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GPS WITHOUT SATELLITES

by

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Falling in love is not at all the most stupid thing that people do, but gravitation cannot be held responsible for it.

Albert Einstein

I believe in intuition and inspiration. Imagination is more important than knowledge. For knowledge is limited, whereas imagination embraces the entire world, stimulating progress, giving birth to evolution. It is, strictly speaking, a real factor in scientific research.

Albert Einstein

Never memorize what you can look up in books.

Albert Einstein

GPS WITHOUT SATELLITES

Abstract

by Marc Vergés Grau

Nowadays, there have been great advances in the location technology. The personal positioning offers a very interesting field of research because the user walking has an unpredictable behaviour and it is difficult to assume predefined routes or to take into account other implemented location techniques for vehicles or robots.

An approach for integration between inertial navigation systems (INS) and GPS is presented. GPS is a navigation aid accurate and reliable but susceptible to interference like multipath. An INS is very accurate over short periods, but its errors drift unbounded over time. Blending INS with GPS can remedy the performance issues of both.

GPS is often combined with other sensors like accelerometers, gyroscopes or magnetometers. The data fusion from these sensors is very important because they allow us to calculate the position and orientation constantly. In this project we are interested in analysing the system behaviour when the signal GPS is unavailable as when the signal is blocked or in indoor environments. The analysis will be carried out through the assessment of a Dead Reckoning algorithm to improve the position information. The system was tested both indoor and outdoor of the Thales building. The personal positioning system is made up of: a receiver GPS, an electronic compass, and the IMU.

There are many types of integration methods, and sensors vary greatly, from the complex and expensive, to the simple and inexpensive, in this project it has been used low cost sensors in a loosely coupled approach.

A Kalman filter for closed loop integration between GPS and INS is done. The filter propagates and estimates the error states, which are fed back to the INS for correction of the internal navigation states. The integration algorithm has been implemented on Matlab. The algorithm receives the GPS and inertial measurements via serial port to later process all the data. The algorithm has been used to experimentally test and compare navigation performance.

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Abbreviations

ADC	**A**nalog to **d**igital **c**onverter
CIM	**C**ommunication **I**nformation **M**odule
D2S2	**D**utch **D**ismounted **S**oldier **S**ystem
DCM	**D**irection **C**osine **M**atrix
DGPS	**D**ifferential GPS
DR	**D**ead **r**eckoning
ECEF	**E**arth **C**entered **E**arth **F**ixed
EKF	**E**xtended **K**alman **F**ilter
ENU	**E**ast-**n**orth-**u**p (coordinates)
GNSS	**G**lobal **N**avigation **S**atellite **S**ystem
GPS	**G**lobal **P**ositioning **S**ystem
IMU	**I**nertial **M**easurement **U**nit
INS	**I**nertial **N**avigation **S**ystem
IRU	**I**nertial **r**eference **u**nit
KF	**K**alman **F**ilter
MEMS	**M**icro**E**lectro**M**echanical **S**ystems
NED	**N**orth-**e**ast-**d**own
PDR	**P**edestrian **r**eckoning
UTC	**U**niversal **T**ime **C**oordinated (Coordinate Universal Time)
WGS-84	**W**orld **G**eodetic **S**ystem 1984
ZUPT	**Z**ero **V**elocity **u**pdates

Chapter 1

Thales

1.1 Thales Group

Thales is an organisation with a solid footing in three core markets: Aerospace and Space, Defence and Security. A worldwide Group with more than 68,000 employees and presence in 50 countries.

As a world leader in mission-critical information systems for Aerospace and Space, Defence and Security markets, Thales builds on proven capabilities in large and complex software systems to meet the requirements of civil and military customers. The company plays an important role in making the world a safer place, with its experience in dual technologies, exceptional technological potential and expertise spanning the entire value chain from prime contracting to systems integration, equipment and services.

Building on proven capabilities in large-scale software systems, Thales is stepping up to the security challenges of its customers in an increasingly interconnected, technology-driven world. Civil and military systems benefit from many of the same technologies and innovations. Developing these dual technologies has been a long tradition for Thales, with its global network of 25,000 high-level researchers and engineers. Thales plays a pivotal role in making the world a safer place.

Thales mission is to make the world a safer place by providing a comprehensive range of integrated solutions to meet three objectives: make reliable and secure, monitor and control, protect and defend. Thales potential for technological innovation is outstanding, with more than 20,000 highly skilled R&D engineers on the payroll, and Thales are Europe's most powerful force in software engineering and complex systems. Across the organisation, Thales draws on a coherent set of technologies and serve a coherent customer base made up of governments, large organisations and major infrastructure operators. The geographic footprint of the operations also brings to Thales a level of coherence that helps to drive overall performance: worldwide centres of excellence develop and produce the equipment in the countries where research and manufacturing are the most efficient, while incountry integration centres directly address the needs of local customers. Thales operations in more than 50 countries benefit from the dynamic global marketing and sales organisation. Last but not least, the company benefits from an extremely capable workforce: Thales employees all over the world share the same values and a corporate culture based on teamwork and sharing, performance and empowerment. "Our potential for technological innovation is outstanding."

Making the world a safer place while growing the business and improving profitability is an ambitious and exciting mission. Across the organisation, Thales brings together all the strengths it needs to succeed in this mission, for the benefit of its customers, employees and shareholders.

1.1.1 Serving the customer, enhancing security

Thales designs, develops and deploys integrated solutions and advanced products and services to meet security requirements in the Aerospace and Space, Defence and Security markets.

The globalised, open world has enabled people, commodities, capital, services and information to flow more freely. But a more open world is inevitably more vulnerable. Transport, energy and data networks form the backbone of our society and are vulnerable to failures within the system and exposed to risks and threats from outside, such as trafficking, terrorism, cyber attacks, failed states and asymmetric armed conflict.

To meet rising demand for security and seize growth opportunities across all its markets, Thales combines expertise in mission-critical information systems, secure communications, supervision systems and sensors. The company offers a complete range of technologies and solutions to address the specific requirements of government and institutional customers, considering long-term relationships based on trust and local presence as key success factors in the complex security projects. As well as military and civil defence forces, customers include public administrations, large-scale operators of critical infrastructure and major commercial aircraft manufacturers.

Thales is a leading defence contractor and a major player on civil and commercial markets around the world. Its businesses are organised by market segment Aerospace and Space, Defence and Security and operate as a single organisation, sharing advanced technologies and drawing on complementary capabilities across the company to meet the specific requirements of each customer.

1.1.2 One company serving three complementary markets

1.1.2.1 Aerospace and space

Aerospace markets offer a vivid illustration of the benefits of dual civil/military technologies. Thales is also the only company in the world with leadership positions in both onboard equipment and ground equipment. The company equips all types of aircraft, commercial airliners, military aircraft and helicopters and is the first partner of the world's leading manufacturers

In air traffic management, Thales's capabilities span the entire flight plan surveillance and security chain, from departure gate through route control to arrival gate in complex and saturated air transport environments. It is one of the few companies in the world with the know-how and experience to provide complete and effective support for air traffic controllers.

The company has also emerged as a major player in Space programmes around the world through some structural alliances. Thales is now the European leader in satellite systems and services for a broad range of applications: commercial telecommunications, navigation, radar and optical observation, meteorology and oceanography, scientific research and military communications and observation. Together, the company's Aerospace and Space divisions employ 20,500 people.

1.1.2.2 Defence

Thales is the European leader in defence systems, providing high-tech products and systems to meet the requirements of air, land, naval and joint forces. It develops the solutions ISTAR[1], C4ISR[2] and UAV[3] systems needed to counter new threats as they emerge. Thales has completely renewed its radar product portfolio and unveiled next generation product families such as the software radio sets. A unique multidomestic development strategy has brought Thales local player status in numerous European countries, as well as in North America and high-growth markets in Asia and the Middle East. More than 23,000 people from the Aerospace, Air Systems, Naval and Land & Joint Systems divisions work in the Defence sector.

1.1.2.3 Security

As civil security markets restructure to meet burgeoning demand, Thales has carved out strong strategic positions in several key segments: security and safety systems for ground transportation (rail, urban, road), supervision and protection for infrastructures (airports, harbours, land and maritime borders, energy supply and transport infrastructures), solutions and services providing personal data protection for the civil administration (encryption, detection, localisation, simulation), and providing management and control of critical information systems for industry and finance. The Thales Security Solutions & Services division employs more than 20,000 people in 35 countries.

1.2 Thales Netherlands

Thales began its expansion outside France in 1990 when it acquired the Dutch company Signaal, one of the world's leading suppliers of naval defence systems and a first contractor to the Royal Netherlands Navy. Building on this experience, the Dutch entity has become one of the company's main centres of naval excellence for radars and combat management systems.

[1] ISTAR stands for Intelligence, Surveillance, Target Acquisition, and Reconnaissance.
[2] C4ISR concept of Command, Control, Communications, Computers, Intelligence, Surveillance and Reconnaissance
[3] UAV refers to Unmanned aerial vehicle

Thales Research & Technology's local research team is housed on the campus of Delft University as a joint operation by Delft and Amsterdam universities, TNO[4] and Thales.

Thales Nederland can be found in Hengelo (HQ), Delft, Huizen and Eindhoven. The Thales Nederland organization performs its own industrial and commercial activities here. However, the company also acts as local point of contact for foreign Thales entities dealing with Dutch customers. This is in line with the Thales Multi-Domain, Multi-Domestic philosophy, to optimize the communications with the customer.

Thales Netherlands has several divisions like, electro-optics, cryogenics and communication products. The communication products include the futuristic Digitised Soldier System where this project takes places.

Supporting the customers, Thales Nederland provides comprehensive integrated training and service to customers in more than 40 countries around the globe. Support is becoming more and more a vital aspect in any program and Thales closes customized agreements with each and every customer in this respect.

1.2.1 History

Thales started in 1922 as NV Hazemeyer's Fabriek van Signaalapparaten in the city of Hengelo

Since its foundation the company has been known under the names of: NV Hazemeyer's Fabriek van Signaalapparaten, Hollandse Signaalapparaten, Thomson CSF Signaal and since 2000 as Thales Nederland.

Through the years the main expertise of Thales Nederland has been the development of fire control and radar systems for many navies around the world, as well as the development of battelfield management systems. In the Thales Group, Thales Nederland is the "Centre of Excellence" in the field of radar technology.

1922 - 2000 Thales Nederland B.V. goes back a long way

In 1922 the basis of Thales Nederland was established under the name of NV Hazemeyer's Fabriek van Signaalapparaten in the city of Hengelo.

[4]TNO is a Dutch Organisation for Applied Scientific Research

First Fire Control System for Royal Neth. Navy

Hazemeyer started the company to produce fire control equipment for two new ships of the Royal Netherlands Navy: Hr.Ms. Sumatra and Hr.Ms. Java. The company grew rapidly and welcomed customers from Sweden, Spain and Greece.

1940 Wartime interrupted common life

In 1940 the factory was captured virtually intact by the invading German Army. A large number of the staff were able to escape to the United Kingdom and continued to work there on radar and fire control systems. When they returned to their home country, they found the factory pillaged and deserted.

1945 Back in bussines: Hollandse Signaalapparaten B.V.

The Dutch government was aware of the importance of a good defence industry and bought the factory. The company continued under the name N.V. Hollandsche Signaalapparaten (Signaal for short). New buildings and facilities were erected and new staff was hired. In these years a lot of techniques and systems were developed, such as radar, fire control for the army, computers and air traffic control equipment.

In 1956 Philips bought a large part of the shares from the government and became the main shareholder. The company flourished and opened plants in several cities across the country. Near the end of the eighties Signaal employed over 5.000 people and served customers in over 35 countries.

1990 Signaal became Thompson CSF Signaal

As the Cold War ended, the political theatre changed dramatically. Large cuts in defence budgets forced Signaal to reorganise, leading to a staff reduction. Meanwhile Philips decided that "Defence and Control Systems" were not part of its core-business and in 1990 Signaal was taken over by Thomson-CSF (now Thales). The reorganisation and the merger with Thomson-CSF brought the company a new driving force. New systems were designed, taking a leap in defence equipment and combat management.

In December 2000 Thomson-CSF changed its name to THALES. Being a member of this group, Thomson-CSF Signaal changed its name to THALES NEDERLAND.

1.3 Thales Huizen

Thales Land & Joint Systems Netherlands (TNL), based in Huizen, has 300 employees (of which 150 engineers) and is the world market leader in multimedia communication networks for vehicles and small platforms. The company is supplier of high quality integrated communications systems for both commercial organisations and armed forces.

Thales Land & Joint Systems Division in Huizen is the leading defence communications company in the Netherlands and a supplier of high quality integrated radio and network communications systems, command and control, battlefield information management and optronics providing added value solutions and services to the Armed Forces in the Netherlands.

FIGURE 1.1: Thales Huizen

The company, with its 300 employees, applies the latest tools to develop, produce and maintain communication equipment as well as integrated communications systems for both armed forces and commercial organisations with requirements for multimedia networks.

As a market leader for intercom systems and multimedia communications, its product portfolio comprises the advanced intercom systems SOTAS[5], SOTASM2[6] and SOTASIP[7] which are used worldwide in a wide range of vehicle types. The systems offer complete tactical internet integration for battlefield where all different C4[8] applications are present.

Thales designs, manufactures and sells state of the art electrooptical components and systems for defence and OEM use. The company supplies system integrators and end-users all over the world. The activities include optomechanical and electrooptical systems and products. Some of them are night vision systems (image intensifier), thermal infrared cameras, advanced CCD cameras for observation, tracking and aiming applications, and head-mounted display systems for soldier and pilot applications. Thales has a

[5]SOTAS is a digital intercom system, developed for use in armoured vehicles. It provides crystal clear communication between crew members inside a vehicle and externally over combat net radios and field cables.

[6]SOTASM2 is a multimedia vehicle communications system designed for the digital battlespace

[7]SOTASIP its an Integrated Vehicle Communications System coverging all the vehicle communication requirements, integratin Ethernet, VoIP phones, Ip router, data links, radio gateways

[8]Command, Control, Communications, and Computers

wide experience in observation and aiming systems for both the individual soldier and platforms (vehicles, naval and airborne).

In September 2006, Thales in the Netherlands opened its Battlespace Transformation Centre - BTC. The new Transformation & Integration Centre - TIC[9] of the Land & Joint Systems Division in Huizen together with the TIC in Hengelo have joined the Thales collaborative centres of expertise in France, United Kingdom, Germany, Australia, Greece and Singapore.

The BTC offers the possibility to exercise the process of Concept, Development & Experimentation in the world of Network Enabled Capabilities. Within BTC, policy and practice are brought closer in order to identify potential goals, detect and analyse risks and explore migration paths.

By combining operational analysis, system engineering and architectures with extensive Battlelab facilities, all parties to transformation are working together to identify and evaluate transformation options and therefore provide a smooth and affordable transition from legacy to Network Enabled Capabilities.

Furthermore, Thales in Huizen represents the Thales Group's portfolio in the Netherlands. Thales has delivered satellite, radio direction finding and navigation aids and propagation prediction systems, always keeping a close track of the latest developments and maintaining a high level of expertise.

The company obtained the international quality certificates ISO[10]-9001:2000 /ISO-14001:1994/AQAP-2110/150 and IPC[11]/EIA J-STD-001C, as well as CMMi[12] level 2 for programs and process development.

[9] Technologies of Information and Communication
[10] ISO: International Organization for Standardization,
[11] IPC: Integrated Procurement Control
[12] CMMi: Capability Maturity Model Integration

Chapter 2

Objectives

2.1 Context

Thales Land & Joint Systems in Huizen has been prime contractor for the development of an essential part of the Dutch Dismounted Soldier System *'D2S2'* designing the Communication and Information Module *'CIM'*. CIM can be compared to a GPS door to door navigation system for soldiers.

The CIM, according to Thales specifications has been designed to fit soldiers and civil requirements therefore there are two versions available of it. The civil version is mainly designed for police and firemen. It is in the civil version of CIM where this project takes place.

Often emergency and police services are faced with missions inside buildings where the GPS signal reception inside those buildings is usually weak or completely blocked. Therefore, other ways of obtaining position information must be developed.

2.2 Dutch Dismounted Soldier System

On 2007, the Organisation for Applied Scientific Research in the Netherlands (TNO), an independent public body established by law in 1932, presented a Dutch prototype of the soldier of the future. The Dutch model was developed in close cooperation with the Netherlands Defence Ministry.

Thales would be entrusted with production of the Command and control (C2) communication systems. The Dutch future soldier, like its European counterparts, incorporates the idea of modularity, C2 communication systems, weapons systems, protection and clothing adapted to different operations.

The Dutch Dismounted Soldier System or D2S2 will be a suite of integrated equipment designed to enhance the efficiency and survivability of the Dutch dismounted soldier.

The Netherlands Ministry of Defence is using an open approach to design and create the Dutch Dismounted Soldier System. This approach allows a continuous integration of new technology, non-stop processing of new developments, on-time responses to changes in a similar way in other countries, and where possible, cooperation with other nations. Forecast International believes the Dutch Dismounted Soldier System will consist of components that fall into five categories or modules: clothing, equipment, communications and information, armament, and energy supply.

Thales has developed and delivered prototype systems that will be the key components of a communications infrastructure for the Dutch Dismounted Soldier System. Prototype systems to be produced by Thales include soldier radios, a central processing friendly control unit, a computer with a user dedicated soldier and commander interactive displays. This communications infrastructure will be linked to the Netherlands Armys command and control battle management system.

The ability of the dismounted infantry group to make and successfully implement some tactical decisions, in the face of big uncertainty and danger, is key determinant. The goal of the group is to be able to deliver the required operational effectiveness is heavily dependent on its perception of the environment, mobility, survivability and sustainability. In all these domains, technology will provide opportunities for significant enhancements. However, there are limits to the cost, weight and availability of electrical power within which such competing enhancements can be accommodated. An integrated system, at group level, is therefore required to maximise the dismounted soldiers operational effectiveness, an optimum balance has to be achieved. This means a complex trade-off dependent on the mission, situation and environment has to be analysed. C4I means: command, control, communication, computing and information. It is also recognised that soldier modernisation will be a process that will provide capability to the soldier as technology evolves and incorporates the changing requirements that soldiers needs.

The Dutch Dismounted Soldier System (D2S2) comprises the following modules:

1. Clothing
2. Equipment
3. Command, control, communication, computing and information
4. Arms
5. Energy
6. Armament and Sensors

FIGURE 2.1: Soldier System

2.2.1 Communication and Information Module

The Communication and Information Module, CIM, is an essential part of the Dutch Dismounted Soldier. CIM is the on-board computer that integrates all the other peripherals like radios, gps, displays and any kind of sensor that could be useful.

In the uncertainty of an operation it is necessary to command soldiers in such a way they achieve the goals laid upon them. For this reason, they need not only good training and doctrine but also the right equipment to perform the given task. Part of this equipment is the Communication and Information Module (CIM), with which Command and Control (C2) and Situation Awareness (SA) will be brought to each individual soldier facilitated by modern ICT.

CIM is a lightweight, battery powered system providing situational awareness and communication with team members commander. CIM has been designed as a junction point. CIM is a tool that supports faster and more accurate operation. The reason for being is to increase the effectiveness and protection and eliminate, as much as possible, the current limitations with regards to his movement functions, within the context of the slogan "Effectivity by simplicity". It provides functionality answering the following basic questions in command and control:

- Where am I? Situational awareness
- Where are my colleagues? Blue Force tracking
- What is the problem?
- Where is the problem? Red Force tracking
- What is the solution?

The CIM is a modular system mainly designed for dismounted groups, soldiers, fireman, police, etc... Here there is a summary of its capabilities:

- Civil & military GPS
- Digital Magnetic Compass
- Data Transmission
- Radio
- Computer
- Power unit
- Display and control
- Download maps
- Real-time situational awareness
- Lightweight
- Configurable
- Very slim unit.
- Low Power usage

The current equipment of a military soldier was developed at the time for a single task and a unique type of environment and mainly consists of a series of independent pieces of equipment with a low degree of compatibility. As a result of changes in task assignment, operations and areas of deployment, additional pieces of equipment were added in the past in order to provide the additional required functionality. Consequently, today a soldier is confronted with an array of equipment that consists of dozens of separate components for which there is barely any free space on a soldier's body, that are impossible to take along in a responsible ergonomic way and which together weight a great deal.

For orientation and navigation purposes each soldier has a map and each group has a PLGR (Precise Lightweight GPS Receiver). The inability to have an overview of the terrain (urbanised region), as well as conditions involving poor visibility, darkness and the threat of randomly operating antagonists make impossible to distinguish the front line, so the direction of the unit becomes more difficult.

Furthermore, without access to night vision equipment, it is impossible to operate around the clock. Communication at the group level is currently only possible on the basis of field signals provided that visual contact is guaranteed or on the basis of verbal commands, provided tactical conditions permit this. Not every soldier has reduced visibility equipment that allows observation during darkness as well. The individual soldier does not have equipment that allows him to determine his own position and his group members. Aside from visual recognition, he does not have equipment that allows him to distinguish his own troops from potential antagonists. He is not connected to the network and once dismounted from a vehicle, helicopter or landing craft no longer has any means for obtaining or sending additional information, for example via the Battlefield Management System (BMS). The dismounted unit as well as the higher echelon are consequently deprived of essential information. These directly affects the decisions on the operational times.

Technical developments make possible to completely or partially remedy a number of the mentioned problems in the area of information exchange and command and control support. This includes the integration of position identification technology (GPS), wireless voice/data communication technologies and portable C2 systems.

2.3 Goals

The goal of this project is to generate a state of the art study on available techniques for positioning in indoors and demonstrate how accurate such a system can be by developing a working demo software. The demonstration will be with real hardware and software.

Technology has made available in the market integrated platforms with gyroscopes and accelerometers, which will provide the platform to develop the demo application.

The main goal is provide GPS-less navigation. This means try to provide indoor navigation when the GPS signal is not reliable or there is not signal at all. Consequently, the idea is to improve the navigation precision in bad GPS coverage.

The objective is to combine GPS information with other sensor information like accelerometers, gyroscopes, magnetometers. The main focus is on high yield solution: low cost, big improvement. There is no need to be perfect but it must be small, low powered system that can be integrated in the CIM. Therefore, a bulky expensive module (black box) is not an option.

The final development should be a real-time low cost positioning system by integrating an Inertial Measurement Unit (IMU) with CIM. The system must be:

- Compact
- Light weight
- Fast
- Accurate calculations

Inertial Navigation Systems (INS) provide high data rates but drift over time. In contrast GPS provides high-accuracy position data but at lower data rates ($\tilde{1}$ Hz). Also INS is completely self-contained and does not require external input for operation. GPS is susceptible to data loss and GPS signals do not penetrate into the ground neither most of the buildings. Integration of INS & GPS through a Kalman Filter will result in high-accuracy position. We want to get the best of each part to create a new system.

An Inertial Navigation System provides information of motion, sensing accelerations and turns, from a known docking location translates these information into a precise grid coordinate location. INS provides real time information any time & any where. In order to start, INS needs an initial position from another source, for example a GPS, thereafter it will be able to compute its own updated position and velocity by integrating information received from the motion sensors. It uses acceleration (changes in speed & direction) instead of speed itself.

Position coordinates information is important but it is not the only information we can get from an INS/GPS system. Sensors data may also give extra information like attitude, speed. All this extra information may be easily obtained and may be really useful for CIM architecture.

All components of the final system must be commercially available and their integration into CIM architecture must be possible. In addition, the final location of the sensors is important. Actually the location of the sensors is already defined due to the CIM is attached on the chest, our INS must be designed to be in this position. This is important because there are several approaches on inertial systems, some of them are using pedestrian reckoning, this means locate the sensors in a boot and try to detect the step length. But this is not even possible in our project.

Chapter 3

Indoor Navigation

3.1 Introduction and Background

This chapter will give a general overview of the most common indoor navigation technologies, focusing in the major advantages and disadvantages of each technology.

There are three technologies commonly used for indoor location systems: ultrasonics, infrared and radio frequency. These can be supplemented by inertial systems which are generally used for prediction. Infrared systems tend to rely on the user taking explicit actions to identify his presence. Rf-systems require sophisticated (and often heavy) aerials. Ultrasonics offer a low cost solution which can operate without any user interaction. The disadvantages of an ultrasonic system are: loss of signal due to obstruction, false signals due to reflections and interference from high frequency sounds. These disadvantages can be minimised and it is possible to find some systems produced by commercial suppliers that have successfully implemented ultrasonic positioning with good results. In contrast these commercial systems are too expensive for a real use. Here there is a list of the indoor technologies:

- GPS, Global Positioning System
  - DGPS, Differential GPS
  - AGPS, Assisted GPS
- GSM, Global System for Mobile communications
- Pseudolites

- WLAN, Wireless local area network
- UWB, Ultra-wide band,
- INS, Inertial Navigation System

3.2 Wireless solutions

3.2.1 GPS signal

The use of GPS indoor has recently been more viable by the development of high sensitivity receiver GPS technology that allows the acquisition and tracking of weak signals.

This technology is based on a longer integration of GPS signals that allows the tracking of signals strongly attenuated. The performance of HSGPS[1] indoors has been reported in several papers and it shows better availability indoors compared to conventional GPS, but with a poorer accuracy.

Satellite based positioning is a well established method due to the wide use of the GPS. The US defence administration stopped interfering in the accuracy of the signal for civilian use since spring 2000 by turning off the Selective Availability (SA), this means that in good conditions the accuracy of GPS can be 10 meters.

European Union together with European Space Agency (ESA) is planning Galileo, a satellite positioning system that has already launched the pilot phase. In Russia works the GLONASS system with capabilities equivalent to GPS system, but because the system is focused on the military the number of devices in the market is still too small for mass using.

Traditionally it has been possible to improve the accuracy by transmitting in real-time correctional information measured at fixed ground reference stations. The correctional information has been broadcasted via link towers or satellites in radio frequencies (WAAS and EGNOS). But last two methods not exact global system and could be refer to hybrid techniques.

The problem with satellite based positioning is the difficulty to receive the signal in urban and indoor environments. A big problem for GPS devices has been the high

[1]HSGPS: High sensitivity GPS

power drainage. Assisted GPS, where assistance information is received via the cellular network, is believed to solve the problems related to the power drain, slow positioning and also increase the signal sensitivity. These problems can be overcome by providing e.g. the satellite position data stream via the cellular network, as for the traditional GPS receivers it takes a relative long time to receive this signal uninterruptedly from each visible satellite. Assisted GPS improves the availability of satellite based positioning, and improves the accuracy like DGPS.

However, A-GPS alone is not enough to make GPS work indoors, as the signal is attenuated when it passes through buildings. The sensitivity of the receiver must be increased, and this can be achieved by adding a very large amount of correlating devices, with which they detect and measure the encoded signal, into the receiver. The assisted approach requires in any case, that the cellular network is used for transmitting the assistance data, cellular phone and GPS receiver must be integrated or connected.

New devices equipped with GPS receivers are required for GPS to become more common. The barrier so far has been the price of the device.

3.2.2 GSM location

If there is a GSM or UMTS network out of the building is possible to estimate one's position. But this position usually rely on significant errors. Network based positioning systems are available as products provided by small and specialized companies as well as the network vendors. The mobile operators have made some efforts to implement location servers and they continue to do so. There is quite limited experience on the use of mobile location based services, and there aren't any public records on the number of users or usage.

The cellular network constantly maintains the location information of a phone that has power switched on at a base station group level, and for a connected phone at base station level. A location server enables the positioning of a cellular phone without a connection such as SMS transmission or transport of GPRS-packet. There are limits to the capacity of network-based positioning and therefore continuous tracking would require considerable resources from the network and is not viable as a commercial service. The most common network-based positioning methods are:

- Cell-id based positioning (CI)
- Direction of arrival (DOA)
- Time of arrival based positioning (TOA)
- Enhanced observed time difference based positioning (E-OTD)

Cell identification (Cell-id): The cell-id also called cell of origin method is the main technique to provide location services for applications. The method uses the characteristic of the mobile networks which can identify the approximate position of a mobile by knowing which cell is using. The main benefit of the technology is that it is already in use today and can be supported by all mobiles. However, the accuracy of the method is generally low (in the range of 200 meters) depending on cell size. This accuracy is higher in dense covered areas (for example, urban places) and much lower in rural environments.

