



Title	Application of Chaos Theory to Engine Systems
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Citation	, 2008-32-0010
Issue Date	2008-09
Doc URL	<a href="http://hdl.handle.net/2115/36646">http://hdl.handle.net/2115/36646</a>
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Type	article
Note	2008-32-0010 (SAE) / 20084710 (JSAE)
File Information	2008-32-0010.pdf



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# Application of Chaos Theory to Engine Systems

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## ABSTRACT

We focus on the control issue for engine systems from the perspective of chaos theory, which is based on the fact that engine systems have a low-dimensional chaotic dynamics. Two approaches are discussed: controlling chaos and harnessing chaos, respectively. We apply Pyragas' chaos control method to an actual engine system. The experimental results show that the chaotic motion of an engine system may be stabilized to a periodic motion. Alternatively, harnessing chaos for engine systems is addressed, which regards chaos as an essential dynamic mode for the engine.

## 1. INTRODUCTION

It has been reported that cycle-to-cycle combustion fluctuation in spark ignition engines exist. This fluctuation, on the one hand, is usually regarded as a malfunctioning. On the other hand, motor bikers tend to compare the engine vibration with the heart beat of mammals. Since we think that there is a certain relationship between such a sensation and the combustion fluctuation, we first pay attention to the question of whether this phenomenon of the combustion fluctuation stems from a stochastic or deterministic process [1-3].

We examined a motorcycle single cylinder spark ignition engine in order to study its behavior. We measured several time series such as combustion pressure, intake manifold pressure, and exhaust gas rate for an engine in its idle state, and analyzed them in terms of nonlinear dynamics. From the results of our analysis presented in a previous paper, it is clarified that cycle-to-cycle combustion fluctuation in spark ignition engines can result from the interplay of a low-dimensional chaotic dynamics of engine systems and stochastic processes [4].

In the following, we stress an important new aspect of our result. In general, it is plausible to consider molecules inside the cylinder during the combustion process to act randomly on the microscopic level. However, a macroscopic behavior as engine rotation or combustion pressure has an order, underpinned by the fact that the engine system has a low-dimensional dynamics. It can be considered that the macroscopic order emerges from a collective behavior of the microscopic motions through self-organization of the engine system.

On the basis of the above thought, we tackle the control problem for the engine system. Two approaches to this issue are feasible. The first one is the suppression of the observed motion in the engine system, i.e., the stabilization to a periodic orbit. This is the framework of "controlling chaos". In an alternative approach, we regard the engine systems to be comparable to living systems. Instead of applying chaos control to the engine systems, we now strive to harmonize the chaotic motion with the drivers very much like a rider and a horse can be seen as an embodied system. We refer to this strategy as "harnessing chaos".

As a first step, we adopt the former approach, i.e., we apply the Pyragas' control method to the investigated engine system. We choose the intake manifold pressure as an observable time series and the throttle valve as feedback. By using the experimental result, we argue that this method might stabilize the chaotic motion of an engine system to a periodic one.

As a contrast, harnessing chaos of engine systems is discussed. Although this research is still in a somewhat speculative state, we pay attention to the significance of the fact that engine systems have chaotic dynamics and discuss which role chaos might play for engine systems, particularly for motorcycles.

The organization of this paper is as follows. In Section 2, typical long-term fluctuation is briefly introduced. In Section 3, we explain the notion of controlling chaos and present experimental results of applying Pyragas' method to an engine system. In Section 4, we address the control issue from the perspective of harnessing chaos. In Section 5, we present a conclusion. Section 6 is devoted to discussions.

## 2. LONG-TERM FLUCTUATION OF ENGINE SYSTEMS

Figures 1(a) and 1(b) show the cycle data of the number of rotations and the maximum combustion pressure, respectively, each for 40000 cycles. Both data were obtained from the same experiment. This experiment was carried out under conditions in which the amount of fuel consumption, the fuel injection timing, the ignition timing and the throttle valve were all fixed. In spite of these conditions, we can observe the long-term fluctuation of the engine system. Both measured entities should be constant in the ideal case. One has the immediate visual impression that the fluctuation is not just noise but contains a complex pattern (order). The emergence of this pattern is of deterministic origin. The low-frequency components and the high-frequency components of the fluctuation result from the chaotic dynamics with long-term correlations and the stochastic process, respectively. The data of low-frequency components have a positive Lyapunov exponent, which is an indication of chaos. It is reported that this fluctuation is caused by the interplay of a low-dimensional chaotic dynamics of engine systems and a stochastic process [4].

