Wavelet-transform analysis of spectral shearing interferometry for phase reconstruction of femtosecond optical pulses

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Abstract: We introduce a novel method for retrieving the phase from a spectral shearing interferogram, based on wavelet-transform technique. We demonstrate with both theoretical and experimental data that this technique provides an alternative and reliable technique for phase retrieval, particularly for highly structured pulse spectra.

OCIS codes: (320.7090) Ultrafast Lasers, (100.5070) Phase retrieval, (120.5050) Phase measurement, (100.7410) Wavelets

References and links

1. K. Yamane, T. Kito, R. Morita and M. Yamashita, "2.8-fs transform-limited optical-pulse generation in the monocycle region," in *Conf. Lasers and Electro-Optics* (Optical Society of America, Washington, D.C., 2004), postdeadline paper PDC2.
2. M. Takeda, H. Ina, and S. Kobayashi, "Fourier-transform method of fringe-pattern analysis for computer-based topography and interferometry." J. Opt. Soc. Am. **72**, 156-160 (1982).
3. C. Iaconis and I. A. Walmsley, "Self-referencing spectral interferometry for measuring ultrashort optical pulses", IEEE J. Quantum Electron. **35**, 501-509 (1999).
4. M. E. Anderson, L. E. E. de Araujo, E. M. Kosik, I. A. Walmsley, "The effects of noise on ultrashort-optical-pulse measurement using SPIDER," Appl. Phys. B **70**, S85-S93 (2000).
5. S. Jensen and M. E. Anderson, "Measuring ultrashort optical pulses in the presence of noise: an empirical study of the performance of spectral phase interferometry for direct electric field reconstruction," Appl. Opt. **43**, 883-893 (2004).
6. B. Schenkel, J. Biegert, and U. Keller, C. Vozzi, M. Nisoli, G. Sansone, S. Stagira, S. De Silvestri, and O. Svelto, "Generation of 3.8-fs pulses from adaptive compression of a cascaded hollow fiber supercontinuum," Opt. Lett. **28**, 1987-1989 (2003).
7. C. P. Hauri , W. Kornelis, F. W. Helbing, A. Heinrich, A. Couairon, A. Mysyrowicz, J. Biegert, and U. Keller, "Generation of intense, Carrier-envelope phase-locked few-cycle laser pulses through filamentation," Appl. Phys. B **79**, 673-677 (2004).
8. K. Yamane, Z. Zhang, K. Oka, R. Morita, M. Yamashita, and A. Suguro, "Optical pulse compression to 3.4 fs in the monocycle region by feedback phase compensation," Opt. Lett. **28**, 2258-2260 (2003).
9. C. K. Chui ed., *An introduction to wavelets* (Academic Press, Boston, 1992).
10. B. Telfer and H. H. Szu, "New wavelet transform normalization to remove frequency bias," Opt. Eng. **31**, 1830-1834 (1992).

11. Y. Morimoto, Y. Imamoto, "Application of wavelet transform to displacement and strain measurement by grid method," in *Proc. of 1995 SEM Spring Conf. on Exp. Mech.* (Society for Experimental Mechanics, MI, 1995), 898-903.
12. L. R. Watkins, S. M. Tan, and T. H. Barnes, "Determination of interferometer phase distributions by use of wavelets," Opt. Lett. **24**, 905-907 (1999).
13. Y. Q. Deng, X. H. Ji, Y. W. Qin, J. L. Chen, "Application of wavelet-transform to measurement of phase of isodyne fringe," in *Proc. SPIE - The International Society for Optical Engineering*, **5286**, 277-281 (2003).
14. J. -C. Hong and Y. Y. Kim, "Determination of the optimal Gabor wavelet shape for the best time-frequency localization using the entropy concept," Society for Exp. Mech. **44**, 387-395 (2004).
15. D. J. Kane and R. Trebino "Single-shot measurement of the intensity and phase of an arbitrary ultrashort pulse by using frequency-resolved optical gating," Opt. Lett. **18**, 823-825 (1993).
16. K. W. DeLong, R. Trebino, J. Hunter, and W. E. White, "Frequency- resolved optical gating with the use of second-harmonic generation", J. Opt. Soc. Am. B **11**, 2206-2215 (1994).