Direction or Angle of Arrival (DOA or AOA): The idea is to point a directional antenna beam until the direction of maximum signal strength of coherent phase is detected. The required directivity to achieve accurate measurements is often obtained by means of antenna arrays. The first measurement produces a straight-line from the base station to the mobile phone. The second measurement will yield a second straight line and the intersection of the two lines will give the position fix for this system. In satellite mobile systems there is an extra problem because the position accuracy depends on the distance from the transmitter, but it is not possible to mount the required antenna on the satellite. To solve this in satellite mobile systems is needed an extra measurement, the Doppler shift component.

Time delay: Since electromagnetic waves travel at constant speed (speed of light) in free space, the distance between two points can be easily estimated by measuring the time delay of a radio wave transmitted between them. This method is well suited for satellite systems. There are two types of time delay methods that can be identified:

- Time of Arrival: The position is calculated from the absolute time of a wave to travel between a transmitter and receiver. This implies the exact time for transmission must be known. An extension of the approx is to measure a round-trip time of a signal transmitted from a source to a destination and the echo back.

- Differential Time of Arrival: Having a precise synchronized clocks at the transmitter and receiver is a problem. To avoid this problem are used several transmitters synchronized to a common time base, and measuring the arrival time difference at the receiver. To understand this method imagine a two dimensional system in which a line is drawn joining all points that have the same time difference, this line will be an hyperbola. Then using the measured differential time of arrival a position can be defined.

E-OTD operates on the same principles as A-GPS, except instead of a signal originating from a satellite, signals are sent from mobile base stations and are then picked up by specialized mobile phones that receive the signals and calculate a location.

3.2.3 Pseudolites

Pseudolite is a contraction of the term ”pseudo-satellite”, used to refer to something that is not a satellite which performs a function commonly in the domain of satellites. Pseudolites are most often small transceivers that are used to create a local, ground-based GPS alternative. The range of each transceiver's signal is dependent on the power available to the unit.

Being able to deploy one's own positioning system, independent of the GPS, can be useful in situations where the normal GPS signals are either blocked/jammed (military conflicts), or simply not available (exploration of other planets).

The main reason against the pseudolite-based and building dependent positioning systems like WLAN is the obvious requirement of building dependent infrastructure. The aim of the building dependent location systems is the location of a person on a specific floor or in a room. The pseudolite-based indoor positioning systems are also limited in their use due to severe multipath and losses in propagation through concrete and brick walls. This causes significant positioning errors and very limited applicability when high accuracy indoor navigation is required.

3.2.4 Wlan

For an indoor environment, several systems based on various technologies such as infrared (IR), ultrasound, video surveillance and radio signal are emerging. But the wireless local area network (IEEE 802.11, also named WiFi), a signal based positioning system have an attention in recent years. A WLAN based positioning system has distinct advantages over all other systems.

A WLAN network is an economic solution because it usually exists already as part of the communications infrastructure. For a notebook computer, PDA[2], or other mobile devices equipped with WLAN capability, the positioning system can be implemented simply in software. This location software reduces costs with respect to dedicated architectures. Comparing WLAN and other indoor positioning systems WLAN may cover a large area. The WLAN system may work in large buildings or even across them. It is a stable system with a robust RF signal propagation. Other systems like Video or IR-based location systems are subject to restrictions, such as line of sight limitations or poor performance with fluorescent lighting or direct sunlight.

WLAN systems rely on collecting the signals to train a signal distribution map, applying a position determination model that can be used to determine the device position. Since the propagation of indoor signals are heavily affected by multipath effects, dead spots, noise and interference the signals appear in an irregular pattern. Therefore, creating an efficient and accurate positioning system for indoor environments is a challenging task.

3.2.5 UWB

Short range wireless technologies such as Radio Frequency technologies (WLAN, Bluetooth, and RFID) Ultrasound and IrDa can be used to provide position determination. But coverage area and precision determination for these technologies are rather better than for outdoor techniques. General comparison of accuracy and coverage applicability are shown.

Of course this list is not complete, especially for indoor techniques and technologies. Accuracy and applicability of indoor technologies now are wider and number of systems

[2]PDA: Personal Digital Assistant

which use technologies like Ultrasonic, RFID and IrDA become more and more. Infrared sensors mounted along a building, for example in the ceilings, can be used to detect a person wearing a device, for example a badge, that periodically emits an ID code via an infrared transmitter. The location of the badge is determined by proximity to the sensor receiving the ID. It requires visual line of sight to work, and normally does not have very high accuracy. Moreover, it can not work when the device is in a pocket.

Ultrasound transmitters also called beacons send signals to a receiver. Those signals allow the receiver to calculate its own location based on proximity. To achieve a higher accuracy these systems send reference radio signals and using timing differences between the ultrasound and radio signals they can improve the accuracy and in some cases calculate the attitude of the target.

Bluetooth is a standard for short range ad-hoc networking to support personal area network. Although mainly created to replace the wires (e.g. between headphones and a portable music player), such systems could also be used for proximity-based location services when a Bluetooth device comes within range of a service point.

3.3 Inertial solution

In this section, Inertial navigation system is briefly described focusing on the main ideas.

Human motion capture using inertial sensors has forbidden due to the weight and size of the sensors, but improvements in the performance of small and lightweight MEMS[3] inertial sensors have made the application of inertial techniques to such problems possible. This has resulted in an increased interest in the topic of inertial navigation. In next chapter inertial navigation is introduced, focusing on strapdown systems based on MEMS devices.

An Inertial Navigation System (INS) is a navigation aid that uses a computer and motion sensors to continuously track the position, orientation, and velocity (direction and speed of movement) of a moving object without the need for external references.

[3]MEMS:Micro-Electro-Mechanical Systems

3.3.1 Overview

An inertial navigation system includes at least a computer and a platform or module containing accelerometers, gyroscopes or other motion sensors. The INS is initially provided with its position and velocity from another source (a human operator, a GPS satellite receiver, etc.), and thereafter computes its own updated position and velocity by integrating information received from the motion sensors. The advantage of an INS is that it requires no external references in order to determine its position, orientation, or velocity once it has been initialized.

An INS can detect a change in its geographical position (a move east or north, for example), a change in its velocity (speed and direction of movement) and a change in its orientation (rotation about an axis). It does this by measuring the linear and angular accelerations applied to the system. Since it requires no external reference (after initialization), it is immune to jamming and deception.

Inertial navigation systems are used in many different moving objects, including vehicles, aircraft, submarines, spacecraft, and guided missiles. However, their cost and complexity depends on the environments in which they are used.

Gyroscopes measure the angular velocity of the system in the inertial reference frame. By knowing the original orientation of the system in the inertial reference frame as the initial condition and integrating the angular velocity, the system's current orientation is known at all times. This can be thought as the ability of a blindfolded passenger in a car to feel the car turn left and right or tilt up and down as the car ascends or descends hills. Buy using only this information, he knows what direction the car is facing but not how fast or slow it is moving, or whether it is sliding sideways.

Accelerometers measure the linear acceleration of the system in the inertial reference frame. Since the accelerometers are fixed to the system, rotate with it and do not know their orientation, they can only measure the accelerations relative to the moving system. This can be thought as the ability of a blindfolded passenger in a car to feel himself pressed back into his seat as the vehicle accelerates forward or pulled forward as it slows down; and feel himself pressed down into his seat as the vehicle accelerates up a hill or rise up out of his seat as the car passes over the crest of a hill and begins to descend. Based on this information alone, he knows how the vehicle is moving relative to itself, that is going forward, backward, left, right, up or down measured relative to the car, but not the direction relative to the Earth, since he did not know what was the car's direction relative to the Earth when he felt the accelerations.

However, by tracking both the current linear acceleration and the current angular velocity of the system measured relative to the moving system, it is possible to determine the linear acceleration of the system in the inertial reference frame. Carrying out an integration on the inertial accelerations (knowing the initial velocity) using the kinematic equations yields the inertial velocities of the system, and integrating again (using the original position as the initial condition) yields the inertial position. In our example, if the blindfolded passenger knew where the car was pointed, what was its velocity before he was blindfolded and he is able to keep track of the car turns and its accelerations and deceleration, he can accurately know the current orientation, position, and velocity of the car at any time.

All inertial navigation systems suffer the so called integration drift. The integration drift is due to the small errors in the measurement of acceleration and angular velocity are integrated into progressively larger errors in velocity, which is became into still greater errors in position. This is a problem that is inherent in every open loop control system.

The propagation of orientation errors caused by noise perturbing gyroscope signals is identified as the main cause of such drift. White noise is the main reason of such drift. Sensor fusion can be used to reduce drift in INS. For example, it is possible to use magnetometers to correct the orientation from gyroscopes. Another way to keep this drift small is from time to time update the velocity to zero. This way the position will remain precise for a longer time. This technique is called zero velocity update.

Inertial navigation may also be used to supplement other navigation systems, providing a higher degree of accuracy than is possible using only one navigation system. To combine information from several sensors. Kalman filtering is the theoretical tool to perform their combination. Usually the most common sensor fusion is done by combining the information from an INS and the GPS system (INS/GPS). INS can be used as a short term positioning system while GPS signals are not available.

Chapter 4

Inertial systems

This chapter starts with the basic definitions about inertial navigation which will be essential for a good understanding of the following chapters. Followed by the main components description of an inertial system, subsequently, the references coordinates systems are presented as the description of some Earth models. We will step into the way to represent the attitude of a body using the Euler angles and direction cosine matrix. Then the inertial equations are described and, at the end of the chapter, the errors of the INS systems are showed.

4.1 Description of inertial navigation system

The main idea for inertial navigation comes from the physics learned in high-school:

The second integral of acceleration is position.

Inertial systems are made from several hardware and software components. Thus, next lines will introduced to some of the most important concepts.

Inertial sensors measure inertial accelerations and rotations, both of which are vector variables. Using these sensors to measure the three acceleration and rotation components over time and given the initial value for the position and velocity, the calculation of the second integral seems to be straightforward, but as usually in the real world the final solution will be far from easy.

Inertia is the propensity of bodies to maintain constant translational and rotational velocity unless is disturbed by an external force (Newton's first law).

An *inertial reference frame* is a coordinate frame in which Newton's laws of motion are valid. This means the frame does not accelerate or rotate. But Earth it is not an inertial reference frame due to the Earth's rotation and gravity. Therefore, these effects must be taken into account.

As said before, the output of an inertial system is to calculate the position and velocity of the tracking person/object. The system will try to give a reliable position & velocity information with high availability in real-time. But, where inertial system comes from? It may be said that an inertial system is the improved version of 'dead reckoning'.

Dead reckoning (DR) is the process of estimating one's current position based on a previously determined position, fix[1], and advancing that position based on known or estimated speeds over elapsed time, and course. While traditional methods of dead reckoning are no longer considered primary means of navigation, modern inertial navigation systems, which also depend on dead reckoning, are widely used. A disadvantage of dead reckoning is that since new positions are only calculated from previous positions, the errors of the process are cumulative, so the error in the position fix grows with time.

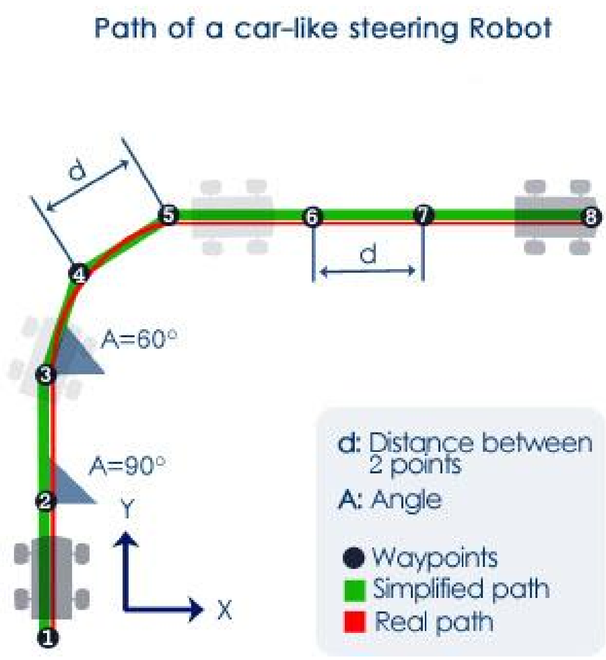


FIGURE 4.1: Dead reckoning

The name dead reckoning has nothing to do with death, it comes from deduced. Dead reckoning allows a navigator to determine his present position by projecting his past courses and speed from a known past position.

[1]A position fix or simply a fix is a term used in position fixing in navigation to describe a position derived from measuring external reference points.

It can also be used to determine his future position by projecting an estimation course and speed from the known present position.

An **inertial sensor assembly** (ISA) is the physical assemble of inertial sensors mounted on a object.

Inertial measurement unit (IMU) is the main component of inertial navigation systems. An IMU sense motion: type, rate and direction of that motion using a combination of accelerometers and gyroscopes. The data collected from these sensors allows a computer to track the position, using dead reckoning. An IMU works by detecting the current rate of acceleration, as well as changes in rotational attributes, including pitch, roll and yaw angles. This data is then fed into the INS computer which calculates the current speed and position. Inertial sensors measure inertial accelerations and rotations, which are vector variables.

For example imagine an IMU installed in an airplane. If the IMU detects a craft acceleration eastward, resulting in a calculated constant speed of 500 km per hour, and it does not detect any other accelerations for 1 hour, then the guidance computer would deduce that the plane must be 500 km east of its initial position. Then the navigation system using some maps can show to the pilot where the plane is located geographically, similar to a GPS navigation system but without the need to communicate with any outside components, such as satellites.

An ***Inertial Navigation System*** (INS) is a navigation aid that uses motion sensors (accelerometers & gyroscopes) and a computer to continuously track the position, orientation, and velocity (direction and speed of movement) of a moving object without the need for external references. It is largely used on vehicles such as ships, aircraft, submarines, guided missiles and spacecraft.

An inertial navigation system includes at least a computer and a platform or module containing accelerometers and gyroscopes, or other motion sensors. The INS is initially provided with its position and velocity from another source (a human operator, a GPS satellite receiver, etc.), and thereafter computes its own updated position and velocity by integrating information received from the motion sensors. The advantage of an INS is that it requires no external references in order to determine its position, orientation, or velocity once it has been initialized.

One of the major disadvantages of INS is that they suffer from accumulated error. Because the navigation system is continuously adding detected changes to its previously calculated positions (see dead reckoning), any errors in measurement, however small, are accumulated from point to point. This leads to 'drift' or an increasing difference between where the system thinks it is located and the real location.

Attitude is the orientation with respect to a defined frame of reference. For example, the attitude of an object can be given by providing a vector. One can think about the attitude as: How many degrees the object is tilted and rolled and also, Where is the north? As it will be described later, one of the most common ways to depict the attitude of a body is by using the Euler angles (roll, pitch, yaw).

Gyroscope is a device for measuring rotation. Commonly called *gyros*, they are mainly used to calculate and/or maintain the attitude, based on the principles of angular momentum. It measures the angular velocity (degrees per second) of the system in the inertial reference frame. INS depend on gyros to know how the accelerometers are oriented in inertial and navigational coordinates.

Accelerometers are sensors to measure accelerations (m/s^2). They detect magnitude and direction of the acceleration. Accelerometers can be used in several ways, for example to sense orientation, vibration, shocks and what is more important for us: motion. Accelerometers are already present in many portable electronic devices and video game controllers, including mobile phones, iPod's and Nintendo's Wii Remote.

Accelerometers may be used in some cases to calculate the attitude by using the measured acceleration produced by gravity forces. This is usually done when the object is not moving. While staying in rest it is possible to detect the gravity vector and therefore calculate the attitude of the object.

Accelerometers are also used in inertial navigation systems where the exact gravity is unknown. Therefore an estimation of the gravity based on the position/altitude is used. Thanks to this estimation one can still use the output to track the changes in velocity and then calculate a position using dead reckoning.

Magnetometers is an instrument that measures the strength and/or direction of a magnetic field. Magnetometers can be used for several things such as geophysical applications (find iron deposits) or in our case to sense the Earth's magnetic field. In fact magnetometers are not inertial sensors (they do not measure any inertial movement) but they are widely used in conjunction with the gyroscopes and accelerometers to keep tracking of the attitude. Usually INS are using magnetometers to calculate the north (yaw angle) and correct the drift on it.

4.2 Approaches

The main goal is to create a personal inertial navigation system where the output gives the position and velocity of the tracking person. This will be performed by using inertial sensors (gyroscopes, accelerometers).

In theory, one will be able to measure the accelerations and the calculate the distance and direction to finally estimate the position. But the first question that comes to our mind is: Where should sensors be physically located? by means of where and how to attach the sensors to the person. Therefore the location of the sensors in the body will make a big difference. In next lines some of the approaches that can be found are described.

- *Belt*: When one starts thinking on how to attach the sensors to a body, the first place to think on it is the belt because the belt is more or less in the middle of the body and then it will be able to detect all the movements. The main problem of locating the sensors on the belt is that they will not only detect the motion information but also detect the shakes of pelvis when walking.
- *Head*: It is a good place to locate the sensors. One will detect all the movements, shakes due to the walking movement will be reduced. It may be also possible to calculate where the person is looking at. But here the problem is a physical: How to attach sensors on the head? Not only the sensors are needed, but also a wire to connect the sensors to the main processor to perform all the calculations that usually is located somewhere in the vest.
- *Chest*: Locating the sensors here one will detect all the motion information while the walking side effect are minimized. Attaching the sensors on the chest one can easily find a place to hold the sensor in the vest of a fireman/policeman/soldier. Therefore, this is going to be our choice.

In this project as mentioned in the second chapter the final location of the sensors is going to be inside the CIM architecture. So while CIM is attached on the chest of a fireman/soldier our system must be designed and developed to fit these requirements.

4.2.1 Pedestrian tracking

It keeps the same goal as the preceding systems but with a new objective. The pedestrian tracking tries to detect and measure the step using inertial sensors. The idea is if one knows the step length and is able to detect the step and its direction then is possible to get a relative position. Of course is also necessary to provide an initialization of the system by providing the initial coordinates. Most of these systems place the sensors on the food.

Pedestrian tracking(PDR) exploits the kinematics of human walking. PDR uses accelerometer signals to detect steps, estimate step length and propagate position using a measured heading (such approach requires that other types of transportation than walking are excluded). Heading can be computed using gyroscopes or a levelled compass.

The PDR algorithms use IMU data in a different way than the classical double integration of acceleration of INS. Before using this system it is necessary to get all step information of the person using it. This means, it is necessary to calibrate the system.

PDR, as INS, it is a relative navigation system and its output accuracy always depends on the initialization accuracy. Initialization requires a highly accurate, if possible GPS, position information. The minimum initialization requirements for a PDR are the same as for INS but with one addition, the user's average step-length. This parameter can be calibrated with several techniques. PDR mechanization is less sensitive to errors in these initial values. Also, the largest drawback of PDR systems is that their orientation with respect to a body needs to be known. In order to maintain orientation with respect to the body, the unit has to be rigidly mounted on it. PDR systems are good systems when long tracking times are needed.

4.3 Inertial Navigation Systems

An IMU senses the rotation (gyros) and acceleration(accelerometers). Inertial navigation will use these gyros and accelerometers to calculate an estimation of the position, velocity and attitude of the system. INS is basically the union of an IMU with a microprocessor to perform all the needed calculations. There are mainly two types of INS:

- Gimbaled: The IMU is isolated from rotations of the host vehicle usually with a stable platform.
- Strapdown: In this case, the IMU is not isolated from rotations and it is rigidly attached and aligned with the axis of the body.

In this project the strapdown system will be used, as CIM is rigidly attached to the vest. The reader may find a lot of information about gimbaled system, mainly related with INS on vehicles. This project will focus on the strapdown implementation.

In a strapdown system IMU is not isolated from rotations of its host. Therefore this isolation is replaced by software that uses the gyroscope outputs to calculate the equivalent accelerometer outputs in a stabilized coordinate frame, and integrates them to provide updates of velocity and position. Strapdown systems require more calculations than gimbaled systems, but they are much cheaper.

4.3.1 Strapdown Implementation

In this next figure 4.2 is shown a basic diagram of Inertial Navigation System, where the basic signal processing functions for a strapdown INS are presented.

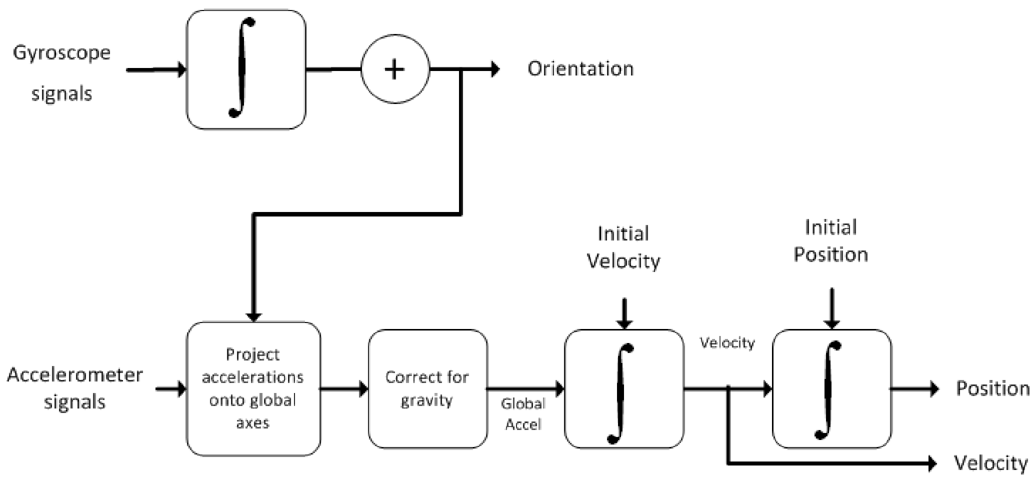


FIGURE 4.2: Basic inertial navigation system

The gyros output gives the 3-axis rotation rate, usually in degrees/second. With this rate is possible to track the system attitude. An initial value for the attitude must be given in order to initialize the integral.

Accelerometers sense the acceleration also in 3-axis. Before one can do the double integration to calculate the estimated position, some transformations are performed because in the strapdown system sensors are rigidly attached to the object. With the calculated attitude from the gyros, the measured accelerations are projected into global axes, so that gravity can be corrected. Finally, one can calculate the double integration to estimate one's current position. Note that also initial values for speed and position must be given.

Another thing to highlight in the diagram is that the main output of the INS is the position, but there are intermediate results available without any extra cost of computing. Velocity is the most clear example, but also the attitude and the heading.

As said, this is a general diagram of INS but there are a few questions that still must be solved before one can build an INS:

- **Initialization** of the integrals for position, velocity and attitude. Usually the GPS and/or compass are used in order to perform this initialization.
- **Calibration** techniques to compensate sensor errors in raw measurements.

- How to calculate the **attitude integration**. Because rotation operations are not commutative, so attitude integration is not as straightforward as acceleration. In some systems Earth's rate must be taken into account.

Alignment: Initialization of the attitude has to be done before INS can start operating, in order to initialize the attitude integral. Alignment refers on how is the IMU relative to the navigation coordinates. There are several techniques to perform this initialization, usually this alignment must be done when the INS is stationary, then with the help of a compass, magnetometers or accelerometers it is possible to estimate the initial values for attitude.

Considering that the host is stationary, one can use the accelerometers to determine the direction of the local vertical. Then, with the magnetometers it is possible to get the North's direction. Also it might be possible to use the gyros to determine the rotation of the Earth to find the North.

The INS position is initialized by knowing where you are, which is commonly done by using a GPS. The error in the position initially is very small, but it tends to increase with time due to the influence of noise in the sensors, the double integration of accelerations is the major source of this growth.

4.4 Coordinates and Reference frames

A coordinate is a way to give a position in our three dimension world. There are many ways of representing this position. Next lines describe the most important coordinates systems used in INS and GPS navigation. There are several reference frames and depending on the type of motion we are studying, it will be more interesting for us to use one or another.

4.4.1 Inertial reference frame, I-frame

This is the only reference frame where Newton laws are valid. It is a non-rotating and non-accelerating frame relative to inertial space. In practice, it is impossible to find this system but one can use some valid approaches. For example, it is possible to use "fixed" stars as the relative reference frame and neglecting the motion of the Earth around the Sun.

In this frame the center is assumed at the Earth's mass center, where z-axis is Earth's spin axis, x and y are on the Earths's equatorial plane with x-axis pointing towards a star.

4.4.2 Earth-centered inertial frame ECI i-frame

This system has its origin at the Earth's mass center and it is supposed not to rotate with respect the inertial frame (because inertial frame also rotates with the Earth). This is just an approach because the Earth is also rotating with respect of the Sun which is also self-rotating. But this system is going to be supposed as inertial. The system axes are aligned with the stars. Z-axis is aligned with the polar star, and the perpendicular plane is taken with Equator, X-axis and Y-axis are pointing to the Vernal equinox[2]. This system is important because the Newton's law are almost certain and then it is easy to change from i-frame to another reference frame.

[2]Equinox derives from the Latin meaning "equal night" and refers to the time when the sun crosses the equator. At such times around the equinox, the night and day are approximately equally long.

4.4.3 Earth-centered, Earth-fixed, ECEF e-frame

Also known as geocentric reference frame, it has the origin at the Earth's mass center and its axes are rotating with the Earth. Z-axis is pointing to North, X-axis and Y-axis are in the Equatorial plane with X pointing to Greenwich meridian (0° latitude, 0° longitude) and Y 90° towards East.

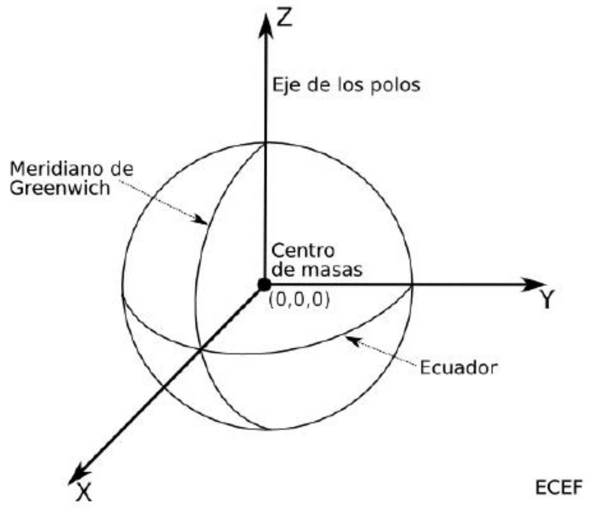


FIGURE 4.3: ECEF frame

4.4.4 Body b-frame

It is an orthogonal system that has its origin at the mass center of the body. This frame is typically used in strapdown approaches. The axis are defined using the axes of accelerometers, for example in the case of an aircraft x-axis is pointing to the nose (roll axis), the y-axis is pointing the right wing(pitch axis) and z-axis is pointing down (yaw axis)

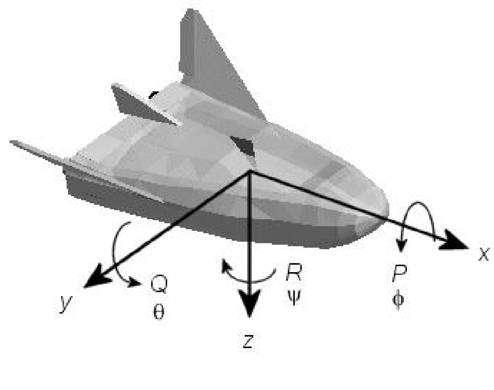


FIGURE 4.4: Body frame

4.4.5 ENU/NED Coordinates, n-frame

This is a local reference frame, similar to b-frame, but one axis is defined in the local geodetic frame. The X and Y axes are tangent to the Earth. Usually X is pointing to the North, Y to East and Z to the Earth's center. There are two types for navigation frames with similar characteristics, called NED and ENU. The only different between them is the direction of their axes.

