tension on equilibrium bond portfolios and reduce the degree of risk sharing implied by bonds.

Our work suggests that allowing for more sources of uncertainty can potentially improve the performance of this class of models in accounting for the consumption-real exchange rate anomaly while generating realistic portfolio positions provided that they satisfy certain conditions. First of all, these additional shocks should imply a low correlation between relative consumption and real exchange rate in the zero-portfolio solution (non-contingent bond economy) to start with. Because, as long as optimal portfolios are chosen to minimise deviations from risk sharing as in our set-up and most of the recent portfolio literature, the unconditional correlation between relative consumption and real exchange rate in the presence of endogenous portfolio cannot be lower than the non-contingent bond economy outcome. Secondly, different shocks should pull portfolios towards different directions. If hedging against all sources of uncertainty in the model require a similar portfolio position, risk sharing would be high even if there are fewer assets than shocks. Finally, these additional shocks should be empirically relevant and should not have counterfactual implications for other business cycle statistics. Our experiments with other shocks such as i-pod shocks and investment shocks suggest that finding shocks that satisfy these properties is a tedious task that might not have much value-added.

Nevertheless, one direction for further research might be to introduce portfolio choice in an estimated DSGE model with many shocks and look at the portfolio implications and consumption-real exchange rate correlations in such a set-up. Another direction is to introduce asset market imperfections alongside market incompleteness to limit asset trade and the degree of risk sharing as in Kollmann (2009).[48]

Finally, it might be important to consider alternative explanations of the anomaly that do not rely on market incompleteness, but on non-separable preferences. Raffo (2010), Karabarbounis (2010), Stathopoulos (2010) and Colacito and Croce (2010)[49] are examples to papers that follow this approach without considering portfolio choice. These models suggest that relative consumption and real exchange rate can be negatively correlated under complete markets. This strand of literature can be reconciled with the general equilibrium portfolio literature that is successful in accounting for the observed portfolio positions in models which do not display large deviations from risk sharing.

[48] Chapter 2 allows for portfolio transaction costs along the lines of Tille and van Wincoop (2010) and shows that they can be instrumental in matching the observed portfolios alongside a negative correlation between relative consumption and real exchange rate.

[49] Chapter 2 looks at the implications of introducing recursive utility as in Colacito and Croce (2010).

Appendix to Chapter 1

1.5.1 Loading factors that determine optimal portfolio under domestic tradable price stabilisation

To understand the hedging motives behind the optimal bond portfolio, we use the partial equilibrium expression in (1.29) to decompose (1.34). We show how excess returns (terms of trade in this case) load on different components of relative consumption risk, namely relative income risk in tradable and non-tradable sectors and real exchange rate risk.

Loadings of terms of trade on relative income risk in the tradable sector

$$\beta_Y^T \equiv \frac{Cov_t[\Lambda_{Y,t+1}^T, \hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} = [4\theta\nu(1-\nu)-1] + \frac{\kappa(2\nu-1)^2}{\gamma\kappa\rho+(1-\gamma)} > 0 \text{ if } \theta > \frac{1}{4\nu(1-\nu)} \tag{1.57}$$

$\theta > \frac{1}{4\nu(1-\nu)}$ is a sufficient condition for $\beta_Y^T > 0$.[50] In other words, for sufficiently large θ, an increase in relative tradable income is associated with a terms of trade depreciation. Intuitively, when the price elasticity of tradables is high, relative price of home goods falls to increase home and foreign demand for home tradables goods and clear the excess supply of tradables in the market. $\beta_Y^T > 0$ implies that foreign bonds pay relatively more when relative tradable income is high, making it optimal to have a short position in foreign bonds as a hedge against tradable income risk.

Loadings of terms of trade on relative income risk in the non-tradable sector

$$\beta_Y^N \equiv \frac{Cov_t[\Lambda_{Y,t+1}^N, \hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} = -\frac{\gamma(2\nu-1)(\kappa\rho-1)}{\gamma\kappa\rho+(1-\gamma)} \gtrless 0 \text{ iff } \kappa\rho \lessgtr 1 \tag{1.58}$$

Note that $\beta_Y^N = 0$ if $\nu = \frac{1}{2}$ or $\kappa\rho = 1$. For $\nu > \frac{1}{2}$, the sign of β_Y^N depends on the sign of $(\kappa\rho - 1)$. In other words, assuming tradables consumption is biased towards home goods, when tradables and non-tradables are gross complements, i.e. $\kappa\rho < 1$, terms of trade depreciates in the states of the nature where relative non-tradable income is high, implying a short position in foreign bonds. On the other hand, when tradable and non-tradable goods are gross substitutes, i.e. $\kappa\rho > 1$, terms of trade appreciates

[50] For the case of no consumption home bias, i.e. $\nu = \frac{1}{2}$, $\theta > 1$ is necessary and sufficient for $\beta_Y^T > 0$. For $\nu > \frac{1}{2}$, $\frac{1}{4\nu(1-\nu)} > 1$, so θ should be sufficiently larger than 1 to have $\beta_Y^T > 0$.

when relative non-tradable income is high, making it optimal to have a long position in foreign bond.

To build intuition for the result note that relative non-tradable income, $\hat{Y}^R_{N,t}$, consists of two components: the relative supply and the relative price of non-tradable goods, i.e. $\hat{Y}^R_{N,t} \equiv \hat{Y}_{N,t} - \hat{Y}^*_{N,t} - \hat{P}^N_t$ where $\hat{P}^N_t = \hat{P}^*_{N,t} + \hat{S}_t - \hat{P}_{N,t}$. Since the terms of trade is independent of non-tradable endowment shocks under the assumption that $\delta_N = 1$,(see equation (1.33)), excess return only loads on the non-tradable income risk coming from tradable endowment shocks, which affect $\hat{Y}^R_{N,t}$ through $\hat{P}^N_t -$ the relative price of non-tradables. Now, consider a positive shock to home tradables endowment that depreciates the terms of trade and increases the consumption of tradables. For $\kappa\rho < 1$, consumption of non-tradables also increase because of the complementarity between the two goods. This in turn implies that the relative price of home non-tradables goes up, increasing the value of the fixed endowment of non-tradable goods. Therefore, for $\kappa\rho < 1$, a rise in relative non-tradable income is associated with a terms of trade depreciation, i.e. a rise in excess return, making it optimal to short foreign bonds.

Loadings of terms of trade on real exchange rate risk

$$\beta_Q \equiv \frac{Cov_t[\Lambda_{Q,t+1}, \hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} = \frac{\gamma\kappa\rho(2\nu - 1)}{\gamma\kappa\rho + (1-\gamma)} \gtrless 0 \text{ iff } \nu \gtrless \frac{1}{2}$$

When there is home bias in tradables consumption, $\nu > \frac{1}{2}$, terms of trade and real exchange rate are positively correlated. Thus, foreign bonds pay more in the states of the nature where home consumption basket is cheaper, making it optimal to have a short position in foreign bonds.

To summarise, under the conditions $\nu > \frac{1}{2}$, $\theta > \frac{1}{4\nu(1-\nu)}$ and $\kappa\rho < 1$, different hedging motives all require a short position in foreign bonds and there is no tension between different hedging motives.

1.5.2 Loading factors that determine optimal portfolio under consumer price stabilisation

To have a better understanding of the hedging motives behind the optimal bond position , we decompose the relative consumption risk generated by tradable and non-tradable shocks into (sectoral) relative income risk and real exchange rate risk components according to the partial equilibrium formulation given in equation (1.29).

Loadings of terms of trade on relative income risk in the tradable sector

$$\beta_Y^T \equiv \frac{Cov_t[\Lambda_{Y,t+1}^T, \hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} = -\frac{(\gamma\kappa\rho + 1 - \gamma)}{(1-\gamma)} \frac{(\theta-1)^2}{\theta^2} \frac{\sigma_T^2}{\sigma_N^2} < 0$$

Excess return, i.e. real exchange rate, and relative tradable income risk are negatively correlated for all possible parameter values. An increase in the relative supply of tradables makes home agents relatively wealthier and appreciates the relative price of non-tradables and therefore the real exchange rate. Therefore, the optimal hedge against the tradables income risk arising from tradable sector shocks is to have a long position in foreign bonds. The presence of non-tradable shocks limit this position, because under the parameter restrictions we impose, relative tradables income is independent of non-tradable supply shocks (because terms of trade is independent). Having a zero bond position is therefore the optimal hedge against non-tradable sector shocks. In other words, taking a long position in foreign bonds to hedge against the tradables income risk caused by shocks to tradable endowment makes the agents vulnerable to non-tradable endowment shocks, which would have no effect on relative tradables income for a zero bond portfolio. This explains why β_Y^T, is decreasing in σ_N^2/σ_T^2 in absolute value terms.

Loadings of terms of trade on relative income risk in the non-tradable sector

$$\beta_Y^N \equiv \frac{Cov_t[\Lambda_{Y,t+1}^N, \hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} = -\frac{(\rho-1) - \gamma(\kappa\rho - 1)}{\rho(1-\gamma)} \lesseqgtr 0 \text{ iff } \kappa\rho - 1 \lesseqgtr \frac{\rho - 1}{\gamma}$$

For $\rho > 1$, a sufficient condition for $\beta_Y^N < 0$ is $\kappa\rho < 1$. If these conditions are satisfied, real exchange rate depreciates (foreign bonds pay higher) when relative non-tradables income is low, making it optimal to have a long position in foreign bonds. To see this, again consider the effects of tradable and non-tradable supply shocks on relative non-tradables income, i.e. $\hat{Y}_{N,t}^R \equiv \hat{Y}_{N,t} - \hat{Y}_{N,t}^* - \hat{P}_t^N$.

An increase in the supply of home tradable goods appreciates the relative price of non-tradables due to wealth effects and raise the value of home non-tradable income compared to foreign. Therefore, conditional on tradable endowment shocks, relative non-tradables income and real exchange rate are negatively correlated, making it optimal to have a long position in foreign bonds.

Now, consider an increase in the supply of home non-tradable goods. If tradables and non-tradables are complements, home agents want to increase the consumption of tradables alongside the consumption of non-tradables. Given fixed supply of tradables, non-tradables price will have to fall even more to clear the excess supply of non-tradables. In this case, relative non-tradable income will fall and relative non-tradables price will depreciate in the home country ($\Lambda^N_{Y,t+1} \downarrow$ and $\hat{P}^N_t \uparrow$) , making it optimal to have a long position in foreign bonds. But when the substitutability between tradables and non-tradables is sufficiently high, an increase in the relative supply of non-tradables only require a small depreciation in relative non-tradables price ($\Lambda^N_{Y,t+1} \uparrow$ and $\hat{P}^N_t \uparrow$), implying a short position in foreign bonds.

Loadings of terms of trade on the real exchange rate risk

$$\beta_Q \equiv \frac{Cov_t[\Lambda_{Q,t+1}, \hat{r}_{x,t+1}]}{Var_t[\hat{r}_{x,t+1}]} = 1 > 0$$

By definition, excess returns load perfectly on real exchange rate risk, therefore for $\rho > 1$, it is optimal to short foreign bonds to hedge against real exchange risk. When home consumption is more expensive ($\Lambda_{Q,t+1} \downarrow$), home bonds are a better hedge as home currency is more valuable in real terms ($\hat{r}_{x,t+1} \downarrow$).

To summarise; under the conditions $\nu = \frac{1}{2}, \rho > 1$ and $\kappa\rho < 1$, relative income risk in each sector require a long position in foreign bonds, whereas the real exchange rate risk requires a short position. But $\kappa\rho < 1$ ensures that optimal portfolio is a long position (see equation (1.39)), which in turn implies that relative income risk dominates the real exchange rate risk under these conditions.[51]

1.5.3 Preference (i-pod) shocks

As an alternative demand shock, we introduce preference shocks as in Coeurdacier et al.(2007) by modifying the consumption of tradables in the following way:

$$C_{T,t} = \left[\nu^{\frac{1}{\theta}}(\Psi_{H,t}C_{H,t})^{\frac{\theta-1}{\theta}} + (1-\nu)^{\frac{1}{\theta}}(\Psi_{F,t}C_{F,t})^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{1.59}$$

where $\Psi_{H,t}$ and $\Psi_{F,t}$ are shocks that reflect changes in world preferences for home and foreign produced tradable goods, respectively. As also mentioned by the authors, these shocks can also be thought as capturing changes in the quality of home and foreign goods, which is more of a supply-side interpretation. The tradables price

[51]Note that for the same restrictions, i.e. $\nu = \frac{1}{2}$, $\rho > 1$ and $\kappa\rho < 1$, there are no conflicting hedging motives when relative bond returns are given by the terms of trade. Relative consumption risk is driven only by the relative income risk in the tradable sector, $\beta_{Y,T}$, which implies a short position in foreign currency for $\theta > 1$.

index that is consistent with the modified tradables consumption is the following:

$$P_{T,t} = \left[\nu(P_{H,t}/\Psi_{H,t})^{1-\theta} + (1-\nu)(P_{F,t}/\Psi_{F,t})^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{1.60}$$

Foreign consumption of tradables and the associated price index are affected by $\Psi_{H,t}$ and $\Psi_{F,t}$ in a similar way as the home variables. Goods market clearing conditions in the tradables sector change accordingly:

$$\begin{aligned} Y_{H,t} &= \Psi_{H,t}^{\theta-1}\left\{\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\theta}\left(\frac{P_{T,t}}{P_t}\right)^{-\kappa}\gamma\nu C_t + \left(\frac{P^*_{H,t}}{P^*_{T,t}}\right)^{-\theta}\left(\frac{P^*_{T,t}}{P^*_t}\right)^{-\kappa}\gamma(1-\nu)C^*_t\right\} \\ Y^*_{F,t} &= \Psi_{F,t}^{\theta-1}\left\{\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\theta}\left(\frac{P_{T,t}}{P_t}\right)^{-\kappa}\gamma(1-\nu)C_t + \left(\frac{P^*_{F,t}}{P^*_{T,t}}\right)^{-\theta}\left(\frac{P^*_{T,t}}{P^*_t}\right)^{-\kappa}\gamma\nu C^*_t\right\} \end{aligned} \tag{1.61}$$

I-pod shocks are assumed to follow AR(1) processes similar to endowment shocks:

$$\log\Psi_{H,t} = \delta_\Psi \log\Psi_{H,t-1} + u_{\Psi,t}, \qquad \log\Psi_{F,t} = \delta_\Psi \log\Psi_{F,t-1} + u^*_{\Psi,t}$$

Since these preference shocks affect tradable goods prices in each country, they affect the consumer prices and hence the real exchange rate. Log-linearisation of the price indices and the decomposition of the real exchange rate shows this clearly:

$$\hat{Q}_t = \gamma(2\nu-1)\left[\widehat{TOT}_t + (\hat{\Psi}_{H,t} - \hat{\Psi}_{F,t})\right] + (1-\gamma)\hat{P}^N_t \tag{1.62}$$

As before, real exchange rate depreciates following a depreciation in the terms of trade for $\nu > \frac{1}{2}$, or a depreciation in the relative non-tradables price for $0 < \gamma < 1$. But now it also depends on relative ipod shocks: for a given $\widehat{TOT}_t$ and $\hat{P}^N_t$, real exchange rate depreciates when there is a positive quality shock in the home country.

Coeurdacier et al.(2007), and Coeurdacier and Gourinchas (2009) note that due to difficulties in measuring quality changes, the observed real exchange rate might be different from the welfare-based real exchange rate given by equation (1.62). Here we present some analytical results assuming that these shocks are perfectly measured as in Coeurdacier et al.(2007).

$$\begin{aligned}\hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho} &= \frac{\Gamma_3}{\kappa\rho(1+2\nu(\theta-1))}\left[(\hat{Y}_{H,t} - \hat{Y}_{F,t}) + (\hat{\Psi}_{H,t} - \hat{\Psi}_{F,t})\right] \qquad (1.63)\\ &+ \frac{(1-\gamma)(\kappa\rho-1)}{\kappa\rho}(\hat{Y}_{N,t} - \hat{Y}^*_{N,t})\\ &+ \frac{(1-\beta)\Gamma_1}{\gamma\kappa\rho(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t}\end{aligned}$$

$$\hat{r}_{x,t} = \widehat{TOT}_t - E_{t-1}\widehat{TOT}_t = \frac{1}{1+2\nu(\theta-1)}(u_{H,t} - u_{F,t}) \qquad (1.64)$$

$$-\frac{2\nu(\theta-1)}{1+2\nu(\theta-1)}(u_{\Psi h,t} - u_{\Psi f,t}) - \frac{(1-\beta)(2\nu-1)}{\gamma(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t} \qquad (1.65)$$

As shown in equation (1.63), an increase in the world demand for home goods affects the real exchange rate adjusted relative consumption in the same way as a positive supply shock. As discussed above, $\hat{C}_t - \hat{C}_t^* - \frac{\hat{Q}_t}{\rho}$ moves in the same way in response to a positive supply or demand shock for $\theta > \theta_3^*$ ($\Gamma_3 > 0$) or $\theta < 1 - \frac{1}{2\nu}$. On the other hand, the response of the terms of trade to supply and demand shocks goes in opposite ways provided that $\theta > 1$. That is, an increase in the world preference for home goods ($\hat{\Psi}_{H,t} \uparrow$) implies an appreciation of domestic terms of trade and thus a fall in the excess return on foreign bonds if the elasticity of substitution between home and foreign tradables, θ, is greater than 1. Therefore, for $\theta > Max(1, \theta_2^*)$, it is optimal to have a long position in foreign bonds to hedge against preference shocks, but a short position to hedge against tradable endowment shocks. As before, relative variance of the two shocks will determine the optimal foreign currency position:

$$\tilde{\alpha}_F = -\frac{\gamma(1-\nu)\Gamma_3}{(1-\beta)(\gamma\kappa\rho+(1-\gamma))\left(1 - \frac{2\nu(\theta-1)\Gamma_1\frac{\sigma_\Psi^2}{\sigma_T^2}}{(\gamma\kappa\rho+(1-\gamma))\left(2\nu(\theta-1)\frac{\sigma_\Psi^2}{\sigma_T^2}-1\right)}\right)}$$

For $\theta > 1$, $\frac{\sigma_\Psi^2}{\sigma_T^2} < \frac{1}{2\nu(\theta-1)}$ is a sufficient condition to ensure that optimal portfolio is a short position in foreign bonds as in the case of only supply shocks (see equation (1.34)). But for sufficiently large $\frac{\sigma_\Psi^2}{\sigma_T^2}$, optimal bond portfolio switches sign as in the case with news shocks.

Real exchange rate-relative consumption correlations in the presence of i-pod shocks

$$\begin{aligned}\hat{C}_t - \hat{C}_t^* &= \psi_c^{nc}\widehat{NFA}_{t-1} + \frac{\gamma(2\theta\nu-1)}{1+2\nu(\theta-1)}\left[(\hat{Y}_{H,t}-\hat{Y}_{F,t}) + (\hat{\Psi}_{H,t}-\hat{\Psi}_{F,t})\right] \\ &\quad +(1-\gamma)(\hat{Y}_{N,t}-\hat{Y}_{N,t}^*) + \frac{4\theta\nu(1-\beta)}{(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t}\end{aligned} \quad (1.66)$$

$$\begin{aligned}\hat{Q}_t &= -\psi_q^{nc}\widehat{NFA}_{t-1} - \left[\tfrac{(1-\gamma)(2\theta\nu-1)-\kappa(2\nu-1)}{\kappa(1+2\nu(\theta-1))}\right]\left[(\hat{Y}_{H,t}-\hat{Y}_{F,t}) + (\hat{\Psi}_{H,t}-\hat{\Psi}_{F,t})\right] \\ &\quad +\tfrac{1-\gamma}{\kappa}(\hat{Y}_{N,t}-\hat{Y}_{N,t}^*) - \tfrac{(1-\beta)[4\theta\nu(1-\nu)(1-\gamma)+\kappa(2\nu-1)^2]}{\gamma\kappa(1-\nu)(1+2\nu(\theta-1))}\tilde{\alpha}_F\hat{r}_{x,t}\end{aligned} \quad (1.67)$$

Just like news shocks, preference shocks can reduce the effectiveness of nominal bonds in hedging against supply shocks. If preference shocks are sufficiently large such that the optimal portfolio is a long position in foreign bonds, relative consumption and real exchange rate can be negatively correlated conditional on tradable sector supply shocks.

Chapter 2

Portfolio Allocation and International Risk Sharing with Trade in Equities, Transaction Costs and Recursive Preferences

2.1 Introduction

In the first chapter, we start from a model which can account for the consumption-real exchange rate anomaly when international asset trade is restricted to a single bond and analyse the implications of introducing a second internationally traded bond. When relative bond returns are given by the terms of trade, trade in two bonds spans the uncertainty caused by shocks to tradables and non-tradables in each country and brings the equilibrium very close to the one under complete markets. This chapter extends this analysis in three dimensions.

First extension is to allow for international trade in equities alongside a single non-contingent bond. We show that trade in equities has similar risk sharing implications as trade in bonds when uncertainty is driven by shocks to the supply of tradable and non-tradable goods. The optimal portfolio that achieves full risk sharing is biased towards foreign equity. Hence, this set-up can neither generate home equity bias nor account for the anomaly. Trade in equities can complete the markets because relative equity returns, just like relative bond returns, do not load strongly on shocks to non-tradables. This implies that equity portfolio can be chosen to hedge against shocks to tradables, which generate larger fluctuations in relative marginal utilities of consumption compared to shocks to non-tradables when uninsured.

Introducing shocks to the share of income that accrues to shareholders (redistributive shocks as in Coeurdacier et al., 2007) helps generate home equity bias and lowers the cross-correlation between relative consumption and real exchange rate. Choosing a home-biased equity portfolio to hedge against redistributive shocks would imply unwanted valuation effects which amplify fluctuations in relative marginal utilities of consumption conditional on tradable shocks.

Naturally, it becomes harder to have meaningful market incompleteness as we increase the number of assets that can be traded internationally. We show that a financial market set-up with trade in bonds and equities can be reconciled with a low degree of risk sharing provided that relative bond returns are equal to the real exchange rate as opposed to the terms of trade and relative equity returns are subject to redistributive shocks. Such a model can generate a long position in foreign bonds and home bias in equity in line with the international portfolio data for industrial countries alongside a low consumption-real exchange rate correlation shown in Table 1.1.

As a second extension, we specify portfolio transaction costs as in Tille and van Wincoop (2010) and Devereux and Yetman (2010) to limit portfolio diversification. We assume that investing assets abroad entails a fixed iceberg cost which might be different for bonds and equities. Due to the presence of transaction costs, agents cannot take a portfolio position that would minimise fluctuations in relative marginal utilities of consumption across countries, which in turn hampers international risk sharing. In a simple calibration exercise, we show that transaction costs can be instrumental in generating realistic international portfolio positions alongside a low correlation between relative consumption and real exchange rate: For values of transaction costs around 11% of the variance of tradable sector shocks, foreign currency holdings are around 37% of GDP, while the share of home equity held domestically is about 89%, both of which are consistent with the data on international portfolios for developed countries. With these portfolio positions, the implied correlation between relative consumption and real exchange rate is around zero.

In a third and final extension, we allow for recursive utility as in Epstein and Zin (1989) and explore the role of time-non-separability for optimal portfolios and consumption-real exchange rate correlations. Under recursive utility, stochastic discount factor has a component that depends on the difference between realised and risk-adjusted expected utility, which we refer to as the 'intertemporal uncertainty factor'. Hence, optimal portfolio has an additional determinant that depends on the covariance between relative intertemporal uncertainty factors and excess return.

This means that optimal portfolio will not be solely focused on minimising real exchange rate adjusted relative consumption as in the case of separable preferences, which could potentially help account for the anomaly in the presence of endogenous portfolio choice. In this framework, intertemporal uncertainty affects the equilibrium of the model through its effect on equilibrium portfolios. Our results show that this effect is not strong enough to make a significant difference for equilibrium portfolios and consumption-real exchange rate correlations in the baseline model with trade in two bonds and stationary shocks.

2.2 International trade in equities and bonds

Chapter 1 shows that it is hard to account for the consumption-real exchange rate anomaly when agents can trade in two bonds instead of one. In this section, we analyse whether international trade in equities has similar risk sharing implications as international trade in bonds.

2.2.1 The model

The model is a two-country two-sector endowment model as described in Chapter 1, modified to allow for international trade in equity and redistributive shocks. For completeness, we also describe the parts of the model that are exactly the same as in Chapter 1.

The representative agent in home country maximises the expected present discounted value of the utility:

$$U_t = E_t \sum_{s=t}^{\infty} \delta_s \frac{C_s^{1-\rho}}{1-\rho}, \tag{2.1}$$

where C is consumption and δ_s is the discount factor, which is determined as follows:

$$\delta_{s+1} = \delta_s \beta(C_{As}), \;\; \delta_0 = 1, \tag{2.2}$$

where C_A is aggregate home consumption and $0 < \beta(C_A) < 1$. To achieve stationarity under incomplete market specification, we assume $\beta_C(C_A) \leq 0$, which implies that agents discount the future more as aggregate consumption increases, i.e. agents bring consumption forward when aggregate consumption is high. Following Devereux and Sutherland (2011), we assume that the individual takes C_A as given when optimising and specify the discount factor as follows:

$$\beta(C_A) = \omega C_A^{-\eta}, \tag{2.3}$$

with $0 \leq \eta < \rho$ and $0 < \omega \bar{C}_A^{-\eta} < 1$ (for $\eta = 0$ we have the constant discount factor).

C represents a consumption index defined over tradable C_T and non tradable C_N consumption:

$$C_t = \left[\gamma^{\frac{1}{\kappa}} C_{T,t}^{\frac{\kappa-1}{\kappa}} + (1-\gamma)^{\frac{1}{\kappa}} C_{N,t}^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}}, \tag{2.4}$$

where κ is the elasticity of intratemporal substitution between C_N and C_T and γ is the weight that the households assign to tradable consumption. The tradable component of the consumption index, in turn, is a CES aggregate of home and foreign tradable consumption goods, C_H and C_F:

$$C_{T,t} = \left[\nu^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-\nu)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{2.5}$$

where θ is the elasticity of intratemporal substitution between C_H and C_F and ν is the weight that agents assign to domestic tradable consumption. We allow for a home bias in tradable goods by assuming $\nu > \frac{1}{2}$. We adopt a similar preference specification for the foreign country except that variables are denoted with an asterisk. The consumption price index (CPI), which is defined as the minimum expenditure required to purchase one unit of aggregate consumption for the home agent is given by:

$$P_t = \left[\gamma P_{T,t}^{1-\kappa} + (1-\gamma) P_{N,t}^{1-\kappa} \right]^{\frac{1}{1-\kappa}}, \tag{2.6}$$

Meanwhile, the traded goods price index, which is defined as the minimum expenditure required to purchase one unit of a traded good is given by:

$$P_{T,t} = \left[\nu P_{H,t}^{1-\theta} + (1-\nu) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{2.7}$$

We assume that the law of one price holds, i.e. $P_{H,t}^* = P_{H,t}/S_t$, and $P_{F,t} = P_{F,t}^* S_t$, where S_t denotes the nominal exchange rate defined as the price of foreign currency in terms of domestic currency. The presence of non-tradable goods and home bias in tradables consumption leads to deviations from purchasing power parity. We define the real exchange rate as $Q = \frac{SP^*}{P}$.

In each country agents can invest their nominal wealth, denoted by W_t, in two riskless bonds denominated in home and foreign currency and two risky assets, which are claims on a fraction of home and foreign countries' tradable sector endowments.[1] The fraction of tradable endowments that accrue to shareholders is denoted by $k_{i,t}$ ($i = H, F$). Thus, $(1 - k_{i,t})P_{i,t}Y_{i,t} + P_{N,t}Y_{N,t}$ represents the non-financial income received by home agents. The budget constraint of the home agent can be written as:

$$W_t = R_{H,t}B_{H,t-1} + R^*_{F,t}S_tB_{F,t-1} + x_{1,t-1}P_{H,t}[V_{H,t} + k_{H,t}Y_{H,t}] + x_{2,t-1}S_tP^*_{F,t}[V^*_{F,t} + k_{F,t}Y^*_{F,t}] \tag{2.8}$$

$$+(1 - k_{H,t})P_{H,t}Y_{H,t} + P_{N,t}Y_{N,t} - P_tC_t,$$

where W_t is net nominal wealth defined as,

$$W_t \equiv B_{H,t} + S_tB_{F,t} + x_{1,t}P_{H,t}V_{H,t} + x_{2,t}S_tP^*_{F,t}V^*_{F,t}. \tag{2.9}$$

$B_{H,t}$ denotes home holdings of internationally traded home bond, while $B_{F,t}$ denotes home holdings of internationally traded foreign bond, $x_{i,t}$ $(i = 1, 2)$ denote the shares of domestic and foreign tradable sector equity held by the home agent. $V_{H,t}, V^*_{F,t}$ are equity prices for holding claims on the tradable sector endowments in each country while $Y_{H,t}, Y^*_{F,t}, Y_{N,t}$ and $Y^*_{N,t}$ represents the stochastic endowments (dividends). $R_{H,t}$ and $R^*_{F,t}$ represent the returns for holding home and foreign currency bonds denominated in respective currency units.

Gross nominal equity returns and bond returns in home currency units are as follows:

$$R_{1,t} \equiv \frac{P_{H,t}[V_{H,t} + k_{H,t}Y_{H,t}]}{P_{H,t-1}V_{H,t-1}},\ R_{2,t} \equiv \frac{S_tP^*_{F,t}[V^*_{F,t} + k_{F,t}Y^*_{F,t}]}{S_{t-1}P^*_{F,t-1}V^*_{F,t-1}} \tag{2.10}$$

$$R_{3,t} \equiv R^*_{F,t}\frac{S_t}{S_{t-1}},\ R_{4,t} \equiv R_{H,t}.$$

V_i and Y_i are in units of the index good i for $i = H, F^*$.[2].

[1] To simplify the analysis, we assume that agents cannot trade claims to foreign non-tradable endowments. See Hnatkovska (2010) for references to empirical studies that find support for limited tradability of foreign non-tradable sector equities.

[2] Real equity returns will depend on the relative price of each index good with respect to the overall price level P_t, i.e. $r_{i,t} = R_{i,t}\frac{P_{t-1}}{P_t} \equiv \frac{\frac{P_{i,t}}{P_t}[V_{i,t}+Y_{i,t}]}{\frac{P_{i,t-1}}{P_{t-1}}V_{i,t-1}}$ for $i = H, F^*$.

We can express home agent's asset holdings as shares of wealth:

$$\begin{gathered} x_{1,t}P_{H,t}V_{H,t} \equiv \alpha_{1,t}W_t,\ x_{2,t}S_tP^*_{F,t}V^*_{F,t} \equiv \alpha_{2,t}W_t, \\ S_tB_{F,t} \equiv \alpha_{3,t}W_t,\ B_{H,t} \equiv \alpha_{4,t}W_t, \\ \sum_{i=1}^{4}\alpha_{i,t} = 1. \end{gathered} \tag{2.11}$$

Using the definition of gross nominal returns given by (2.10) and the shares of wealth defined by (2.11), we can rewrite the budget constraint in terms of excess returns over the return on the home bond, $R_{xi} = R_{i,t} - R_{4,t}$ for $i = 1, 2, 3$:

$$\begin{gathered} W_t = [R_{x1,t}\alpha_{1,t-1} + R_{x2,t}\alpha_{2,t-1} + R_{x3,t}\alpha_{3,t-1} + R_{4,t}]W_{t-1} \\ +(1-k_{H,t})P_{H,t}Y_{H,t} + P_{N,t}Y_{N,t} - P_tC_t. \end{gathered} \tag{2.12}$$

2.2.2 Equilibrium

Optimality conditions related to assets allocation for domestic and foreign households are given by the following set of equations:

$$\begin{gathered} E_t\left[m_{t+1}R_{i,t+1}\right] = 1 \qquad i = 1, ..., 4. \\ E_t\left[m^*_{t+1}R_{i,t+1}\frac{S_t}{S_{t+1}}\right] = 1 \qquad i = 1, ..., 4. \end{gathered} \tag{2.13}$$

where $m_{t+1} = \beta(C_t)\frac{P_t}{P_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}$ and $m^*_{t+1} = \beta(C^*_t)\frac{P^*_t}{P^*_{t+1}}\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\rho}$. Stock prices are determined by substituting the definition of gross equity returns into the Euler equations for equities. Forward iteration together with no-bubble condition gives the tradable sector stock prices in home and foreign countries:

$$\begin{aligned} V_{H,t} &= \sum_{s=0}^{\infty}E_t\left\{\omega^{s+1}\Psi_{t,s}\frac{C_{t+s+1}^{-\rho}\frac{P_{H,t+s+1}}{P_{t+s+1}}}{C_t^{-\rho}\frac{P_{H,t}}{P_t}}k_{H,t+s+1}Y_{H,t+s+1}\right\}, \\ V^*_{F,t} &= \sum_{s=0}^{\infty}E_t\left\{\omega^{s+1}\Psi^*_{t,s}\frac{C_{t+s+1}^{*-\rho}\frac{S_{t+s+1}P^*_{F,t+s+1}}{P_{t+s+1}}}{C_t^{*-\rho}\frac{S_tP_{F,t}}{P_t}}k_{F,t+s+1}Y_{F,t+s+1}\right\}, \end{aligned} \tag{2.14}$$

where $\Psi_{t,s} \equiv \prod_{j=0}^{s}C_{t+s}^{-\eta}$ and $\Psi^*_{t,s} \equiv \prod_{j=0}^{s}C_{t+s}^{*-\eta}$ (see equation (2.3)).

Equilibrium in asset markets requires home and foreign shares of stock sum up to one and home and foreign holdings of bonds are equal to zero:

$$\begin{gathered} x_{1,t} + x^*_{1,t} = 1, \; x_{2,t} + x^*_{2,t} = 1, \\ B_{F,t} + B^*_{F,t} = 0, \; B_{H,t} + B^*_{H,t} = 0. \end{gathered} \tag{2.15}$$

Equation (2.15) together with (2.11) and its foreign counterpart imply the following:

$$\begin{gathered} \alpha_{1,t} W_t + \alpha^*_{1,t} S_t W^*_t = P_{H,t} V_{H,t}, \; \alpha_{2,t} \frac{W_t}{S_t} + \alpha^*_{2,t} W^*_t = P^*_{F,t} V^*_{F,t} \\ \alpha_{3,t} \frac{W_t}{S_t} + \alpha^*_{3,t} W^*_t = 0, \; \alpha_{4,t} W_t + \alpha^*_{4,t} S_t W^*_t = 0 \\ W_t + S_t W^*_t = P_{H,t} V_{H,t} + S_t P^*_{F,t} V^*_{F,t} \end{gathered} \tag{2.16}$$

Equilibrium in good markets requires:

$$\begin{gathered} Y_{H,t} = \left(\frac{P_{H,t}}{P_{T,t}} \right)^{-\theta} \left(\frac{P_{T,t}}{P_t} \right)^{-\kappa} \gamma \nu C_t + \left(\frac{P^*_{H,t}}{P^*_{T,t}} \right)^{-\theta} \left(\frac{P^*_{T,t}}{P^*_t} \right)^{-\kappa} \gamma (1-\nu) C^*_t, \\ Y^*_{F,t} = \left(\frac{P_{F,t}}{P_{T,t}} \right)^{-\theta} \left(\frac{P_{T,t}}{P_t} \right)^{-\kappa} \gamma (1-\nu) C_t + \left(\frac{P^*_{F,t}}{P^*_{T,t}} \right)^{-\theta} \left(\frac{P^*_{T,t}}{P^*_t} \right)^{-\kappa} \gamma \nu C^*_t, \\ Y_{N,t} = \left(\frac{P_{N,t}}{P_t} \right)^{-\kappa} (1-\gamma_H) C_t, \quad Y^*_{N,t} = \left(\frac{P^*_{N,t}}{P^*_t} \right)^{-\kappa} (1-\gamma_F) C^*_t. \end{gathered} \tag{2.17}$$

As in Chapter 1, the model is closed by considering two simple policy rules. In the first one, policy authorities stabilise domestic tradable prices ($P_{H,t} = 1$, and $P_{F^*,t} = 1$) whereas in the second one they stabilise domestic consumer prices ($P_t = 1$, and $P^*_t = 1$). Relative bond return (nominal exchange rate) is equal to the terms of trade in the former and the real exchange rate in the latter.

