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PARAMETRIC ENVELOPE IN LPC SPEECH CODERS

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Abstract:

During the last decade, many efforts have been devoted to the relative importance of associated functions like magnitude and phase of Fourier Transforms in image and signal bandwidth reduction. The reported work deals with the importance of the real envelope and instantaneous frequency in signal analysis/sintesis problems. In this paper authors show a method to parametrize the envelope and instantaneous frequency of a real signal. This method is very closed to spectral analysis methods in the sense that with an appropiate study, time domain and frequency domain can be analyced in a similar way.

I. INTRODUCTION

Signal processing techniques are characterised in the frequency domain by means of the magnitude and the phase of the associated Fourier transform of the signals under analysis. It is clear that there are not similarities between time domain and frequency domain signal representations, in the sense that one domain uses a real signal and the other two signals. Many authors have reported some interesting results which deals with this topic and facing the question of redundancy in the representation of a real signal in the frequency domain -1- ,-2- .

In some way we can recognize that regardless of the previosly mentioned work, in signal processing, we are very familiar with magnitude/phase representation in the frequency domain instead of real and imaginary parts of the complex Fourier transform. In other words, no matter redundancy, a designer can always recognize better, a low pass filter in a magnitude--phase plot than from the curves corresponding to their real and imaginary part. The point is, up to what degree the real part of the analityc signal, the given signal x(t), provides a better representation of the phenomena under study. Following the frequency domain experience, in a very heuristical way, the conclussion will be that magnitude and instantaneous phase are a good information support for the designer. In other words, envelope and instantaneous frequency, looks like, they deserve the same importance in the time domain that magnitude and phase in the frequency domain.

In this paper, the authors will explore the potential of such representation and the difficulties around handling it in signal processing problems. To be more concise, envelope and instantaneous frequency are computed for simulated signals using two approaches and the problems which arise are reported. Also, and dealing with a problem of bandwidh reduction in a signal communication system, it is shown the advantages of well-known spectral estimation procedures in parametric models for envelope and instantaneous phase.

The paper is organized as follows: Section II presents an introduction of the general concepts of interest about analityc signals. Section III reports the relationship between spectral estimation procedures and envelope smoothing. After Section III, the next one will show fundamental questions about envelope and instantaneous frequency. Finally, some preliminary results are introduced.

II ANALITYC SIGNAL

Any signal x(t) has an associated complex signal $a_x(t)$ being its Fourier transform twice the rigth hand side of the original X(w).

$$A_x(w)=\begin{cases} 2\ X(w) & ;\ w>0 \\ 0 & ;\ w<0 \end{cases} \qquad (1)$$

Clearly, $a_x(t)$ is a complex signal, and its time formulation includes as real component the original data signal x(t) and as imaginary component the so-called Hilbert Transform of it.

$$a_x(t) = x(t) + j\ h_x(t) \qquad (2)$$

where

$$h_x(t)=(1/\pi)\int_{\infty}^{\infty}(x(t')/(t-t'))dt' \qquad (3)$$

This last formula steems from the definition of the analytic signal in the frequen-

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cy domain. More concretly, derived from the causal condition of $A_x(w)$. It is easy to conclude that $a_x(t)$ plays the same role in the time domain that $X(w)$ does in the frequency domain.

However, the main concern of this work are not $x(t)$ and $h_x(t)$, but the alternative representation shown in (4).

$$a_x(t) = x(t)+jh_x(t) = e_x(t) \exp \phi_x(t) \quad (4)$$

being $e_x(t)$ and $\phi_x(t)$ the already refered envelope and instantaneous phase.

$$e_x^2(t) = x^2(t)+h_x^2(t) = |a_x(t)|^2 \quad (5.a)$$

$$\Phi_x(t) = \tan^{-1}(h_x(t)/x(t)) = \text{Phase of } a_x(t) \quad (5.b)$$

It is worthwhile to mention that given $x(t)$ there is not an uniquiness in finding functions $e_x(t)$ and $\Phi_x(t)$. To check this, just add some residual only to the imaginary part of (2). The uniquiness of $e_x(t) \cos(\Phi_x(t))$ in representing $x(t)$ lies with the minimum phase condition of $a_x(t)$. Exactly, the envelope/phase representation is conected with the analitycity of $a_x(\xi)$ in the upper hand side of the ξ plane. Thus no poles can be inside the upper hand side of plane ξ in order to guarantee the causality constraint for $A_x(w)$ holds.

With respect the importance of both parameters, note that the envelope provides information concerning to the time energy distribution and the instantaneous frequency with zero-crossing information.

