Studies of the Mechanical Properties of Sea Ice XI

The Flexural Strength of Sea Ice *in situ**, **

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Abstract

The flexural strength of sea ice *in situ* was measured by a cantilever method. The measurement was carried out every February from 1963. The thickness of ice was 23∼30 cm. The width of the beam was 30∼40 cm. The length was 1.5∼2 m. The surface temperature of the beam was −2.0∼−3.7°C, the mean temperature was −0.8∼−2.5°C and the chrolinity was 2.6∼7.5‰.

Both upward and downward bending forces were applied manually by a wooden lever with various speeds. The force was measured by an electric load-cell and was recorded with an oscillograph. The acceleration of the bending at the free end of the beam was also measured with an accelometer and also recorded. The deflection of the beam was obtained by integrating the acceleration. The increasing rate of the tensile stress at the surface of the supporting end of the beam was 0.1∼30 kg/cm²·sec. The flexural strength and Young's modulus were 0.8∼5 kg/cm² and 1∼12×10³ kg/cm² respectively.

The results are summarized as follows. In the case of the increasing rate of stress $\dot{\sigma}$ less than 1 kg/cm², the beam was deflected plastically, and in the case of $\dot{\sigma}$ larger than 1 kg/cm², the beam was deflected elastically and broke in the same manner as brittle material. In the latter case, the flexural strength increased remarkably with the increasing rate of stress. It can be said the flexural strength of sea ice is dependent upon the increasing rate of stress, ice temperature, the chlorinity (or salinity) and the structure.

I. Introduction

Studies of the strength of small samples of sea ice removed from natural sea ice sheets have been made by many writers. Such results indicate clearly that the strength of sea ice depends upon its crystal structure, temperature profile and salinity. Butkovich (1956), Weeks and Anderson (1958) measured the strength of sea ice by running a large number of in place cantilever tests, and showed that the flexural strength of sea ice becomes weaker with the increase in its salinity and with the ascending of temperature. In a concurrent paper, Anderson and Weeks (1958) explained that the flexural strength of young sea ice depends upon brine inclusions within the sea ice. Assur (1958) extended this work and showed that, at high temperatures above −8.2°C, the strength of sea ice decreased with the increase of the square root of the volume of brine.

Now sea ice is a visco-elastic substance, and is well known that the strength of visco-elastic material, or even that of elastic material, is dependent upon the increasing rate of the applied force. Therefore, it may be expected that the strength of sea ice

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will depend also upon the increasing rate of the applied force. Actually one of the present authors Tabata (1960, 1967) and Peyton (1964) found the fact that the strength of sea ice increases with the increasing rate of the applied force.

The present study is confined only to in-place cantilever sea ice beam tests made on thin (less than 30 cm), shore fast young ice. The field tests were carried out at Mombetsu harbour located on the Okhotsk Sea coast of Hokkaido (44°21′ N, 143°21′ E). The initial ice-formation on this coast takes place in the middle of January. The thickness of ice reaches about 40 or 50 cm in the middle of March. The measurements were carried out in the middle of February during the last four winters, 1963～1966 (Tabata and Fujino, 1964, 1965; Tabata, 1966).

The objective of the present study is to determine the relations between the flexural strength of young sea ice *in situ* and such parameters as the increasing rate of applied force, temperature and salinity.

II. Method of Measurement

A U-shaped channel was cut in the ice sheet using an ice chain saw and a hand saw. A cantilever ice beam with one end attached to the ice sheet was made with this channel. The width of the beam was 30～40 cm and the length was 1.5～2.5 m. The thickness of the ice sheet was 22～29 cm. A very thin ice beam (8～10 cm) 1 m long was also tested. The ice beam was bent mainly in two directions, vertically downward and vertically upward. In February 1965 alone, the beam was also bent in a horizontal direction.

The bending force, F, was applied manually. Downward force was applied with a vertical wooden lever attached to the horizontal wooden lever. Upward force was applied by pulling up a chain which was placed around the free end of the ice beam and connected to the horizontal wooden lever. The procedure of the measurement is shown schematically in Fig. 1. In horizontal bending, a chain was placed around the free end of the ice beam and was pulled manually. The applied force was measured with an electric load-cell connected to the wooden vertical lever or chain and recorded with an oscillograph.

