LASER NOISE IMPACT IN THE DESIGN OF AN HETERODYNE OPTICAL PSK RECEIVER

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SUMMARY

In this paper an evaluation of the performance of a coherent optical receiver for binary PSK has been carried out. Both the AM and FM noise due to transmitter and LO semiconductor lasers have been taken into account. Also the nonlinear dependence between the phase noise variance and the laser linewidth is considered.

1. INTRODUCTION

Coherent lightwave transmission systems show great advantages compared to intensity modulation and direct detection systems. This is due, in the first place, to an improvement in the receiver sensitivity because of the modulation techniques used. Moreover, the use of local oscillator (LO) high power levels reduces the thermal noise and the photodetector dark current noise contributions. In this case the signal to noise ratio improves to the theoretical shot noise limit (assuming an ideal LO) allowing the design of larger links [1].

Nevertheless, SC lasers generate quantum noise giving place to AM and FM fluctuations. It is necessary to evaluate the degradation introduced by a non ideal LO and transmitter as is done in this work.

The paper is structured as follows:

In Part 1 there is a short explanation of the different physical phenomena that generate the laser quantum noise. A characterization of the electrical field emitted is obtained through the study of the power spectral densities of the AM and FM noise.

In Part 2 an heterodyne receiver is studied in terms of its output SNR taking into account AM noise and both thermal and shot noises. An optimum value for the LO power injected on the fiber coupler is found.

In Part 3 the required SNR to achieve a BER of 10^{-9} is studied considering the FM noise in both SC lasers. The use of general expressions allows the comparation with the results obtained when considering, as a simplified model, a linear relation between phase noise variance and linewidth. As far as we know this study has been carried out only for ASK transmitted signals.

Finally, in Part 4, there is a summary of the main conclusions obtained in this work.

2. CHARACTERIZATION OF THE AM AND FM NOISE

The electric field in a laser cavity can be studied as a plane wave propagating in a dielectric medium between two planar mirrors separated a distance L. Taking into account the active medium and the fluctuations in the refraction index due to the spontaneous emission, it is possible to determine the rate equations for the intensity or photon number, I(t), the phase, $\phi(t)$, and the carrier density, N(t).

In order to evaluate the fluctuations of the former parameters it is necessary to solve the rate equations for small signal analysis and add the Langevin fluctuation terms [3]. By so doing and calculating the autocorrelation and the Fourier transform of $\Delta I(t)$ and $\Delta\dot{\phi}(t)$ (fluctuations around the stationary values I_o and ϕ_o), the power spectral densities of the AM and FM noises ($S_{\Delta I}(f)$ and $S_{\Delta\dot{\phi}}(f)$ respectively) are obtained [4] as a function of the operating conditions and physical parameters of the laser

This characterization allows the electric field to be expressed as

$$E(t) = \left[E_o + e(t)\right] \cos\left[w_o t + \Delta\phi_o(t)\right] \qquad (1)$$

where

- E_o is the stationary amplitude and e(t) a random variable with zero mean value which represents the fluctuations around E_o associated to $\Delta I(t)$. Taking into account that $[E_o+e(t)]$ is proportional to $[I_o+\Delta I(t)]^{1/2}$, we can express the power spectral density of the electric field amplitude fluctuations as $S_e(f) = RIN(f)E_o^2/4$, being $S_{\Delta I}(f)/I_o^2 = RIN(f)$ (relative intensity noise).

- $\Delta\phi_o(t)$ is a random variable of zero mean value which stands for the fluctuations in the instantaneous signal frequency.

3 RECEIVER SENSITIVITY: AM NOISE INFLUENCE

In optical systems with heterodyne detection, the electric field coming from the fiber is combined with the one coming from a LO whose center frequency is shifted $\Delta f=f_I$ from the received signal carrier. If both the received and LO signals were stable in amplitude and frequency, only the shot noise associated to the photodetection process would be present. In that case, the SNR would attain a theoretical maximum known as the quantum limit. Nevertheless, the performance is degraded when we take into account the AM and FM noises.