2.2.3 Solving the model

Given the paths of the stochastic processes for the exogenous state variables $\{Y_{H,t}$, $Y_{F,t}$, $Y_{N,t}$, $Y^*_{N,t}$, $k_{H,t}$, $k_{F,t}\}$ and the policy rules, the full solution to the model is described by the sequence $\{C_t$, C^*_t, m_t, m^*_t, $P_{H,t}$,$P^*_{F,t}$,$P_{N,t}$,$P^*_{N,t}$,$P_{T,t}$,$P^*_{T,t}$, P_t, P^*_t, S_t, Q_t, $R_{1,t}$, $R_{2,t}$, $R_{3,t}$, $R_{4,t}$, $V_{H,t}$, $V_{F,t}$, W_t, W^*_t $\}$ and the vector $\{\alpha_{1,t}$, $\alpha^*_{1,t}$, $\alpha_{2,t}$, $\alpha^*_{2,t}$, $\alpha_{3,t}$, $\alpha^*_{3,t}$, $\alpha_{4,t}$, $\alpha^*_{4,t}\}$ which satisfy equations (2.6), (2.7) and their foreign counterparts, equations (2.10), (2.12), (2.13), definitions of stochastic discount factors as well as the asset and good market clearing conditions given by (2.16) and (2.17).

To solve the model in the presence of endogenous portfolio choice under incomplete markets, we use the approximation techniques developed by Devereux and Sutherland (2011) and Tille and van Wincoop (2010). We approximate our model around the symmetric steady-state described below.

Steady-state

We choose steady-state endowments and share of capitalisable income such that $\bar{k}_H = \bar{k}_F^*$, $\bar{Y}_H = \bar{Y}_F^*$ and $\bar{Y}_N = \bar{Y}_N^*$ and $\frac{\bar{Y}_H}{\bar{Y}_N} = \frac{\bar{Y}_F^*}{\bar{Y}_N^*} = \frac{1-\gamma}{\gamma}$, which imply $\bar{C} = \bar{C}^*$. This together with the assumption of zero steady-state inflation gives $\bar{m} = \bar{m}^* = \bar{\beta}$ and $\overline{R}_1 = \overline{R}_2 = \overline{R}_3 = \overline{R}_4 = \bar{\beta}^{-1}$ through home and foreign Euler equations. From the definition of nominal returns on assets (2.10), we obtain $\frac{\bar{V}_H}{\bar{k}_H \bar{Y}_H} = \frac{\bar{V}_F^*}{\bar{k}_F \bar{Y}_F^*} = \frac{\bar{\beta}}{1-\bar{\beta}}$. All relative prices are equal to 1 and $\bar{S} = \bar{Q} = 1$. Goods market equilibrium conditions given in (2.17) pin down steady-state consumption relative to tradable and non-tradable sector endowments as $\frac{\bar{C}}{\bar{Y}_H} = \frac{\bar{C}^*}{\bar{Y}_F^*} = \gamma$ and $\frac{\bar{C}}{\bar{Y}_N} = \frac{\bar{C}^*}{\bar{Y}_N^*} = 1 - \gamma$.

As the initial wealth distribution is not determined, we set $\bar{W} = \overline{SW}^*$ as in Benigno and Nisticò (2009). This assumption together with the our normalisation that $\bar{k}_H = \bar{k}_F^*$ and $\bar{Y}_H = \bar{Y}_F^*$, implies that the steady-state wealth in each country is equal to the total value of tradable sector equity, i.e. $\bar{W} = \bar{P}_H \bar{V}_H$ and $\bar{W}^* = \bar{P}_F^* \bar{V}_F^*$. Hence, steady-state wealth shares and equity shares are equivalent, i.e. $\bar{x}_1 = \bar{\alpha}_1$ and $\bar{x}_2 = \bar{\alpha}_2$ as implied by equation (2.11).

There is home bias in equity if the fraction invested by home agents in home tradable sector, $\bar{x}_1$, exceeds the share of home country's stock market capitalisation in world stock market capitalisation, which is given by $\bar{V}_H/(\bar{V}_H + \bar{S}\bar{V}_F^*) = 1/2$ at the symmetric steady-state.

Optimal portfolio solution

Taking a second order approximation to optimal portfolio choice equations given by (2.13) and using the property of the model that expected excess returns are zero up to a first order approximation, i.e. $E_t\left[\hat{R}_{x,t+1}\right] = 0 + O(\varepsilon^2)$, gives an orthogonality condition between excess returns and the relative stochastic discount factors which pins down optimal steady-state portfolios shares $\tilde{\alpha}' = [\frac{\bar{\alpha}_1}{\beta}\ \frac{\bar{\alpha}_2}{\beta}\ \frac{\bar{\alpha}_3}{\beta}]$:

$$Cov_t\left[(\hat{m}_{t+1} - \hat{m}^*_{t+1} + \Delta\hat{S}_{t+1}), \hat{R}_{x,t+1}\right] = 0 + O(\varepsilon^3), \tag{2.18}$$

where $\hat{R}'_{x,t+1} = [\hat{R}_{1,t} - \hat{R}_{4,t},\ \hat{R}_{2,t} - \hat{R}_{4,t},\ \hat{R}_{3,t} - \hat{R}_{4,t}]$.[3] As shown by Devereux and Sutherland (2011), to evaluate (2.18) and determine the portfolio shares, it is sufficient to take a first order approximation to the remaining equilibrium conditions for which the only aspect of portfolio behaviour that matters is the vector of steady-state portfolio shares, $\tilde{\alpha}'$. To see this consider a first order approximation of the budget constraint (2.12):

$$\begin{aligned} \widehat{W}_t &= \frac{1}{\beta}\widehat{W}_{t-1} + \widehat{R}_{x1,t}\tilde{\alpha}_1 + \widehat{R}_{x2,t}\tilde{\alpha}_2 + \widehat{R}_{x3,t}\tilde{\alpha}_3 + \frac{1}{\beta}\widehat{R}_{4,t} \\ &\quad + \left(1 - \bar{k}_H\right)\frac{\bar{P}_H\bar{Y}_H}{\bar{W}}(\hat{P}_{H,t} + \hat{Y}_{H,t}) - \bar{k}_H\frac{\bar{P}_H\bar{Y}_H}{\bar{W}}\hat{k}_{H,t} + \frac{\bar{P}_N\bar{Y}_N}{\bar{W}}(\hat{P}_{N,t} + \hat{Y}_{N,t}) \\ &\quad - \frac{\overline{PC}}{\overline{W}}(\widehat{P}_t + \widehat{C}_t) \end{aligned} \tag{2.19}$$

where $\widetilde{\alpha}_i = \frac{\overline{\alpha_i}}{\beta}$ for $i = 1, ..., 3$, $\frac{\bar{P}_H\bar{Y}_H}{\bar{W}} = \frac{\bar{Y}_H}{\bar{V}_H} = \frac{1-\bar{\beta}}{\bar{\beta}\bar{k}_H}$, $\frac{\bar{P}_N\bar{Y}_N}{\bar{W}} = \frac{\bar{P}_N\bar{Y}_N}{\bar{P}_H\bar{Y}_H}\frac{\bar{P}_H\bar{Y}_H}{\bar{W}} = \frac{\gamma}{1-\gamma}\frac{1-\bar{\beta}}{\bar{\beta}\bar{k}_H}$ and $\frac{\overline{PC}}{\overline{W}} = \frac{\bar{P}\bar{C}}{\bar{P}_H\bar{Y}_H}\frac{\bar{P}_H\bar{Y}_H}{\bar{W}} = \gamma\frac{1-\bar{\beta}}{\bar{\beta}\bar{k}_H}$.

It is possible to express optimal portfolio shares as a function of the loadings of excess returns on relative non-financial income risk and real exchange rate risk by substituting in partial equilibrium solutions for the components of the orthogonality condition (2.18).[4]

Calibration

We use a symmetric calibration for home and foreign economies. In choosing the parameters of utility function, we set steady state discount factor, $\overline{\beta}$, equal to 1/1.04 and the Uzawa convergence parameter equal to 0.01 which is similar to Devereux and Yetman (2010). As in Stockman and Tesar (1995) the coefficient of constant relative risk aversion (the inverse of the intertemporal elasticity of substitution), ρ, is set to 2.

We calibrate the parameters pertaining to the consumption basket in the following way. The share of tradable goods in final consumption, γ, is 0.55, while the share of home goods in tradable consumption, ν, is 0.72. The calibration of these parameters is in line with Benigno and Thoenissen (2008) and Corsetti et al.(2008).

We assume an elasticity of substitution between home and foreign traded goods, θ, of 2.5 and an elasticity of substitution between traded and non-traded goods, κ, of 0.41, similar to what is suggested by Stockman and Tesar (1995). Given $\rho = 2$, this

[3] For any variable X, $\hat{X} = \log(X/\bar{X})$ unless stated otherwise.
[4] See Coeurdacier and Gourinchas (2009) and Benigno and Nisticò (2009).

implies that utility is non-separable between traded and non-traded goods. Indeed, traded and non-traded goods are complements in our benchmark calibration since $\kappa\rho < 1$.

Sectoral endowments are described by the following first-order autoregressive processes:

$$\begin{aligned}
\log Y_{H,t} &= (1-\delta_T)\log \bar{Y}_H + \delta_T \log Y_{H,t-1} + \upsilon_{H,t}, \qquad (2.20)\\
\log Y^*_{F,t} &= (1-\delta_T)\log \bar{Y}^*_{F,t} + \delta_T \log Y^*_{F,t} + \upsilon_{F,t},\\
\log Y_{N,t} &= (1-\delta_N)\log \bar{Y}_N + \delta_N \log Y_{N,t-1} + \upsilon_{N,t}, \qquad (2.21)\\
\log Y^*_{N,t} &= (1-\delta_N)\log \bar{Y}^*_N + \delta_N \log Y^*_{N,t-1} + \upsilon^*_{N,t},
\end{aligned}$$

where $0 \leq \delta_T < 1$, $0 \leq \delta_N < 1$, and $\upsilon_{H,t}, \upsilon^*_{F,t}, \upsilon_{N,t}, \upsilon^*_{N,t}$ are i.i.d. shocks with a variance-covariance matrix $V(\upsilon)$.

We rely on Benigno and Thoenissen (2008) to calibrate sectoral endowment shocks. We set the persistence of shocks higher, $\delta_T = 0.96$ and $\delta_N = 0.34$, and use the same variance-covariance matrix:

$$V(\upsilon) = \begin{bmatrix} 0.0376 & 0.0159 & 0.0072 & 0.0044 \\ 0.0159 & 0.0376 & 0.0044 & 0.0072 \\ 0.0072 & 0.0044 & 0.0051 & 0.0021 \\ 0.0044 & 0.0072 & 0.0021 & 0.0051 \end{bmatrix}.$$

Redistributive shocks follow AR(1) processes of the form:

$$\begin{aligned}
\log k_{H,t} &= (1-\delta_K)\log \bar{k}_H + \delta_K \log k_{H,t-1} + \upsilon_{KH,t},\\
\log k^*_{F,t} &= (1-\delta_K)\log \bar{k}^*_{F,t} + \delta_K \log k^*_{F,t} + \upsilon_{KF,t}.
\end{aligned}$$

where $0 \leq \delta_K < 1$, and $\upsilon_{KH,t}, \upsilon_{KF,t}$ are i.i.d. shocks with a variance-covariance matrix $V(\upsilon_K)$. The mean capital share in tradable sector is set as $\bar{k}_H = 0.4$.[5] We assume that redistributive shocks are equally persistent as tradable endowment shocks, i.e. $\delta_K = \delta_T$, and that they are equally large, i.e. $Var(\upsilon_{KH,t}) = Var(\upsilon_{H,t})$ and $Var(\upsilon_{KF,t}) = Var(\upsilon_{F,t})$.[6] We set the correlation between home and foreign redistributive shocks equal to the correlation between home and foreign tradable endowments and assume

[5] This is consistent with Coeurdacier et al. (2007) who compute the steady-state capital share to be 40% using data for G7 countries.

[6] Coeurdacier et. al. (2007) compute the standard deviations of capital share and real GDP growth for G7 countries and set the size of redistributive shocks slightly larger than endowment shocks.

that redistributive shocks are not correlated with sectoral endowment shocks.

After solving the model in terms of the state variables, we use these autoregressive processes to generate simulated time series of length T (T=600) for the variables of interest. We repeat this procedure J (J =200) times and then compute the average of the moments from HP-filtered series.

2.2.4 Risk sharing implications of international trade in equities versus bonds

Table 2.1 reports optimal portfolio allocation and the correlation between relative consumption and real exchange rate under alternative asset markets with trade in equities and bonds. For completeness, we also report other key moments such as the cross-country correlations of consumption and output and the volatility of consumption and real exchange rate.

We first consider a version of the model where shocks to tradable and non-tradable good endowments are the only sources of uncertainty as in Benigno and Thoenissen (2008) and Corsetti et al. (2008). Starting from a simple asset market structure where international asset trade is restricted to a single non-contingent bond, we introduce portfolio choice between two assets at a time - either home and foreign currency bonds or home and foreign tradable sector equities. The aim is to compare trade in bonds with trade in equities in terms of their risk sharing implications in an incomplete market set-up where agents do not have enough assets to span the shocks hitting each economy.[7] The set-up with international trade in bonds and equities represents the complete market benchmark (see Table 2.1).

The first three columns under the heading of 'Endowment Shocks' in Table 2.1, illustrate the main conclusion of Chapter 1 that the consumption-real exchange anomaly is back when we allow for one additional bond to be internationally traded. Trade in two nominal bonds achieves the complete market outcome when relative bond returns are given by the terms of trade (This case corresponds to domestic tradable good price stabilisation and is denoted by II in Table 2.1). The optimal portfolio that leads to a perfect correlation between relative consumption and real

[7]Budget constraints described in the previous section nest all the alternative asset market structures we consider. For example, when agents in each country can hold shares in home and foreign tradable sector equities as well as a single non-contingent bond, and all tradable income is diversifiable, the budget constraint given by equation (2.8) can be modified in the following way:

$$W_t = R_{H,t}B_{H,t-1} + x_{1,t-1}P_{H,t}[V_{H,t} + Y_{H,t}] + x_{2,t-1}S_tP^*_{F,t}[V^*_{F,t} + Y^*_{F,t}] + P_{N,t}Y_{N,t} - P_tC_t, \quad (2.22)$$

where $W_t \equiv B_{H,t} + x_{1,t}P_{H,t}V_{H,t} + x_{2,t}S_tP^*_{F,t}V^*_{F,t}$.

exchange rate is a short position in foreign bonds. Chapter 1 discusses these results in detail.

The new result is that allowing portfolio choice between home and foreign tradable sector equities instead of home and foreign currency bonds also completes the markets (See column 6 in Table 2.1). In this case, the optimal portfolio that achieves full risk sharing is biased towards foreign equity. Because trade in two equities already completes the markets, allowing agents to trade home and foreign bonds as well equities does not make a difference for the moments of variables and changes the equity portfolio only slightly.

	Data	Endowment Shocks					
		Single Bond	Two Bonds		Single Bond & Two equities	Two Bonds & Two equities	
			I	II		I	II
$\bar{B}_F/\bar{Y}$	0.47	-	7.62	-4.67	-	0.02	-0.80
$\bar{x}_1$	0.89	-	-	-	0.39	0.39	0.49
$corr(C-C^*,Q)$	-0.45	-0.11	0.11	1.00	1.00	1.00	1.00
$Std(C)/Std(Y)$	0.76	0.91	0.90	0.87	0.87	0.87	0.87
$Std(Q)/Std(Y)$	6.16	0.76	0.51	0.68	0.69	0.69	0.69
$corr(C,Y)$	0.78	0.97	0.95	0.91	0.91	0.91	0.91
$corr(Y,Y^*)$	0.35	0.46	0.46	0.46	0.46	0.46	0.46
$corr(C,C^*)$	0.06	0.78	0.79	0.92	0.92	0.92	0.92
$corr(Q,TOT)$	0.32	-0.59	-0.42	0.47	0.47	0.47	0.47

Table 2.1: Optimal portfolios and international risk sharing with shocks to the endowment of tradables and non-tradables

Notes: $\bar{B}_F/\bar{Y}$ denotes home country's steady-state foreign bond holdings as a share of home GDP, $\bar{x}_1$ denotes the steady-state share of home tradable equity held by home agents. Data column contains data and statistics calculated for the U.S.: Data for $\bar{B}_F/\bar{Y}$ is from Lane and Shambaugh (2010) and data for $\bar{x}_1$ is from Coeurdacier et. al. (2007). Data on business cycle statistics are from Benigno and Thoenissen (2008) and Corsetti et al.. (2008). I and II correspond to the two simple policy rules: I is for the case of CPI stabilisation, where relative bond returns are given by surprises in the real exchange rate; II is for the case of domestic tradable price stabilisation, where relative bond returns are given by surprises in the terms of trade. Policy rules do not matter for the solution unless both home and foreign currency bonds are traded.

Figure 2.1 illustrates why trade in two equities can span the risks caused by tradable and non-tradable endowment shocks and complete the markets. First of all, as discussed in Chapter 1, deviations from efficient risk sharing are larger conditional on tradable endowment shocks in the absence of any ex-ante insurance (See the circled lines denoted by NC Bond in the upper right panel of Figure 2.1). Secondly, relative equity returns do not load on non-tradable endowment shocks. Hence, equity portfolio can be chosen to hedge against tradable sector shocks without implying unwanted valuation effects conditional on non-tradable sector shocks. This is the same intuition that explains why trade in bonds ensures higher risk sharing when relative bond returns are given by the terms of trade as opposed to the real exchange rate as discussed in Chapter 1. Note that the larger is the response of excess return to a non-tradable endowment shock, the larger are the deviations from full risk sharing in response to both shocks.

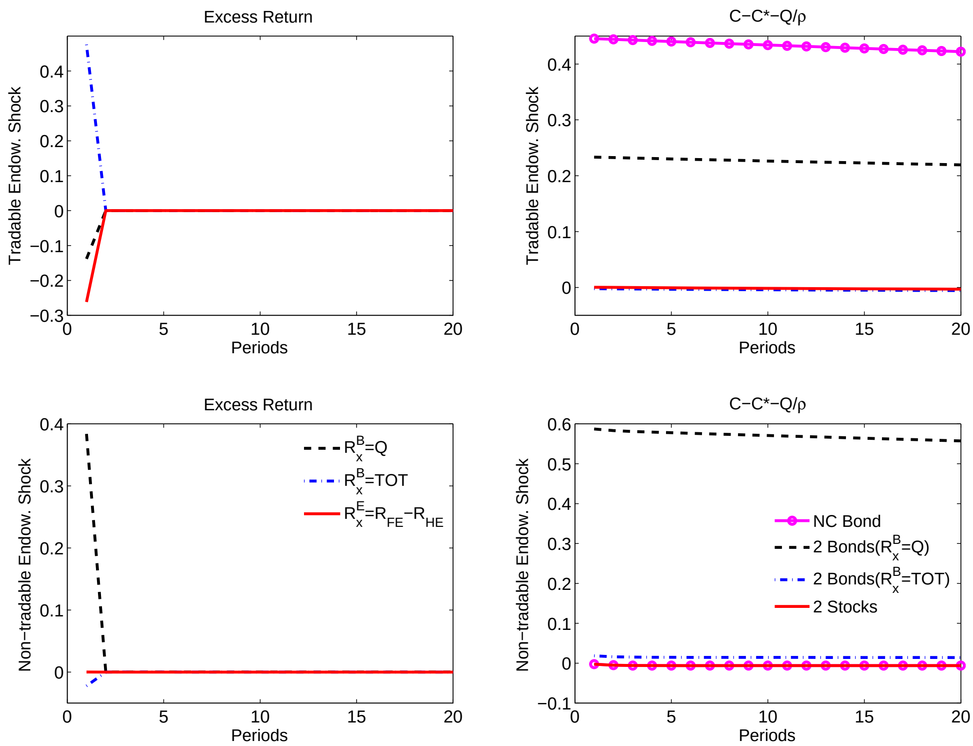


Figure 2.1: Risk sharing properties of bonds versus equities conditional on tradable and non-tradable endowment shocks

Notes: This figure plots the impulse responses of relative bond ($\hat{R}^B_x = \hat{R}_{F,t} - \hat{R}_{H,t}$) and equity returns ($\hat{R}^E_x = \hat{R}^E_{F,t} - \hat{R}^E_{H,t}$) and deviations from efficient risk sharing ($\hat{C} - \hat{C}^* - \hat{Q}/\rho$) conditional on endowment shocks under alternative asset markets: i) trade in single non-contingent bond (NC Bond), ii) trade in two bonds under CPI stabilisation, (2 Bonds, $\hat{R}^B_x = Q$), iii) trade in two bonds under domestic tradable price stabilisation, (2 Bonds, $\hat{R}^B_x = TOT$), and iv) trade in two equities (x-axis:periods, y-axis:percentage deviations from steady-state).

Excess return on foreign equity depends on the terms of trade and relative dividends (endowments). Under the baseline calibration with a relatively high trade elasticity ($\theta = 2.5$), an increase in home tradable endowment is associated with an increase in home equity return relative to foreign because the depreciation in home terms of trade remains limited and the increase in relative dividends dominate. Given that home relative consumption (adjusted by the real exchange rate) is higher in these states of nature, it is optimal to have a foreign bias in equity.

When we allow for redistributive shocks as described in the previous section, the performance of the model with trade in two equities improves along both dimensions: optimal equity portfolio becomes home biased ($\bar{x}_1 = 0.71$) and the correlation between relative consumption and real exchange rate becomes 0.03 instead of 1 as shown in Table 2.2.[8] Hence, in the presence of redistributive shocks, trade in equities cannot complete the markets and the equivalence between trade in bonds (under domestic tradable price stabilisation) and equities no longer holds.

	Data	Endowment and Redistributive Shocks			
		Single Bond	Single Bond Two equities	Two Bonds Two equities	
				I	II
$\bar{B}_F/\bar{Y}$	0.47	-	-	5.56	-4.67
$\bar{x}_1$	0.89	-	0.71	0.54	1.00
$corr(C-C^*,Q)$	-0.45	-0.11	0.03	0.22	1.00
$Std(C)/Std(Y)$	0.76	0.91	0.90	0.90	0.87
$Std(Q)/Std(Y)$	6.16	0.76	0.75	0.54	0.69
$corr(C,Y)$	0.78	0.97	0.96	0.95	0.91
$corr(Y,Y^*)$	0.35	0.46	0.46	0.46	0.46
$corr(C,C^*)$	0.06	0.78	0.81	0.80	0.92
$corr(Q,TOT)$	0.32	-0.59	-0.35	-0.22	0.47

Table 2.2: Optimal portfolios and international risk sharing with endowment and redistributive shocks

Notes: See notes under Table 2.2.

Hence, if we consider a model where relative bond returns are given by the real exchange rate and relative equity returns are subject to redistributive shocks, we can still have meaningful market incompleteness (a low correlation between relative consumption and real exchange rate) even when agents can trade both bonds and equities. The fifth column in Table 2.2 shows that such a model can generate a long position in foreign bonds, home bias in equity and a relative consumption-real exchange rate correlation of 0.22.

As can be expected in an endowment model, other business cycle statistics ob-

[8]See Heathcote and Perri (2007), Coeurdacier et al. (2007), Engel and Matsumoto (2009), Coeurdacier et al. (2010), Rahbari (2009) and references in these papers for explanations of home equity bias based on non-financial income risk. Coeurdacier and Rey (2010) provides a comprehensive review of the literature on equity home bias.

tained from the model do not match their empirical counterparts very well. For all cases, consumption is more volatile than in the data and the volatility of real exchange rate is very low because we assume that the law of one price holds for traded goods and trade elasticity is set at a relatively high value. Also, the cross-correlation between consumption levels is higher compared to the cross-correlation between GDPs. Comparing the last two columns of Table 2.2 with each other shows that market incompleteness goes in the right way as it implies a lower cross-country consumption correlation with respect to complete markets (0.80 versus 0.92) although it does not fix the problem. The mechanism that generates a low correlation between relative consumption and real exchange rate implies a negative correlation between the real exchange rate and terms of trade.

2.3 Portfolio transaction costs

In this section, we specify exogenous portfolio transaction costs as in Tille and van Wincoop (2010) and Devereux and Yetman (2010) to limit portfolio diversification. The aim is to see if portfolio transaction costs can help the model generate realistic portfolio positions alongside a low correlation between relative consumption and the real exchange rate.[9]

We assume that investing in assets abroad entails a fixed iceberg cost. We consider the general case where transaction costs can be different for bonds and equities. For each unit invested in foreign equities (bonds), domestic agents receive $e^{-\tau_e}$ $(e^{-\tau_b})$ times the gross return on these assets. These costs are assumed to be second-order (proportional to the variance of structural shocks) to ensure that expected excess returns are zero up to a first order approximation. The rest of the model is specified as in the previous section.

Home and foreign Euler equations (2.13) are modified in the following way:

$$\begin{array}{ll} E_t m_{t+1} R_{1,t+1} = 1, & E_t m^*_{t+1} \frac{S_t}{S_{t+1}} e^{-\tau_e} R_{1,t+1} = 1, \\ E_t m_{t+1} e^{-\tau_e} R_{2,t+1} = 1, & E_t m^*_{t+1} \frac{S_t}{S_{t+1}} R_{2,t+1} = 1, \\ E_t m_{t+1} e^{-\tau_b} R_{3,t+1} = 1, & E_t m^*_{t+1} \frac{S_t}{S_{t+1}} R_{3,t+1} = 1, \\ E_t m_{t+1} R_{4,t+1} = 1, & E_t m^*_{t+1} \frac{S_t}{S_{t+1}} e^{-\tau_b} R_{4,t+1} = 1. \end{array} \tag{2.23}$$

where $R_{i,t}$ for $i = 1, ..., 4$ are defined as in equation (2.10).

Home and foreign portfolio choice equations can be written in a more compact

[9] See Martin and Rey (2005) and Coeurdacier and Guibaud (2008) for a detailed discussion of the effect of financial market imperfections on international portfolios, asset prices and asset returns.

form by treating home bond as the numeraire asset:

$$\begin{aligned}
&E_t m_{t+1}\left(R_{1,t+1}-R_{4,t+1}\right)=0, && E_t m^*_{t+1}\tfrac{S_t}{S_{t+1}}\left(e^{-\tau_e}R_{1,t+1}-e^{-\tau_b}R_{4,t+1}\right)=0,\\
&E_t m_{t+1}\left(e^{-\tau_e}R_{2,t+1}-R_{4,t+1}\right)=0, && E_t m^*_{t+1}\tfrac{S_t}{S_{t+1}}\left(R_{2,t+1}-e^{-\tau_b}R_{4,t+1}\right)=0,\\
&E_t m_{t+1}\left(e^{-\tau_b}R_{3,t+1}-R_{4,t+1}\right)=0, && E_t m^*_{t+1}\tfrac{S_t}{S_{t+1}}\left(R_{3,t+1}-e^{-\tau_b}R_{4,t+1}\right)=0.
\end{aligned} \tag{2.24}$$

Given that τ_b and τ_e are second-order by assumption, the first order approximation to the portfolio choice equations given in (2.23) implies $E_t[\hat{R}_{i,t+1}-\hat{R}_{4,t+1}]=0+o(\varepsilon^2)$ for $i=1,2,3$.

Second-order approximation to home and foreign portfolio choice equations (2.24) gives the modified orthogonality condition which pins down the steady-state portfolio positions:

$$Cov_t\left[(\hat{m}_{t+1}-\hat{m}^*_{t+1}+\Delta\hat{S}_{t+1}),\hat{R}_{x,t+1}\right]=\tau+O(\varepsilon^3), \tag{2.25}$$

where $\hat{R}'_{x,t+1}=[\hat{R}_{1,t}-\hat{R}_{4,t},\ \hat{R}_{2,t}-\hat{R}_{4,t},\ \hat{R}_{3,t}-\hat{R}_{4,t}]$ as before and $\tau'=[\tau_b-\tau_e,\ \tau_b+\tau_e,\ 2\tau_b]$. The appendix shows how the formula for optimal steady-state portfolios derived in Devereux and Sutherland (2011) needs to be modified to reflect the presence of transaction costs.

The first panel of Figure 2.2 plots equilibrium foreign bond position and the corresponding correlation between relative consumption and real exchange rate for different values of portfolio transaction costs in the model with two bonds where relative bond returns are equal to the real exchange rate and uncertainty is driven by shocks to tradable and non-tradable endowments. We let transaction costs to take values up to 20 percent of the variance of tradable endowment shocks. When transaction cost is equal to zero, optimal steady-state foreign bond position, $\bar{B}_F/\bar{Y}$, is equal to 7.62 as in the third column of Table 2.1. As we increase the cost of investing in foreign bonds, the long position in foreign bonds becomes smaller, which implies a lower correlation between relative consumption and real exchange rate. When τ_b takes a value around 11% (of the variance of tradable endowment), foreign bond holdings of domestic agents are around 50% of GDP (as opposed to 762% for when $\tau_b=0$), which is in line with the foreign currency exposure data for the U.S. as reported in Lane and Shambaugh (2010a). As the size of the foreign currency position shrinks due to higher transaction costs, the correlation between relative consumption and real exchange rate falls to -0.05.

The second panel of Figure 2.2 repeats the same exercise for the model where there is international trade in home and foreign tradable sector equities as well as a single non-contingent bond and the only sources of uncertainty are sectoral endowment

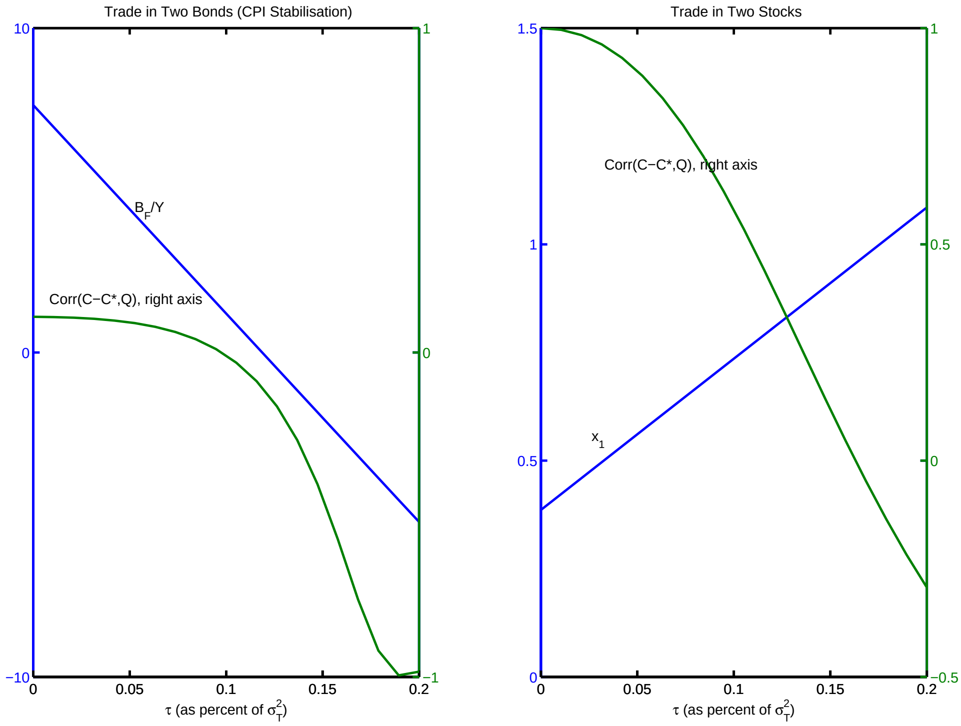


Figure 2.2: Optimal portfolios and relative consumption-real exchange rate correlations for different values of portfolio transaction costs in the model with endowment shocks and trade in bonds or equities.

Notes: The left panel shows the model with portfolio choice between two bonds under CPI stabilisation, $\hat{R}_x^B = \hat{Q}$ and the right panel shows the model with portfolio choice between two tradable sector equities. B_F/Y denotes the steady-state foreign bond portfolio as a share of GDP and is depicted on the left axis. $Corr(C - C^*, Q)$ denotes the correlation between relative consumption and real exchange rate and is depicted on the right axis.

shocks. When transaction costs are equal to zero, the share of home equity held domestically, $\bar{x}_1$, is equal to 0.39, i.e. optimal portfolio exhibits a foreign bias, and the correlation between relative consumption and real exchange rate is equal to 1 consistent with the fifth column of Table 2.1. As we increase the cost of investing in foreign equity, equity portfolio becomes home biased and the correlation between relative consumption and real exchange rate falls. For τ_e around 17% of the variance of the tradable endowment shock, 97% of home equity is held domestically and the correlation between relative consumption and real exchange rate is around -0.05. Hedging against real exchange rate risk implies a foreign bias in equity but due to transaction costs agents bias their portfolio towards home equity. This implies adverse valuation effects conditional on both tradable and non-tradable endowment shocks, which in turn hampers international risk sharing.

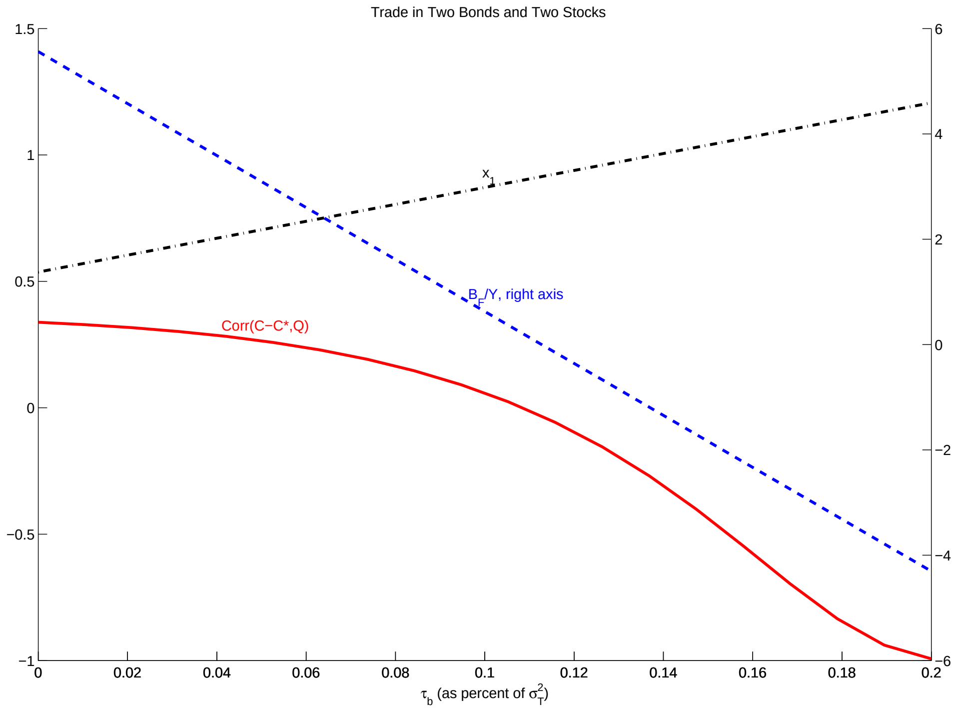


Figure 2.3: Optimal portfolios and relative consumption-real exchange rate correlations for different values of portfolio transaction costs in the model with endowment and redistributive shocks and trade in two bonds and two equities.

Notes: B_F/Y denotes the steady-state foreign bond portfolio as a share of GDP and is depicted on the right axis. x_1 denotes the steady-state share of domestic equity held by domestic agents, $Corr(C - C^*, Q)$ denotes the correlation between relative consumption and real exchange rate. x_1 and $Corr(C - C^*, Q)$ are depicted on the left axis. x-axis shows the value of the transaction costs on foreign bonds, τ_b, as a percent of the variance of tradable endowment shock, σ_T^2. We assume that $\tau_e = 1.1\tau_b$.

