Pilot Symbol Design for Channel Estimation in OFDM with Null Subcarriers

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***Abstract*—The problem of training symbol placement and power for the channel estimation of OFDM over block-fading channel in the presence of null subcarriers is considered. We use convex optimization techniques to find a pilot design that results in near-optimal channel estimation. We design OFDM preamble and then, based on the obtained preambles, we design pilot symbols and their placements for pilot aided channel estimation using our proposed iterative algorithm. Several examples based on the IEEE 802.16 are provided to demonstrate the efficacy of our proposal.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is becoming widely applied in wireless communication systems due to its high rate transmission capability with high bandwidth efficiency and its robustness with regard to multi-path fading and delay. Since the receiver performance strongly depends on the accuracy of the estimated instantaneous channel, the channel coefficients should be estimated with minimum error. This poses challenges for efficient channel estimation schemes necessary to obtain channel state information (CSI) required to compensate for channel distortion.

Preamble channel estimation is developed under the assumption of slow fading channel, Thus, if the channel remains constant over several OFDM symbols, channel estimation by an OFDM preamble may be sufficient for symbol detection. But when the channel changes even from one OFDM block to the subsequent one, in order to contend with channel variation effects, pilot symbols have to be inserted into certain subcarriers of each OFDM symbol to enable channel estimation. This is known as pilot symbol-assisted modulation (PSAM) [1], [2], which allows tracking of the channel variation. For PSAM, an analytical approach to the design of pilot assisted transmissions is presented [1].

When all subcarriers are available, OFDM preambles and pilot symbols have been well designed to enhance the channel estimation accuracy [3]. They can be optimally designed in terms of: i) minimizing the channel mean square estimation error [4]; ii) minimizing the bit-error rate when symbols are detected with channel estimates by pilot symbols [5]; iii) maximizing the lower bound on channel capacity with channel estimate [6]. It has been found that equi-distant and equi-powered pilot symbols are optimal with respect to several performance measures. Pilot symbols are also designed for OFDM systems with multiple antennas [7], [8], [9].

However, in practice, not all the subcarriers are available for transmission. It is often the case that null subcarriers are set on both edges of the allocated bandwidth to mitigate the interferences from/to adjacent bands [10]. For example, IEEE 802.16e standard has 256 subcarriers among which 56 subcarriers at DC component and at the edges of the band are set to be null. The presence of these null subcarriers complicates the design of both training preamble for channel estimation and pilot symbols for pilot-aided channel estimation. Null subcarriers render equi-distant and equi-powered pilot symbols impossible to use.

In [8], equi-powered pilot symbols are studied for channel estimation in multiple antenna OFDM system with null subcarriers. But they are not always optimal even for point-to-point OFDM system. Pilot sequences designed to reduce the channel mean square error (MSE) in multiple antenna OFDM system are also reported in [9] but they are not necessarily optimal. In [11], pilot symbols are designed using convex optimization. Also in [12], a proposal was made that uses cubic parameterizations of the pilot subcarriers in conjunction with convex optimization algorithm to produce pilot designs. However, the convex optimization problem of the method in [12] uses the approximated objective function to minimize the channel MSEs, which may not accurately encapsulate the system performance. Furthermore, the accuracy of cubic function based optimizations in [12] depends on many parameters to be selected for every channel/subcarriers configuration, which complicate the design.

In this paper, we propose the pilot design based on the l_∞ norm of channel/symbol estimate MSE. Also, we show that the pilot design in [11] which is based on l_2 norm of the channel estimate MSE can be further improved by using our proposed algorithm to select pilot subcarriers. In [12], it is stated that l_2 norm of the subcarrier channel-estimate MSE in [11] may not accurately encapsulate the system performance, however, we verify through simulations that choosing one of these norms over the other is not an important choice as the two gives a close performance under different performance measures.

For fixed pilot subcarriers, we formulate our l_∞ based design problem as a semidefinite programming (SDP) [13], which enables numerical solutions. Then, we propose an algorithm for the selection of pilot subcarriers, that removes symmetrically a certain number of insignificant subcarriers in the preamble and optimize the remaining subcarriers iteratively. Several design examples under the same setting as IEEE 802.16 are presented to verify that our pilot symbols have the reasonable channel and symbol estimate MSE. The designed preamble and pilot symbols when used for channel and symbol estimation outperform both the pilot symbol estimate and the long preamble of IEEE 802.16e.