- NED (North, East, Down)
- ENU (East, North, Up)

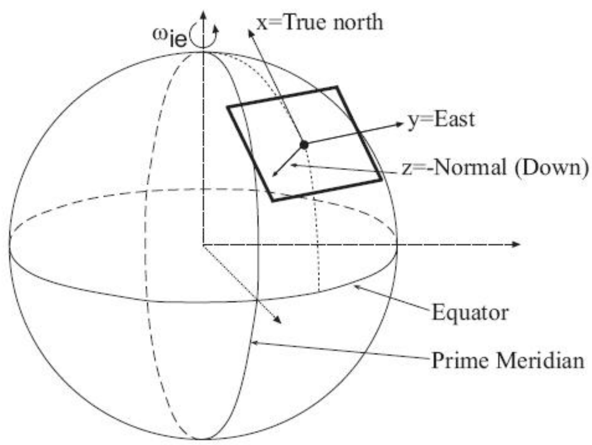


FIGURE 4.5: NED frame

4.4.6 Wander Frame

It is a coordinate frame defined on the basis of the local level frame, but it differs from the n-frame in that x-axis is not slaved to the North point. The wander frame rotates with respect to the local level frame around its z-axis. The angle between north and the x-axis is called wander angle, (α). The local level frame is then precessed about the vertical axis to maintain the level axes pointing North and East; however, the amount of L frame precession becomes very large at high latitudes. The wander frame was derived to avoid this large precession rate if the Earth's poles are traversed. This reference frame is widely used on inertial mechanization systems.

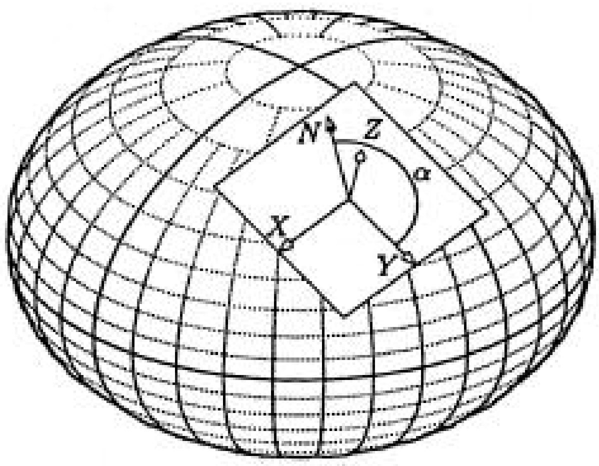


FIGURE 4.6: Wander frame

4.5 Earth models

The inertial navigation system will mainly work on Earth's surface, therefore Earth must be modelled in order to be able to correct and adjust the sensed data and the coordinates. Here is a short description of some of the models used. Inertial navigation and satellite navigation require models for the shape, gravity and rotation of Earth.

4.5.1 Earth rotation

Earth is the mother of all clocks. It has been used to calculate days, hours, minutes and seconds. Although there are some imperfections in the way to use Earth as clock, it is still used as the primary time reference, adding or subtracting leap seconds to atomic clocks to keep them synchronized to the rotation of the earth. These time variations are significant for GNSS navigation, but not for inertial navigation.

4.5.1.1 WGS84 Earthrate

It tries to model the value of Earth's rate in the World Geodetic System 1984 (WGS84) Earth model used by GPS is about 15.04109 degrees/h. This is its sidereal rotation rate with respect to distant stars. It means rotation rate with respect to the nearest star (our sun). Averaged over one year an viewed from the Earth, the rotating is 15 degrees/h.

4.5.2 Gravity models

The gravity models are necessary because our INS will operate on the Earth's surface, therefore the sensed data should be corrected to remove gravity effects.

4.5.3 Ellipsoid models

Because the Earth is not spherical, shape models are used to determine the shape of the Earth. Often ellipsoids are used as approximations of this shape. Then latitude on a reference ellipsoid is measured in terms of the angle between the equator and the normal to the ellipsoid surface. The most common ellipsoid is the WGS84 Ellipsoid The World Geodetic System (WGS) is an international standard used for GPS.

WGS84 approximates mean sea level with an ellipsoid with the axis on the rotation axis of the earth, its center at Earth's mass center, and its prime meridian through Greenwich. Its semimajor axis (equatorial radius) is defined to be 6,378,137 m, and its semiminor axis (polar radius) is defined to be 6,356,752.3142 m.

4.6 Attitude representation and transformation between reference frames

There are several different ways to represent the attitude of an object but the most common way to mathematically represent it is by using the Euler Angles. Here it is described some ways to give attitude and also some of the transformations between the reference frames.

4.6.1 Euler Angles

Euler angles were developed to describe the orientation of a rigid body in a 3 dimensional space. Imagine one would like to give a specific orientation to an object. To do so, three rotations are needed, these rotations are described by the Euler angles. Another way of thinking about this is to decompose the rotation matrix in three elemental rotations.

The Euler angles are phi ϕ, theta Θ and psi ψ usually known in aviation as roll (ϕ), pitch (Θ) and yaw (ψ) angles.

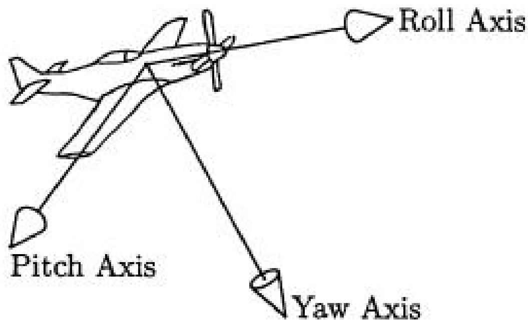


FIGURE 4.7: Euler angles

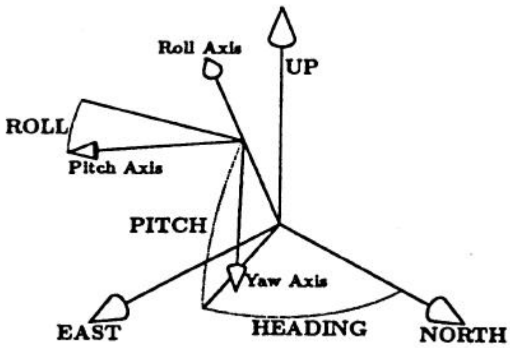


FIGURE 4.8: Euler angles2

4.6.2 Direction Cosine Matrix, DCM

The DCM is a way to perform the transformation from one reference frame to another. DCM is a 3x3 matrix, where the columns represent unit vectors in body axes, projected along the reference axes. The element in the ith row and the jth column is the cosine of the angle between the i axis of the reference frame and j axis of the body frame.

$$\mathbf{A} = \begin{bmatrix} \hat{\mathbf{u}}_x & \hat{\mathbf{v}}_x & \hat{\mathbf{w}}_x \\ \hat{\mathbf{u}}_y & \hat{\mathbf{v}}_y & \hat{\mathbf{w}}_y \\ \hat{\mathbf{u}}_z & \hat{\mathbf{v}}_z & \hat{\mathbf{w}}_z \end{bmatrix}$$

4.6.2.1 b-frame to n-frame transformation

The origin of this transformation is the mass center of the sensors platform. It is important to define the direction of the Euler angles in order to avoid problems.

$$C_b^n = C_1^T C_2^T C_3^T =$$

$$\begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\Theta) & 0 & \sin(\Theta) \\ 0 & 1 & 0 \\ -\sin(\Theta) & 0 & \cos(\Theta) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} =$$

$$\begin{bmatrix} \cos(\Theta)\cos(\psi) & -\cos(\phi)\sin(\psi)+\sin(\phi)\sin(\Theta)\cos(\psi) & \sin(\phi)\sin(\psi)+\cos(\phi)\sin(\Theta)\cos(\psi) \\ \cos(\Theta)\sin(\psi) & \cos(\phi)\cos(\psi)+\sin(\phi)\sin(\Theta)\sin(\psi) & \sin(\phi)\cos(\psi)+\cos(\phi)\sin(\Theta)\sin(\psi) \\ sin(\Theta) & \sin(\phi)cos(\Theta) & \cos(\phi)cos(\Theta) \end{bmatrix}$$

The three rotations are expressed as three separated matrices, where ψ represents rotation around the z axis,θ around the y axis,and ϕ around the x axis.

$$C_1 = \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} C_2 = \begin{bmatrix} \cos(\Theta) & 0 & -\sin(\Theta) \\ 0 & 1 & 0 \\ \sin(\Theta) & 0 & \cos(\Theta) \end{bmatrix} C_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{bmatrix}$$

Therefore, the transformation may be expressed as the product of the separate transformations.

$$C_b^n = C_b^{nT} = C_3 C_2 C_1$$

In order to do the transformation from one reference frame to another :

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}^n = C_b^n \begin{bmatrix} x \\ y \\ z \end{bmatrix}^b$$

For small rotation angles, next approximations are valid: $\sin(\psi) \to \psi, sin(\theta) \to \theta$ and $sin(\phi) \to \phi$. This is going to be our case because changes in attitude between two consecutive updates will be small. Then DCM can be expressed as a skew symmetric matrix as follows:

$$C_b^n = \begin{bmatrix} 1 & -\psi & \Theta \\ \psi & 1 & -\phi) \\ -\Theta & \phi) & 1 \end{bmatrix} \tag{4.1}$$

Some properties:

$$Det(C_b^n) = 1$$
$$(C_b^n)^{-1} = (C_b^n)^T = C_n^b$$

4.6.2.2 e-frame to n-frame transformation

We have seen how to do the transformation from b-frame to a local reference frame. Here is presented a second transformation in order to transform from the local reference frame to an Earth-centered, Earth-fixed coordinate system. In this coordinates system, the x-axis is pointing out of the intersection of the prime meridian and the equator; the z-axis points along the north pole and the y-axis completes right-hand coordinate system. The locally level frame is a cartesian coordinate system with x-y-z axes pointing along NED.

It is a 3x3 direction cosine matrix providing the transformation from the Earth frame to the local-level frame

$$\begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} = C_e^n \begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} \tag{4.2}$$

$$C_e^n = C_1 C_2 = \begin{bmatrix} \cos(\delta) & \sin(\delta) & 0 \\ -\sin(\delta) & \cos(\delta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\lambda) & 0 & -\sin(\lambda) \\ 0 & 1 & 0 \\ \sin(\lambda) & 0 & \cos(\lambda) \end{bmatrix} = \tag{4.3}$$

$$\begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} = \begin{bmatrix} \cos(\delta)\cos(\lambda) & \cos(\delta)\sin(\lambda) & \sin(\delta) \\ -\sin(\delta) & \cos(\lambda) & 0 \\ \sin(\delta)\cos(\lambda) & -\sin(\delta)\sin(\lambda) & \cos(\delta) \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} \tag{4.4}$$

4.7 Sensors

In this section, the sensors and their technologies used in the INS platform are described. Attitude of an object may be sensed by different technologies, usually gyros are used in conjunction with magnetometers (magnetocompass) to correct gyro's drift. Also a way to calculate the yaw angle from the magnetometers will be shown. Then accelerometers are presented and finally some remarks about GPS are shown, like the NMEA protocol.

4.7.1 Gyroscopes

Gyros measure the rotation speed or rate without any external reference. This rotation is mainly measured in degrees per second ($°/sec$). A single gyro measures rotation on a single plane, but a triad gyro monitors the three possible rotations in 3-D space. There are many types of gyros available, ranging in price and stability. Gyros can be gimbaled or strapdown, where gimbaled gyros maintain a fixed orientation in an inertial frame and strapdown gyros measure rotation on a fixed plane with respect to the vehicle. Therefore, the strapdown gyros in the heading axis do not only sense the entire rotation in heading, but also sense rotations in pitch and roll. In table 4.1 is summarized the main gyro technologies available in the market. One thing to remark about the gyroscopes is the advantage above magnetic compasses measuring the attitude because they are immune to ferromagnetic interferences that affect compasses.

Gyroscope				
Physical effect used	Conservation of angular momentum	Coriollis effect	Sagnac effect	Gyroscopic precession
Sensor implementation methods	Angular displacement Torque rebalance	Vibration Rotation	Ring laser Fiberoptic	Angular displacement Torque rebalance

TABLE 4.1: Gyroscopes

Here the MEMS[3] gyros are described. The most common are the vibratory coriolis gyroscopes using the Electrostatic or piezoelectric forcing. Rotational Vibratory Coriolis Gyroscopes use a momentum wheel coupled to a torsion spring and driven by a rotational electrostatic "comb drive" at resonance to create sinusoidal angular momentum in the wheel. When the device is turned about any axis, the coriolis effect will introduce sinusoidal tilting. This tilting is then sensed by capacitor sensors and the electronics extract the angular rate measurements.

The fiber optic and laser gyros use the Sagnac Effect. This sensor works by measuring the difference in transit time between two light waves passing through the same optical path but in opposite directions.

The most important parameters in a gyro are the stability and accuracy. In Table 4.2 is presented a relation between the cost and stability.

Gyro type	Principle of operation	Cost(€)	Stability ($^\circ/h$)
Rotating	Conservation angular momentum	10-1000	1-100+
Fiberoptic	Sagnac Effect	50-1000	5-100+
Vibrating Piezoelectric	Coriolis Effect	10-200	50-100+

Table 4.2: Comparison of gyro technologies

4.7.2 Accelerometers

An accelerometer measures the acceleration it experiences. The measurement is typically expressed in IS units $meters/second^2$ (m/s^2) or in terms of g-force. In practice, to find the acceleration of an object with respect to the Earth, such as for use in an inertial navigation system, the correction due to gravity along the vertical axis must be done.

Modern accelerometers are often MEMS and they operate by measuring the displacement of a small mass, called proof or seismic mass, constrained in an accelerating case. This constraining device returns a signal proportional to the displacement of the proof mass. A good alternative of using seismic mass is by integrating piezoresistors in the springs to detect spring deformation. Many MEMS accelerometers are Open-Loop,

[3]MEMS: Microelectromechanical systems

in the sense that no feedback control is used on the proof mass. If using a spring, the acceleration can be described by:

$$F = ma = m(\frac{\partial^2 x}{t^2}) + c(\frac{\partial x}{t}) + Kx$$

where F is the applied force, m is the mass of the proof mass, c is the damping coefficient, K is the spring stiffness and x is the displacement of the spring relative to rest position.

The available accelerometers in the market can sense a single axis or multiple axis and it is possible to find a wide variety on range measurement(min. and max. detectable acceleration) and sensitivity. Accelerometers are typically divided into two classes:

- Guidance accelerometers: For measure accelerations of rigid bodies, for example a car.
- Vibratory or seismic accelerometers: To measure sinusoidal accelerations, for example earthquake, structural testing.

The accelerometer can be used to detect impacts (air bag), inclination through the Earths gravity. In our case we want to measure the acceleration, which can be integrated to obtain the velocity, and integrated again to obtain the travelled distance. As it will be shown in next chapters, due to the sensor drift it requires zero velocity updates (ZUPTs) for periods without an external reference to try to bound the errors.

4.7.3 Magnetometers

This sensor senses the Earth magnetic field in 3-axis. With this vector information is then possible to obtain the yaw angle. Heading or yaw angle is the most important attitude parameter in navigation because its influence on dead-reckoning. Since yaw angle is pointing the direction of the displacement, errors in the yaw angle become in important position errors. For this reason, sensors which provide a measure of heading are extremely important in navigation. The intensity of a magnetic field is measured in Gauss (G).

The strength of the Earth's magnetic field is around 0.5 to 0.6 Gauss, and it depends on where one is sensing the field. Also there is a difference between true North and magnetic north, known as declination. The declination is not a fixed parameter, it varies with time and geographical location. While the true North is at the earth rotational axis, magnetic north may not. At some locations, the magnetic north and true north can differ by $\pm 25°$, so a correction must be made in the form of declination. Another drawback in magnetic compass is the hostile magnetic environment that can be found during navigation, specially inside buildings.

There are several types of magnetometers:

- Rotating coil magnetometer
- Hall effect magnetometer
- Proton precession magnetometer
- Fluxgate magnetometer
- Overhauser magnetometer
- Spin-exchange-relaxation-free (SERF) atomic magnetometers
- SQUID magnetometer

Disturbances induced by cars or the magnetic field generated by high voltage lines can create big disturbances on the local magnetic field. These disturbances are usually identified by checking that total measured field strength does not exceed a threshold.

The azimuth (heading) can be calculated by using the X and Y components in a horizontal plane, but usually sensors are not levelled, and the attitude of the system may be changing. Therefore, it is more difficult to determine the azimuth since the compass is not always horizontal to the surface. The error introduced by these tilt angles can be large. A way to correct this is by using an inclinometer, to determine the roll and pitch angles. Then the magnetic readings can be transformed back to the horizontal plane (XH, YH) by applying the rotational equations shown below. A method to calculate the heading with inclination information is:

Algorithm 1 Magnetometer

$xh = x * \cos(pitch) - y * \sin(pitch) * \sin(roll) - z * \sin(pitch) * \cos(roll)$

$yh = y * \cos(roll) + z * \sin(roll)$

if $(xh == 0 \; \&\& \; yh < 0)$ **then**

  Yaw $= \pi/2$

end if

if $(xh == 0 \; \&\& \; yh > 0)$ **then**

  Yaw $= 3 * \pi/2$

end if

if $(xh < 0)$ **then**

  Yaw $= \pi - \arctan(yh/xh)$

end if

if $(xh > 0 \; \&\& \; yh < 0)$ **then**

  Yaw $= -\arctan(yh/xh)$

end if

if $(xh > 0 \; \&\& \; yh > 0)$ **then**

  Yaw $= 2 * \pi - \arctan(yh/xh)$

end if

4.7.4 GPS

The Global Positioning System, GPS, is a satellite navigation system of the USA's DoD[4]. The system has three parts: 24 to 32 satellites, 4 control and monitoring stations on Earth, and the GPS receiver. The satellites are sorted in 6 orbital plans, tilted 55° with respect equatorial plane.

These satellites send navigation signals and provide a global navigation system. A device on the Earth's surface needs at least *vision* to 4 satellites, then the user will be able to determine the unknown parameters: x, y, z and time. The system is based on the synchronization and precision of the clocks and the position of each satellite.

In order for a user to determine its position, satellites send (each one using a different frequency) information in the ephemerides[5]. These ephemeris data have the health and

[4] DoD: Department of Defence

[5] An ephemeris: is a table of values that gives the positions of astronomical objects in the sky at a given time or times.

exact location data that GPS receivers then use (together with the signal's elapsed travel time to the receiver) to calculate their own location on Earth using trilateration.

A receiver calculates its position by precisely timing the signals transmitted by satellites. Each of the satellites continuously transmits messages including:

- Time the message was sent
- Precise orbital information(the ephemeris)
- General system health and rough orbits of all GPS satellites(almanac)

The receiver calculates the distance to each satellite by measuring the transit time of a message. The distances of different satellites is then combined with the satellites locations to obtain the receiver position.

Usually 4 satellites are required to estimate the position instead of three. This is because of the use of low cost oscillators in the receivers and also because any small clock error multiplied by the very large speed of light results in a large positional error.

The error on the GPS is a combination of noise, bias, and blunders:

- Noise errors are the combined effect of code noise (around 1 meter) and noise within the receiver noise (around 1 meter).
- Other bias error sources include clock errors and errors due to atmospheric effects.
- Blunders can result in errors of hundreds of kilometers and can be caused by control segment mistakes, and human mistakes.

Atmosphere conditions also influence in the accuracy due to the ionosphere and troposphere refract the GPS signals. This affects on speed of the GPS signal to differ from the speed of light in free space. Therefore, the distance calculated from "Signal Speed x Time" will be different.

4.7.4.1 NMEA protocol

All standard GPS hardware communications use the NMEA-0183[6] standard for marine electronic devices. Most GPS hardware also supports a variety of additional proprietary

[6]NMEA: National Marine Electronics Association

communication protocols, but NMEA-0183 is the standard that is supported by most software libraries. The NMEA-0183 standard specifies that communication between devices takes place through a standard serial link running at 4800 bps. The NMEA-0183 protocol consists of ASCII "sentences" which are sent repeatedly by the GPS receiver. These sentences always start with the character $ and end with a carriage return/newline (CRNL) sequence. The format of the sentence is:

${talker id}{sentence id},{comma separated list of fields...}{optional checksum}

The talker id is a two-letter code that indicates the type of device sending the message. This will always be "GP" when reading data from a GPS receiver. The sentence id is a three-letter code that indicates the type of information being sent and the format of the following data fields. Some of the sentences types are:

- GGA
- GLL
- RMC

which are explained using examples:
$GPRMC,225446,A,4916.45,N,12311.12,W,000.5,054.7,191194,020.3,E*68

225446	Time of fix 22:54:46 UTC
A	Navigation receiver warning A = OK, V = warning
4916.45,N	Latitude 49 deg. 16.45 min North
12311.12,W	Longitude 123 deg. 11.12 min West
000.5	Speed over ground, Knots
054.7	Course Made Good, True
191194	Date of fix 19 November 1994
020.3,E	Magnetic variation 20.3 deg East

$GPGGA,123519,4807.038,N,01131.324,E,1,08,0.9,545.4,M,46.9,M, , *42

123519	Fix taken at 12:35:19 UTC
4807.038,N	Latitude 48 deg 07.038' N
01131.324,E	Longitude 11 deg 31.324' E
1	Fix quality: 0 = invalid
	1 = GPS fix
	2 = DGPS fix
08	Number of satellites being tracked
0.9	Horizontal dilution of position
545.4,M	Altitude, Metres, above mean sea level
46.9,M	Height of geoid (mean sea level) above WGS84 ellipsoid
(empty field)	time in seconds since last DGPS update
(empty field)	DGPS station ID number

4.8 Principles of inertial navigation systems

In this section is presented the equations for a standalone Inertial Navigation System. Attention is focused on the process and combination of the raw data acquired from the accelerometers and gyroscopes. It is analysed all the steps in order to transform this raw data to navigation coordinates. It is also detailed how to update the attitude and position of the object. At the end of this section one will be able to built a system where its outputs are the position, velocity and attitude.

4.8.1 INS mechanization

It is assumed that the system has three gyroscopes and three accelerometers each of them sensing one axis with all the sensors rigidly attached to the body.

The body attitude is calculated by integrating the angular rate. Then, this information is used to change the reference frame of the measured acceleration as drawn in the Figure 4.9. A gravity model will be needed to provide an estimation of the gravity components in the reference frame. This gravity estimation is merged together with the sensed accelerations to determine the true accelerations. Finally this accelerations are double integrated to obtain the body velocity and position.

With all the basic steps in mind we will proceed to develop the equations.

4.8.1.1 General analysis

Imagine a body on a non-accelerating and non-rotating reference frame. In this situation the measured accelerations can be double integrated to obtain the velocity and position.

$$\mathbf{a_i} = \left.\frac{\partial^2 \mathbf{r}}{\partial t^2}\right|_i \tag{4.5}$$

with a being the sensed acceleration r distance and the subindex $_i$ indicating the reference frame of the parameter.
Now one should compensate the gravity in order to obtain the real acceleration of the body.

$$\mathbf{f} = \left.\frac{\partial^2 \mathbf{r}}{\partial t^2}\right|_i - \mathbf{g} \tag{4.6}$$

where g represents the gravity vector and f is the real force or acceleration applied to the body. One can rewrite 4.6 as:

$$\left.\frac{\partial^2 \mathbf{r}}{\partial t^2}\right|_i = \mathbf{f} + \mathbf{g} \tag{4.7}$$

The Equation 4.7 is the navigation equation because is possible to first obtain the velocity and with the second integration we obtain the position in that frame.

$$\mathbf{v}_i = \left.\frac{\partial \mathbf{r}}{\partial t}\right|_i \tag{4.8}$$

4.8.1.2 Navigation with respect a rotating frame

Usually, the system will not fit on the assumptions made on the last equations. In practice the position and velocity estimations will be with respect of a rotating reference frame, like the Earth. Therefore, some modifications on the navigation equations must be done. v_e is the ground speed of the body, and it may be expressed using the Coriolis theorem as:

$$\mathbf{v}_e = \left.\frac{\partial \mathbf{r}}{\partial t}\right|_e = \mathbf{v}_i - \omega_{ie} \times \mathbf{r} \tag{4.9}$$

with $\omega_{ie} = \begin{bmatrix} 0 & 0 & \Omega \end{bmatrix}^T$ being the Earth turn rate with respect of the i-frame.

The accelerometers provide the data in the body frame, $\mathbf{f^b}$. We have to resolve the accelerations components in the chosen inertial reference frame.

$$\mathbf{f^i} = \mathbf{C}_b^i \mathbf{f}^b \tag{4.10}$$

with $\mathbf{C}_b^i$ the 3x3 matrix defining the attitude of the body frame with respect to the i-frame. As seen before, $\mathbf{C}_b^i$ is the direction cosine matrix. Once the DCM is initialized, it is possible to update it from the angular rate measurements using:

$$\dot{\mathbf{C}}_b^i = \mathbf{C}_b^i \mathbf{\Omega^i}_b \tag{4.11}$$

with $\mathbf{\Omega}_b^i$ being the skew matrix:

$$\mathbf{\Omega}_b^i = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \tag{4.12}$$

This skew matrix is built from the vector $\mathbf{\Omega}_{ib}^b = \begin{bmatrix} p & q & r \end{bmatrix}^T$,this vector is the turn rate of the body with respect to the i-frame. p,q, r are the turn rates measured by the gyros.

Reviewing all the equations we have and combining them, the navigation equation could be expressed in i-frame coordinates as:

$$\left.\frac{\partial^2 \mathbf{r}}{\partial t^2}\right|_i = \mathbf{f}^i + \mathbf{g}^i = \mathbf{C}_b^i \mathbf{f}^b + \mathbf{g}^i \tag{4.13}$$

4.8.1.3 Inertial Navigation Equations

We have seen the navigation equations to work on the inertial reference frame and how to compensate the gravity and the Earth rate. The next figure presents the complete strapdown inertial navigation system operating on the local geographic reference frame:

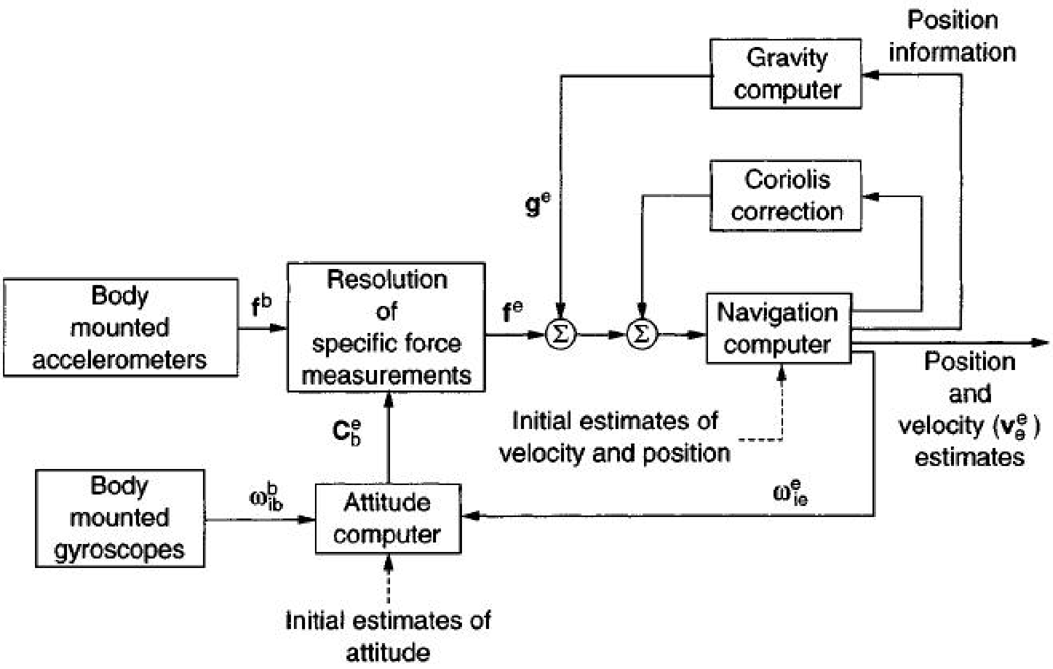


FIGURE 4.9: Strapdown inerital navigation sytem

Our system must be able to navigate all around the world, where the position is usually needed in terms of latitude and longitude:

$$\left.\frac{\partial \mathbf{v}_e}{\partial t}\right|_n = \left.\frac{\partial \mathbf{v}_e}{\partial t}\right|_i - [\omega_{ie} + \omega_{en}] \times \mathbf{v}_e \tag{4.14}$$

with:

$$\left.\frac{\partial \mathbf{v}_e}{\partial t}\right|_i = \mathbf{f} - \omega_{ie} \times \mathbf{v}_e + \mathbf{g} \tag{4.15}$$

then:

$$\left.\frac{\partial \mathbf{v}_e}{\partial t}\right|_n = \mathbf{f} - [2\omega_{ie} + \omega_{en}] \times \mathbf{v}_e + \mathbf{g} \tag{4.16}$$

expressed in navigation axes:

$$\dot{\mathbf{v}}_e^n = \mathbf{f}^n - (2\omega_{ie}^n + \omega_{en}^n) \times \mathbf{v}_e^n + \mathbf{g}^n \tag{4.17}$$

with $\mathbf{v}_e^n$ being the velocity with respect to the Earth expressed in the local geographic frame.

$$\mathbf{v}_e^n = \begin{bmatrix} v_N & v_E & v_D \end{bmatrix}^T \tag{4.18}$$

$\mathbf{f^n}$ is force vector measured by the accelerometers and transformed to the local geographic frame:

$$\mathbf{f}^n = \begin{bmatrix} f_N & f_E & f_D \end{bmatrix}^T \tag{4.19}$$

ω_{ie}^n is the Earth turn rate also in the local geographic reference frame.

$$\omega_{ie}^n = \begin{bmatrix} \Omega\cos(L) & 0 & -\Omega\sin(L) \end{bmatrix}^T \tag{4.20}$$

ω_{en}^n is the turn rate of the local geographic frame with respect to the Earth-fixed frame. It can be expressed in terms of change of latitude and longitude as:

$$\omega_{en}^n = \begin{bmatrix} \dot{l}\cos(L) & -\dot{L} & -\dot{l}\sin(L \end{bmatrix}^T \tag{4.21}$$

with $\dot{l} = v_E/(R_0 + h)cosL$ and $\dot{L} = v_N/(R_0 + h)$ then 4.21 can be rewritten as:

$$\omega_{en}^n = \begin{bmatrix} \frac{v_E}{R_0+h} & -\frac{v_N}{R_0+h} & -\frac{v_E\tan(L)}{R_0+h} \end{bmatrix}^T \tag{4.22}$$

where R_0 is the Earth radius and h the height above the Earth's surface.