Figure 2.3 shows the effect of portfolio transaction costs in a model where agents can trade two bonds and two equities to hedge against sectoral endowment and redistributive shocks. We set transaction costs on equity to be 10% higher than transaction costs on bonds, i.e. $\tau_e = 1.1\tau_b$, and vary τ_b between zero and 20% of the variance of tradable endowment shocks. In the baseline calibration with zero transaction costs, optimal portfolio is long in foreign bonds and has a small bias towards home equity, i.e. $\bar{B}_F/\bar{Y} = 5.6$ and $\bar{x}_1 = 0.54$, while the implied correlation between relative consumption and real exchange rate is around 0.33 (Table 2.2 and Figure 2.3). As we increase τ_b and τ_e, the size of the foreign currency portfolio shrinks and the share of home tradable sector equity held domestically increases. For τ_b around 11%, foreign currency holdings are around 37% of GDP, while the share of home tradable sector equity held domestically rises to 89%, both of which are well within the range of values reported in Table 1.1 regarding the international portfolios of industrial countries. The corresponding consumption-real exchange rate correlation is around 0.02.

These results suggest that transaction costs can be instrumental in generating realistic international portfolios alongside a low correlation between relative consumption and real exchange rate.[10] They are effective as they generate a wedge between desired and actual portfolio positions. The larger the transaction cost, the further away the actual portfolio is from the optimal portfolio and the lower is the correlation between relative consumption and real exchange rate.

2.4 Recursive utility

Recent contributions like Raffo (2010), Karabarbounis (2010), Stathopoulos (2010) and Colacito and Croce (2010) have emphasised the role of non-separable preferences in accounting for the consumption-real exchange rate anomaly. With non-separable preferences, cross-country consumption patterns might differ considerably even if stochastic discount factors are closely related due to international trade in assets. While Raffo (2010) and Karabarbounis (2010) focus on the non-separabilities between consumption and leisure, Stathopoulos (2010) and Colacito and Croce (2010) analyse the role of time-non-separabilities using habits and recursive utility, respectively. All these papers provide different mechanisms that can generate a low (or negative) correlation between relative consumption and real exchange rate under complete markets without considering endogenous portfolio choice.

Here we focus on recursive preferences as in Colacito and Croce (2010) and analyse the implications of non-time-separability for equilibrium portfolios and consumption-real exchange rate correlations in a simpler version of the two-sector endowment model with incomplete markets described above.

We replace the utility function in equation (2.1) by Epstein and Zin (1989) preferences:

$$V_t = \left[C_t^{1-\rho} + \beta(C_{A,t}) \left(E_t (V_{t+1})^{1-\psi} \right)^{\frac{1-\rho}{1-\psi}} \right]^{\frac{1}{1-\rho}}, \tag{2.26}$$

where ρ is the inverse of the intertemporal elasticity of substitution and ψ is the coefficient of risk aversion. As before, C is home consumption and $\beta(C_{A,t})$ is the

[10]However, we should note that transaction costs do not yield more realistic bond portfolios when excess returns are given by the terms of trade as higher transaction costs imply a larger long position in home bonds.

endogenous discount factor as defined in equation (2.3). These preferences nest the expected utility framework for $\rho = \psi$.[11] For $\rho < \psi$, agents prefer an early resolution of uncertainty.

Consumption and price indices are as defined in equations (2.4) to (2.7). To illustrate the role of recursive preferences in the simplest possible way, we consider a set-up where agents can only trade in two bonds. Hence home budget constraint is simply given by:

$$W_t = R_{H,t}B_{H,t-1} + R^*_{F,t}S_tB_{F,t-1} + P_{H,t}Y_{H,t} + P_{N,t}Y_{N,t} - P_tC_t, \tag{2.27}$$

where $W_t = B_{H,t} + S_tB_{F,t}$. Preferences and the budget constraint are defined similarly for the foreign country. The stochastic processes for tradable and non-tradable endowments are as described in (2.20) and (2.21). Given that there are not enough assets to span all shocks, markets are incomplete.

As before, optimality conditions with respect to home and foreign bonds can be written in terms of the stochastic discount factors:

$$\begin{aligned} E_tm_{t+1}R_{H,t+1} = 1, \quad E_tm^*_{t+1}\tfrac{S_t}{S_{t+1}}R_{H,t+1} = 1, \\ E_tm_{t+1}R_{F,t+1} = 1, \quad E_tm^*_{t+1}\tfrac{S_t}{S_{t+1}}R_{F,t+1} = 1, \end{aligned} \tag{2.28}$$

where $R_{H,t+1}$ and $R_{F,t+1}$ denote gross nominal returns on home and foreign bonds in units of the home currency and the real stochastic discount factors are defined as:

$$m_{t+1} \equiv \beta(C_t)\left(\frac{V_{t+1}}{\left(E_tV^{1-\psi}_{t+1}\right)^{\frac{1}{1-\psi}}}\right)^{\rho-\psi}\frac{C^{-\rho}_{t+1}}{C^{-\rho}_t}\frac{P_t}{P_{t+1}}, \tag{2.29}$$

$$m^*_{t+1} \equiv \beta(C^*_t)\left(\frac{V^*_{t+1}}{\left(E_tV^{*1-\psi}_{t+1}\right)^{\frac{1}{1-\psi}}}\right)^{\rho-\psi}\frac{C^{*-\rho}_{t+1}}{C^{*-\rho}_t}\frac{P^*_t}{P^*_{t+1}}. \tag{2.30}$$

Under expected utility, $\rho = \psi$, and the stochastic discount factors given by (2.29) and (2.30) reduce to the usual expected utility forms. Let the ratio of realised utility to the risk-adjusted expected utility be denoted by $g_{t+1} \equiv \left(\frac{V_{t+1}}{\left(E_tV^{1-\psi}_{t+1}\right)^{\frac{1}{1-\psi}}}\right)^{\rho-\psi}$ and $g^*_{t+1} \equiv \left(\frac{V^*_{t+1}}{\left(E_tV^{*1-\psi}_{t+1}\right)^{\frac{1}{1-\psi}}}\right)^{\rho-\psi}$, where risk adjustment is governed by the risk aversion

[11]For $\rho = \psi$, the transformation $\frac{V_t^{1-\rho}}{1-\rho}$ gives the utility function, U_t, defined in equation (2.1).

parameter ψ. Stochastic discount factors under recursive utility differ from the ones under expected utility by the factors g_{t+1} and g^*_{t+1}, which we refer to as 'intertemporal uncertainty factors'. For $\rho < \psi$, agents prefer an early resolution of uncertainty. This means that if utility turns out to be lower than its risk-adjusted expected value, g_{t+1} increases, which would increase agents appetite for wealth, m_{t+1}, keeping everything else constant.

Provided that fluctuations in g_{t+1} and g^*_{t+1} are sufficiently large, stochastic discount factors can differ significantly from the growth rate of the marginal utility of consumption, which can potentially help generate lower correlation between relative consumption and real exchange rate. However, in a first-order approximation, intertemporal utility risk does not affect the international risk sharing condition as the expected value of the intertemporal uncertainty factors, $E_t g_{t+1}$ and $E_t g^*_{t+1}$, are equal to zero.[12]

To see this, consider a first-order approximation to the stochastic discount factors (2.29) and (2.30):

$$\begin{aligned} \hat{m}_{t+1} &= \underbrace{-\eta \hat{C}_t}_{\hat{\beta}_t} + \underbrace{(\rho - \psi)(\hat{V}_{t+1} - E_t \hat{V}_{t+1})}_{\hat{g}_{t+1}} - \rho \Delta \hat{C}_{t+1} - \pi_{t+1}, \\ \hat{m}^*_{t+1} &= \underbrace{-\eta \hat{C}^*_t}_{\hat{\beta}^*_t} + \underbrace{(\rho - \psi)(\hat{V}^*_{t+1} - E_t \hat{V}^*_{t+1})}_{\hat{g}^*_{t+1}} - \rho \Delta \hat{C}^*_{t+1} - \pi^*_{t+1}, \end{aligned}$$

Hence, combining home and foreign Euler equations with respect to either of the two internationally traded bonds leads to the same risk sharing condition under expected and recursive utility:

$$E_t \left(\Delta \hat{C}_{t+1} - \Delta \hat{C}^*_{t+1} \right) - \frac{1}{\rho} E_t \Delta \hat{Q}_{t+1} + \frac{\eta}{\rho} \left(\hat{C}_t - \hat{C}^*_t \right) = O\left(\varepsilon^2\right). \tag{2.31}$$

Equation (2.31) links the expected growth rate of relative marginal utilities of consumption across countries to the real exchange rate depreciation adjusted by an additional term that reflects the differences between endogenous discount factors. Although, recursive preferences do not change the risk sharing condition, they affect the first-order solution through their impact on equilibrium portfolios. To see this, consider a second-order approximation to the equations in (2.28), which gives the

[12] Intertemporal uncertainty factors, g_{t+1} and g^*_{t+1}, only enter as second or higher-order terms in higher order approximations of the risk sharing condition.

orthogonality condition that pins down optimal foreign bond portfolio:

$$Cov_t \left[(\hat{m}_{t+1} - \hat{m}^*_{t+1} + \Delta\hat{S}_{t+1}), \hat{R}_{x,t+1} \right] = 0 + O(\varepsilon^3), \tag{2.32}$$

where $\hat{R}_{x,t+1} = \hat{R}_{F,t+1} - \hat{R}_{H,t+1} = \hat{S}_{t+1} - E_t\hat{S}_{t+1}$. Recursive preferences would affect steady-state portfolios as $\hat{m}_{t+1} - \hat{m}^*_{t+1} + \Delta\hat{S}_{t+1}$ depends on the difference between realised and expected values of the utility denoted by $\hat{g}_{t+1}$ and $\hat{g}^*_{t+1}$:

$$\hat{m}_{t+1} - \hat{m}^*_{t+1} + \Delta\hat{S}_{t+1} = \hat{g}_{t+1} - \hat{g}^*_{t+1} - \rho(\Delta\hat{C}_{t+1} - \Delta\hat{C}^*_{t+1}) + \Delta\hat{Q}_{t+1} - \eta(\hat{C}_t - \hat{C}^*_t) + O\left(\varepsilon^2\right).$$

Given that $E_t\hat{R}_{x,t+1} = 0 + O(\varepsilon^2)$, the orthogonality condition in equation (2.32) can be written as:[13]

$$Cov_t \left[\frac{\hat{g}^*_{t+1} - \hat{g}_{t+1}}{\rho}, \hat{R}_{x,t+1} \right] + Cov_t \left[\hat{C}_{t+1} - \hat{C}^*_{t+1} - \frac{\hat{Q}_{t+1}}{\rho}, \hat{R}_{x,t+1} \right] = 0 + O(\varepsilon^3), \tag{2.33}$$

Hence, with recursive utility optimal portfolio has an additional determinant that depends on the covariance between relative intertemporal uncertainty factors and excess return. This means that optimal portfolio will not be solely focused on minimising real exchange rate adjusted relative consumption as in the case of separable preferences, which could potentially help account for the anomaly in the presence of endogenous portfolio choice. Because intertemporal uncertainty affects equilibrium portfolios, it will affect equilibrium quantities and prices in a first-order solution.[14]

Figure 2.4 shows how the components of the portfolio orthogonality condition (2.33) covary in response to tradable and non-tradable sector shocks under recursive preferences when the degree of risk aversion, ψ, equals 10. In the zero-portfolio solution depicted by circled lines, an increase in home tradable endowment leads to a larger increase in the relative intertemporal uncertainty factor, $\frac{\hat{g}^*_{t+1} - \hat{g}_{t+1}}{\rho}$, compared to

[13] Benigno and Nisticò (2009) derives partial equilibrium expressions for equilibrium bond and equity portfolios in the presence of model uncertainty which is equivalent to having recursive preferences under log-utility. They show that recursive utility amplifies the importance of real exchange rate risk relative to the non-financial income risk in the determination of equilibrium portfolios.

[14] Recursive utility has a first-order effect on equilibrium quantities also under complete markets as the intertemporal uncertainty factors enter the risk sharing condition directly:

$$\begin{aligned} \hat{m}^*_{t+1} - \hat{m}_{t+1} &= \Delta\hat{S}_{t+1}, \\ \Delta\hat{C}_{t+1} - \Delta\hat{C}^*_{t+1} - \frac{1}{\rho}\Delta\hat{Q}_{t+1} &= \hat{g}^*_{t+1} - \hat{g}_{t+1}. \end{aligned}$$

See Benigno et. al. (2011) for a discussion of the implications of recursive preferences for international risk sharing under complete markets.

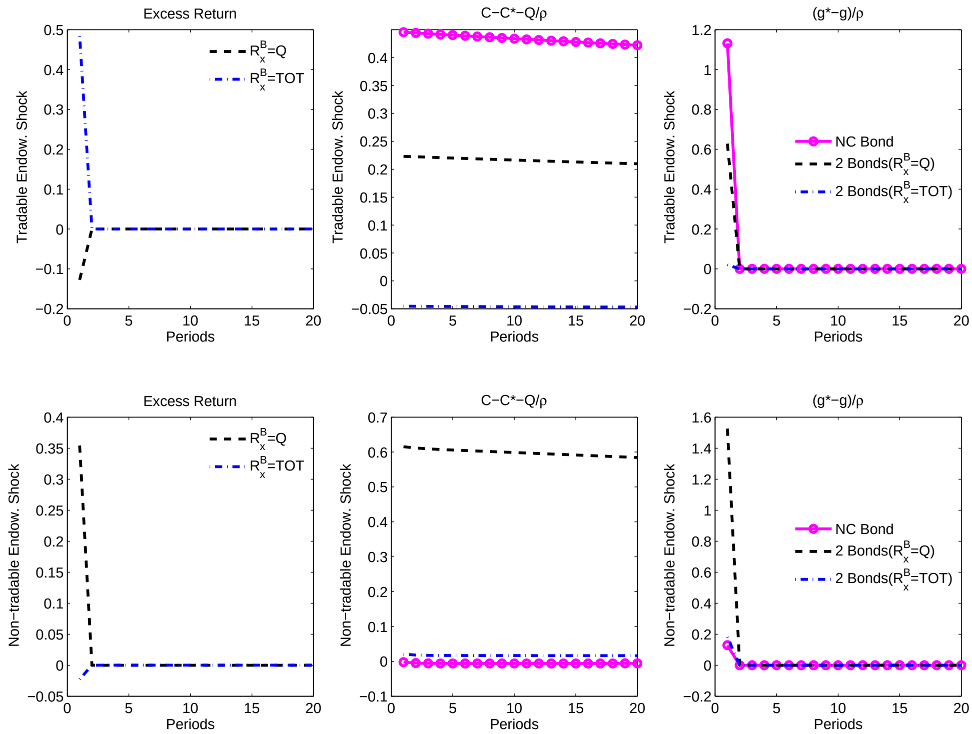


Figure 2.4: Determinants of steady-state foreign bond portfolio with recursive preferences

Notes: This figure plots the impulse responses of relative bond returns ($\hat{R}_x^B$), deviations from efficient risk sharing under expected utility ($\hat{C} - \hat{C}^* - \hat{Q}/\rho$) and relative intertemporal uncertainty factors ($(\hat{g} - \hat{g}^*)/\rho$)) conditional on endowment shocks under alternative asset markets: i) trade in single non-contingent bond (NC Bond), ii) trade in two bonds under CPI stabilisation, (2 Bonds, $\hat{R}_x^B = Q$), iii) trade in two bonds under domestic tradable price stabilisation, (2 Bonds, $\hat{R}_x^B = TOT$) (x-axis:periods, y-axis:percentage deviations from steady-state).

the real exchange rate adjusted relative consumption, $\hat{C} - \hat{C}^* - \hat{Q}/\rho$. Hence, under recursive utility, the optimal hedge against tradable endowment shocks is a larger long (short) position in foreign bonds when relative bond returns are equal to the real exchange rate (terms of trade). Having a larger bond portfolio to hedge against fluctuations in intertemporal uncertainty implies that there will be larger fluctuations in real exchange rate adjusted relative consumption. For example, when relative bond return is equal to the terms of trade, trade in bonds ensures that $\hat{C} - \hat{C}^* - \hat{Q}/\rho$ is not affected by a tradable endowment shock under expected utility (see the upper right panel in Figure 2.1). However, under recursive utility, $\hat{C} - \hat{C}^* - \hat{Q}/\rho$ falls in response to a positive tradable shock (see the upper middle panel in Figure 2.3). In other words, with endogenous portfolio choice, marginal utility gap is larger under recursive utility due to the presence of intertemporal risk. But as we show below, this effect is not strong enough to make a significant difference for equilibrium portfolios and consumption risk sharing.

Figure 2.5 shows the optimal bond portfolio and the consumption-real exchange rate correlation for different degrees of risk aversion under recursive utility. As ψ increases, optimal bond portfolio becomes larger in size to reflect hedging against intertemporal uncertainty and the correlation becomes lower. But this effect is not quantitatively important and makes equilibrium portfolio positions even bigger compared to the case of expected utility.

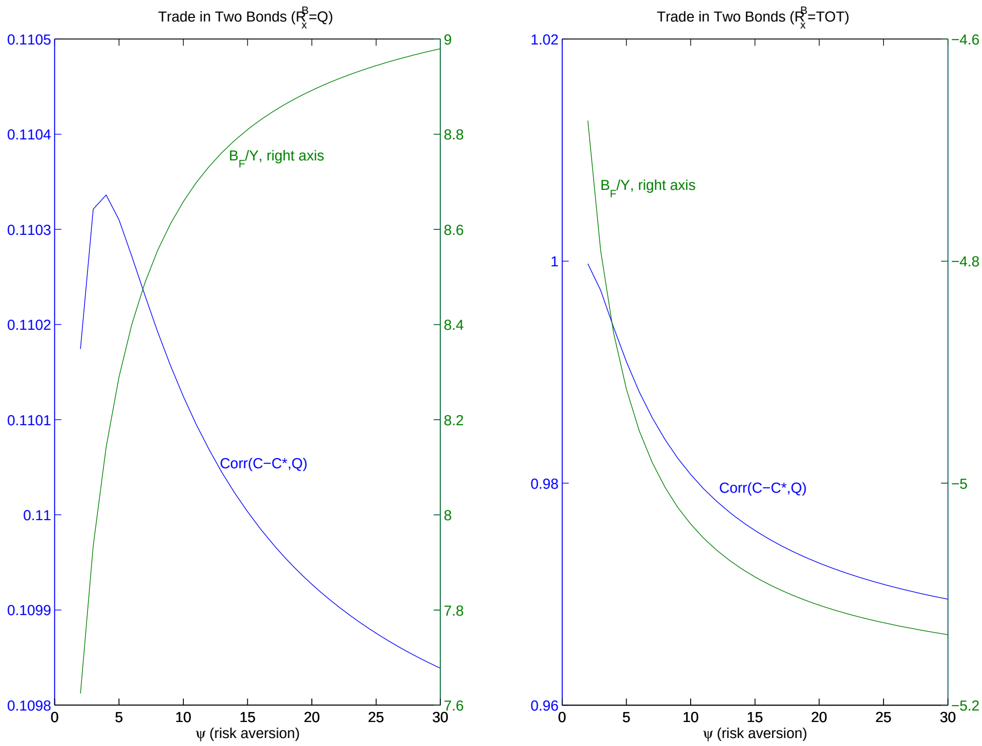


Figure 2.5: Optimal foreign bond portfolio and relative consumption-real exchange rate correlation for different values of risk aversion, ψ, with recursive preferences.

Notes: The left and right panels show, respectively, the model with trade in two bonds under CPI stabilisation, $(\hat{R}_x^B = Q)$, and under domestic tradable price stabilisation, $(\hat{R}_x^B = TOT)$. B_F/Y denotes the steady-state foreign bond portfolio as a share of GDP and is depicted on the right axis. $Corr(C - C^*, Q)$ denotes the correlation between relative consumption and real exchange rate and is depicted on the left axis.

This analysis shows that recursive preferences on their own are not sufficient to make a meaningful difference compared to time-separable preferences in this framework. However, one drawback of the analysis here is that recursive preferences affect the non-portfolio parts of the model only through their effect on steady-state portfolios since we solve the model using a first-order approximation where intertemporal uncertainty does not matter. Solving the incomplete market model with endogenous portfolio choice up-to a second-order is more complicated because one also needs to determine the first-order component of portfolio shares which is pinned down in a third-order approximation to portfolio choice equations as described in Devereux and Sutherland (2010a). Hence, we leave this analysis for future work but our sense is that this would not make an important difference. Indeed, we checked for the non-contingent bond and complete market set-ups that the second-order solution to the model with recursive preferences is almost the same as the first-order solution in terms of their implications for the correlation between relative consumption and real exchange rate and other moments.

We know from a vast literature following Bansal and Yaron (2004) that recursive preferences can account for many asset market facts when combined with 'long-run risks', which refer to the slow-moving long-run predictable component in consumption growth rates. Colacito and Croce (2010) applies the closed economy framework of Bansal and Yaron (2004) to a two-country two-good endowment economy with complete markets where endowments in each country follow a unit root process and have a small predictable component that is stochastic and highly correlated across countries. In their model, a positive shock to the predictable component of the growth rate of home endowment implies a big improvement in home utility due to recursive preferences. Efficient risk sharing requires that resources are transferred from home to foreign. As a result, home consumption falls while real exchange rate depreciates, which helps the model account for the consumption-real exchange rate anomaly. These findings suggest that specifying permanent shocks with a predictable component might be key for time-non-separability to matter for portfolio allocation and international risk sharing as it would imply larger fluctuations in intertemporal uncertainty factors, g_{t+1} and g^*_{t+1}. Chapter 1 discusses the role of news shocks for portfolio allocation and consumption-real exchange rate correlations for expected utility. We find that allowing for recursive preferences and a higher degree of risk aversion does not make an important difference for the results even in the presence of news shocks, hence we do not report the results here. We leave the analysis of recursive preferences with permanent shocks for future work.

2.5 Conclusion

This chapter extends the analysis in Chapter 1 in three dimensions. We show that allowing for international trade in two equities instead of two bonds also completes the markets when uncertainty is due to shocks to tradables and non-tradables in each country. Optimal equity portfolio that achieves full risk sharing in this set-up is foreign biased. The similarity between the risk sharing implications of trade in bonds and equities no longer holds in the presence of redistributive shocks, which pull optimal portfolio towards home equity and hamper risk sharing conditional on tradable sector shocks. In a second extension, we explore the role of portfolio transaction costs. When there are transaction costs, agents cannot choose the portfolio that minimises the fluctuations in relative marginal utilities of consumption across countries, which in turn lowers the correlation between relative consumption and real exchange rate. Finally, we allow for recursive utility as in Epstein and Zin (1989) and explore the effect of non-time-separability on optimal portfolios and consumption-real exchange rate correlations. Under recursive utility, optimal portfolio has an additional determinant that depends on the covariance between relative intertemporal uncertainty and excess return. This means that portfolios will not be solely focused on hedging against the fluctuations in relative marginal utilities of consumption across countries as in the case of expected utility, which could potentially help account for the anomaly in the presence of endogenous portfolio choice. Our results show that this effect is not strong enough to make a significant difference for equilibrium portfolios and consumption-real exchange rate correlations in the baseline model with stationary shocks. We leave the analysis of long-run risks for future work.

Appendix to Chapter 2

2.5.1 Formula for optimal portfolio shares in the presence of second-order transaction costs

To derive a formula for $\tilde{\alpha}$ using equation (2.25), we follow the steps in Devereux and Sutherland (2011). First, we express the first-order solutions for $\hat{m}_{t+1}$, $\hat{m}^*_{t+1}$, $\Delta\hat{S}_{t+1}$ and $\hat{R}_{x,t+1} = \hat{R}_{H,t+1} - \hat{R}_{F,t+1}$ as a function of $\tilde{\alpha}$. Note that τ does not enter the model up to a first order approximation since it is assumed to be a second order term.

We treat the realised excess return on the portfolio as an exogenous mean-zero i.i.d. random variable denoted by ξ_{t+1}, $\xi_{t+1} \equiv \tilde{\alpha}'\hat{R}_{x,t+1}$.The first-order solution for excess returns can be expressed as follows:

$$\hat{R}_{x,t+1} = R_1\xi_{t+1} + R_2\epsilon_{t+1} + o(\varepsilon^2) \tag{2.34}$$

Substituting this expression into $\xi_{t+1} = \tilde{\alpha}'\hat{R}_{x,t+1}$, we get the excess return on the portfolio in terms of the exogenous shocks:

$$\xi_{t+1} = \tilde{H}\epsilon_{t+1}, \quad \text{where } \tilde{H} = \frac{\tilde{\alpha}'R_2}{1-\tilde{\alpha}'R_1} \tag{2.35}$$

Using equation (2.35) with (2.36) we get:

$$\hat{R}_{x,t+1} = \tilde{R}\epsilon_{t+1} + o(\varepsilon^2) \quad \text{where } \tilde{R} = R_1\tilde{H} + R_2 \tag{2.36}$$

The solution of relative stochastic discount factor in terms of model innovations and excess return on the portfolio can be written as:

$$\hat{m}_{t+1} - \hat{m}^*_{t+1} + \Delta\hat{S}_{t+1} = F_1\xi_{t+1} + F_2\epsilon_{t+1} + ... + o(\varepsilon^2) \tag{2.37}$$

Substituting the excess return on the portfolio using (2.35) we get:

$$\hat{m}_{t+1} - \hat{m}^*_{t+1} + \Delta\hat{S}_{t+1} = \tilde{F}\epsilon_{t+1} + ... + o(\varepsilon^2) \;\; \text{where } \tilde{F} = F_1\tilde{H} + F_2 \tag{2.38}$$

Then we use (2.36) and (2.38) to calculate the covariance between excess returns and relative stochastic discount factors:

$$Cov_t[(\hat{m}_{t+1} - \hat{m}^*_{t+1} + \Delta\hat{S}_{t+1}), \hat{R}_{x,t+1}] = \tilde{R}\Sigma\tilde{F}' + o(\varepsilon^3) \tag{2.39}$$

This equation together with (2.25) gives the equality that pins down optimal portfolio shares $\tilde{\alpha}$:

$$\tilde{R}\Sigma\tilde{F}' = \tau \tag{2.40}$$

Substituting the expressions for $\tilde{R}$ and $\tilde{F}'$ and solving for $\tilde{\alpha}$ yields the following modified formula:

$$\tilde{\alpha} = [R_2\Sigma F_2' R_1' - F_1 R_2 \Sigma R_2']^{-1}(R_2\Sigma F_2' - \tau) \tag{2.41}$$

where $\tau' = [\tau_b - \tau_e,\ \tau_b + \tau_e,\ 2\tau_b]$.

Chapter 3

Monetary policy rules and foreign currency positions[1]

3.1 Introduction

Over the past decade, international financial markets have become increasingly integrated. This process of financial globalisation is reflected in the rapid expansion of the external balance sheets of countries which records cross-border ownership of assets and liabilities (see Lane and Milesi-Ferretti (2006) and Lane and Shambaugh (2010a)). In this world of interlinked balance sheets, exchange rate movements can give rise to large valuation effects. In fact, recent shifts in US and UK external positions have been attributed to currency movements (see Higgins et al. (2007) and Astley et al. (2009)). This chapter looks at the interplay between monetary policy rules and foreign asset positions in two ways. First, it examines how different monetary policy regimes can lead to different foreign currency positions in external balance sheets. Second, it explores how these different foreign currency positions affect the valuation effect of monetary shocks.

Lane and Shambaugh (2010b) present evidence that the covariance between nominal exchange rates and output fluctuations is an important determinant of foreign currency exposure. In particular, they find that countries where domestic currency tends to depreciate in bad times are associated with longer foreign currency positions in their external balance sheets. At the same time, Clarida and Waldman (2007) show how monetary policy regimes affect the covariance between exchange rates and infla-

[1]This chapter draws on a joint work with Bianca De Paoli and Jens Søndergaard. We have equal part in determining the subject and contents. I have carried out most of the analytical and numerical analyses. My share in writing up the text is around 80%.

tion and hence the comovements between inflation and output. They find evidence that in response to bad news about inflation, domestic currency tends to appreciate in inflation-targeting countries, but depreciate in non inflation targeters. Arguably, these two pieces of evidence would suggest an indirect link between monetary policy regimes and external positions. In particular, together these facts would indicate that non inflation targeting countries are inclined to have longer positions in foreign currency than inflation-targeting countries.

The main contribution of this work is to formalise this link between monetary policy and foreign asset holdings, emphasising the role of monetary policy regimes in determining the cyclical properties of nominal exchange rates. Consistent with the conjecture above, in our framework countries in which monetary policy does not focus solely on inflation stabilisation will tend to hold a portfolio weighted towards foreign currency denominated bonds. For example, if the central bank is assumed to target money growth, agents would choose a portfolio that is short in domestic bonds and long in foreign bonds. This is because, with a money-growth rule, any adverse real country-specific shocks will be associated with a nominal depreciation of the domestic currency. Holding domestic currency denominated assets is therefore a bad hedge. On the other hand, when the central bank conducts policy through a Taylor-type rule that responds only to inflation, the same adverse shock will trigger a nominal domestic currency appreciation. So holding domestic currency denominated assets is a good hedge and agents will choose to hold an optimal portfolio that is overweight in home bonds.

These results are shown analytically in a two-country flexible price model with incomplete markets, where there is international asset trade in nominal bonds. We also consider an extension where we allow for international trade in equities as well as bonds. We show numerically that the model's link between monetary policy and foreign currency positions is robust provided that we add an extra source of risk to keep the financial markets incomplete. In addition, we demonstrate that the results also hold in a model where prices are sticky - irrespective of whether exports are invoiced in local currency or in the currency of the producer. Moreover, the portfolio shares in the sticky price set-up are quantitatively similar to that in the flexible price model. Finally, we show that the results obtained under a money-growth rule also hold under a Taylor rule that puts weight on stabilising output growth. So the crucial determinant of portfolios in our analysis is whether policy is sufficiently focused on inflation stabilisation.

Nominal bond portfolios have been analysed before by, among others, Devereux and Sutherland (2008) and Engel and Matsumoto (2009).[2] In a model where monetary policy is specified as a Taylor rule that reacts to PPI inflation, Devereux and Sutherland (2008) finds a negative position in foreign bonds under incomplete markets.[3] Our results show that this finding is overturned if the central bank follows a money-growth rule, or a 'passive' monetary policy. On the other hand, Engel and Matsumoto (2009), under a similar money-growth rule find that the negative foreign currency position would still be optimal when asset markets are complete. So overall, our results highlight the importance of both the asset market structure and the policy rule specification as determinants of foreign currency positions.

We also examine how foreign currency positions affect the valuation effect of monetary shocks. In our model, a domestic monetary loosening which depreciates the domestic currency will have positive or negative valuation effects depending on the country's position in the foreign currency market. Under an inflation-targeting Taylor-type rule, it is optimal to be short in foreign currency so a domestic currency depreciation generates a decrease in net external wealth of domestic agents. Hence, valuation effects of monetary policy shocks are beggar-thy-self. Conversely, if the domestic portfolio is long in foreign bonds, as under a money-growth rule, then a domestic monetary policy loosening would trigger an increase in net external wealth and international valuations effects are beggar-thy-neighbour.

The valuation channel of monetary policy has been explored in earlier literature. For instance, Dornbusch and Fischer (1980) and Svensson (1989) and later Kim (2002) examine the implications of net foreign asset positions for the transmission mechanism of monetary policy in a setting in which portfolio positions are exogenous.[4] Until recently, the analysis of optimal portfolio choice was mostly restricted to partial equilibrium models. But new methodological contributions (Devereux and Sutherland (2011), Tille and van Wincoop (2010) and Evans and Hnatkovska (2007)) have now allowed us to analyse optimal portfolio choice in general equilibrium models. Therefore, we revisit the old insights from Dornbusch and Fischer (1980) and Svensson (1989) in a two-country general equilibrium model where agents can choose optimally among home and foreign nominal bonds.

[2] Devereux and Sutherland (2011), Benigno and Nisticò (2009) and Coeurdacier and Gourinchas (2009) are other examples of open economy DSGE models with endogenous nominal bond portfolios.

[3] This is true under the regularity condition which ensures that a positive technology shock at home deteriorates the home terms of trade.

[4] Neumeyer (1998) analyses how a monetary union affects welfare by changing the hedging properties of currencies in an incomplete market setting with nominal securities and mean-variance preferences. Doepke and Schneider (2006) look at the effects of inflation on the redistribution of wealth between old and young generations in a closed economy model with trade in nominal assets.

In our analysis, as in Devereux and Sutherland (2008), monetary policy shocks can have real effects even when all prices are fully flexible as long as financial markets are incomplete. In a set-up where agents optimally choose to hold a portfolio of nominal bonds that are either denominated in domestic or foreign currency, monetary shocks will generate endogenous currency movements that trigger international valuation effects.[5] These valuation effects on international bond portfolios that work through unanticipated nominal exchange rate depreciation would still be present when trade in real assets (equities) is also allowed, provided that there is an extra source of risk in the model such that trade in equities and bonds cannot complete the markets.

Quantitatively, our results points to small valuation effects in that, changes in net external wealth due to the valuation channel have a small effect on consumption and other real variables. This result is consistent with both empirical and earlier theoretical literature. Labhard et al.(2005) and Fair (2004) find that a 1% change in aggregate wealth has less than 0.03% effect on steady-state consumption in the United Kingdom and United States. In addition, Chari et al. (2002), Baxter and Crucini (1995) and Betts and Devereux (2001) show that international wealth transfers also tend to be small in theoretical models. But we also find that the valuation channel becomes more important in an economy that is subject to more persistent shocks. Increasing the persistence of shocks means that agents are exposed to more country-specific risks which they want to hedge against by holding a larger gross portfolio position. Then monetary shocks become more potent and can trigger larger valuation effects when gross positions are very large - perhaps even unrealistically large. We do not think it is necessarily desirable to have a set-up which implies very strong wealth effects given the aforementioned empirical literature. Thus, what we aim to achieve is a good understanding of the valuation channel, acknowledging (and demonstrating) that its quantitative importance is small relative to other channels such as the one coming from sticky prices.[6]

The remainder of the chapter is structured as follows. Section 3.2 presents the flexible price model with trade in nominal bonds. Section 3.3 focuses on the deriva-

[5] We focus on valuation effects caused by *unanticipated* movements in the nominal exchange rate. Devereux and Sutherland (2010b) show that anticipated valuation effects (risk premia) are small.

[6] The cross-border implication of monetary policy has already been thoroughly examined in the New Open Economy Macro literature. Corsetti and Pesenti (2001), Corsetti et al. (2000) showed that monetary expansions can be beggar-thy-self in a micro-founded version of the Mundell-Fleming model. When prices are sticky, currency depreciations coming from such shocks lead to a deterioration in the country's purchasing power. So this literature focuses on the cross-border implications of policy shocks coming from expenditure-switching effects that would only be present under sticky prices. Our work, on the other hand, concentrates on the wealth effects which are present even under flexible prices.

tion of the optimal foreign currency position under different monetary policy regimes, and in Subsection 3.3.3 we analyse valuation effects of monetary policy shocks implied by these positions. Section 3.4 is devoted to our quantitative analysis and model extensions. We start by illustrating our results under flexible prices numerically (Subsection 3.4.1). The model is then extended to allow for trade in equities (Subsection 3.4.2). Finally, in Subsection 3.4.3, we consider the case in which prices are sticky. Section 3.3.5 concludes.