II. CHANNEL ESTIMATION IN OFDM

Let us consider a point-to-point wireless OFDM system with N subcarriers. We assume that our discrete-time baseband equivalent channel has coefficients $h_0, \dots, h_{L-1}$ of maximum length L, and remains constant in at least one OFDM symbol.

The received baseband frequency-domain signal is expressed as

$$Y_k = H_k s_k + W_k, \tag{1}$$

for $k \in [0, N-1]$, where H_k is the channel frequency response at frequency $2\pi k/N$ given by $H_k = \sum_{l=0}^{L-1} h_l e^{-j\frac{2\pi kl}{N}}$ and W_k are i.i.d. circular Gaussian with zero mean and variance σ_w^2.

Let $\mathcal{K}$ be the set of active subcarriers and $|\mathcal{K}|$ be the number of elements in $\mathcal{K}$. For channel estimation, we place $N_p(\leq |\mathcal{K}|)$ pilot symbols $\{p_1, \dots, p_{N_p}\}$ at subcarriers $k_1 < k_2 < \cdots < k_{N_P} \in \mathcal{K}$, which is known to the receiver. We assume that $N_p \geq L$ so that the channel can be perfectly estimated if there is no noise and denote the index set of pilot symbols as

$$\mathcal{K}_p = \{k_1, \dots, k_{N_P}\}. \tag{2}$$

In a long training OFDM preamble, all subcarriers in $\mathcal{K}$ can be utilized for pilot symbols so that $\mathcal{K}_p = \mathcal{K}$. On the other hand, in pilot-assisted modulation (PSAM) [1], a few known pilot symbols are embedded in one OFDM symbol. In PSAM, a set of data subcarriers is $\mathcal{K}_s = \mathcal{K} \setminus \mathcal{K}_p$, where $\setminus$ denotes set difference.

Let us define an $N \times N$ DFT matrix as $\boldsymbol{F}$, whose $(m+1, n+1)$th entry is $e^{-j2\pi mn/N}$. We denote an $N \times L$ matrix $\boldsymbol{F}_L = [\boldsymbol{f}_0, \dots, \boldsymbol{f}_{N-1}]^{\mathcal{H}}$ consisting of N rows and first L columns of DFT matrix $\boldsymbol{F}$, where $\mathcal{H}$ is the complex conjugate transpose operator. We also define an $N_p \times L$ matrix $\boldsymbol{F}_p$ having $\boldsymbol{f}_{k_n}^{\mathcal{H}}$ for $k_n \in \mathcal{K}_p$ as its nth row.

Let diag$(\boldsymbol{a})$ be a diagonal matrix with the vector $\boldsymbol{a}$ on its main diagonal. Collecting the received signals having pilot symbols as $\tilde{\boldsymbol{Y}} = [Y_{k_1}, \dots, Y_{k_{N_p}}]^T$, we obtain

$$\tilde{\boldsymbol{Y}} = \boldsymbol{D}_p \boldsymbol{F}_p \boldsymbol{h} + \tilde{\boldsymbol{W}}, \tag{3}$$

where the diagonal matrix $\boldsymbol{D}_p$ and channel vector $\boldsymbol{h}$ are respectively defined as $\boldsymbol{D}_p = \text{diag}[p_1, \dots, p_{N_p}]$, and $\boldsymbol{h} = [h_0, \dots, h_{L-1}]^T$.

From (3), the LS estimate $\hat{H}_k$ of H_k is found to be

$$\hat{H}_k = \boldsymbol{f}_k^{\mathcal{H}} (\boldsymbol{F}_p^{\mathcal{H}} \boldsymbol{\Lambda}_p \boldsymbol{F}_p)^{-1} (\boldsymbol{D}_p \boldsymbol{F}_p)^{\mathcal{H}} \tilde{\boldsymbol{Y}}, \tag{4}$$

with

$$\boldsymbol{\Lambda}_p = \boldsymbol{D}_p^{\mathcal{H}} \boldsymbol{D}_p = \text{diag}\left(\lambda_1, \dots, \lambda_{N_p}\right). \tag{5}$$

Let us denote the mean squared error (MSE) of the channel gain at the kth subcarrier as

$$r_k = E\{|\hat{H}_k - H_k|^2\}. \tag{6}$$

In [11], the preamble and pilot symbols are designed to minimize the l_2 norm of MSE, i.e., the sum $\sum_{k \in \mathcal{K}_s} r_k$ of MSE, while in [12], the l_∞ norm of MSE, i.e., $\max_{k \in \mathcal{K}_s} r_k$. However, the selection of pilot subcarriers in [11] does not necessary work for some subcarrier/channel length configurations and the object function in [12] is not always equal to the l_∞ norm as it uses objective function which is an approximate of the channel estimate MSE.