Referencing to the initial navigation equation seen in 4.17 its last parameter is the local gravity vector $\mathbf{g}_l^n$. In this vector is combined the effects of the mass attraction $\mathbf{g}$ and the centripetal acceleration cause by the Earth's rotation $\omega_{ie} \times \omega_{ie} \times \mathbf{R}$.

$$\mathbf{g}_l^n = \mathbf{g} - \omega_{ie} \times \omega_{ie} \times \mathbf{R} = \mathbf{g} - \frac{\Omega^2(R_0 + h)}{2} \begin{pmatrix} \sin(2L) \\ 0 \\ 1 + \cos(2L) \end{pmatrix} \tag{4.23}$$

With all the parameters one can rewrite 4.17 in a component form as:

$$\begin{aligned} \dot{v}_N &= f_N - v_E(2\Omega + \dot{l})sin(L) + v_D\dot{L} + g_x = \\ &= f_N - 2\Omega v_E sin(L) + \frac{v_N v_D - v_E^2 tan(L)}{R_0 + h} + g_N \end{aligned} \tag{4.24}$$

$$\begin{aligned}\dot{v}_E &= f_E + v_N(2\Omega + \dot{l})sin(L) + \dot{v}_D(2\Omega + \dot{L})cos(L) - g_E = \\ &= f_E + 2\Omega(v_N sin(L) + v_D cos(L) + \frac{v_E}{R_0 + h}(v_D + v_N tan(L)) - g_E\end{aligned} \tag{4.25}$$

$$\begin{aligned}\dot{v}_D &= f_D - v_E(2\Omega + \dot{l})cos(L) - v_N\dot{L} + g = \\ &= f_D - 2\Omega v_E cos(L) - \frac{v_E^2 + v_N^2}{R_0 + h} + g\end{aligned} \tag{4.26}$$

Finally, assuming the Earth is spherical the latitude, longitude and height on the Earth's surface are given by:

$$\dot{L} = \frac{v_N}{R_0 + h} \tag{4.27}$$

$$\dot{l} = \frac{v_E \sec(L)}{R_0 + h} \tag{4.28}$$

$$\dot{h} = -v_D \tag{4.29}$$

4.8.2 Modelling the Earth

Here it is rapidly described the way to model the Earth. The reader may find plenty of literature about it. The spherical model used for the last equations is far from efficient, a better approach is the Ellipsoid model. Here there are the main parameters:

R,	length of the semi-major axis
r=R(1-f)	length of the semi-minor axis
f=(R-r)/R	flattening of the ellipsoid
$e = [f(2-f)]^{1/2}$	major eccentricity

the model commonly used is the WGS-84 Model, which is the model used in the GPS system, this model has the parameters set at:

R =	6378137m
r =	6356752.3145m
e =	0.0818191908426
Ω=	$7.292115 \times 10^{-5} rad/s = 15.041067°/h$

then is possible to define a meridian radius R_N and a transverse radius R_E:

$$R_N = \frac{R(1-e^2)}{(1 - e^2 sin(L)^2)^{3/2}} \tag{4.30}$$

$$R_E = \frac{R}{(1 - e^2 sin(L)^2)^{1/2}} \tag{4.31}$$

then is possible to rewrite 4.32 using the new ellipsoid model as:

$$\dot{L} = \frac{v_N}{R_N + h} \tag{4.32}$$

$$\dot{l} = \frac{v_E \sec(L)}{R_N + h} \tag{4.33}$$

$$\dot{h} = -v_D \tag{4.34}$$

4.9 Attitude computation

The common approach of determining the attitude is using the direction cosine matrix, DCM, to relate the body frame to the coordinate frame. The main problem of computing each time a DCM is that one usually have to update 100 times per second the matrix, this becomes in many operations are needed. Therefore an alternative formulation made by Bortz [1] is used. Here is presented how to update a direction cosine matrix.

$$\mathbf{C}_{l+1} = \mathbf{B}_l \mathbf{C}_l \tag{4.35}$$

where $\mathbf{B_l}$ is the DCM relating the the navigation axes from time t_l to t_{l+1}. B_l is expressed as:

$$\mathbf{B}_l = \mathbf{I} + \frac{sin(\sigma)}{\sigma}[\sigma x] + \frac{1 - cos(\sigma)}{\sigma^2}[\sigma x]^2 \tag{4.36}$$

where σ is the rotation vector magnitude and $[\sigma x]$ is the skew matrix of the rotating vector.

4.10 INS errors

In a practical implementation, the accuracy of an inertial navigation system is hardly limited because of the noisy data of its sensors. The sources of errors may be classified as:

- initial alignment errors, errors in the initial position, speed and attitude information.
- inertial sensor errors, refers to the noise on sensors 4.10
- computational errors, refers to approximations on the equations and also to some decimal truncations.

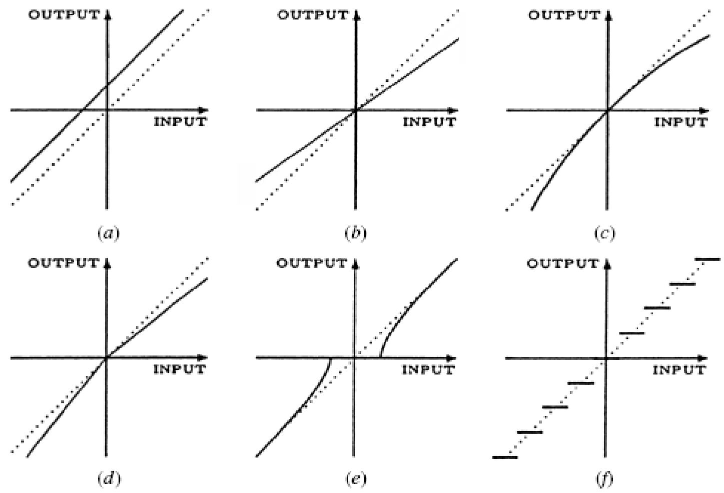


FIGURE 4.10: Errors in sensors

a: bias; b: scale Factor; c: nonlinearity; d: asymmetry; e: dead zone; f: quantization.

Any measurement error when using dead reckoning is passed from one estimation to another. INS performance is characterised by a growing error of its outputs. This section presents the error equations of an INS, which later will be used on the Kalman filter to finally create the INS GPS system.

4.10.1 Attitude errors

As explained earlier in this chapter, the attitude of a body may be expressed using the DCM Section 4.6.2. Here we are trying to study the error on the body attitude, so it is logical to relate the estimated DCM $\hat{\mathbf{C}}_b^n$ and the real one $\mathbf{C}_b^n$ using the error between them:

$$\hat{\mathbf{C}}_b^n = \mathbf{B}\mathbf{C}_b^n \tag{4.37}$$

The **B** matrix is the transformation from the true DCM to the estimated DCM. If the errors are small the B matrix can be approximated by a skew matrix:

$$\mathbf{B} = [\mathbf{I} - \mathbf{\Psi}] \tag{4.38}$$

with **I** the identity matrix and $\mathbf{\Psi}$ being:

$$\mathbf{\Psi} = \begin{pmatrix} 0 & -\delta\gamma & \delta\beta \\ \delta\gamma & 0 & -\delta\alpha \\ -\delta\beta & \delta\alpha & 0 \end{pmatrix} \tag{4.39}$$

Operating with the above presented equations one arrives to 4.41

$$\hat{\mathbf{C}}_b^n = [\mathbf{I} - \mathbf{\Psi}]\mathbf{C}_b^n \tag{4.40}$$

$$\mathbf{\Psi} = \mathbf{I} - \hat{\mathbf{C}}_b^n \mathbf{C}_b^{nT} \tag{4.41}$$

First differentiating the equation and a later development [2] one arrives to express Ψ as follows:

$$\mathbf{\Psi} = -\omega_{in}^n \times \mathbf{\Psi} + \delta\omega_{in}^n - \mathbf{C}_b^n \delta\omega_i b^b \tag{4.42}$$

where $\dot{\mathbf{\Psi}} = \begin{bmatrix} \delta\alpha & \delta\beta & \delta\gamma \end{bmatrix}^T$

$$\mathbf{\Psi}\times = \mathbf{\Psi} \qquad \omega_{in}^n \times = \mathbf{\Omega}_{in}^n \qquad \delta\omega_{in}^n \times = \delta\mathbf{\Omega}_{in}^n \qquad \delta\omega_{ib}^b \times = \delta\mathbf{\Omega}_{ib}^b$$

4.10.2 Velocity and position errors

The velocity and the position error equations will be of high interest on the preceding chapter. In the same way as the preceding section, only the most characteristic equations are presented. As can be observed in 4.17, the velocity may be expressed as:

$$\dot{\mathbf{v}} = C_b^n f^b - (2\omega_{ie}^n + \omega_{en}^n) \times \mathbf{v} + g_l \tag{4.43}$$

in the same way the estimated velocity can be expressed as:

$$\dot{\hat{\mathbf{v}}} = \hat{C}_b^n \hat{f}^b - (2\hat{\omega}_{ie}^n + \hat{\omega}_{en}^n) \times \hat{\mathbf{v}} + \hat{g}_l \tag{4.44}$$

Differencing these equations we have:

$$\begin{aligned} \delta\dot{\mathbf{v}} &= \dot{\hat{v}} - \dot{v} \\ &= \hat{\mathbf{C}}_b^n \hat{\mathbf{f}}^b - \mathbf{C}_b^n \mathbf{f}^b - (2\hat{\omega}_{ie}^n + \hat{\omega}_{en}^n) \times \hat{v} + (2\omega_{ie}^n + \omega_{en}^n) \times v + \hat{g}_l - g_l \end{aligned} \tag{4.45}$$

ignoring the error on the Coriolis and the gravity terms, and later perform the following substitutions:

$$\hat{C}_b^n = [\mathbf{I} - \mathbf{\Psi}]\mathbf{C}_b^n \tag{4.46}$$

$$\hat{f}^b - f_b = \delta f_b \tag{4.47}$$

$$\hat{v} - v = \delta v \tag{4.48}$$

one arrives to:

$$\delta\dot{v} = [f^n \times]\, \Psi + C_b^n \delta f^b \tag{4.49}$$

Finally the position error $\delta\dot{p}$ is expressed as:

$$\delta\dot{p} = \delta v \tag{4.50}$$

Chapter 5

Kalman Filter

5.1 Kalman Filter

Kalman filter is an optimal recursive data processing algorithm which incorporates all the available information, process this measurements to finally estimate the unknown variables. For example, imagine we want to determine the speed of a plane, and we can measure its speed from several different ways, using a radar, an INS inside the plane and the relative wind speed. How can one determine the plane's speed? The best way is to use all the available information to built a Kalman filter to combine all the data to generate the best estimation.

A Kalman filter does not require all previous data in order to operate(just last estimation), every time a new measurement is taken is processed with no need to keep or save the previous measurements. This is really important in our system because must be on real time.

5.1.1 Kalman Filter Basics

The Kalman filter was designed to estimate a discrete-time process. The desired variables are hold in the so called state vector x:

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & ... & x_n \end{bmatrix}^T \tag{5.1}$$

All the measurements are grouped in the vector z:

$$\mathbf{z} = \begin{bmatrix} z_1 & z_2 & z_3 & ... & z_l \end{bmatrix}^T \tag{5.2}$$

Notice that the state vector and measurements vector do not have to be on the same dimension. State vector is an array of n values and the measurement vector of l. In the following lines is shown how these two arrays are related.

Kalman filter was designed to estimate a process governed by the linear stochastic difference equation:

$$\mathbf{x}_k = \mathbf{\Phi}\mathbf{x}_{k-1} + \mathbf{B}\mathbf{u}_{k-1} + \mathbf{w}_{k-1} \tag{5.3}$$

where $\mathbf{x}_k$ is the state array, $\mathbf{x}_{k-1}$ the previous process state and $\mathbf{z}_k$ is the measurement. With $\mathbf{z}_k$ expressed in the form:

$$\mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k \tag{5.4}$$

The w_k is the process noise and v_k is the measurement noise which is assumed they are independent, white and with normal probability distribution. $\mathbf{H}$ is a $m \times n$ matrix relating the state to the measurement z_k.

$$p(w) \sim N(0, \mathbf{Q}) \tag{5.5}$$

$$p(v) \sim N(0, \mathbf{R}) \tag{5.6}$$

where $\mathbf{Q}$ is the process noise covariance matrix and $\mathbf{R}$ the measurement noise covariance matrix.

$$\mathbf{R} = E\{v_k v_k'\} = diag\begin{pmatrix} \sigma_{z_1}^2 & \sigma_{z_2}^2 & \sigma_{z_3}^2 & ... & \sigma_{z_l}^2 \end{pmatrix} \tag{5.7}$$

$$\mathbf{Q} = E\{w_k w_k'\} = diag\begin{pmatrix} \sigma_{x_1}^2 & \sigma_{x_2}^2 & \sigma_{x_3}^2 & ... & \sigma_{x_n}^2 \end{pmatrix} \tag{5.8}$$

The kalman filter tries to find the lineal estimator without bias, with the minimum variance error and without a prior knowledge of the state to be estimated. Using the equation 5.4 is possible to perform this estimation.

$$\mathbf{x}_k = \mathbf{\Phi}\mathbf{x}_{k-1} + \mathbf{B}\mathbf{u}_{k-1} + \mathbf{w}_{k-1}$$

Φ is a $n \times n$ matrix which relates the state at the previous time step to the state at the current step, in the absence of either a driving function or process noise. $\mathbf{\Phi}$ is the solution of the difference equation 5.4.

And the matrix $\mathbf{B}$ with dimension $n \times l$ relates the optional control input u to the state x. In our case, the $\mathbf{B}$ and u vector are not used, so we can rewrite 5.4 as:

$$\mathbf{x}_k = \mathbf{\Phi}\mathbf{x}_{k-1} + \mathbf{w}_{k-1} \tag{5.9}$$

The first step for the optimal estimation is to solve 5.9. We are going to change to the continuous time in order to solve it:

$$\frac{\partial}{\partial t}x(t) = \mathbf{F}(t)\mathbf{x}(t) + \mathbf{w}(t) \tag{5.10}$$

with

$$\Phi(t,\tau) = e^{F(t-\tau)} = \sum_{i=0}^{\infty} \frac{(t-\tau)^i}{i!} F^i \tag{5.11}$$

$$\Phi_{k-1} = exp\left(\int_{t_{k\_1}}^{t_k} \mathbf{F}(s)ds\right) \tag{5.12}$$

The matrix Φ_{k-1} is the discrete-time model of state transition matrix for the dynamic system defined by $\mathbf{F}$. Doing F time-invariant and taking a first order linearisation:

$$\Phi = I + \Delta t\mathbf{F} \tag{5.13}$$

Now following the development the covariance noise matrix $\mathbf{Q}$ may be expressed like:

$$\mathbf{Q}_k = E\{w_k w_k'\} = \Phi_k Q \Phi_k' \Delta t \tag{5.14}$$

Since now it has been presented the basis of the Kalman Filter. Our goal is to find the minimum square error estimator. Following the development [3] in the following lines is presented the Kalman algorithm.

The Kalman filter has two different phases: Predict and Update. The predict phase uses the state estimate from the previous time step to produce an estimation of the state at the current time step. In the update phase, measurement information at the current time step is used to refine this prediction to reach at a new and more accurate estimation state, again for the current times step.

Predict:

$$\text{Predicted state: } \hat{\mathbf{x}}(-)_k = \mathbf{\Phi}_k \hat{\mathbf{x}}_{k-1}(+) \tag{5.15}$$

$$\text{Predicted estimate covariance: } \mathbf{P}_k(-) = \mathbf{\Phi}_k \mathbf{P}_{k-1}(+) \mathbf{\Phi}_k^{\mathrm{T}} + \mathbf{Q}_{k-1} \tag{5.16}$$

Update:

$$\text{Kalman gain: } \mathbf{K}_k = \mathbf{P}_k(-)\mathbf{H}_k^{\mathrm{T}} \left(\mathbf{H}_k \mathbf{P}_k(-) \mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k\right)^{-1} \tag{5.17}$$

$$\text{Updated state estimate: } \hat{\mathbf{x}}_k(+) = \hat{\mathbf{x}}_k(-) + \mathbf{K}_k \left(\mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_k(-)\right) \tag{5.18}$$

$$\text{Updated estimate covariance:} \mathbf{P}_k(+) = \mathbf{P}_k(-) - \mathbf{K}_k \mathbf{H}_k \mathbf{P}_k(-) \tag{5.19}$$

The difference $(\mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_k(-))$ is called the measurement innovation. It reflects the discrepancy between the predicted measurement$\mathbf{H}_k \hat{\mathbf{x}}_k(-)$ and the actual measurement $\mathbf{z}_k$.

The $n \times m$ matrix $\mathbf{K}$ known as Kalman gain minimizes the a posteriori error covariance. The equation 5.24 is the most significant on the algorithm thanks to it is weighted the measured values and the predicted values in order to introduce the minimum error on the solution.

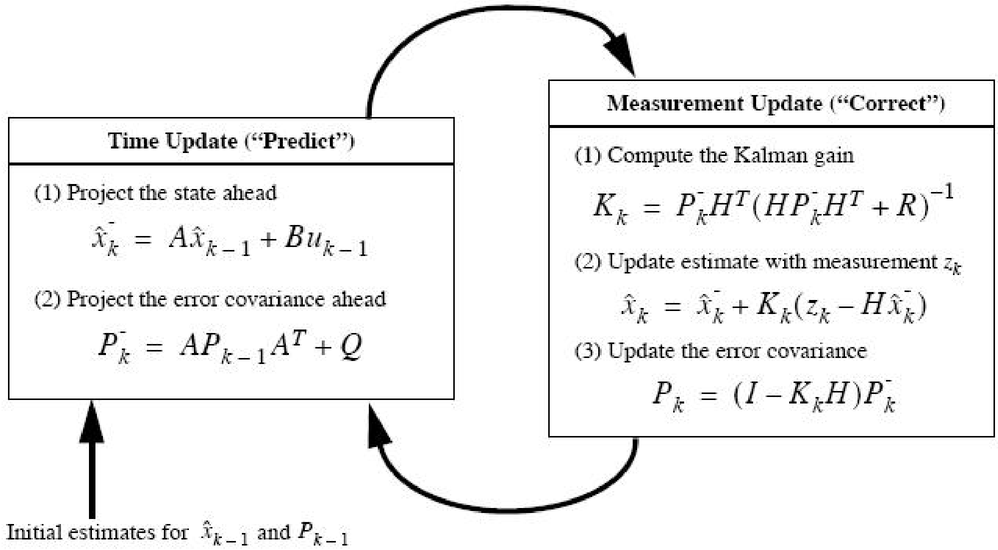


FIGURE 5.1: Kalman Filer diagram

5.1.2 Extended Kalman Filter

In last section, the presented Kalman filter was a lineal version of the filter, where only matrix operations were allowed. The measurement was a linear function of the state, and the next state also was a linear function of the previous state. Therefore, the filter can not handle rotations and projections. That filter was mainly designed for the general problem of trying to estimate the state of a discrete time controlled process governed by a linear stochastic difference equation. When this process is non-linear, appears the Extended Kalman filter (EKF). Now, the equations relating the state of the system are non-lineal.

$$x_k = f(x_{k-1}, u_k, w_{k-1}) \tag{5.20}$$

$$z_k = h(x_k, v_k) \tag{5.21}$$

as before w and v are the process and measurement noise, which we consider Gaussian and with a zero mean. Here are presented the EKF equations:

EKF time update or predict:

$$\text{Predicted state: } \hat{\mathbf{x}}(-)_k = f(\hat{x}_{k-1}, u_k) \tag{5.22}$$

$$\text{Predicted estimate covariance: } \mathbf{P}_k(-) = \mathbf{F}_k \mathbf{P}_{k-1}(+) \mathbf{F}_k^{\mathrm{T}} + \mathbf{W}_k \mathbf{Q}_{k-1} \mathbf{W}_k^T \tag{5.23}$$

EKF measurement update:

$$\text{Kalman gain: } \mathbf{K}_k = \mathbf{P}_k(-) \mathbf{H}_k^{\mathrm{T}} \left(\mathbf{H}_k \mathbf{P}_k(-) \mathbf{H}_k^{\mathrm{T}} + \mathbf{V}_k \mathbf{R}_k \mathbf{V}_k^T \right)^{-1} \tag{5.24}$$

$$\text{Updated state estimate: } \hat{\mathbf{x}}_k(+) = \hat{\mathbf{x}}_k(-) + \mathbf{K}_k \left(\mathbf{z}_k - h\left(\hat{\mathbf{x}}_k(-), 0 \right) \right) \tag{5.25}$$

$$\text{Updated estimate covariance: } \mathbf{P}_k(+) = \mathbf{P}_k(-) - \mathbf{K}_k \mathbf{H}_k \mathbf{P}_k(-) \tag{5.26}$$

with

- **F** the Jacobian matrix of partial derivatives of f respect x:

$$F = \frac{\partial f}{\partial x}(\hat{x}_{k-1}, u_k, 0)$$

- W the Jacobian matrix of partial derivatives of f respect w:

$$W = \frac{\partial f}{\partial w}(\hat{x}_k, 0)$$

- V the Jacobian matrix of partial derivatives of h with respect v:

$$V = \frac{\partial h}{\partial v}(\hat{x}, 0)$$

And here is shown the EKF diagram:

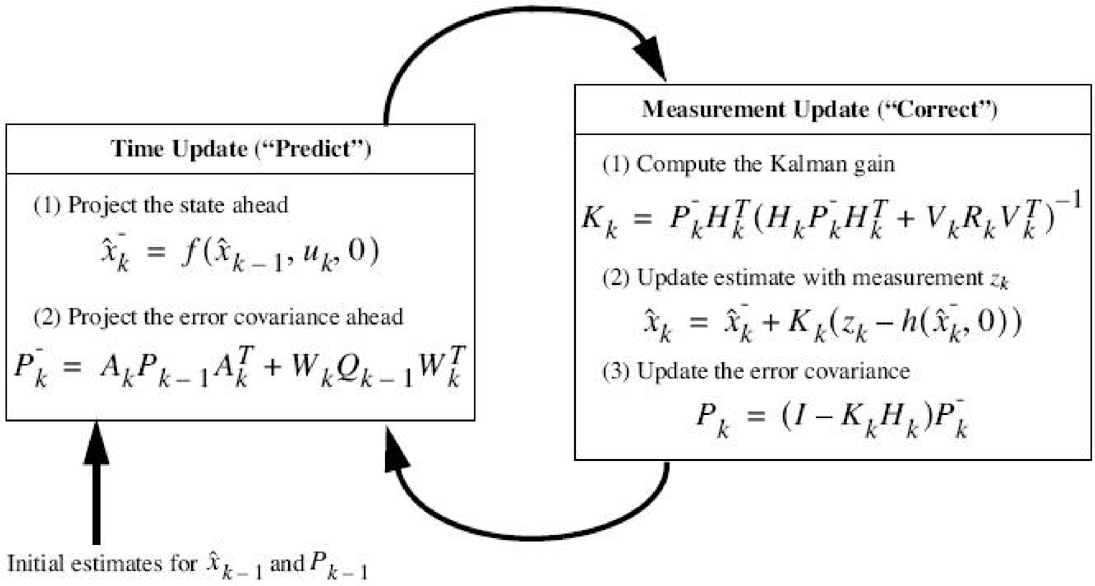


FIGURE 5.2: Extended Kalman Filer diagram

5.1.2.1 Linearising

In order to solve the presented equations in chapter 4 with a Kalman filter system, it is necessary to linearities them. This linearisation or approximation is usually made about a nominal set of states commonly refereed as a nominal trajectory $\tilde{x}$. Therefore this approximation will only be valid for short time after which the system will have to be linearised again. The technique to linearise is usually the truncation of the Taylor series:

$$f(x) = f(\tilde{x}) + \left.\frac{\partial f}{\partial t}\right|_{\tilde{x}} (x - \tilde{x}) + \left.\frac{\partial f}{\partial t^2}\right|_{\tilde{x}} \frac{(x - \tilde{x})^2}{2} + ... \quad (5.27)$$

This nominal trajectory can be defined as:

$$\tilde{x}_k = f(\tilde{x}_{k-1}, k-1) \quad (5.28)$$

$$\frac{\partial \tilde{x}}{\partial t} = f(\tilde{x})\tilde{x} \quad (5.29)$$

$$z = h(\tilde{x})\tilde{x} \quad (5.30)$$

doing:

$$\delta x_k = x_k - \tilde{x} \quad (5.31)$$

$$\delta z_k = z_k - h\left(\tilde{x}, k\right) \quad (5.32)$$

and subtracting from the original equations one arrives to:

$$\frac{\partial \delta x}{\partial t} = \left.\frac{\partial f}{\partial t}\right|_{\tilde{x}} \delta x + \left.\frac{\partial d}{\partial t}\right|_{\tilde{x}} \delta w \tag{5.33}$$

$$\partial z = \left.\frac{\partial h}{\partial t}\right|_{\tilde{x}} \delta x + v \tag{5.34}$$

We can rewrite the preceding functions as:

$$\delta x_k = \Phi \delta x_{k-1} + w_{k-1} \tag{5.35}$$

with Φ being:

$$\Phi = \frac{\partial f(x, k-1)}{\partial x} = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} \tag{5.36}$$

One can develop **H** to obtain:

$$\delta z_k = \mathbf{H} \delta x_k + v_k \tag{5.37}$$

Now, focusing again in the inertial navigation, the Kalman filter equations must be updated at each measurement interval. Each time one has to update the kalman filter due to there is a new measurement of the GPS or from the gyros or accelerometers, then the linearisation has to be done about the nominal trajectory. This trajectory is usually the latest estimation of the states (one will not know the real ones).

When the linearisation is done, next step is to calculate the transition matrix and all the other matrices of the filter. Calculate the prediction step 5.22 and obtaining the Kalman gain will let the algorithm to calculate the predict step as shown in 5.25.

5.2 INS/GPS Kalman filter equations

In lasts sections the theory about the Kalman filter in general terms has been introduced. Here we will focus on INS/GPS integration using the Kalman filter. This real implementation of the INS/GPS system may be done by several different models. Next lines give a brief description of them.

