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# Estimation of Volume Fraction in a Three Species Bed

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**Abstract**

A model for estimating the volume fraction of a three species bed of spherical particles was derived by extending a model for a two species bed. By the proposed model, the volume fraction of a bed can be computed with each diameter, mixing ratio and volume fraction of three particles.

The validity of this model was verified by comparing the computed volume fractions with the experimental results of three different size glass bead mixtures.

## 1. Introduction

The volume fraction is one of the most important properties of a bed of particles and is of interest in various areas of science and engineering. In water treatment engineering, it is used for hydrodynamical studies of sedimentation and stratification of granular filters, ion exchange resin beds and activated carbon beds.

Methods for estimating the volume fraction or void fraction have been proposed, however, many of them are applicable only to a bed of uniform particles. Since most of bed materials have a diameter distribution, it is important to estimate the volume fraction of a different size particle bed.

Okazaki et al[1]. and Tanaka et al[2]. proposed models for a two species bed void fraction. For a bed of three species, Suzuki et al[3]. proposed a model based on the co-ordinate number. The void fractions computed by this model are less than the experimental results when the hypothesis that a bed is homogeneous does not hold.

In this paper, a model for a three species bed volume fraction is derived by extending the one proposed by Okazaki et al. for a two species bed.

## 2. A model for a three species bed volume fraction

### 2−1 Definition

Consider the bed comprising coarse, medium and fine particles. Let the coarse, medium and fine particles be the species c, m and f, their diameters be $d_c$, $d_m$ and $d_f$ and mixing ratios $x_c$, $x_m$ and $x_f$, respectively.

In the case of a three species bed, a triad of mixing ratios $x_{(1,2,3),1}$, $x_{(1,2,3),2}$, $x_{(1,2,3),3}$ among the three species gives the maximum volume fraction. And a pair of mixing ratios $x_{(1,2),1}$ and $x_{(1,2),2}$ between the two species gives the maximum volume fraction in a two species bed.

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These combinations of mixing ratios are denoted as the maximum volume fraction volume mixing ratios (later written as MVFMR's) and will be calculated in sections 2−3 and 2−5.

One approach to the prediction of a three species (c, m, f) bed volume fraction is to assume that the bed would consist of three zones such as follows ;

Zone (c, m, f) : A zone comprising three species c, m and f. Their mixing ratios $x_{(c,m,f),c}$, $x_{(c,m,f),m}$ and $x_{(c,m,f),f}$ coincide with MVFMR's of a three species bed. The volume fraction is $\phi_{(c,m,f),T}$.

Zone (i, j) : A zone of two species i and j. Their mixing ratios $x_{(i,j),i}$ *and* $x_{(i,j),j}$ coincide with MVFMR's of two species bed. Species i and j are residuals of c, m and f after packing zone (c, m, f). The volume fraction is $\phi_{(i,j),T}$.

Zone (k) : A zone of one species k which is a residual of species of c, m and f after packing zone (c, m, f) and zone (i, j). Volume fraction $\phi_{sk}$ in this zone is that of a bed of species k alone.

This three zone approach will be referred to as the 'three bed model'.

## 2−2 Calculation of a three species bed volume fraction by the three bed model

Fig. 1 illustrates a schematic diagram of the three bed model, where zone (i, j) comprises c and f and zone (k) camprises species f. If $V_p$ is the total particle volume in a bed and if $V_{(c,m,f),K}$, $V_{(c,f),K}$ and $V_{(f),K}$ are void volumes in zone (c, m, f), zone (c, f) and zone (f) respectively, the volume fractions of the bed $\phi_{sT}$ can be written as ;

$$\phi_{sT}=\frac{V_p}{V_p+V_{(c,m,f),K}+V_{(c,f),K}+V_{(f),K}} \quad (1)$$

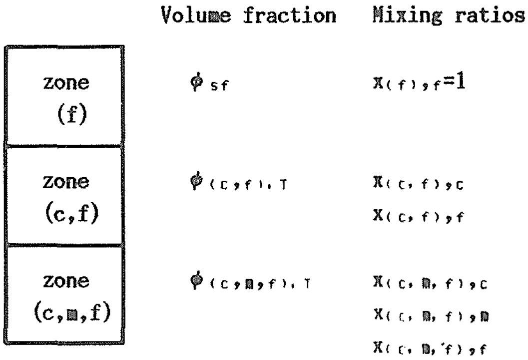


**Fig. 1** Schematic diagram of the three bed model

In Eq. (1), $V_{(c,m,f),K}/V_p$, $V_{(c,f),K}/V_p$ and $V_{(f),K}/V_p$ can be written as a function of volume fractions and MVFMR's in three zones and mixing ratios of the bed $x_c$, $x_m$ and $x_f$ (See Appendix), then $\phi_{sT}$ becomes ;