The electric field coming from the fiber and from the LO for a PSK signal can be expressed in the following way:

$$x_s(t) = E_s \cos(w_s t + \phi_s + \Delta\phi_s(t))$$

$$x_o(t) = \left[E_o + e(t)\right] \cos(w_o t + \Delta\phi_o(t)) \qquad (2)$$

E_s and E_o are the field amplitudes in the stationary state and ϕ_s is the modulation. $e(t)$ and $\Delta\phi_o(t)$ are terms associated to the AM and FM LO noises, respectively, and $\Delta\phi_s(t)$ is the FM noise due to x_s. In the analysis we assume that the amplitude fluctuations of x_s are negligible.

The signal incident on the photodetector is

$$x(t) = p^{1/2}\, x_s(t) + K^{1/2}\,(1-p)^{1/2} x_o(t) \qquad (3)$$

In this expression we have considered the power transmission factor of the combiner, p, as well as the losses in the LO emitted signal, K. The electrical current generated by the photodiode is proportional to $x^2(t)$ and can be expressed as:

$$i(t) = i_{sg}(t) + i_{AM}(t) + i_{shot}(t) \qquad (4)$$

$$i_{sg}(t) = R\sqrt{p(1-p)K}\; E_o E_s \cos\left(\omega_I t + \phi_s + \phi(t)\right) \qquad (5)$$

$$i_{AM}(t) = R\, K\, (1-p)\, E_o\, e(t) \;\; ; \; R = q\eta/hf \qquad (6)$$

and $\phi(t) = \Delta\phi_s(t) - \Delta\phi_o(t)$.

where we have eliminated the terms outside the photodiode bandpass and the ones at zero frequency than can be filtered.
In order to evaluate the thermal noise due to the receiver we have used the same model as in [5] and so, the voltage at the equalizer output is:

$$v_o(t) = G^{1/2} i(t) * h_i(t) + v_{th}(t) \qquad (7)$$

where $h_i(t)$ stands for the impulsional response of the convolution of the amplifier plus photodiode frontend, the bandpass filter and equalizer responses. If we assume that the receiver preserve the input pulse shape:

$$|H_i(f)| = \begin{cases} Z & -B_{IF}/2 \le f - f_I \le B_{IF}/2 \\ 0 & \text{outside the bandpass} \end{cases} \qquad (9)$$

where B_{IF} is the IF bandwidth.

In the following we will calculate each noise contribution. With a development similar to the one described in [5], and using the same parameters, we have for the shot and thermal noises, respectively:

$$P_{shot} = 2\, q\, Z^2\, G\, R\, K\, (1-p)\, P_o\, B_{FI} \qquad (10)$$

$$P_{th} = 2\,(2\pi C_{eq})^2\, G\, S_{ea} \int_{-\infty}^{+\infty} |H_i(f)|^2 f^2\, df + 2G\left[\frac{S_{ea}}{R_{eq}^2} + S_{ia} + \frac{2K\theta}{R_b}\right] \int_{-\infty}^{+\infty} |H_i(f)|\, df^2 = 2G\, Z^2 S_T \qquad (11)$$

The expression for the AM noise power is:

$$P_{AM} = 2\, G \int_{-\infty}^{+\infty} S_{AM}(f)\, |H_i(f)|^2\, df \qquad (12)$$

where $S_{AM}(f)$ is the power spectral density of the current fluctuations associated to the LO AM noise. This density is calculated as the Fourier transform of:

$$\langle i_{AM}(t)\, i_{AM}(t+\tau) \rangle = R^2 K^2 (1-p)^2 E_o^2\, R_e(\tau) \qquad (13)$$

where $R_e(\tau)$ is the autocorrelation of the amplitude fluctuations. The relationship between $S_e(f)$ and the power spectral density of the photon number fluctuation has been given in the previous part, so

$$P_{AM} = 2GZ^2R^2K^2(1-p)^2 \frac{E_o^4}{4} \int_{-B_{IF}/2}^{+B_{IF}/2} RIN(f)\, df \qquad (14)$$

Finally we have:

$$SNR = \frac{p\, P_s}{\frac{q}{R} B_{IF} + (1-p) P_L \int_{B_{FI}} RIN(f)\, df + \frac{S_T}{R^2 (1-p) P_L}} \qquad (15)$$

where $P_L = KP_o$

Starting from (15) we can obtain the minimum detectable power for a given value of the SNR. It is then obvious the existance of an optimum value for the LO injected power, P_{Lopt}, which can be expressed as:

$$P_{Lopt} = \sqrt{\frac{S_T}{\frac{1}{R^2}\,(1-p)^2 \int_{B_{FI}} RIN(f)\, df}} \qquad (16)$$

4 BER CALCULATION: FM NOISE INFLUENCE

We are now going to determine the degradation introduced by the FM noise in the system bit error rate (BER). The output receiver voltage (7) can be rewritten as:

$$v(t) = A \cos(w_I t + \phi_s + \phi(t)) + n(t) \qquad (17)$$

where $n(t)$ stands for all the considered noise terms and $A = ZG^{1/2}R\sqrt{p(1-p)K}\, E_o E_s$. We assume that $n(t)$ is a gaussian noise of zero mean value and variance given by the sum of the different noise contributions, this is: shot, thermal and AM noises.

