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Localization-delocalization transition in one-dimensional electron systems with long-range correlated disorder

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We investigate localization properties of electron eigenstates in one-dimensional (1D) systems with long-range correlated diagonal disorder. Numerical studies on the localization length ξ of eigenstates demonstrate the existence of the localization-delocalization transition in 1D systems and elucidate nontrivial behavior of ξ as a function of the disorder strength. The critical exponent ν for localization length is extracted for various values of parameters characterizing the disorder, revealing that every ν disobeys the Harris criterion $\nu > 2/d$.

I. INTRODUCTION

Spatial correlation of disorder often causes an unexpected phenomenon in quantum disordered systems. Most intriguing is the breakdown of Anderson localization in one-dimensional (1D) systems induced by correlated disorder. Breakdown of the localization in 1D systems has been predicted for a random dimer model,[1] wherein the on-site potential $\{\varepsilon_i\}$ has a binary distribution with a *short-range* spatial correlation. Subsequently, a discrete number of extended eigenstates was found numerically in the random dimer model.[2,3] It was examined experimentally using transmission measurements on semiconductor superlattices.[4] These findings have motivated the studies of the nature of 1D systems with *long-range* correlated disorder.[5–12] Particularly noteworthy is the system in which the sequence $\{\varepsilon_i\}$ has a power-law spectral density of the form $S(k) \propto k^{-p}$; the function S(k) is the Fourier transform of the spatial correlation function $\langle \varepsilon_i \varepsilon_j \rangle$. For exponents p greater than 2.0, there is a finite range of energy values with extended eigenstates.[6] This fact indicates the presence of the localization-delocalization transition in 1D disordered systems against the conclusion of the well-known scaling theory.[13,14] The emergence of extended eigenstates was also observed in harmonic chains with random couplings[15] and in those with randoms masses.[16] Note that diagonal disorder treated in Ref. 16 is characterized by the power-law spectral density denoted above.

The randomness of the long-range correlated potentials $\{\varepsilon_i\}$ is characterized by two quantities. The first is the exponent of the power-law spectral density p, determining the roughness of potential landscapes, as shown in Fig. 1. The second quantity is the distribution width W defined by the relation $\varepsilon_i \in [-W/2, W/2]$, which characterizes the amplitude of the potential.[17] Effects of p on the localization properties of eigenstates have been examined,[6] but those of W remain unclarified. If the disorder is spatially *uncorrelated* ($p=0$), an increase in W trivially induces strongly localized wave functions because all eigenstates are localized exponentially.[14,20] On the other hand, when the disorder is sufficiently long-range correlated to yield extended eigenstates ($p > 2.0$), the system shows a critical point W_c separating localized and delocalized phases. However, there is no attempt to quantitatively determine the value of W_c. Calcu-
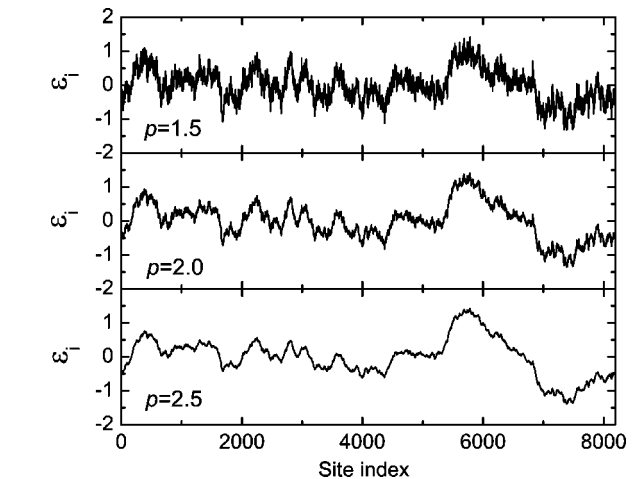

FIG. 1. Landscapes of spatially correlated on-site potentials. The exponent p of the power-law spectral density $S(k) \propto k^{-p}$ is varied as displayed in the figure. The roughness of the energy landscape is gradually reduced with increasing p.

lations for W_c for various values of p allow us to establish the phase diagram in the W-p space, thereby engendering better understanding of localization properties of the system. We are also interested in the value of the critical exponent ν for the localization length of eigenstates. Those values can be obtained accurately using finite-size scaling analysis.

