New Logical Perspectives on Ceteris Paribus Preference

Fenrong Liu

Tsinghua University and University of Amsterdam

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Outline

Example: Choosing a hotel

Bob is going to attend the SOCREAL 2013 workshop and is thinking of which hotel he would like to stay. He found two alternatives *a* and *b* to choose from. *a* is cheaper than *b*, but *b* is closer to the workshop venue than *a*. Since he considers cost more important, he prefers *a* over *b*.

General feature of preference: ceteris paribus

- We form our preference or need to make decisions constantly, just as illustrated in such examples.
- Our preference has a crucial feature of ceteris paribus. Bob prefers *a* over *b* in the sense of everything else being equal.
- "Everything else being equal" highlights a context, which the preference depends on. For instance, in this example:
  (a) Bob considers the aspects of cost and the distance to the workshop venue.
  (b) Cost is more important than the distance.

Hidden dynamics: two scenarios

In other words, if there is any change concerning (a) and (b), Bob's preference would differ. Consider

Scenario 1: Bob realizes that having a swimming pool is very important to him, he reconsiders and now prefers *b* over *a*!

Scenario 2: Bob's doctor just told him before he leaves for Sapporo that his leg has some problem, and it is better not to walk much. He no longer considers cost more important. Now he prefers *b* over *a*!

Importance of ceteris paribus

- Preference logic: central feature.
  Already crucial in von Wright's work 1963.

  $$\varphi > \psi \leftrightarrow ((\varphi \wedge r) > (\psi \wedge r)) \wedge ((\varphi \wedge \neg r) > (\psi \wedge \neg r))$$

- Semantics of natural language: default expressions.
- Philosophy of science: CP laws

Different readings: equality vs. normality

- "all other things being equal": identifies facts to be kept constant in judging preference relations.
- Ceteris paribus as "all other things being normal" is taken to mean that under *normal* conditions, something ought to be the case.
  Example: A preference for red wine over white wine, unless one is eating fish.

Equality vs. normality

- Normality reading would lead to default logic, non-monotonic logics. (e.g. Boutilier 1994)
- Our line of equality reading has received a mathematical interpretation in Doyle and Wellman's work. Van Benthem et al developed this further.
- I will focus on the equality reading in this talk.

Modal logic approach by van Benthem, et al

(Van Benthem, Girard and Roy 2009) introduce new modalities of the form $[\Gamma]\varphi$ in modal language. Γ is an arbitrary set of formulas, semantically, definining what is equal in the following:

$$\mathcal{M}, w \models [\Gamma]\varphi \textit{ iff for all } v, \textit{ if } w =_\Gamma v, \textit{ then } \mathcal{M}, v \models \varphi.$$

They discuss to what extent it formalises von Wright's idea mentioned above and provide a complete axiomatization for the case of finite Γ.

Priorities-based preference: my own work

- Preferences have reasons which are represented as ordered priorities.
- The mathmatical theory of priority graphs by (Andreka et al 2002) applies to preference.
- Dynamical changes can be studied at the two levels of preference: world order and reasons.

These ideas are now being applied in economics, for instance, Dietrich and List 2012.

New ideas for this lecture

(a) Richer structure needed than the "flat sets" (Γ) in the modalities $[\Gamma]\varphi$ of van Benthem, Girard and Roy.

(b) Dynamics is essential to understand ceteris paribus.

Modal preference logic

Definition

A *modal preference model* $\mathcal{M} = (W, \preceq, V)$ has a non-empty set of worlds W, $\preceq$ is a reflexive and transitive relation (pre-order), and V is a valuation for proposition letters.

If $s \preceq t$ but not $t \preceq s$, then t is *strictly preferable* than s ('$s \prec t$').

Definition

The *modal preference language* over propositional variables `Prop` is given by the following inductive syntax rule:

$\varphi := p \mid \neg\varphi \mid \varphi \wedge \psi \mid \langle\leq\rangle\varphi \mid \langle<\rangle\varphi \mid E\varphi.$

Truth definition

Definition

Truth conditions for the atomic propositions and Boolean combinations are standard. Modalities work like this:

- $\mathcal{M}, s \models \langle\leq\rangle\varphi$ iff for some t wih $s \preceq t$, $\mathcal{M}, t \models \varphi$.
- $\mathcal{M}, s \models \langle<\rangle\varphi$ iff for some t with $s \prec t$, $\mathcal{M}, t \models \varphi$.
- $\mathcal{M}, s \models E\varphi$ iff for some world t in W, $\mathcal{M}, t \models \varphi$.