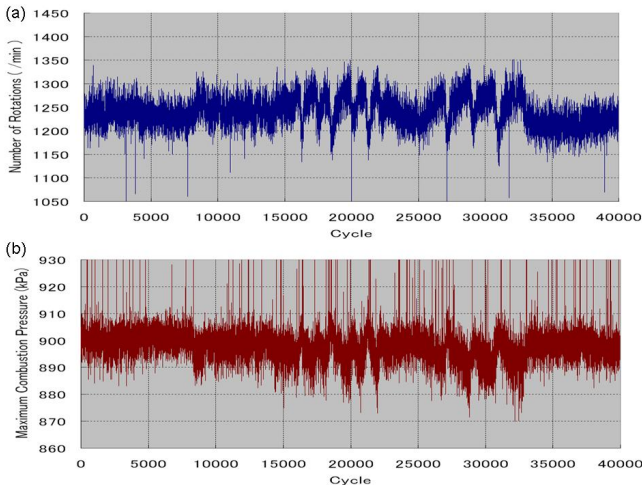


Fig. 1: Long-term fluctuation of an engine system. (a) Number of rotations vs. cycle. (b) Maximum combustion pressure vs. cycle.

## 3. CONTROLLING CHAOS FOR ENGINE SYSTEMS

The presence of deterministic components in the engine system in principle implies that the behavior of the system can be controlled by using the underlying dynamics. The notion of controlling chaos essentially refers to the stabilization of the chaotic behavior to a periodic behavior, because there exist

infinitely many unstable periodic orbits within the chaotic attractor. As for the subject of controlling chaos, the Ott-Grebogi-York (OGY) method [5] and Pyragas' method [6] are commonly used. The main advantage of Pyragas' method is that even if one does not explicitly know the equations of motion, one is able to obtain a periodic orbit merely by using an observed time series. One only has to supply the system with a feedback of the difference between the current value and a delayed value of the observed time series. Recently, we made use of this advantage and theoretically outlined how to apply Pyragas' method to the engine system [4]. We also made experiments of controlling chaos of an engine system [7]. In the following, we briefly explain the theoretical background of Pyragas' method and the results of these experiments.

### AN EXAMPLE OF PYRAGAS' METHOD

In order to explain this method, we give a numerical example using the Rössler system, which reads

$$\begin{aligned}\dot{x}(t) &= -y(t) - z(t) \\ \dot{y}(t) &= x(t) + 0.2y(t) + F(t) \\ \dot{z}(t) &= 0.2 + z(t)(x(t) - 5.7).\end{aligned}\quad (1)$$

Hereby, the externally added forcing term  $F(t) = K (y(t - \tau) - y(t))$  is the delayed feedback term with a feedback gain,  $K$ , and a delay time,  $\tau$ .  $K=0$  leads to the original unforced system. If  $K$  is unequal to 0, the original dynamics will be changed as a result of the difference between the actual state  $y(t)$  and the control state  $y(t - \tau)$ . Thereby, the control state  $y(t - \tau)$  is taken from the same system, however, measured at an earlier instant of time,  $t - \tau$ . In case  $y(t)$  is an exactly periodic state variable with period  $\tau$ , the forcing term vanishes. In case of some irregularity in the period, the resulting difference  $y(t - \tau) - y(t)$  forces the system back to a periodic behavior. A chaotic motion is aperiodic by definition. However, it contains periodic unstable sub-orbits which can be effectively stabilized using Pyragas' control method.

The effect of the Pyragas' method is illustrated in Figs. 2(b)-(c) and contrasted with the case of non-feedback shown in Fig. 2(a). The delayed feedback started at approximately  $t = 70$  shown in Fig. 2(b) and has been continuously applied thereafter. Figure 2(c) shows an obtained periodic orbit that appears shortly after the onset of the feedback. While the original system shows a chaotic orbit depicted in Fig. 2(a), the delayed feedback with parameter values  $K = 0.2$  and  $\tau = 5.9$  leads to a periodic orbit shown in Figs. 2(b)-(c). As can be seen in Fig. 2(b), the perturbation  $F(t)$  converges to zero, which indicates that the periodic relation  $y(t - \tau) = y(t)$  is satisfied for all  $t$ . Thus, the period of the obtained periodic orbit is exactly equal to the designated delay time  $\tau = 5.9$ . This result tells us that an unstable periodic orbit with period  $\tau = 5.9$  inherent in the original Rössler attractor is stabilized. Note that this principle works very well using only one variable of the three-dimensional system that is controlled. Assume that the engine fluctuation is also a chaotic motion that still contains a periodic orbit in the sense of an unstable orbit. Then it should be possible to use Pyragas' chaos control