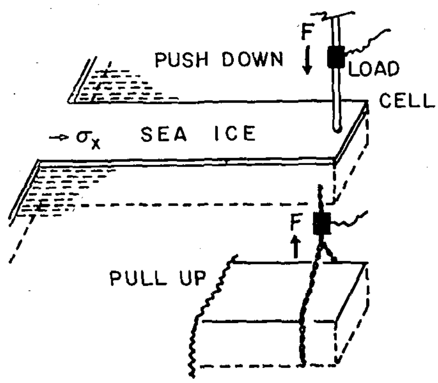

Fig. 1. Schematic diagram of experimental procedure

In applying the bending force, this was done in such a way that the applied force would increase with a constant rate to the breaking point. Moreover, to obtain the relation between the flexural strength and the rate of loading, the increasing rate of the applied force was changed purposely with every ice beam.

The acceleration of the bending of the beam, a, was also measured with a small electric accelometer placed on the free end of the beam, this was also recorded with an oscillograph. The deflection of the beam, d, was obtained by integrating the recorded acceleration.

After failure, the broken beam was immediately pulled up from the water and the length, l, the width, b, and the thickness, h, were measured.

A series of thermocouples were frozen vertically into the ice sheet in the test area at several centimeter intervals and recordings were made during the period of the tests. The temperature profile of the tested beam was determined by this temperature record. During the measurement, several vertical cores of 7 or 5 cm in diameter were taken from the ice sheet in the test area and stored in tightly sealed bag. These ice samples were melted and the chlorinity of melted water was determined by Mohr's $AgNO_3$–method of titration.

To observe the crystallographic structure of the ice sheet, a vertical ice plate, of 1 cm in thickness and about 25 cm in width, was sawed out from the ice sheet and was allowed to float horizontally on the sea surface. By this method, the structure of the ice sheet, such as, thickness of slush ice and of skelton layer, the approximate dimension of ice grain was easily estimated.

III. Ice Condition

Throughout the test period during four winters, the temperature of the ice sheet was relatively high. Ice conditions during the measurement are summarized in Table 1.

The surface and the bottom layers of the ice sheet had a higher chlorine content than that of the middle part of the ice sheet.

Table 1. Ice condition

	Total ice thickness (cm)	Thick. of slush ice (cm)	Mean ice temp. (°C)	Mean chlorinity (Salinity) (‰)	Type* & N. of test	Date (Feb.)
1963	26		−1.9∼−2.5	5.8 (10.4)	D–21 H–20	13–18
1964	8∼23	0∼ 6	−1.2∼−1.5	3.2 (5.9)	D–50 U–20	15–19
1965	19∼22	15∼20	−1.0∼−1.6	2.2 (4.1)	D–11 U–25	18–22
1966	21∼29	11∼16	−2.0∼−2.6	4.2 (7.5)	D–27 U–17	21–27

* D: Downward bending U: Upward bending H: Horizontal bending

IV. Results of Measurement

Examples of the record of the changes in the bending force, F, and in the acceleration, a, are shown in Fig. 2. It was found that though the bending force was applied manually, the force increased linearly. The moment of failure is easily recognized from the curves of the force and the acceleration. The bending force at the moment of failure, F_c, ranged from 8 to 80 kg. The time required to break the beam was between 0.1 to 8.4 sec. As is the usual case of a cantilever beam, almost all tested ice beams were also broken along the vertical plane at the supported end.

The flexural strength of the beam, σ_c, is determined primarily by the tensile strength of the outermost surface at the supporting end and is obtained from

$$\sigma_c = 6F_c/wh^2 , \tag{1}$$

where, F_c is the force applied to the free end of the ice beam at the moment of failure,

