特集論文

Frequent Closed Item Set Mining Based on Zero-suppressed BDDs

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Summary

Frequent item set mining is one of the fundamental techniques for knowledge discovery and data mining. In the last decade, a number of efficient algorithms for frequent item set mining have been presented, but most of them focused on just enumerating the item set patterns which satisfy the given conditions, and it was a different matter how to store and index the result of patterns for efficient data analysis. Recently, we proposed a fast algorithm of extracting all frequent item set patterns from transaction databases and simultaneously indexing the result of huge patterns using Zero-suppressed BDDs (ZBDDs). That method, ZBDD-growth, is not only enumerating/listing the patterns efficiently, but also indexing the output data compactly on the memory to be analyzed with various algebraic operations. In this paper, we present a variation of ZBDD-growth algorithm to generate frequent closed item sets. This is a quite simple modification of ZBDD-growth, and additional computation cost is relatively small compared with the original algorithm for generating all patterns. Our method can conveniently be utilized in the environment of ZBDD-based pattern indexing.

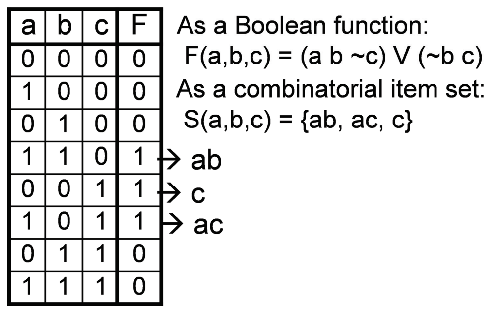
1. Introduction

Frequent item set mining is one of the fundamental techniques for knowledge discovery and data mining. Since the introduction by Agrawal et al.[Agrawal 93], the frequent item set mining and association rule analysis have been received much attentions from many researchers, and a number of papers have been published about the new algorithms or improvements for solving such mining problems[Goethals 03a, Han 04, Zaki 00]. However, most of such item set mining algorithms focused on just enumerating or listing the item set patterns which satisfy the given conditions and it was a different matter how to store and index the result of patterns for efficient data analysis.

Recently, we proposed a fast algorithm[Minato 06] of extracting all frequent item set patterns from transaction databases, and simultaneously indexing the result of huge patterns on the computer memory using Zero-suppressed BDDs. That method, called *ZBDD-growth*, does not only enumerate/list the patterns efficiently, but also indexes the output data compactly on the memory. After mining, the result of patterns can efficiently be analyzed by using algebraic operations.

The key of the method is to use BDD-based data structure for representing sets of patterns. BDDs[Bryant 86] are graph-based representation of Boolean functions, now widely used in VLSI logic design and verification area. For the data mining applications, it is important to use Zero-suppressed BDDs (ZBDDs)[Minato 93], a special type of BDDs, which are suitable for handling large-scale sets of combinations. Using ZBDDs, we can implicitly enumerate combinatorial item set data and efficiently compute set operations over the ZBDDs.

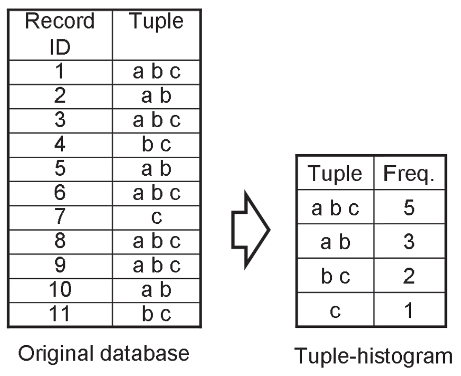
In this paper, we present an interesting variation of ZBDD-growth algorithm to generate frequent closed item sets. Closed item sets are the subset of item set patterns each of which is the unique representative for a group of sub-patterns relevant to the same set of transaction records. Our method is a quite simple modification of ZBDD-growth. We inserted several operations in the recursive procedure of ZBDD-



a	b	c	F	
0	0	0	0	
1	0	0	0	
0	1	0	0	
1	1	0	1	→ ab
0	0	1	1	→ c
1	0	1	1	→ ac
0	1	1	0	
1	1	1	0	

As a Boolean function:
F(a,b,c) = (a b ~c) V (~b c)
As a combinatorial item set:
S(a,b,c) = {ab, ac, c}

Fig. 1 A Boolean function and a combinatorial item set.