$$\phi_{sT}=\frac{1}{f_c x_c+f_m x_m+f_f x_f} \quad (2-1)$$

where

$$f_c=\frac{1}{x_{(c,f),c}}\left[\frac{1}{\phi_{(c,f),T}}-\frac{x_{(c,f),f}}{\phi_{sf}}\right] \quad (2-2)$$

$$f_m=\frac{1}{x_{(c,m,f),m}}\left[\frac{1}{\phi_{(c,m,f),T}}-\frac{x_{(c,m,f),c}}{\phi_{(c,f),T}\,x_{(c,f),c}}-\frac{x_{(c,m,f),f}-x_{(c,m,f),c}}{\phi_{sf}}\right] \quad (2-3)$$

$$f_f=1/\phi_{sf} \quad (2-4)$$

## 2−3 Calculation of MVFMR's in zone (c, m, f)

The difference among three species diameters is expressed by two diameter ratios ; $r_{c,m}$ $(=d_m/d_c)$, $r_{m,f}$ $(=d_f/d_m)$. In the following four cases of a pair of $r_{c,m}$ and $r_{m,f}$, MVFMR can be calculated theoretically.

(1) case 1 : $r_{c,m}=0$ and $r_{m,f}=0$

$$x_{(c,m,f),c}=x_{<0,0>,c}=\frac{\phi_{sc}}{\phi_{sc}+(1-\phi_{sc})\phi_{sm}+(1-\phi_{sc})(1-\phi_{sm})\phi_{sf}} \quad (3-1)$$

$$x_{(c,m,f),m}=x_{<0,0>,m}=\frac{(1-\phi_{sc})\phi_{sm}}{\phi_{sc}+(1-\phi_{sc})\phi_{sm}+(1-\phi_{sc})(1-\phi_{sm})\phi_{sf}} \quad (3-2)$$

$$x_{(c,m,f),f}=x_{<0,0>,f}=\frac{(1-\phi_{sc})(1-\phi_{sm})\phi_{sf}}{\phi_{sc}+(1-\phi_{sc})\phi_{sm}+(1-\phi_{sc})(1-\phi_{sm})\phi_{sf}} \quad (3-3)$$

where $\phi_{sc}$, $\phi_{sm}$ and $\phi_{sf}$ are volume fractions in the bed of each species alone, respectively.

(2) case 2 : $r_{c,m}=1$ and $r_{m,f}=1$

$$x_{(c,m,f),c}=x_{(c,m,f),m}=x_{(c,m,f),f}=x_{<1,1>}=1/3 \quad (4)$$

(3) case 3 : $r_{c,m}=1$ and $r_{m,f}=0$

$$x_{(c,m,f),c}=x_{<1,0>,c}=\frac{\phi_{sm}}{2\phi_{sm}+\phi_{sf}-\phi_{sm}\phi_{sf}} \quad (5-1)$$

$$x_{(c,m,f),m}=x_{<1,0>,m}=x_{<1,0>,c} \quad (5-2)$$

$$x_{(c,m,f),f}=x_{<1,0>,f}=\frac{\phi_{sf}-\phi_{sm}\phi_{sf}}{2\phi_{sm}+\phi_{sf}-\phi_{sm}\phi_{sf}} \quad (5-3)$$

(4) case 4 : $r_{c,m}=0$ and $r_{m,f}=1$

$$x_{(c,m,f),c}=x_{<0,1>,c}=\frac{\phi_{sc}}{\phi_{sc}+2\phi_{sm}-2\phi_{sc}\phi_{sm}} \quad (6-1)$$

$$x_{(c,m,f),m}=x_{<0,1>,m}=\frac{\phi_{sm}-\phi_{sc}\phi_{sm}}{\phi_{sc}+2\phi_{sm}-2\phi_{sc}\phi_{sm}} \quad (6-2)$$