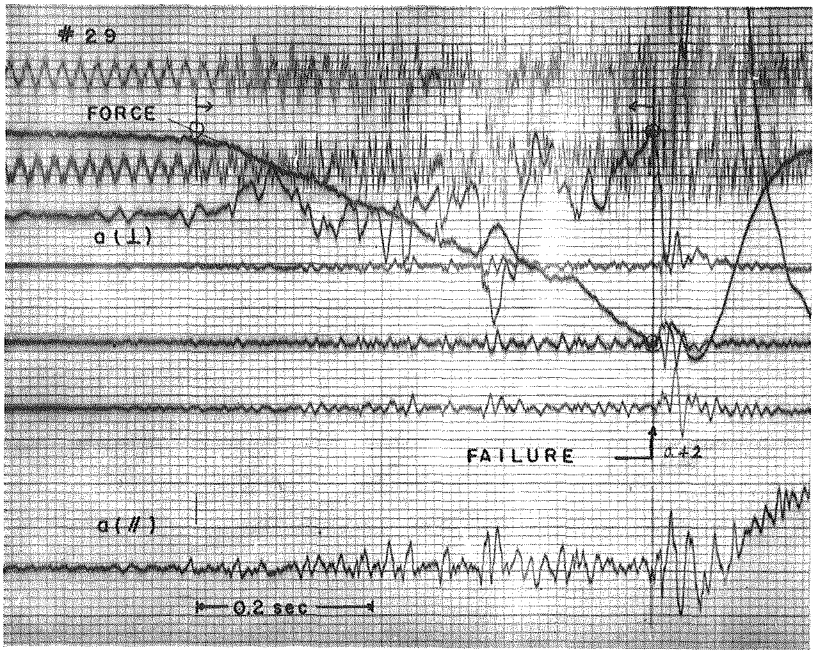

Fig. 2. An example of a recorded curve

l, the distance between the point of application of the force and the point of breakage, w, the width of the beam and, h, the thickness of the beam. Since the bending force F increases linearly, the tensile stress within the beam also increases linearly. The increasing rate of the maximum tensile stress of the surface at the breakage point, $\dot{\sigma}$, was also computed by

$$\dot{\sigma} = 6\dot{F}\,l/wh^2 . \tag{2}$$

$\dot{\sigma}$ was constant for each beam and ranged from 0.1 to 40 kg/cm²·sec throughout the measurements.

The obtained results of the strength were classified into four groups with the mean ice temperature, namely, -1.0 ± 0.2, -1.5 ± 0.2, -2.0 ± 0.2°C and less than -2.3°C. Ex-

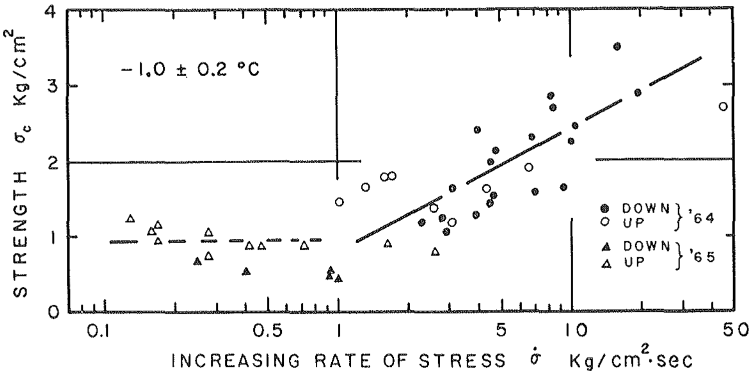

Fig. 3. The relation between the flexural strength of sea ice σ_c and the increasing rate of applied stress $\dot{\sigma}$ at -1.0°C

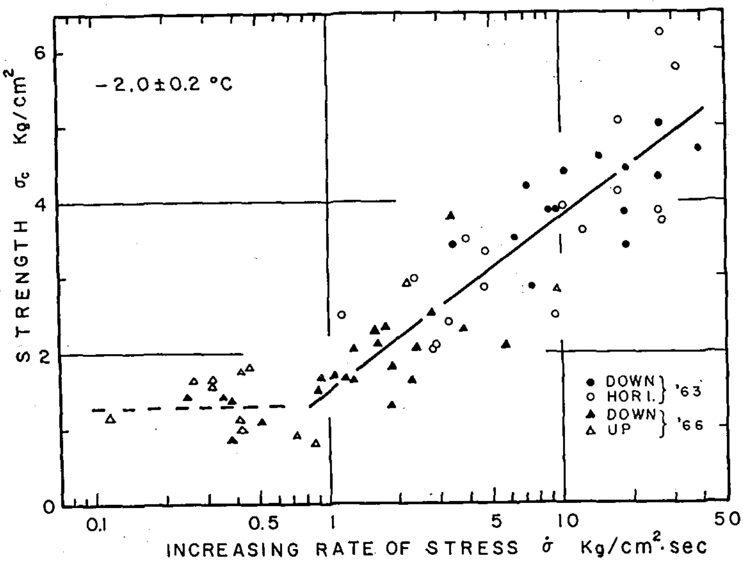

Fig. 4. The relation between the flexural strength of sea ice σ_c and the increasing rate of applied stress $\dot{\sigma}$ at -2.0°C