III. PREAMBLE AND PILOT DESIGN WITH SDP

Similar to the minimization of the l_2 norm of MSE [11], for given pilot and data subcarrier set, we formulate the minimization of the l_∞ norm as a convex optimization, which can be solved numerically.

We utilize the notation $\boldsymbol{A} \succeq 0$ for a symmetric matrix $\boldsymbol{A}$ to indicate that $\boldsymbol{A}$ is positive semi-definite and the notation $\boldsymbol{a} \succeq 0$ for a vector to signify that all entries of $\boldsymbol{a}$ are greater than or equal to 0. We normalize the pilot power such as

$$\boldsymbol{p}^{\mathcal{H}} \boldsymbol{p} = 1 \tag{7}$$

and define

$$\boldsymbol{\lambda} = [\lambda_1, ...\lambda_{N_p}]^T \tag{8}$$

We would like to minimize the largest r_k in the data subcarrier set $\mathcal{K}_s$ subject to (7), i.e.,

$$\min_{\boldsymbol{\lambda}} \max_{k \in \mathcal{K}_s} r_k. \tag{9}$$

The minimization is equivalent to

$$\min_{\boldsymbol{\lambda}, \nu} \nu \tag{10}$$

subject to

$$[1, \dots, 1]\boldsymbol{\lambda} \leq 1, \boldsymbol{\lambda} \succeq 0, \tag{11}$$

and for all $k \in \mathcal{K}_s$,

$$r_k \leq \nu. \tag{12}$$

Since r_k can be expressed for $\sigma_w^2 = 1$ as

$$r_k = \boldsymbol{f}_k^{\mathcal{H}} \left(\boldsymbol{F}_p^{\mathcal{H}} \boldsymbol{\Lambda}_p \boldsymbol{F}_p \right)^{-1} \boldsymbol{f}_k, \tag{13}$$

by using Schur's complement, (12) can be written as

$$\begin{bmatrix} \sum_{n=1}^{N_P} \lambda_n \tilde{\boldsymbol{f}}_n \tilde{\boldsymbol{f}}_n^{\mathcal{H}} & \boldsymbol{f}_k \\ \boldsymbol{f}_k^{\mathcal{H}} & \nu \end{bmatrix} \succeq 0, \tag{14}$$

where $\tilde{\boldsymbol{f}}_n^{\mathcal{H}}$ is the nth row of $\boldsymbol{F}_p$.

Since the object function is linear in the arguments and the constraints are expressed by their linear matrix inequalities (LMI), the minimization is described as an SDP problem [13], which lies in a convex optimization, whose global solution can be efficiently and numerically found by the existing routines.

A. Pilot Subcarrier Selection

We have to select pilot subcarriers from $\mathcal{K}$ active subcarriers for numerical optimization. In [12], a cubic parameterization of the pilot subcarriers is proposed. Here, we take a heuristic approach, assuming even N.

First, we use a designed optimal preamble with SDP and denote its λ_k as $\lambda_1^{(0)}, ...\lambda_{|\mathcal{K}|}^{(0)}$, then we iteratively remove symmetrically N_r subcarriers with N_r minimum power and optimize the remaining subcarriers. For the ith iteration, after removing a certain number of subcarrier index set corresponding to N_r minimum $\lambda_k^{(i-1)}$ we optimize pilot power for the remaining set again with SDP. When the iterative algorithm is completed, we will remain with only $\mathcal{K}_p$ optimal subcarrier indexes and its corresponding optimal power.

The optimal placement and optimal power design procedure is as outlined by a pseudo-code algorithm below:

1) *Obtain the optimal preamble using convex optimization and initialize temporary set* $\mathcal{K}_t = \mathcal{K}$
2) *Save the obtained position and power of the subcarriers* $|\mathcal{K}_t|$
3) *If* $N_p < |\mathcal{K}_t|$, *remove symmetrically* N_r *minimum subcarriers, else go to step 6*
4) *Update* $\mathcal{K}_t$ $(|\mathcal{K}_t| = |\mathcal{K}_t| - N_r)$.
5) *Optimize the power of the remaining subcarriers using SDP and go to step 2*
6) *Exit*

B. Power Allocation

We express the received signal as

$$Y_k = \hat{H}_k s_k + V_k, \tag{15}$$

where V_k is the effective noise given by $V_k = (H_k - \hat{H}_k)s_k + W_k$. The variance of V_k is found to be $E\{|V_k|^2\} = r_k\sigma_s^2 + \sigma_w^2$, where σ_s^2 is the variance of transmitted symbols.

