# SIMULATION STUDIES OF OPTIMAL DECENTRALIZED LOAD-FREQUENCY REGULATORS

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Abstract. On appliying the theory of the Optimal Decentralized Regulator (ODR) to the control of an Electric Power System (EPS), due attention has not been paid to some aspects such as the importance of the design model structure, the feasibility of reducing the generating unit efforts by penalizing the transient loop of the speed regulators deviations, and the application of the ODR only to an area with low capacity relative to the pool. In this paper, the importance of the external equivalent load in an area ODR design structure is studied, both analitically and by simulation, comparing the Calovic / Siljak structure and the Hiyama structure. Taking into account the real state of the EPS, the classical assumption that an ODR is implemented in each area is rejected; instead it is applied only in a low capacity area, using for the rest PI regulators. In a second part of this paper, the feasibility of reduce the generating efforts is considered by including in the model the transient loop ("dash-pot") of the speed regulators. Finally the effect of a multiple variety of penalizations of the area state variable deviations (in the $\underset{\sim}{Q}$-matrix of the ODR) is studied by simulation, using a model elaborated with experimental and compiled values from the Catalan Power System and the directly coupled systems (Spanish and French ones).

Keywords. Optimal Control. Large-Scale Systems. Power Systems Modelling and Control.

## INTRODUCTION

The study of the ODR application in the Load-Frequency Control (LFC) problem has normally started from design structures composed of simetrical areas, using the same model for the design of the ODR of each area. In this paper, taking into account the real state of the EPS, this classical assumption is rejected, and the implementation of an ODR only in a low capacity area is studied.

Due to the low capacity of this area, the kinetic energy delivered by the external load can be relativelly important compared with the area generation capacity. In this way, the Calović, Cuk and Djorović (1977) design structure (where only the internal states of the area and those of the power links area needed) and the Hiyama (1982) structure (enlarged with an external load equivalent) are compared, and the feasibility of an area ODR capable of reducing the external area frequency deviations and also capable of combining the area internal generation with the external load delivered energy, is studied. In the first place, this study is done analitically, fitting the Calović and Siljak (1978) structure to the Elgerd and Fosha (1970) scheme of interconnected areas and comparing the sub-optimality index of two different design structures.

In a second part, the ODR for an area is designed (using for the remaining areas PI regulators), and the importance of the external load and the effect of a multiple variety of penalizations of the area state variable deviations are studied by simulation. These simulation studies are carried out using a model elaborated with experimental results from the Catalan Power System and with compiled values from the directly coupled systems (Spanish and French ones).

## AUTONOMOUS AREA OLR FORMULATION

In this section, starting from the Calović (1977) and Siljak (1978) formulation and fitting it to the Elgerd and Fosha (1970) model of two interconnected areas, a general formulation for the autonomous area OLR is established.

Given an EPS described by:

$$\dot{\underset{\sim}{x}}(t) = \underset{\sim}{A}\ \underset{\sim}{x}(t) + \underset{\sim}{B}\ \underset{\sim}{u}(t) + \underset{\sim}{F}\ \underset{\sim}{z}(t)$$

$$\underset{\sim}{y}(t) = \underset{\sim}{C}^T\ \underset{\sim}{x}(t) \qquad [1]$$

where:

- $\underset{\sim}{x}(t)$ = states of the areas.
- $\underset{\sim}{u}(t)$ = control actions (control laws).
- $\underset{\sim}{z}(t)$ = perturbations.
- $\underset{\sim}{y}(t)$ = measurable outputs.

the purpose is to design a local OLR for every area.

Decomposing [1] for every subsystem (area), the following equation is obtained:

$$\dot{\underset{\sim}{x}}_i(t) = \underset{\sim}{A}_i\underset{\sim}{x}_i(t) + \underset{\sim}{B}_i\underset{\sim}{u}_i(t) + \underset{\sim}{F}_i\underset{\sim}{z}_i(t) + \underset{\sim}{V}_i\Delta p_i(t) \qquad [2]$$

being $\Delta \underset{\sim}{p}_i$ the power interchanged through the tie lines, and $\underset{\sim}{V}_i$ a constant vector that weighs the coupling degree between the tie lines of the different areas.