1. Introduction

Ultrashort laser pulses in visible and near infrared have broken through 3fs barrier with precise phase compensation [1]. In generation of such pulses, the precision measurement of the spectral phase is the key when using measured phase for feedback compensation. Based on Fourier-transform (FT) technique [2], spectral phase interferometry for direct electric-field reconstruction (SPIDER) has been shown to be a reliable technique for phase measurement of ultra-broadband femtosecond pulses [3-5]. In conventional SPIDER procedure, the frequency domain interferogram is Fourier-transformed to a time domain. This procedure yields a function of three peaks. The AC component peaked at $t = +\tau$ is selected using a filter (e.g. Gaussian function) and then inversely Fourier-transformed to the frequency domain. A concatenation procedure is used to recover the phase [3]. The reliability of this procedure was examined by Anderson and Jensen *et al*. [4, 5]. They proved that SPIDER is robust against noise for a large range of filter windows. However, their analysis was performed for only very smooth and narrow spectra (10.3 THz bandwidth [4]), and relatively broad temporal pulses (50fs [5]) respectively, while the mono-cycle pulses could have a bandwidth of as broad as 500 THz structured spectra. In few-cycle and mono-cycle regime, the pulse spectrum becomes extremely broad and complicated. This is because the conventional ultrashort pulse lasers are unable to produce enough bandwidth to support such pulses; the spectrum broadening is usually obtained by passing intense femtosecond pulses through gas filled hollow fibers [6], or through filamentary propagation [7]. Those processes are usually associated with complicated structures. Furthermore, when the phase compensation is made by a liquid crystal spatial light modulator (LC-SLM), the pulse spectrum could display very sharp modulations if the applied phase difference between the adjacent pixels is over the maximum phase a pixel can offer, for example, 2π [8]. To our experience, because the modulation period is somewhere similar to the fringe spacing, the transform of this modulation will be close to the AC signal, and a temporal filter cannot exactly exclude such noise. Therefore, the retrieved phase, and as a result the pulse profile, is more or less dependent on the width of the temporal filter.

In this paper, we introduce a novel technique for extracting the phase from a SSI [3], based on wavelet-transform (WT) [9, 10], for the first time to the best of our knowledge. We show the calculation procedure of phase reconstruction through SSI signal by WT, with both theoretical and experimental SSI data from SPIDER measurements, and discuss some important features of WT graphics. We demonstrate that this technique provides an alternative and reliable technique for phase retrieval from SSI, particularly for highly structured pulse spectra. This technique should be useful in generation and characterization of ultrashort and monocycle pulses.

2. Mathematics description of wavelet-transform

Wavelet-transform, as a rising branch of mathematics, has become a powerful tool for signals analysis, especially for analysis of signal with complicated frequency components. In principle, WT converts one-dimensional time signal into two-dimensional time-frequency topography, which displays the time and frequency information of the signal in an apparent time-frequency plane. When a complex wavelet is used as mother wavelet, WT of the signal is also complex; therefore, there come two topographies of WT: magnitude topography and phase topography. Magnitude topography reflects the frequency intensities at each time point; and phase topography reflects phase of frequencies at corresponding time point. All the time and frequency information of the signal shows readily apparent on the two topographies; therefore, one could directly read out the phase at the corresponding frequency in the phase topography. This phase retrieval technique has been verified in reference [11-13].

Here we describe the mathematics of complex WT and show how it is applied in phase retrieval from SSI.

The basis functions of WT called daughter wavelets, which are generated by dilation and translation from a mother wavelet $\psi(\omega)$. In order to apply WT technique on the SSI analysis, the WT of the SSI function $f(\omega')$ can take the following form:

$$W(\Delta\omega,\omega)=\frac{1}{\Delta\omega}\int_{-\infty}^{+\infty}f(\omega')\psi^*(\frac{\omega'-\omega}{\Delta\omega})d\omega' \tag{1}$$

where $\psi^*[(\omega'-\omega)/\Delta\omega]/\Delta\omega$ is the complex conjugation of daughter wavelet $\psi[(\omega'-\omega)/\Delta\omega]/\Delta\omega$, $\Delta\omega$ is a dilation factor that represents a variable width of the daughter wavelet, for measuring the spacing of the interferogram, and ω the translation factor that shifts the peak of the daughter wavelet along the interferogram.