In a PSK transmission system a phase locked loop (PLL) is required. The phase shift at its output will be:

$$\Delta\phi(t) = \phi(t)-\phi(t-\tau) \; ; \; \tau = \text{PLL delay} \qquad (18)$$

The IF signal is demodulated, translated to baseband and sampled giving:

$$x = A/2 \cos(\Delta\phi) + n_{if} \qquad (19)$$

where $n_{if} = n(t) \cos(w_I t)$; $\sigma^2_{n_{if}} = \sigma^2_n/4$

The received signal has phase noise and an additive amplitude noise, so x can be expressed as the sum of two random variables, w and n, being $w = A/2 \cos(\Delta\phi)$. Assuming that the phase noise can be characterized by a gaussian random variable we can derive the PDF (probability density function) of w. We can then express the BER of a binary PSK system with heterodyne detection as [6]:

$$P(e) = \frac{2}{\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{h(x)} e^{-x^2} e^{-y^2} \, dy \, dx \qquad (20)$$

where $h(x) = -\sqrt{SNR}\cos(\sqrt{2}\,\sigma_{\Delta\phi}\, x)$ and $SNR = A^2/(2\sigma^2)$ which agrees with expression (15).

Fig.I shows the SNR as a function of $\sigma_{\Delta\phi}$ taking the BER as parameter. We can observe that for small phase variance (high power emitted by the laser) the BER depends critically on the SNR. However, a high phase variance can be the limiting factor to achieve a given BER, producing a dramatic drop of sensitivity.

For obtaining the receiver sensitivity as a function of P_o it is necessary to determine the expression for $\sigma^2_{\Delta\phi}(\tau)$. Assuming that the phase noise is a random variable with zero mean value, the variance is:

$$\sigma^2_{\Delta\Phi}(\tau) = \frac{1}{\pi}\int_{-\infty}^{+\infty} S_{\Delta\dot\phi}(w) \, \frac{2\sin^2 w\tau/2}{w^2} \, dw \qquad (21)$$

After some calculation we obtain (Fig.II):

$$\sigma^2_{\Delta\phi} = 2\pi\Delta\nu\tau + \frac{R\alpha^2\left[\cos 3\delta - e^{\gamma_e/2\tau}\cos(\Omega_o\tau-3\delta)\right]}{2I_o\gamma_e\cos\delta} \qquad (22)$$

where $\Delta\nu$ is the laser linewidth, R the spontaneous emission rate, γ_e is the damping coefficient, α the quotient between real and imaginary variation of the refraction index due to spontaneous emission, Ω_o is the relaxation oscillation frequency and $\cos\delta = \Omega_o/w_R$ [3].

Most of the authors take into account only the first term in the expression above given and neglect the term of damping oscillations. This simplifies the analysis but implies a considerable error, specially at low LO powers and high bit rate. This degradation is showed in Fig.III where the quotient between the exact formula and the linear aproximation has been represented. Also in Fig.IV the sensitivity for both cases has been represented for a fixed bit rate (v_t = 560 Mbit/s) and different LO emitted powers. Finally in Fig.V the sensitivity considering the general expression for $\sigma^2_{\Delta\phi}(\tau)$ is given for different bit rate.

5 CONCLUSION

It has been shown the importance of taking into account the effects of the AM and FM noise in order to study the receiver sensitivity in a binary PSK coherent optical system. An accurate evaluation of the penalty in the sensitivity due to the laser quantum noise is obtained.

We show that the AM noise due to the laser working as local oscillator degrades the expected SNR. The sensitivity of the receiver can be optimized by choosing an appropriate LO injection level on the combiner.