5.2.1 INS GPS models

5.2.1.1 Open loop, Closed loop

Open loop models refers to an implementation without any corrections. This means the system has just one way of operation(see Figure 5.3 and 5.4). Each part or block in the system does his job to later feed the next block, but there are no feedbacks between INS block neither GPS block. Usually, this model is used when there is no other information available than the one from INS. Here the Kalman filter may run external to the INS but an extended Kalman filter is needed.

The **closed loop** model refers to those implementations which have some corrections or some feedback between blocks. Depending on the integration degree between INS and GPS signals more specific names are used: loosely coupled, tightly coupled.

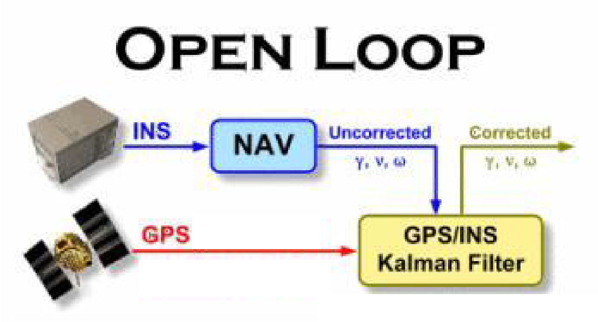


FIGURE 5.3: Open loop

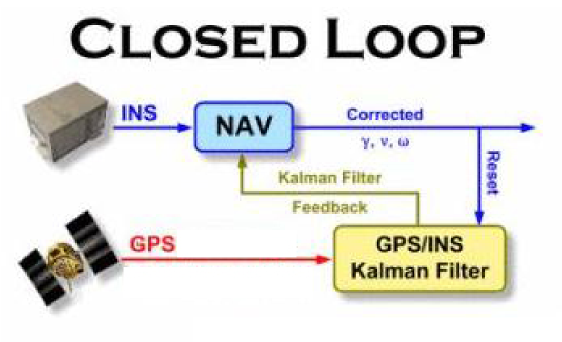


FIGURE 5.4: Cosed loop

5.2.1.2 Loosely coupled

Figure 5.6 represents a *loosely coupled* INS/GPS integration. The inertial navigation block calculates position, velocity and attitude from raw inertial sensor measurements, in parallel GPS calculates position and velocity. Then using an external Kalman filter the system output is computed. A benefit of a loosely coupled system in front of a tighter models is GPS receiver can be used as a black box which makes the Kalman filter design simpler. This Kalman filter uses the GPS position and velocity to calibrate INS errors. There are a couple ways to implemented this model: *forward* 5.5 and *feedback* 5.6.

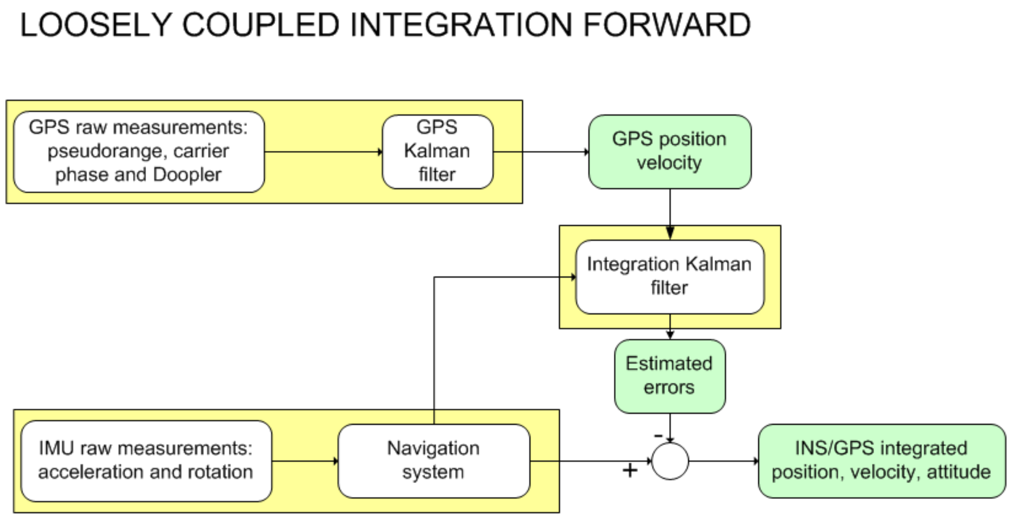


FIGURE 5.5: Loosely coupled forward

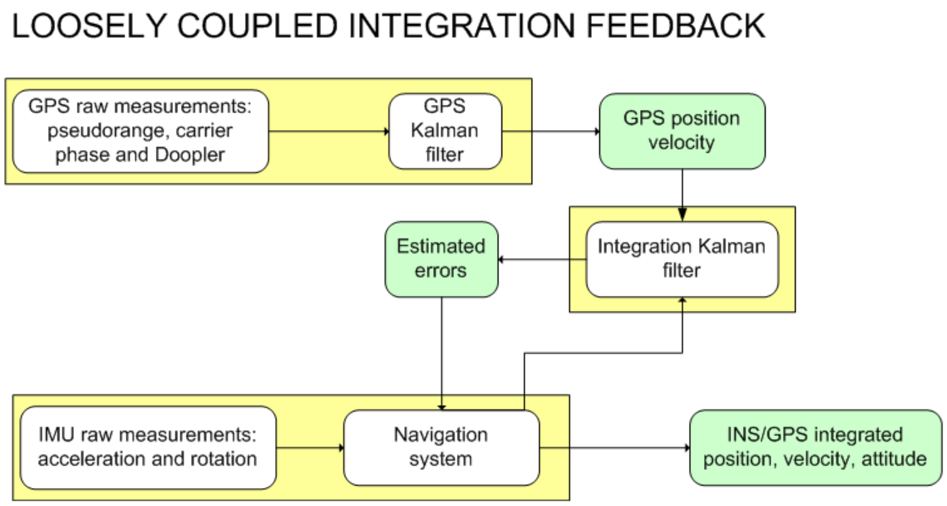


FIGURE 5.6: Loosely coupled feedback

It is important to remark in the *forward* implementation that once the GPS stops providing measurements then INS/GPS filter stops as well. One way to solve this is using the feedback model. Because in the *forward* model there is no correction on the INS block which may lead to some instabilities on the filter.

5.2.1.3 Tightly coupled

A more sophisticated but also complicated design is the *tightly coupled* integration. In this system, the Kalman filter becomes the navigation algorithm. It receives raw GPS measurements like pseudo-ranges and the raw IMU measurements to process everything together and finally give the desired position and velocity. In this integration one can use GPS measurement even if there are less than four satellites available.

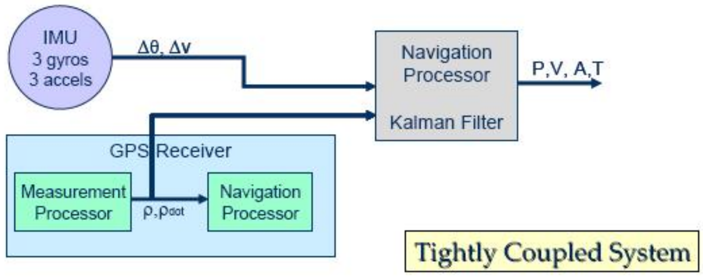


FIGURE 5.7: Tightly coupled

The tightly integration can be done on different levels. For example use INS outputs to aid GPS in faster signal acquisition or interference rejection. On the other hand there is a big difference on the complexity of this model and the others presented.

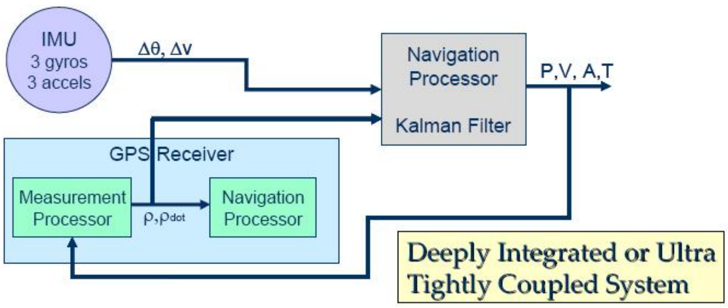


FIGURE 5.8: Tighter coupled

In next table is summarized the different models:

Implementation	Advantages	Disadvantages
Open loop	• KF may be run external to INS. • Used when only navigation solution from INS available.	• Non linear error model due to large second order effect. • Extended KF needed.
Closed loop	• Inertial system errors, linear model is enough. • Suitable for integration at software level.	• More complex processing. • Blunders in GPS may affect INS performance
Loosely coupled	• Flexible, modular combination. • Small KF, faster processing. • Suitable for parallel processing.	• Sub optimal processing. • Unrealistic covariance. • Four satellites needed for stable solution. • INS data not used for ambiguity estimation.
Tightly coupled	• One error state model. • Optimal solution. • GPS measurements can be used with less than 4 satellites. • Direct INS aiding throughout GPS outages. • Faster ambiguity estimation.	• More complex processing. • Large size of error state model.

TABLE 5.1: INS/GPS models

5.2.2 Kalman filter equations

In this project it has been implemented the *loosely feedback* model. The model has been chosen because gives us a lot flexibility. Our model may operate in three different configurations depending on the situation:

- INS/GPS: The main mode, here GPS and INS are working together helping each other to improve the position measurements.
- INS: When GPS is not available, the system may continue working just using the INS block.
- GPS: In order to save power, one may switch off the INS (also KF) and use GPS in the normal way.

Here are presented the Kalman filter equations. The state vector used is:

$$x = \begin{bmatrix} \delta x & \delta y & \delta z & \delta v_x & \delta v_y & \delta v_z & \delta\Phi & \delta\Theta & \delta\Psi & \delta\dot{\Phi} & \delta\dot{\Theta} & \delta\dot{\Psi} \end{bmatrix} \tag{5.38}$$

where: δx δy δz are the position error.
δv_x δv_y δv_z are the velocity error.
$\delta\Phi$ $\delta\Theta$ $\delta\Psi$ are the attitude errors(roll,pitch,yaw)
$\delta\dot{\Phi}$ $\delta\dot{\Theta}$ $\delta\dot{\Psi}$ are the turning rates

The F matrix is:

$$F = \begin{bmatrix} 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix} \tag{5.39}$$

then it is possible to obtain Φ as:

$$\Phi = I + \Delta t\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{5.40}$$

A linear error has been used to relate one step to the next one. A way to interpret the matrix is that the error in the velocity becomes and error in the position. In the same way, the error on the attitude comes from the error on the turning rates.

The P matrix is initialized as an identity matrix which will be updated on both steps "Predict" and "Correct".

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{5.41}$$

Due to time constrictions it has been decided to use a linear error model for the Kalman filter. The equations used in the Kalman filter are not complicated because all the calculations and transformations are done in the INS block, so the Kalman filter block is only receiving the processed measurements from both blocks, GPS and INS.

5.2.2.1 GPS update

Every time a GPS is giving a new measurement, next equations are used to update the Kalman filter states. Here it is the measurement vector z when a news GPS measurement is available:

$$z = \begin{pmatrix} x_{INS} - x_{GPS} \\ y_{INS} - y_{GPS} \\ z_{INS} - z_{GPS} \\ v_{x_{INS}} - v_{x_{GPS}} \\ v_{y_{INS}} - v_{y_{GPS}} \\ v_{z_{INS}} - v_{z_{GPS}} \end{pmatrix} \tag{5.42}$$

and the H matrix relating the measurement vector z with the state vector x:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5.43}$$

5.2.2.2 Magnetometer update

In the same way, every time there is new data from the magnetometer, the state array is update. The measurement vector z when there is a new yaw measurement:

$$z = \left(yaw_{INS} - yaw_{MAG} \right) \tag{5.44}$$

and the H matrix:

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{5.45}$$

5.2.3 Kalman filter code

The Kalman filter code used is presented as pseudocode in last section of Chapter 6.

Chapter 6

Developing the solution

In this chapter is explained the real implementation that has been done. First is described all the used components, then is presented a description of the hardware and the software part.

6.1 Overview

In the project, it has been used a board which includes a 3-axes accelerometer, 3-axes gyroscope and a 3-axes magnetometer. But due to the poor design of the board, magnetometer measurements were too noisy to be useful (see next section). For these reason an electronic compass has been used as a substituted of the magnetometers.

6.1.1 Issues

In the real implementation of the INS/GPS system it has been found some problems that must be described:

- *Temperature*: The first important problem has been the temperature on the board, in theory the board was temperature compensated, but may be due to the cage we had some temperature effects. This means that during a test, the calibration of the raw measurements in accelerometers and gyroscopes were fluctuating a lot, see Figure 6.2.

- *Magnetometers*: In theory the board was providing information about the magnetic field. The idea was to use this important information in the kalman filter in order to avoid the gyro drift.
  Here apart of the temperature problem, we have found a problem on the sensor. Due to the poor design of the IMU board, the measurements of the magnetometer were useless. As it is possible to see in Figure 6.1 one can't distinct between signal and noise.

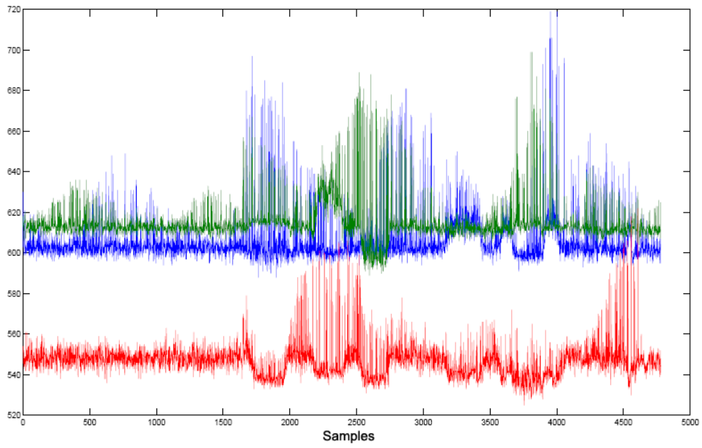


FIGURE 6.1: Noise in the magnetos

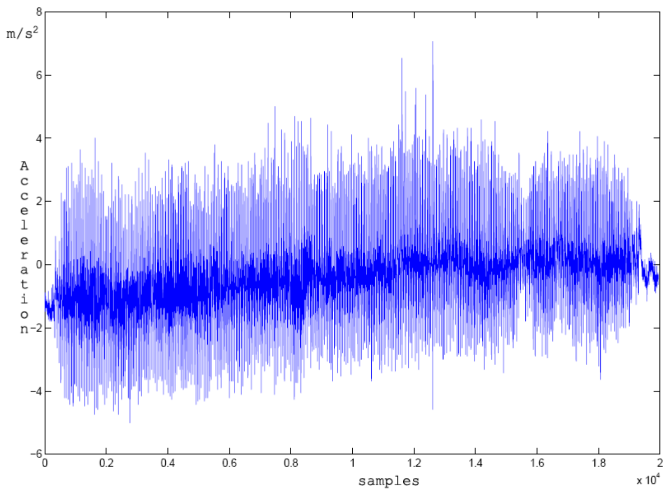


FIGURE 6.2: Temperature effects in accelerometer

Due to the magnetometer problem it has been finally decided to use a separated magneto-compass to provide the yaw angle to our system.

6.2 Hardware description

In the project has been used an accelerometer, gyroscope and initially a magnetometer. As commented due to the poorly design of the board the magnetometer measurements are too noise to be able to use, for these reason an electronic compass has been used as a substituted of the magnetometers.

6.2.1 IMU board

The board holding the sensors is the 6DOF v4 Inertial Measurement Unit from www.sparkfun.com. It has 3 axes of acceleration data, 3 axes of gyroscopic data and 3 axes of magnetic data. This data is sent using a TTL line or via Bluetooth(configurable). The data sampling frequency is user selectable from 1Hz to 300Hz.

The data sent by the board can be in ASCII or in a binary format, which may be good for a realtime implementation. The core of the board is an ARM processor LPC2138. Here it is the the sensor list of the board:

- Freescale MMA7260Q triple-axis accelerometer, settable to 1.5 g, 2 g, 4 g or 6 g sensitivity
- 2 InvenSense IDG300 500 degree/second gyros
- Honeywell HMC1052L and HMC1051Z magnetic sensors

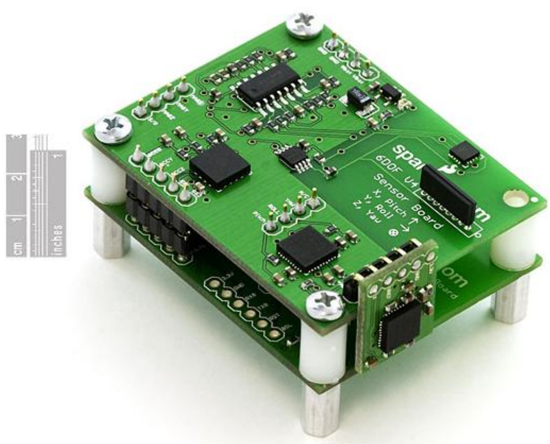


FIGURE 6.3: IMU Board

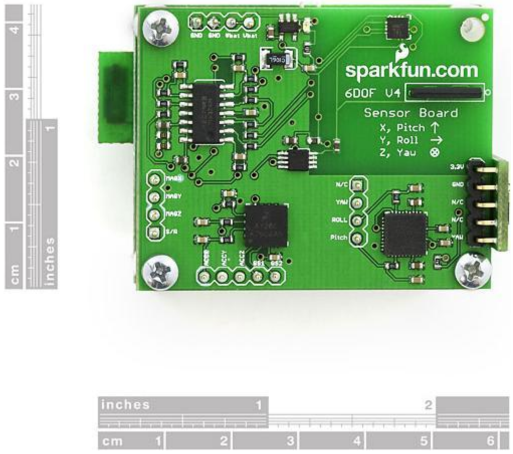


FIGURE 6.4: IMU Board top view

A technical specification of the IMU:

- Input voltage: 4.2V to 7V DC
- Current consumption: less that 150mA
- Frequency response:
  - Magnetic sensors: 312Hz
  - IDG300 Gyros: 120Hz
- MMA7260Q Accelerometer:
  - 350Hz, X and Y axes
  - 150Hz, Z axis

6.2.1.1 Data structure

Here is shown the data structure of the IMU board in ASCII and binary mode. In both modes, the data from all active channels starts with an A (ASCII 65) and has a Z (ASCII 90) at the end. The data sent has this format:

Sample Count	X-Magneto	Y-Magneto	Z-Magneto	X-Accel	Y-Accel	Z-Accel	X-Gyro	Y-Gyro	Z-Gyro

An example of a ASCCI sample data:

A 602 617 529 511 481 694 507 435 507 Z

An example of a binary sample data presented in hexadecimal:

410000025902640224020901E602B301F101B301F85A

Here is presented an algorithm to parse this binary data:

```
while (a! = 65) do {Looking for starting 'A' }
  a=fread(serialobject,1,char);
end while
RawBuffer=fread(serialobject,20,char); %Read all the 20 bytes of the packet
z=fread(serialobject,1,'char');
if z == 90 then {Check last character is the Z}
  b=2;
  for j= 1 to 9 do {Parse the data}
    Buffer(j)=RawBuffer(j+b)*256+RawBuffer(j+b+1);
    b=b+1;
  end for
end if
```

6.2.2 Compass board

The OceanServer Compass is a 3-Axis Tilt Compensated compass which provides a good performance and easy interface. Here it is presented its features:

- Precision compass accuracy, 0.5 degrees nominal, 0.1 resolution
- Roll & Pitch full rotation (< 1 degree (0-60 degrees))
- Pitch Angles +/-90 degrees, Roll Angles +/- 180 degrees
- Tilt-compensated (electronically gimbaled)
- Low Power Consumption, <30ma @3.3V
- Hard and soft-iron compensation routines
- Serial Interface:
  - RS232, USB or TTL
  - Baud rate programmable 4,800 to 115,000 baud
- ASCII sentence output, in several formats, NMEA checksum
- High Data Update Rate to 40HZ
- Support for True or Magnetic North Output

- Precision components
  - 3 Axis magnetic sensors from Honeywell
  - 3 Axis Accelerometers from ST Microelectronics
  - 24 bit differential Analog to Digital converters

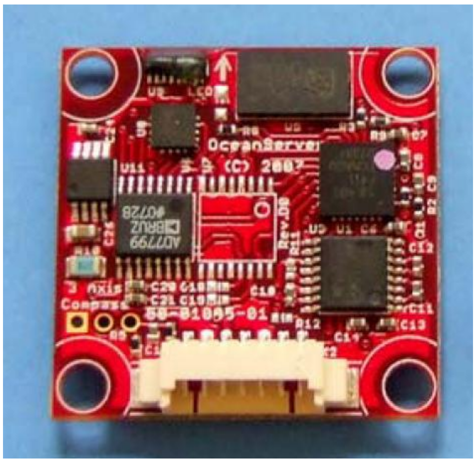


FIGURE 6.5: Compass

6.2.2.1 Data structure

The compass has a USB serial interface to receive the ASCII strings. This string has the following format:

$Chhh.hPpp.pRrr.rTtt.tMx0.000My0.000Mz0.000Ax000.0Ay000.0Az000.0*cc

$C212.4P2.5R-14.0T28.4Mx107977.90My-79422.00Mz173.27Ax0.045Ay-0.245Az0.977*3A

- Hhh.h: Heading in degrees, corrected for Declination if one is entered
- Ppp.p: Pitch angle, P precedes the pitch angle in degrees
- Rrr.rr Roll angle, R precedes the roll angle in degrees
- Ttt.t: Temperature of the compass board.
- Mx,My,Mz Magnetic field strength reported on each sensor, in mGuass
- Ax,Ay,Az Acceleration measured on all three sensors in G
- *cc Is the HEX X-OR sum of the character between the $ and the *.

6.3 Software description

6.3.1 Files

Here are listed all the files that has been developed:

TABLE 6.1: Develop files

interface.m interface.fig	The MAIN file, containing the main algorithm and the user interface
INS Mechanization.m	File with the INS mechanization algorithm
INS GPS Kalman Filter.m	File with the INS GPS Kalman filter algorithm
log data.m	Algorithm to log all the data synchronized
calculate yaw magentos.m	Function to calculate the YAW angle from magnetic data
gps parser.m	Function to connect and read GPS NMEA data
Read Compass Data File.m	Function to read the yaw angle from the Compass samples
NMEA to GPX.m	Function to convert from NMEA data to the track file GPX
Save to GPX file.m	Not to be called directly
index.php	PHP file to plot position data in realtime on google maps.

6.3.2 Files format

Many types of data must be saved on each test. Not just the raw data is needed but also is important to store the sampling frequency of each device, the initial coordinates, initial attitude, time, date... Therefore, it was decided to create a simple XML at the beginning of each logfile containing all the needed information. Next lines present this XML files:

6.3.2.1 IMU FILE

The IMU file contains all raw data from accelerometers and gyroscopes, it also stores on the XML the sampling frequency, the accelerometer sensitivity, initial attitude, as one can see here:

```
<Conf>
  <Freq>100</Freq> %Sampling frequency
  <Accel>2</Accel> %Accelerometer sensitivity
  <Comment>Write your on comment</Comment> %To write a comment
```

```
   <Attitude> %Initial attitude
      <Roll>0</Roll>
      <Pitch>0</Pitch>
      <Yaw>0</Yaw>
  </Attitude>
  <Date>06082009</Date> %The date
  <Time>124117</Time> %Time of the simulation
  <Position> %Initial position
    <Lat>52.305025</Lat>
    <Lon>5.241806</Lon>
  </Position>
  <Calibration> %Calibration for the sensors, to correct the sensor bias
    <MagX>1.6</MagX>
    <MagY>1.6</MagY>
    <MagZ>1.6</MagZ>
    <AccelX>1.6</AccelX>
    <AccelY>1.55</AccelY>
    <AccelZ>1.64</AccelZ>
    <GyroX>1.73</GyroX>
    <GyroY>1.485</GyroY>
    <GyroZ>1.7</GyroZ>
  </Calibration>
</Conf>
```

6.3.2.2 Compass FILE

The compass file contains the raw data from compass (strings presented in last section) and the sampling frequency. Here is shown the XML configuration:

```
<Conf>
  <Freq>10</Freq>  %Sampling frequency
</Conf>
```

6.3.2.3 MetaFile

What we call MetaFile is just a XML text file that holds all the files of each test. The aim of this file is to make things easier when running the INS/GPS algorithm. Imagine each time you want run the INS/GPS algorithm you must write the name and path of the file containing the GPS data, the IMU data and finally the compass data. This file holds all the files needed for the algorithm in order to run.

```
<METAFILEGPSINS>
    <GPS>GPS.TXT</GPS> %File contain the NMEA strings
    <COMPASS>Compass.TXT</COMPASS> %File with Compass data
    <IMU>IMU.TXT</IMU> %File with IMU data
</METAFILEGPSINS>
```

6.3.3 Logging the data

After running some tests we realise the importance of having a good synchronization between all the devices and much more due to the difference of sampling frequencies of each device. Therefore a program has been developed in order to log all the data in a synchronised way. Here is presented the pseudocode of the program:

```
Open Serial Port GPS
Open Serial Port COMPASS
Open Serial Port IMU
Wait For User Key Press To start logging the data
Flush all buffers
While(!stop)
        If(GPS data ready)
                Read GPS Data
        endif
        If(Compass data ready)
                Read compass Data
        endif
        If(IMU data ready)
                Read IMU Data
        endif
endwhile
Close serial ports
Save data files
```

6.3.4 User Interface

It has been designed a user interface to host the algorithms and provide a friendly way to manage the system. In next picture is shown a screenshot of it.

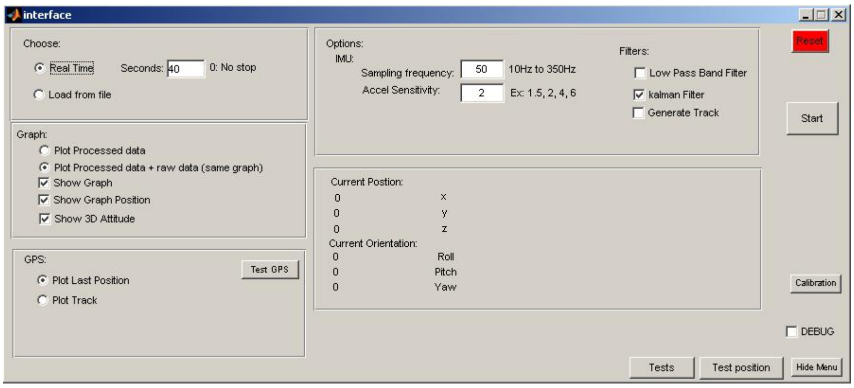


FIGURE 6.6: Interface

As said before, the file interface.m is the MAIN function. This file holds all the calls to the algorithms and to the INS/GPS System. It can work on REALTIME or using LOGFILES. Depending on the operation mode it can do

In the OFFLINE mode:

- Reads ALL the log files
- It may do some filtering (low pass)
- It uses KalmanFilter to calculate positions
- It finally creates the track

In the REALTIME (not working due to problems):

- Connects Matlab with IMU board throught dongle bluetooth
- Connects Matlab with GPS usign another bluetooth dongle
- Connects Matlab with the Compass using a USB cable
- It reads ALL information
- It may do some filtering (low pass)
- It uses KalmanFilter
- It may plot all the information in real time on google maps.

