Real option valuation of social systems

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I. INTRODUCTION

We introduce, in this paper, the method that identifies net present values of a queue by using a real option approach. The queueing model which we employ here is the one dimensional ASEP with open boundaries. This model shows various phenomena even though their interaction rules are simple. In particular, the ASEP forms a shock front (domain wall) under restricted boundary conditions. Motion of the domain wall follows a biased random walk [1]. Hence we apply the concepts of a real option approach[2–4] to estimate length change of a queue. Our approach make it possible to give a quantitative worth of present congestions.

II. ANALOGY BETWEEN THE ASEP AND REAL OPTION CONCEPT

We consider, in this article, the evaluation of congestion. For simplicity, we employ the one-dimensional ASEP as the model of a queuing system and apply this to pricing of a queue. At the boundaries of this system, particles are supplied at the left end with α, and removed at the right end with β. According to Ref. [1], motion of the domain wall to the left at rate α or to the right at rate β, so that the domain wall does a biased random walk with drift velocity $V = \beta - \alpha$ and diffusion coefficient $D = (\alpha + \beta)/2$. Positive (negative) values of velocity are defined that the domain wall moves to the left (right) boundary

Assume that the stochastic process $X(t)$, the length of a queue, follows an arithmetic Brownian motion:

$$dX = \mu dt + \sigma dz, \tag{1}$$

where μ is the drift of the domain wall per unit time, σ is the instantaneous standard deviation per unit time, and dz is the increment of a standard Wiener process. In the domain wall framework, μ and σ corresponds to $V = \beta - \alpha$ and $D = (\alpha + \beta)/2$ respectively. It means that values of a queue tail primarily depend both on entering and exit rates at boundaries. Note that our main interest is on changes of a queue length itsself, not on percentage changes (not on an geometric Brownian motion).

We define degree of human clogging as $\pi = X(t) - I$. The value π is a indicator which shows congestion (free flow) when its values are positive (negative). The variable I fixed at 0.5 which means a threshold queue length equal to half of the system size. In other word, we defined a queue is crowded when π is positive. Therefore, instant clogging from a queue, which we consider, is given by

$$\pi(X) = \max[X - I, 0]. \tag{2}$$

The value of a queue at time t can be expressed as the sum of the instantaneous clogging over the interval (t, dt) and the continuation value beyond $t + dt$. The value of queue is

$$J(x) \;=\; \pi(X)dt + e^{-\rho dt}E[J(X + dX)]. \tag{3}$$

To express weight of time, J is discounted at the rate ρ. Expanding the right-hand side using Ito's Lemma, we have

$$J(x) \;=\; \pi(X)dt + (1 - \rho dt + o(dt))[J(x) + J'(x)\mu dt + \frac{\sigma^2}{2}J''(x)dt + o(dt)], \tag{4}$$

where $o(dt)$ collects terms that go to zero faster than dt. Simplifying, dividing by dt, and $dt \to 0$, we get the differential equation

$$\frac{1}{2}\sigma J(X)'' + \mu J(X)' - \rho J(X) + \pi(X) = 0. \tag{5}$$

The solution of Eq. (5) can be written as where The solution of Eq. (5) can be written as

$$J(X) \;=\; \begin{cases} \frac{C_3 I}{\rho}\exp\Big(-\frac{\mu}{\sigma^2}(X - I)\Big)\sinh\Big(C_2 X\Big) & X < I \\ \Big[\frac{C_3 I}{\rho}\sinh\Big(C_2 I\Big) - \frac{\mu}{\rho^2}\Big]\exp\Big[-(C_1 + C_2)(X - I)\Big] + \frac{X-I}{\rho} + \frac{\mu}{\rho^2} & X \geq I \end{cases}$$

where

$$C_1 \;=\; \frac{\mu}{\sigma^2},\; C_2 = \frac{\sqrt{\mu^2 + 2\rho\sigma^2}}{\sigma^2},$$

$$C_3 \;=\; \Big\{\frac{\sqrt{\mu^2 + 2\rho\sigma^2}}{\sigma^2}\exp\Big(\frac{\sqrt{\mu^2 + 2\rho\sigma^2}}{\sigma^2}I\Big) + \Big(\frac{\sqrt{\mu^2 + 2\rho\sigma^2} + \mu}{\sigma^2}\Big)\frac{\mu}{\rho^2}\exp\Big(\frac{\mu I}{\sigma^2}\Big)\Big\}^{-1}.$$

Note that the term $[(X_0 - I)/\rho] + \mu/\rho$ in the solution is the expected value of stream π when initial level is X_0. This is the net present value (NPV) of a queue obtained by

$$\mathrm{NPV} = \int_0^\infty e^{-\rho t}\Big(X(t) - I\Big)dt = \frac{X_0 - I}{\rho} + \frac{\mu}{\rho}. \tag{6}$$

III. NONLINEAR PROPERTY OF REAL OPTION EVALUATION

To examine the relationship between J and the parameters, α and β dependences of J are plotted in Figs. 1(a) and 1(b). Figures show J does not have linear dependence on both parameters. For example, in Fig. 1(b), J does not monotonically decreases with β even if an exit rate of the system are increased. It means that a clogging of human flow (cumulative jamming signals) has intrinsically nonlinear properties.

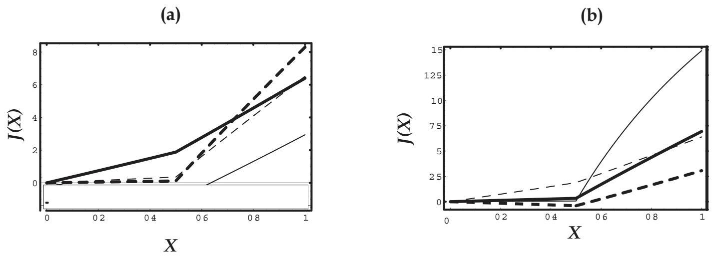

FIG. 1: (a)Dependence of $J(X)$ on α when (ρ, β, I)=(0.1, 0.25, 0.5). Thin solid line, thick solid line, thin dotted line and thick dotted line correspond to $\alpha = 0.2, 0.25, 0.3$ and 0.4. (b)Dependence of $J(X)$ on β when (ρ, α, I)=(0.1, 0.25, 0.5). Thin solid line, thick solid line, thin dotted line and thick dotted line correspond to $\beta = 0.1, 0.2, 0.25$ and 0.35

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[1] Kolmeisky, A.B., Schütz, G.M., Kolmeisky, E.B., Straley, J. P.: Phase diagram pf one-dimensional driven lattice gases with open boundaries. J. Phys. A: Math. Gen. 31, 6911-6919 (1998)

[2] Hull, J.: Options, Futures and Other Derivatives, 6th ed. Princentice-Hall, Englewood Cliffs, New Jersey (2005)

[3] Dixit, A.K., Pindyck, R.S.: Investment under uncertainty, Princeton Univ. Press (1994)

[4] Øksendal, B.: Stochastic Differential Equations: An Introduction with Applications, 6th ed. Springer (2003)