The present work is intended to reveal critical properties of electron eigenstates in 1D systems with long-range correlated disorder. Numerical studies on the localization lengths ξ have demonstrated the nontrivial behavior of ξ as a function of W and p. A series of critical widths W_c and that of critical exponents ν are determined by finite-size scaling analysis. Remarkably, the results of ν disobey the Harris criterion, $\nu > 2/d$ (Refs. 21,22), which is believed to be satisfied in general disordered systems. Our findings present prospects for the study of Anderson transition in 1D systems with correlated disorder.

This paper is organized as follows. Section II describes the long-range correlated on-site potentials to be considered. It presents a numerical algorithm for calculating the localization length of eigenstates. Section III analyzes the localization length as a function of W and p. The transition point W_c

and the critical exponent ν are extracted by finite-size scaling analysis. Conclusions and discussion are presented in Sec. IV.

II. MODEL AND METHOD

A. Long-range correlated potentials

We consider noninteracting electrons in 1D disordered systems within a tight-binding approximation. The Schrödinger equation of the system is expressed as

$$\varepsilon_i \phi_i + t(\phi_{i+1} + \phi_{i-1}) = E\phi_i, \qquad (1)$$

where ϕ_i is the amplitude of the wave function at the ith site of the lattice. The hopping energy t is taken as a unit of energy hereafter. A sequence of long-range correlated potential $\{\varepsilon_i\}$ is produced by the Fourier filtering method.[23–25] This method is based on a transformation of the Fourier components of a random number sequence. The outline of the method is as follows: (i) A sequence $\{u_i\}$ of uncorrelated random numbers with a Gaussian distribution is prepared. (ii) Its Fourier components $\{u_q\}$ are computed using the fast-Fourier transformation method. (iii) A sequence $\{\varepsilon_q\}$ is generated for a given p using the relation $\varepsilon_q = q^{-p/2} u_q$. (iv) Finally, the objective $\{\varepsilon_i\}$ is obtained as the inverse Fourier transform of $\{\varepsilon_q\}$. The resulting potentials ε_i are spatially correlated and produce the power-law spectral density $S(k) \propto k^{-p}$. In the following, the mean value $\langle \varepsilon_i \rangle$ is set to be zero and the periodic boundary condition is imposed.

Figure 1 displays the landscapes of long-range correlated potentials ε_i that are generated by the procedure above. The system size $L=2^{13}$ and the distribution width $W=3.0$ are fixed. An increase in p markedly reduces the roughness in potential landscapes. We have confirmed that the sequences $\{\varepsilon_i\}$ appearing in Fig. 1 produce the power-law spectral density, $S(k) \propto k^{-p}$, for all values of p.

B. Localization lengths

Localization lengths of eigenstates in a potential field $\{\varepsilon_i\}$ are computed easily using the conventional transfer-matrix method.[26] The Schrödinger equation (1) is expressed by the following matrix equation:

$$\begin{pmatrix} \phi_{i+1} \\ \phi_i \end{pmatrix} = M_i \begin{pmatrix} \phi_i \\ \phi_{i-1} \end{pmatrix}, \quad M_i \equiv \begin{pmatrix} E - \varepsilon_i & -1 \\ 1 & 0 \end{pmatrix}. \qquad (2)$$