amples of the relation between the increasing rate of the maximum tensile stress $\dot{\sigma}$ and the strength σ_c are shown in Figs. 3 and 4.

From these figures, the stress rate dependence of the strength is clearly recognized. It is obvious that, even in such a high temperature range, if the increasing rate of stress $\dot{\sigma}$ is larger than 1 kg/cm²·sec, the strength of sea ice increases remarkably with $\dot{\sigma}$, and for the smaller $\dot{\sigma}$, the strength seems to be constant. It is also obvious that the strength increases with lowering the temperature.

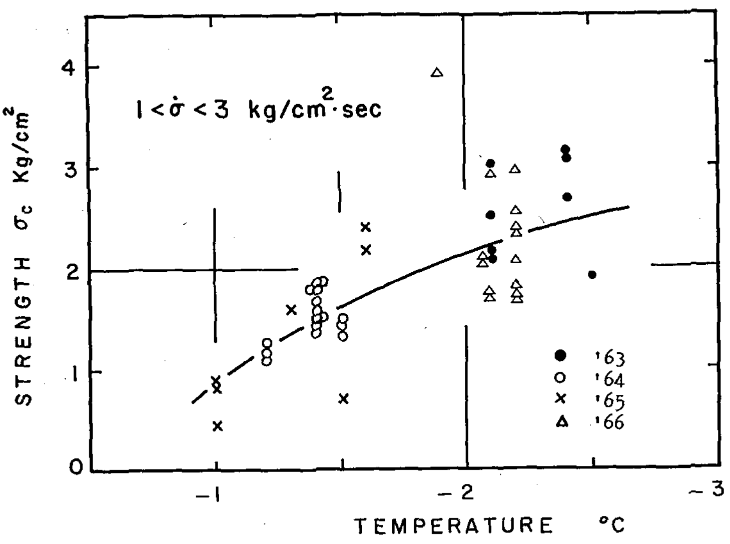

Fig. 5. The relation between the strength of sea ice σ_c and temperature in the stress rate range $1<\dot{\sigma}<3$ kg/cm²·sec

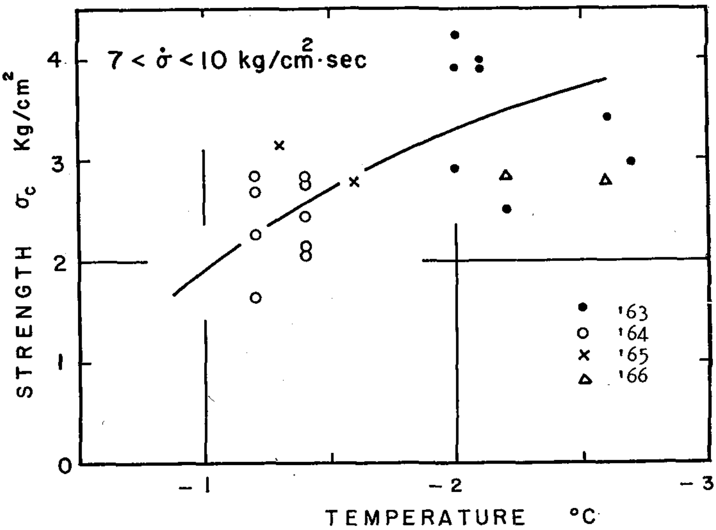

Fig. 6. The ralation between the strength of sea ice σ_c and temperature in the stress rate range $7<\dot{\sigma}<10\,\mathrm{kg/cm^2 \cdot sec}$