Original database

Record ID	Tuple
1	a b c
2	a b
3	a b c
4	b c
5	a b
6	a b c
7	c
8	a b c
9	a b c
10	a b
11	b c

Tuple-histogram

Tuple	Freq.
a b c	5
a b	3
b c	2
c	1

Fig. 3 Example of tuple-histogram.

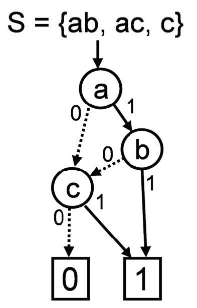
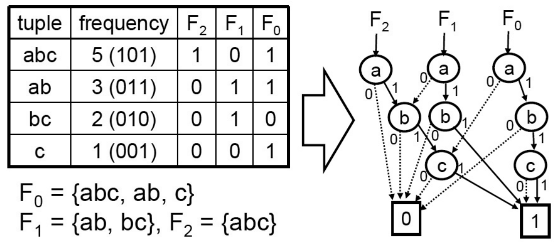


Fig. 2 An example of ZBDD.



tuple	frequency	F_2	F_1	F_0
abc	5 (101)	1	0	1
ab	3 (011)	0	1	1
bc	2 (010)	0	1	0
c	1 (001)	0	0	1

F_0 = {abc, ab, c}
F_1 = {ab, bc}, F_2 = {abc}

Fig. 4 ZBDD vector for tuple-histogram.

growth, to filter the closed patterns from all frequent patterns. The experimental result shows that the additional computation cost is relatively small compared with the original algorithm for generating all patterns. Our method can conveniently be utilized in the environment of ZBDD-based data mining and knowledge indexing.

2. ZBDD-based item set representation

As the preliminary section, we describe the methods for efficiently indexing item set data based on Zero-suppressed BDDs.

2·1 Combinatorial item set and ZBDDs

A combinatorial item set consists of the elements each of which is a combination of a number of items. There are 2^n combinations chosen from n items, so we have 2^{2^n} variations of combinatorial item sets. For example, for a domain of five items a, b, c, d, and e, we can show examples of combinatorial item sets as: $\{ab, e\}$, $\{abc, cde, bd, acde, e\}$, $\{1, cd\}$, 0. Here "1" denotes a combination of null items, and "0" means an empty set. Combinatorial item sets are one of the basic data structure for various problems in computer science, including data mining.

A combinatorial item set can be mapped into Boolean space of n input variables. For example, Figure 1 shows a truth table of Boolean function: $F = (a\,b\,\overline{c}) \vee (\overline{b}\,c)$, but also represents a combinatorial item set $S = \{ab, ac, c\}$. Using BDDs for the corresponding Boolean functions, we can implicitly represent and manipulate combinatorial item set. In addition, we can enjoy more efficient manipulation using "Zero-suppressed BDDs" (ZBDD)[Minato 93], which are special type of BDDs optimized for handling combinatorial item sets. An example of ZBDD is shown in Figure 2.

The detailed techniques of ZBDD manipulation are described in the articles[Minato 93]. A typical ZBDD package supports cofactoring operations to traverse 0-edge or 1-edge, and binary operations between two combinatorial item sets, such as union, intersection, and difference. Our ZBDD package generates new ZBDD nodes in the main memory, as the results of these algebraic operations. The computation time for each operation is almost linear to the number of ZBDD nodes related to the operation. We can also delete a ZBDD which has become useless, and such garbage ZBDD nodes are efficiently collected to be used for another new ZBDD.

2·2 Tuple-Histograms and ZBDD vectors

A *Tuple-histogram* is the table for counting the number of appearance of each tuple in the given database. An example of tuple-histogram is shown in Figure 3. This is just a compressed table of the database to combine the same tuples appearing more than once into one line with the frequency.