In SSI, the relationship between time and spacing of the interferogram can be expressed as:

$$t=2\pi/\Delta\omega \tag{2}$$

Therefore, WT of SSI can take another form:

$$W(t,\omega)=\frac{t}{2\pi}\int_{-\infty}^{+\infty}f(\omega')\psi^*[\frac{(\omega'-\omega)\cdot t}{2\pi}]d\omega' \tag{3}$$

Unlike FT, the basis of WT is not unique. In this research, we choose Gabor wavelet (also called Gaussian wavelet) as the mother wavelet. The reason is that Gabor wavelet has the least spread in both frequency and time domain; therefore it has the best time-frequency resolution [14]. The expression of the Gabor wavelet is:

$$\psi(\omega')=e^{(-\omega'^2/2\sigma^2+i2\pi\omega')}/(\sigma^2\pi)^{1/4} \tag{4}$$

where $\sigma=(2\ln 2)^{-1/2}$ is a constant.

3. Phase extraction procedure and graphics of wavelet-transform

To demonstrate the phase analysis procedure of SSI signal using WT, we constructed a SSI signal with an assumed femtosecond pulse at a central wavelength of 800nm and a bandwidth of 100nm, for a *zero phase*. The spectral shear and the time delay between two replicas for this SSI are of Ω=25×10^{12}rad/s and τ=0.7ps respectively. Then we performed WT of such a SSI through Eq. (1). The time delay is discretized from 0.1ps to 1.3ps, at a step of 0.033ps, and ω is discretized in the space of 0.1THz.

As mentioned above, the complex WT of SSI produces simultaneously two graphs: the magnitude topography (also called WT trace) and the phase topography, which are shown in Fig. 1(a) and Fig. 1(b). In Fig. 1(a), the case of the SSI analysis with WT, the frequency means the frequency of the sum waves (not the fundamental wave) and the vertical axis is its relative delay (not the real time). In Fig. 1(b), the vertical axis is the wrapped phase. The two-dimensional information is more easily and intuitively interpreted for the second-order dispersion and the higher-order dispersion of the spectral phase in time-frequency space.

#6316 - $15.00 US Received 13 January 2005; revised 25 February 2005; accepted 12 March 2005

To extract this phase, a program is applied to search for the maximum magnitude of the WT trace for each carrier frequency point, which results in a straight line (pink colored) in Fig. 1(a). The phase along the line in Fig. 1(b) that is projected from Fig. 1(a) are the wrapped phase $\varphi(\omega)-\varphi(\omega+\Omega)+\omega\tau$. The rest procedure for extracting the spectral phase $\varphi(\omega)$ will be exactly the same as the standard SPIDER concatenation algorithm. As expected, this procedure results in a flat phase. This calculation procedure does not require a filter and the second transform.

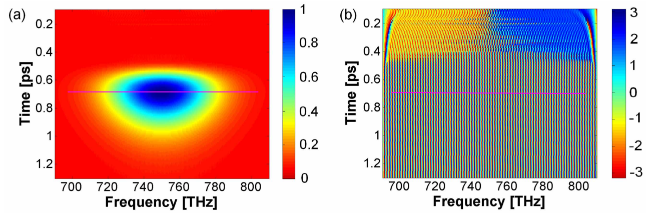
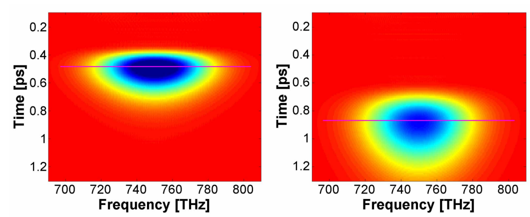
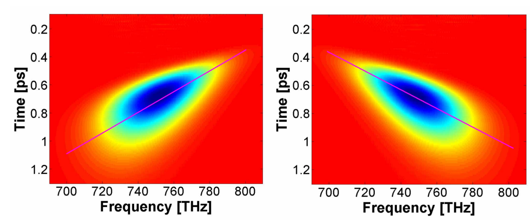


Fig. 1. WT graphics for zero phases: (a) magnitude topography. (b) phase topography.