A general view: PDL-programs

Definition

Preference change programs are built from tests for modal preference formulas, weak and strict basic order relations R, $R^<$, and the universal relation $\top$, using arbitrary unions and sequential compositions:

$\pi := ?\varphi \mid R \mid R^< \mid \top \mid ; \mid \cup$

These are interpreted as the standard *PDL* program operations of *test* $?\varphi$, *sequential composition* ; and *choice* $\cup$. Many further relation transformers can be defined in *PDL* format.

Example: radical revision

Definition

Given any modal preference model $(\mathcal{M}, s)$ and formula φ, the *radical revision* $(\mathcal{M}_{\Uparrow\varphi}, s)$ is the model with relations defined as follows in *PDL*-format:

$\Uparrow\varphi(R) := (?\varphi; R; ?\varphi) \cup (?\neg\varphi; R; ?\neg\varphi) \cup (?\neg\varphi; \top; ?\varphi)$.

Here $\top$ denotes the universal relation. Under this transformation, all φ-worlds become better than all $\neg\varphi$-worlds, whether or not they were better before, and within these two zones, the old ordering remains.

Ceteris paribus context

Definition

A *ceteris paribus context* (CP context) $\mathcal{G} = \langle P, < \rangle$ is a strictly partially ordered set of propositions in a language L. It is called a priority graph in (Andreka et al 2002).

Definition

Let $\mathcal{G} = \langle P, < \rangle$ be a CP context, and $\mathcal{M}$ a model in which the language L defines properties of objects. The *preference based on the CP context* $\preceq_{\mathcal{G}}$ is defined as follows:

$y \preceq_{\mathcal{G}} x := \forall P {\in} \mathcal{G}((Py \rightarrow Px) \vee \exists P' {<} P(P'x \wedge \neg P'y))$.

Lexicographic ordering

For *total orders* $\mathscr{G}$, this reduces to lexicographic ordering:

$y \preceq^{lin}_{\mathscr{G}} x$:=
$\forall P \in \mathscr{G}(Px \leftrightarrow Py) \vee \exists P' \in \mathscr{G}(\forall P < P'(Px \leftrightarrow Py) \wedge (P'x \wedge \neg P'y))$.

Representation theorem

Theorem

Let $\mathcal{M} = (W, \preceq, V)$ be any modal model, without constraints on its relation. The following two statements are equivalent:

(*a*) *The relation $\preceq$ is a reflexive and transitive order,*

(*b*) *There is a CP context $\mathscr{G} = (P, <)$ such that, for all worlds $x, y \in W$, $y \preceq x$ iff $y \preceq_{\mathscr{G}} x$.*

For any given preference order, one can find a CP context.

Basic operations on CP context

Two basic operations on CP contexts $\mathscr{G}, \mathscr{G}'$:

- the *sequential composition* $\mathscr{G};\mathscr{G}'$ adds the graph $\mathscr{G}$ on top of $\mathscr{G}'$ in the order: all nodes in the first come before all those in the second,
- the *parallel composition* $\mathscr{G}\|\mathscr{G}'$ is the disjoint union of the graphs $\mathscr{G}$ and $\mathscr{G}'$, without any order links between them.

Basic context updates

Definition

Let 'A' be the CP context with one single node A. The set $\alpha(\mathscr{G}, A)$ of *basic context updates* is defined by:

$$\alpha(\mathscr{G}, A) := A \mid \mathscr{G}_1;\mathscr{G}_2 \mid \mathscr{G}_1\|\mathscr{G}_2.$$

Algebraic equations for CP contexts

Fact

1. $\mathscr{G};\mathscr{G} \equiv \mathscr{G}$.
2. $\mathscr{G} \parallel \mathscr{G} \equiv \mathscr{G}$.
3. $\mathscr{G}_1 \parallel \mathscr{G}_2 \equiv \mathscr{G}_2 \parallel \mathscr{G}_1$.
4. $(\mathscr{G}_1 \parallel \mathscr{G}_2)^< \equiv (\mathscr{G}_1^< \parallel \mathscr{G}_2) \cup (\mathscr{G}_1 \parallel \mathscr{G}_2^<)$.
5. $(\mathscr{G}_1;\mathscr{G}_2)^< \equiv (\mathscr{G}_1^< \cup (\mathscr{G}_1 \parallel \mathscr{G}_2^<))$.