$$x_{(c,m,f),f}=x_{<0,1>,f}=x_{<0,1>,m} \quad (6-3)$$

In the case $0<r_{c,m}<1$ and $0<r_{m,f}<1$, the MVFMR is obtained by the following linear approximation as ;

$$x_{(c,m,f),l}=(1-r_{c,m})x_{<0,0>,l}+(r_{c,m}-r_{m,f})x_{<1,0>,l}+r_{m,f}x_{<1,1>} \quad (r_{c,m}\geqq r_{m,f}) \quad (7-1)$$

$$x_{(c,m,f),l}=(1-r_{m,f})x_{<0,0>,l}+(r_{m,f}-r_{c,f})x_{<0,1>,l}+r_{c,m}x_{<1,1>} \quad (r_{c,m}<r_{m,f}) \quad (7-2)$$

where l=c, m and f.

### 2−4 Calculation of volume fraction in zone (c, m, f)

Assuming that the void of species c is occupied by species m at a rate $\eta(r_{c,m})$ and the residual void of c is occupied by f at a rate $\eta(r_{m,f})$ and on the supposition that the void of m is occupied by f at a rate $\eta(r_{m,f})$, then the volume fraction $\phi_{(c,m,f),T}$ in zone (c, m, f) can be written as ;

$$\phi_{(c,m,f),T}=\left[1+x_{(c,m,f),c}\frac{1-\phi_{sc}}{\phi_{sc}}\{\eta(r_{c,m})-\eta(r_{m,f})+\eta(r_{c,m})\eta(r_{m,f})\}+x_{(c,m,f),m}\frac{1-\phi_{sm}}{\phi_{sm}}\{1-\eta(r_{m,f})\}+x_{(c,m,f),f}\frac{1-\phi_{sf}}{\phi_{sf}}\right]^{-1} \quad (8)$$

where $\eta(r_{c,m})$ is assumed to be expressed by a following equation derived by Okazaki et al. ;

$$\eta(r_{c,m})=\left[\frac{1-r_{c,m}}{1+r_{c,m}}\right]^{1.5} \tag{9}$$

and

$$\eta(r_{m,f})=\left[\frac{1-r_{m,f}}{1+r_{m,f}}\right]^{1.5} \tag{10}$$

### 2−5 MVFMR and volume fraction in zone (c, f)

MVFMR and volume fraction $\phi_{(c,f),T}$ in zone (c, f) are calculated by the following equations derived by Okazaki et al. for a two species bed.

$$x_{(c,f),c}=\frac{\phi_{sc}}{\phi_{sc}+\phi_{sf}-\phi_{sc}\phi_{sf}}-\left[\frac{\phi_{sc}}{\phi_{sc}+\phi_{sf}-\phi_{sc}\phi_{sf}}-0.5\ r_{c,f}\right] \tag{11−1}$$

$$x_{(c,f),f}=1-x_{(c,f),c} \tag{11−2}$$

and

$$\phi_{(c,f),T}=\frac{\phi_{sc}\phi_{sf}}{\phi_{sc}+(\phi_{sc}-\phi_{sf})x_{(c,f),c}-(1-\phi_{sc})\phi_{sf}x_{(c,f),c}\eta(r_{c,f})} \tag{11−3}$$

where $r_{c,f}=d_f/d_c$.

## 3. Comparison between computed volume fraction by the three bed model and experimental results

A 5cm i. d. and 30cm length tube was used as a packing bed. Three species mixtures were fed in the tube and tapped. The top surface height was read from a rule on the outside wall of the tube. The bed materials used were glass beads. Relevant properties of these are listed in Table 1. The particles are all spherical and closely graded. Before a three species run, the volume fraction in a single species bed was measured as summarized in Table 1.

In Figs. 2−8, the computed volume fractions are compared with the experimental results for various three species mixtures beds. The computed volume fractions follow the experimental results well.

## 4. Conclusion

For the estimation of a three species bed volume fraction, the three bed model was derived by extending the model proposed by Okazaki et al. for a two species bed. By this model the volume fraction can be computed with three species diameters, mixing ratios and volume fractions of each species bed.

The volume fractions computed by this model were compared with the experimental results of three different size spherical glass bead mixtures. The comparison between the computed and the experimental results confirmed the validity of the model.