3.2 The model

We develop a basic two-country open economy model with tradable endowments. There is a home and a foreign country, each endowed with its own tradable good. Households maximise utility over infinite horizon and they can trade in home and foreign nominal bonds; one-period risk-free bonds that pay one unit of the currency they are issued in.[7]

3.2.1 Consumers

The representative agent in the home economy maximises the expected present discounted value of the utility:

$$U_t = E_t \sum_{s=t}^{\infty} \delta_s u\left(C_s, \frac{M_s}{P_s}\right), \tag{3.1}$$

with

$$u\left(C_s, \frac{M_s}{P_s}\right) = \frac{C_s^{1-\rho}}{1-\rho} + \chi \log\left(\frac{M_s}{P_s}\right), \tag{3.2}$$

where C is consumption, $\frac{M}{P}$ is real money holdings.[8] δ_s is the discount factor, which is determined as follows:

$$\delta_{s+1} = \delta_s \beta(C_{As}),\ \delta_0 = 1, \tag{3.3}$$

[7] See Section 3.4.2 for a brief account of the case where equities are traded alongside bonds.

[8] While agents' preferences towards different bonds are determined through an endogenous portfolio choice problem, preferences toward currency (or cash) are exogenously imposed in the utility function. Our specification is equivalent to the one in which agents can only do transactions with (domestic) currency – that is, they face cash-in advance constraint. And these constraints directly determine the demand for money. Although this is out of the scope of this chapter, one could think of an alternative specification in which the choice of money holdings is also an outcome of a portfolio decision.

where C_A is aggregate home consumption and $0 < \beta(C_A) < 1$. To achieve stationarity under incomplete market specification, we assume $\beta_C(C_A) \leq 0$, which implies that agents discount the future more as aggregate consumption increases, i.e. agents bring consumption forward when aggregate consumption is high.[9] We assume that the individual takes C_A as given when optimising and we follow Devereux and Sutherland (2011) in assuming:

$$\beta(C_A) = \omega C_A^{-\eta}, \tag{3.4}$$

with $0 \leq \eta < \rho$ and $0 < \omega \bar{C}_A^{-\eta} < 1$ (as in the constant discount factor).

C represents a consumption index defined over C_H and C_F, home and foreign produced goods, respectively.

$$C_t = \left[\nu^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-\nu)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{3.5}$$

where θ is the elasticity of intratemporal substitution between C_H and C_F and ν is the weight that the household assigns to home consumption. The consumption price index, defined as the minimum expenditure required to purchase one unit of aggregate consumption for the home agent is given by:

$$P_t = \left[\nu P_{H,t}^{1-\theta} + (1-\nu) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{3.6}$$

We adopt a similar preference specification for the foreign country except that variables are denoted with an asterisk.

In each country agents can invest in two nominal bonds denominated in the home and foreign currency. The budget constraint of the home agent in real terms is given by:

$$\alpha_{H,t} + \alpha_{F,t} + \frac{M_t}{P_t} = \alpha_{H,t-1} r_{H,t} + \alpha_{F,t-1} r_{F,t} + \frac{M_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} + \frac{P_{H,t} Y_t}{P_t} - C_t - T_t, \tag{3.7}$$

where Y is the endowment received by home agents, C is consumption of home agents, T represents real taxes minus transfers. The role of T_t will be to allow for variations in the nominal supply of money, with $-T_t = \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t}$. $\alpha_{H,t-1}$ and $\alpha_{F,t-1}$ are the real holdings of home and foreign bonds expressed in units of home consumption good, purchased at the end of period $t-1$ for holding into period t.[10] $r_{H,t}$ and $r_{F,t}$

[9] Another way of ensuring stationarity is to assume, following Turnovsky (1985), that the international trade of foreign currency denominated bonds is subject to intermediation costs.

[10] A similar budget constraint holds for the foreign agent, where foreign variables are denoted with an asterisk, *. $\alpha^*_{H,t-1}$ and $\alpha^*_{F,t-1}$ denote the foreign country's real holdings of home and foreign

are gross real returns in home units:

$$\begin{aligned} r_{H,t+1} &= \frac{1/P_{t+1}}{Z_{H,t}}, \\ r_{F,t+1} &= r^*_{F,t+1}\frac{Q_{t+1}}{Q_t} = \frac{1/P^*_{t+1}}{Z^*_{F,t}}\frac{Q_{t+1}}{Q_t}, \end{aligned} \tag{3.8}$$

where Z_H and Z^*_F are bond prices in terms of home and foreign consumption baskets, respectively. Q_t is the real exchange rate defined as $\frac{P^*_t S_t}{P_t}$. Nominal returns (in home currency) for each of these assets will be given by $R_{i,t} = r_{i,t}\frac{P_t}{P_{t-1}}$ for $i = H, F$.

Endowments in each country follow an AR(1) process:

$$\begin{aligned} \log Y_t &= \zeta_Y \log Y_{t-1} + \varepsilon_{Y,t},\ E_{t-1}[\varepsilon_{Y,t}] = 0,\ Var[\varepsilon_{Y,t}] = \sigma^2_Y, \\ \log Y^*_t &= \zeta_Y \log Y^*_{t-1} + \varepsilon_{Y^*,t},\ E_{t-1}[\varepsilon_{Y^*,t}] = 0,\ Var[\varepsilon_{Y^*,t}] = \sigma^{*2}_Y. \end{aligned}$$

Defining $NFA_t \equiv \alpha_{H,t} + \alpha_{F,t}$ as the total net claims of home agents on the foreign country at the end of period t (i.e. the net foreign assets of home agents) and $r_{x,t} = r_{F,t} - r_{H,t}$ as the excess return of foreign bond on home bond, we write the home budget constraint as follows:[11]

$$NFA_t = NFA_{t-1}r_{H,t} + \alpha_{F,t-1}r_{x,t} + \frac{P_{H,t}Y_t}{P_t} - C_t + \frac{M_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} - \frac{M_t}{P_t} - T_t \tag{3.9}$$

Note that once α_F is determined, α_H, α^*_H and α^*_F will also be determined as $\alpha_H \equiv NFA - \alpha_F$ by definition and $\alpha^*_H = -\alpha_H$, $\alpha^*_F = -\alpha_F$ from market clearing conditions. Thus, we let $\alpha_F \equiv \alpha$ and only focus on α in what follows.

3.2.2 Policy rules

To examine how the choice of monetary policy regime affects foreign currency positions and valuation effects, we consider two extreme policy specifications: a Taylor rule that only responds to inflation and a money-growth rule with no feedback to another variable. The former represents an 'active' central bank, which sets interest rates to offset the effects of shocks on inflation. The latter, on the other hand, represents a 'passive' central bank, which does not respond to inflation or any other

bonds, expressed in units of home consumption good. Bonds are assumed to be in net zero supply in each country. Thus, equilibrium in asset market requires that total bond holdings of home and foreign agents should equal zero, i.e. $\alpha_{H,t} + \alpha^*_{H,t} = 0$ and $\alpha_{F,t} + \alpha^*_{F,t} = 0$.

[11] Net foreign assets of home agent is defined as net claims of home country on foreign country assets, i.e. $NFA_t \equiv \alpha_{F,t} - \alpha^*_{H,t}$. Since bonds are assumed to be in net zero supply, $\alpha_{H,t} = -\alpha^*_{H,t}$. It follows that $NFA_t \equiv \alpha_{H,t} + \alpha_{F,t}$.

variable at all. We also consider Taylor rules which feedback to domestic output as well as inflation and show in Section 3.5 that a Taylor rule which puts sufficient weight on output stabilisation has similar implications for foreign currency positions and valuation effects as a 'passive' money-growth rule.

Under the inflation-targeting Taylor rule, the central bank sets the nominal interest rate on domestic bonds in response to CPI inflation (inflation target assumed to be zero):

$$\begin{aligned} R_{H,t+1} &= \bar{\beta}^{-1}\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi}\exp(\varepsilon_{R,t}),\ E_{t-1}[\varepsilon_{R,t}]=0,\ Var[\varepsilon_{R,t}]=\sigma_R^2, \\ R^*_{F,t+1} &= \bar{\beta}^{-1}\left(\frac{P^*_t}{P^*_{t-1}}\right)^{\phi_\pi}\exp(\varepsilon^*_{R,t}), E_{t-1}[\varepsilon_{R^*,t}]=0,\ Var[\varepsilon_{R^*,t}]=\sigma_R^{*2}. \end{aligned}$$

Under the money-growth rule, the central bank sets the rate of growth of the money supply.[12]

$$\begin{aligned} \mu_t &= \zeta_M\mu_{t-1}+\varepsilon_{M,t},\ E_{t-1}[\varepsilon_{M,t}]=0,\ Var[\varepsilon_{M,t}]=\sigma_m^2, \\ \mu^*_t &= \zeta^*_M\mu^*_{t-1}+\varepsilon_{M^*,t},\ E_{t-1}[\varepsilon_{M^*,t}]=0,\ Var[\varepsilon_{M^*,t}]=\sigma_m^2. \end{aligned}$$

where $\mu_t = \log\left(\frac{M_t}{M_{t-1}}\right)$ and $\mu^*_t = \log\left(\frac{M^*_t}{M^*_{t-1}}\right)$. Monetary shocks differ with respect to the monetary policy specification we are considering.

3.2.3 Equilibrium

Given our assumption on preferences in 3.2, the Euler equations are given by:

$$C_t^{-\rho} = \beta(C_t)E_tC_{t+1}^{-\rho}r_{i,t+1},\ i=H,F \tag{3.10}$$

where $\beta(C_t)=\omega C_t^{-\eta}$ from equation (3.4) since in equilibrium aggregate consumption, $C_{A,t}$, is equal to individual consumption, C_t. Money demand depends negatively on the opportunity cost of holding money, which is equal to $\frac{R_{i,t+1}-1}{R_{i,t+1}}$ in terms of gross returns.

$$\frac{M_t}{P_t} = \chi C_t^\rho\left(1-R_{i,t+1}^{-1}\right)^{-1}. \tag{3.11}$$

Equilibrium bond prices, Z_H and Z^*_F, are obtained by substituting $r_{H,t+1}$ and

[12] Under both rules, steady-state inflation is zero. But our results would go through with non-zero steady-state inflation.

$r^*_{F,t+1}$ from equation (3.8) into home and foreign Euler equations:

$$\begin{aligned} Z_{H,t} &= \beta(C_t)E_t\frac{C_{t+1}^{-\rho}}{C_t^{-\rho}}\frac{1}{P_{t+1}}, \\ Z^*_{F,t} &= \beta(C^*_t)E_t\frac{C_{t+1}^{*-\rho}}{C_t^{*-\rho}}\frac{1}{P^*_{t+1}}. \end{aligned} \tag{3.12}$$

Goods market clearing implies:

$$\begin{aligned} Y_t &= \nu\left(\frac{P_{H,t}}{P_t}\right)^{-\theta}C_t + (1-\nu)\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\theta}C^*_t, \\ Y^*_t &= \nu\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\theta}C^*_t + (1-\nu)\left(\frac{P_{F,t}}{P_t}\right)^{-\theta}C_t. \end{aligned}$$

We assume that law of one price holds, i.e. $P^*_{H,t} = \frac{P_{H,t}}{S_t}$ and $P_{F,t} = P^*_{F,t}S_t$.

3.2.4 Approximated solution

To solve the model we use the approximation techniques proposed in Devereux and Sutherland (2011) and Tille and van Wincoop (2010). We approximate our model around the symmetric steady state in which steady-state inflation rates are assumed to be zero.

To determine the portfolio allocation, it is useful to rewrite the home portfolio choice equation given in equation (3.10) and its foreign counterpart as follows:

$$\begin{aligned} E_t\left[\psi_{t+1}r_{x,t+1}\right] &= 0, \\ E_t\left[\psi^*_{t+1}r_{x,t+1}\frac{Q_t}{Q_{t+1}}\right] &= 0, \end{aligned}$$

where home and foreign stochastic discount factors are given by $\psi_{t+1} = \beta(C_t)\frac{C_{t+1}^{-\rho}}{C_t^{-\rho}}$ and $\psi^*_{t+1} = \beta(C^*_t)\frac{C_{t+1}^{*-\rho}}{C_t^{*-\rho}}$, respectively, and $r_{x,t+1}$ is the excess return on foreign nominal bond, taking home bond as a reference.

These two sets of conditions imply the following equation that characterises optimal portfolio choice up to a second order:

$$E_t\left[(\hat{\psi}_{t+1} - \hat{\psi}^*_{t+1} + \Delta\hat{Q}_{t+1})\hat{r}_{x,t+1}\right] = 0 + O(\varepsilon^2).$$

This is an orthogonality condition between excess returns in domestic currency and the difference in the stochastic discount factors evaluated in the same currency. Since expected excess returns are zero up to a first-order approximation, i.e. $E_t\left[\hat{r}_{x,t+1}\right] = 0 + O(\varepsilon^2)$, this condition can be expressed as:

$$Cov_t(\hat{\psi}_{t+1} - \hat{\psi}^*_{t+1} + \Delta\hat{Q}_{t+1}, \hat{r}_{x,t+1}) = 0 + O(\varepsilon^2). \tag{3.13}$$

As shown by Devereux and Sutherland (2011), to evaluate equation (3.13) and determine the portfolio shares, it is sufficient to take a first-order approximation of the remaining equilibrium conditions for which the only aspect of portfolio behaviour that matters is the steady-state foreign bond portfolio, $\bar{\alpha}$.

3.3 Optimal foreign currency position and valuation effects

In this section we derive the optimal foreign currency portfolio position and valuation effects of a monetary expansion under different assumptions as to how monetary policy is conducted. To understand the hedging motives of investors in our economy we first derive a partial equilibrium expression for their foreign bond position. We then analyse the determinants of the portfolio orthogonality condition under the different policy regimes. This allows us to understand the investors' optimal portfolio allocation. Finally, a full general equilibrium solution to these portfolios is derived. [13]

3.3.1 Partial equilibrium analysis of foreign currency position

Investors in our model can choose to hold both domestic as well as foreign currency denominated nominal bonds. In what follows, we will show that this portfolio choice depends on the hedging characteristics of both types of bonds. Following Benigno and Nisticò (2009), we derive a partial equilibrium solution for foreign bond holdings. Specifically, we use the first-order approximation to the model equations to evaluate the portfolio orthogonality condition coming from the second-order approximation to the portfolio choice equations.

Using the definition of stochastic discount factors, equation (3.13) can be written in terms of relative consumption growth adjusted for the change in the real exchange

[13] Throughout this section we set $\eta = 0$, and use the standard constant discount factor rather than Uzawa preferences to characterise analytical solutions.

rate:

$$Cov_t\left(\Delta\hat{C}^R_{t+1}-\Delta\frac{\hat{Q}_{t+1}}{\rho},\hat{r}_{x,t+1}\right)=0+O(\varepsilon^2). \tag{3.14}$$

Thus to get a partial equilibrium solution for portfolios, we need to express $\Delta\hat{C}^R_{t+j+1}-\Delta\frac{\hat{Q}_{t+1}}{\rho}$ as a function of prices, relative endowments and excess returns on the steady-state foreign bond portfolio using the equations for the accumulation of net foreign assets and the combined Euler equations.

As shown in the appendix, we can write the relative net foreign asset ($\widehat{NFA}^R_t \equiv \widehat{NFA}_t-\widehat{NFA}^*_t$) accumulation as:

$$\widehat{NFA}^R_t=\frac{1}{\beta}\widehat{NFA}^R_{t-1}+2\tilde{\alpha}\hat{r}_{x,t}+\hat{Y}^R_t-\hat{C}^R_t+\hat{Q}_t, \tag{3.15}$$

where $\tilde{\alpha}=\frac{\bar{\alpha}}{\beta\bar{Y}}$ is the steady-state foreign bond portfolio of home agents normalised by income, $\hat{r}_{x,t}\equiv\hat{r}_{F,t}-\hat{r}_{H,t}$ is the excess return on foreign bonds expressed in home currency, $\hat{Y}^R_t\equiv\hat{P}_{H,t}+\hat{Y}_{H,t}-(\hat{P}^*_{F,t}+\hat{S}_t+\hat{Y}^*_{F,t})$ is the relative non-financial income measured in domestic currency.

We combine the domestic and foreign Euler equations to get:

$$E_t\hat{C}^R_{t+1}-\hat{C}^R_t=\frac{1}{\rho}\left(E_t\hat{Q}_{t+1}-\hat{Q}_t\right). \tag{3.16}$$

The appendix shows that solving equations (3.15) and (3.16) forward for $N\hat{F}A^R_t$ and $\hat{C}^R_t$, we can express the first component of the covariance in equation (3.14) as:

$$\begin{aligned}\Delta\hat{C}^R_{t+1}-\Delta\frac{\hat{Q}_{t+1}}{\rho} &= 2(1-\beta)\tilde{\alpha}\hat{r}_{x,t+1} \\ &\quad +E_{t+1}\sum_{j=0}^{\infty}\beta^j\Delta\hat{Y}^R_{t+1+j}-E_t\sum_{j=0}^{\infty}\beta^j\Delta\hat{Y}^R_{t+1+j} \\ &\quad +\left(1-\frac{1}{\rho}\right)\left(E_{t+1}\sum_{j=0}^{\infty}\beta^j\Delta\hat{Q}^R_{t+1+j}-E_t\sum_{j=0}^{\infty}\beta^j\Delta\hat{Q}^R_{t+1+j}\right).\end{aligned} \tag{3.17}$$

Now define $\Lambda_{Y,t+1}$ as the 'news at time $t+1$' about relative non-financial income. That is, $\Lambda_{Y,t+1}$ is the net present value of the relative non-financial income (expressed in the same currency), i.e. $\Lambda_{Y,t+1}\equiv E_{t+1}\sum_{j=0}^{\infty}\beta^j\Delta\hat{Y}^R_{t+1+j}-E_t\sum_{j=0}^{\infty}\beta^j\Delta\hat{Y}^R_{t+1+j}$. Similarly, we define $\Lambda_{Q,t+1}$ as 'news at time $t+1$' about the future value of real exchange rates, i.e. $\Lambda_{Q,t+1}\equiv E_{t+1}\sum_{j=0}^{\infty}\beta^j\Delta\hat{Q}_{t+1+j}-E_t\sum_{j=0}^{\infty}\beta^j\Delta\hat{Q}_{t+1+j}$. Using equations (3.17) and

(3.14), we can derive an expression for the foreign currency portfolio, $\tilde{\alpha}$:

$$\tilde{\alpha} = -\frac{1}{2(1-\beta)} \left(\frac{Cov_t(\Lambda_{Y,t+1}, \hat{r}_{x,t+1})}{Var_t(\hat{r}_{x,t+1})} + \left(1 - \frac{1}{\rho}\right) \frac{Cov_t(\Lambda_{Q,t+1}, \hat{r}_{x,t+1})}{Var_t(\hat{r}_{x,t+1})} \right). \tag{3.18}$$

Equation (3.18) shows that the foreign currency portfolio, $\tilde{\alpha}$, depends on two covariance-variance ratios, which represent the two risks that agents want to hedge against irrespective of the special characteristics of the model: the relative non-financial income risk given by $\Lambda_{Y,t+1}$ and the real exchange rate risk given by $\Lambda_{Q,t+1}$. As put forth by Coeurdacier and Gourinchas (2009) and Benigno and Nisticò (2009), these covariance-variance ratios can be interpreted as asset return loadings on risk - the regression coefficient when you regress risk on excess return.

The excess return on foreign bonds relative to home bonds is given by surprises in home currency depreciation as the UIP holds in a first-order approximation to the model:

$$\hat{r}_{x,t+1} = \hat{S}_{t+1} - E_t \hat{S}_{t+1}. \tag{3.19}$$

Then, according to equations (3.18) and (3.19), it is optimal to take a long position in foreign currency (i.e. $\tilde{\alpha} > 0$) if the home currency depreciates in periods when non-financial income is lower at home than abroad, i.e. $Cov_t(\Lambda_{Y,t+1}, r_{x,t+1}) < 0$ and/or when home consumption basket is more expensive, i.e. $Cov_t(\Lambda_{Q,t+1}, r_{x,t+1}) < 0$. In other words, domestic investors would prefer to hold foreign over domestic bonds if foreign bonds yield an excess return (foreign currency appreciates) in periods when domestic income is relatively low or when the domestic consumption basket rises in price relative to the foreign (a domestic real appreciation).[14] This in turn, would depend crucially on the monetary policy regime as we illustrate in the general equilibrium solution below.

3.3.2 General equilibrium solution for optimal foreign currency position

It is possible to characterise closed-form solutions for optimal foreign bond portfolios given the relatively simple structure of our model.[15]Since optimal portfolios are pinned down by the portfolio orthogonality condition given in equation (3.14), it is useful

[14] Note that under log utility ($\rho = 1$) there is no real exchange rate hedging motive but investors are still facing relative non-financial income risk. This is also true if real exchange rate is constant, which corresponds to $\nu = 0.5$ in our model.

[15] Even in this simple endowment economy the expressions are quite complicated. Thus, for ease of exposition, we set the persistence of endowment shocks to 1 in this section.

to analyse the components of this covariance, $\hat{C}^R_{t+1} - \frac{\hat{Q}_{t+1}}{\rho}$ and $\hat{r}_{x,t+1}$ (or $\hat{S}_{t+1}$) to understand the equilibrium foreign currency position.

First, we consider the solution for the real exchange rate adjusted consumption differential. We rewrite equation (3.17) as a function of the structural shocks and the excess return on foreign bond holdings in the following way:[16]

$$\begin{aligned} \hat{C}^R_{t+1} - \frac{\hat{Q}_{t+1}}{\rho} &= \frac{2\nu(\theta\rho - 1) - (\rho - 1)}{\rho(1 + 2\nu(\theta - 1))}(\varepsilon_{Y,t+1} - \varepsilon_{Y^*,t+1}) \\ &+ \frac{(1-\beta)(1 + 4\nu(1-\nu)(\theta\rho - 1))}{(1-\nu)\rho(1 + 2\nu(\theta - 1))}\tilde{\alpha}\hat{r}_{x,t+1}. \end{aligned} \tag{3.20}$$

Equation (3.20) shows that if agents did not have any foreign currency position, that is $\tilde{\alpha} = 0$, the real exchange rate adjusted relative consumption would depend only on the relative supply shock. This is because, without valuation effects coming from movements in the exchange rate, monetary policy has no effect on real variables under flexible prices. So, if agents were only faced with monetary policy shocks, the optimal portfolio would imply having no foreign currency position, as this ensures perfect smoothing in the adjusted relative consumption. But equation (3.20) also shows that the zero-portfolio position, i.e. $\tilde{\alpha} = 0$, would not insure agents against endowment shocks. So the relative importance of the different shocks will pin down how far from the zero portfolio agents will choose to be.

Money-growth rule

Consider now that the central bank follows a money-growth rule, as specified in Section 3.2.2. Taking the difference of money demand equations in each country yields:

$$\hat{M}^R_t = \rho\hat{C}^R_t - \frac{\bar{\beta}}{1-\bar{\beta}}(\hat{R}_{H,t+1} - \hat{R}^*_{F,t+1}) + \hat{P}_t - \hat{P}^*_t, \tag{3.21}$$

where $\hat{M}^R_t = \hat{M}_t - \hat{M}^*_t$. Substituting equation (3.19) into (3.21), it is possible to express the nominal exchange rate as a function of relative money supplies and consumption differential adjusted by the real exchange rate:

$$\frac{1}{1-\bar{\beta}}(\hat{S}_t - \bar{\beta}E_t\hat{S}_{t+1}) = \hat{M}^R_t - \rho(\hat{C}^R_t - \frac{\hat{Q}_t}{\rho}). \tag{3.22}$$

[16] Note that we ignore other state variables in the solution as they do not matter when evaluating the conditional covariance given in equation 3.14.

Note that under complete markets where $\hat{C}_t - \hat{C}^*_t = \frac{\hat{Q}_t}{\rho}$, the nominal exchange rate only depends on relative money supplies. But in our incomplete markets setting, solving equation (3.22) forward for the nominal exchange rate, and assuming a no-bubbles solution, gives:

$$\hat{S}_t = E_t \sum_{j=0}^{\infty} \beta^j \left(\Delta \hat{M}^R_{t+j} - \rho \left(\Delta \hat{C}^R_{t+j} - \frac{\Delta \hat{Q}_{t+j}}{\rho} \right) \right). \tag{3.23}$$

Equation (3.23) shows that for a given level of relative money supply growth, the domestic currency depreciates when the economy is hit by adverse real shocks that decrease relative consumption (adjusted by the real exchange rate). The intuition is that an adverse domestic real shock implies a decline in the demand for money. Given a fixed supply of money, domestic interest rates fall relative to foreign which - via the UIP condition - would entail a domestic currency depreciation. Hence, this partial equilibrium equation already illustrates how assuming a money-growth rule implies that the domestic currency will be negatively correlated with relative consumption in the face of real shocks. It, thus, suggests that a long position in foreign currency would help investors hedge against such shocks.

In fact, the general equilibrium solution for excess returns in terms of the shocks and steady-state portfolio is given by the following expression:

$$\begin{aligned} \hat{r}_{x,t+1} &= \hat{S}_{t+1} - E_t \hat{S}_{t+1} \\ &= \varkappa_{mg} \left[-\frac{(2\nu(\theta\rho - 1) - (\rho - 1))}{1 + 2\nu(\theta - 1)} (\varepsilon_{Y,t+1} - \varepsilon_{Y^*,t+1}) + (\varepsilon_{M,t+1} - \varepsilon_{M^*,t+1}) \right], \end{aligned} \tag{3.24}$$

where

$$\varkappa_{mg} = \left[1 + \frac{\tilde{\alpha}_{mg}(1-\beta)(1 + 4\nu(1-\nu)(\theta\rho - 1))}{(1-\nu)(1 + 2\nu(\theta - 1))} \right]^{-1}.$$

If agents did not have any foreign bond holdings, i.e. $\tilde{\alpha}_{mg} = 0$, a fall in relative home endowment would lead to a nominal exchange rate depreciation. At the same time, equation (3.20) demonstrates that, in the presence of home bias and when goods are substitutes in the utility (i.e. $\nu > 1/2$ and $\theta\rho > 1$),[17] a fall in relative home endowment decreases relative home consumption. This suggests that agents would want to have a long position in foreign bonds, i.e. have $\tilde{\alpha}_{mg} > 0$, to hedge against relative endowment shocks.

But if $\tilde{\alpha}_{mg} \neq 0$, relative consumption is also subject to relative money supply

[17] The presence of home bias and substitute goods is a sufficient condition. The necessary condition is $2\nu(\theta\rho - 1) > (\rho - 1)$.

shocks. However, regardless of the sign of the foreign currency position, relative consumption and excess returns will be positively correlated ($\hat{C}^R_{t+1} - \frac{\hat{Q}_{t+1}}{\rho}$ depends positively on $\tilde{\alpha}_{mg}\hat{r}_{x,t+1}$ as shown in equation (3.20)). Hence, the presence of monetary shocks limits the size of the bond portfolio, but it does not influence the sign.

Using equations (3.20) and (3.24) in the second-order portfolio orthogonality condition (3.14), we get the following analytical expression for steady-state foreign bond holdings under the money-growth rule:

$$\tilde{\alpha}_{mg} = \frac{(1-\nu)(2\nu(\theta\rho-1)-(\rho-1))^2}{(1-\beta)(1+2\nu(\theta-1))(1+4\nu(1-\nu)(\theta\rho-1))}\frac{\sigma^2_Y}{\sigma^2_M}. \tag{3.25}$$

Equation (3.25) confirms that, under the money-growth rule, it is optimal to have a long position in foreign bonds (regardless of the value of structural parameters). The expression above also confirms that the size of the bond position decreases as the relative variance of monetary shocks increases.

Inflation-targeting Taylor rule

Consider next the case where monetary policy is characterised by a Taylor rule that only responds to movements in inflation. We can derive the relative stance of monetary policies in the two countries by taking the difference of the linearised Taylor rules:

$$\hat{R}_{H,t+1} - \hat{R}^*_{F,t+1} = \phi_\pi(\hat{P}_t - \hat{P}_{t-1}) - \phi_\pi(\hat{P}^*_t - \hat{P}^*_{t-1}) + \varepsilon_{R,t} - \varepsilon_{R^*,t}.$$

Substituting the condition for excess returns (3.19) we get:

$$-(\hat{S}_t - E_t\hat{S}_{t+1}) = \phi_\pi(\hat{\pi}_t - \hat{\pi}^*_t) + \varepsilon_{R,t} - \varepsilon_{R^*,t}. \tag{3.26}$$

Equation (3.26) shows that the domestic currency appreciates when domestic inflation increases relative to foreign. The intuition is that higher inflation at home requires the domestic central bank to raise interest rates which would trigger a domestic nominal appreciation. In the words of Clarida and Waldman (2007), any bad news about inflation is 'good news for the exchange rate'. Note that, in our model, for most parameter values, a decline in relative domestic endowment is associated with an increase in domestic inflation. Hence, assuming a Taylor rule implies that adverse endowment shocks are associated with both a nominal appreciation of the domestic currency and a decline in the excess return on foreign bonds ($\hat{r}_{x,t+1}$). This suggests that foreign currency denominated bonds are a poor hedge in the face of endowment shocks.

To demonstrate this point formally, and derive an analytical expression for the optimal portfolio, we obtain a general equilibrium solution for excess returns:

$$\begin{aligned}\hat{r}_{x,t+1} &= \hat{S}_{t+1} - E_t\hat{S}_{t+1} \\ &= \varkappa_{tr}\left[\frac{2\nu-1}{1+2\nu(\theta-1)}(\varepsilon_{Y,t+1}-\varepsilon_{Y^*,t+1}) - \frac{1}{\phi_\pi}(\varepsilon_{R,t+1}-\varepsilon_{R^*,t+1})\right],\end{aligned} \tag{3.27}$$

where

$$\varkappa_{tr} = \left[1 + \frac{\tilde{\alpha}_{tr}(1-\beta)(2\nu-1)^2}{(1-\nu)(1+2\nu(\theta-1))}\right]^{-1}.$$

Let us consider the zero-portfolio solution (i.e. $\tilde{\alpha}_{tr} = 0$) in a specification of the model that features consumption home bias and assumes that domestic and foreign goods are substitutes in the utility. In this case, a negative relative endowment shock at home leads to a unexpected appreciation in the home currency. And, as shown in equation (3.20), this shock also decreases relative consumption. So, in the face of such shocks, agents would want to hold a short position in foreign bonds, i.e. $\tilde{\alpha}_{tr} < 0$.

However, for $\tilde{\alpha}_{tr} \neq 0$, monetary shocks affect consumption through the valuation channel. But again, as illustrated by equations (3.20) and (3.27), this relative consumption risk cannot be diversified away (that is, relative consumption and excess returns move in the same direction in response to relative monetary shocks for any value of $\widetilde{\alpha}_{tr} \neq 0$).

Evaluating equations (3.14), (3.20) and (3.27) we can obtain an analytical expression for steady-state foreign bond holdings under the Taylor rule:

$$\tilde{\alpha}_{tr} = -\frac{(1-\nu)(2\nu-1)(2\nu(\theta\rho-1)-(\rho-1))}{(1-\beta)(1+2\nu(\theta-1))\left[(1+4\nu(1-\nu)(\theta\rho-1))\frac{\sigma_R^2/\phi_\pi^2}{\sigma_Y^2} + (2\nu-1)^2\rho\right]}. \tag{3.28}$$

This shows that for $\nu > \frac{1}{2}$ and $\theta\rho > 1$, it is optimal to have a short position in foreign bonds under the Taylor rule. As with a money-growth rule, the size of the bond portfolio decreases as the relative variance of monetary shocks increases. Also, the bigger the response to inflation in the Taylor rule, the bigger the size of the bond portfolio. This is because with a stronger response to inflation the monetary authority offsets the effect of monetary shocks on excess returns (as shown in equation (3.27) and pointed out by Devereux and Sutherland (2008)).

As illustrated above and emphasised by Benigno and Benigno (2008) and Clarida and Waldman (2007), different monetary regimes change the cyclical properties of the exchange rate. Moreover, these different cyclical properties of the exchange rate will determine the hedging characteristic of domestic over foreign bonds - that is, whether the domestic currency depreciates or appreciates in periods of low domestic income determines whether investors take a long or a short position in the foreign currency. Therefore, the agents' optimal portfolio position crucially depends on the choice of policy rule.

3.3.3 Valuation effects of monetary policy

Having demonstrated how different monetary policy rules affect the optimal currency positions, we now turn to the international transmission of monetary shocks. In our set-up, monetary policy shocks generate endogenous currency movements. Since agents hold a portfolio of both foreign and domestic-currency denominated bonds, any shifts in the nominal exchange rate will trigger international valuation effects. As defined below, these valuation effects depend on the excess return $\hat{r}_{x,t}$ as well as the foreign bond position $\tilde{\alpha}$.

$$\underbrace{\widehat{NFA}_t - \frac{1}{\beta}\widehat{NFA}_{t-1}}_{\Delta NFA_t} = \underbrace{\hat{P}_{H,t} + \hat{Y}_{H,t} - \left(\hat{P}_t + \hat{C}_t\right)}_{CA_t} + \underbrace{\tilde{\alpha}(\hat{r}_{x,t})}_{VAL_t} \tag{3.29}$$

To see how the valuation effect changes across monetary policy regimes, we use the expressions for excess returns (3.24) and (3.27) to obtain:

$$VAL_t^{mg} = \tilde{\alpha}_{mg}\varkappa_{mg}\left[-\frac{(2\nu(\theta\rho-1)-(\rho-1))}{1+2\nu(\theta-1)}(\varepsilon_{Y,t+1}-\varepsilon_{Y^*,t+1}) + (\varepsilon_{M,t+1}-\varepsilon_{M^*,t+1})\right], \tag{3.30}$$

$$VAL_t^{tr} = \tilde{\alpha}_{tr}\varkappa_{tr}\left[\frac{2\nu-1}{1+2\nu(\theta-1)}(\varepsilon_{Y,t+1}-\varepsilon_{Y^*,t+1}) - \frac{1}{\phi_\pi}(\varepsilon_{R,t+1}-\varepsilon_{R^*,t+1})\right], \tag{3.31}$$

where $\tilde{\alpha}_{mg}$ and $\tilde{\alpha}_{tr}$ are given by equations (3.25) and (3.28), respectively.

In our model, an exogenous domestic monetary expansion always depreciates the domestic currency.[18] Given that $\hat{r}_{x,t} = \hat{S}_t - E_{t-1}\hat{S}_t$, this shock increases the excess return on the foreign bond. If domestic investors are long in foreign currency, as is the case under the money-growth rule (i.e. $\tilde{\alpha}_{mg} > 0$), then this domestic monetary expansion will give rise to a positive valuation effect in the domestic economy as shown by equation (3.30). Therefore, monetary policy will be *beggar-thy-neighbour.*

[18]Note that a relative monetary expansion is an increase in relative money supplies ($\varepsilon_{M,t+1} - \varepsilon_{M^*,t+1} > 0$) under the money-growth rule, and a fall in relative nominal interest rates ($\varepsilon_{R,t+1} - \varepsilon_{R^*,t+1} < 0$) under the Taylor rule.

In contrast, if the same investors are short in foreign currency, as is the case under the Taylor rule (i.e. $\tilde{\alpha}_{tr} < 0$), monetary policy will be *beggar-thy-self.* [19]

Although we are mainly interested in the valuation effects implied by monetary policy shocks, it is worth noting that supply shocks also create portfolio valuation effects through their effect on the nominal exchange rate as shown in equations (3.30) and (3.31). Whether a positive supply shock at home implies positive or negative valuation effect would also depend on monetary policy regime.

3.4 Numerical results and robustness checks

The previous section provided some analytical results for the foreign currency portfolios and the valuation effects of monetary policy. In this section, we present numerical results for our model, and discuss how the valuation channel of monetary policy works under flexible prices using impulse response functions. We look at the sensitivity of these results to different values of shock persistence, relative variance and monetary policy stance. We then provide more robustness checks by allowing for trade in equities in addition to bonds in the flexible price model and by considering a sticky price model with trade in nominal bonds. When prices are sticky we also allow for a more general specification of the Taylor rule.