The BER calculation carried out taking into account FM noise and the expression for the SNR obtained in Part 2, allows the determination of the minimum signal power necessary at the receiver input to obtain a desired error probability. The sensitivity values shown in the figures for low LO emitted power are too low to be of interest. This is due to the fact that the lasers used for the computer simulation have a high spectral linewidth and it increases when P is decreased.

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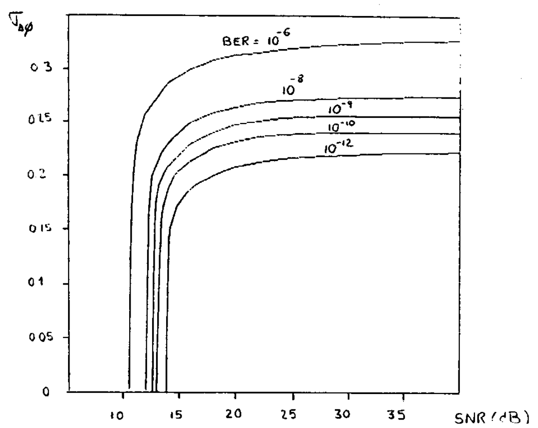

Figure I

Phase noise dev.as function of SNR for different BER ($\tau = 0.1/v_t$)

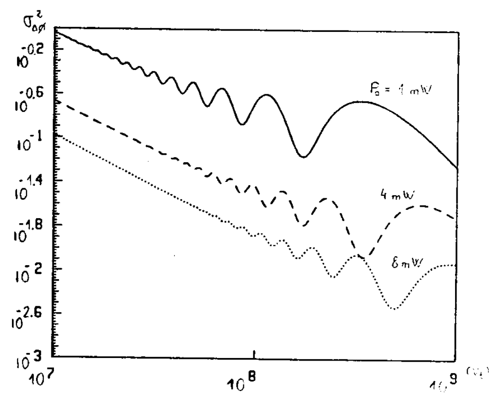

Figure II

Phase noise variance as function of the bit rate for different LO emitted powers.

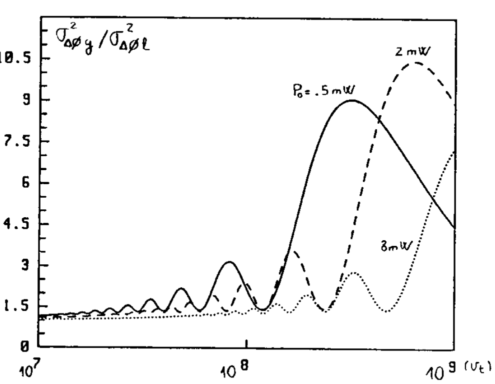

Figure III

Relation between the general and linear expressions for the phase nose variance as function of the bit rate for different LO emitted powers.

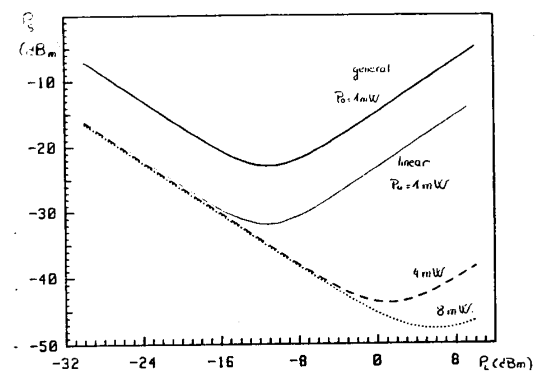

Figure IV

Sensitivity for both, general and linear aproximation, as a function of the LO injected power.(bit rate = 560 Mbit/s) and considering AM and FM noises.

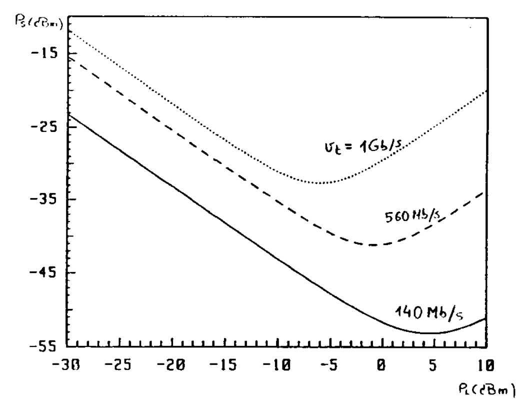

Figure V

Sensitivity in the general case as function of P_L and for different bit rate. (AM and FM noise degradation are included)