Using the Elgerd and Fosha (1970) model for the interconnection lines between the areas i and j,

$$P_{ij}(s) = \frac{2\pi T^\circ_{ij}}{s}\ (\Delta f_i(s) - \Delta f_j(s))$$

$$T^\circ_{ij} = \frac{|V^\circ_i|\,|V^\circ_j|}{X}\ \cos(\delta^\circ_i - \delta^\circ_j)$$

$$2\pi T^\circ_{ij} = T'_{ij} \qquad [3]$$

where $\delta^\circ_i$ and $\delta^\circ_j$ are the angles of the voltages $V^\circ_i$ and $V^\circ_j$ in the tie line terminals, $f_i$ the nominal frequency of the area i and X the reactance of the tie line, one can write for a system composed of s areas:

$$\Delta \dot{p}_i(t) = \sum_{\substack{j=1\\ j=i}}^{s} (T_{ii}\Delta f_i(t) - T_{ij}\Delta f_j(t)), \qquad i=1,2,\dots,s \qquad [4]$$

Defining two vectors $\underset{\sim}{m}_{ii}$ and $\underset{\sim}{q}_{ij}$, with only an element different than zero (unitary), in the way that:

$$\underset{\sim}{m}_{ii}\ \underset{\sim}{x}_i(t) = \Delta f_i(t)$$

$$\underset{\sim}{q}_{ij}\ \underset{\sim}{x}_j(t) = \Delta f_j(t) \qquad [5]$$

the expression [2] is extended:

$$\dot{\underset{\sim}{x}}_i(t) = \underset{\sim}{A}_i\underset{\sim}{x}_i(t) + \underset{\sim}{B}_i\underset{\sim}{u}_i(t) + \underset{\sim}{F}_i\underset{\sim}{z}_i(t) + \underset{\sim}{V}_i\Delta p_i(t)$$

$$\Delta \dot{p}_i(t) = T_{ii}\underset{\sim}{m}_{ii}\underset{\sim}{x}_i(t) - \sum_{\substack{j=1\\ j=i}}^{s} T_{ij}\underset{\sim}{q}_{ij}\underset{\sim}{x}_j$$

$$y_i(t) = \underset{\sim}{C}_i^T\ \underset{\sim}{x}_i(t) \qquad [6]$$

being n the dimension of the state vector $\underset{\sim}{x}_i$.

Following the Calović line, g generating units are assumed, being t the thermal and h the hydroelectrical ones. Its participation levels to the total generated power of the area is established by means of a participation vector $\underset{\sim}{\gamma}_i$. On the other hand, the economic dispatch can fit another vector $\underset{\sim}{\mu}_i$, with the purpose of authorizing the transmission of the control actions $\underset{\sim}{u}_i$ only if the obtained value in every computation is significatively different with respect to the last value transmitted to the power plants. With these vectors, the vector of transmitted set-points is:

$$\underset{\sim}{u}_i = \underset{\sim}{\gamma}_i\underset{\sim}{\mu}_i\ \underset{\sim}{w}_i \qquad [7]$$

where $\underset{\sim}{w}_i$ is the optimal control law vector obtained from the Riccati equation (OLR design).

Assuming that the perturbations z(t) are constant and including the ACE (Area Control Error) and the vectors $\underset{\sim}{\gamma}_i$ and $\underset{\sim}{\mu}_i$, the expression [6] is writen in the perturbational form:

$$\dot{\hat{\underset{\sim}{x}}}_i(t) = \hat{\underset{\sim}{A}}_i\hat{\underset{\sim}{x}}_i(t) + \hat{\underset{\sim}{B}}_i\hat{\underset{\sim}{w}}_i(t) + \sum_{\substack{j=1\\ j=i}}^{s} \hat{\underset{\sim}{E}}_{ij}\hat{\underset{\sim}{x}}_j \qquad [8]$$

being, for the i area:

$$\hat{\underset{\sim}{x}}_i = \left[ (\underset{\sim}{x}_i - \underset{\sim}{x}_{iss})^T,\ v_i - v_{iss},\ \Delta p_i \right]^T \quad ;\text{dim.}=n+2$$

$$\dot{v}_i = ACE_i = \underset{\sim}{d}_i^T\ \underset{\sim}{x}_i + \Delta p_i$$

$$\underset{\sim}{d}_i^T\ \underset{\sim}{x}_i = \beta_i\ \Delta f_i \quad (\beta = \text{biass coefficient}).$$

$$\hat{\underset{\sim}{u}}_i = (\underset{\sim}{u}_i - \underset{\sim}{u}_{iss}) = \underset{\sim}{\gamma}_i\underset{\sim}{\mu}_i(\underset{\sim}{w}_i - \underset{\sim}{w}_{iss}) = \underset{\sim}{\gamma}_i\underset{\sim}{\mu}_i\ \hat{\underset{\sim}{w}}_i$$

$$\hat{\underset{\sim}{A}}_i = \begin{bmatrix} \underset{\sim}{A}_i & 0 & \underset{\sim}{V}_i \\ \underset{\sim}{d}_i^T & 0 & 1 \\ T_{ii}\underset{\sim}{m}_{ii} & 0 & 0 \end{bmatrix}$$

$$\hat{\underset{\sim}{B}}_i = \begin{bmatrix} \underset{\sim}{B}_i\underset{\sim}{\gamma}_i\underset{\sim}{\mu}_i \\ 0 \\ 0 \end{bmatrix}$$

$$\hat{E}_{ij} = \begin{bmatrix} 0 & 0 \cdots 0 \\ 0 & 0 \cdots 0 \\ -\sum_{\substack{j=1 \\ j=i}}^{s} T_{ij} q_{ij} & 0 \cdots 0 \end{bmatrix} \qquad [9]$$

$\left.\begin{matrix} x_{ss} \\ v_{ss} \end{matrix}\right\}$ steady state values.