The line which represents the stress rate dependence of the strength in Fig. 4 has a much larger gradient than that in Fig. 3. It may be concluded, therefore, that the stress rate dependence of the strength increases with the lowering of the mean ice temperature. The temperature dependence of the strength is shown in Figs. 5 and 6. To avoid the stress rate effect upon the strength, Figs. 5 and 6 are written by the data obtained in a narrow range of the stress rate $1>\dot{\sigma}<3$ and $7<\dot{\sigma}<10\,\mathrm{kg/cm^2 \cdot sec}$ respectively. The temperature dependence of the strength in high temperature range is evident. The sudden decrease in the strength near freezing temperature may be due to the sudden increase of the brine volume and weakened strength of pure ice. From Figs. 3~6, if the increasing rate of the stress is larger than $1\,\mathrm{kg/cm^2 \cdot sec}$, it can be concluded that the strength of sea ice is largely dependent upon both the increasing rate of stress and the temperature.

In Figs. 3 and 4, the type of the test, upward bending, downward bending and horizontal bending, is classified in the legend. The strength of the beam is determined by the tensile strength of the extreme surface at the fixed end of the beam. In upward bending, it is the bottom surface and in downward bending, it is the upper surface and for horizontal bending test, the vertical plane. The bottom surface of the ice sheet is known as the weakest part because of its high salinity and high temperature. Therefore, upward bending is expected to give the weakest strength among them. It is surprising, however, as may be seen in these figures and as was already stated by Weeks and Anderson (1958) and the authors (Tabata and Fujino, 1964), that the upward bending, downward bending and horizontal bending give essentially the same strength. This result indicates, insofar as this paper is concerned that sea ice may be considered as a homogeneous material for bending test. Therefore, it can be said that an application of eq. (1) for computing the strength is approximately correct.

An example of the acceleration of bending at the free end of the beam during the

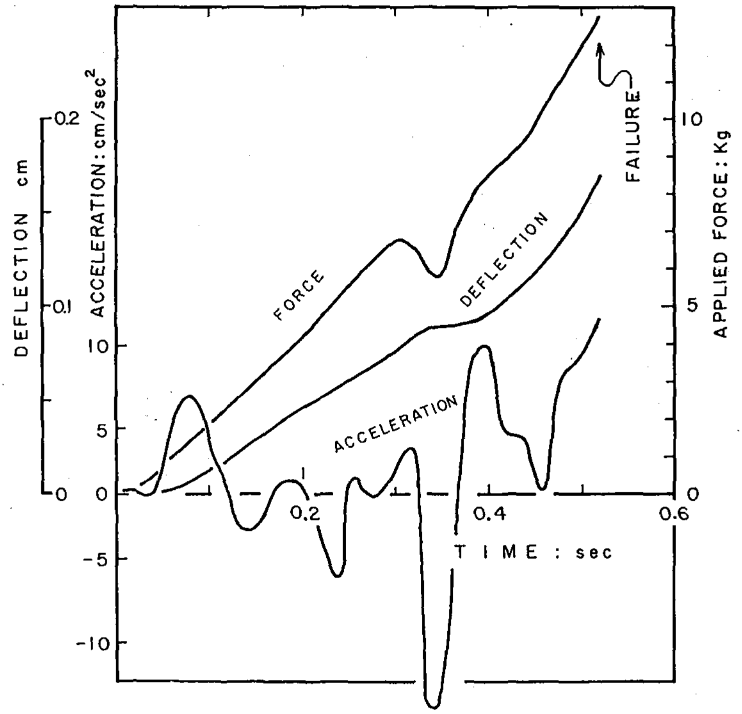

Fig. 7. An example of acceleration a and deflection d at the free end of the beam