Our item set mining algorithm manipulates ZBDD-based tuple-histogram representation as the internal data structure. Here we describe how to represent tuple-histograms using ZBDDs. Since ZBDDs are representation of sets of combinations, a simple ZBDD distinguishes only existence of each tuple in the database. In order to represent the numbers of tuple's appearances, we decompose the number into m-digits of ZBDD vector $\{F_0, F_1, \ldots, F_{m-1}\}$ to represent integers up to $(2^m - 1)$, as shown in Figure 4. Namely, we encode the appearance numbers into binary digital code, as F_0 represents a set of tuples appearing odd times (LSB = 1), F_1 represents a set of tuples whose appearance number's second lowest bit is 1, and similar way we define the set of each digit up to F_{m-1}.

In the example of Figure 4, The tuple frequencies are decomposed as: $F_0 = \{abc, ab, c\}$, $F_1 = \{ab, bc\}$, $F_2 = \{abc\}$, and then each digit can be represented by a simple ZBDD. The three ZBDDs shares their sub-graphs each other.

Now we explain the procedure for constructing a ZBDD-based tuple-histogram from original database. We read a tuple data one by one from the database, and accumulate the single tuple data to the histogram. More concretely, we generate a ZBDD of T for a single tuple picked up from the database, and accumulate it to the ZBDD vector. The ZBDD of T can be obtained by starting from "**1**" (a null-combination), and applying "Change" operations several times to join the items in the tuple. Next, we compare T and F_0, and if they have no common parts, we just add T to F_0. If F_0 already contains T, we eliminate T from F_0 and carry up T to F_1. This ripple carry procedure continues until T and F_k have no common part. After finishing accumulations for all data records, the tuple-histogram is completed.

Using the notation F.add(T) for addition of a tuple T to the ZBDD vector F, we describe the procedure of generating tuple-histogram H for given database D.

```
  $H = \mathbf{0}$
  forall $T \in D$ do
      $H = H$.add($T$)
  return $H$
```

When we construct a ZBDD vector of tuple-histogram, the number of ZBDD nodes in each digit is bounded by total appearance of items in all tuples. If there are many partially similar tuples in the database, the subgraphs of ZBDDs are shared very well, and compact representation is obtained. The bit-width of ZBDD vector is bounded by $\log S_{max}$, where S_{max} is the appearance of most frequent items.

Once we have generated a ZBDD vector for the tuple-histogram, various operations can be executed efficiently. Here are the instances of operations used in our pattern mining algorithm.

- H.factor0(v): Extracts sub-histogram of tuples without item v.
- H.factor1(v): Extracts sub-histogram of tuples including item v and then delete v from the tuple combinations. (also considered as the quotient of H/v)
- $v \cdot H$: Attaches an item v on each tuple combinations in the histogram F.
- $H_1 + H_2$: Generates a new tuple-histogram with sum of the frequencies of corresponding tuples.
- H.tuplecount: The number of tuples appearing at least once.

These operations can be composed as a sequence of ZBDD operations. The result is also compactly represented by a ZBDD vector. The computation time is bounded by roughly linear to total ZBDD sizes.

3. ZBDD-growth Algorithm

Recently, we developed a ZBDD-based algorithm[Minato 06], ZBDD-growth, to generate "all" frequent item set patterns. Here we describe this algorithm as the basis of our method for "closed" item set mining.

ZBDD-growth is based on a recursive depth-first search over the ZBDD-based tuple-histogram representation. The basic algorithm is shown in Figure 5, where H is a ZBDD for the tuple-histogram of the given database, and α is a given number of minimum support threshold.

In this algorithm, we choose an item v used in H, and compute the two sub-histograms H_1 and H_0. (Namely, $H = (v \cdot H_1) \cup H_0$.) Since we always choose v at the highest position in the ZBDD vector, H_1 and H_0 can be obtained just by referring the 1-edge and 0-edge of the highest ZBDD-node, and the computation time for factoring each digit of ZBDD is a constant

```
ZBDDgrowth($H, \alpha$)
{
   if($H$ has only one item $v$)
      if($v$ appears more than $\alpha$ )
         return $v$ ;
      else return "0" ;
   $F \leftarrow$ Cache($H$) ;
   if($F$ exists) return $F$ ;
   $v \leftarrow H.top$ ; /* Top item in $H$ */
   $H_1 \leftarrow H$.factor1($v$) ;
   $H_0 \leftarrow H$.factor0($v$) ;
   $F_1 \leftarrow$ZBDDgrowth($H_1, \alpha$) ;
   $F_0 \leftarrow$ZBDDgrowth($H_0 + H_1, \alpha$) ;
   $F \leftarrow (v \cdot F_1) \cup F_0$ ;
   Cache($H$) $\leftarrow F$ ;
   return $F$ ;
}
```

Fig. 5 ZBDD-growth algorithm.

to the ZBDD size.