3.4.1 Numerical solution of the flexible price model

In this section, we calculate the optimal steady-state portfolio holdings numerically and analyse the model up to a first-order approximation around that particular steady-state. We compare the simulations from the case of a money-growth rule with that for the Taylor rule. While the analytical results in the previous section explained how portfolio choice affects the international transmission of monetary shocks, the aim of the simulation results in this section is to quantify the importance of the international valuation effects.

The calibration parameters are summarised in the top panel in Table 3.1 while the bottom panel contains the steady-state portfolio shares and percentage change in

[19] Under the conditions for which $\tilde{\alpha}_{tr} < 0$, namely for $v > \frac{1}{2}$ and $\theta\rho > 1$, $VAL_t^{tr} < 0$ in a home monetary expansion ($\varepsilon_R - \varepsilon_{R^*} < 0$). This becomes clear when we substitute the expression for $\tilde{\alpha}_{tr}$ from equation 3.28 into equation 3.31, which yields:

$$VAL_t^{tr} = -\Omega[(2\nu - 1)\phi_\pi(\varepsilon_{Y,t+1} - \varepsilon_{Y^*,t+1}) - (1 + 2\nu(\theta - 1))(\varepsilon_{R,t+1} - \varepsilon_{R^*,t+1})]$$

where $\Omega = \frac{(1-\nu)(2\nu(\theta\rho-1)-(\rho-1))(2\nu-1)\phi_\pi V_Y}{(1-\beta)(1+4\nu(1-\nu)(\theta\rho-1))(1+2\nu(\theta-1))^2 V_M+(2\nu-1)^2\phi_\pi^2 V_Y}$.

net foreign asset position implied by a 1% exchange rate depreciation for the case of a money-growth rule as well as for the Taylor-rule case.

Parameter	Description	Money Growth Rule	Taylor Rule
$\bar{\beta} = \omega\bar{C}^{-\eta}$	Steady-state discount factor	0.99	0.99
η	Uzawa convergence parameter	0.01	0.01
ρ	CRRA	2	2
θ	Elas. of subs. across dom. and foreign goods	2.5	2.5
ν	Preference for domestic goods in consumption	0.72	0.72
ζ_Y	Persistence of endowment shocks	0.96	0.96
ζ_M	Persistence of monetary shocks	0	0
ϕ_π	Reaction to inflation	-	1.2
σ_y^2/σ_m^2	Relative size of endow. shocks wrt mon. shocks	1	1
$\bar{\alpha}/\beta\bar{Y}$	Steady-state foreign bond position rel. to GDP	2.0297	-1.4103
$\frac{\bar{\alpha}}{\beta\bar{Y}}\hat{r}_{x,t}$	Valuation effect of 1% unexpected nominal exchange rate depreciation	2% of GDP	-1.4% of GDP

Table 3.1: Baseline calibration and steady-state foreign bond position

The calibration parameters are fairly standard. The steady-state discount factor, $\bar{\beta}$, equals 0.99 while the Uzawa convergence parameter equals 0.01 which is similar to Devereux and Yetman (2010). The coefficient of relative risk aversion (CRRA), ρ, equals 2 and the elasticity of substitution across domestic and foreign goods, θ, equals 2.5. This is slightly higher than the values chosen by Heathcote and Perri (2002) but below that typically chosen in the New Open Economy Macroeconomics literature (see for instance De Paoli (2009)). The home bias in consumption parameter, ν, is equal to 0.74 which implies an import share of 26%. The persistence of endowment process, ζ_Y, is set equal to 0.96 while monetary shocks are assumed to be i.i.d. Our benchmark calibration presumes that the volatility of monetary shocks is similar to that of real shocks but our sensitivity analysis considers the case where real shocks are more volatile than monetary shocks.

Monetary shocks and a money-growth rule

Under a money-growth rule, we calculate that the steady-state foreign bond position relative to GDP is 203%. Hence, agents find it optimal to go short in their home currency (and long in the foreign currency). This is consistent with the analytical results in subsection 3.3.2. With a money-growth rule, agents realise that adverse real shocks (which lower their income and consumption) are associated with a foreign currency appreciation and hence a positive excess return from holding foreign currency denominated bonds.

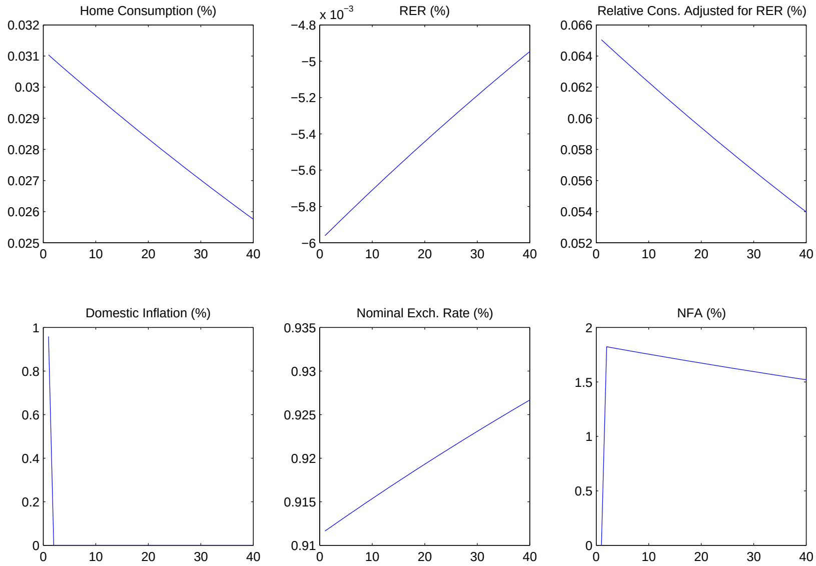


Figure 3.1: Impulse responses following a monetary shock under a money-growth regime.

Notes: x-axis shows periods, y-axis shows percentage deviations from steady-state.

Figure 3.1 illustrates the effects of a positive domestic monetary shock when monetary policy is conducted via a money-growth rule. On impact, the domestic currency depreciates in nominal terms. Given that domestic investors are long in foreign currency denominated bonds, the endogenous currency movement generates an increase in the domestic currency value of the country's net foreign asset (NFA) position (shown as a 1.7% jump in the NFA position in Figure 3.1). This valuation effect allows domestic consumers to increase their consumption. Since part of the domestic consumption is imported, the domestic currency has to appreciate in real terms to satisfy the higher demand for imported consumption. This adjustment in real exchange rates comes about via higher domestic prices.

Thus, because agents are long in foreign bonds, monetary policy shocks trigger international valuation effects that are *beggar-thy-neighbour*. While the effects are quantitatively fairly small (a 1.7% rise in net external wealth is associated with a 0.03% increase in consumption), the size of these are roughly in line with the estimates of wealth effects on consumption found in the empirical literature (see Labhard et al. (2005) and Fair (2004)).

Monetary shocks and an inflation-targeting Taylor rule

We now turn to the case when monetary policy is assumed to follow a Taylor rule. Here the steady-state foreign bond position to GDP is calculated to be -141% which implies that agents want to go short in foreign currency (and long in domestic currency). This is because adverse real shocks which lower consumption are now associated with a foreign currency depreciation and hence a negative excess return from holding foreign currency denominated bonds. So agents prefer to hold domestic bonds over foreign bonds. Figure 3.2 illustrates the valuation effects from a positive domestic monetary policy shock when the central bank follows a Taylor rule. We assume this is an exogenous shock to the domestic Taylor rule function (i.e. a rise in $\varepsilon_{R,t}$) where the size of the shock is standardised to give a one standard deviation increase in domestic money growth. This shock implies a jump in the nominal exchange rate. Since agents hold a portfolio that is short in foreign currency denominated bonds, this nominal appreciation of the foreign currency causes a decline in the domestic net foreign wealth (measured in domestic currency). The loss in wealth triggers a decline in domestic consumption and as a result an excess supply of domestic goods. The domestic currency has to depreciate in real terms to ensure that this excess supply is eliminated.

In contrast to our previous experiment, monetary policy shocks with a Taylor rule cause international valuation effects that are *beggar-thy-self*. Agents have chosen a portfolio position to optimally hedge themselves against the consumption risks caused by real shocks. So in the case of the Taylor rule, agents prefer to hold a positive position in domestic bonds and a negative position in foreign bonds. The side-effect of having these optimally chosen portfolios is that monetary shocks can cause negative valuation effects. As in the case with money-growth rules, the size of these wealth effects are quantitatively small (a 1.46% fall in wealth is associated with a 0.02% decline in consumption).

Sensitivity analysis

This section first examines the sensitivity of our results using the model where monetary policy is conducted via a Taylor rule where the central bank targets inflation. As previously shown, with an inflation-targeting Taylor rule investors will choose an optimal portfolio that is overweight in home bonds.

Varying the shock persistence

Baxter and Crucini (1995) have shown how more persistent shocks in an incom-

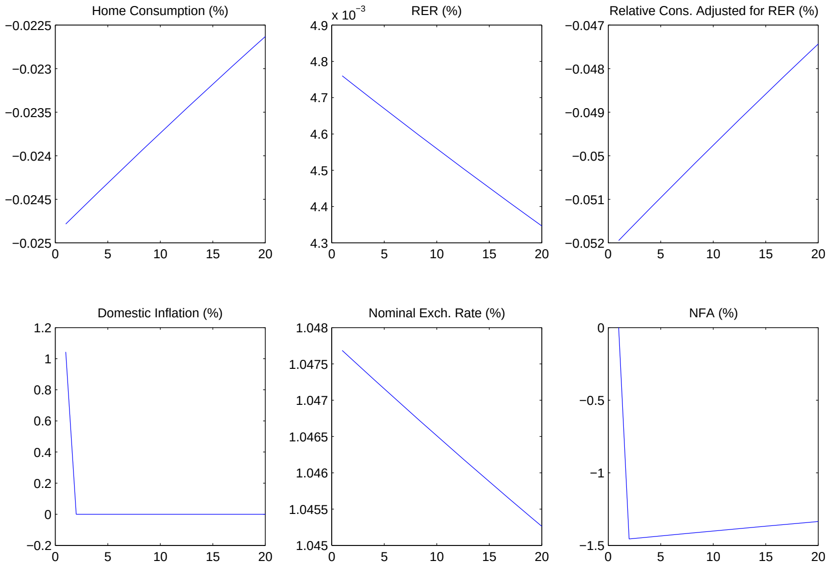


Figure 3.2: Impulse responses following a monetary shock under an inflation-targeting Taylor rule.

Notes: x-axis shows periods, y-axis shows percentage deviations from steady-state. The shock to the Taylor rule is standardised to give one standard deviation increase in money growth to make it compatible with Figure 3.1.

plete markets model imply greater changes in relative wealth across countries. The first panel in Figure 3.3 shows the effects of a 1% domestic monetary expansion (1% fall in home policy rates) on the steady-state foreign bond position, the impact response on the nominal exchange rate as well as on domestic consumption for different degrees of shock persistence. The figures show that making the endowment shocks more persistent exposes agents to larger exchange rate movements and thus more risk - which they want to hedge against by holding a larger gross portfolio position. A higher gross portfolio position amplifies the valuation effects of monetary policy shocks. As illustrated in Figure 3.3, domestic consumption declines by more (on impact) following a domestic monetary expansion, the more persistent the endowment process.

Varying the relative variance of real shocks

Agents in our model choose a portfolio to hedge themselves against the consumption risk caused by real shocks. As equation 3.28 shows, the size of the bond portfolio increases as the relative variance of real shocks increases. This analytical result is confirmed by the second panel in Figure 3.3. When real shocks are assumed to be ten times more volatile than monetary shocks, agents will choose a steady-state foreign

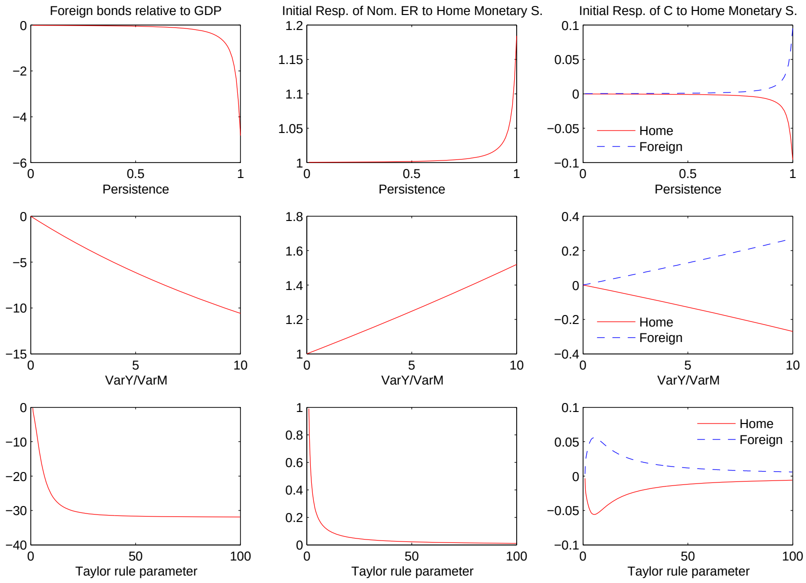


Figure 3.3: Sensitivity analysis for the endowment model with an inflation-targeting Taylor rule

Notes: Columns show the steady-state foreign bond position relative to GDP, impact response of nominal exchange rate to a domestic monetary expansion and impact response of consumption to a home monetary shock, respectively. Rows show sensitivity with respect to persistence of endowment shocks, relative variance of endowment shocks and the weight of inflation in the Taylor rule.

bond position relative to GDP equal to minus 10! In other words, investors will *significantly* short foreign bonds and go long in domestic bonds. Again, a higher gross portfolio position increases the potency of monetary shocks. This implies a greater nominal depreciation in the domestic currency (on impact), triggering greater valuation effects and a larger decline (on impact) in domestic consumption (as shown in the second panel in Figure 3.3).

Varying the weight of inflation in the Taylor rule

The implication of a larger Taylor rule response to inflation (i.e. a larger ϕ_π) for the valuation effect of monetary shocks is twofold: First, the higher ϕ_π the smaller is the effect of the money shock on inflation and the exchange rate. But this increases the importance of real shocks relative to monetary shocks. So this increases the investors' hedging motives and thus the size of their portfolio positions. The first effect - the smaller exchange rate response-would diminish the valuation effect of monetary shocks. But the second effect - the increase in portfolio position - would amplify it. As shown in the bottom panel of Figure 3.3 for values of ϕ_π between 0 and 20, the second effect dominates. But for larger values of ϕ_π the endogenous monetary policy response completely offsets the effects of monetary shocks. This is consistent with Devereux and Sutherland (2008), who show that the central bank can complete the markets and ensure full risk sharing by pursuing a price stability objective.

3.4.2 Trade in equities in the flexible price model

In this part, we allow for international trade in equities as well as bonds and show that the main implication of our model about the link between monetary policy and foreign currency positions is robust provided that we continue to have incomplete markets. As shown by Engel and Matsumoto (2009) and Devereux and Sutherland (2008), monetary policy does not have a significant effect on equilibrium portfolios under complete markets. If there are only supply shocks and monetary shocks in each country, allowing for trade in equities in addition to bonds will complete the markets. In this case, the optimal bond position becomes zero irrespective of the monetary policy regime. This is because real shocks will be hedged by equities whose returns are not affected by nominal shocks under flexible prices, unlike nominal bonds. Then, having a zero bond portfolio will insulate the economy from monetary shocks. However, the optimal equity portfolio in this case will exhibit a foreign bias: when output is higher at home, the home consumption basket becomes cheaper (real exchange rate depreciates), while the returns on home equity relative to foreign goes up because of the increase in output (dividend).

We know from the existing literature on general equilibrium portfolio models that one possible way to derive home bias in equity is to have a negative correlation between relative non-financial income and relative home equity returns.[20] Thus, when we allow for trade in equities, we assume that only a part of total endowment in each country can be diversified away by holding equities. This part of the income is generally referred to as financial income (or capital income) in the literature. The rest of the income, which is not subject to equity returns, is the non-financial income, which can be thought of as labour income in a production economy. We allow for both capital income and non-financial income to be stochastic as in Devereux and Sutherland (2010b). These shocks work as the redistributive shocks introduced by Coeurdacier et al. (2007).

The linearised net foreign asset accumulation equation with equities can be written

[20]See Heathcote and Perri (2007), Coeurdacier et al. (2007), Engel and Matsumoto (2009), Coeurdacier et al. (2010) and the references in these papers.

as follows:

$$\widehat{NFA}_t = \frac{1}{\beta}\widehat{NFA}_{t-1} + \frac{\bar{\alpha}_F}{\beta\bar{Y}}(\hat{r}_{F,t} - \hat{r}_{H,t}) + \frac{\bar{\alpha}_F^E}{\beta\bar{Y}}(\hat{r}_{F,t}^E - \hat{r}_{H,t}^E) + \hat{P}_{H,t} - \hat{P}_t + \hat{Y}_t - \hat{C}_t, \quad (3.32)$$

where $\bar{\alpha}_F$ is the steady-state foreign currency (bond) portfolio as before and $\bar{\alpha}_F^E$ is the steady-state foreign equity holdings.[21] $\hat{r}_{F,t} - \hat{r}_{H,t}$ is the excess return on foreign bonds relative to home bonds as before while $\hat{r}_{F,t}^E - \hat{r}_{H,t}^E$ gives the excess return on foreign equities relative to home equities expressed in terms of home good. Another key difference here relative to the model with only bonds is that $Y_t = Y_{K,t} + Y_{L,t}$ and $Y_t^* = Y_{K,t}^* + Y_{L,t}^*$, where Y_K and Y_L represent financial and non-financial incomes as explained above.[22] The stochastic processes for Y_K and Y_L are specified as in Devereux and Sutherland (2010b). A similar structure exists for the foreign country with symmetric parameters.

$$\log Y_{K,t} = (1-\zeta_K)\log \bar{Y}_K + \zeta_K Y_{K,t-1} + \varepsilon_{K,t}$$
$$\log Y_{L,t} = (1-\zeta_L)\log \bar{Y}_L + \zeta_L Y_{L,t-1} + \varepsilon_{L,t}$$

where $E_{t-1}[\varepsilon_{K,t}] = 0,\ Var[\varepsilon_{K,t}] = \sigma_K^2,\ Cov[\varepsilon_K, \varepsilon_L] = \sigma_{KL}$ and $E_{t-1}[\varepsilon_{L,t}] = 0$, $Var[\varepsilon_{L,t}] = \sigma_L^2$.

How does international trade in equities affect optimal bond positions within this framework? When equity trade is allowed, nominal bonds hedge the part of the risk that is not hedged by equities. Thus, what matters for the bond portfolio in this case is the conditional covariance-variance ratios. The partial equilibrium solution for foreign bond position presented earlier, in equation (3.18), changes in the following way:

$$\tilde{\alpha}_F = -\frac{1}{2(1-\beta)}\left(\frac{Cov_t(\Lambda_{Y_L,t+1}, \hat{r}_{x,t+1}|\hat{r}_{x,t+1}^E)}{Var_t(\hat{r}_{x,t+1}|\hat{r}_{x,t+1}^E)} + \left(1-\frac{1}{\rho}\right)\frac{Cov_t(\Lambda_{Q,t+1}, \hat{r}_{x,t+1}|\hat{r}_{x,t+1}^E)}{Var_t(\hat{r}_{x,t+1}|\hat{r}_{x,t+1}^E)}\right). \quad (3.33)$$

where $\Lambda_{Y_L,t+1}$ is the news at time $t+1$ about the net present value of the relative non-financial income and $\Lambda_{Q,t+1}$ is the news at time $t+1$ about the net present value of real exchange rates as before. As shown in Coeurdacier and Gourinchas (2009),

[21] Here, equity holdings are defined as gross real holdings, not as shares of total equity stock but it is possible to express these gross positions as shares of stock as we do later when reporting numerical results. We use the assumption that all assets are in net zero supply when writing equation 3.32.

[22] Note that Y_K is what is paid out every period by domestic equity (dividend). In the model without equities, all endowment is non-financial income because bonds do not represent any claims on income, i.e. $Y_{K,t} = 0$ and $Y_t = Y_{L,t}$.

non-financial income risk will be mainly hedged by equities, while nominal bonds will be used to hedge against the real exchange rate risk since real exchange rates are correlated more with relative bond returns than with relative equity returns. Thus, there will still be a role for nominal assets in providing consumption risk sharing across countries in the presence of equities provided that equities do not complete the markets. What is more, the sign of the conditional covariance-variance ratios given in equation (3.33) will still be determined by the monetary policy regime in the same way as in the bonds only case. On the other hand, equity portfolios will not be affected by monetary policy much. The important factors in determining optimal equity portfolio are the share of capital income and the correlation between financial and non-financial incomes. In Table 3.2, we give some numerical results regarding the model with equities. We only report the calibration for the new parameters, the calibration for the rest of model parameters is the same as in Table 3.1.

Parameter	Description	Money Growth Rule	Taylor Rule
ζ_K	Persistence of financial income shocks	0.96	0.96
ζ_L	Persistence of non-financial income shocks	0.96	0.96
$\bar{Y}_K/\bar{Y}_L$	Steady-state financial income share	0.36	0.36
σ_K^2/σ_M^2	Rel. size of financial income wrt monetary shocks	2	2
σ_L^2/σ_K^2	Rel. size of fin. income wrt non-fin. income shocks	1	1
$Corr(\varepsilon_K, \varepsilon_L)$	Correlation of fin. and non-fin. income shocks	-0.2	-0.2
$\bar{x}_H$	Steady-state share of dom. stock held by home agents	0.9056	0.9091
$\bar{\alpha}_F/\bar{\beta}\bar{Y}$	Steady-state foreign bond position rel. to GDP	1.8067	-1.2611
$\frac{\bar{\alpha}_F}{\bar{\beta}\bar{Y}}\hat{r}_{x,t}$	Valuation effect of 1% unexpected nominal exchange rate depreciation	1.8% of GDP	-1.3% of GDP

Table 3.2: Steady-state foreign bond position and domestic equity shares when there is trade in bonds and equities

Our calibration for the parameters that determine the stochastic properties of capital and labour income shocks follow closely that of Devereux and Sutherland (2010b). Accordingly, we set the steady-state financial income share equal to 0.36 and the correlation between financial and non-financial income shocks to -0.2, which is a less negative number than is required to derive equity home bias in a one-good model without bonds. We take the relative variance of financial and non-financial income shocks to be equal, while we lower the size of monetary policy shocks relative to income shocks to get bond positions closer in size to the bond positions we obtain under the bonds only case.[23]

With the calibration in Table 3.2, we get that 90% of domestic equity is held by home agents, which is in line with the observed home equity bias. Note that the equity portfolio is independent of the monetary policy rule as expected. On the other hand, the sign of bond portfolios continue to depend crucially on the policy rule, even when relative bond returns are conditioned on relative equity returns. When bond portfolios are mainly hedging against real exchange rate appreciations, a negative relative non-financial income shock that appreciates the real exchange rate will require a long position in foreign bonds if the home currency depreciates in response to the negative non-financial income shock as is the case under a passive monetary policy regime. *Vice versa* is true for the inflation-targeting Taylor rule. These results justify analysing only bonds to study international valuation effects created by monetary policy.

With trade in equities, there will also be valuation effects coming from the capital gains and losses on equity positions. Taking the difference in net foreign assets using equation (3.32) and decomposing it into current account and valuation terms as we did before in equation (3.29), we can define the valuation effect with trade in equities as follows:

$$VAL_t = \frac{\bar{\alpha}_F}{\bar{\beta}\bar{Y}}(\underbrace{\hat{r}_{F,t} - \hat{r}_{H,t}}_{r_{x,t}}) + \frac{\bar{\alpha}^E_F}{\bar{\beta}\bar{Y}}(\underbrace{\hat{r}^E_{F,t} - \hat{r}^E_{H,t}}_{r^E_{x,t}}),$$

where the first term is what we analyse as valuation effects throughout this chapter. Using the definitions of bond and equity returns and the property that expected excess returns are zero in a first order approximation, we can further write this as follows:

$$\begin{aligned} VAL_t &= \tilde{\alpha}_F(\hat{S}_t - E_{t-1}\hat{S}_t) \\ &\quad + \tilde{\alpha}^E_F \left[\begin{array}{c} \hat{TOT}_t - E_{t-1}\hat{TOT}_t - (1-\bar{\beta})(\hat{Y}^R_{K,t} - E_{t-1}\hat{Y}^R_{K,t}) \\ +\bar{\beta}(\hat{Z}^*_{F,t} - E_{t-1}\hat{Z}^*_{F,t}) - \bar{\beta}(\hat{Z}_{H,t} - E_{t-1}\hat{Z}_{H,t}) \end{array} \right]. \end{aligned} \tag{3.34}$$

where TOT is the terms of trade defined as the price of imports relative to exports, $Y^R_{K,t}$ is the difference between home and foreign capital incomes and Z_H and Z^*_F are the real prices of home and foreign equity defined in terms of home and foreign goods, respectively.

The first important point is that relative equity returns only depend on real vari-

[23]When monetary shocks are the same size as (capital) income shocks, steady–state foreign bond positions are -0.6411 under Taylor rule and 0.9016 under money-growth rule.

ables like the terms of trade, relative income and relative equity prices as shown in equation (3.34). Thus, even though we are operating under incomplete markets and monetary policy shocks have real effects through home bond portfolios, the effect of monetary policy shocks on $\hat{r}^E_{F,t} - \hat{r}^E_{H,t}$ is negligible. Thus, in our flexible price set-up, a currency depreciation caused by a monetary policy expansion affects VAL_t mainly through relative bond returns, which is what we have analysed so far.

3.4.3 Sticky prices

Our analysis so far only considers fully flexible prices. A natural question that comes to mind is whether our results on the link between policy regimes and foreign bond positions, and consequently on the valuation channel of monetary policy, would go through in a sticky price environment. To answer this question, we introduce price rigidities in the model with only bonds and show that the sign of the foreign currency portfolio continues to be affected in a similar way as in the flexible price model for reasonable calibrations of the model.

Optimal portfolios under sticky prices have been analysed before in Devereux and Sutherland (2008) and Engel and Matsumoto (2009). In a model where monetary policy is specified as a Taylor rule that reacts to PPI inflation, Devereux and Sutherland (2008) find a negative position in foreign bonds under incomplete markets. The main difference of our model with Devereux and Sutherland (2008) is that we analyse portfolios under 'passive' monetary policy regimes as well as inflation targeting regimes and show how the optimal foreign bond portfolio might switch sign depending on this classification in an incomplete market setting. Engel and Matsumoto (2009), on the other hand, assume a money-growth rule for monetary policy and show that the optimal foreign currency position will be negative whether or not money supplies are allowed to respond to productivity shocks under a complete market setting where there is also trade in equities.[24] Therefore, market incompleteness is crucial for our result on the link between policy and portfolio currency shares under sticky prices.

The basic features of the model are as follows: Price stickiness is modelled à la Calvo. In each period, a fraction $\kappa \in [0, 1)$ of randomly selected firms in each country cannot change their prices. The remaining $1 - \kappa$ fraction of firms chooses prices optimally to maximise the expected discounted value of future profits. Each firm pro-

[24] The intuition in Engel and Matsumoto is as follows: due to price stickiness, home labour income and relative returns on home equity are negatively correlated for a given exchange rate. Thus, it is optimal to have home bias in equity. When exchange rate depreciates, home revenues increase in home currency terms. Given that equity portfolios are home biased, this leads to an increase in relative consumption which can be hedged by having a short position in foreign currency.

duces a single variety of the domestic consumption good according to a production function that is linear in labour, i.e. $Y_t = A_t L_t$ for the home country, where A_t is an AR(1) productivity shock common across all home firms. A similar production function and productivity shock exist for the foreign country. The representative agent in each country now also gets disutility from work as well as utility from consumption and real money holdings. The model is otherwise similar to the model presented in Section 3.2. We present the important equations of the sticky price model in the appendix.

We know from the literature that the currency in which export prices are set is crucial for the international transmission of monetary shocks under sticky prices.[25] Therefore, we consider two different price-setting assumptions for exports: producer currency pricing (PCP) and local currency pricing (LCP). Under PCP, producers in each country set export prices in their own currency, whereas under LCP, export prices are set in the currency of the buyer.[26] When prices are set according to LCP, the law of one price no longer holds. In this case, real exchange rate fluctuations reflect both the presence of home bias in consumption and deviations from the law of one price.

We solve the model numerically to analyse the interaction between policy regimes and bond portfolios under sticky prices. We use the same parameter values as in the flexible price model given in Table 3.1, where applicable. The rest of the model parameters are calibrated as follows. The elasticity of substitution across varieties in each country, ϕ, is set to 10, which is consistent with a price mark-up of 11%. The price stickiness parameter κ equals 0.75, so that prices are set for a year at a time. The inverse of the Frisch elasticity of labour supply, ϖ, is set to 2$-$the same value as the coefficient of relative risk aversion. The persistence of the productivity shock in each country, ζ_A, equals 0.96, the same value as the persistence of the endowment process in the flexible price model. Monetary shocks are assumed to be i.i.d as before. Our benchmark calibration presumes that the volatility of monetary shocks is equal to that of real shocks but we also report the results for different values of relative variance.

Table 3.3 gives home country's optimal steady-state foreign bond holdings in proportion to GDP for two different policy regimes and three different price-setting assumptions under the benchmark calibration described above.[27] We see that the

[25] See for example Betts and Devereux (2000, 2001), Devereux and Engel (2003), Corsetti and Pesenti (2005).

[26] Optimal price for each case is given in the appendix.

[27] We obtain the flexible price solution by letting $\kappa \to 0$. Note that the flexible price solution for

	Money-Growth Rule			Taylor Rule		
$\bar{\alpha}/\bar{\beta}\bar{Y}$	Flex	PCP	LCP	Flex	PCP	LCP
$\sigma_A/\sigma_M = 1$	1.9821	1.0327	2.0037	-1.9014	-2.0975	-1.3250
$\sigma_A/\sigma_M = 0.5$	0.9890	0.1317	1.1104	-0.9743	-1.3870	-0.6490
$\sigma_A/\sigma_M = 0.1$	0.1975	-0.5867	0.3982	-0.1988	-0.8003	-0.0885
$\frac{\bar{\alpha}}{\beta\bar{Y}}\hat{r}_{x,t}$	Flex	PCP	LCP	Flex	PCP	LCP
$\sigma_A/\sigma_M = 1$	2%	1%	2%	-1.9%	-2.1%	-1.3%

Table 3.3: Steady-state foreign bond position and implied valuation effects under the baseline calibration

Notes: Cells in the upper panel gives the values of $\bar{\alpha}/\bar{\beta}\bar{Y}$, steady-state foreign bond position relative to GDP, and cells in the lower panel gives the values of $\frac{\bar{\alpha}}{\beta}\bar{Y}\hat{r}_{x,t}$, the valuation effect of 1% nominal exchange rate depreciation as a percent GDP.

link between monetary policy regimes and foreign bond portfolios under sticky prices is similar to that under flexible prices. That is, money-growth targeting is associated with a long position in foreign bonds, while the opposite is true for strict inflation targeting irrespective of the currency in which export prices are set. To understand the portfolio positions given in Table 3.3 and explain the differences across different policy regimes and price-setting assumptions, we look at the covariance of real exchange rate adjusted relative consumption and excess returns conditional on relative supply shocks and monetary shocks under the zero-portfolio solution (ignoring the valuation effects). Table 3.4 shows how each component of the portfolio orthogonality condition given in equation (3.13) responds to these shocks under the benchmark calibration.

First of all, just as in the case of flexible prices, the covariance of real exchange rate adjusted relative consumption, $\hat{C}_t^R - \hat{Q}_t/\rho$, and excess returns, $\hat{r}_{x,t} = \hat{S}_t - E_{t-1}\hat{S}_t$, conditional on relative productivity shock is negative under money-growth rule and positive under inflation-targeting Taylor rule - irrespective of the price-setting assumption. Indeed, whether export prices are set in producer's or buyer's currency does not affect relative consumption and excess returns significantly in the face of productivity shocks.[28] Thus, hedging against the relative consumption risk coming from productivity shocks requires a long position in foreign currency under money-growth rule and a short position under strict inflation targeting just as in the case

this model is different than the one for the endowment model, because here labour supply is elastic. As $\varpi \rightarrow \infty$, labour supply becomes infinitely inelastic and the model collapses to the endowment economy case.

[28] See Benigno and Benigno (2008) and Clarida and Waldman (2007) for the analysis of nominal exchange rate dynamics under different policy rules in a sticky price model with PCP, and Benigno (2004) for an LCP model.

Zero-portfolio solution	Money-Growth Rule		Taylor Rule	
	$\varepsilon^R_{A,t}$	$\varepsilon^R_{M,t}$	$\varepsilon^R_{A,t}$	$-\varepsilon^R_{R,t}$
Flexible Price				
$\hat{C}^R_t - \frac{\hat{Q}_t}{\rho}$	+	0	+	0
$\hat{r}_{x,t}$	−	+	+	+
Sticky price with PCP				
$\hat{C}^R_t - \frac{\hat{Q}_t}{\rho}$	+	+	+	+
$\hat{r}_{x,t}$	−	+	+	+
Sticky price with LCP				
$\hat{C}^R_t - \frac{\hat{Q}_t}{\rho}$	+	−	+	−
$\hat{r}_{x,t}$	−	+	+	+

Table 3.4: Conditional covariance of real exchange rate adjusted relative consumption and excess returns under zero-portfolio solution

Notes: This table shows the signs of the impact responses of the components of the portfolio orthogonality condition given by equation (3.13) to relative technology and monetary shocks under alternative policy rules.

of flexible prices. The difference in the sign of the covariance across policy regimes comes from the different response of nominal exchange rates. The intuition for this result is as described in Section 3.2.

The most important implication of sticky prices for bond positions is that now agents use bonds *also* to hedge against the relative consumption risk coming from monetary shocks. Table 3.4 shows that the covariance of relative consumption and nominal exchange rate, conditional on monetary shocks, does not change sign across different policy regimes, but across different price-setting assumptions for a given a policy regime. That is, a monetary expansion leads to a rise in adjusted relative consumption, as well as a depreciation in home currency, i.e. $\hat{C}^R_t - \hat{Q}_t/\rho \uparrow$ and $\hat{r}_{x,t} \uparrow$, under both policy regimes if prices are set in producer's currency. However, if prices are set in buyer's currency, a monetary expansion at home actually depresses relative consumption, while depreciating domestic currency, i.e. $\hat{C}^R_t - \hat{Q}_t/\rho \downarrow$ and $\hat{r}_{x,t} \uparrow$.

What explains this difference in the response of adjusted relative consumption, $\hat{C}^R_t - \hat{Q}/\rho$, to a home monetary expansion across PCP and LCP? Under PCP, a domestic monetary expansion that depreciates the nominal exchange rate also worsens the terms of trade given that import and export prices are sticky in the currency of the producer. [29]Depreciation in the terms of trade shifts the world demand towards the cheaper home good. Consumption increases in both countries, but by more in

[29]Under PCP, $TOT_t = \hat{P}^*_{F,t} + \hat{S}_t - \hat{P}_{H,t}$. Since $\hat{P}_{H,t}$ and $\hat{P}^*_{F,t}$ are sticky in home and foreign currencies, respectively, home terms of trade depreciates following a monetary expansion that depreciates the nominal exchange rate.

the home country. Real exchange rate depreciates due to home bias in consumption. Overall, given the strong expenditure-switching effects created by a home currency depreciation, consumption of home agents increase by more compared to foreign agents in purchasing power terms under PCP.