The localization length ξ at a given energy E is defined by the relation[26]

$$\xi^{-1} = \lim_{L \to \infty} \frac{1}{L} \ln \frac{|\Pi_{i=1}^N M_i z(0)|}{|z(0)|} \qquad (3)$$

with a generic initial condition

$$z(0) = \begin{pmatrix} \phi_1 \\ \phi_0 \end{pmatrix}. \qquad (4)$$

Equation (3) gives the single value of ξ only for the infinite system size $L \to \infty$. However, when L is finite, the calculated result of Eq. (3)—denoted as ξ_L—depends on the choice of
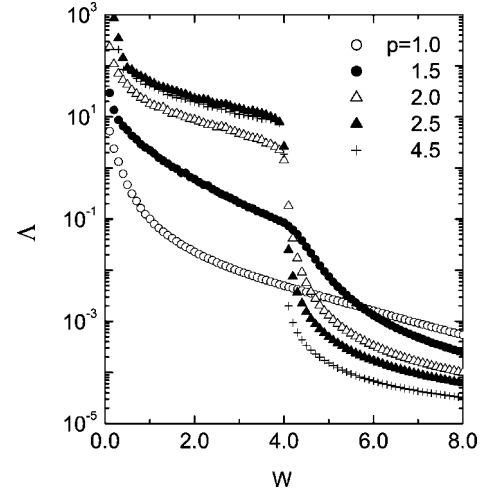

FIG. 2. The normalized localization length $\Lambda \equiv \xi_L/L$ as a function of the distribution width W. Several values of p are taken as denoted in the figure. The system size L is fixed to be 2^{16} for all values of p.

the potential field $\{\varepsilon_i\}$. To obtain a typical value of ξ_L for a given L, we take a geometrical mean of ξ_L on more than 10^4 samples. Energy E is fixed at the band center $E=0$ throughout this paper.

A critical point $W=W_c$ (and $p=p_c$) can be deduced from the dependence of the normalized localization length $\Lambda \equiv \xi_L/L$ on the system size L. The typical values of ξ_L increase with L for delocalized states, where the growth of ξ_L is faster than that in L. This causes the quantity Λ to be an increasing function of L. On the other hand, Λ vanishes for sufficiently large L for localized states because ξ_L approaches a constant value. Therefore, at the localization-delocalization transition, Λ must be invariant for the change in the system size L. Values of W_c are obtained accurately using the finite-size scaling method, as explained in Sec. III.

III. NUMERICAL RESULTS

A. The W-dependence for the function $\Lambda(W)$

Figure 2 plots the normalized localization length Λ as a function of the distribution width W. The exponent p of the power-law spectral density $S(k)$ is increased from $p=1.0$ to $p=4.5$, incrementally. The system size $L=2^{16}$ is fixed for all p. When p equals unity or less, $\Lambda(W)$ is a monotonous function of W. For larger p, on the other hand, curves of Λ show a kink at $W=4.0$, which sharpens as p increases.

We find two striking features in Fig. 2. The first is a peculiar p-dependence of $\Lambda(W)$. The p-dependence of $\Lambda(W)$ in the region where $W<4.0$ differs completely from that in the region where $W>4.0$. For $W<4.0$, the values of Λ rise with increasing p indicating that the growth of p for $W<4.0$ causes an increase in the localization length ξ_L of eigenstates. In contrast, the values of Λ for $W>4.0$ systematically decrease with increasing p (except for the data $p=1.0$). Hence, growth of p for $W>4.0$ produces strongly localized eigenstates. This difference in the effect of increasing p on ξ_L is nontrivial because the increase in p simply smoothes poten-
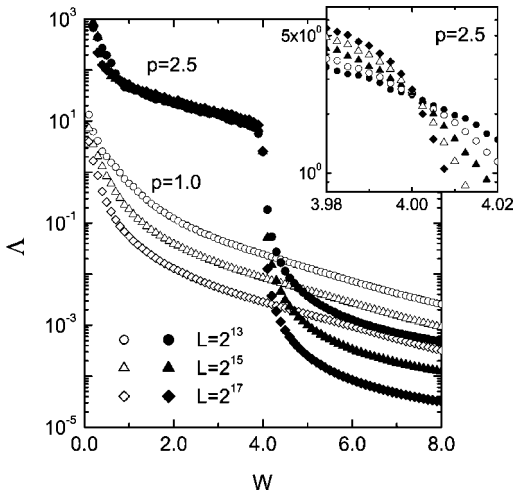
FIG. 3. The L-dependence for $\Lambda(W)$ with setting $p=1.0$ (open symbols) and $p=2.5$ (solid symbols). Inset: Enlargement of $\Lambda(W)$ in the vicinity of $W=4.0$. The system size L is varied to $L=2^{13}$ (solid circles), 2^{14} (open circles), 2^{15} (solid triangles), 2^{16} (open triangles), and 2^{17} (diamonds).
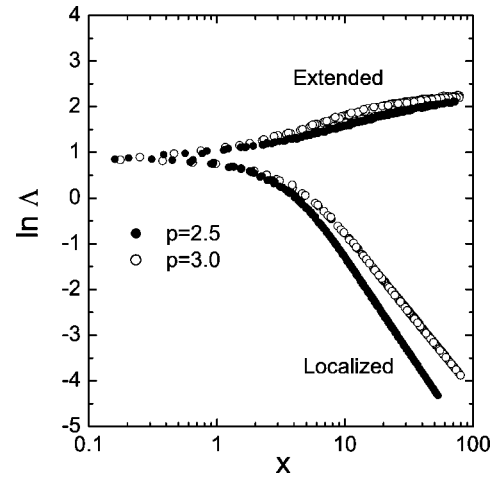