Including the ACE as a state variable of the system, and assuming $x_{ss} = v_{ss} = 0$, the state equation of the decoupled i area is:

$$\begin{bmatrix} \dot{x}_i \\ \dot{v}_i \\ \Delta \dot{p}_i^m \end{bmatrix} = \begin{bmatrix} A_i & 0 & V_i & x_i \\ d_i^T & 0 & 1 & v_i \\ T_{ii} m_{ii} & 0 & 1 & \Delta p_i^m \end{bmatrix} + \begin{bmatrix} B_i \gamma_i \mu_i \\ 0 \\ 0 \end{bmatrix} w_i$$

$$\dot{x}_i = A_i x_i + V_i \Delta p_i + B_i \gamma_i \mu_i w_i$$

$$\dot{v}_i = d_i x_i + \Delta p_i = ACE$$

$$\Delta \dot{p}_i^m = T_{ii} m_{ii} x_i \qquad [10]$$

The variables $x_i$ and $\Delta p_i$ are physical variables of the system, whereas the variable $v_i$ ($\int ACE\ dt$) is obtained artificially from the frequency and the interchanged power samples. On the other hand, $\dot{v}_i$ is a discrete time variable, which means a bound in the classical studies in the continuous domain (Bertran, 1985). However, in this paper, we shall use the continuous domain in order to facilitate the comparison of results.

Starting from the following expression of the decoupled i area:

$$\dot{\hat{x}}_i = \hat{A}_i \hat{x}_i + \hat{B}_i \hat{w}_i \qquad [11]$$

the purpose is to design an OLR that minimizes the integral index:

$$J = \int_0^{\infty} e^{2\alpha_0 t} (\hat{x}_i^T Q_i \hat{x}_i + \hat{w}_i^T R_i \hat{w}_i) dt \qquad [12]$$

Using the substitutions:

$$\hat{\hat{x}}_i = e^{\alpha_0 t} x_i \quad \text{and} \quad \hat{\hat{w}}_i = e^{\alpha_0 t} w_i \qquad [13]$$

and choosing a quasi-diagonal $Q_i$ matrix

$$Q_i = \begin{bmatrix} Q_{ai} & — & 0 \\ | & q_{Ii} & | \\ 0 & — & q_{pi} \end{bmatrix} \qquad [14]$$

where $Q_{ai} = Q_{ai}^T$ is a n x n positive definite matrix that wheigs the internal state deviations of the i area, $q_{Ii}$ is the ACE deviations penalty and $q_{Pi}$ is the power interchanged penalty. As Calović, Cuk and Djorović (1977) have demonstrated, with this choice of the $Q_i$ matrix it is possible to find a Riccati matrix $P$ such that the optimal control law is:

$$\hat{w}_i^o = -K_p x_i - K_I \int ACE\ dt - K_T \Delta p_i^m \qquad [15]$$

where :

$$K_p = R_i^{-1} B_i^T P_{11} \qquad K_T = R_i^{-1} B_i^T p_{13}$$

$$K_I = R_i^{-1} B_i^T p_{12}$$

being $p_{ij}$ the elements of the Riccati matrix.

Considering the vectors $\gamma_i$ and $\mu_i$, the last control action to be transmitted to the generating units of the i area is:

$$\hat{u}_i^o = \gamma_i \mu_i \hat{w}_i^o \qquad [16]$$

## STUDY OF THE STRUCTURE OF A TWO-AREA EPS

### Introduction

The typical study of the decentralized LFR is based on the decoupling of the large-scale system (LSS) through the power links of the areas, designing a sub-optimal regulator for every area. In this case, only the internal states of the area and those of the power links are needed for the design of the ODR (Siljak,1978). We shall denote this design structure as STR.I.

In a two-area EPS, with a low capacity area (Area 1) relative to the pool, the power interchanges between both areas are established from the kinetic energy delivered by the load in Area 2 providing that the perturbation level (in Area 1) be low; this is due to the bigger capacity of Area 2 and the nullification of the regulation of this area, which is due to the ACE dead-band and transducer's insensibility). In this case, Hiyama (1982) has satisfactorily experimented a design structure (STR.II) for the regulators of Area 1 based on a model composed of this area and its external equivalent load.