## References

1) Okazaki M. et al : Kagakukougaku Dai 49 Nenkai Kouenyoshi-shu, I104, 1984
2) Tanaka Z. et al : Funtaikougakukaishi, 19,457, 1982
3) Suzuki M. et al : Kagakukougaku ronbunshu, 10, 6, 1984

**Table 1** Properties of particles used

| Mark | Mean Diameter ($\mu$m) | Volume fraction in a bed (−) |
|---|---|---|
| G920 | 923.1 | 0.633 |
| G500 | 508.8 | 0.630 |
| G370 | 377.7 | 0.627 |
| G170 | 170.4 | 0.625 |
| G80 | 86.8 | 0.627 |

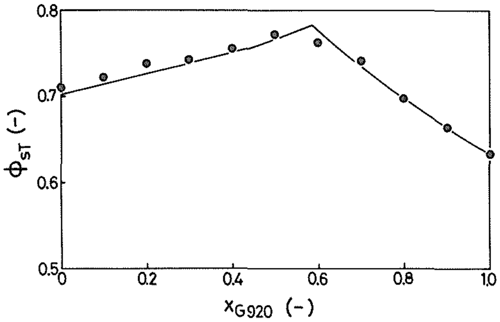


**Fig. 2** Comparison between computed and experimental results-1 (Species : G920, G500, G80, $x_{G500}=x_{G80}=(1-x_{G920})/2$)

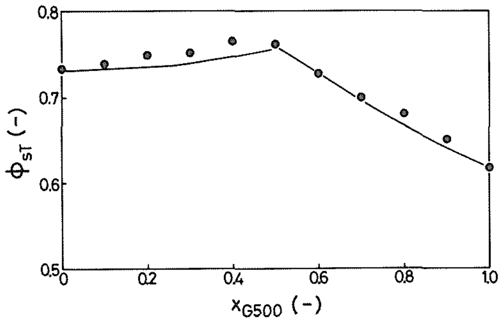


**Fig. 3** Comparison between computed and experimental results-2 (Species : G920, G500, G80, $x_{G920}=x_{G80}=(1-x_{G500})/2$)

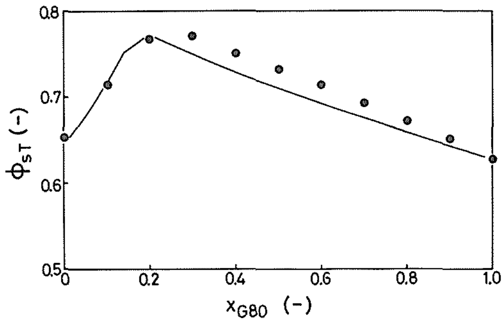


**Fig. 4** Comparison between computed and experimental results-3 (Species : G920, G500, G80, $x_{G920}=x_{G500}=(1-x_{G80})/2$)

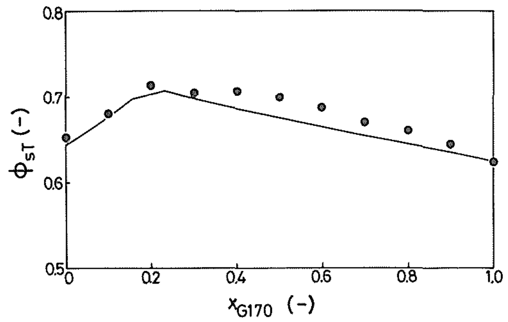


**Fig. 5** Comparison between computed and experimental results-4 (Species : G920, G500, G170, $x_{G920}=x_{G500}=(1-x_{G170})/2$)

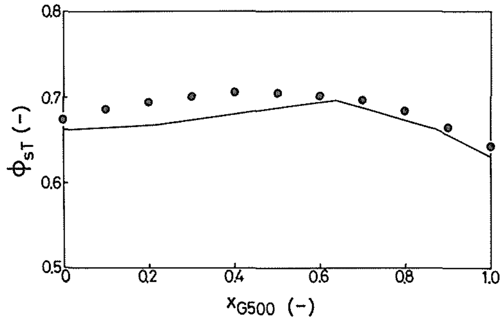


**Fig. 6** Comparison between computed and experimental results-5 (Species : G920, G500, G170, $x_{G320}=0,25$, $x_{G170}=1-x_{G920}-x_{G500}$)