III SPA METHODS IN ENVELOPE REPRESENTATION

From the definition of envelope for a given analytical signal it can be infered that the square of the envelope can be viewed as a time domain periodogram. In other words, if we know the Fourier Transform of the analytic signal $A_x(w)$, we can obtain the envelope in the same way as we obtain the Periodogram for a given data signal.

$$A_x(w) \xrightarrow[\text{Transform}]{\text{Fourier}} a_x(t) \rightarrow |\cdot|^2 \rightarrow \text{envelope}$$
$$x(t) \xrightarrow[\text{Transform}]{\text{Fourier}} X(w) \rightarrow |\cdot|^2 \rightarrow \text{Periodogram} \quad (6)$$

From the previous similarity we can conclude that the most familiar procedures applied in parametric spectral estimation could also be applied over $A_x(w)$ to obtain an estimation of $e_x^2(t)$.

Concretely, the most popular maximum entropy technique can be used over a sampled version of the causal signal $A_x(w)$. If $A_x(w)$ is given as a data register lenght of N/2 samples with index l (i.e. $A_x(l)$; $l = 0, N/2-1$), then a linear predictor of coeficients $\alpha(q)$ ($q = 1,Q$) can be designed by minimizing the square error $\varepsilon(l)$. Thus the linear predictor is:

$$\tilde{A}_x(l) = \sum_{q=1}^{Q} (q)\, A_x(l-q) \quad (7)$$

The predictor residual $\varepsilon(l)$ is defined as $A_x(l)-\tilde{A}_x(l)$ and the quantity to be minimized is:

$$\sum_{l=N_1}^{N_2} |\varepsilon(l)|^2 \quad (8)$$

There are many well-known procedures in the literature to solve (8) depending on the choice for N_1 and N_2. We select the procedure of correlation, regardless it is not recommended when the signal under analysis is deterministic in nature, as it is the case due to the causal character of $A_x(l)$. Anyway, the consequences derived from the use of Levinson algorithms in the minimization of (8) will be greater degree of smoothing or low resolution, in terms of spectral estimation, of the resulting parametric envelope representation. Other procedures can be carried over the problem previosly stated in (7) and (8) (see for example -5-). As concerns with this paper, it is not very relevant the spectral estimation procedure selected and the reader could change it accordingly with the desired features in the resulting envelope estimate.

The envelope estimate, once coeficients $\alpha(.)$ have been obtained, can be derived assuming the white character of the residual sequence (l).

With this assumption the square magnitude of the inverse discrete Fourier transform e(n) is constant, so that;

$$\varepsilon(l) = \alpha(.) * A_x(.)$$
$$e(l) = \beta(.) \cdot a_x(n) \quad (9)$$

and because the white character of $\varepsilon(l)$ is assumed,

$$|a_x(n)|^2 \cdot |\beta(n)|^2 = k_0 \quad (10)$$

being k_0 the average power of $\varepsilon(l)$ (i.e. the minimum of the objective in the design process (8))

In summary,

$$K_0 = E\left[\,|\varepsilon(l)|^2\right] \quad (11.a)$$

$$\hat{e}_x^2(n) = K_0/|\beta(n)|^2 \quad (11.b)$$

where

$$\beta(n) = 1+\sum_{q=1}^{Q} \alpha(q) \exp(j2\pi qn/N) \quad (12)$$

The estimate shown in (11.b) have been applied succesfully to voice speech records. In Fig 1 the reader can see such envelope estimate compared with the actual envelope and the data signal record. The data length was 32 ms and the linear prediction order 20 . It can be viewed in this plot that the LP estimate provides a smoothed version of the actual envelope. The replica obtained with the estimate results accurate when the order Q is almost double than the number of periods included in the original signal.

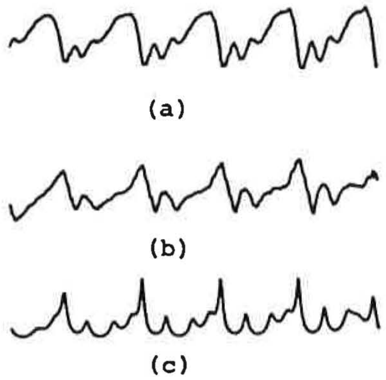

Fig.1. (a) Voiced speech signal, (b) envelope,(c) parametric envelope.

Note that the corresponding analytic signal to the so-called parametric envelope is always a minimum phase signal in the sense that $a_x(\xi)$ has no zeros in the upper hand side of the complex plane ξ . This property guarantees that the logaritm of the estimate envelope and the phase of $a_x(n)$ are a Hilbert Transform pair.

IV INSTANTANEOUS FREQUENCY FROM PARAMETRIC ENVELOPE.

In the preceding section it is introduced how envelope information could be smoothed by using well-known LPC techniques. This point is very interesting as concerns with data rate reduction for signal transmission purposes. The question which remains is that, a non minimum phase signal needs also, to be recovered at the receiver, instantaneous phase information.