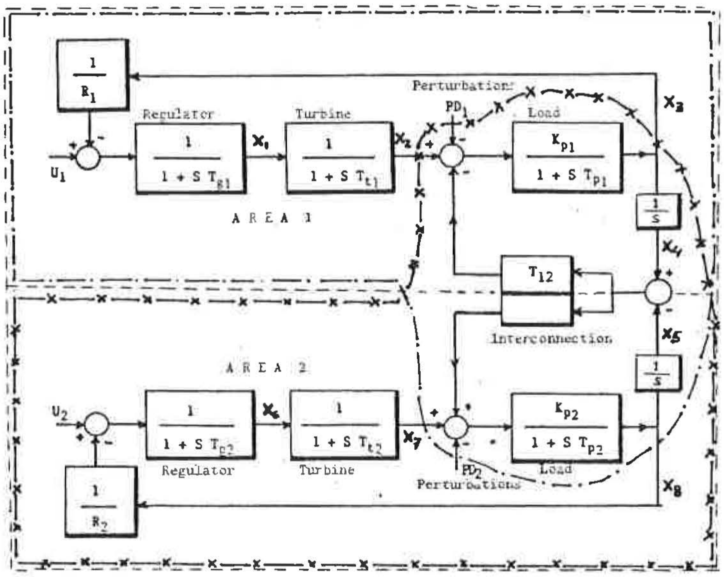
In this section, the Calović / Siljak structure (with the areas decoupled by the power links) and the Hiyama structure (based on the area model and an external equivalent load) are analitically compared and the external load importance is studied.

## Suboptimality studies

Some first considerations about the optimality level of an ODR based on each of the two design structures introduced above are here studied, by using the classical model of two interconnected areas because of its simplicity and its pessimistic forecast.

The suboptimality of an ODR when the design model is decoupled by the interconnection link (STR.I) is first studied by taking as state variables the following set of physical variables (Fig.1):

$x_1$, $x_6$ = valves opening
$x_2$, $x_7$ = generated power
$x_3$, $x_8$ = frequency deviations
$x_4$, $x_5$ = phase deviations.




- - - - - Areas 1 and 2 ODR design stricture I (STR.I)
—·—·— Area 1 ODR design structure II (STR.II)
—x—x— Area 2 ODR design structure II (STR.II)

Fig.1.- Two-area model.

By using the perturbational expression [8] and assuming the perturbations (PD) in Fig.1 as constant, the following model is obtained:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -T_{g1}^{-1} & 0 & -R_1^{-1}T_{g1}^{-1} & 0 \\ T_{t1}^{-1} & -T_{t1}^{-1} & 0 & 0 \\ 0 & K_{p1}T_{p1}^{-1} & -T_{p1}^{-1} & -T_{12}K_{p1}T_{p1}^{-1} \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} T_{g1}^{-1} \\ 0 \\ 0 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ T_{12}K_{p1}T_{p1}^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} \quad [17]$$

The suboptimality index $\varepsilon$ defined by Siljak and Sundareshan (1976) is reformulated for a system with an ODR in each area and without structural perturbations as follows:

$$\sum_{i=1}^{s} \sum_{j=1}^{s} \xi_{ij} \leqslant \frac{1}{2} \frac{\varepsilon}{1+\varepsilon} \frac{\min_i \lambda_m (\underset{\sim}{W}_i(t))}{\max_i \lambda_M (\underset{\sim}{P}_i(t))} \qquad i = 1,\dots,s \quad [18]$$

where:

$\underset{\sim}{P}_i$ = Riccati matrix for the subsistem i.

$\underset{\sim}{W}_i = \underset{\sim}{P}_i \underset{\sim}{B}_i \underset{\sim}{R}_i^{-1} \underset{\sim}{B}_i^T \underset{\sim}{P}_i + \underset{\sim}{Q}_i$

$\lambda_m(.)$ = minimum eigenvalue of the matrix (.)

$\lambda_M(.)$ = maximum eigenvalue of the matrix (.)

$\xi_{ij} = \lambda_M^{1/2} (\underset{\sim}{H}_{ij}^T \underset{\sim}{H}_{ij})$

being $\underset{\sim}{H}_{ij} = \hat{\underset{\sim}{E}}_{ij}$ when the control law of the i-area is based only on the internal variables of the area, and $\underset{\sim}{H}_{ij} = \hat{\underset{\sim}{E}}_{ij} - \hat{\underset{\sim}{B}}_i \underset{\sim}{k}_{ij}^g$ when there exists a gain vector $\underset{\sim}{k}_{ij}^g$ that relates the control law of the i-area with its interconnection variables.