Let us assume that channel coefficients are complex Gaussian and normalize the sum of their variances to be one. Then, H_k is also complex Gaussian with unit variance. The effective SNR at the receiver is given by

$$\frac{(1-r_k)\sigma_s^2}{r_k\sigma_s^2 + \sigma_w^2}. \tag{16}$$

Suppose that the transmission power of one OFDM symbol is $N\mathcal{E}$. We allocate $\alpha N\mathcal{E}$ to the information symbols and $(1-\alpha)N\mathcal{E}$ to the pilots for $0 < \alpha < 1$. In this case, $\sigma_s^2 = \alpha N\mathcal{E}/|\mathcal{K}_s|$ and the error covariance is given by $r_k/[(1-\alpha)N\mathcal{E}]$. Then, the effective SNR is expressed by

$$\mathrm{SNR}_k == \frac{(1-\alpha)\alpha N\mathcal{E} - \alpha r_k}{\alpha r_k + (1-\alpha)|\mathcal{K}_s|\mathcal{N}_0}, \tag{17}$$

where $\sigma_w^2 = \mathcal{N}_0$. Then, for given $\mathcal{E}$, r_k and $\mathcal{N}_0$, SNR_k is a function of α, which should be determined correctly.

Suppose QPSK modulation. If the signal is detected with the channel estimate $\hat{H}_k$, then for a given $\hat{H}_k$ the BER is $Q(\sqrt{\mathrm{SNR}_k})$, where $Q(\cdot)$ is the complementary error function such that $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-t^2/2}dt$. Then, the average BER of the kth subcarrier is given by

$$E\{Q(\sqrt{\mathrm{SNR}_k})\} = \frac{1}{2}\left(1 - \sqrt{\frac{\mathrm{SNR}_k/2}{1+\mathrm{SNR}_k/2}}\right). \tag{18}$$

Thus, the average BER of one OFDM symbol is

$$\frac{1}{|\mathcal{K}_s|}\sum_{k\in\mathcal{K}_s}\left(1 - \sqrt{\frac{\mathrm{SNR}_k/2}{1+\mathrm{SNR}_k/2}}\right). \tag{19}$$

If the noise variance at the receiver is available at the transmitter, one can compute the average BER (19) as a function of α to obtain the optimal power allocation ratio α.

IV. PILOT DESIGN OPTIMIZATION EXAMPLE

Pilot designs under the same setting as IEEE 802.16, OFDM transmission frame [14, p.429] is considered. In a data-carrying symbol 200 subcarriers of the $N = 256$ subcarrier window are used for data and pilots. Of the other 56 subcarriers, 28 are null in the lower-frequency guard band, 27 are nulled in the upper frequency guard band and one is the DC subcarrier which is nulled. Of the 200 used subcarriers, 8 are allocated as pilots, while the remaining 192 are used for data transmission. For each channel length L, to design pilot tones, we construct an index set corresponding to the N_p subcarriers with significant pilot power and reasonable position in the optimal OFDM preamble.

The pilot positions specified by the standard are $\{\pm 13, \pm 38, \pm 63, \pm 88\}$, and all contain the same amount of power, while the proposed pilot design places the 8 significant pilots at $\{\pm 15, \pm 45, \pm 74, \pm 100\}$. For $N_p > L$ the proposed method places $N_p = L$ pilots necessary to estimate the channel to nearly equi-space within the in-band region and then the rest are placed symmetrically but not necessarily equally spaced. This suggests that the symbol/channel MSE can be achieved even with the minimum number of pilot symbols necessary for symbol/channel estimation.

To obtain the channel estimate MSE for both preamble and the proposed pilot, we varied the channel length L, from 2 to 16. Fig. 1 presents the normalized frequency-domain channel MSE $\sum_{k\in\mathcal{K}_s} r_k$, where the additive variance is set as 1. The three curves correspond to the standard OFDM preamble and the l_2 and the l_∞ based optimized OFDM preamble when all the subcarriers in $\mathcal{K}$ are utilized.