To realice how instantaneous phase or frequency can be represented by a finite set of parameters, it will be interesting to explore how classical frequency discriminators work to detect instantaneous frequency.
Considering the linear system depited in Fig.2, if the input signal is given with $e_x(t)$ almost constant, the output signal will be:

$$\int_{-\infty} h(t')\, e_x(t-t') \cos \phi_x(t-t')dt' \simeq$$
$$\simeq e_{x(t)} \int_{-\infty} h(t') \cos \phi_{x(t-t')}\, dt' \qquad (13)$$

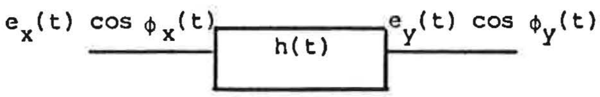

Fig 2. Instantaneous frequency though linear systems.

Thus, assuming that two terms of Taylor's series for the phase term is adequate to represent it in the convolution integral,

$$\phi_x(t-t') = \phi_x(t) - t' \dot{\phi}_x(t) \qquad (14)$$

the output signal will be:

$$y(t) \simeq e_x(t) \; \mathrm{Re}\{ \exp(j\,\phi_x)\; H(\dot{\phi}_x) \} \qquad (15)$$

being H(.) the transfer response of the linear system (i.e. the Fourier Transform of h(t)).

From (15) it is easy to conclude that, being $e_x(t)$ almost constant, the envelope of y(t) is given from transfer response and the instantaneous frequency $\dot{\phi}_x$ of the input signal. This reasoning do not preclude the analitycity of $e_x(t).H(\dot{\phi}_x)\exp(j\phi_x)$, thus to avoid this assumption the above computations will be carried over the complex signal $\exp(j\phi_x)$.

The mentioned complex signal is obtained from the quotien of the given signal $a_x(t)$ and its actual envelope obtained with a Hilbert transform.

$$x_m(t) = a_x(t)/e_x(t) \qquad (16)$$

Note that $x_m(t)$ is not analytic in general. Only in the case of $e_x(t)$ band limited to w_0 and cos $(\phi_x(t))$ no spectraly overlapped in this band, $x_m(t)$ will have an analytic character. This case is not longer true for many practical signals x(t). Regardless of the analytic condition of $x_m(t)$, if it is applied to a linear system h(t), the output signal will be:

$$y_m(t) = H(\dot{\phi}_x) \exp(j\phi_x) \qquad (17)$$

As a consequence, we could use procedures to smooth $E\{|y_m|\}^2$ in the same fashion we use them in the previous section. Furthermore if H(.) is a derivative in the time domain the magnitude of $|y_m(t)|$ will be aproximately $|\dot{\phi}_x(t)|$. In summary, as in a classic discriminator, using a linear system with constant slope frequency response we can obtain the instantaneous frequency as the magnitude of a complex signal.

Note that to recover exactly the instantaneous frequency it must be positive. This problem could be avoided by adequate scaling and carrire modulation. Anyway the designer must guarantee the basic assumption of no abrupt phase changes in the instantaneous phase (low instantaneous frequency) to derive the previous results.

To check out the resulting performance of the procedure, a 128 data sample record with instantaneous frequency evolution as is shown bellow, was used.

$$f_i(n) \begin{cases} 0.125 & ; n = 1, 32 \\ \text{linear} & ; n = 33, 95 \\ 0.25 & ; n = 96,128 \end{cases}$$

The linear system was implemented by a transfer function that varies lineary whith the frequency.
The instantaneous frequency wich results can be viewed in Fig.3 together with the original signal x(n).

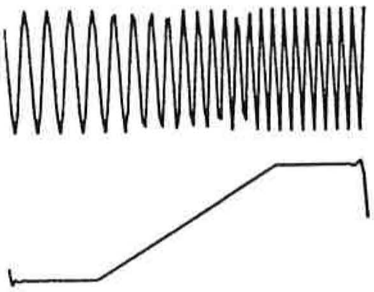

Fig 2. Top, original signal. Bottom, instantaneous frequency detected.

It can be viewed some side effects due to the window effects in the detected instantaneous frequency. It is espected that this distorsion wich also appears in the envelope will not appear in a continous processing of the signal under analysis. Anyway, from the results obtained by the authors the approximation in the centra zone, in a block processing fashion, is accourate enough to reproduce the original signal.

CONCLUSIONS

The main contribution of this work resides in revealing the interest of these functions associated to any signal to be processed. No essential efforts in the past have been devoted to such representation of signals, just in modulation problems some authors reported interesting results in such field . Currently, time-frequency representations exhibit a new look of the problem. We really believe that this functions deserve more attention, because it is well recognized that essential information is involved inside the envelope or the instantaneous phase evolution.

In this paper it is reported how envelope and instantaneous frequency are suitable for parametric methods which are familiar in spectral estimation problems.

Nevertheless the main guidelines to work over these associated fuctions have been shown, further work must be devoted to signal processing tools in order to alleviate uncertainly in envelope/phase representation of signals.Also side effects or differents ways to obtain envelope and phase will be explored in the future.

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