method to reduce the chaotic motion and stabilize the previously unstable periodic orbit. Of course, we do not have the differential equation in the case of an engine. However, we can control the dynamics through the application of the forcing term to the mechanism that determines the measured variables, which is explained in the following.

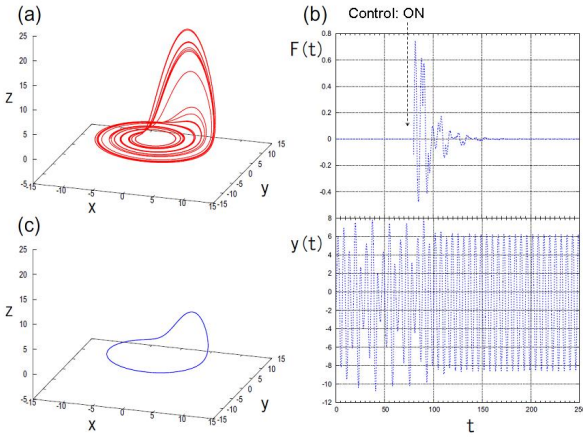


Fig. 2: Effect of Pyragas' method on the Rössler system. (a) Chaotic orbit with  $F(t)=0$  (non-feedback). (b) Perturbation  $F(t)$  (top) and time series  $y(t)$  (bottom) with  $K = 0.2$  and  $\tau = 5.9$ . (c) Stabilized periodic orbit obtained after some time from the onset of the feedback shown in (b).

## APPLICATION OF PYRAGAS' METHOD TO ENGINE SYSTEMS

As we described above, engine systems have a chaotic dynamics. In other words, it is considered that the long term fluctuation as in the engine rotation and the combustion pressure is caused by the dynamics. According to [1], cycle-to-cycle fluctuation in spark ignition engines entails the difficulty of controlling engine systems. It is known that the elimination of the fluctuation would lead to a 10 % increase in power output of engines [3]. From this viewpoint, the purpose of the experiment performed in the framework of controlling chaos is to stabilize the long term fluctuation, i.e., the chaotic motion to a periodic one and to eliminate the fluctuation.

We examined a four-stroke, single-cylinder 250cc engine for motorcycles. The experiments are conducted in the idle state around 1380 /min without load. The data are sampled as a function of crank angle. The amount of fuel consumption, the fuel injection timing and the ignition timing of each cycle are all fixed. The calculation of delayed feedback is made within the ECU. The feedback term is given by  $K ( y ( \theta - 720 \cdot M ) - y(\theta) )$ , where feedback gain is denoted as  $K$ , crank angle as  $\theta$ , delay time as  $M$  and intake manifold pressure as  $y(\theta)$ . The number 720 refers to two rotations of the engine, because one cycle consists of two rotations in four-stroke engines. The important ingredients with respect to the application of Pyragas' method are: (1) the observed data, (2) the choice of the value for the delay time, (3) characteristics of the

controlled object, and (4) the choice of the value of the feedback gain  $K$ .

First, we choose the intake manifold pressure as an observable variable, because its behavior shows a long-term fluctuation that contains low-dimensional chaotic dynamics. Second, the choice of delay time depends on the dominant periodic components of the system. As a result of calculating the autocorrelation function of the intake manifold time series, which was sampled against crank angle, its first local maximum appeared at approximately 1000 cycles. Therefore, we choose the value of delay time as  $M=1000$ . Third, we intend to stabilize the system's dynamics to be periodic by affecting the intake manifold pressure. However, we can not directly access this variable because it is impossible, in principle, to operate the pressure itself. Therefore, we instead use the throttle valve as the controlled object. The change of the throttle valve angle inversely corresponds to the change of the intake manifold pressure. Fourth, it is difficult to determine the proper value of the feedback gain analytically. The value depends on the experimental environment. Therefore, the adjustment of  $K$  is subject to a trial-and-error procedure.