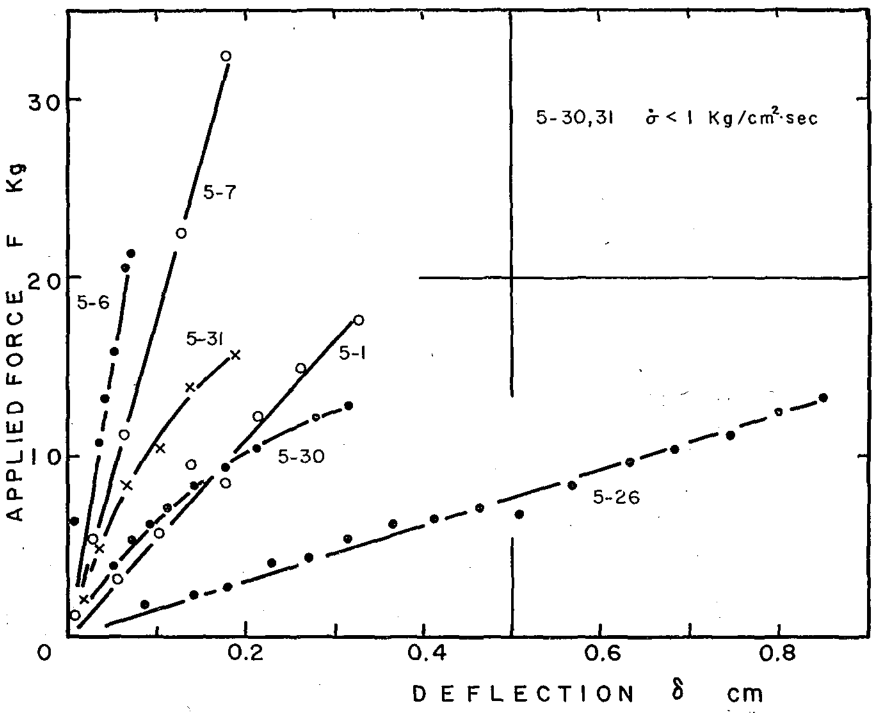

Fig. 8. The relation between the applied force F and deflection of the beam d

bending and the deflection which is obtained by integrating the acceleration are shown in Fig. 7. The deflection at the moment of failure is about 0.17 cm in the figure. Throughout the measurement, the deflection of the beam was very small and ranged from 0.06 to 1.0 cm. From Fig. 7, it is obvious that the deflection d increases in proportion to the increasing of the applide force F. Examples of the relation between the force F applied to the beam and the deflection d of the beam is shown in Fig. 8. The results obtained when the increasing rate of the stress is less than 1 kg/cm²·sec are identified in the legend. It is obvious from the figure, only in the case when the stress rate $\dot{\sigma}$ is larger than 1 kg/cm²·sec, that the deflection increases in linear proportion to the applied force F, that is to say, the beam deflected in such the same manner as an elastic substance and broke brittlly. Young's modulus for the sea ice beam may, therefore, be obtained from the equation,

$$E=\frac{4Fl^3}{dbh^3}\,. \tag{3}$$

It must be noted, however, since no recovery was observed in the deflected beam when the applied force was removed, that the authors cannot conclude that the sea ice beams are truly elastic. On the contrary, when $\dot{\sigma}$ is less than 1 kg/cm²·sec the beam deflected and broke plastically.

The obtained Young's modulus E is plotted against the stress rate $\dot{\sigma}$ in Figs. 9 and 10. Figure 9 is the data obtained in a temperature range of -1.0 ± 0.2°C and Fig. 10 is that obtained at -1.5 ± 0.2°C. From these figures, the increasing of the Young's modulus E due to the increasing of the stress rate $\dot{\sigma}$ is obviously recognised.

The reason of increase of Young's modulus with the increase of the stress rate is a no uncommon occurrence and was considered as follows. Since sea ice is a typical visco-elastic substance, the mechanical property of sea ice can be represented by a combination of many Maxwell-models of various time constant. Therefore, with the increase of the increasing rate of the stress, the time constant of the model which acts in response to the applied stress becomes shorter. In other words, the model of higher frequency, that is to say, the spring of larger Young's modulus comes to act in response to the stress. Thus, the observed Young's modulus was considered to increase with the increasing rate of the stress.