The number of appearance of v in H can be calculated by seeing the total number of records in H_1. This computation is done in a linear time to the total ZBDD size of H_1. The heaviest operation in ZBDD-growth algorithm is to generate ZBDD for $(H_0 + H_1)$. This requires union and intersection operations for each corresponding digit of H_0 and H_1, and the result of ZBDD sometimes grows large depending on the data property. Anyway, the total computation time is linearly bounded by the size of ZBDDs related to the operations.

The algorithm consists of the two recursive calls, one of which computes the subset of patterns including v, and the other computes the patterns excluding v. The two subsets of patterns can be obtained as a pair of pointers to ZBDDs, and then the final result of ZBDD is computed. This procedure may require an exponential number of recursive calls, however, we prepare a hash-based cache, Cache(H), to store the result of each recursive call. Each entry in the cache is formed as pair (H, F), where H is the pointer to the ZBDD vector for a given tuple-histogram, and F is the pointer to the result of ZBDD. On each recursive call, we check the cache to see whether the same histogram H has already appeared, and if so, we can avoid duplicate processing and return the pointer to F directly. By using this technique, the computation time becomes almost linear to the final (may be largest) ZBDD size.

In our implementation, we use some simple techniques to prune the search space. For example, if H_1 and H_0 are equivalent, we may skip to compute F_0. For another case, we can stop the recursive calls if total frequencies in H is no more than α. There are some other elaborate pruning techniques, but they needs additional computation cost for checking the conditions, so sometimes effective but not always.

4. Frequent closed item set mining

In frequent item set mining, we sometimes faced with the problem that a huge number of frequent patterns are extracted and hard to find useful information. Closed item set mining is one of the techniques to filter important subset of patterns. In this section, we present a variation of ZBDD-growth algorithm to generate frequent closed item sets.

4·1 Closed item sets

Closed item sets are the subset of item set patterns each of which is the unique representative for a group of sub-patterns relevant to the same set of tuples. For more clear definition, we first define the *common item set* $Com(S_T)$ for the given set of tuples S_T, such that $Com(S_T)$ is the set of items commonly included in every tuple $T \in S_T$. Next, we define *occurence* $Occ(D, X)$ for the given database D and item set X, such that $Occ(D, X)$ is the subset of tuples in D, each of which includes X. Using these notations, if an item set X satisfies $Com(Occ(D, X)) = X$, we call X is a closed item set in D.

For example, let us consider the database D as shown in Figure 3. Here, all item set patterns with threshold $\alpha = 1$ is: $\{abc, ab, ac, a, bc, b, c\}$, but closed item sets are: $\{abc, ab, bc, b, c\}$. In this example, "ac" is eliminated from a closed pattern because $Occ(D, \text{“}ac\text{”}) = Occ(D, \text{“}abc\text{”})$.

In recent years, many researchers discuss the efficient algorithms for closed item set mining. One of the remarkable result is *LCM* algorithm[Uno 03] presented by Uno et. al. LCM is a depth-first search algorithm to extract closed item sets. It features that the computation time is bounded by linear to the output data length. Our ZBDD-based algorithm is also based on a depth-first search manner, so, it has similar properties as LCM. The major difference is in the data structure of output data. Our method generates ZBDDs for the set of closed patterns, ready to go for more flexible analysis using ZBDD operations.

```
$P$.permit($Q$)
{
    if($P$ =“0” or $Q$ =“0”) return “0” ;
    if($P = Q$) return $P$ ;
    if($P$ =“1”) return “1” ;
    if($Q$ =“1”)
        if($P$ include “1” ) return “1” ;
        else return “0” ;
    $R \leftarrow$ Cache($P, Q$) ;
    if($R$ exists) return $R$ ;
    $v \leftarrow$TopItem($P, Q$) ; /* Top item in $P, Q$ */
    $(P_0, P_1) \leftarrow$factors of $P$ by $v$ ;
    $(Q_0, Q_1) \leftarrow$factors of $Q$ by $v$ ;
    $R \leftarrow (v \cdot P_1$.permit$(Q_1))$
            $\cup (P_0$.permit$(Q_0 \cup Q_1))$ ;
    Cache($P, Q$) $\leftarrow R$ ;
    return $R$ ;
}
```