We show the control system configuration in Fig. 3. The measured intake manifold pressure  $y(\theta)$  is sent to the ECU. Within the ECU, the delayed feedback term  $W$  is calculated, and then the target angle is set based on the value of the term. The ECU commands the throttle valve to reach the target angle.

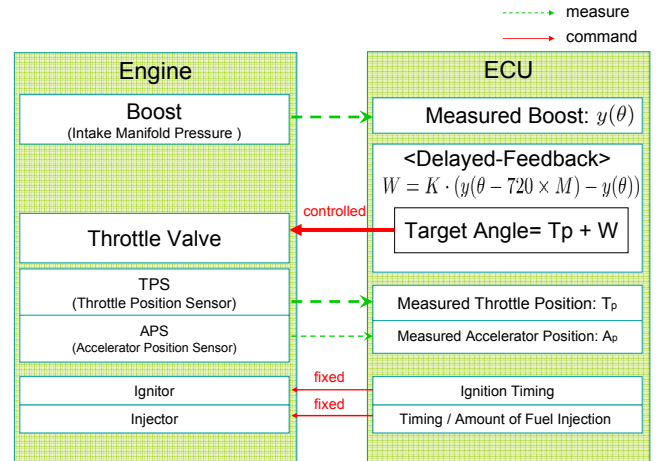


Fig. 3: Control System Configuration.

## EXPERIMENTAL RESULTS

We set the average throttle valve angle so that the average engine rotation became 1380 /min. The adaptation of the angle through the control mechanism was limited at each control step to an interval of  $\pm 0.5$  degrees around the current angle in order to keep the engine rotation within the range of 1300 /min to 1700 /min. In general, within this range, the increase of the throttle valve angle causes a decrease of the intake manifold pressure. Therefore, the value of the feedback gain has to be negative. Eventually, we fixed  $K = -50$ .

Figures 4(a) and 4(b) show both the cycle data for the number of rotations for 20000 cycles without control and with control, respectively. Figures 5(a) and 5(b) show the autocorrelation function corresponding to the data of Fig. 4. We can see from Fig. 5 that a periodic component, i.e., a 1000-cycle appears, which corresponds to the designated delay time chosen for our experiment with the delayed feedback. The average engine rotations in the experiment without control and with control, respectively, are 1386 /min and 1367 /min. Analogously, the standard deviations are 36 /min and 52 /min, respectively. From this result, one might conclude that the delayed feedback control amplifies the fluctuation of the engine rotation. However, in the experiment with control, the long-term fluctuation vanishes. Instead the fluctuation of high-frequencies becomes dominant. The dominance of the high-frequency components is due to the relationship between the throttle valve angle and the intake manifold pressure. Actually, it turned out that there is no one-to-one correspondence between these variables within the region of the engine rotation in this experiment. In other words, the opening of the valve does not always lead to a decrease of intake manifold pressure. In the average, however, a decrease can be clearly observed. To be more specific, opening the valve during compression stroke leads to a decrease of pressure, whereas, opening the valve during exhaust stroke leads to an increase of pressure. This means that in future studies we have to find a more appropriate observable variable or controlled object in order to effectively stabilize the chaotic behavior to a periodic one using Pyragas' method. In order to support our approach, we removed the high-frequency components by means of a moving average over 200 cycles for both the data of the uncontrolled and controlled system. Then we calculated the corresponding standard deviations, which result in 17 /min and 9/min. This indicates successful control and that the method is actually worth to be investigated in more detail in future studies.

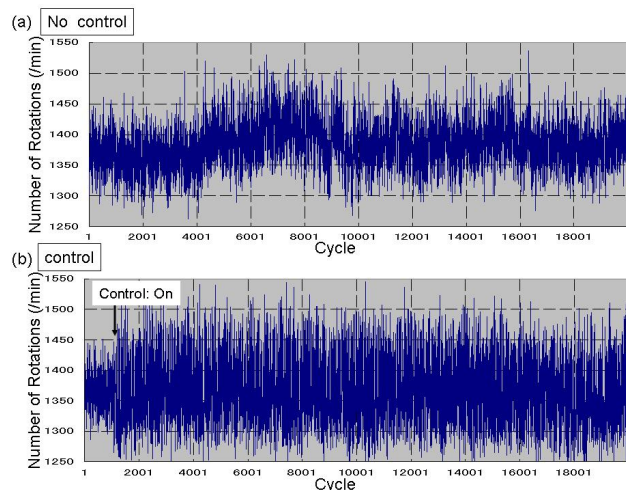


Fig. 4: Number of rotations vs. cycle. (a) No control. (b) Control.