Under LCP, on the other hand, a domestic monetary expansion that depreciates the nominal exchange rate does not generate expenditure-switching effects like it does under PCP, because foreign currency price of home good does not fall. Revenues of home exporters increase, which improves the home terms of trade and raises the value of home output.[30]Thus, home consumption increases by much more than in the PCP case and foreign consumption falls due to the worsening in foreign terms of trade. So, under LCP relative consumption, $\hat{C}_t^R = \hat{C}_t - \hat{C}_t^*$, increases by much more in response to a domestic monetary expansion than under PCP. But, the real exchange rate also depreciates by much more under LCP compared to the PCP case - so much that relative consumption adjusted for purchasing power, $\hat{C}_t^R - \hat{Q}/\rho$, slightly falls in the face of a positive monetary shock. [31] This result is in line with the literature which shows that the presence of pricing to market increases exchange rate volatility relative to the case in which law of one price holds.[32]

Given the discussion on the comovement of relative consumption and excess returns under different price-setting assumptions, the optimal hedge against monetary shocks under LCP is to go long in the foreign bond irrespective of the policy regime, while the opposite is true for PCP.[33] Consequently, for a money-growth rule, it is optimal to have a *bigger* long position in foreign bonds under LCP compared to flexible prices or PCP, because in this case hedging against monetary shocks also requires a long position in foreign currency, reinforcing the effect coming from productivity shocks. However, under PCP, monetary shocks affect the covariance between relative consumption and excess returns in the opposite way as productivity shocks and therefore a *smaller* long position in FX is optimal. A similar reasoning applies to an

[30] Under LCP, $TOT_t = \hat{P}_{F,t} - \hat{P}^*_{H,t} - \hat{S}_t$. Since $\hat{P}_{F,t}$ and $\hat{P}^*_{H,t}$ are sticky in home and foreign currencies, respectively, home terms of trade appreciates following a monetary expansion that depreciates the nominal exchange rate.

[31] To understand the real exchange rate response under LCP, it is useful to decompose it in terms of the deviation from the law of one price in each country and home terms of trade:

$$\hat{Q}_t = \nu(\hat{P}^*_{H,t} + \hat{S}_t - \hat{P}_{H,t}) + \nu(\hat{P}^*_{F,t} + \hat{S}_t - \hat{P}_{F,t}) + (2\nu - 1)(\hat{P}_{F,t} - \hat{P}^*_{H,t} - \hat{S}_t)$$

Despite the fact that the home terms of trade improves following an expansionary monetary shock, real exchange rate depreciates because of the deviation from the law of one price in each country, which are equal because of symmetry.

[32] For example, see Betts and Devereux (2000).

[33] Note that in the flexible price case, the optimal hedge against monetary shocks is to have zero portfolio as discussed in Section 3.2.

inflation-targeting Taylor rule. For this regime, the short position in FX is magnified under PCP and reduced under LCP (Tables 3.3 and 3.4).

The result presented above implies that, for a sufficiently large relative variance of monetary shocks, optimal foreign bond position might be negative under a passive policy when prices are sticky in the producer's currency and positive under an inflation-targeting policy when prices are sticky in the buyer's currency. Table 3.3 shows how portfolios change when monetary shocks are twice and ten times more volatile than productivity shocks, i.e. $\sigma_A^2/\sigma_M^2 = 0.5$, and $\sigma_A^2/\sigma_M^2 = 0.1$. First of all, bond portfolios shrink in size as monetary shocks become more important. Secondly, increasing the size of monetary shocks affects portfolios less under a Taylor rule compared to a money-growth rule because the reaction to inflation in the Taylor rule partially offsets the increase in the variance.[34] Therefore, foreign bond position does not switch sign under Taylor rule with LCP even when monetary shocks are ten times more volatile than productivity shocks. All in all, for reasonable calibrations of the relative variance, the link between policy regimes and foreign bond positions is robust to the introduction of sticky prices in the model.

Valuation effects of monetary policy shocks under sticky prices

Under the benchmark calibration, a monetary expansion that generates 1% depreciation in domestic currency creates a positive valuation effect up to 2% of GDP under a money-growth rule and a negative valuation effect up to -2% of GDP under a Taylor rule depending on the price-setting assumption (Table 3.3). However, this valuation effect on international bond positions created by monetary policy shocks is small when compared with the conventional effect coming from the slow adjustment of prices. For example, the negative valuation effect that follows a monetary expansion under a Taylor rule is not enough to make monetary policy beggar-thy-self as in the case of flexible prices (where there is no other channel for the transmission of monetary shocks except the valuation channel). Figure 3.4 shows how the valuation channel works in the case of a Taylor rule with PCP by comparing the impulse responses from a non-contingent bond economy (zero-portfolio solution) with those from the model with trade in two nominal bonds. The difference between the dashed line and the solid line is due to the valuation channel.

As can be seen from the impulse response of NFA in Figure 3.4, the valuation effect of a monetary expansion is negative, but the response of home and foreign consumption, real exchange rate and other variables are not much affected by the fall in NFA, because this channel is dominated by the conventional sticky-price channel.

[34] See equation 3.28 and the related discussion in Section 3.3 for the flexible price argument.

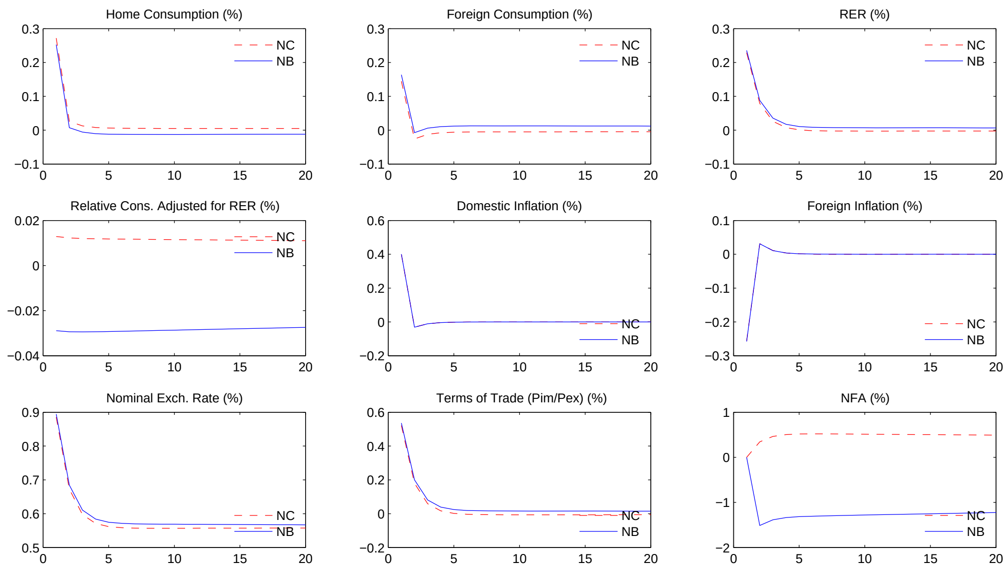


Figure 3.4: Impulse responses following an expansionary monetary shock under Taylor rule with sticky prices and PCP

Notes: x-axis shows periods, y-axis shows percentage deviations from steady-state.

The only variable that is affected most significantly by the valuation channel is relative home consumption adjusted for the real exchange rate, which shows deviations from perfect risk sharing.

A more general specification of the Taylor rule

A standard Taylor rule (Taylor (1993)) specifies that interest rates respond to divergences of inflation from target and output from trend. Here we consider a set-up where the central bank responds to inflation and output growth. Arguably, a rule that responds to output growth rather than the output gap (defined as the deviation of actual output from its efficient level) might not be consistent with a central bank that cares about social welfare. We allow for such specification in order to consider the case that the monetary policy has some preference for stabilising output itself. The analysis of this rule will be an alternative way of assessing the implications of having a more accommodative policy.[35]

[35] Note that a specification of the Taylor rule that responds to output gap - defined as deviations of actual output from its flexible price allocation - would not change our results (at least in qualitative terms). This is because the objective of targeting the flexible price allocation is no different than the objective of targeting stable inflation. We actually run this sensitivity analysis and find that the presence of an output gap in the Taylor rule would make the foreign currency position even more

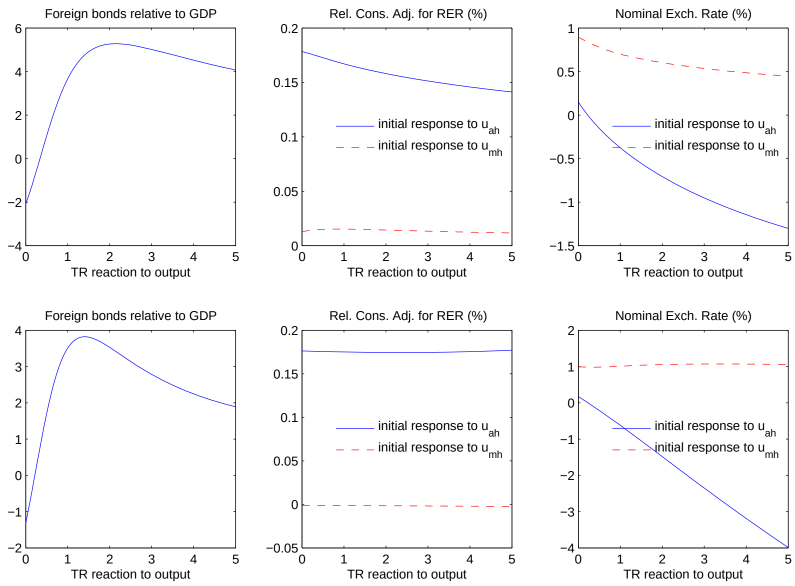


Figure 3.5: Sensitivity analysis with respect to the weight of output in the Taylor rule

Notes: Columns show the plots of the steady-state foreign bond position relative to GDP and the impact responses of portfolio determinants, i.e. real exchange rate adjusted relative consumption and nominal exchange rate, to productivity (u_{ah}) and monetary (u_{mh}) shocks under PCP (top row) and LCP (bottom row).

Figure 3.5 examines the effects of increasing the weight on output stabilisation in the Taylor rule ϕ_y. The case of $\phi_y = 0$ corresponds to the set-up where the central bank only targets inflation and is analysed above. In this special case, the domestic currency depreciates in response to a positive endowment shock and agents would want to hold a portfolio that is short in foreign bonds (i.e. the steady share foreign bond position is negative). Expansionary domestic monetary policy shocks cause a domestic nominal depreciation which triggers negative valuation effects at home.

However, as Figure 3.5 illustrates, this result is overturned as ϕ_y increases. Then positive endowment shocks that raise output would require the central bank to raise interest rates, appreciating the domestic currency in nominal terms. This different exchange rate dynamic has been emphasised by Clarida and Waldman (2007) and Benigno and Benigno (2008). In response to this new exchange rate dynamic, agents would now choose to hold less domestic and more foreign bonds. As a consequence of this change in the international portfolio position, domestic monetary shocks can trigger positive valuation effects.

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negative.

Note that in our set-up, the weight on output stabilisation, ϕ_y, only needs to be slightly positive to overturn the results obtained under a pure inflation-targeting Taylor rule. So the case of a Taylor rule with a small weight on output stabilisation has similar implications for the international transmission of monetary policy as the results obtained under money-growth rule. Therefore our results suggests that the relevant classification of policy rules distinguishes between those that focus solely on inflation stabilisation and those that do not.

3.5 Conclusion

Over recent decades, the external balance sheets of countries have grown in size. At the same time, the currency denomination of those balance sheets has changed. While some countries have most of their foreign debt liabilities denominated in domestic currency, in others most debt liabilities are in foreign currency (Lane and Shambaugh (2010a)). In our model, the optimal portfolio currency shares are directly linked to exchange rate dynamics. Whether the domestic currency depreciates or appreciates in periods of low domestic income determines whether investors take a long or a short position in the foreign currency. The key insight of our analysis is that different monetary regimes change the cyclical properties of the exchange rate which then affects the hedging characteristic of domestic over foreign bonds. Specifically, if the central bank is assumed to target money growth, or follows a Taylor rule which puts weight on output stabilisation, agents would choose a portfolio that is short in domestic bonds and long in foreign bonds. This is because, with such rules, any adverse real country-specific shocks will be associated with a nominal depreciation of the domestic currency. Holding domestic currency denominated assets is therefore a bad hedge. On the other hand, when the central bank conducts policy through an inflation-targeting Taylor-type rule, the same adverse shock will trigger a nominal domestic currency appreciation. So holding domestic currency denominated assets is a good hedge and agents will choose an optimal portfolio that is overweight in home bonds.

We also show how the endogenous portfolio choice determines the cross-border transmission of monetary policy shocks via a valuation channel. In the case of money-growth rules, agents are long in foreign bonds and monetary policy shocks then trigger international valuation effects that are *beggar-thy-neighbour*. In contrast, monetary policy shocks with a Taylor rule cause international valuation effects that are *beggar-thy-self* since agents are holding a portfolio that is short in foreign bonds.

Our results on the optimal portfolio under different monetary policy regimes appear in line with the findings of Clarida and Waldman (2007) and Lane and Shambaugh (2010b). When put together, the empirical evidences from these papers tend to suggest that inflation-targeting countries are inclined to hold relatively more foreign debt liabilities denominated in foreign currency than non inflation targeting countries. But we believe that a clear look at the data on the link between policy regimes, hedging motives and portfolio positions would be an interesting avenue for future research.

Appendix to Chapter 3

3.5.1 Deriving conditions determining optimal shares

We use the first-order approximation to the model equations to evaluate the portfolio orthogonality condition coming from the second-order approximation to the portfolio choice equations.

The first-order approximation to the accumulation of net foreign assets given by equation (3.9) in the text and its foreign counterpart can be written as follows:[36]

$$\widehat{NFA}_t = \frac{1}{\beta}\widehat{NFA}_{t-1} + \tilde{\alpha}(\hat{r}_{F,t} - \hat{r}_{H,t}) + \hat{P}_{H,t} - \hat{P}_t + \hat{Y}_{H,t} - \hat{C}_t, \tag{3.35}$$

$$\widehat{NFA}^*_t = \frac{1}{\beta}\widehat{NFA}^*_{t-1} - \tilde{\alpha}(\hat{r}_{F,t} - \hat{r}_{H,t}) + \hat{P}^*_{F,t} - \hat{P}^*_t + \hat{Y}^*_{F,t} - \hat{C}^*_t. \tag{3.36}$$

Taking the difference of two asset accumulation equations we get equation (3.15) in the text:

$$\widehat{NFA}^R_t = \frac{1}{\beta}\widehat{NFA}^R_{t-1} + 2\tilde{\alpha}\hat{r}_{x,t} + \hat{Y}^R_t - \hat{C}^R_t + \hat{Q}_t,$$

where $\widehat{NFA}^R_t = \widehat{NFA}_t - \widehat{NFA}^*_t$, $r_{x,t} = r_{F,t} - r_{H,t}$ (excess return on foreign bonds expressed in home currency), $\hat{Y}^R_t = \hat{P}_{H,t} + \hat{Y}_{H,t} - (\hat{P}^*_{F,t} + \hat{S}_t + \hat{Y}^*_{F,t})$ (relative non-financial income adjusted for the terms of trade), $\hat{C}^R_t = \hat{C}_t - \hat{C}^*_t$ and $\hat{Q}_t = \hat{P}^*_t + \hat{S}_t - \hat{P}_t$.

The second dynamic equation we have is the combined Euler equations, i.e. equation (3.16) in the text:

$$E_t\hat{C}^R_{t+1} - \hat{C}^R_t = \frac{1}{\rho}\left(E_t\hat{Q}_{t+1} - \hat{Q}_t\right).$$

We can solve equations (3.15) and (3.16) forward for NFA^R_t and C^R_t as a function of $NFA_{t-1}, \tilde{\alpha}\hat{r}_{x,t}, \hat{Y}^R_t$ and $\hat{Q}_t$:

$$\widehat{NFA}^R_t = \widehat{NFA}^R_{t-1} - E_t\sum_{j=0}^{\infty}\beta^{j+1}\Delta\hat{Y}^R_{t+1+j} - \left(1 - \frac{1}{\rho}\right)E_t\sum_{j=0}^{\infty}\beta^{j+1}\Delta\hat{Q}^R_{t+1+j} + 2\beta\tilde{\alpha}\hat{r}_{x,t}, \tag{3.37}$$

[36] Note that in Section 3.2 of the text we defined $\alpha = \alpha_F$ and mentioned that we can define all other portfolio shares in terms of α using the definition of NFA and the asset market clearing conditions.

$$\hat{C}_t^R = \frac{1-\beta}{\beta}\widehat{NFA}_{t-1}^R + E_t\sum_{j=0}^{\infty}\beta^{j+1}\Delta\hat{Y}_{t+1+j}^R + \left(1-\frac{1}{\rho}\right)E_t\sum_{j=0}^{\infty}\beta^{j+1}\Delta\hat{Q}_{t+1+j}^R$$
$$+2(1-\beta)\tilde{\alpha}\hat{r}_{x,t} + \hat{Y}_t^R + \hat{Q}_t. \quad (3.38)$$

We get the equation for $\Delta\hat{C}_{t+1}^R - \Delta\frac{\hat{Q}_{t+1}}{\rho}$ given in equation (3.17) in the text by using (3.37) and (3.38):

$$\begin{aligned}\Delta\hat{C}_{t+1}^R - \Delta\frac{\hat{Q}_{t+1}}{\rho} &= 2(1-\beta)\tilde{\alpha}\hat{r}_{x,t+1} \\ &+E_{t+1}\sum_{j=0}^{\infty}\beta^j\Delta\hat{Y}_{t+1+j}^R - E_t\sum_{j=0}^{\infty}\beta^j\Delta\hat{Y}_{t+1+j}^R \\ &+\left(1-\frac{1}{\rho}\right)\left(E_{t+1}\sum_{j=0}^{\infty}\beta^j\Delta\hat{Q}_{t+1+j}^R - E_t\sum_{j=0}^{\infty}\beta^j\Delta\hat{Q}_{t+1+j}^R\right).\end{aligned}$$

3.5.2 Equations of the sticky price model

Here, we present the important equations of the sticky price model discussed in Section 3.4.3 in the main text.

Consumers

The representative agent in the home economy now also gets disutility from work as well as utility from consumption and real money holdings. So the expected present discounted value of utility given in equations (3.1) and (3.2) changes as follows:

$$U_t = E_t\sum_{s=t}^{\infty}\delta_s u\left(C_s, \frac{M_s}{P_s}, L_s\right), \quad (3.39)$$

with

$$u\left(C_s, \frac{M_s}{P_s}, L_s\right) = \frac{C_s^{1-\rho}}{1-\rho} + \chi\log\left(\frac{M_s}{P_s}\right) - K\frac{L_s^{1+\varpi}}{1+\varpi}, \quad (3.40)$$

where $\varpi > 0$, $K > 0$ and L_s is the hours worked.

The consumption index and the associated price index are given in equations (3.5) and (3.6) with the elasticity of substitution between home and foreign goods denoted by θ. C_H and C_F are now composite indices of domestic and foreign varieties. The elasticity of substitution between varieties produced within a country is $\phi > 1$.

The budget constraint and the definition of asset returns are as before. Note that total income that accrues to the home agent, $P_{H,t}Y_t$ in equation (3.7), is now the sum of profits and wage income, i.e. $P_{H,t}Y_t = P_t\Pi_t + W_tL_t$, where Π denotes real profits of home firms and W_t is nominal wage.

Firms

Firms have market power over the supply of their products. Each firm produces a single variety of the home consumption good according to the production technology:

$$Y_t = A_t L_t,$$

where A_t is a common stochastic productivity shock that follows an AR(1) process.

$$\log A_t = \zeta \log A_{t-1} + u_t,\ 0 \leq \zeta \leq 1,\ u_t \text{is i.i.d with} E_{t-1}[u_t] = 0, Var_{t-1}[u_t] = \sigma_u^2.$$

Prices change at random intervals à la Calvo. At each period a fraction $\kappa \in [0, 1)$ of randomly selected firms cannot change their prices. The remaining $1 - \kappa$ fraction of firms chooses prices optimally to maximise expected discounted value of future profits.

Each firm in home country chooses the optimal home and foreign market price, $\tilde{P}_{H,t}$ and $\tilde{P}^*_{H,t}$, respectively to maximise expected value of discounted future profits from selling at home and abroad:

$$E_t \sum_{j=0}^{\infty} \kappa^j \Psi_{t+j} \left[\tilde{P}_{H,t} \tilde{Y}_{H,t+j} + S_{t+j} \tilde{P}^*_{H,t} \tilde{Y}^*_{H,t+j} - \frac{W_{t+j}}{A_{t+j}} (\tilde{Y}_{H,t+j} + \tilde{Y}^*_{H,t+j}) \right] \tag{3.41}$$

where Ψ is the stochastic discount factor and $\tilde{Y}_{H,t}$ and $\tilde{Y}^*_{H,t}$ are the demand for home good from the home market and the demand for home good from the foreign market, respectively and are given by the following expressions:

$$\tilde{Y}_{H,t+j} = \left(\frac{\tilde{P}_{H,t}}{P_{H,t+j}} \right)^{-\phi} \nu \left(\frac{P_{H,t+j}}{P_{t+j}} \right)^{-\theta} C_{t+j}, \tag{3.42}$$

$$\tilde{Y}^*_{H,t+j} = \left(\frac{\tilde{P}^*_{H,t}}{P^*_{H,t+j}} \right)^{-\phi} (1-\nu) \left(\frac{P^*_{H,t+j}}{P^*_{t+j}} \right)^{-\theta} C^*_{t+j}. \tag{3.43}$$

The optimal price for the home good sold in the home market is given by:

$$\tilde{P}_{H,t} = \frac{\phi}{\phi - 1} \frac{E_t \sum_{j=0}^{\infty} \kappa^j \Psi_{t+j} \frac{W_{t+j}}{A_{t+j}} \tilde{Y}_{H,t+j}}{E_t \sum_{j=0}^{\infty} \kappa^j \Psi_{t+j} \tilde{Y}_{H,t+j}}. \tag{3.44}$$

The foreign currency price can be set in two ways depending on the currency in which home exports are invoiced. Under producer currency pricing (PCP), exporters in both countries set export prices in their own currency. Home firms maximise equation (3.41) with respect to $S_{t+j}\tilde{P}^*_{H,t}$ and the optimal export price is given by:

$$\tilde{P}^*_{H,t}\underset{pcp}{|} = \frac{\tilde{P}_{H,t}}{S_t}. \tag{3.45}$$

On the other hand, under local currency pricing (LCP), export prices are set in the destination market's currency. Maximising equation (3.41) with respect to $\tilde{P}^*_{H,t}$ gives the following equation for optimal export price under LCP:

$$\tilde{P}^*_{H,t}\underset{lcp}{|} = \frac{\phi}{\phi-1}\frac{E_t\sum_{j=0}^{\infty}\kappa^j\Psi_{t+j}\frac{W_{t+j}}{A_{t+j}}\tilde{Y}^*_{H,t+j}}{E_t\sum_{j=0}^{\infty}\kappa^j\Psi_{t+j}S_{t+j}\tilde{Y}^*_{H,t+j}}. \tag{3.46}$$

Under Calvo price-setting, the price indices $P_{H,t}$ and $P^*_{H,t}$ can be written as follows:

$$\begin{aligned} P_{H,t} &= \left[(1-\kappa)\tilde{P}_{H,t}^{1-\phi} + \kappa P_{H,t-1}^{1-\phi}\right]^{\frac{1}{1-\phi}}, \\ P^*_{H,t} &= \left[(1-\kappa)\tilde{P}_{H,t}^{*1-\phi} + \kappa P_{H,t-1}^{*1-\phi}\right]^{\frac{1}{1-\phi}}. \end{aligned}$$

Optimal prices for the foreign goods sold in the domestic market and abroad, $\tilde{P}^*_{F,t}$ and $\tilde{P}_{F,t}$ as well as $P^*_{F,t}$ and $P_{F,t}$ are derived in a similar way.

Policy rules are as described in Section 3.2.2 and equilibrium conditions are as in Section 3.2.3. In addition to the Euler equations and the money demand equation given by (3.10) and (3.11), there is a new household first-order condition that determines optimal labour supply:

$$KL_t^{\varpi} = C_t^{-\rho}\frac{W_t}{P_t}. \tag{3.47}$$

The law of one price no longer holds in the sticky price model with LCP, but it continues to hold if exports are priced according to PCP.

Chapter 4

International Transmission of Uncertainty Shocks: The Role of Financial Market Integration

4.1 Introduction

Time-varying volatility is an important feature of macroeconomic data. Recent works have shown that models which allow for stochastic volatility on structural shocks fit the data better than models with homoscedastic structural shocks.[1] Hence, it is important to incorporate uncertainty shocks in macro models and understand the way they affect agents' economic decisions and the economy as a whole. This chapter focuses on the cross-border effects of country-specific uncertainty shocks and investigates the role of financial market integration.

The extent of financial market integration is important for the international transmission of uncertainty shocks because it affects the extent of precautionary savings, which in turn affects the net foreign asset accumulation. In a world with complete financial markets, there is no precautionary saving motive because agents can pool all risks. In the other extreme of financial autarky, there is no role for precautionary savings either because agents cannot buy foreign bonds to increase their savings in response to increased uncertainty.[2] For precautionary savings to increase in response to higher uncertainty, financial market structure should be such that agents can borrow

[1]See Fernandez-Villaverde and Rubio-Ramirez (2007) and Justiniano and Primiceri (2008). Fernandez-Villaverde and Rubio-Ramirez (2010) offer a comprehensive review of the literature on the estimation and modelling of time varying volatility.

[2] We abstract from capital accumulation. If agents can invest in domestic capital stock, there will be precautionary savings even if there is no access to international financial markets.

and lend internationally but cannot insure perfectly against country-specific shocks.

We use a two-country endowment model with non-tradable goods in which agents have access to a single international bond subject to some portfolio adjustment costs that represent the trading frictions across countries.[3] We allow for recursive preferences and specify stochastic volatility in the endowments of tradable and non-tradable goods. To analyse the importance of financial market linkages for the international transmission of uncertainty shocks, we compare the implications of this model to the complete markets model. We also investigate the effects of varying the scale of portfolio adjustment costs which we interpret as varying the degree of financial market integration. When portfolio adjustment costs go to infinity the model corresponds to the financial autarky model.

The baseline model with international trade in a single bond is able to generate an improvement in the net foreign asset position, a fall in real interest rate differential and a depreciation in real exchange rate following an increase the volatility of domestic tradable endowment in line with the empirical evidence provided by Fogli and Perri (2006, 2010), Guerrón-Quintana (2009) and Benigno et al. (2011) regarding the effects of higher uncertainty.[4] This model can also account for the Backus and Smith (1993) and Kollmann (1995) evidence on the negative correlation between relative consumption and real exchange rate conditional on both level and volatility shocks. However, it cannot generate meaningful deviations from the uncovered interest rate parity (UIP) condition because in this model real exchange rate appreciates in good states where consumption is higher, which helps to account for the Backus-Smith-Kollmann evidence but has a counterfactual implication for the foreign exchange risk premium and for the relation between the real interest rate differential and expected depreciation rate.

The complete market model is not able to match the empirical observations related to the responses of net foreign assets and real exchange rate to increased uncertainty but it can generate an increase in the foreign exchange premium following an increase in tradable income risk, because it implies that real exchange rate depreciates in good states where consumption is higher.

[3]This model is capable of generating a low degree of risk sharing conditional on level shocks as shown in Benigno and Thoenissen (2008) and Corsetti et al. (2008) and discussed in Chapter 1.

[4]Fogli and Perri (2006, 2010) provide evidence for the U.S. and a set of OECD countries that an increase in the macroeconomic uncertainty of a country with respect to the rest of the world leads to higher net foreign asset accumulation in that country due to higher precautionary savings. Guerrón-Quintana (2009) and Benigno et al. (2011) estimate VAR models for the U.S. economy with respect to a number of developed economies and find evidence that an increase in the uncertainty about the U.S. economy leads to a real depreciation of the U.S. dollar and a fall in the U.S. interest rate.

We find that increasing the degree of financial market imperfections brings the model close to the complete market model in terms of its implications for the transmission of uncertainty shocks. When there are high portfolio adjustment costs, and no other means to save, agents cannot increase precautionary savings following an increase in income uncertainty, hence consumption and real exchange rate responses to uncertainty shocks remain limited as in the case of complete insurance. This is an interesting result as it implies that international risk sharing conditional on uncertainty shocks might be higher under financial autarky compared to the single bond economy.

This chapter is closely related to the literature that studies the open economy implications of time-varying volatility. Theoretical works that analyse the impact of uncertainty on external imbalances, like Ghosh and Ostry (1997), Fogli and Perri (2006, 2010) and Fernandez-Villaverde et al. (2009) consider one-good models and stay silent on the behaviour of international relative prices in response to increased precautionary savings following uncertainty shocks. On the other hand, models that are built to understand the dynamics of exchange rates and excess returns in the presence of stochastic volatility, like Gavazzoni (2009) and Benigno et al. (2011) assume complete financial markets and abstract from the adjustment in current account and net external positions, which in turn have a bearing on the determination of international relative prices.[5] The incomplete market model proposed in this chapter provides a useful framework to think about the joint behaviour of the net foreign asset position, exchange rate and the currency risk premium in the face of uncertainty shocks.

This chapter is also related to the literature that studies the effects of financial market structure for international business cycles. The main finding of this literature is that a standard incomplete market set-up which allows for international trade in a single non-contingent bond brings the equilibrium close to the one that arises under complete markets conditional on stationary level shocks (Cole and Obstfeld, 1991, Baxter and Crucini, 1995, Heathcote and Perri, 2002 and Chari et al., 2002). The results presented in this chapter, on the other hand, suggest that the single bond and complete market set-ups have completely different implications for the international transmission of stationary volatility shocks.

The chapter is organised as follows. Section 4.2 describes the two-country two-

[5]Indeed, most of the papers that aim to account for deviations from the UIP relying on time-varying foreign exchange premium such as Verdelhan (2010), Bansal and Shaliastovich (2009), Colacito and Croce (2010), Backus et al. (2001, 2010), assume full financial integration and treat exchange rates as a shadow price.

sector endowment model with recursive preferences and stochastic volatility. Section 4.3 presents the impulse responses to shocks to the level and volatility of endowments under alternative asset markets and discusses the results. Section 4.4 concludes.

4.2 A two-country two-sector endowment model

There is a home and a foreign country, each endowed with a tradable and a non-tradable good. Endowment processes in each country are subject to stochastic volatility. Households maximise utility over infinite horizon under different asset market structures. In the baseline model, international asset trade is restricted to a single non-contingent bond. We compare the implications of this model with that of the complete markets. The structure of the model is related to the production economies described in Benigno and Thoenissen (2008) and Corsetti et al. (2008).

4.2.1 Preferences and good markets

Preferences are recursive as in Epstein and Zin (1989) and Weil (1990):

$$V_t = \left[C_t^{1-\rho} + \beta \left(E_t (V_{t+1})^{1-\psi} \right)^{\frac{1-\rho}{1-\psi}} \right]^{\frac{1}{1-\rho}}, \tag{4.1}$$

where C is consumption, β is the discount factor. ρ is the inverse of the intertemporal elasticity of substitution and ψ is risk aversion. These preferences nest the expected utility framework for $\rho = \psi$. For $\rho < \psi$, agents prefer an early resolution of uncertainty. C represents a consumption index defined over tradable C_T and non-tradable C_N consumption:

$$C_t = \left[\gamma^{\frac{1}{\kappa}} C_{T,t}^{\frac{\kappa-1}{\kappa}} + (1-\gamma)^{\frac{1}{\kappa}} C_{N,t}^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}}, \tag{4.2}$$

where κ is the elasticity of intratemporal substitution between C_N and C_T and γ is the weight that the households assign to tradable consumption. The tradable component of the consumption index is in turn a CES aggregate of home and foreign tradable consumption goods, C_H and C_F:

$$C_{T,t} = \left[\nu^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-\nu)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{4.3}$$

where θ is the elasticity of intratemporal substitution between C_H and C_F and ν is the weight that households assign to domestic tradable consumption. Preferences are biased towards domestic goods, i.e. $\nu > \frac{1}{2}$. The consumption price index (CPI), which is defined as the minimum expenditure required to purchase one unit of aggregate consumption is given by:

$$P_t = \left[\gamma P_{T,t}^{1-\kappa} + (1-\gamma) P_{N,t}^{1-\kappa}\right]^{\frac{1}{1-\kappa}}, \tag{4.4}$$

for the home agent. Meanwhile, the traded goods price index, which is defined as the minimum expenditure required to purchase one unit of a traded good is defined as:

$$P_{T,t} = \left[\nu P_{H,t}^{1-\theta} + (1-\nu) P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{4.5}$$

Consumption and price indices for the foreign country are similarly defined and are denoted with an asterisk. The law of one price holds for home and foreign tradable goods, i.e. $P^*_{H,t} = P_{H,t}$, and $P_{F,t} = P^*_{F,t}$. The presence of non-tradable goods and home bias in tradables consumption leads to deviations from purchasing power parity. Real exchange rate is defined as $Q = \frac{P^*}{P}$.

Good market clearing requires $Y_{H,t} = C_{H,t} + C^*_{H,t}$, $Y^*_{F,t} = C^*_{F,t} + C_{F,t}$, $Y_{N,t} = C_{N.t}$ and $Y^*_{N,t} = C^*_{N,t}$ where C_H and C_F (C^*_F and C^*_H) should satisfy the intratemporal optimisation decisions of home (foreign) households.

Endowments have the following stochastic processes:

$$\begin{aligned}
\log Y_{H,t+1} &= (1-\delta_T)\log \bar{Y}_H + \delta_T \log Y_{H,t} + u_{H,t}\varepsilon_{H,t+1}, \\
\log Y^*_{F,t+1} &= (1-\delta_T)\log \bar{Y}^*_F + \delta_T \log Y^*_{F,t} + u_{F,t}\varepsilon^*_{F,t+1}, \\
\log Y_{N,t+1} &= (1-\delta_N)\log \bar{Y}_N + \delta_N \log Y_{N,t} + u_{N,t}\varepsilon_{N,t+1}, \\
\log Y^*_{N,t+1} &= (1-\delta_N)\log \bar{Y}^*_N + \delta_N \log Y^*_{N,t} + u^*_{N,t}\varepsilon^*_{N,t+1},
\end{aligned} \tag{4.6}$$

where $0 \leq \delta_T < 1$, $0 \leq \delta_N < 1$, $\varepsilon_{H,t}, \varepsilon^*_{F,t}, \varepsilon_{N,t}, \varepsilon^*_{N,t}$ are i.i.d. white-noise.

Time-varying volatility is modelled through linear processes for the variances following Benigno et al. (2011):

$$\begin{aligned}
u^2_{H,t+1} &= (1-\rho_T)\sigma^2_T + \rho_T u^2_{H,t} + \sigma^2_\zeta \zeta_{H,t+1} \\
u^2_{F,t+1} &= (1-\rho_T)\sigma^2_T + \rho^*_T u^2_{F,t} + \sigma^2_\zeta \zeta_{F,t+1} \\
u^2_{N,t+1} &= (1-\rho_N)\sigma^2_N + \rho_N u^2_{N,t} + \sigma^2_\zeta \zeta_{N,t+1} \\
u^2_{N^*,t+1} &= (1-\rho_N)\sigma^2_N + \rho^*_N u^{*2}_{N,t} + \sigma^2_\zeta \zeta^*_{N,t+1}
\end{aligned} \tag{4.7}$$

in which all the ζ_t are i.i.d white-noise processes and $0 \leq \rho_T, \rho_N \leq 1$ with $\sigma^2_T, \sigma^2_N, \sigma^2_\zeta > 0$.

4.2.2 Asset markets

Non-contingent bond economy

Home agents can trade two real bonds, one that pays in units of home consumption basket and the other in units of foreign, each with one-period maturity. The flow budget constraint of the representative home country consumer can be written as:

$$B_{H,t}+Q_tB_{F,t} = R_{H,t}B_{H,t-1}+R^*_{F,t}Q_tB_{F,t-1}+\frac{P_{H,t}}{P_t}Y_{H,t}+\frac{P_{N,t}}{P_t}Y_{N,t}-C_t-\frac{\Phi}{2}\left(Q_tB_{F,t}-\bar{b}\right)^2, \tag{4.8}$$

where $B_{H,t}$ and $B_{F,t}$ are home agent's holding of one-period real bonds denominated in home and foreign consumption units, respectively and $R_{H,t}$ and $R^*_{F,t}$ denote the gross real interest rates on these bonds. There is a quadratic cost of changing the foreign asset position with respect to the steady-state value of foreign assets, denoted by $\bar{b}$. Specifying convex portfolio adjustment costs is one way of ensuring stationarity in incomplete market models as discussed by Schmitt-Grohe and Uribe (2003) and Bodenstein (2011) for small and large open economies, respectively. The scale of this portfolio adjustment cost is determined by a non-negative parameter, Φ, which is often interpreted as a measure of the degree of financial market integration as in Sutherland (1996) and Benigno (2009). The higher the value of Φ, the less integrated is the home country with international financial markets.