The nullification of $\varepsilon$ is possible if there exists a non-zero vector $\underset{\sim}{k}_{ij}^g$ such as:

$$\underset{\sim}{k}_{ij}^g = (\hat{\underset{\sim}{B}}_i^T \hat{\underset{\sim}{B}}_i)^{-1} \hat{\underset{\sim}{B}}_i^T \hat{\underset{\sim}{E}}_{ij} \quad [19]$$

Noting that $\underset{\sim}{k}_{12}^g = \underset{\sim}{k}_{21}^g = 0$ for the system [17], the following results have been obtained (Bertran, 1985):

$$\xi_{12} = T_{12} K_{p1} T_{p1}^{-1}$$
$$\xi_{21} = T_{12} K_{p2} T_{p2}^{-1} \quad [20]$$

and,

$$T_{12} K_{p1} T_{p1}^{-1} + T_{12} K_{p2} T_{p2}^{-1} \leqslant \frac{\varepsilon}{1+\varepsilon} \frac{\min_i \lambda_m (\underset{\sim}{W}_i)}{\max_i \lambda_M (\underset{\sim}{P}_i)} \quad , i=1,2. \quad [21]$$

The minimum eigenvalue of $\underset{\sim}{W}_i$ and the maximum of $\underset{\sim}{P}_i$ not only depends on the system structure, but also the $\underset{\sim}{R}$ and $\underset{\sim}{Q}$ matrices ([12]) take part in their determination. For this reason it's only possible to conclude that there is a dependency between the state of the interconnection ($T_{12}$) and that of the load of the two areas ($K_p$, $T_p$) in the determination of $\varepsilon$. This suboptimality index has a random component, owing to aspect t-variant of the load.

The study of the Hiyama structure (STR.II) is more complex since the expression [18] is only appliable to isolated subsytems, and the load of every area is now a common part of the design model of each ODR. For this reason the feedback gain $k_8^\circ$ corresponding to the optimal gain vector that relates the state variable $x_8$ (Fig.1) with the control input $U_1$ has been included. In this case the perturbational formulation of Area 1 is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -T_{g1}^{-1} & 0 & -R_1^{-1}T_{g1}^{-1} & 0 \\ T_{t1}^{-1} & -T_{t1}^{-1} & 0 & 0 \\ 0 & K_{p1}T_{p1}^{-1} & -T_{p1}^{-1} & -T_{12}K_{p1}T_{p1}^{-1} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} T_{g1}^{-1} \\ 0 \\ 0 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 & 0 & 0 & k_8^\circ T_{g1}^{-1} \\ 0 & 0 & 0 & 0 \\ T_{12}K_{p1}T_{p1}^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} \quad [22]$$

From this expression and [19] a coordination vector is possible,

$$\underset{\sim}{k}_{12}^{g} = (\hat{\underset{\sim}{B}}_1^T \hat{\underset{\sim}{B}}_1)^{-1} \hat{\underset{\sim}{B}}_1^T \hat{\underset{\sim}{E}}_{12} = (0 \quad 0 \quad 0 \quad k_8^\circ) \quad [23]$$

with which the expression [21] is also valid. Without this coordination vector,

$$\mathcal{F}_{12} = \mathrm{MAX}\,(k_8^\circ T_{g1}^{-1},\ T_{12}K_{p1}T_{p1}^{-1}) \quad [24]$$

and the expression [18] becomes:

$$\mathrm{MAX}\,(k_8^\circ T_{g1}^{-1},\ T_{12}K_{p1}T_{p1}^{-1}) + T_{12}K_{p2}T_{p2}^{-1} \leqslant \frac{\varepsilon}{1+\varepsilon}\ \frac{\min_i \lambda_m(\underset{\sim}{W}_i)}{\max_i \lambda_M(\underset{\sim}{P}_i)} \quad i=1,2 \quad [25]$$

Comparing [21] and [25], if

$$k_8^\circ T_{g1}^{-1} > T_{12}K_{p1}T_{p1}^{-1} \quad [26]$$

the left term of the expression [21] is less than the one of the left of [25], that means a worse optimality of the ODR based on the Hiyama structure if the gain $k_8^\circ$ exists and it is a high one. On the other hand, the random aspect of $\varepsilon$ is now reduced.

Using typical parametric values in the model of Fig.1 (Bertran,1985), it is concluded that the importance of the $\underset{\sim}{Q}$ and $\underset{\sim}{R}$ matrices is bigger than the design structure.

## SIMULATION STUDIES

### Introduction

In this section the importance of the $\underset{\sim}{Q}$ and $\underset{\sim}{R}$ matrices in the design of the ODR of Area 1 is studied by simulation. Taking into account the real state of the EPS, the classical assumption that an ODR is implemented in each area is rejected; instead it is applied only to a low capacity area (Area 1) relative to the pool, using for the rest (Area 2) typical PI regulators. The ODR design is made following the two design structures of the preceding section.