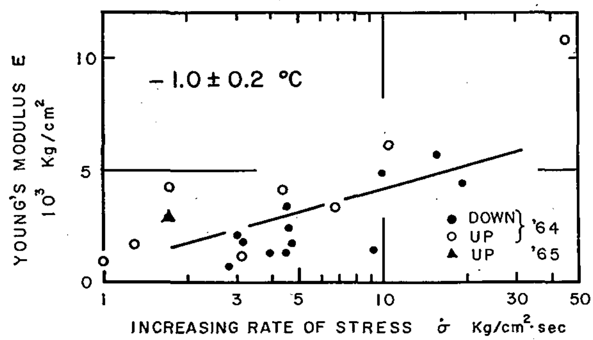

Fig. 9. The relation between the Young's modulus E and the increasing rate of applied stress $\dot{\sigma}$ at -1.0°C

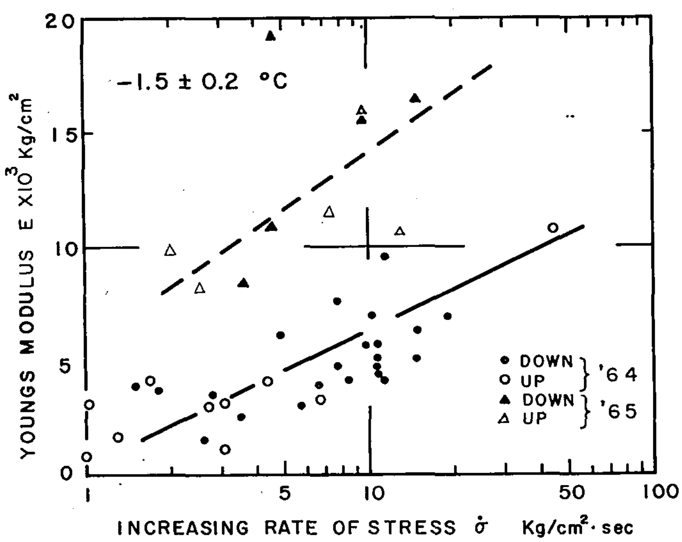

Fig. 10. The relation between the Young's modulus E and the increasing rate of applied stress $\dot{\sigma}$ at $-1.5°C$

In Fig. 10, data obtained in 1965 show extremely high values of Young's modulus. In this winter, as seen in Table 1, the thickness of slush ice was much thicker as compared with that of other winters. The slush ice is formed by freezing of the mixture of snow accumulated on the ice sheet and flooded sea water, and is composed of granular ice crystals. The Young's modulus of such a granular sea ice is usually larger than that of normal sea ice (Tabata, 1958). The thickness of such slush ice was very thick in 1965 as seen in Table 1. The observed extremely high value of Young's modulus may be caused by this large thickness of slush ice.

The Young's modulus reaches a magnitude of 10^4 kg/cm^2 in the case of a high stress rate, which is equivalent to that obtained by ultra-sonic methods. (Brown, 1963;

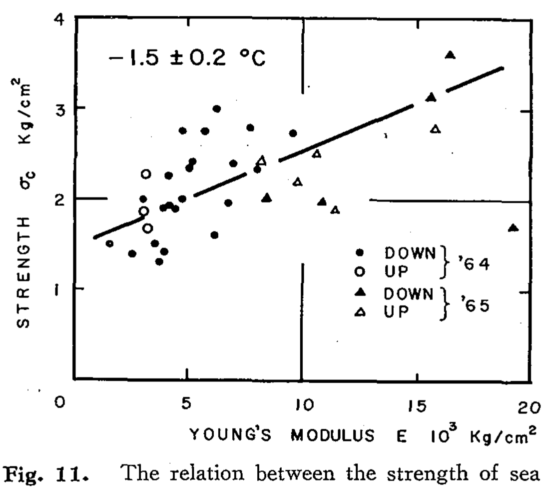

Fig. 11. The relation between the strength of sea ice σ_c and the Young's modulus E

Langleben and Pounder, 1963). The least value of the Young's modulus is almost equal to that given by a static method. (Butkovich, 1956; Tabata, 1960).

As found in Figs. 3 and 4, the strength σ_c also increases with the stress rate $\dot{\sigma}$. Increasing of the strength with $\dot{\sigma}$, therefore, may be caused by increasing of Young's modulus with $\dot{\sigma}$. Figure 11 is an example of the relation between the strength σ_c and the Young's modulus E. The tendency of increasing of σ_c with E is obvious.

A scattering in the values is seen in Figs. 3 to 11 which represent the results of the measurement. In these figures, the strength and the Young's modulus is summarized only as the function of the mean ice temperature and the increasing rate of stress. Actually, however, besides these parameters, the strength and the Young's modulus of sea ice are also dependent upon its chlorinity (or salinity), crystallographic structure and air inclusion. Concerning the relation between the structure of sea ice and the mechanical properties, Tabata (1960) alone observed higher strength in slush ice than that in normal sea ice, however, no precise work is known in this relation.