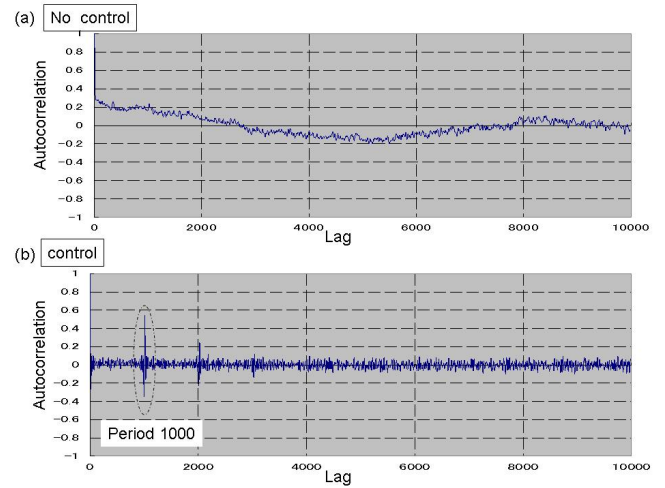


Fig. 5: Autocorrelation for the corresponding data in Fig.4. (a) No control. (b) Control.

#### 4. HARNESSING CHAOS FOR ENGINE SYSTEMS

Engineers and industrial designers often face the problem of competing strategies. On the one hand they have to maximize measures of entertainment and on the other hand optimize energetic and environmental conditions. In many cases the engineers eventually have to follow a certain policy. Harnessing chaos is not a patent remedy to resolve such problems. It has to be understood as an attempt to shift the focus on the engineering level from “control thinking” to “systemic thinking”. In the specific case of chaotic dynamics this shifted perspective means to regard chaos not a priori as spurious and disturbing but instead take seriously into consideration that chaos may be constituent for the dynamics on a global level. The following has to be understood as qualitative contribution in the sense of raising a hypothesis and the quest for experimental evidence in future studies.

We want to refer to the fact that aesthetic pleasure is obviously related to a certain degree of complexity [8]. There is experimental evidence for visual and acoustic phenomena. In short, pleasure arises between full order and full randomness. Chaos plays a prominent role in aesthetics. So far, there is lack in testing the impact of chaos in tactile or haptic experience where the whole human body is in resonance with some vibrating phenomenon. Strangely, there is the metaphor of “feeling vibration” and “movement” when people qualitatively refer to such experiences but rarely any experimental evidence beyond visual and acoustic phenomena. We here raise the hypothesis that the chaotic part of a system with which we deal is constituent for enjoyment. Moreover, even beyond entertainment we suggest to shift the focus to harnessing chaos whenever we deal with systems that exhibit chaotic dynamics. It is not always the best solution to make systems stationary with brute force but find strategies to synchronize in a nonlinear way with these systems when encountering chaotic dynamics.

Both harnessing chaos and controlling chaos utilize the underlying dynamics to control the system [9]. However, the concept of harnessing chaos crucially differs from that of controlling chaos, although it is not yet uniquely defined. To formulate it in an illustrative way, one might say that controlling chaos kills chaos in order to stabilize the controlled system to a periodic behavior, whereas harnessing chaos attempts to harmonize the user's behavior with the system's chaotic dynamics without killing chaos. The notion of harnessing chaos metaphorically refers to the unity of a horse and its rider. The rider does not suppress the horse, but rather allows for a certain degree of freedom of the horse's behavior and strives to harmonize with the horse. In brief, one can say that harnessing chaos is a control method that does not intend to suppress a "contingent" dynamics that emerges as a complex macroscopic order, it rather enhances this order.