In order to reduce the generating efforts, an alternative way to the use of non-linear elements is studied, including in the model the transient loop ("dash-pot") of the speed regulators, and imposing a penalty to this loop deviations in the $\underset{\sim}{Q}$-matrix. On the other hand, the effect of a multiple variety of penalizations of the Area 1 state variables is studied by simulation.

### Model formulation

A set of physical variables has been selected for the design model formulation from the reduced small signal hydro-electrical model of Fig.2 In this figure Area 1 equivalent speed regulator has been modelled according to the IEEE scheme and Area 2 speed regulators with the Elgerd and Fosha scheme. The parametric values have been obtained from experimental results in the Catalan Power System -Area 1- and from compiled values of the remaining directly coupled system (Spanish and French, Area 2) -Bertran, 1985. The inertia of the thermal units has been included in the load model of each area.

This set of physical variables is as follows:

- $x_1$, $x_8$ : gate openings.
- $x_3$, $x_7$ : generated power.
- $x_4$, $x_6$ : frequency.
- $x_5$ : interchanged power.
- $x_2$ : transient loop ("dash-pot") displacement
- $v$ : ACE integral.

The perturbational design model for the design of the ODR of Area.1 is, according to the STR.I:

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{v} \\ \dot{x}_5 \end{bmatrix} =
\begin{bmatrix}
-\sigma T_{g1}^{-1} & -T_{g1}^{-1} & 0 & -T_{g1}^{-1} & 0 & 0 \\
-\sigma\delta T_{g1}^{-1} & -(\delta T_{g1}^{-1}+T_r^{-1}) & 0 & -\delta T_{g1}^{-1} & 0 & 0 \\
\frac{2}{T_{w1}}+2\sigma T_{g1}^{-1} & 2T_{g1}^{-1} & -\frac{2}{T_{w1}} & 2T_{g1}^{-1} & 0 & 0 \\
0 & 0 & K_{p1}T_{p1}^{-1} & -T_{p1}^{-1} & 0 & -K_{p1}T_{p1}^{-1} \\
0 & 0 & 0 & \beta_1 & 0 & 1 \\
0 & 0 & 0 & T_{12} & 0 & 0
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ v \\ x_5 \end{bmatrix}
+
\begin{bmatrix} T_{g1}^{-1} \\ \delta T_{g1}^{-1} \\ -2T_{g1}^{-1} \\ 0 \\ 0 \\ 0 \end{bmatrix} u_1
\qquad [27]
$$

And, including the load of Area 2 (STR.II):

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{v} \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} =
\begin{bmatrix}
-\sigma T_{g1}^{-1} & -T_{g1}^{-1} & 0 & -T_{g1}^{-1} & 0 & 0 & 0 \\
-\sigma\delta T_{g1}^{-1} & -(\delta T_{g1}^{-1}+T_r^{-1}) & 0 & -\delta T_{g1}^{-1} & 0 & 0 & 0 \\
\frac{2}{T_{w1}}+2\sigma T_{g1}^{-1} & 2T_{g1}^{-1} & -\frac{2}{T_{w1}} & 2T_{g1}^{-1} & 0 & 0 & 0 \\
0 & 0 & K_{p1}T_{p1}^{-1} & -T_{p1}^{-1} & 0 & -K_{p1}T_{p1}^{-1} & 0 \\
0 & 0 & 0 & \beta_1 & 0 & 1 & 0 \\
0 & 0 & 0 & T_{12} & 0 & 0 & -T_{12} \\
0 & 0 & 0 & 0 & 0 & K_{p2}T_{p2}^{-1} & -T_{p2}^{-1}
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ v \\ x_5 \\ x_6 \end{bmatrix}
+
\begin{bmatrix} T_{g1}^{-1} \\ \delta T_{g1}^{-1} \\ -2T_{g1}^{-1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u_1
\qquad [28]
$$