FIG. 4. Scaling plots of $\ln \Lambda = f(x)$ for $p=2.5$ (solid circles) and for $p=3.0$ (open circles). Upper and lower branches correspond to extended and localized phases, respectively.

tial landscapes, as shown in Fig. 1. Therefore our results suggest that the effect of potential roughness on the localization length ξ_L depends strongly on the value of W.

The second notable feature is a shoulder structure of $\Lambda(W)$—a sharp bent of $\Lambda(W)$—at around $W=4.0$, engendering a plateaulike shape within the region $0.5<W<4.0$. The shoulder of $\Lambda(W)$ appears for exponents $p>2.0$, i.e., for p large enough to yield extended eigenstates.[6] That fact implies that localization properties of eigenstates for $W<4.0$ (and $p>2.0$) differ substantially from those for $W>4.0$. The value $W=4.0$ is a critical disorder strength separating a localized and delocalized phase, as explained later. Section III B presents a demonstration that eigenstates are delocalized within the plateau region, under conditions $W<4.0$ and $p>2.0$.

Values of the critical width W_c can be estimated from the dependence of $\Lambda(W)$ on system size L. Figure 3 shows the L-dependence for $\Lambda(W)$, where the system size is increased from $L=2^{13}$ (circles) up to 2^{17} (diamonds). Open and solid symbols correspond to the exponent $p=1.0$ and $p=2.5$, respectively. For $p=1.0$, the magnitude of $\Lambda(W)$ declines with increasing L such that the eigenstates are localized for any W. On the other hand, for $p=2.5$, the L-dependence for $\Lambda(W)$ is rather complicated. The inset of Fig. 3 displays detailed behavior of Λ in the vicinity of $W=4.0$, where the system size is varied from $L=2^{13}$ to 2^{17} as denoted in the figure caption. All curves intersect on a single point at $W=4.0$, suggesting the presence of the Anderson transition at $W=W_c=4.0$ for $p=2.5$.

We have confirmed that, when $p \gtrsim 2.0$, $\Lambda(W)$ shows the same L-dependence as presented in the inset of Fig. 3. Intriguingly, all data of $\Lambda(W)$ for $p>2.0$ provide an identical value of $W_c=4.0$. Therefore, we conclude that the critical distribution width $W_c=4.0$ is independent of p whenever $p \gtrsim 2.0$.

B. Finite-size scaling analysis

Finite-size scaling analysis allows the determination of critical properties of the transition for $L \to \infty$ from data for finite L.[26] This method stems from the hypothesis that the normalized localization length Λ close to the transition obeys the scaling law expressed by

$$\ln \Lambda = f\left(\frac{L}{\xi_\infty}\right), \qquad (5)$$

where ξ_∞ is the localization length of eigenstates in an infinite system. The argument L/ξ_∞ becomes much smaller than unity because ξ_∞ diverges with obeying the form $\xi_\infty \propto |W-W_c|^{-\nu}$ near the transition point $W=W_c$. This allows expansion of the scaling function as

$$\ln \Lambda = a_0 + a_1|W-W_c|L^{1/\nu} + \cdots + a_n|W-W_c|^n L^{n/\nu}, \quad (6)$$

terminating the expansion at the order n. Fitting the numerical data of $\ln \Lambda$ for various values of W and L to Eq. (6), we obtain W_c and ν with great accuracy. Note that the optimal value of constants $a_0, a_1, \cdots, a_n$ for $W>W_c$ are different from those for $W<W_c$ because ξ_∞ is a function of the absolute value of $W-W_c$.