Fig. 6 Permit operation.

```
ZBDDgrowthC($H, \alpha$)
{
    if($H$ has only one item $v$)
        if($v$ appears more than $\alpha$ )
            return $v$ ;
        else return “0” ;
    $F \leftarrow$ Cache($H$) ;
    if($F$ exists) return $F$ ;
    $v \leftarrow H.top$ ; /* Top item in $H$ */
    $H_1 \leftarrow H$.factor1($v$) ;
    $H_0 \leftarrow H$.factor0($v$) ;
    $F_1 \leftarrow$ZBDDgrowthC($H_1, \alpha$) ;
    $F_0 \leftarrow$ZBDDgrowthC($H_0 + H_1, \alpha$) ;
    $F \leftarrow (v \cdot F_1) \cup$
        $(F_0 - (F_1 - F_1$.permit$(H_0)))$ ;
    Cache($H$) $\leftarrow F$ ;
    return $F$ ;
}
```

Fig. 7 ZBDD-growthC algorithm.

4·2 Eliminating non-closed patterns

Our method is a quite simple modification of ZBDD-growth shown in Figure 5. We inserted several operations in the recursive procedure of ZBDD-growth, to filter the closed patterns from all frequent patterns. The ZBDD-growth algorithm is starting from the given tuple-histogram H, and computes the two sub-histograms H_1 and H_0, such that $H = (v \cdot H_1) \cup H_0$. Then ZBDD-growth($H_1$) and ZBDD-growth($H_1 + H_0$) is recursively executed.

Here, we consider the way to eliminate non-closed patterns in this algorithm. We call the new algorithm ZBDD-growthC(H). It is obvious that ($v\cdot$ ZBDD-growthC(H_1)) generates (a part of) closed patterns for H each of which includes v, because the occurrence of any closed pattern with v is limited in $(v \cdot H_1)$, thus we may search only for H_1. Next, we consider the second recursive call ZBDD-growthC($H_1 + H_0$) to generate the closed patterns without v. Important point is that some of patterns generated by ZBDD-growthC($H_1 + H_0$) may have the same occurrence as one of the patterns with v already found in H_1. The condition of such duplicate pattern is that it appears only in H_1 but irrelevant to H_0. In other words, we eliminate the patterns from ZBDD-growthC($H_1 + H_0$) such that the patterns are already found in ZBDD-growthC(H_1) but not included in any tuples in H_0.

For checking the condition for closed patterns, we can use a ZBDD-based operation, called *permit* operation by Okuno et al.[Okuno 98].*1 P.permit(Q) returns a set of combinations in P each of which is a subset of some combinations in Q. For example, when $P = \{ab, abc, bcd\}$ and $Q = \{abc, bc\}$, then P.permit(Q) returns $\{ab, abc\}$. The permit operation is efficiently implemented as a recursive procedure of ZBDD manipulation, as shown in Figure 6. The computation time of permit operation is almost linear to the ZBDD size.

Finally, we describe the ZBDD-growthC algorithm using the permit operation, as shown in Figure 7. The difference from the original algorithm is only one line, written in the frame box.

5. Experimental Results

Here we show the experimental results to evaluate our new method. We used a Pentium-4 PC, 800MHz, 1.5GB of main memory, with SuSE Linux 9. We can deal with up to 40,000,000 nodes of ZBDDs in this machine.