Choice of the penalizations

The following choices of the parameter $\alpha_0$ and the $\tilde{Q}$ and $\tilde{R}$ matrices [12] have been studied:

| case | $\alpha_0$ | $\tilde{R}$ | $\tilde{Q}$ |
|---|---|---|---|
| A | 0 | $\tilde{I}$ | diag(0,5;0,5;0,5;0,5;0,5;0,5) |
| B | 0 | $\tilde{I}$ | diag(0,5;0,5;0,5;0,5;0,5;0,5;0,5) |
| C | 0 | $\tilde{I}$ | diag(0,5;0,5;0,5;1;0,5;0,5;0,5) |
| D | 0 | $\tilde{I}$ | diag(0,5;0,5;0,5;0,1;0,5;0,5;0,5) |
| E | 0 | $\tilde{I}$ | diag(0,5;0,5;0,5;0,5;0,5;0,5;1) |
| G | 0 | $\tilde{I}$ | diag(0,5;0,5;0,5;0,5;0,5;0,1;0,5) |
| I | 0 | $\tilde{I}$ | diag(0,5;0,5;0,5;0,5;1;0,5;0,5) |
| K | 0 | $\tilde{I}$ | diag(0,5;0,02;0,5;0,5;1;0,5;0,5) |
| L | 0 | $\tilde{I}$ | diag(0,5;0,5;1;0,5;1;0,5;0,5) |
| N | 0 | 0,5$\tilde{I}$ | diag(0,5;0,5;0,5;0,5;1;0,5;0,5) |
| Q | 0,02 | $\tilde{I}$ | diag(0,2;0,5;0,2;0,8;1;0,9;0,8) |

Case A corresponds to the design structure STR.I, whereas the remaining cases correspond to STR.II for different penalizations of the deviations of the states of Area 1. Solving the Riccati equation, the optimal gain vectors (Fig.2) of Table I have been obtained.

TABLE 1 : Optimal gain vectors.

| CASE | $K_1$ | $K_2$ | $K_3$ | $K_4$ | $K_5$ | $K_6$ | $K_7$ |
|---|---|---|---|---|---|---|---|
| A | 3,282 | -0,131 | 0,3401 | 0,1137 | 0,7068 | 0,0398 | - |
| B | 3,535 | -0,1378 | 0,3536 | 0,0954 | 0,706 | 0,0558 | 0,019 |
| C | 3,7629 | -0,1443 | 0,3627 | 0,0899 | 0,7066 | 0,0848 | 0,0252 |
| D | 3,35 | -0,1325 | 0,3463 | 0,0998 | 0,7064 | 0,0325 | 0,0141 |
| E | 3,778 | -0,1444 | 0,3607 | 0,0878 | 0,7065 | 0,0706 | 0,0276 |
| G | 3,5477 | -0,138 | 0,348 | 0,0912 | 0,7064 | 0,0643 | 0,0223 |
| I | 4,6268 | -0,1895 | 0,5061 | 0,1455 | 0,9996 | 0,0534 | 0,023 |
| K | 4,512 | -0,283 | 0,51 | 0,1497 | 0,999 | 0,048 | 0,0283 |
| L | 4,644 | -0,1898 | 0,5024 | 0,1453 | 0,9996 | 0,0537 | 0,023 |
| N | 6,6425 | -0,2279 | 0,7286 | 0,1973 | 1,4141 | 0,1046 | 0,0466 |
| Q | 6,6244 | -0,2947 | 0,7124 | 0,198 | 1,3919 | 0,0906 | 0,04 |

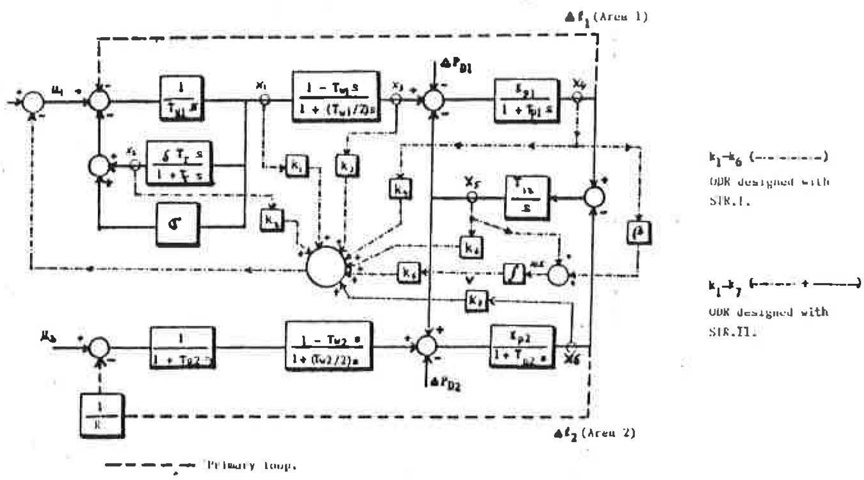



Fig. 2.- Two - area model.