In this context, consider the following two categories of systems: complicated (decomposable) systems and complex (non-decomposable) systems. For a complicated system, constituent elements are autonomously prepared in order to obtain a system by combining these elements. Each element has a specific function, and the characteristic of the system is determined by the combination of the elements. An example is a (humanoid) robot. When the robot is constructed, one prepares elements such as a head, arms and so on, and mounts the elements to obtain the robot. Each element has a characteristic function and it does not lose this function even if one decomposes the system. Therefore, the re-constructed system shows exactly the same characteristics as before the decomposition. On the contrary, a complex system substantially differs from a complicated system. A complex system emerges as a unit entity and is, therefore, essentially historic. An example is an embryo. Through embryogenesis, its functions are differentiated by a self-organization of the system. It is no longer ensured that there exist uniquely determined relationships between parts of the system and functions. Once a complex system is decomposed into parts, one can not re-construct it without losing an essential characteristic of the whole system. For sure, living systems belong to the class of complex systems.

Engine systems can be considered as complicated systems in the sense that they are composed of many elements. However, what does it mean that the engine system has a low-dimensional chaotic dynamics, as described above? In a synergetic framework, this chaotic dynamics emerges from a collective behavior of the microscopic motion through a self-organizing process of the engine system. Thus, the engine system achieved this order spontaneously in an autopoietic way. Seen from this more process-based perspective, we tend to regard the engine as a complex system with characteristics close to a living system. Due to the contingent aspect of the complex order, it is justified to speak of an individuality of the engine system. From this viewpoint, it is also plausible to apply control not in a suppressive way but in the direction of enhancing individuality.

How to apply harnessing chaos to engine systems is still an open problem. A possible approach could be to incorporate a learning unit into engine systems such as neural networks in

order to increase combustion efficiency, for instance, following the model of living systems. It is already known that neural networks can acquire the time evolution mechanism of chaotic dynamical systems [10-11]. It should also be possible to learn the dynamics of an engine system using neural networks. As a further step, we want to stimulate the near future investigation of artificial intelligent units that also include the adaptation to the dynamics of interacting drivers of motorcycles or automobiles. Such a learning system may enable us to build a motorcycle (or automobile) in such a way that the individuality of the extended system that comprises motorcycle and driver is preserved or even emphasized. Beyond these engineering strategies we suggest to learn from the experiments performed in the fields of visual and acoustic entertainment in order to design proper experiments for the performative (embodied) interaction with engine systems. Neuro-aesthetics is a promising candidate to cast new light into the area of harnessing chaos. Hereby, pleasure and arousal significantly correlate with sympathetic and parasympathetic physiological signals like skin-resistance and pulse. Such kind of experiments using these signals along with the investigation of rating variables of test persons are extensively performed to optimize industrial design in the visual and partially in the acoustic areas but rarely for the "whole-body interaction" with systems.

## 5. CONCLUSION

We described the control issue for engine systems by comparing controlling chaos and harnessing chaos. As an example of controlling chaos, we applied Pyragas' method to an actual engine system. The experimental results indicate that the chaotic motion of the engine system could be stabilized to a periodic motion, although slight modifications are needed, including the choice of the observable variable and the controlled object. As an alternative approach, we introduced the concept of harnessing chaos applied to engine systems. Although our explanation remained qualitative and somewhat speculative, we nevertheless believe that it will fruitfully contribute to development of engine systems.

## 6. DISCUSSION

Modern control techniques are rather precise. For example, the availability of electronic fuel injection systems precisely controls the amount of fuel consumption. Furthermore, the development of an ECU led to the accuracy of the ignition timing and the throttle valve angle. The dynamics of the engine system is thereby suppressed, and the degree of freedom in the engine system behavior decreases.

At that point, we change our point of view and instead promote an engine system with more freedom. In the past, carburetor engines have been commonly used. A carburetor is a device to mix air with fuel. Unlike fuel injections, carburetors can not precisely supply a desired amount of fuel to the combustion chamber, because it depends on the state of the engine system due to the Venturi effect. This means that the amount of supplied fuel fluctuates and can be regarded as a variable, rather than a parameter. Within our metaphoric language of a living system, this fluctuation can be compared

with the respiration of the engine system. We regard the following questions as an outline for future studies.

Can the formula “less control and more freedom leads to self-organizing phenomena in engine systems” be scientifically justified? If so, can engine systems spontaneously improve their performances in a self-organized way which may lead to an increase of combustion efficiency and reduction of emissions? According to [12], there is a clear indication for the presence of chaos in dynamics of living systems as, for example, heart beat or brain activity. A periodic dynamics observed in such systems usually indicates malfunctioning or unhealthy states, like epilepsy or heart insufficiency. It is reasonable to assume that “liveliness” of a living system is strongly connected to chaos. The joy of riding motorcycles may likewise be related to liveliness induced by chaos. How can this be reconciled with the demand for environmental care?