Table 1 shows the time and space for generating ZBDD vectors of tuple-histograms for the FIMI2003 benchmark databases[Goethals 03b]. This table shows the computation time and space for providing input data for ZBDD-growth algorithm. In this table, #T shows the number of tuples, $total|T|$ is the total of tuple sizes (total appearances of items), and |ZBDD| is the number of ZBDD nodes for the tuple-histograms. We can see that tuple-histograms can be constructed

*1 Permit operation is basically same as *SubSet* operation by Coudert et al.[Coudert 93], defined for ordinary BDDs.

Table 1 Generation of tuple-histograms[Minato 06].

Data name	$\#T$	$total\|T\|$	|ZBDD Vector|	Time(s)
T10I4D100K	100,000	1,010,228	552,429	55.6
T40I10D100K	100,000	3,960,507	3,396,395	155.73
chess	3,196	118,252	40,028	0.98
connect	67,557	2,904,951	309,075	33.19
mushroom	8,124	186,852	8,006	1.09
pumsb	49,046	3,629,404	1,750,883	167.10
pumsb_star	49,046	2,475,947	1,324,502	125.17
BMS-POS	515,597	3,367,020	1,350,970	1,317.07
BMS-WebView-1	59,602	149,639	46,148	19.55
BMS-WebView-2	77,512	358,278	198,471	184.83
accidents	340,183	11,500,870	3,877,333	128.47

Table 2 Result of the original ZBDD-growth[Minato 06].

Data name: Min. freq. α	#Frequent patterns	(output) |ZBDD|	Time(sec)
mushroom: 5,000	41	11	0.06
1,000	123,277	1,417	2.60
200	18,094,821	12,340	8.60
50	198,169,865	36,652	9.11
16	1,176,182,553	53,804	6.59
4	3,786,792,695	59,970	3.18
1	5,574,930,437	40,557	0.68
T10I4D100K: 5,000	10	10	20.51
1,000	385	382	75.81
200	13,255	4,288	223.78
50	53,385	20,364	365.52
16	175,915	89,423	500.09
4	3,159,067	1,108,723	602.83
BMS-WebView1: 1,000	31	31	2.43
200	372	309	5.39
50	8,191	3,753	28.55
40	48,543	12,176	53.15
36	461,521	34,790	84.11
35	1,177,607	47,457	93.14
34	4,849,465	64,601	102.55
33	69,417,073	80,604	111.69
32	1,531,980,297	97,692	115.42
31	8,796,564,756,112	117,101	119.80
30	35,349,566,550,691	152,431	125.58

for all instances in a feasible time and space. The ZBDD sizes are almost same or less than $total|T|$.

After generating ZBDD vectors for the tuple-histograms, we applied ZBDD-growth algorithm to generate frequent patterns. Table 2 show the results of the original ZBDD-growth algorithm[Minato 06] for the selected benchmark examples, "mushroom," "T10I4D100K," and "BMS-WebView-1." The execution time does not include the time for generating the initial ZBDD vectors for tuple-histograms.

The results shows that the ZBDD size is exponentially smaller than the number of patterns for "mushroom." This is a significant effect of using the ZBDD data structure. On the other hand, no remarkable reduction is seen in "T10I4D100K." "T10I4D100K" is known as an artificial database, consists of randomly generated combinations, so there are almost no relationship between the tuples. In such cases, ZBDD nodes cannot be shared well, and only the overhead factor is revealed. For the third example, "BMS-WebView-1," the ZBDD size is almost linear to the number of patterns when the output size is small, however, an exponential factor of reduction is observed for the cases of generating huge patterns.

Next, we show the experimental results of frequent closed pattern mining using ZBDD-growthC algorithm. In Table 3, we show the results for the same examples as used in the experiment of the original ZBDD-growth. The last column $Time_{(closed)}/Time_{(all)}$ shows the ratio of computation time between the ZBDD-growthC and the original ZBDD-growth algorithm. We can observe that the computation time is almost the same order as the original algorithms for "mushroom" and "BMS-WebView-1," but some additional factor is observed for "T10I4D100K." Anyway, filtering closed item sets has been regarded as not a easy task. We can say that the ZBDD-growthC algorithm can generate closed item sets with a relatively small additional cost from the original ZBDD-growth.