Modal logic of context-induced preferences

Definition

Consider a set `Prop` of propositional variables p, and a set `Nom` of nominals n. Let $\mathbb{G}$ be a set of CP context $\mathscr{G}$. The *modal CP preference language* is defined by the following syntax rule:

$\varphi := n \mid p \mid \neg\varphi \mid \psi \wedge \varphi \mid \langle \mathscr{G} \rangle^{\leq} \varphi \mid \langle \mathscr{G} \rangle^{<} \varphi \mid E\varphi$.

$\mathscr{G} := \mathscr{G}_1;\mathscr{G}_2 \mid \mathscr{G}_1 \parallel \mathscr{G}_2$.

(Girard 2008) axiomatizes this modal graph logic.

Definition

Let α: $(\mathscr{G}, A) \to \mathscr{G}'$, with $\mathscr{G}, \mathscr{G}'$ priority graphs, and A a new proposition which is not in $\mathscr{G}$. Let σ: $(\preceq, A) \to \preceq'$ be a map with $\preceq$ and $\preceq'$ betterness relations over worlds. We say that α *induces* σ, if:

$$\sigma(\preceq_{\mathscr{G}}, A) \quad = \quad \preceq_{\alpha(\mathscr{G},A)}$$

We call *the operation* α *PDL-definable* if it induces a relation transformer σ that is *PDL*-definable in the format afore-mentioned.

Summary

- Representing CP **structure**: Priority graphs
- **Dynamics** of CP context

Earlier scenario 1 revisited

Bob is going to attend the SOCREAL 2013 workshop and is thinking of which hotel he would like to stay. He found two alternatives *a* and *b* to choose from. *a* is cheaper than *b*, but *b* is closer to the workshop venue than *a*. Since he considers cost more important, he prefers *a* over *b*.

$\mathscr{G} = \{C > D\}$

Later Bob realizes that having a swimming pool is very important to him, he reconsiders and now prefers *b* over *a*!

"Put a new consideration on top of the CP context: $A; \mathscr{G}$".

Scenario 2 revisited

Bob's doctor just told him before he leaves for Sapporo that his leg has some problem, and it is better not to walk much. He no longer considers cost more important. Now he prefers *b* over *a*!

"Switch the order of two consecutive priorities".

Our framework can also deal with it, see Chapter 9 of Liu 2011.

Correspondence result

The CP context change also induces preference change, as shown by the fact:

Fact

Prefixing a new proposition A to a CP context $(\mathscr{G}, <)$ induces the radical upgrade operation $\Uparrow A$ on possible worlds models. More precisely, the following diagram commutes:

$$\begin{array}{ccc} \langle \mathscr{G}, < \rangle & \xrightarrow{A;\mathscr{G}} & \langle (A;\mathscr{G}), < \rangle \\ \downarrow & & \downarrow \\ \langle W, \preceq \rangle & \xrightarrow{\Uparrow A} & \langle W, \Uparrow A(\preceq) \rangle \end{array}$$

Basic context updates

Theorem

Basic context updates induce PDL-preference transformers.

We prove by brute enumeration:

Lemma

All basic graph updates reduce to a finite set of cases.

Up to graph equivalences, all basic graph updates reduce to the five cases A, $\mathscr{G}$, $A;\mathscr{G}$, $\mathscr{G};A$, and $A \parallel \mathscr{G}$. They are closed under operations ; and $\parallel$. All these operations indeed induce *PDL*-definable preference transformers.

- Add knowledge and belief. Chapter 5 of (Liu 2011).
- Incoporate normality views of CP. (e.g. Boutilier 1994).
- Connect the current account to other approaches:

  (a) Philosophical analysis in Hansson 1996

  (b) The broader notion of dependence in Väänänen 2007

  (c) Philosophy of science (e.g. Fodor 1991, Schiffer 1991)

  (d) Preference handling in AI and computer science. (e.g. IJCAI workshop 2013)

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Thanks!