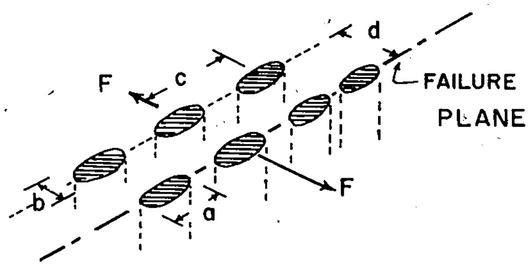

Fig. 12. Schematic diagram of brine cylinders.

To obtain an approximate relationship between the strength of normal sea ice (mosaic ice) and slush ice (granular ice), a well known simplified model of sea ice shown in Fig. 12 will be used. In the figure, the brine cell is assumed to have an elliptical section. When sea ice breaks under tension, it breaks primarily along these brine cells. It can be assumed, as stated by Anderson and Weeks (1958) and Assur (1958), that the strength of sea ice depends on the actual cross sectional area of ice itself normal to the tension. In Fig. 12, brine volume ν in a unit volume of sea ice is represented by $\nu=\pi ab/4cd$, where a and b is the length of the long and short axis of the ellipse respectively, c is the cylinder separation in the failure plane, d is the cylinder separation perpendicular to the failure plane. The minimum line density of the hole in the failure line is $a/c=\nu d/\pi b$. Spacing d is about 0.5 mm and the maximum value of b is about 0.07 mm (Anderson and Weeks, 1958). Therefore, $a/c \fallingdotseq 9\nu = n\nu$ in the case of normal sea ice. Slush ice is composed of granular ice grain and it has a random distribution of brine pockets or air inclusions. Therefore, the line density in any line in the failure plane is always equal to the brine volume ν. Thus, the tensile strength of normal sea ice σ_n and of slush ice σ_s may represented in the form

$$\sigma_n = k\sigma(1-n\nu), \qquad \sigma_s = k\sigma(1-\nu),$$

where, k is a stress concentration factor and σ is the strength of pure ice. Thus, $\sigma_n/\sigma_s \fallingdotseq 1-(n-1)\nu$. The value of ν is less than 0.15 (Anderson and Weeks, 1958). If the brine cylinder is an elliptical cylinder, n is estimated as nine. However in fact, the geometrical form of brine cylinder substantially differs from an elliptical cylinder, and in that case, the value n is reduced. In any case, the ratio σ_n/σ_s is smaller than unity, in other words the slush ice is expected to have a larger strength than that of normal sea ice.

It is widely known that the strength and the Young's modulus of sea ice decreases with the increase of the chlorinity, because the relative volume of liquid brine within sea ice increases. But in this paper the above two factors were not considered. Actually, however, as shown in Table 1, the chlorine content of sea ice and the thickness of slush ice are different from each other for each winter. It is considered that owing to these reasons, a scattering in the value is seen in the figures.

V. Summary

The flexural strength of sea ice *in situ* was measured by a cantilever method. Because of the complexity of the properties of sea ice tested, the obtained results are scattered, however, they give the following results.

(1) Upward, downward and horizontal bendings give essentially the same strength.

(2) Even in such a high temperature range as -1.0 to -2.2°C, the flexural strength of sea ice is largely dependent upon the increasing rate of the applied stress. If the stress rate exceeds an order of magnitude 1 kg/cm^2·sec, the sea ice deforms as in the case of elastic substances and broke brittly, and both the Young's modulus and the strength increase with the stress rate. In the case of a lesser stress rate, the sea ice deforms plastically and the strength seems to be constant.

(3) In the case of a given increasing rate of applied stress, the strength of sea ice is dependent upon its temperature.

(4) The flexural Young's modulus, at a given temperature, also increases with the increase of the increasing rate of applied stress.

Besides the increasing rate of applied stress, the temperature and the chlorine content, the density and the crystallographic structure also have an affect on the strength and the Young's modulus of sea ice. In the present study, these effects were not ascertained.

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