- It saves all the sensed data for later analysis

Here is presented the algorithm of the interface in pseudocode

```
if(OFFLINE MODE)
        OPEN METAFILE
        OPEN IMUFILE
        OPEN GPSFILE
        OPEN COMPASSFILE
        Convert ADC values from IMU to physic values
        if(filtering)
          Filter using low pass filter
        endif
        %Calculate position
        if(UseKalman)
           for(ALL DATA)
              Position=INS_GPS_Kalman_Filter(Data);
           endfor
        else
           for(ALL DATA)
               Position=INS_Mechanization(Data);
           endfor
        endif
        Generate the GPX track
elseif(ONLINE MODE)
        Open Serial Port IMU
        Open Serial Port GPS
        Open Serial Port Compass
        Configure IMU \& Compass
        while(!stop)
           Buffer=Read All available serial data;
           Convert ADC values from IMU to physic values
           if(filtering)
               Filter using low pass filter
           endif
           %Calculate position
           if(UseKalman)
              for(ALL DATA)
                 Position=INS_GPS_Kalman_Filter(Data);
              endfor
           else
              for(ALL DATA)
                  Position=INS_Mechanization(Data);
              endfor
           endif
           Movement detector algorithm
           Show 3d Box attitude
           Show strings with Position
```

```
            Plot position
        endwhile
    Close_Serial_Port(Serial_Object);
endif
```

6.3.5 INS mechanization

Here is presented the INS mechanization in pseudocode:

```
Update_Direction_Cosine_Matrix
Body to local frame accelerations transformation
Compute local gravity vector
Navigation update
Get Position estimation
Get Velocity estimation
```

6.3.6 INS GPS Kalman Filter

Here is presented the INS & GPS mechanization in pseudocode:

```
% Propogate the INS states
INS_Mechanization;
% Generate the Phi matrix
Phi = PhiGen(dt);
P = Phi*P*Phi' + Q;
if(gps_update) %GPS update once per second
    ins_position_error = errINSGPS(gps,ins); % Calculate the error position ins-gps
    Z = [ins_pos_err_enu;ins_vel_err];
    R = diag([PosSD PosSD PosSD VelSD VelSD VelSD]);
    H = [eye(6,6)  zeros(6,6)]; %Create H matrix
    K = P*H'*inv(H*P*H' + R); %Compute the kalman gain
    X = X + K*(Z - H*X); %Update the state vector
    P = P - K*H*P; % update the coverience matrix
end
if(compass_update) % Compass update 5 times per second
    R=diag([CompSD CompSD CompSD]);
    z=[0;0;YawMag];
    H=[zeros(3,6) eye(3,3) zeros(3,3)];
    K = P*H'*inv(H*P*H' + R); %Compute the kalman gain
    Xest = Xest + K*(z - H*Xest); %Update the state vector
    P = P - K*H*P; %Update the coverience matrix
end
DCMbn Kalman update
DCMel Kalman update
```

Chapter 7

Test solution

Several tests have been done all along the project. Initially the tests were focused on the sensors to find out the raw measurements of the IMU board. In these earlier tests it was discovered the problem with the magnetometers. A significant amount of time was inverted on solving the magneto problems, then it was decided not to use the magnetometer raw data and use a magneto compass instead. In parallel with these first steps it was decided the INS/GPS integration model to comply with the requirements.

The final INS/GPS system was started by building the INS-standalone block, using the presented equations in Chapter 4. With the INS system working and tested, focus was pointed on the design and implementation of the Kalman filter.

With all the system built in Matlab a walking-test was done. First issue during the test was the data synchronization. Attention was focused to find a proper way to log the data from GPS, accelerometers, gyroscopes and the compass which finished with the development of the program on Section 6.3.3.

In this chapter is presented the latest tests done. Each test consists on the study of the results obtained from a real person walking with the sensors rigidly attached on his body. Each test has different characteristics, couple of them are run outside the Thales building, one starts outside to later walk into the building and the last test is a short walk around the laboratory facilities.

7.1 Early tests

One of the very first tests was to cover gyros measurements. Small program was built to demonstrate the gyroscopes behaviour. This small program reads the turning rates and draws a 3D-cube on the screen, the cube follows the same turns as the real board. With the program we were able to see the so-called drift. We have seen that after some operating time the accumulated error was so big that the real attitude of the board and the simulated cube was completely different.

7.1.1 Logging the data

As shown in last chapter, it was designed a program to log the data. The Matlab program saves all raw data without any conversion or correction in the measurements. Later on, the algorithm will perform the magnitude transformations and the needed corrections.

Something to remark about the synchronizing program is the fact that the program logs the data in a synchronized way in the suppose that each board has no timing-error. That is, it is supposed the GPS is plotting the position at 1Hz, the readings from the IMU board are correctly set up to a specific sampling rate, and in the compass is also set up to a specific sampling rate. The problem with this method is that it can not figure out about and error in the sampling rate of the IMU board, compass or GPS.

7.2 Solution tests

The sensors are rigidly attached to the chest of the person. It has been used a self-made cage and a belt to do it. CIM is not firmly attached to the chest of the person, it is usually hold in the soldier/fireman vest. Therefore our tests try to be as close as possible to the real situations and don't avoid some shakes on the sensor cage while walking.

The algorithm's outputs are three tracks:

- GPS stand-alone track.
- INS stand-alone track.

- INS/GPS track.

All tracks are generated following GPX file format. Due to time constraints it was not feasible to perform detailed studies on the signals and the generated tracks.

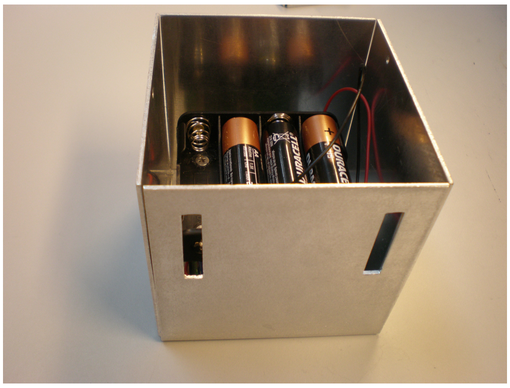
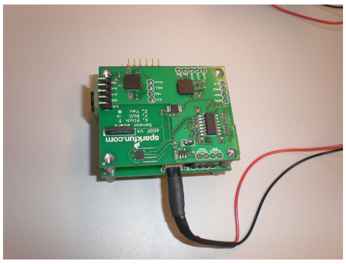


FIGURE 7.1: IMU cage

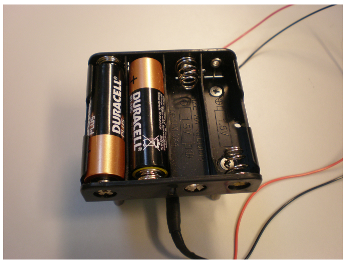
The connection between the laptop with GPS was done by Bluetooth, in the same way the connection IMU-laptop was Bluetooth, and the connection compass-laptop was done with a USB cable.

FIGURE 7.2: GPS

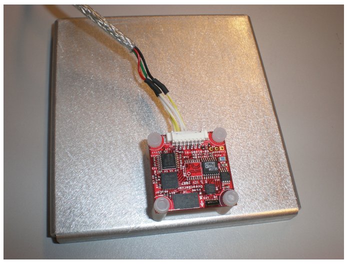
Before starting with the tests, it is presented some real photos of the system.



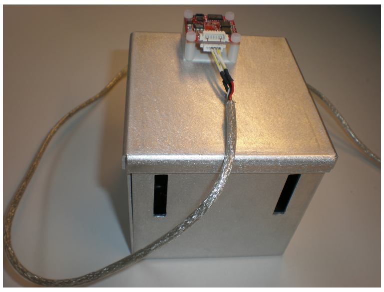
(a) IMU BOARD



(b) IMU BOARD WITH BATTERIES



(c) COMPASS



(d) ALL THE SYSTEM

FIGURE 7.3: Systesm

7.2.1 Assumptions and approximations

The algorithm presented and programmed does not do any assumption. Once this is said, after the firsts tests it was found a problem due to the walking shakes. The shakes were mainly affecting to the vertical channel(z-axis). In order to avoid this issue at every time update the height is forced to be 0.

Another assumption is done in the velocity. When INS block works as stand-alone, with no GPS help, the error in the speed grows unbounded. So some mechanism must be used/found to try to bound the velocity errors. In this project it has been used the following approximation: Every 20 seconds the actual speed is reset to 0. Another way to see this is like every twenty seconds the person is supposed to be still.

Here, resetting the velocity to zero is an approximation, but in the scope of Pedestrian Reckoning this is not an approximation and is commonly used. This mechanism is called "Zero Updates" or ZUPTS (see Section 4.2.1 Pedestrian tracking).

Note: The sampling rate used is 150 Hz for the accelerometers and gyros, 10 Hz for the compass and 1Hz for the GPS.

7.2.2 Test 1

This first tests was done outside the Thales building, the track is a 2 minutes walk around the parking place next to Thales. Thus this is the first test, here it will presented some of the raw measurements obtained from the sensors. On later tests only final results are shown.

7.2.2.1 The walking path

In this first test it was tried to check all the system. In figure 7.4 is presented the theoretical path I walked with the sensors.

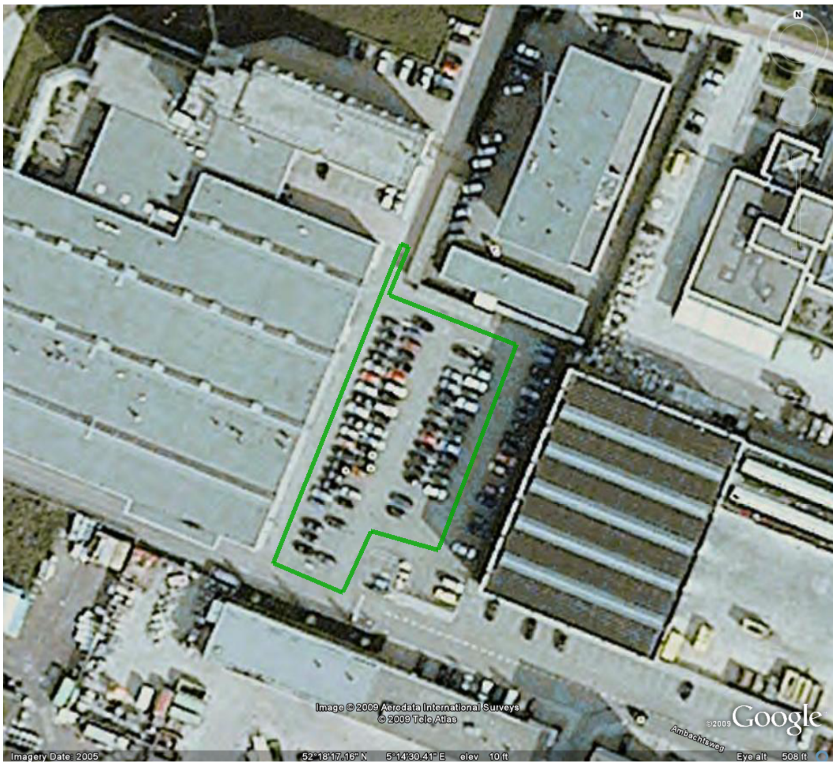


Figure 7.4: Test 1, Hand-made track

7.2.2.2 Measurements

Here is presented the raw measurements obtained from the accelerometers and gyros sensors.

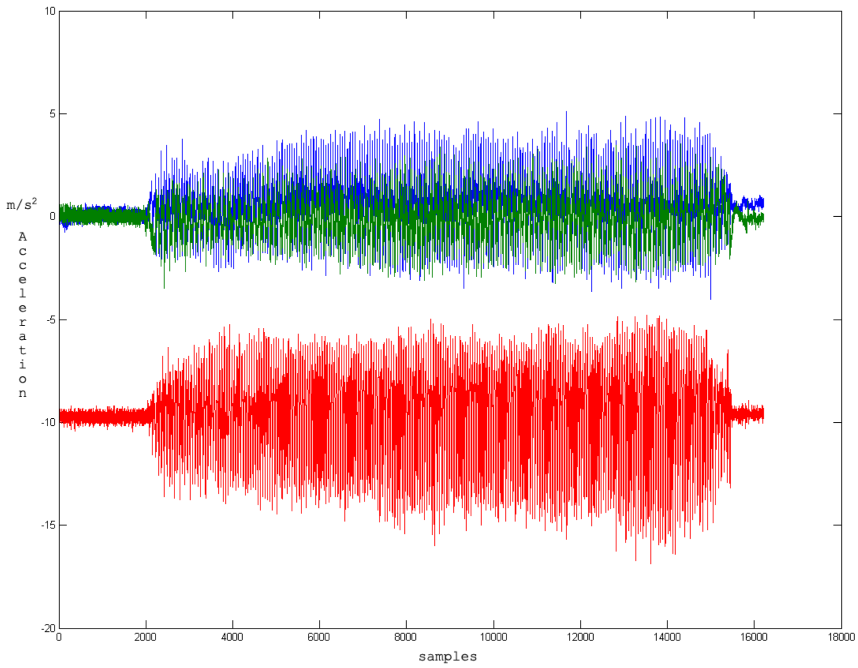


FIGURE 7.5: Test 1, Accelerations

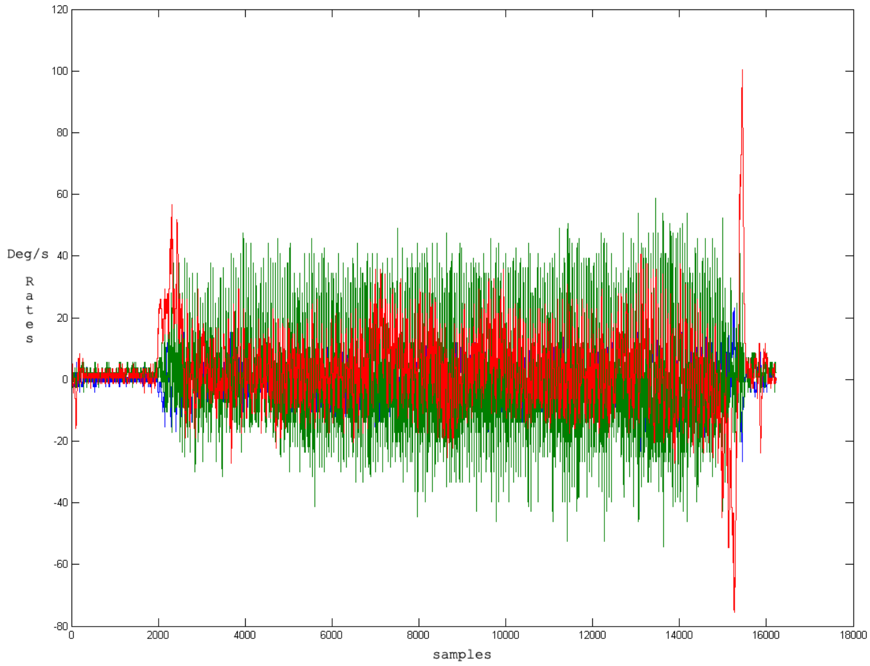


FIGURE 7.6: Test 1, Gyro measurements

7.2.2.3 Results

With all raw data from accelerometers, gyros, compass and GPS, the algorithm has processed the information. In next figures all the results are presented.

In this first image GPS track is shown.

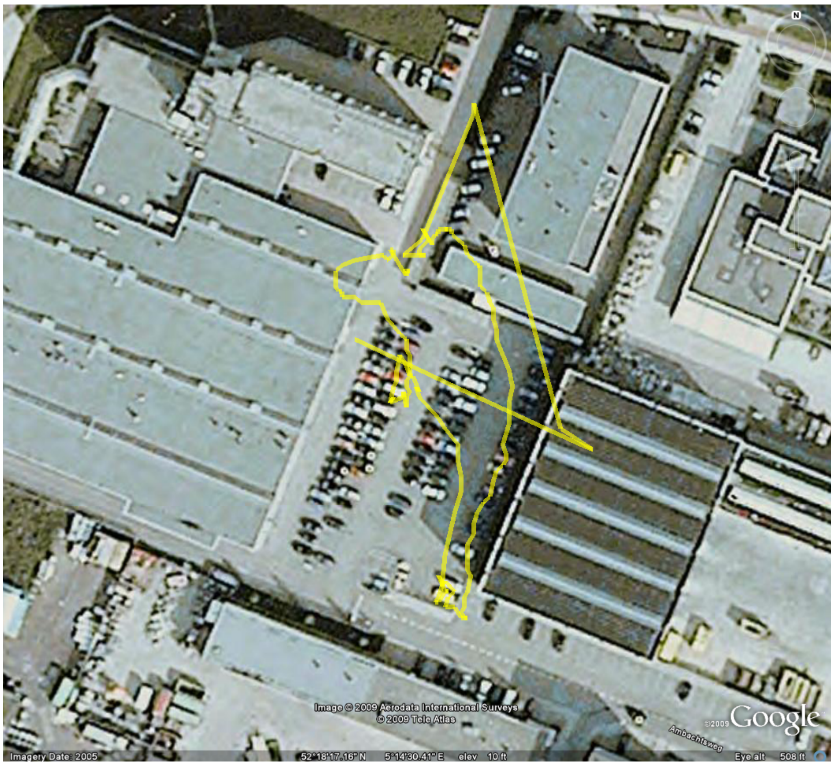


Figure 7.7: Test 1,GPS track

As one can see the GPS track is not fine. There is a great mismatch between the real track and the GPS track, the shape of the GPS and real track are quiet different. These errors may occur due to reflections or due to a poor GPS signal reception.

Here it is presented the INS standalone solution.

FIGURE 7.8: Test 1,INS track

The inertial system was provided with the initial coordinate and then it has computed all the positions without any other help. Although the track is not perfect because there are some errors, the shape of the walk is correct. The maximum error is around 23m-25m.

Next Figure presents the complete INS/GPS solution.

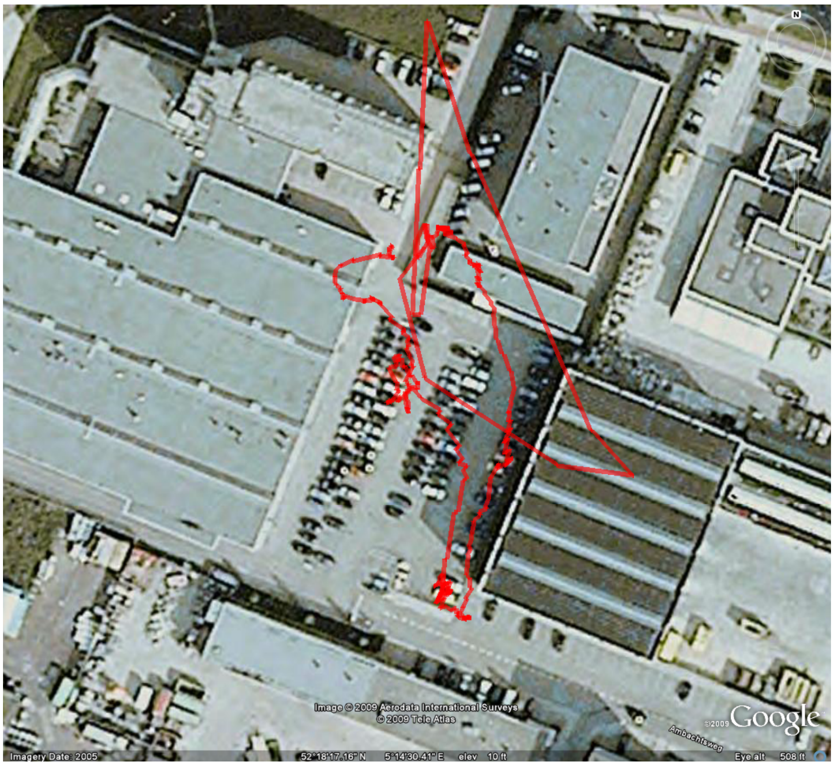


FIGURE 7.9: Test 1,INS/GPS track

And finally here is presented all the tracks in the same picture.

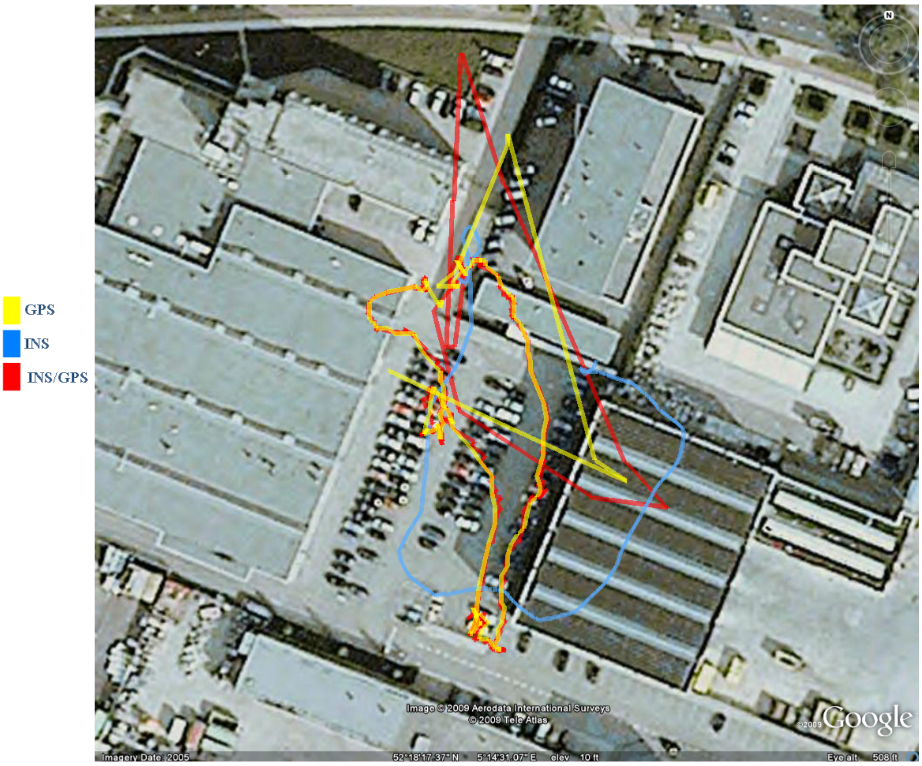


FIGURE 7.10: Test 1,All tracks

Due to our system trusts the GPS, it thinks the received position from the GPS is a good position but it is not. It has not been implemented any mechanism to check the GPS error. So when using the INS/GPS integration the output track is close to the GPS track.

The system feeds back the INS block with the calculated Kalman filter estimations every time there is new GPS position to provide the corrections. In the project it has not been implemented the mechanism to detect blunders or measure the GPS error in order to use this information in the Kalman filter.

Roll and pitch angles are close to zero only the yaw angle is shown. Heading is the most important attitude parameter because it gives a valuable information. Just using the figure 7.11 is almost possible to imagine the walk path shape. First it points to north to later head to the South-West and so on.

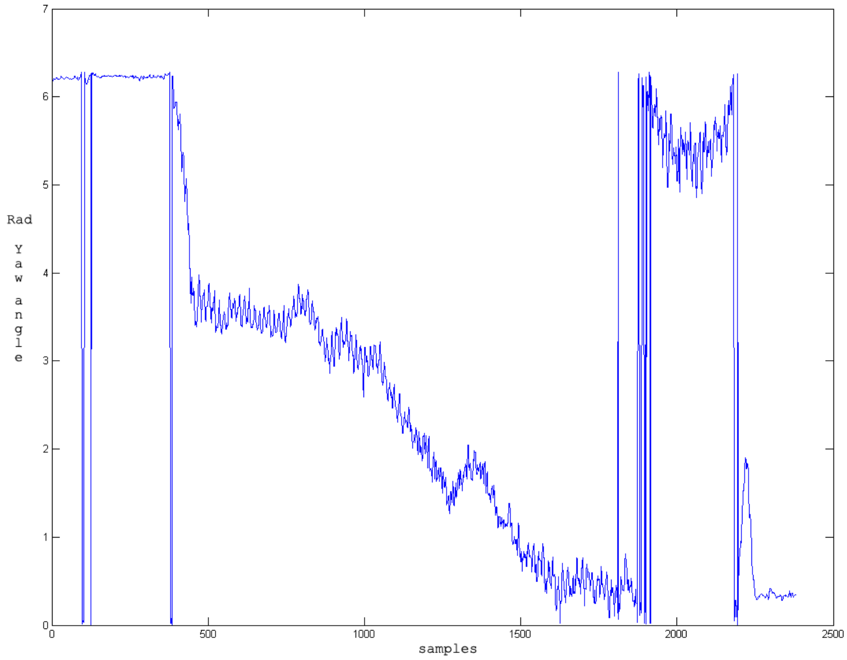


FIGURE 7.11: Test 1, Yaw angle from compass

Here is plotted the calculated speeds for X and Y axis in NED frame without doing a zero updated.

The signals may look nice but the problem of these signals is their magnitude. A person usually walks at around 3, 4 or 5 km/h, if running the speed can go up to $17km/h$. In the next figure we can see the speed goes more or less upto $20m/s$ which is $72km/h$. This behaviour is due to the accumulated INS errors integrating the accelerations.

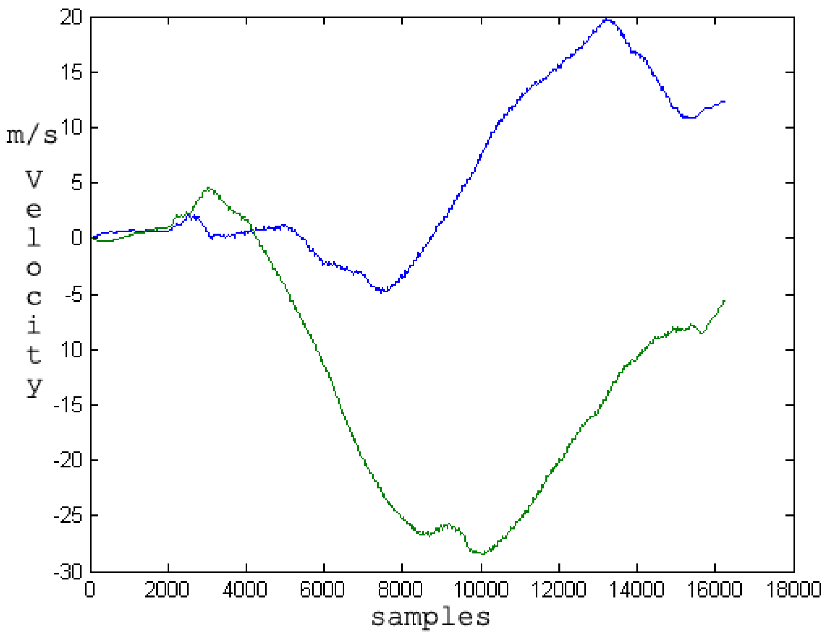


Figure 7.12: Test 1, INS standalone velocity X and Y axis in NED frame, no zero updates

As pointed before a mechanism to bound the error must be used when the INS is working standalone. Here is plotted the calculated speeds of X and Y axis in NED frame using the zero update mechanism.

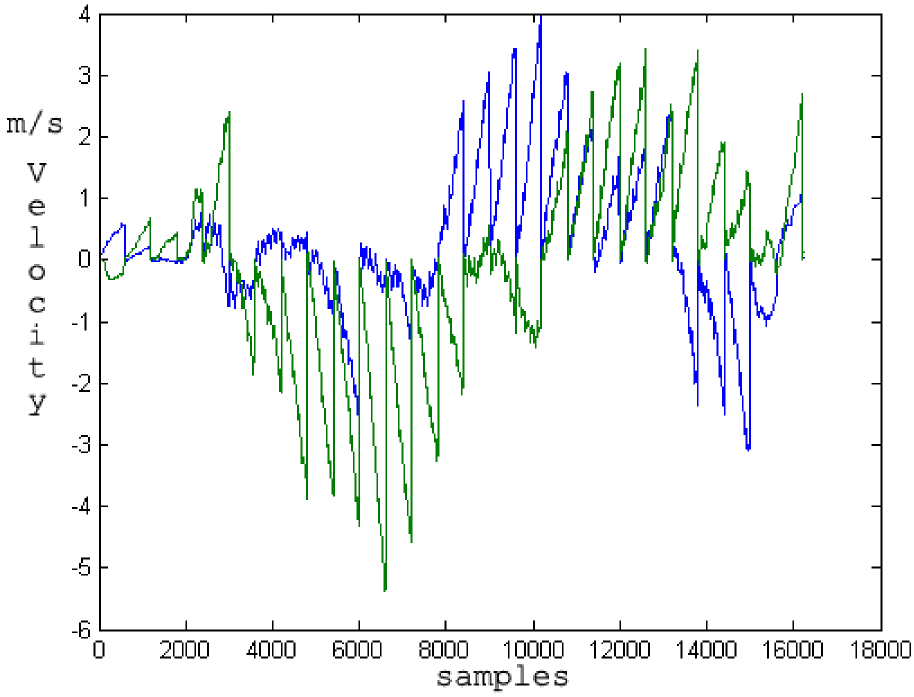


FIGURE 7.13: Test 1, INS standalone velocity X and Y in NED frame

On last figure is easy to identify the effect of ZUPT's on the speeds. Every defined seconds the speed was reset to zero.

Next Figure plots the calculated speeds in the INS/GPS system.