Foreign agents can only trade foreign bonds by assumption and they do not face any adjustment costs. Portfolio adjustment costs incurred by home agents are transferred to foreign agents:

$$B^*_{F,t} = R^*_{F,t}B^*_{F,t-1}+\frac{P^*_{F,t}}{P^*_t}Y_{F,t}+\frac{P^*_{N,t}}{P^*_t}Y^*_{N,t}-C^*_t+TR^*_t,$$

where $B^*_{F,t}$ denotes foreign agent's holding of foreign riskless bond and $TR^*_t = \frac{\Phi}{2}\left(Q_tB_{F,t}-\bar{b}\right)^2$.

Given that bonds are in net zero supply and foreign agents cannot hold any domestic bonds, market clearing requires $B_{H,t} = 0$ and $B_{F,t} + B^*_{F,t} = 0$. Defining $NFA_t \equiv Q_tB_{F,t}$ and $NFA^*_t \equiv B^*_{F,t}$, asset market clearing implies $\frac{NFA_t}{Q_t} + NFA^*_t = 0$. Hence, we can just focus on the flow budget constraint of home agents which can be written as:

$$NFA_t = R_{F,t}NFA_{t-1}+\frac{P_{H,t}}{P_t}Y_{H,t}+\frac{P_{N,t}}{P_t}Y_{N,t}-C_t-\frac{\Phi}{2}\left(NFA_t-\bar{b}\right)^2, \tag{4.9}$$

where $R_{F,t}$ is the foreign real interest rate in units of home consumption good, defined as $R_{F,t} \equiv 1 + r_{F,t} \equiv R^*_{F,t}\frac{Q_t}{Q_{t-1}}$. Imposing the market clearing conditions, i.e. $Y_N = C_N$ and $Y_H = C_H + C^*_H$ and using the definition of total consumption expenditure, i.e. $C \equiv \frac{P_H}{P}C_H + \frac{P_F}{P}C_F + \frac{P_N}{P}C_N$, gives the familiar current account equation for the home country:

$$CA_t \equiv \Delta NFA_t = r_{F,t}NFA_{t-1} + \underbrace{\frac{P_{H,t}}{P_t}C^*_{H,t} - \frac{P_{F,t}}{P_t}C_{F,t}}_{NEX_t}.$$

Home agent's optimality conditions with respect to the home and foreign bonds can be written in terms of the stochastic discount factors[6],

$$E_t M_{t+1} R_{H,t+1} = 1, \tag{4.10}$$

$$E_t M_{t+1} R^*_{F,t+1}\frac{Q_{t+1}}{Q_t} = 1 + \Phi(NFA_t - \bar{b}), \tag{4.11}$$

where

$$M_{t+1} \equiv \beta \left(\frac{V_{t+1}}{\left(E_t V_{t+1}^{1-\psi}\right)^{\frac{1}{1-\psi}}} \right)^{\rho-\psi} \frac{C_{t+1}^{-\rho}}{C_t^{-\rho}}. \tag{4.12}$$

As explained in Schmitt-Grohe and Uribe (2003), the Euler equation of the home agent with respect to the foreign bond states that to save an additional unit in foreign bonds, home agents should give up one unit of consumption plus the marginal cost of adjusting their portfolio, $\Phi(NFA_t - \bar{b})$.

Foreign agent's optimality condition with respect to foreign bonds is given by:

$$E_t M^*_{t+1} R^*_{F,t+1} = 1, \tag{4.13}$$

where

$$M^*_{t+1} \equiv \beta \left(\frac{V^*_{t+1}}{\left(E_t V_{t+1}^{*1-\psi}\right)^{\frac{1}{1-\psi}}} \right)^{\rho-\psi} \frac{C_{t+1}^{*-\rho}}{C_t^{*-\rho}}. \tag{4.14}$$

For $\rho = \psi$, equation (4.1) gives the expected utility specification, where present

[6]Derivation of first order conditions with recursive preferences and portfolio adjustment costs is shown in the appendix.

discounted value of life-time utility is given by $U_t \equiv \frac{V_t^{1-\rho}}{1-\rho}$.

$$\begin{aligned} V_t^{1-\rho} &= C_t^{1-\rho} + \beta E_t V_{t+1}^{1-\rho}, \\ U_t &\equiv \frac{V_t^{1-\rho}}{1-\rho} = \sum_{j=0}^{\infty} \beta^j \frac{C_{t+j}^{1-\rho}}{1-\rho}. \end{aligned}$$

Under expected utility, stochastic discount factor gives the ratio of the marginal utilities of consumption across time $t+1$ and t. Letting $M_{t+1}^{EU} \equiv \beta \frac{\frac{\partial U_{t+1}}{\partial C_{t+1}}}{\frac{\partial U_t}{\partial C_t}} = \beta \frac{C_{t+1}^{-\rho}}{C_t^{-\rho}}$ we can express M_{t+1} in terms of this expected utility stochastic discount factor and the "intertemporal uncertainty factor", g_{t+1} :

$$M_{t+1} \equiv M_{t+1}^{EU} \left(\frac{V_{t+1}}{\left(E_t V_{t+1}^{1-\psi} \right)^{\frac{1}{1-\psi}}} \right)^{\rho-\psi} = M_{t+1}^{EU} g_{t+1}. \tag{4.15}$$

g_{t+1} denotes the ratio of realised utility, V_{t+1}, to the risk-adjusted expected utility, $\left(E_t V_{t+1}^{1-\psi} \right)^{\frac{1}{1-\psi}}$, where risk adjustment is governed by the risk aversion parameter ψ. For $\rho < \psi$, agents prefer an early resolution of uncertainty. This means that if utility turns out to be lower than its risk-adjusted expected value, g_{t+1} increases, which would increase agents appetite for wealth, M_{t+1}, for a given M_{t+1}^{EU}.

A similar expression can be written for the foreign country:

$$M_{t+1}^{*} \equiv M_{t+1}^{*EU} \left(\frac{V_{t+1}^{*}}{\left(E_t V_{t+1}^{*1-\psi} \right)^{\frac{1}{1-\psi}}} \right)^{\rho-\psi} = M_{t+1}^{*EU} g_{t+1}^{*}, \tag{4.16}$$

where $M_{t+1}^{*EU} \equiv \beta \frac{\frac{\partial U_{t+1}^{*}}{\partial C_{t+1}^{*}}}{\frac{\partial U_t^{*}}{\partial C_t^{*}}} = \beta \frac{C_{t+1}^{*-\rho}}{C_t^{*-\rho}}$.

A first-order approximation to equations (4.11) and (4.13) shows the effect of portfolio adjustment costs on the international risk sharing condition:

$$E_t \hat{M}_{t+1}^{*} - E_t \hat{M}_{t+1} - E_t \Delta \hat{Q}_{t+1} = \Phi \widehat{NFA}_t + O\left(\epsilon^2\right). \tag{4.17}$$

where variables with hats denote log deviations from steady-state, i.e. $\hat{X} = \log(X/\bar{X})$ for any variable X unless stated otherwise, and $O\left(\epsilon^2\right)$ is a residual which contains all terms of order two and higher.

In a first-order approximation, intertemporal utility risk does not affect the risk sharing condition as the expected value of the intertemporal uncertainty factors, $E_t g_{t+1}$ and $E_t g^*_{t+1}$, are equal to zero.[7] To see this, consider a first-order approximation to (4.12) and (4.14) gives:

$$\begin{aligned}
\hat{M}_{t+1} &= \hat{g}_{t+1} + \hat{M}^{EU}_{t+1} = (\rho - \psi)(\hat{V}_{t+1} - E_t\hat{V}_{t+1}) - \rho\Delta\hat{C}_{t+1}, \\
\hat{M}^*_{t+1} &= \hat{g}^*_{t+1} + \hat{M}^{*EU}_{t+1} = (\rho - \psi)(\hat{V}^*_{t+1} - E_t\hat{V}^*_{t+1}) - \rho\Delta\hat{C}^*_{t+1}.
\end{aligned}$$

Taking expectations and substituting in equation (4.17) gives the familiar risk sharing condition which links the expected growth rate of relative marginal utilities of consumption across countries to the real exchange rate depreciation:

$$\begin{aligned}
E_t\hat{M}^{*EU}_{t+1} - E_t\hat{M}^{EU}_{t+1} - E_t\Delta\hat{Q}_{t+1} &= \Phi\widehat{NFA}_t + O\left(\epsilon^2\right), \qquad (4.18) \\
E_t\left(\Delta\hat{C}_{t+1} - \Delta\hat{C}^*_{t+1}\right) - \frac{1}{\rho}E_t\Delta\hat{Q}_{t+1} &= \Phi\widehat{NFA}_t + O\left(\epsilon^2\right).
\end{aligned}$$

The presence of portfolio adjustment costs can potentially weaken the link between relative marginal utilities of consumption and real exchange rate depreciation, though it is generally assumed to be quite small to have an important effect. (Benigno and Thoenissen, 2008).

Portfolio adjustment costs also have implications for the UIP relationship. Foreign bonds pay an excess return to compensate for the cost of changing the foreign asset position. Hence, there will be deviations from the UIP even under certainty equivalance. Taking a first-order approximation to equations (4.10) and (4.11) illustrates this point:

$$\hat{R}^*_{F,t+1} - \hat{R}_{H,t+1} + \Delta E_t\hat{Q}_{t+1} = \Phi\widehat{NFA}_t + O\left(\epsilon^2\right). \qquad (4.19)$$

Foreign exchange risk premium comes into the picture in a second-order approximation to home country Euler equations[8]:

$$\hat{R}^*_{F,t+1} - \hat{R}_{H,t+1} + E_t\Delta\hat{Q}_{t+1} = \Phi\widehat{NFA}_t \underbrace{- \frac{Var_t\left(\Delta\hat{Q}_{t+1}\right)}{2} - Cov_t(\hat{M}_{t+1}, \Delta\hat{Q}_{t+1})}_{\equiv fxp^H_t} + O\left(\epsilon^3\right). \qquad (4.20)$$

The first term in fxp^H_t is due to Jensen's inequality while the second term states that foreign bonds should command a premium if home bond provides a better hedge, i.e. home currency appreciates in real terms ($\Delta Q_{t+1} \downarrow$) in states of the nature when home

[7] In an incomplete market model, intertemporal uncertainty factors, g_{t+1} and g^*_{t+1}, only enter as second or higher-order terms in higher order approximations of the risk sharing condition.

[8] See appendix for the derivation.

agent's risk appetite is higher ($\hat{M}_{t+1} \uparrow$).

Equations (4.18) and (4.19) show that deviations from efficient risk sharing and the UIP are closely related each other through portfolio adjustment costs. Later, we analyse whether this link also arises endogenously in a higher solution to the model: Is the foreign exchange risk premium more volatile when there is a larger gap between the expected relative marginal utilities and real exchange rate depreciation? Is it easier to account for the UIP puzzle in a model that is consistent with Backus-Smith-Kollmann evidence?

Complete markets

When markets are complete, agents in each country can trade in a full set of state-contingent assets, which implies that the following risk sharing condition holds in terms of the stochastic discount factors:

$$M^*_{t+1} = \frac{Q_{t+1}}{Q_t} M_{t+1}. \tag{4.21}$$

Substituting the definitions for M^*_{t+1} and M_{t+1} using (4.15) and (4.16) gives:

$$\frac{\frac{\partial U^*_{t+1}}{\partial C^*_{t+1}}}{\frac{\partial U^*_t}{\partial C^*_t}} = \frac{Q_{t+1}}{Q_t} \frac{\frac{\partial U_{t+1}}{\partial C_{t+1}}}{\frac{\partial U_t}{\partial C_t}} \frac{g_{t+1}}{g^*_{t+1}}. \tag{4.22}$$

Under expected utility, i.e. $\rho = \psi$, $g_{t+1} = g^*_{t+1} = 1$ so that equation (4.22) boils down to the familiar risk sharing condition that equates the ratio of the growth rates of marginal utility of consumption across countries to the rate of real exchange rate depreciation.

For $\rho \gtrless \psi$, this relationship between the ratio of the marginal utility growth rates and the real exchange rate depreciation is tilted by the ratio of intertemporal uncertainty factors, $\frac{g_{t+1}}{g^*_{t+1}}$. Hence, provided that the surprises in utility realisations are sufficiently different across countries, i.e. $\frac{g_{t+1}}{g^*_{t+1}}$ is sufficiently different from 1, the complete market model with recursive preferences could generate a low correlation between relative consumption levels and the real exchange rate and help account for the Backus-Smith-Kollmann puzzle as suggested by Colacito and Croce (2010).

Because the relation between the marginal utilities of consumption and the real exchange rates is directly affected by the ratio $\frac{g_{t+1}}{g^*_{t+1}}$, choosing the initial allocation of state-contingent securities to equate the initial values of home and foreign marginal utilities of consumption, i.e. choosing initial wealth such that $\frac{\partial U_0}{\partial C_0} = \frac{\partial U^*_0}{\partial C^*_0} \frac{1}{Q_0}$, does not guarantee the equalisation of these marginal utilities in all states and dates due to the fluctuations in $\frac{g_{t+1}}{g^*_{t+1}}$.[9]

Let us define the deviation from complete risk sharing under expected utility as the 'marginal utility gap' and denote it by $MUgap_t \equiv \frac{\frac{\partial U_t}{\partial C_t}}{\frac{\partial U^*_t}{\partial C^*_t}\frac{1}{Q_t}}$. Then, we can re-write equation (4.22) to show how the marginal utility gap evolves over time under complete markets with recursive preferences:

$$MUgap_{t+1} = \frac{g^*_{t+1}}{g_{t+1}} MUgap_t. \tag{4.23}$$

A first-order approximation of equation (4.23) shows the role of recursive preferences for consumption risk sharing:

$$\begin{aligned} \Delta\hat{C}_{t+1} - \Delta\hat{C}^*_{t+1} - \frac{1}{\rho}\Delta\hat{Q}_{t+1} &= \hat{g}^*_{t+1} - \hat{g}_{t+1} \qquad (4.24) \\ &= (\psi-\rho)\left[(\hat{V}_{t+1} - E_t\hat{V}_{t+1}) - (\hat{V}^*_{t+1} - E_t\hat{V}^*_{t+1})\right] + O\left(\epsilon^2\right). \end{aligned}$$

Hence, under complete markets intertemporal uncertainty factors have a first-order effect on risk sharing. On the other hand, under incomplete markets, g_{t+1} and g^*_{t+1} only have only second and higher-order effects on the relationship between expected consumption growth rates and real exchange depreciation. This suggests that intertemporal uncertainty about utility might play a more important role for international consumption allocations under complete markets relative to incomplete markets.

There are no portfolio adjustment costs under complete markets. Foreign exchange risk premium can be calculated from the perspective of home,

$$fxp^H_t \equiv \hat{R}^*_{F,t+1} - \hat{R}_{H,t+1} + E_t\Delta\hat{Q}_{t+1} = -\frac{Var_t\left(\Delta\hat{Q}_{t+1}\right)}{2} - Cov_t(\hat{M}_{t+1}, \Delta\hat{Q}_{t+1}), \tag{4.25}$$

and foreign agents,

$$fxp^F_t \equiv \hat{R}^*_{F,t+1} - \hat{R}_{H,t+1} + E_t\Delta\hat{Q}_{t+1} = \frac{Var_t\left(\Delta\hat{Q}_{t+1}\right)}{2} - Cov_t(\hat{M}^*_{t+1}, \Delta\hat{Q}_{t+1}), \tag{4.26}$$

using home and foreign Euler equations with respect to the two real bonds.

Taking an average of the two equations (4.25) and (4.26), we get the familiar equation for the foreign exchange risk premium which holds in any financial market

[9]See the discussion in Benigno et al. (2011).

structure where both agents can trade in home and foreign bonds:

$$fxp_t = -Cov_t\left(\frac{\hat{M}_{t+1} + \hat{M}^*_{t+1}}{2}, \Delta\hat{Q}_{t+1}\right). \tag{4.27}$$

Equation (4.27) shows that foreign exchange risk premium is determined by how good a hedge is foreign exchange to stabilise fluctuations in home and foreign appetites for wealth. This covariance will be time-varying in a second order approximation provided that structural shocks are subject to stochastic volatility.

4.3 International transmission of volatility shocks

The main aim of the chapter is to understand how the structure of international financial markets affects the transmission of uncertainty shocks across the border. Does an incomplete market model do a better job in matching the empirical findings related to the dynamics of net foreign assets, real exchange rate and foreign exchange risk premium in response to volatility shocks? Is an economy with a lower degree of financial market integration affected more by uncertainty shocks? We first discuss the complete market case and then move on to the analysis of the single international bond economy with varying degrees of financial market integration.

4.3.1 Solving the model

Given the processes for exogenous state variables $\{Y_{H,t}, Y_{F,t}, Y_{N,t}, Y^*_{N,t}\}$ described by equations (4.6) and (4.7), the full solution to the model is given by the sequence $\{C_t$, C^*_t, $\frac{P_{H,t}}{P_t}, \frac{P^*_{F,t}}{P^*_t}, \frac{P_{N,t}}{P_t}, \frac{P^*_{N,t}}{P^*_t}, \frac{P_{T,t}}{P_t}, \frac{P^*_{T,t}}{P^*_t}$, Q_t, $R_{H,t}$, $R^*_{F,t}$, NFA_t, V_t, V^*_t M_{t+1}, $M^*_{t+1}\}$ which satisfies equations (4.1), (4.4), (4.5) and their foreign counterparts, and equations (4.9), (4.10), (4.11), (4.12), (4.13), (4.14) as well as the good market clearing conditions.

Methodology

We use the methodology developed by Benigno et al. (2010) to solve the model with stochastic volatility.[10] They argue that it is sufficient to consider a second-order accurate solution to the model to have a distinct and separate role for uncertainty shocks. Their methodology relies on the assumption that exogenous state variables follow conditionally-linear stochastic processes where the variance or standard deviation of

[10]We modify the codes they provide on their web-site to solve and simulate the model.

the shocks to the exogenous state variables follow linear stochastic processes.[11]

Because structural shocks have stochastic volatility, foreign exchange risk premium evaluated using the first-order accurate solutions to the components of the second moments in equations (4.20) and (4.27) are going to be time-varying. When shocks are homoscedastic, risk premia are time-varying only at third and higher order approximations. Stochastic volatility helps generate deviations from the UIP at a lower degree of approximation.

Calibration

The calibration of the model follows closely Chapters 1 and 2. The discount factor, β, is equal to 1/1.04, consistent with a steady-state real interest rate of 4% per year. Coefficient of relative risk aversion, ψ, is 5 as in Benigno et al. (2011) while the inverse of the elasticity of intertemporal substitution is 2. The parameters pertaining to the consumption basket are set in the following way. The share of tradable goods in final consumption, γ, is 0.55, while the share of home goods in tradable consumption, ν, is 0.72. The elasticity of substitution between traded and non-traded goods, κ, is 0.44, as suggested by Stockman and Tesar (1995). For $\rho = 2$, this implies that utility is non-separable between traded and non-traded goods. Given that $\kappa\rho < 1$, our benchmark calibration implies traded and non-traded goods are complements.

The persistence of tradable and non-tradable endowment shocks, δ_T and δ_N are set as 0.88 and 0.30 respectively, while the persistence of volatility shocks, ρ_T and ρ_N, are equalised to 0.5. We scale the variance of tradable and non-tradable shocks as in Benigno and Thoenissen (2008) such that $\sigma_T^2 = 0.00376$ and $\sigma_N^2 = 0.00051$.We set the standard deviation of volatility shocks using the data presented in Fogli and Perri (2006). They report a percentage standard deviation of 0.96 for the U.S. real GDP between 1984 and 2005. Hence we set $\sigma_\zeta^2 = 0.00096$.

4.3.2 Complete markets

Before moving on to the international transmission of uncertainty shocks under complete financial markets, we briefly discuss the effects of level shocks to illustrate the workings of the model.

Figure 4.1 shows the impulse responses to tradable and non-tradable endowment shocks computed for the baseline calibration using a second-order solution to the

[11] See Benigno et al. (2010, 2011) for a detailed description of the solution method.

model.[12] Efficiency requires that consumption should be higher in the country where it is cheaper to consume. Hence, in response to both tradable and non-tradable endowment shocks, relative consumption rises in favour of the home country, which benefits from the fall in the price of the good that becomes more abundant. Due to recursive preferences, marginal utilities of consumption converted into home consumption units are not equalised across countries, i.e. the marginal utility gap defined in equation (4.23) increases in response to both shocks, but relative consumption and real exchange rate are still positively correlated because the effect of intertemporal uncertainty is not strong enough.[13] There are no deviations from UIP because i) there are no financial frictions, ii) the covariance between stochastic discount factors and the real exchange rate depends only on the variance of endowments, which is constant conditional on level shocks.

The solid red (dashed black) lines in Figure 4.2 show the impulse responses to an increase in the *volatility* of tradable (non-tradable) sector endowments under complete markets. Qualitatively, responses to increased uncertainty about tradable and non-tradable good endowments are quite similar. Due to recursive utility and a preference for early resolution of risk, an increase in volatility is a bad surprise for both home and foreign agents' utilities, i.e. g_{t+1} and g^*_{t+1} defined in equations (4.15) and (4.16) both increase, leading to a rise in M_{t+1} and M^*_{t+1}.[14] Because home agents are affected more negatively by an increase in home uncertainty, efficiency requires that funds are transferred from foreign to home agents, leading to an increase in home consumption matched with a decrease in foreign consumption. Relatively higher consumption demand at home leads to higher home prices, which leads to a real appreciation in the terms of trade and the real exchange rate. Although these responses are small, these patterns contradict the empirical findings in the literature which show that uncertainty shocks have contractionary effects for the domestic economy and are associated with a real exchange rate depreciation.[15]

Foreign exchange risk premium, i.e. the expected excess return of investing in the foreign bond relative to home bond, increases with an increase in endowment volatility. To understand this, we need to look at the first-order accurate solutions

[12]Impulse responses show the log-deviations from steady-state in response to a one standard deviation shock normalised by the size of the shock.

[13]Indeed, impulse responses to level shocks are almost identical across expected utility and recursive preferences.

[14]In this set-up with complete markets and flexible prices, uncertainty shocks do not have any effect under expected utility.

[15]Relative consumption and real exchange rate are negatively correlated conditional on uncertainty shocks, which might potentially help in accounting for the Backus-Smith-Kollmann puzzle, but the direction of the responses are not in line with the empirical findings referred above.

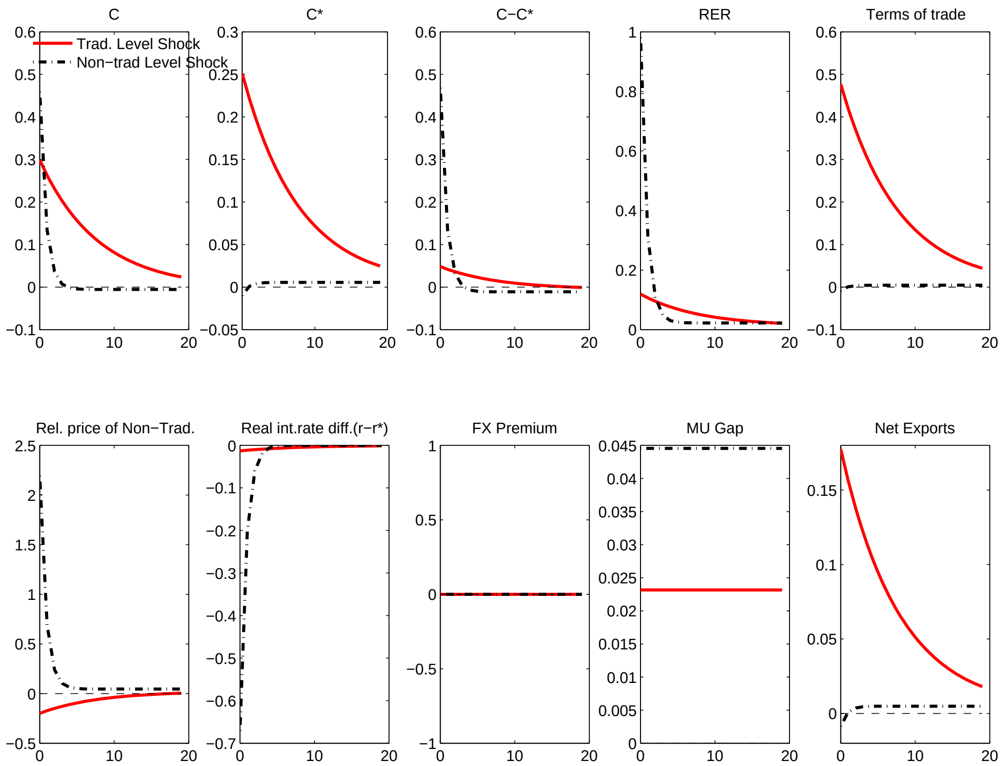

Figure 4.1: Impulse responses to level shocks under complete markets.

Notes: Impulse responses to a one standard deviation shock to the level of tradables (solid line) or non-tradables (dashed line). x-axis shows periods, y-axis shows log-deviations from steady-state normalised by the standard deviation of the shock.

to the components of the covariance terms given in equation (4.27). The sign of the covariance between home and foreign stochastic discount factors and the real exchange rate conditional on level shocks will determine whether foreign exchange risk premium will increase or decrease given an increase in the variance of endowments. Real exchange rate appreciates when home and foreign appetite for wealth is high, so home bonds are a good hedge on average, which means that foreign bonds have to command a risk premium in international financial markets. Thus, investing in the bond of the country with the higher real interest rate (foreign country) pays a positive excess return in line with the carry trade evidence. To put it differently, real exchange rate of the low interest rate country (home) is expected to depreciate rather than appreciate in line with the data. But, this set-up cannot account for the fact that low interest rate currency has a more depreciated real exchange rate on impact (Engel, 2011). This is related to the observation that this set-up cannot generate a

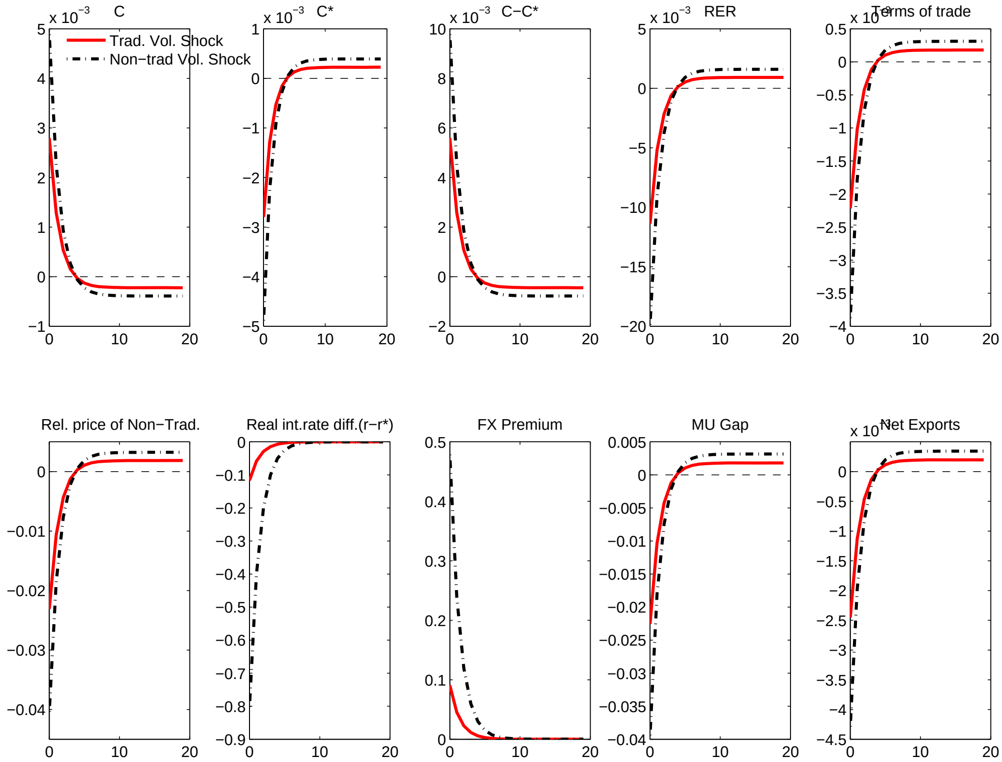

Figure 4.2: Impulse responses to volatility shocks under complete markets.

Notes: Impulse responses to a one standard deviation shock to the variance of tradables (solid line) or non-tradables (dashed line). x-axis shows periods, y-axis shows log-deviations from steady-state normalised by the standard deviation of the shock.

real exchange rate depreciation in response to increased uncertainty.

4.3.3 Incomplete financial markets

How does the international transmission of uncertainty shocks change when agents can only trade in a single non-contingent bond and portfolio adjustment is costly? We start with the discussion of the baseline calibration with small portfolio adjustment costs ($\Phi = 0.01$). Later, we vary the parameter Φ to see the role of financial market integration in dampening or amplifying the responses to increased uncertainty.

Again, for completeness, we first discuss shocks to the level of endowments. Impulse responses to tradable endowment shocks are given by the solid red lines in Figure 4.3. An increase in home tradable endowment leads to an increase in consumption in both countries, more so in the home country. Due to market incompleteness, home agents become wealthier and increase their demand for both tradable and non-tradable goods. Given that home and foreign goods are highly substitutable in the baseline calibration, the supply effect dominates the demand effect in the tradable goods market and home terms of trade depreciates. Higher demand for non-tradables in the home country increases the relative price of non-tradable goods, leading to a real exchange rate appreciation (fall in the real exchange rate). This is the mechanism that can account for the consumption-real exchange rate anomaly in Benigno and Thoenissen (2008). Home country increases its exports of tradable goods and net foreign asset position improves. Comparing the red solid lines in Figures 4.1 and 4.3 shows the importance of market incompleteness for the international transmission of tradable endowment shocks in this set-up.

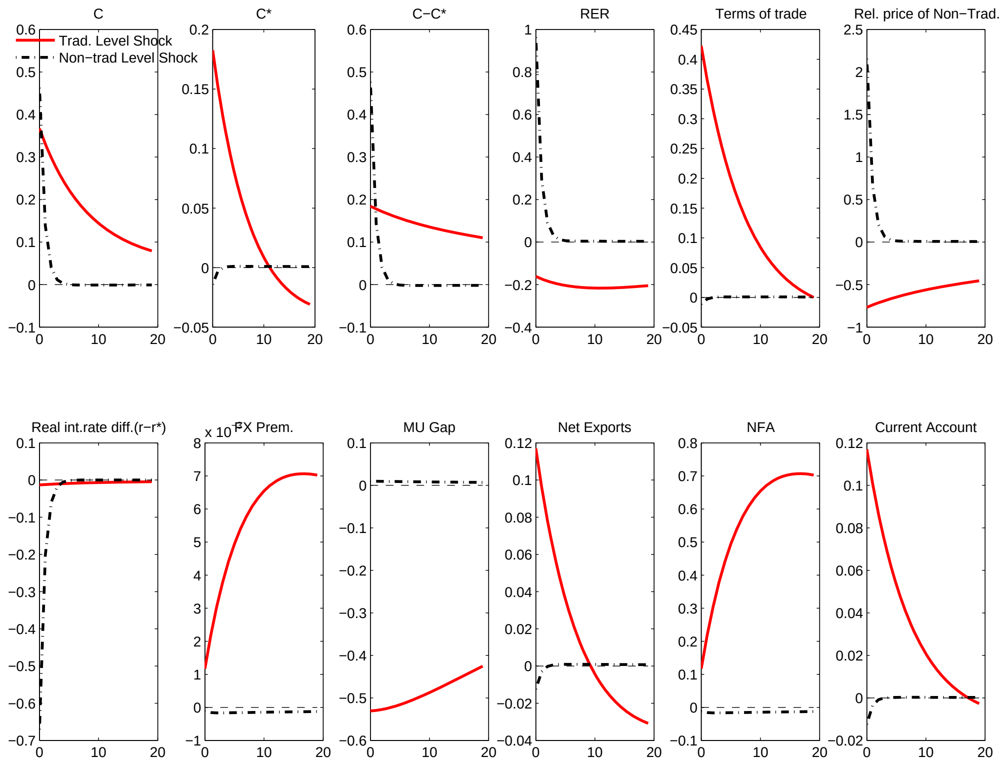

Figure 4.3: Impulse responses to level shocks under incomplete markets.

Notes: Impulse responses to a one standard deviation shock to the level of tradables (solid line) or non-tradables (dashed line). x-axis shows periods, y-axis shows log-deviations from steady-state normalised by the standard deviation of the shock.

Responses to a non-tradable endowment shock are almost identical across complete and incomplete markets (comparing the dashed black lines across Figures 4.1

and 4.3). In response to an increase in home non-tradable endowment, home consumption increases while foreign consumption falls independently of the asset market structure. Because tradable and non-tradable goods are complements in the baseline calibration, demand for tradable goods increases along with the demand for non-tradable goods, as a consequence home (foreign) terms of trade improves (worsens). Relative price of non-tradable goods fall as home non-tradable goods become more abundant, which leads to a real exchange rate depreciation.

Deviations from uncovered interest rate parity reflect the small portfolio adjustment costs that depend on the net foreign asset position as shown in equation (4.19). Tradable endowment shocks generate a larger marginal utility gap hence larger wealth transfers cross the border, which in turn imply that deviations from the UIP will be larger conditional on these shocks for a given Φ. As agents accumulate net foreign assets following a positive tradable sector shock, marginal cost of adjusting the portfolio position, ΦNFA_t, increases, hence foreign bonds pay a higher return to compensate for this cost.

Figure 4.4 shows the impulse responses to an increase in the volatility of tradable and non-tradable endowments. Increased uncertainty about tradable goods endowment leads to a fall in home consumption matched by a rise in foreign consumption - shown by the solid red lines. Faced with higher uncertainty, home agents increase their precautionary savings, i.e. they lend to foreigners by buying foreign bonds. Higher saving in home country implies lower consumption on impact, but as the net foreign asset position improves, home consumption increases towards its long-run value. On the other hand, higher borrowing by foreign agents implies higher foreign consumption. Given that preferences are biased towards domestic goods, the fall in the relative consumption of home agents depreciates both the terms of trade and the relative price of non-tradables and home consumption basket becomes cheaper compared to foreign. The rise in the net foreign asset position and the depreciation in the real exchange rate following a shock to uncertainty are consistent with the patterns that arise in the data as mentioned in the introduction. In this sense, the incomplete market model generates a more realistic transmission of uncertainty shocks compared to the complete market model. Also, comparing the red solid lines in Figures 4.2 and 4.4 shows that macroeconomic volatility in response to uncertainty shocks is significantly higher under incomplete markets.