$T_{g1}$ = 26,51 p.u.cm ó V /p.u.MWócm
$\delta$ = 5,3518 Hz/p.u.MW
$\sigma$ = 12,97 Hz/p.u.MW
$T_r$ = 3,66981 seg.
$T_{w1}$ = 0,964 seg.
$K_{p1}$ = 66,6666 Hz/p.u.MW
$T_{p1}$ =10,666 seg.
$T_{12}$ = 1,53 p.u.MW x seg / Hz.
$R^{-1}$ = 1,777 p.u.MW/Hz
$T_{g2}$ = 33,86 seg.
$T_{w2}$ = 1 seg.
$K_{p2}$ = 5,0625 Hz/p.u.MW
$T_{p2}$ = 0,81 seg.

$\beta_1$ = 0.092 p.u. MW/Hz.
$\beta_2$ = 1,9745 p.u. MW/Hz.

## Simulation results

With these values, and fitting the secondary regulation of Area 2 (ACE regulation) as

$$u_2 = K_2 \int ACE\ dt$$

with: $K_2 = 0{,}02$, the simulation results of Fig.3 and Table II have been obtained (with the CSMP language).

| CASE | $\int_0^{160} ACE\ dt$ | $\int_0^{160} x_3^2(t)\ dt$ | $\int_0^{160} x_5^2(t)\ dt$ |
|---|---|---|---|
| A | -0,2388 | 0,0061 | 0,0013 |
| B | -0,2386 | 0,0061 | 0,0013 |
| C | -0,242 | 0,0061 | 0,0013 |
| E | -0,2422 | 0,0061 | 0,0013 |
| G | -0,2394 | 0,0061 | 0,0013 |
| I | -0,1812 | 0,0063 | 0,001 |
| N | -0,1439 | 0,0064 | 0,0008 |
| Q | -0,1459 | 0,0064 | 0,0008 |
| PI regulator | -0,1622 | 0,0065 | 0,001 |

TABLE II: Cost functions.

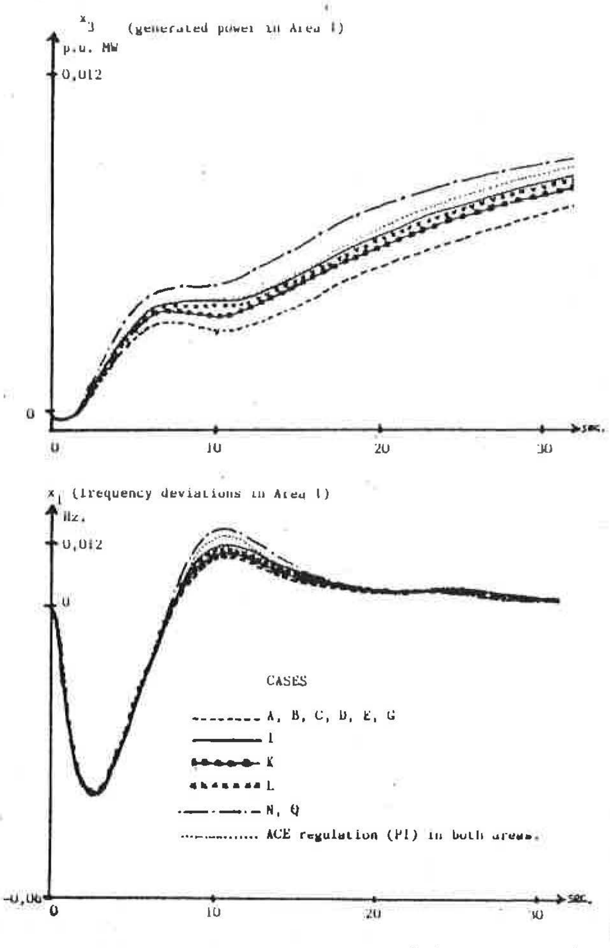



Fig.3.- Simulated responses for a 0.01p.u.MW load increase in Area 1.

## Conclusions

Comparing the optimal gain vectors and the simulation results of cases A and B, the neglegibility of the effect of the inclusion of the external load in the design model is deduced.

In cases C, D and E, where the frequency deviations have been penalized, the optimal gain vector and the simulated system dynamics are very similar among them and with respect to cases A and B. The same situation occurs in case G, where power interchanges are penalized.

When ACE deviations are penalized in case I, the system behaviour is sensitive to this penalization. The accumulated interchanged power is less than in preceding cases and the frequency deviations are bigger, although these values are directly related to $\beta$. The ACE improvement in this case means a bigger generation in Area 1, and the lower power interchanges are obtained at the cost of high generation efforts in Area 2 during the first seconds of the transient, which implies a considerable spinning reserve in Area 2.

In case K the penalization of the transient loop deviations in the speed regulator reduces the unit generating efforts. These results are similar to those obtained in case L, where the generated power is penalized.

In case N ($\underset{\sim}{R}$ decreasing) a major control effort is allowed; that improves the system behaviour at the cost of an important spinning reserve in Area 1. A similar situation occurs in case Q, where a minimum stability degree is imposed.

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