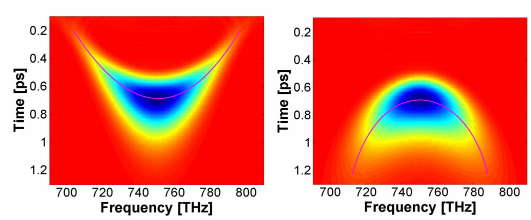
The same procedure can be applied to more complicated phase. Some typical WT traces with high order phases are shown in Fig. 2 (Phase topographies are not plotted here). The pink colored lines or curves superimposed in the figures indicate the maximum magnitude of the WT traces.



(a) WT traces for quadratic phase: left: $-4\times10^3 fs^2$; right: $+4\times10^3 fs^2$.



(b) WT traces for cubic phase: left: $-6\times10^3 fs^3$; right: $+6\times10^3 fs^3$.



(c) WT traces for quartic phase: left: $-2\times10^4 fs^4$; right: $+2\times10^4 fs^4$.

Fig. 2. WT graphics for high order phases.

Figure 2(a) is for the phase of negative and positive quadratic phase $\mp 4\times10^3 fs^2$. The pink colored line reveals that the negative and positive phases are shifted to higher or lower positions with respect to zero phase τ =0.7ps. In Fig. 2(b), the WT traces and the lines for

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maximum magnitude are tilted showing the cubic phase ($\mp 6\times10^3$fs^3). Notice that they are tilted in opposite directions. Subsequently, the quadratic curves in Fig. 2(c) represent quartic phase ($\mp 2\times10^4$fs^4). Therefore, the phase of a femtosecond pulse can be identified from the position, slope, or curvature of WT traces.

We operate the phase retrieval procedure described before. For all assumed phases, RMS error with the assumed phase is of <0.02rad, which is no more than the error with Fourier technique.

4. Wavelet-transform reconstruction with practical SSI signal

Having theoretically proved the validity of this technique, we attempted the reconstruction of the phase for a practical SSI. The laser amplifier and the measurement system are exactly the same as described in reference [8]. We first measured the phase of a femtosecond pulse through a hollow fiber filled with 3atm argon gas, using a standard SPIDER setup, in which the spectral shear and the delay of pulse replicas were Ω =24.5×10^{12}rad and τ =1.006ps, respectively. The measured phase was then sent to the LC-SLM, as feedback, so that the phase is nearly flat. Figure 3 is a measured SSI of when feedback is applied. It can be seen that the SSI fringes are highly structured.

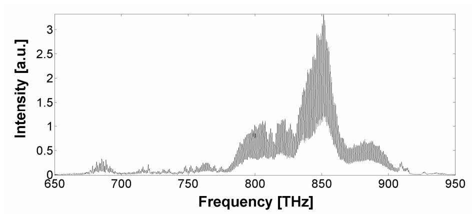

Fig. 3. Measured spectral shearing interferogram.

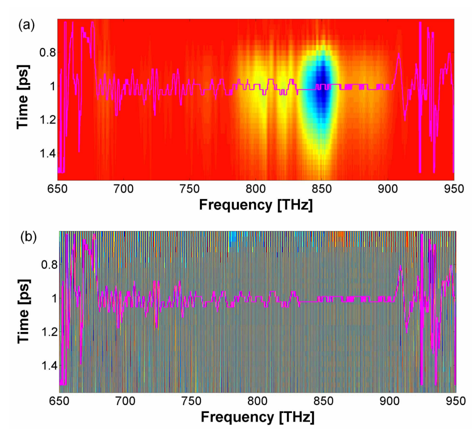

Fig. 4. Results of WT: (a) Magnitude topography. (b) Phase topography. (The pink colored curve represents the maximum value of the magnitude.)