The optimized OFDM preambles exhibit lesser frequency-domain channel MSE than the IEEE 802.16 OFDM preamble with equi-powered pilot symbols at all data subcarriers in $\mathcal{K}$. From the results it is clear that, channel estimate MSE of the two objective functions, l_2 [11] and l_∞ outperform the IEEE 802.16 standard with a small margin.

Since the difference in pilot placement and power allocation may not be well encapsulated in the channel estimate MSE [12], we made a comparison of channel MSE at each subcarrier between the IEEE 802.16 standard and our pilot design for $L = 1, 4$ and 8.

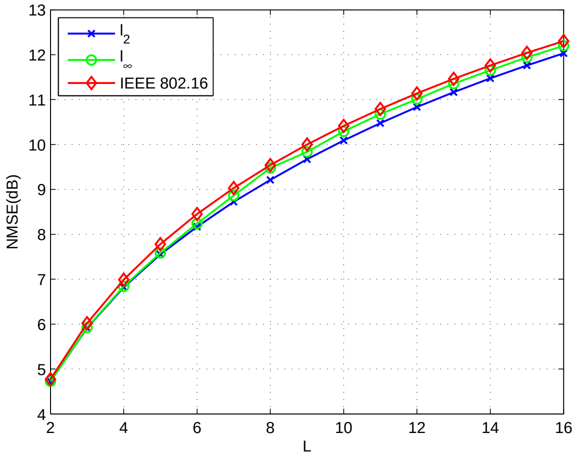

Fig. 1. Frequency domain channel MSE of preamble

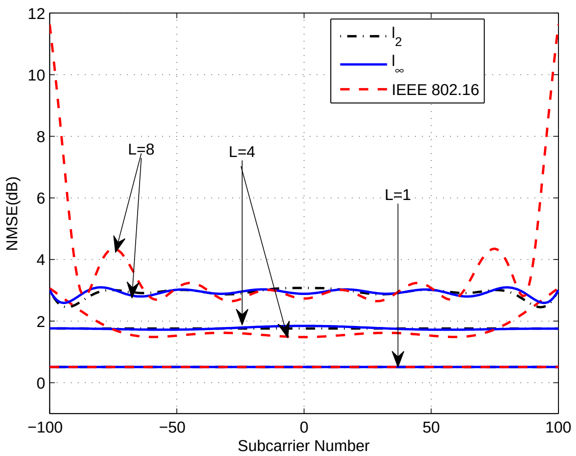

Fig. 2. NMSE performance of IEEE 802.16 versus the designed pilots. $|\mathcal{K}_p| = 8$ in all cases.

The MSE r_k is depicted in Fig. 2. It is clear that our proposed methods provide nearly constant MSE over all data subcarriers while for the reference standard (IEEE 802.16) there is a variation especially at the edge of the inband. Our constant MSE over all data subcarriers guarantee the better quality of service.

From the results of both channel and symbol estimate MSE's, it is clear that, the proposed objective functions outperform the IEEE 802.16 standard preamble and pilot aided symbol/channel estimation which justifies the efficiency of our Preamble and pilot-aided symbol/channel estimation.

Fig.3 is a plot of the BER vs α for $L = N_p = 8$ (cf. (19)). The optimal values of α lies between 0.6 and 0.8 for 10dB and 20dB respectively. For small values of α, the BER performance of the three are almost the same. However, close to optimal values, our proposed design outperforms the IEEE 802.16 standard, which implies that the BER performance of our proposed method is superior to the existing standard. The results of l_2 and l_∞ norm are exactly the same for both 10dB and 20dB, which suggests that choosing one of these norms over the other is not an important choice.

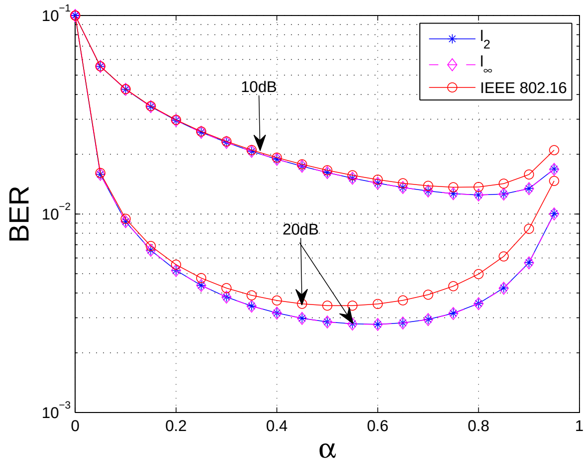

Fig. 3. BER versus α for 10dB and 20dB SNR

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