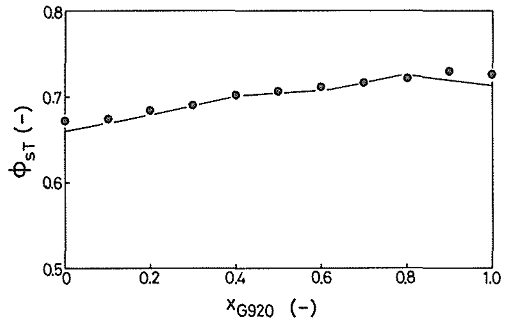


**Fig. 7** Comparison between computed and experimental results-6 (Species : G920, G500, G170, $x_{G170}=0.25$, $x_{G500}=1-x_{G920}-x_{G170}$)

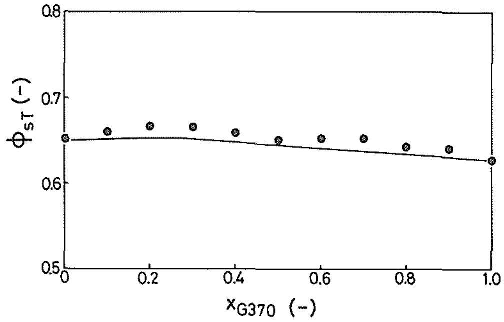


**Fig. 8** Comparison between computed and experimental results-7 (Species : G920, G500,G370, $x_{G920}=x_{G500}=(1-x_{G370})/2$)

### Appendix Calculation $V_{(c,m,f),K}$, $V_{(c,f),K}$ and $V_{(f),K}$

Let $V_{(c,m,f),c}$ $V_{(c,m,f),m}$ and $V_{(c,m,f),f}$ be particle volumes of species c, m and f in zone (c, m, f), $V_{(c,f),c}$ and $V_{(c,f),f}$ be volumes in zone (c, f) and $V_{(f),f}$ be volume in zone (f), then $V_{(c,m,f),k}$, $V_{(c,f),K}$ and $V_{(f),K}$ can be written as follows ;

$$V_{(c,m,f),K}=\frac{1-\phi_{(c,m,f),T}}{\phi_{(c,m,f),T}}(V_{(c,m,f),c}+V_{(c,m,f),m}+V_{(c,m,f),f}) \quad (A-1)$$

$$V_{(c,f),K}=\frac{1-\phi_{(c,f),T}}{\phi_{(c,f),T}}(V_{(c,f),c}+V_{(c,f),f}) \quad (A-2)$$

$$V_{(f),K}=\frac{1-\phi_{sf}}{\phi_{sf}}\ V_{(f),f} \quad (A-3)$$

The following relations between particle volumes and MVFMR's hold in each zone ;

$$x_{(c,m,f),c}:x_{(c,m,f),m}:x_{(c,m,f),f}=V_{(c,m,f),c}:V_{(c,m,f),m}:V_{(c,m,f),f} \quad (A-4)$$

$$x_{(c,f),c}:x_{(c,f),f}=V_{(c,f),c}:V_{(c,f),f} \quad (A-5)$$

And particle volume balance in the bed requires the following relations ;

$$V_p x_c = V_{(c,m,f),c}+V_{(c,m),c} \quad (A-6)$$

$$V_p x_m = V_{(c,m,f),m} \quad (A-7)$$

$$V_p x_f = V_{(c,m,f),f}+V_{(c,m),f}+V_{(f),f} \quad (A-8)$$

Particle volumes in the three zones can be calculated by Eqs. (A−4)−(A−8) and Eqs. (A−1)−(A−3) become as follows ;

$$V_{(c,m,f),K}=\frac{1-\phi_{(c,m,f),T}}{\phi_{(c,m,f),T}}\times\frac{V_p x_m}{x_{(c,m,f),m}} \quad (A-9)$$

$$V_{(c,f),K}=\frac{1-\phi_{(c,f),T}}{\phi_{(c,f),T}}\left[\frac{V_p x_c}{x_{(c,f),c}}-\frac{V_p\,x_m x_{(c,m,f),c}}{x_{(c,f),c}x_{(c,m,f),m}}\right] \quad (A-10)$$

$$V_{(f),K}=\frac{1-\phi_{sf}}{\phi_{sf}}\times\left[V_p x_f-\frac{x_{(c,f),f}}{x_{(c,f),c}}V_p x_c-\frac{x_{(c,f),c}x_{(c,m,f),f}-x_{(c,f),f}x_{(c,m,f),c}}{x_{(c,f),c}x_{(c,m,f),m}}V_p x_m\right] \quad (A-11)$$