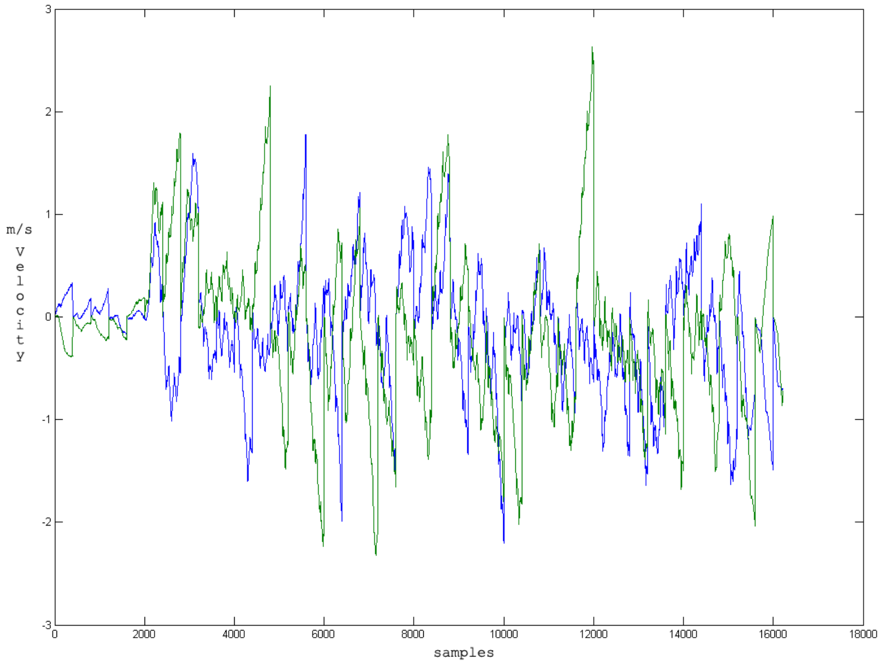


Figure 7.14: Test 1, INS/GPS velocity X and Y in NED frame

To clearly see the differences between each track next figure shows the tracks:

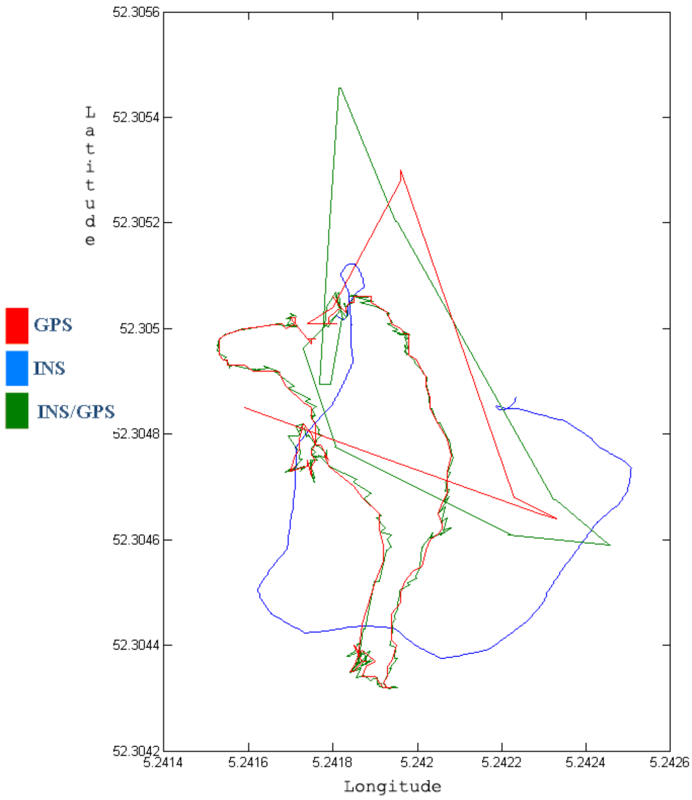


FIGURE 7.15: Test 1, Tracks

In previous Google Earth screen shots it may seem INS/GPS track and GPS are the same. In this next figure is possible to clearly see the differences between them. This figures represent the evolution of the latitude(left) and longitude(right) along time.

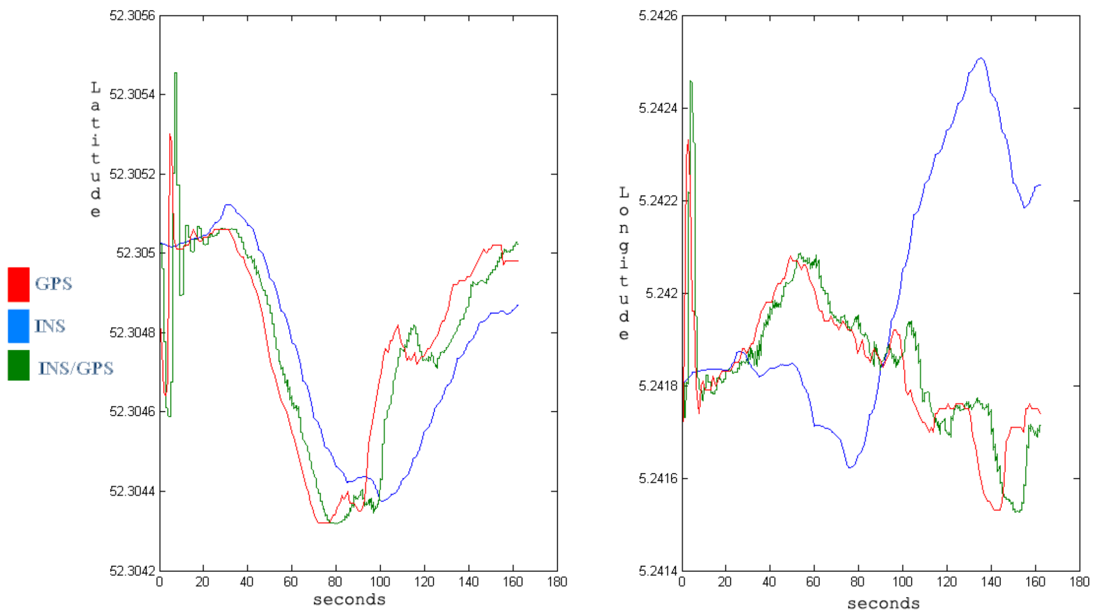


Figure 7.16: Test 1,Latitude and longitude

On the latitude figure, is possible to see that the INS/GPS track is somewhere between the GPS track and the INS track.

7.2.3 Test 2

In this second test the walked path was the same as the previous test. Figure 7.17 shows the GPS track. In this test the track has a better precision than in Test 1. Here the GPS error is around 4-5 meters.

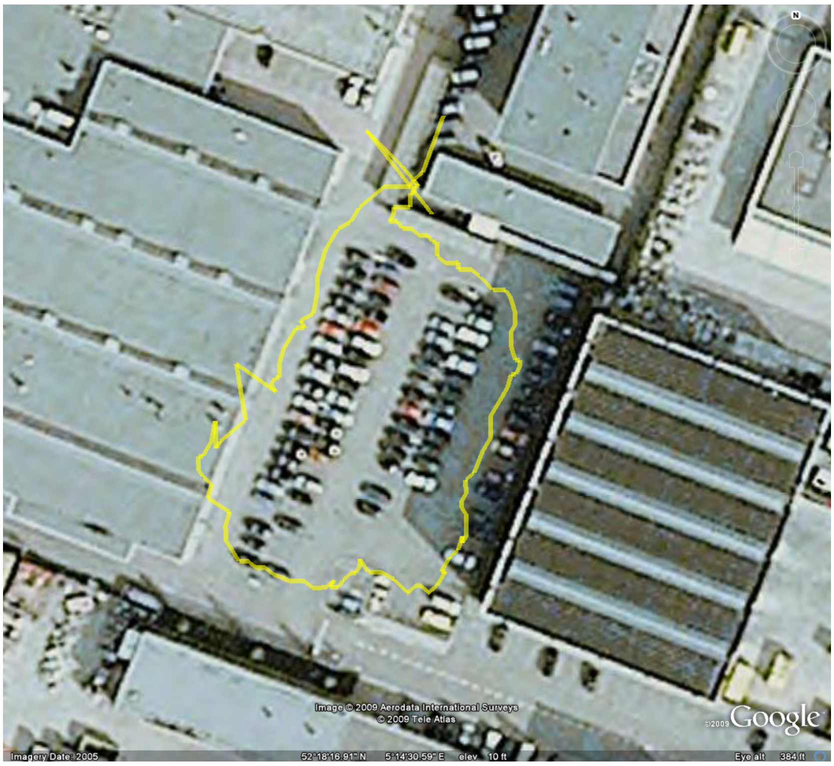


FIGURE 7.17: Test 2,GPS track

The job of the inertial system standalone is similar to the Test 1. It has some errors but it defines the shape of the walked path.

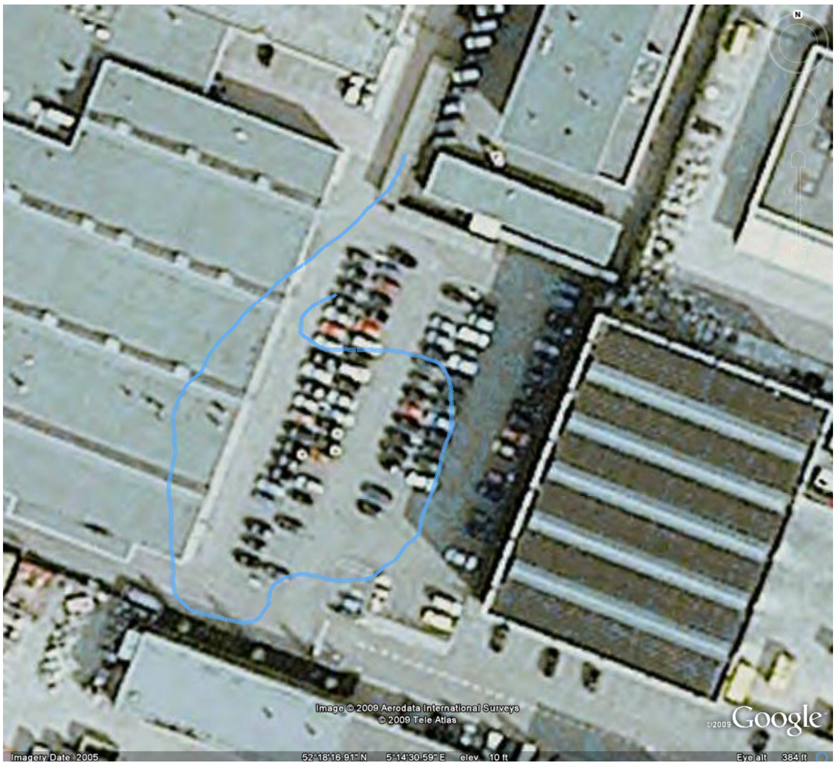


FIGURE 7.18: Test 2,INS track

And finally the track made for the INS/GPS system. In this track is clearly identified some sharps due to the GPS updates. Each time a new update of GPS is available the "Update" step on the Kalman filter is calculated and then the INS is corrected using the state vector.

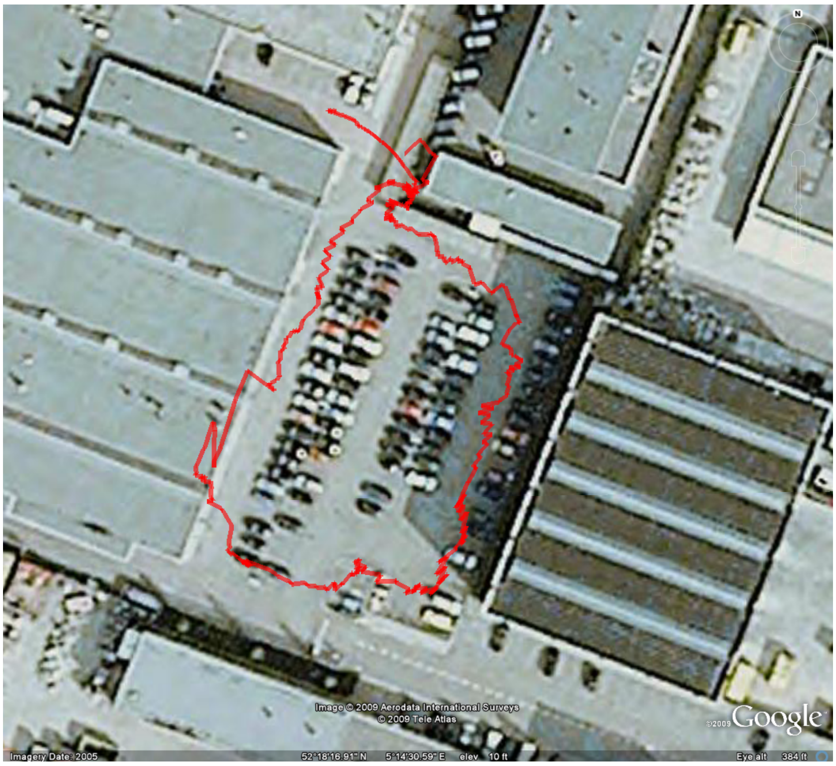


Figure 7.19: Test 2,INS/GPS track

Drawing all the tracks in the same image.

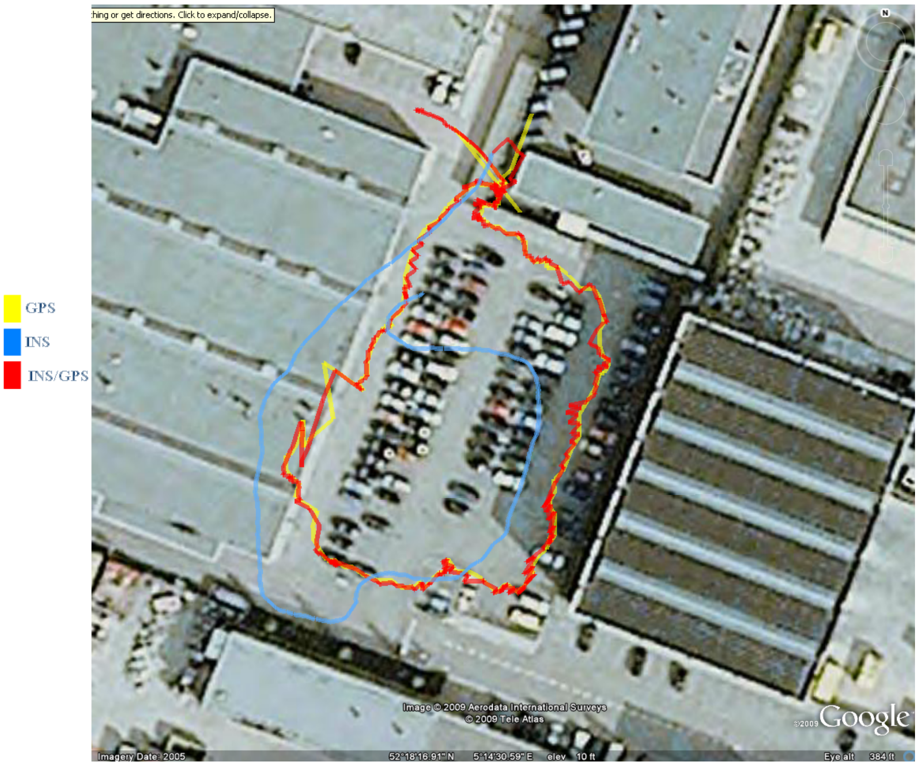


FIGURE 7.20: Test 2,All tracks

Within this test we see INS system standalone is always doing the shape of the path but it drift over time. Here GPS track has smaller error, so the INS/GPS will perform much better than in test 1. As explained the INS/GPS relies on the GPS to correct the errors on INS.

Here are presented the same tracks with Matlab.

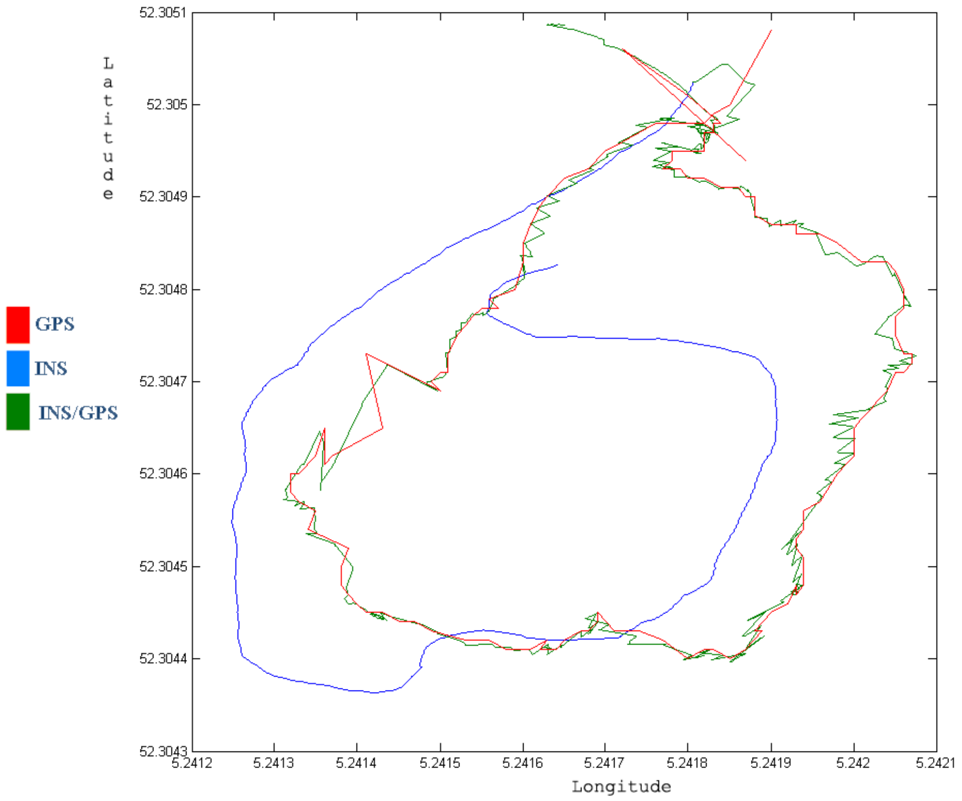


FIGURE 7.21: Test 2, Tracks

Next picture shows the evolution of the latitude and latitude along time.

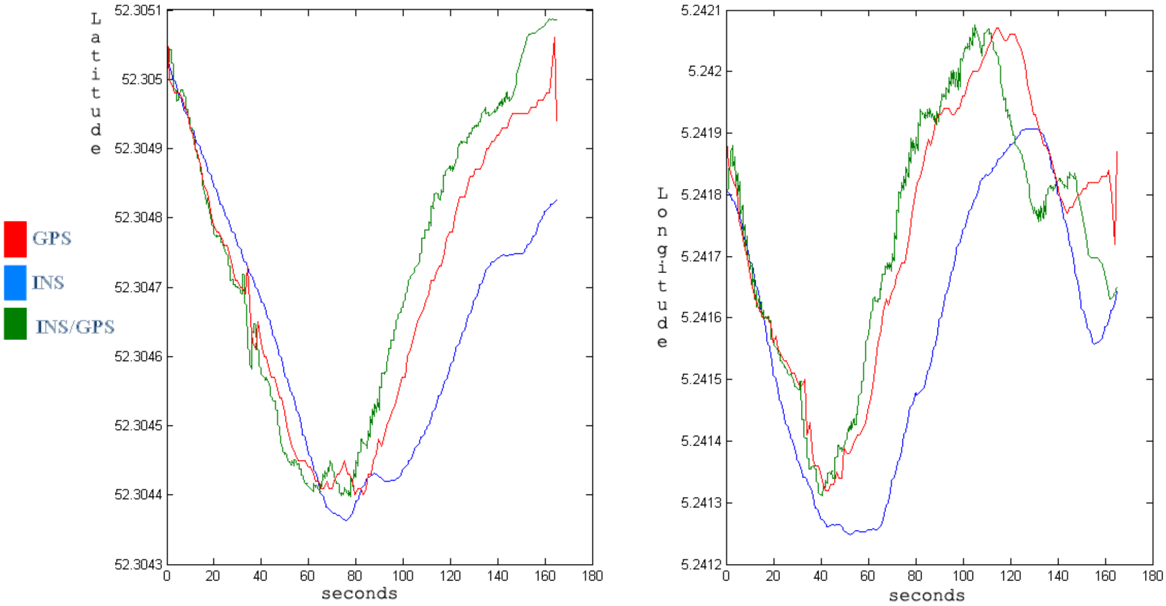


FIGURE 7.22: Test 2,Latitude and longitude

7.2.4 Test 3

This test was entirely done inside the building. GPS was not able to receive any GPS signal, therefore it has not been used this test. The test starts in the center of laboratory to later walk around it. The track it was made was:

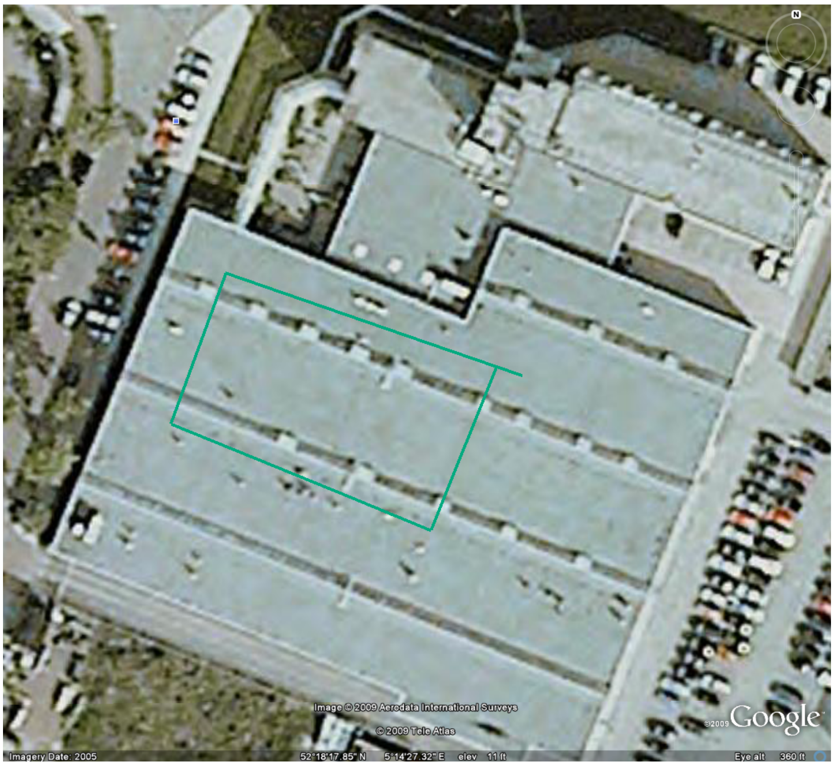


FIGURE 7.23: Test 3,Hand-made track

The INS track as in the other tests keeps the shape of the path but has some errors.

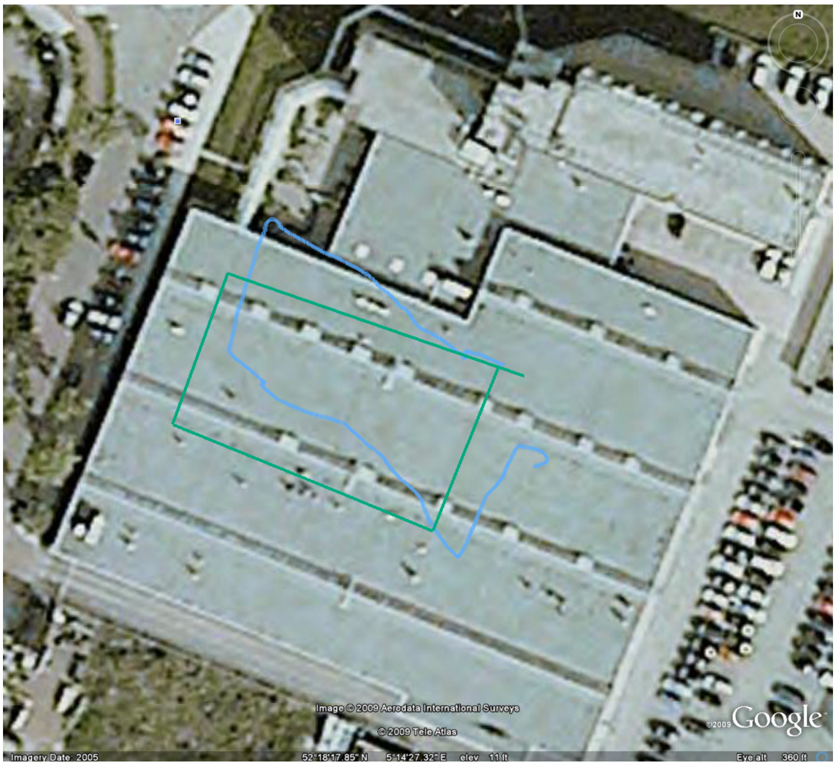


FIGURE 7.24: Test 3,INS track

Representing the INS with the matlab.

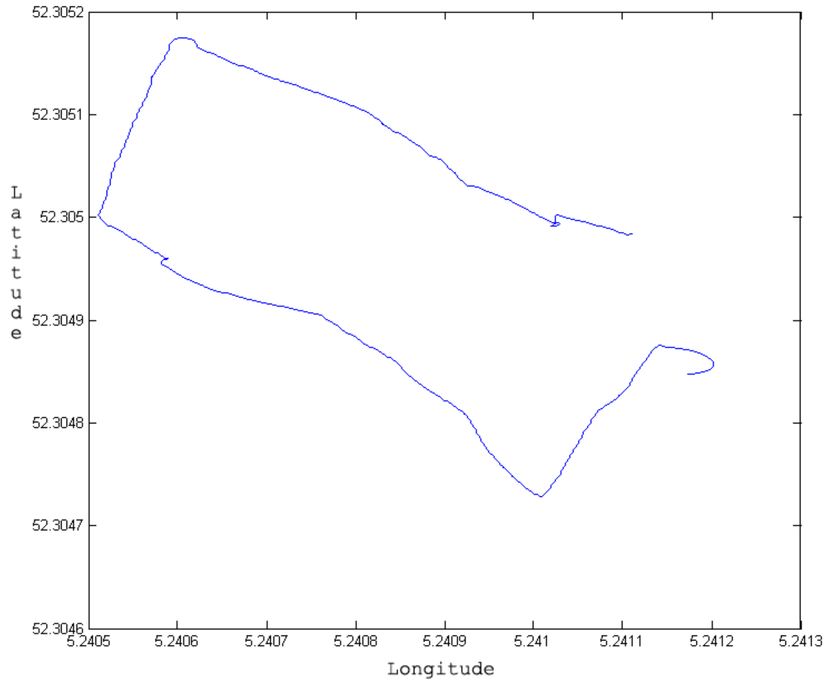


FIGURE 7.25: Test 3,All tracks

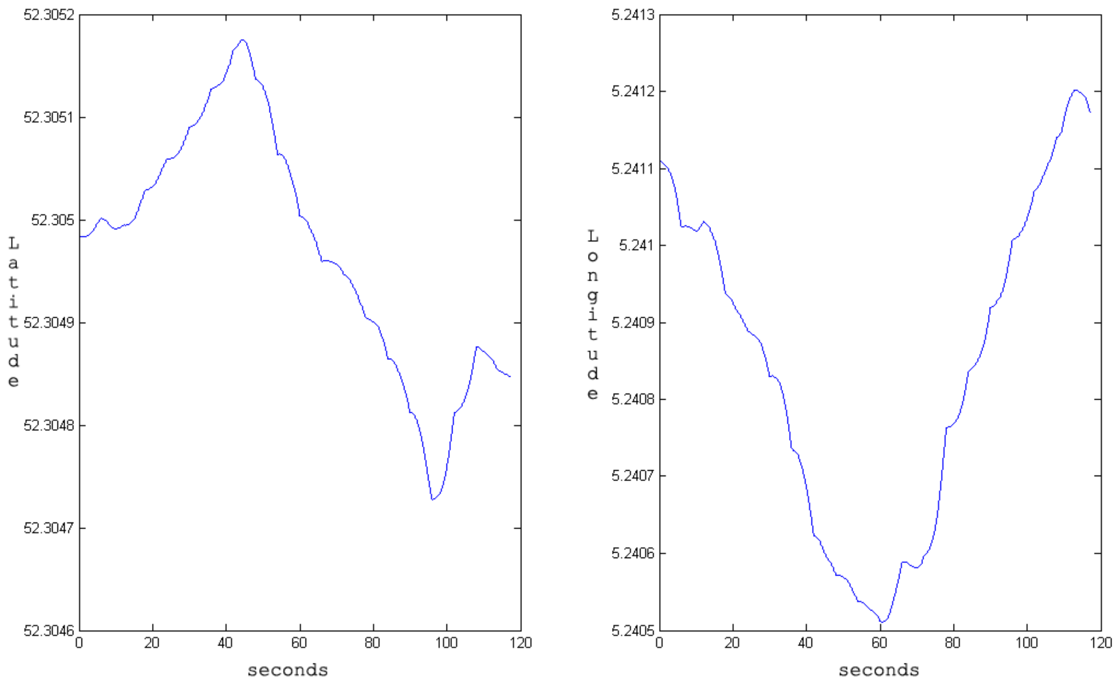


FIGURE 7.26: Test 3,Latitude and longitude

In this test is not possible to compare INS with GPS because inside the building GPS was not able to give a position. Due to this is a loosely coupled solution, GPS is used as a black box, so if it does not have 4 satellites it can't provide with a position. But, tighter approaches may use some GPS information even if the GPS has 2 satellites.