Nowadays it is common practice to produce noiseless and vibration-less automobiles and motorcycles. Needless to say, particularly in the case of motorcycles this leads to a reduction of joy riding them. It is, therefore, a legitimate question of how joy and an environmental reasonability can be reconciled. Chaos is a prominent candidate to ensure contingency within a deterministic dynamic framework [13] because of a fundamental uncertainty in observation. According to [13], when we encounter with contingency, we can definitely experience something new, which is not captured by our current representation. This contingency might enable us to have an experience which we have never had. We argue that this experience may contribute to our amusement.

As a matter of course, we have to take care of the earth’s environment. The above thoughts concerning harnessing chaos are understood as a proposition to harmonize the environmental concern with spirit and purpose of riding a motorcycle or driving a car. Needless to say, the factor of entertainment is more emphasized for the case of motorcycles. We have to devise a new way in which both environmental conservation and our entertainment are compatible. In this respect, we conclude with an emphasis of the importance of harnessing chaos for “iron horses” (motorcycles). In our view, chaos is not regarded as spurious but rather as a “life-giving” element. This attitude is in line with a generally emergent rethinking with respect to the control of complex systems that include human beings [13]. Instead of reifying the user we advocate an embodied and more process-based approach to engineering.

## ACKNOWLEDGMENTS

One of the authors (K. M.) acknowledges the Institute for New Media (INM) in Frankfurt for hospitality in his three-month residence. He is grateful to the “Deutscher Akademischer Austauschdienst” (DAAD) for a financial support during his stay in Germany (ID No.: A/07/98966).

## REFERENCES

1. M. Wendeker, G. Litak, J. Czarnigowski, and K. Szabelski, “Nonperiodic oscillations of pressure in a spark ignition combustion engine,” *Int. J. Bifurcation and Chaos*, 14, 5 (2004) 1801-1806.
2. G. Litak, T. Kamiński, R. Rusinek, J. Czarnigowski, and M. Wendeker, “Patterns in the combustion process in a spark ignition engine,” *Chaos, Solitons & Fractals*, 35 (2008) 578-585.
3. M. Wendeker, J. Czarnigowski, G. Litak, and K. Szabelski, “Chaotic combustion in spark ignition engines,” *Chaos, Solitons & Fractals*, 18 (2003) 803-806.
4. K. Matsumoto, I. Tsuda, and Y. Hosoi, “Controlling engine system: a low-dimensional dynamics in a spark ignition engine of a motorcycle,” *Z. Naturforsch.*, 62a (2007) 587-595.
5. E. Ott, C. Grebogi, and J. A. Yorke, “Controlling chaos,” *Phys. Rev. Lett.* 64, 11 (1990) 1196-1199.
6. K. Pyragas, “Continuous control of chaos by self-controlling feedback,” *Phys. Lett. A*, 170 (1992) 421-428.
7. K. Matsumoto, I. Tsuda, and Y. Hosoi, “Controlling Engine System from the Perspectives of Chaos Theory,” *JSAE Annual Congress (Spring)*, Yokohama, (2008) (to be held).
8. J. Casti and A. Karlqvist (Eds.), “Art and Complexity,” Elsevier Science, 2003.
9. M. Yamaguti ed., “Towards the Harnessing of Chaos: A Collection of Contributions Based on Lectures Presented at the Seventh Toyota Conference, Mikkabi, Shizuoka, Japan,” Elsevier, (1994).
10. M. Sato, and Y. Murakami, “Learning nonlinear dynamics by recurrent neural networks”, *Proceedings of the Symposium on Some Problems on the Theory of Dynamical Systems in Applied Sciences*, Singapore: WorldScientific, (1991) 49-63.
11. I. Tokuda, R. Tokunaga, and K. Aihara, “Back-propagation learning of infinite dimensional dynamical systems,” *Neural Networks*, 16 (2003) 1179–1193.
12. I. Tsuda, T. Tahara, and H. Iwanaga, “Chaotic Pulsation in Human Capillary Vessels and its Dependence on Mental and Physical Conditions,” *International J. of Bifurcation and Chaos* 2 (1992) 313-324.
13. Hans H. Diebner, “Performative Science and Beyond - Involving the Process in Research,” Springer Verlag, Wien, NY, (2006).

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