Figure 4 shows scaling plots of $\ln \Lambda$ for $p=2.5$ (solid) and $p=3.0$ (open). Here we define $x \equiv |W-W_c|L^{1/\nu}$ and set $n=4$. Each upper and lower branch in the figure corresponds to the extended phase ($W<W_c$) and the localized phase ($W>W_c$), respectively. All data of $\ln \Lambda$ for various values of W and L fit well onto two branches. Resulting values of the critical exponent ν and the critical distribution width W_c are listed in Table I, where the error is a 95% confidence interval. We see that all values of W_c are almost identical to 4.0, as expected. On the other hand, values of ν exhibit a discrepancy for different branches and different ps.[27] This discrepancy of ν contradicts the principle of one-parameter scaling requiring that ν should be independent of a choice of parameters in the Hamiltonian of the system. It is surmised that the discrepancy of ν occurs because of a finite-sized effect that causes a systematic error in scaling plots. A novel technique of scal-

TABLE I. Calculated results of the critical exponent ν and the critical distribution width W_c. The error is a 95 % confidence interval.

	Branch	ν	W_c
$p=2.5$	Extended	1.79 ± 0.08	4.002 ± 0.002
	Localized	1.87 ± 0.04	4.001 ± 0.001
$p=3.0$	Extended	1.58 ± 0.08	4.001 ± 0.002
	Localized	1.51 ± 0.02	4.001 ± 0.001

ing correction (two-parameter scaling)[28–30] would help to solve the problem, as we shall present in a future study.[31]

C. The p-dependence for the function $\Lambda(p)$

Next, we examine Λ behavior as a function of p. Figure 5 shows a plot of the function $\Lambda(p)$ with setting $W=1.0$ and 3.0, where the system size is varied from $L=2^{13}$ to 2^{17}. For all W and L, $\Lambda(p)$ increases with p; it then becomes constant for $p>2.5$. This p-dependence for Λ is consistent with the results displayed in Fig. 2, in which the growth of p for $W<4.0$ causes an increase in Λ. Detailed calculations of $\Lambda(p)$ in the vicinity of $p=2.0$ reveal that all curves of $\Lambda(p)$ that belong to different Ls intersect at the point at $p=2.0$ as long as $W<4.0$. This indicates that localization-delocalization transition occurs at an identical point $p=p_c=2.0$ whenever $W<4.0$. The critical point $p_c=2.0$ estimated above is consistent with the conclusion reported in Ref. 6.

A series of critical points W_c and p_c we have found are summarized in the phase diagram illustrated in Fig. 6. Solid circles express the critical point $W=W_c$ deduced from the finite-size scaling procedure, whereas solid squares express $p=p_c$ defined by the position at which $\Lambda(p)$ is independent of L (see Fig. 5). An extended phase appears in the region surrounded by the two straight lines of $W=4$ and $p=2$. It is noteworthy that values of W_c and p_c equal an *integral* num-

ber: 4 and 2, respectively. The same integral value of $W_c=4$ has been observed in other 1D systems with correlated disorder,[1–3,34–36] as discussed below.

IV. CONCLUDING REMARKS

Two open problems remain with respect to the critical properties of the transition in 1D systems with long-range correlated disorder. First, all results of the critical exponent ν listed in Table I disobey the Harris criterion $\nu>2/d$ (Refs. 21,22). The inequality is widely believed to be satisfied in general disordered systems with any spatial dimension d, thereby determining the lower bound for ν. According to the inequality, ν in 1D systems must be larger than 2, which disagrees with our results. We note here that the Harris criterion was originally derived for a spatially *uncorrelated* disorder.[21] Therefore, the relation $\nu>2/d$ may be violated in systems with long-range *correlated* disorder. In fact, for *classical* percolation model, the inequality must be modified in the presence of long-range correlation in the site or bond occupations.[32,33] To elucidate the lower bound of ν for 1D systems considered, we should generalize the argument in Ref. 22 for the correlated disorder producing the power-law spectral density $S(k)\propto k^{-p}$.