7.2.5 Test 4

This is the last test. This time the test started outside and finishes inside the building. I walked around the parking place and after two minutes before entering the building the GPS was switch off.

The first image is the GPS track. GPS has done a really good job comparing it with the other tests. The error of the track is around 1-2m. As commented, the GPS track finishes at the parking place, because after 2 minutes it was switch off.

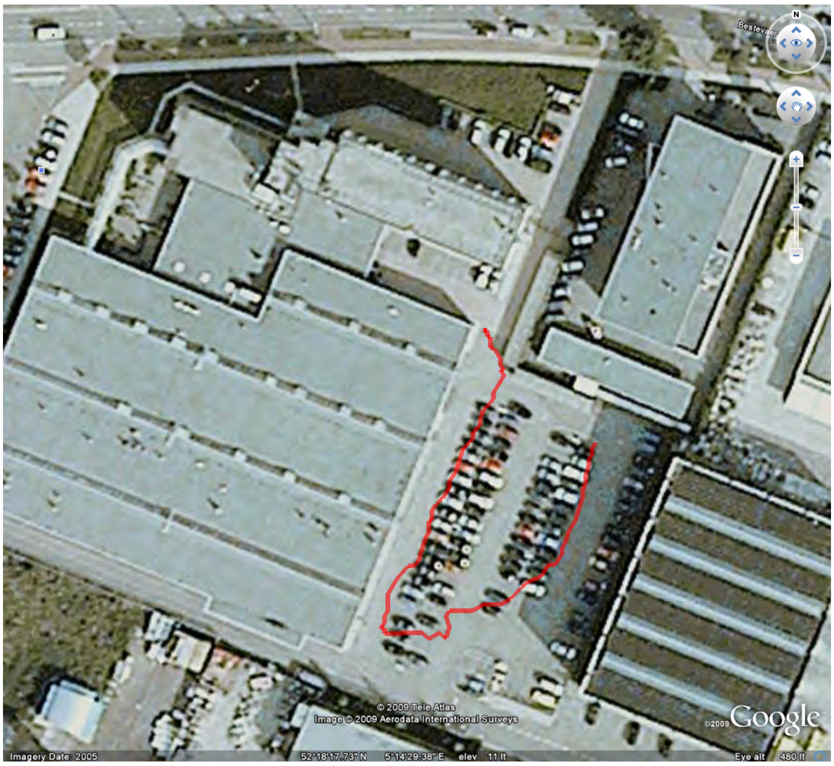


FIGURE 7.27: Test 4,GPS track

The output track from the INS has the shape of the walked path, although it has an increasing position error over time.

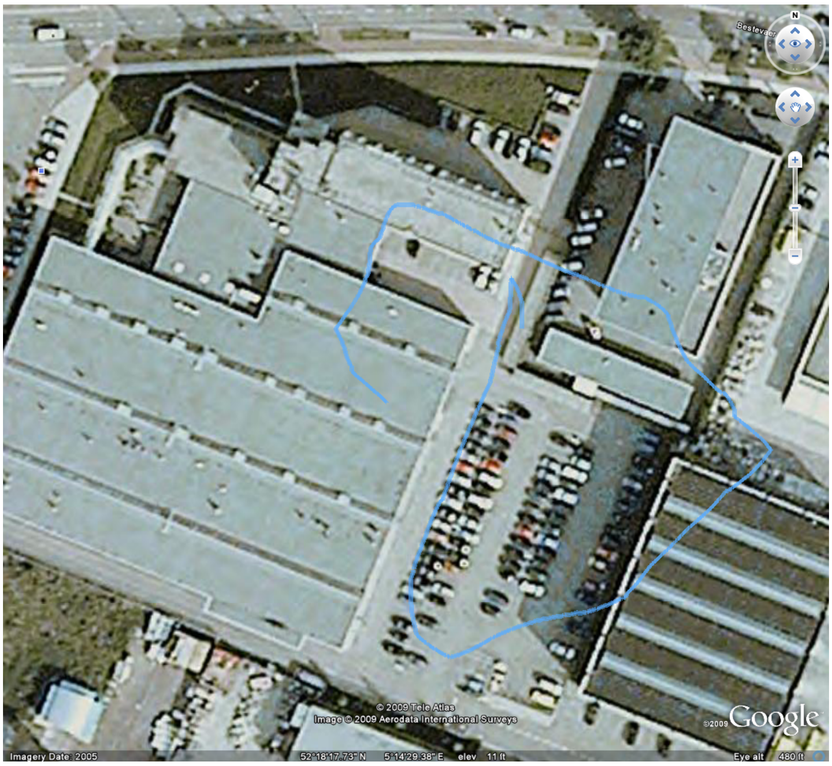


FIGURE 7.28: Test 4,INS track

The INS/GPS system has done a good track. Initially the system is using GPS information to continuously corrected the INS through the Kalman filter. Later on, with the GPS switch off, the INS system has done its best calculating the positions.

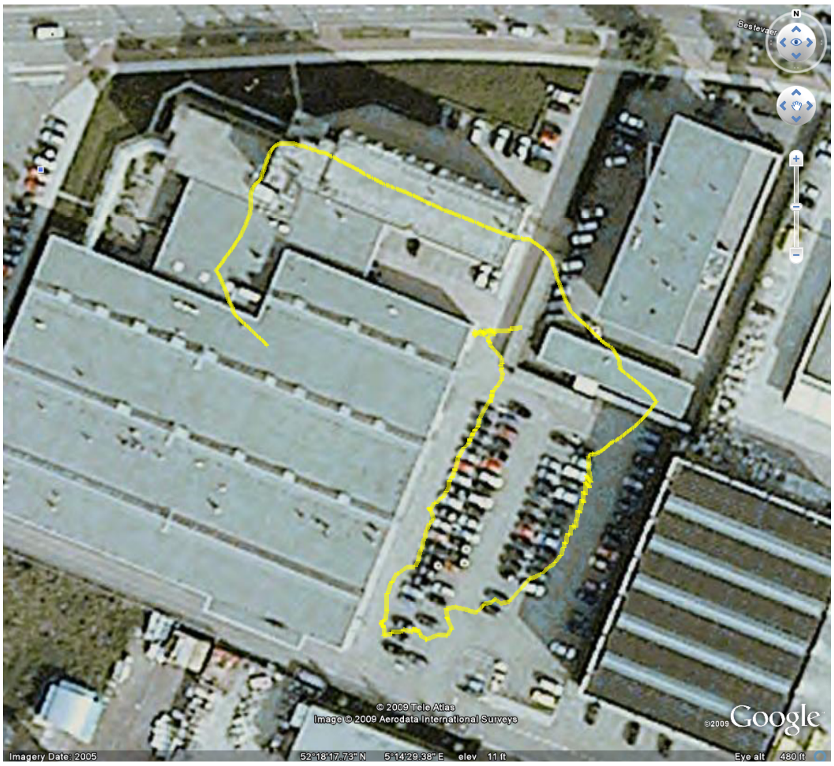


Figure 7.29: Test 4,INS/GPS track

To compare the presented tracks:

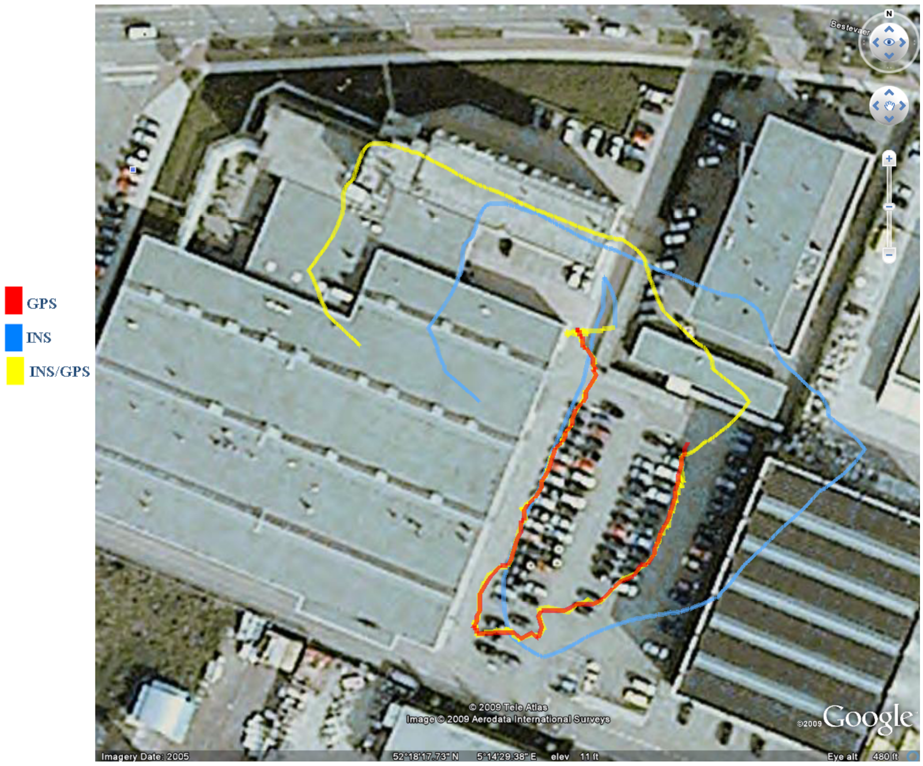


FIGURE 7.30: Test 4,All tracks

GPS track stops after 2 minutes, the INS standalone has a clear drift and the INS/GPS track obtains the best of each part to finally create the yellow track.

Next figure plots the yaw angle for this test.

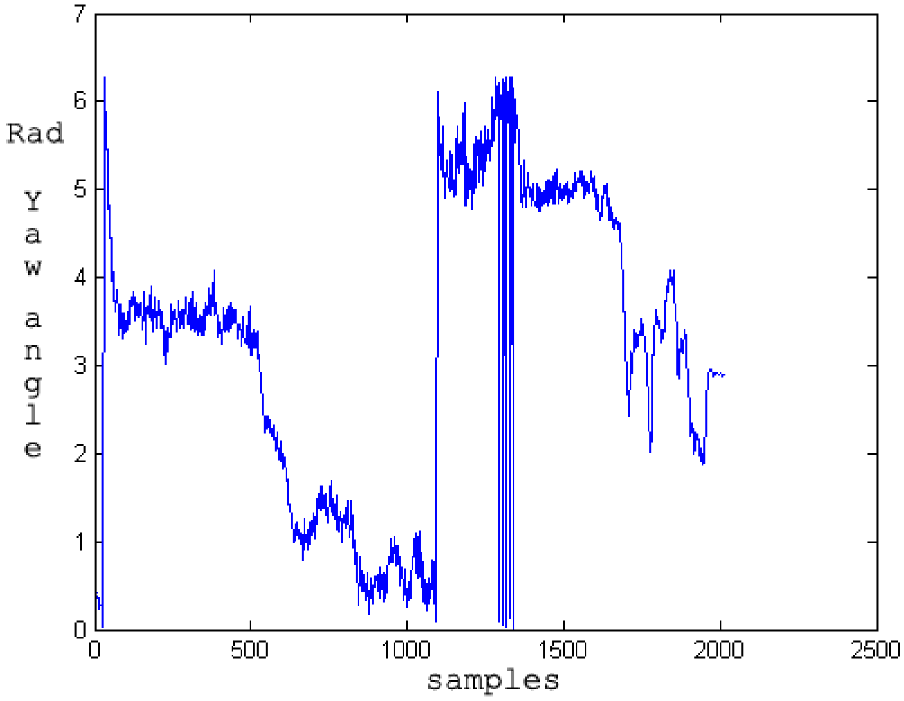


FIGURE 7.31: Test 4,Yaw angle

Here it is possible to see in better details the tracks. INS/GPS track has some sharp movement when working together with GPS this is because of GPS position updates on the Kalman filter and the linear error used to estimated these errors.

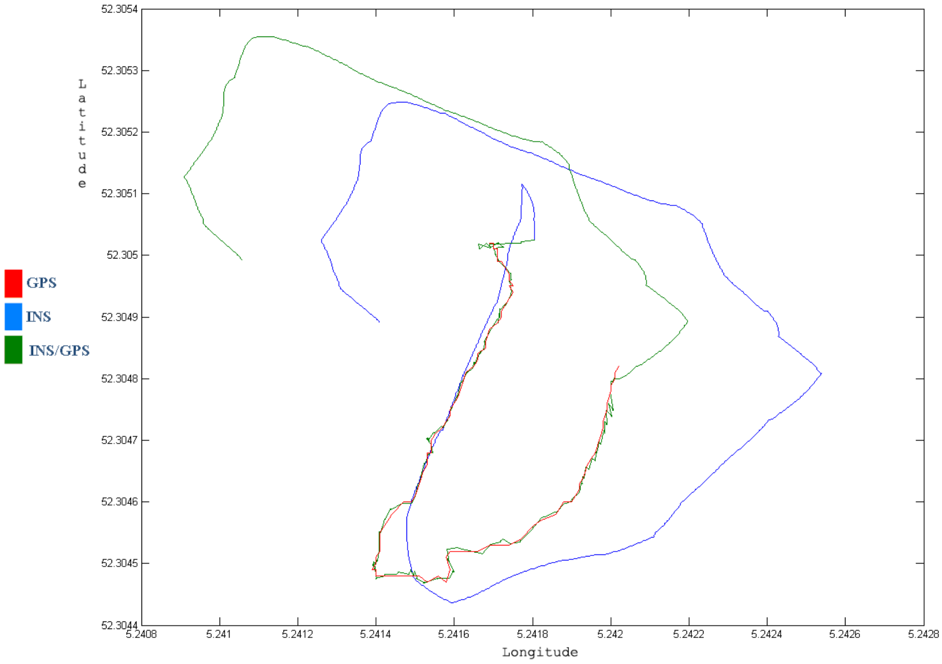


FIGURE 7.32: Test 4,Tracks

Here is presented the evolution of the latitude and longitude along the time.

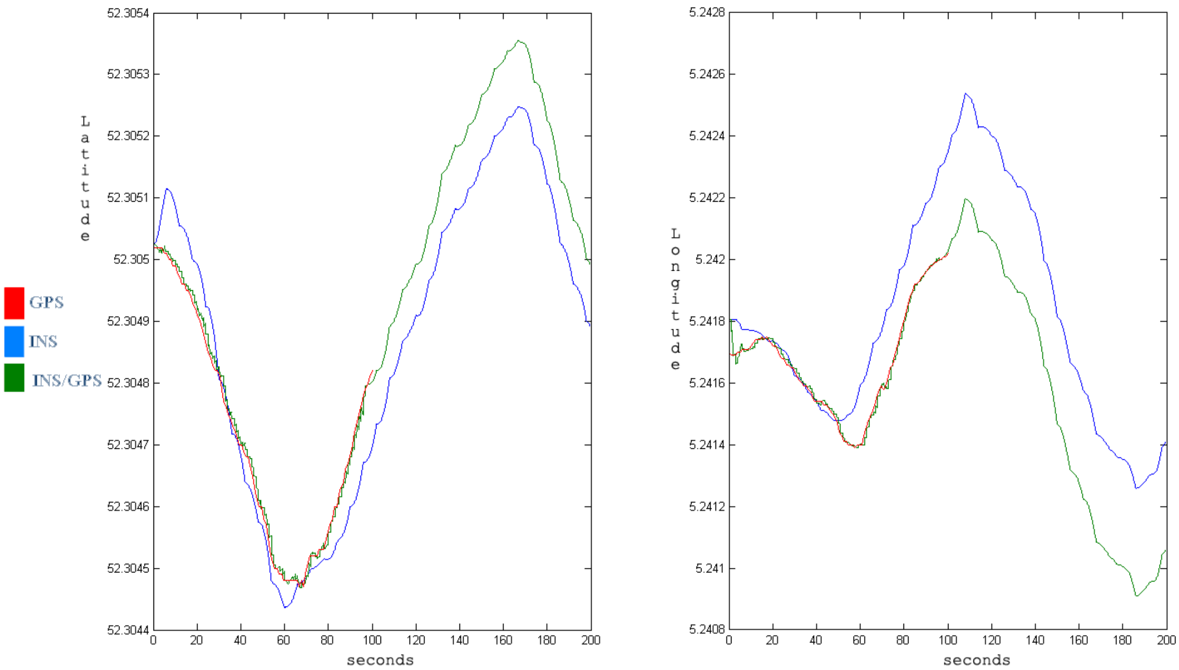


FIGURE 7.33: Test 4,Latitude and longitude

Chapter 8

Conclusion

8.1 Conclusion

This thesis is focused on the development of an inertial navigation system with the capability on standalone operation. Once the system was working the integration with GPS was analysed. Although using an INS/GPS system provides better accuracy the biggest goal has been to obtain the INS standalone system. The performance of both the systems was analysed and compared on chapter 7.

An overview of the GPS, INS and INS/GPS and its several implementations was discussed along with their advantages and drawbacks. It was explained the benefits of GPS with an INS integration for better navigation performance. The basics on Kalman filter focusing on inertial systems were explained in detail. It has been pointed that sensor characterization is important in order to obtain a better and optimal position and attitude estimation using the Kalman filter.

Most of the effort has been inverted on the INS module. Therefore, error models used in Kalman filter can be easily improved by improving all the error equations in the filter. Initialization and calibration are two other aspects that must be improved in order to obtain better precision.

This thesis has demonstrate how accurate an INS can be by developing a working demo software. It has generated a state of the art study on available techniques for indoor positioning, which will be helpful for becoming projects. The main goal, GPS-less navigation and the secondary goal, improve GPS precision by combining INS and GPS data have been reached.

The impact of GPS errors and blunders should be analysed for the INS/GPS system in order to avoid the problems described in test 1. The INS module has been designed to take care of most effects like, Coriolis and Earth's rate. These consideration may be not useful in a pedestrian, but they are of important interest if the project is translated to a car, truck, plane, which can travel large distances. As said it has been designed and implemented a *general* INS.

The INS/GPS presented is improving the GPS position, and by using the *feedback* model the INS errors are continuously corrected while GPS fix are available. It must be said the usually GPS fix are good enough for most applications. Comparing GPS and INS/GPS errors, one does not appreciate a big improvement. May be the most interesting information obtained from INS/GPS integration is the attitude. Attitude can be useful on some applications and the integration with the magnetometer seems to work quiet well.

This project is the first step and approach for processing in real time the Kalman filter to obtain the position inside CIM.

8.2 Further work

There are several ways to continue the work on this project, next lines try to summarize some of those ideas. These next steps are some how an improvement of the developed system. Some are trying to solve the encountered problems during the project, others tries to improve the system with more sophisticate implementations.

But before the improvements one last remark. Due to time constraints it was not feasible to perform detailed studies on the system outputs and generated tracks. In my opinion it is from high interest and can provided good knowledge. Some examples could be: Signal analysis (raw signals,position,velocities...), calculate position errors by calculating distances between the different models, etc.

Here it is a list with some ideas to continue working on inertial navigation systems:

- Physical system re-design
  - Add a barometer to stabilize the vertical channel.
  - Design/buy/integrate in CIM a board with all sensors. An integrated board including all sensors will lead to avoid some synchronization problems.
  - Temperature sensor to calibrate sensors (real time calibration).
  - Use more sensors improve data accuracy (eg: 6 accelerometers).
  - Improve INS initialization algorithms.
- Real implementation on CIM: With the project we have reached a good state of art on indoor navigation with special focus on inertial systems, the first idea to continue the work is to start the real implementation on the CIM.
  - Real time operation: Move code from Matlab to C/C++ and compile project on CIM architecture.
  - Offline: Create a small program and use CIM to log all the data to later use the Matlab project to perform the calculations.
  - Once **inside CIM** one can start playing with the real issues about the power consumption of the system, turning on/off the GPS and INS when are not needed (see Appendix A).
    * System power consumption
    * Turn on/off GPS or INS.
- Algorithms: It is possible to continue working on the algorithms in order to improve them. As explained in the project it has been used the loosely feedback model, but there are plenty of different other models, more tightly approaches, as presented is Table 5.1:
  - Implemented a new state vector(with more variables, to correct more errors, biases and drifts)
  - Use a tighter approach (some works are pointing that the difference between the loosely coupled and the tight does not rely on a big difference).
  - Improve actual approach in order to correct more errors:
    * 1st: Integrate INS equations into the Kalman filter (tighter integration).

    * 2nd (if 1st is reached): Introduce GPS pseudo-ranges with Kalman filter integration. This way GPS can be used with less than 4 satellites.

  - Improve the error model of the sensors, including the dynamics of the movement(walking person), vibrations...

- Tests It must be tested the algorithm in oder conditions: running, and while the person lying on the ground, jumping...

  - Do more tests: running, jumping, rolling...

Appendix A

Diagrams

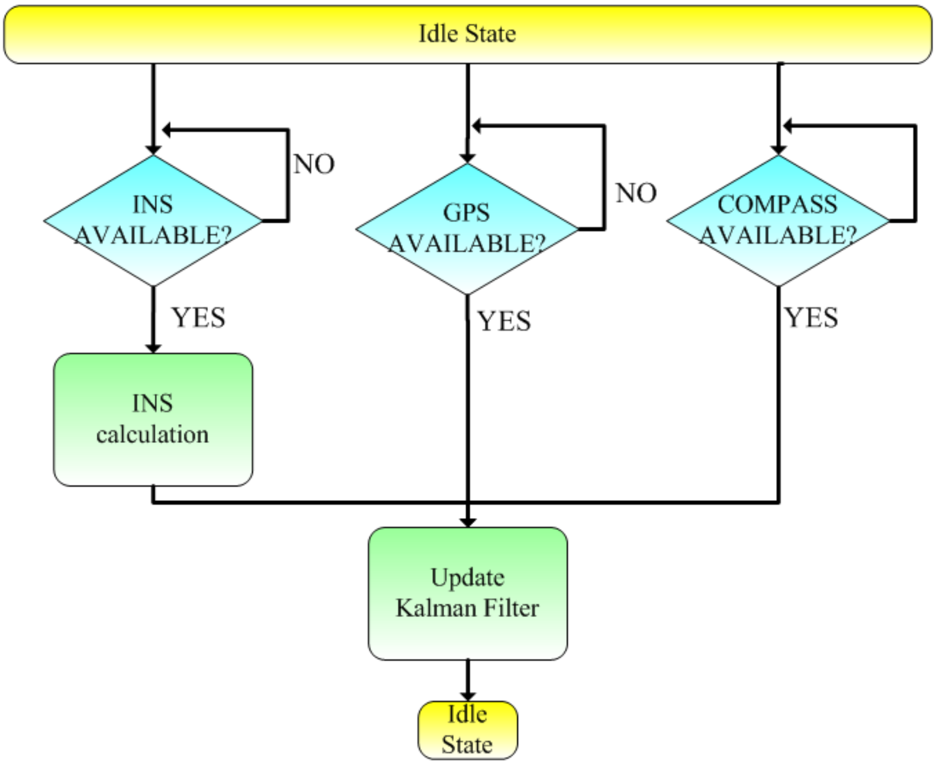


FIGURE A.1: INS/GPS flow diagram

Figure A.1 shows a basic flow diagram of a INS/GPS system.

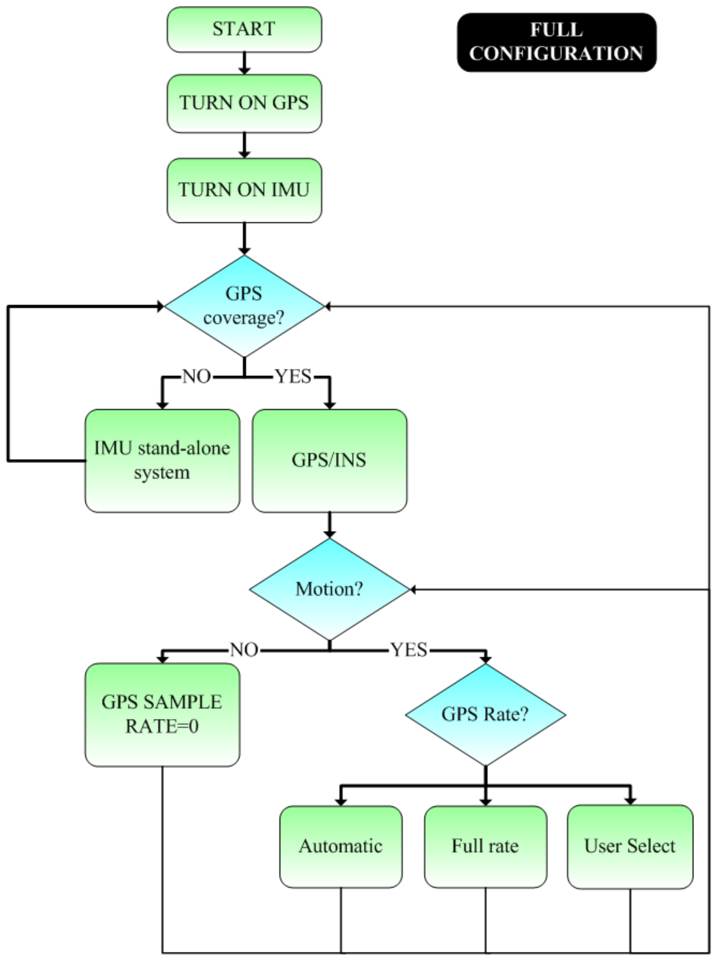


Figure A.2: Extended INS/GPS flow diagram

Figure A.2 shows an INS/GPS diagram, focusing on motion detection, power consumption (power on/off devices when are not needed) and modification the sampling rate to also save power.

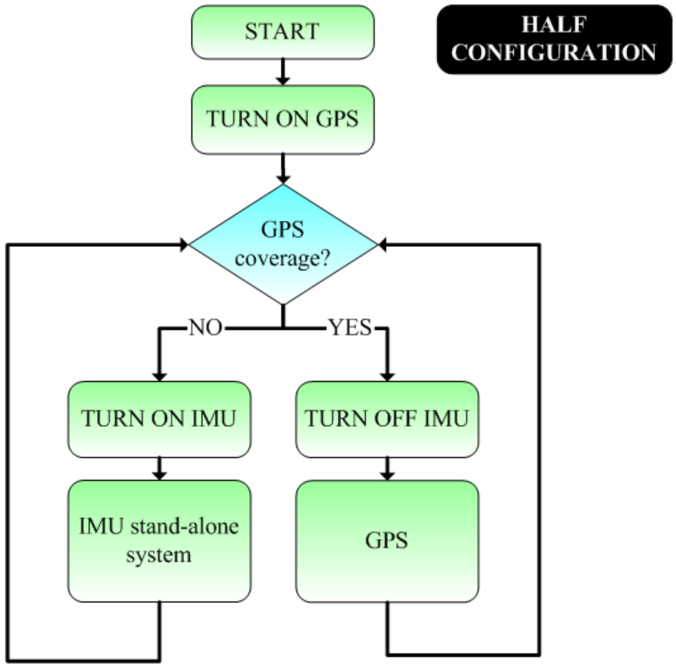


FIGURE A.3: INS/GPS flow diagram

Figure A.3 is a reduced of the latest.

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