Finally, we compared our results with a state-of-the-art implementation of the LCM algorithm[Uno 03] on the same PC. The results are shown in the right most column of Table 3. We can observe that the LCM-based program is more than a hundred times faster than our ZBDD-based program. The LCM algorithm features that the computation time is lin-

Table 3 Results of ZBDD-based closed pattern mining.

Data name: Min. freq. α	#Freq. closed patterns	(output) |ZBDD|	ZBDD-growthC Time(s)	$Time_{(closed)}$ $/Time_{(all)}$	LCM[Uno 03] Time(s)
mushroom: 5,000	16	16	0.06	1.00	0.01
1,000	3,427	1,660	2.75	1.06	0.14
200	26,968	9,826	8.84	1.03	0.32
50	68,468	19,054	11.90	1.31	0.53
16	124,411	24,841	12.24	1.84	0.68
4	203,882	26,325	12.07	3.80	0.77
1	238,709	20,392	11.85	17.43	0.79
T10I4D100K: 5,000	10	10	43.94	2.14	0.06
1,000	385	382	145.03	1.91	0.56
200	13,108	4,312	2,657.40	11.88	0.98
50	46,993	20,581	4,556.90	12.47	1.24
16	142,520	89,185	5,755.32	11.51	1.56
4	1,023,614	691,154	18,529.82	30.74	3.97
BMS-WebView-1: 1,000	31	31	3.95	1.62	0.03
200	372	309	15.22	2.82	0.09
50	7,811	3,796	46.02	1.61	0.15
40	29,489	11,748	87.14	1.64	0.22
36	64,762	25,117	135.39	1.61	0.36
35	76,260	30,011	150.87	1.62	0.43
34	87,982	35,392	168.18	1.64	0.51
33	99,696	40,915	189.42	1.70	0.63
32	110,800	46,424	203.40	1.76	0.78
31	120,190	51,369	229.40	1.91	0.95
30	127,131	55,407	253.15	2.02	1.20

early bounded by the total size of closed item sets. In addition, the implementation is highly optimized for enumerating the number of closed item sets without printing out. On the other hand, ZBDD-based method is especially effective when a huge number of item sets produced and they are compactly represented by small size of ZBDDs. In general, closed item sets are already reduced representation of all item sets, and in such cases, ZBDD-based compression is not very effective.

If our final goal is only enumerating closed item sets, the LCM algorithm would be much better. However, our ZBDD-based method does not only enumerate but also constructs efficient data structures for indexing the result of item sets, and they can be used for various data analysis with ZBDD-based algebraic set operations. Here we show several examples of useful post-processing.

(Extracting all non-closed patterns): After executing ZBDD-growth and -growthC, we can easily obtain a set of non-closed patterns by applying a difference operation between the two ZBDDs of all item sets and closed item sets.

(Filtering closed patterns with sub-patterns): By using "factor1(v)" operation, we can filter the subset of closed patterns including an item v. Repeating this operations, "key-word filtering" of closed patterns can be executed.

(Filtering closed patterns by a permissible set): When a set of permissible patterns are given as a ZBDD, we can filter the closed patterns each of which satisfies the constraint given by the ZBDD. For example, we first generate a ZBDD representing a set of patterns each of which contains exactly three items. Next we generate all closed patterns by ZBDD-growthC. Then, an intersection operation between the two ZBDDs extracts all closed patterns each of which consists of three items.

6. Conclusion

In this paper, we presented a variation of ZBDD-growth algorithm to generate frequent closed item sets. Our method is a quite simple modification of ZBDD-growth. We inserted several operations in the recursive procedure of ZBDD-growth, to filter the closed patterns from all frequent patterns. The experimental result shows that the additional computation cost is relatively small compared with the original algorithm for generating all patterns.

In order to just enumerate the closed patterns, our method is not faster than LCM algorithm, which is an existing state-of-the-art method. The main reason is that the closed patterns are already a kind of reduced information of the frequent patterns. In such cases, the ZBDD-based data compressing power is not very effective, and only the overhead factor reveals.

However, our method is still useful because it does not only enumerate but also constructs an efficient indexing data structure for all set of closed patterns, and we can apply various data analysis tasks using the ZBDD-based algebraic operations. In addition, it will be an interesting future work to deeply combine the techniques of ZBDD manipulation and the LCM algorithm.

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