In this set-up, an increase in tradable sector volatility works just like a negative demand shock. Hence, volatility shocks can potentially help in accounting for the negative correlation between relative consumption and real exchange rate in line with

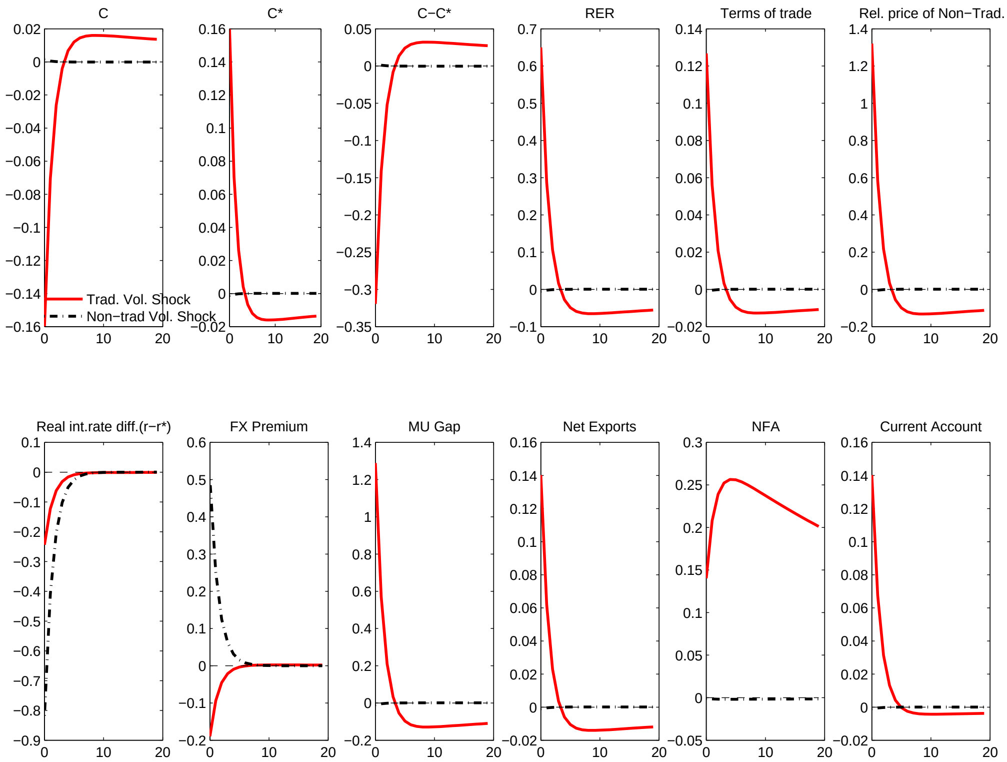

Figure 4.4: Impulse responses to volatility shocks under incomplete markets.

Notes: Impulse responses to a one standard deviation shock to the variance of tradables (solid line) or non-tradables (dashed line). x-axis shows periods, y-axis shows log-deviations from steady-state normalised by the standard deviation of the shock.

the Backus-Smith-Kollmann evidence. This would ultimately depend on the size of these volatility shocks with respect to the other shocks that drive business cycles.

Real interest rate falls in both countries but falls more in the home country consistent with higher savings by home agents, giving rise to a negative interest rate differential. This is also in line with the VAR results reported in Guerrón-Quintana (2009) showing that the U.S. interest rate falls more than foreign interest rate following an an increase in U.S. uncertainty.[16]

The relationship between the real interest rate differential and the real exchange rate conditional on tradable volatility shocks is consistent with the empirical fact that the country with the lower real interest rate has a more depreciated real exchange rate

[16]However, we should note that his results are based on nominal interest rates. Benigno et al. (2011) also show that an increase in the volatility of U.S. productivity leads to a fall in U.S. interest rates.

as emphasised by Engel (2011). However, in this simple set-up, stochastic volatility in the tradable sector is *not* able to account for the negative relationship between real interest differential and expected real depreciation rate that we observe in the data (the UIP puzzle). In response to increased volatility, real interest rate differential falls and real exchange rate depreciates on impact but is expected to appreciate in the short-term. In this case, foreign exchange risk premium does not help with the UIP puzzle, on the contrary, it makes the puzzle worse as it implies that real exchange rate is expected to appreciate by more than what would be implied by the UIP.

Why does the model imply that home bonds pay an excess return in periods of increased uncertainty about tradable endowment? As shown in equation (4.25), foreign exchange risk premium is determined by a simple hedging motive captured by $-Cov_t(\hat{M}_{t+1}, \Delta\hat{Q}_{t+1})$ if we leave aside the Jensen's inequality term which is always negative, and the portfolio adjustment cost, which is small for this calibration. To understand the response of the risk premium, we need to understand the first-order accurate responses of the appetite for wealth and the real exchange rate to shocks to the level of tradable endowment. Home agent's appetite for wealth, i.e. home stochastic discount factor given by $\hat{M}_{t+1}$, is higher in the states where tradable endowment is lower. In those states, demand for non-tradable goods is also lower, which depreciates the real exchange rate by leading to a larger fall in the relative price of non-tradables in the home country in line with the above discussion about Figure 4.3. Hence, $\hat{M}_{t+1}$ and $\Delta\hat{Q}_{t+1}$ are positively correlated conditional on level shocks which implies that foreign exchange is a better hedge for home country, hence a rise in the volatility of tradable endowment will lead to a fall in the foreign exchange rate premium.

It is true that, by using a second-order approximation, we are ignoring the covariance between the stochastic discount factor and the real exchange rate conditional on uncertainty shocks, which by construction is a fourth-order term. However, this would not overturn the response of the FX premium as the appetite for wealth and real exchange rate are positively correlated also conditional on uncertainty shocks.

Market incompleteness does not have an important effect on the international transmission of non-tradable income uncertainty shocks: the dashed black lines in Figures 4.2 and 4.4 are almost identical (but drawn at different scales). Hence, volatility shocks in the non-tradable sector have a much more limited effect on the economy compared to shocks to the volatility of tradable good endowment under incomplete markets. The increase in the volatility of non-tradable endowment affects the marginal utility of home agents much more than the marginal utility of foreign agents as home agents are the sole consumer of these goods. Hence, foreign agents are willing to lend funds to home agents whose appetite for wealth is higher due to increased uncertainty. Relative home consumption increases while real exchange rate appreciates, opposite to what we observe in the data in response to an increase in overall uncertainty.

Given that real exchange rate appreciates when home stochastic discount factor is high, home bonds are a better hedge than foreign bonds against non-tradable sector uncertainty shocks, hence excess return on foreign bonds increase with an increase in home non-tradable income uncertainty and real exchange rate is expected to depreciate.

The analysis so far shows that a flexible price complete market model with tradable and non-tradable goods is not able to match the empirical observations related to the responses of net foreign assets and real exchange rate to increased uncertainty about income. On the other hand, an incomplete market version of the model is able to generate a rise in the net foreign asset position, a fall in interest rate differential and a real exchange rate depreciation following an increase in tradable income uncertainty which is in line with the data. The incomplete market model can also account for the negative correlation between relative consumption and real exchange rate conditional on both level and volatility shocks in the tradable sector. However, it cannot account for the UIP puzzle, because in this set-up real exchange rate appreciates in good states where consumption is higher, which helps to account for the Backus-Smith puzzle but has a counterfactual implication for the foreign exchange risk premium and for the relation between the real interest rate differential and expected depreciation rate. This analysis shows that a two-sector model with incomplete markets along the lines of Benigno and Thoenissen (2008) and Corsetti et al. (2008), which can account for the negative correlation between relative consumption and real exchange rate cannot account for the UIP puzzle even in the presence of stochastic volatility.

The reason why the complete market model is able to generate an increase in the foreign exchange premium following an increase in tradable income risk, is because it implies that real exchange rate depreciates in good states where consumption is higher (appetite for wealth is lower), which makes foreign bonds a worse hedge and hence raises their excess return.

These results suggest that lowering the elasticity of substitution between home and foreign tradable goods, or decreasing the share of non-tradable goods in total consumption, can help the model do a better job in matching the UIP deviations as these would change the sign of the covariance of the stochastic discount factor and the real exchange rate in response to tradable shocks. The question is whether such a model would still generate realistic responses of net foreign asset accumulation and real exchange rates in response to increased uncertainty?[17]To put it differently, would market incompleteness still matter for uncertainty shocks in a model without non-tradable goods?

A model where all goods are tradable

To answer these questions, Figure 4.5 plots impulse responses to an increase in tradable endowment uncertainty for $\gamma = 0.55$ (baseline calibration) and $\gamma = 1$ (all goods tradable). Naturally, an increase in uncertainty about tradable endowment has a larger effect on the economy when all goods are tradable, simply because tradable income risk now affects a bigger part of the consumption basket. Hence, precautionary saving motive is stronger, which leads to higher net foreign asset accumulation at the cost of lower domestic consumption.

With a larger drop in the demand for home tradables, home terms of trade depreciates more but real exchange rate response is smaller due to the lack of fluctuations coming from the relative price of non-tradables. Marginal utility gap across countries becomes bigger while the response of the foreign exchange risk premium switches sign and becomes more volatile. Foreign bonds are expected to pay a higher return following an increase in home uncertainty because in the absence of non-tradable goods, real exchange rate appreciates in bad states where income is lower, making foreign bonds a worse hedge.

While the presence of non-tradable goods is crucial to generate a low degree of international risk sharing conditional on level shocks in the single bond economy, it does not play such a role for the transmission of uncertainty shocks. On the contrary, uncertainty shocks lead to larger deviations from efficient risk sharing (larger marginal utility gap) when all goods are tradable, because the precautionary saving motive becomes stronger which leads to larger differences between consumption patterns across countries. Hence, for the rest of the analysis, we use the model where all goods are tradable.

[17]An alternative way to reconcile the two-sector incomplete market model with the UIP evidence is to consider alternative shocks (like long-run risk shocks or news shocks), which could lead to large fluctuations in the risk adjustment factor g_{t+1} such that the appetite for wealth M_{t+1} increases when consumption growth increases and real exchange rate appreciates. This could account for the Backus-Smith puzzle while generating realistic deviations from the UIP.

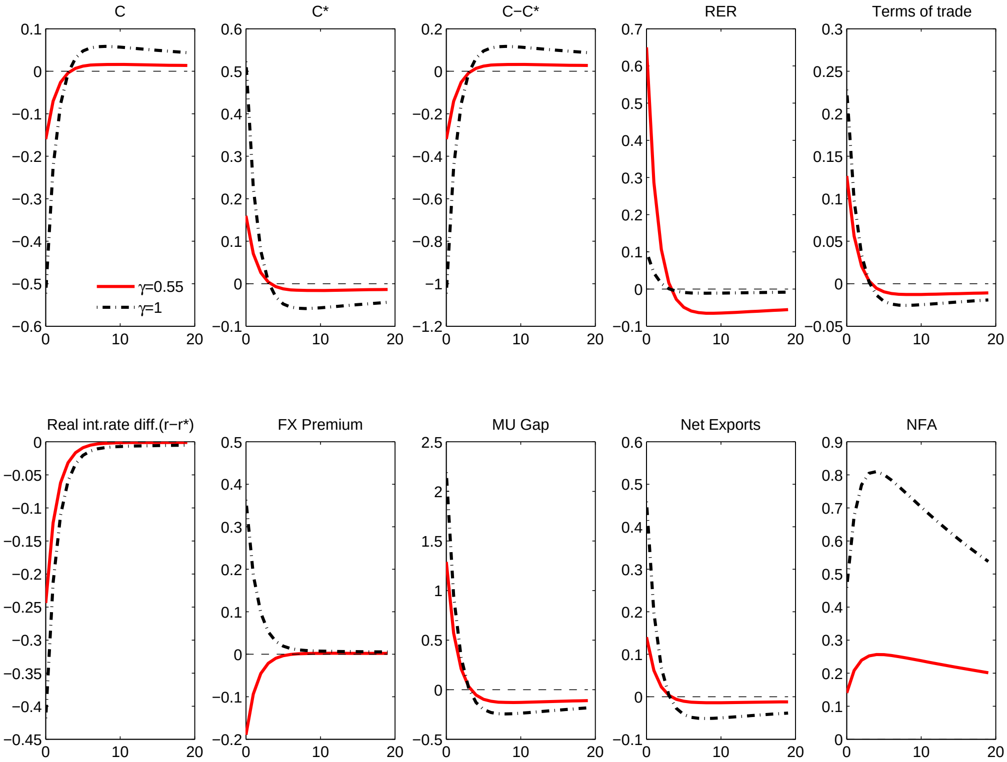

Figure 4.5: Impulse responses to a shock to the volatility of tradable endowment under incomplete markets for $\gamma = 0.55$ and $\gamma = 1$.

Notes: γ denotes the share of tradable goods in final consumption: $\gamma = 0.55$ is the baseline calibration and $\gamma = 1$ corresponds to the model with only tradable goods. x-axis shows periods, y-axis shows log-deviations from steady-state normalised by the standard deviation of the shock.

Imperfect financial market integration

How do agents react to increased uncertainty about their endowment when it is much more costly to adjust their foreign asset holdings? To answer this question, we focus on the model with only tradable goods and compare the impulse responses to an increase in endowment volatility across different degrees of financial market integration, as measured by the parameter Φ (Figure 4.6).

We consider three different values of Φ : $\Phi = 0.01$, $\Phi = 5$ and $\Phi = 100$. The interpretation is that the higher the value of Φ, the less integrated is the home country with international financial markets as in Sutherland (1996) and Benigno (2009). The highest degree of financial market integration is represented by the complete market economy, which is shown by the solid red lines in Figure 4.6.

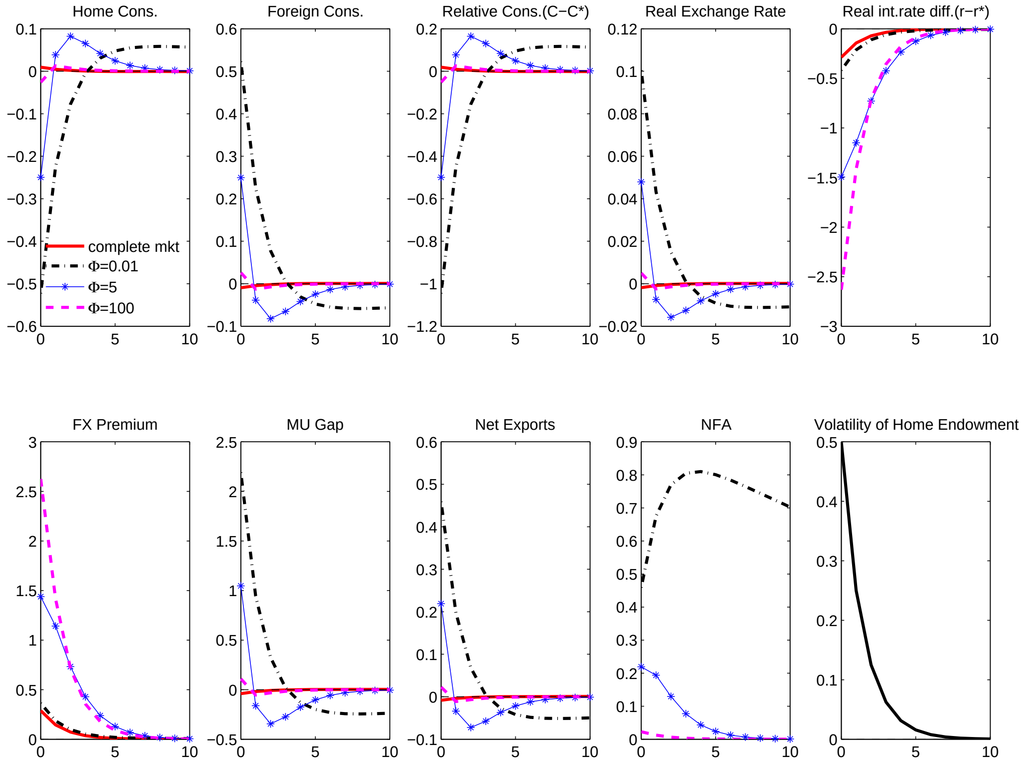

Figure 4.6: Impulse responses to a shock to the volatility of tradable endowment for different degrees of financial market integration measured by Φ.

Notes: γ Impulse responses are obtained for the model where all goods are tradable, $\gamma = 1$. x-axis shows periods, y-axis shows log-deviations from steady-state normalised by the standard deviation of the shock.

The role of higher portfolio adjustment costs is best understood by analysing the response of the net foreign asset position. When Φ is higher, it is more costly to increase savings, hence the increase in net foreign asset position remains limited and the responses of consumption and real exchange rate become less volatile. Due to the convexity of the portfolio adjustment costs, agents increase precautionary savings more in the initial periods and pay a lower cost of adjustment later on by running down these assets, which implies that the effects of uncertainty shocks die out more quickly for higher Φ.

Interestingly, as we increase the degree of financial market imperfections by increasing Φ (as Φ increases the model approaches to financial autarky), we bring the economy closer to the complete market allocation where the responses of consumption and real exchange rate to uncertainty shocks are muted. Comparing the marginal utility gap across different market structures shows this point. Hence, financial autarky and complete market models have similar implications for the transmission of uncertainty shocks due to the lack of precautionary savings in both set-ups, while the single bond set-up significantly differs from the two. This is an interesting result because for the transmission of level shocks the opposite is true: In a two-country two-good business cycle model driven by shocks to productivity levels, Heathcote and Perri (2002) show that allocations in the financial autarky model always differ significantly from those under the single bond or complete market economies, while the latter two are very similar to each other. This result also suggests that precautionary savings might hamper international risk sharing.

Lowering the degree of financial market integration increases the volatility of the foreign exchange risk premium by construction, as increasing Φ increases deviations from the UIP condition, due to increased transaction costs.

Sensitivity: The role of recursive preferences and risk aversion

In a real model with complete markets and expected utility, uncertainty shocks do not affect consumption or the real exchange rate. However, this is not true for an incomplete market model because uncertainty shocks affect the economy also under expected utility.

Figure 4.7 compares impulse responses to an increase in endowment uncertainty for alternative degrees of risk aversion in an incomplete market model where all goods are tradable. For $\psi = 2$, the model boils down to the expected utility model. We compare this with the benchmark risk aversion value, $\psi = 5$ and a much higher risk aversion value, $\psi = 30$.

Under incomplete markets, the degree of risk aversion is important for precautionary net foreign asset accumulation: the higher the risk aversion, the higher the net foreign asset accumulation and the larger the marginal utility gap across countries. For the responses to level shocks, the degree of risk aversion does not have a significant effect.

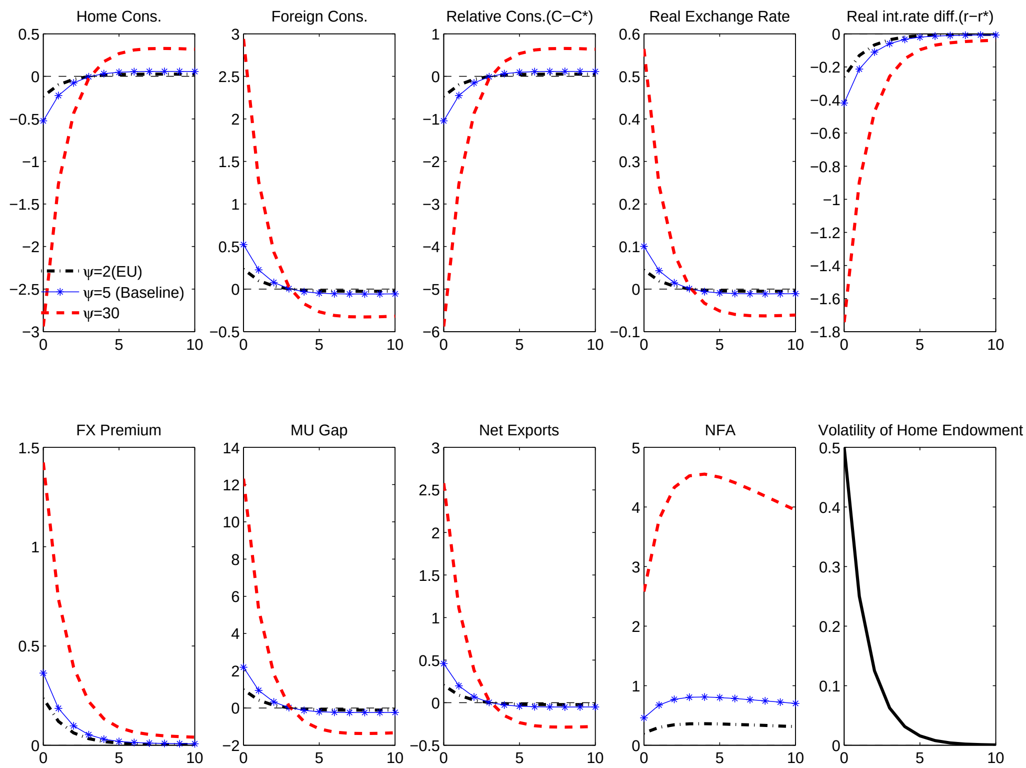

Figure 4.7: Impulse responses to a shock to the volatility of tradable endowment for different degrees of risk aversion measured by ψ.

Notes: Impulse responses are obtained for the model where all goods are tradable, $\gamma = 1$. x-axis shows periods, y-axis shows log-deviations from steady-state normalised by the standard deviation of the shock.

4.4 Conclusion

This chapter studies the effects of financial market integration for the international transmission of country-specific uncertainty shocks. We use a two-country endowment model with non-tradable goods and recursive preferences in which endowment processes are subject to stochastic volatility. In the baseline model, there is international trade in a single bond subject to portfolio adjustment costs. We compare this model with the complete markets model in terms of its implications for the cross-border transmission of domestic volatility shocks. We also investigate the effects of varying the scale of portfolio adjustment costs which we interpret as varying the degree of financial market integration.

We find that an incomplete model with international trade in a single bond performs better than a model with complete financial integration in matching the empirical observations regarding the dynamics of net foreign assets and real exchange rate in response to increased macroeconomic uncertainty. This model can also account for the Backus and Smith (1993) and Kollmann (1995) evidence on the negative correlation between relative consumption and real exchange rate conditional on both level and volatility shocks. However, it cannot generate meaningful deviations from the uncovered interest rate parity (UIP) condition because in this model real exchange rate appreciates in good states where consumption is higher, which implies a fall in the foreign exchange risk premium following an increase in domestic volatility.

We also show that increasing the degree of financial market imperfections brings the model close to the complete market model in terms of its implications for the transmission of uncertainty shocks. When there are high portfolio adjustment costs, and no other means to save, agents cannot increase precautionary savings following an increase in income uncertainty, hence consumption and real exchange rate responses to uncertainty shocks remain limited as in the case of complete insurance.

An obvious extension is to consider production economies with capital accumulation. If agents can save by investing in the domestic capital stock, net foreign asset position would be affected less by an increase in uncertainty. But to the extent that uncertainty shocks make domestic capital more risky, agents would still buy foreign assets for precautionary purposes. In the presence of endogenous capital accumulation, financial autarky model might have different implications compared to the complete market model because agents would be able to increase precautionary savings by investing in the domestic capital stock even if they cannot invest in international financial markets, while there will be no precautionary saving motive under complete markets. We leave the analysis of production economies with endogenous capital accumulation for future research.

Another interesting extension is to allow for endogenous portfolio choice and analyse the dynamics of international portfolios following volatility shocks. With time-varying volatility, the zero-order component of equilibrium portfolios, which is determined by the covariance between the real exchange rate adjusted relative marginal utilities of consumption and excess returns will be time-varying. It would be interesting to analyse the allocation of precautionary savings among domestic and foreign bonds and stocks in an incomplete market framework.

Appendix to Chapter 4

4.4.1 Deriving the first-order condition for optimal bond holdings under incomplete markets

$$
\begin{aligned}
\max_{C_t,B_{H,t},B_{F,t}} V_t(B_{H,t-1},B_{F,t-1}) &= \left[C_t^{1-\rho}+\beta\left(E_t(V_{t+1}(B_{H,t},B_{F,t})^{1-\psi}\right)^{\frac{1-\rho}{1-\psi}}\right]^{\frac{1}{1-\rho}} \\
&\quad -\lambda_t\left[\begin{array}{c} C_t+B_{H,t}+Q_tB_{F,t}+\frac{\Phi}{2}\left(Q_tB_{F,t}-\bar{b}\right)^2 \\ -R_{H,t}B_{H,t-1}-R^*_{F,t}Q_tB_{F,t-1}-\frac{P_{H,t}}{P_t}Y_{H,t}-\frac{P_{N,t}}{P_t}Y_{N,t}\end{array}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial V_t}{\partial C_t} &= \underbrace{\left[C_t^{1-\rho}+\beta\left(E_tV_{t+1}^{1-\psi}\right)^{\frac{1-\rho}{1-\psi}}\right]^{\frac{\rho}{1-\rho}}}_{\equiv V_t^{\rho}}C_t^{-\rho}-\lambda_t=0 \\
\frac{\partial V_t}{\partial C_t} &= \lambda_t=C_t^{-\rho}V_t^{\rho},
\end{aligned}
$$

where λ_t is the marginal utility of consumption.

$$
\begin{aligned}
\frac{\partial V_t}{\partial B_{H,t}} &= V_t^{\rho}\beta\left(E_tV_{t+1}^{1-\psi}\right)^{\frac{-\rho+\psi}{1-\psi}}E_tV_{t+1}^{-\psi}\frac{\partial V_{t+1}}{\partial B_{H,t}}-\lambda_t=0 \\
\frac{\partial V_t}{\partial B_{F,t}} &= V_t^{\rho}\beta\left(E_tV_{t+1}^{1-\psi}\right)^{\frac{-\rho+\psi}{1-\psi}}E_tV_{t+1}^{-\psi}\frac{\partial V_{t+1}}{\partial B_{F,t}}-\lambda_tQ_t\left(1+\Phi(Q_tB_{F,t}-b)\right)=0
\end{aligned}
$$

Using envelope conditions,

$$
\begin{aligned}
\frac{\partial V_t}{\partial B_{H,t-1}} &= \lambda_tR_{H,t} \\
\frac{\partial V_t}{\partial B_{F,t-1}} &= \lambda_tR^*_{F,t}Q_t
\end{aligned}
$$

first order conditions become:

$$\beta V_t^{\rho}\left(E_t V_{t+1}^{1-\psi}\right)^{\frac{-\rho+\psi}{1-\psi}} E_t V_{t+1}^{-\psi}\lambda_{t+1}R_{H,t+1} = \lambda_t$$

$$\beta V_t^{\rho}\left(E_t V_{t+1}^{1-\psi}\right)^{\frac{-\rho+\psi}{1-\psi}} E_t V_{t+1}^{-\psi}\lambda_{t+1}R_{F,t+1}^{*}Q_{t+1} = \lambda_t Q_t\left(1+\Phi(Q_t B_{F,t}-b)\right)$$

Plug in $\lambda_t = C_t^{-\rho}V_t^{\rho}$:

$$E_t\beta\left(\frac{V_{t+1}}{\left(E_t V_{t+1}^{1-\psi}\right)^{\frac{1}{1-\psi}}}\right)^{\rho-\psi}\frac{C_{t+1}^{-\rho}}{C_t^{-\rho}}R_{H,t+1} = 1,$$

$$E_t\beta\left(\frac{V_{t+1}}{\left(E_t V_{t+1}^{1-\psi}\right)^{\frac{1}{1-\psi}}}\right)^{\rho-\psi}\frac{C_{t+1}^{-\rho}}{C_t^{-\rho}}R_{F,t+1}^{*}\frac{Q_{t+1}}{Q_t} = 1+\Phi(NFA_t-b).$$

4.4.2 Calculating foreign exchange risk premium

In the non-contingent bond economy, we can only calculate foreign currency risk premium from the perspective of home agent as foreign agents can only trade in foreign bonds. For ease of exposition we assume portfolio adjustment costs are zero.[18]

- Second order approximation of home euler equation with respect to (riskless) home bond:

$$\begin{aligned} E_t\left[M_{t+1}\right]R_{H,t+1} &= 1 \\ E_t M_{t+1} &= R_{H,t+1}^{-1} \end{aligned}$$

Define $\frac{X-\bar{X}}{\bar{X}} = \hat{X}_t + \frac{1}{2}\hat{X}_t^2$

$$R_{H,t+1}^{-1} \simeq (\bar{R}_H)^{-1} - (\bar{R}_H)^{-2}(R_{H,t+1}-\bar{R}_H) + (\bar{R}_H)^{-3}(R_{H,t+1}-\bar{R}_H)^2$$

$$E_t M_{t+1} \simeq \bar{M} + \bar{M}E_t\left(M_{t+1}-\bar{M}\right)$$

[18]For these derivations, I benefited from Bianca De Paoli's notes titled "Second-order approximation to asset pricing conditions", internal Bank of England document.

At the steady-state: $\bar{M} = \bar{R}_H^{-1}$

$$\begin{aligned} E_t\hat{M}_{t+1} + \frac{E_t\hat{M}_{t+1}^2}{2} &= -\left(\hat{R}_{H,t+1} + \frac{\hat{R}_{H,t+1}^2}{2}\right) + \hat{R}_{H,t+1}^2 \\ \hat{R}_{H,t+1} &= -E_t\hat{M}_{t+1} - \frac{E_t\hat{M}_{t+1}^2}{2} + \frac{\hat{R}_{H,t+1}^2}{2} \end{aligned}$$

Square and ignore terms with order higher than 2:

$$\hat{R}_{H,t+1}^2 = \left[E_t\hat{M}_{t+1}\right]^2$$

Substitute:

$$\begin{aligned} \hat{R}_{H,t+1} &= -E_t\hat{M}_{t+1} - \frac{1}{2}\left(E_t\hat{M}_{t+1}^2 - \left[E_t\hat{M}_{t+1}\right]^2\right) \\ &= -E_t\hat{M}_{t+1} - \frac{Var_t\hat{M}_{t+1}}{2} \end{aligned} \tag{4.28}$$

- Second order approximation of home euler equation with respect to foreign bond (risky asset):

$$E_t\left[M_{t+1}R_{F,t+1}\right] = 1$$

$$M_{t+1}R_{F,t+1} \simeq \bar{M}\bar{R}_F + \bar{M}\left(R_{F,t+1} - \bar{R}_F\right) + \bar{R}_F\left(M_{t+1} - \bar{M}\right) + (M_{t+1} - \bar{M})(R_{F,t+1} - \bar{R}_F)$$

$$\bar{M}\bar{R}_F = 1$$

$$E_t\left(R_{F,t+1} - \bar{R}_F\right) + E_t\left(M_{t+1} - \bar{M}\right) + E_t(M_{t+1} - \bar{M})(R_{F,t+1} - \bar{R}_F) = 0$$

$$E_t\hat{R}_{F,t+1} + \frac{E_t\hat{R}_{F,t+1}^2}{2} + E_t\hat{M}_{t+1} + \frac{E_t\hat{M}_{t+1}^2}{2} + E_t\hat{M}_{t+1}\hat{R}_{F,t+1} = 0$$

$$E_t\hat{R}_{F,t+1} = -\frac{E_t\hat{R}_{F,t+1}^2}{2} - E_t\hat{M}_{t+1} - \frac{E_t\hat{M}_{t+1}^2}{2} - E_t\hat{M}_{t+1}\hat{R}_{F,t+1}$$

Take the square of $E_t\hat{R}_{F,t+1}$:

$$\left[E_t\hat{R}_{F,t+1}\right]^2 = \left[E_t\hat{M}_{t+1}\right]^2$$

Premultiply $E_t\hat{R}_{F,t+1}$ by $E_t\hat{M}_{t+1}$:

$$\left[E_t\hat{M}_{t+1}\right]\left[E_t\hat{R}_{F,t+1}\right] = -\left[E_t\hat{M}_{t+1}\right]^2$$

Substitute these in:

$$E_t\hat{R}_{F,t+1} = -\frac{E_t\hat{R}^2_{F,t+1}}{2} - E_t\hat{M}_{t+1} - \frac{E_t\hat{M}^2_{t+1}}{2} \mp \frac{\left[E_t\hat{M}_{t+1}\right]^2}{2} - E_t\hat{M}_{t+1}\hat{R}_{F,t+1} \mp \left[E_t\hat{M}_{t+1}\right]\left[E_t\hat{R}_{F,t+1}\right]$$

Note that $\left[E_t\hat{R}_{F,t+1}\right]^2 = \left[E_t\hat{M}_{t+1}\right]^2$

$$\begin{aligned}
E_t\hat{R}_{F,t+1} = & -\left(\frac{E_t\hat{R}^2_{F,t+1}}{2} - \frac{\left[E_t\hat{M}_{t+1}\right]^2}{2}\right) - E_t\hat{M}_{t+1} - \frac{E_t\hat{M}^2_{t+1}}{2} - \frac{\left[E_t\hat{M}_{t+1}\right]^2}{2} \\
& - E_t\hat{M}_{t+1}\hat{R}_{F,t+1} + \left[E_t\hat{M}_{t+1}\right]\left[E_t\hat{R}_{F,t+1}\right] + \left[E_t\hat{M}_{t+1}\right]^2 \\
= & -\frac{Var_t\hat{R}_{F,t+1}}{2} - E_t\hat{M}_{t+1} - \left(\frac{E_t\hat{M}^2_{t+1}}{2} - \frac{\left[E_t\hat{M}_{t+1}\right]^2}{2}\right) \\
& - \left(E_t\hat{M}_{t+1}\hat{R}_{F,t+1} - \left[E_t\hat{M}_{t+1}\right]\left[E_t\hat{R}_{F,t+1}\right]\right)
\end{aligned}$$

$$E_t\hat{R}_{F,t+1} + \frac{Var_t\hat{R}_{F,t+1}}{2} = -E_t\hat{M}_{t+1} - \frac{Var_t\hat{M}_{t+1}}{2} - Cov_t\left(\hat{M}_{t+1}, \hat{R}_{F,t+1}\right) \tag{4.29}$$

- Foreign Exchange Risk Premium from the perspective of home investor

To calculate expected excess returns on foreign bond in terms of home currency subtract (4.29) from (4.28)

$$\begin{aligned}
E_t\hat{R}_{F,t+1} - \hat{R}_{H,t+1} &= -E_t\hat{M}_{t+1} - \frac{Var_t\hat{M}_{t+1}}{2} - Cov_t\left(\hat{M}_{t+1}, \hat{R}_{F,t+1}\right) - \frac{Var_t\hat{R}_{F,t+1}}{2} \\
& + E_t\hat{M}_{t+1} + \frac{Var_t\hat{M}_{t+1}}{2} \\
&= -\frac{Var_t\hat{R}_{F,t+1}}{2} - Cov_t\left(\hat{M}_{t+1}, \hat{R}_{F,t+1}\right)
\end{aligned}$$

Note that $\hat{R}_{F,t+1} = \hat{R}^*_{F,t+1} + \hat{S}_{t+1} - \hat{S}_t$. Hence,

$$\begin{aligned} \frac{Var_t \hat{R}_{F,t+1}}{2} &= \frac{Var_t\left(\hat{R}^*_{F,t+1} + \hat{S}_{t+1} - \hat{S}_t\right)}{2} = \frac{Var_t\left(\Delta S_{t+1}\right)}{2} \\ Cov_t\left(\hat{M}_{t+1}, \hat{R}_{F,t+1}\right) &= Cov_t\left(\hat{M}_{t+1}, \hat{R}^*_{F,t+1} + \hat{S}_{t+1} - \hat{S}_t\right) \\ &= Cov_t(\hat{M}_{t+1}, \Delta\hat{S}_{t+1}) \end{aligned}$$

and

$$fxp_t \equiv \hat{R}^*_{F,t+1} - \hat{R}_{H,t+1} + E_t \Delta\hat{S}_{t+1} = -\frac{Var_t\left(\Delta S_{t+1}\right)}{2} - Cov_t(\hat{M}_{t+1}, \Delta\hat{S}_{t+1}) \quad (4.30)$$

When foreign agents are allowed to trade home bonds as well as foreign bonds, we can also calculate a foreign exchange risk premium from the perspective of foreign agent following similar steps as above. Taking a second order approximation to foreign Euler equations with respect to home and foreign currency bonds gives:

$$fxp_t \equiv \hat{R}^*_{F,t+1} - \hat{R}_{H,t+1} + E_t \Delta\hat{S}_{t+1} = \frac{Var_t\left(\Delta S_{t+1}\right)}{2} - Cov_t(\hat{M}^*_{t+1}, \Delta\hat{S}_{t+1}) \quad (4.31)$$

Taking an average of the two equations (4.30) and (4.31), we get the familiar equation for foreign exchange risk premium which also holds under complete markets: (see Engel, 2011 and Benigno et al., 2011).

$$fxp_t = -Cov_t(\frac{M_{t+1} + M^*_{t+1}}{2}, \Delta S_{t+1}) \quad (4.32)$$

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