Figure 4(a) and Fig. 4(b) show the WT trace and phase topography obtained from the SSI in Fig. 3, respectively. The time delay selected as from 0.6ps to 1.5ps, at a step of 0.03ps. The pink colored curve in Fig. 4(a) is the maximum magnitude detected for each frequency point, and the one in Fig. 4(b) is the projection of that in Fig. 4(a). The wrapped phase was obtained from Fig. 4(b) at the corresponding points. The spectral phase was then retrieved following the standard concatenation algorithm. The same procedure was also applied using FT technique with different width of rectangular shaped filter. The results with both techniques are shown in Fig. 5.

Figure 5(a) demonstrates that, with FT technique, the different filter width indeed led to differences in retrieved phase, which modifies the profile of the reconstructed pulse (Fig. 5(b)).

#6316 - $15.00 US Received 13 January 2005; revised 25 February 2005; accepted 12 March 2005

While the phase and pulse profile calculated with WT technique are unique, and they at least align with one of the results obtained with the standard FT procedure with a “suitable” filter width.

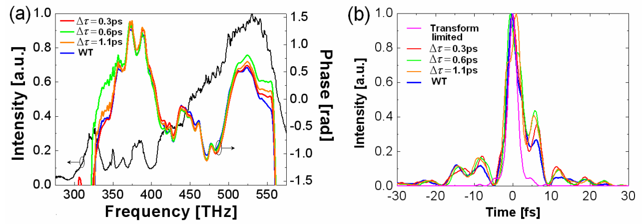

Fig. 5. (a) Spectrum and reconstructed phase with WT and FT techniques. (b) Reconstructed pulse profiles in comparison with the transform-limited pulses.

5. Discussions

As shown above, one of the practical advantages of using WT technique is that it does not need to choose filter type or filter width for a particularly structured SSI, which may benefit the automatic feedback control. This is because substituting the procedure of filter, WT algorithm read the phase along the maximum value curve in magnitude topography, thus the phase noise introduced from filter window is avoided.

Another interesting feature of application of WT technique in phase extraction is that WT traces may analogy to FROG traces [15]. Indeed, WT traces look very much like FROG [15] or SHG FROG traces [16]. However, they have at least two differences:

1) WT trace is basically the derivative of the FROG trace (the second order derivative of the phase). Therefore the position in the temporal axis defines the sign and value of the quadratic phase; while in FROG the temporal axis is relevant.

2) In WT algorithm, the phase is extracted directly from the corresponding position in the associated phase topography; while FROG does not have such a phase trace; therefore it has to apply the iteration program.

Compared with the FT algorithm, the phase extraction of WT is more straightforward. In FT technique, signal information need transform between two domains: spectral interferogram should be transformed into pseudo time domain to slice out the AC component; to recover the phase in frequency domain, a second Fourier-transform is needed. However, WT of spectral interferogram reflects information associated with time domain and frequency domain on an apparent plane, on which spectral phase at each time delay (or period) can be read directly. Therefore no second inverse transform is needed to transfer information between the two domains.

In FT, different width of filter window may introduce different phase noise, which results in uncertainty of retrieved phase. One may propose that the modulation noise can be excluded by choosing larger delay τ. However, the practical limitation of large τ is the resolution of the spectrometer, since larger τ means less fringe spacing and less points for each fringe.

It appears that the WT produces artifacts when applied near the edges of signals; however, it should have no much effect on the reconstructed pulse, because they usually happen at far edges of the spectrum.

6. Conclusions

We have described an application of wavelet-transform based phase extraction algorithm of ultrashort pulse characterization, using as an example SSI, and demonstrated its certain advantages of the proposed technique over the conventional Fourier transform based algorithm. The results proved this new technique viable in characterization of ultra-broadband pulses and pulses with highly structured spectrum. This technique should be best to use for highly structured pulse spectra. Further investigation is being conducted for further reduction of the phase noise for highly structured spectra of monocycle optical pulses.

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