Second, the critical disorder width W_c is exactly equal to an *integral* number $W_c=4$. Furthermore, the value of W_c is independent of the potential roughness characterized by p. In
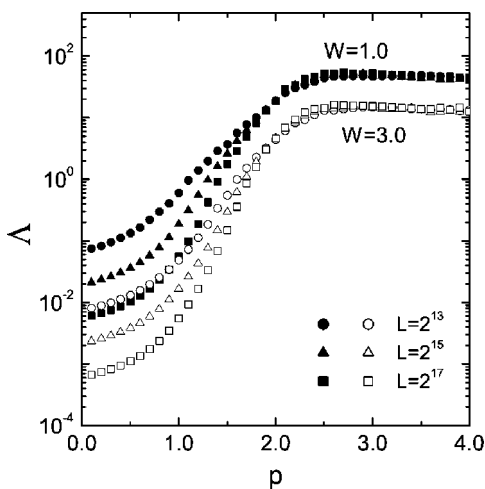

FIG. 5. The L-dependence for $\Lambda(p)$ with setting $W=1.0$ (solid symbols) and $W=3.0$ (open symbols), respectively. The system size L is varied as 2^{13} (circle), 2^{15} (triangle), and 2^{17} (square).
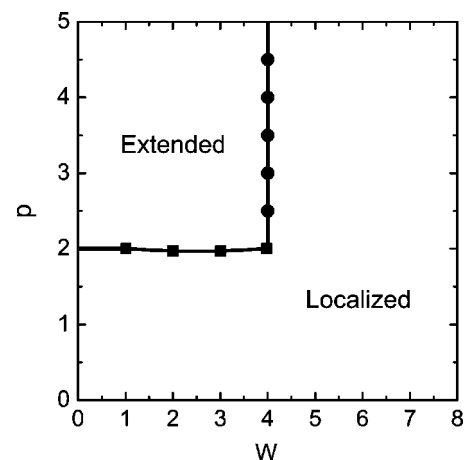

FIG. 6. A phase diagram of the system in the W-p space. The critical line separating the localized and delocalized phase consists of two straight lines: $W=W_c=4.0$ and $p=p_c=2.0$.

fact, the same value of $W_c=4$ has been found in other correlated potential models. One example is the random dimer model,[1–3] where the site energies ε_a and ε_b are assigned in pairs. The disorder width W of the system is defined by the relation $\varepsilon_a - \varepsilon_b \in [-W/2, W/2]$. If W is less than the critical value $W_c=4$, the localization length diverges obeying the form $\xi_\infty(E) \propto E^{-2}$, yielding the delocalized eigenstate at $E=0$. Another example is the Harper model,[34,35] in which site energies are described by a periodic function $\varepsilon_i = (W/2)\cos(2\pi i\omega)$ with an irrational number ω. This model undergoes the localization-delocalization transition at $W=4$. That is, all eigenstates become critical at $W=4$.[36] We conjecture that the coincidence of the value of W_c in the three different models is evidence of an unexplored universality of W_c in 1D systems with spatially correlated disorders.[37]

In conclusion, we have investigated critical properties of 1D electron systems subject to long-range correlated disorder. The normalized localization length Λ of eigenstates shows a nontrivial behavior as a function of W and p, which indicates a peculiarity of 1D systems exhibiting the Anderson transition. Detailed calculations for $\Lambda(W,p)$ reveal that the transition point $W_c=4$ is invariant to the change of the potential roughness. Moreover, the critical exponent ν in the considered system is proven to violate the Harris criterion $\nu > 2/d$. We hope that our findings enlighten the study of quantum phase transition in manifold disordered systems.

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